Title: Economic Inequality

Date: Oct 23, 2014 02:45 PM

URL: http://pirsa.org/14100109

Abstract:


$400 = 100,000,000$
There is an elephant in the room: the richest 400 A
ricans own
as much as the 100,000,000 poorest.
Suncus etruscus
Matteo Smerlak
Thermodynamics
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400 = 100, 000, 000
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Suncus etruscus
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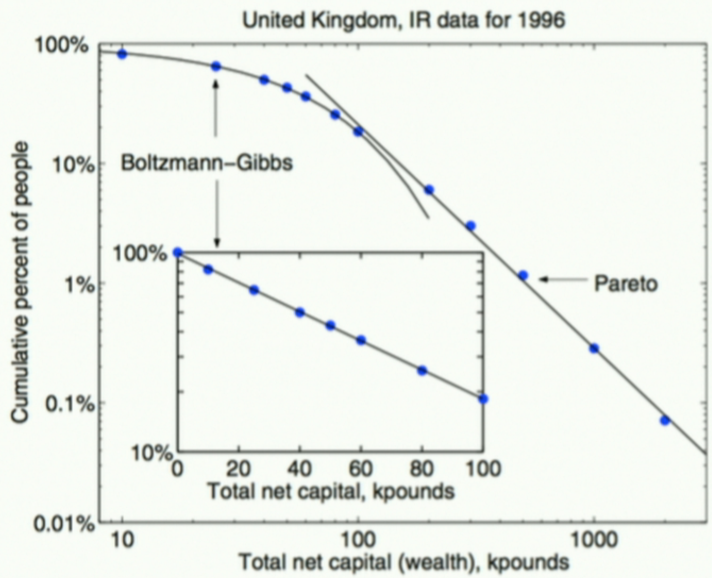
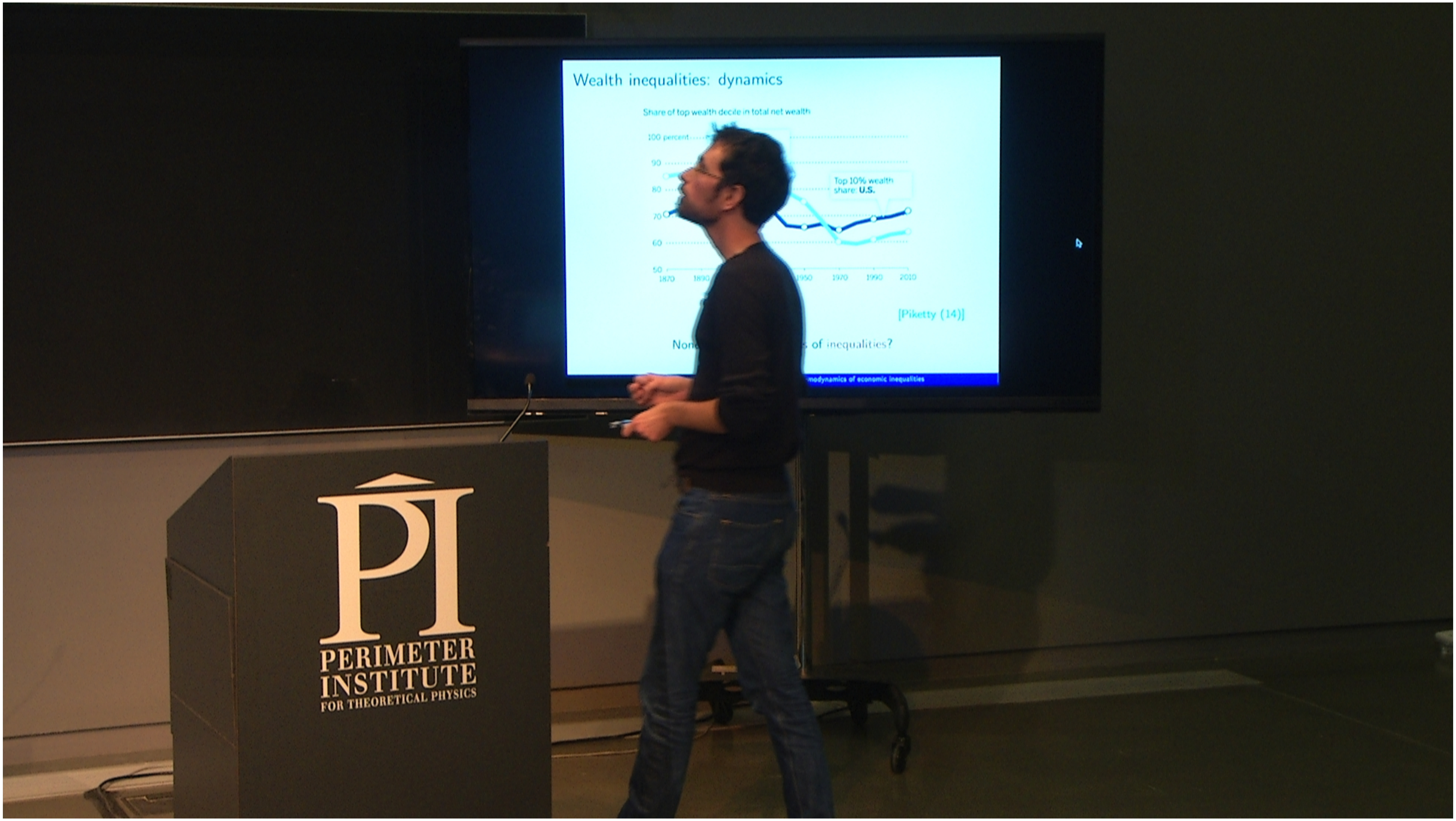
Wealth inequalities: structure



[Dragulescu, Yakovenko (01)]

Easy to understand: capital returns are multiplicative!


Wealth inequalities: dynamics
Share of top wealth decile in total net wealth
100 percent
90
80
70
60
50
1870
1890
1950
1970
1990
2010
Top 10% wealth share: U.S.
[Piketty (14)]
of inequalities?
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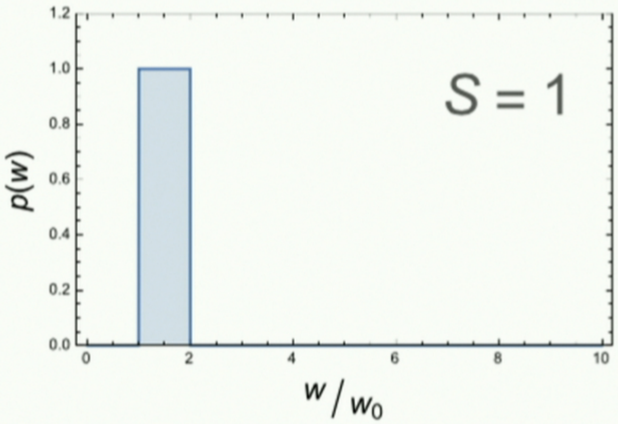
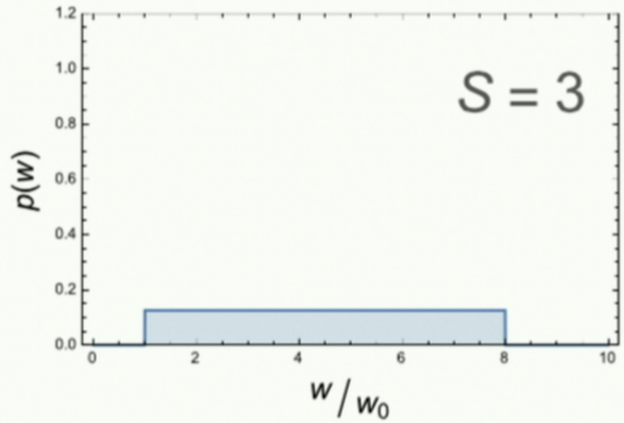
Stratification index

The statistical entropy

$$S = -\int dw\, p(w) \log_2 [w_0 p(w)]$$

counts the number of strata $[2^i w_0, 2^{i+1} w_0]$ in the distribution.





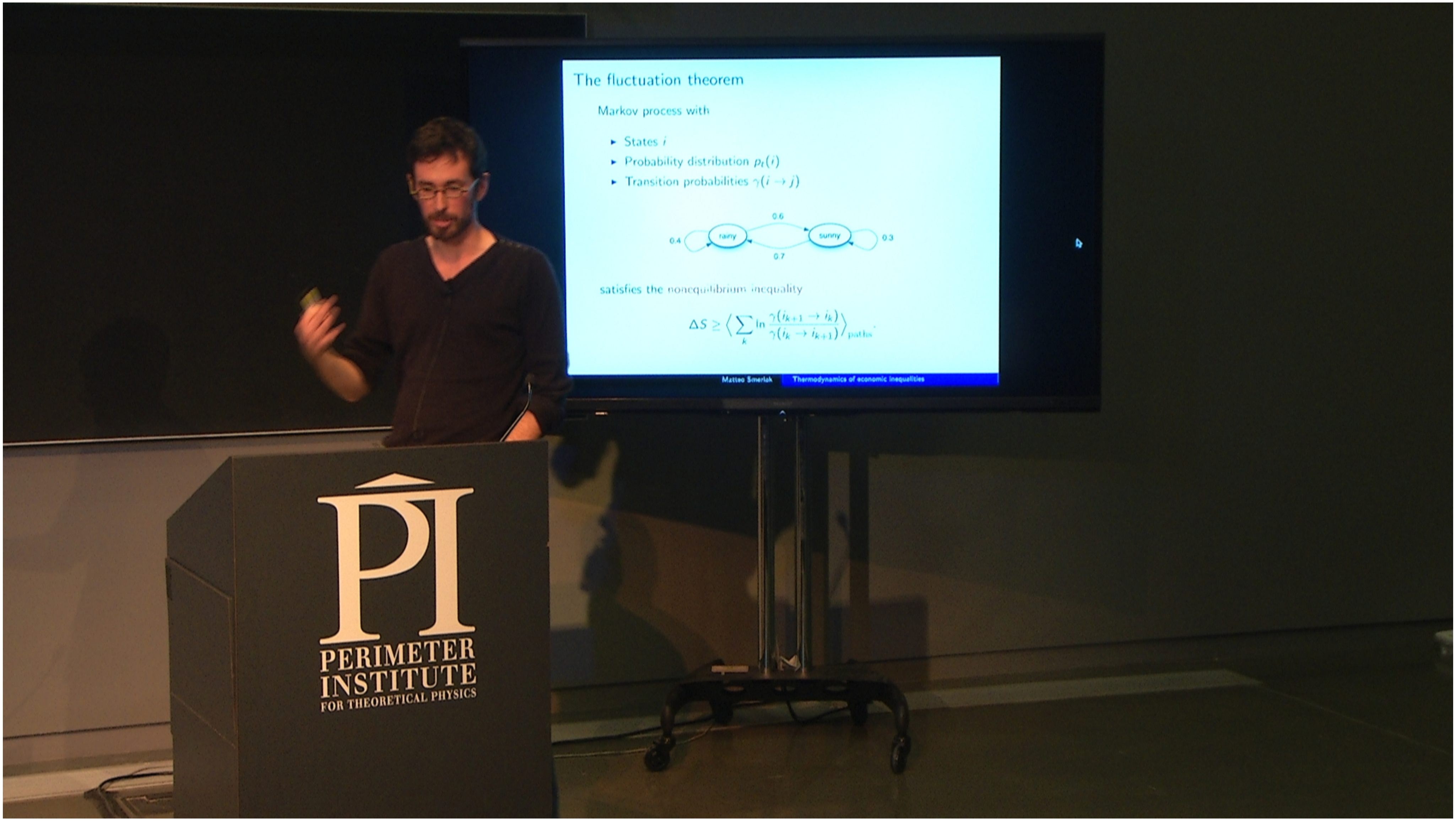
Second law of thermodynamics

In physics we know that, for an open system,

$$\Delta S \geq \beta Q.$$

This identifies one dynamical causes for entropy growth— heat fluxes.

What is (heat)/(temperature) for stratification?


The fluctuation theorem
Markov process with
States i
Probability distribution $p_t(i)$
Transition probabilities $\gamma(i \to j)$
0.6
0.4
rainy
sunny
0.3
0.7
satisfies the nonequilibrium inequality
$\Delta S \geq \left\langle \sum_k \ln \frac{\gamma(i_{k+1} \to i_k)}{\gamma(i_k \to i_{k+1})} \right\rangle_{\text{paths}}.$
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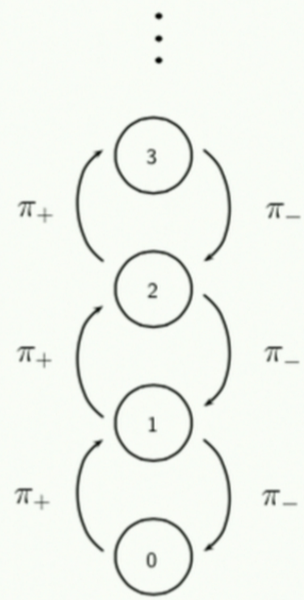
Social ladder model I

Assume

- A discrete number of "classes" k
- Prob. to go one step up the ladder π_+
- Prob. to go one step down the ladder π_-



We have a "second law of social ladders":

$$\Delta S \geq \underbrace{\ln(\pi_-/\pi_+)}_{\text{precariousness}} \underbrace{\langle \Delta k \rangle}_{\text{mobility}}$$

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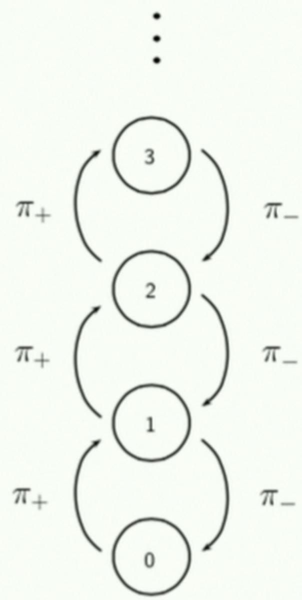

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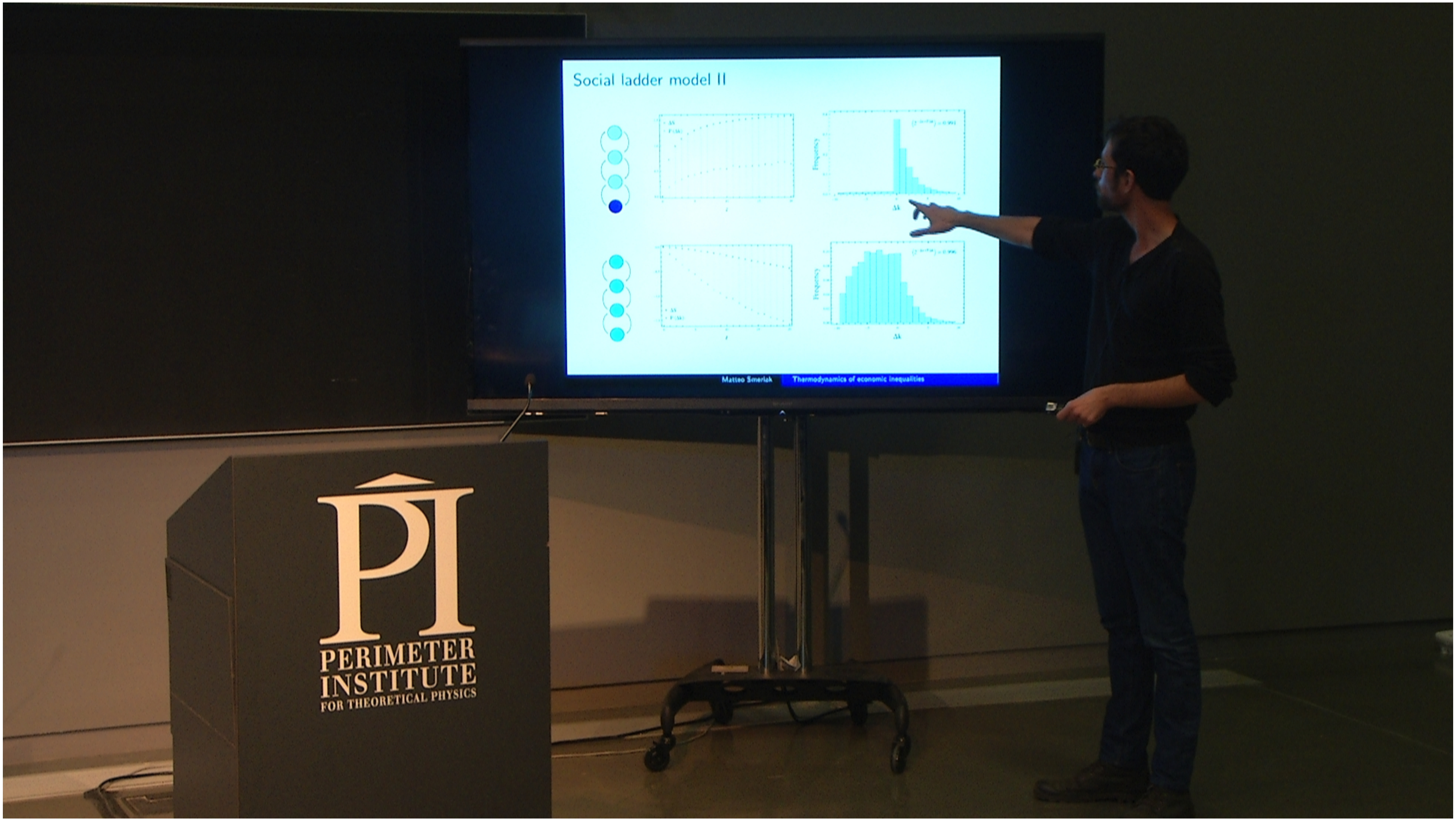
Social ladder model II
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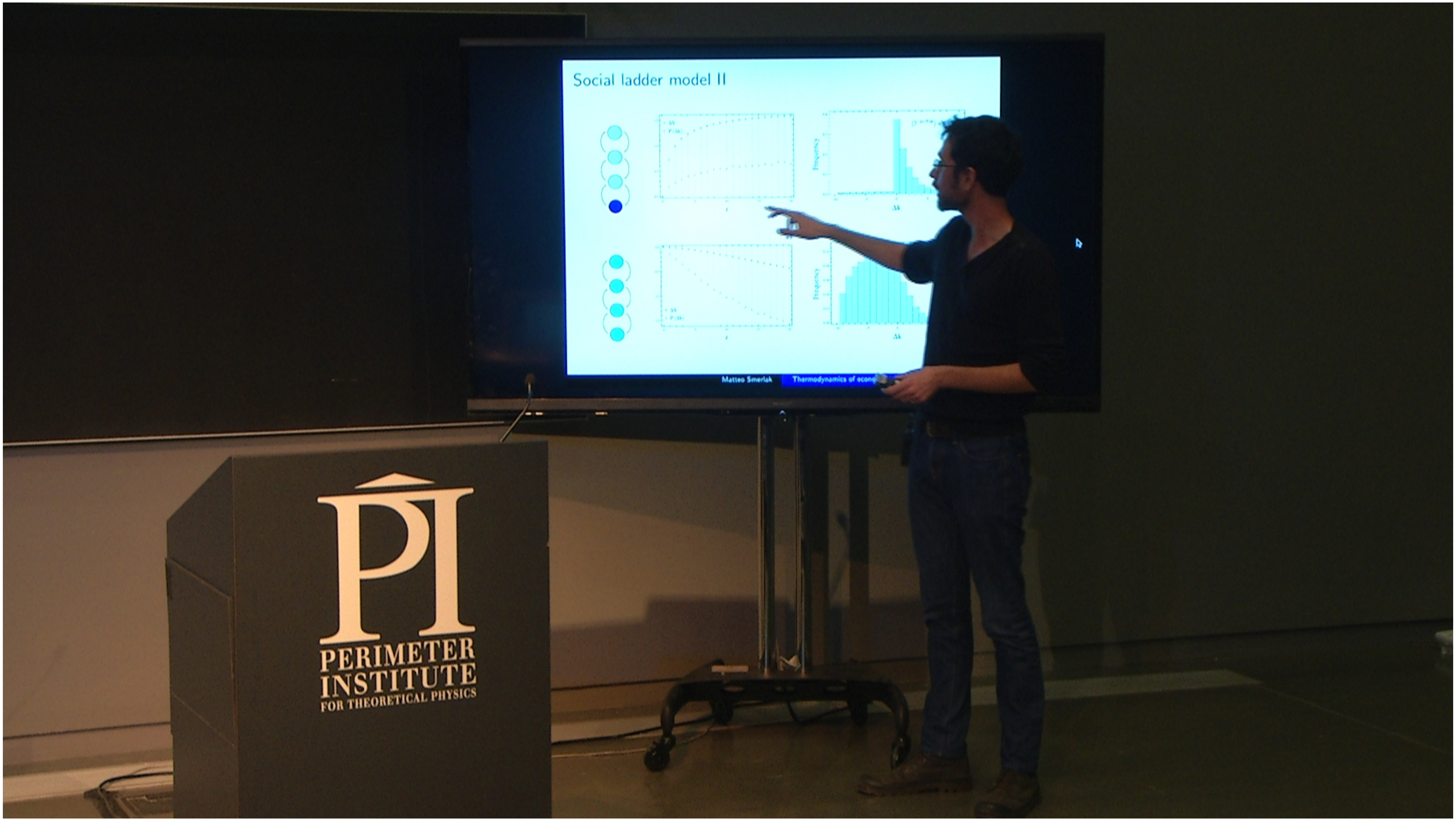
Social ladder model II
Frequency
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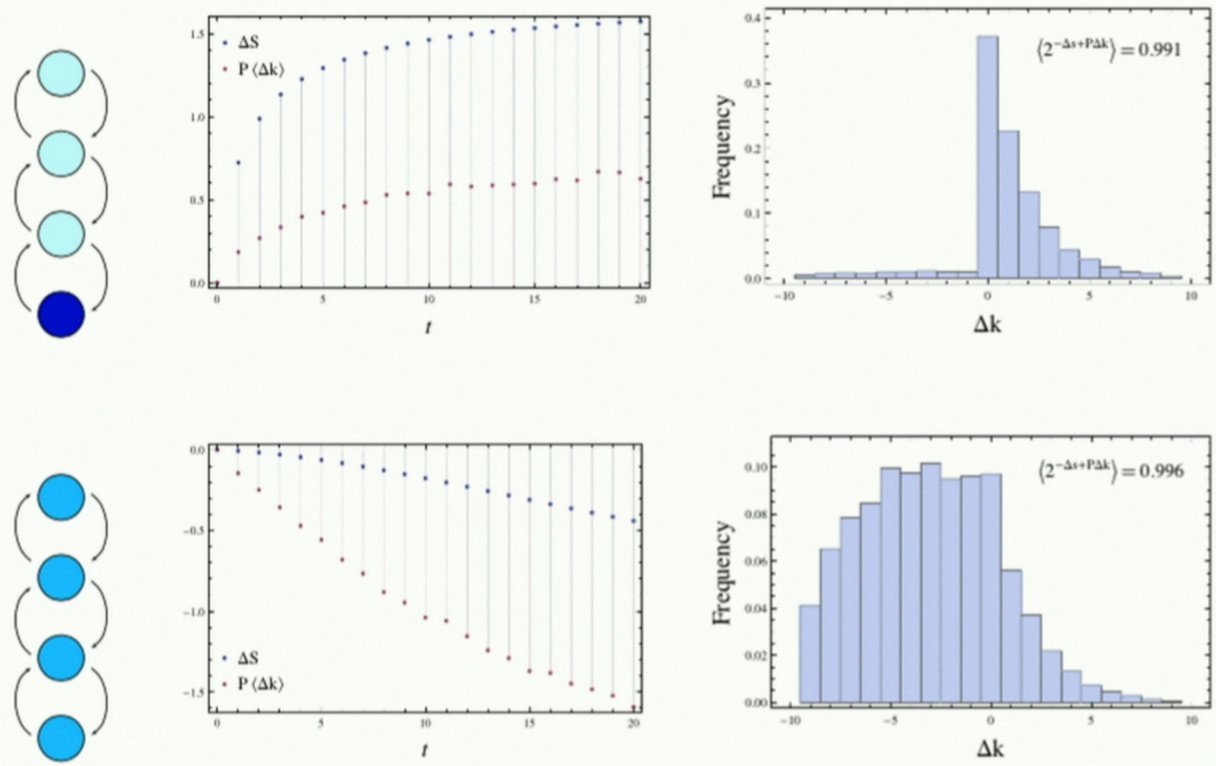
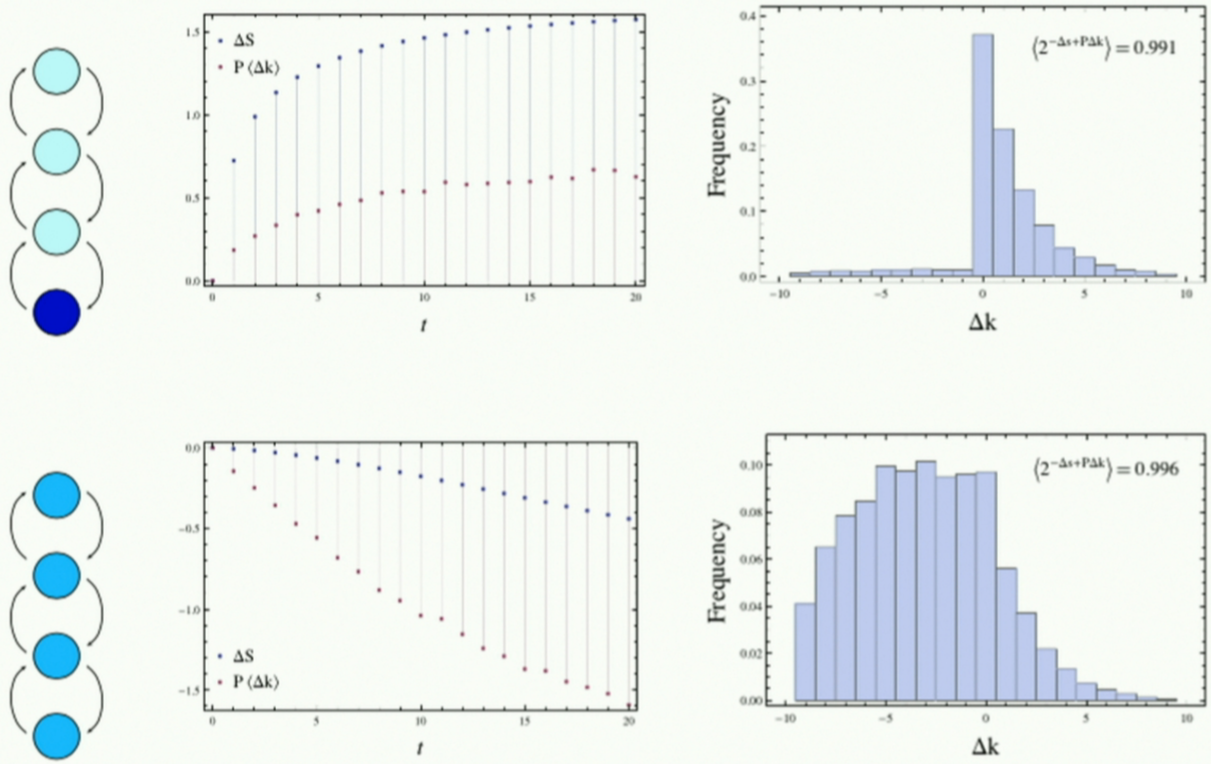


Social ladder model II



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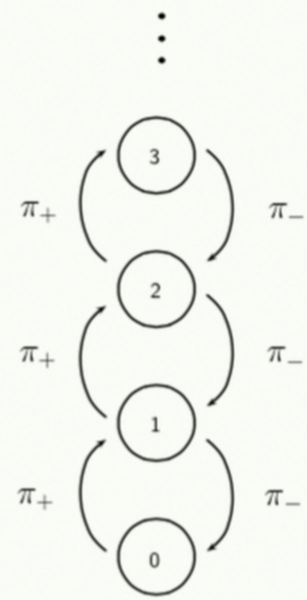

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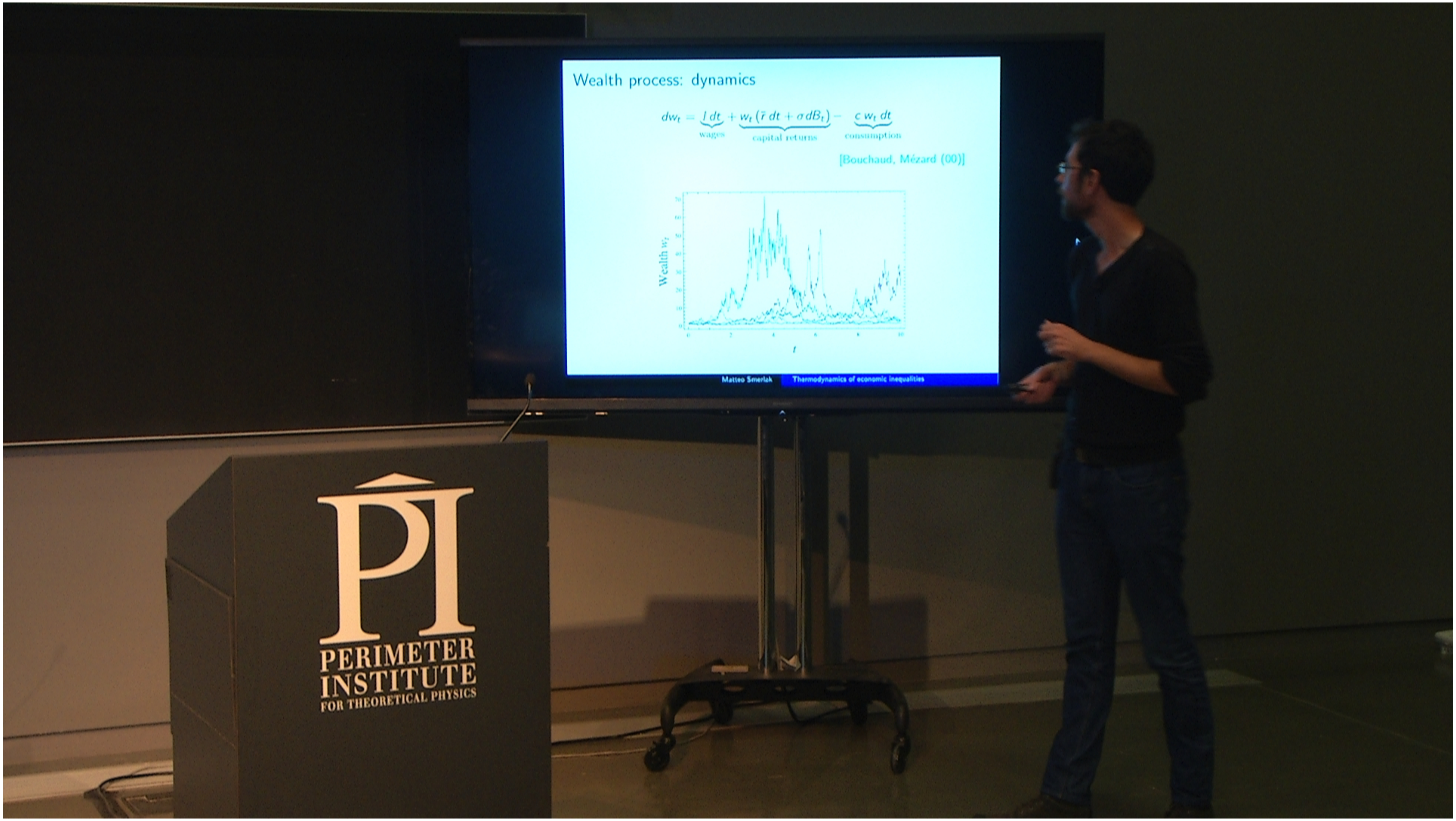

Wealth process: dynamics
$dw_t = \underbrace{l\,dt}_{\text{wages}} + \underbrace{w_t\,(\bar{r}\,dt + \sigma\,dB_t)}_{\text{capital returns}} - \underbrace{c\,w_t\,dt}_{\text{consumption}}$
[Bouchaud, Mézard (00)]
Wealth w_t
t
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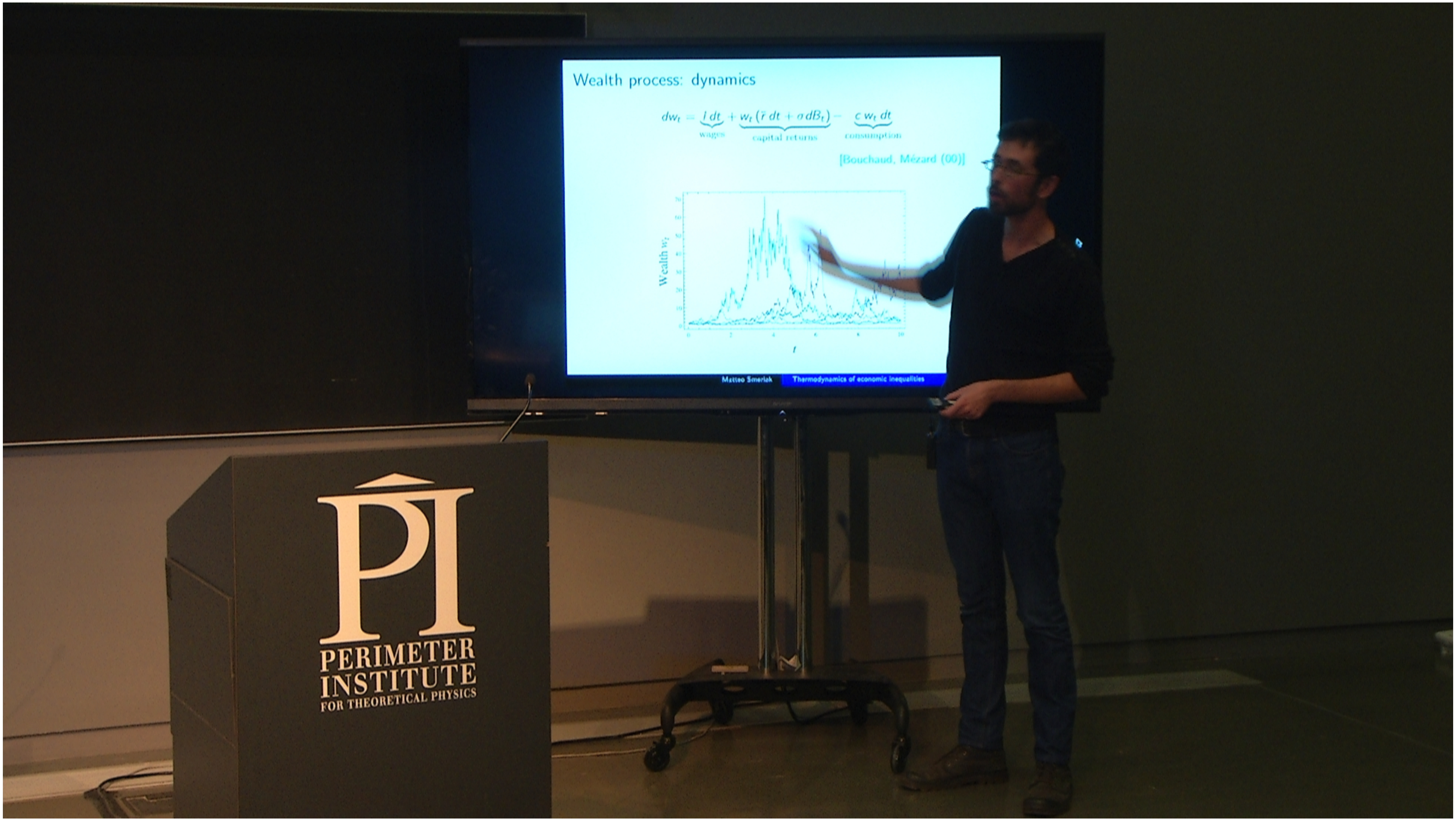
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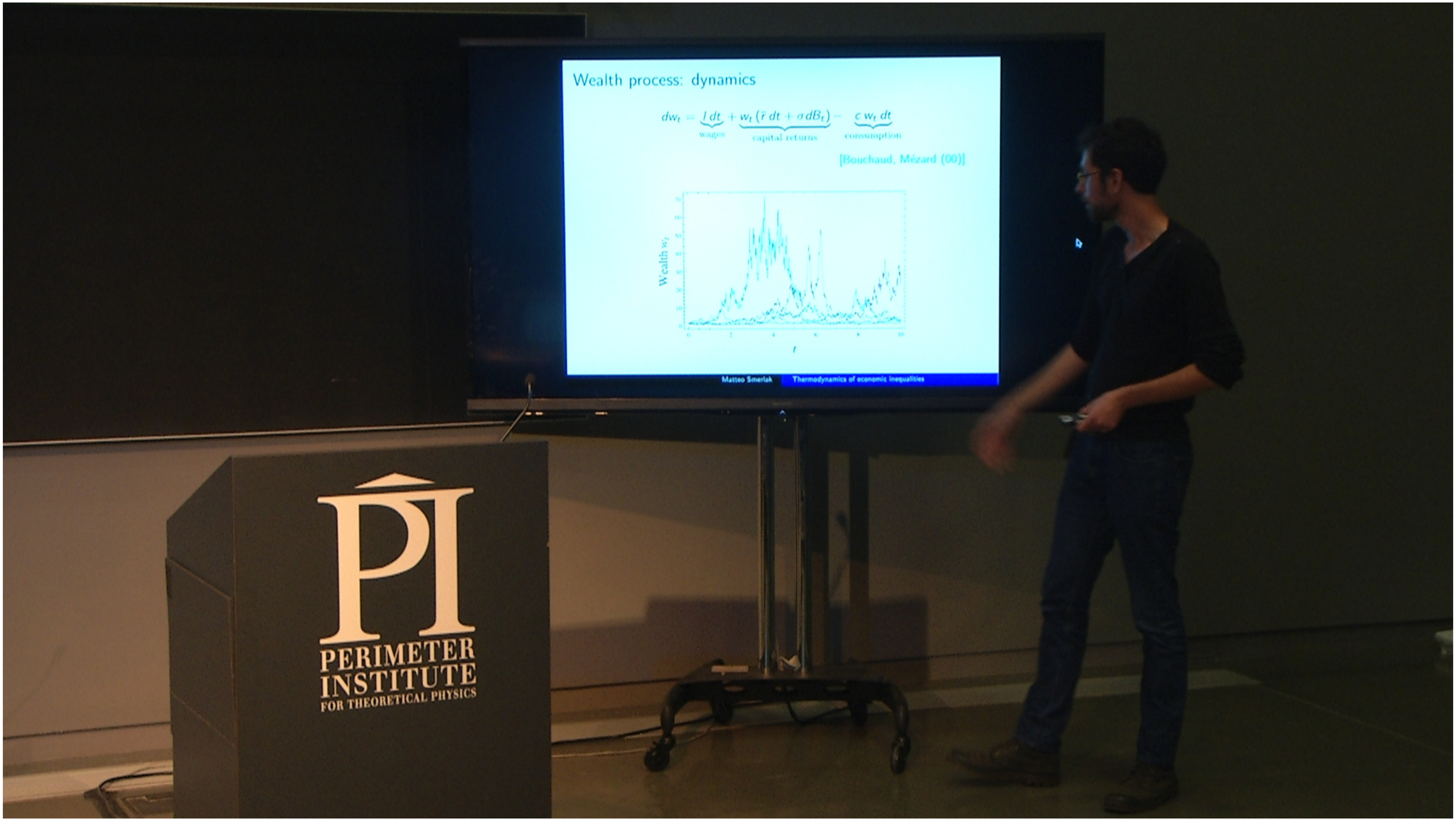

Wealth process: dynamics
$dw_t = \underbrace{I\,dt}_{\text{wages}} + \underbrace{w_t\,(\bar{r}\,dt + \sigma\,dB_t)}_{\text{capital returns}} - \underbrace{c\,w_t\,dt}_{\text{consumption}}$
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Wealth w_t
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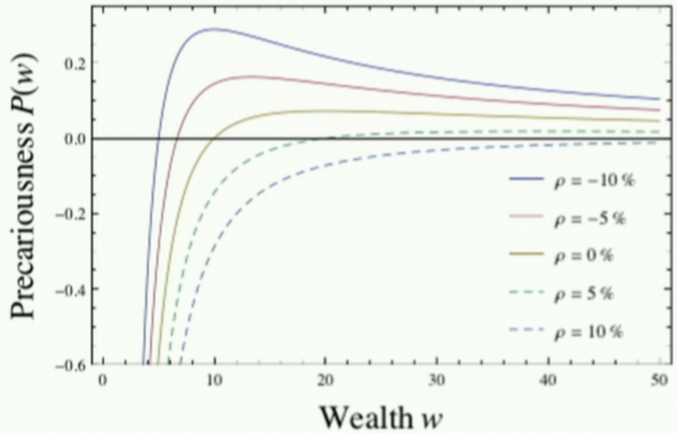
Wealth process: second law

Second law for the wealth process:

$$\frac{dS_t}{dt} \geq \int P(w)\, j_t(w)\, dw$$

where the precariousness function changes sign at $w = I/(\sigma^2 - \rho)$.



$$P(w) = \frac{2}{\ln 2}\left(\frac{\sigma^2 - \rho}{\sigma^2 w} - \frac{I}{\sigma^2 w^2}\right)$$

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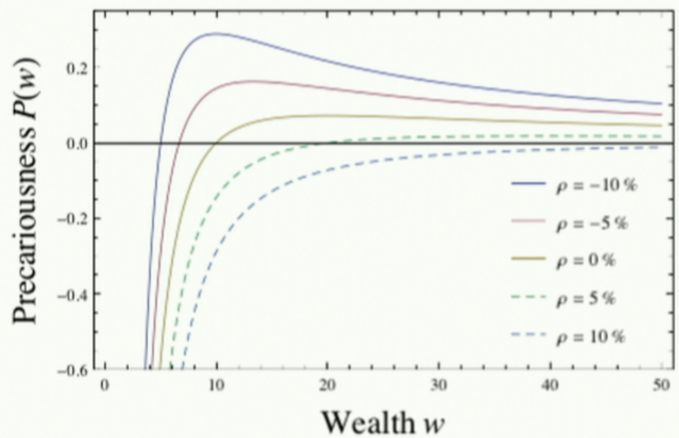

$$P(w) = \frac{2}{\ln 2}\left(\frac{\sigma^2 - \rho}{\sigma^2 w} - \frac{I}{\sigma^2 w^2}\right)$$

Wealth process: relaxation time
"How many years will it take for the top 10% to change
to that value if $\bar{r} - g$ changes from x% to y%?"
T. Piketty [private communication]
Relax
where
of volatility!
Relaxation time τ
Pareto tail exponent $\alpha = 1 - 2\rho/\sigma^2$
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Conclusions
Thermodynamics is most interesting when ap
equilibrium.
An analogue of the second law holds for
distribution, which displays precariousnes
Can estimate time scales; importance of
Turn this into a proper economic
Ref: MS, "Thermodynamics of inequal
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Thermodynamics is most interesting when applied out of equilibrium.
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dynamics of inequalities", arXiv:1406.6441.
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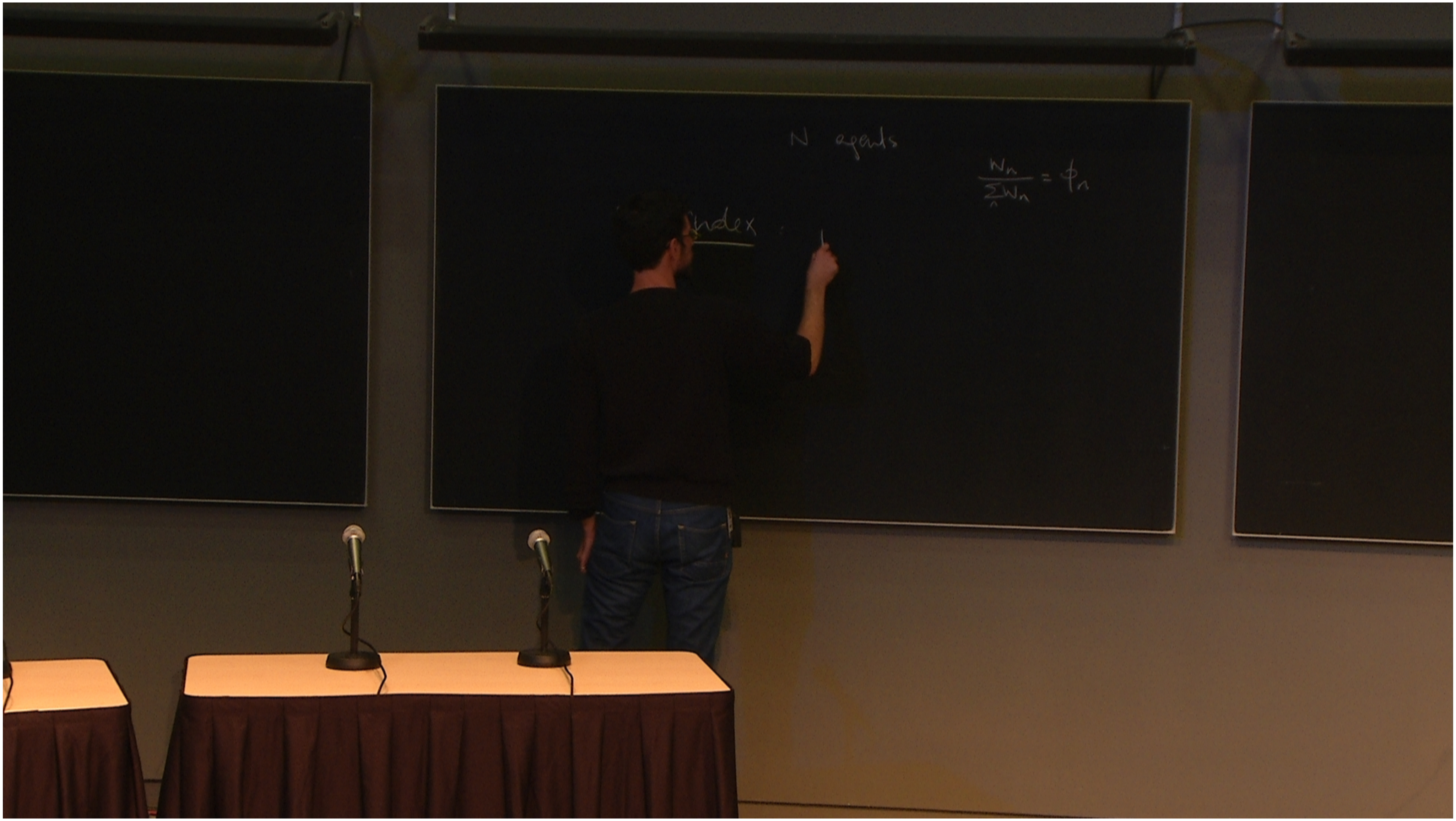
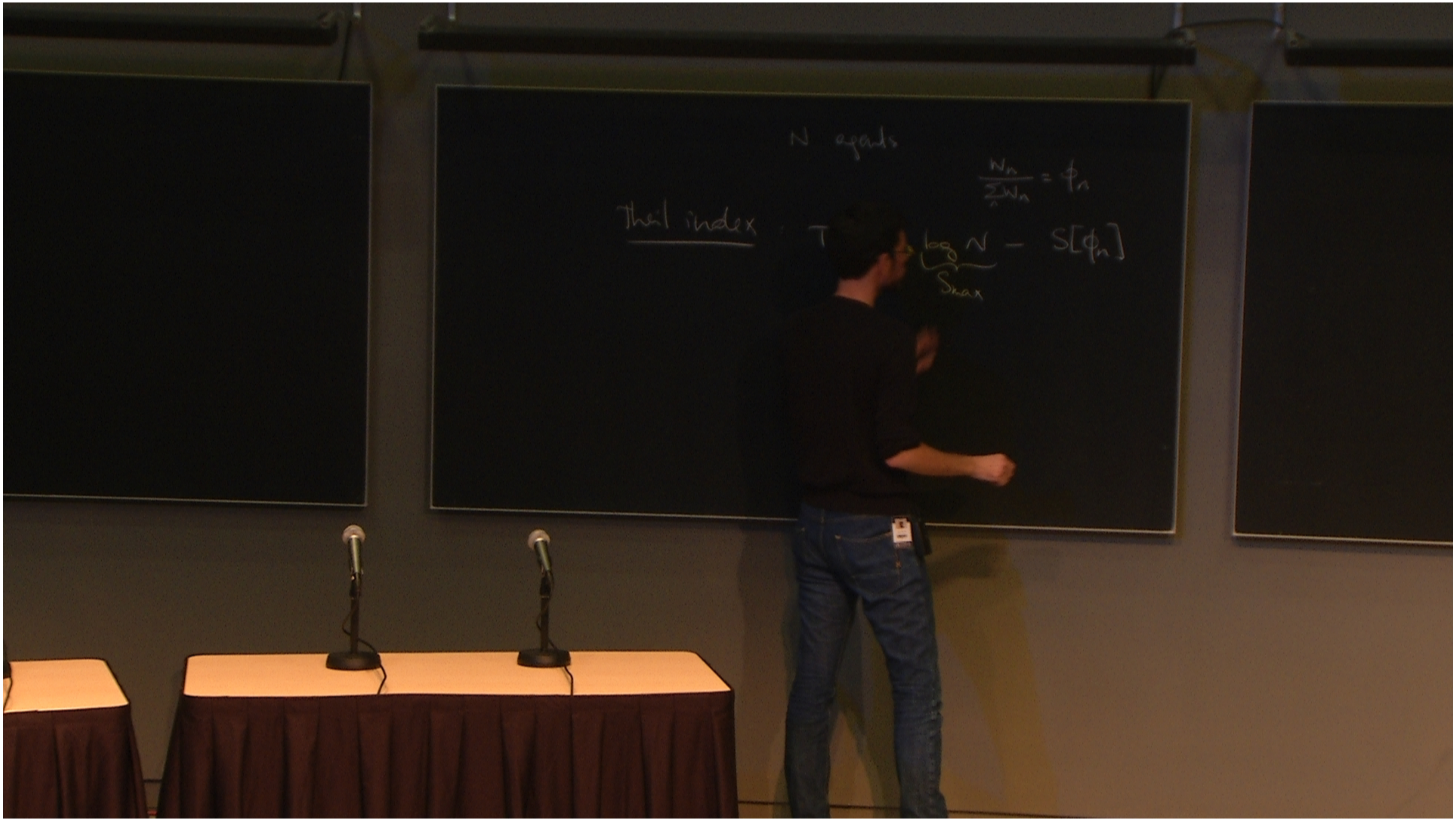

N agents
$\frac{W_n}{\sum_n W_n} = \phi_n$
Index :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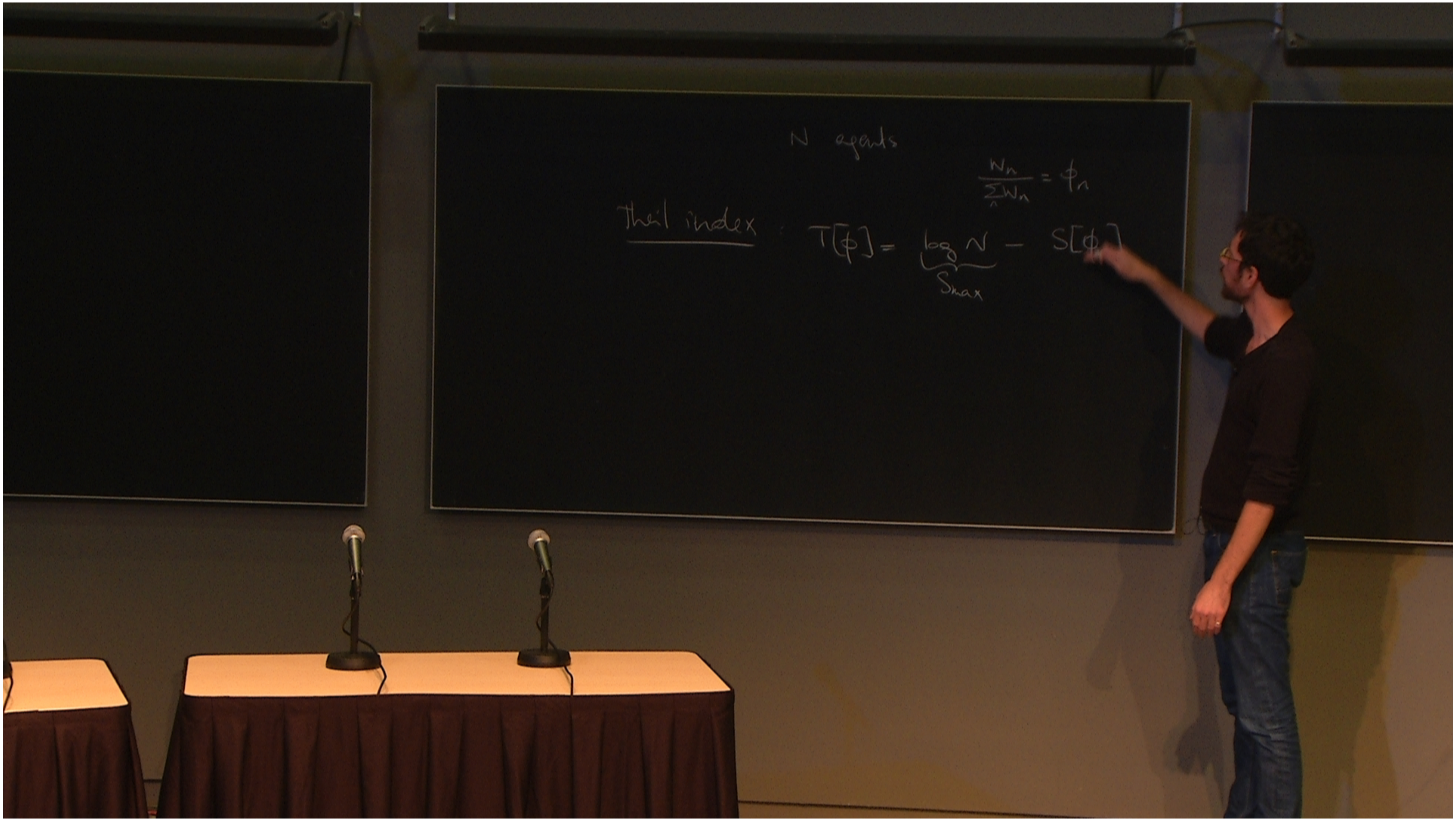
Theil index
$\log N - S[\phi_n]$
S_{max}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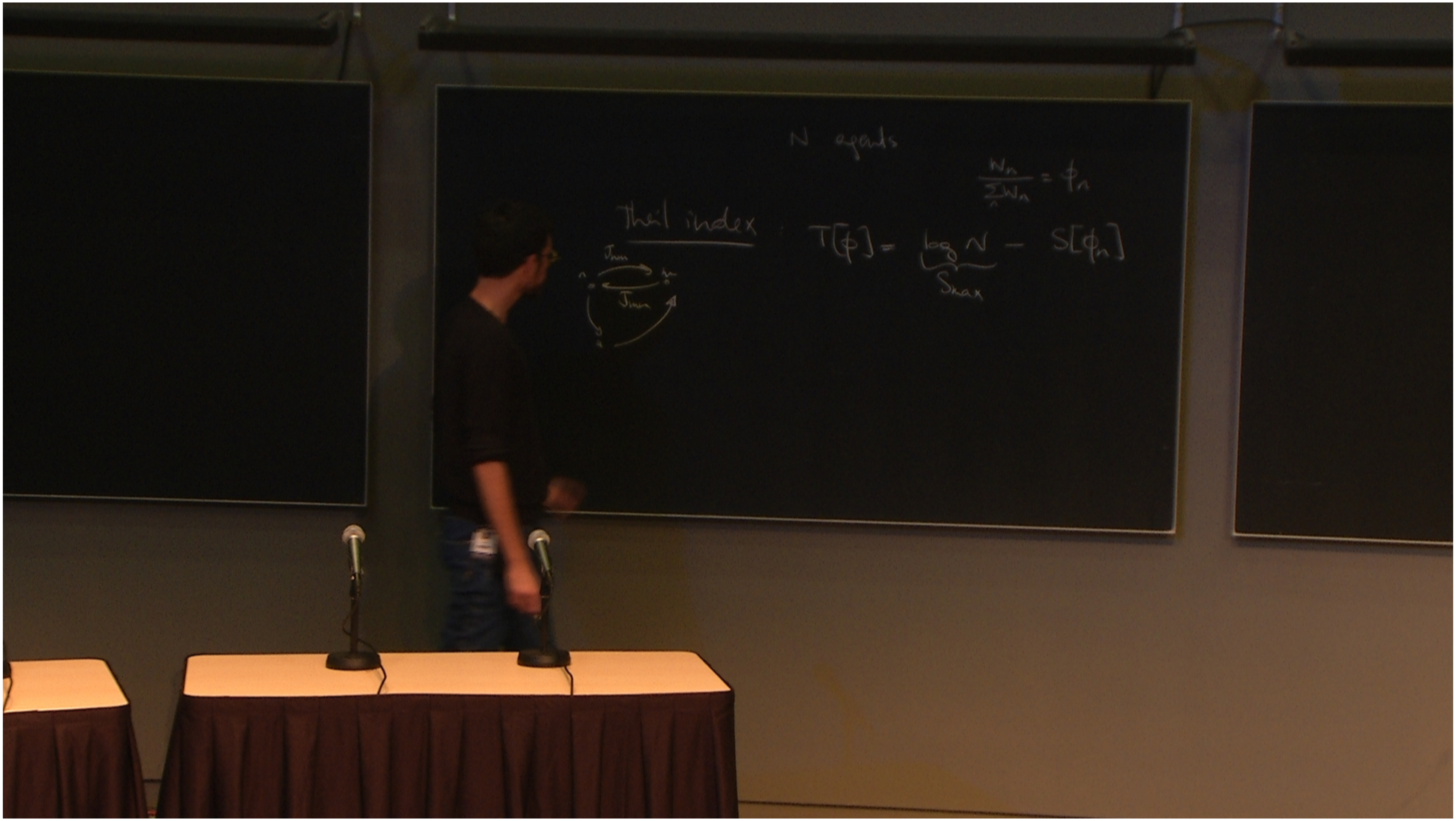
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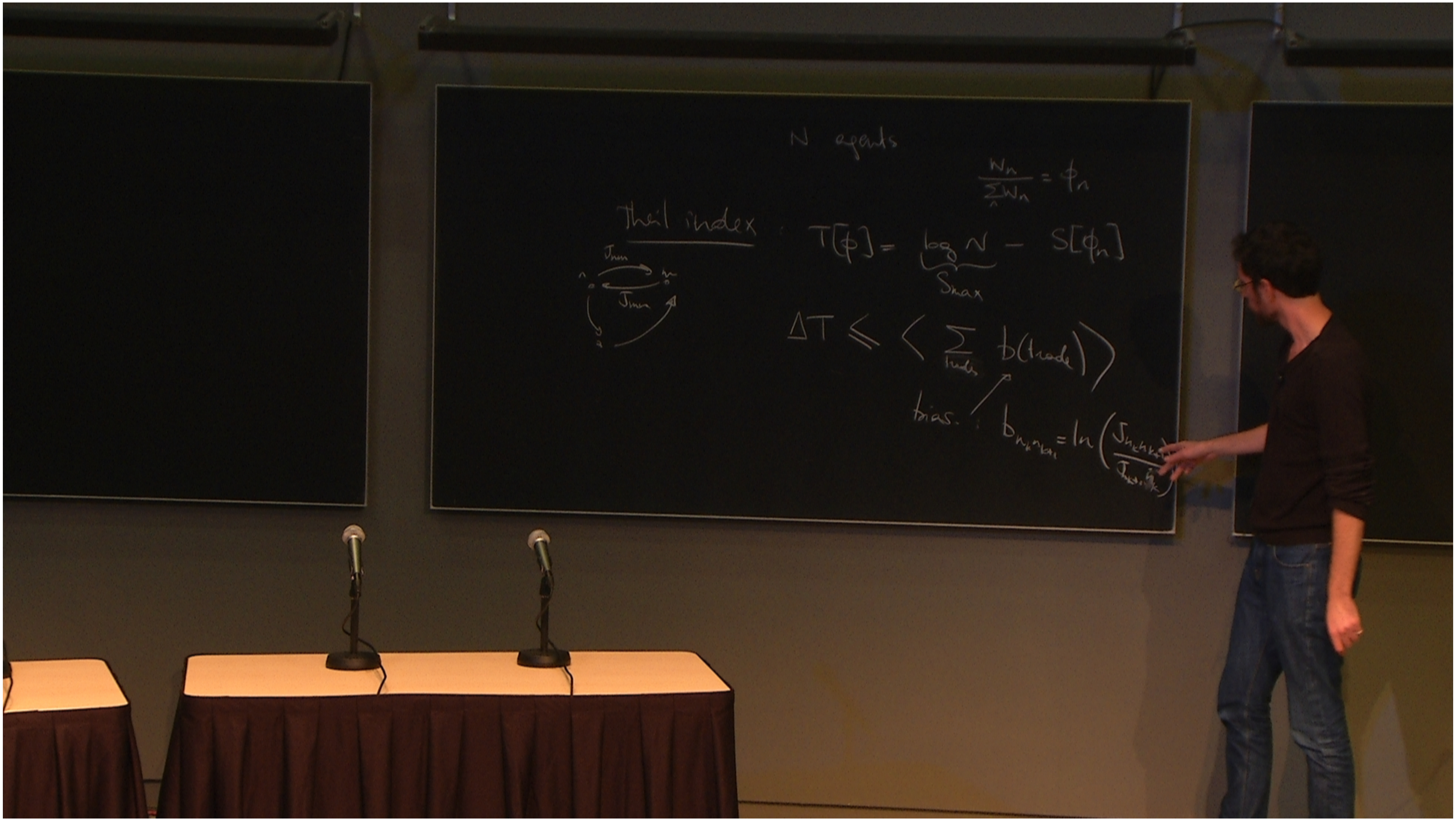
J_{nm}
J_{mn}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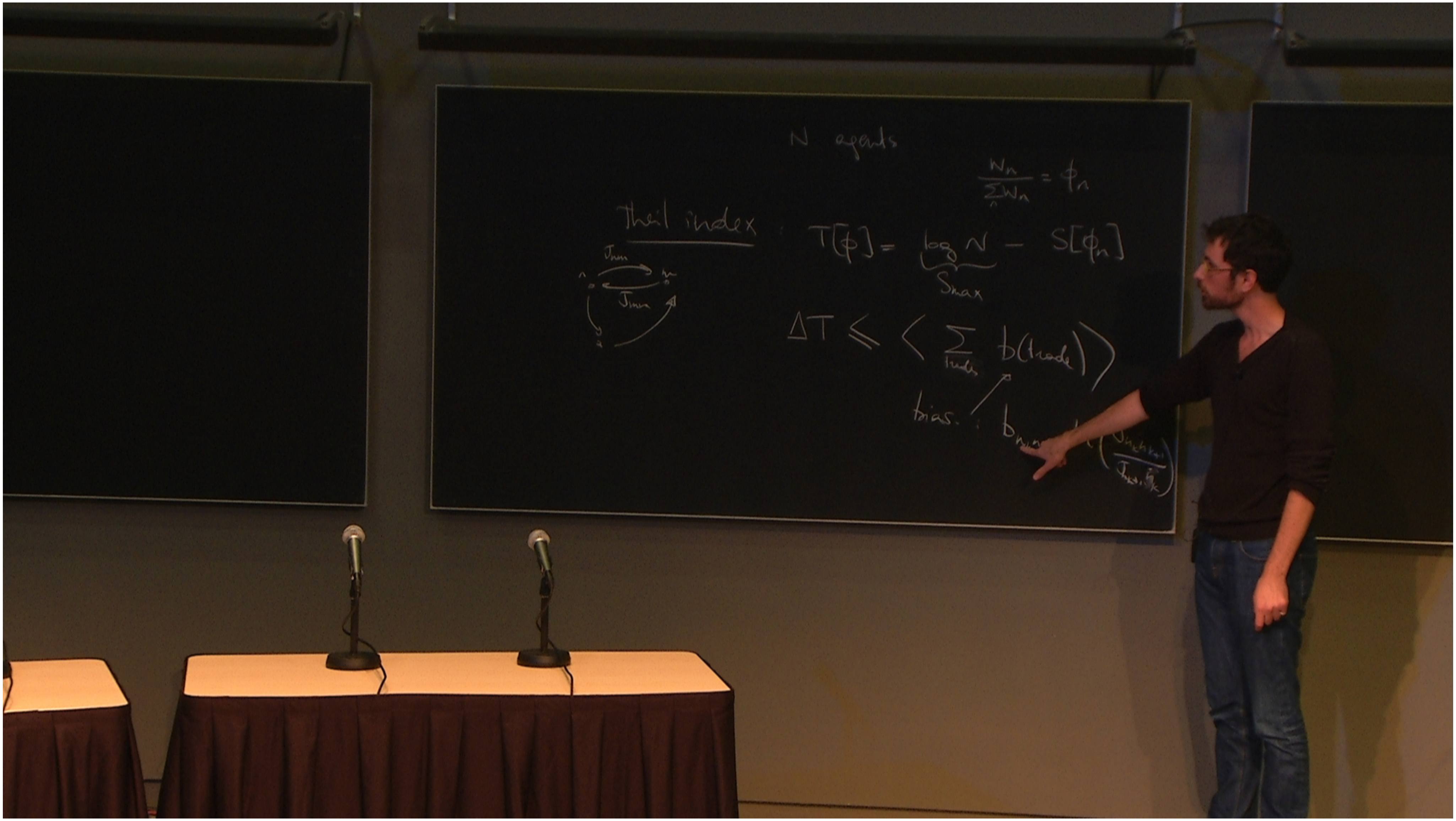
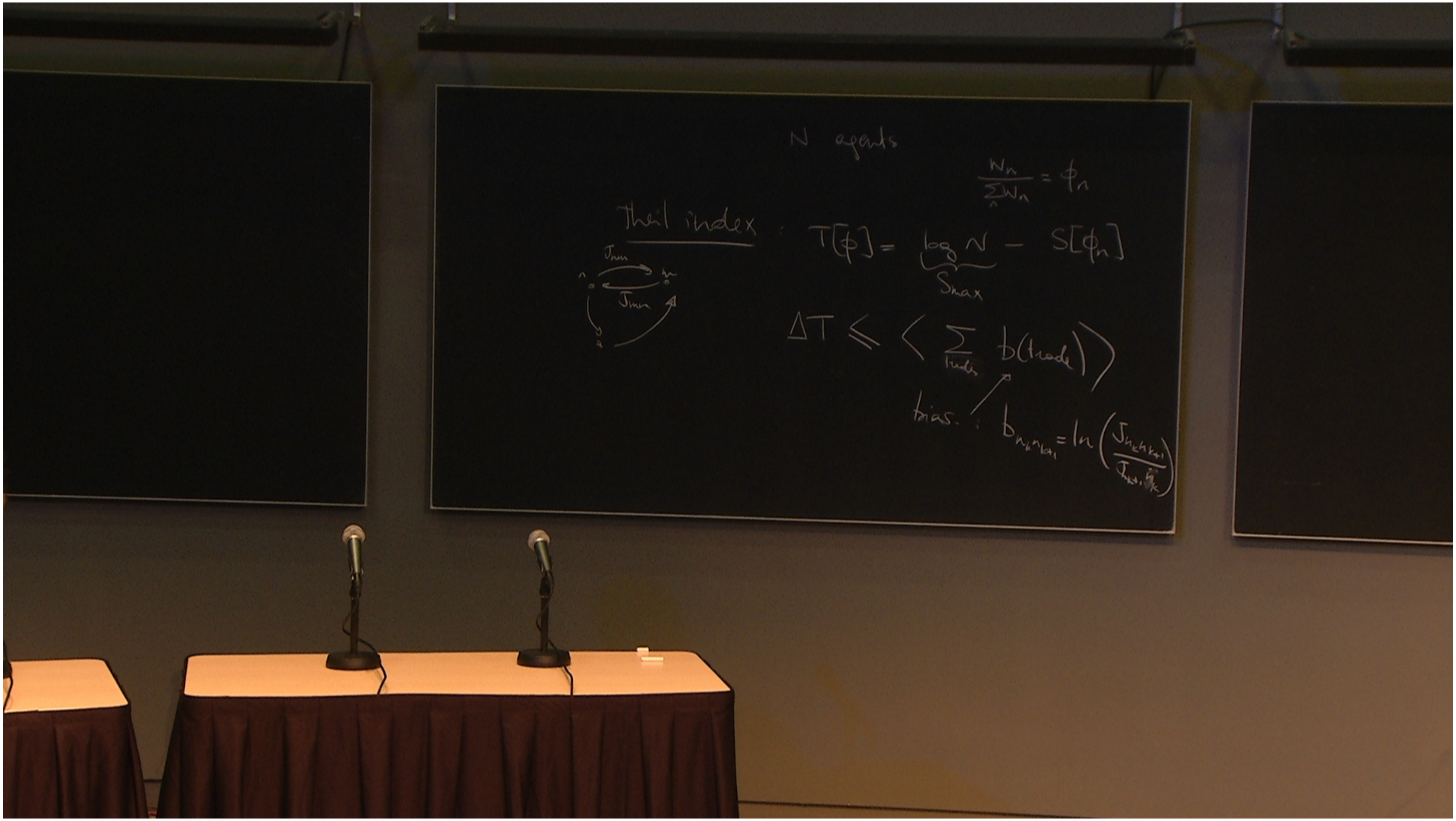
Theil index : $T[\phi] = \underbrace{\log N}_{S_{max}} - S[\phi_n]$
J_{nm}
J_{mn}
$\Delta T \leqslant \left\langle \sum_{trades} b(trade) \right\rangle$
bias :
$b_{n_k n_{k+1}} = \ln\left(\frac{J_{n_k n_{k+1}}}{J_{n_{k+1} n_k}}\right)$


N agents
Theil index


N agents
Theil index
T[φ] = log N − S[φ_n]
S_max
ΔT ≤ ⟨ Σ b(trade) ⟩
bias