Title: Holographic Thermalization and Chaos

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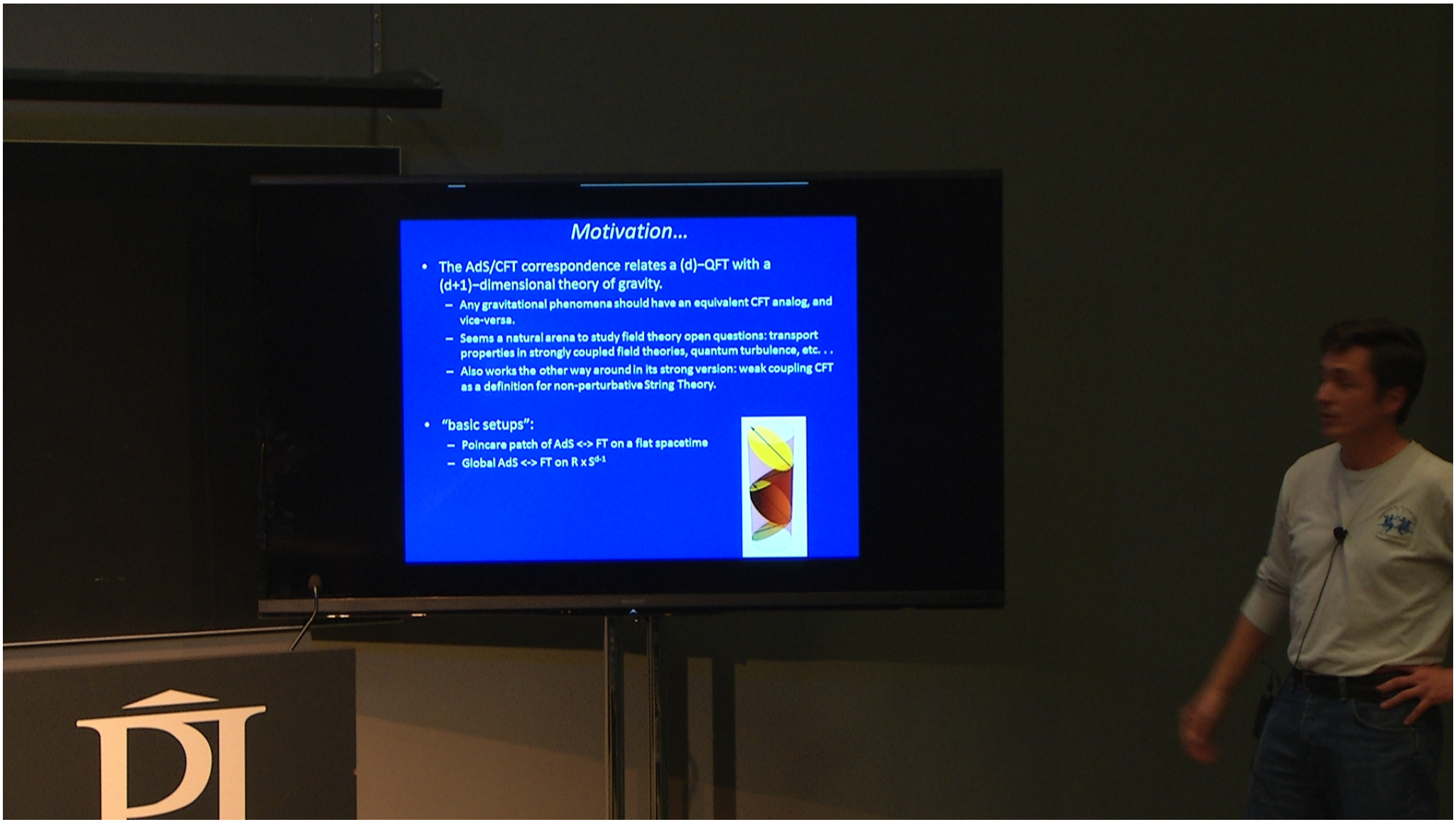
URL: http://pirsa.org/14100108

Abstract:

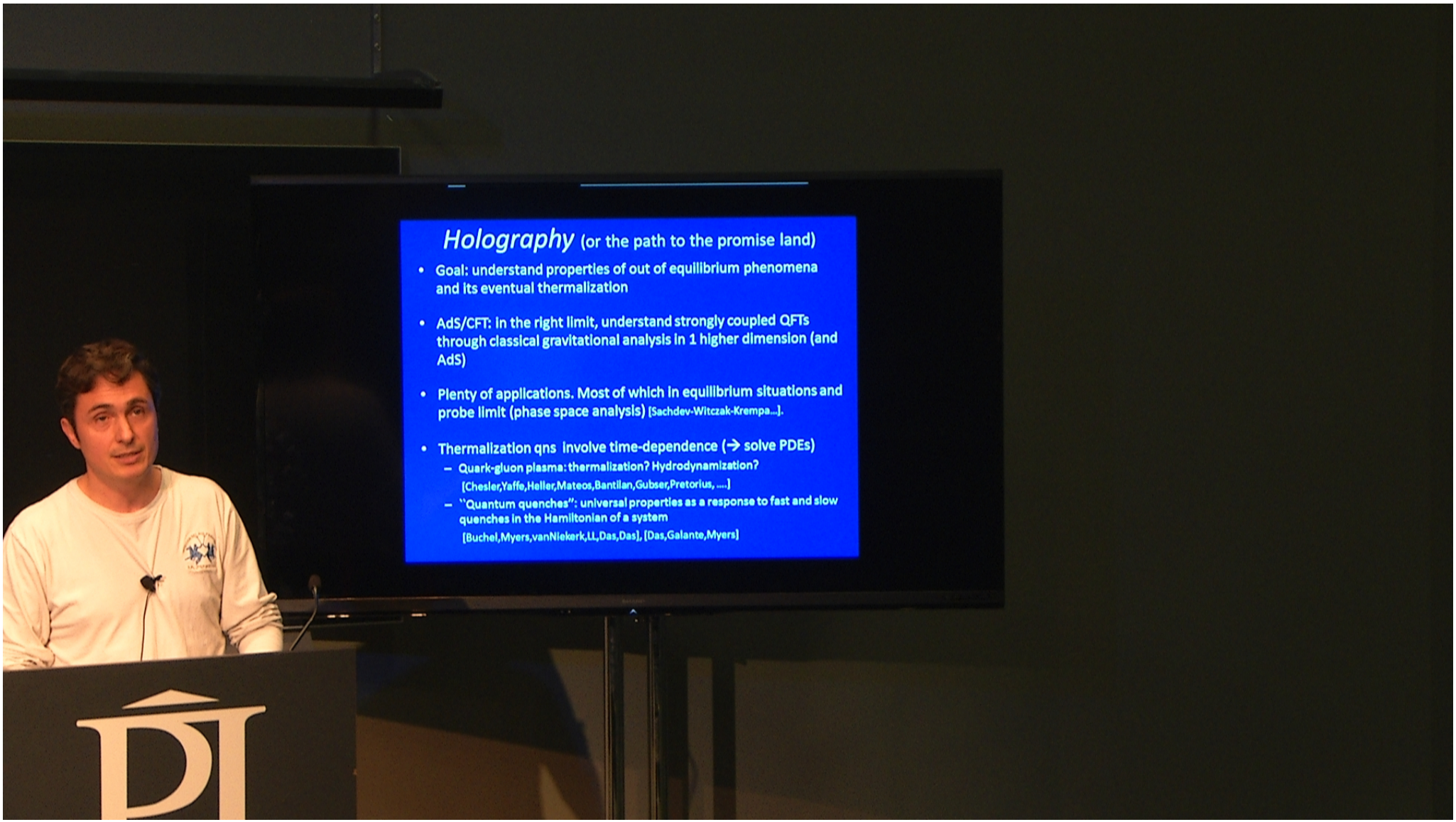

Motivation...
The AdS/CFT correspondence relates a (d)–QFT with a (d+1)–dimensional theory of gravity.
Any gravitational phenomena should have an equivalent CFT analog, and vice-versa.
Seems a natural arena to study field theory open questions: transport properties in strongly coupled field theories, quantum turbulence, etc. . .
Also works the other way around in its strong version: weak coupling CFT as a definition for non-perturbative String Theory.
"basic setups":
Poincare patch of AdS <-> FT on a flat spacetime
Global AdS <-> FT on R x S^{d-1}

Holography (or the path to the promise land)

- Goal: understand properties of out of equilibrium phenomena and its eventual thermalization
- AdS/CFT: in the right limit, understand strongly coupled QFTs through classical gravitational analysis in 1 higher dimension (and AdS)
- Plenty of applications. Most of which in equilibrium situations and probe limit (phase space analysis) [Sachdev-Witczak-Krempa...].
- Thermalization qns involve time-dependence (→ solve PDEs)
  - Quark-gluon plasma: thermalization? Hydrodynamization?

    [Chesler,Yaffe,Heller,Mateos,Bantilan,Gubser,Pretorius, ...]
  - ``Quantum quenches'': universal properties as a response to fast and slow quenches in the Hamiltonian of a system

    [Buchel,Myers,vanNiekerk,LL,Das,Das], [Das,Galante,Myers]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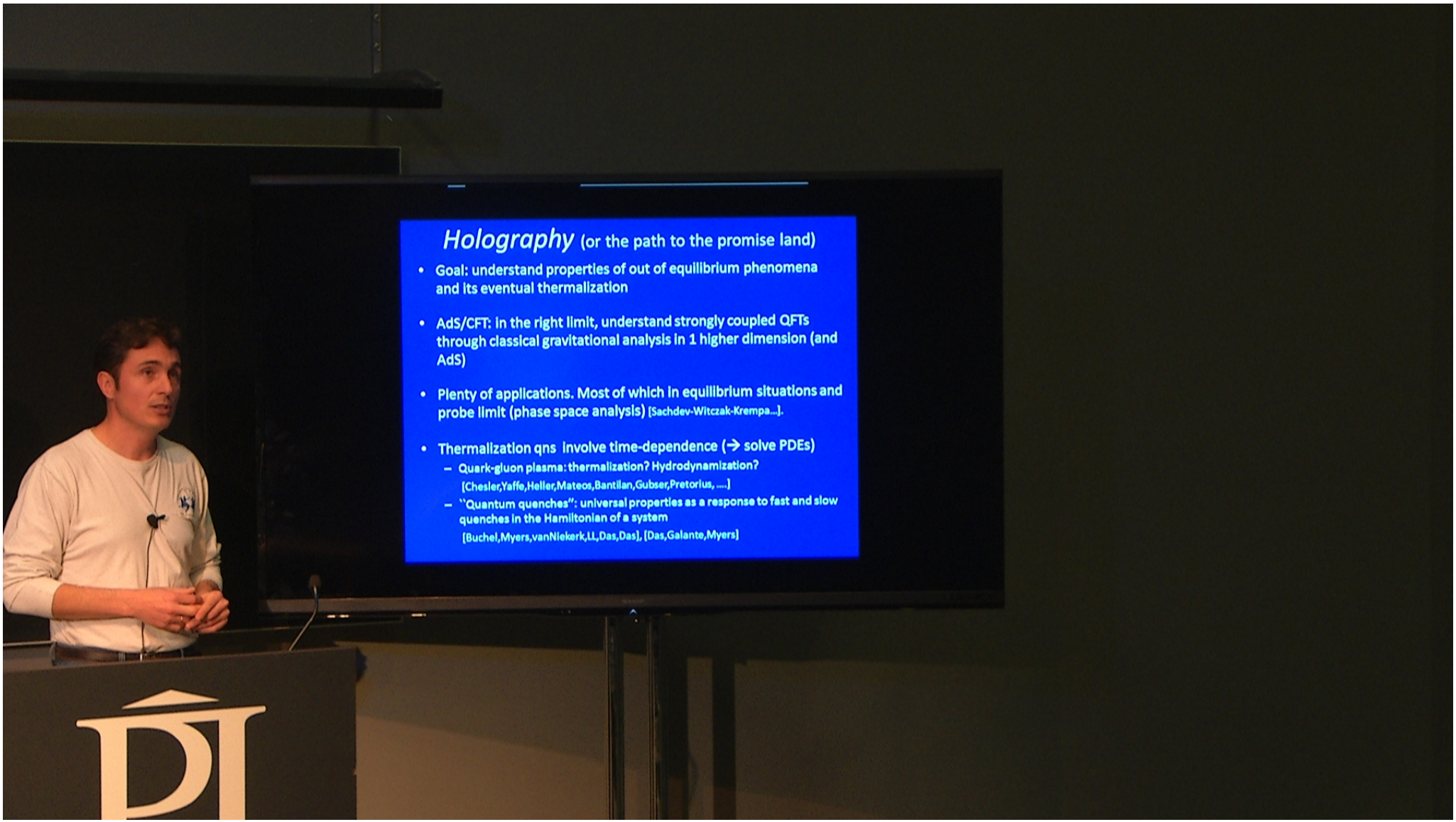
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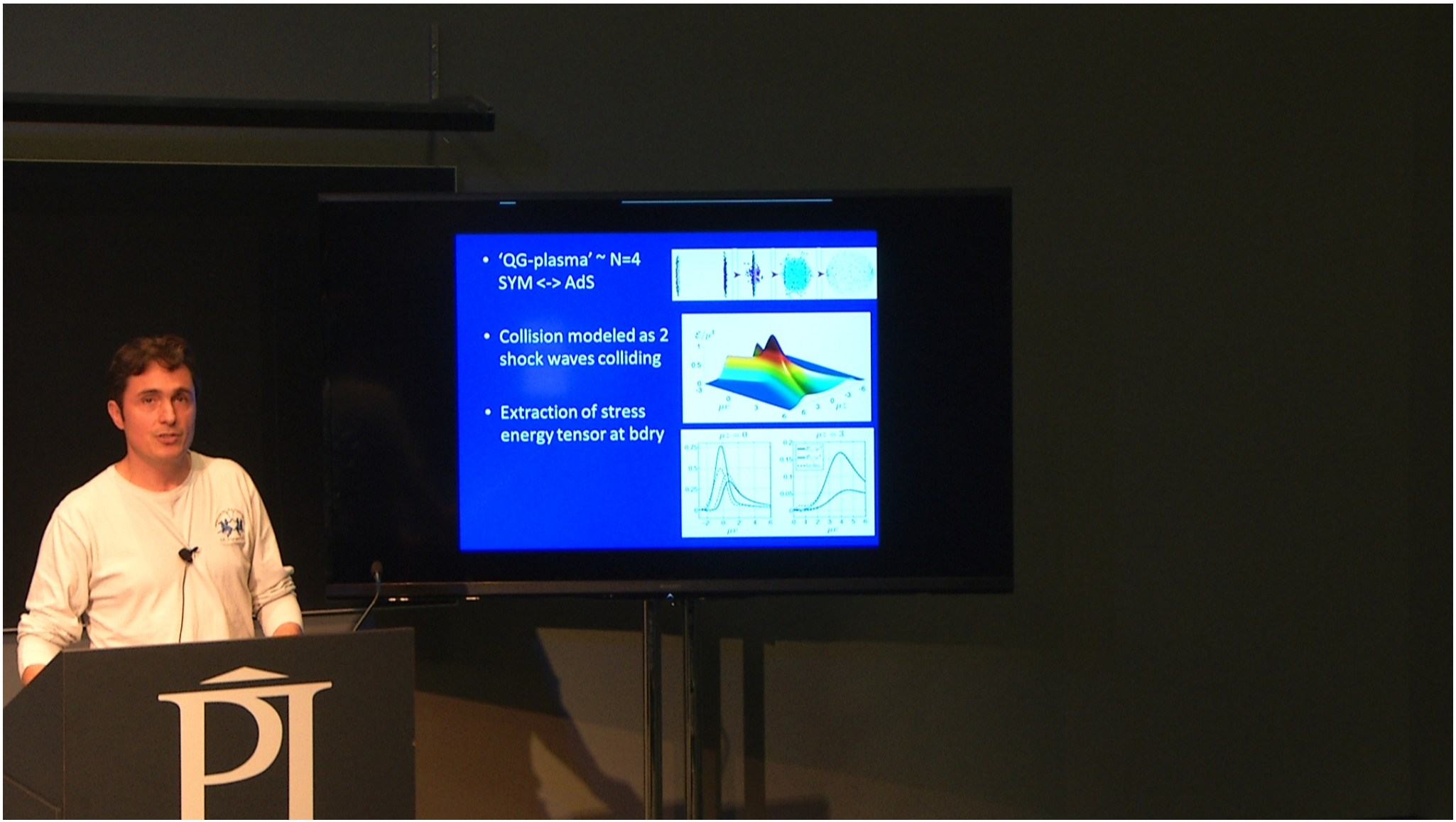
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• Collision modeled as 2 shock waves colliding
• Extraction of stress energy tensor at bdry

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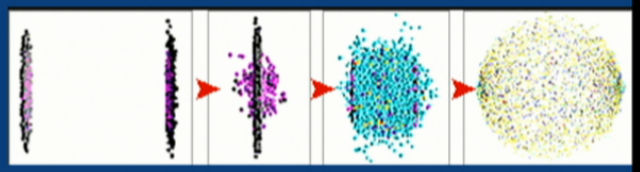

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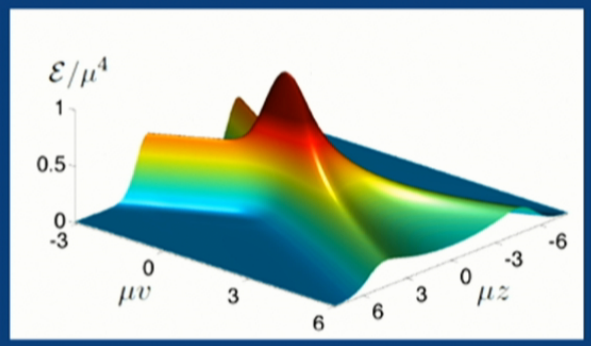

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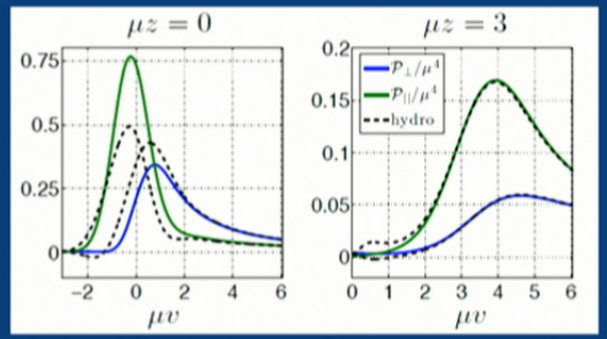

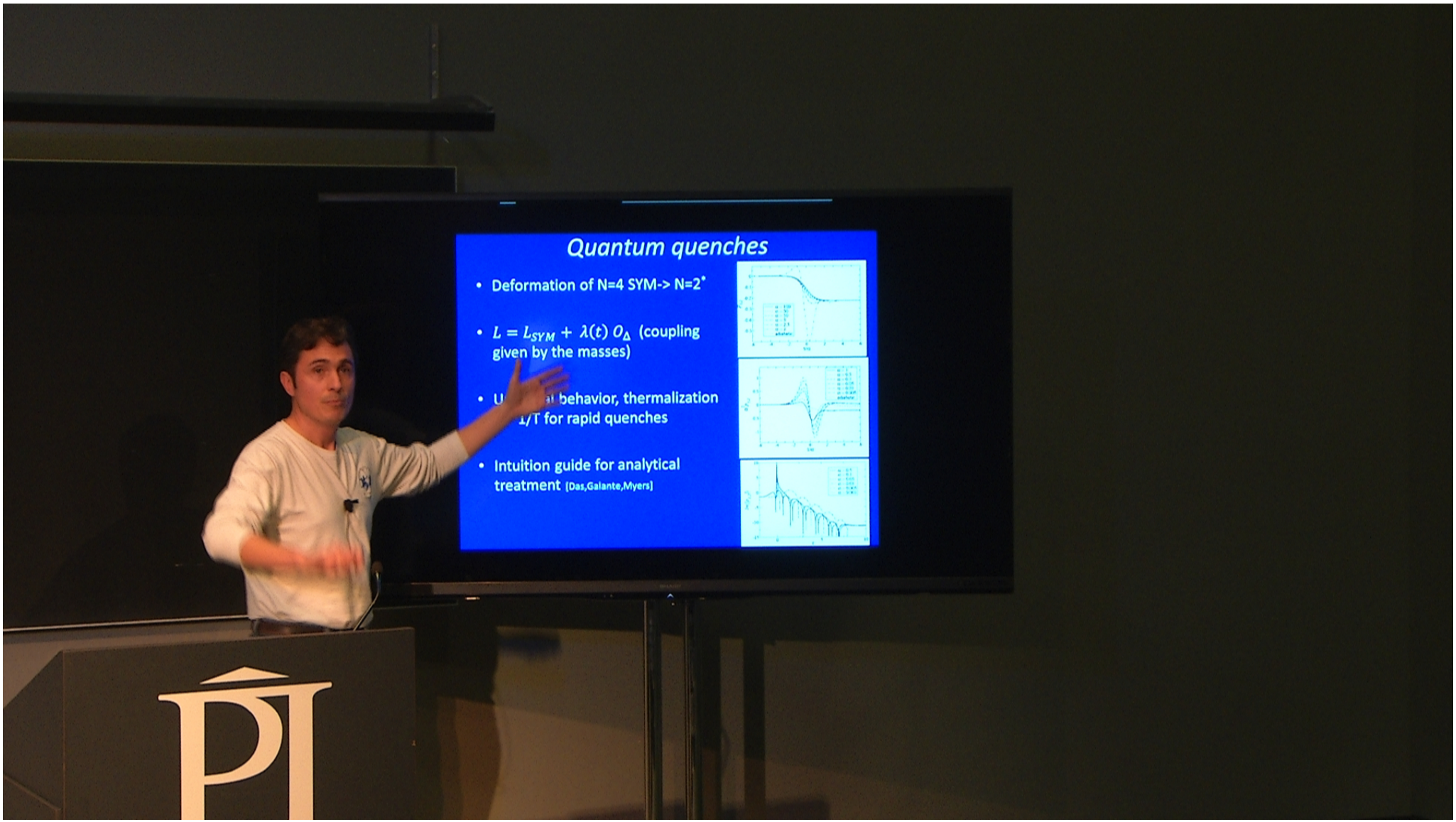
Quantum quenches
• Deformation of N=4 SYM-> N=2*
• $L = L_{SYM} + \lambda(t)\, O_{\Delta}$ (coupling given by the masses)
• U behavior, thermalization 1/T for rapid quenches
• Intuition guide for analytical treatment [Das,Galante,Myers]

Some observations

- Starting with a perturbed thermal state, thermalization time scale given by : perturbation time scale (adiabatic case), 1/T scale consistent with the slowest –triggered- black hole QNM (abrupt case) [Note: many problems are essentially the same in gravitational terms]

- And so, one might be tempted in running with this… however,
  - QNMs aren't a basis
  - QNMs for all relevant cases not known (e.g. d=4,5 Kerr-AdS [Cardoso,Dias,Santos,Harnet,LL dec 2013])
  - Kerr-AdS (and even Schwarzschild-AdS) is not known to be stable [in fact arguments for the opposite]. Further. if not 'linearly-stable', can we use QNMs in a straightforward way?
  - `pure' AdS is not known to be stable. E.g. if it doesn't form a black hole? What's the path for a thermal state?

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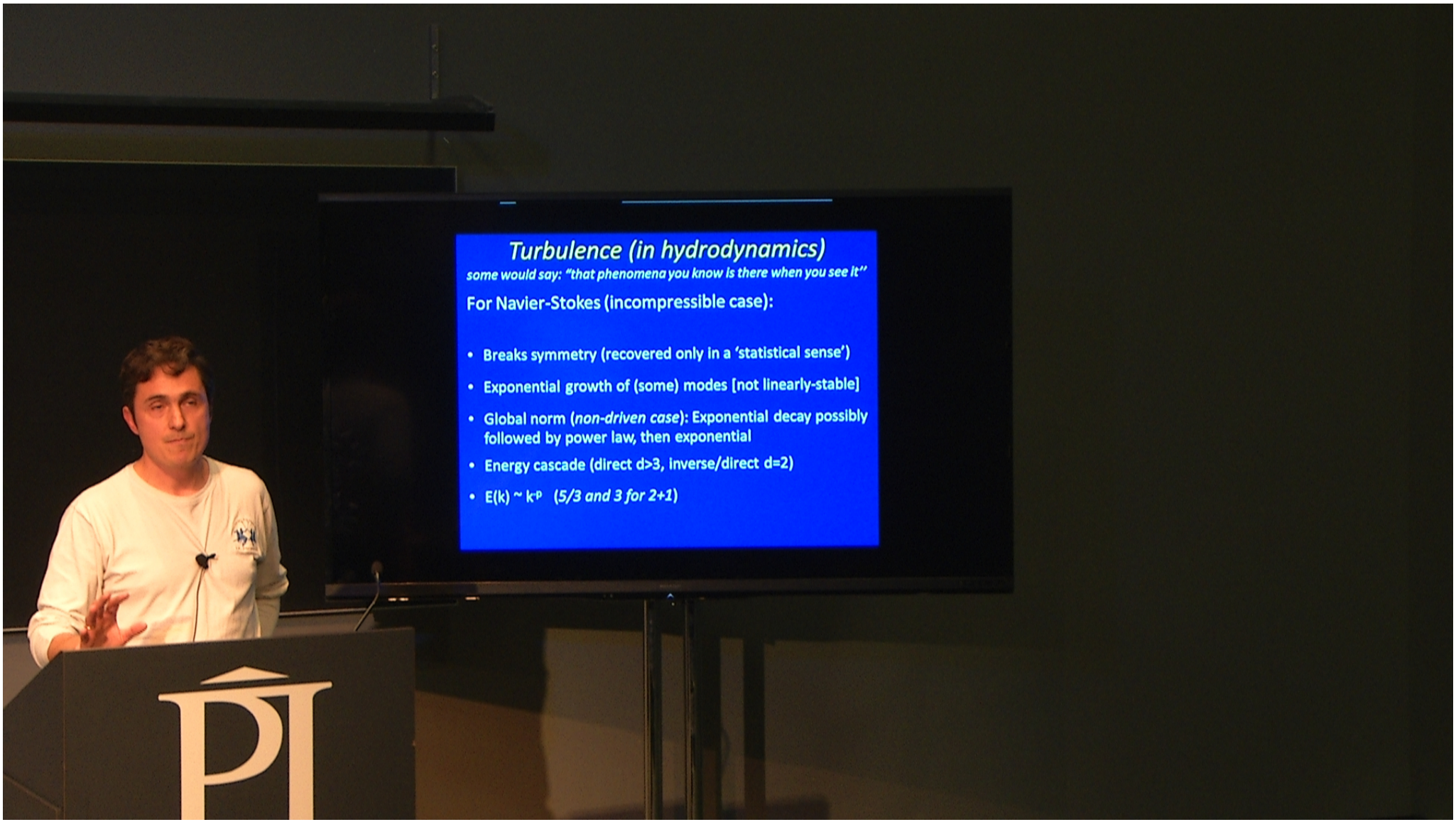
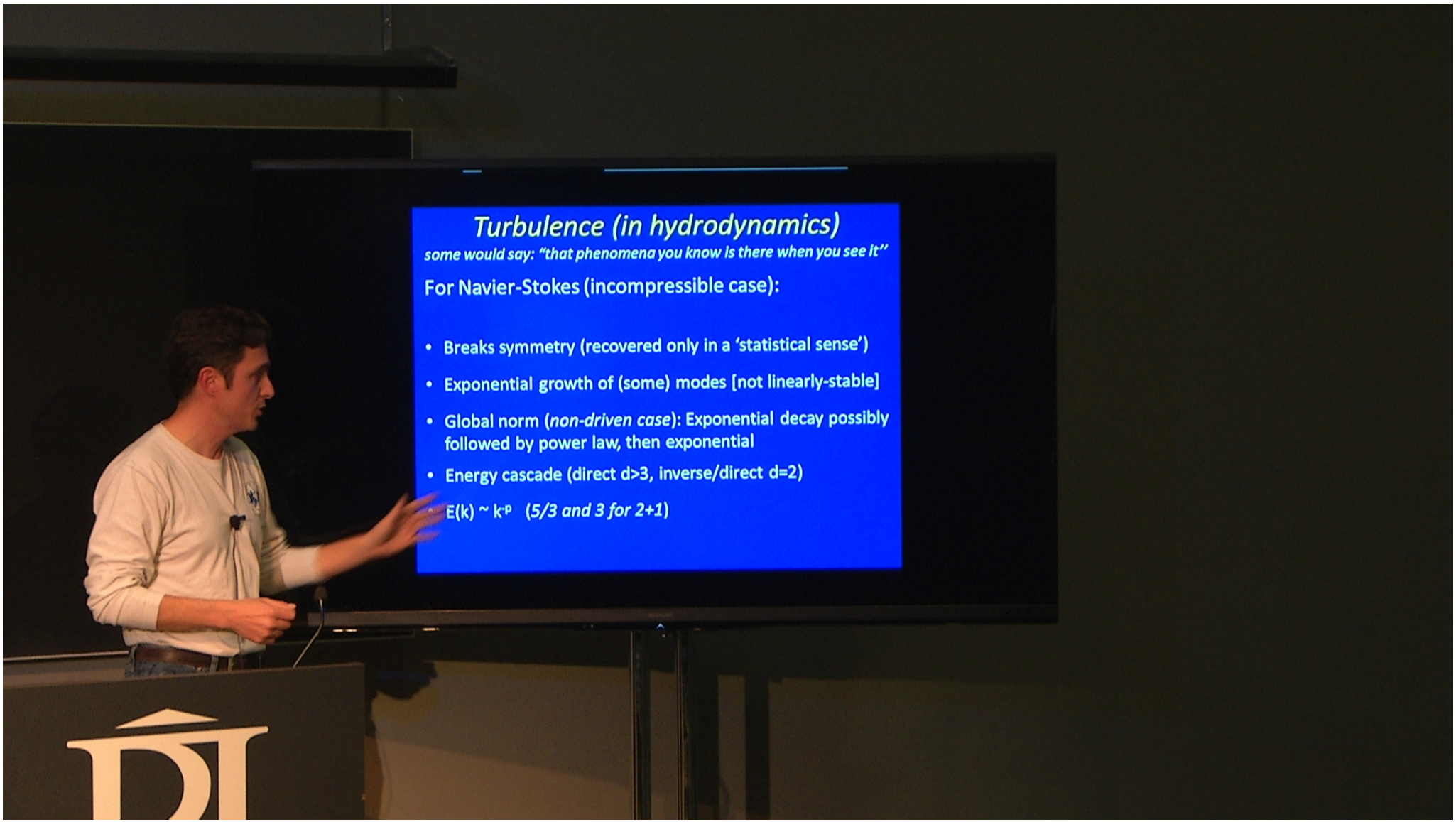

Turbulence (in hydrodynamics)
some would say: "that phenomena you know is there when you see it"
For Navier-Stokes (incompressible case):
• Breaks symmetry (recovered only in a 'statistical sense')
• Exponential growth of (some) modes [not linearly-stable]
• Global norm (non-driven case): Exponential decay possibly followed by power law, then exponential
• Energy cascade (direct d>3, inverse/direct d=2)
• E(k) ~ k^p (5/3 and 3 for 2+1)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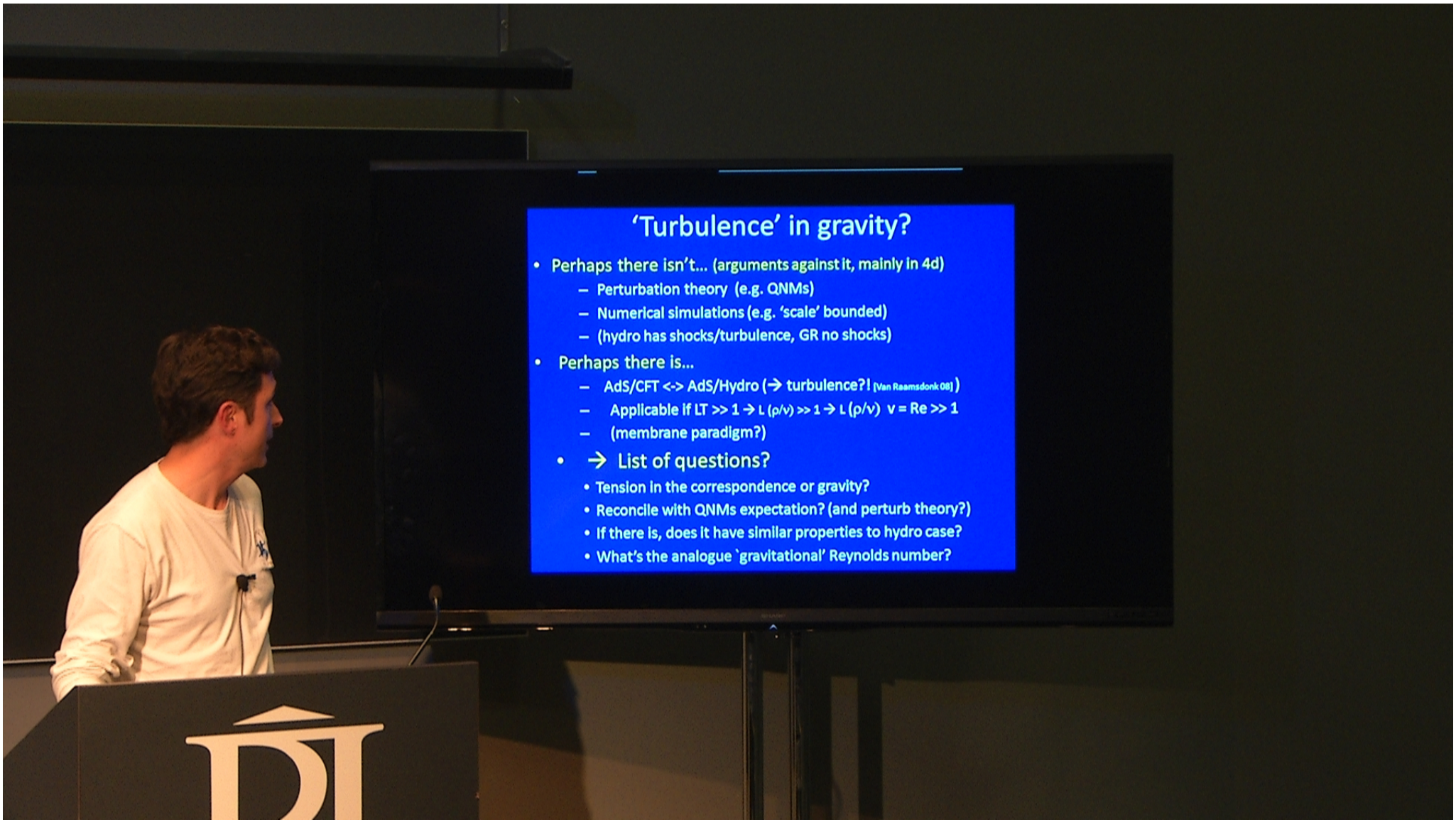
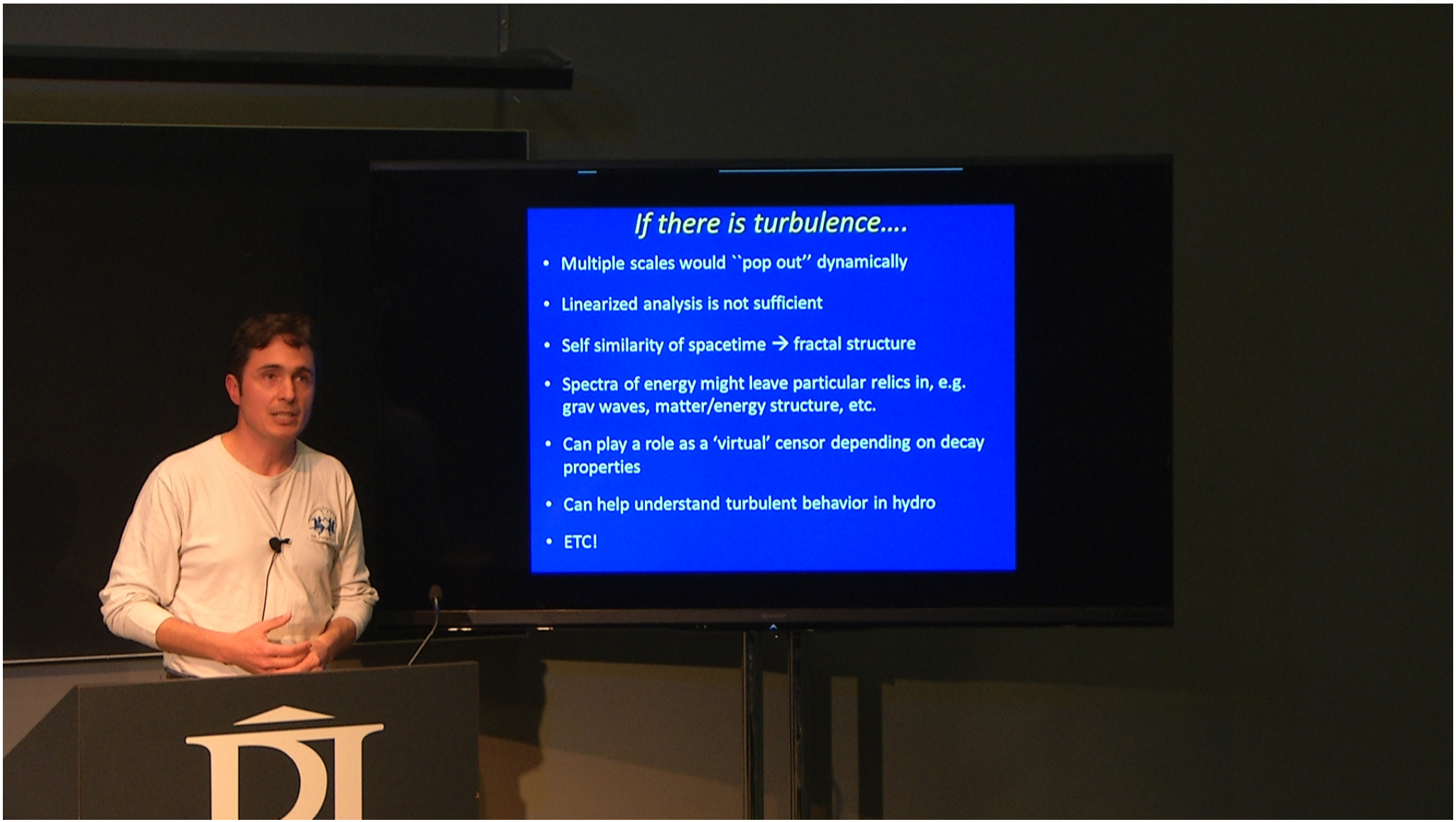
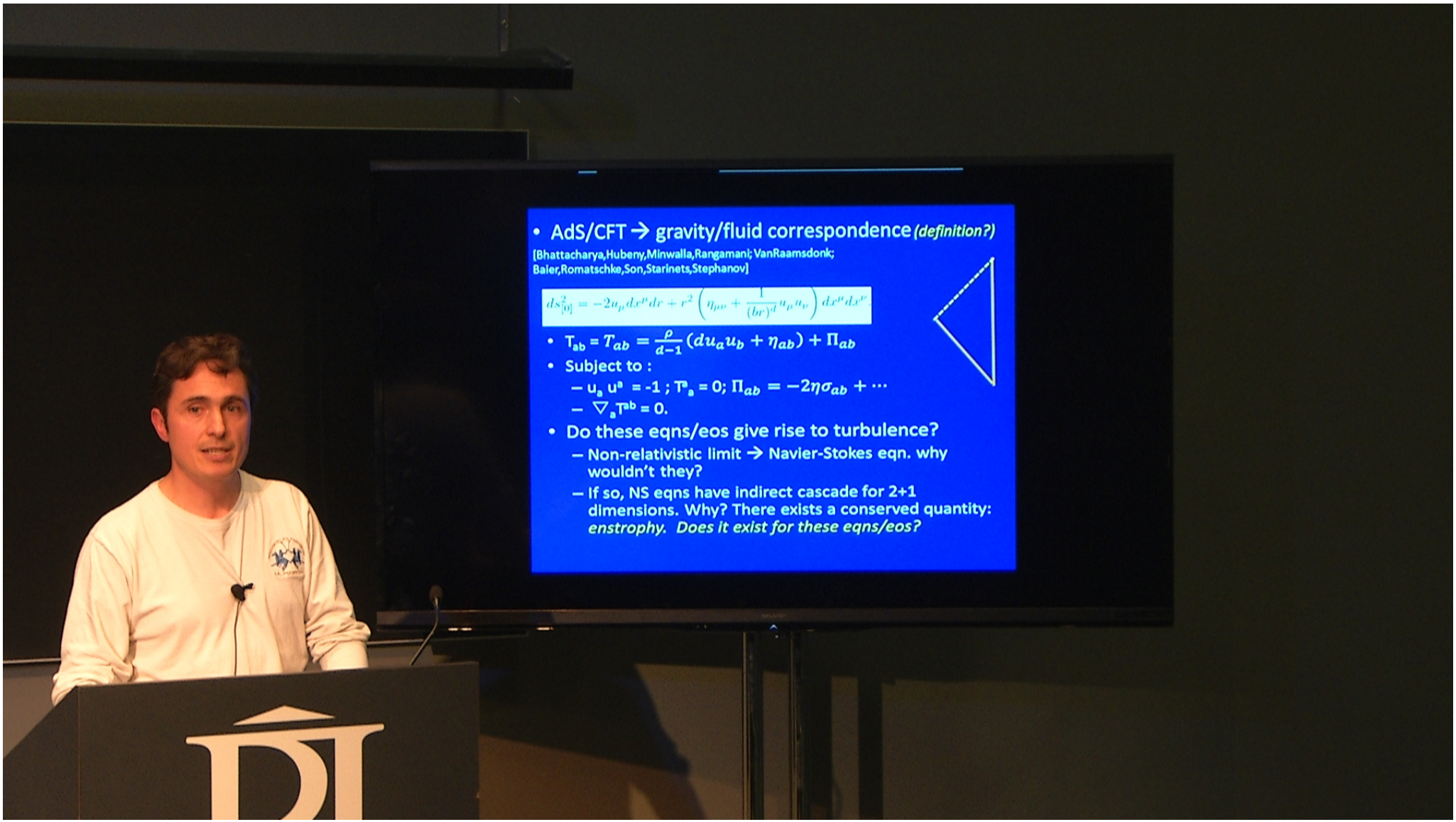
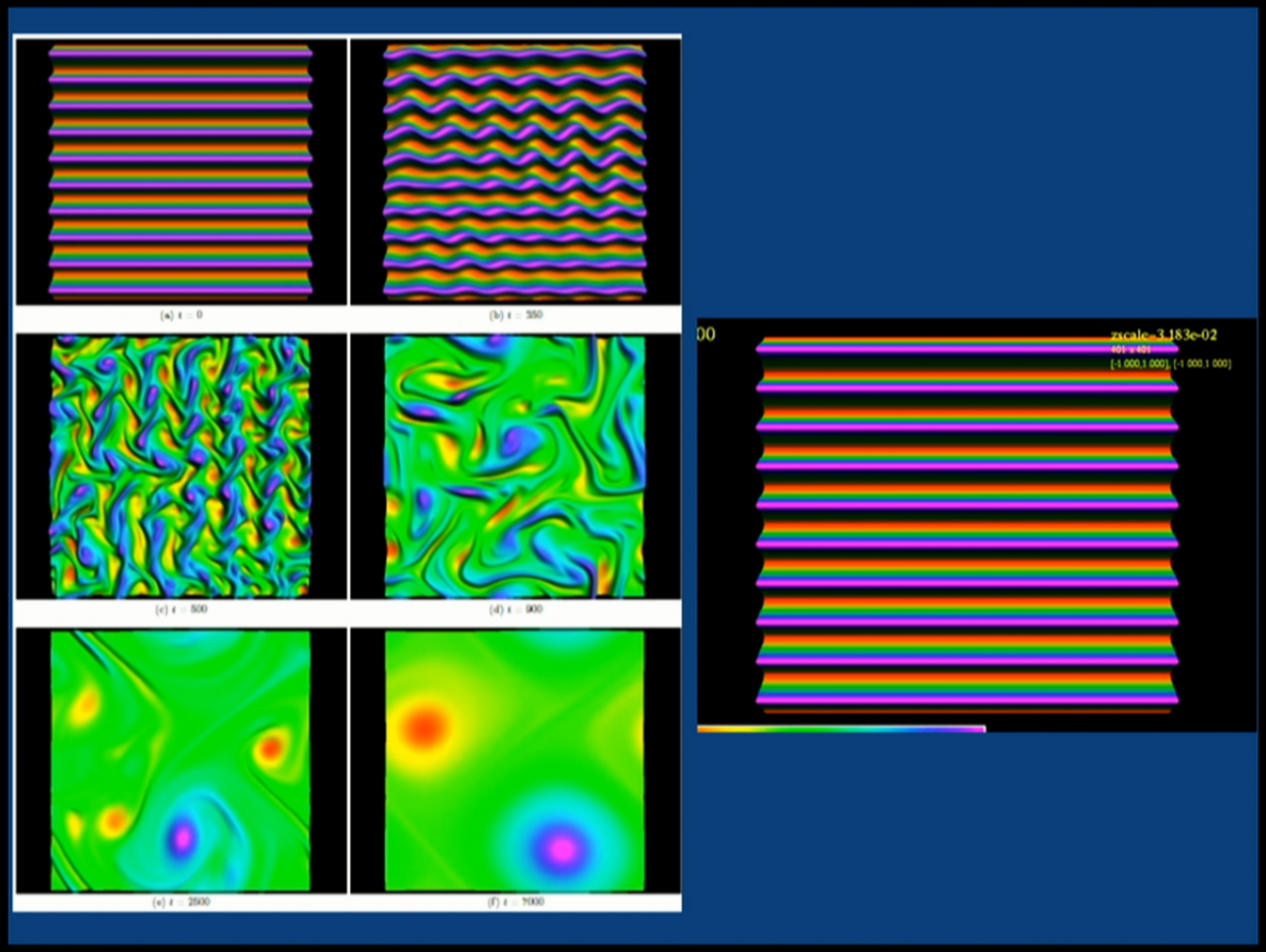

'Turbulence' in gravity?
• Perhaps there isn't... (arguments against it, mainly in 4d)
– Perturbation theory (e.g. QNMs)
– Numerical simulations (e.g. 'scale' bounded)
– (hydro has shocks/turbulence, GR no shocks)
• Perhaps there is...
– AdS/CFT <-> AdS/Hydro (→ turbulence?! [Van Raamsdonk 08])
– Applicable if LT >> 1 → L (ρ/ν) >> 1 → L (ρ/ν) ν = Re >> 1
– (membrane paradigm?)
• → List of questions?
• Tension in the correspondence or gravity?
• Reconcile with QNMs expectation? (and perturb theory?)
• If there is, does it have similar properties to hydro case?
• What's the analogue `gravitational' Reynolds number?


If there is turbulence....
• Multiple scales would ``pop out'' dynamically
• Linearized analysis is not sufficient
• Self similarity of spacetime → fractal structure
• Spectra of energy might leave particular relics in, e.g. grav waves, matter/energy structure, etc.
• Can play a role as a 'virtual' censor depending on decay properties
• Can help understand turbulent behavior in hydro
• ETC!


• AdS/CFT → gravity/fluid correspondence (definition?)
[Bhattacharya,Hubeny,Minwalla,Rangamani; VanRaamsdonk; Baier,Romatschke,Son,Starinets,Stephanov]
$ds^2_{[0]} = -2u_\mu dx^\mu dr + r^2\left(\eta_{\mu\nu} + \frac{1}{(br)^d}u_\mu u_\nu\right)dx^\mu dx^\nu$
• $T_{ab} = T_{ab} = \frac{\rho}{d-1}(du_a u_b + \eta_{ab}) + \Pi_{ab}$
• Subject to :
– $u_a u^a = -1$; $T^a{}_a = 0$; $\Pi_{ab} = -2\eta\sigma_{ab} + \cdots$
– $\nabla_a T^{ab} = 0$.
• Do these eqns/eos give rise to turbulence?
– Non-relativistic limit → Navier-Stokes eqn. why wouldn't they?
– If so, NS eqns have indirect cascade for 2+1 dimensions. Why? There exists a conserved quantity: *enstrophy. Does it exist for these eqns/eos?*


(a) t = 0
(b) t = 550
(c) t = 500
(d) t = 900
(e) t = 2500
(f) t = 7000
00
zscale=3.183e-02
401 x 401
[-1.000,1.000], [-1.000,1.000]


Onto global AdS
• Dual to Kerr-AdS black hole is a rotating fluid on RxS²
• EOMs=> stress energy conservation on a sphere
0.00e+00
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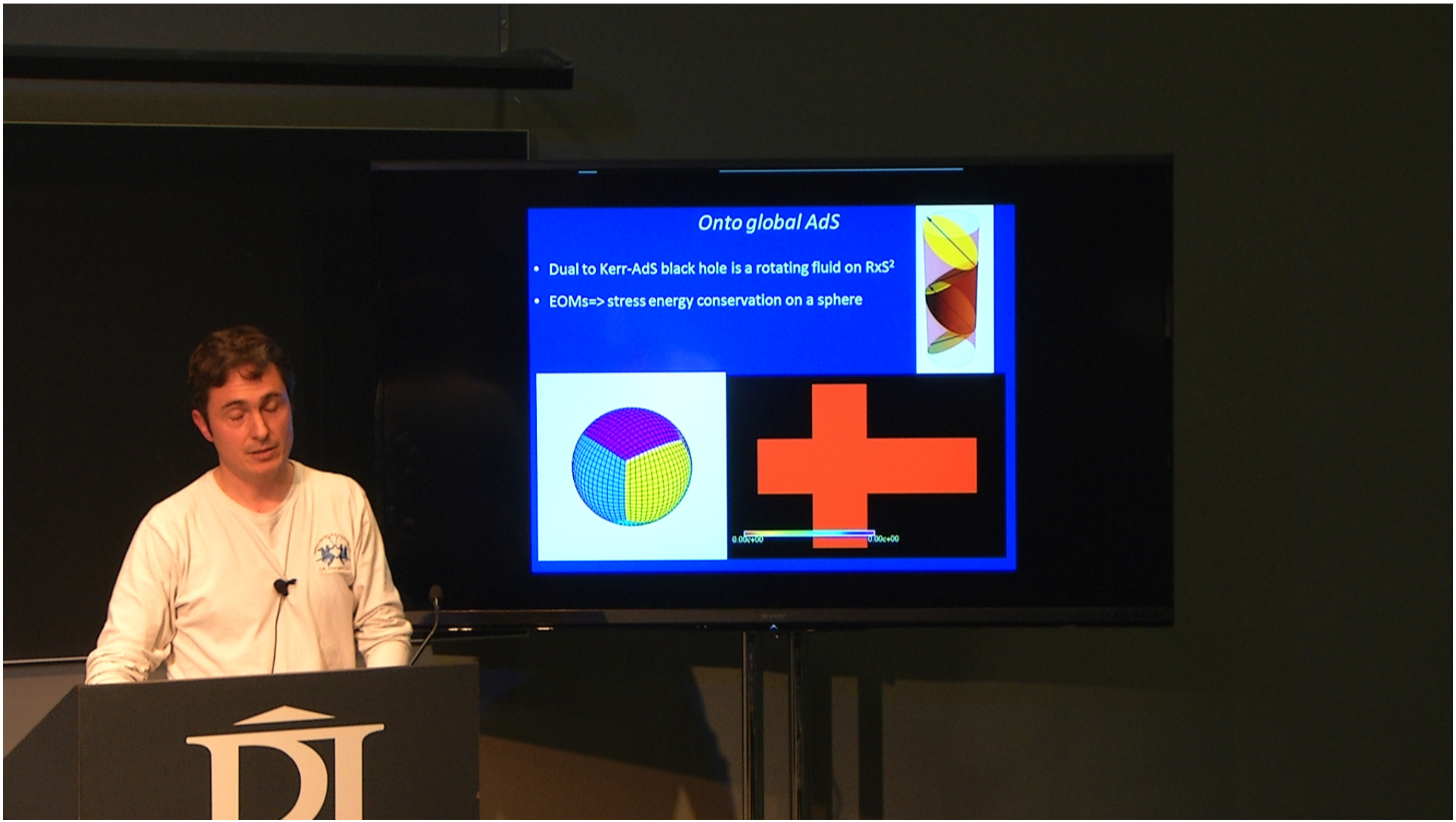
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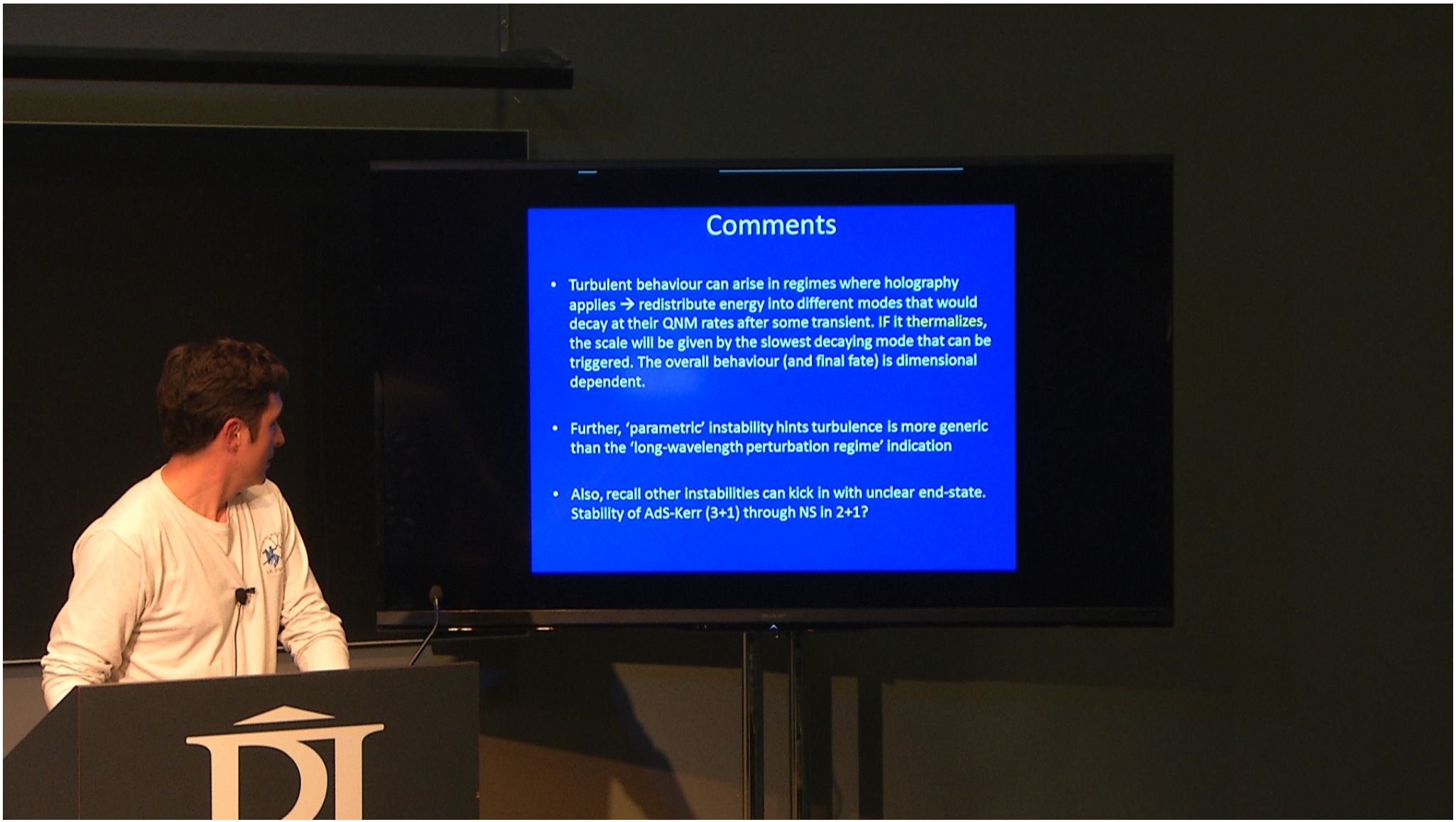
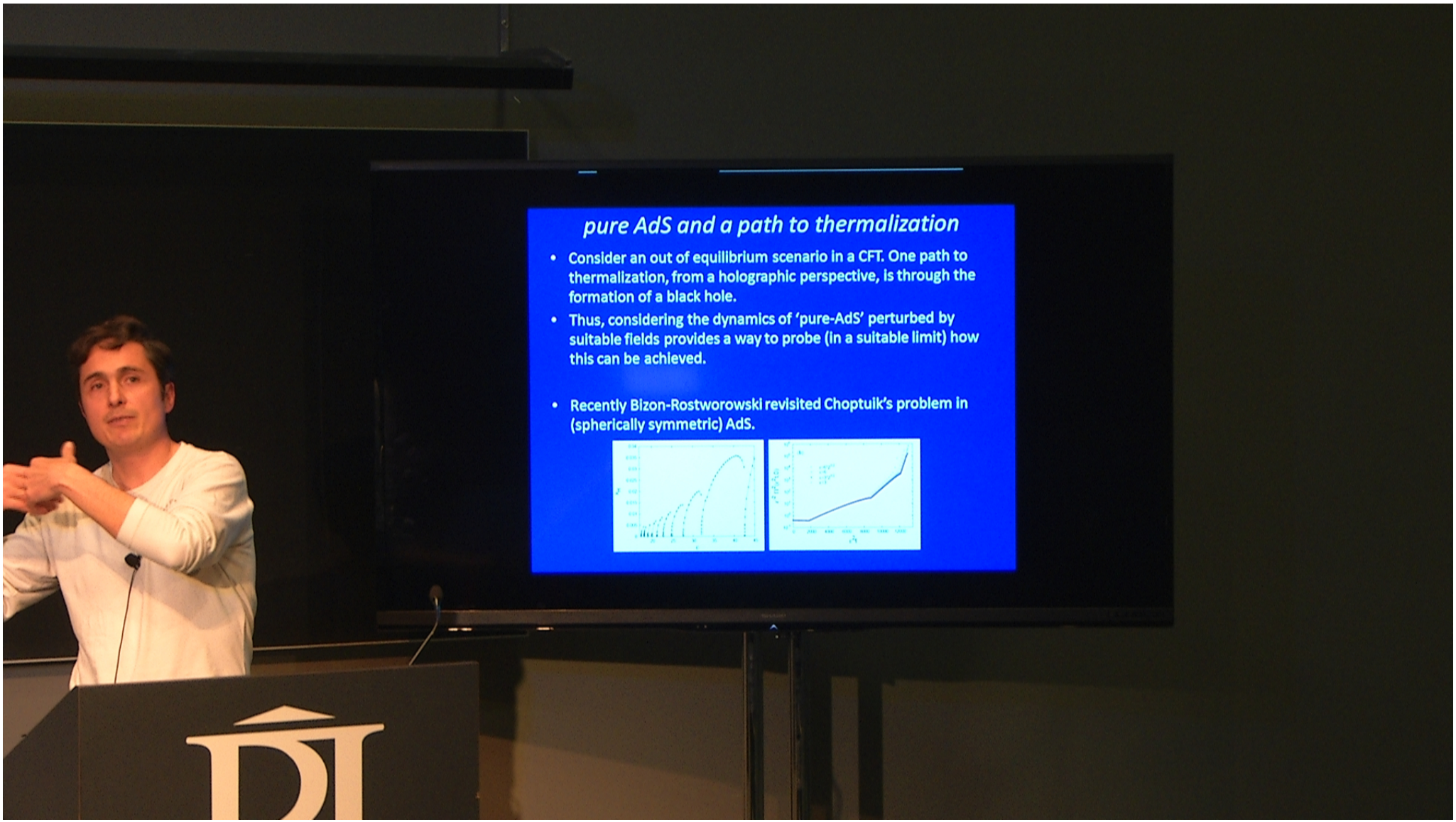
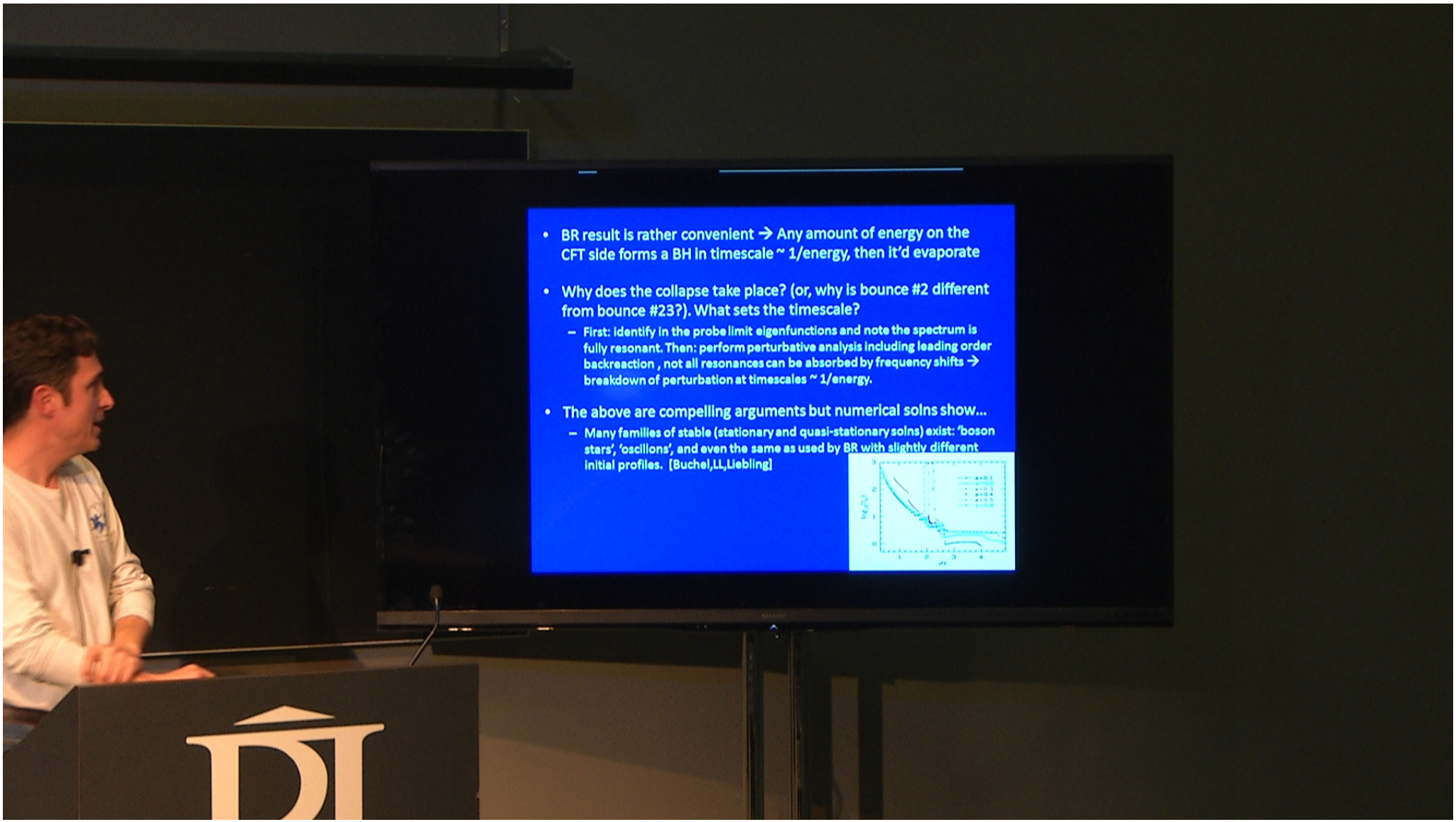

Comments
• Turbulent behaviour can arise in regimes where holography applies → redistribute energy into different modes that would decay at their QNM rates after some transient. IF it thermalizes, the scale will be given by the slowest decaying mode that can be triggered. The overall behaviour (and final fate) is dimensional dependent.
• Further, 'parametric' instability hints turbulence is more generic than the 'long-wavelength perturbation regime' indication
• Also, recall other instabilities can kick in with unclear end-state. Stability of AdS-Kerr (3+1) through NS in 2+1?


pure AdS and a path to thermalization
Consider an out of equilibrium scenario in a CFT. One path to thermalization, from a holographic perspective, is through the formation of a black hole.
Thus, considering the dynamics of 'pure-AdS' perturbed by suitable fields provides a way to probe (in a suitable limit) how this can be achieved.
Recently Bizon-Rostworowski revisited Choptuik's problem in (spherically symmetric) AdS.


BR result is rather convenient → Any amount of energy on the CFT side forms a BH in timescale ~ 1/energy, then it'd evaporate
Why does the collapse take place? (or, why is bounce #2 different from bounce #23?). What sets the timescale?
First: identify in the probe limit eigenfunctions and note the spectrum is fully resonant. Then: perform perturbative analysis including leading order backreaction , not all resonances can be absorbed by frequency shifts → breakdown of perturbation at timescales ~ 1/energy.
The above are compelling arguments but numerical solns show...
Many families of stable (stationary and quasi-stationary solns) exist: 'boson stars', 'oscillons', and even the same as used by BR with slightly different initial profiles. [Buchel,LL,Liebling]

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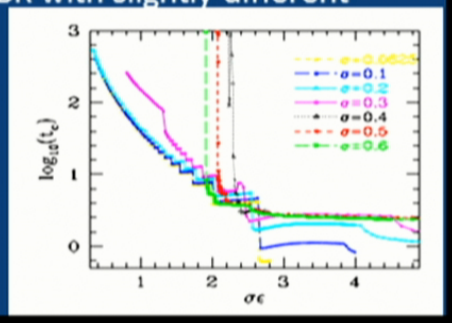

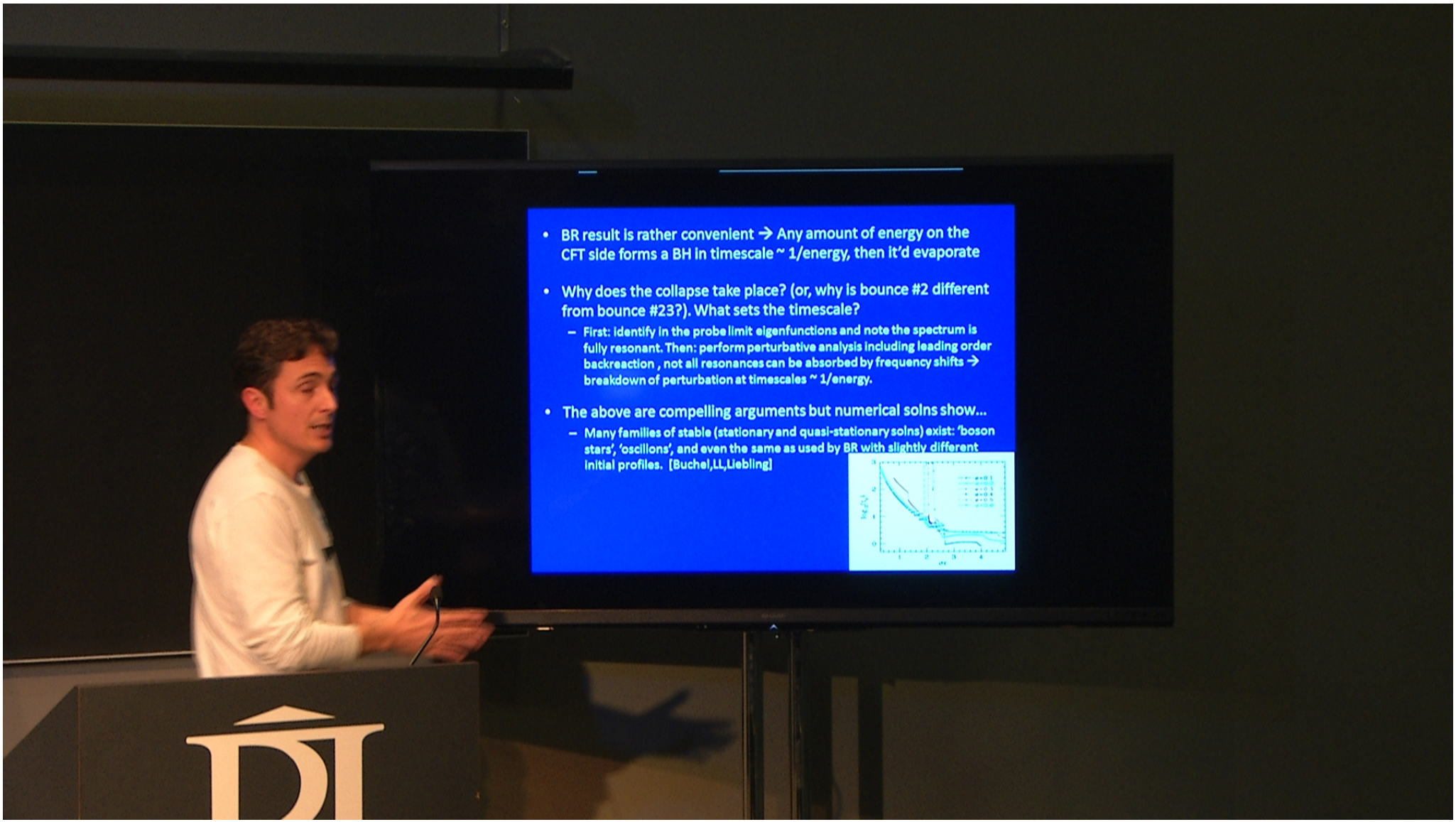
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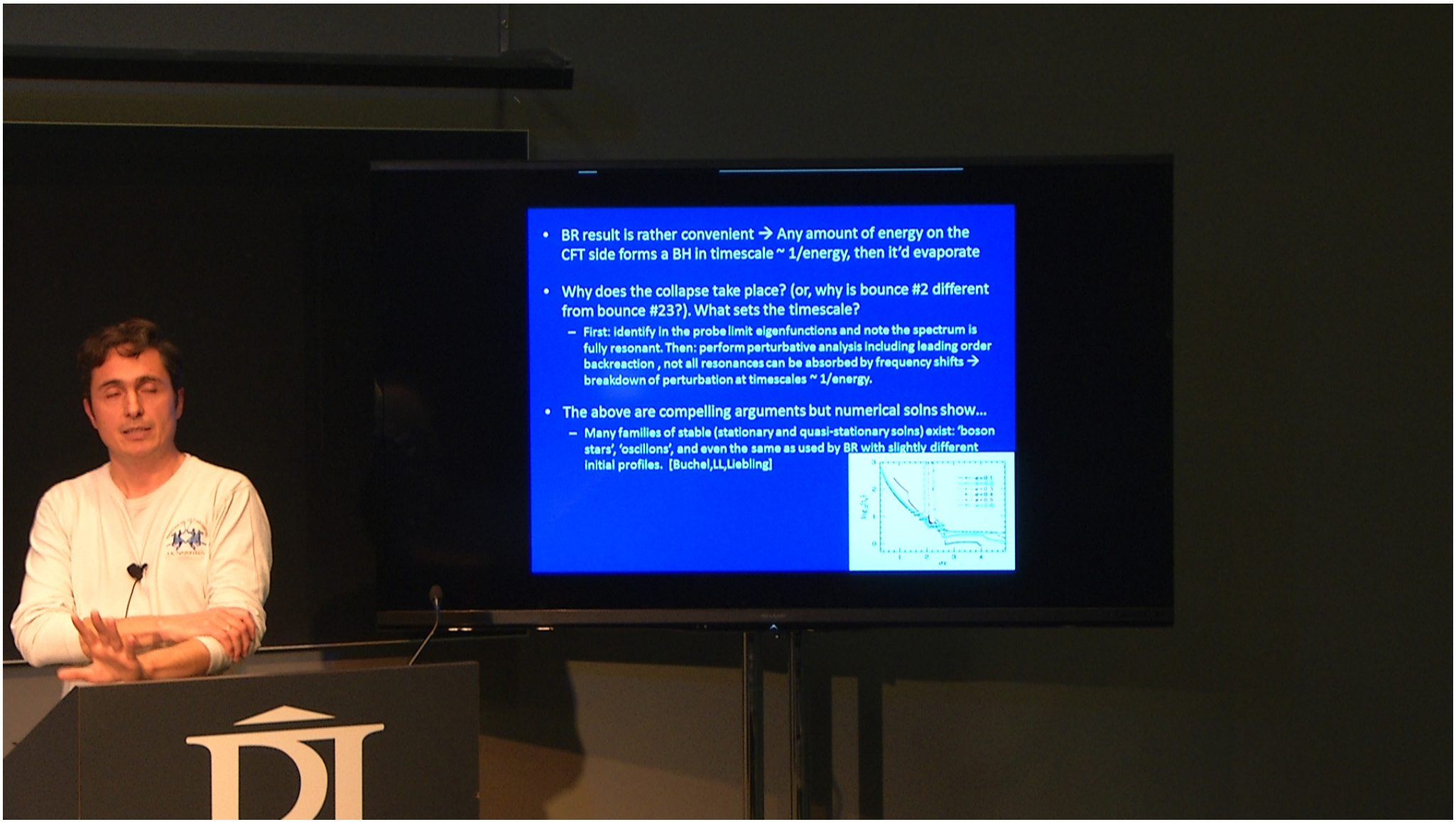
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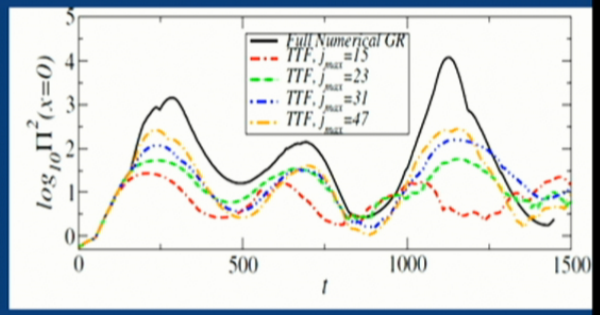
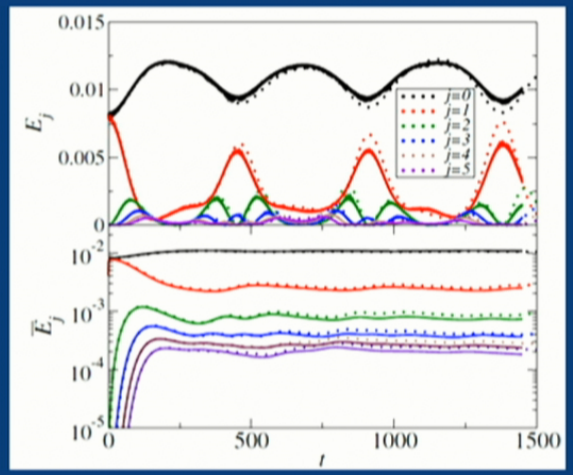
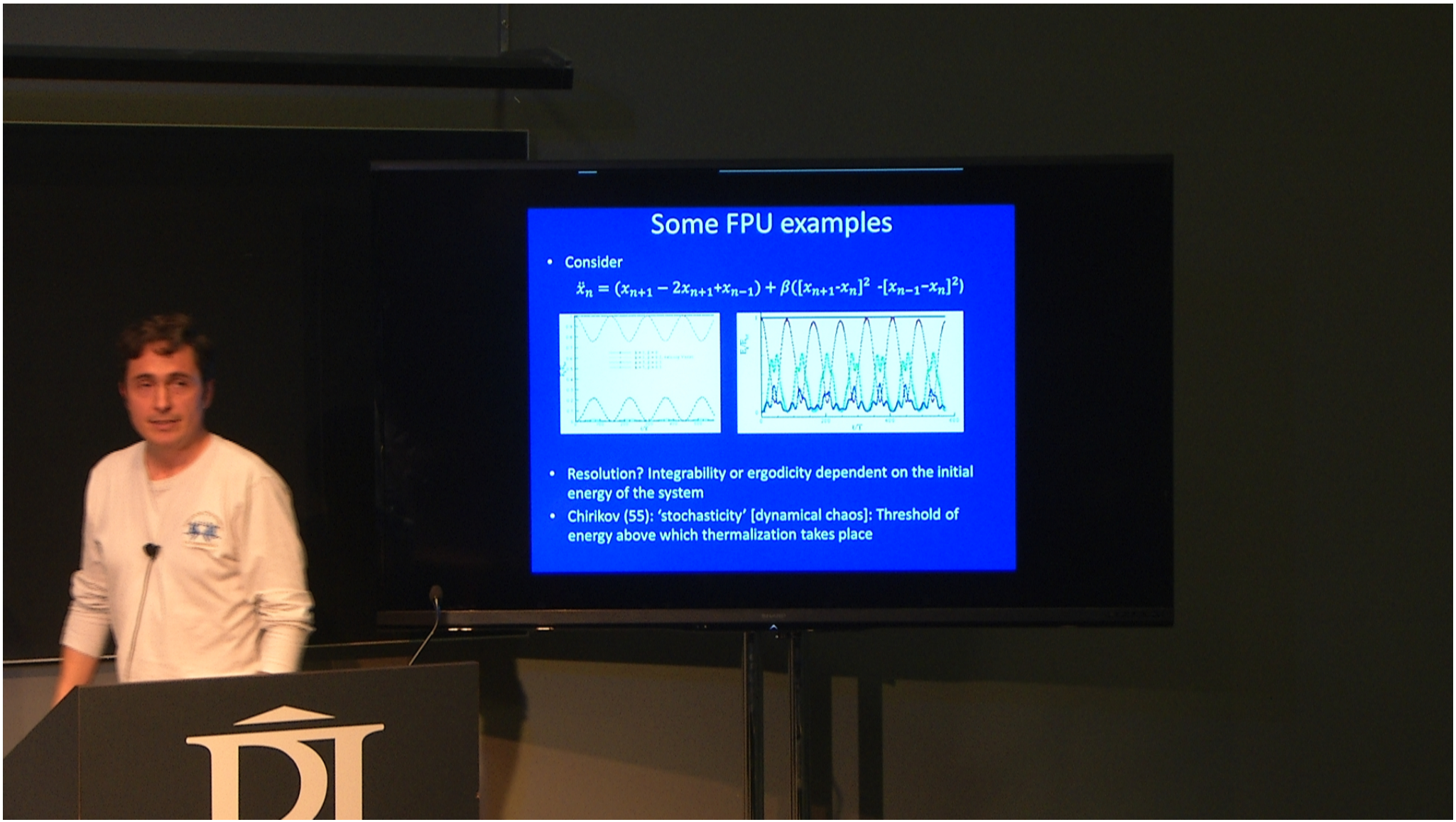
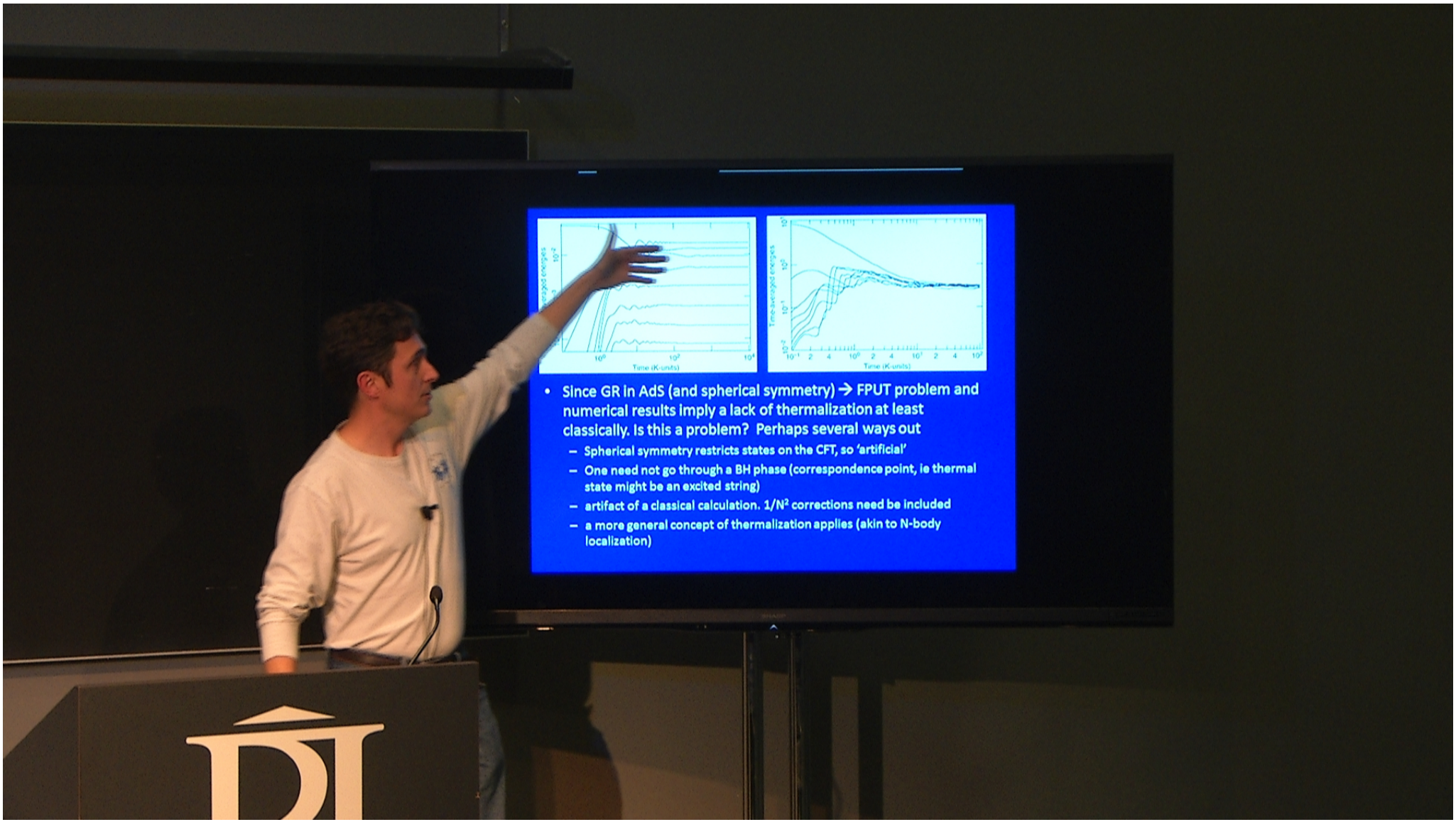
- 'Stable' solutions for small enough amplitude → perturbative analysis should capture the behavior. An improved perturbative analysis shows:
  - Resonances can be completely removed
  - More importantly: eoms are the same as the 'Fermi-Pasta-Ulam (Tsinglou)' problem
  - FPU problem (1953-55): N coupled oscillators with nonlinear interactions which does not display ergodic behavior! [first example of numerical solns opening up a new field]





[Balasubramanian,Buchel,Green,LL,Liebling]


Some FPU examples
• Consider
$\ddot{x}_n = (x_{n+1} - 2x_{n+1} + x_{n-1}) + \beta([x_{n+1} - x_n]^2 - [x_{n-1} - x_n]^2)$
• Resolution? Integrability or ergodicity dependent on the initial energy of the system
• Chirikov (55): 'stochasticity' [dynamical chaos]: Threshold of energy above which thermalization takes place


Time (K-units)
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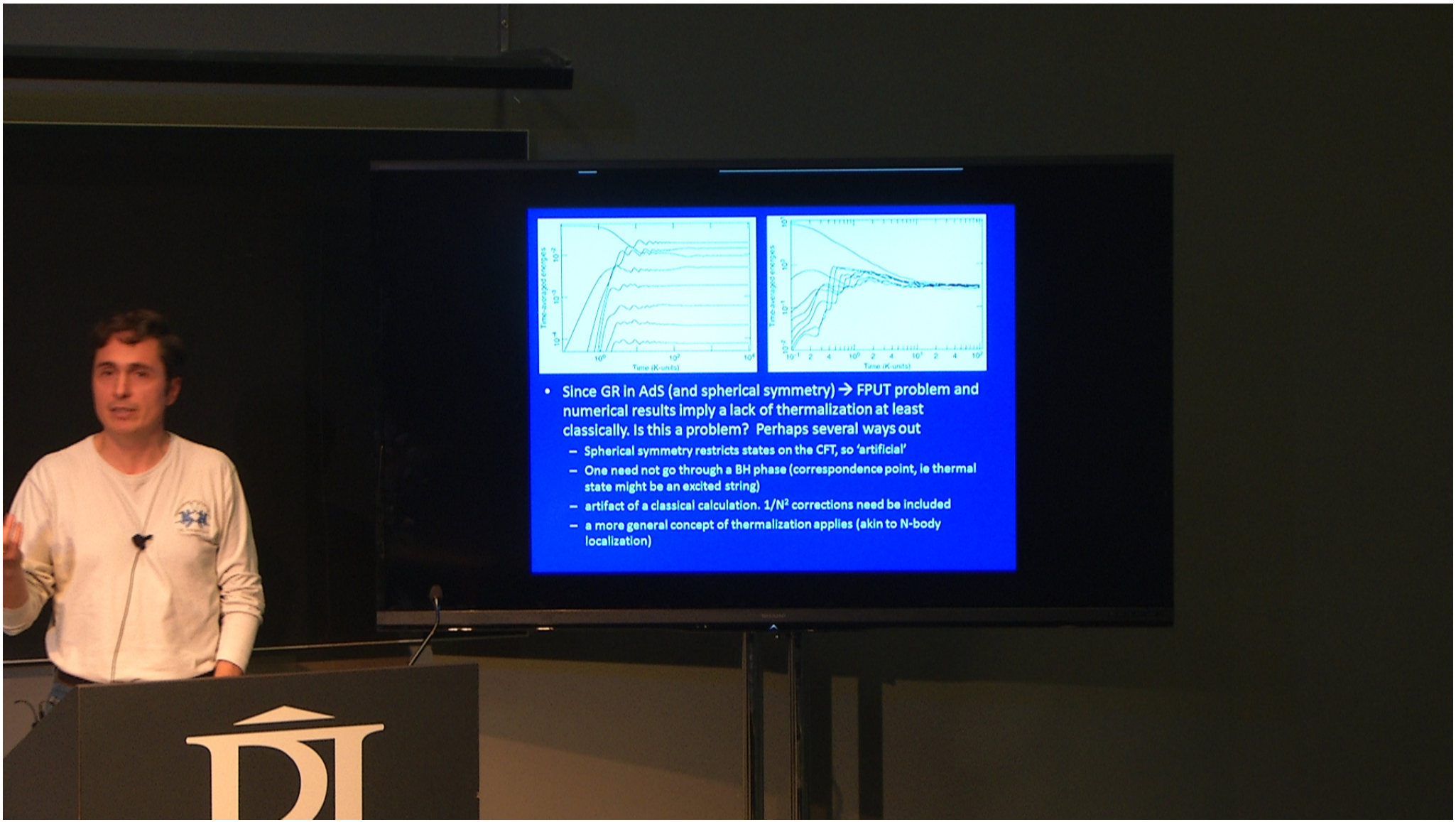
• Since GR in AdS (and spherical symmetry) → FPUT problem and numerical results imply a lack of thermalization at least classically. Is this a problem? Perhaps several ways out
– Spherical symmetry restricts states on the CFT, so 'artificial'
– One need not go through a BH phase (correspondence point, ie thermal state might be an excited string)
– artifact of a classical calculation. 1/N² corrections need be included
– a more general concept of thermalization applies (akin to N-body localization)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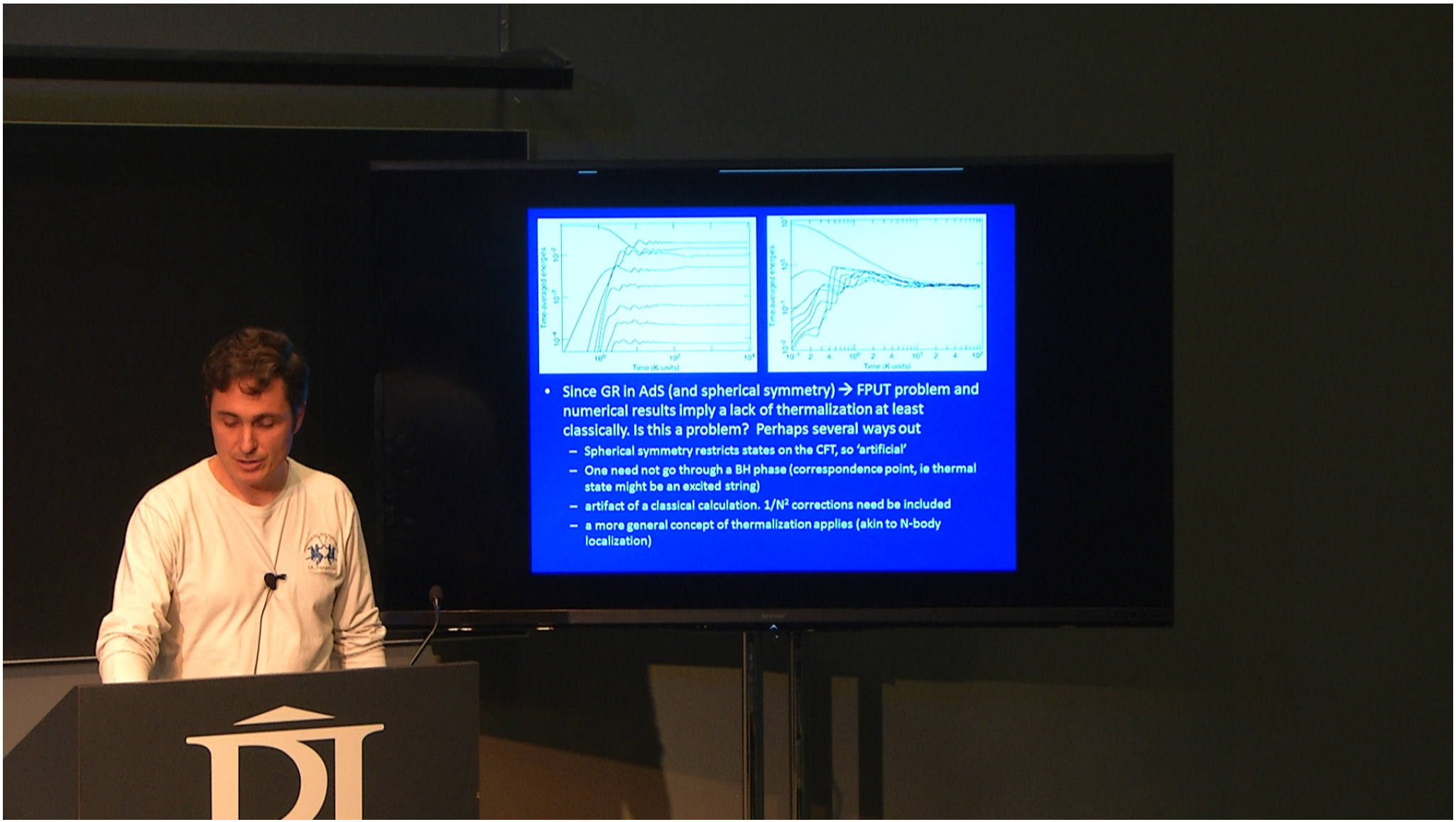
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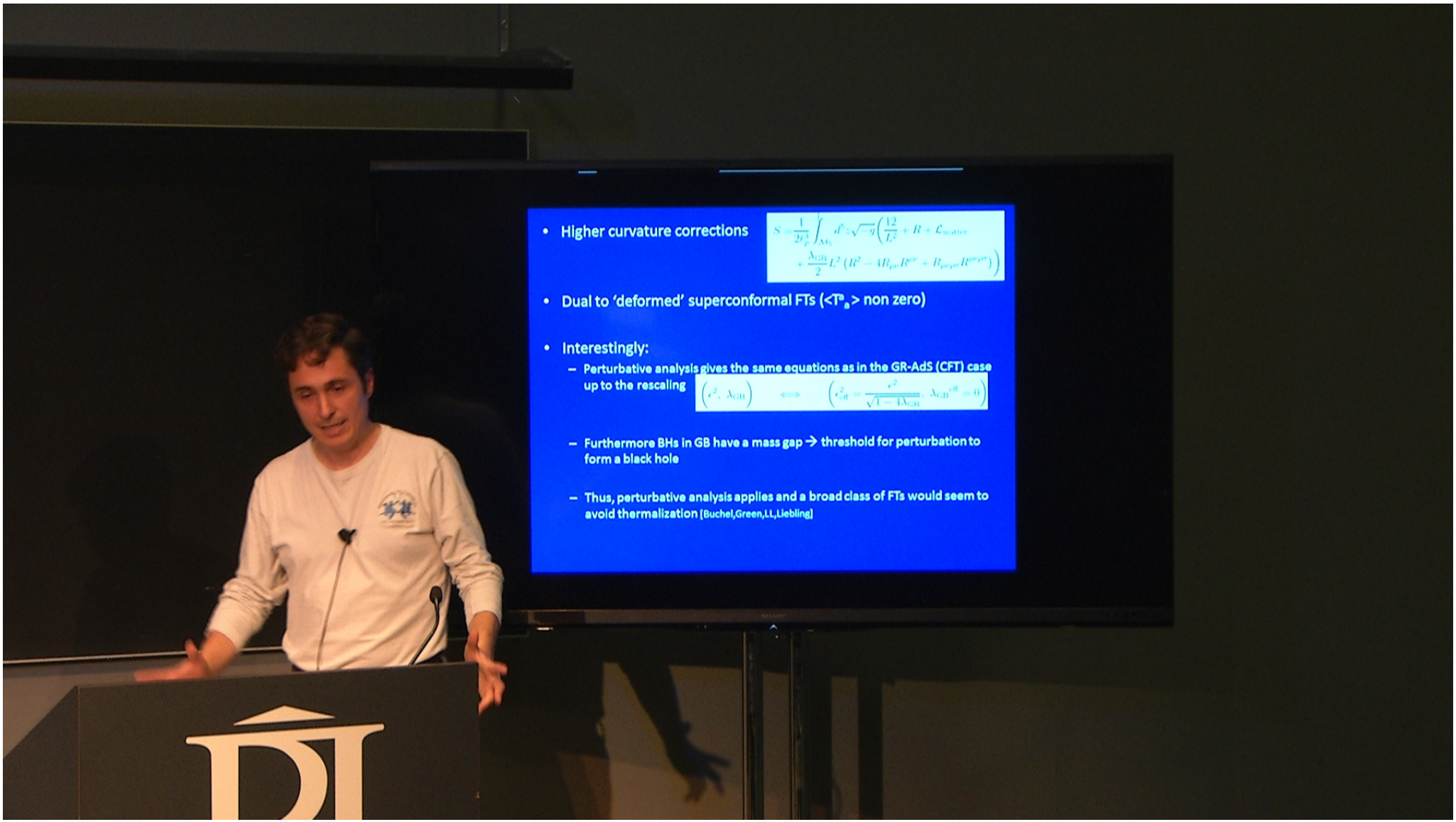
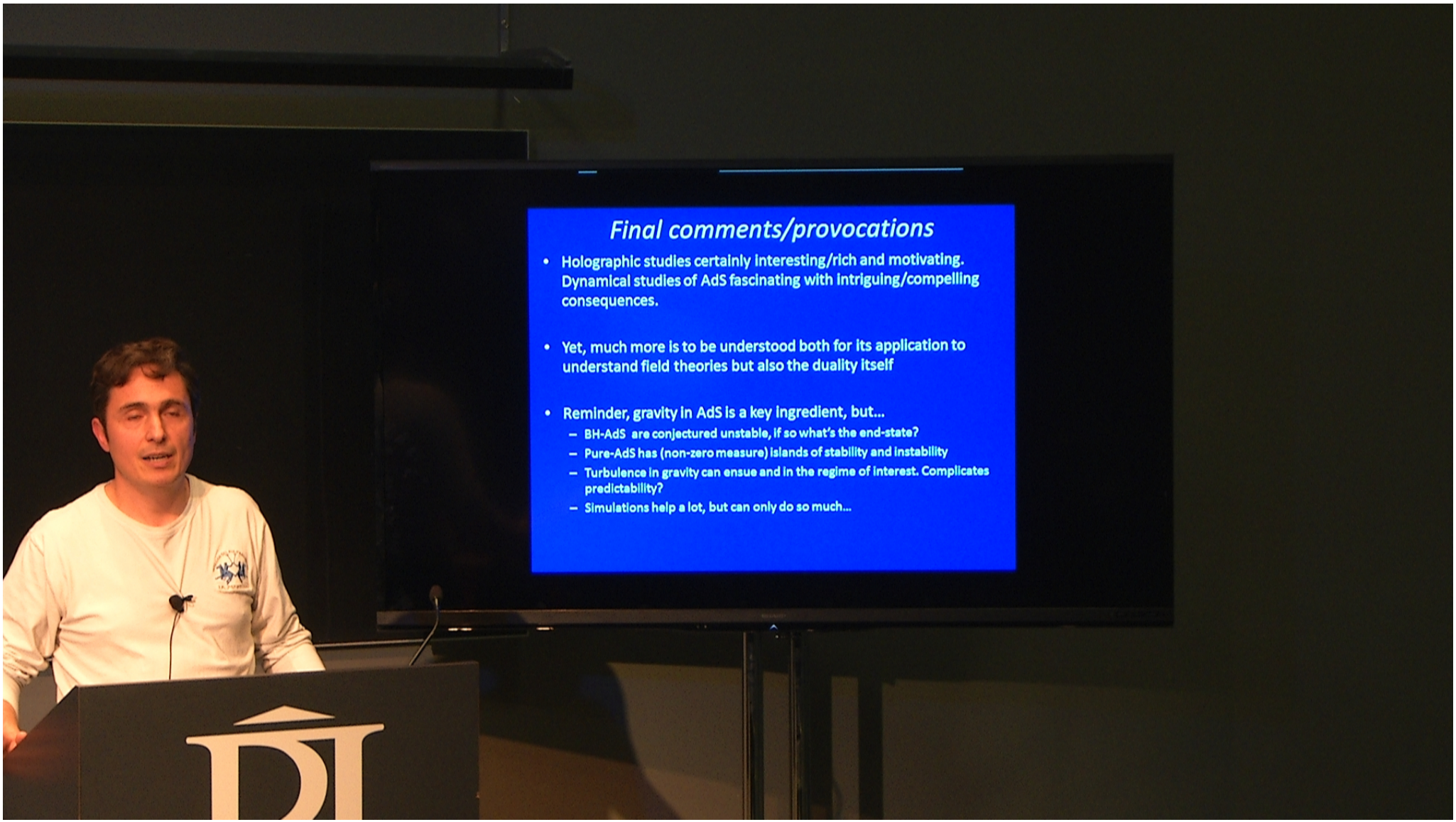

• Higher curvature corrections
$S = \frac{1}{2\ell_p^3}\int_{M_5} d^5x\sqrt{-g}\left(\frac{12}{L^2} + R + \mathcal{L}_{matter} + \frac{\lambda_{GB}}{2}L^2\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right)$
• Dual to 'deformed' superconformal FTs (<T^a_a> non zero)
• Interestingly:
– Perturbative analysis gives the same equations as in the GR-AdS (CFT) case up to the rescaling
$\left(\epsilon^2, \lambda_{GB}\right) \longleftrightarrow \left(\epsilon^2_{eff} = \frac{\epsilon^2}{\sqrt{1-4\lambda_{GB}}}, \lambda_{GB}^{eff} = 0\right)$
– Furthermore BHs in GB have a mass gap → threshold for perturbation to form a black hole
– Thus, perturbative analysis applies and a broad class of FTs would seem to avoid thermalization [Buchel,Green,LL,Liebling]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• Yet, much more is to be understood both for its application to understand field theories but also the duality itself
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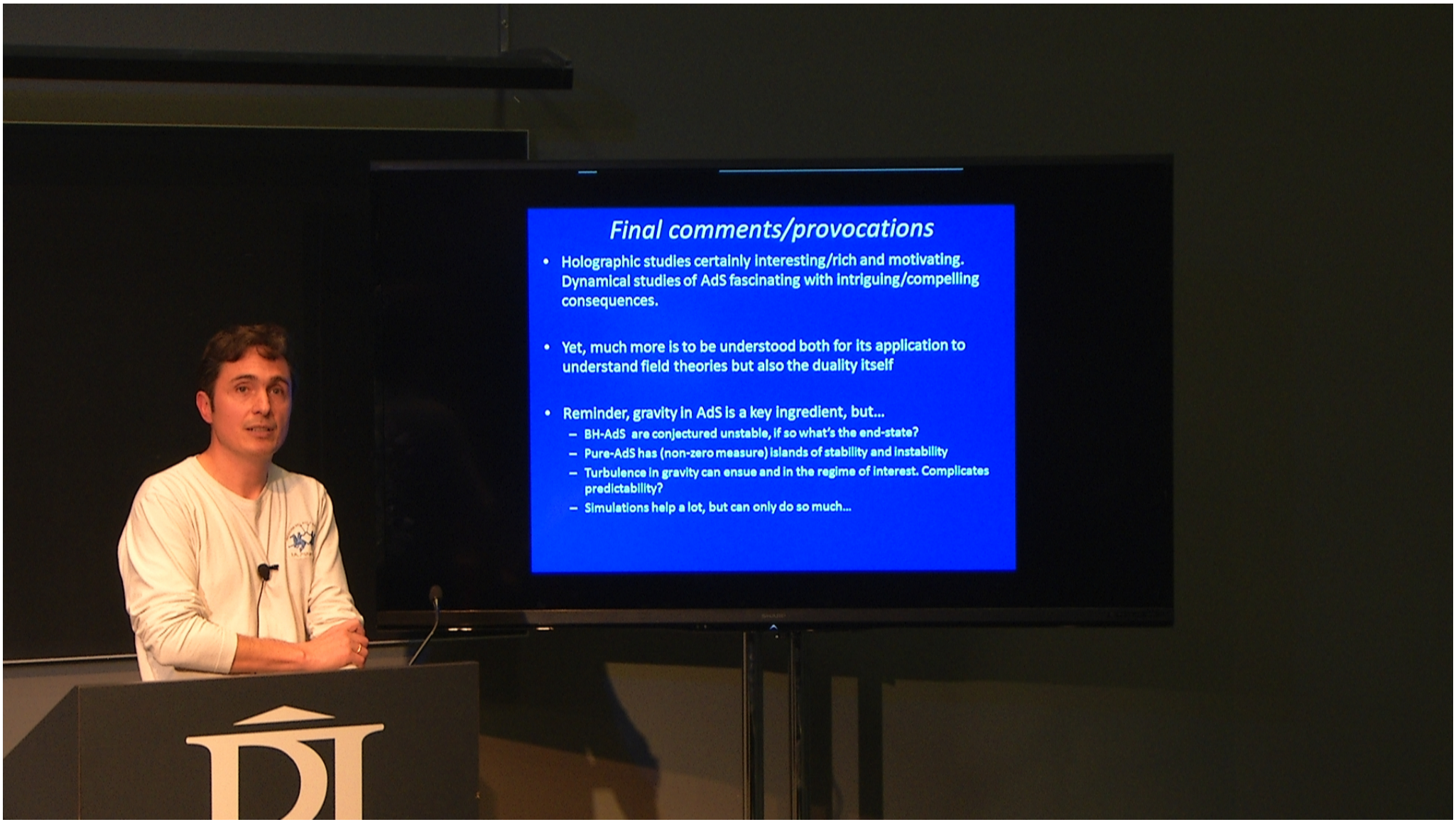
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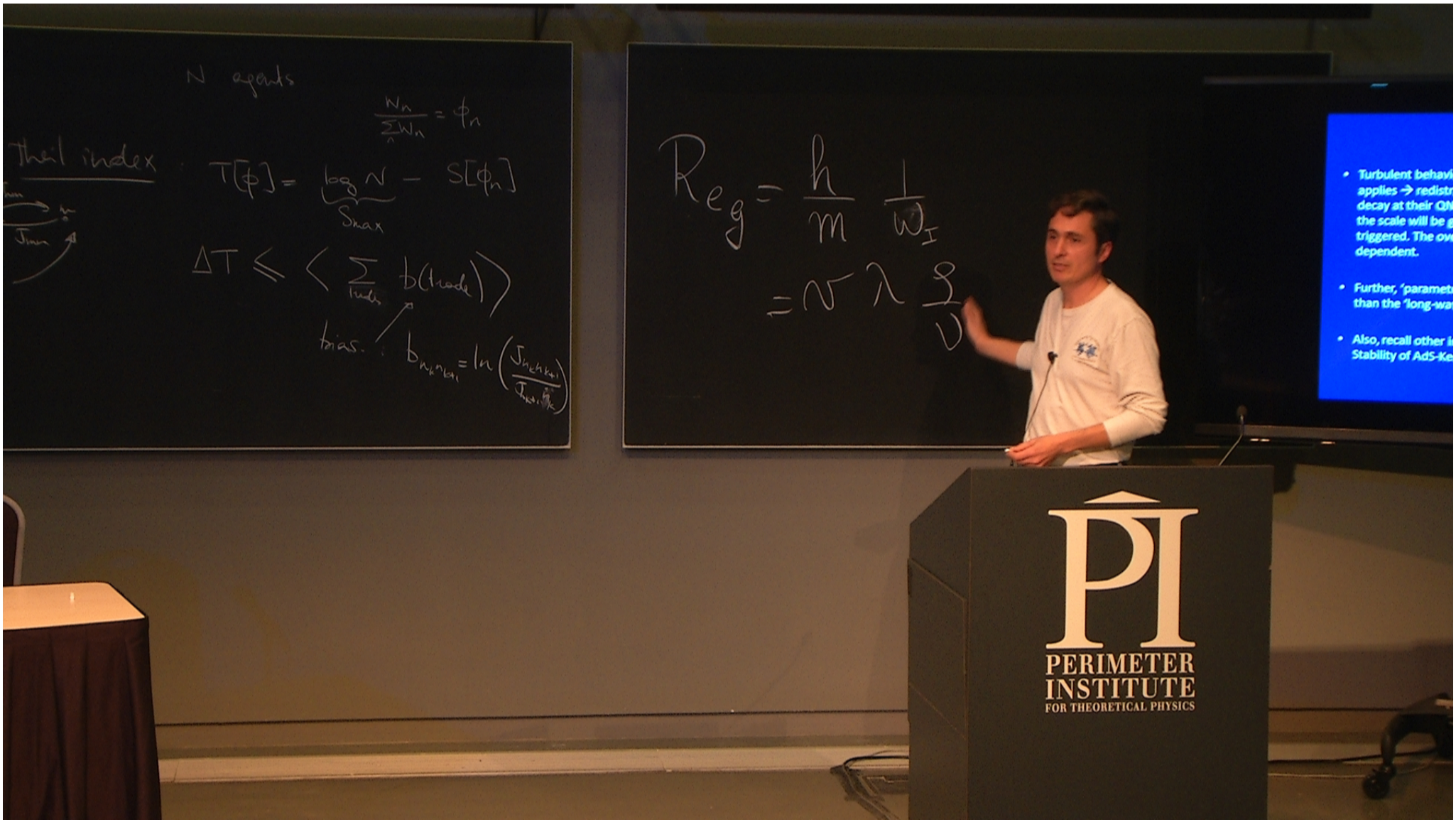
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