

Title: 14/15 PSI - Statistical Mechanics - Lecture 14

Date: Oct 24, 2014 10:45 AM

URL: <http://pirsa.org/14100102>

Abstract:

Imaginary-time correlation function:

$\tau > 0$

$$G_{ij}(\tau) = \langle \sigma_i^z(\tau) \sigma_j^z(0) \rangle = \frac{1}{Z} \text{Tr} [e^{-\beta H} \sigma_i^z(\tau) \sigma_j^z(0)] = \frac{1}{Z} \overline{\sigma_i^z \sigma_j^z}$$

$$\langle \sigma_j^z(\tau) \sigma_j^z(0) \rangle = \frac{1}{Z} \sum_{n,m} \langle n | e^{-\beta H} | n \rangle \langle n | \sigma_j^z(\tau) | m \rangle \langle m | \sigma_j^z(0) | n \rangle =$$

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$$\sigma_i^z(\tau) = e^{H\tau} \sigma_i^z e^{-H\tau}$$

$$\sigma_i^z(t) = e^{iHt} \sigma_i^z e^{-iHt}$$

$$\langle \sigma_i^z(t) \sigma_j^z(0) \rangle = \frac{1}{Z} \sum_{n,m} \langle n | e^{-\beta H} | n \rangle \langle n | \sigma_i^z(t) | m \rangle \langle m | \sigma_j^z(0) | n \rangle =$$

$$\langle n | \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle$$

Imaginary-time correlation function:

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$$E_{mn} = E_m - E_n$$

$$\sigma_i^z(\tau) = e^{H\tau} \sigma_i^z e^{-H\tau}$$

$$\sigma_i^z(t) = e^{iHt} \sigma_i^z e^{-iHt}$$

$$\langle \sigma_i^z(t) \sigma_j^z(0) \rangle = \frac{1}{Z} \sum_{n,m} \langle n | e^{-\beta H} | n \rangle \langle n | \sigma_i^z(t) | m \rangle \langle m | \sigma_j^z(0) | n \rangle =$$

$$\langle \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle = \frac{1}{Z} \sum_n e^{-\beta E_n} e^{-t E_{mn}} \langle n | \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle$$

$$G_{ij}(i\omega_n) = \int_0^{\beta} dt G_{ij}(t) e^{i\omega_n t} = \frac{1}{Z} \sum_{mn} e^{-\beta E_n} \int_0^{\beta} dt e^{(i\omega_n - E_m) t}$$

$$e^{iW_n T} = \frac{1}{Z} \sum_{mn} e^{-\beta E_n} \int_0^\beta dt e^{(iW_n - E_{mn})t} \langle n | \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle =$$

$$G_{ij}(i\omega_n) = \int_0^{\beta} dt G_{ij}(t) e^{i\omega_n t} = \frac{1}{Z} \sum_{mn} e^{-\beta E_n} \int_0^{\beta} dt e^{(i\omega_n -$$

$$= \frac{1}{Z} \sum_{mn} e^{-\beta E_n} \frac{1}{i\omega_n - E_{mn}} \left[e^{\beta(i\omega_n - E_{mn})} - 1 \right] \langle n | \sigma_i$$

$(W_n - E_{mn})T$

$$\langle n | \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle =$$

$$\langle \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle = \frac{1}{Z} \sum_{mn} e^{-\beta E_n} \left(e^{-\beta E_{mn}} - 1 \right) \frac{1}{i(W_n - E_{mn})} \langle n | \sigma_i^z | m \rangle \cdot \langle m | \sigma_j^z | n \rangle$$

Spectral density:

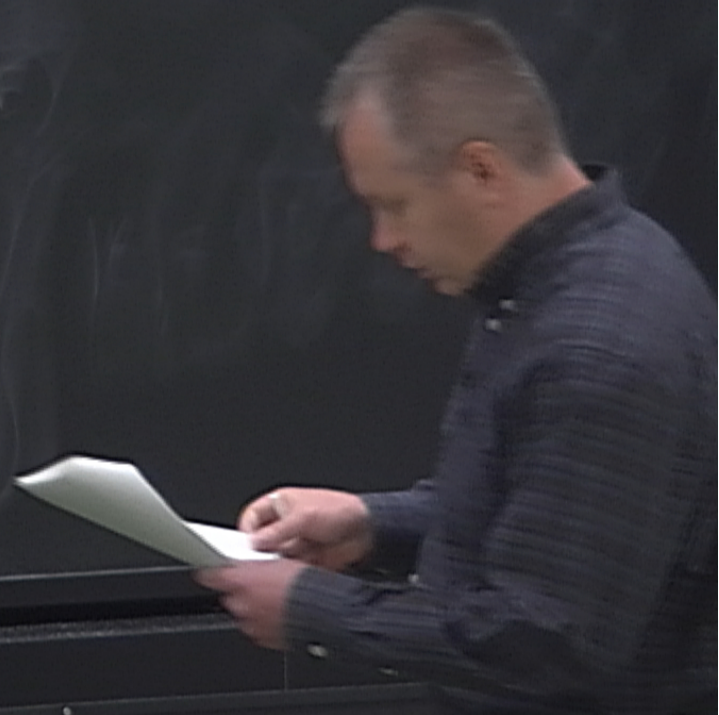
$$\rho_{ij}(\Omega) = \frac{2\pi}{Z} \sum_{m,n} e^{-\beta E_n} (1 - e^{-\beta E_{mn}}) \langle n | \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle$$

$$G_{ij}(\Omega) = -$$

Spectral density:

$$\rho_{ij}(\Omega) = \frac{2\pi}{Z} \sum_{m,n} e^{-\beta E_n} (1 - e^{-\beta E_{mn}}) \langle n | \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle$$

$$G_{ij}(i\omega_n) = - \int_{-\infty}^{\infty} \frac{d\Omega}{2\pi} \frac{\rho_{ij}(\Omega)}{i\omega_n - \Omega}$$



Spectral density:

$$P_{ij}(\Omega) = \frac{2\pi}{Z} \sum_{mn} e^{-\beta E_n} (1 - e^{-\beta E_{mn}}) \langle n | \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle$$

$$G_{ij}(i\omega_n) = - \int_{-\infty}^{\infty} \frac{d\Omega}{2\pi} \frac{P_{ij}(\Omega)}{i\omega_n - \Omega}$$

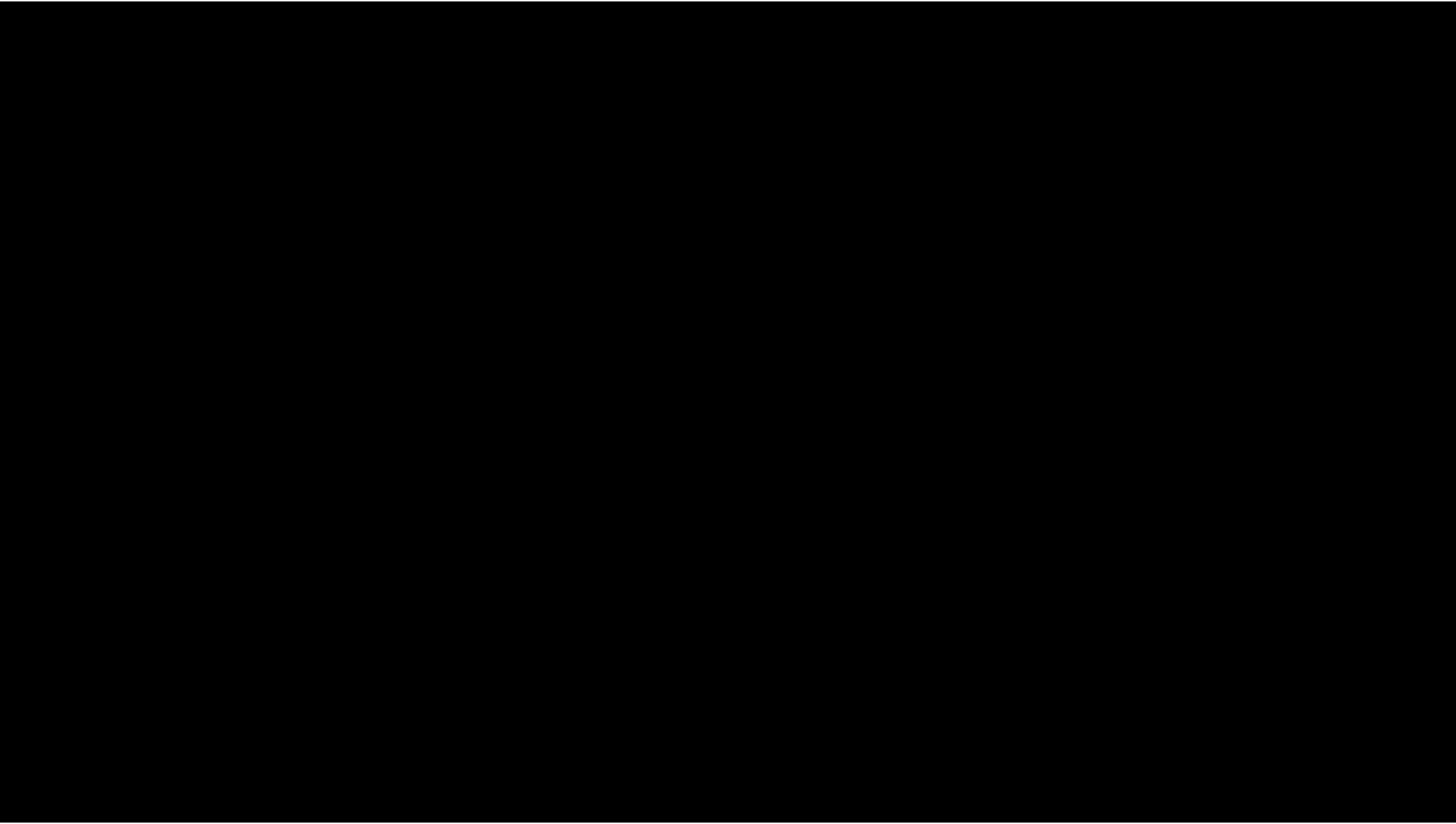
Real-time response.

$$H = H_0 - \sum_i h_i(t) \sigma_i^z = H_0 + V(t)$$

$$i \frac{\partial}{\partial t} |\psi\rangle = H |\psi\rangle$$

Interaction representation:

$$\sigma_i^z(t) = e^{iH_0 t} \sigma_i^z e^{-iH_0 t}$$



Real-time response.

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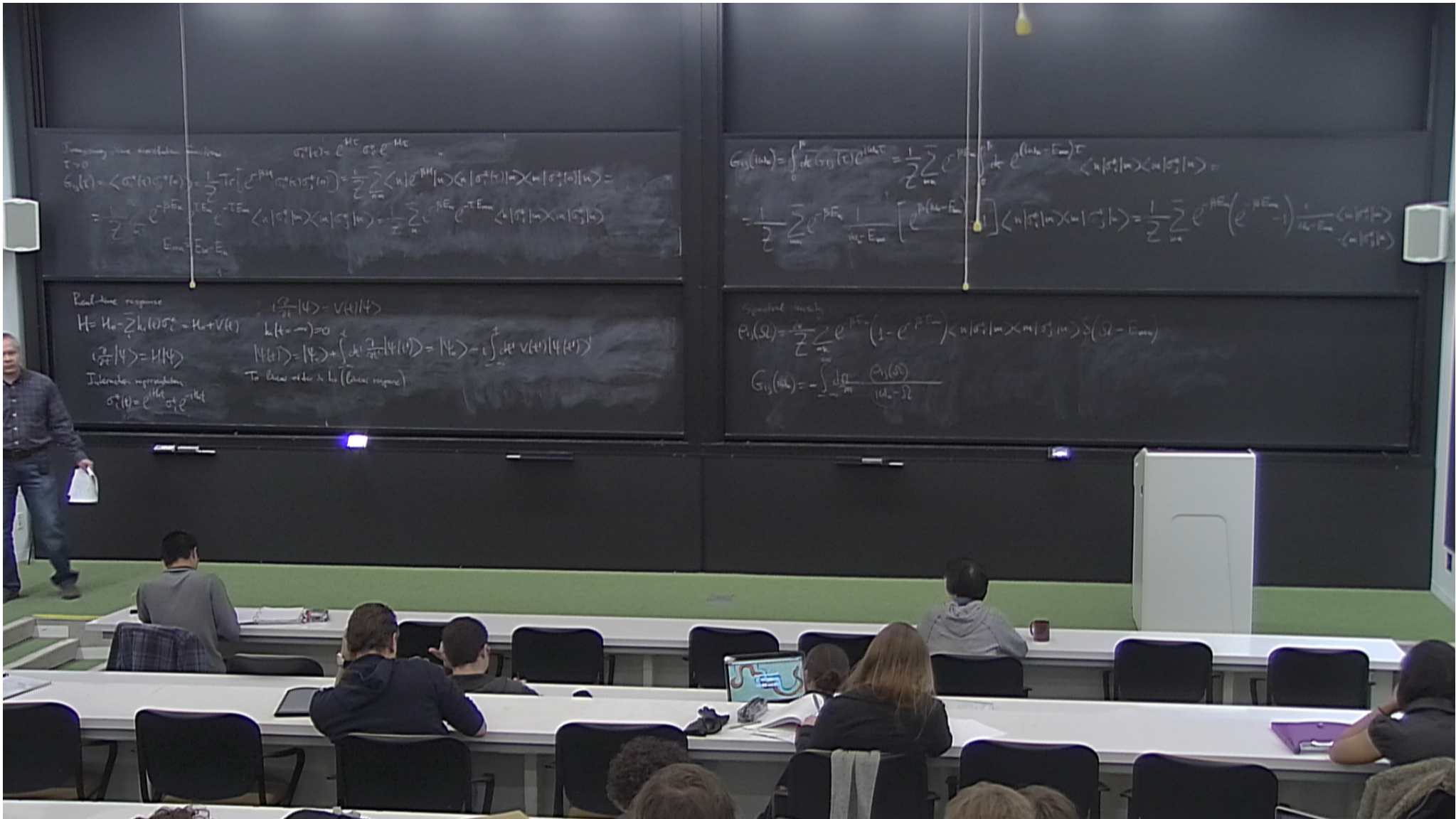
$$h_i(t \rightarrow -\infty) = 0$$

$$|\psi(t)\rangle = |\psi_0\rangle$$

$$i \frac{\partial}{\partial t} |\psi\rangle = V(t) |\psi\rangle$$

$$\lim_{t \rightarrow -\infty} |\psi\rangle = |\psi_0\rangle$$

$$|\psi(t)\rangle = |\psi_0\rangle + \int_{-\infty}^t dt' \frac{\partial}{\partial t'} |\psi(t')\rangle = |\psi_0\rangle - i \int_{-\infty}^t dt' V(t') |\psi(t')\rangle$$



Imaginary time evolution function
 $\sigma^*(t) = e^{Ht} \sigma^* e^{-Ht}$
 $G_{ij}(t) = \langle \sigma_i^*(t) \sigma_j^*(t) \rangle = \frac{1}{2} \text{Tr} [e^{i\beta H} \sigma_i^*(t) \sigma_j^*(t)] = \frac{1}{2} \sum_n \langle n | e^{i\beta H} | n \rangle \langle n | \sigma_i^*(t) | n \rangle \langle n | \sigma_j^*(t) | n \rangle =$
 $= \frac{1}{2} \sum_n e^{-\beta E_n} e^{-iE_n t} \langle n | \sigma_i^* | n \rangle \langle n | \sigma_j^* | n \rangle = \frac{1}{2} \sum_n e^{-\beta E_n} e^{-iE_n t} \langle n | \sigma_i^* | n \rangle \langle n | \sigma_j^* | n \rangle$
 $E_{nn} = E_n$

$$G_{ij}(i\omega) = \int_0^\beta dt G_{ij}(t) e^{i\omega t} = \frac{1}{2} \sum_n \int_0^\beta dt e^{i(\omega - E_n)t} \langle n | \sigma_i^* | n \rangle \langle n | \sigma_j^* | n \rangle =$$

$$= \frac{1}{2} \sum_n \int_0^\beta dt e^{-\beta E_n} \frac{1}{i(\omega - E_n)} [e^{i(\omega - E_n)\beta} - 1] \langle n | \sigma_i^* | n \rangle \langle n | \sigma_j^* | n \rangle = \frac{1}{2} \sum_n e^{-\beta E_n} \frac{(e^{-\beta E_n} - 1)}{i(\omega - E_n)} \langle n | \sigma_i^* | n \rangle \langle n | \sigma_j^* | n \rangle$$

Real-time response
 $H = H_0 + \sum_k h_k(t) \sigma_k^* = H_0 + V(t)$ $(\frac{\partial}{\partial t} |\psi\rangle = V(t) |\psi\rangle)$
 $h_k(t \rightarrow \infty) = 0$ $h_k(t \rightarrow -\infty) = 0$
 $|\psi(t)\rangle = |\psi\rangle + \int_{-\infty}^t dt' V(t') |\psi(t')\rangle$
 To linear order in h_k (linear response)
 Interacting representation
 $\sigma_i^*(t) = e^{iH_0 t} \sigma_i^* e^{-iH_0 t}$

Spectral density
 $P_i(\omega) = -\frac{1}{2} \sum_n e^{-\beta E_n} (1 - e^{-\beta E_n}) \langle n | \sigma_i^* | n \rangle \langle n | \sigma_i^* | n \rangle \delta(\omega - E_n)$
 $G_{ij}(i\omega) = -\int_{-\infty}^{\infty} \frac{d\Omega}{2\pi} \frac{P_{ij}(\Omega)}{i\omega - \Omega}$

$$|\psi\rangle; |\psi_0\rangle = |\psi(t = -\infty)\rangle$$

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = |\psi_0\rangle - i \int_{-\infty}^t dt' V(t') |\psi(t')\rangle \approx$$

(linear response)

$$\approx |\psi_0\rangle - i \int_{-\infty}^t dt' V(t') |\psi_0\rangle$$

$$\delta \langle \sigma_i^z(t) \rangle = \langle \psi(t) | \sigma_i^z | \psi(t) \rangle - \langle \psi_0 | \sigma_i^z | \psi_0 \rangle =$$

$$\langle \psi(t) | \sigma_i^z | \psi(t) \rangle - \langle \psi_0 | \sigma_i^z | \psi_0 \rangle = \left[\langle \psi_0 | + i \langle \psi_0 | \int_{-\infty}^t dt' V(t') \right] \sigma_i^z$$

$$\delta \langle \sigma_i^z(t) \rangle = \langle \psi(t) | \sigma_{i(t)}^z | \psi(t) \rangle - \langle \psi_0 | \sigma_{i(t)}^z | \psi_0 \rangle = \left[\langle \psi_0 | + i \right]$$

$$\delta \langle \sigma_i^z(t) \rangle = \langle \psi(t) | \sigma_i^z(t) | \psi(t) \rangle - \langle \psi_0 | \sigma_i^z(t) | \psi_0 \rangle = \left[\langle \psi_0 | + i \langle \right.$$

$$- \langle \psi_0 | \sigma_i^z(t) | \psi_0 \rangle = i \int_{-\infty}^t dt' \langle \psi_0 |$$

$$\begin{aligned}
 & \langle \psi_0 | \sigma_i^z(t) | \psi(t) \rangle - \langle \psi_0 | \sigma_i^z(t) | \psi_0 \rangle = \left[\langle \psi_0 | + i \langle \psi_0 | \int_{-\infty}^t dt' V(t') \right] \sigma_i^z(t) \\
 & = i \int_{-\infty}^t dt' \langle \psi_0 | V(t') \sigma_i^z(t) - \sigma_i^z(t) V(t') | \psi_0 \rangle =
 \end{aligned}$$

$$\begin{aligned}
 & \langle \psi_0 | \sigma_i^z(t) | \psi(t) \rangle - \langle \psi_0 | \sigma_i^z | \psi_0 \rangle = \left[\langle \psi_0 | + i \langle \psi_0 | \int_{-\infty}^t dt' V(t') \right] \sigma_i^z(t) | \psi_0 \rangle \\
 & = i \int_{-\infty}^t dt' \langle \psi_0 | V(t') \sigma_i^z(t) - \sigma_i^z(t) V(t') | \psi_0 \rangle = i \int_{-\infty}^t dt' \langle \psi_0 | \left[-\sum_j h_{ij}(t) \right] | \psi_0 \rangle
 \end{aligned}$$

$$\begin{aligned}
 & \left[1 + i \langle \psi_0 | \int_{-\infty}^t dt' V(t') \right] \sigma_i^z(t) \left[|\psi_0\rangle - i \int_{-\infty}^t dt' V(t') |\psi_0\rangle \right] - \\
 & i \int_{-\infty}^t dt' \langle \psi_0 | \left[-\sum_j h_j(t') \sigma_j^z(t'), \sigma_i^z(t) \right] |\psi_0\rangle =
 \end{aligned}$$

$$\delta \langle \sigma_i^z(t) \rangle = \langle \psi(t) | \sigma_i^z(t) | \psi(t) \rangle - \langle \psi_0 | \sigma_i^z(t) | \psi_0 \rangle = \left[\langle \psi_0 | + i \langle \right.$$

$$- \langle \psi_0 | \sigma_i^z(t) | \psi_0 \rangle = i \int_{-\infty}^t dt' \langle \psi_0 | v(t') \sigma_i^z(t) - \sigma_i^z(t) v(t') | \psi_0 \rangle$$

$$= i \int_{-\infty}^t dt' \sum_j \langle \psi_0 | [\sigma_i^z(t), \sigma_j^+(t')] | \psi_0 \rangle h_j(t')$$

Retarded response function:

$$\chi_{ij}(t-t') = i\theta(t-t') \langle [\sigma_i^z(t), \sigma_j^z(t')] \rangle$$

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$$\delta \langle \sigma_i^z(t) \rangle = \int_{-\infty}^{\infty} dt' \sum_j \bar{\chi}_{ij}(t-t') h_j(t')$$

At finite temperature:

$$\langle \dots \rangle = \frac{1}{Z} \sum_n e^{-\beta E_n} \langle n | \dots | n \rangle$$

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$$\chi_{ij}(t-t') = i\theta(t-t') \frac{1}{Z} \sum_n e^{-\beta E_n} \langle n | [\sigma_i^z(t), \sigma_j^z(t')] | n \rangle$$

$$e^{-\beta E_n} \langle n | [\sigma_i^z(t), \sigma_j^z(t')] | u \rangle = i\theta(t-t') \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} \langle n | \sigma_i^z(t) |$$

$$\chi_{ij}(t) = i\theta(t) \frac{1}{Z} \sum_n e^{-\beta E_n} \langle n | [\sigma_i^z(t), \sigma_j^z(0)] | n \rangle$$

$$- \langle n | \sigma_j^z(0) | m \rangle \langle m | \sigma_i^z(t) | n \rangle = i\theta(t) \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} [$$

$$\begin{aligned}
 \langle n | [\sigma_i^z(t), \sigma_j^z(0)] | n \rangle &= i\theta(t) \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} \langle n | \sigma_i^z(t) | m \rangle \langle m | \sigma_j^z | n \rangle \\
 i\theta(t) \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} & \left[e^{iE_n t} e^{-iE_m t} \langle n | \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle - e^{-iE_n t} e^{iE_m t} \langle m | \sigma_j^z | n \rangle \langle n | \sigma_i^z | m \rangle \right]
 \end{aligned}$$

$$|u\rangle = i\theta(t) \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} \left[\langle n | \sigma_i^z(t) | m \rangle \langle m | \sigma_j^z(0) | u \rangle - \right. \\ \left. e^{iE_n t} e^{-iE_m t} \langle n | \sigma_i^z | m \rangle \langle m | \sigma_j^z | u \rangle - e^{-iE_n t} e^{iE_m t} \langle n | \sigma_j^z | m \rangle \langle m | \sigma_i^z | u \rangle \right]$$

$$\begin{aligned}
 \chi_{ij}(t) &= i\theta(t) \frac{1}{Z} \sum_n e^{-\beta E_n} \langle n | [\sigma_i^z(t), \sigma_j^z(0)] | n \rangle \\
 &= i\theta(t) \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} \left[e^{iE_n t} \langle n | \sigma_j^z(0) | m \rangle \langle m | \sigma_i^z(t) | n \rangle - e^{-iE_n t} \langle n | \sigma_i^z(0) | m \rangle \langle m | \sigma_j^z(t) | n \rangle \right] \\
 &= i\theta(t) \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} \left[e^{-iE_n t} \langle n | \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle - e^{iE_n t} \langle n | \sigma_j^z | m \rangle \langle m | \sigma_i^z | n \rangle \right]
 \end{aligned}$$

$$\langle n | [\sigma_i^z(t), \sigma_j^z(0)] | n \rangle = i\theta(t) \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} \langle n | \sigma_i^z(t) | m \rangle \langle m |$$

$$(t) \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} \left[e^{iE_n t} e^{-iE_m t} \langle n | \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle - e^{-iE_n t} e^{iE_m t} \langle n | \sigma_j^z | m \rangle \langle m | \sigma_i^z | n \rangle \right]$$

$$X_{ij}(\omega) = \int_{-\infty}^{\infty} dt X_0(t) e^{i\omega t} \quad ; \quad \int_{-\infty}^{\infty} dt \theta(t) e^{i\omega t} e^{i\omega t}$$

$$X_{ij}(\omega) = \int_{-\infty}^{\infty} dt X_0(t) e^{i\omega t} ; \int_{-\infty}^{\infty} dt \theta(t) e^{iE_{ij}t} e^{i\omega t}$$

$$(t) e^{iE_m t} e^{i\omega t} = \int_0^\infty dt e^{i(\omega + E_m)t}$$

$$(t) e^{iE_m t} e^{i\omega t} = \lim_{\eta \rightarrow 0^+} \int_0^{\infty} dt e^{i(\omega + E_m + i\eta)t} = \frac{1}{i(\omega + E_m + i\eta)} [e^{i(\omega + E_m + i\eta)t}]_0^{\infty} = -\frac{i}{\omega + E_m + i\eta}$$

$$\begin{aligned} &= \lim_{\eta \rightarrow 0^+} \int_0^{\infty} dt e^{i(\omega + E_{nn} + i\eta)t} \\ &= \frac{1}{i(\omega + E_{nn} + i\eta)} \left[e^{i(\omega + E_{nn} + i\eta)t} \right]_0^{\infty} = \frac{i}{\omega + E_{nn} + i\eta} \end{aligned}$$

$\eta = 0^+$

$$\chi_{ij}(\omega) = \int_{-\infty}^{\infty} dt \chi_0(t) e^{i\omega t} \quad ; \quad \int_{-\infty}^{\infty} dt \theta(t) e^{iE_m t} e^{i\omega t} = \lim_{\eta \rightarrow 0^+} \int_{-\infty}^{\infty} dt \theta(t) e^{i(E_m + \omega - i\eta)t}$$

$$\chi_{ij}(\omega) = -\frac{1}{Z} \sum_{n,m} e^{-\beta E_n} \left[\frac{\langle n | \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle}{\omega + E_m - i\eta} \right]$$

$$\int_{-\infty}^{\infty} dt \theta(t) e^{iE_{mn}t} e^{i\omega t} = \lim_{\eta \rightarrow 0^+} \int_0^{\infty} dt e^{i(\omega + E_{mn} + i\eta)t} = \frac{1}{i(\omega + E_{mn} + i\eta)}$$

$$\left[\frac{\langle n | \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle}{\omega + E_{mn} + i\eta} - \frac{\langle n | \sigma_j^z | m \rangle \langle m | \sigma_i^z | n \rangle}{\omega - E_{mn} + i\eta} \right] \quad \eta = 0^+$$

$$\chi_{ij}(\omega) = \int_{-\infty}^{\infty} dt \chi_{ij}(t) e^{i\omega t} \quad ; \quad \int_{-\infty}^{\infty} dt \theta(t) e^{iE_{nm}t} e^{i\omega t} = \dots$$

$$\chi_{ij}(\omega) = -\frac{1}{Z} \sum_{nm} e^{-\beta E_n} \left[\frac{\langle n | \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle}{\omega + E_{nm} + i\eta} - \frac{\langle n | \sigma_j^z | m \rangle \langle m | \sigma_i^z | n \rangle}{\omega - E_{nm} + i\eta} \right]$$

Dynamic structure factor: $S_{ij}(\omega) = \frac{2\pi}{Z} \sum_{nm} e^{-\beta E_n} \langle n | \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle \delta(\omega - E_{nm})$

$$S_{ij}(\omega) = \frac{P_{ij}(\omega)}{1 - e^{-\beta\omega}}$$

$$S_{ij}(\omega) = \frac{P_{ij}(\omega)}{1 - e^{-\beta\omega}}$$

$$X_{ij}(\omega) = - \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left[\frac{S_{ij}(\omega')}{\omega - \omega'} \right]$$

$$S_{ij}(\omega) = \frac{P_{ij}(\omega)}{1 - e^{-\beta\hbar\omega}} \quad ; \quad \chi_{ij}(\omega) = - \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left[\frac{S_{ij}(\omega')}{\omega - \omega' + i\eta} - \dots \right]$$

$$) = - \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left[\frac{S_{ij}(\omega')}{\omega - \omega' + iy} - \frac{S_{ji}(\omega')}{\omega + \omega' + iy} \right] = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left[S_{ij}(\omega') - S_{ji}(-\omega') \right]$$

$$S_{ij}(\omega) = \frac{P_{ij}(\omega)}{1 - e^{-\beta\omega}} ; \chi_{ij}(\omega) = - \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left[\frac{S_{ij}(\omega')}{\omega - \omega' + i\eta} - \frac{S_{ij}(\omega')}{\omega - \omega' - i\eta} \right]$$

$$S_{ij}(\omega) = e^{\beta\omega} S_{ji}(-\omega) ; \chi_{ij}(\omega) = - \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} S_{ij}(\omega') (1 - e^{-\beta\omega'})$$

$$\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left[\frac{S_{ij}(\omega')}{\omega - \omega' + iy} - \frac{S_{ji}(\omega')}{\omega + \omega' + iy} \right] = - \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left[S_{ij}(\omega') - S_{ji}(-\omega') \right] \frac{1}{\omega - \omega'}$$

$$\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} S_{ij}(\omega') (1 - e^{-\beta\omega'}) \frac{1}{\omega - \omega' + iy} = - \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{P_{ij}(\omega')}{\omega - \omega' + iy}$$

$$\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left[\frac{S_{ij}(\omega')}{\omega - \omega' + iy} - \frac{S_{ji}(\omega')}{\omega + \omega' + iy} \right] = - \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left[S_{ij}(\omega') - S_{ji}(-\omega') \right] \frac{1}{\omega - \omega'}$$

$$\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} S_{ij}(\omega') (1 - e^{-\beta\omega'}) \frac{1}{\omega - \omega' + iy} = - \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{P_{ij}(\omega')}{\omega - \omega' + iy}$$

$$= - \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{P_{ij}(\omega')}{i\omega_n - \omega'} \quad , \quad G_{ij}(i\omega_n \rightarrow \omega + iy) = K_{ij}(\omega)$$

$$\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left[\frac{S_{ij}(\omega')}{\omega - \omega' + iy} - \frac{S_{ji}(\omega')}{\omega + \omega' + iy} \right] = - \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left[S_{ij}(\omega') - S_{ji}(-\omega') \right] \frac{1}{\omega - \omega'}$$

$$\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} S_{ij}(\omega') (1 - e^{-\beta\omega'}) \frac{1}{\omega - \omega' + iy} = - \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{P_{ij}(\omega')}{\omega - \omega' + iy}$$

$$= - \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{P_{ij}(\omega')}{i\omega_n - \omega'} \quad , \quad G_{ij}(i\omega_n \rightarrow \omega + iy) = K_{ij}(\omega)$$

$$G_{ij}(t) = \langle \sigma_i^z(t) \sigma_j^z(0) \rangle = \frac{1}{Z} \text{Tr} [e^{-\beta H} \sigma_i^z(t) \sigma_j^z(0)]$$

$$0 \leq t \leq \beta$$

$$= \frac{1}{Z} \text{Tr} [e^{-(\beta-t)H} \dots]$$

$$G_{ij}(\tau) = \langle \sigma_i^z(\tau) \sigma_j^z(0) \rangle = \frac{1}{Z} \text{Tr} [e^{-\beta H} \sigma_i^z(\tau) \sigma_j^z(0)]$$

$0 \leq \tau \leq \beta$

$$= \frac{1}{Z} \text{Tr} [e^{-(\beta-\tau)H} \sigma_i^z e^{-\tau H} \sigma_j^z] = \frac{1}{Z}$$

$$G_{ij}(t) = \langle \sigma_i^z(t) \sigma_j^z(0) \rangle = \frac{1}{Z} \text{Tr} [e^{-\beta H} \sigma_i^z(t) \sigma_j^z(0)]$$

$$0 \leq t \leq \beta$$

$$= \frac{1}{Z} \text{Tr} [e^{-(\beta-t)H} \sigma_i^z e^{-tH} \sigma_j^z] = \frac{1}{Z} \sum_{\langle \sigma^z(t_n) \rangle} \dots$$

$$S[\varphi] = \int_0^{\beta} dt \int d^d x \left[\frac{1}{2} (\partial_t \varphi)^2 + \frac{1}{2} (\vec{\nabla} \varphi)^2 + \frac{m^2}{2} \varphi^2 + \frac{\lambda}{4!} \varphi^4 \right]$$

$$S[\varphi] = \int_0^{\beta} dt \int d^d x \left[\frac{1}{2} (\partial_t \varphi)^2 + \frac{1}{2} (\vec{\nabla} \varphi)^2 + \frac{\epsilon}{2} \varphi^2 + \frac{\lambda}{4!} \varphi^4 \right]$$

$$G(\vec{x}, t) = \langle \varphi(\vec{x}, t) \varphi(0, 0) \rangle \quad T \gg T_c$$

$$S[\varphi] = \int_0^{\beta} dt \int d^d x \left[\frac{1}{2} (\partial_t \varphi)^2 + \frac{1}{2} (\vec{\nabla} \varphi)^2 + \frac{r}{2} \varphi^2 + \frac{u}{4!} \varphi^4 \right]$$

$$G(\vec{x}, t) = \langle \varphi(\vec{x}, t) \varphi(0, 0) \rangle \quad T \gg T_c \Rightarrow r > 0 \Rightarrow \text{ignore } \varphi^4 \text{ term}$$

$$S[\varphi] = \frac{1}{V\beta} \sum_{\vec{k}, \omega_n} \frac{1}{2} (k^2 + \omega_n^2 + r) |\varphi(\vec{k}, \omega_n)|^2, \quad G(\vec{k}, i\omega_n)$$

$$\left[\frac{\epsilon}{2} \varphi^2 + \frac{\lambda}{4!} \varphi^4 \right]$$

\Rightarrow ignore φ^4 term

$$G(\vec{k}, i\omega_n) = \frac{1}{k^2 + \omega_n^2 + r}$$

$$\begin{aligned} \chi(\vec{k}, \omega) &= G(\vec{k}, i\omega_n \rightarrow \omega + i\eta) = \\ &= \frac{1}{k^2 + r - (\omega + i\eta)^2} = \frac{1}{\sqrt{k^2 + r}} \end{aligned}$$

$$\left[\frac{\Gamma}{2} \varphi^2 + \frac{\Gamma}{4!} \varphi^4 \right]$$

\Rightarrow ignore φ^4 term

$$G(\vec{k}, i\omega_n) = \frac{1}{k^2 + \omega_n^2 + \Gamma}$$

$$\begin{aligned} \chi(\vec{k}, \omega) &= G(\vec{k}, i\omega_n \rightarrow \omega + i\eta) = \\ &= \frac{1}{k^2 + \Gamma - (\omega + i\eta)^2} = \frac{1}{(\sqrt{k^2 + \Gamma} - \omega - i\eta)(\sqrt{k^2 + \Gamma} + \omega + i\eta)} \end{aligned}$$

$$\left[\frac{\epsilon}{2} \varphi^2 + \frac{\lambda}{4!} \varphi^4 \right]$$

\Rightarrow ignore φ^4 term

$$G(\vec{k}, i\omega_n) = \frac{1}{k^2 + \omega_n^2 + \Gamma}$$

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$$\operatorname{Im} \frac{1}{x+iy} = \operatorname{Im} \frac{x-iy}{x^2+y^2} = -\frac{y}{x^2+y^2}$$



$$\operatorname{Im} \frac{1}{x+iy} = \operatorname{Im} \frac{x-iy}{x^2+y^2} = -\frac{y}{x^2+y^2} = -\pi \delta(x)$$

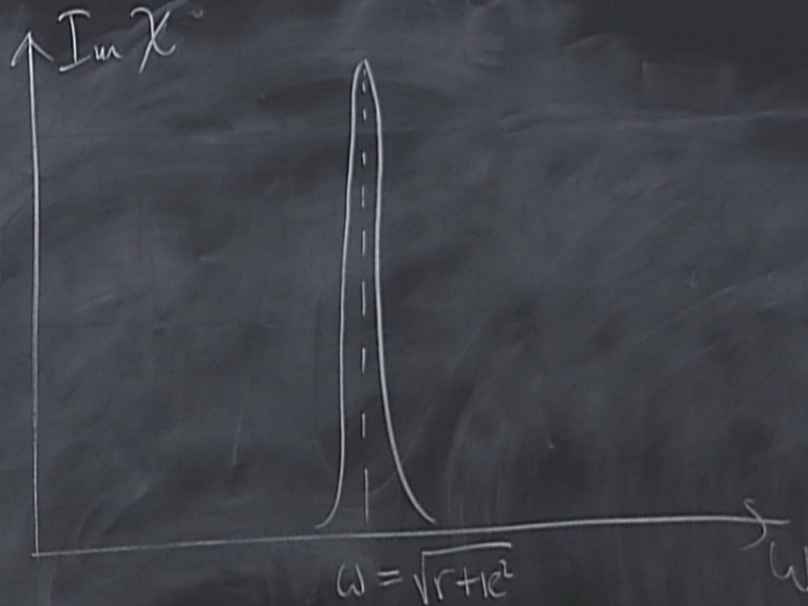
$$\operatorname{Im} \frac{1}{x+iy} = \operatorname{Im} \frac{x-iy}{x^2+y^2} = -\frac{y}{x^2+y^2} = -\pi \delta(x)$$

$$\operatorname{Im} \chi(k, \omega) = \frac{\pi}{2\sqrt{r+e^2}} \left[\delta(\omega - \sqrt{r+e^2}) - \delta(\omega + \sqrt{r+e^2}) \right]$$

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$$\sqrt{r+e^2}$$



$\text{Im } \chi$



$$\omega = \sqrt{r+k^2}$$

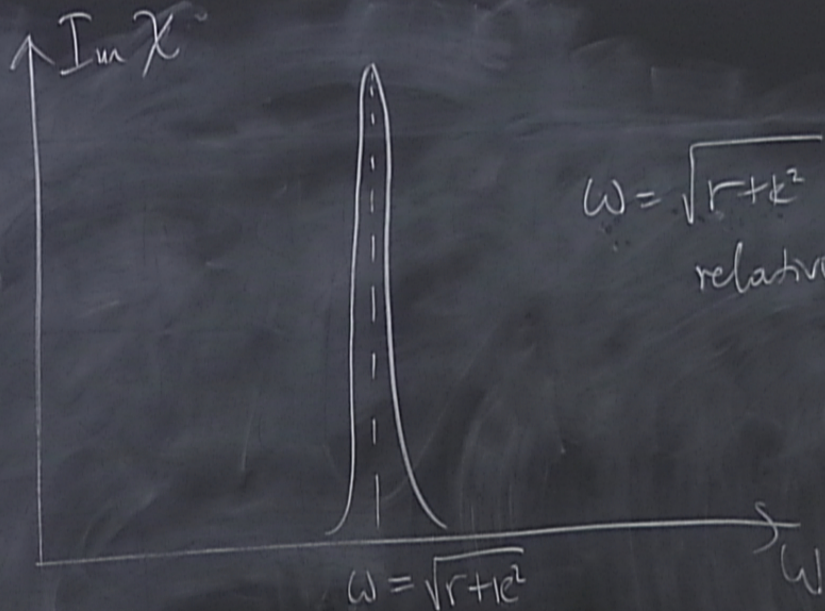
$\omega = \sqrt{r+k^2}$ - dispersion of a relativistic particle of mass $m = \sqrt{r}$

$\text{Im } \chi$



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Correlation function when $\frac{h}{\lambda} \gg 1$ $|GS\rangle = |\rightarrow \rightarrow \rightarrow \rightarrow$

At the critical point:

$$G(x) \sim \frac{1}{x^{D-2+\eta}} = \frac{1}{x^{d-1+\eta}}$$

$$G(\vec{k}) = \int d^D x e^{-i\vec{k}\cdot\vec{x}} G(\vec{x}) \sim \frac{1}{k^{D-1+\eta}}$$

At the critical point:

$$G(x) \sim \frac{1}{x^{D-2+\gamma}} = \frac{1}{x^{d-1+\gamma}}$$

$$G(\vec{k}) = \int d^D x e^{-i\vec{k}\cdot\vec{x}} G(\vec{x}) \sim \frac{1}{k^{d-1+\gamma}}$$

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$$G(\vec{k}) = \int d^D x e^{-i\vec{k}\cdot\vec{x}} G(\vec{x}) \sim \frac{1}{k^{2-\eta}} ; G(E, \omega_n) \sim \frac{1}{(E^2 - \omega_n^2)}$$

$$G(\vec{k}) = \int d^D x e^{-i\vec{k}\cdot\vec{x}} \quad G(\vec{x}) \sim \frac{1}{k^{2-\epsilon}} \quad ; \quad G(\vec{k}, i\omega_n) \sim \frac{1}{(k^2 + \omega_n^2)^{\frac{2-\epsilon}{2}}}$$

$$\chi(\vec{k}, \omega) \sim \frac{1}{(k^2 - \omega^2)^{\frac{2-\epsilon}{2}}} \quad , \quad \text{Im} \chi(\vec{k}, \omega) = \theta(\omega - k)$$

$$G(\vec{x}) \sim \frac{1}{k^{2-\gamma}} \quad ; \quad G(E, \omega) \sim \frac{1}{(k^2 + \omega^2)^{\frac{2-\gamma}{2}}}$$

$$X(E, \omega) \sim \frac{1}{(k^2 - \omega^2)^{\frac{2-\gamma}{2}}} \quad ; \quad \text{Im} X(E, \omega) = \theta(\omega - k) \text{Im} \left[\frac{e^{-i\pi \left(\frac{2-\gamma}{2}\right)}}{(\omega^2 - k^2)^{\frac{2-\gamma}{2}}} \right]$$

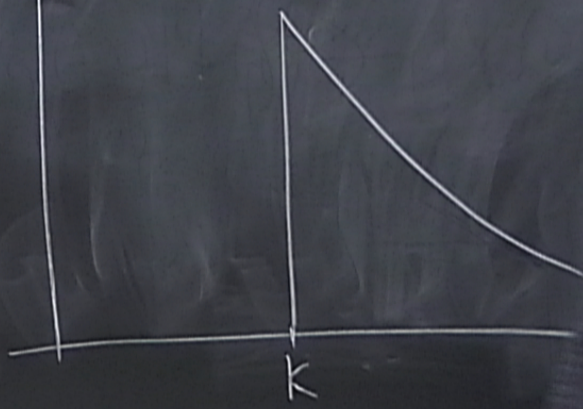
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$$G(\vec{x}) \sim \frac{1}{k^{2-\eta}} ; G(E, \omega)$$

↑ Im x



$$G(E, \omega) \sim \frac{1}{(k^2 - \omega^2)^{\frac{2-\eta}{2}}}$$

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$$G(\vec{x}) \sim \frac{1}{k^{2-\eta}} ; G(E, \omega)$$



$$\chi(E, \omega) \sim \frac{1}{(k^2 - \omega^2)^{\frac{2-\eta}{2}}}$$

Quasiparticles disappear at the critical point