

Title: A Mathematician Looks at Supergravity and AdS/CFT

Date: Oct 22, 2014 02:45 PM

URL: <http://pirsa.org/14100104>

Abstract:

SUSY field theory on \mathbb{R}^n

Fermionic symmetries Q_i
Satisfy

$$Q_i Q_j + Q_j Q_i = \text{a translation}$$

Susy field theory on \mathbb{R}^n

Fermionic symmetries Q_i
Satisfy

$$Q_i Q_j + Q_j Q_i = \text{a translation}$$

Idea (Witten)
If we choose a fermionic sym. Q
with $Q^2 = 0$

theory on \mathbb{R}^n
symmetries Q_i
 $P_i = \text{a translation}$

Idea (Witten)
If we choose a fermionic sym. Q
with $Q^2 = 0$
then computing Q -invariant quantities
is easy.

Nice Q :
 \Rightarrow correlators of Q

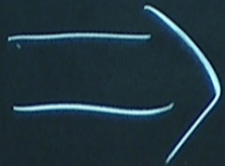


Nice Q :

\Rightarrow correlators of Q -inv. operators are independent of position

Generally

\rightarrow 1 loop diagrams don't contribute to Q -invariant calculations



Nice \mathcal{Q} :

\Rightarrow correlators of \mathcal{Q} -inv. operators are independent of position

Generally

\rightarrow 1 loop diagrams don't contribute to \mathcal{Q} -invariant calculations

\Rightarrow Many exact calculations.

Twisted field theory has \mathbb{R}^n
 Q -inv. operators of original theory
Operators like $Q \cdot \Theta$

Twisted field theory has R^n
 Q -inv. operators of original theory

Operators like $Q \cdot \theta$

(Note:

$$\langle Q \cdot \theta \rangle = 0 \text{ as}$$

Q preserves functional measure.

Example

Z ($N=2$ Yang-Mills)
in 4d

can be expressed in terms of diff. geometry

Count # Yang-Mills instantons
(Donaldson invariants)

1) Twisted supergravity

2) \uparrow can be quantized using
standard perturbative techniques

3) Gauge-Gravity duality

Ghosts:

YM theory field $A \in \Omega^1(\mathbb{R}^4, \mathfrak{g})$

Redundancy:

$$X: \mathbb{R}^4 \rightarrow G, \quad A \rightarrow XAX^{-1} + dA = A^X$$

indistinguishable.

Faddeev-Popov: Deal with redundancy using ghosts

Fermionic field
 $\chi \in \Omega^0(\mathbb{R}^4, g)$
Corr. to infinitesimal
gauge symmetries.

Fermionic field

$$\chi \in \Omega^0(\mathbb{R}^4, g)$$

Corr. to infinitesimal
gauge symmetries.

A functional \int over
both A and χ

removes redundancy
of A in description

Einstein-Hilbert:

Fields of gravity are
metric tensor g

Redundancy given by
changes of coord

Introduce ghosts

Fermions in $\text{Vect}(\mathbb{R}^4)$
→ vector fields

Supergravity.

Fields

$g_{\mu\nu}$ + other
gravitinos

fields including a
 $\psi \in \Omega^1(\mathbb{R}^4, S)$



Supergravity.

Fields

g , + other
gravitino

fields including a
 $\psi \in \Omega^1(\mathbb{R}^4, S)$

Redundancy:

- Changes of coord.

- Also "super" change of
coords.

Fermionic gauge symmetry $Q(x)$, depending
on $x \in \mathbb{R}^n$, $Q \in C^\infty(\mathbb{R}^n, \mathfrak{S})$ (a spinor field)

Ghosts for SUGRA

Fermionic
 $\text{Vect}(\mathbb{R}^n)$

Bosonic
 $C^\infty(\mathbb{R}^n, \mathfrak{S})$

Twisted sugra
= SUGRA
with in a background
non-zero bosonic ghost.

Twisted sugra

= SUGRA
with

in a background
non-zero bosonic ghost

Equations of motion

Assume that non-zero fields
are g, Q bosonic ghost

EOM

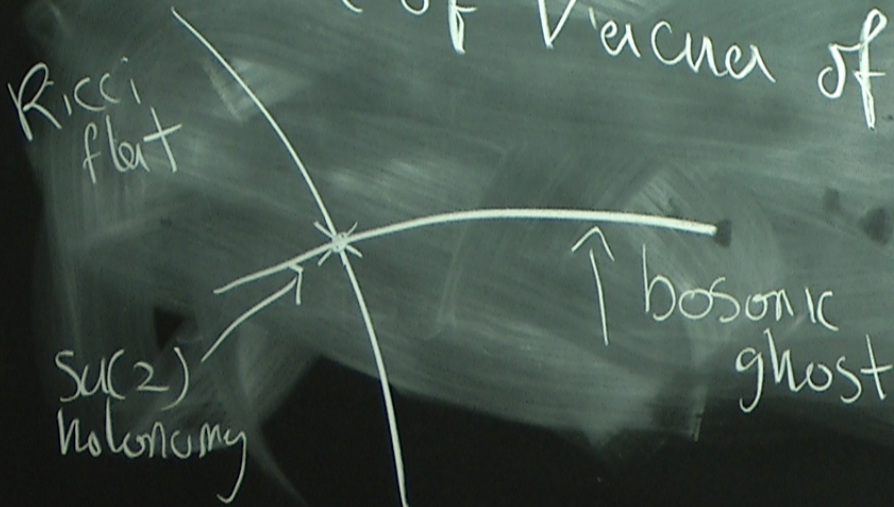
$$1) \nabla_g Q = 0 \quad 2) Q^2 = 0$$

$N=1$ SUGRA in 4d

$\nabla_g \mathcal{Q} = 0 \Rightarrow$ Holonomy fixes \mathcal{Q}

\Rightarrow $SU(2)$ holonomy

Picture of Vacuum of $N=1$ SUGRA in 4d



Example

$N=2$ SUGRA in 4d
has a "Donaldson twist"

Perturbation theory only goes to 1-loop

$Z = \#$ of gravitational instantons
 $=$ manifolds with $SU(2)$ holonomy

Example

$N=2$ SUGRA in 4d
has a "Donaldson twist"

Perturbation theory only goes to 1-loop

$Z = \#$ of gravitational instantons
= manifolds with $SU(2)$ holonomy

IIB SUGRA in 10d

Twisted SUGRA, metric satisfy

$$\nabla_g Q = 0, \Rightarrow \text{SU}(10) \text{ holonomy}$$

"Calabi-Yau"

Guess: Gravitational theory on CYs

1996: Bershadsky et. al proposed a gravity theory of CY manifold;

Conjecture

Twist of $\mathbb{I}B$ SUGRA

= BCWV theory

↗ Berchaudsky.

Conjecture

Twist of $\mathbb{I}B$ SUGRA
= BCOW theory
↑ Berchadsky.

Theorem (C. Si Li)

BCOV is naively non-renormalizable.
Even so, \exists a unique quantization compatible with
a 10d gauge theory.

Counterterms
in gravity



Anomalies
in gauge
theory

Anomaly free gauge theory fixes all counterterms