

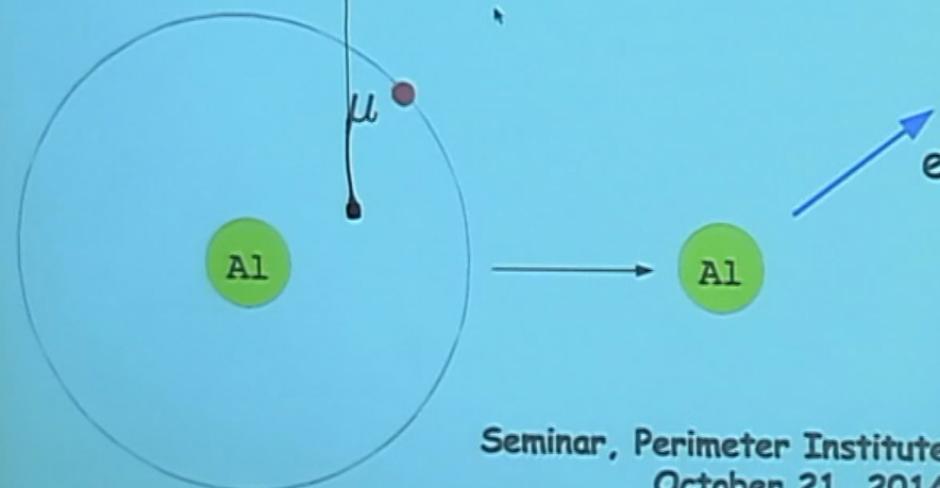
Title: Fundamental physics with muons

Date: Oct 21, 2014 01:00 PM

URL: <http://pirsa.org/14100115>

Abstract: <span>I will review applications of the muon as a probe for new phenomena. Topics to be discussed include the free muon decay and the determination of the Fermi constant; the anomalous magnetic moment of the muon; and searches for lepton flavor violation such as  $\mu \rightarrow e + \gamma$ ,  $\mu \rightarrow 3e$ , and the muon-electron conversion, with special emphasis on the modification of the muon decay by the atomic binding.</span>

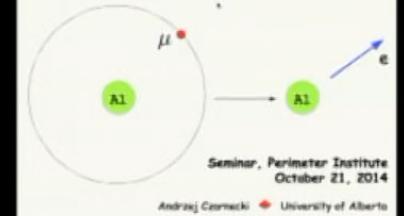
## Fundamental Physics with Muons



Seminar, Perimeter Institute  
October 21, 2014

Andrzej Czarnecki University of Alberta

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## Outline

Three promising directions in low-energy searches:

- \* anomalous magnetic moments (muon vs electron)
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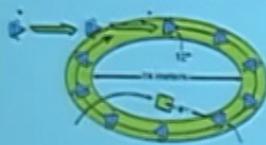
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## The puzzle of the muon magnetic moment

The 3.6 sigma discrepancy persists,



$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = 287(80) \times 10^{-11}$$

PRD 86, 095009 (2012)

This is rather large when compared with other bounds on New Physics:

Muon MDM  $d_{\mu} \sim \frac{e}{2m_{\mu}} a_{\mu}^{\text{NP}} \sim 3 \cdot 10^{-22} e \cdot \text{cm}$

Muon-electron transition moment  $|d_{\mu \rightarrow e}| < 4 \cdot 10^{-27} e \cdot \text{cm}$  MEG 2013

Electron EDM  $|d_e| < 8.7 \cdot 10^{-29} e \cdot \text{cm}$  ACME 2014

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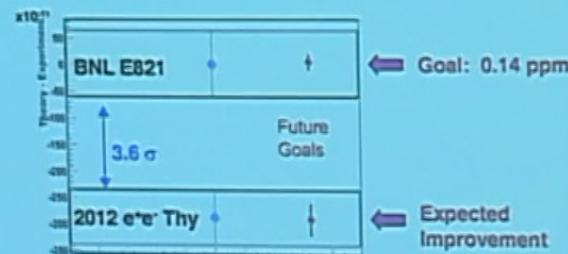
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New experiment at Fermilab



New experimental concept at J-PARC

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## New approach to $g_\mu - 2$ at J-PARC

Slower muons 300 MeV (instead of the "magic" 3.1 GeV)

Ultracold muons; no electric focusing!

Smaller ring  $r = 33$  cm (instead of 7 m)

$$r \text{ [in meters]} \approx \frac{\gamma}{3B \text{ [in Tesla]}}$$

Strong, very precisely controlled magnetic field.

~10 times more muons than at Fermilab (compensates shorter lifetime).

	Brookhaven	Fermilab	J-PARC
Muon momentum	3.09 GeV/c	0.3 GeV/c	
gamma	29.3	3	
Storage field	$B=1.45$ T	$3.0$ T	
Focusing field	Electric quad	None	$\approx \sqrt{\frac{2\pi}{\alpha}}$
# of detected $\mu^+$ decays	5.0E9	1.8E11	1.5E12
# of detected $\mu^-$ decays	3.6E9	-	-
Precision (stat)	0.46 ppm	0.1 ppm	0.1 ppm

From N. Saito

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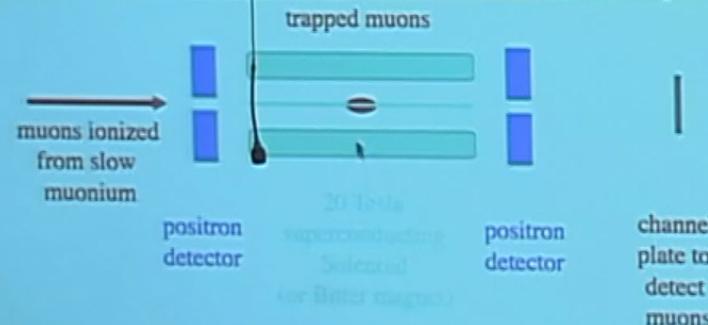
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USM2013

Gabrielse

### Measure Muon Magnetic Moment in a Trap



Advantages: Use muons to measure the magnetic field in situ  
Less expensive  
Different systematics

Challenge: 2.2 microsecond muon lifetime

Thanks to A. Mills for helpful conversations

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Measured with relative error  $25 \cdot 10^{-11}$

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Provides the fine structure constant with the same precision,

$$\alpha^{-1} (a_e) = 137.035\,999\,1736(331)(86)$$

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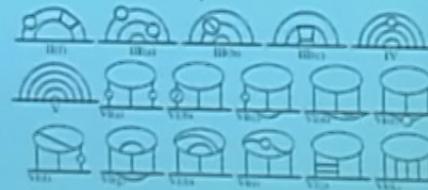
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If the muon anomaly is due to New Physics, the expected effect for the electron is likely smaller by

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$$\Delta a_\mu \sim 287 \cdot 10^{-11} \rightarrow \Delta a_e \sim 7 \cdot 10^{-14}$$

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from atomic spectroscopy

$$R_\infty = \frac{m_e c \alpha^2}{2 h}$$

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(but is it  
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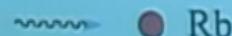
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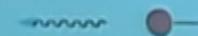
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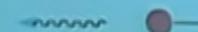
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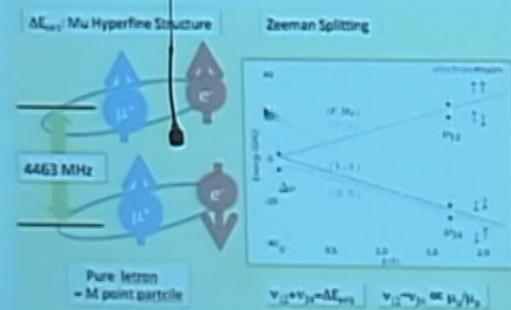
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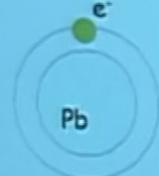


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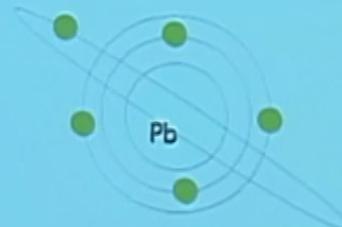


## Another source of alpha: highly-charged ions

$$g \approx 2 - \frac{2(Z\alpha)^2}{3} \quad \xrightarrow{\hspace{1cm}} \quad \frac{\delta\alpha}{\alpha} \sim \frac{1}{(Z\alpha)^2} \sqrt{(\delta g_{\text{exp}})^2 + (\delta g_{\text{th}})^2} \quad \text{large } Z \text{ favorable}$$



Hydrogen-like lead



Boron-like lead

There is a combination of  $g$ -factors in both ions where the sensitivity to the nuclear structure largely cancels, but the sensitivity to alpha remains.

Much interesting theoretical work remains to be done!

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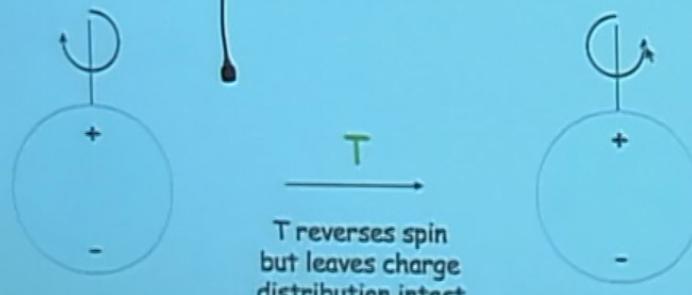
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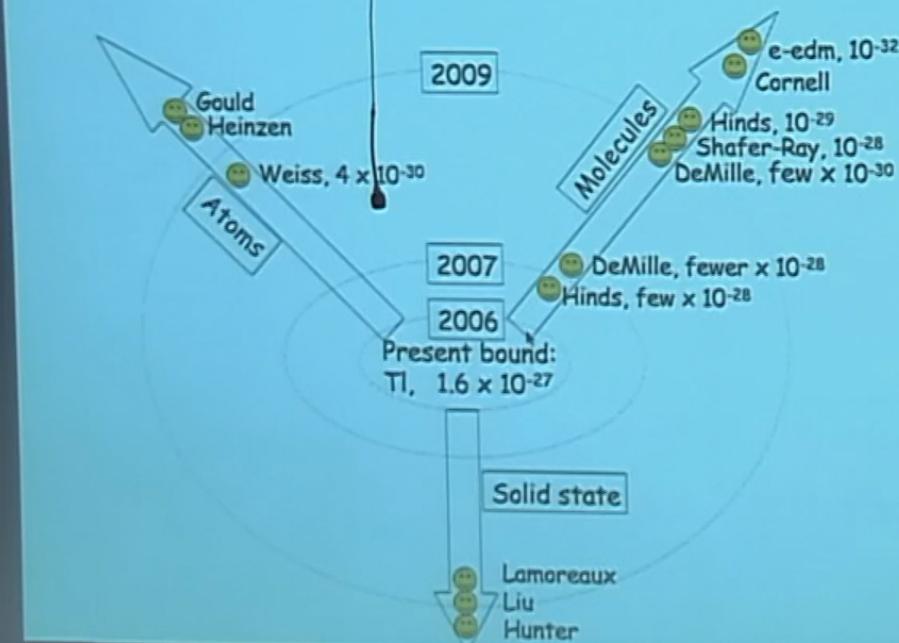


T reverses spin  
but leaves charge  
distribution intact

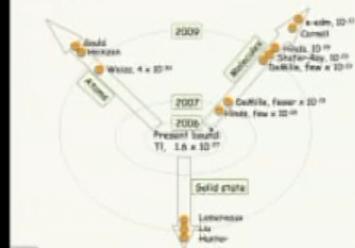
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## Electron EDM: what used to be expected (2006)

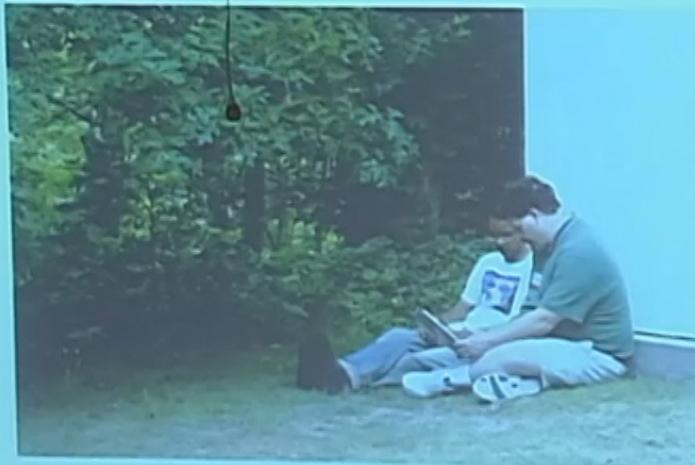


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Are they hiding new results?



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## Recent great progress

2011: YbF @ Imperial

$$|d_e| < 10.5 \times 10^{-28} \text{ e cm}$$

LETTER

doi:10.1038/nature12154

### Improved measurement of the shape of the electron

J. J. Hudson<sup>1</sup>, G.-M. Kam<sup>1</sup>, J. J. Smallman<sup>1</sup>, B. E. Sauer<sup>2</sup>, M. R. Tarbutt<sup>1</sup> & E. A. Hinds<sup>1</sup>

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2014: ThO by ACME

$$|d_e| < 8.7 \times 10^{-29} e\cdot\text{cm}$$

Science

### Order of Magnitude Smaller Limit on the Electric Dipole Moment of the Electron

The ACME Collaboration,\* J. Baron,<sup>1</sup> W. C. Campbell,<sup>2</sup> D. Dettling,<sup>3</sup> J. M. Doyle,<sup>2†</sup> G. Gabrielse,<sup>2,4</sup> Y. V. Gurevich,<sup>2,5</sup> P. W. Hegg,<sup>6</sup> N. R. Huelga,<sup>2</sup> E. Kitchin,<sup>2,9</sup> I. Kleppergen,<sup>2,1</sup> B. R. O'Leary,<sup>2</sup> C. D. Panda,<sup>1</sup> M. F. Parsons,<sup>2</sup> E. S. Petrie,<sup>2</sup> B. Spann,<sup>2</sup> A. C. Wallra,<sup>2</sup> A. D. West<sup>2</sup>

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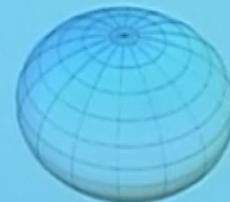
## What will be on the cover when $d_e$ discovered?

Analogous discussion about the shape of the earth (18<sup>th</sup> century): which theory of gravity is correct?

Descartes:



Newton:



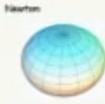
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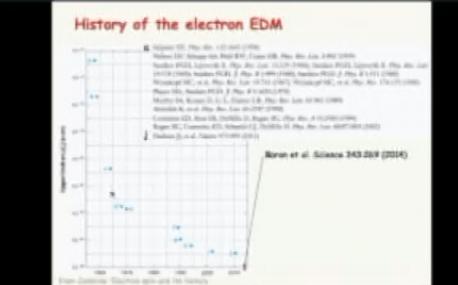
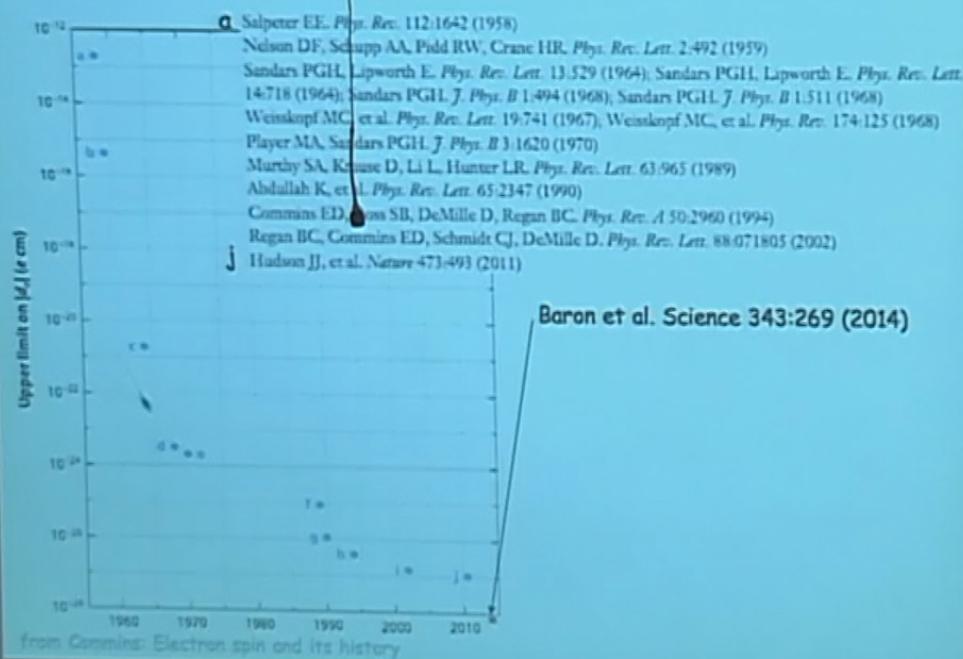
## When will we get there? Plans for $d_e$ :

	Near future	Ultimate goal	
Molecular ion	< ACME	$< 10^{-29}$	Ye Cornell
Atoms in an optical lattice	< ACME	$\text{few} \cdot 10^{-30}$	Weiss
YbF	$10^{-29}$	$\sim 10^{-30}$	Sauer
Future ACME	$10^{-29}$	$< 10^{-30}$	DeMille Gabrielse

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Molecular ion	< ACME	$< 10^{-29}$	Ye Cornell
Atoms in an optical lattice	< ACME	$\text{few} \cdot 10^{-30}$	Weiss
YbF	$10^{-29}$	$\sim 10^{-30}$	Sauer
Future ACME	$10^{-29}$	$< 10^{-30}$	DeMille Gabrielse

## History of the electron EDM



## Remarks on $d_e$

Molecules: great new player; but enhancement factors very tricky  
One way to check: hyperfine structure ( $\text{YbF}$ )

Molecules are a very new field; many improvements possible  
(sources!)

An even newer field: molecular ions.

Challenge for theorists: "enhancement factors": this is a new frontier for quantitative studies in molecules.

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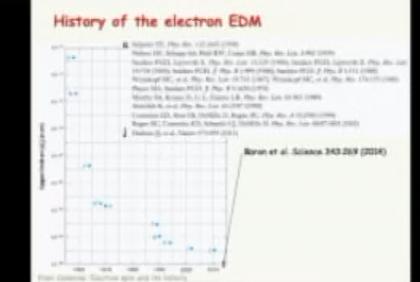
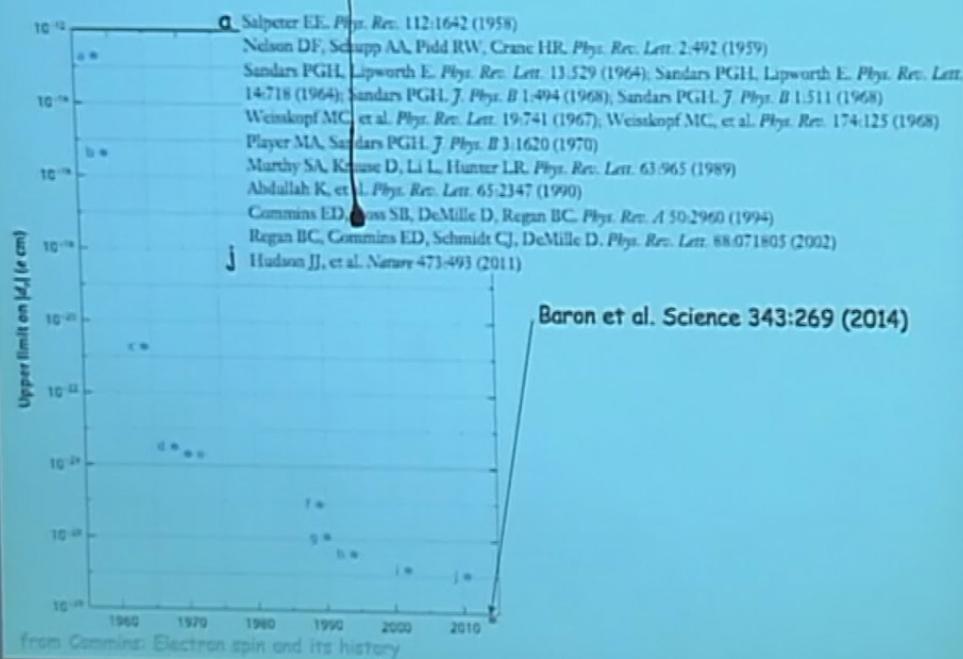
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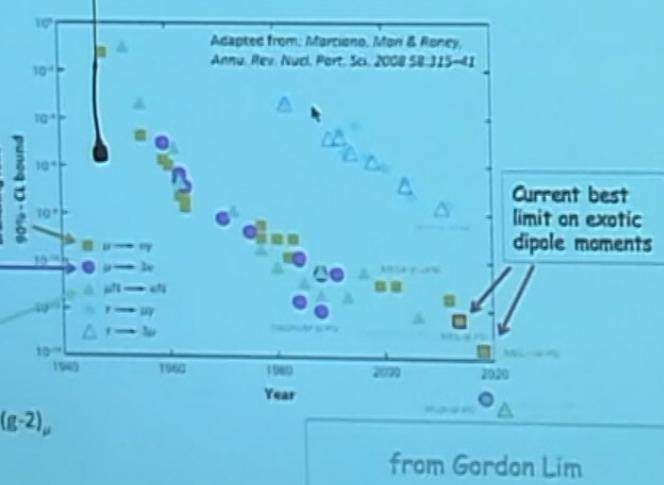
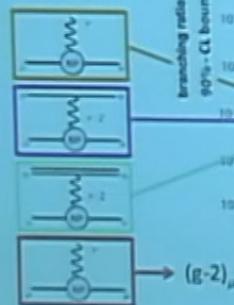
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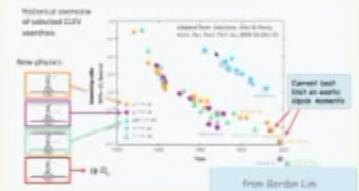
## Efforts in LFV searches

Historical overview  
of selected CLFV  
searches:

New physics:



## Efforts in LFV searches

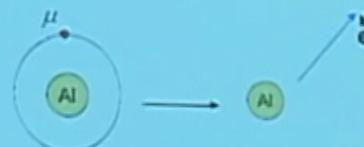


## Conversion: probes also non-dipole interactions

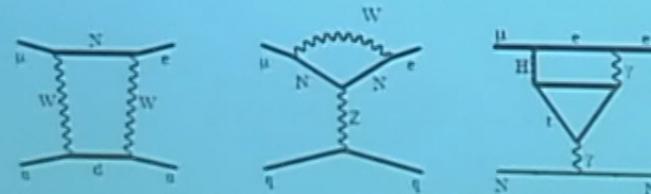
So far, we have only talked about dipole interactions.  
There are also vectors and scalars.

They are not (directly) probed by processes with external photons,  
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Variety of mechanisms:



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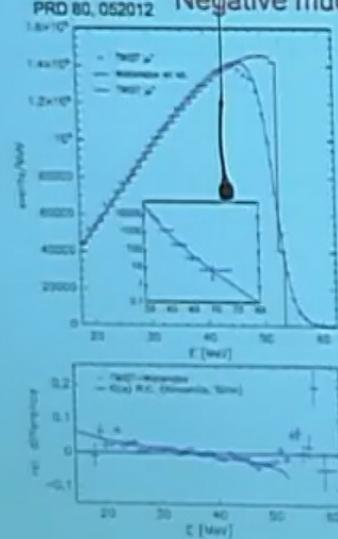
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Variety of mechanisms:



## Bound muon decay has been measured: TWIST/TRIUMF

### Negative muon decay-in-orbit

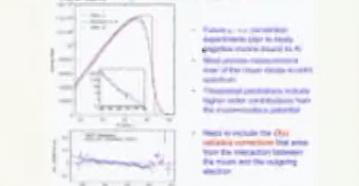


- Future  $\mu \rightarrow e$  conversion experiments plan to study negative muons bound to Al
- Most precise measurement ever of the muon decay-in-orbit spectrum
- Theoretical predictions include higher-order contributions from the muon+nucleus potential
- Need to include the  $O(\alpha)$  radiative corrections that arise from the interaction between the muon and the outgoing electron

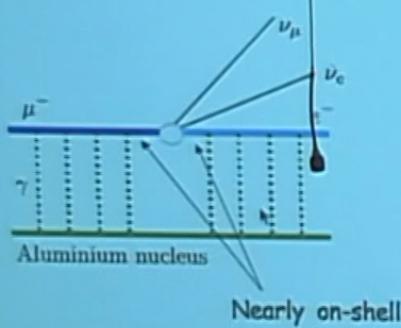
From Carl A. Gagliardi/TWIST

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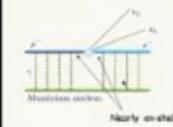
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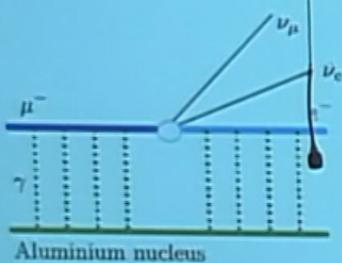
## Decay of a muon bound in aluminium



Decay of a muon bound in aluminium



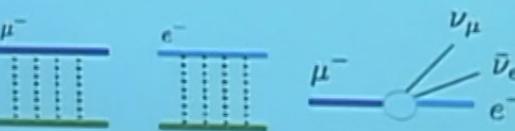
## Decay of a muon bound in aluminium



factorization

Free muon decay rate,  
with all corrections!

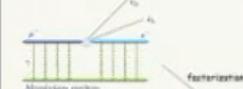
$$\frac{d\Gamma}{dE_e} = \int d\lambda s(\lambda) \frac{d\Gamma_{\text{free}}}{dz} \left. \frac{dz}{dE_e} \right|_{z \rightarrow z(\lambda)}$$



$$z(\lambda) = \frac{2(E_e + \lambda) + (Z\alpha)^2 m_\mu}{m_\mu + \lambda}$$

A.C., M. Dowling, X. Garcia i Tormo, W. Marciano, R. Sacfran

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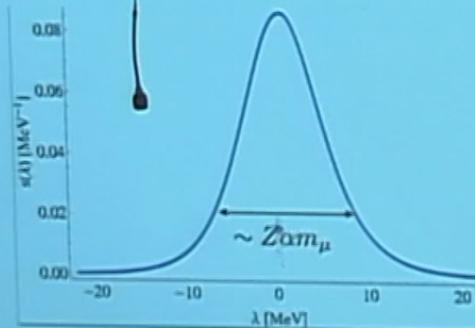


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## Shape function

$$s(\lambda) = \int \frac{d^3k}{(2\pi)^3} \psi_g^*(\vec{k}) \delta(\lambda + \vec{n} \cdot \vec{k}) \psi_g(\vec{k})$$

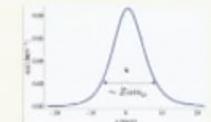


Previously used in heavy mesons, where it cannot be computed from first principles, but can be experimentally accessed.  
Manohar, Neubert, Bigi, Shifman, Uraltsev, Vainshtein

AC, M. Dowling, X. Garcia i Tormo, W. Marciano, R. Szafron

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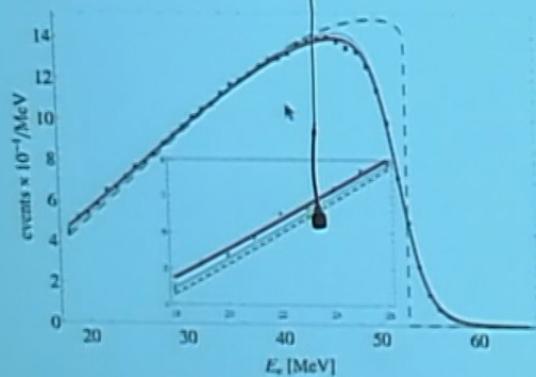
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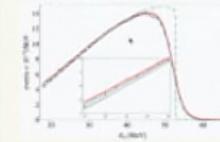
AC, M. Dowling, X. Garcia i Tormo, W. Marciano, R. Szafron

## Result: spectrum of the bound muon decay



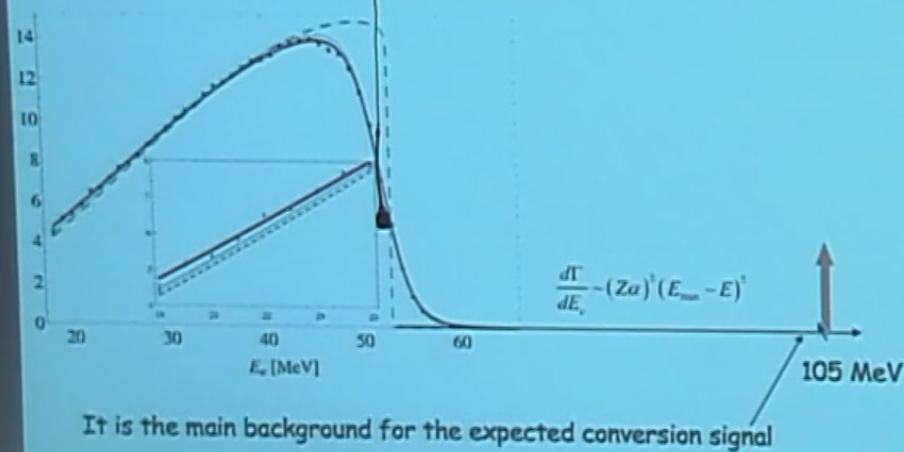
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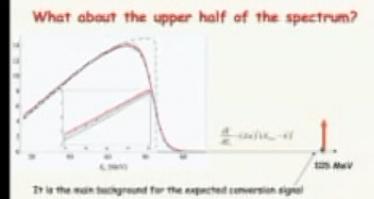


A.C., M. Dowling, X. Garcia i Tormo, W. Marciano, R. Szcfran

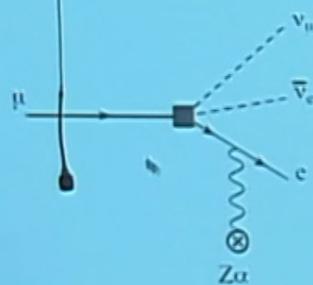
What about the upper half of the spectrum?



It is the main background for the expected conversion signal



## Next step: radiative corrections to the electron spectrum



Competing effects:

- vacuum polarization in the hard photon; and
- self-energy and real radiation

Ultimate goal: smooth matching of all energy regions,  
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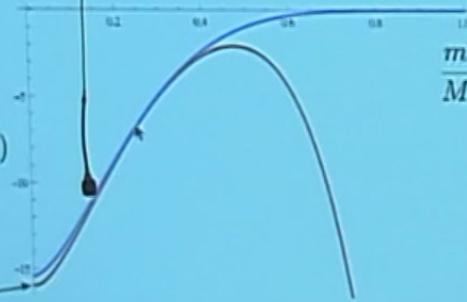
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$\alpha^2$  correction to  
 $\Gamma(\mu(M) \rightarrow e(m) \nu \bar{\nu})$

exact



Note: the blue curve is designed for  $m \sim M$ ,  
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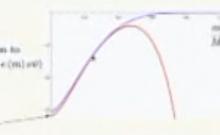
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M. Dowling, with J. Piclum, AC, M. Czakon

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## Conclusions

Great opportunities for precise theoretical studies in low-energy physics:

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