Title: Disturbance in weak measurements and the difference between quantum and classical weak values

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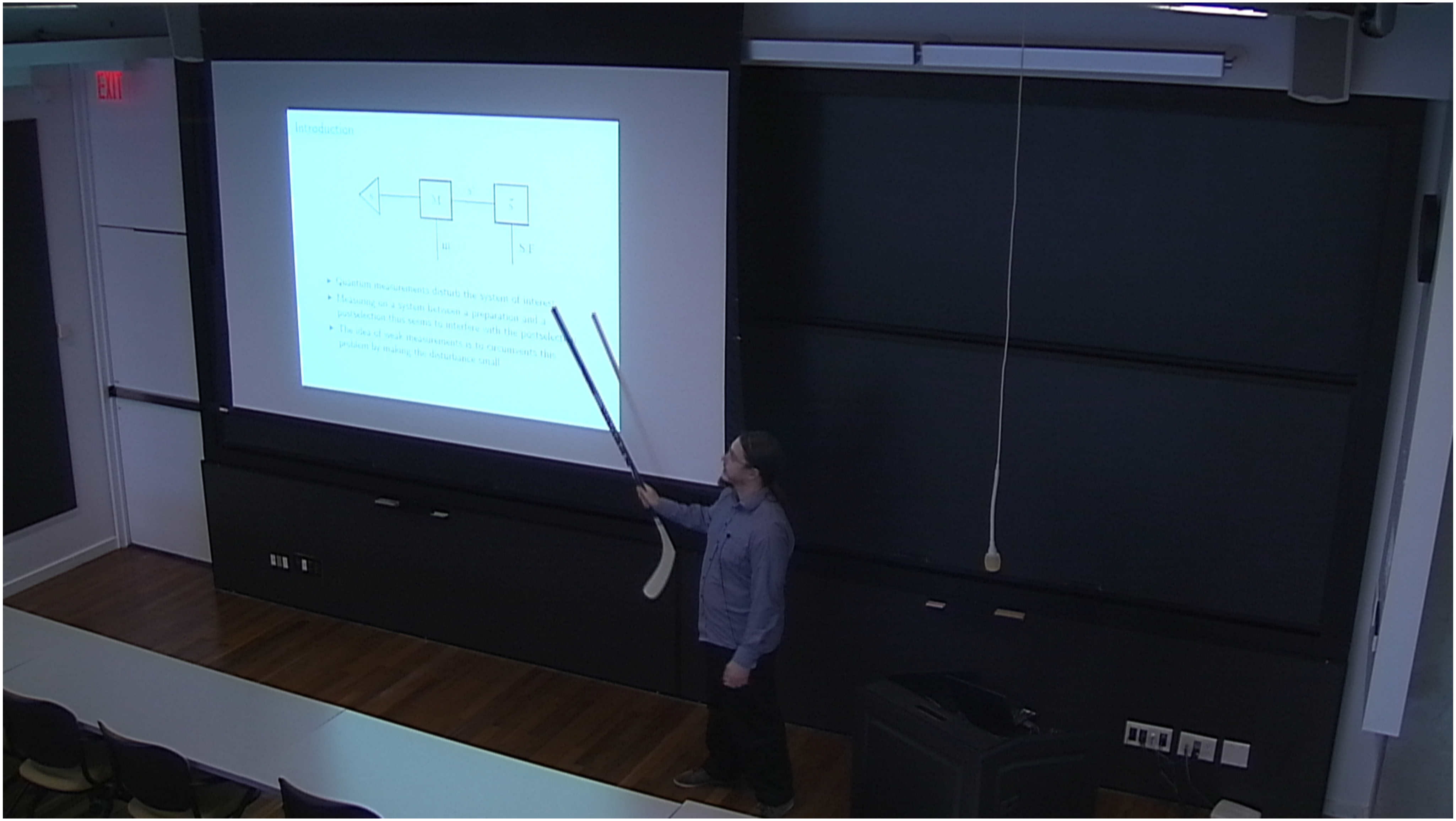
Abstract: The role of measurement induced disturbance in weak measurements is of central importance for the interpretation of the weak value. Uncontrolled disturbance can interfere with the postselection process and make the weak value dependent on the details of the measurement process. Here we develop the concept of a generalized weak measurement for classical and quantum mechanics. The two cases appear remarkably similar, but we point out some important differences. A priori it is not clear what the correct notion of disturbance should be in the context of weak measurements. We consider three different notions and get three different results: (1) For a `strong' definition of disturbance, we find that weak measurements are disturbing. (2) For a weaker definition we find that a general class of weak measurements are non-disturbing, but that one gets weak values which depend on the measurement process. (3) Finally, with respect to an operational definition of the `degree of disturbance', we find that the AAV weak measurements are the least disturbing, but that the disturbance is still non-zero.



Introduction
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Quantum measurements disturb the system of interest
Measuring on a system between a preparation and a
postselection thus seems to interfere with the postselect
The idea of weak measurements is to circumvents this
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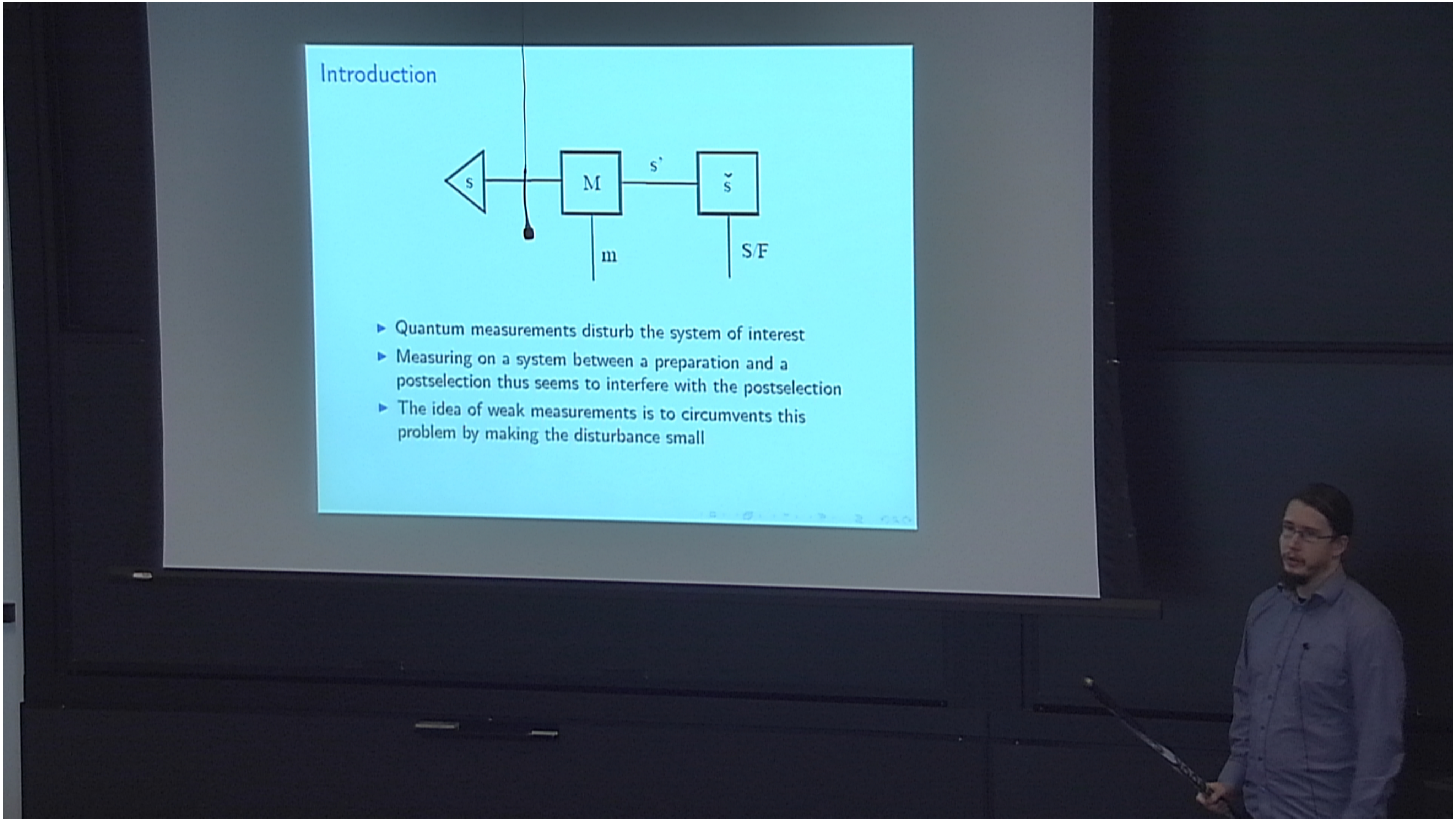
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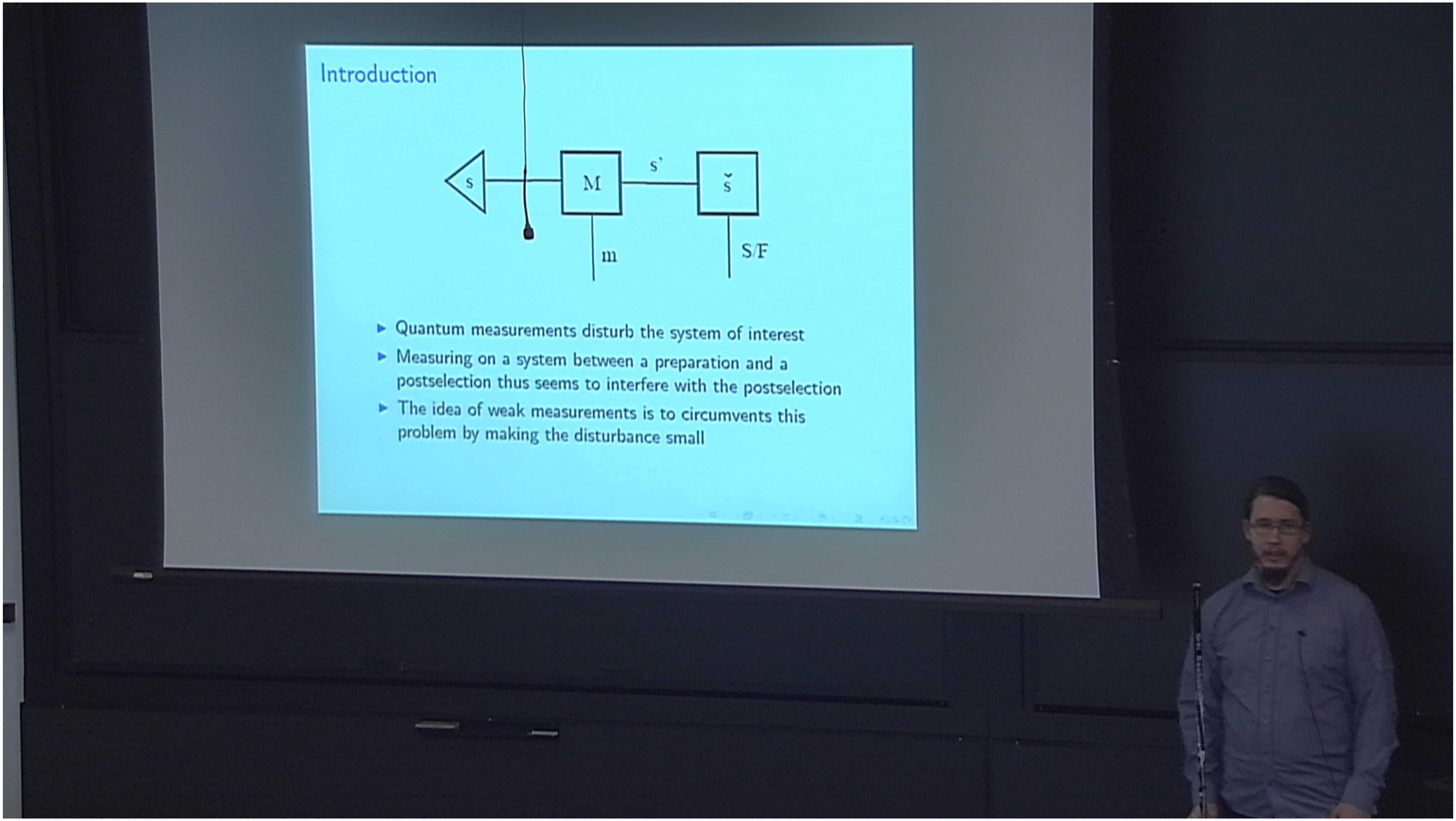
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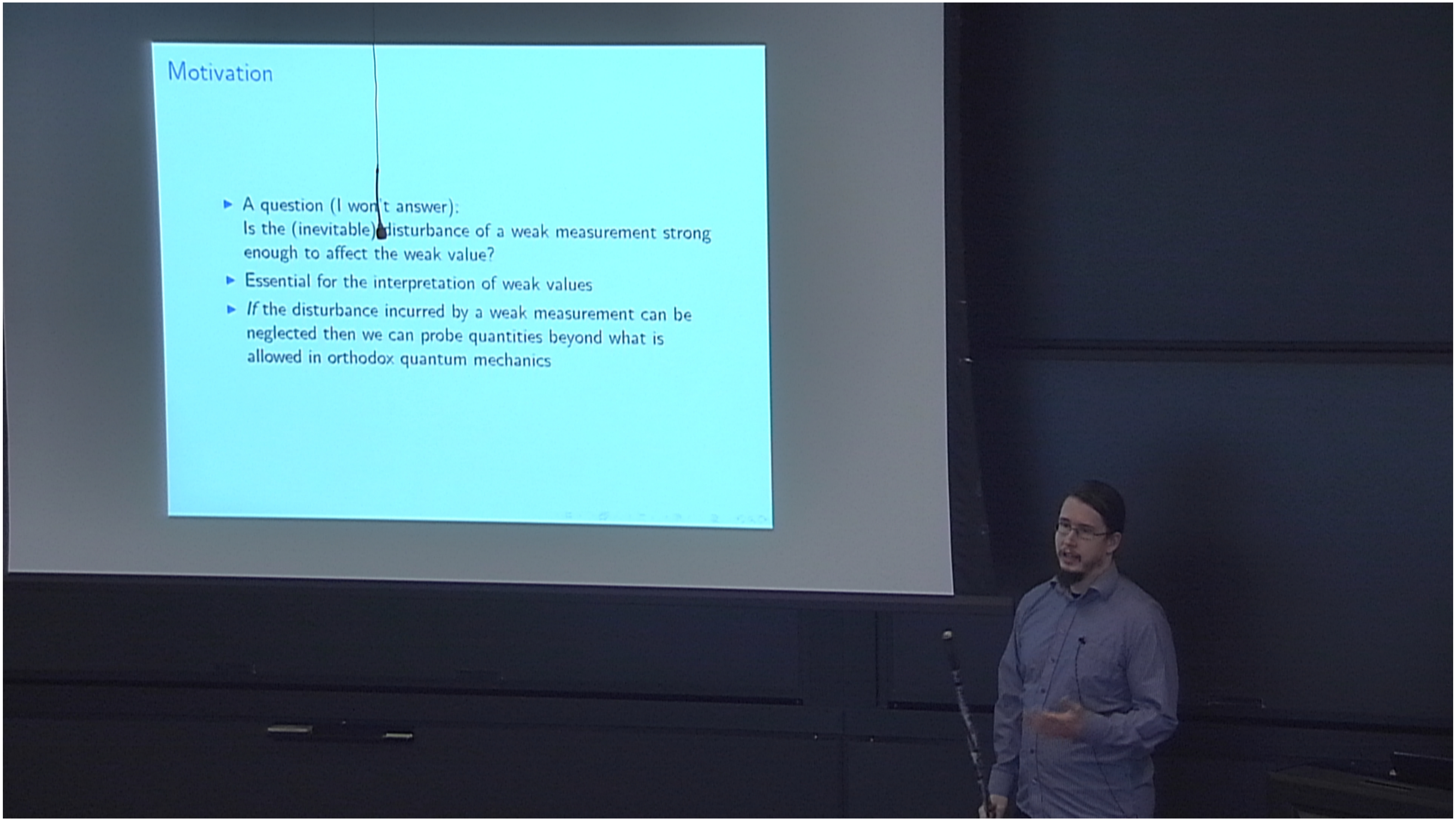
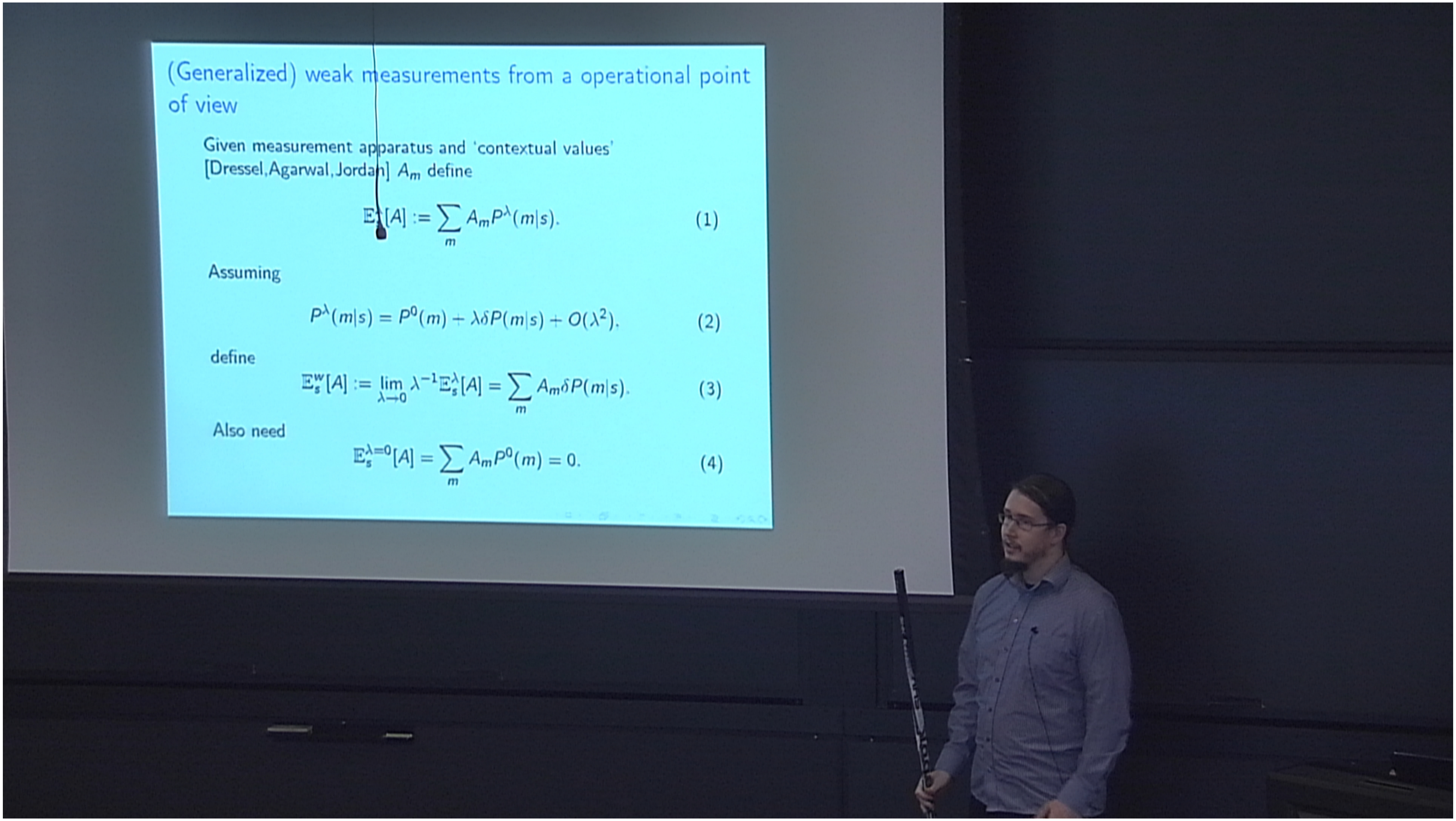
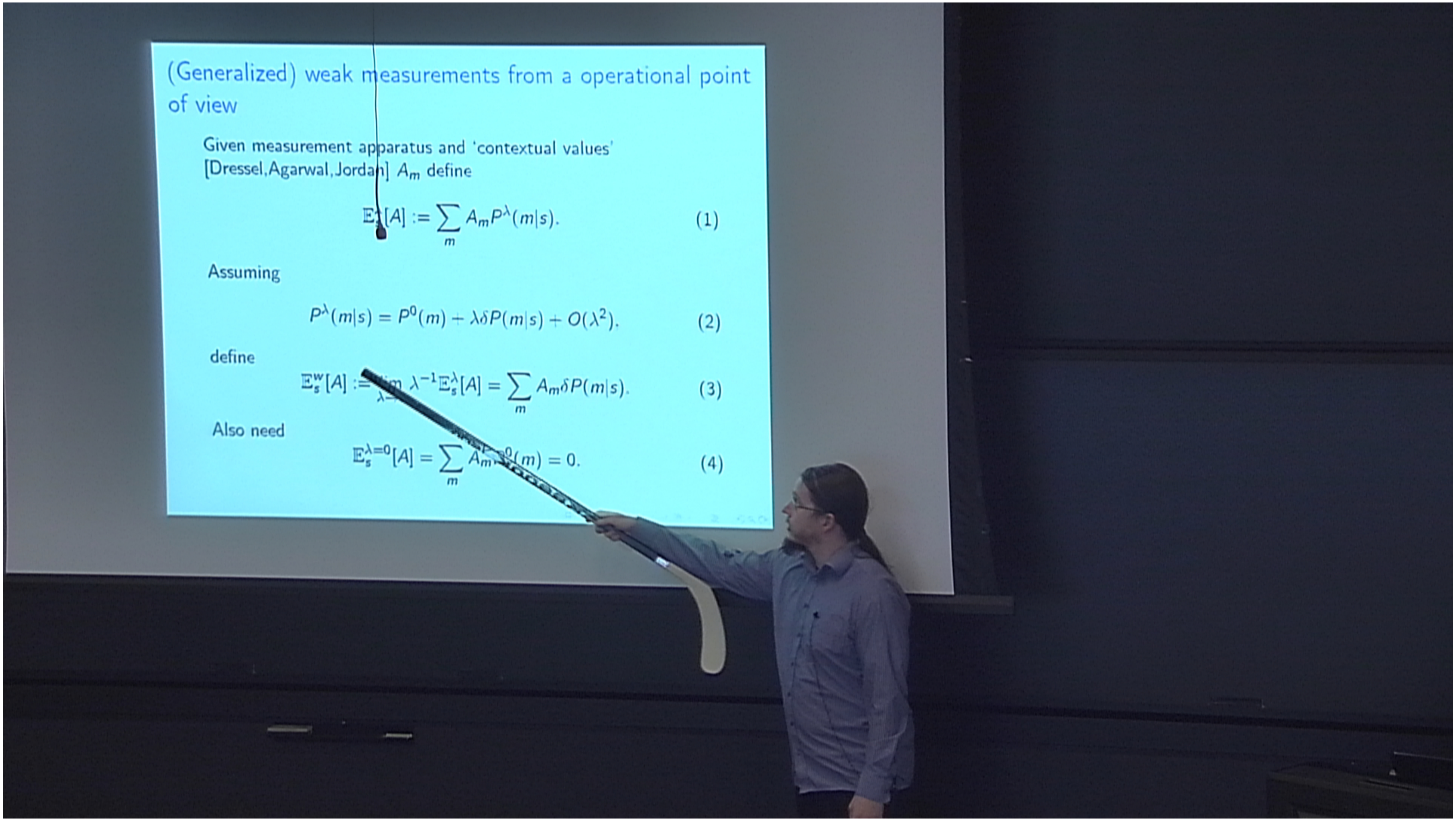

Motivation
A question (I won't answer):
Is the (inevitable) disturbance of a weak measurement strong enough to affect the weak value?
Essential for the interpretation of weak values
If the disturbance incurred by a weak measurement can be neglected then we can probe quantities beyond what is allowed in orthodox quantum mechanics


(Generalized) weak measurements from a operational point of view
Given measurement apparatus and 'contextual values' [Dressel,Agarwal,Jordan] A_m define
$\mathbb{E}_s^{\lambda}[A] := \sum_m A_m P^{\lambda}(m|s).$ (1)
Assuming
$P^{\lambda}(m|s) = P^0(m) + \lambda \delta P(m|s) + O(\lambda^2).$ (2)
define
$\mathbb{E}_s^{w}[A] := \lim_{\lambda \to 0} \lambda^{-1} \mathbb{E}_s^{\lambda}[A] = \sum_m A_m \delta P(m|s).$ (3)
Also need
$\mathbb{E}_s^{\lambda=0}[A] = \sum_m A_m P^0(m) = 0.$ (4)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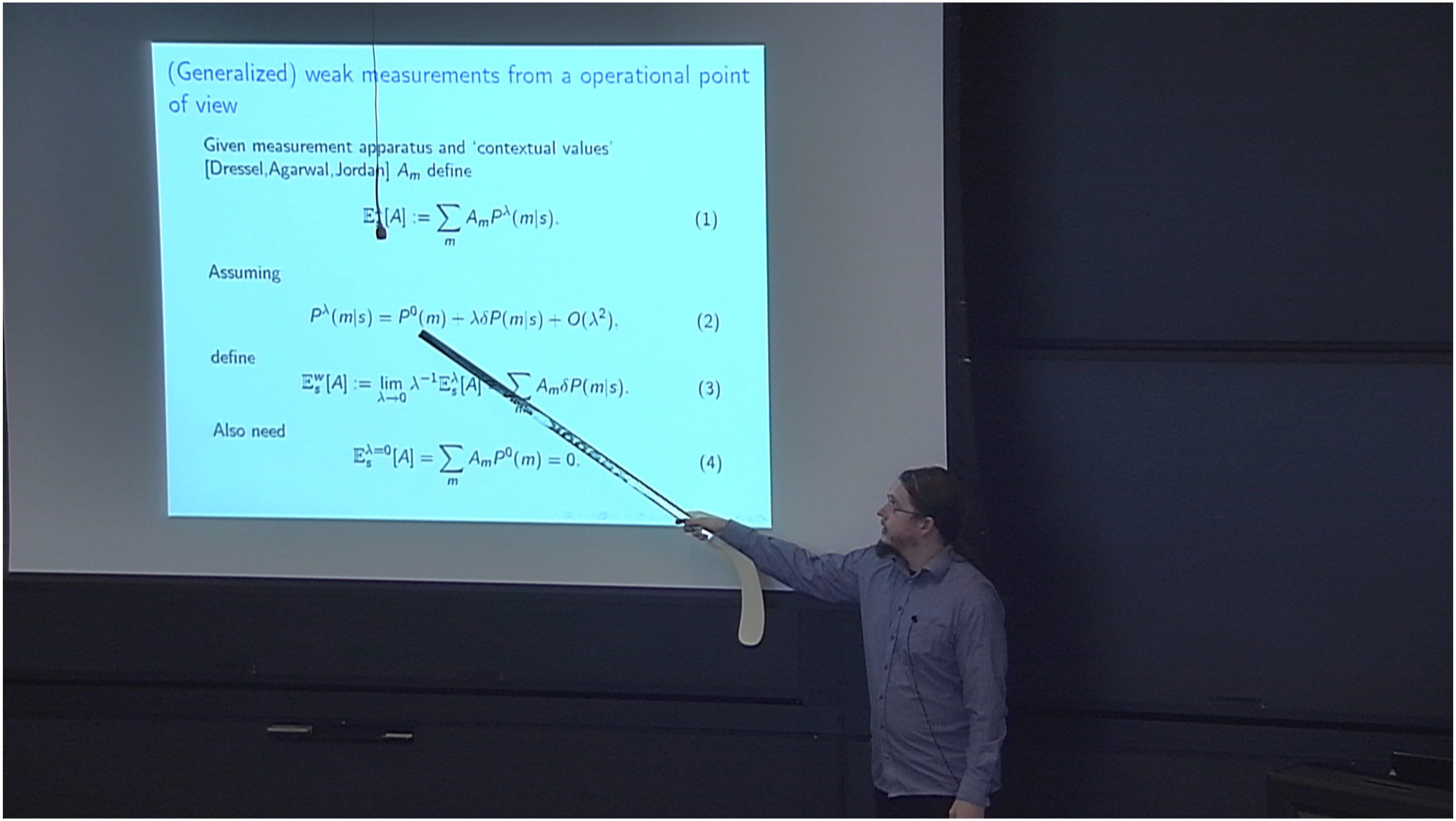
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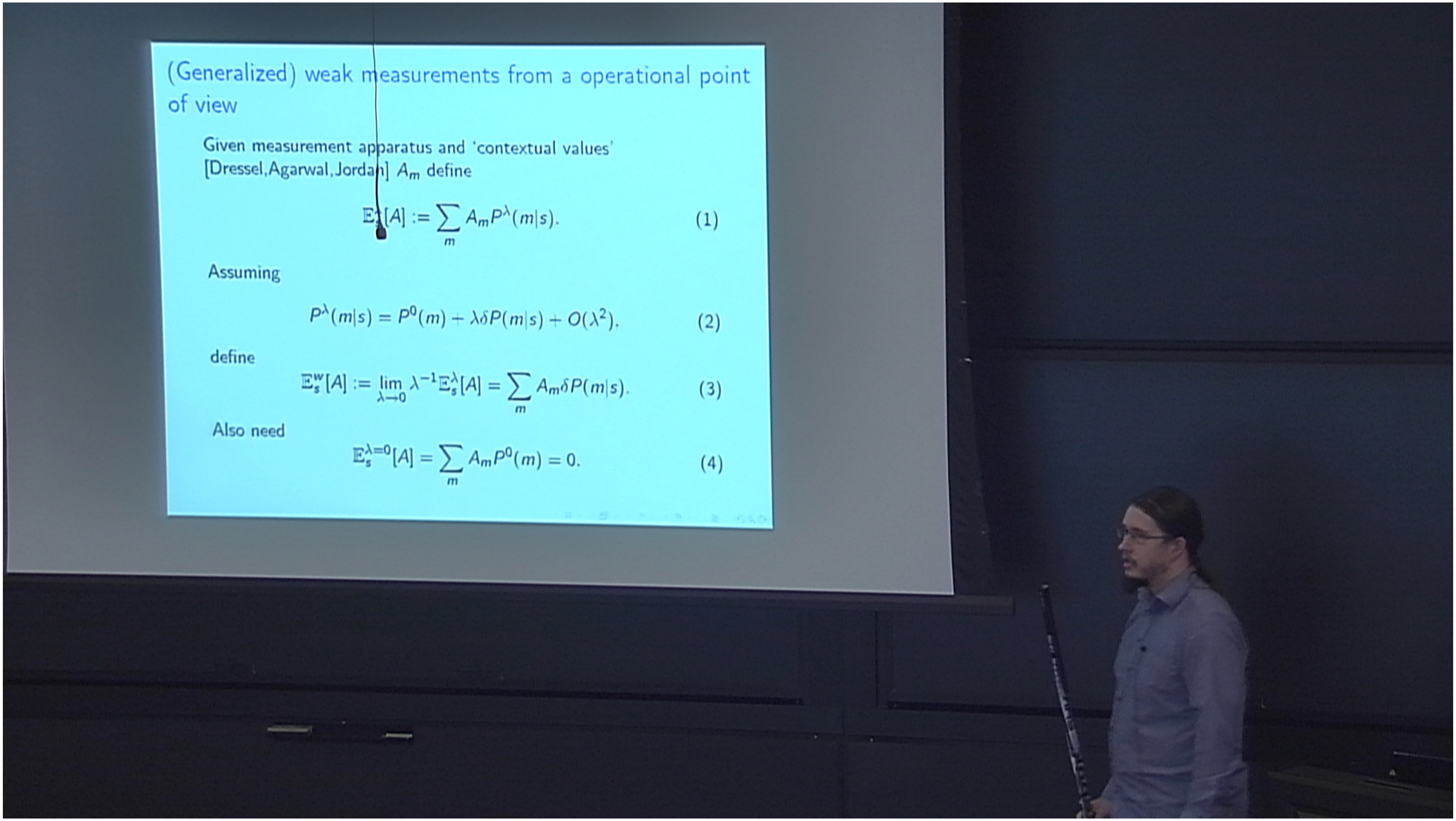
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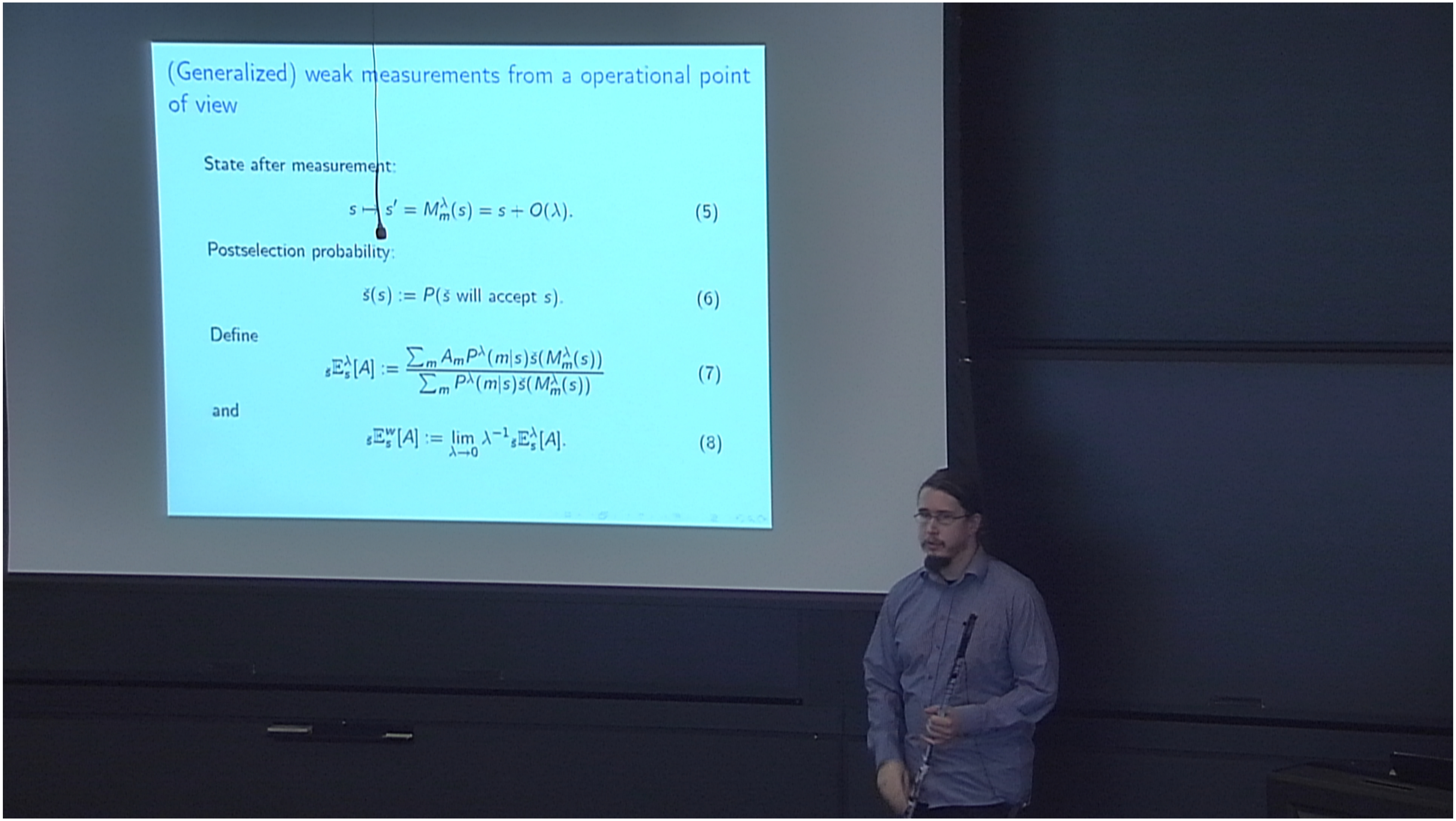
(Generalized) weak measurements from a operational point of view
State after measurement:
$s \mapsto s' = M_m^\lambda(s) = s + O(\lambda).$ (5)
Postselection probability:
$\check{s}(s) := P(\check{s} \text{ will accept } s).$ (6)
Define
${}_{\check{s}}\mathbb{E}_s^\lambda[A] := \frac{\sum_m A_m P^\lambda(m|s)\check{s}(M_m^\lambda(s))}{\sum_m P^\lambda(m|s)\check{s}(M_m^\lambda(s))}$ (7)
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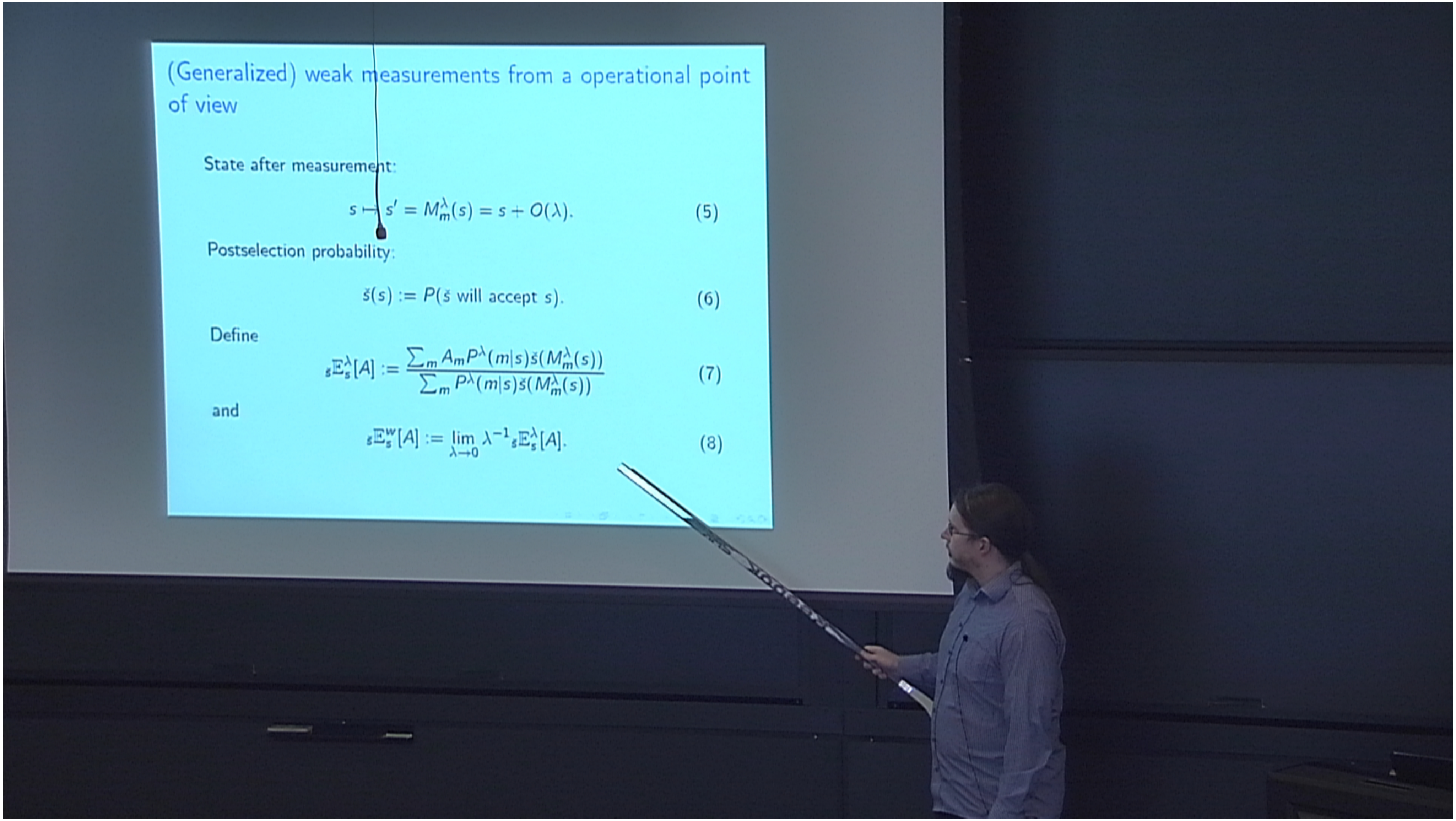
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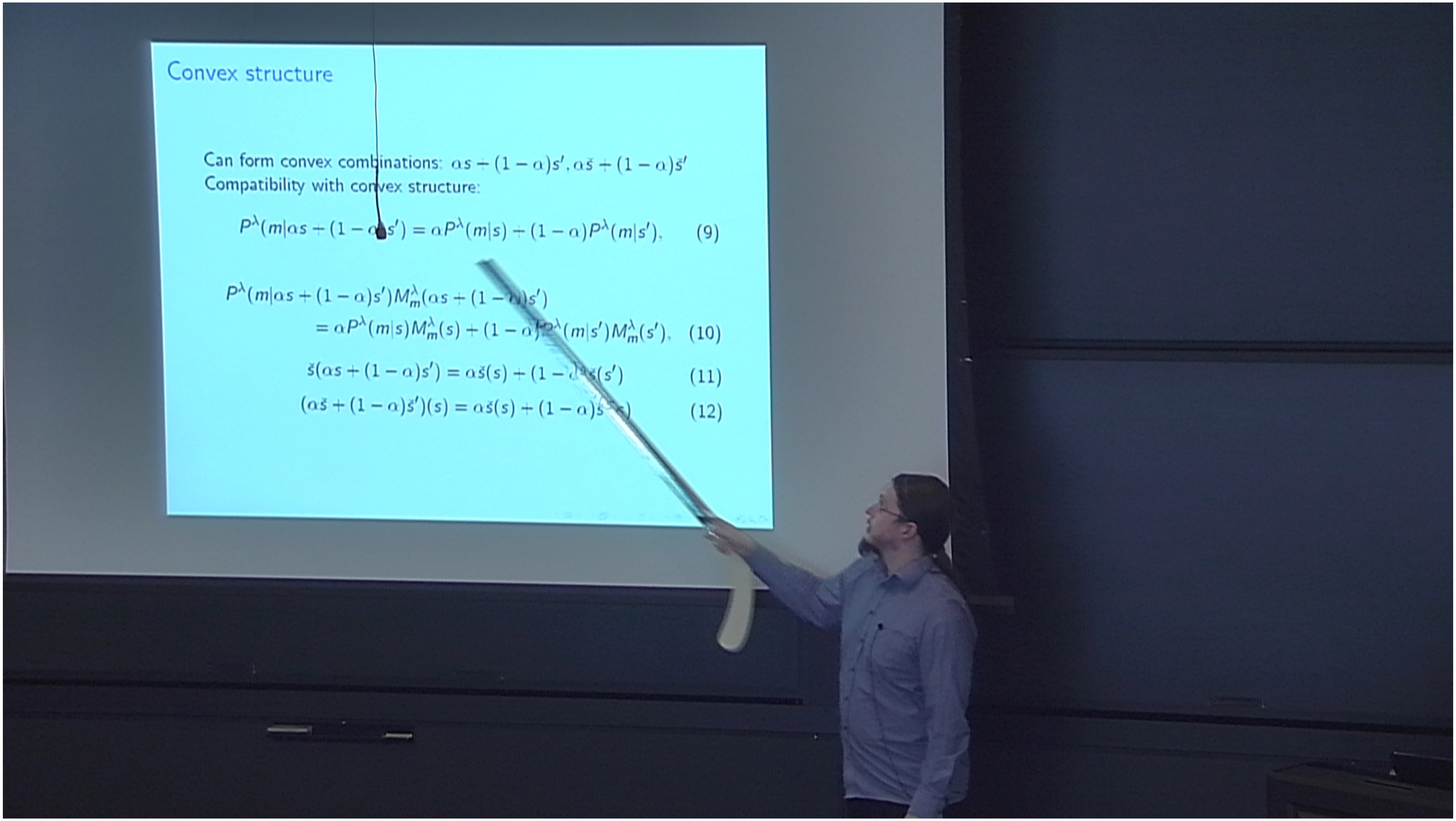
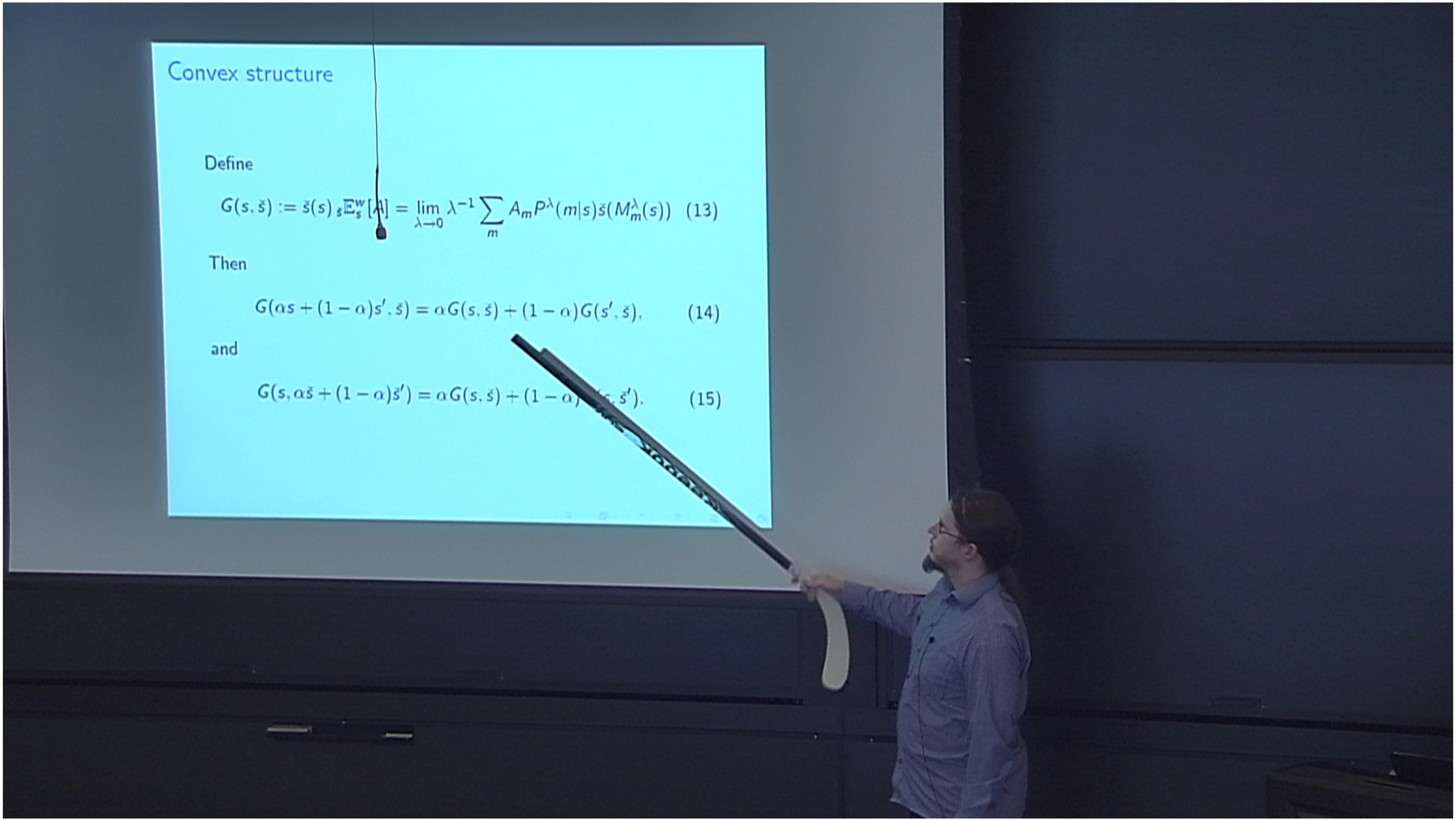
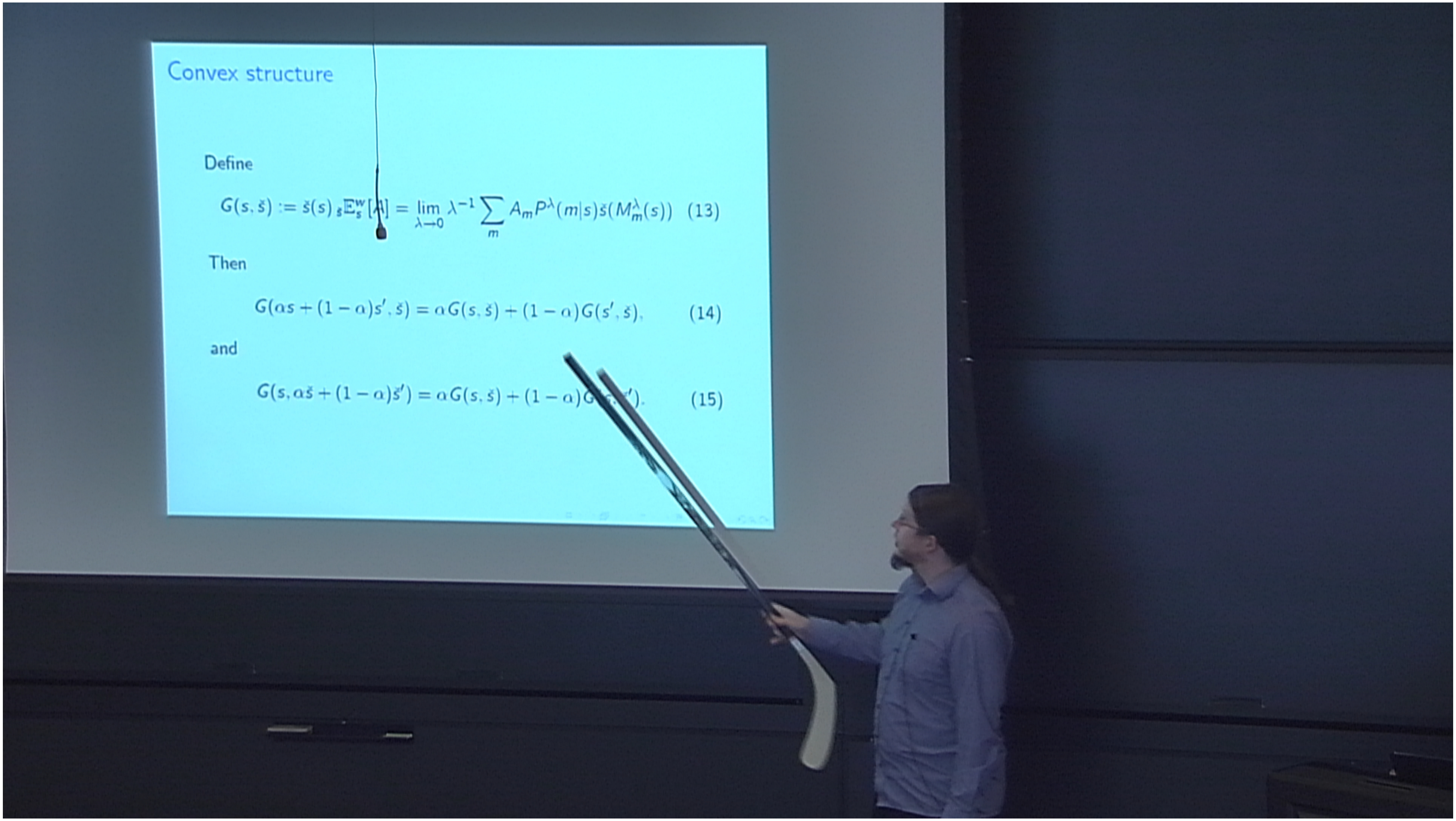

Convex structure
Can form convex combinations: $\alpha s + (1-\alpha)s', \alpha\check{s} + (1-\alpha)\check{s}'$
Compatibility with convex structure:
$P^\lambda(m|\alpha s + (1-\alpha)s') = \alpha P^\lambda(m|s) + (1-\alpha)P^\lambda(m|s'), \quad (9)$
$P^\lambda(m|\alpha s + (1-\alpha)s')M^\lambda_m(\alpha s + (1-\alpha)s') = \alpha P^\lambda(m|s)M^\lambda_m(s) + (1-\alpha)P^\lambda(m|s')M^\lambda_m(s'), \quad (10)$
$\check{s}(\alpha s + (1-\alpha)s') = \alpha\check{s}(s) + (1-\alpha)\check{s}(s') \quad (11)$
$(\alpha\check{s} + (1-\alpha)\check{s}')(s) = \alpha\check{s}(s) + (1-\alpha)\check{s}'(s) \quad (12)$


Convex structure
Define
$G(s,\check{s}) := \check{s}(s)\,{}_s\mathbb{E}^w_s[A] = \lim_{\lambda\to 0}\lambda^{-1}\sum_m A_m P^\lambda(m|s)\check{s}(M^\lambda_m(s))$ (13)
Then
$G(\alpha s + (1-\alpha)s', \check{s}) = \alpha G(s,\check{s}) + (1-\alpha)G(s',\check{s}),$ (14)
and
$G(s, \alpha\check{s} + (1-\alpha)\check{s}') = \alpha G(s,\check{s}) + (1-\alpha)G(s,\check{s}').$ (15)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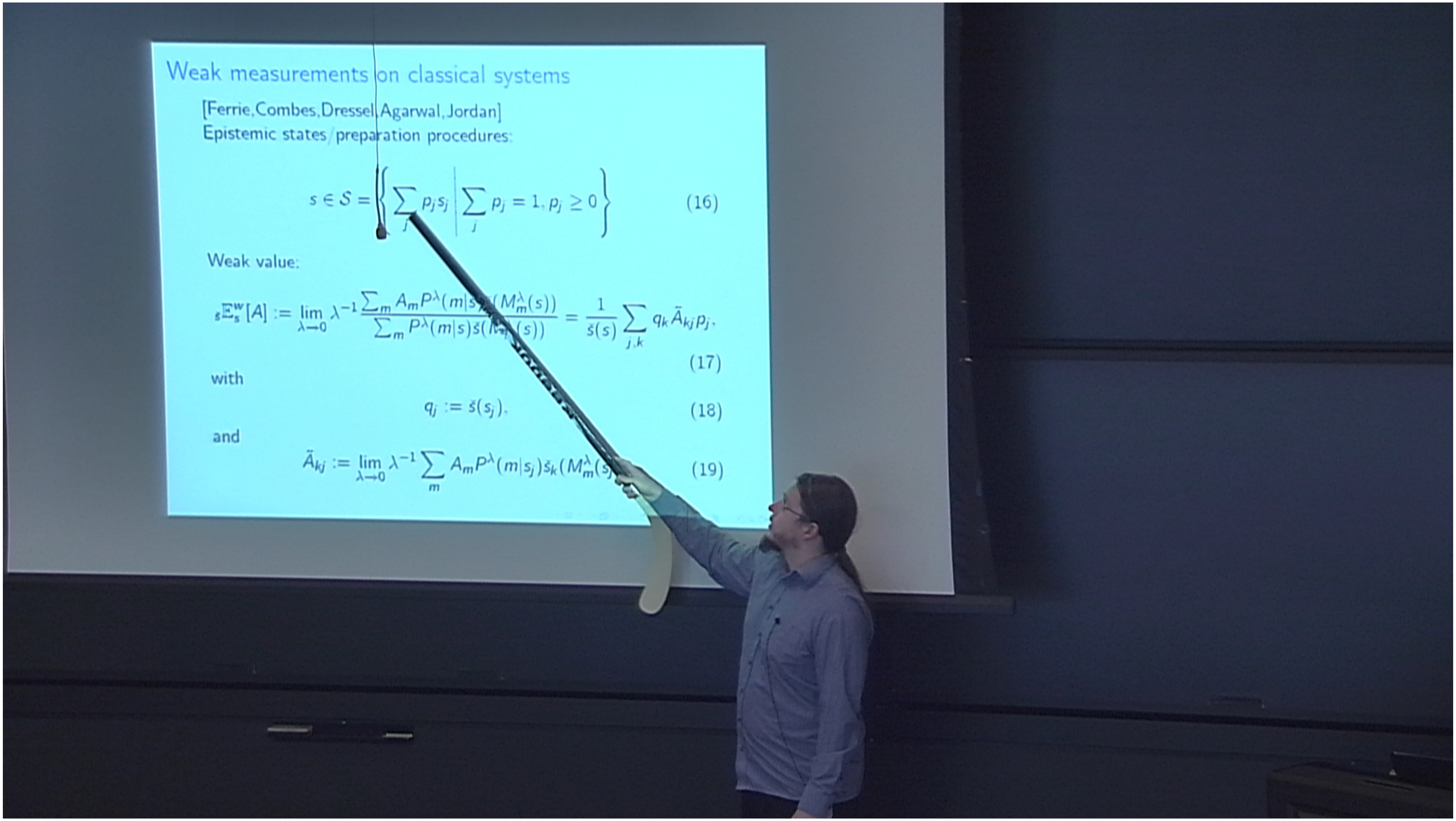
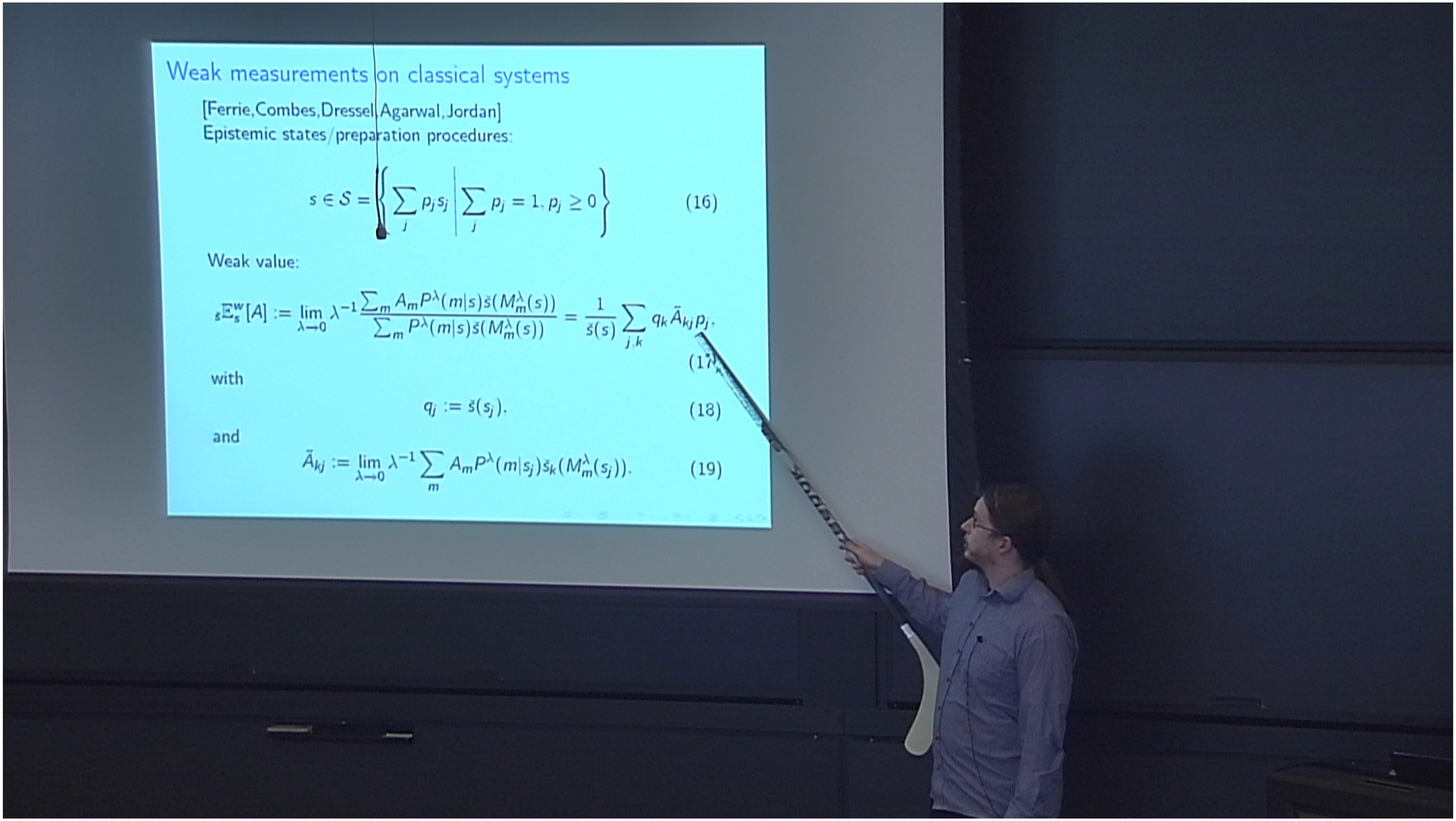
and
$G(s, \alpha\check{s} + (1-\alpha)\check{s}') = \alpha G(s, \check{s}) + (1-\alpha) G$ (15)


Weak measurements on classical systems
[Ferrie,Combes,Dressel,Agarwal,Jordan]
Epistemic states/preparation procedures:
$s \in \mathcal{S} = \left\{ \sum_j p_j s_j \,\middle|\, \sum_j p_j = 1, p_j \geq 0 \right\}$ (16)
Weak value:
(17)
with
$q_j := \check{s}(s_j),$ (18)
and
(19)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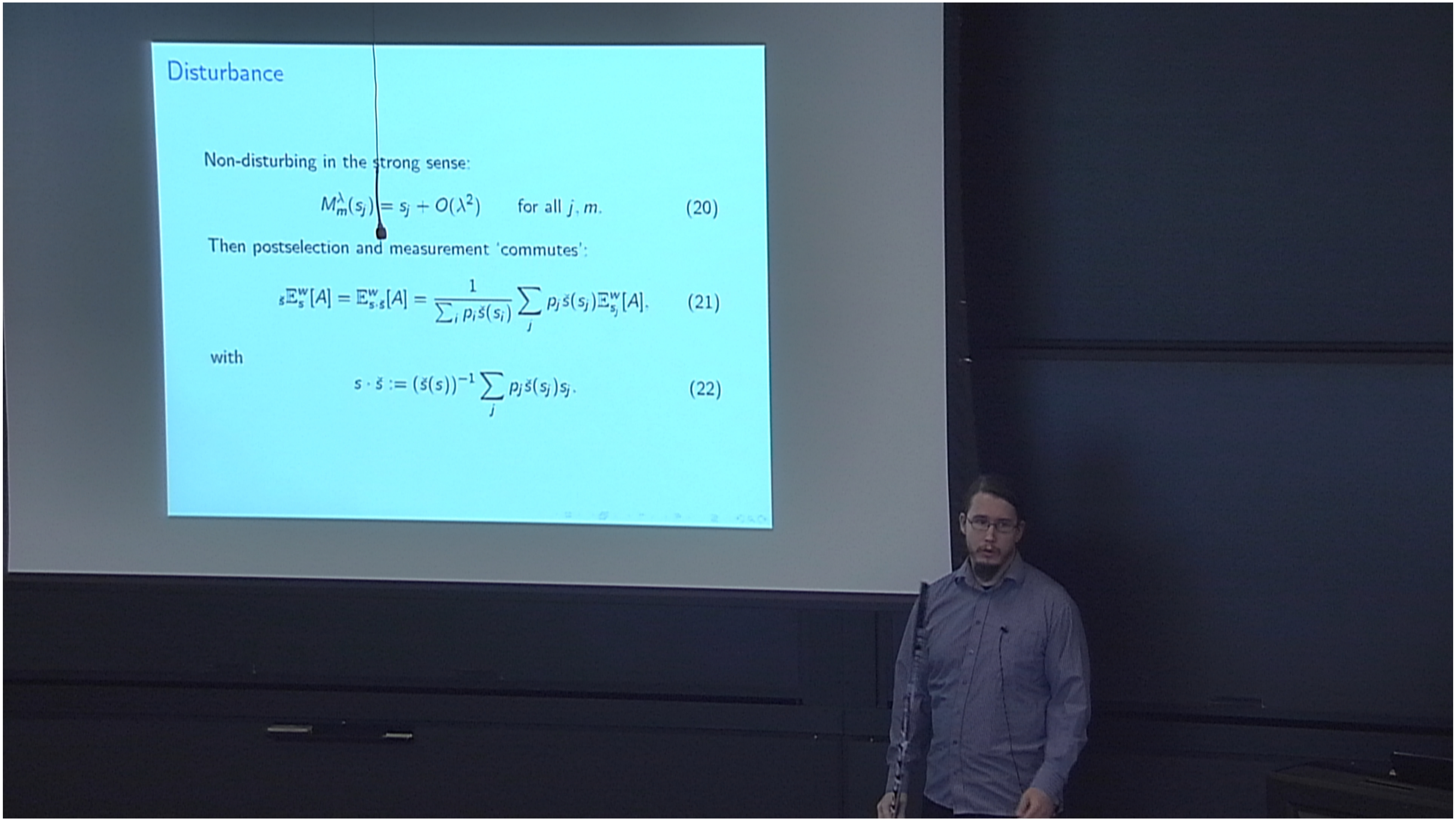
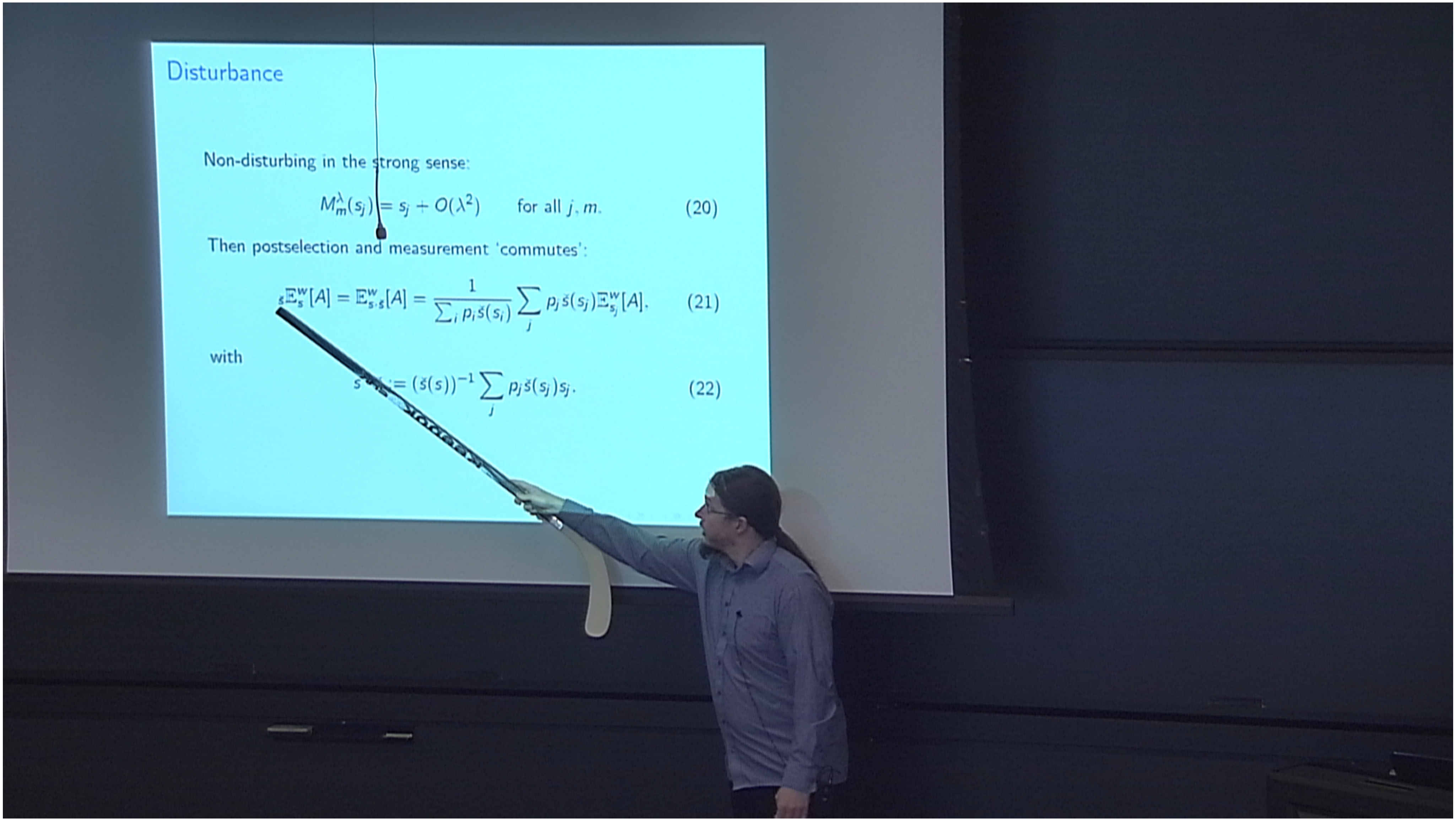
with
$q_j := \check{s}(s_j).$ (18)
and
$\bar{A}_{kj} := \lim_{\lambda \to 0} \lambda^{-1} \sum_m A_m P^\lambda(m|s_j)\check{s}_k(M^\lambda_m(s_j)).$ (19)


Disturbance
Non-disturbing in the strong sense:
$M_m^\lambda(s_j) = s_j + O(\lambda^2)$ for all j, m. (20)
Then postselection and measurement 'commutes':
${}_{\check{s}}\mathbb{E}_s^w[A] = \mathbb{E}_{s\cdot\check{s}}^w[A] = \frac{1}{\sum_i p_i \check{s}(s_i)} \sum_j p_j \check{s}(s_j) \mathbb{E}_{s_j}^w[A].$ (21)
with
$s \cdot \check{s} := (\check{s}(s))^{-1} \sum_j p_j \check{s}(s_j) s_j.$ (22)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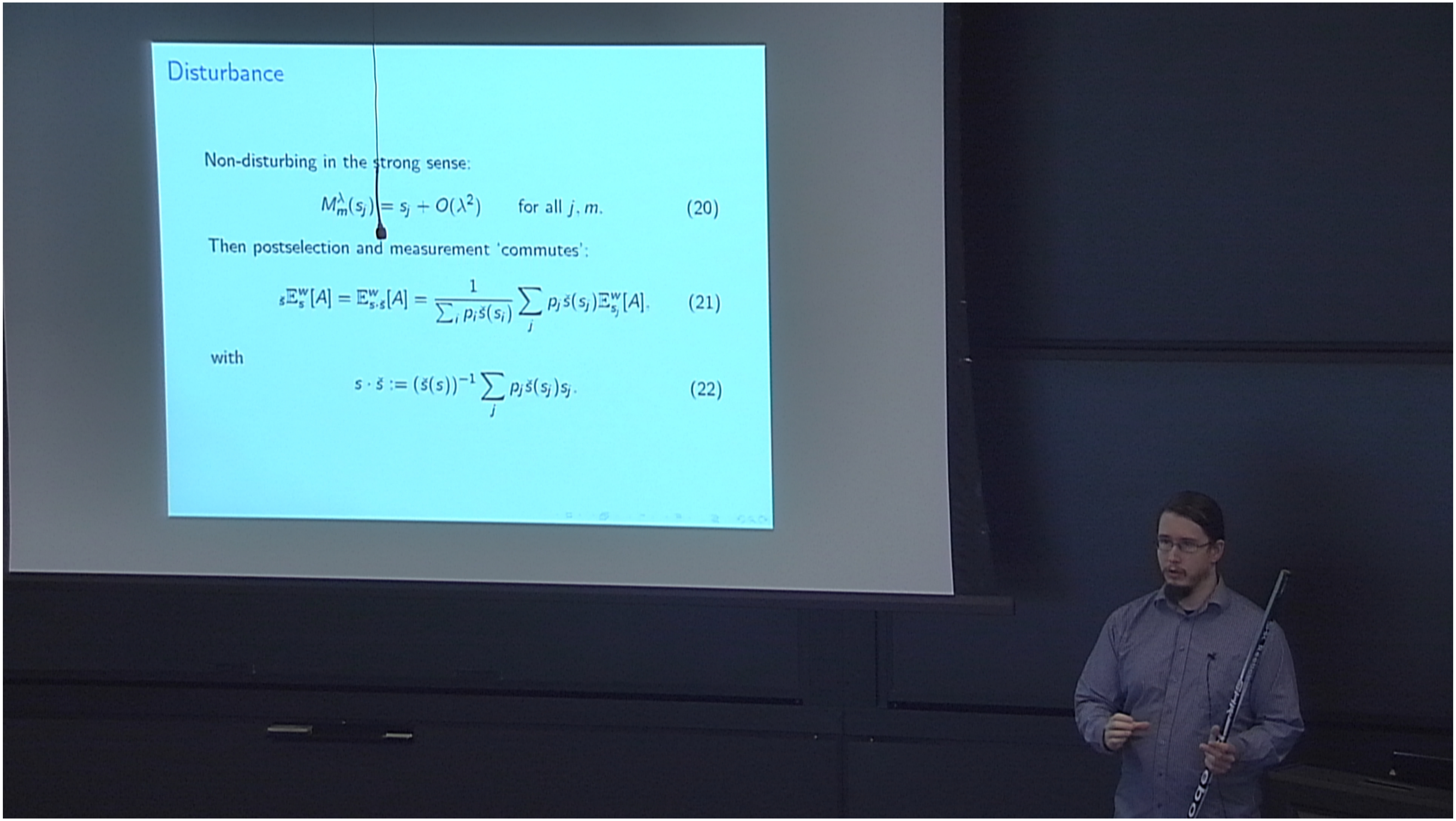
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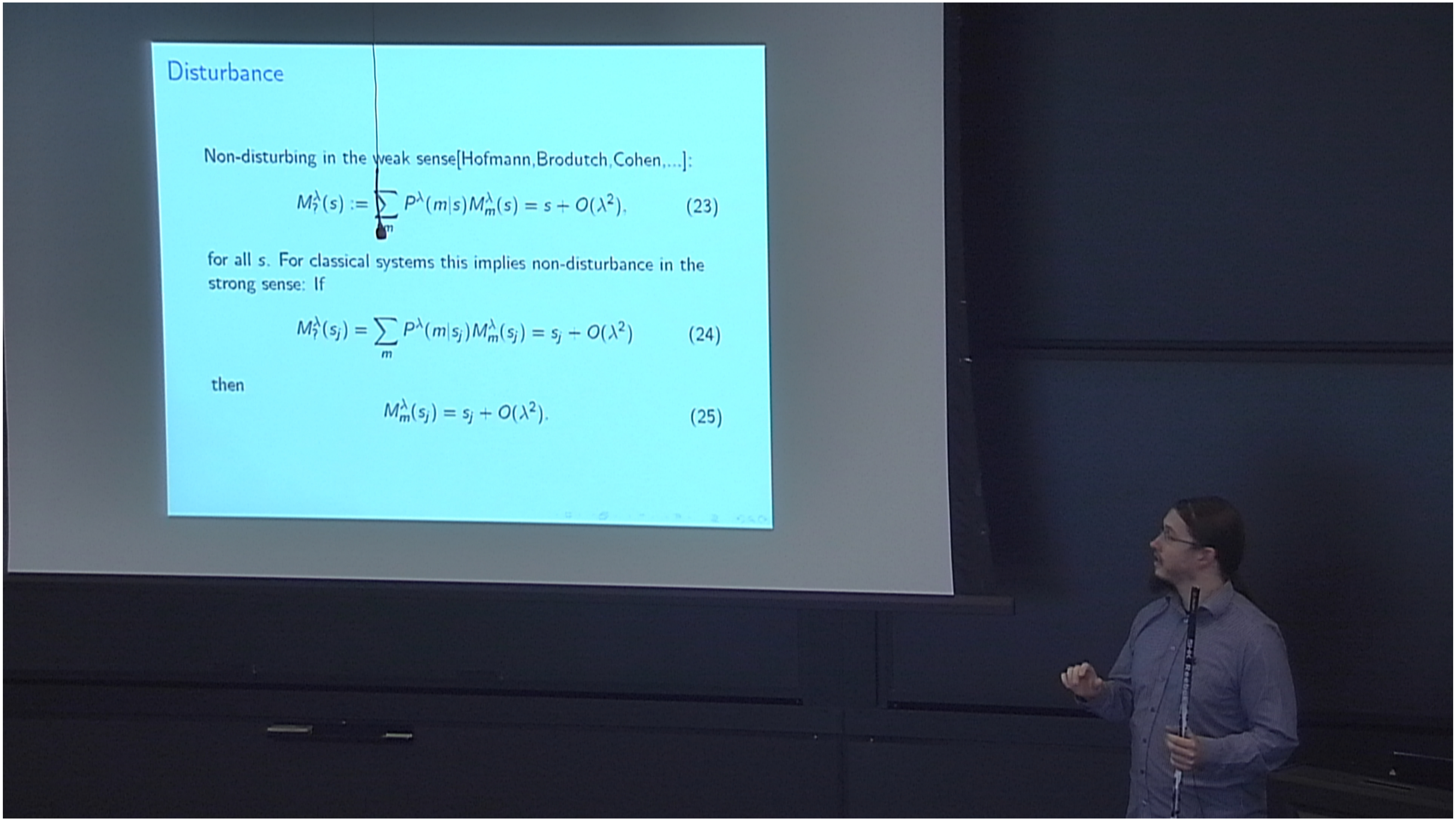
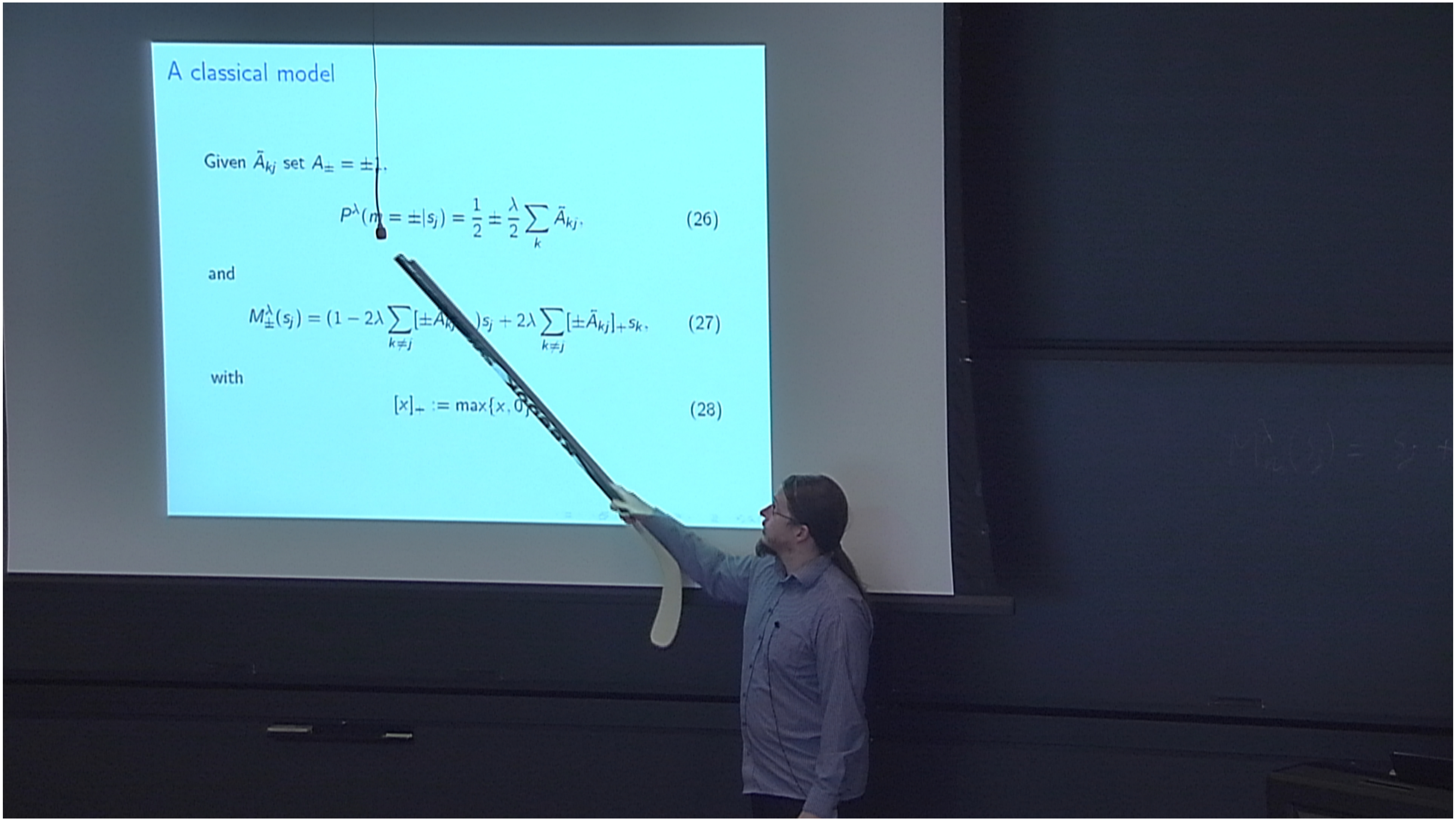
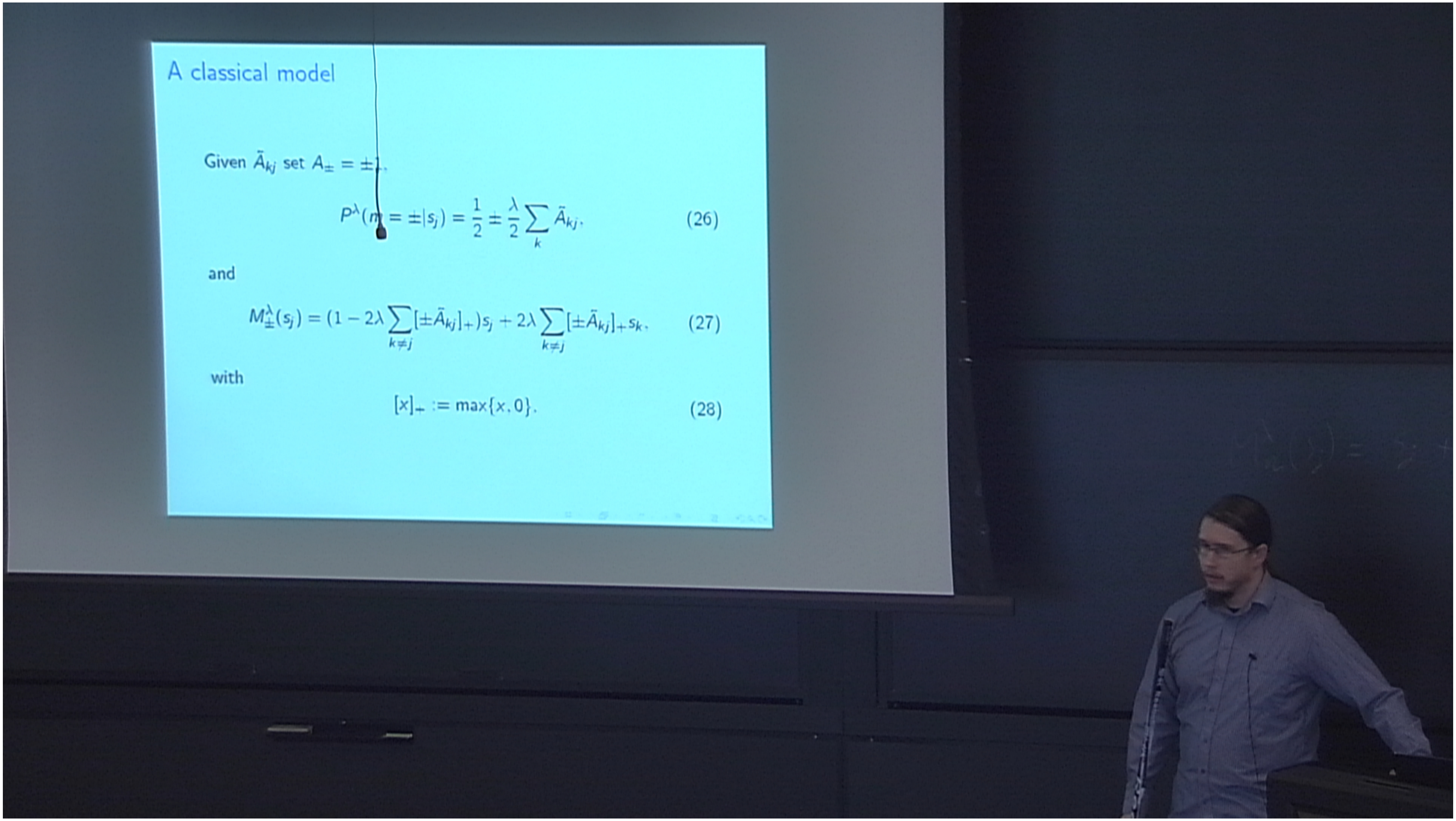

Disturbance
Non-disturbing in the weak sense[Hofmann,Brodutch,Cohen,...]:
$M_?^\lambda(s) := \sum_m P^\lambda(m|s) M_m^\lambda(s) = s + O(\lambda^2)$, (23)
for all s. For classical systems this implies non-disturbance in the strong sense: If
$M_?^\lambda(s_j) = \sum_m P^\lambda(m|s_j) M_m^\lambda(s_j) = s_j + O(\lambda^2)$ (24)
then
$M_m^\lambda(s_j) = s_j + O(\lambda^2)$. (25)


A classical model
Given $\tilde{A}_{kj}$ set $A_\pm = \pm 1$.
$P^\lambda(m = \pm|s_j) = \frac{1}{2} \pm \frac{\lambda}{2} \sum_k \tilde{A}_{kj}$, (26)
and
$M^\lambda_\pm(s_j) = (1 - 2\lambda \sum_{k \neq j} [\pm \tilde{A}$ $)s_j + 2\lambda \sum_{k \neq j} [\pm \tilde{A}_{kj}]_+ s_k$, (27)
with
$[x]_+ := \max\{x, 0$ (28)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and
$M^\lambda_\pm(s_j) = (1 - 2\lambda \sum_{k \neq j} [\pm \bar{A}_{kj}]_+) s_j + 2\lambda \sum_{k \neq j} [\pm \bar{A}_{kj}]_+ s_k$. (27)
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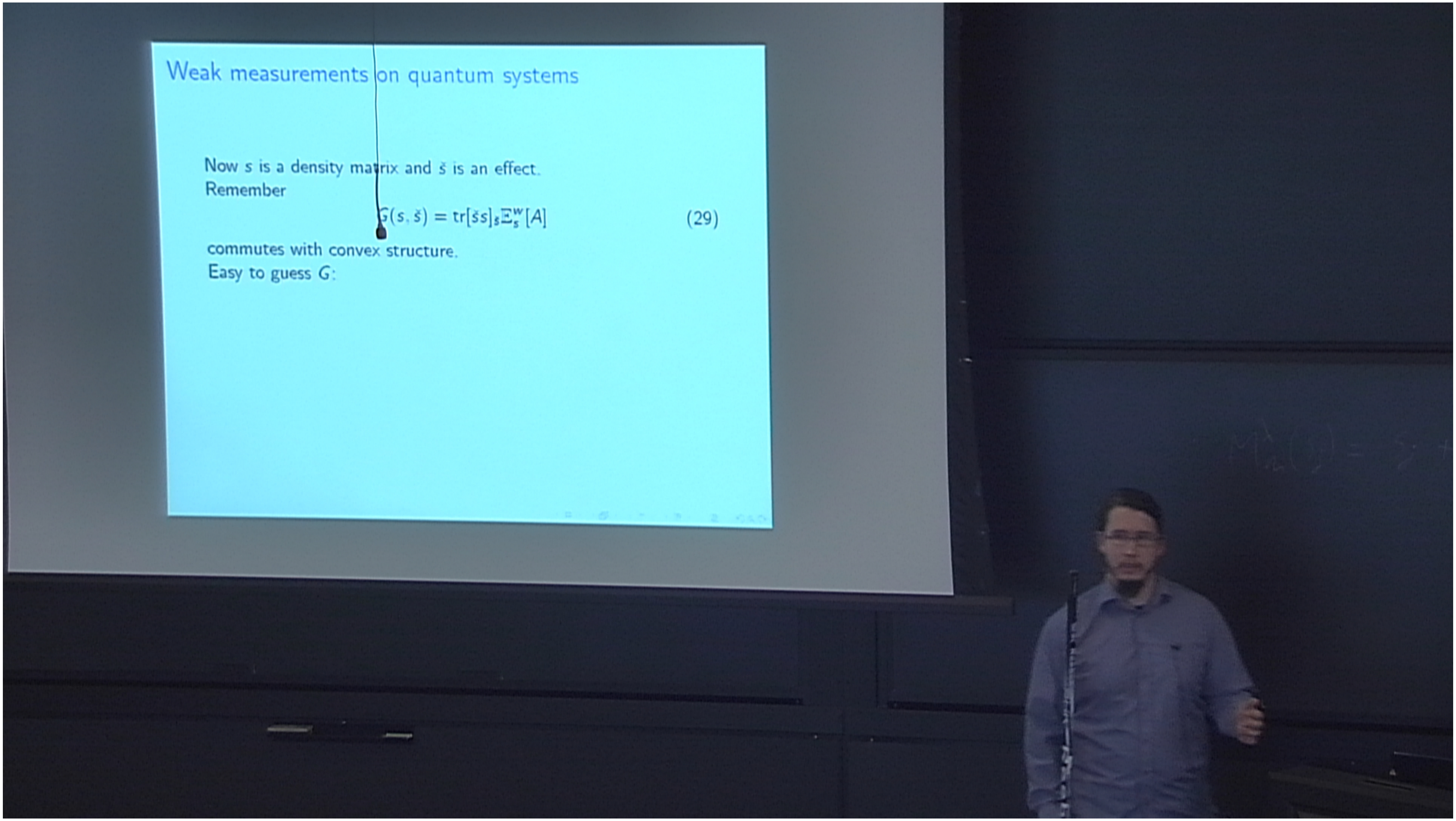
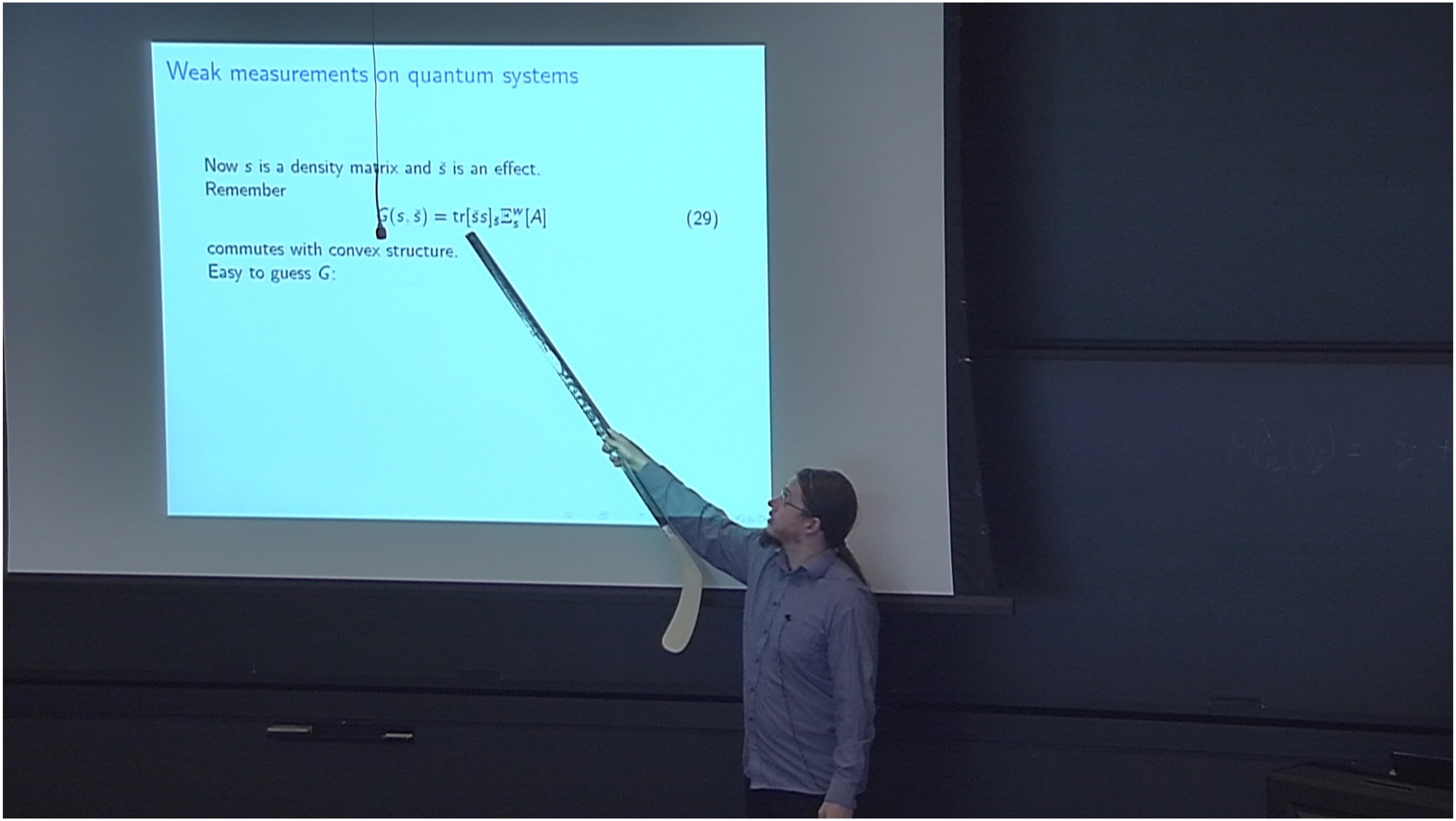

Weak measurements on quantum systems
Now s is a density matrix and $\check{s}$ is an effect.
Remember
$G(s, \check{s}) = \mathrm{tr}[\check{s}s]_s \Xi^w_s[A]$ (29)
commutes with convex structure.
Easy to guess G: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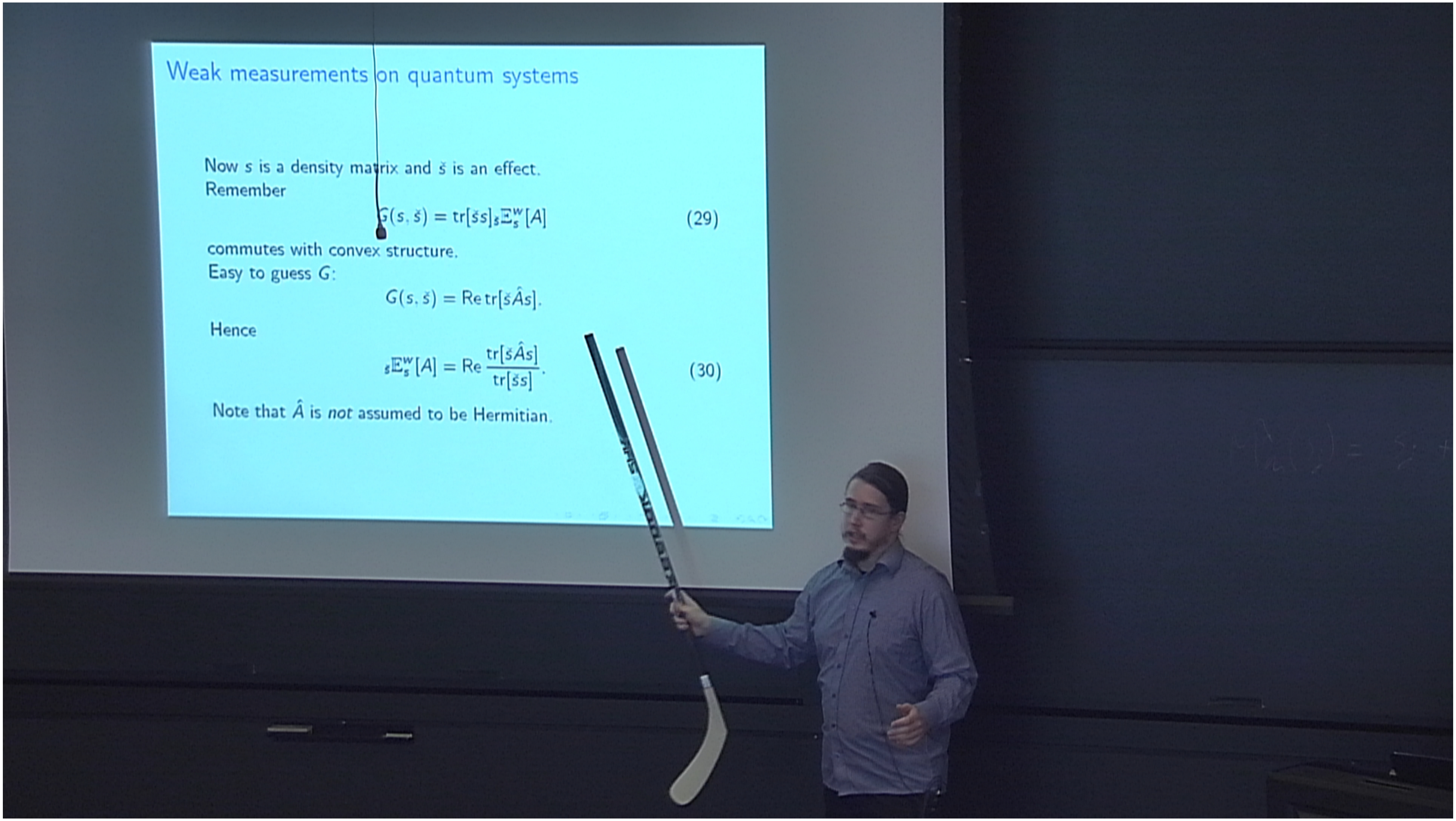
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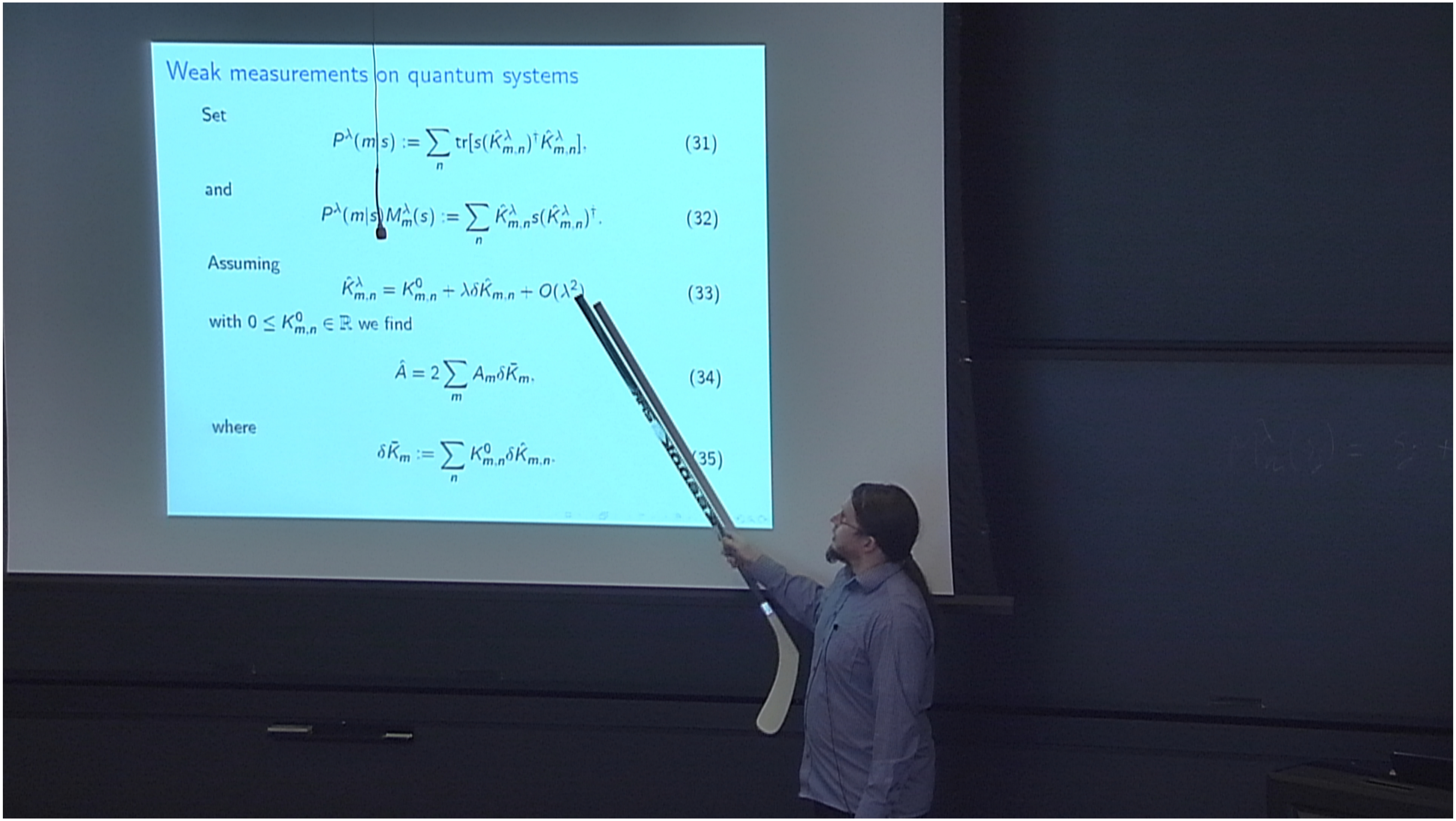
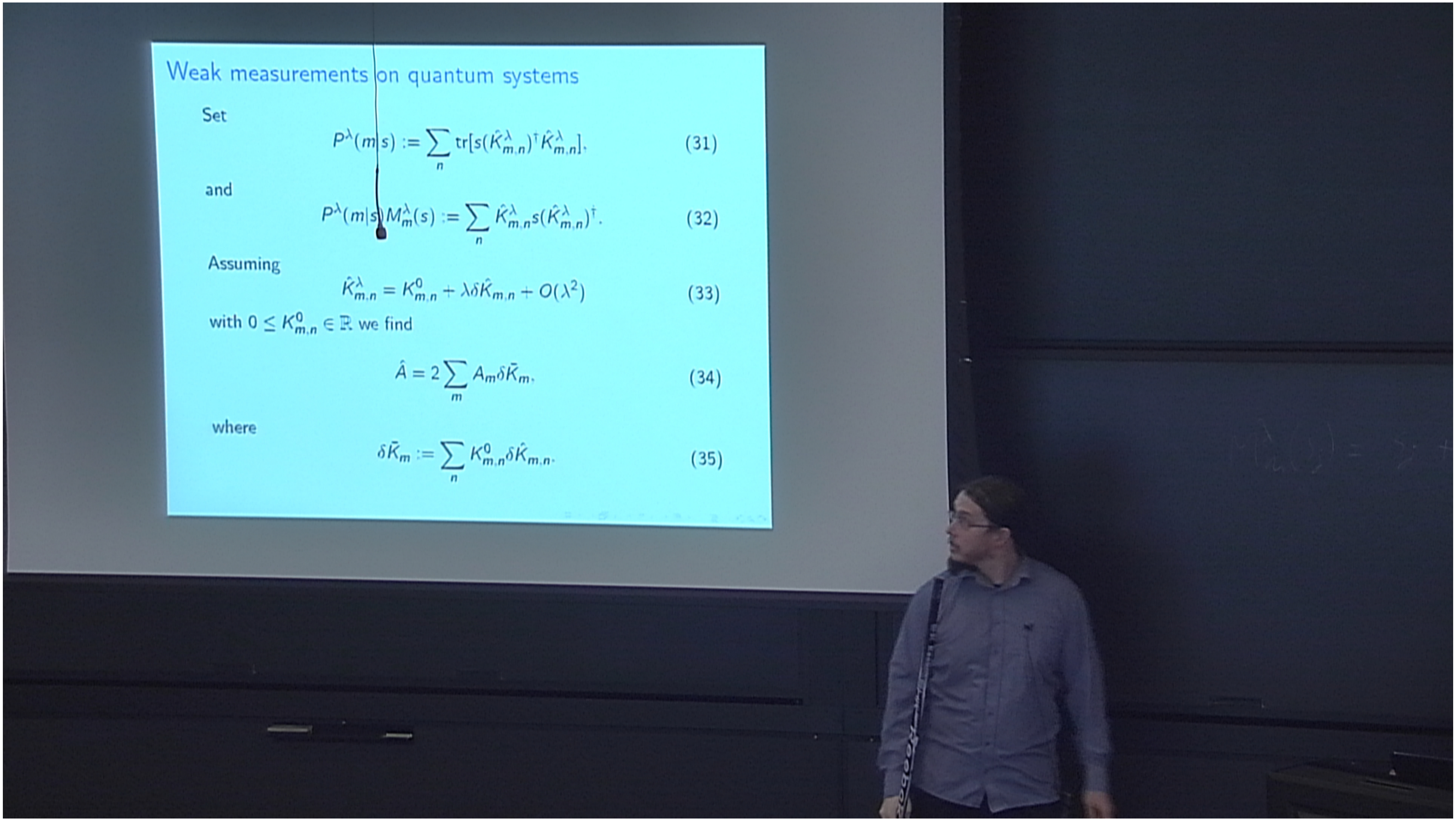
commutes with convex structure.
Easy to guess G:
$G(s, \check{s}) = \mathrm{Re}\,\mathrm{tr}[\check{s}\hat{A}s]$.
Hence
${}_s\mathbb{E}^w_{\check{s}}[A] = \mathrm{Re}\,\frac{\mathrm{tr}[\check{s}\hat{A}s]}{\mathrm{tr}[\check{s}s]}$. (30)
Note that $\hat{A}$ is *not* assumed to be Hermitian.


Weak measurements on quantum systems
Set
$P^\lambda(m|s) := \sum_n \mathrm{tr}[s(\hat{K}^\lambda_{m,n})^\dagger \hat{K}^\lambda_{m,n}],$ (31)
and
$P^\lambda(m|s) M^\lambda_m(s) := \sum_n \hat{K}^\lambda_{m,n} s (\hat{K}^\lambda_{m,n})^\dagger.$ (32)
Assuming
$\hat{K}^\lambda_{m,n} = K^0_{m,n} + \lambda \delta \hat{K}_{m,n} + O(\lambda^2)$ (33)
with $0 \leq K^0_{m,n} \in \mathbb{R}$ we find
$\hat{A} = 2 \sum_m A_m \delta \bar{K}_m.$ (34)
where
$\delta \bar{K}_m := \sum_n K^0_{m,n} \delta \hat{K}_{m,n}.$ (35)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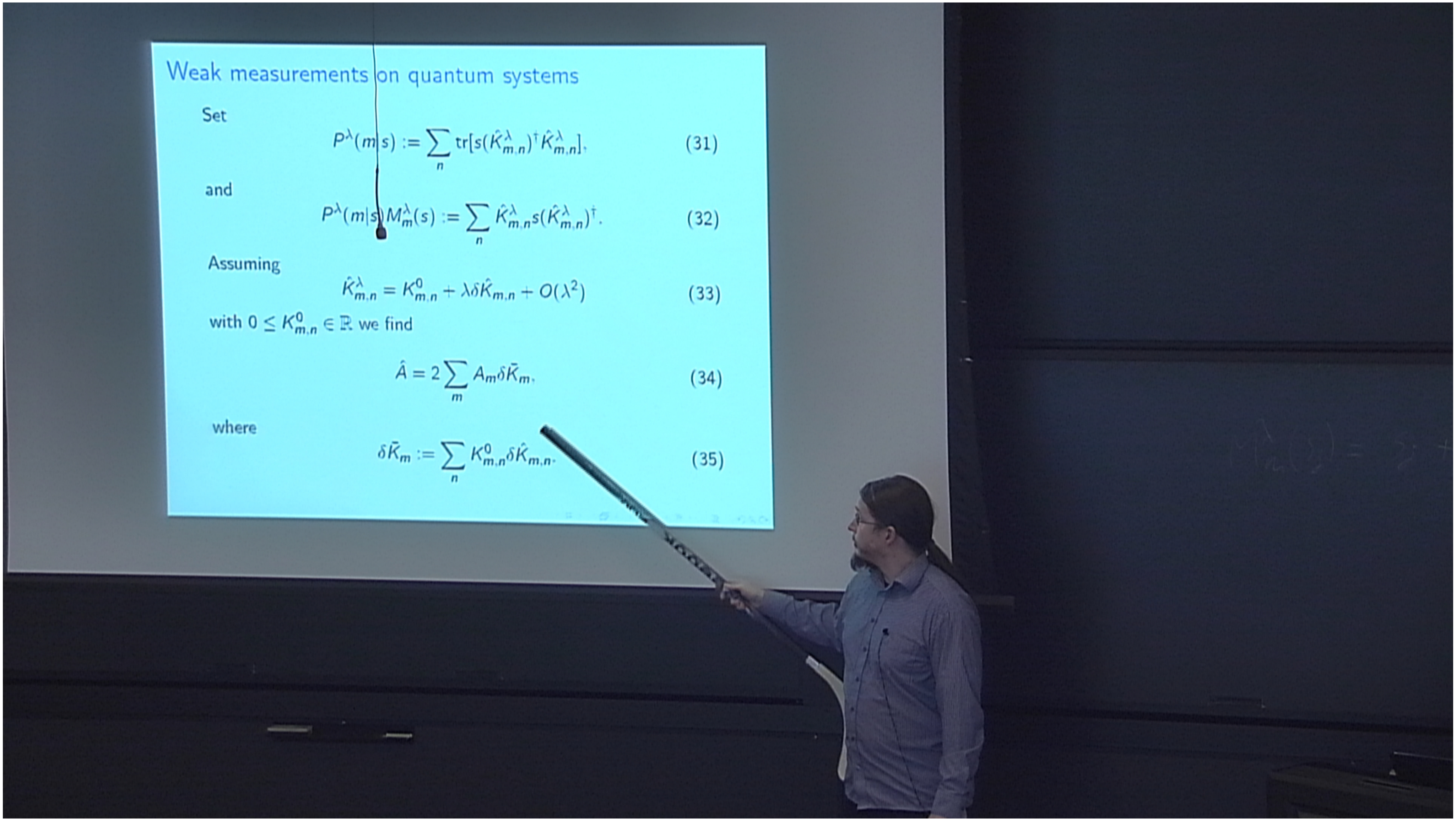
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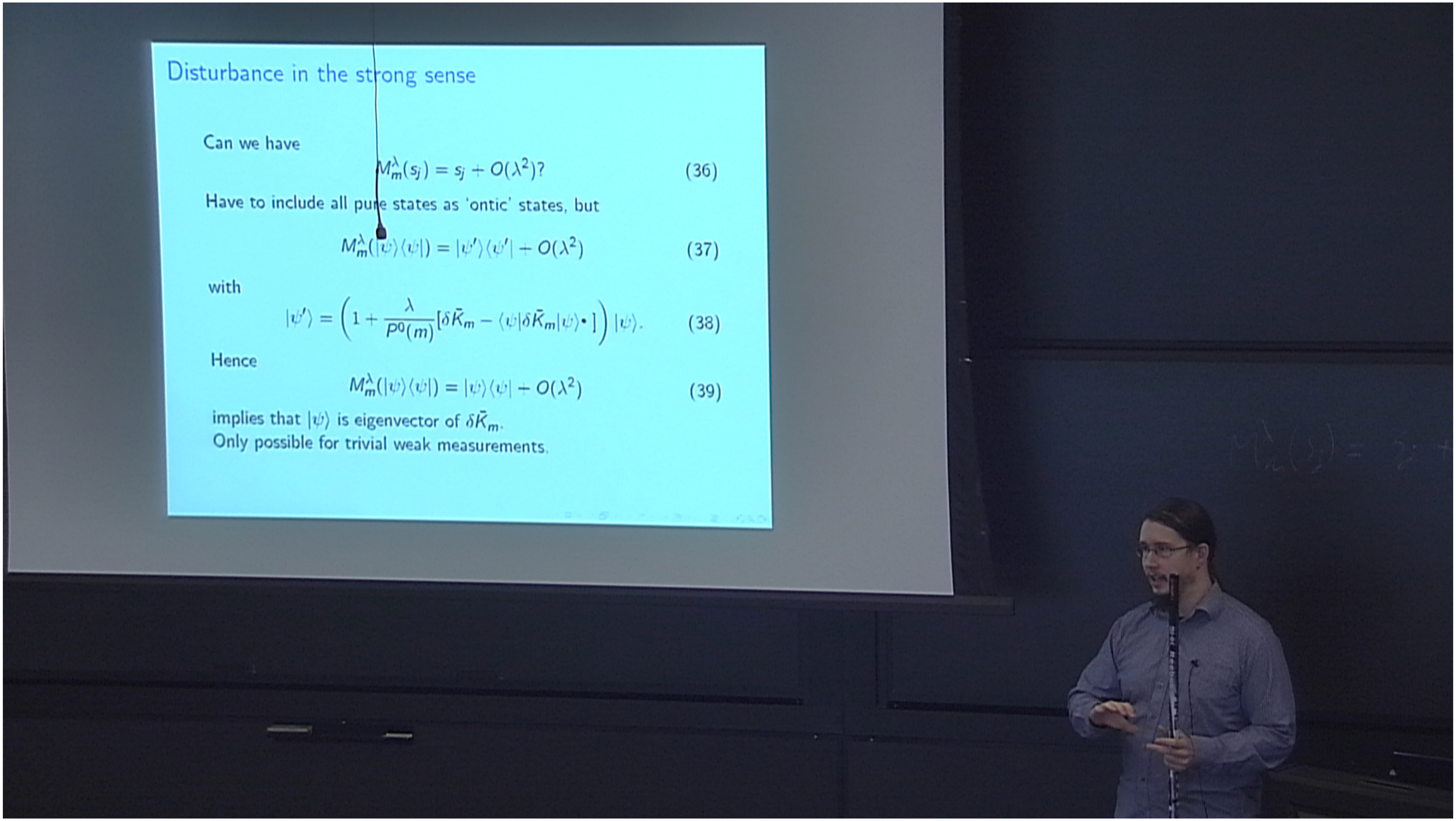
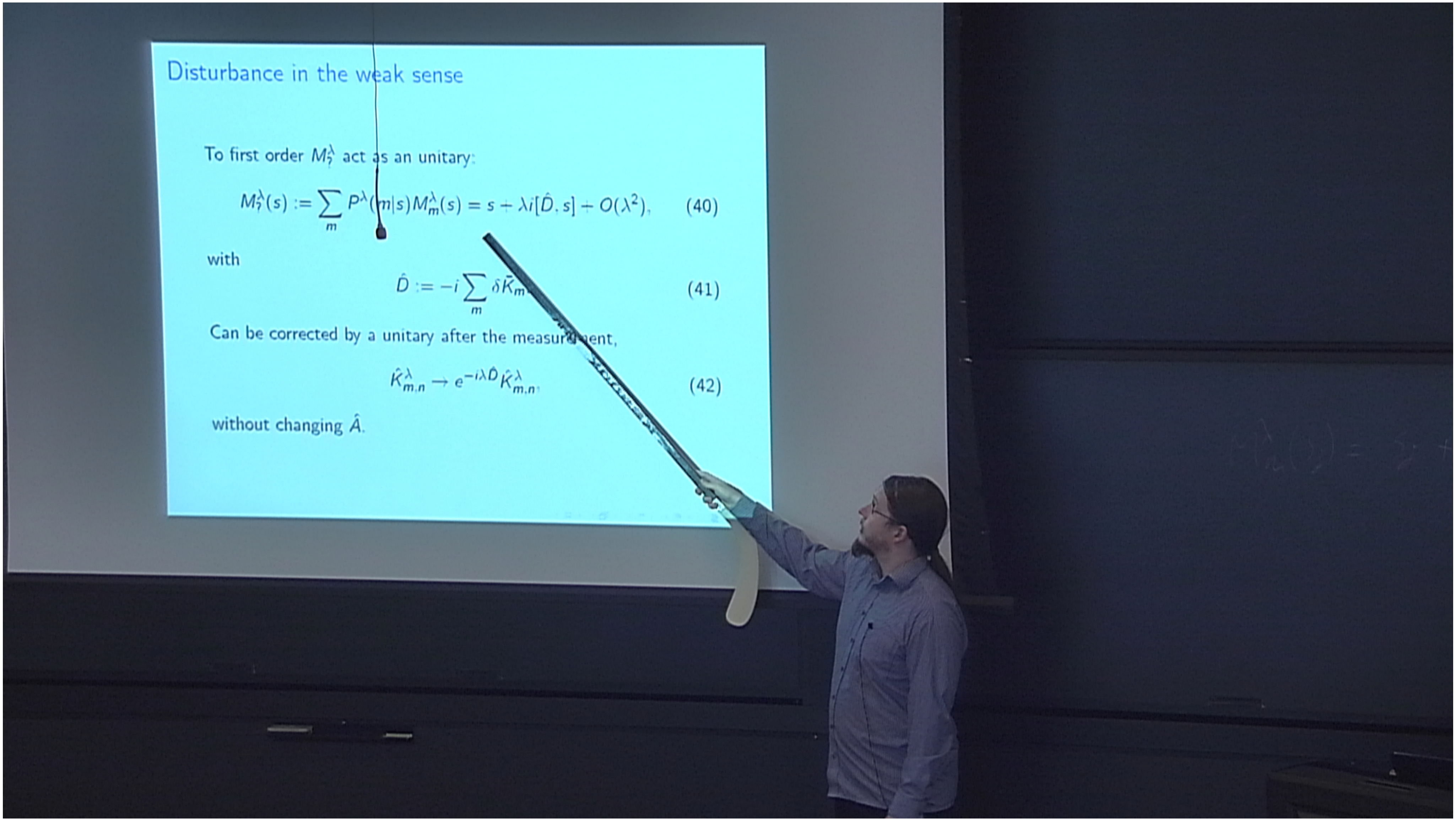
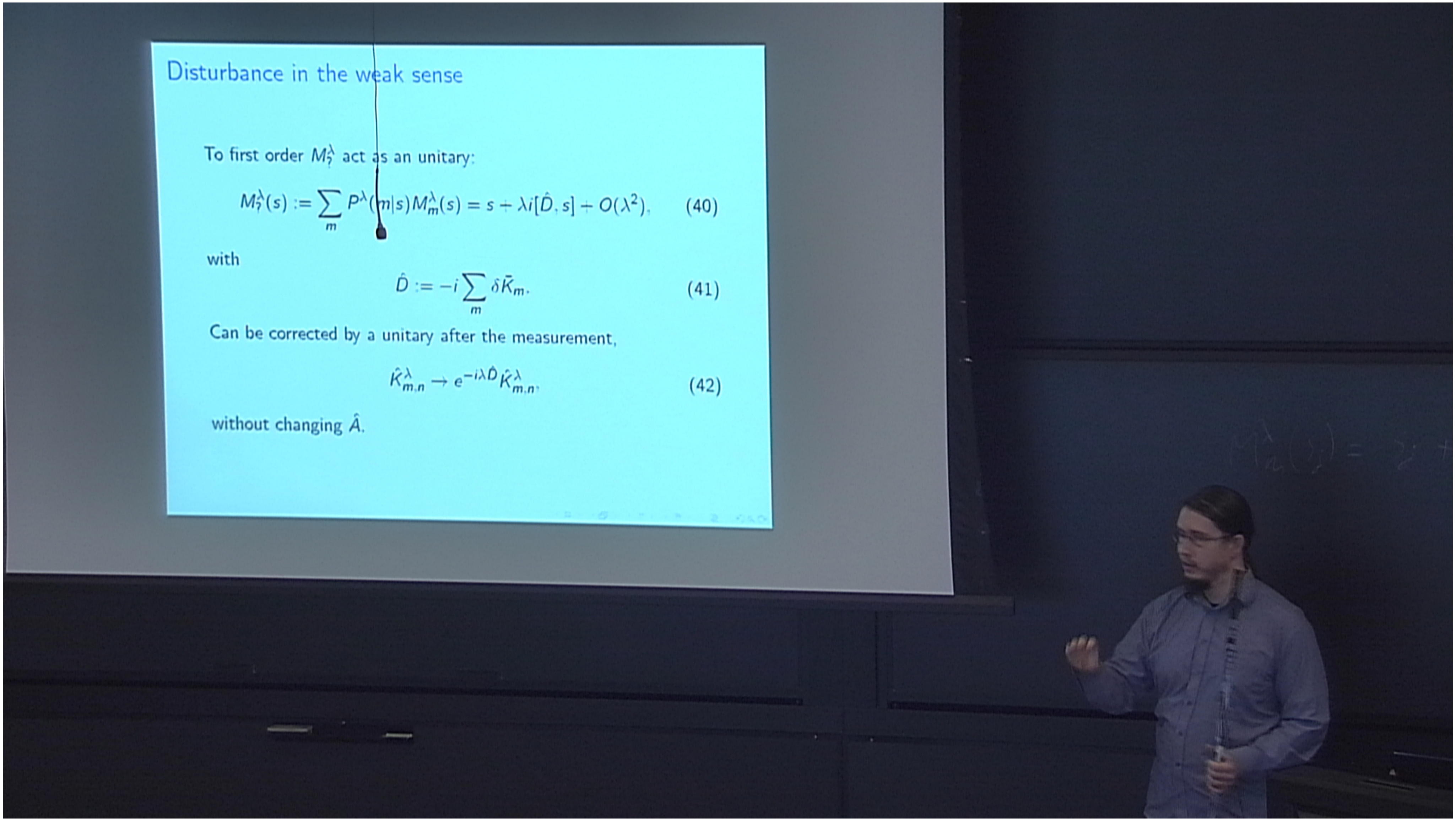

Disturbance in the strong sense
Can we have
$M_m^\lambda(s_j) = s_j + O(\lambda^2)?$ (36)
Have to include all pure states as 'ontic' states, but
$M_m^\lambda(|\psi\rangle\langle\psi|) = |\psi'\rangle\langle\psi'| + O(\lambda^2)$ (37)
with
$|\psi'\rangle = \left(1 + \frac{\lambda}{P^0(m)}[\delta\bar{K}_m - \langle\psi|\delta\bar{K}_m|\psi\rangle\bullet]\right)|\psi\rangle.$ (38)
Hence
$M_m^\lambda(|\psi\rangle\langle\psi|) = |\psi\rangle\langle\psi| + O(\lambda^2)$ (39)
implies that $|\psi\rangle$ is eigenvector of $\delta\bar{K}_m$.
Only possible for trivial weak measurements.


Disturbance in the weak sense
To first order $M_?^\lambda$ act as an unitary:
$M_?^\lambda(s) := \sum_m P^\lambda(m|s) M_m^\lambda(s) = s + \lambda i[\hat{D}, s] + O(\lambda^2)$, (40)
with
$\hat{D} := -i \sum_m \delta \bar{K}_m$ (41)
Can be corrected by a unitary after the measurement,
$\hat{K}_{m,n}^\lambda \to e^{-i\lambda\hat{D}} \hat{K}_{m,n}^\lambda$ (42)
without changing $\hat{A}$.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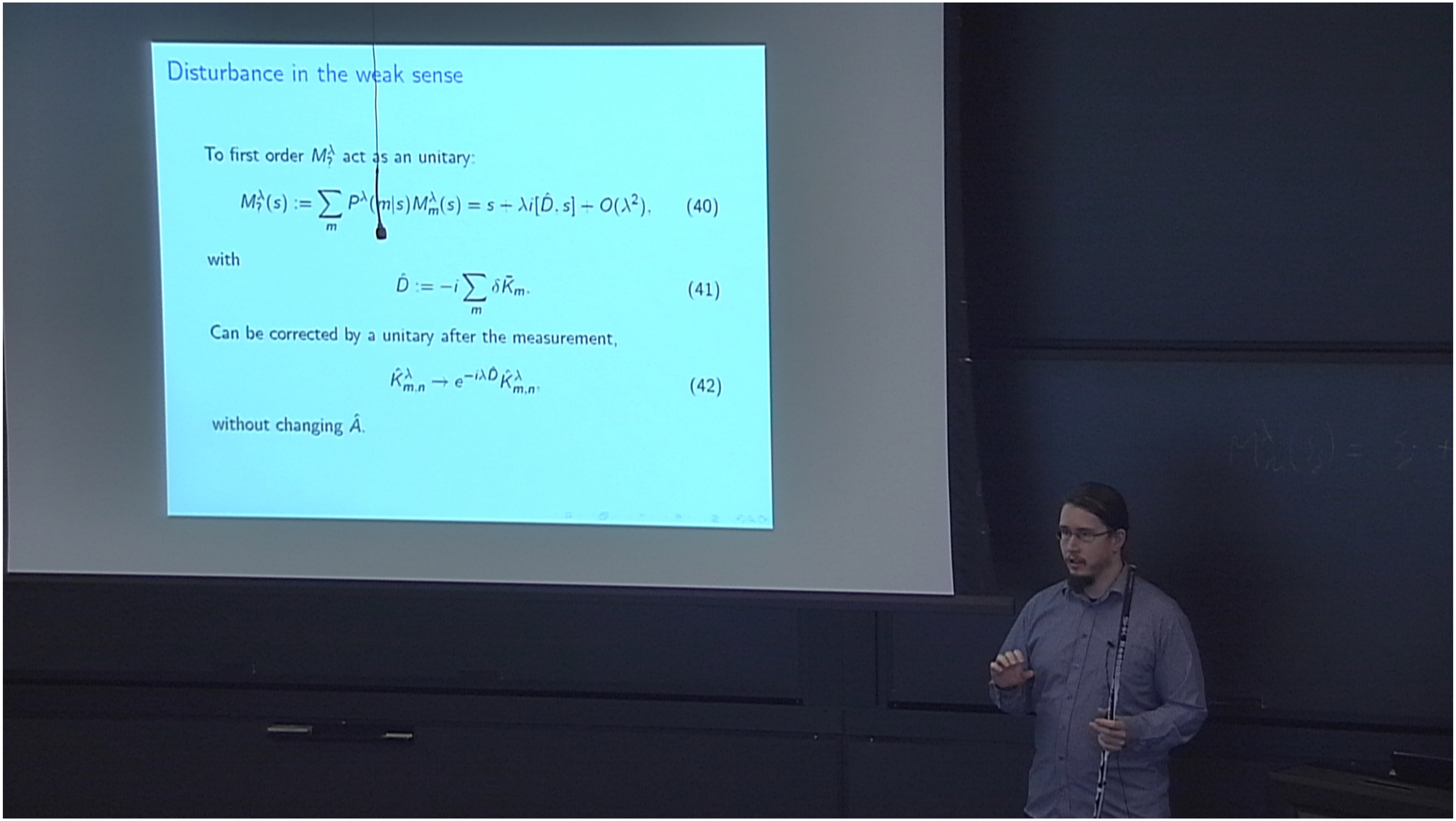
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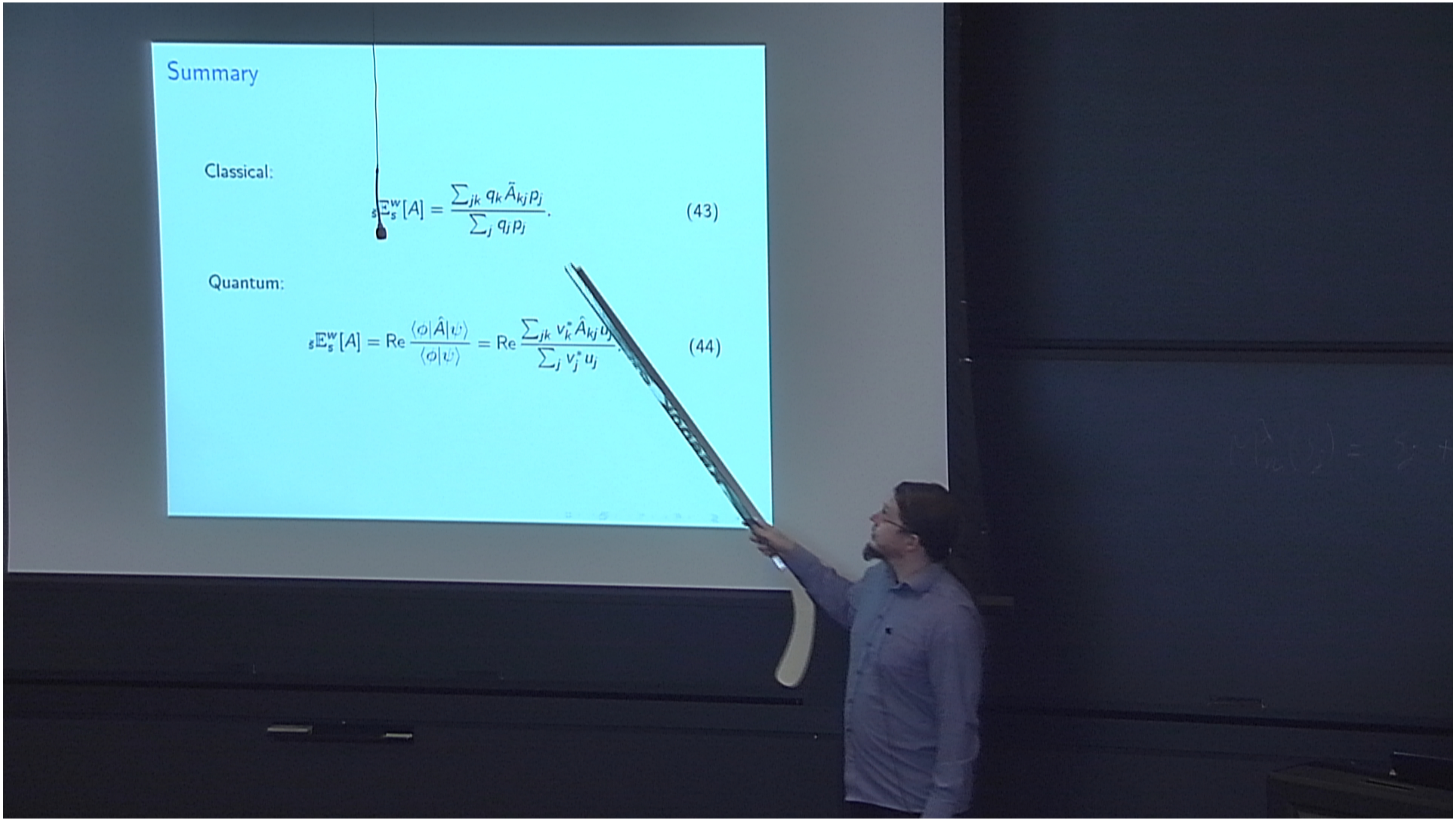
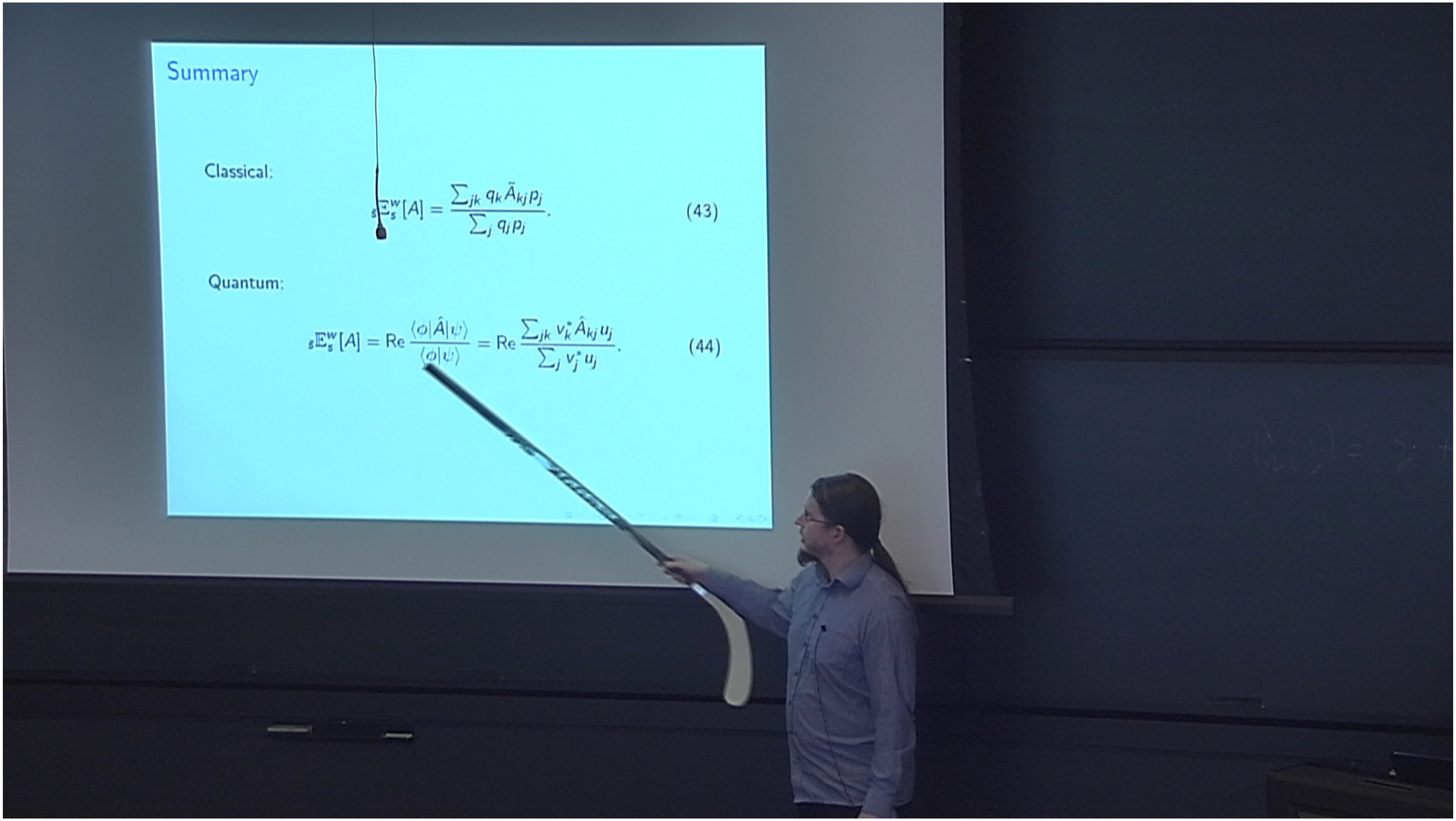

Summary
Classical:
$${}_s\mathbb{E}^w_s[A] = \frac{\sum_{jk} q_k \tilde{A}_{kj} p_j}{\sum_j q_j p_j}. \quad (43)$$
Quantum:
$${}_s\mathbb{E}^w_s[A] = \mathrm{Re}\,\frac{\langle\phi|\hat{A}|\psi\rangle}{\langle\phi|\psi\rangle} = \mathrm{Re}\,\frac{\sum_{jk} v_k^* \hat{A}_{kj} u_j}{\sum_j v_j^* u_j}. \quad (44)$$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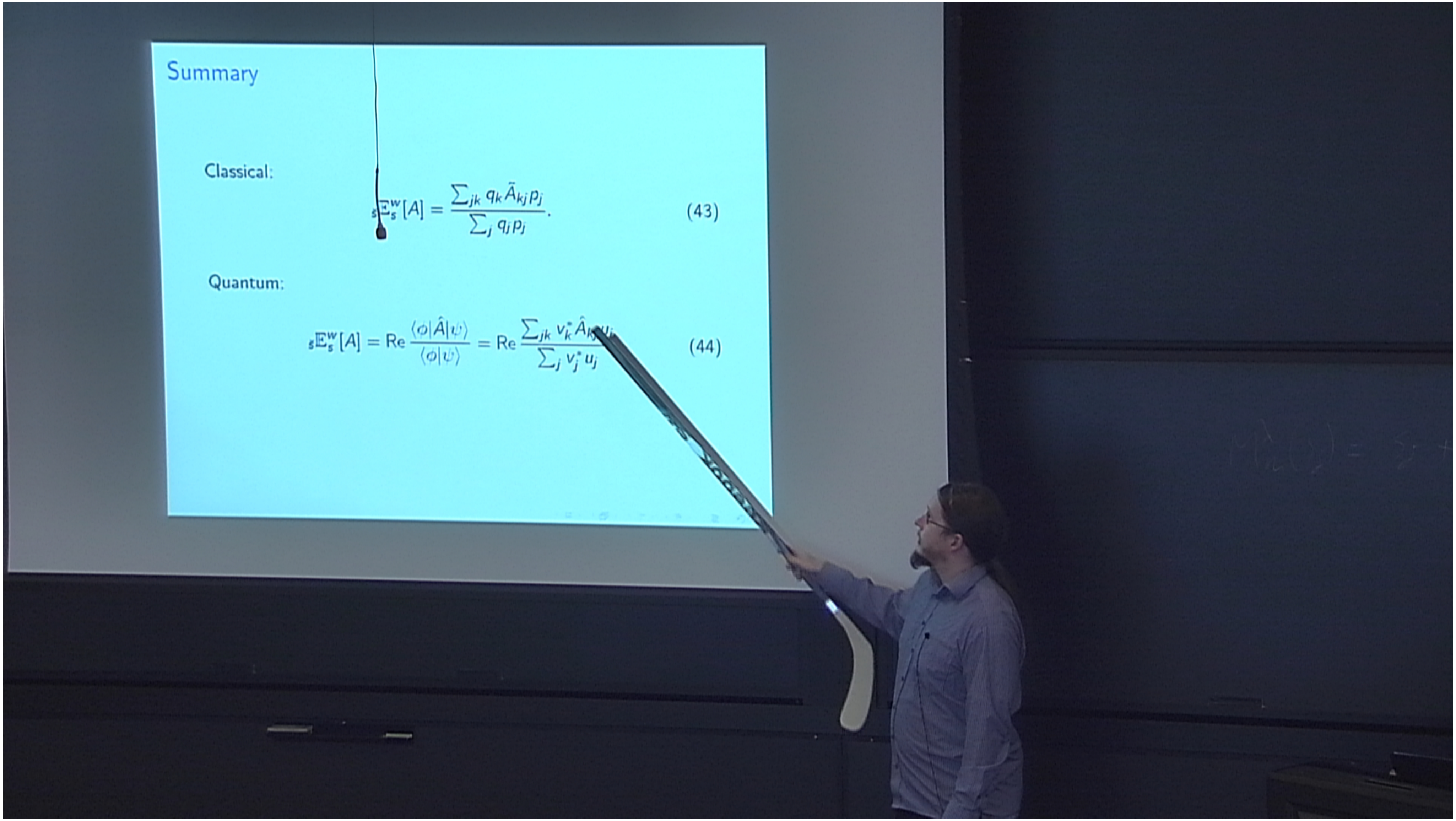
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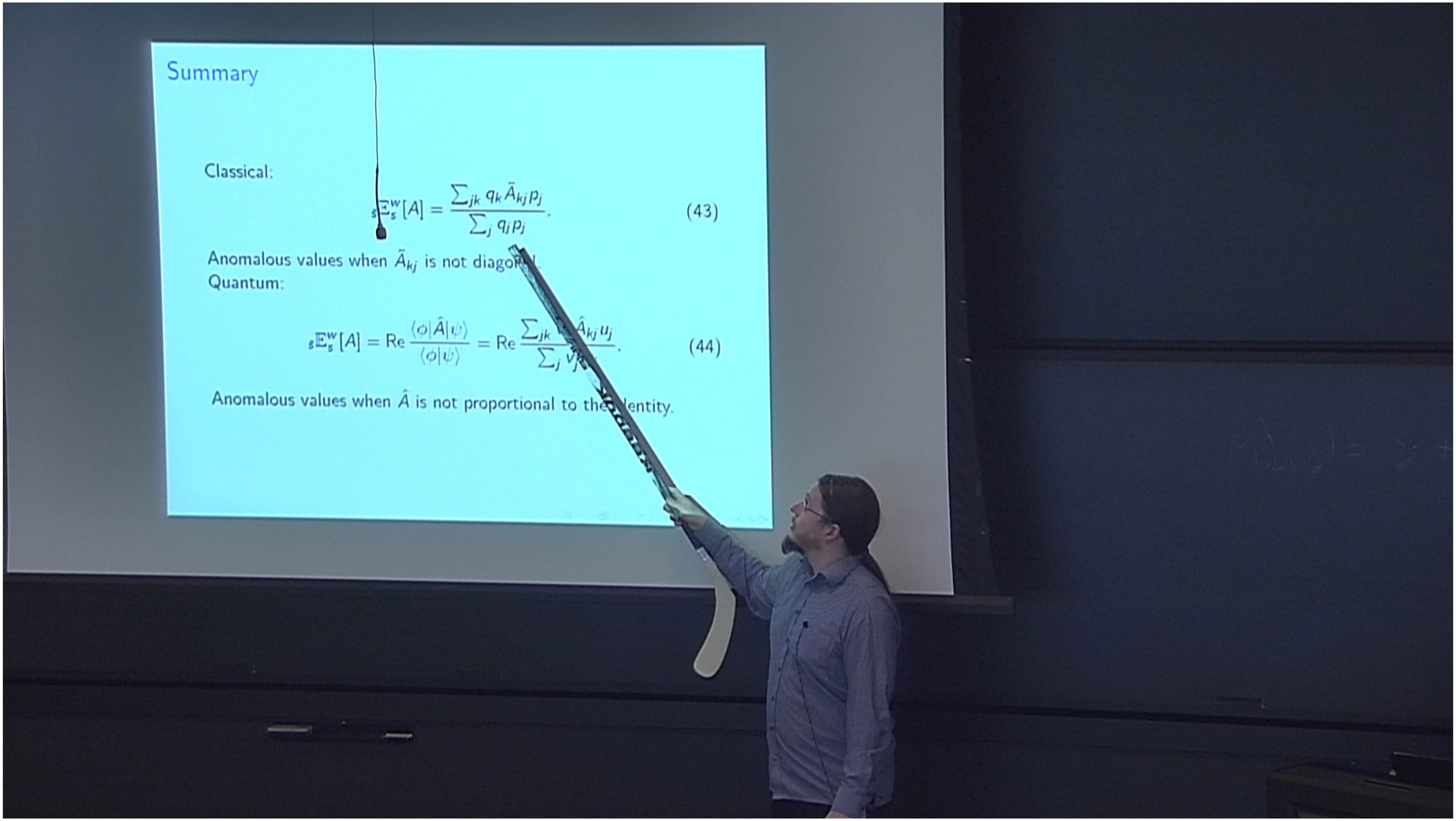
Summary
Classical:
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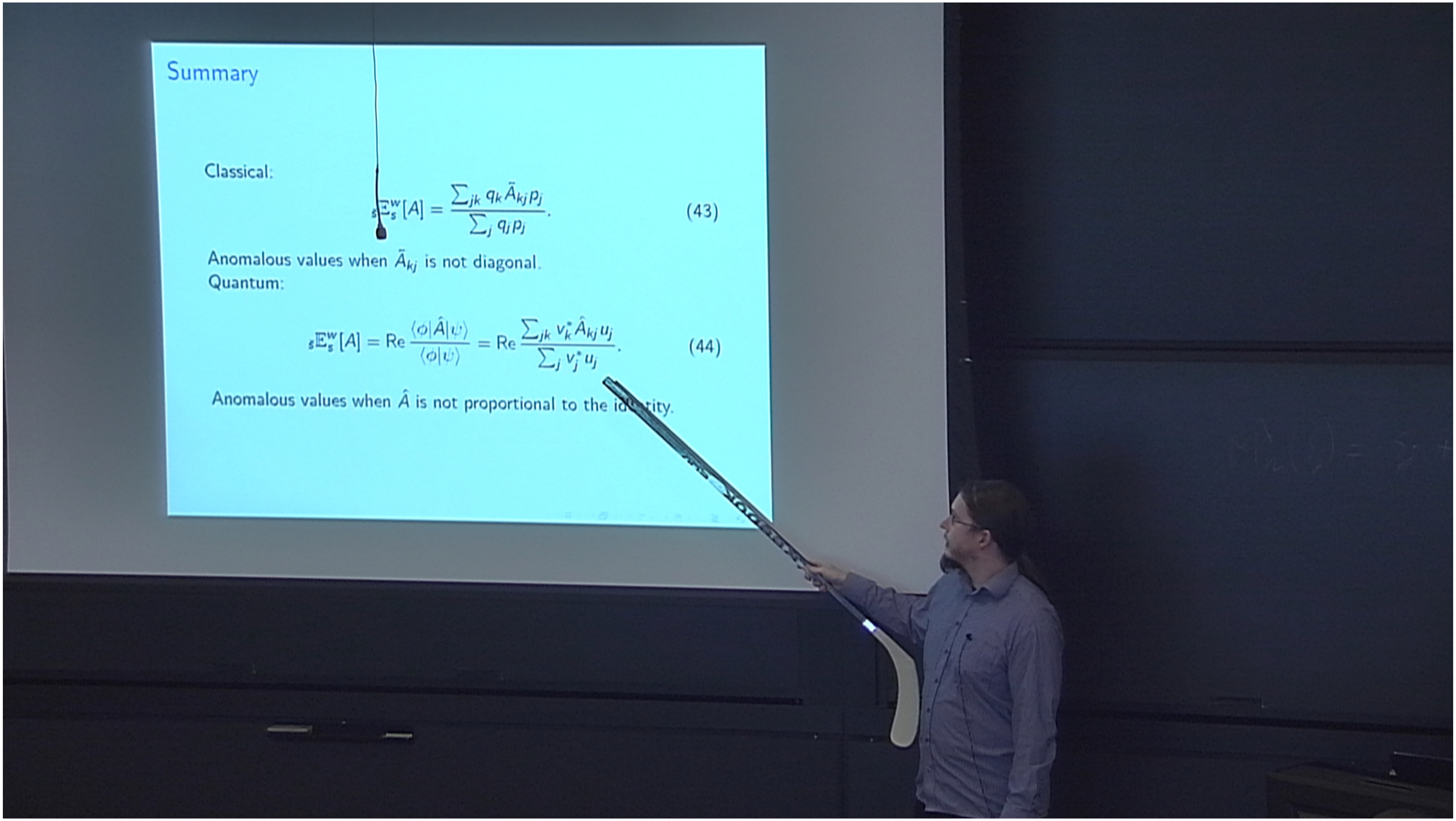
(43)
Anomalous values when $\bar{A}_{kj}$ is not diago
Quantum:
$_s\mathbb{E}_s^w[A] = \mathrm{Re}\,\frac{\langle\phi|\hat{A}|\psi\rangle}{\langle\phi|\psi\rangle} = \mathrm{Re}$
(44)
Anomalous values when $\hat{A}$ is not proportional to the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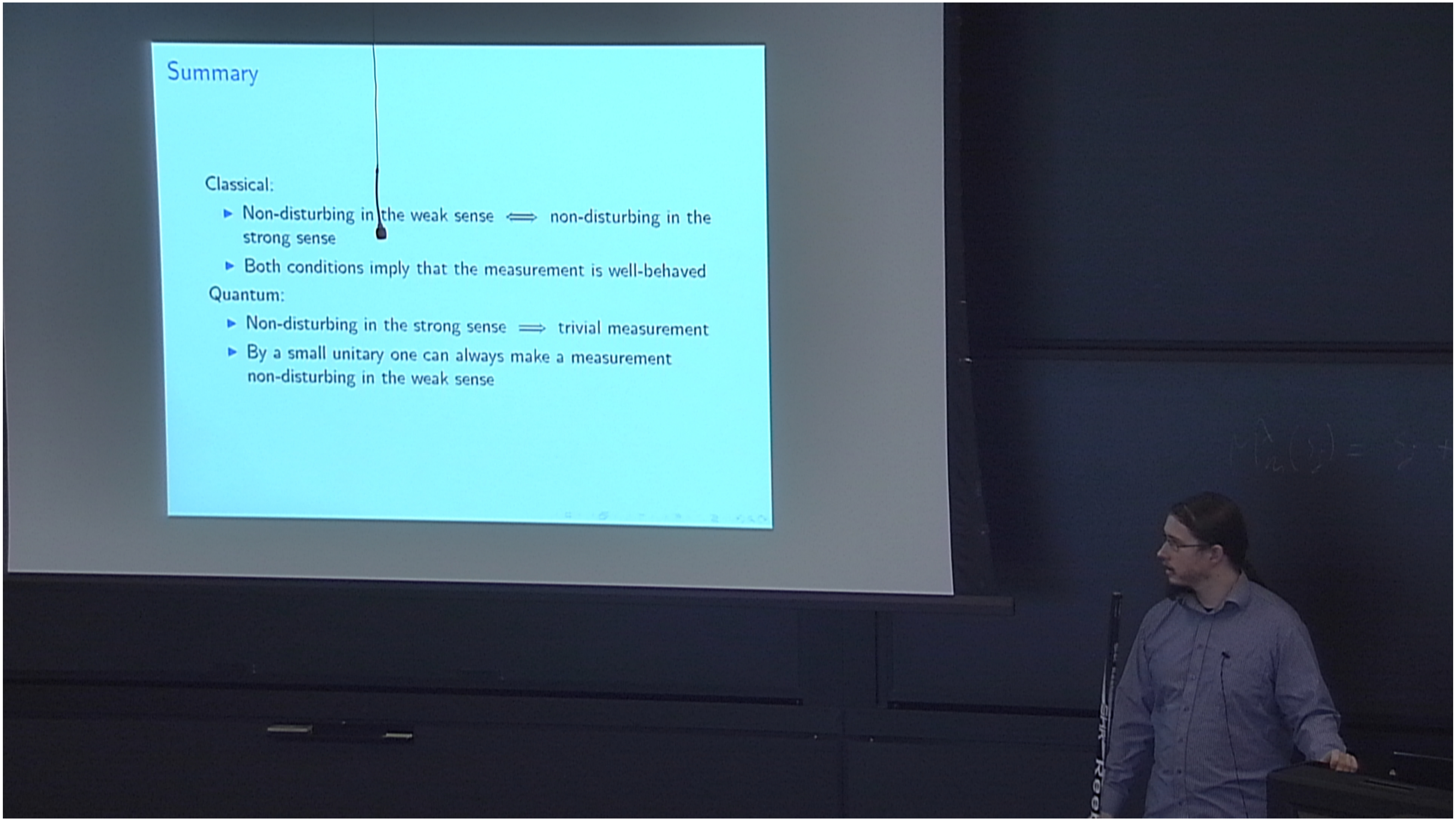
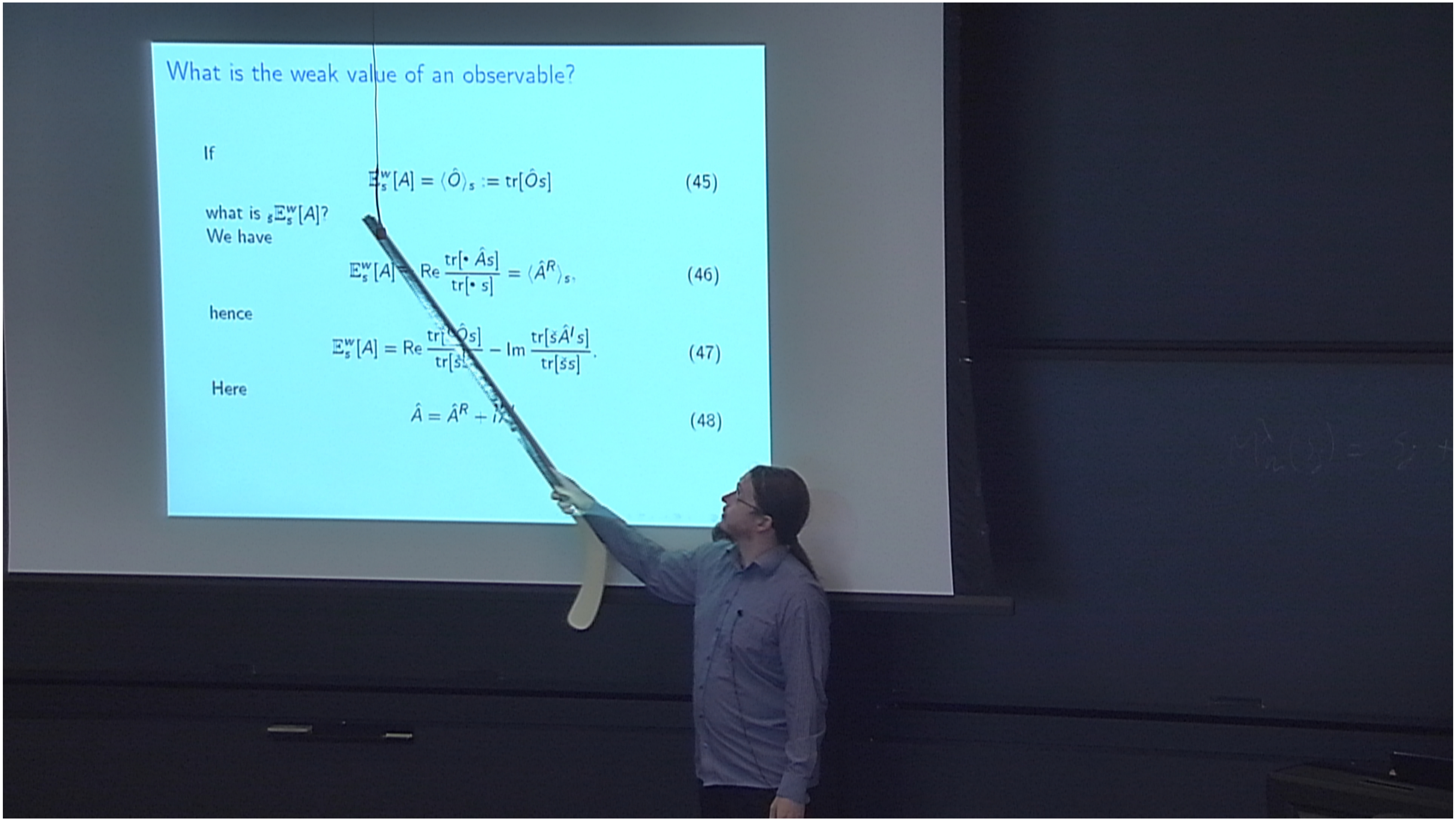

Summary
Classical:
Non-disturbing in the weak sense ⟺ non-disturbing in the strong sense
Both conditions imply that the measurement is well-behaved
Quantum:
Non-disturbing in the strong sense ⟹ trivial measurement
By a small unitary one can always make a measurement non-disturbing in the weak sense


What is the weak value of an observable?
If
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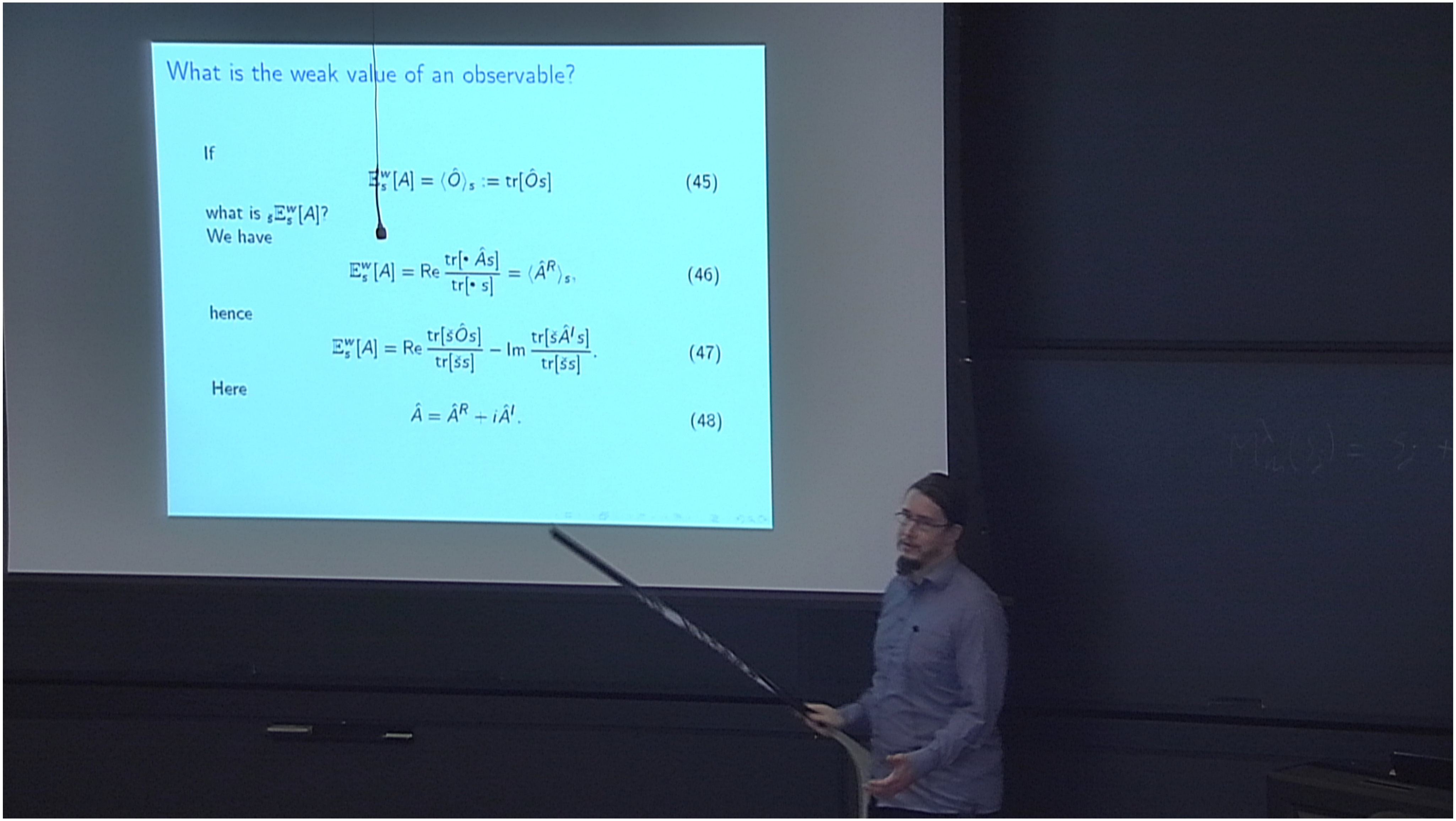
We have
$\mathbb{E}^w_s[A] = \mathrm{Re}\,\frac{\mathrm{tr}[\bullet\,\hat{A}s]}{\mathrm{tr}[\bullet\, s]} = \langle \hat{A}^R \rangle_s,$ (46)
hence
$\mathbb{E}^w_s[A] = \mathrm{Re}$ … $- \mathrm{Im}\,\frac{\mathrm{tr}[\check{s}\hat{A}^I s]}{\mathrm{tr}[\check{s}s]}.$ (47)
Here
$\hat{A} = \hat{A}^R + i$ … (48)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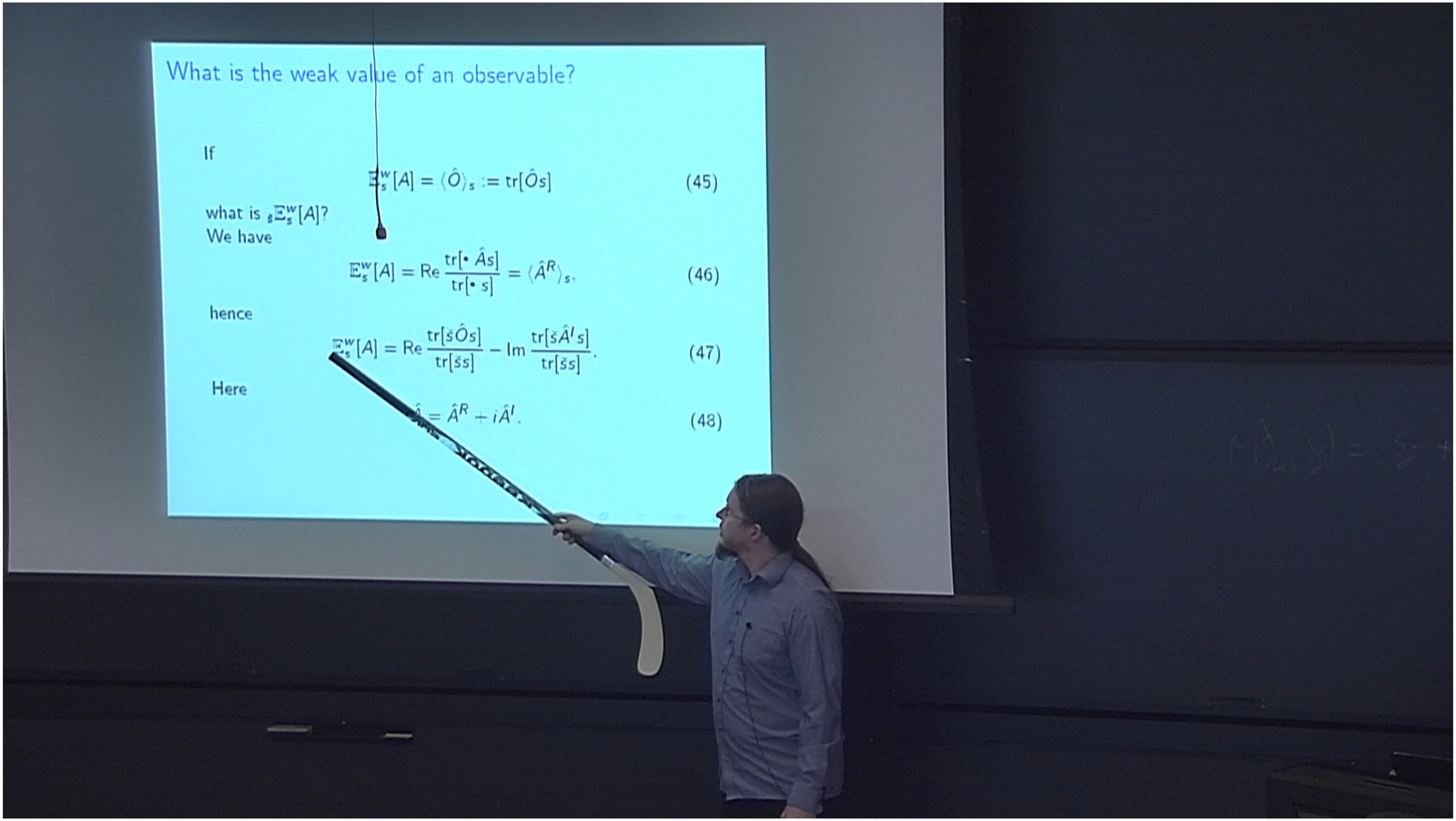
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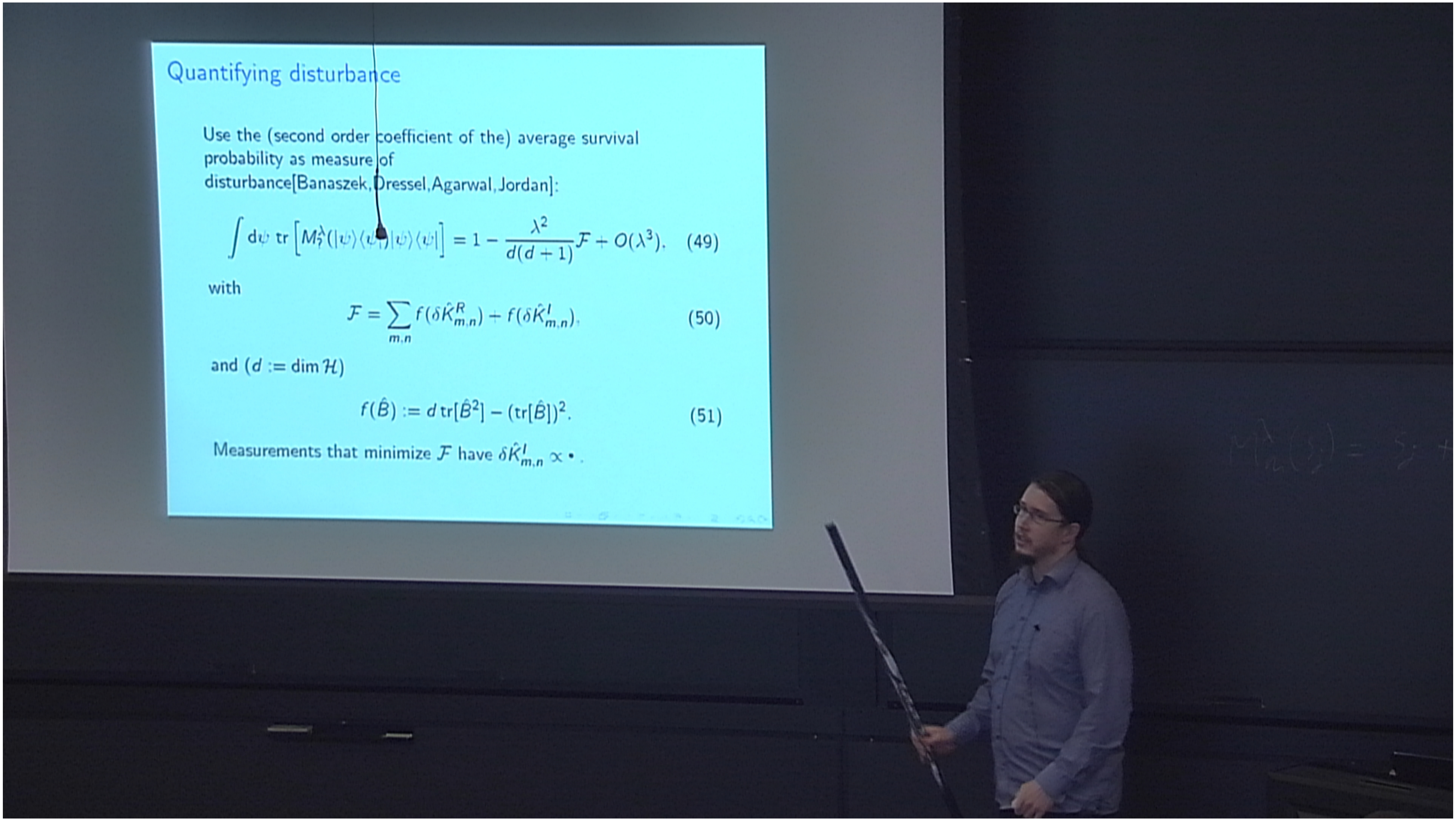
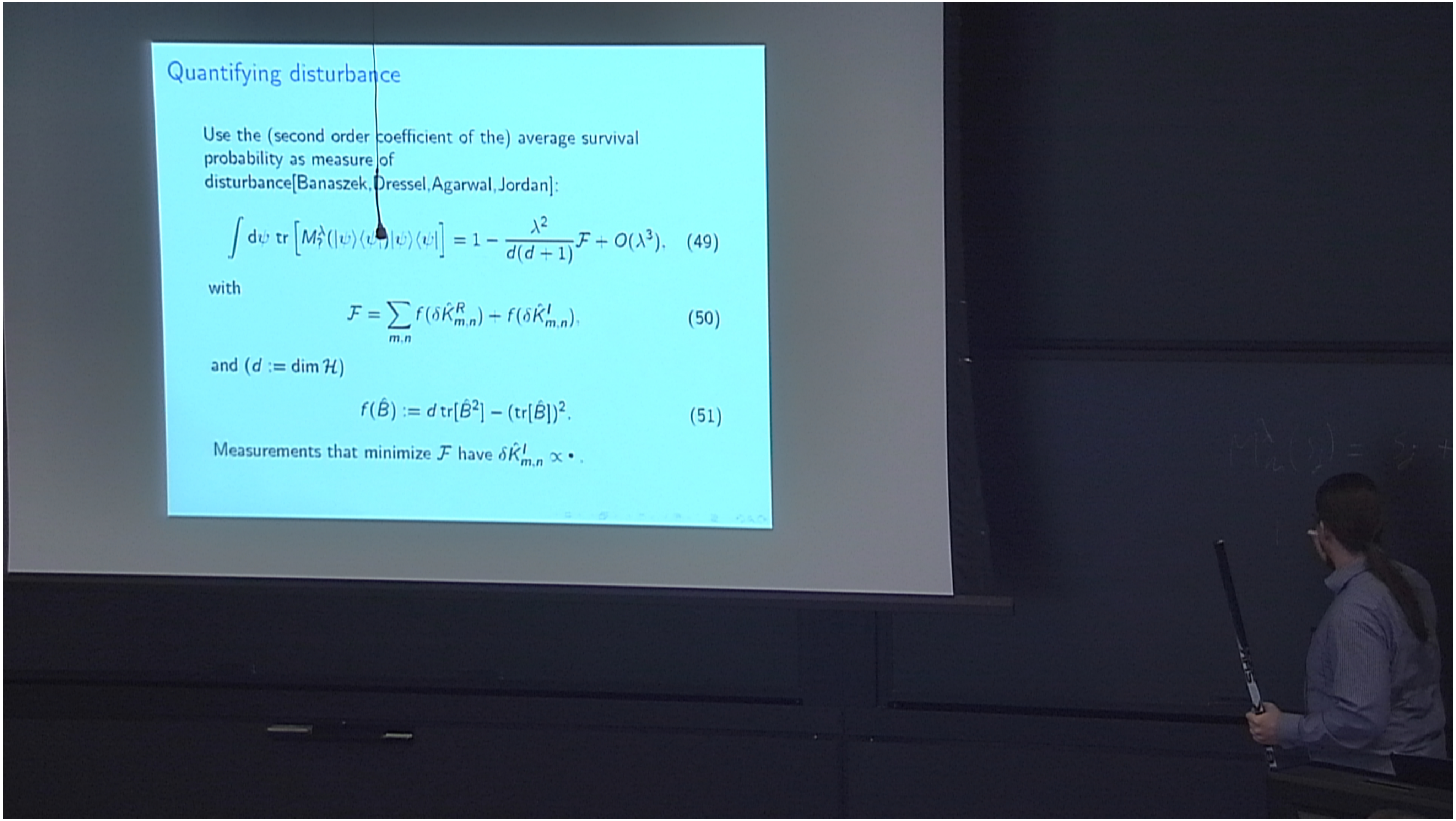

Quantifying disturbance
Use the (second order coefficient of the) average survival probability as measure of disturbance[Banaszek,Dressel,Agarwal,Jordan]:
$\int d\psi \, \mathrm{tr}\left[M^{\lambda}_{?}(|\psi\rangle\langle\psi|)|\psi\rangle\langle\psi|\right] = 1 - \frac{\lambda^2}{d(d+1)}\mathcal{F} + O(\lambda^3).$ (49)
with
$\mathcal{F} = \sum_{m,n} f(\delta\hat{K}^R_{m,n}) + f(\delta\hat{K}^I_{m,n}),$ (50)
and ($d := \dim\mathcal{H}$)
$f(\hat{B}) := d\,\mathrm{tr}[\hat{B}^2] - (\mathrm{tr}[\hat{B}])^2.$ (51)
Measurements that minimize $\mathcal{F}$ have $\delta\hat{K}^I_{m,n} \propto \bullet$.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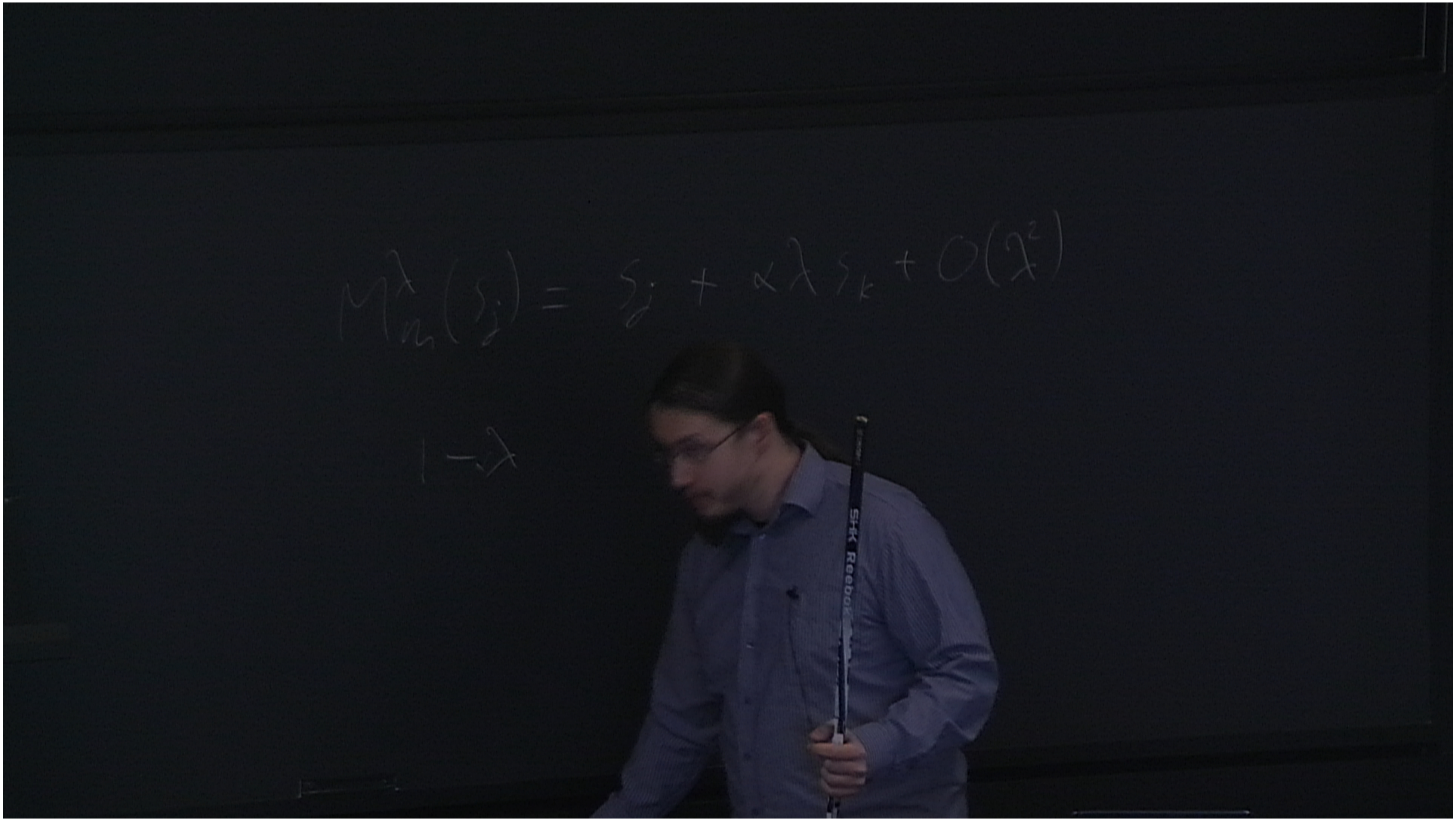
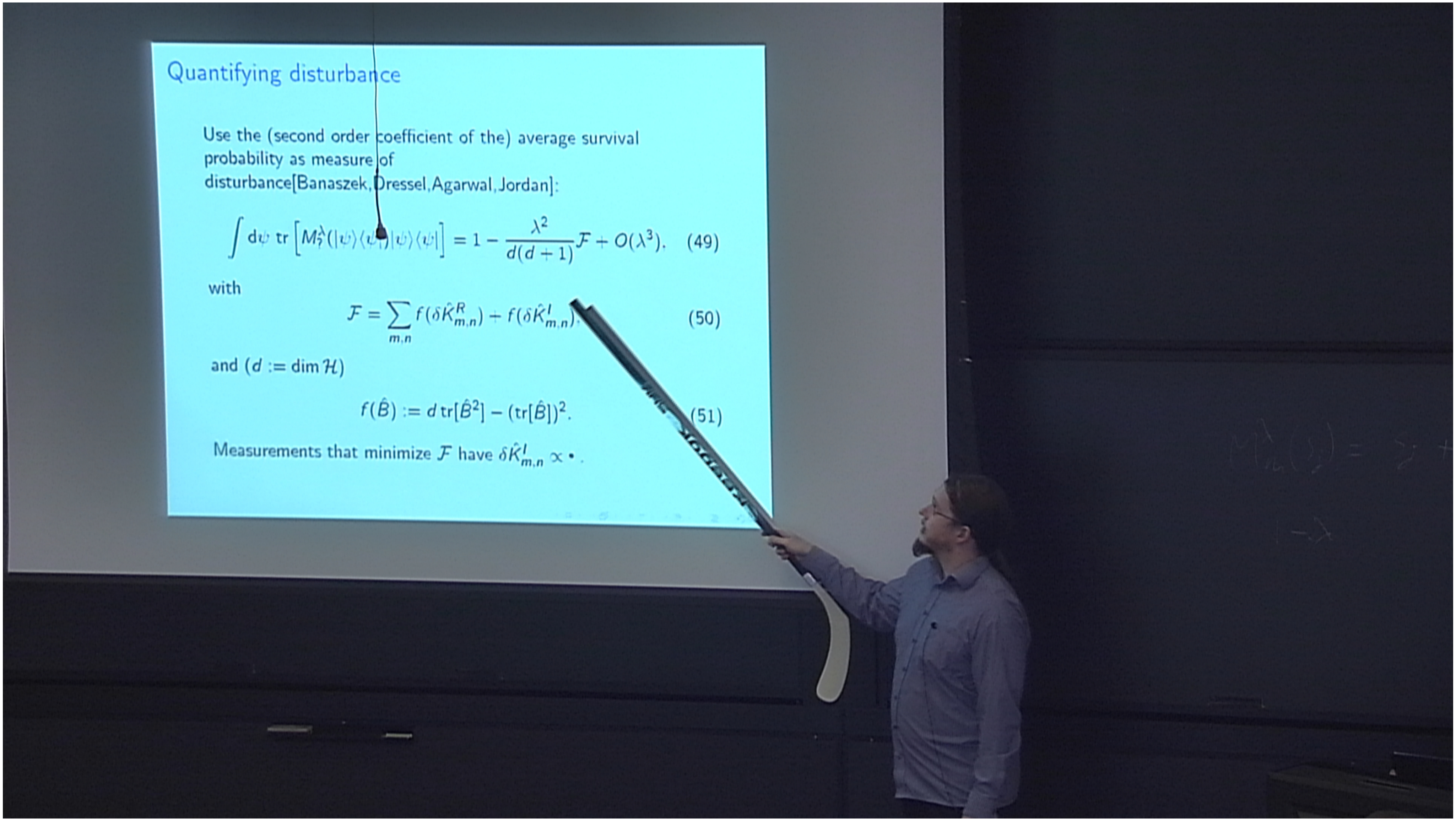

$M^{\lambda}_{m}(s_j) = s_j + \alpha\lambda s_k + O(\lambda^2)$
$1 - \lambda$
SHK Reebok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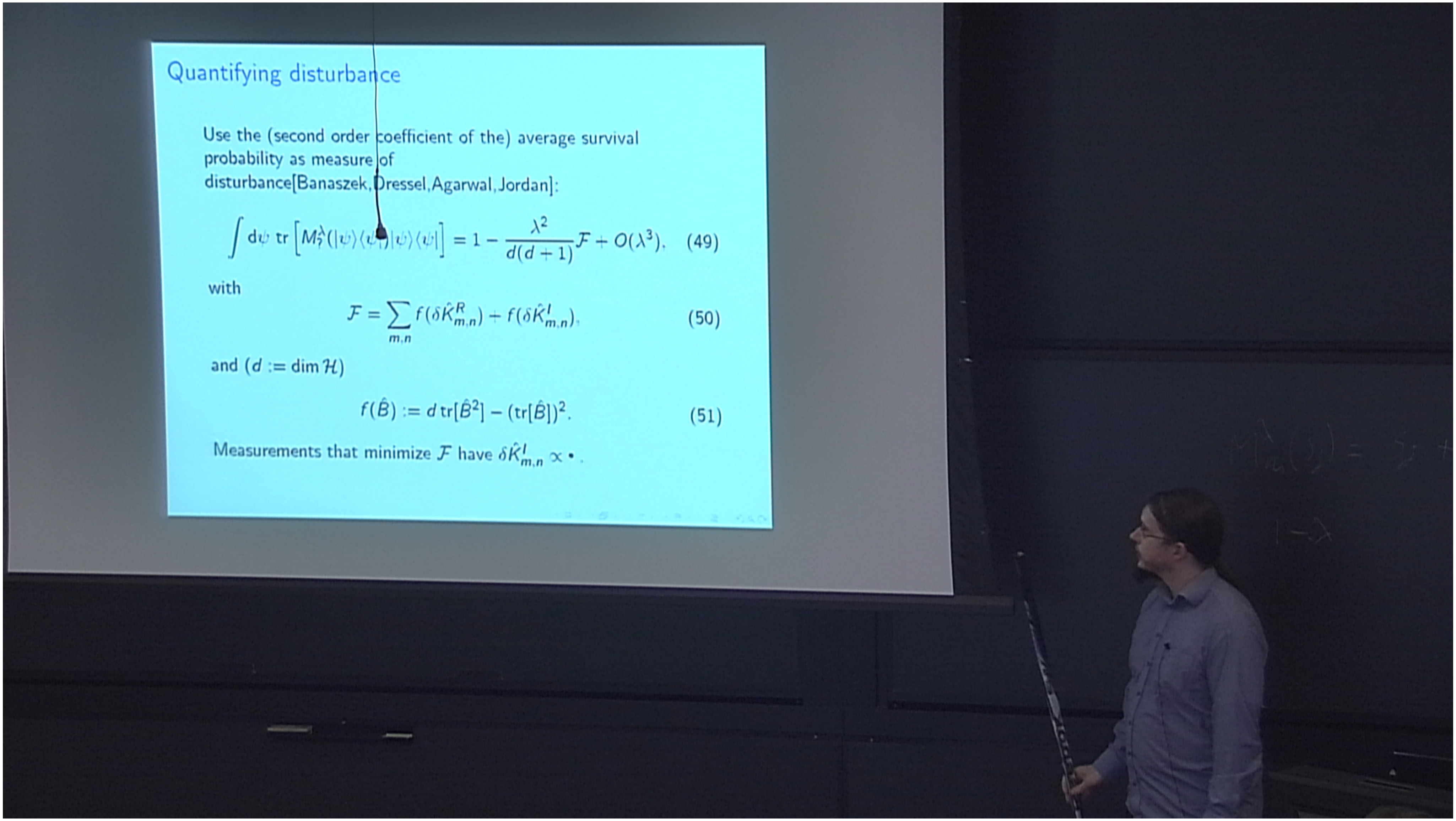
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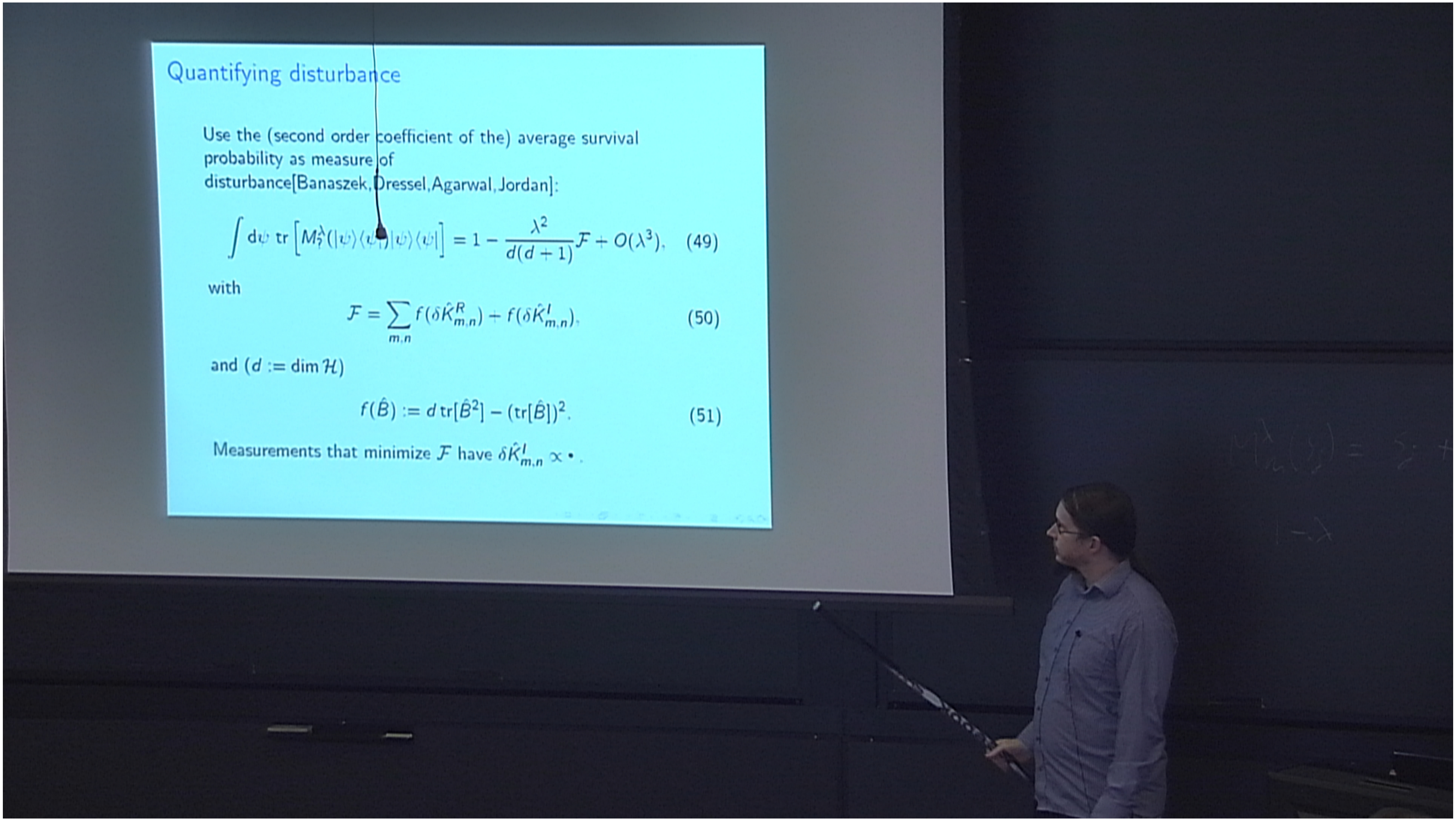
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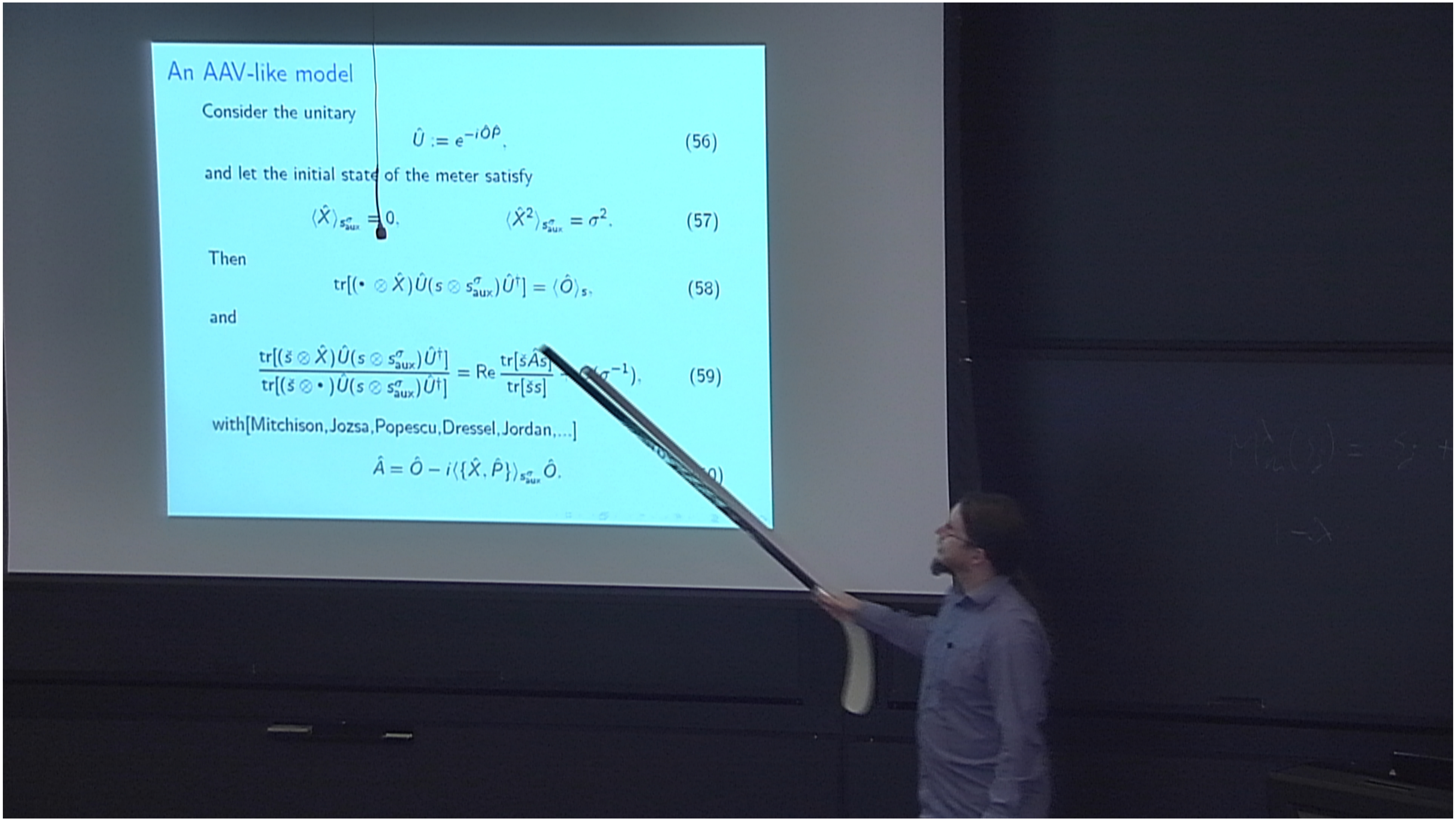
An AAV-like model
Consider the unitary
$\hat{U} := e^{-i\hat{O}\hat{P}},$ (56)
and let the initial state of the meter satisfy
$\langle\hat{X}\rangle_{s^\sigma_{aux}} = 0, \quad \langle\hat{X}^2\rangle_{s^\sigma_{aux}} = \sigma^2.$ (57)
Then
$\mathrm{tr}[(\bullet \otimes \hat{X})\hat{U}(s \otimes s^\sigma_{aux})\hat{U}^\dagger] = \langle\hat{O}\rangle_s,$ (58)
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$\frac{\mathrm{tr}[(\breve{s} \otimes \hat{X})\hat{U}(s \otimes s^\sigma_{aux})\hat{U}^\dagger]}{\mathrm{tr}[(\breve{s} \otimes \bullet)\hat{U}(s \otimes s^\sigma_{aux})\hat{U}^\dagger]} = \mathrm{Re}\,\frac{\mathrm{tr}[\breve{s}\hat{A}s]}{\mathrm{tr}[\breve{s}s]}$ $(\sigma^{-1}),$ (59)
with[Mitchison,Jozsa,Popescu,Dressel,Jordan,...]
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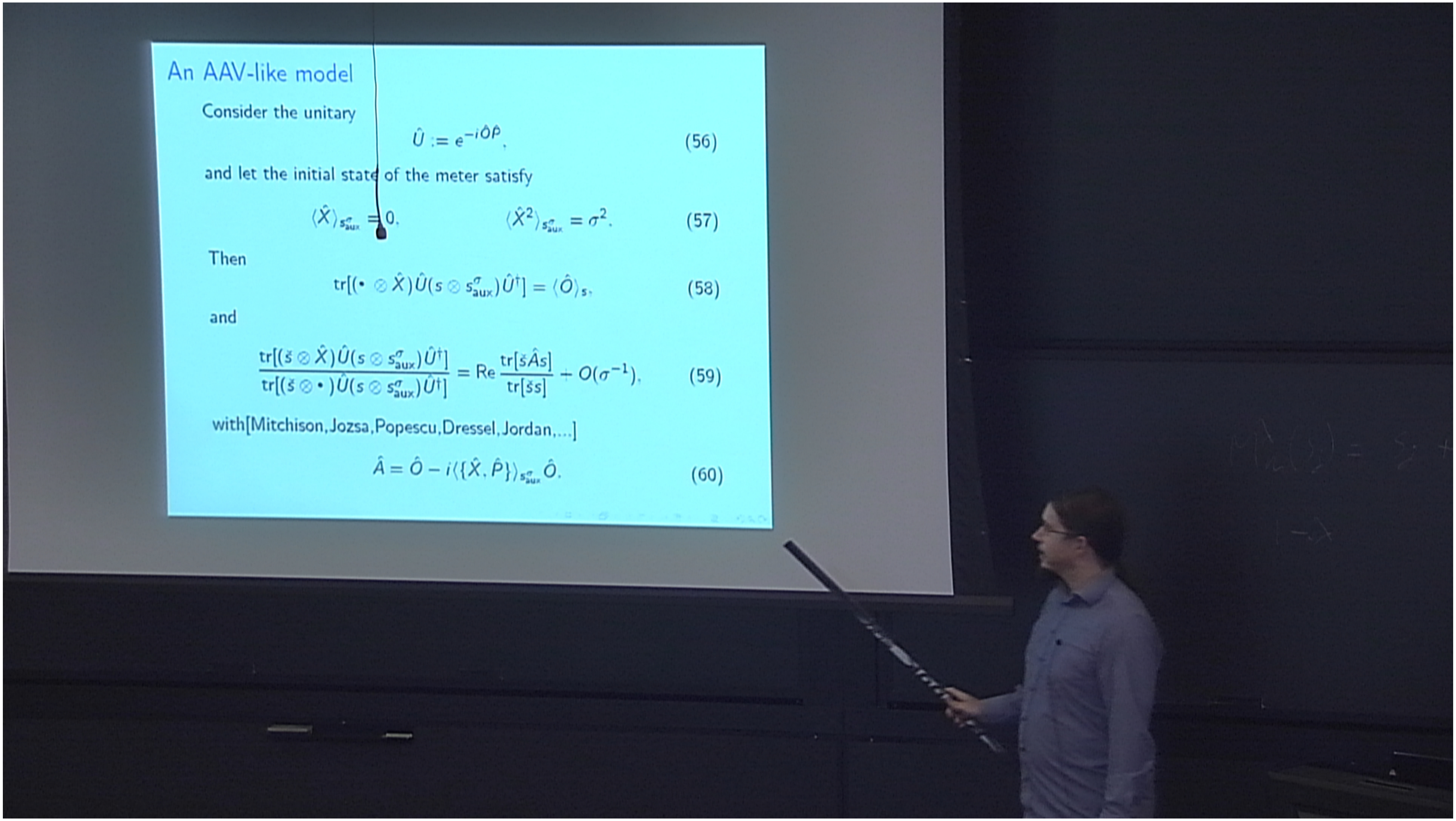
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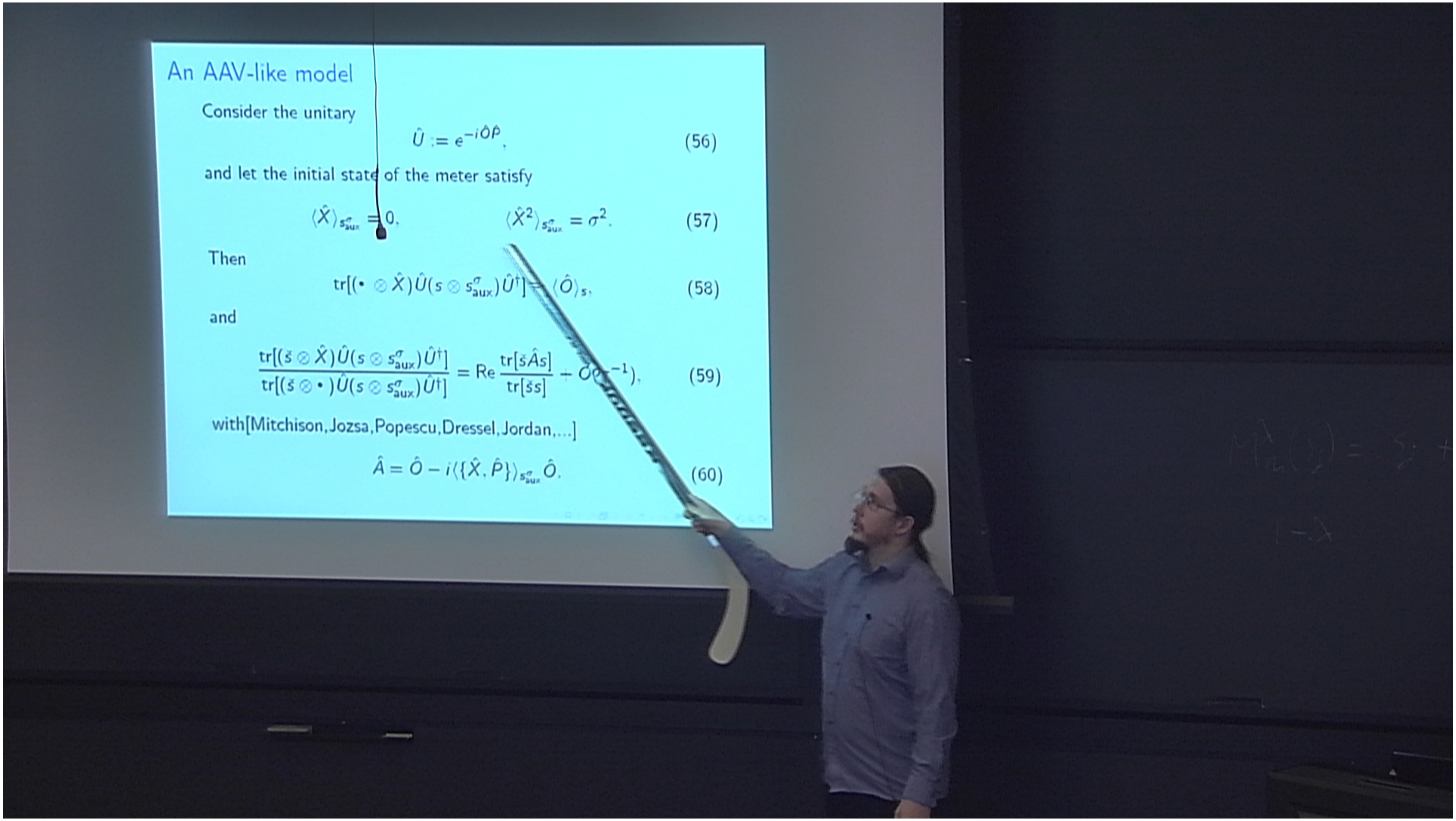
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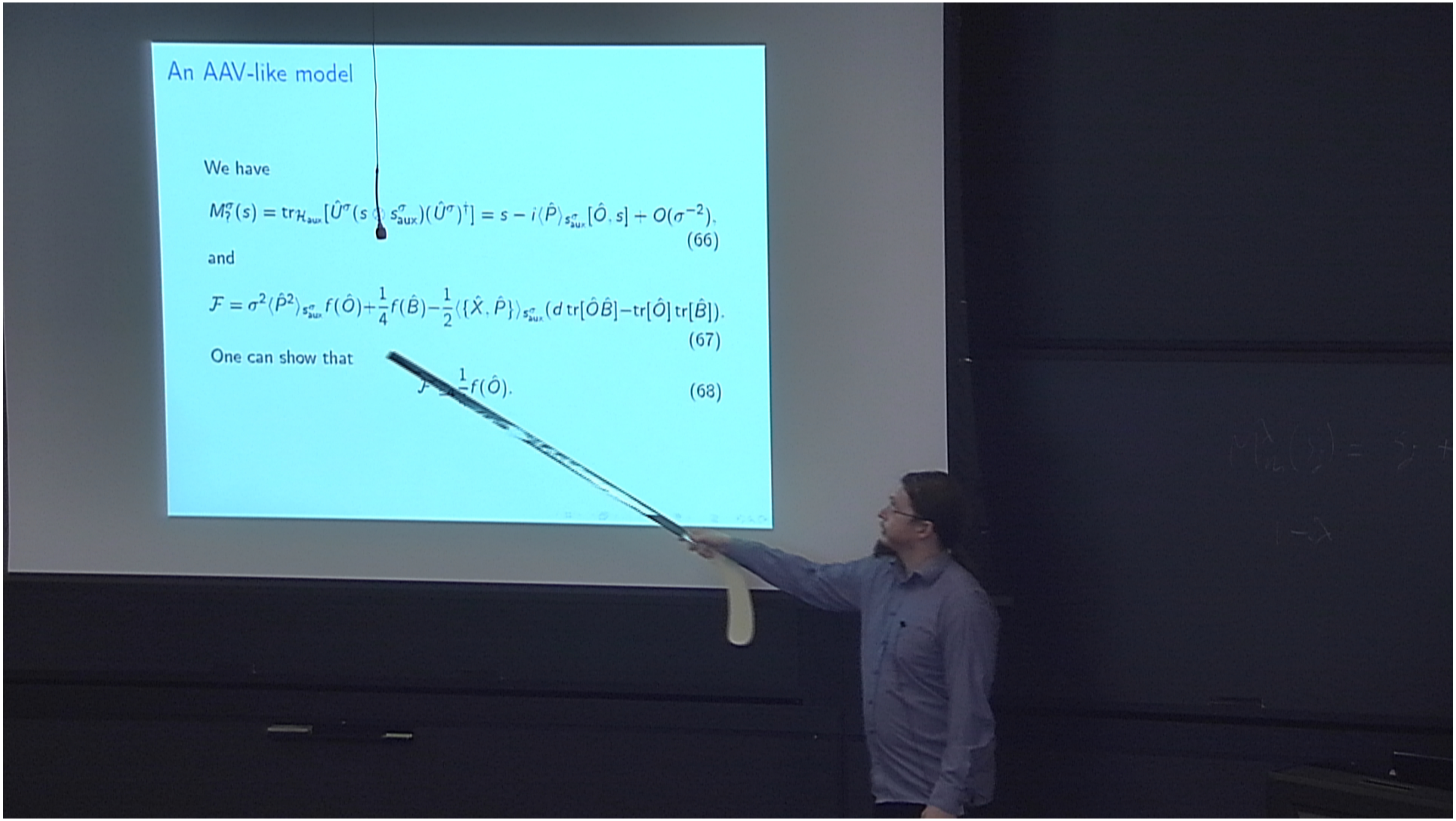
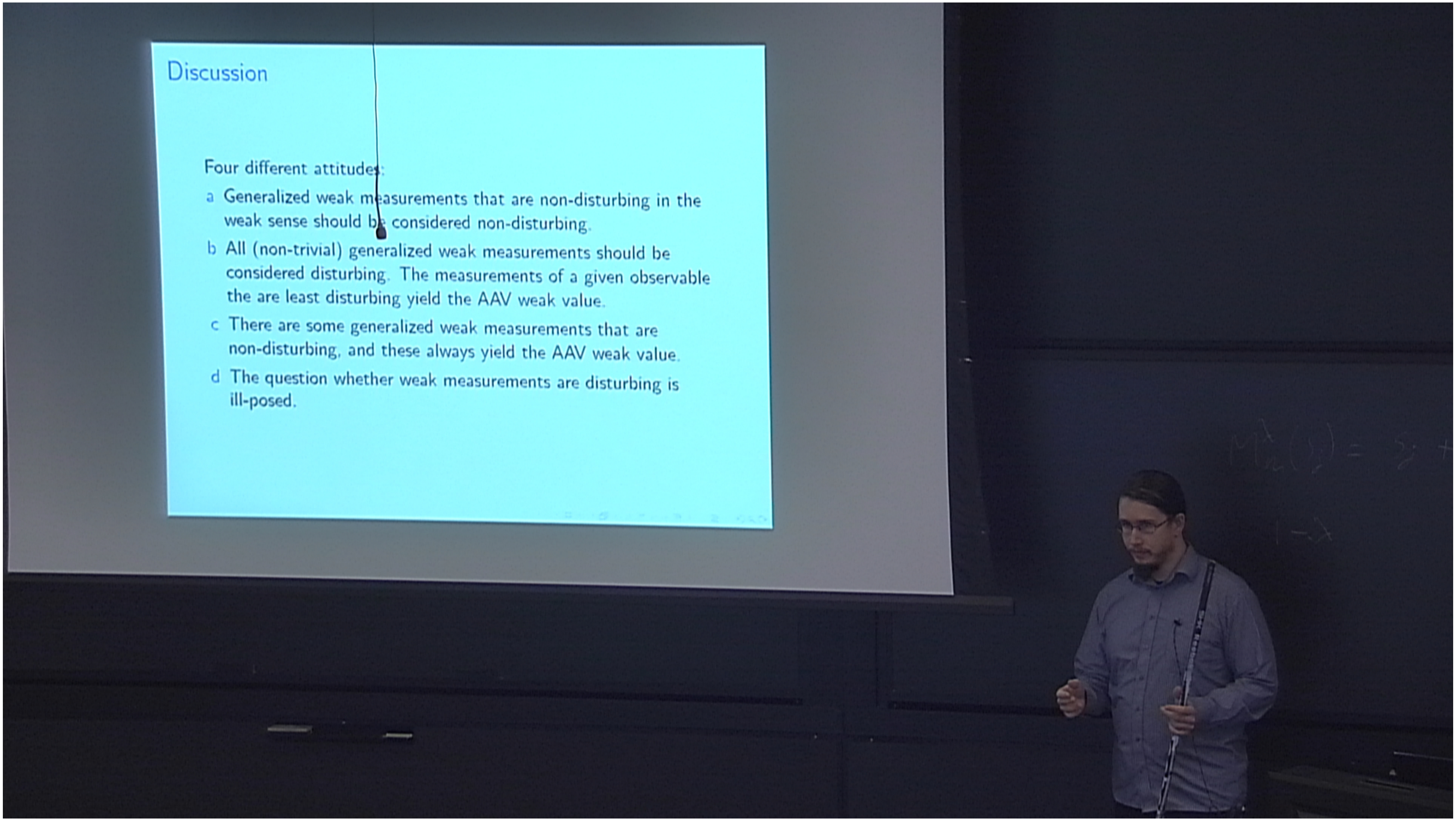
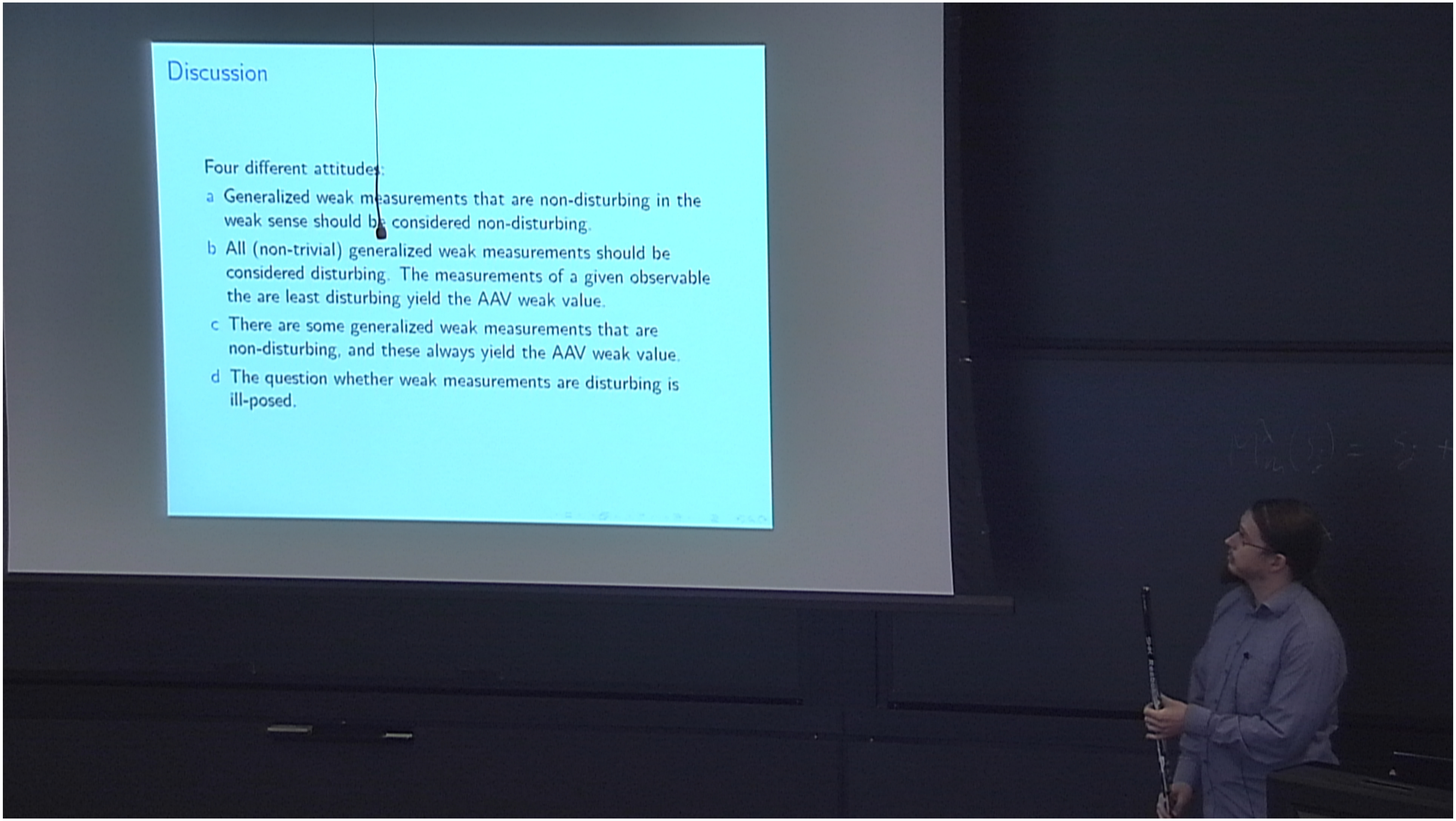

An AAV-like model
We have
$= \mathrm{tr}_{\mathcal{H}_{aux}}[\hat{U}^\sigma(s$
$s^\sigma_{aux})(\hat{U}^\sigma)^\dagger] = s - i\langle \hat{P}\rangle_{s^\sigma_{aux}}[\hat{O}, s] + O(\sigma^{-2}).$ (66)
and
$\mathcal{F} = \sigma^2\langle \hat{P}^2\rangle_{s^\sigma_{aux}} f(\hat{O}) + \frac{1}{4}f(\hat{B}) - \frac{1}{2}\langle\{\hat{X}, \hat{P}\}\rangle_{s^\sigma_{aux}}(d\,\mathrm{tr}[\hat{O}\hat{B}] - \mathrm{tr}[\hat{O}]\,\mathrm{tr}[\hat{B}]).$ (67)
One can show that
$f(\hat{O}).$ (68)


Discussion
Four different attitudes:
a Generalized weak measurements that are non-disturbing in the weak sense should be considered non-disturbing.
b All (non-trivial) generalized weak measurements should be considered disturbing. The measurements of a given observable the are least disturbing yield the AAV weak value.
c There are some generalized weak measurements that are non-disturbing, and these always yield the AAV weak value.
d The question whether weak measurements are disturbing is ill-posed.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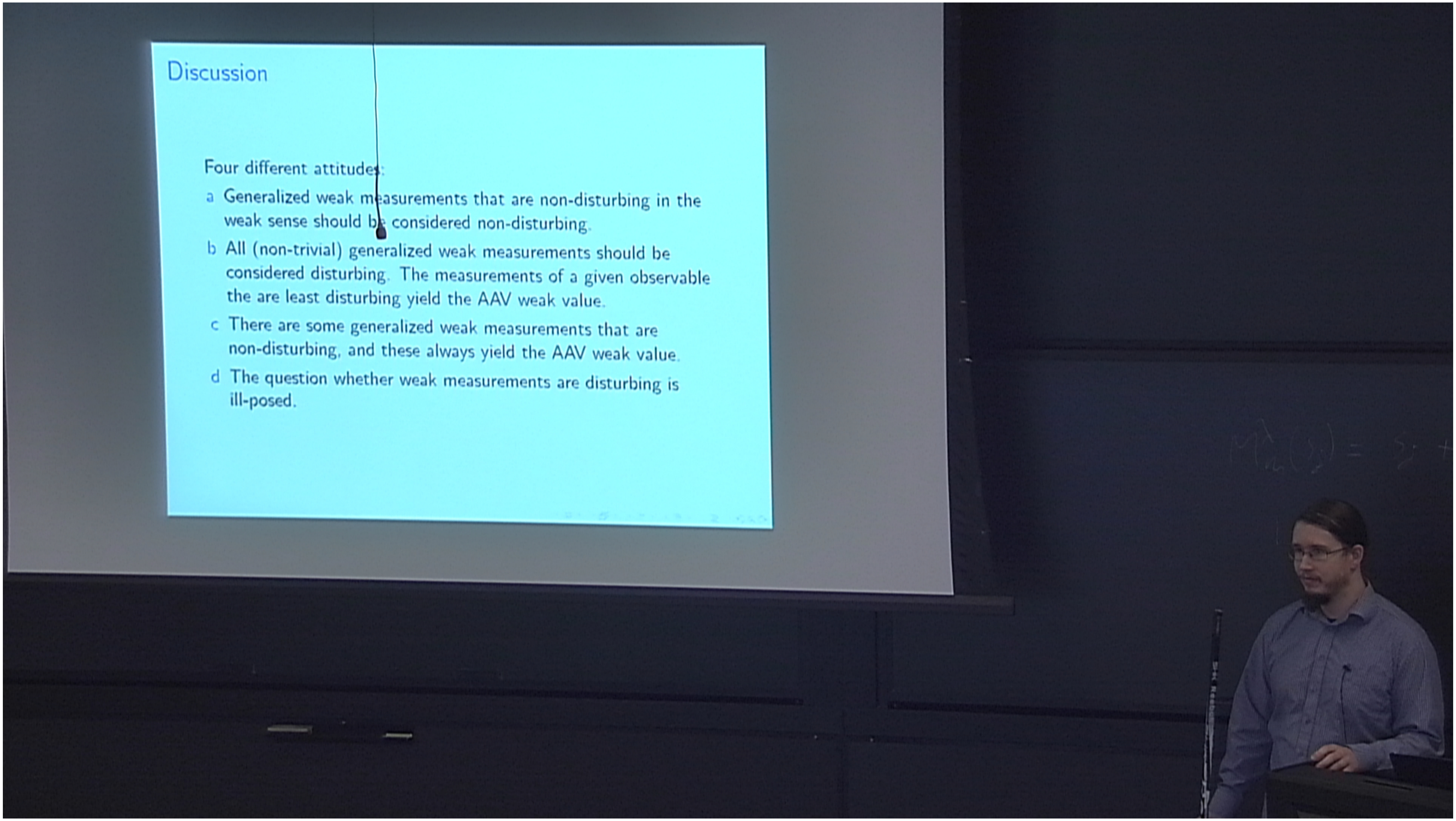
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