

Title: Non-Equilibrium Thermodynamics of a Gravitational Screen

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URL: <http://pirsa.org/14100110>

Abstract:

Relativistic Hydrodynamics and gravitational screens

Laurent Freidel Pl.

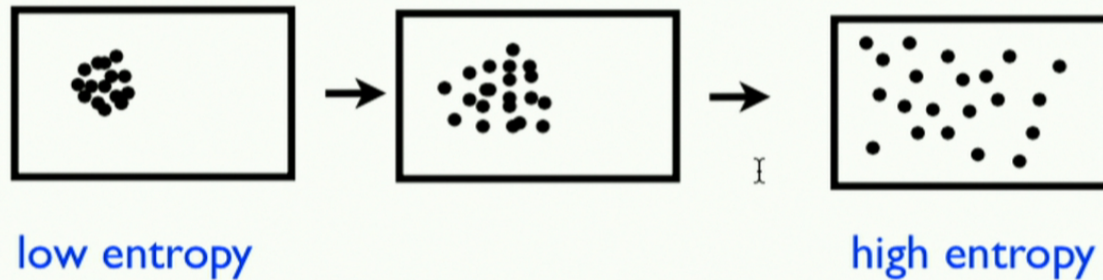
PI-Day 14 I

[arXiv:1312.1538](#), [1405.4881](#), ...

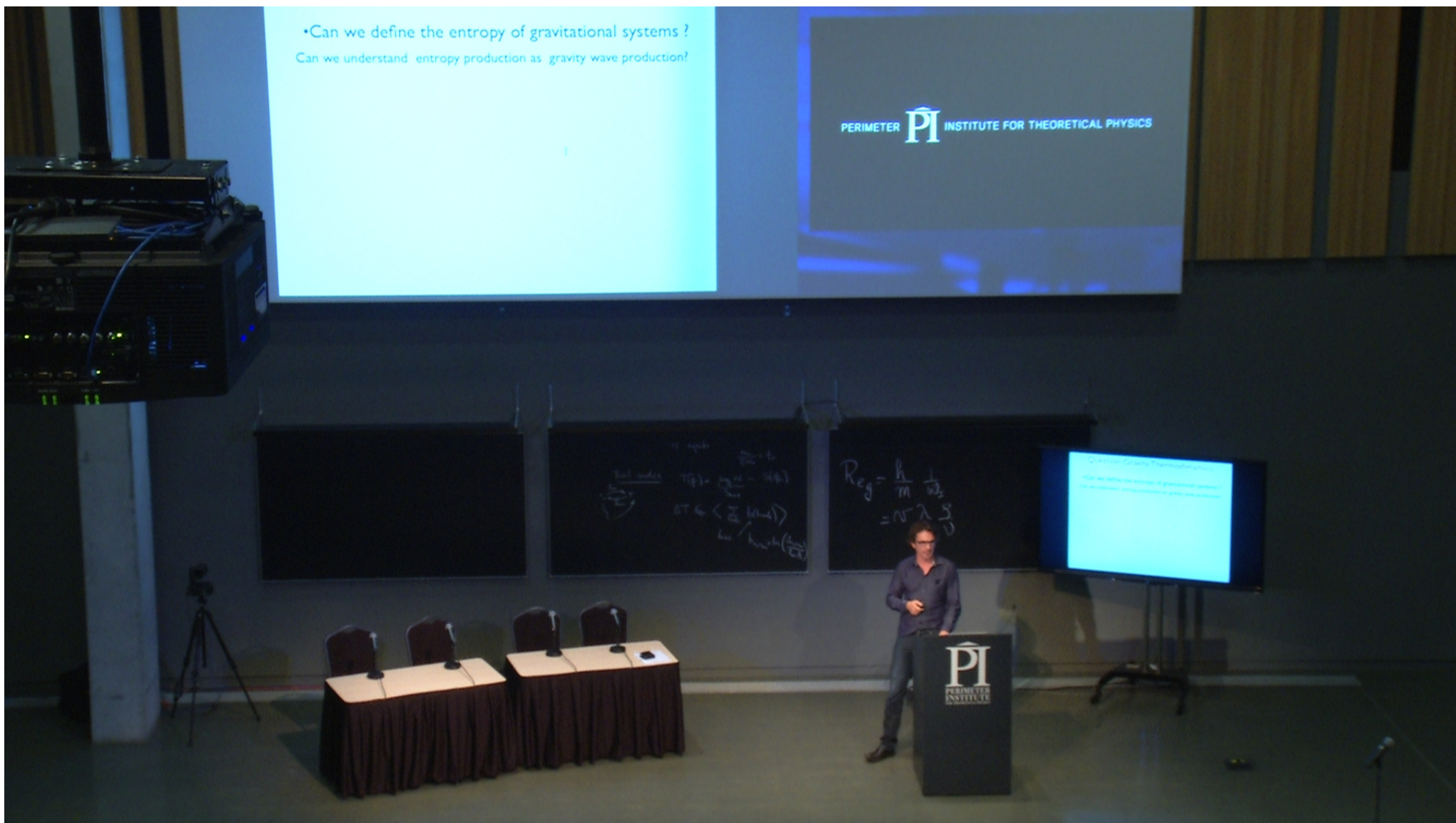
The Questions ?

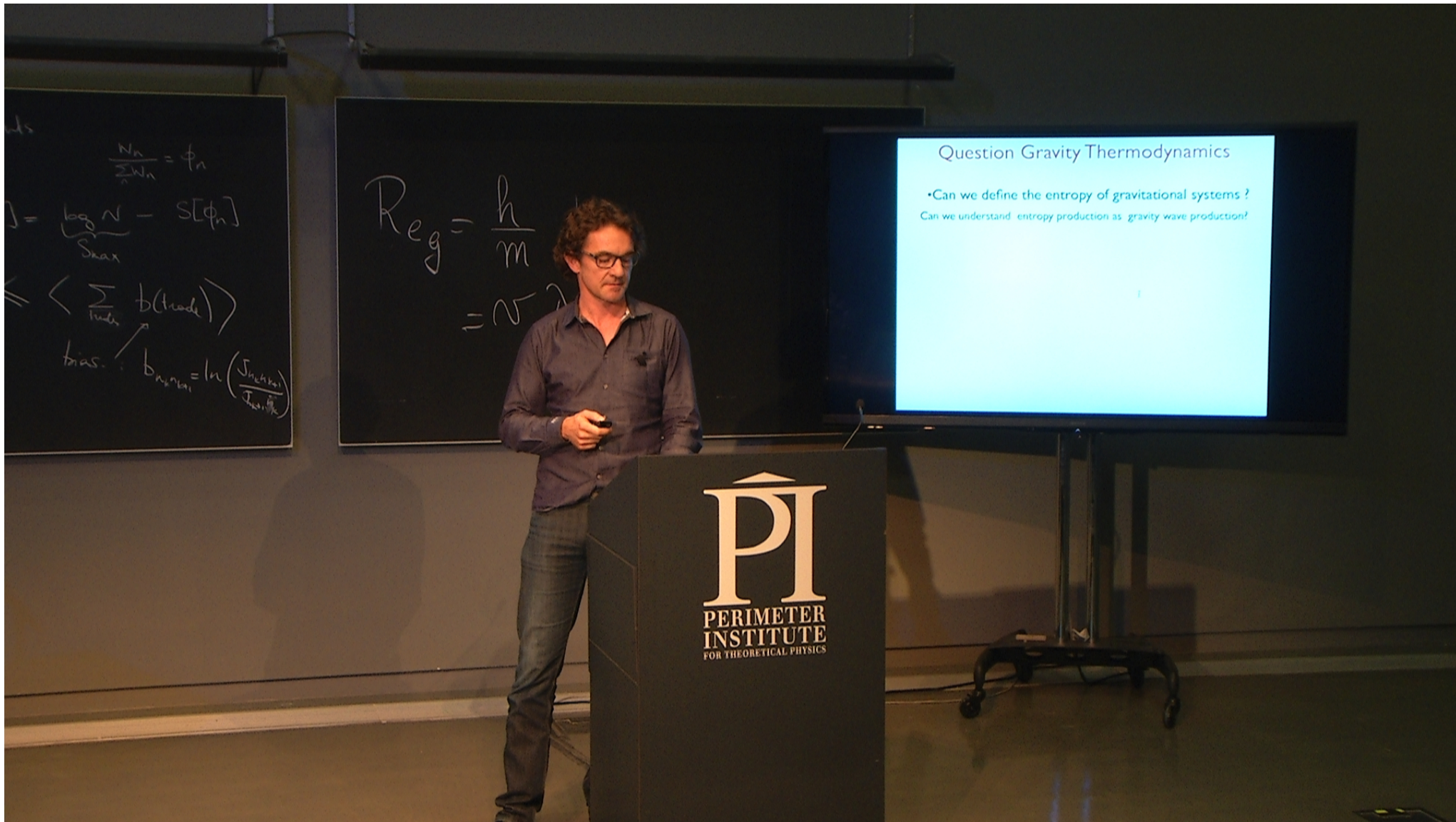
- Can we define the entropy of gravitational systems ?

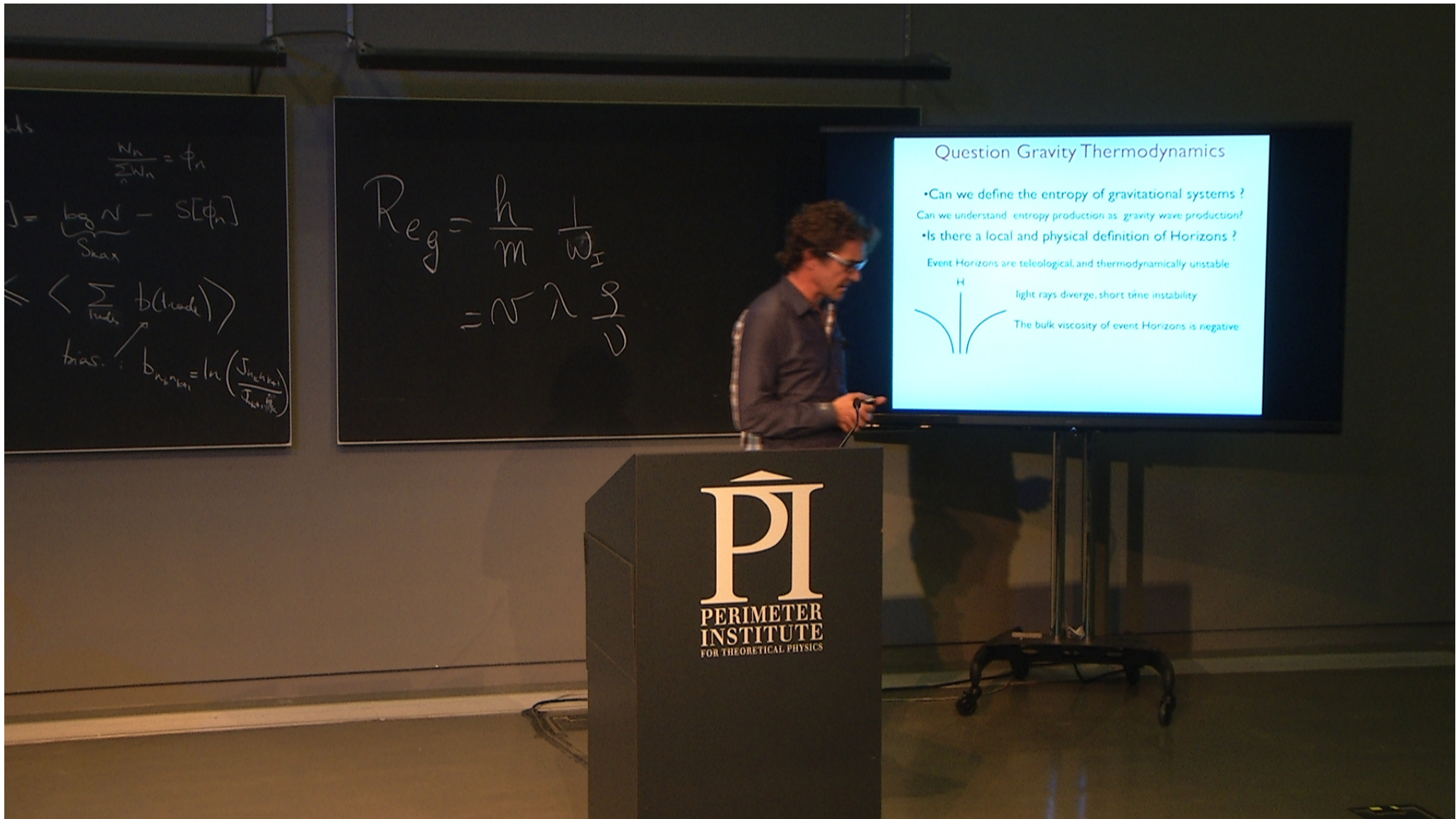
Time evolution of **non** gravitational system











$$\frac{W_n}{\sum_n W_n} = \phi_n$$

$$J = \underbrace{\log N}_{S_{\text{max}}} - S[\phi_n]$$

$$\leq \left\langle \sum_{\text{trials}} b(\text{trials}) \right\rangle$$

bias. $b_{n_{\text{trials}}} = \ln \left(\frac{J_{n_{\text{trials}}}}{J_{\text{max}}/N} \right)$

$$R_{\text{eg}} = \frac{h}{m} \frac{1}{w_I}$$

$$= N \lambda \frac{g}{v}$$

Question Gravity Thermodynamics

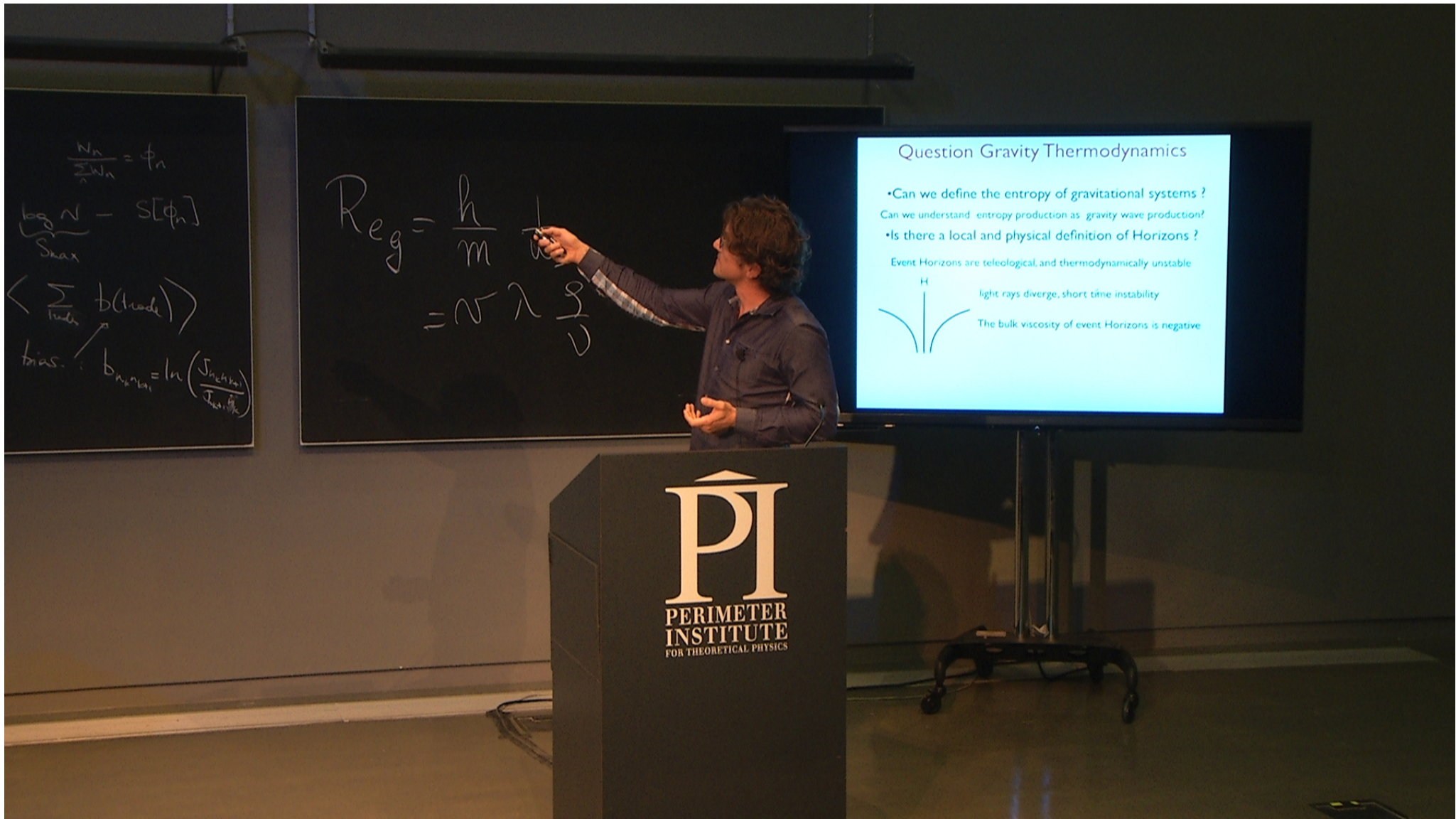
- Can we define the entropy of gravitational systems ?
- Can we understand entropy production as gravity wave production?
- Is there a local and physical definition of Horizons ?

Event Horizons are teleological, and thermodynamically unstable



light rays diverge, short time instability

The bulk viscosity of event Horizons is negative



Question Gravity Thermodynamics

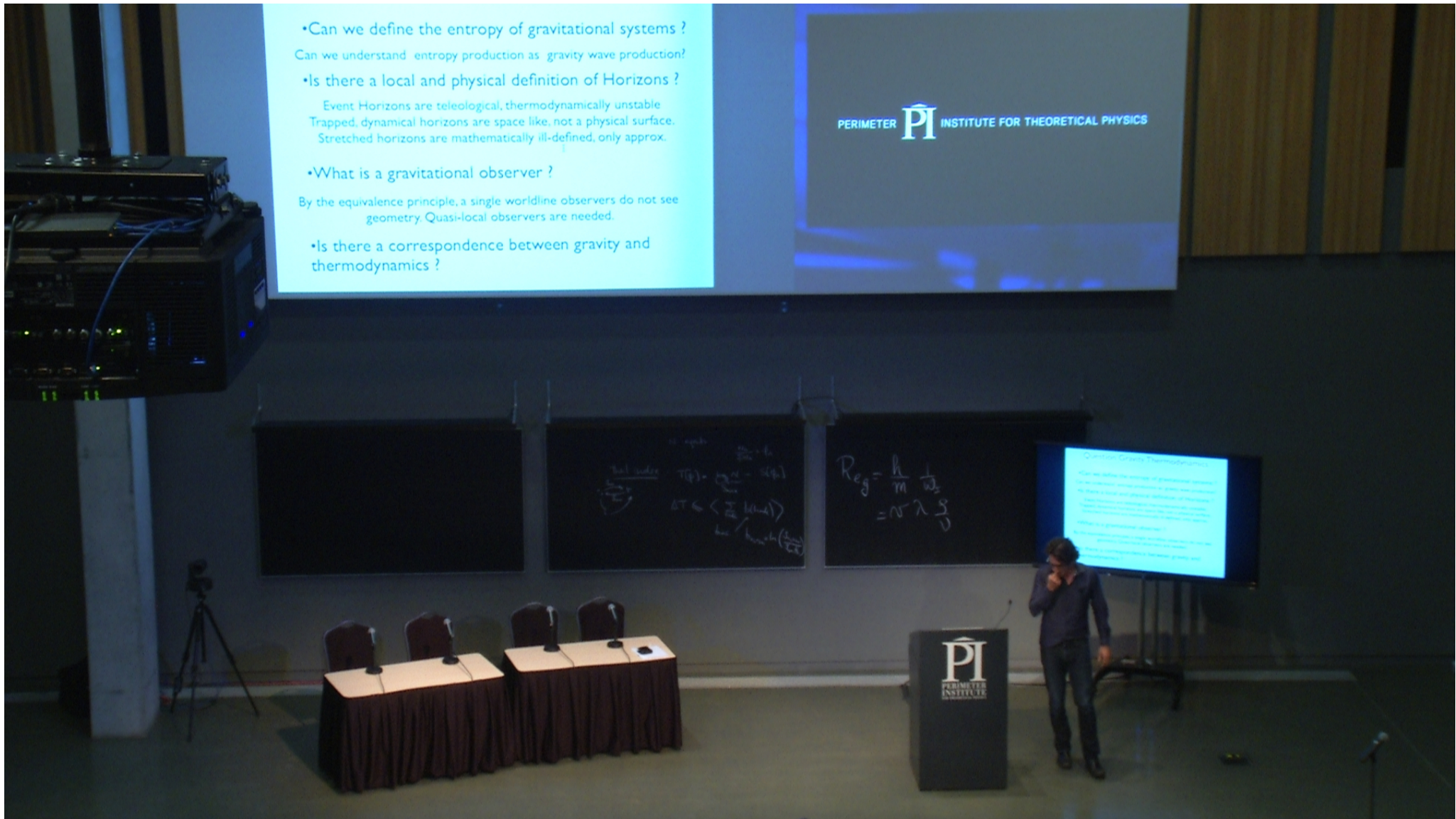
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- Can we define the entropy of gravitational systems ?
Can we understand entropy production as gravity wave production?
- Is there a local and physical definition of Horizons ?
Event Horizons are teleological, thermodynamically unstable
Trapped, dynamical horizons are space like, not a physical surface.
Stretched horizons are mathematically ill-defined, only approx.
- What is a gravitational observer ?
By the equivalence principle, a single worldline observers do not see geometry. Quasi-local observers are needed.
- Is there a correspondence between gravity and thermodynamics ?

PERIMETER **PI** INSTITUTE FOR THEORETICAL PHYSICS

is approx $\frac{dS}{dt} = \frac{1}{4\pi} \int_{\mathcal{H}} \kappa^2 dA$

Real surface $T(\mathcal{H}) = \frac{1}{4\pi} \int_{\mathcal{H}} \kappa^2 dA$

$\Delta T \ll \langle \frac{1}{4\pi} \int_{\mathcal{H}} \kappa^2 dA \rangle$

but $\left(\frac{1}{4\pi} \int_{\mathcal{H}} \kappa^2 dA \right) \ll \left(\frac{1}{4\pi} \int_{\mathcal{H}} \kappa^2 dA \right)$

$$R_{\text{eg}} = \frac{\hbar}{m} \frac{1}{\omega_2}$$
$$= N \lambda \frac{S}{V}$$

Quantum Gravity Thermodynamics

Can we define the entropy of gravitational systems?
Can we understand entropy production as gravity wave production?
Is there a local and physical definition of horizons?
What horizons are thermodynamically unstable?
Trapped, dynamical horizons are space like, not a physical surface.
Stretched horizons are mathematically ill-defined, only approx.

What is a gravitational observer?
By the equivalence principle, a single worldline observers do not see geometry. Quasi-local observers are needed.
Is there a correspondence between gravity and thermodynamics?

Question Gravity Thermodynamics

- Can we define the entropy of gravitational systems ?

Can we understand entropy production as gravity wave production?

- Is there a local and physical definition of Horizons ?

Event Horizons are teleological, thermodynamically unstable
Trapped, dynamical horizons are space like, not a physical surface.
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- What is a gravitational observer ?

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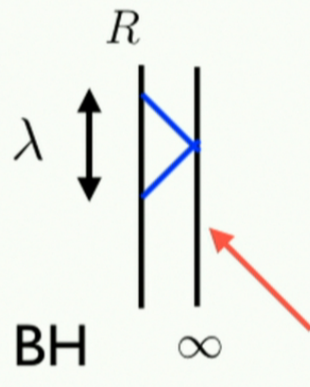
- Is there a correspondence between gravity and thermodynamics ?

Analogy Gravity-Thermodynamics

- There seems to be lots of evidence of a correspondence between Black Hole physics and eq. thermo-Static.
- Is it just an analogy or is there a deeper correspondence? which is valid locally and beyond equilibrium.

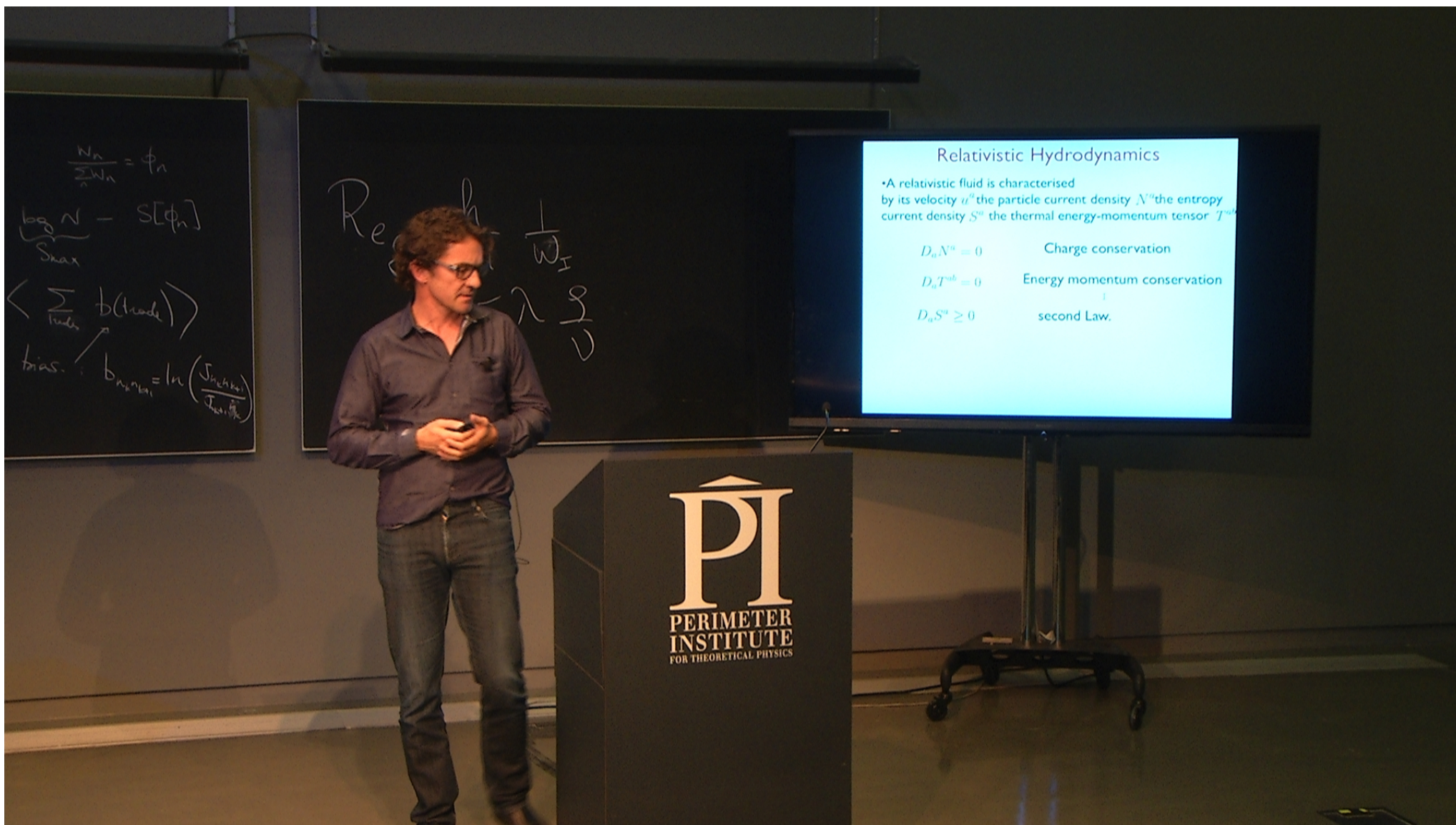
Minwalla- Hubeny

Gravity-Fluid: perturbation of large BH in AdS



λ : Mean free path

$$\text{Kn} = \frac{\lambda}{R} \ll 1 \quad \text{Hydrodynamic limit}$$



Relativistic Hydrodynamics

A relativistic fluid is characterised
 by its velocity u^a the particle current density N^a the entropy
 current density S^a the thermal energy-momentum tensor T^{ab}

$D_a N^a = 0$	Charge conservation
$D_a T^{ab} = 0$	Energy momentum conservation
$D_a S^a \geq 0$	second Law.

$$\frac{Z}{Z_{\text{max}}} = \phi_n$$

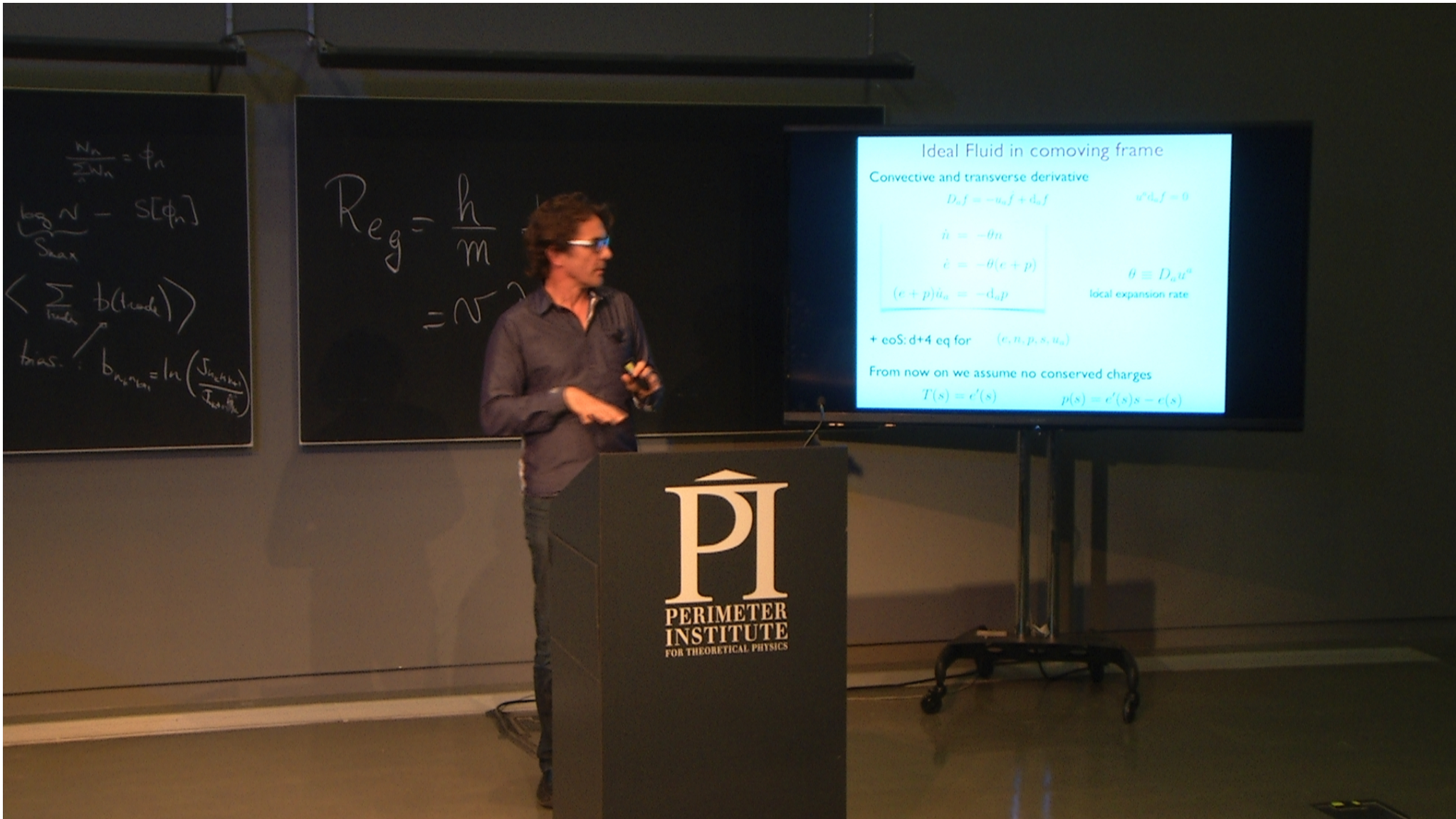
$$\log N = S[\phi_n]$$

$$\langle \sum_{\text{tree}} b(\text{tree}) \rangle$$

bias: $b_{n_1, n_2} = \ln \left(\frac{J_{n_1, n_2}}{J_{n_1, n_2}^{\text{ref}}} \right)$

$$\text{Reg } h = \frac{1}{w_I}$$

$$= \lambda \frac{g}{v}$$



$$\frac{Z}{Z_0} = \phi_n$$

$$\log \frac{Z}{Z_0} = S[\phi_n]$$

$$\left\langle \sum_{\text{tree}} b(\text{tree}) \right\rangle$$

$$\text{bias: } b_{n_1 n_2} = \ln \left(\frac{J_{n_1 n_2}}{J_{n_1} J_{n_2}} \right)$$

$$R_{eg} = \frac{h}{m}$$

$$= \sqrt{}$$

Ideal Fluid in comoving frame

Convective and transverse derivative

$$D_a f = -u_a f + d_a f \quad u^a d_a f = 0$$

$$\dot{n} = -\theta n$$

$$\dot{e} = -\theta(e + p)$$

$$(e + p)\dot{u}_a = -d_a p$$

$\theta \equiv D_a u^a$
local expansion rate

+ eos: d+4 eq for (e, n, p, s, u_a)

From now on we assume no conserved charges

$$T(s) = e'(s) \quad p(s) = e'(s)s - e(s)$$

Ideal Fluid in comoving frame

Convective and transverse derivative

$$D_a f = -u_a \dot{f} + d_a f$$

$$u^a d_a f = 0$$

$$\dot{n} = -\theta n$$

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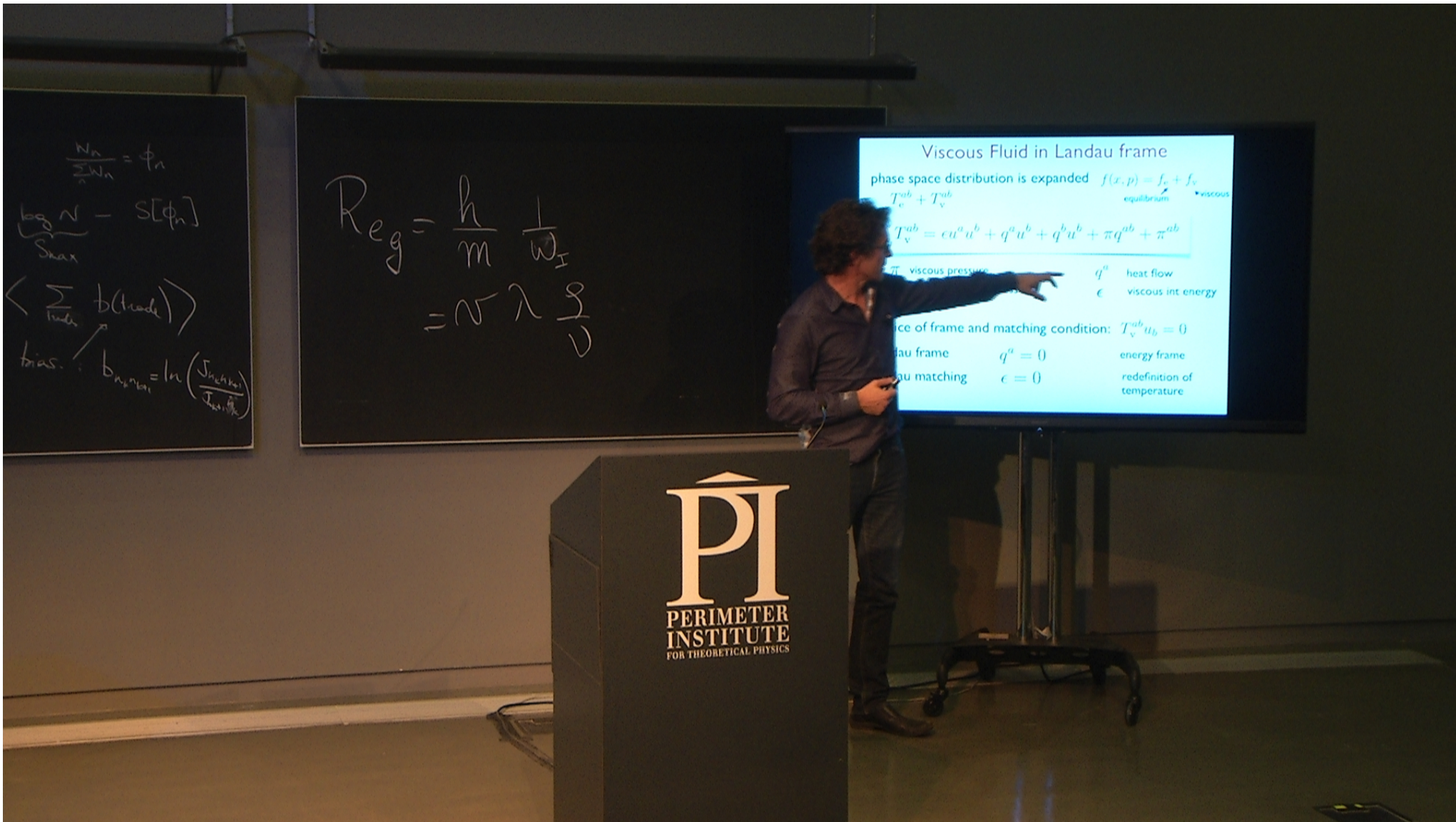
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From now on we assume no conserved charges

$$T(s) = e'(s)$$

$$p(s) = e'(s)s - e(s)$$



$$\frac{N}{N_0} = \phi_n$$

$$\log N - S[\phi_n]$$

$$\langle \sum_{i \in \text{tree}} b(i) \rangle$$

$$\text{bias: } b_{n_1, n_2} = \ln \left(\frac{J_{n_1, n_2}}{J_{n_1, n_2}^0} \right)$$

$$R_{eg} = \frac{h}{m} \frac{1}{w_I}$$

$$= N \lambda \frac{g}{v}$$

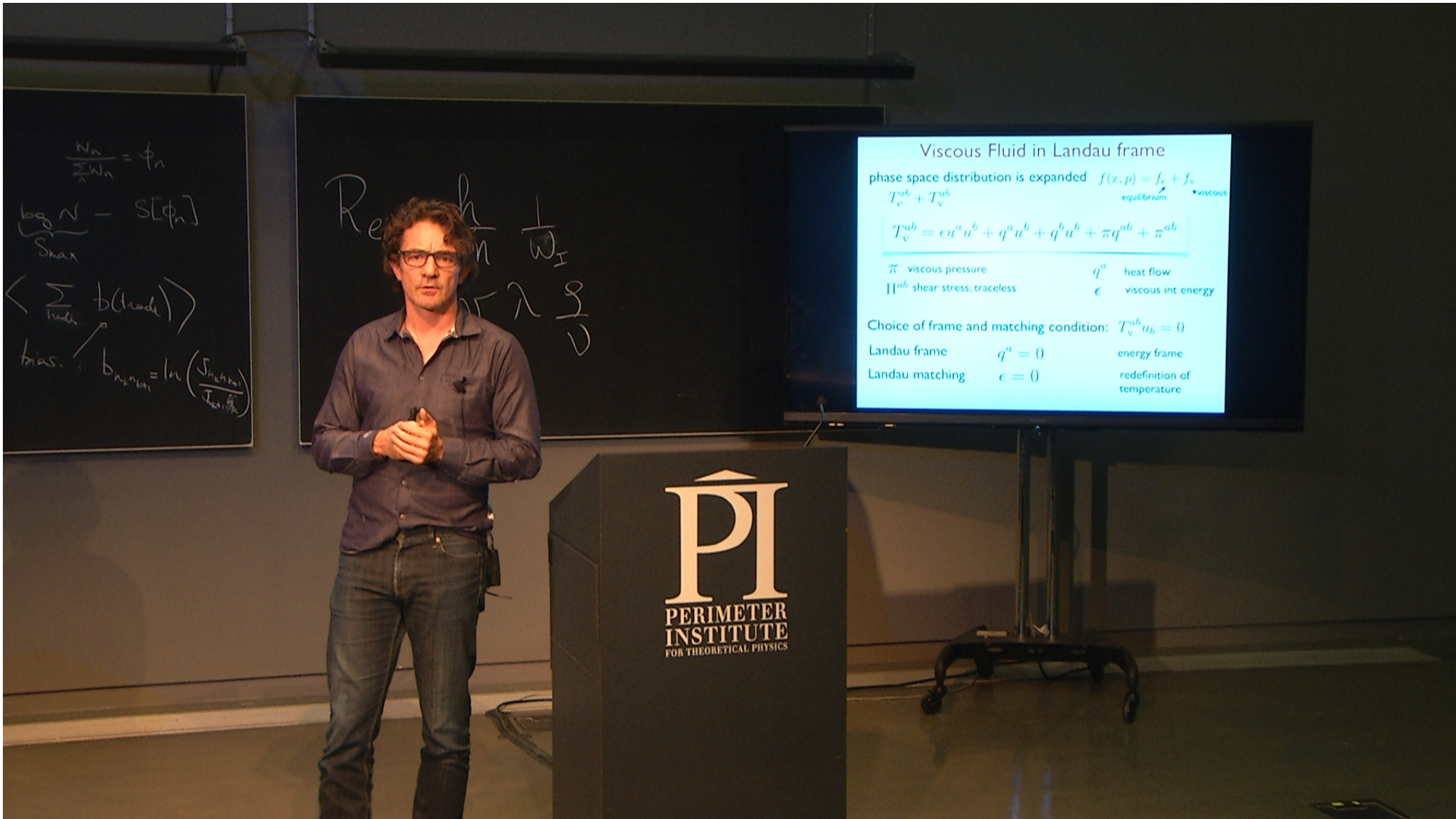
Viscous Fluid in Landau frame

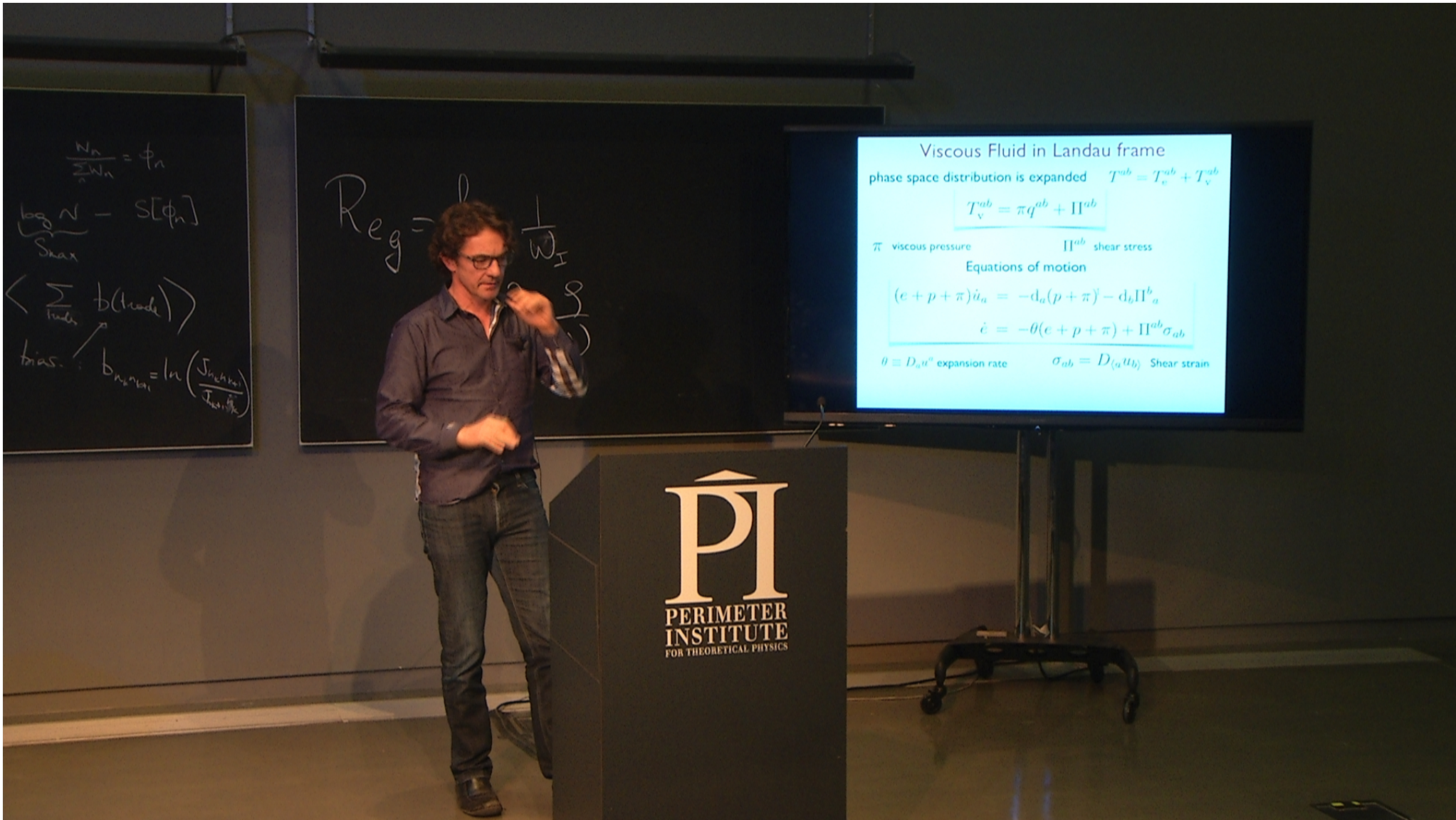
phase space distribution is expanded $f(x, p) = f_e + f_v$
 $T_e^{ab} + T_v^{ab}$
 $T_v^{ab} = \epsilon u^a u^b + q^a u^b + q^b u^a + \pi q^{ab} + \pi^{ab}$

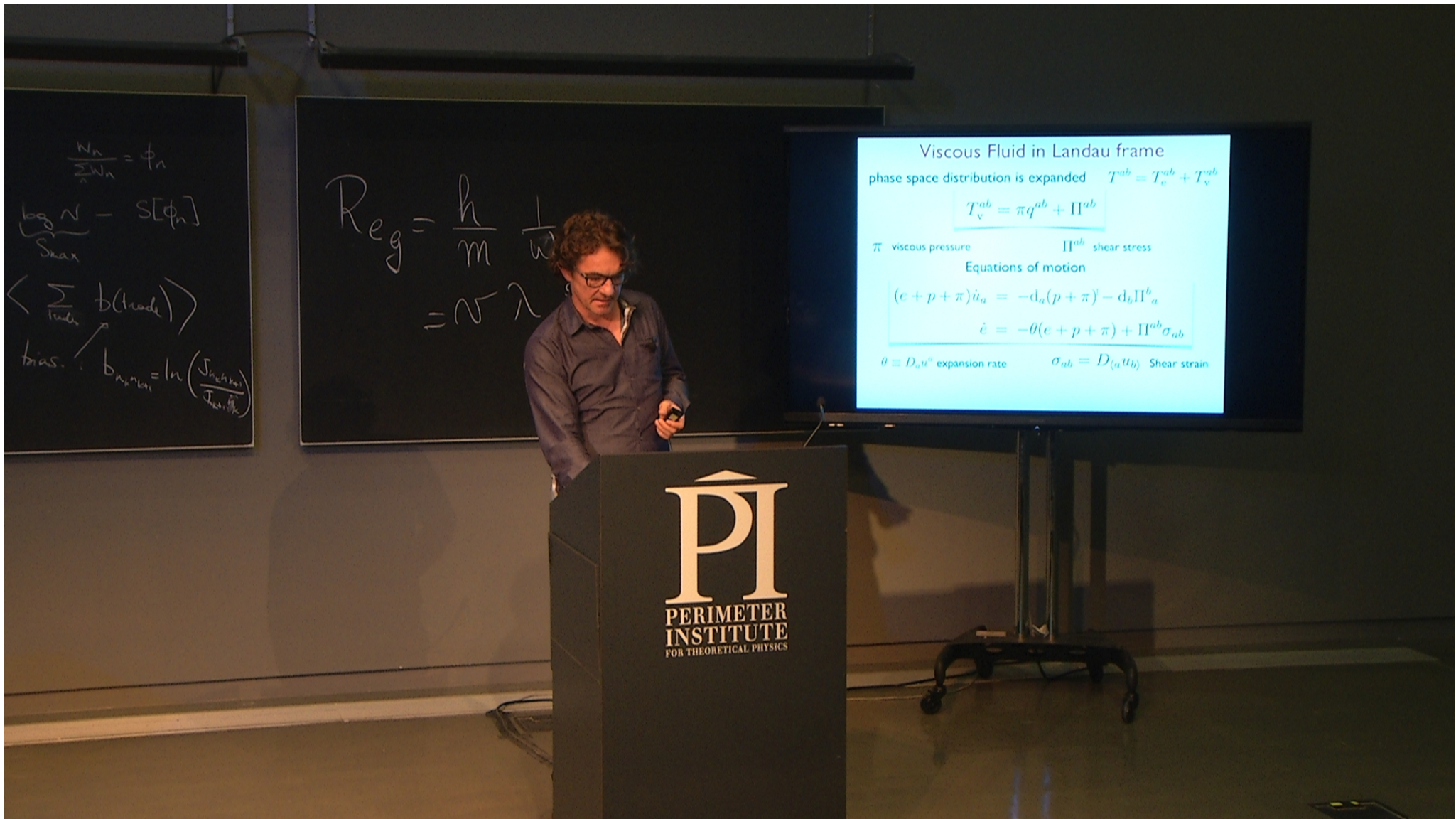
π viscous pressure	q^a heat flow
	ϵ viscous int energy

choice of frame and matching condition: $T_v^{ab} u_b = 0$

Landau frame	$q^a = 0$	energy frame
Landau matching	$\epsilon = 0$	redefinition of temperature







$$\frac{N}{N_0} = \phi_n$$

$$\log N - S[\phi_n]$$

$$\langle \sum_{i \in \text{tree}} b(i) \rangle$$

$$\text{bias: } b_{n_1, n_2} = \ln \left(\frac{J_{n_1, n_2}}{J_{n_1, 0} J_{n_2, 0}} \right)$$

$$R_{eg} = \frac{h}{m} \frac{1}{w}$$

$$= N \lambda$$

Viscous Fluid in Landau frame

phase space distribution is expanded $T^{ab} = T_e^{ab} + T_v^{ab}$

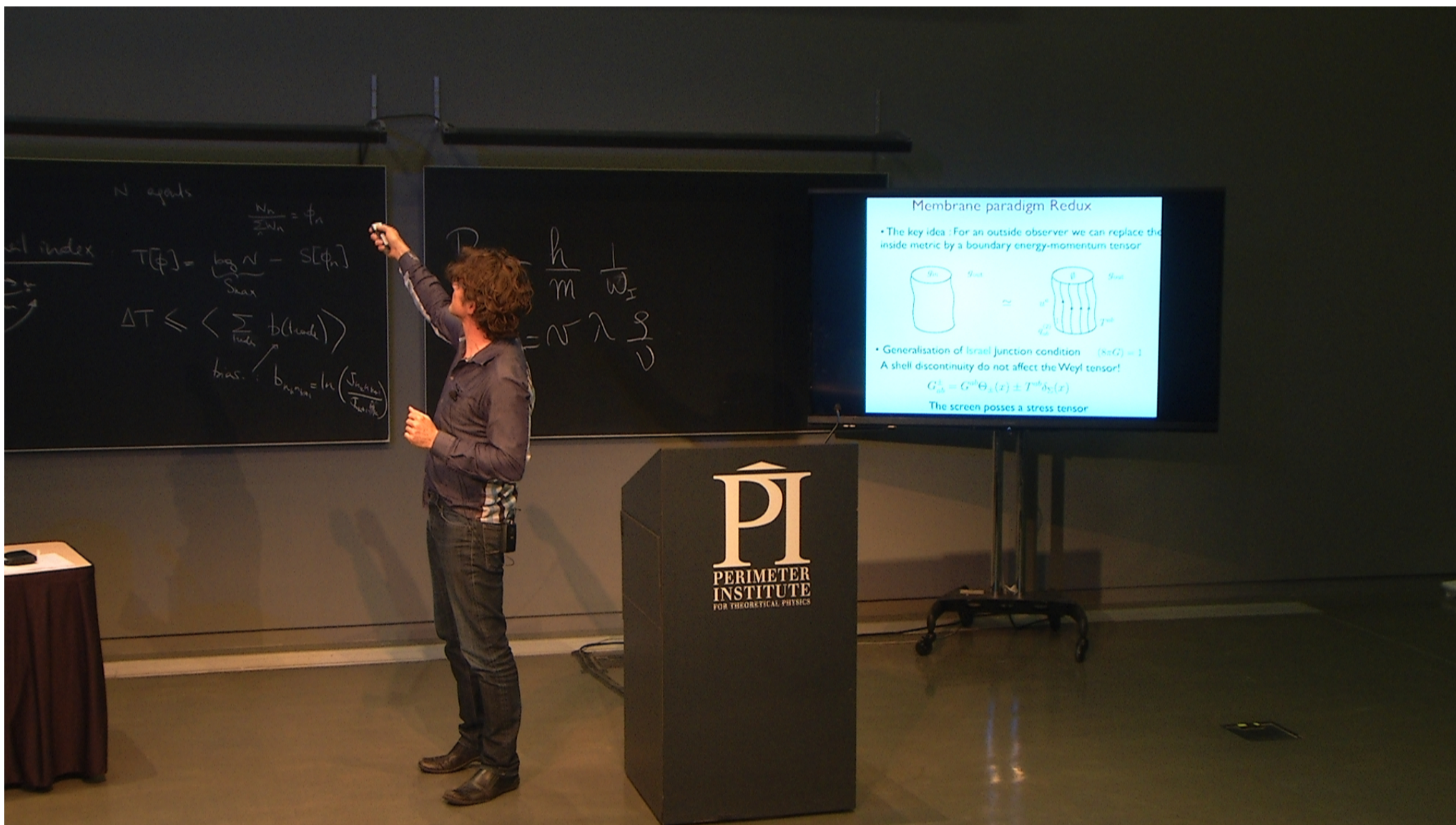
$T_v^{ab} = \pi q^{ab} + \Pi^{ab}$

π viscous pressure Π^{ab} shear stress

Equations of motion

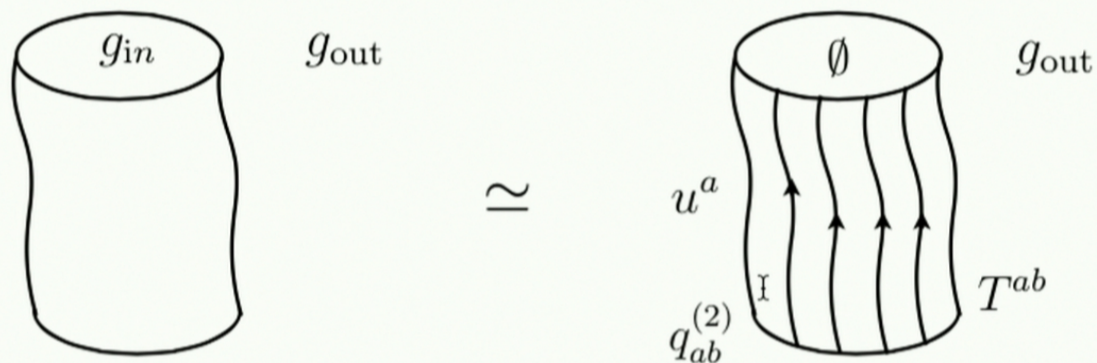
$(e + p + \pi) \dot{u}_a = -d_a(p + \pi) - d_b \Pi^b_a$
 $\dot{e} = -\theta(e + p + \pi) + \Pi^{ab} \sigma_{ab}$

$\theta \equiv D_a u^a$ expansion rate $\sigma_{ab} \equiv D_{(a} u_{b)}$ Shear strain



Membrane paradigm Redux

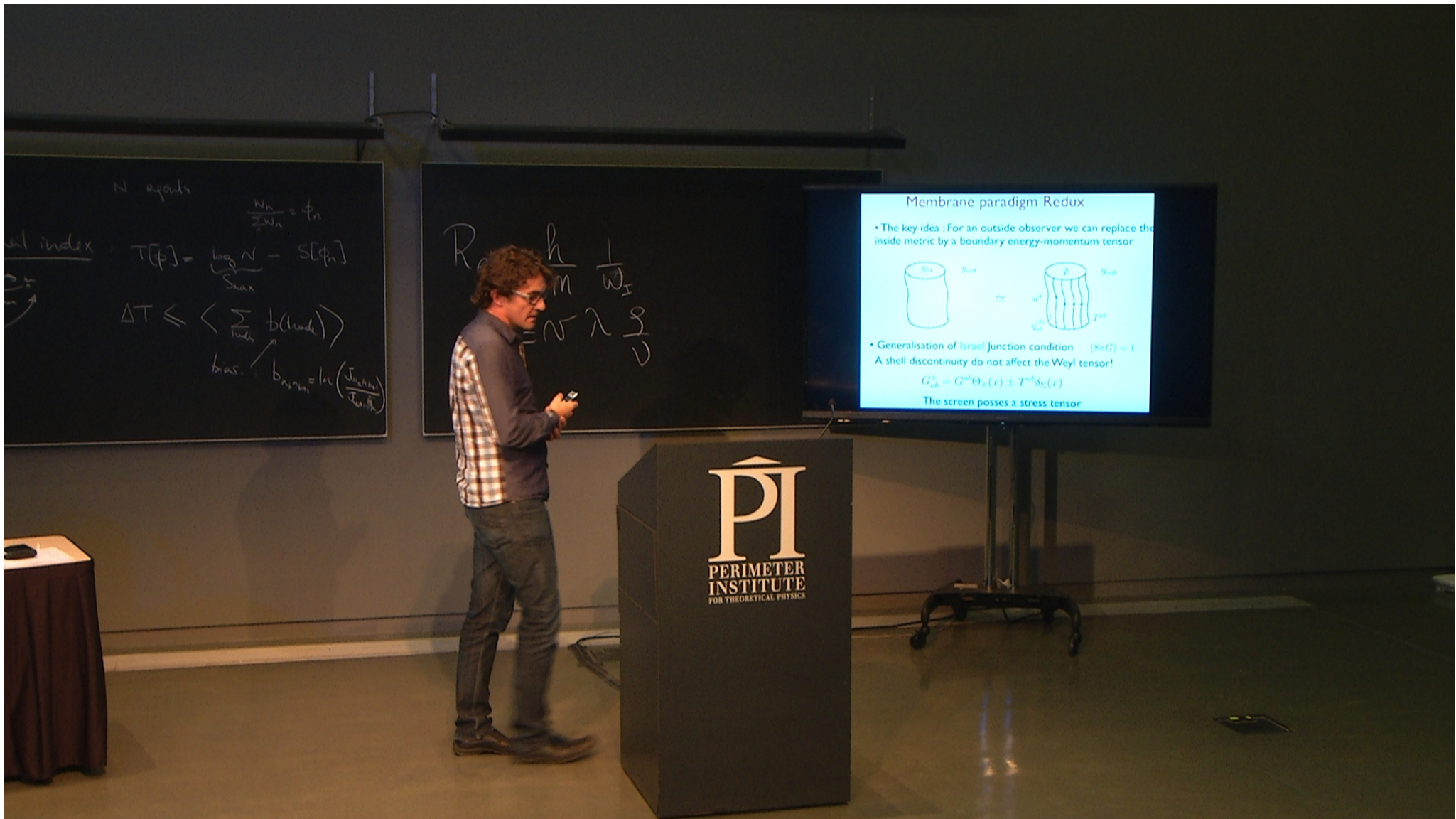
- The key idea : For an outside observer we can replace the inside metric by a **boundary energy-momentum** tensor



- Generalisation of **Israel** Junction condition $(8\pi G) = 1$
A shell discontinuity do not affect the Weyl tensor!

$$G_{ab}^{\pm} = G^{ab} \Theta_{\pm}(x) \pm T^{ab} \delta_{\Sigma}(x)$$

The screen posses a stress tensor



N equals $\frac{N_n}{\frac{1}{2}W_n} = \frac{1}{2}n$

al index : $T(\phi) = \frac{\log N}{S_{\text{max}}} - S[\phi_n]$

$\Delta T \leq \left\langle \sum_{\text{index}} b(\text{index}) \right\rangle$

trans. : $b_{n,n+1} = \ln \left(\frac{J_{n,n+1}}{J_{n,n}} \right)$

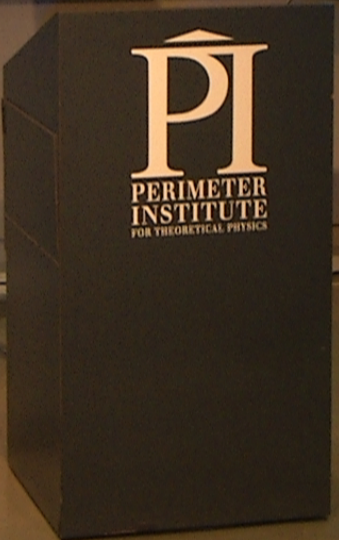
$R = \frac{h}{m} \frac{1}{\omega_I}$

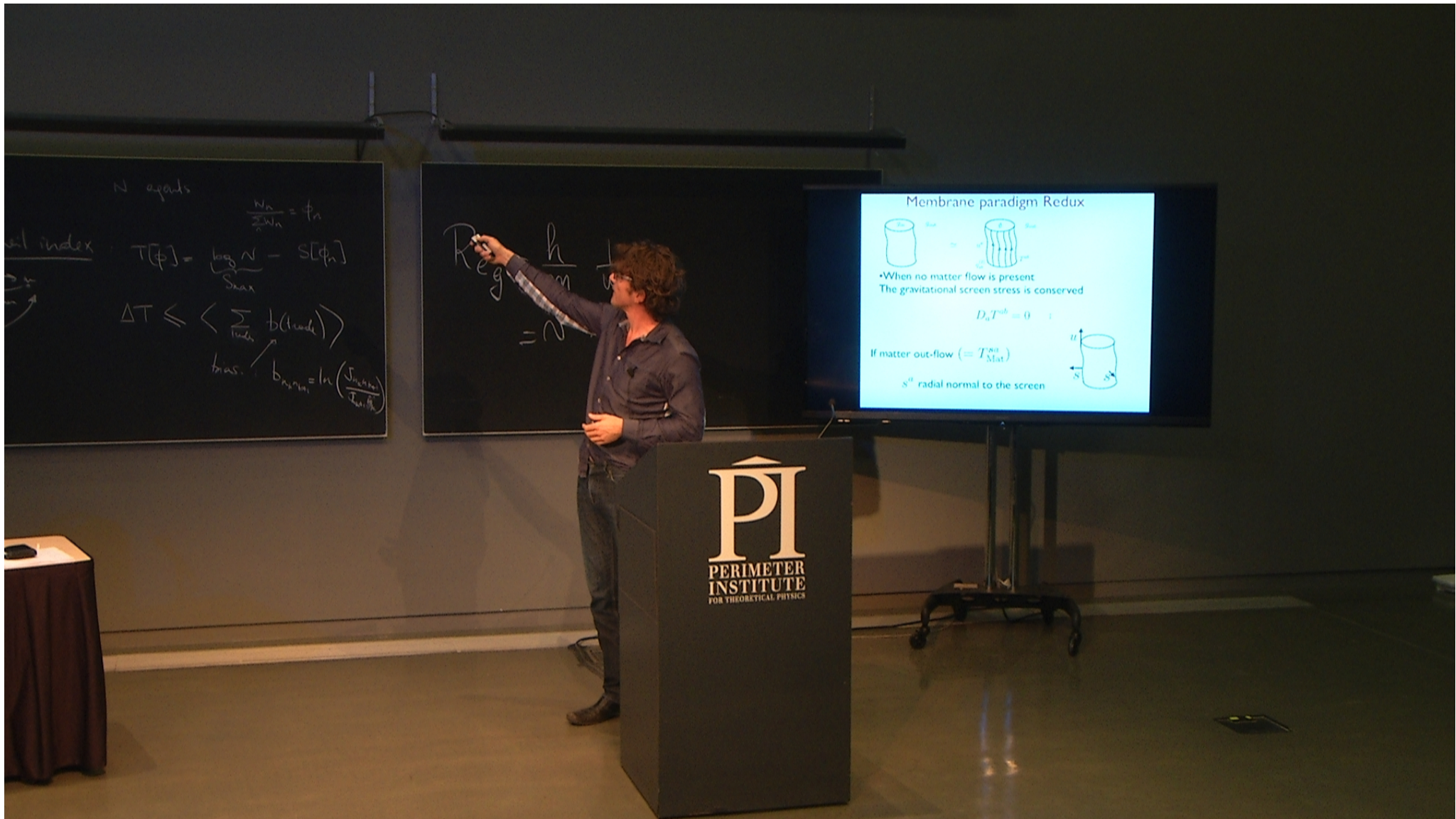
$= N \lambda \frac{g}{v}$

Membrane paradigm Redux

- The key idea : For an outside observer we can replace the inside metric by a boundary energy-momentum tensor

- Generalisation of Israel junction condition $(S \cdot G) = 1$
A shell discontinuity do not affect the Weyl tensor!
 $G_{ab}^+ = G_{ab}^- \Theta_\Sigma(x) \pm T_{ab}^\Sigma \delta_\Sigma(x)$
The screen posses a stress tensor





N eqals

$$\frac{N_n}{\sum W_n} = \phi_n$$

al index: $T(\phi) = \log \frac{N}{S_{max}} - S[\phi_n]$

$$\Delta T \leq \left\langle \sum_{i \in \text{index}} b(i_{\text{index}}) \right\rangle$$

trans. $b_{n_{\text{index}}} = \ln \left(\frac{J_{n_{\text{index}}}}{J_{\text{min}}(h)} \right)$

$$R_{\text{eg}} = \frac{h}{m} \frac{1}{w} = N$$

Membrane paradigm Redux



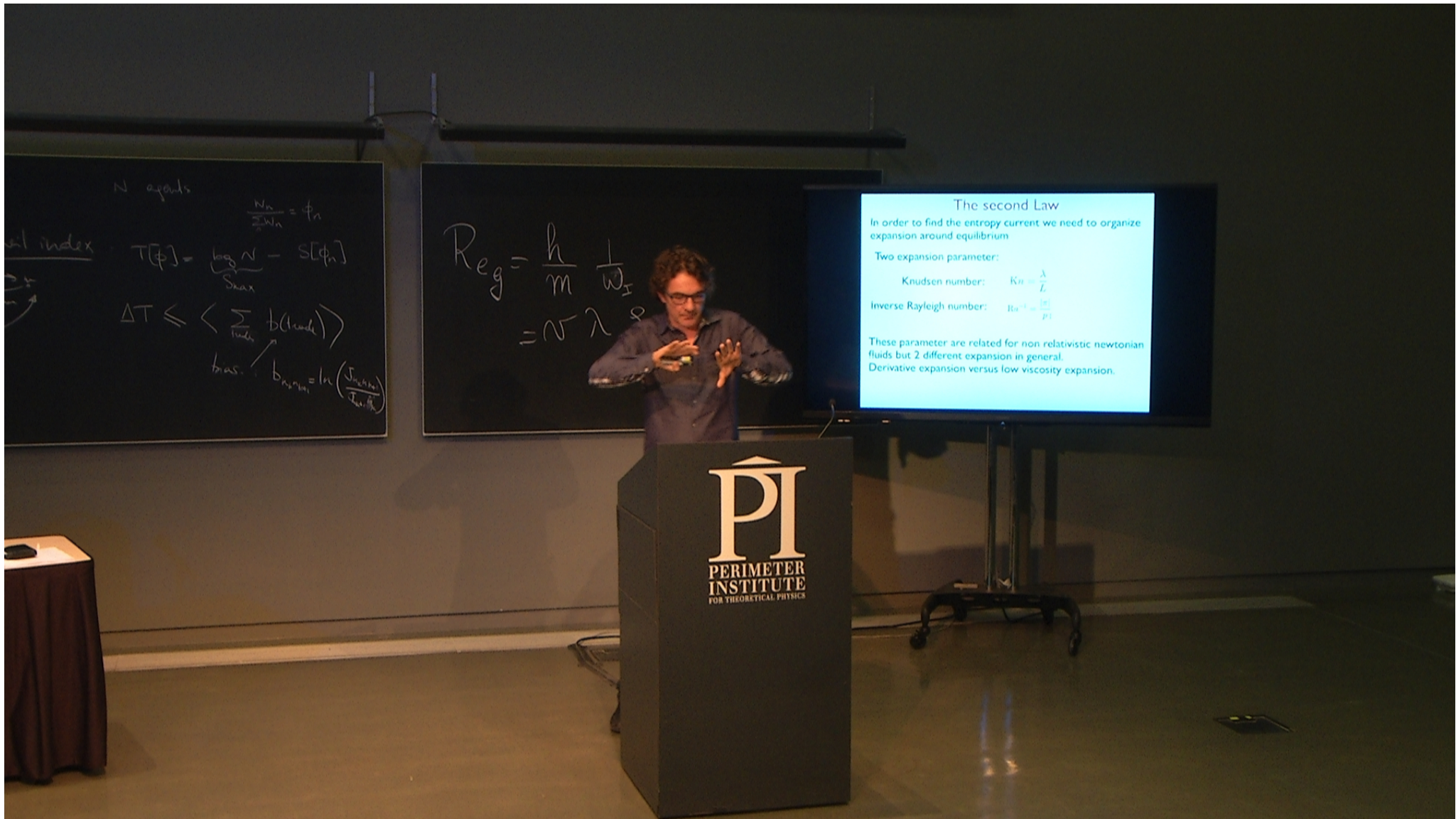
• When no matter flow is present
The gravitational screen stress is conserved

$$D_a T^{ab} = 0$$

If matter out-flow ($= T^{aa}_{\text{Mat}}$)

s^a radial normal to the screen





N eqs

$\frac{N_n}{\sum_n N_n} = \phi_n$

at index: $T(\phi) = \log N - S[\phi_n]$

$\Delta T \leq \langle \sum_{\text{index}} b(\text{index}) \rangle$

trans. $b_{n,n+1} = \ln \left(\frac{J_{n,n+1}}{J_{n,n}} \right)$

$$Re_g = \frac{h}{m} \frac{1}{w_I} = N \lambda^3$$

The second Law

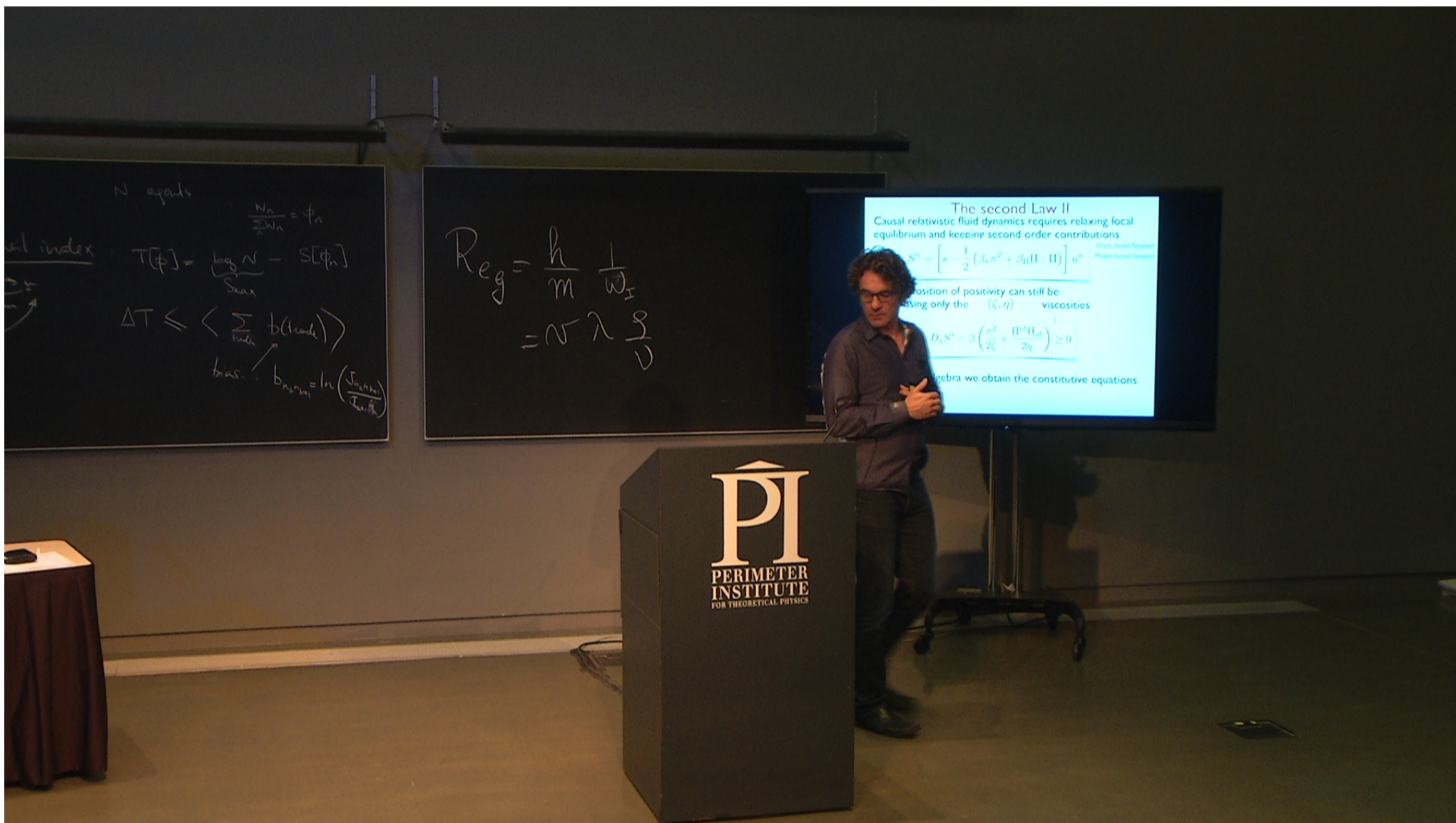
In order to find the entropy current we need to organize expansion around equilibrium

Two expansion parameter:

Knudsen number: $Kn = \frac{\lambda}{L}$

Inverse Rayleigh number: $Ra^{-1} = \frac{\eta}{\rho L}$

These parameter are related for non relativistic newtonian fluids but 2 different expansion in general.
Derivative expansion versus low viscosity expansion.



The second Law II

Causal relativistic fluid dynamics requires relaxing local equilibrium and keeping second order contributions

$S^\alpha = \left[s - \frac{1}{2} (\beta_\pi \pi^2 + \beta_\Pi (\Pi \cdot \Pi)) \right] u^\alpha$

Position of positivity can still be relaxing only the (ζ, η) viscosities

$D_\mu S^\alpha = \beta \left(\frac{\pi^2}{2\zeta} + \frac{\Pi^\alpha \Pi_\alpha}{2\eta} \right) \geq 0$

Algebra we obtain the constitutive equations

N equals

$\frac{N_n}{\sum_n W_n} = \phi_n$

al index: $T(\phi) = \frac{\log N}{S_{\max}} - S[\phi_n]$

$\Delta T \leq \left\langle \sum_{\text{index}} b(\text{index}) \right\rangle$

trans: $b_{n_{\text{max}}+1} = \ln \left(\frac{S_{\max} n_{\text{max}}}{S_{\max} \phi_n} \right)$

$$R_{eg} = \frac{h}{m} \frac{1}{\omega_I}$$

$$= N \lambda \frac{g}{v}$$



The second Law II

Causal relativistic fluid dynamics requires relaxing local equilibrium and keeping second order contributions

$$S^a = \left[s - \frac{1}{2} (\beta_\pi \pi^2 + \beta_\Pi \Pi : \Pi) \right] u^a$$

Grad- Israel-Stewart
Muller-Israel-Stewart

The imposition of positivity can still be done using only the (ζ, η) viscosities

$$D_a S^a = \beta \left(\frac{\pi^2}{2\zeta} + \frac{\Pi^{ab} \Pi_{ab}}{2\eta} \right) \geq 0$$

After some algebra we obtain the constitutive equations

$$R_{eg} = \frac{h}{m} \frac{1}{w_I}$$

$$= \sqrt{\lambda} \frac{g}{v}$$

The second Law and Gravity

Remarkably there exist a choice of coefficients for which the constitutive equation for bulk pressure is equal to a component of Einstein equation

$$\tau_\pi \left(\dot{\pi} + \frac{1}{2} \theta \pi \right) - \lambda \Pi : \sigma = \alpha_\pi \pi$$

$$\lambda = -\tau_\pi/2$$

$$\alpha_\pi = 0$$

Constitutive equation for π determines the rate of in falling gravity waves

$$W_{sA}$$

Constitutive laws for π determine



$$R_{eg} = \frac{h}{m} \frac{1}{\omega_I}$$

$$= N \lambda \frac{g}{v}$$

Conclusion

- Profound correspondence between gravity and non equilibrium thermo: Membrane paradigm redux.
- A gravitational screen can be a viscous relativistic fluid
- Key idea to describe a screen: use a low viscosity expansion in the derivative expansion. Very different from a fluid.
- Remarkably the truncated second law to second order is consistent with what appears as a low viscosity expansion of a Einstein GR = near eq.
- It suggest that the equations of gravity corresponds to fluid mechanics.
- Is it useful?



