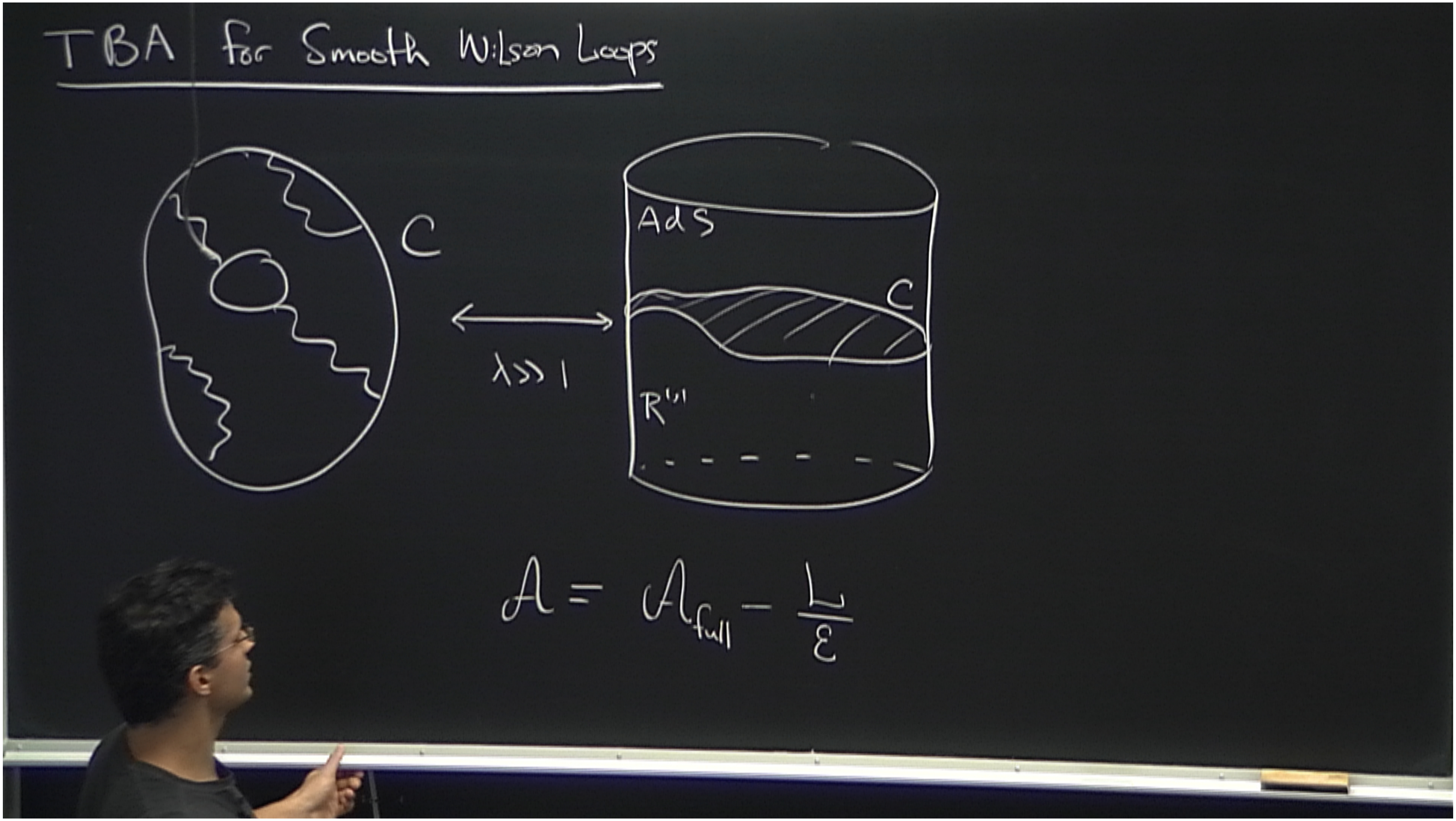
Title: Smooth Wilson loops from the continuum limit of null polygons

Date: Oct 30, 2014 11:00 AM

URL: http://pirsa.org/14100118

Abstract: We present integral equations for the area of minimal surfaces in AdS_3 ending on generic smooth boundary contours. The equations are derived from the continuum limit of the AMSV result for null polygonal boundary contours. Remarkably these continuum equations admit exact solutions in some special cases. In particular we describe a novel exact solution which interpolates between the circle and 4-cusp solutions.


TBA for Smooth Wilson Loops
C
AdS
C
λ>>1
A = A_full − L/ε

• $\boxed{y_{\sigma_1\sigma_2}(\theta)} = y^{circ}_{\sigma_1\sigma_2}\, e^{-4|z_{\sigma_1}-z_{\sigma_2}|\cosh\theta - K\star\left(y_{\tau_1\tau_2}(\theta') - y^{circ}_{\tau_1\tau_2}\right)}$

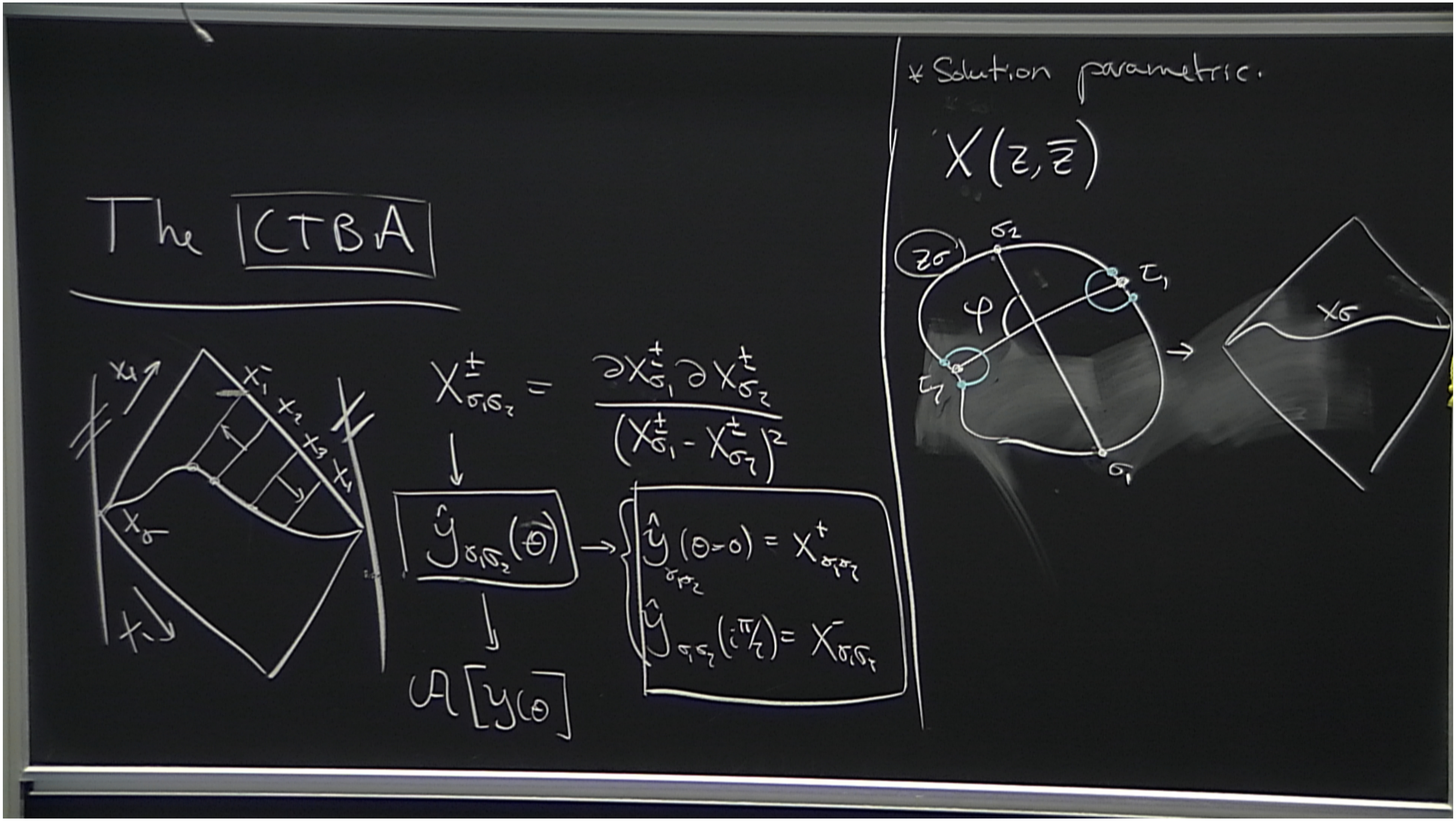
$y^{circ}_{\sigma_1\sigma_2} \downarrow \quad \dfrac{\partial f_{\sigma_1}\,\partial f_{\sigma_2}}{(f_{\sigma_1}-f_{\sigma_2})^2}$

$$K\star f = \int_{\sigma_2}^{\sigma_1+2\pi} d\tau_2 \int_{\sigma_1}^{\sigma_2} d\tau_1 \int d\theta' \, \frac{f_{\tau_1\tau_2}(\theta')}{\sinh[\theta-\theta'+i\varphi_{\sigma_1\sigma_2}-i\varphi_{\tau_1\tau_2}]}$$

$$\varphi_{\sigma_1\sigma_2} = \mathrm{Arg}(z_{\sigma_1}-z_{\sigma_2})$$

• $$\text{Area} = -2\pi - 4A_\Sigma - \frac{1}{\pi}\int_0^{2\pi} d\tau_2 \int_0^{2\pi} d\tau_1 \int d\theta\, |z_{\sigma_1}-z_{\sigma_2}|\, e^{-\theta}\left(y_{\tau_1\tau_2}(\theta) - y^{Kink}_{\tau_1\tau_2}\right)$$

A_Σ ↑ Area enclosed by z_σ



The CTBA
* Solution parametric.
X(z, z̄)

- $\boxed{y_{\sigma_1\sigma_2}(\theta)} = y_{\sigma_1\sigma_2}\, e$

$\downarrow$

$$\frac{\partial f_{\sigma_1}\,\partial f_{\sigma_2}}{(f_{\sigma_1}-f_{\sigma_2})^2}$$

$$K * f = \int_{\sigma_2} d\tau_2 \int_{\sigma_1} d\tau_1 \int d\theta' \,\frac{f_{\tau_1\tau_2}(\theta')}{\sinh[\theta-\theta'+i\varphi_{\sigma_1\sigma_2}-i\varphi_{\tau_1\tau_2}]}$$

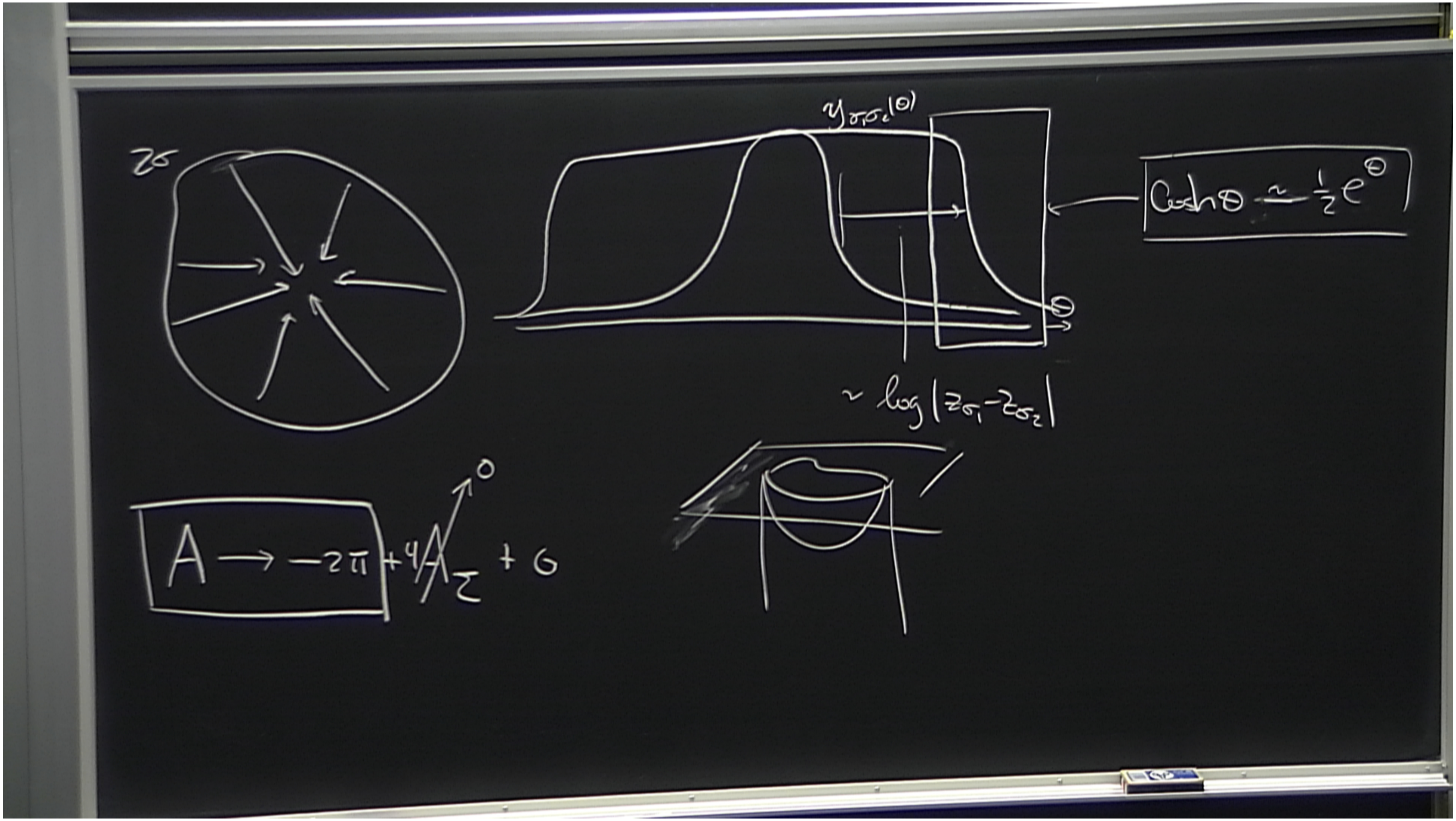
$$\varphi_{\sigma_1\sigma_2} = \mathrm{Arg}\left(z_{\sigma_1}-z_{\sigma_2}\right)$$

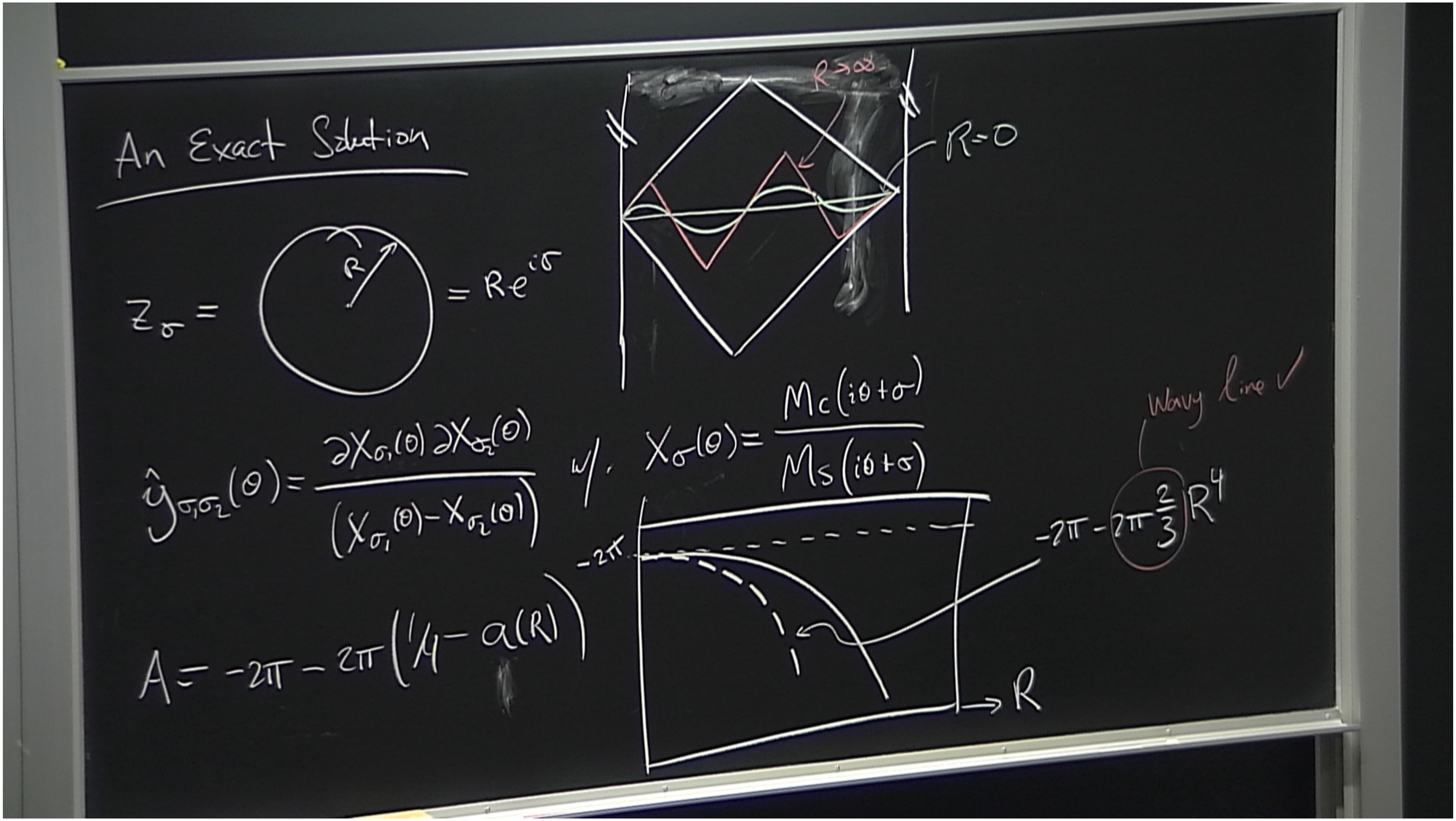
- $$\text{Area} = -2\pi - 4A_\Sigma - \frac{1}{\pi}\int_0^{2\pi} d\tau_2 \int_0^{2\pi} d\tau_1 \int d\theta\, |z_{\sigma_1}-z_{\sigma_2}|\, e^{-\theta}\left(y_{\tau_1\tau_2}(\theta) - y^{\text{Kink}}_{\tau_1\tau_2}\right)$$

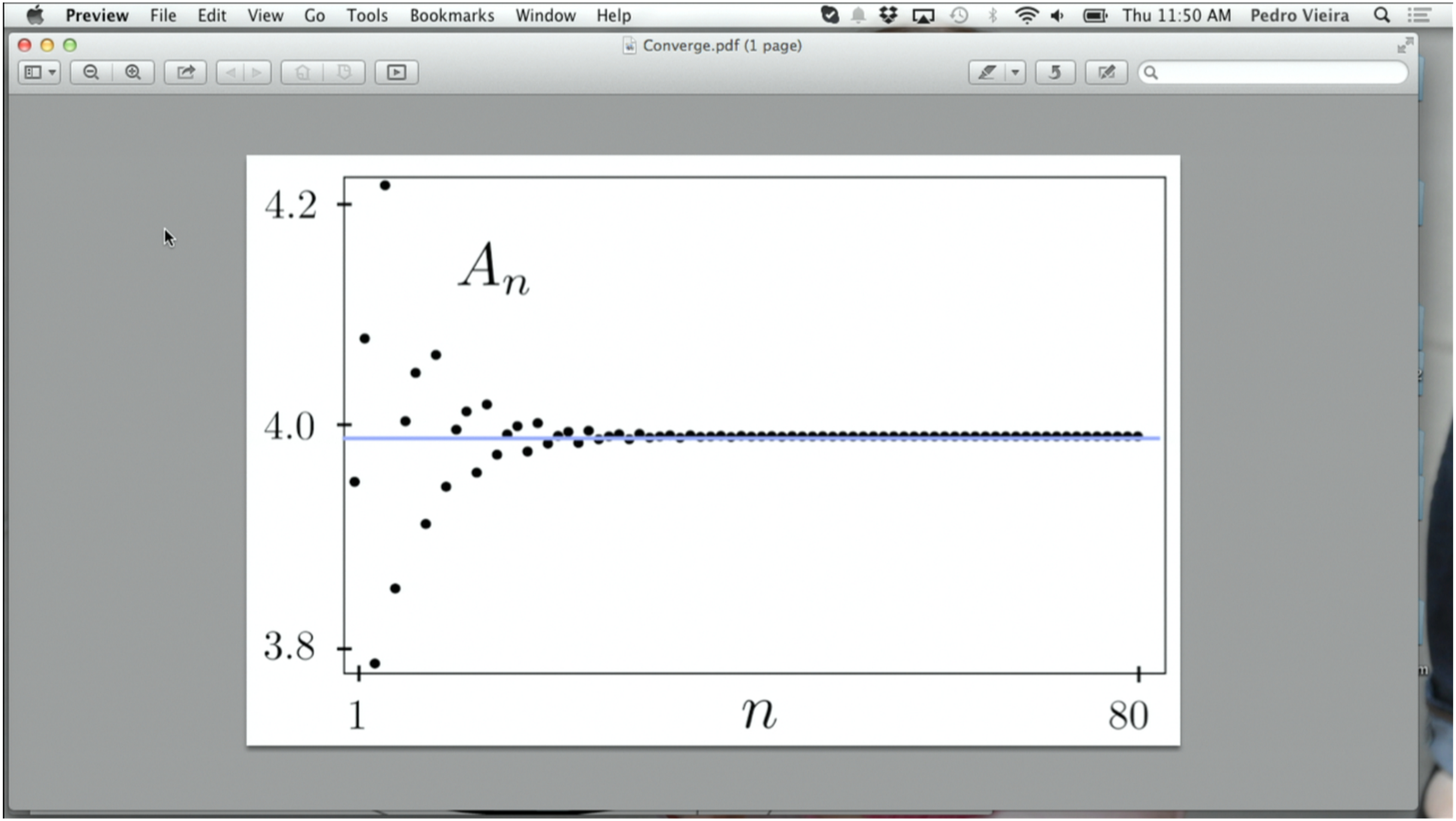
A_Σ: Area Enclosed by z_σ

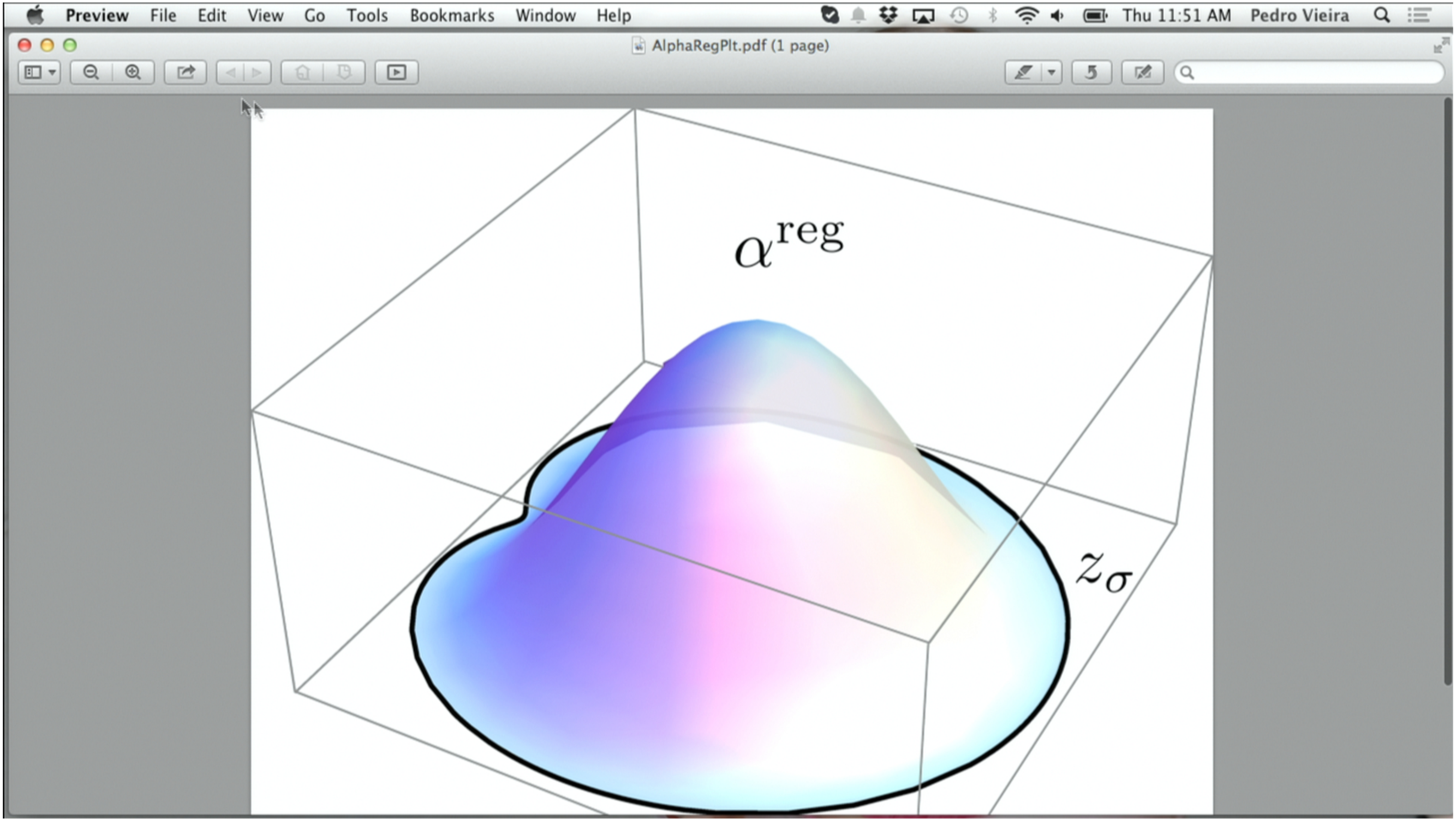
$|z_{\sigma_1}-z_{\sigma_2}|$: $\partial z_{\sigma_1}\,\partial z_{\sigma_2}\, e^{-i(\theta+i\varphi_{\sigma_1\sigma_2})}$

$y^{\text{Kink}}_{\tau_1\tau_2}$: $\mathrm{Csch}^2\left[e^{-\theta}|z_{\sigma_1}-z_{\sigma_2}|\right]$


zσ
yσ1σ2(θ)
cosh θ ~ ½ e^θ
θ
~ log |zσ1 − zσ2|
A → −2π + 4/Σ + G


An Exact Solution
R→∞
R=0
Z_σ = R e^{iσ}
R
Wavy line ✓
ŷ_{σ1σ2}(θ) = ∂X_{σ1}(θ) ∂X_{σ2}(θ) / (X_{σ1}(θ) − X_{σ2}(θ))
w/. X_σ(θ) = Mc(iθ+σ) / Ms(iθ+σ)
−2π − 2π 2/3 R^4
−2π
A = −2π − 2π(1/4 − a(R))
R


Preview File Edit View Go Tools Bookmarks Window Help
Thu 11:50 AM Pedro Vieira
Converge.pdf (1 page)
4.2
4.0
3.8
A_n
1
n
80


Preview File Edit View Go Tools Bookmarks Window Help
Thu 11:51 AM Pedro Vieira
AlphaRegPlt.pdf (1 page)
α^{reg}
z_σ

$$\boxed{|z_{\sigma_1} - z_{\sigma_2}| \longrightarrow e^{i\pi/4} |z_{\sigma_1} - z_{\sigma_2}|}$$

$$p \rightarrow p$$

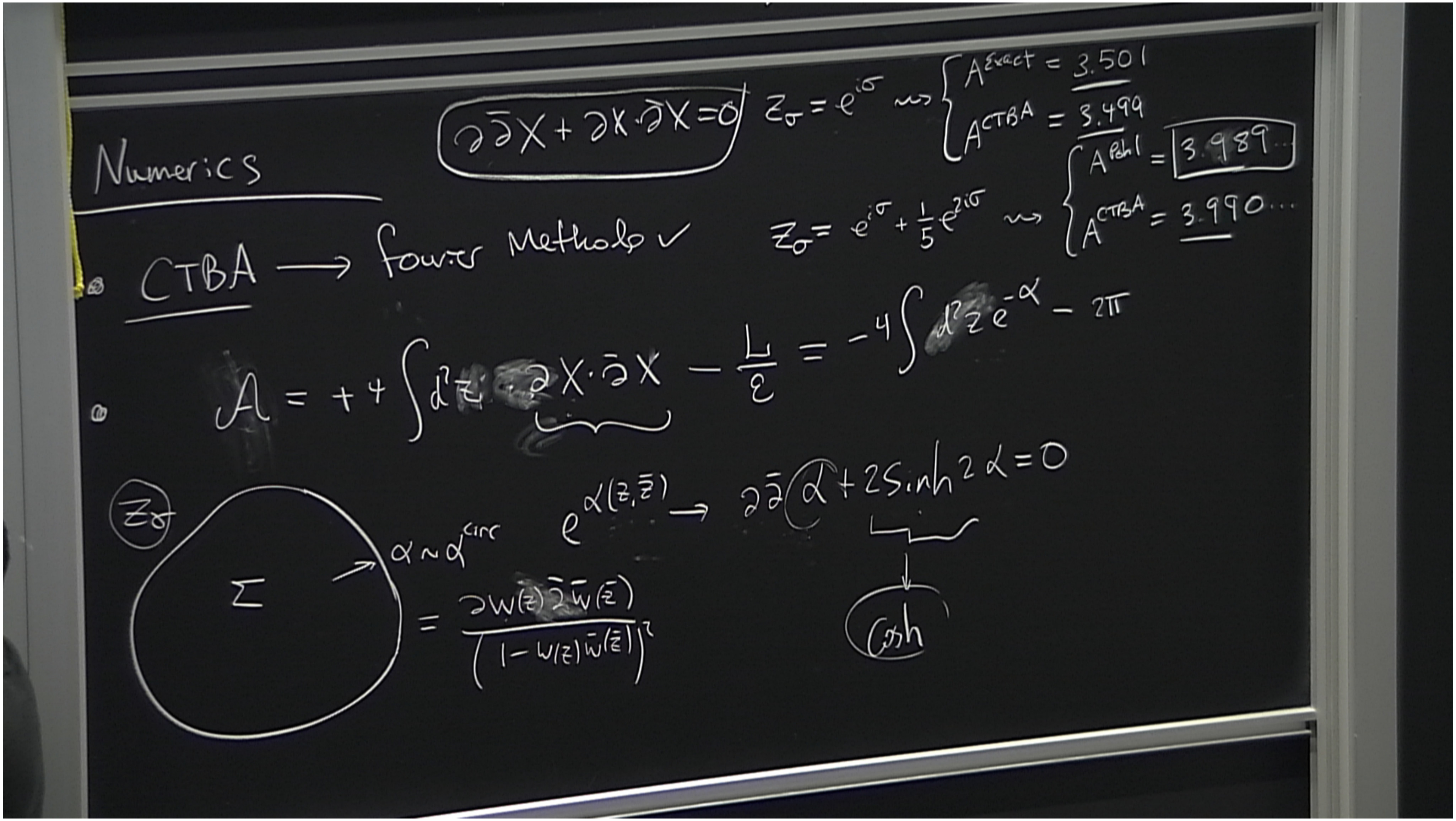
$$\bar{p} \rightarrow -\bar{p}$$

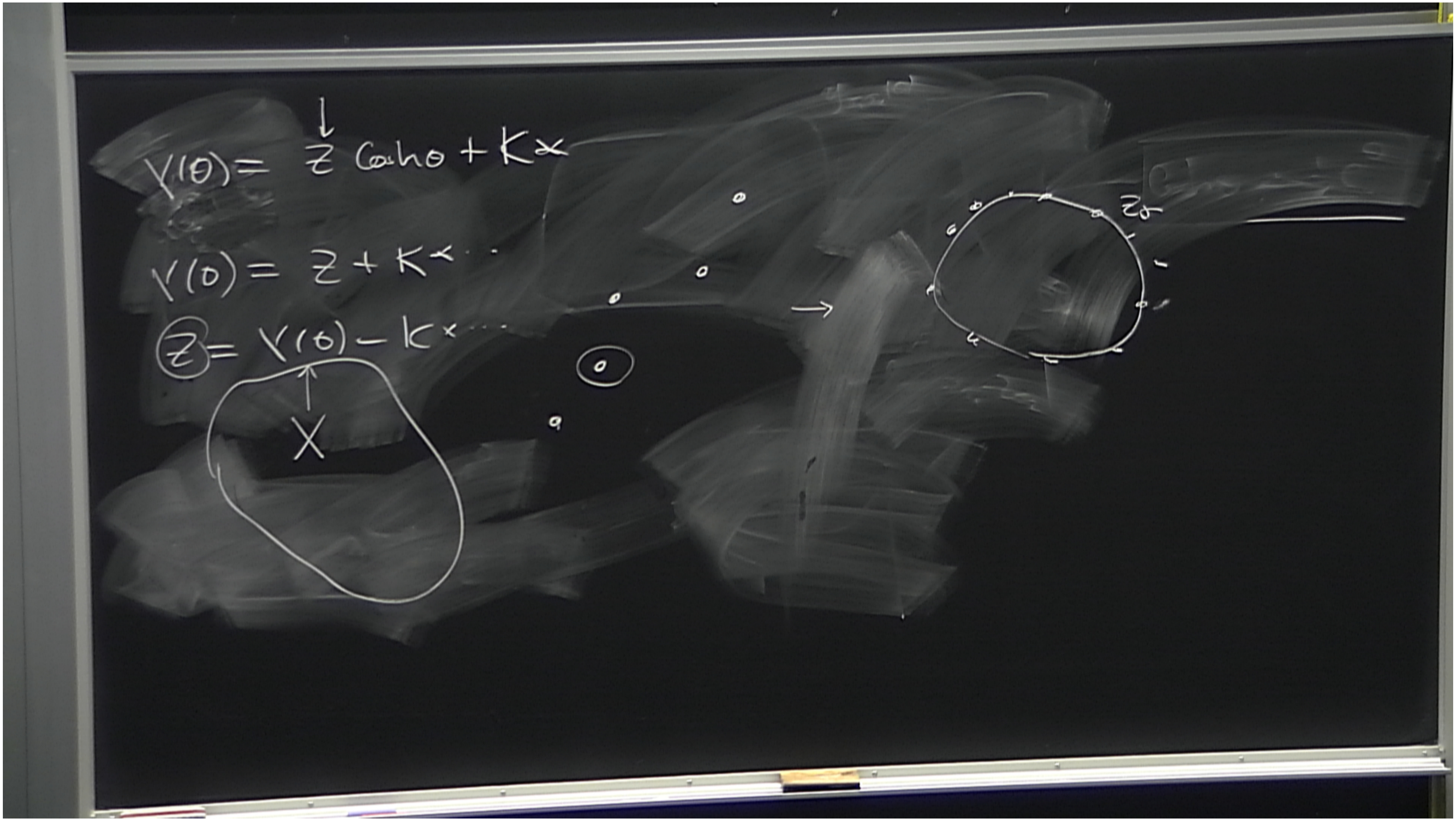
- Theta function sol'ns

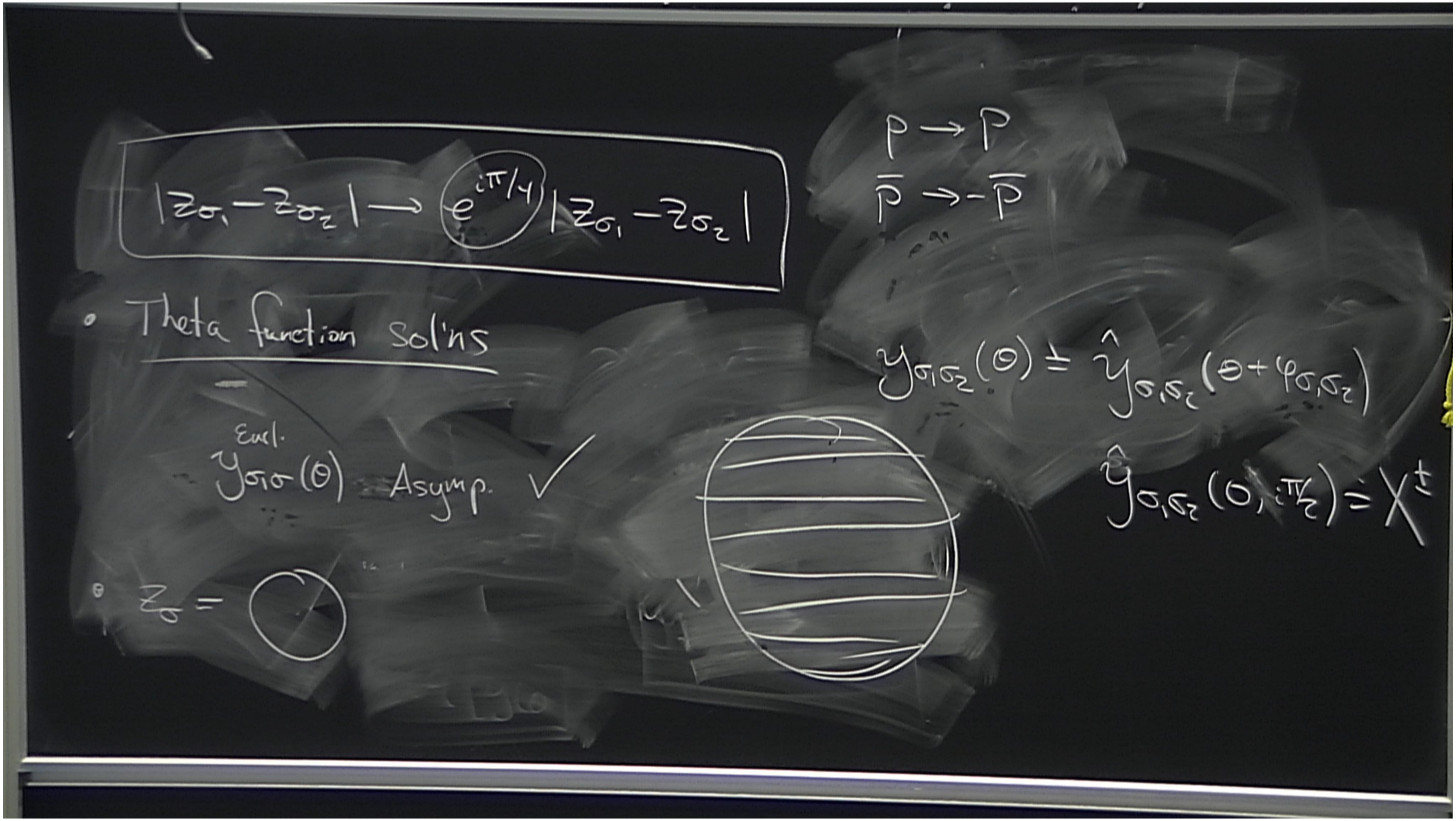
Eucl.

$y_{\sigma\sigma}(\theta)$ Asymp. ✓

- $Z_\sigma =$ 


Numerics
∂∂̄X + ∂X·∂̄X = 0
z_σ = e^{iσ} ⇝ A^{Exact} = 3.501
A^{CTBA} = 3.499
z_σ = e^{iσ} + 1/5 e^{2iσ} ⇝ A^{Pohl} = 3.989...
A^{CTBA} = 3.990...
CTBA → fourier Methods ✓
A = +4 ∫ d²z ∂X·∂̄X − L/ε = −4 ∫ d²z e^{−α} − 2π
z_σ
Σ
α ~ α^{circ}
e^{α(z,z̄)} → ∂∂̄α + 2 sinh 2α = 0
cosh
= ∂w(z) ∂̄w̄(z̄) / (1 − w(z) w̄(z̄))²


Y(θ) = Z Cosh θ + Kα
Y(θ) = Z + Kα ...
Z = Y(θ) − Kα ...
X


$|z_{\sigma_1} - z_{\sigma_2}| \longrightarrow e^{i\pi/4} |z_{\sigma_1} - z_{\sigma_2}|$
Theta function sol'ns
Eucl.
$y_{\sigma\sigma}(\theta)$ Asymp.
$z_\sigma =$
$p \to p$
$\bar{p} \to -\bar{p}$
$y_{\sigma_1\sigma_2}(\theta) \doteq \hat{y}_{\sigma_1\sigma_2}(\theta + \varphi_{\sigma_1\sigma_2})$
$\hat{y}_{\sigma_1\sigma_2}(0, \pm\pi/2) = X^{\pm}$