Title: Contextuality as a Litmus Test for Quantum Weirdness

Date: Oct 22, 2014 04:30 PM

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Abstract:

Contextuality as a litmus test for quantum weirdness

Matthew Pusey

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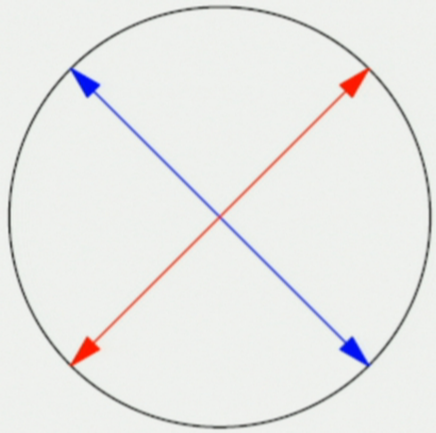
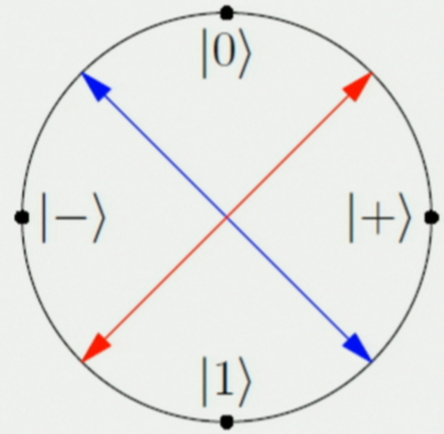
Candidate 1: Incompatibility with diagonal operators



Candidate 2: Incompatibility with non-contextual ontological models



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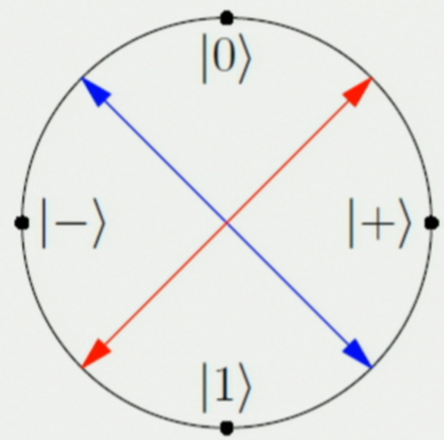
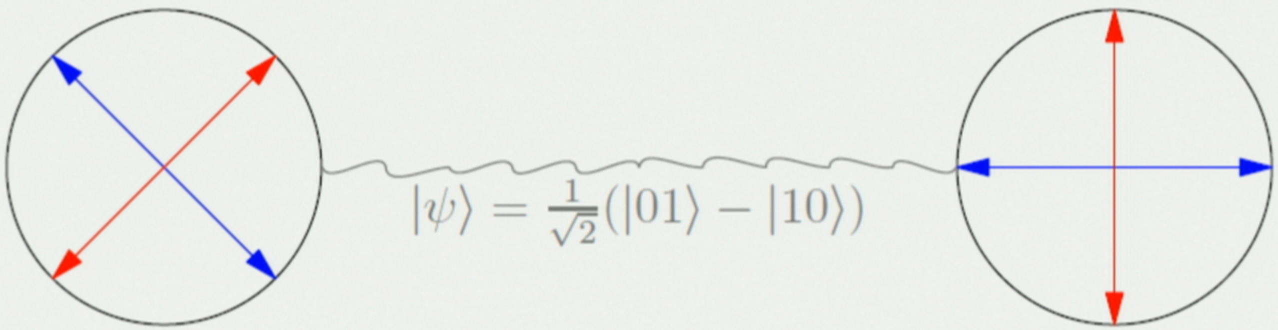


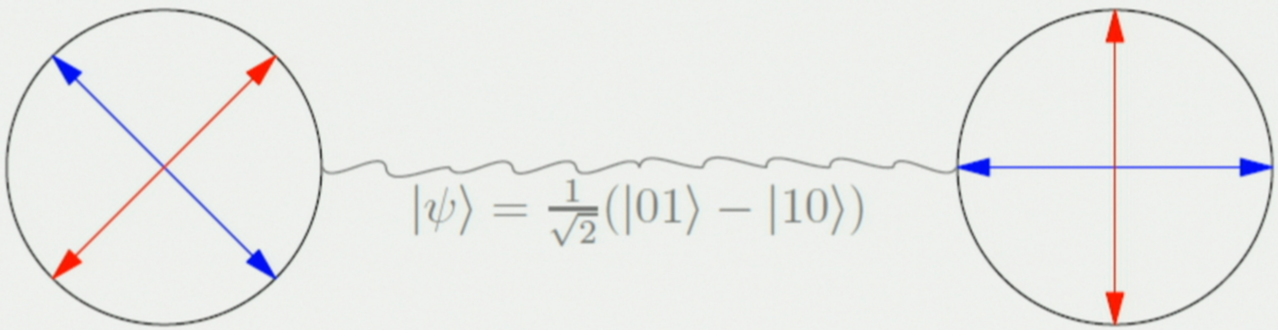
$$|\langle\phi|\psi\rangle|^2 = \int p(\phi|\lambda)p(\lambda|\psi)d\lambda$$

$$\frac{1}{2}p(\lambda|0) + \frac{1}{2}p(\lambda|1) = \frac{1}{2}p(\lambda|+) + \frac{1}{2}p(\lambda|-)$$

Candidate 3: Incompatibility with Bell-local models



Candidate 3: Incompatibility with Bell-local models



A weirdness hierarchy

$$3 \implies 2 \implies 1$$

Comparison

Candidate	1	2	3
Incompatibility with...	Diagonal operators	Non-contextual models	Bell-local models
Jargon	Coherence	Contextuality	Nonlocality

Comparison

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Incompatibility with...	Diagonal operators	Non-contextual models	Bell-local models
Jargon	Coherence	Contextuality	Nonlocality
Shock value	Low[1]	Medium	High

[1] e.g. R. W. Spekkens, PRA 75, 032110

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Abundance	High	Medium	Low
Independent of quantum formalism	No	Yes	Yes

[1] e.g. R. W. Spekkens, PRA 75, 032110

Comparison

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Incompatibility with...	Diagonal operators	Non-contextual models	Bell-local models
Jargon	Coherence	Contextuality	Nonlocality
Shock value	Low[1]	Medium	High
Abundance	High	Medium	Low
Independent of quantum formalism	No	Yes	Yes
Applicability	Very wide	Wide	Narrow

[1] e.g. R. W. Spekkens, PRA 75, 032110

Principle of non-contextuality

No distinctions without a difference.
OR
The identity of indiscernibles.

Principle of non-contextuality

No ontological distinctions without an operational difference.
OR
The ontological identity of operational indiscernibles.

Ontological models of QT

$$\mathrm{Tr}(E\rho) = \int p(E|\lambda, C_{\mathsf{meas}})p(\lambda|\rho, C_{\mathsf{prep}})d\lambda$$

Non-contextual ontological models[2]

$$\mathrm{Tr}(E\rho) = \int p(E|\lambda)p(\lambda|\rho)d\lambda$$

[2] R.W. Spekkens, PRA 71, 052108

E.g. preparation contexts

$$\sum_i p_i \left|\psi_i\right\rangle \left\langle\psi_i\right| = \rho = \sum_j q_j \left|\phi_j\right\rangle \left\langle\phi_j\right|$$

E.g. preparation contexts

$$\sum_i p_i |\psi_i\rangle \langle\psi_i| = \rho = \sum_j q_j |\phi_j\rangle \langle\phi_j|$$

$$p(\lambda|\rho, C_1) = \sum_i p_i p(\lambda|\psi_i)$$

$$p(\lambda|\rho, C_2) = \sum_j q_j p(\lambda|\phi_j)$$

E.g. preparation non-contextuality

$$\sum_i p_i |\psi_i\rangle\langle\psi_i| = \rho = \sum_j q_j |\phi_j\rangle\langle\phi_j|$$

$$\Longrightarrow \sum_i p_i p(\lambda|\psi_i) = p(\lambda|\rho) = \sum_j q_j p(\lambda|\phi_j)$$

The utility of non-contextuality

I claim: non-contextuality is a necessary condition for the explanation offered by an ontological model to be physically plausible.

The utility of non-contextuality

I tentatively claim: non-contextuality is a necessary *and sufficient* condition for the explanation offered by an ontological model to be physically plausible.

Examples: multiplexing

PRL **102,** 010401 (2009) PHYSICAL REVIEW LETTERS week ending 9 JANUARY 2009

Preparation Contextuality Powers Parity-Oblivious Multiplexing

Robert W. Spekkens,[1] D. H. Buzacott,[2,3] A. J. Keehn,[2,3] Ben Toner,[4] and G. J. Pryde[2,3]

[1]*DAMTP, University of Cambridge, Cambridge, CB3 0WA, United Kingdom*
[2]*Centre for Quantum Computer Technology, Griffith University, Brisbane 4111, Australia*
[3]*Centre for Quantum Dynamics, Griffith University, Brisbane 4111, Australia*
[4]*Centrum voor Wiskunde en Informatica, Kruislaan 413, 1098 SJ Amsterdam, The Netherlands*

(Received 12 May 2008; published 5 January 2009)

In a noncontextual hidden variable model of quantum theory, hidden variables determine the outcomes of every measurement in a manner that is independent of how the measurement is implemented. Using a generalization of this notion to arbitrary operational theories and to preparation procedures, we demonstrate that a particular two-party information-processing task, "parity-oblivious multiplexing," is powered by contextuality in the sense that there is a limit to how well any theory described by a noncontextual hidden variable model can perform. This bound constitutes a "noncontextuality inequality" that is violated by quantum theory. We report an experimental violation of this inequality in good agreement with the quantum predictions. The experimental results also provide the first demonstration of 2-to-1 and 3-to-1 quantum random access codes.

DOI: 10.1103/PhysRevLett.102.010401 PACS numbers: 03.65.Ta, 03.67.−a, 42.50.Dv, 42.50.Ex

The Bell-Kochen-Specker theorem [1] shows that the

The NC inequality we derive provides a bound on the

Examples: measurement-based QC

PHYSICAL REVIEW A **88**, 022322 (2013)

Contextuality in measurement-based quantum computation

Robert Raussendorf[*]
Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia V6T 1Z1, Canada
(Received 1 May 2013; revised manuscript received 11 July 2013; published 19 August 2013)

We show, under natural assumptions for qubit systems, that measurement-based quantum computations (MBQCs) which compute a nonlinear Boolean function with a high probability are contextual. The class of contextual MBQCs includes an example which is of practical interest and has a superpolynomial speedup over the best-known classical algorithm, namely, the quantum algorithm that solves the "discrete log" problem.

DOI: 10.1103/PhysRev.

I. INTRODUC

While numerous quantum alg
that offer polynomial or superpoly
classical counterparts [1–3], the pr
origin of this speedup remains
candidates—entanglement [4], sup
[5], and largeness of Hilbert sp
understanding in many situation

PHYSICAL REVIEW A **84**, 062107 (2011)

Generalized Bell-inequality experiments and computation

Matty J. Hoban,[1,2] Joel J. Wallman,[3] and Dan E. Browne[1]
[1]*Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom*
[2]*Department of Computer Science, University of Oxford, Wolfson Building, Parks Road, Oxford OX1 3QD, United Kingdom*
[3]*School of Physics, The University of Sydney, Sydney, New South Wales 2006, Australia*
(Received 25 August 2011; published 6 December 2011)

We consider general settings of Bell inequality experiments with many parties, where each party chooses from a finite number of measurement settings each with a finite number of outcomes. We investigate the constraints that Bell inequalities place upon the correlations possible in local hidden variable theories using a geometrical picture of correlations. We show that local hidden variable theories can be characterized in terms of limited computational expressiveness, which allows us to characterize families of Bell inequalities. The limited computational expressiveness for many settings (each with many outcomes) generalizes previous results about the many-party situation each with a choice of two possible measurements (each with two outcomes). Using this computational picture we present generalizations of the Popescu-Rohrlich nonlocal box for many parties and nonbinary inputs and outputs at each site. Finally, we comment on the effect of preprocessing on measurement data in our generalized setting and show that it becomes problematic outside of the binary setting, in that it allows local hidden variable theories to simulate maximally nonlocal correlations such as those of these generalized Popescu-Rohrlich nonlocal boxes.

DOI: 10.1103/PhysRevA.84.062107 PACS number(s): 03.65.Ud, 03.67.Lx, 02.10.De

Examples: magic state QC

ARTICLE

doi:10.1038/nature13460

Contextuality supplies the 'magic' for quantum computation

Mark Howard[1,2], Joel Wallman[2], Victor Veitch[2,3] & Joseph Emerson[2]

Quantum computers promise dramatic advantages over their classical counterparts, but the source of the power in quantum computing has remained elusive. Here we prove a remarkable equivalence between the onset of contextuality and the possibility of universal quantum computation via 'magic state' distillation, which is the leading model for experimentally realizing a fault-tolerant quantum computer. This is a conceptually satisfying link, because contextuality, which precludes a simple 'hidden variable' model of quantum mechanics, provides one of the fundamental characterizations of uniquely quantum phenomena. Furthermore, this connection suggests a unifying paradigm for the resources of quantum information: the non-locality of quantum theory is a particular kind of contextuality, and non-locality is already known to be a critical resource for achieving advantages with quantum communication. In addition to clarifying these fundamental issues, this work advances the resource framework for quantum computation, which has a number of practical

Anomalous weak values

VOLUME 60, NUMBER 14 PHYSICAL REVIEW LETTERS 4 APRIL 1988

How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman

Physics Department, University of South Carolina, Columbia, South Carolina 29208, and School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel

(Received 30 June 1987)

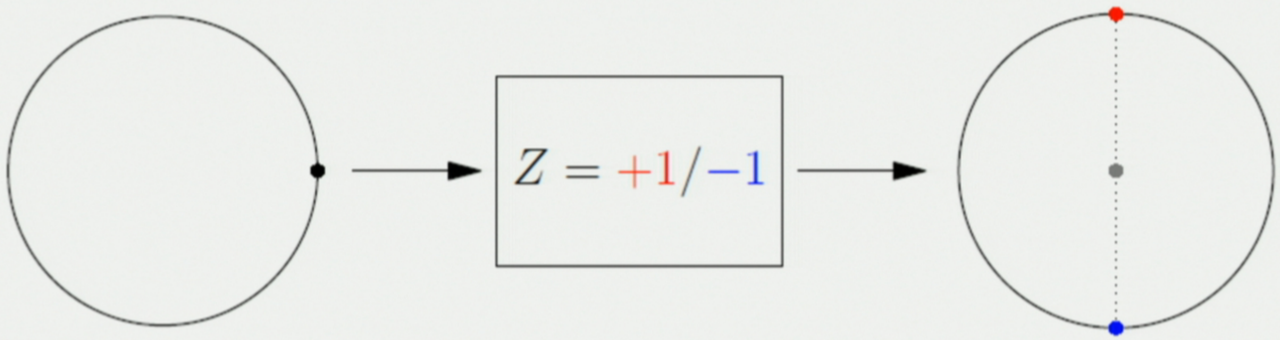
We have found that the usual measuring procedure for preselected and postselected ensembles of quantum systems gives unusual results. Under some natural conditions of weakness of the measurement, its result consistently defines a new kind of value for a quantum variable, which we call the weak value. A description of the measurement of the weak value of a component of a spin for an ensemble of preselected and postselected spin-$\frac{1}{2}$ particles is presented.

PACS numbers: 03.65.Bz

This paper will describe an experiment which measures a spin component of a spin-$\frac{1}{2}$ particle and yields a result which is far from the range of "allowed" values. We shall start with a brief description of the standard

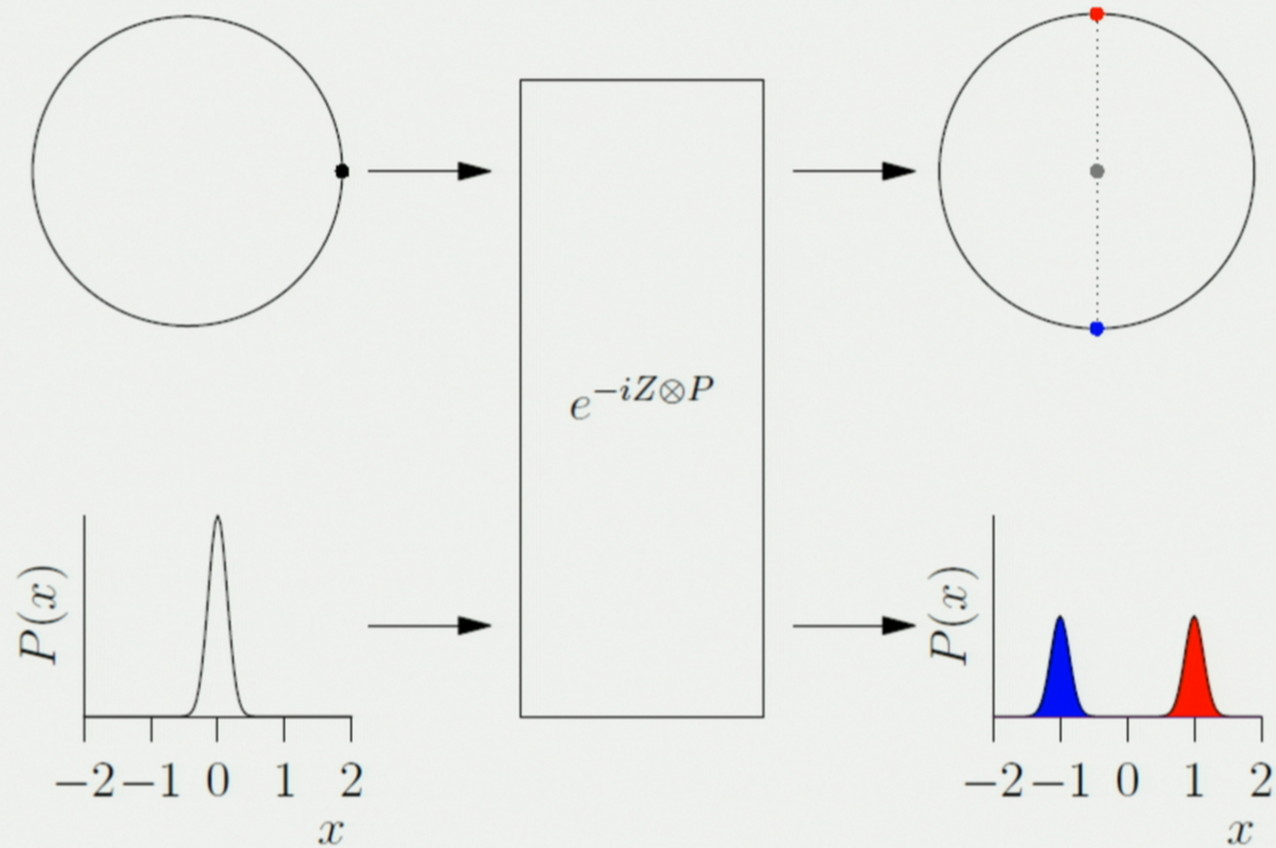
a_i, the final probability distribution will be again close to a Gaussian with the spread $\Delta\pi$. The center of the Gaussian will be at the mean value of A: $\langle A\rangle = \sum_i |\alpha_i|^2 a_i$. One measurement like this will give no information be-

Strong measurements

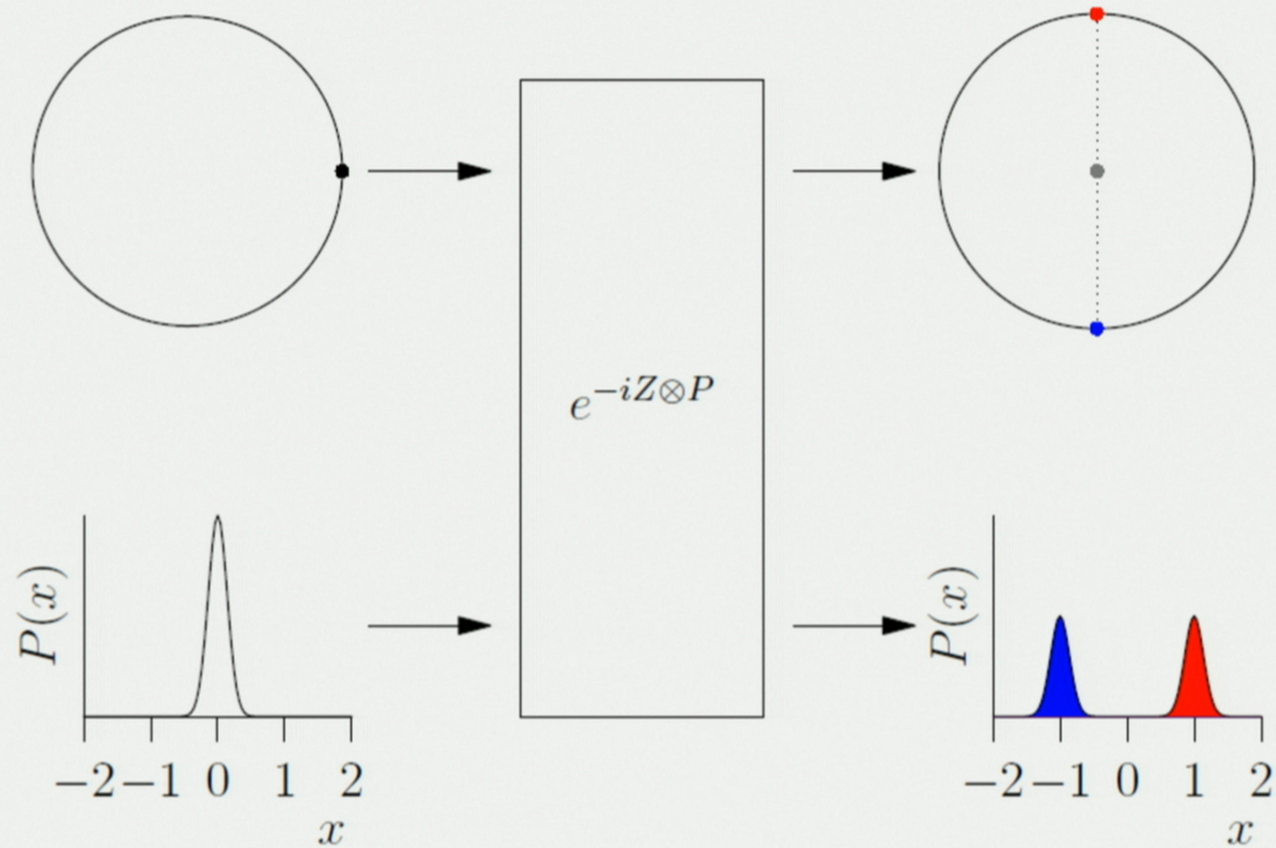


"Pointer"

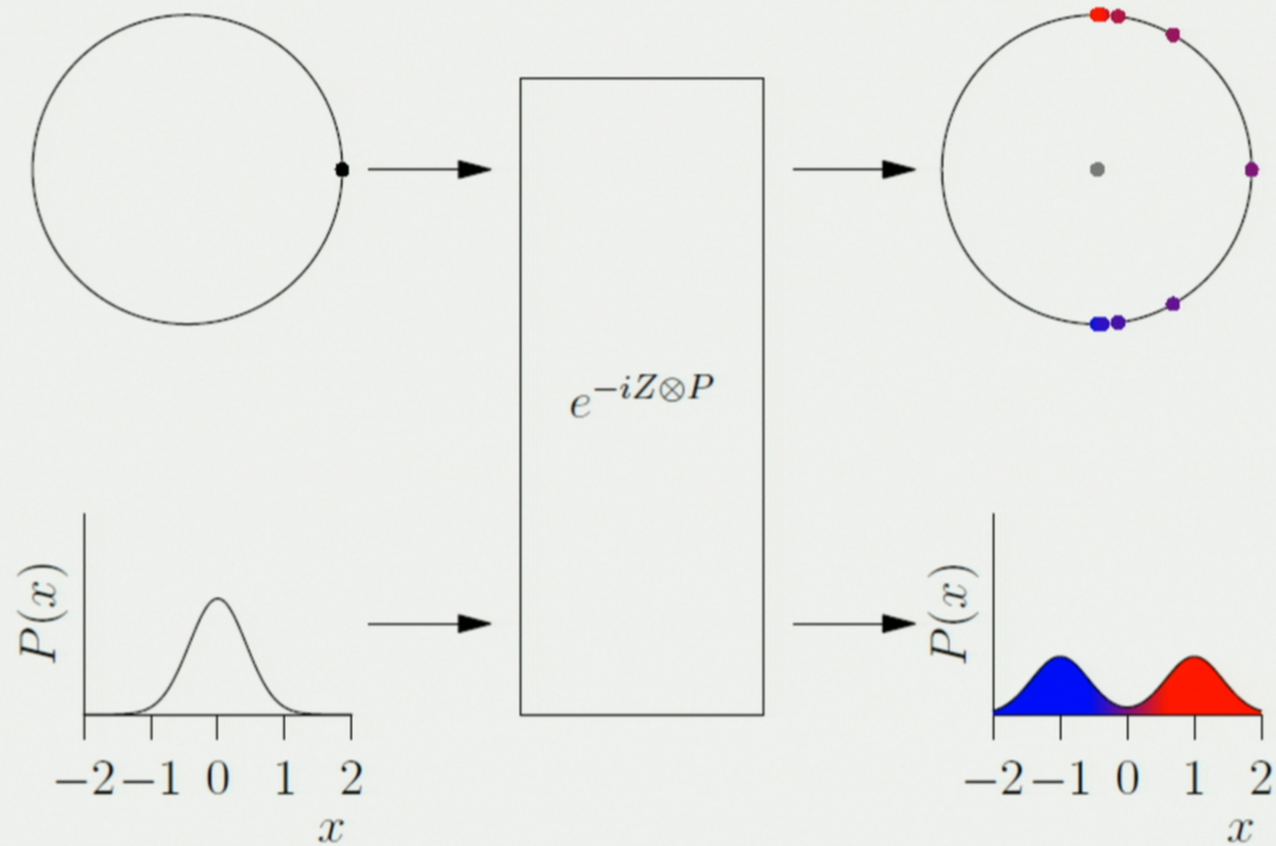
Strong measurements



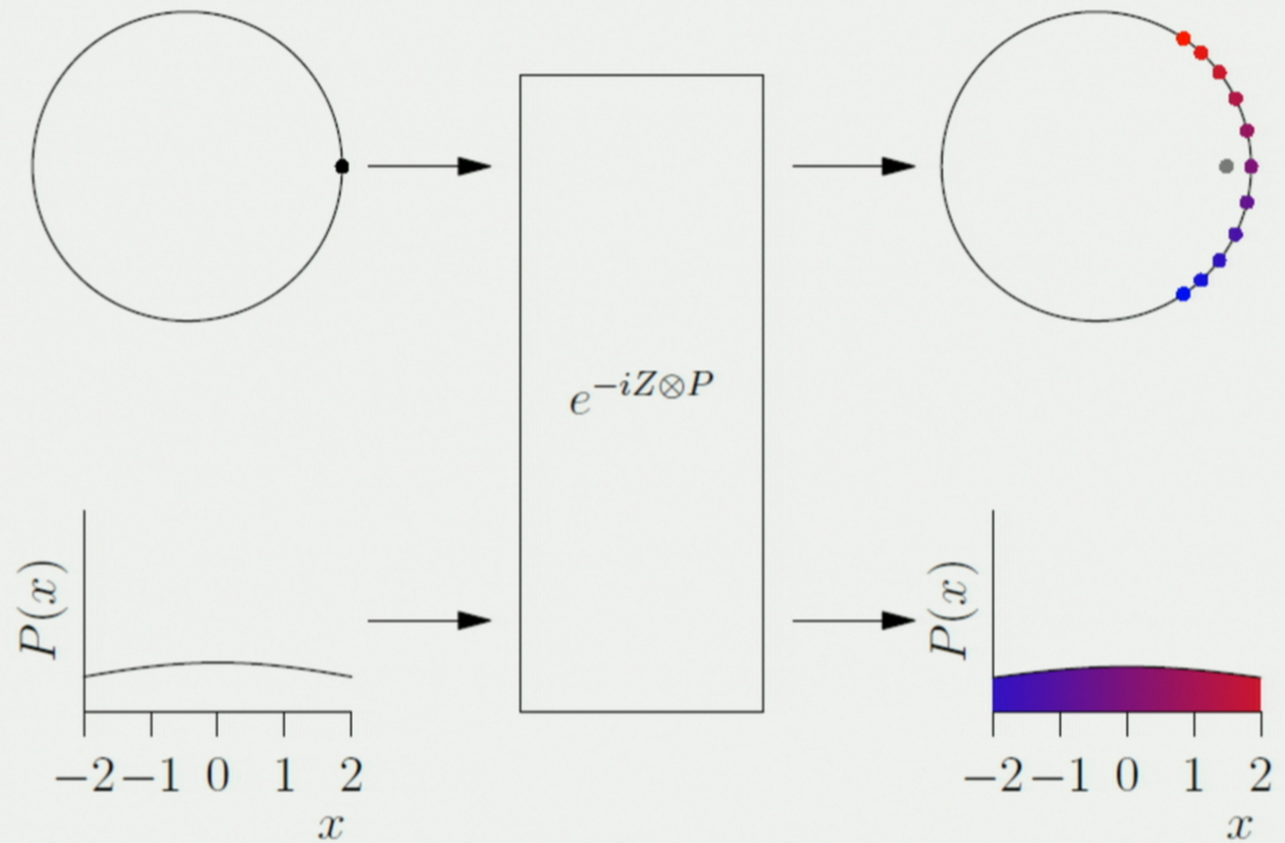
Strong measurements

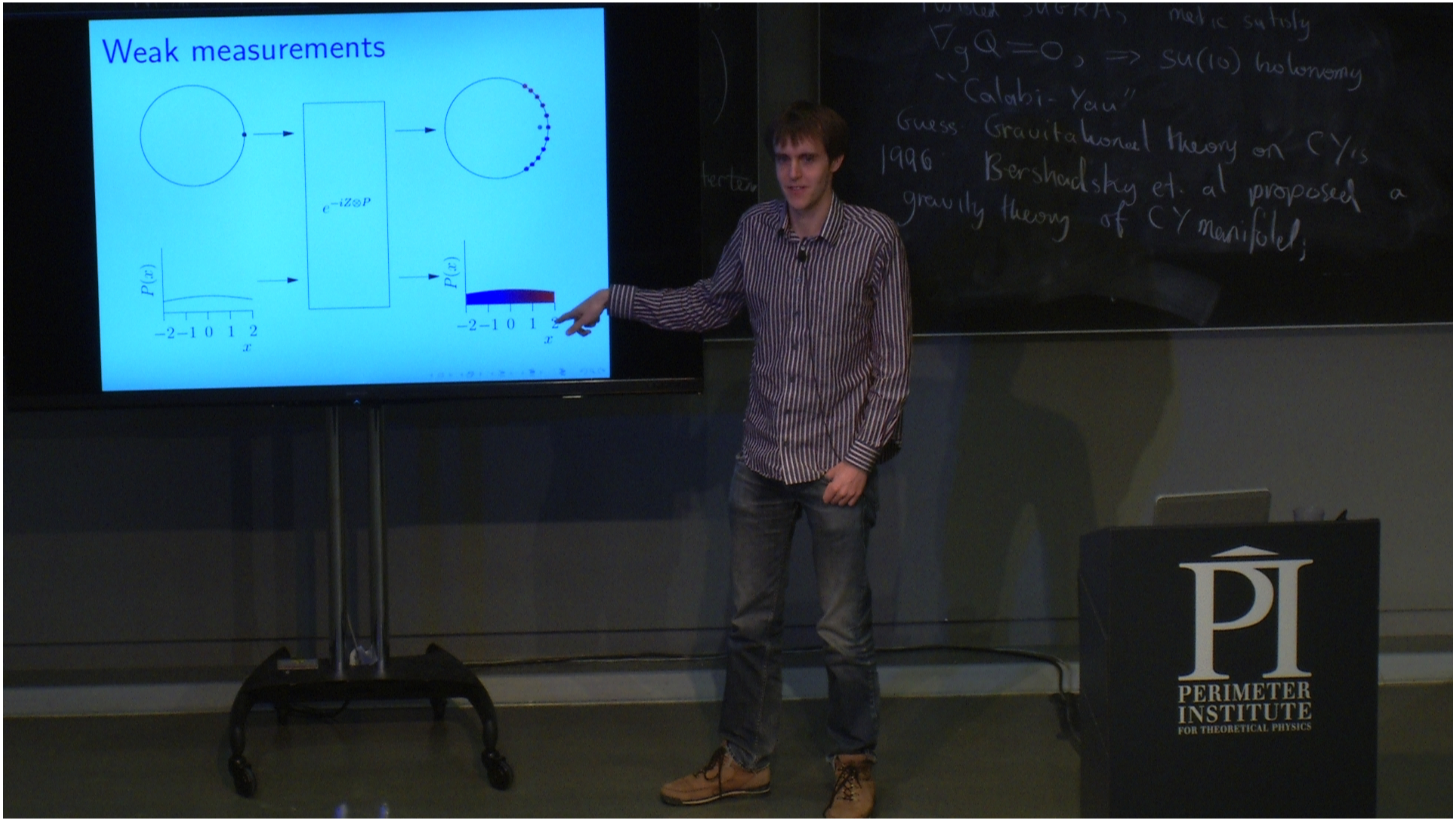


Fairly strong measurements



Weak measurements




Weak measurements
$e^{-iZ\otimes P}$
$P(x)$
−2 −1 0 1 2
x
$P(x)$
−2 −1 0 1 2
x
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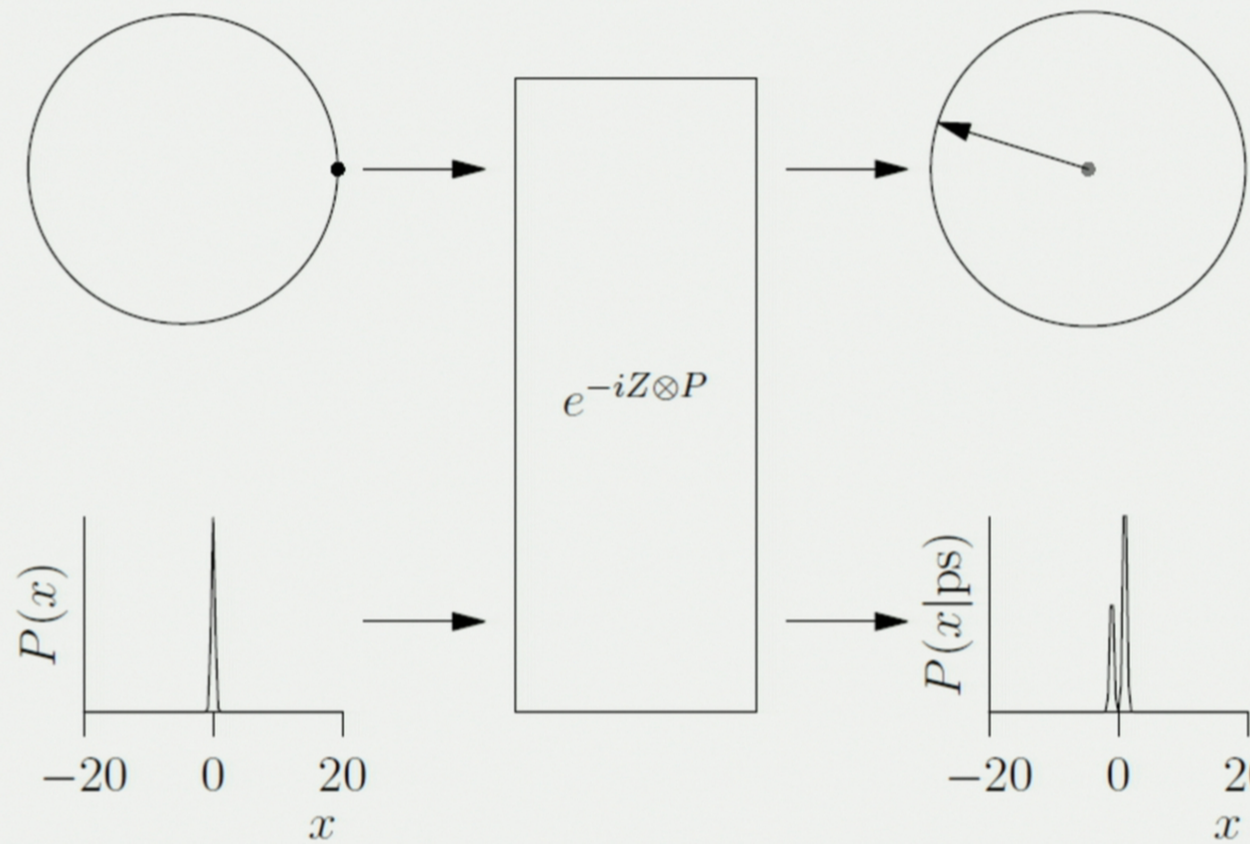
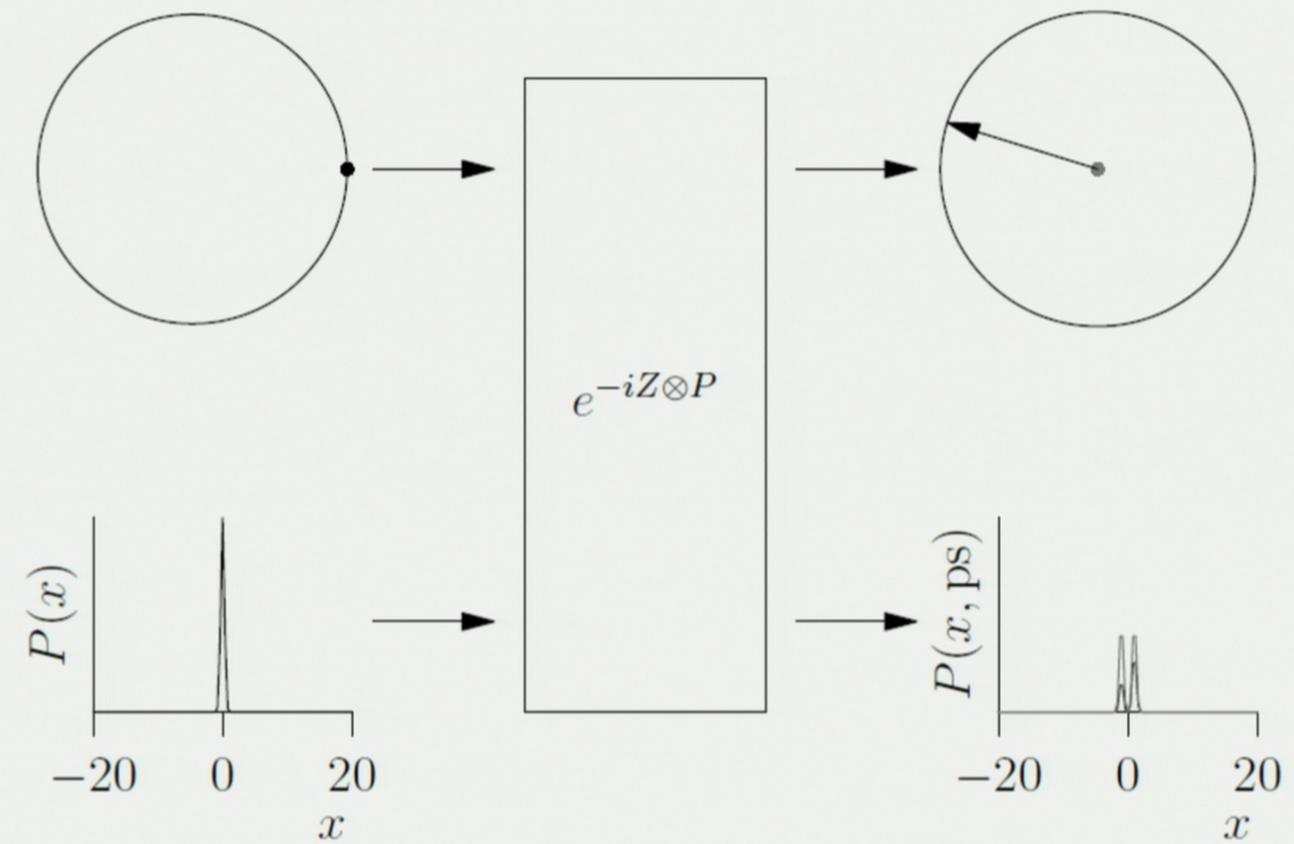
INSTITUTE
FOR THEORETICAL PHYSICS

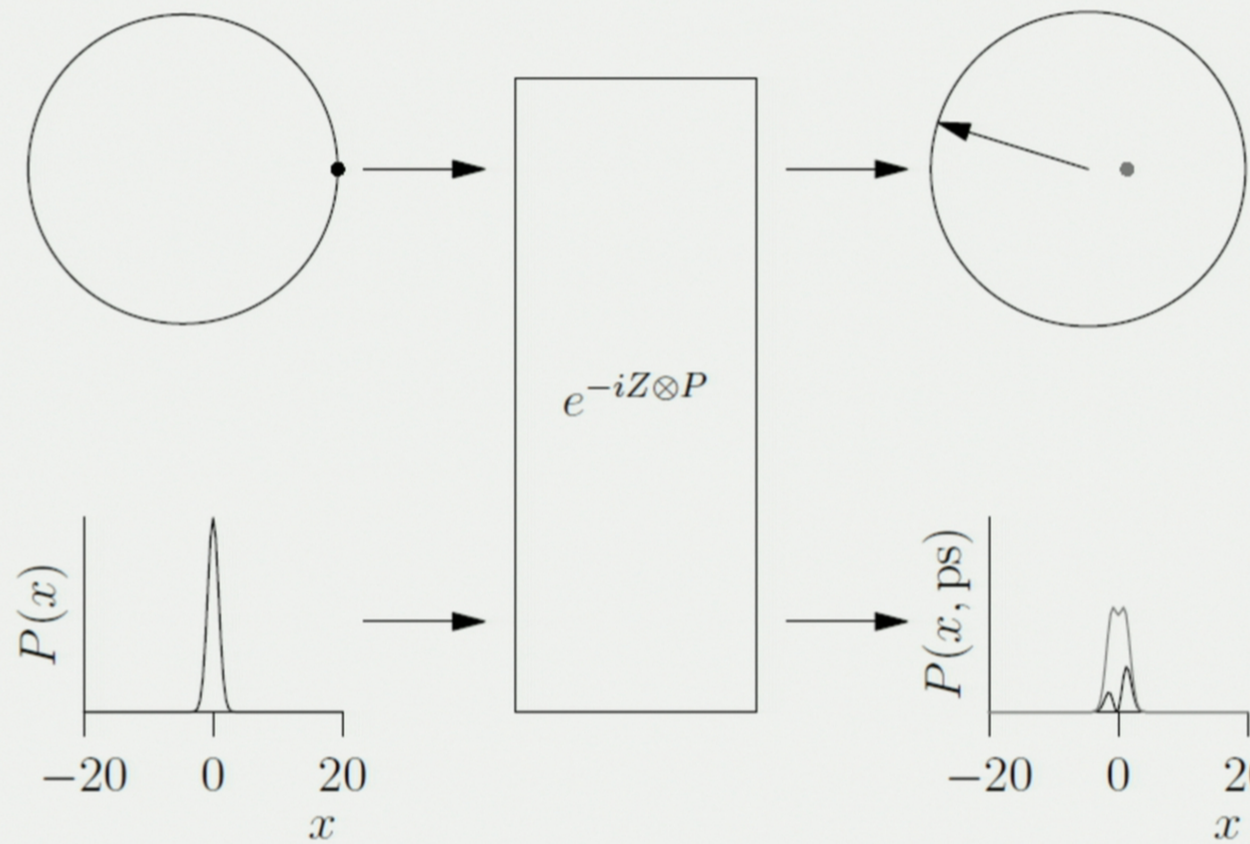
Post-selection



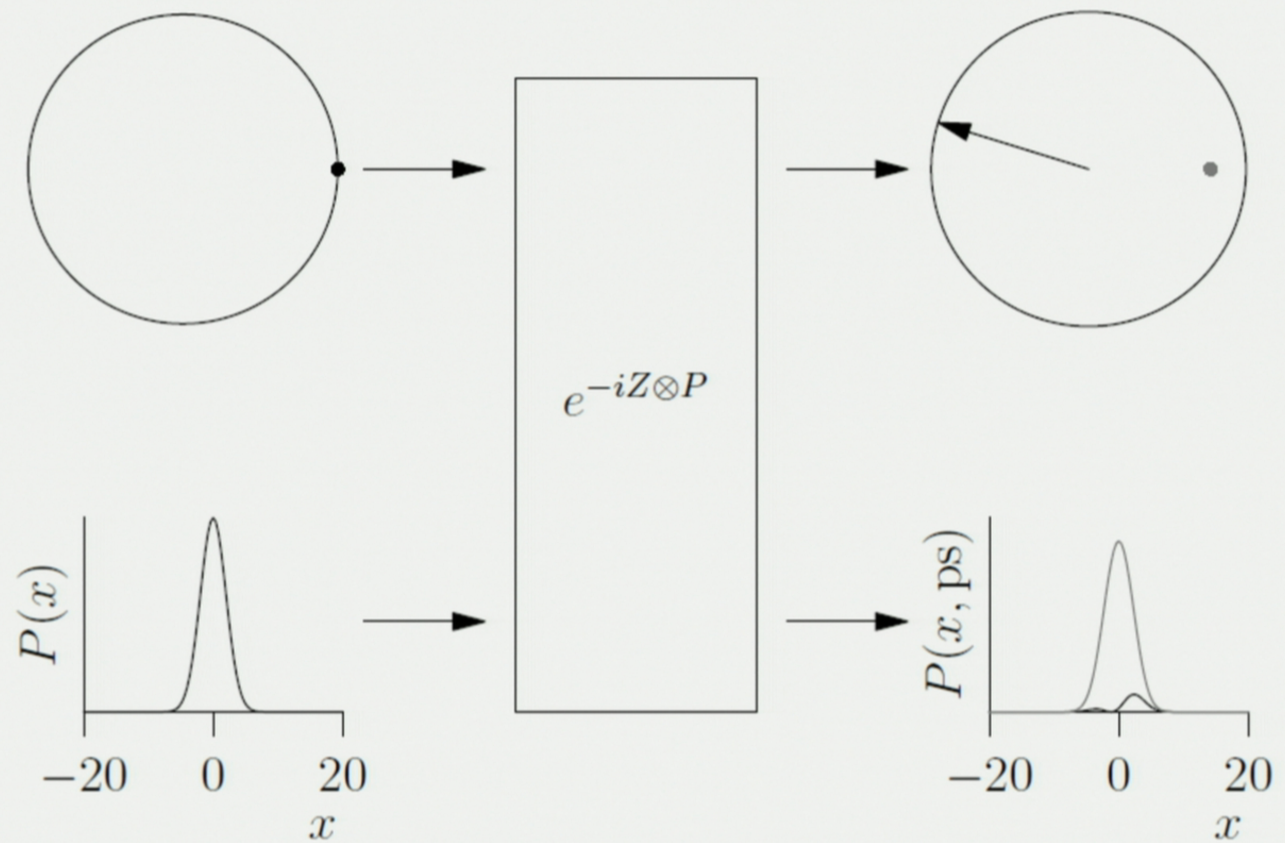
Post-selection



Post-selection



Post-selection



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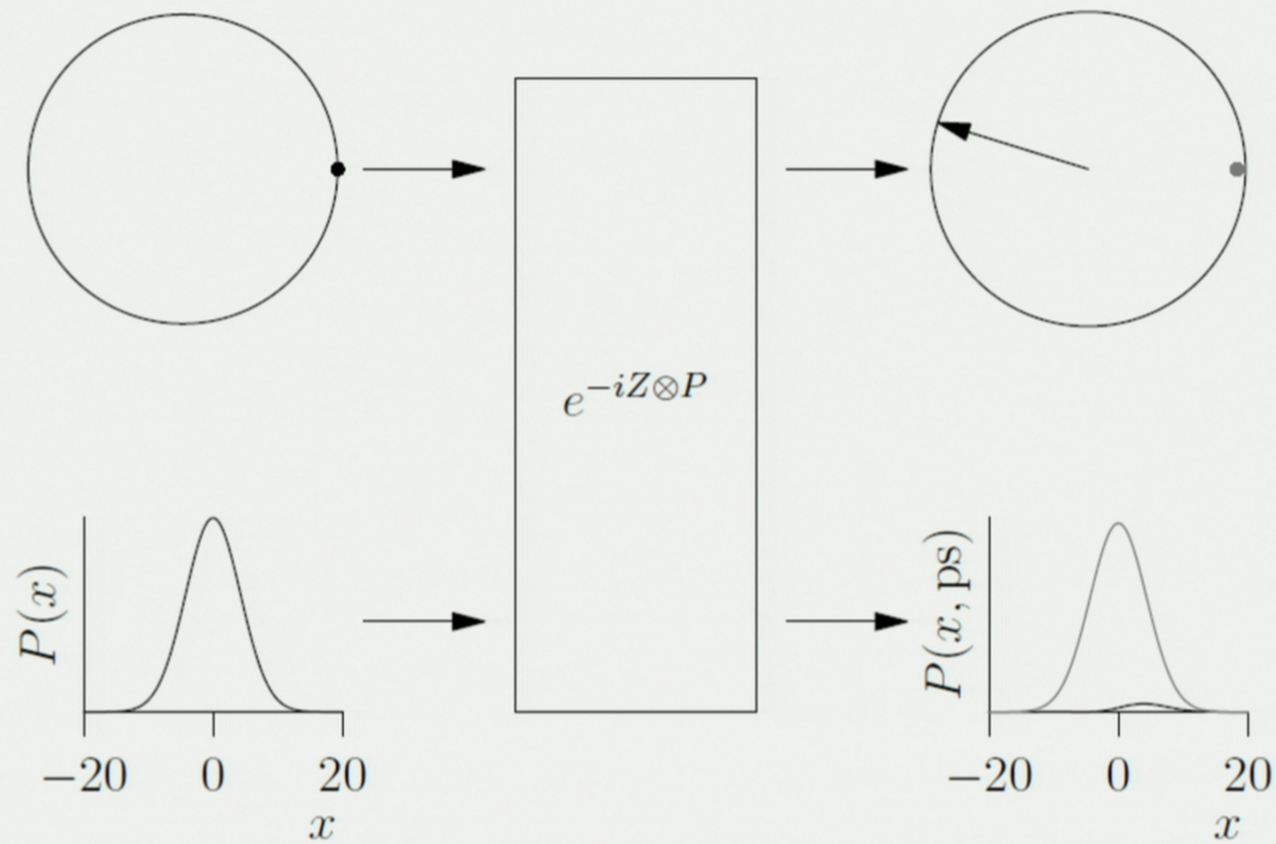


Weak value

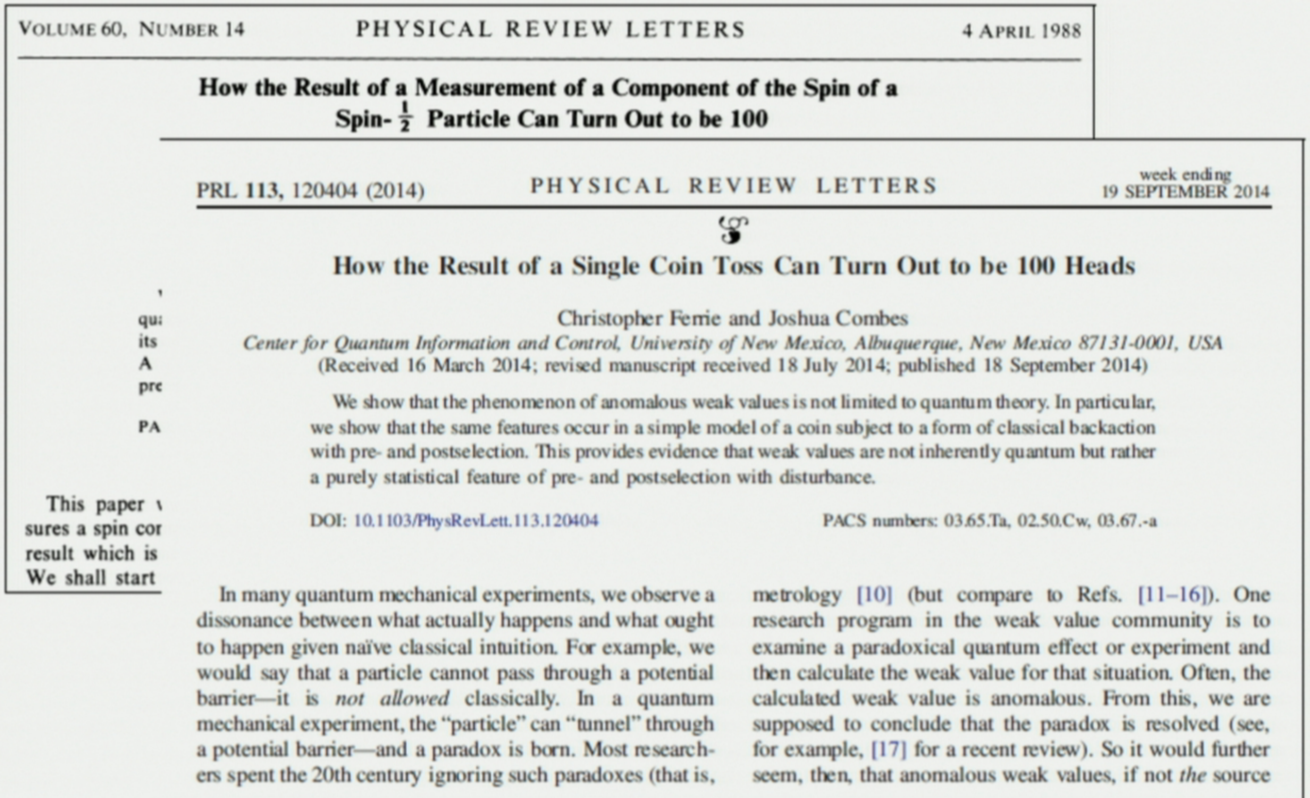
$$\int xP(x|\mathsf{ps})dx \to \mathrm{Re}(A_w)$$

Weak value

$$\int xP(x|\mathsf{ps})dx \to \mathrm{Re}(A_w)$$

$$A_w = \frac{\langle\phi|A|\psi\rangle}{\langle\phi|\psi\rangle}$$

Anomalous weak values


VOLUME 60, NUMBER 14 PHYSICAL REVIEW LETTERS 4 APRIL 1988

How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

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This paper
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result which is
We shall start

PRL 113, 120404 (2014) PHYSICAL REVIEW LETTERS week ending 19 SEPTEMBER 2014

How the Result of a Single Coin Toss Can Turn Out to be 100 Heads

Christopher Ferrie and Joshua Combes
Center for Quantum Information and Control, University of New Mexico, Albuquerque, New Mexico 87131-0001, USA
(Received 16 March 2014; revised manuscript received 18 July 2014; published 18 September 2014)

We show that the phenomenon of anomalous weak values is not limited to quantum theory. In particular, we show that the same features occur in a simple model of a coin subject to a form of classical backaction with pre- and postselection. This provides evidence that weak values are not inherently quantum but rather a purely statistical feature of pre- and postselection with disturbance.

DOI: 10.1103/PhysRevLett.113.120404 PACS numbers: 03.65.Ta, 02.50.Cw, 03.67.-a

In many quantum mechanical experiments, we observe a dissonance between what actually happens and what ought to happen given naïve classical intuition. For example, we would say that a particle cannot pass through a potential barrier—it is *not allowed* classically. In a quantum mechanical experiment, the "particle" can "tunnel" through a potential barrier—and a paradox is born. Most researchers spent the 20th century ignoring such paradoxes (that is, metrology [10] (but compare to Refs. [11–16]). One research program in the weak value community is to examine a paradoxical quantum effect or experiment and then calculate the weak value for that situation. Often, the calculated weak value is anomalous. From this, we are supposed to conclude that the paradox is resolved (see, for example, [17] for a recent review). So it would further seem, then, that anomalous weak values, if not *the* source

Anomalous weak values

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How the Result of a Single Coin Toss Can Turn Out to be 100 Heads

Christopher Ferrie and Joshua Combes
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Anomalous weak values are proofs of contextuality

Matthew F. Pusey*
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(Dated: September 4, 2014)

The average result of a weak measurement of some observable A can, under post-selection of the measured quantum system, exceed the largest eigenvalue of A. The nature of weak measurements, as well as the presence of post-selection and hence possible contribution of measurement-disturbance, has led to a long-running debate about whether or not this is surprising. Here, it is shown that such "anomalous weak values" are non-classical in a precise sense: a sufficiently weak measurement of one constitutes a proof of contextuality. This clarifies, for example, which features must be present (and in an experiment, verified) to demonstrate an effect with no satisfying classical explanation.

In 1988 Aharonov, Albert and Vaidman explained "How the result of a measurement of a component of the spin of a spin-$\frac{1}{2}$ particle can turn out to be 100." [1] Defining the *weak value* of an observable A for a quantum system prepared in state $|\psi\rangle$ and post-selected on giving the first outcome of $\{|\phi\rangle\langle\phi|, I - |\phi\rangle\langle\phi|\}$,

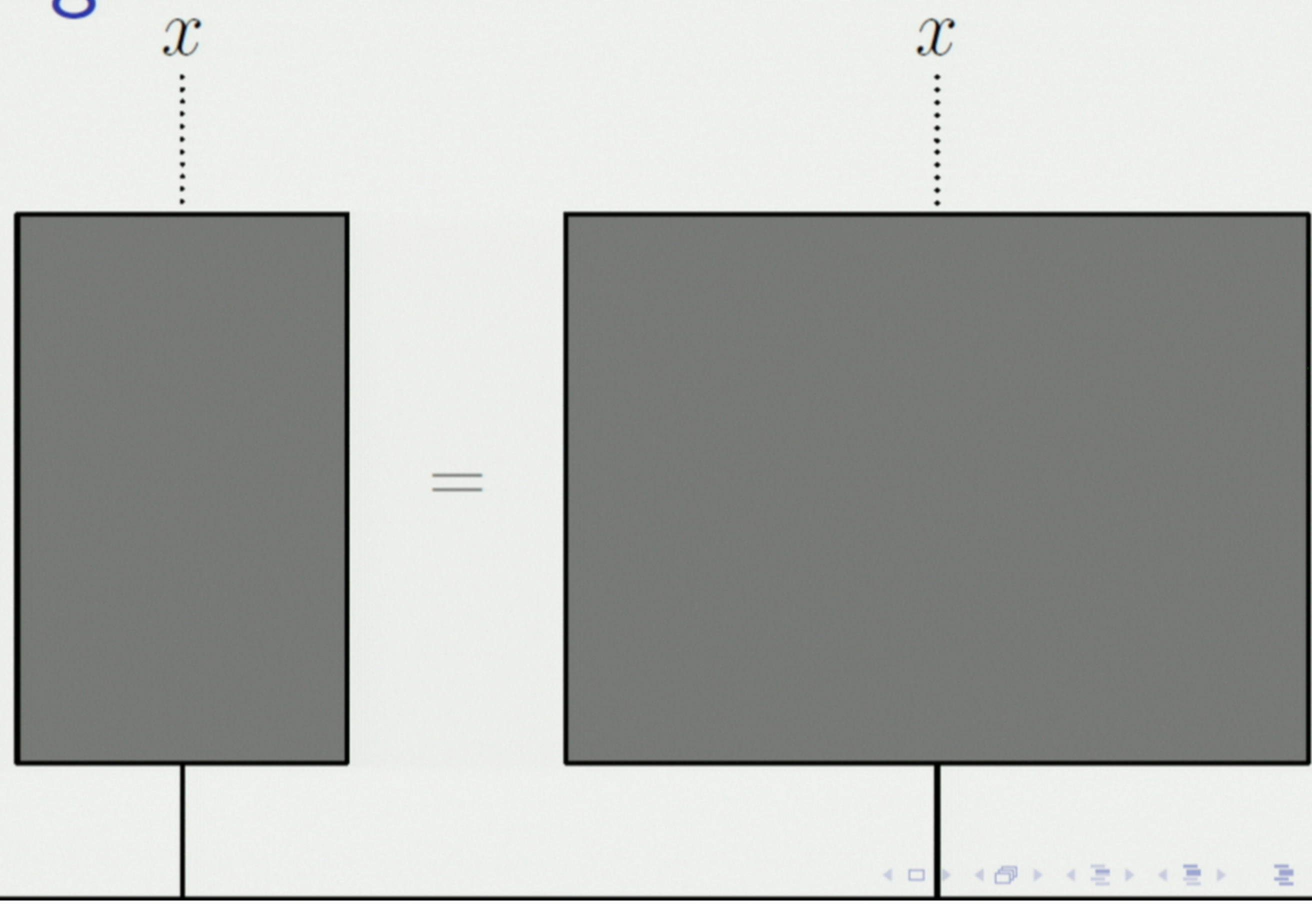
$$A_w = \frac{\langle\phi|A|\psi\rangle}{\langle\phi|\psi\rangle}, \quad (1)$$

Here I will prove it. Interestingly, the proof hinges on two issues already identified in the literature: what do weak measurements measure, and how much do they disturb the system? It transpires that both questions have clear answers in the setting of a non-contextual ontological model, but the particular information-disturbance tradeoff of the weak measurements in quantum theory makes these answers irreconcilable.

Let us begin by specifying exactly what is meant by an

Possibility 1: post-selection does not pick out a set of λ

Possibility 2: some λ "predisposed" to give anomalous results



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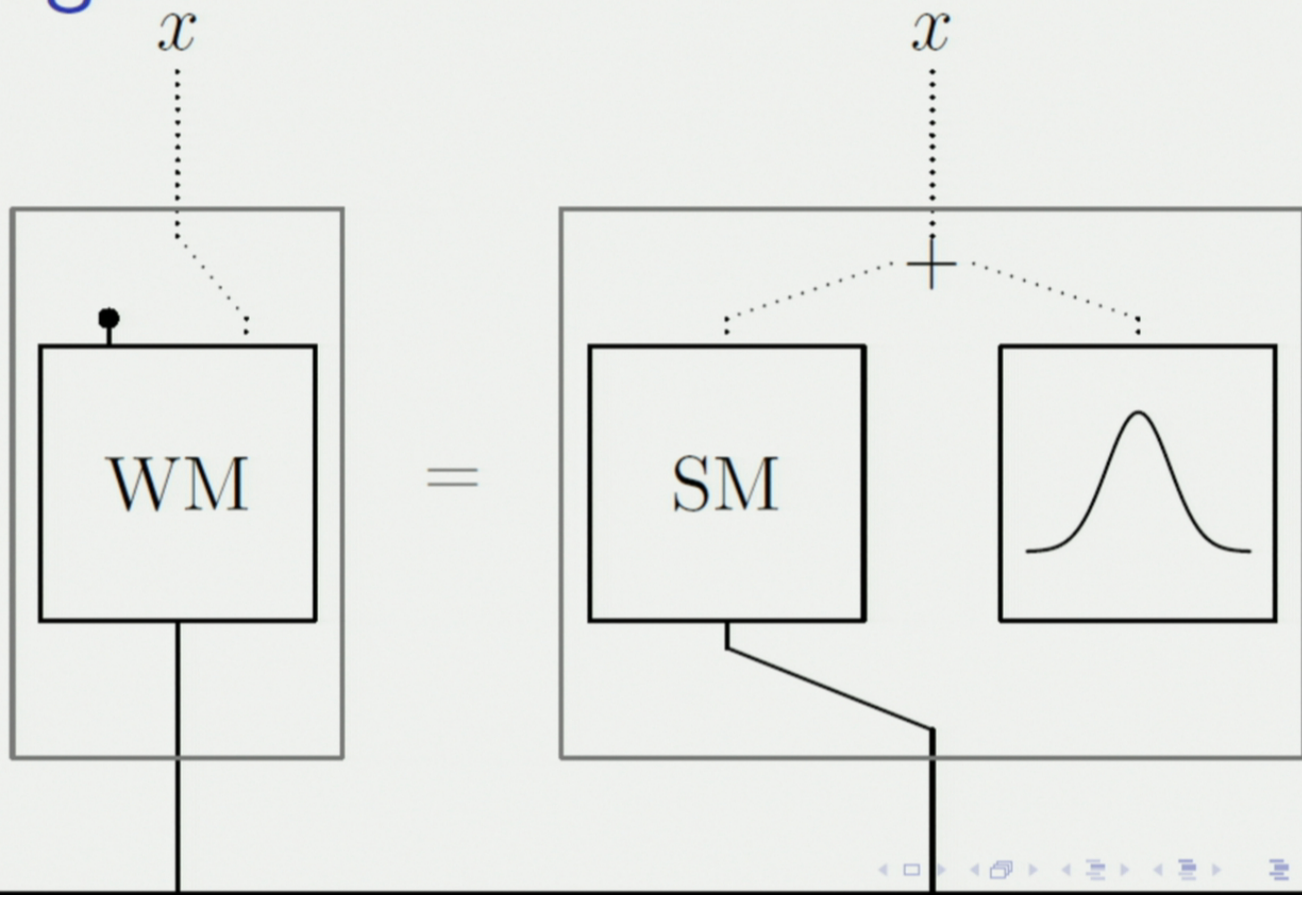
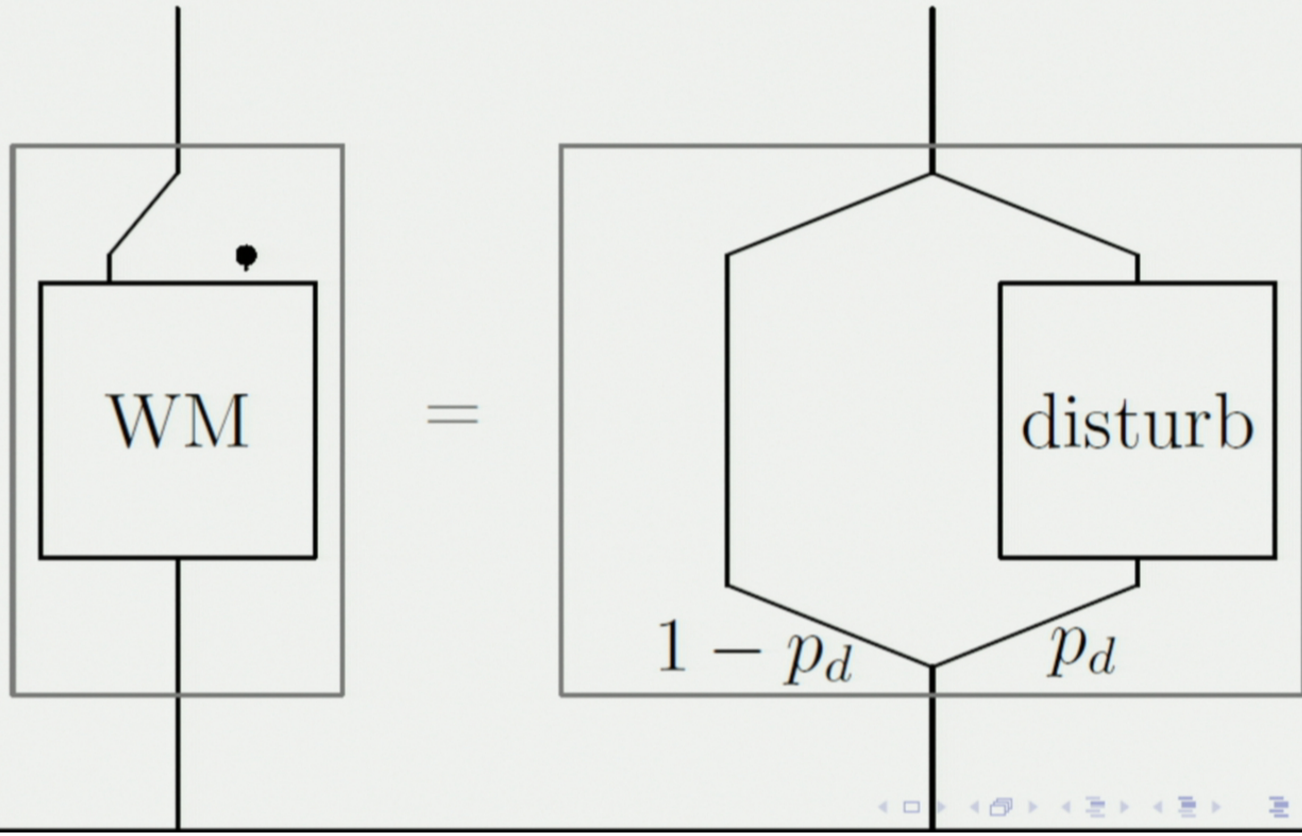


Possibility 3: anomaly is solely due to measuremnt-disturbance



Possibility 3: anomaly is solely due to measuremnt-disturbance

$p_d \sim \lambda^2$, but need $p_d \sim \lambda$ to explain anomaly

Thank you

[“Anomalous weak values are proofs of contextuality” is arXiv:1409.1535]