Title: 14/15 PSI - Statistical Mechanics - Lecture 14

Date: Oct 24, 2014 10:45 AM

URL: http://pirsa.org/14100102

Abstract:

Imaginary-time correlation function:

$\tau > 0$

$$G_{ij}(\tau) = \langle \sigma_i^z(\tau)\,\sigma_j^z(0) \rangle = \frac{1}{Z}\,\mathrm{Tr}\left[ e^{-\beta H}\,\sigma_i^z(\tau)\,\sigma_j^z(0) \right] = \frac{1}{Z}$$

$$\ldots(\tau)\sigma_j^z(0)\rangle = \frac{1}{Z}\sum_{nm}\langle n|e^{-\beta H}|n\rangle\langle n|\sigma_i^z(\tau)|m\rangle\langle m|\sigma_j^z(0)|n\rangle =$$

Imaginary-time correlation function:

$\sigma_i^z(\tau) = e^{H\tau} \sigma_i^z e$

$\tau > 0$

$$G_{ij}(\tau) = \langle \sigma_i^z(\tau)\sigma_j^z(0)\rangle = \frac{1}{Z}\mathrm{Tr}\left[e^{-\beta H}\sigma_i^z(\tau)\sigma_j^z(0)\right] = \frac{1}{Z}\sum_{nm}$$

$$= \frac{1}{Z}\sum_n e^{-\beta E_n} e^{\tau E_n} e^{-\tau E_m}$$

$$\sigma_i^z(\tau) = e^{H\tau}\,\sigma_i^z\,e^{-H\tau}$$

$$\ldots{}^{-\beta H}\,\sigma_i^z(\tau)\,\sigma_j^z(0)\big] = \frac{1}{Z}\sum_{nm}\langle n|e^{-\beta H}|n\rangle\langle n|\sigma_i^z(\tau)|m\rangle\langle m|\sigma_j^z(0)|n\rangle =$$

$$\langle n|\sigma_i^z|m\rangle\langle m|\sigma_j^z|n\rangle$$

Imaginary-time correlation function: $\sigma_i^z(\tau) = e^{H\tau}\sigma_i^z e$

$\tau > 0$

$$G_{ij}(\tau) = \langle \sigma_i^z(\tau)\sigma_j^z(0)\rangle = \frac{1}{Z}\mathrm{Tr}\left[e^{-\beta H}\sigma_i^z(\tau)\sigma_j^z(0)\right] = \frac{1}{Z}\sum_{nm}\langle$$

$$= \frac{1}{Z}\sum_{n} e^{-\beta E_n} e^{\tau E_n} e^{-\tau E_m}\langle n|\sigma_i^z|m\rangle\langle m|\sigma_j^z|n\rangle =$$

$$E_{mn} = E_m - E_n$$

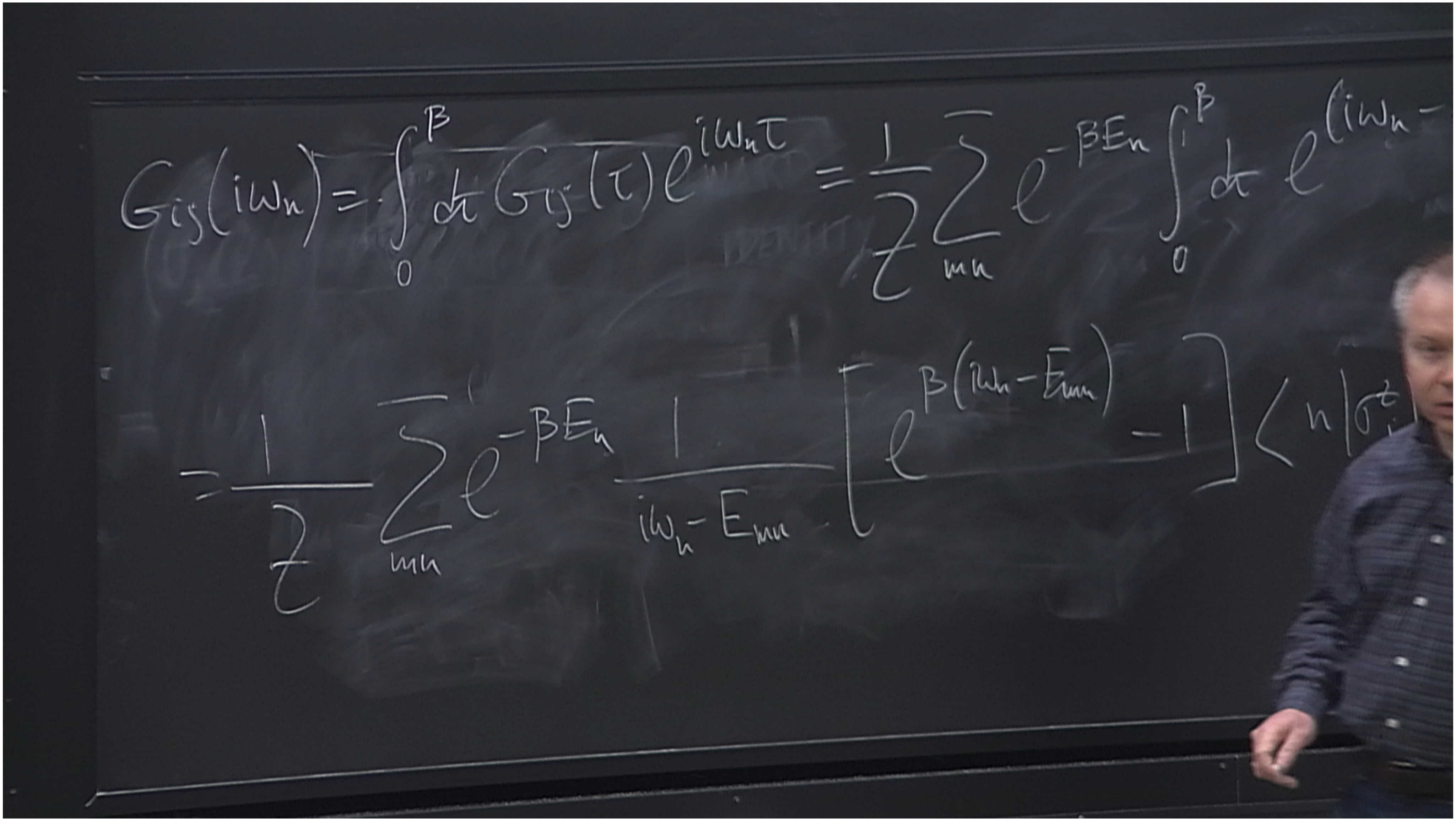
$$\sigma_i^z(\tau) = e^{H\tau}\,\sigma_i^z\,e^{-H\tau}$$

$$\ldots\sigma_i^z(\tau)\,\sigma_j^z(0)\big) = \frac{1}{Z}\sum_{nm}\langle n|e^{-\beta H}|n\rangle\langle n|\sigma_i^z(\tau)|m\rangle\langle m|\sigma_j^z(0)|n\rangle =$$

$$\ldots\sigma_i^z|m\rangle\langle m|\sigma_j^z|n\rangle = \frac{1}{Z}\sum_{n} e^{-\beta E_n}\,e^{-\tau E_{mn}}\,\langle n|\sigma_i^z|m\rangle\langle m|\sigma_j^z|n\rangle$$

$$G_{ij}(i\omega_n) = \int_0^\beta d\tau\, G_{ij}(\tau)\, e^{i\omega_n \tau} = \frac{1}{Z} \sum_{mn} e^{-\beta E_n} \int_0^\beta d\tau\, e^{(i\omega_n - E_{mn}}$$

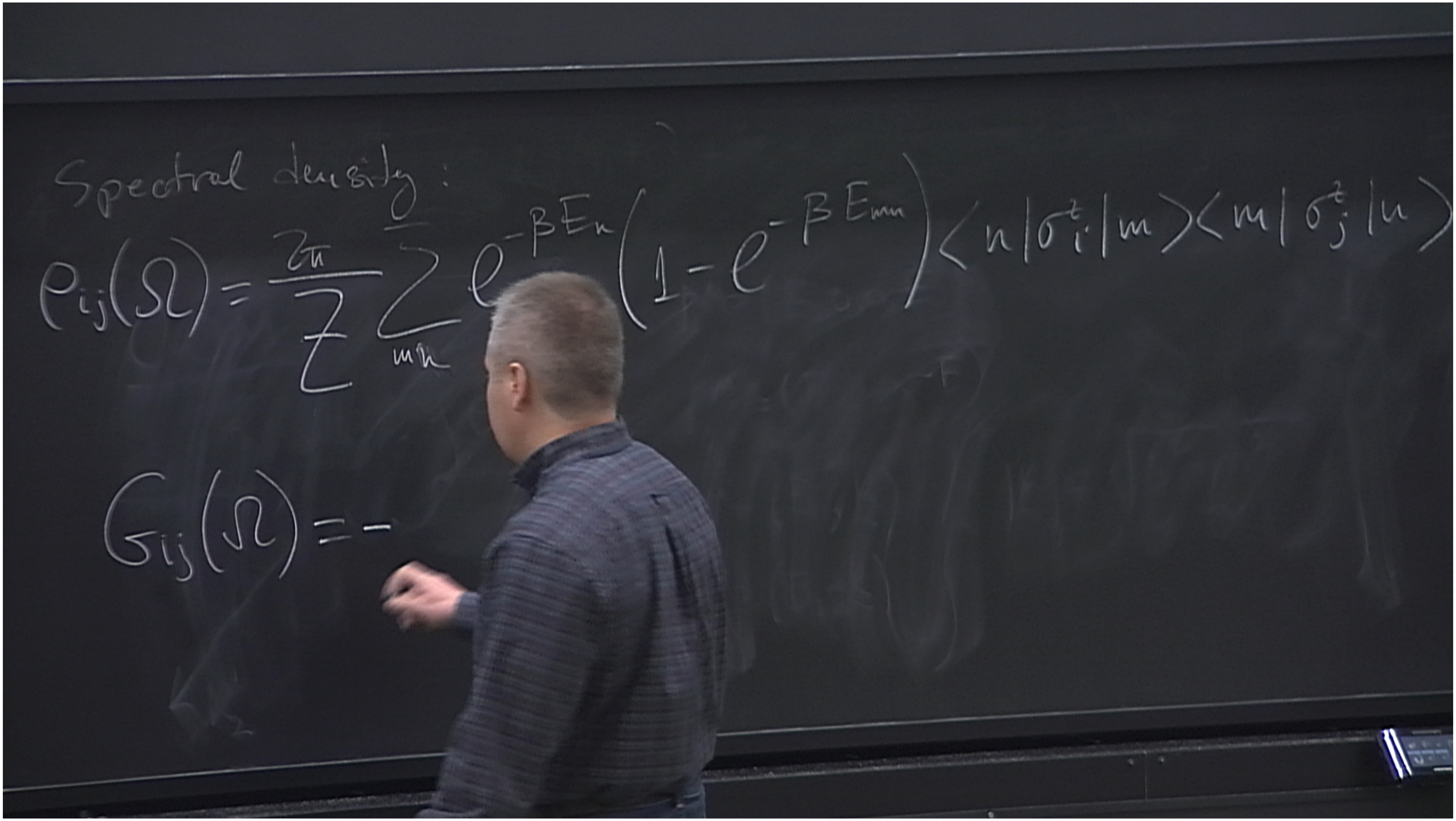
$$e^{i\omega_n \tau} = \frac{1}{Z}\sum_{mn} e^{-\beta E_n}\int_0^\beta d\tau\, e^{(i\omega_n - E_{mn})\tau}\langle n|\sigma^z_i|m\rangle\langle m|\sigma^z_j|n\rangle =$$

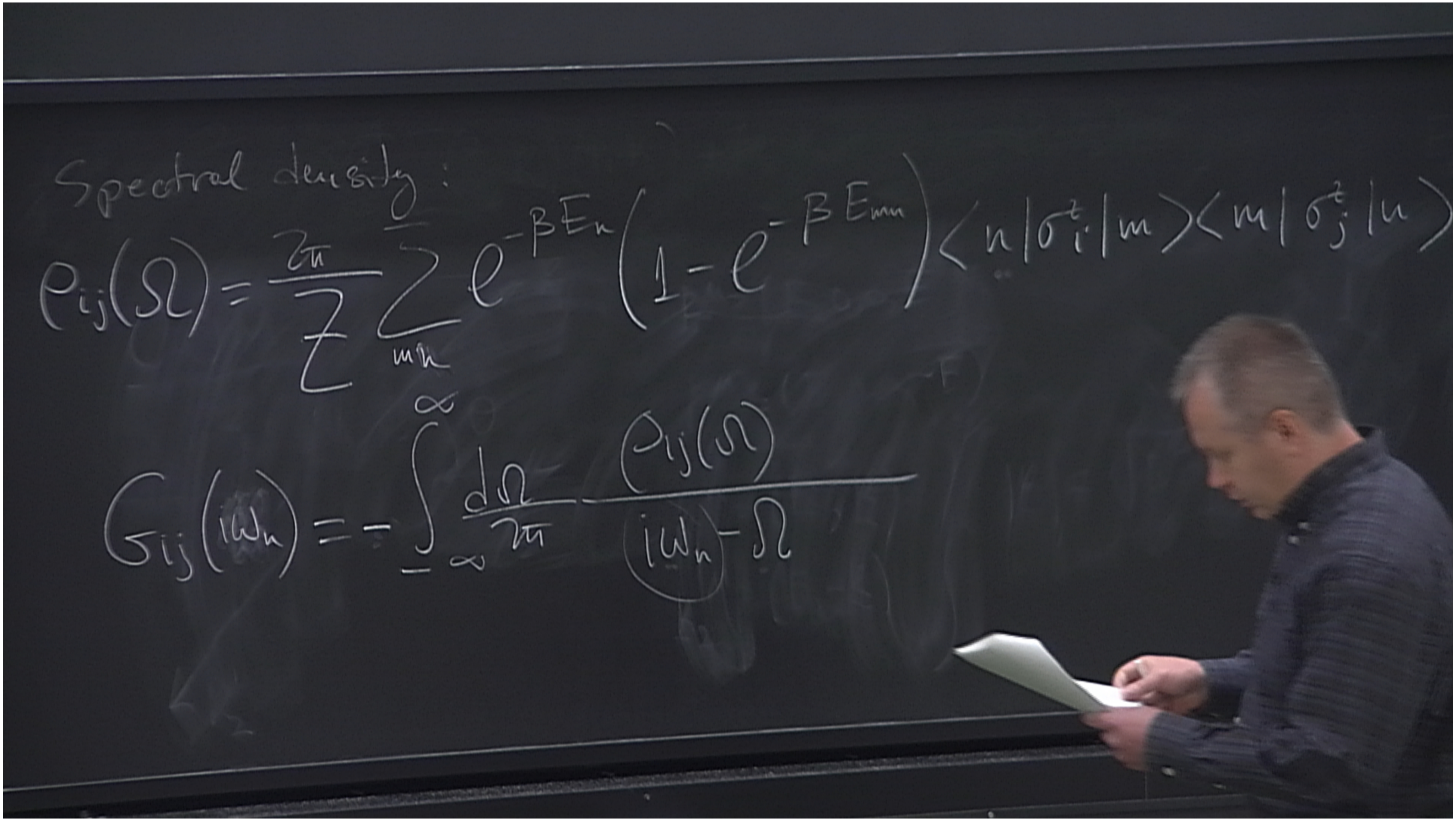

$G_{ij}(i\omega_n) = \int_0^\beta d\tau\, G_{ij}(\tau)\, e^{i\omega_n \tau} = \frac{1}{Z} \sum_{mn} e^{-\beta E_n} \int_0^\beta d\tau\, e^{(i\omega_n -}$
$= \frac{1}{Z} \sum_{mn} e^{-\beta E_n} \frac{1}{i\omega_n - E_{mn}} \left[ e^{\beta(i\omega_n - E_{mn})} - 1 \right] \langle n |$

$$i\omega_n - E_{mn})\tau$$

$$\langle n|\sigma_i^z|m\rangle\langle m|\sigma_j^z|n\rangle =$$

$$|\sigma_i^z|m\rangle\langle m|\sigma_j^z|n\rangle = \frac{1}{Z}\sum_{mn} e^{-\beta E_n}\left(e^{-\beta E_{mn}} - 1\right)\frac{1}{i\omega_n - E_{mn}}\langle n|\sigma_i^z|m\rangle \cdot \langle m|\sigma_j^z|n\rangle$$


Spectral density:
$\rho_{ij}(\Omega) = \frac{2\pi}{Z} \sum_{mn} e^{-\beta E_n} \left(1 - e^{-\beta E_{mn}}\right) \langle n | \sigma^z_i | m \rangle \langle m | \sigma^z_j | n \rangle$
$G_{ij}(\Omega) = -$


Spectral density:
$\rho_{ij}(\Omega) = \frac{2\pi}{Z} \sum_{mn} e^{-\beta E_n}\left(1 - e^{-\beta E_{mn}}\right) \langle n|\sigma^z_i|m\rangle\langle m|\sigma^z_j|n\rangle$
$G_{ij}(i\omega_n) = -\int_{-\infty}^{\infty} \frac{d\Omega}{2\pi} \frac{\rho_{ij}(\Omega)}{i\omega_n - \Omega}$

Spectral density:

$$\rho_{ij}(\Omega) = \frac{2\pi}{Z} \sum_{mn} e^{-\beta E_n}\left(1 - e^{-\beta E_{mn}}\right) \langle n | \sigma^z_i | m \rangle \langle m | \sigma^z_j | n \rangle$$

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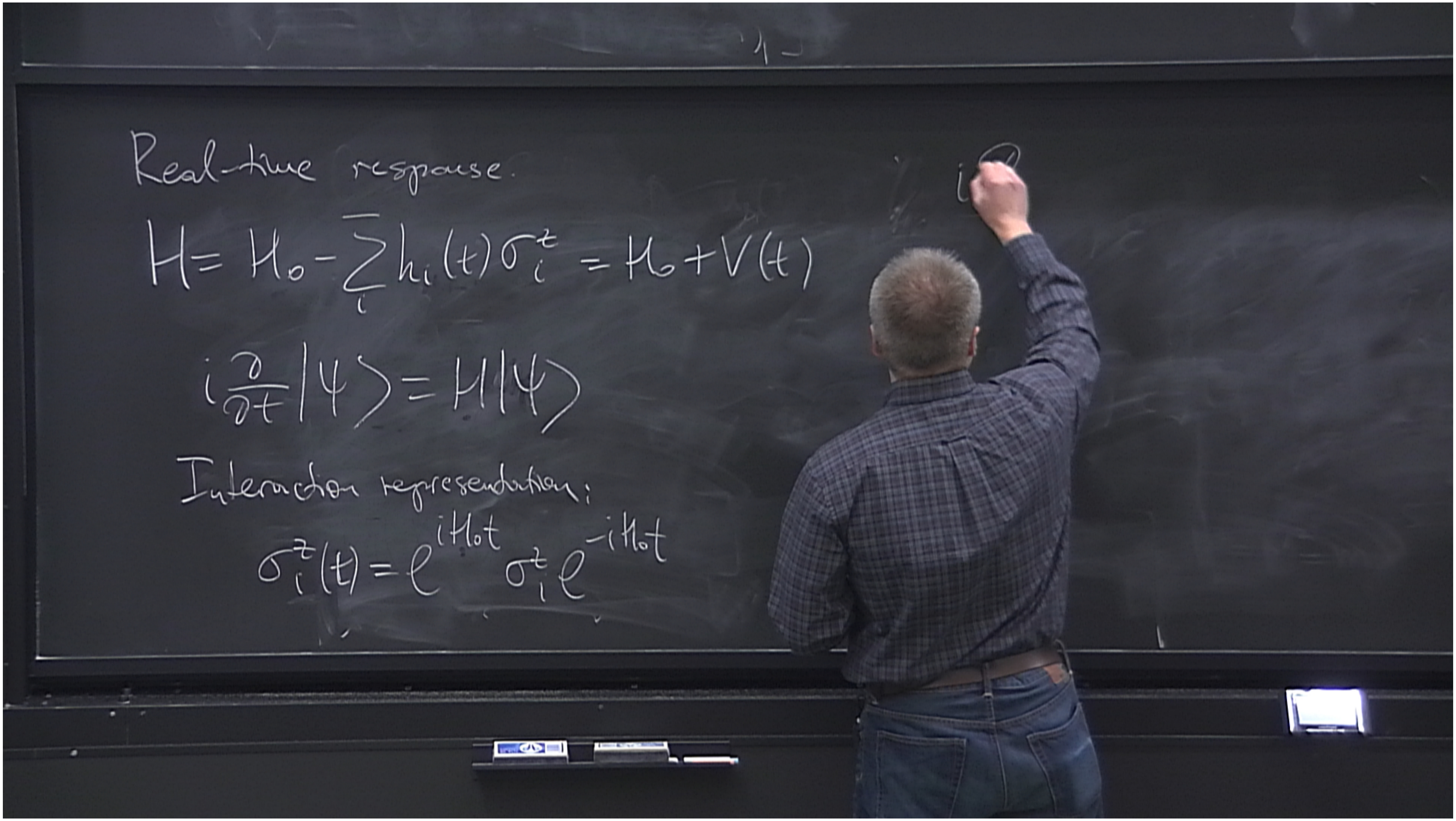
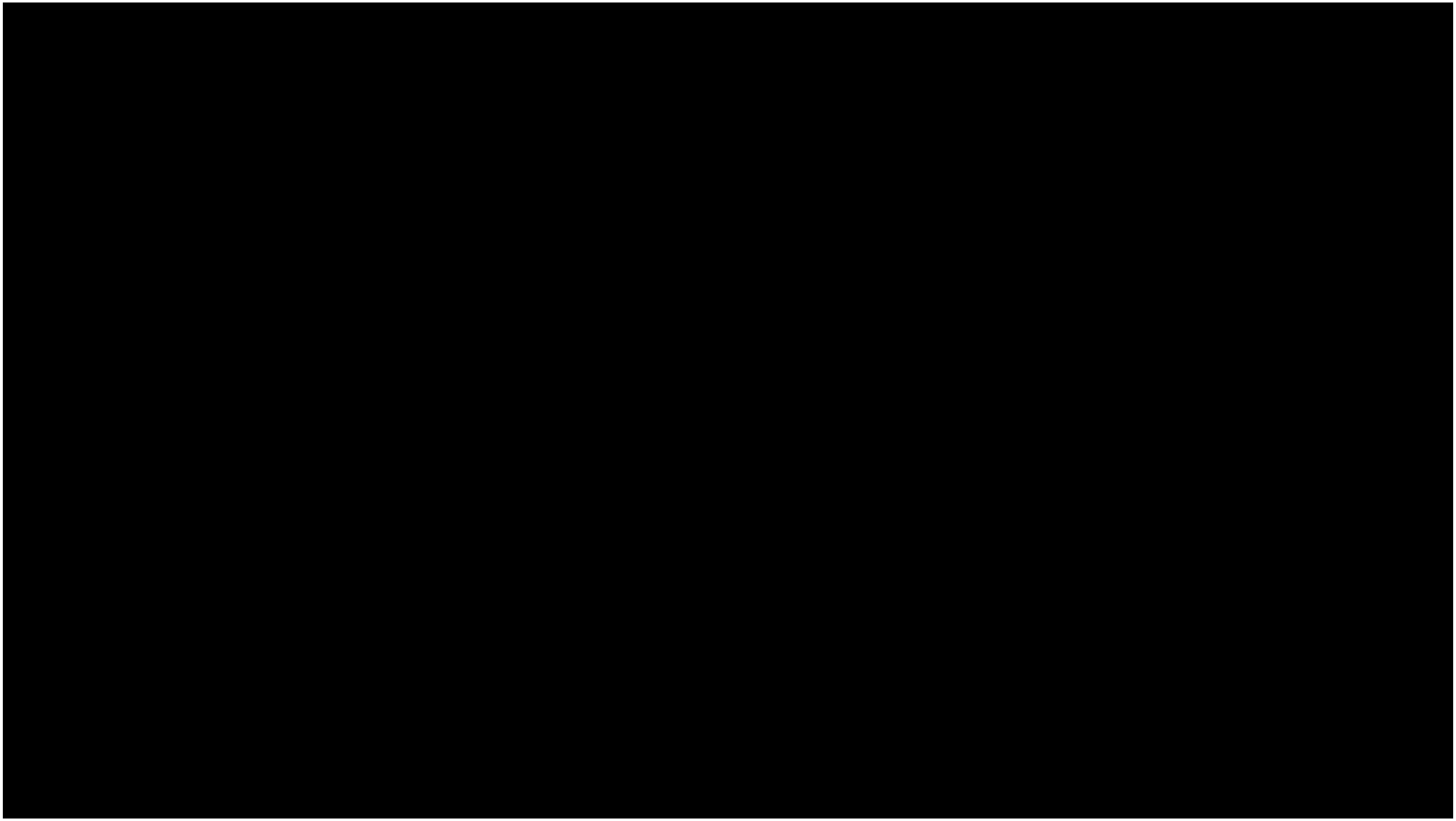
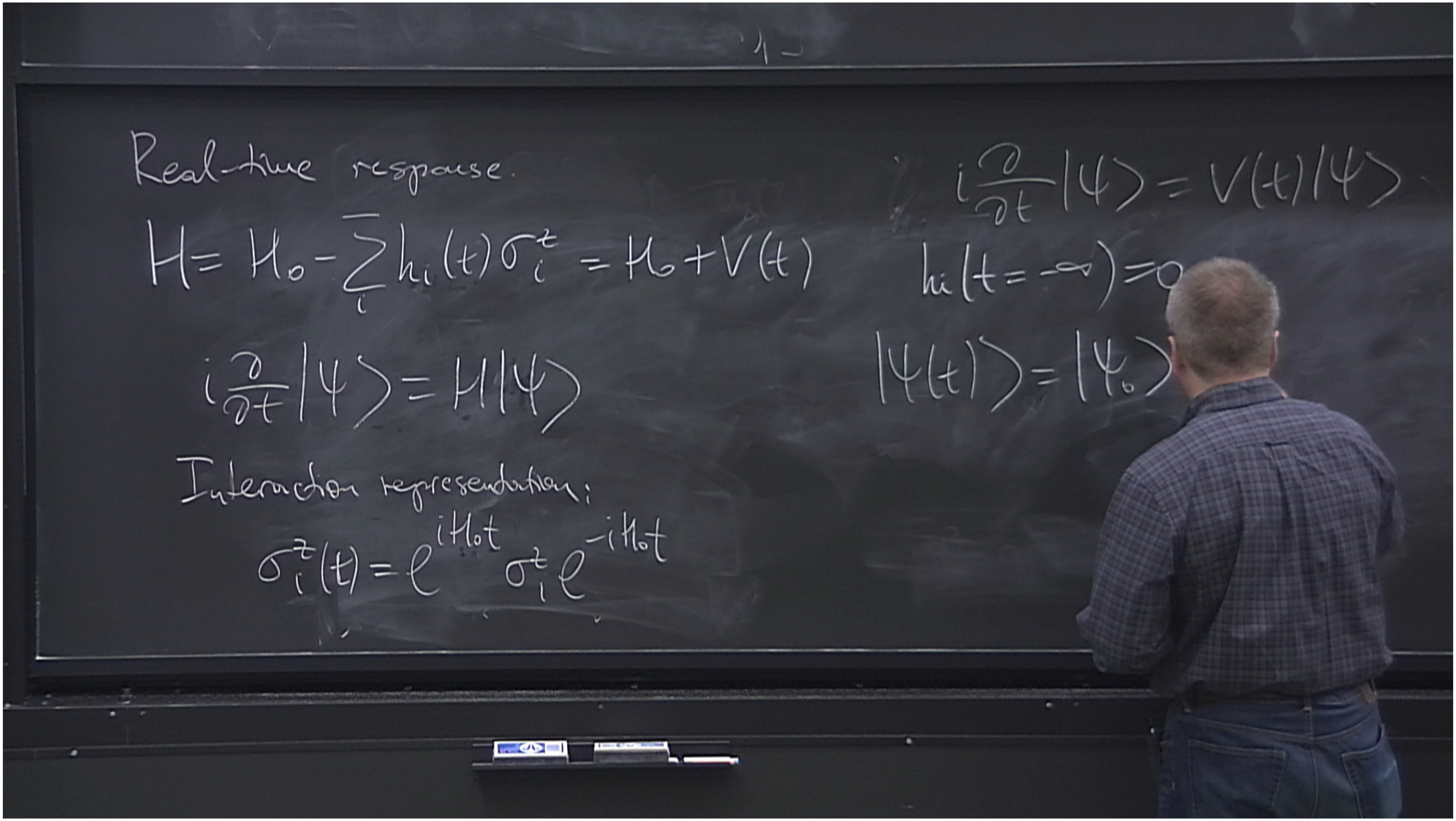

Real-time response.
$H = H_0 - \sum_i h_i(t)\sigma_i^z = H_0 + V(t)$
$i\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$
Interaction representation:
$\sigma_i^z(t) = e^{iH_0 t}\sigma_i^z e^{-iH_0 t}$

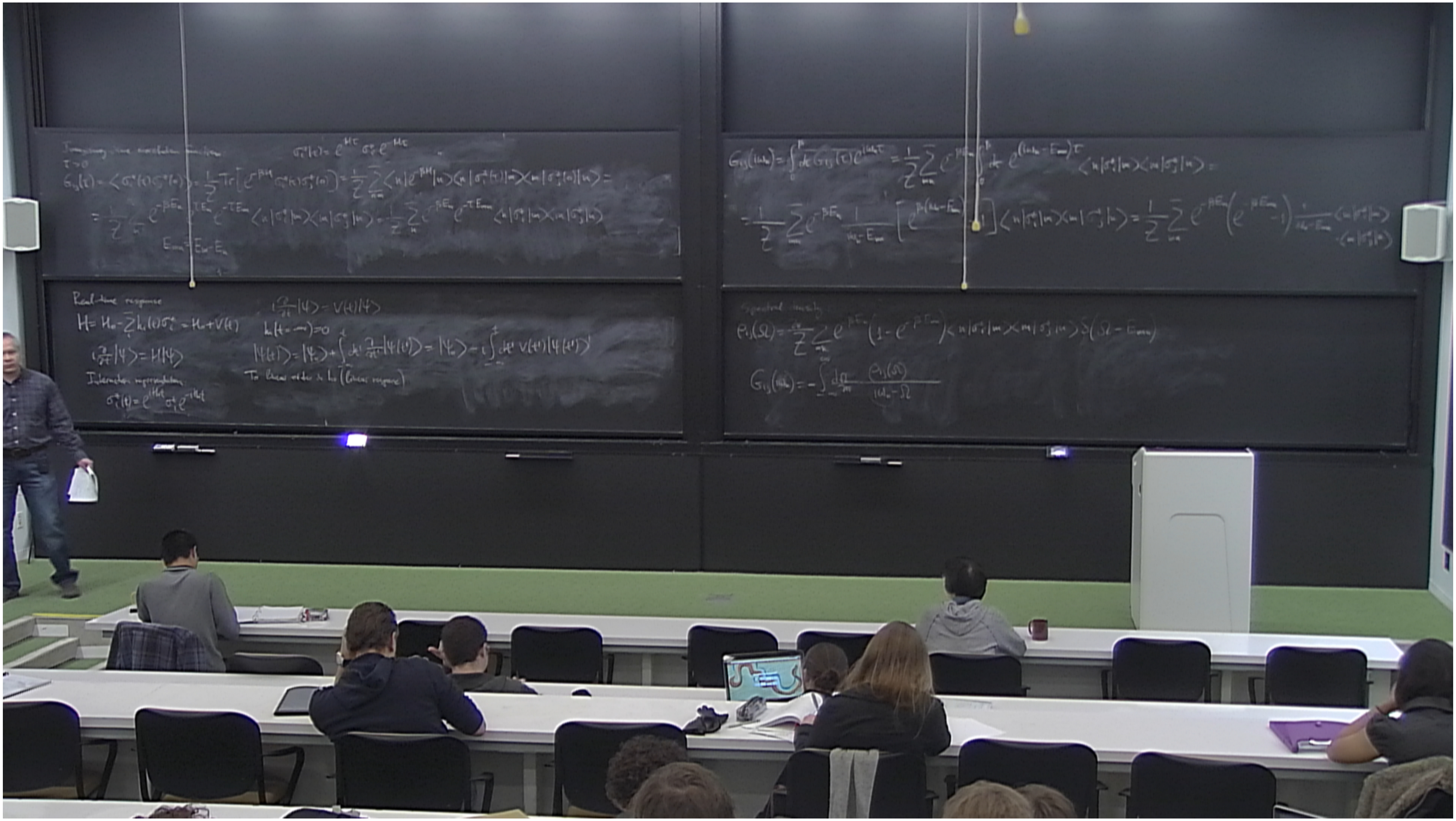


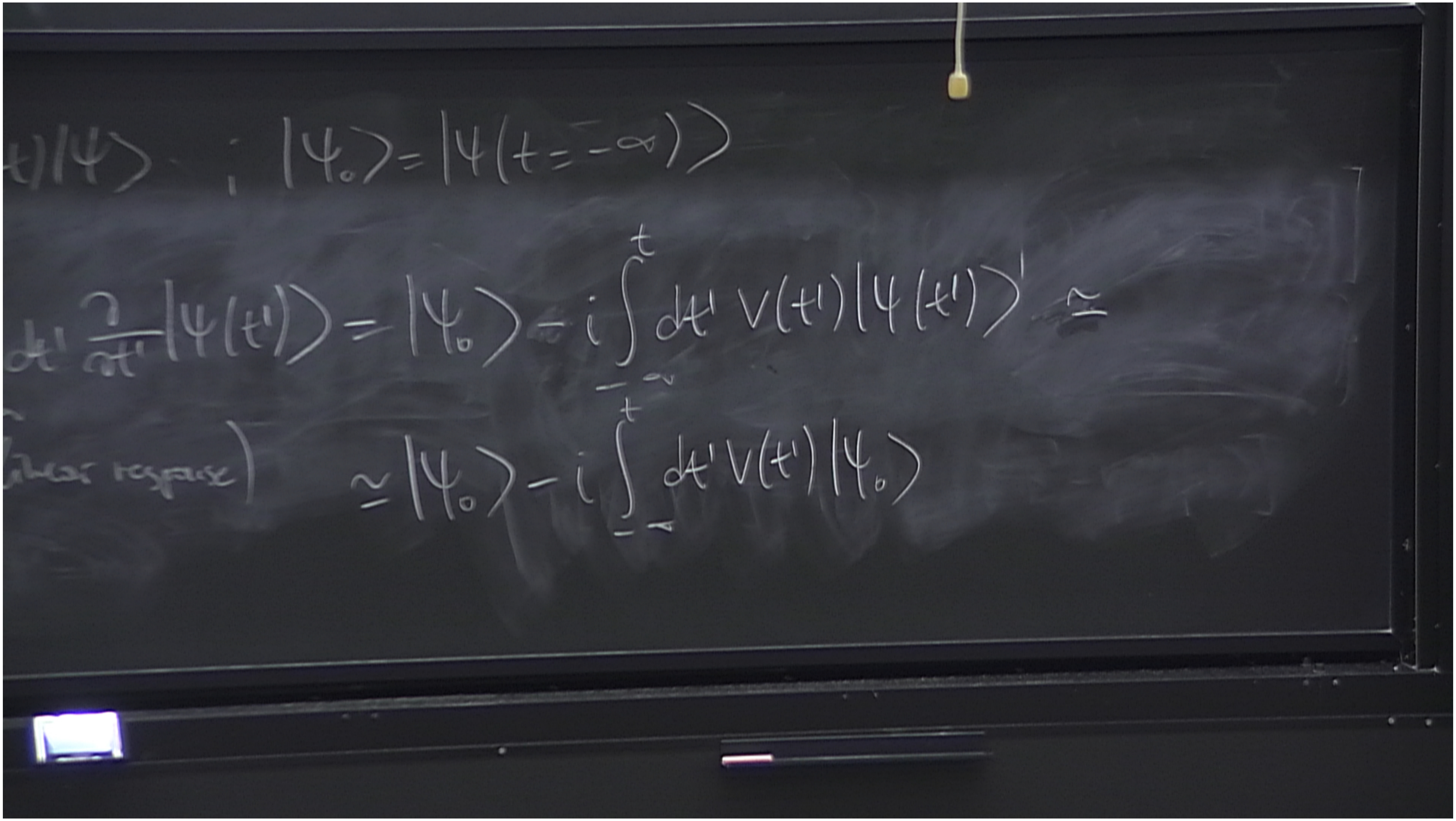

Real-time response.
$H = H_0 - \sum_i h_i(t)\sigma_i^z = H_0 + V(t)$
$i\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$
Interaction representation:
$\sigma_i^z(t) = e^{iH_0 t}\sigma_i^z e^{-iH_0 t}$
$i\frac{\partial}{\partial t}|\psi\rangle = V(t)|\psi\rangle$
$h_i(t=-\infty)=0$
$|\psi(t)\rangle = |\psi_0\rangle$

$$i\frac{\partial}{\partial t}|\psi\rangle = V(t)|\psi\rangle$$

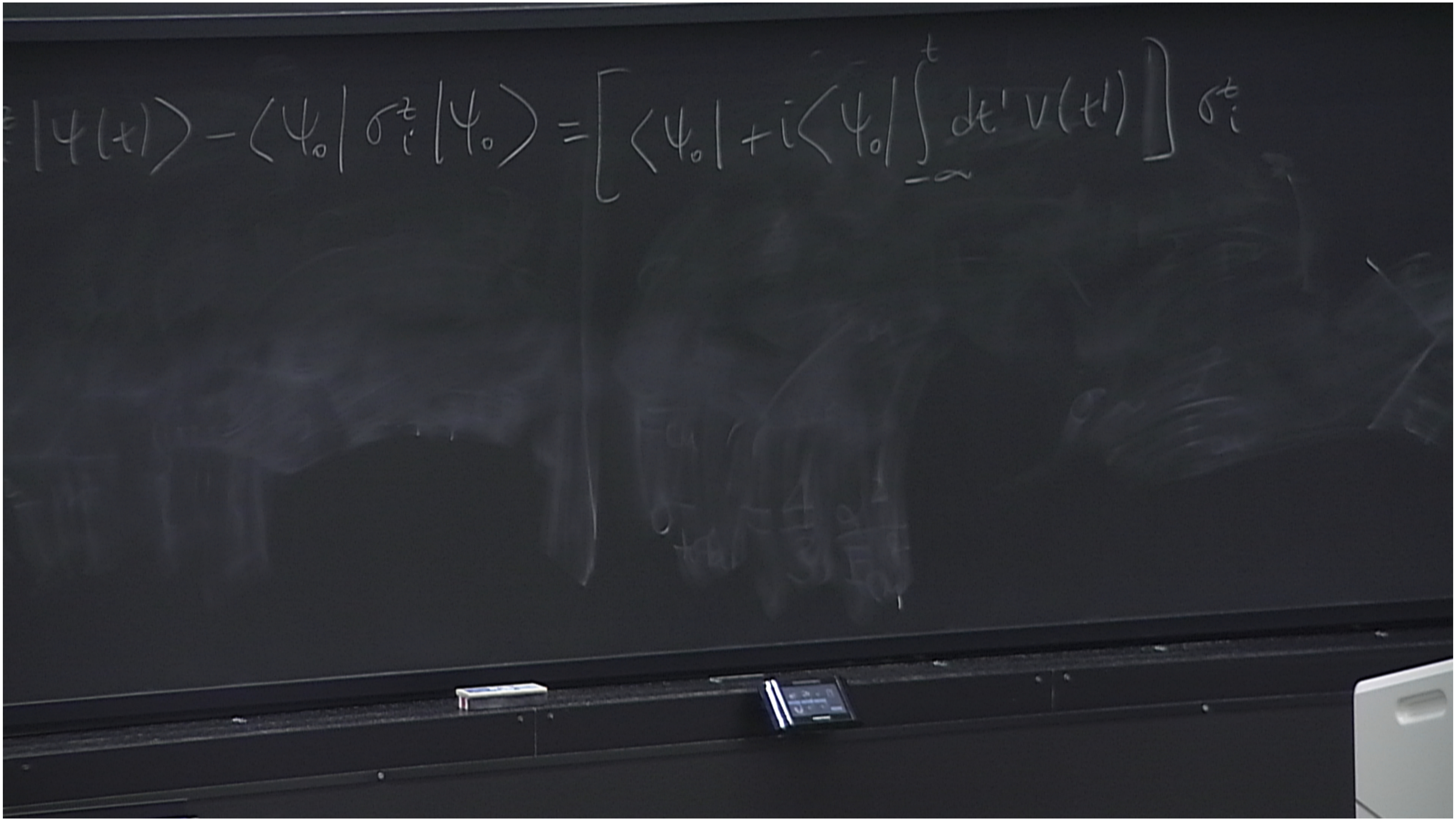
$$h_i(t=-\infty)=0$$

$$|\psi(t)\rangle = |\psi_0\rangle + \int_{-\infty}^{t} dt' \frac{\partial}{\partial t'}|\psi(t')\rangle = |\psi_0\rangle - i\int_{-\infty}^{t} dt'\, V(t')|\psi(t')\rangle$$

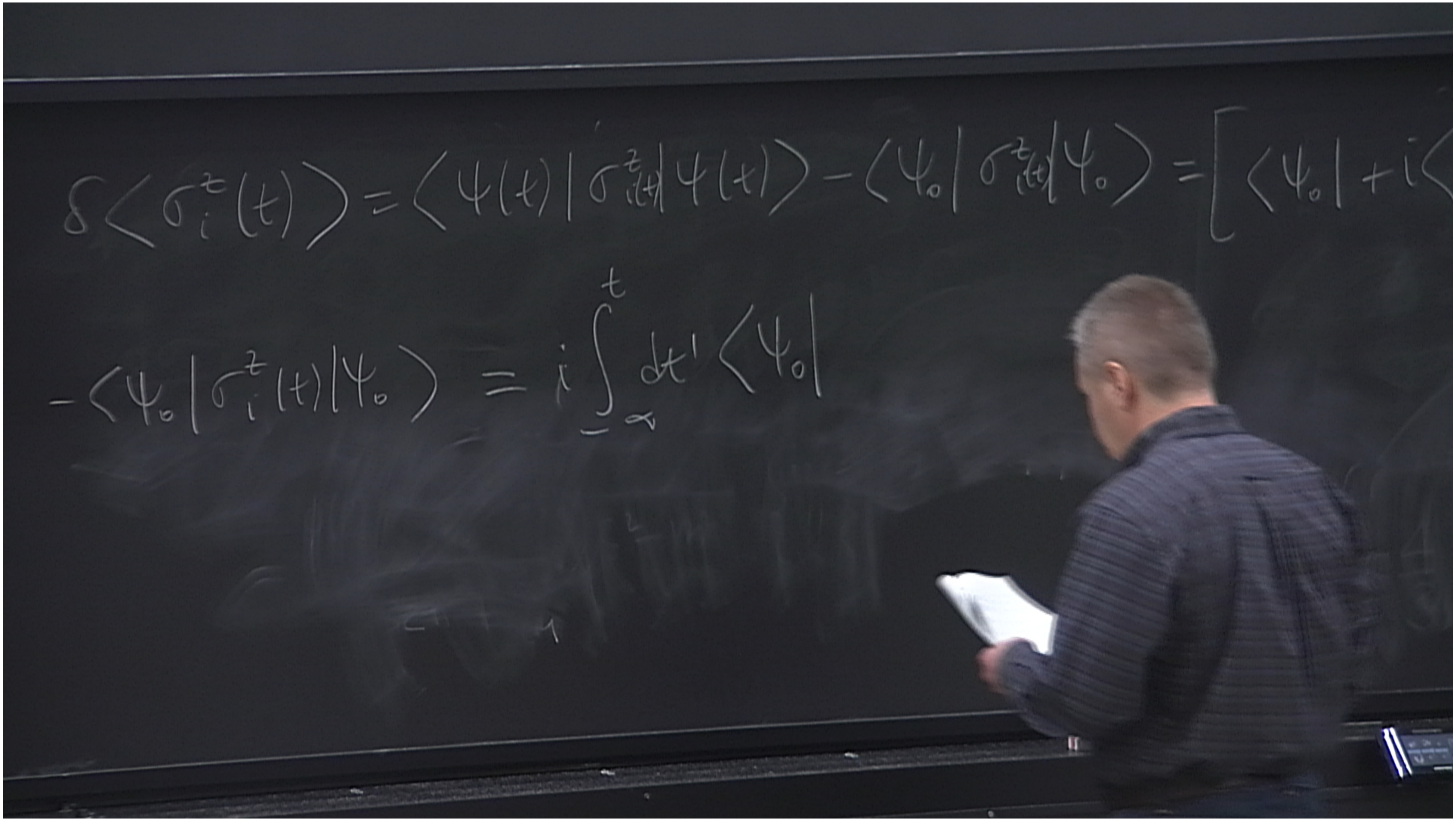


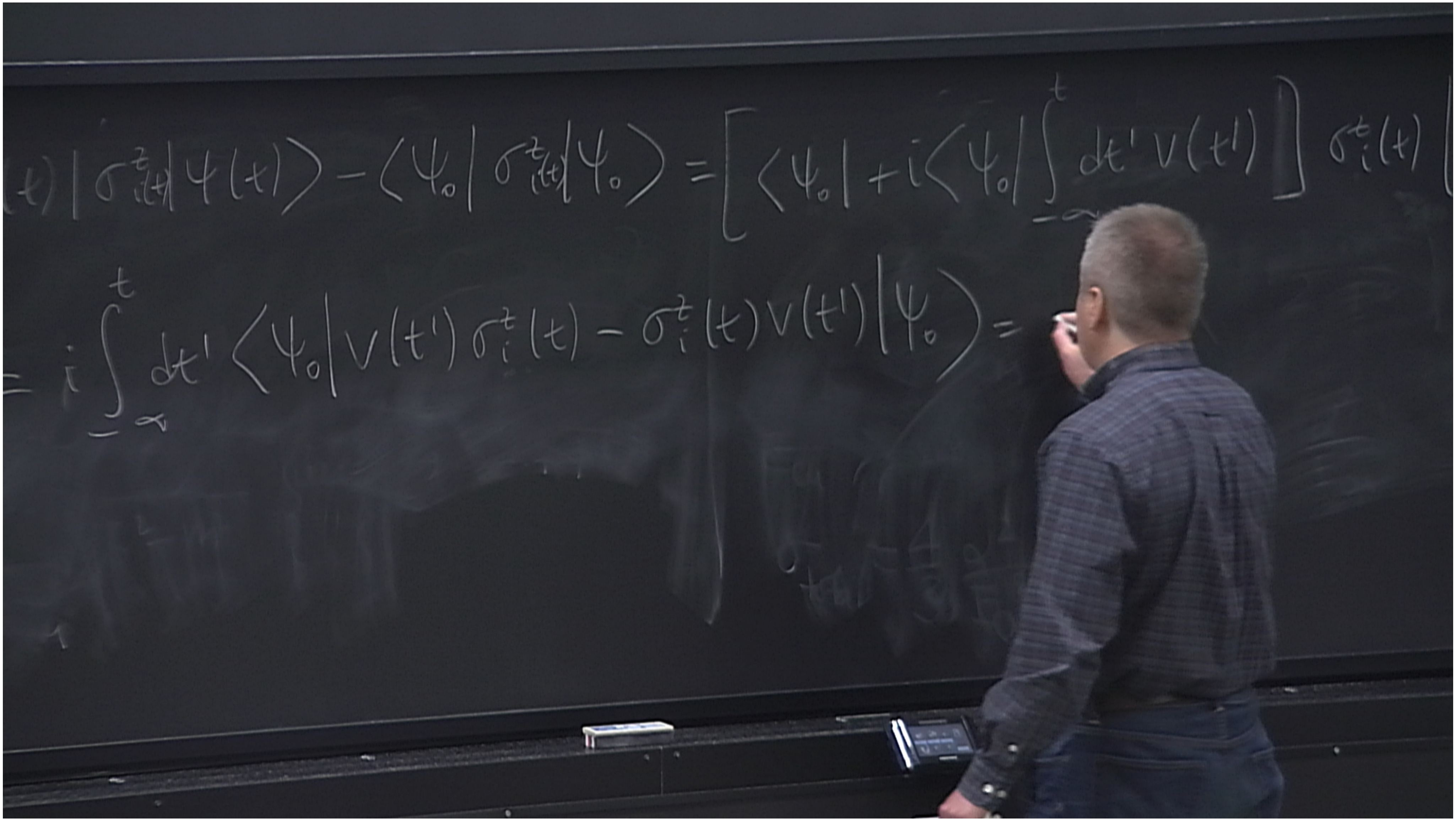

$t)|\psi\rangle \;;\; |\psi_0\rangle = |\psi(t=-\infty)\rangle$
$dt' \frac{\partial}{\partial t'}|\psi(t')\rangle = |\psi_0\rangle - i\int_{-\infty}^{t} dt'\, V(t')|\psi(t')\rangle \simeq$
(linear response)
$\simeq |\psi_0\rangle - i\int_{-\infty}^{t} dt'\, V(t')|\psi_0\rangle$

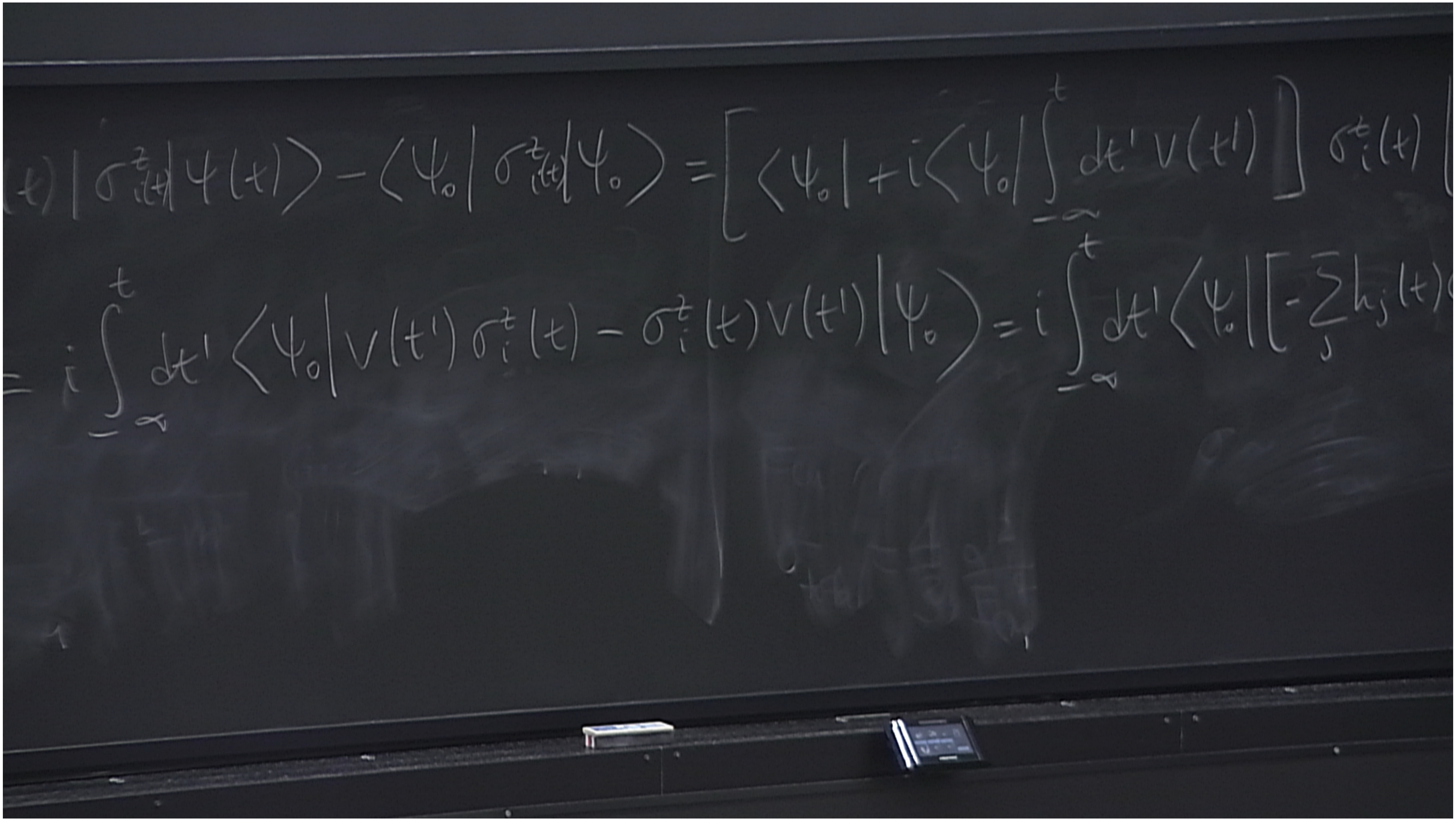
$$\delta\langle \sigma_i^z(t)\rangle = \langle \psi(t)|\sigma_i^z|\psi(t)\rangle - \langle \psi_0|\sigma_i^z|\psi_0\rangle =$$


$|\psi(t)\rangle - \langle\psi_0|\sigma^z_i|\psi_0\rangle = \left[\langle\psi_0| + i\langle\psi_0|\int_{-\infty}^{t} dt' V(t')\right]\sigma^z_i$

$$\delta \langle \sigma_i^z(t) \rangle = \langle \psi(t) | \sigma^z_{i(t)} | \psi(t) \rangle - \langle \psi_0 | \sigma^z_{i(t)} | \psi_0 \rangle = \Big[ \langle \psi_0 | + i$$


$\delta\langle\sigma_i^z(t)\rangle = \langle\psi(t)|\sigma_i^z(t)|\psi(t)\rangle - \langle\psi_0|\sigma_i^z(t)|\psi_0\rangle = \Big[\langle\psi_0| + i\langle$
$-\langle\psi_0|\sigma_i^z(t)|\psi_0\rangle = i\int_{-\infty}^{t} dt'\,\langle\psi_0|$


$(t)|\sigma^z_i(t)|\psi(t)\rangle - \langle\psi_0|\sigma^z_i(t)|\psi_0\rangle = \left[\langle\psi_0| + i\langle\psi_0|\int_{-\infty}^{t} dt'\, V(t')\right]\sigma^z_i(t)\,|$
$= i\int_{-\infty}^{t} dt'\,\langle\psi_0|\,V(t')\,\sigma^z_i(t) - \sigma^z_i(t)\,V(t')\,|\psi_0\rangle =$



$(t)|\sigma^z_{i}(t)|\psi(t)\rangle - \langle\psi_0|\sigma^z_{i}(t)|\psi_0\rangle = \left[\langle\psi_0| + i\langle\psi_0|\int_{-\infty}^{t} dt'\, V(t')\right]\sigma^z_i(t)\,[$
$= i\int_{-\infty}^{t} dt'\,\langle\psi_0|V(t')\,\sigma^z_i(t) - \sigma^z_i(t)\,V(t')|\psi_0\rangle = i\int_{-\infty}^{t} dt'\,\langle\psi_0|\left[-\sum_j h_j(t)\right.$

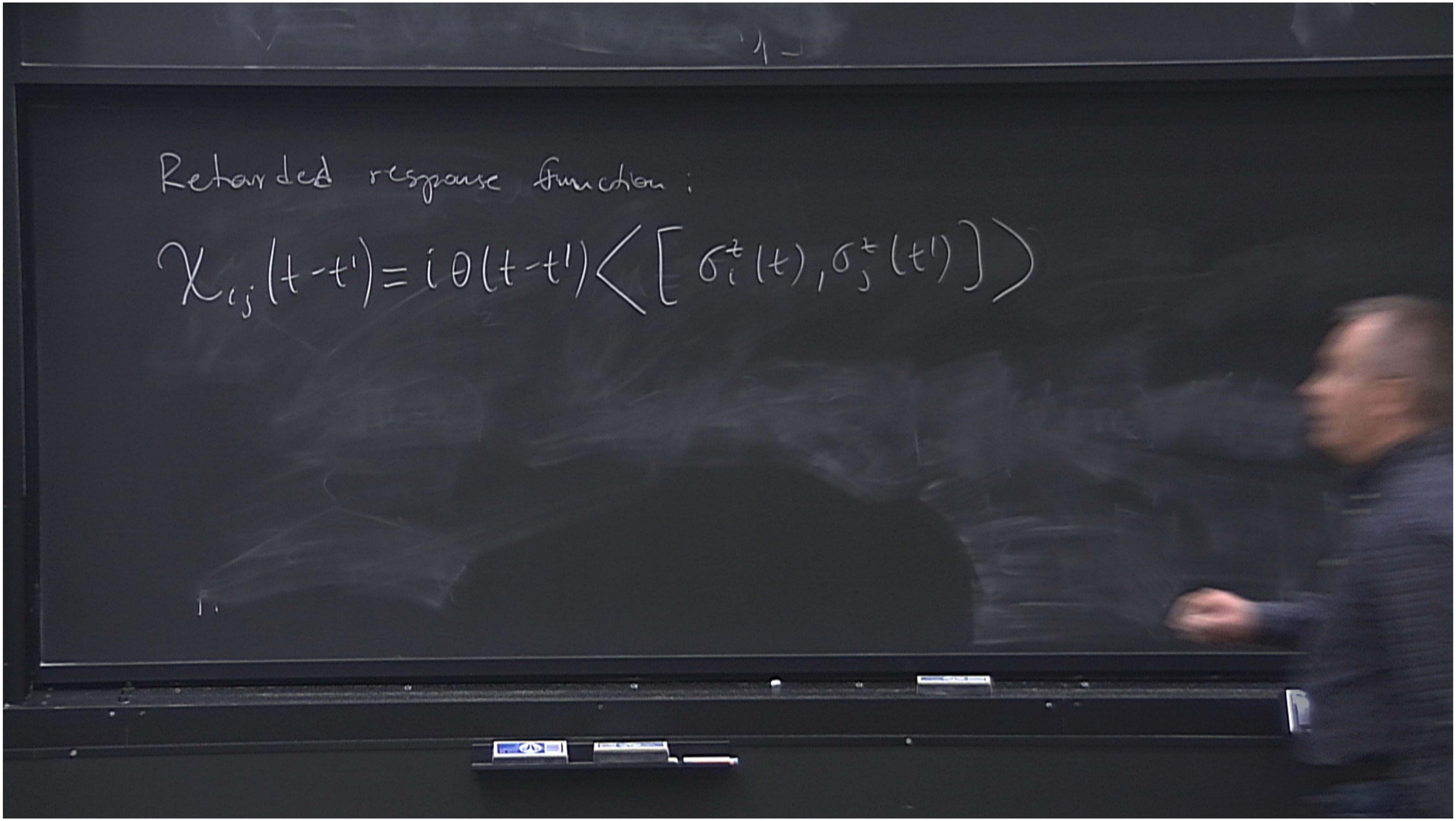
$$\ldots| + i\langle\psi_0|\int_{-\infty}^{t} dt'\, V(t')\Big]\, \sigma_i^z(t)\Big[|\psi_0\rangle - i\int_{-\infty}^{t} dt'\, V(t')|\psi_0\rangle\Big] -$$

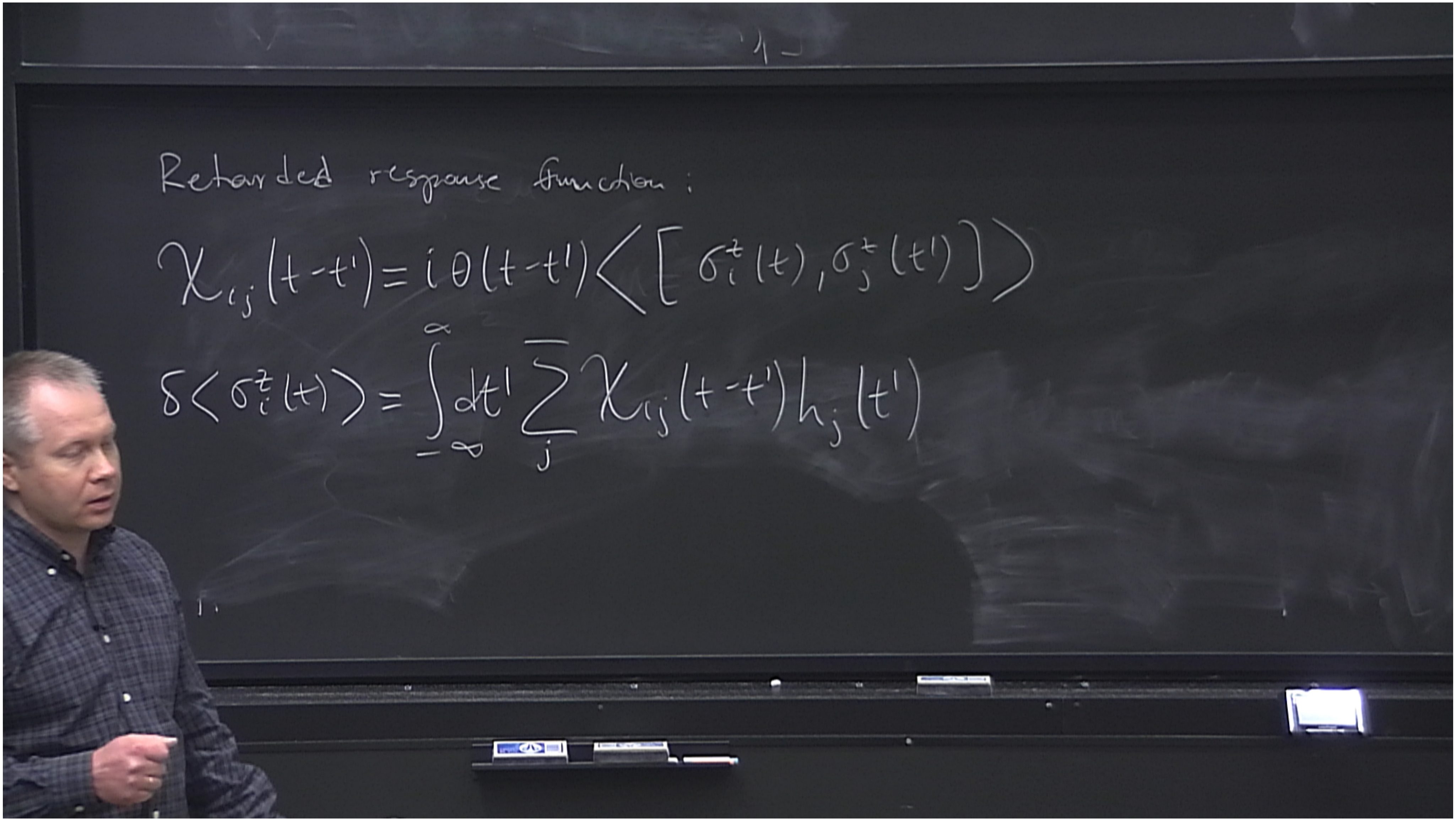
$$\ldots t')|\psi_0\rangle = i\int_{-\infty}^{t} dt'\,\langle\psi_0|\Big[-\sum_j h_j(t')\sigma_j^z(t'),\, \sigma_i^z(t)\Big]|\psi_0\rangle =$$

$$\delta\langle\sigma_i^z(t)\rangle = \langle\psi(t)|\sigma_i^z(t)|\psi(t)\rangle - \langle\psi_0|\sigma_i^z(t)|\psi_0\rangle = \Big[\langle\psi_0| + i\langle$$

$$- \langle\psi_0|\sigma_i^z(t)|\psi_0\rangle = i\int_{-\infty}^{t} dt' \langle\psi_0|V(t')\sigma_i^z(t) - \sigma_i^z(t)V(t')|\psi_0\rangle$$

$$= i\int_{-\infty}^{t} dt' \sum_j \langle\psi_0|[\sigma_i^z(t), \sigma_j^z(t')]|\psi_0\rangle h_j(t')$$


Retarded response function:
$\chi_{ij}(t-t') = i\,\theta(t-t')\langle[\sigma_i^z(t), \sigma_j^z(t')]\rangle$

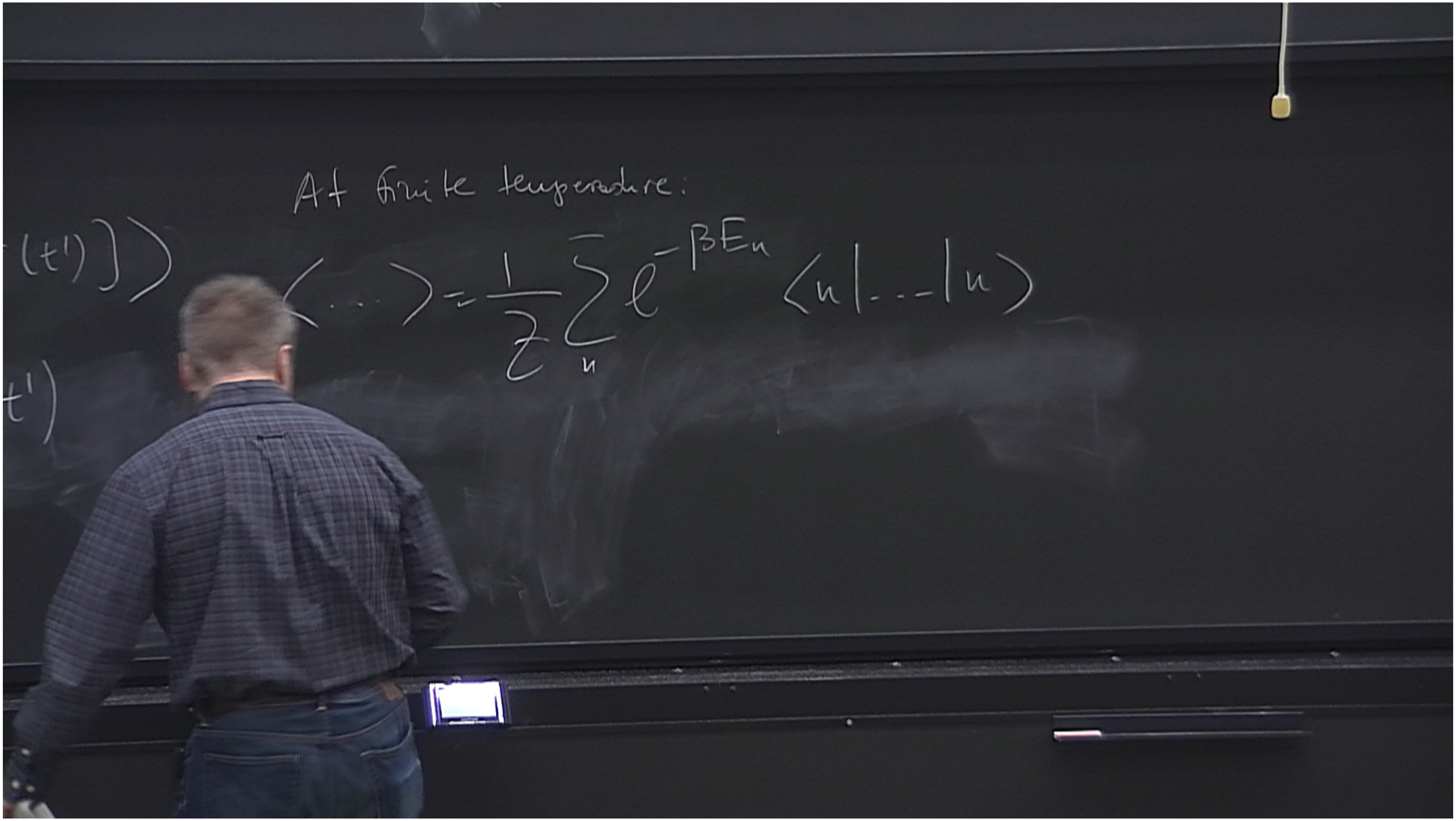

Retarded response function:
$\chi_{ij}(t-t') = i\theta(t-t')\langle[\sigma_i^z(t), \sigma_j^z(t')]\rangle$
$\delta\langle\sigma_i^z(t)\rangle = \int_{-\infty}^{\infty} dt' \sum_j \chi_{ij}(t-t') h_j(t')$

$(t')])$

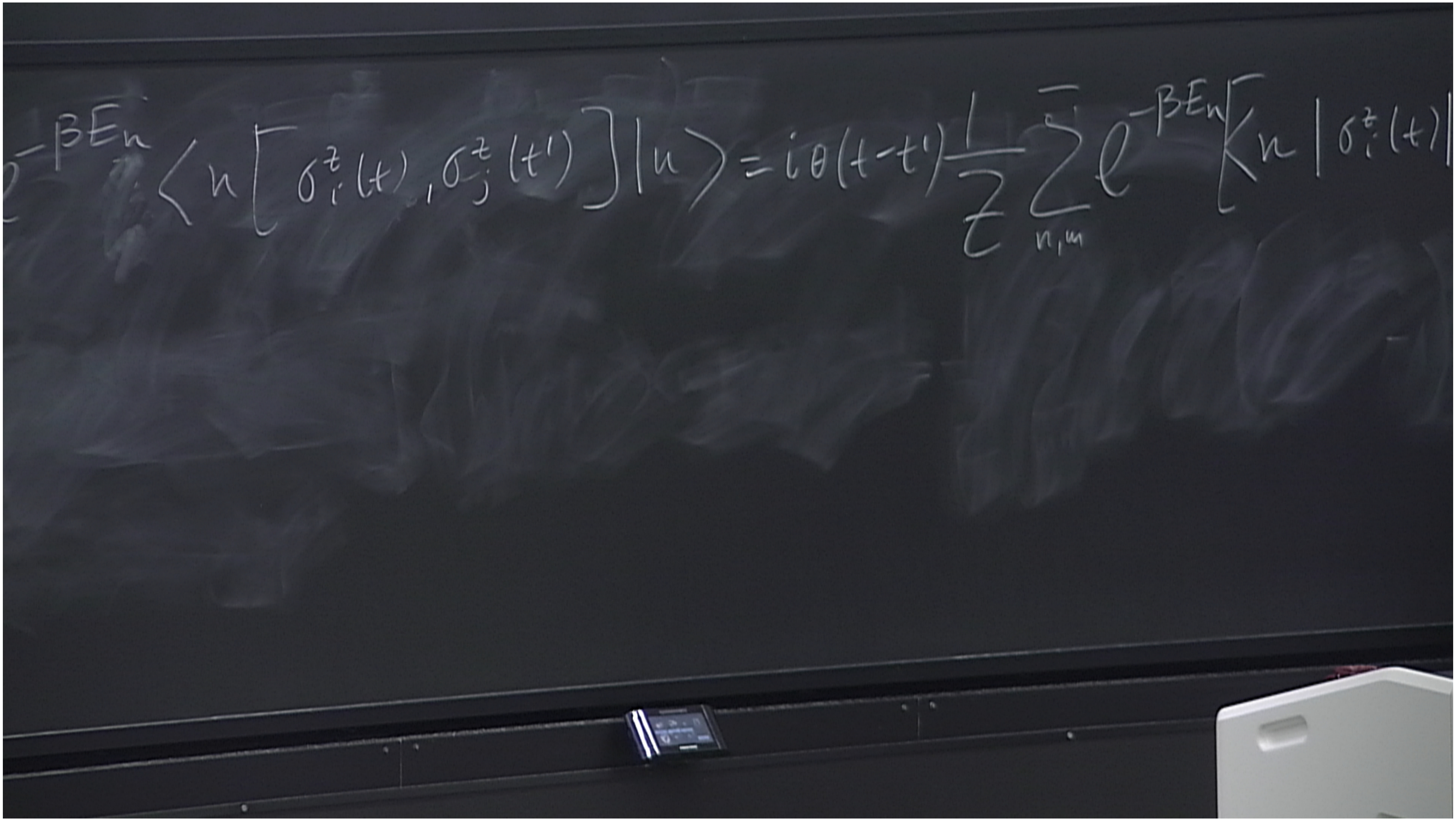
$t')$

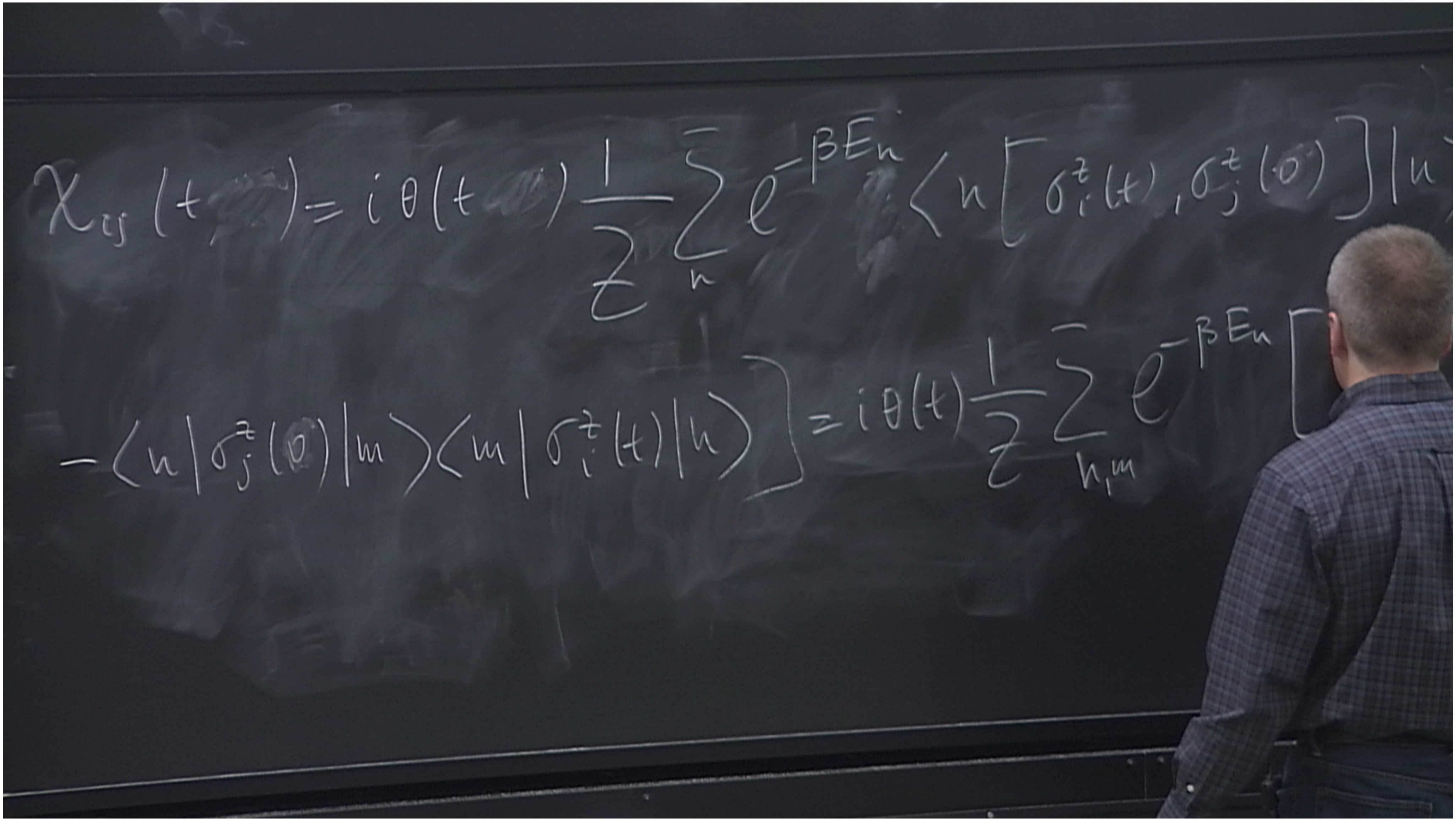
At finite temperature:

$$\langle \ldots \rangle = \frac{1}{Z} \sum_n e^{-\beta E_n} \langle n | \ldots | n \rangle$$


At finite temperature:
$\langle \dots \rangle = \frac{1}{Z} \sum_n e^{-\beta E_n} \langle n | \dots | n \rangle$
(t')])
t')

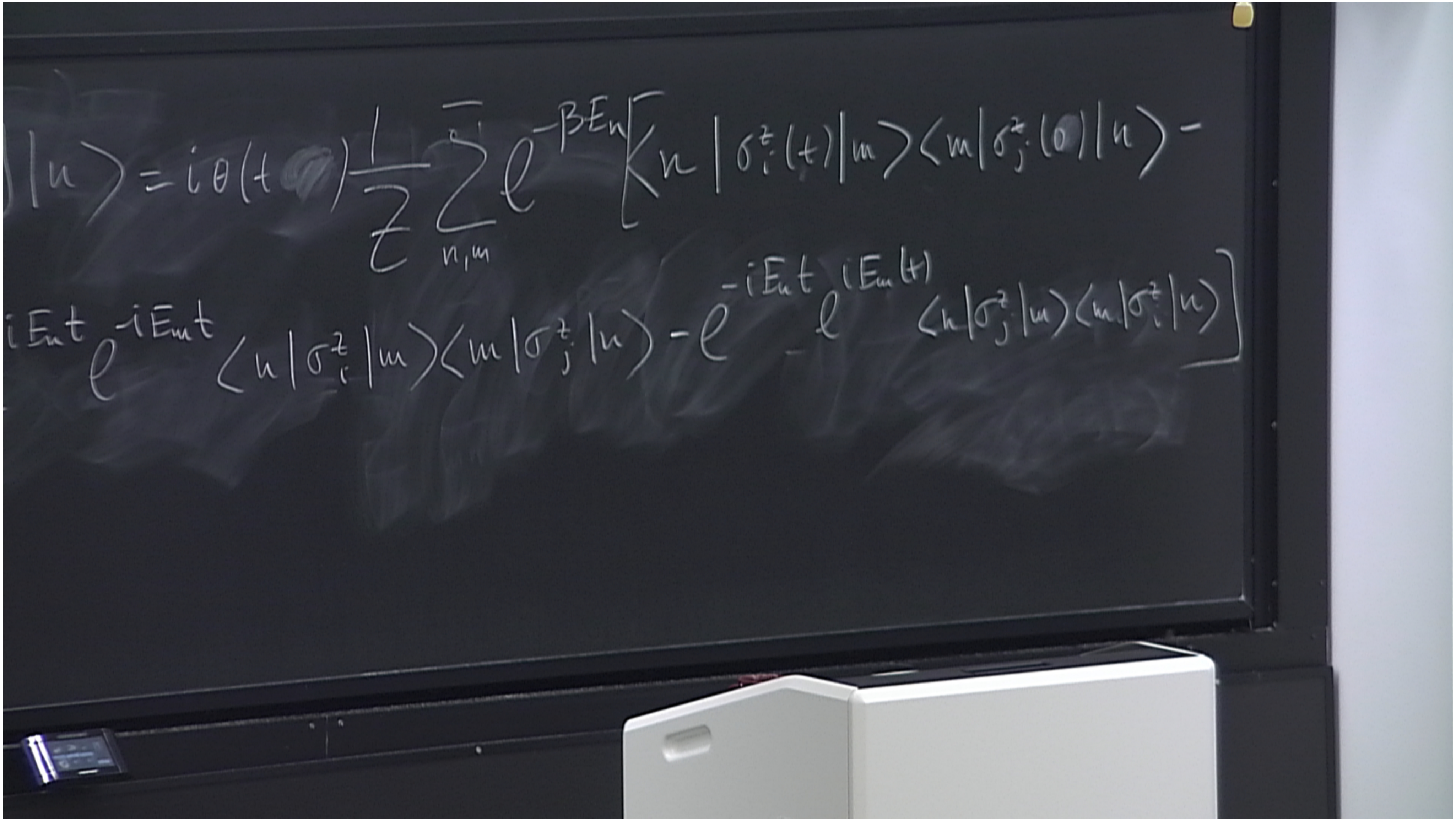
$$\chi_{ij}(t-t') = i\,\theta(t-t')\,\frac{1}{Z}\sum_n e^{-\beta E_n}\langle n[\sigma_i^z(t),\sigma_j^z(t')]|n$$


$e^{-\beta E_n} \langle n [\sigma_i^z(t), \sigma_j^z(t')] | n \rangle = i\theta(t-t') \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} [\langle n | \sigma_i^z(t) |$


$\chi_{ij}(t) = i\theta(t)\frac{1}{Z}\sum_n e^{-\beta E_n}\langle n[\sigma_i^z(t), \sigma_j^z(0)]|n\rangle$
$-\langle n|\sigma_j^z(0)|m\rangle\langle m|\sigma_i^z(t)|n\rangle\Big] = i\theta(t)\frac{1}{Z}\sum_{n,m} e^{-\beta E_n}\Big[$

$$ {}^{E_n} \langle n [ \sigma_i^z(t), \sigma_j^z(0) ] | n \rangle = i\theta(t) \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} [ \langle n | \sigma_i^z(t) | m \rangle \langle $$

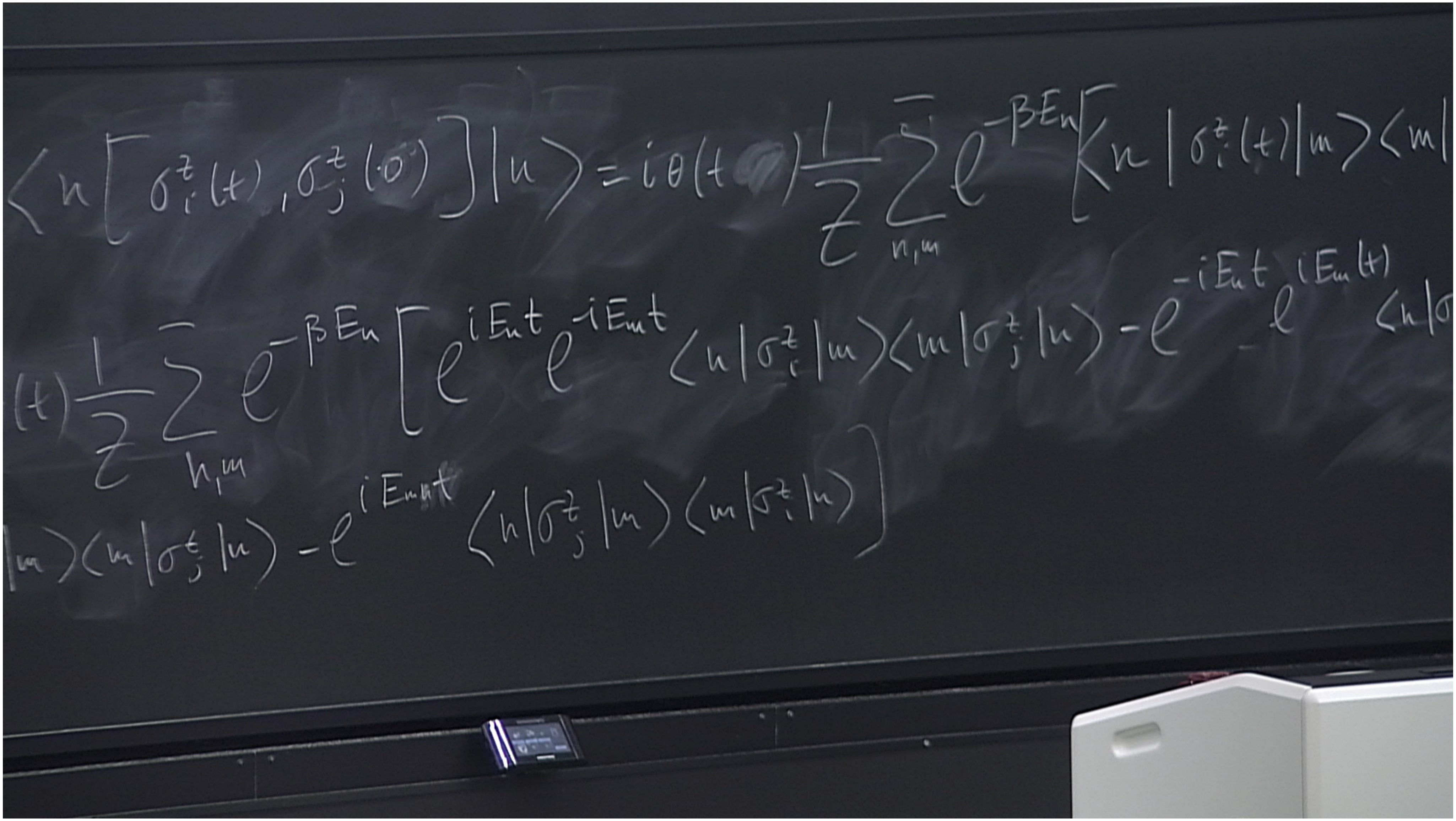
$$ i\theta(t) \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} [ e^{iE_n t} e^{-iE_m t} \langle n | \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle - e^{-iE_n t} e^{iE_m (t)} $$

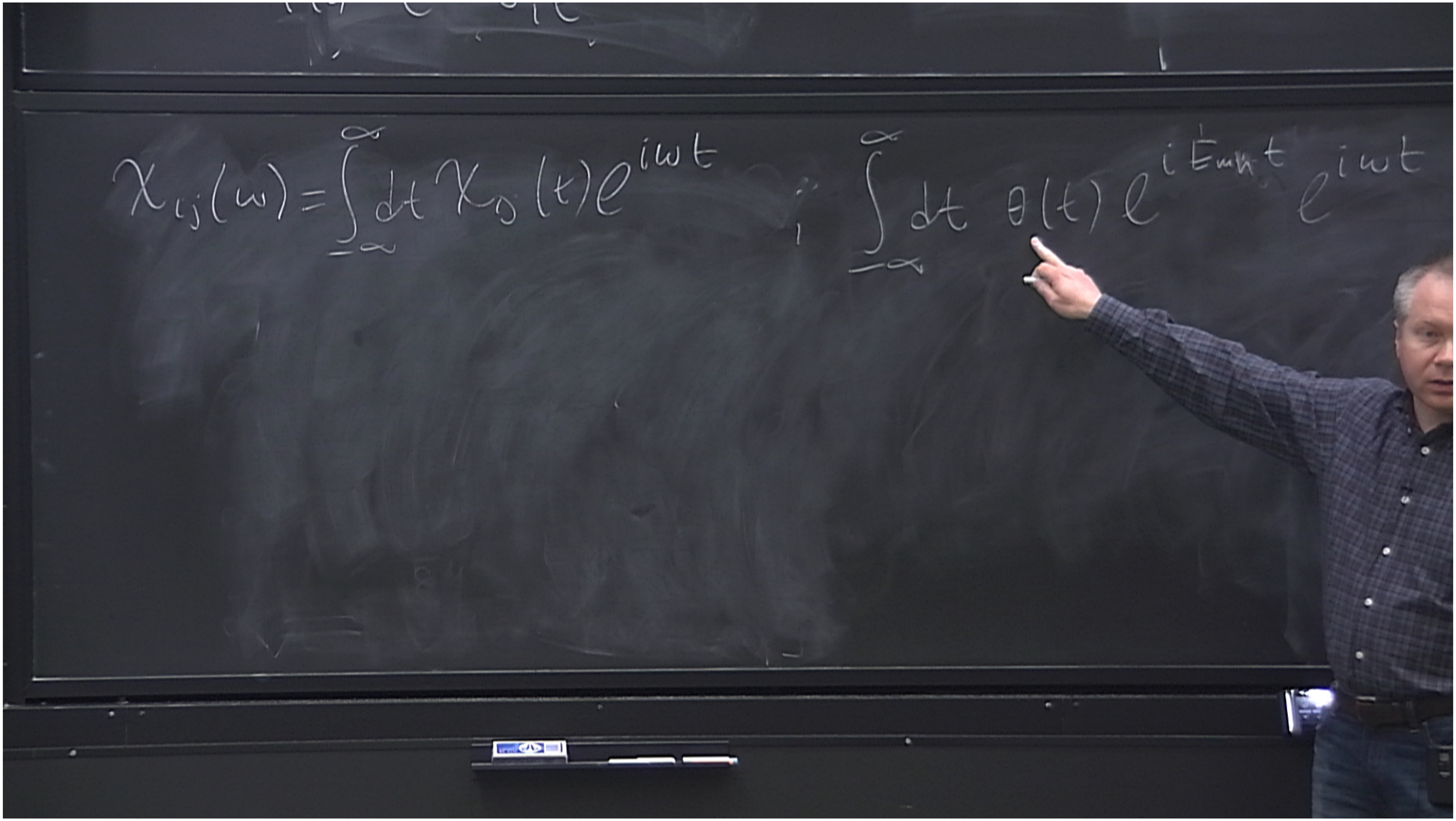

$|n\rangle = i\theta(t)\frac{1}{Z}\sum_{n,m} e^{-\beta E_n}[\langle n|\sigma_i^z(t)|m\rangle\langle m|\sigma_j^z(0)|n\rangle -$
$iE_n t\; e^{-iE_m t}\langle n|\sigma_i^z|m\rangle\langle m|\sigma_j^z|n\rangle - e^{-iE_n t}e^{iE_m(t)}\langle n|\sigma_j^z|m\rangle\langle m|\sigma_i^z|n\rangle]$

$$\chi_{ij}(t, \ ) = i\theta(t \ ) \frac{1}{Z} \sum_n e^{-\beta E_n} \langle n [\sigma_i^z(t), \sigma_j^z(0)] | n$$

$$- \langle n|\sigma_j^z(0)|m\rangle \langle m|\sigma_i^z(t)|n\rangle \Big] = i\theta(t)\frac{1}{Z}\sum_{n,m} e^{-\beta E_n} \Big[ e^{iE_n t}$$

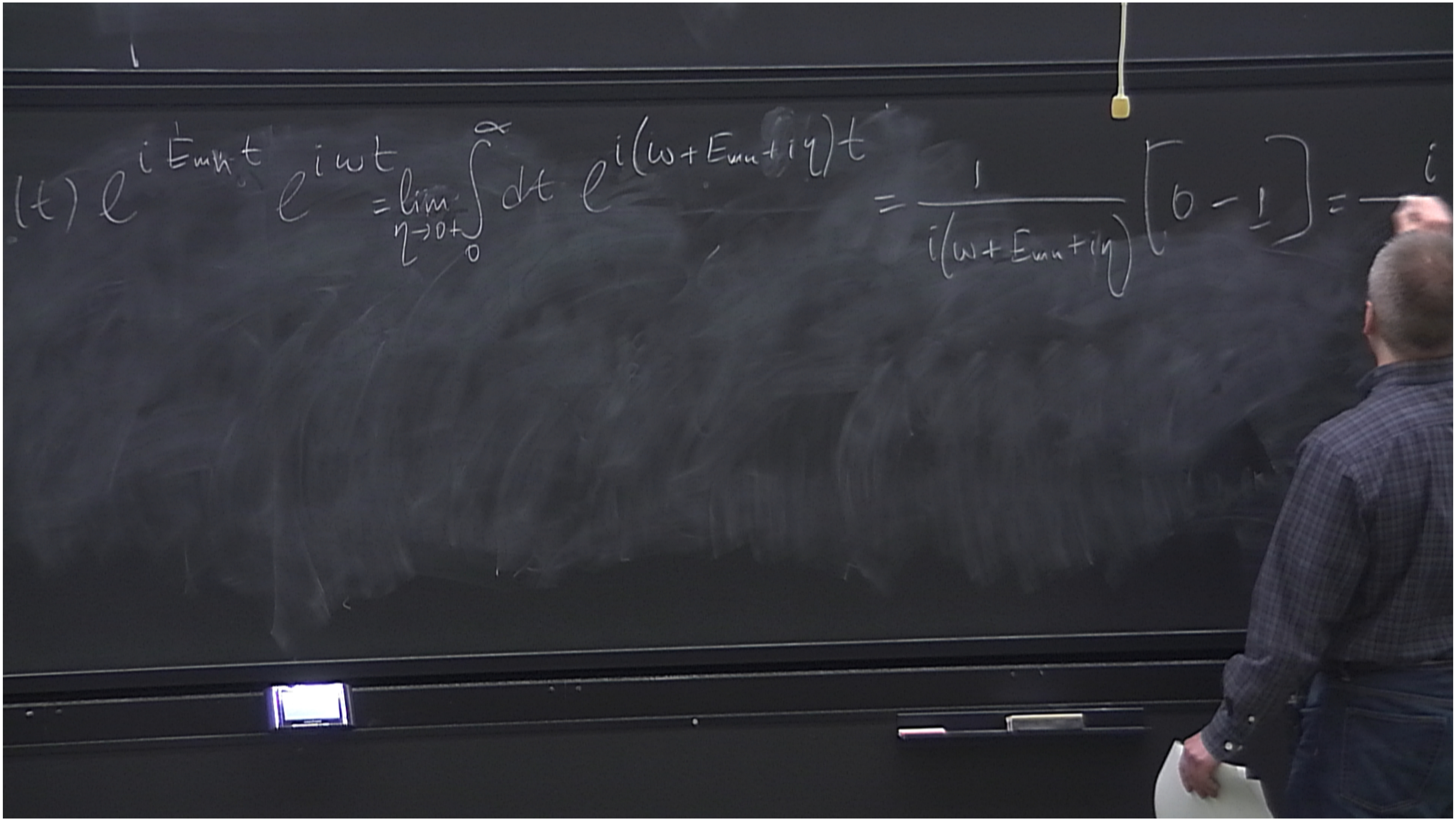
$$= i\theta(t)\frac{1}{Z}\sum_{n,m} e^{-\beta E_n} \Big[ e^{-iE_{mn}t} \langle n|\sigma_i^z|m\rangle\langle m|\sigma_j^z|n\rangle - e^{iE_{mn}t} \ \langle n|\sigma$$

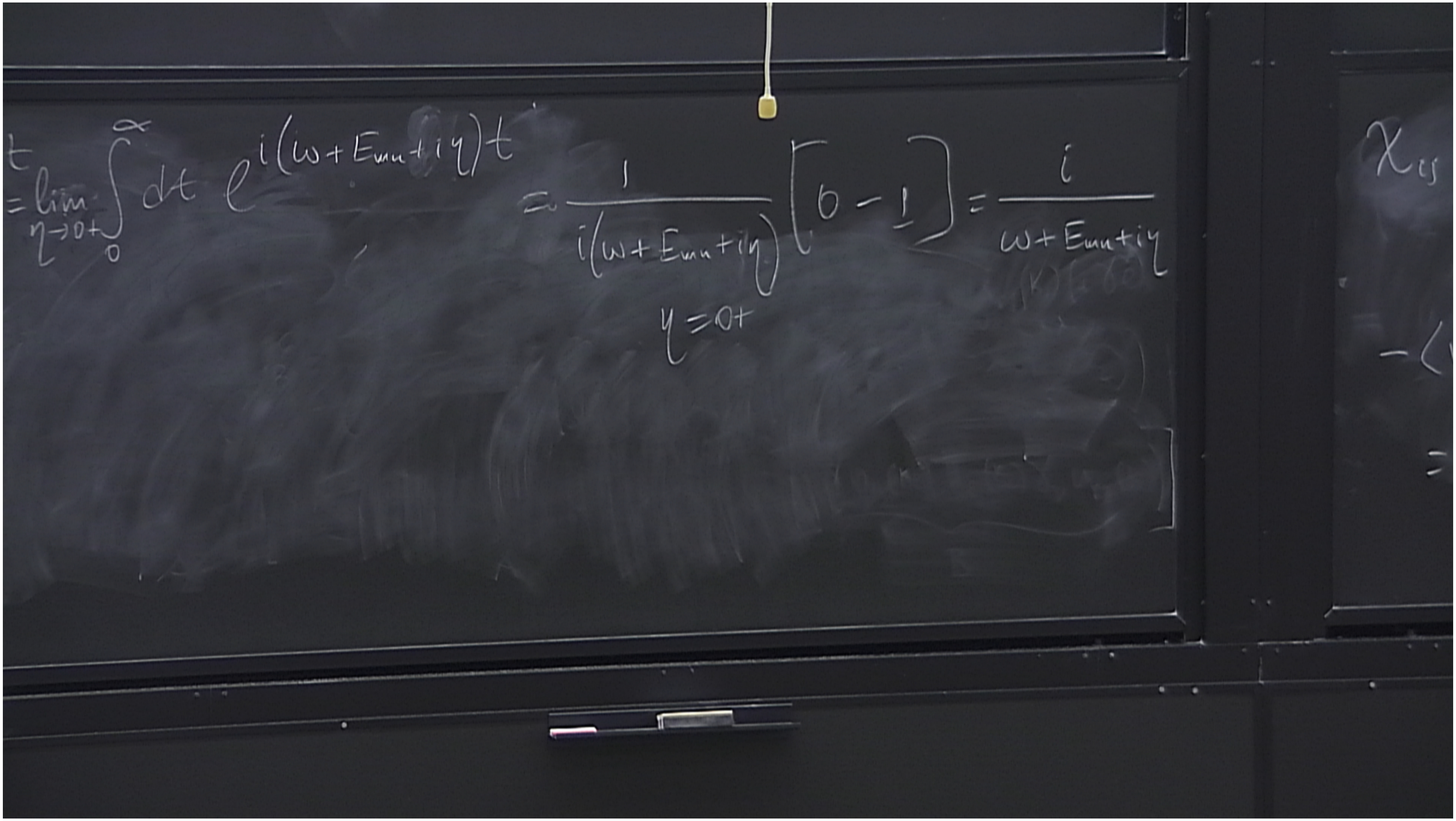

$\langle n [ \sigma_i^z(t), \sigma_j^z(0) ] | n \rangle = i\theta(t) \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} [ \langle n | \sigma_i^z(t) | m \rangle \langle m |$
$e^{-iE_n t} e^{iE_m t} \langle n | \sigma$
$(t) \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} [ e^{iE_n t} e^{-iE_m t} \langle n | \sigma_i^z | m \rangle \langle m | \sigma_j^z | n \rangle - e$
$| m \rangle \langle m | \sigma_j^z | n \rangle - e^{iE_m t} \langle n | \sigma_j^z | m \rangle \langle m | \sigma_i^z | n \rangle ]$


$\chi_{ij}(\omega) = \int_{-\infty}^{\infty} dt\, \chi_0(t) e^{i\omega t}$
$\int_{-\infty}^{\infty} dt\, \theta(t) e^{iE_{mn}t} e^{i\omega t}$

$$\chi_{ij}(\omega) = \int_{-\infty}^{\infty} dt\, \chi_{ij}(t)\, e^{i\omega t} \qquad \int_{-\infty}^{\infty} dt\ \theta(t)\, e^{iE_{mn}t}\, e^{i\omega t}$$

$$(t)\, e^{i E_{mn} t}\, e^{i\omega t} = \int_0^\infty dt\, e^{i(\omega + E_{mn})t}$$


$(t)\, e^{i E_{mn} t}\, e^{i\omega t} = \lim_{\eta \to 0+} \int_0^\infty dt\, e^{i(\omega + E_{mn} + i\eta)t} = \frac{1}{i(\omega + E_{mn} + i\eta)} \left[ 0 - 1 \right] = -\, i$


$= \lim_{\eta \to 0+} \int_0^\infty dt\, e^{i(\omega + E_{mn} + i\eta)t} = \frac{1}{i(\omega + E_{mn} + i\eta)} [0 - 1] = \frac{i}{\omega + E_{mn} + i\eta}$
$\eta = 0+$
$\chi_{ij}$

$$\chi_{ij}(\omega) = \int_{-\infty}^{\infty} dt\, \chi_{ij}(t)\, e^{i\omega t}$$

$$\int_{-\infty}^{\infty} dt\; \theta(t)\, e^{iE_{mn}t}\, e^{i\omega t} = \lim_{\eta \to}$$

$$\chi_{ij}(\omega) = -\frac{1}{Z}\sum_{nm} e^{-\beta E_n}\left[\frac{\langle n|\sigma_i^z|m\rangle\langle m|\sigma_j^z|n\rangle}{\omega + E_{nm} + i\eta}\right.$$

$$t \quad \int_{-\infty}^{\infty} dt\, \theta(t)\, e^{iE_{mn}t}\, e^{i\omega t} = \lim_{\eta\to 0^+} \int_{0}^{\infty} dt\, e^{i(\omega+E_{mn}+i\eta)t} = \frac{1}{i(\omega+E_{mn}+i}$$

$$E_n \left[ \frac{\langle n|\sigma_i^z|m\rangle\langle m|\sigma_j^z|n\rangle}{\omega+E_{nm}+i\eta} - \frac{\langle n|\sigma_j^z|m\rangle\langle m|\sigma_i^z|n\rangle}{\omega-E_{nm}+i\eta} \right] \qquad \eta = 0^+$$

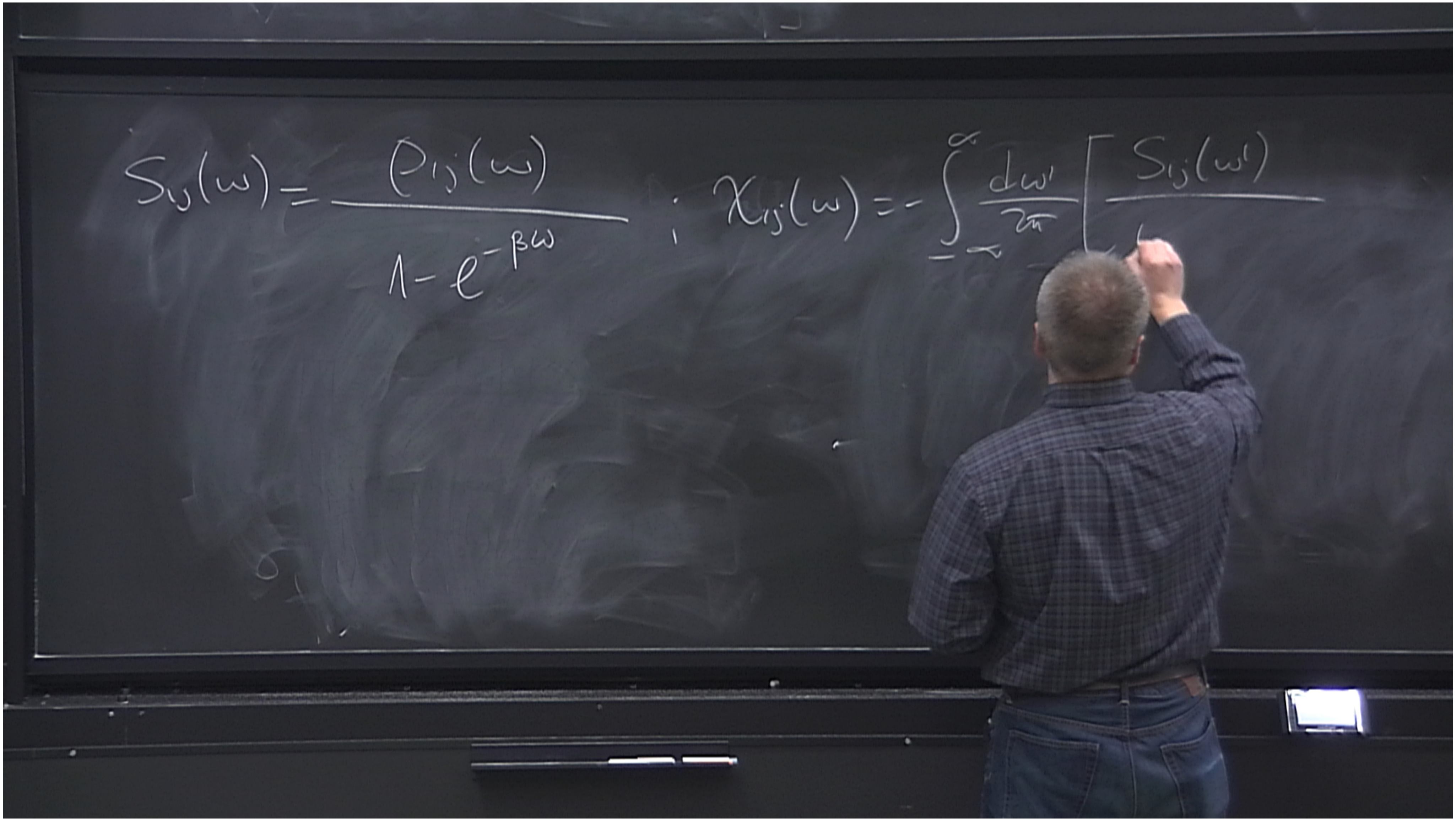
$$\chi_{ij}(\omega) = \int_{-\infty}^{\infty} dt\, \chi_{ij}(t) e^{i\omega t}$$

$$\int_{-\infty}^{\infty} dt\, \theta(t)\, e^{iE_{mn}t}\, e^{i\omega t} = \lim_{\eta \to}$$

$$\chi_{ij}(\omega) = -\frac{1}{Z} \sum_{nm} e^{-\beta E_n} \left[ \frac{\langle n|\sigma_i^z|m\rangle\langle m|\sigma_j^z|n\rangle}{\omega + E_{nm} + i\eta} - \frac{\langle n|\sigma_j^z|}{\omega -} \right.$$

Dynamic structure factor: $S_{ij}(\omega) = \frac{2\pi}{Z} \sum_{nm} e^{-\beta E_n} \langle n|\sigma_i^z|m\rangle\langle m|$

$$S_{ij}(\omega) = \frac{\rho_{ij}(\omega)}{1 - e^{-\beta\omega}}$$


$S_{ij}(\omega) = \frac{\rho_{ij}(\omega)}{1 - e^{-\beta\omega}}$ ; $\chi_{ij}(\omega) = -\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \Big[ \frac{S_{ij}(\omega')}{}$

$$S_{ij}(\omega) = \frac{\rho_{ij}(\omega)}{1 - e^{-\beta\omega}} \; ; \; \chi_{ij}(\omega) = -\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left[ \frac{S_{ij}(\omega')}{\omega - \omega' + i\eta} - \right.$$

$$) = -\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left[ \frac{S_{ij}(\omega')}{\omega - \omega' + i\eta} - \frac{S_{ji}(\omega')}{\omega + \omega' + i\eta} \right] = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left[ S_{ij}(\omega') - S_{ji}(-\omega') \right]$$

$$S_{ij}(\omega) = \frac{\rho_{ij}(\omega)}{1-e^{-\beta\omega}} \quad ; \quad \chi_{ij}(\omega) = -\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi}\left[\frac{S_{ij}(\omega')}{\omega-\omega'+i\eta} - \right.$$

$$S_{ij}(\omega) = e^{\beta\omega} S_{ji}(-\omega) \quad ; \quad \chi_{ij}(\omega) = -\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} S_{ij}(\omega')\left(1-e^{-\beta\omega'}\right)$$

$$\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\left[\frac{S_{ij}(\omega')}{\omega-\omega'+i\eta}-\frac{S_{ji}(\omega')}{\omega+\omega'+i\eta}\right]=-\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\left[S_{ij}(\omega')-S_{ji}(-\omega')\right]\frac{1}{\omega-\omega'}$$

$$-\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}S_{ij}(\omega')\left(1-e^{-\beta\omega'}\right)\frac{1}{\omega-\omega'+i\eta}=-\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\rho_{ij}(\omega')}{\omega-\omega'+i\eta}$$

$$\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi}\left[\frac{S_{ij}(\omega')}{\omega-\omega'+i\eta} - \frac{S_{ji}(\omega')}{\omega+\omega'+i\eta}\right] = -\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\left[S_{ij}(\omega') - S_{ji}(-\omega')\right]\frac{1}{\omega-\omega'}$$

$$\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} S_{ij}(\omega')\left(1-e^{-\beta\omega'}\right)\frac{1}{\omega-\omega'+i\eta} = -\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\rho_{ij}(\omega')}{\omega-\omega'+i\eta}$$

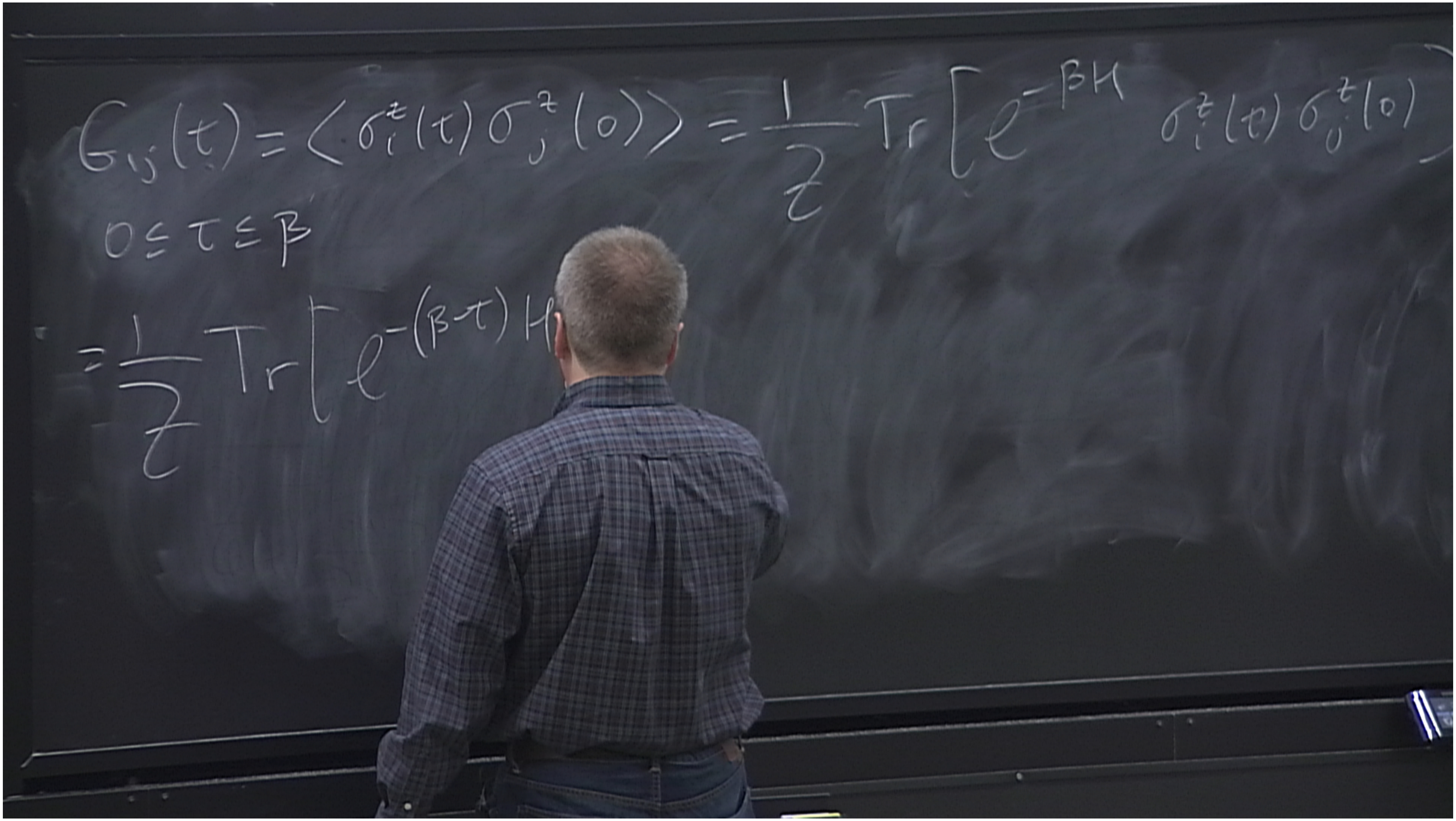
$$= -\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\rho_{ij}(\omega')}{i\omega_n-\omega'}$$

$$G_{ij}(i\omega_n \to \omega+i\eta) = \chi_{ij}(\omega)$$

$$\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left[ \frac{S_{ij}(\omega')}{\omega-\omega'+i\eta} - \frac{S_{ji}(\omega')}{\omega+\omega'+i\eta} \right] = -\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left[ S_{ij}(\omega') - S_{ji}(-\omega') \right] \frac{1}{\omega-\omega'}$$

$$\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} S_{ij}(\omega') \left(1-e^{-\beta\omega'}\right) \frac{1}{\omega-\omega'+i\eta} = -\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\rho_{ij}(\omega')}{\omega-\omega'+i\eta}$$

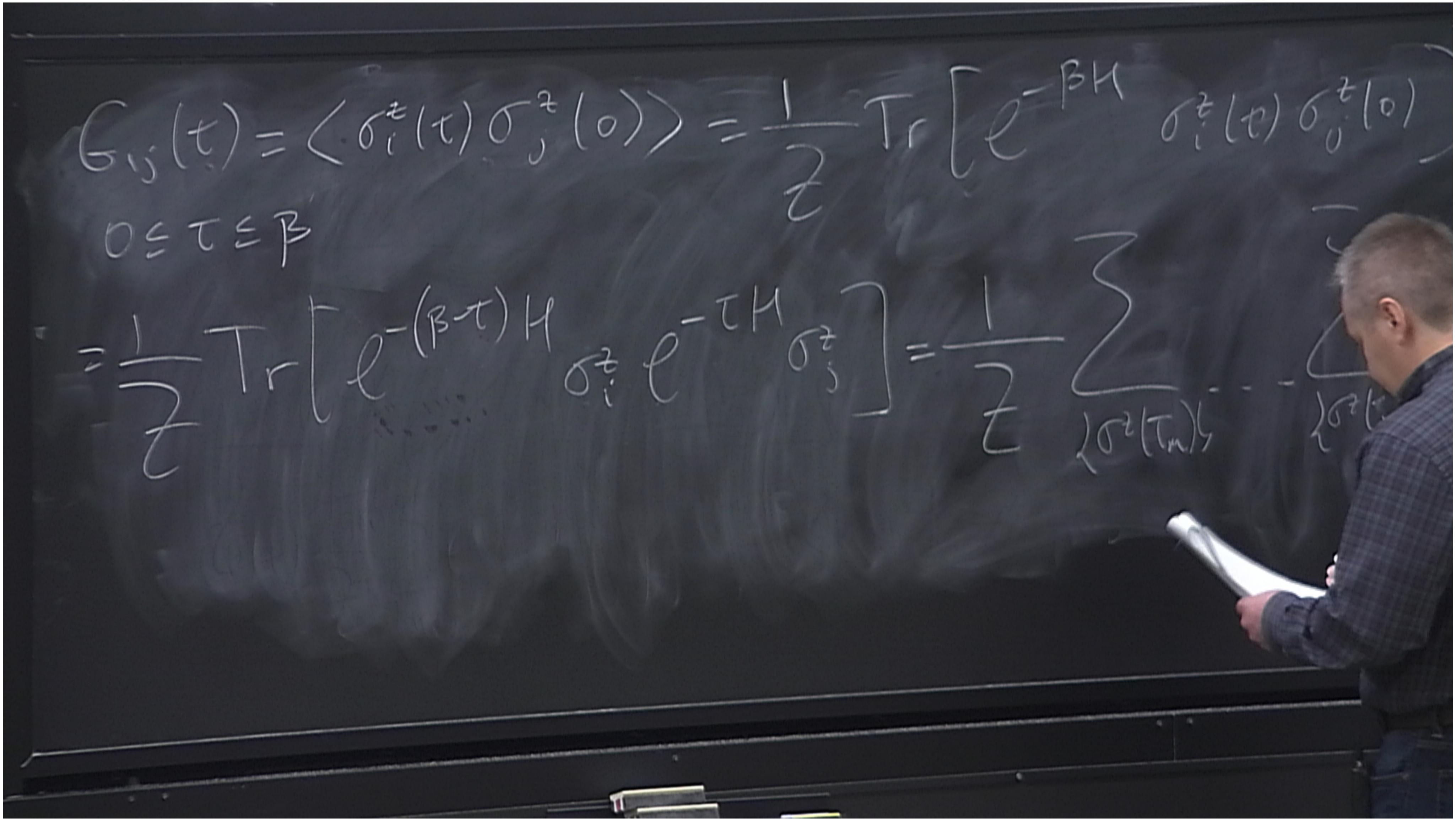
$$= -\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\rho_{ij}(\omega')}{i\omega_n-\omega'} , \quad G_{ij}(i\omega_n \to \omega+i\eta) = \chi_{ij}(\omega)$$


$G_{ij}(\tau) = \langle \sigma_i^z(\tau)\, \sigma_j^z(0) \rangle = \frac{1}{Z} \mathrm{Tr}\left[ e^{-\beta H}\, \sigma_i^z(\tau)\, \sigma_j^z(0) \right]$
$0 \le \tau \le \beta$
$= \frac{1}{Z} \mathrm{Tr}\left[ e^{-(\beta-\tau) H} \right.$

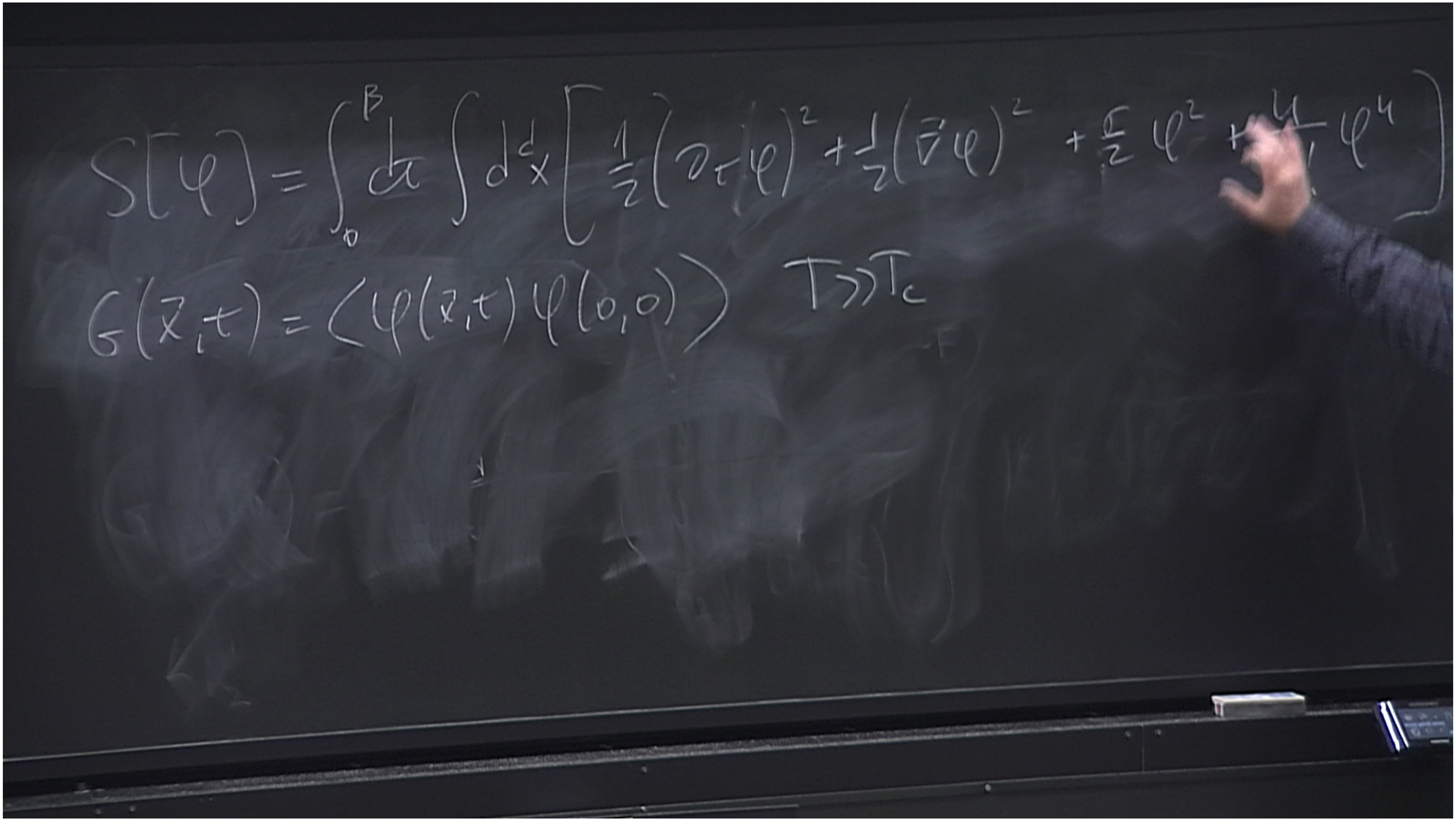
$$G_{ij}(\tau) = \langle \sigma_i^z(\tau)\,\sigma_j^z(0)\rangle = \frac{1}{Z}\,\mathrm{Tr}\left[e^{-\beta H}\,\sigma_i^z(\tau)\,\sigma_j^z(0)\right]$$

$$0 \le \tau \le \beta$$

$$= \frac{1}{Z}\,\mathrm{Tr}\left[e^{-(\beta-\tau)H}\,\sigma_i^z\, e^{-\tau H}\,\sigma_j^z\right] = \frac{1}{Z}$$


$G_{ij}(\tau) = \langle \sigma_i^z(\tau)\,\sigma_j^z(0)\rangle = \frac{1}{Z}\mathrm{Tr}\left[e^{-\beta H}\,\sigma_i^z(\tau)\,\sigma_j^z(0)\right]$
$0 \le \tau \le \beta$
$= \frac{1}{Z}\mathrm{Tr}\left[e^{-(\beta-\tau)H}\,\sigma_i^z\,e^{-\tau H}\,\sigma_j^z\right] = \frac{1}{Z}\sum_{\{\sigma^z(\tau_m)\}}\cdots\sum$

$$S[\varphi] = \int_0^\beta d\tau \int d^d x \left[ \frac{1}{2}(\partial_\tau \varphi)^2 + \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 \right]$$

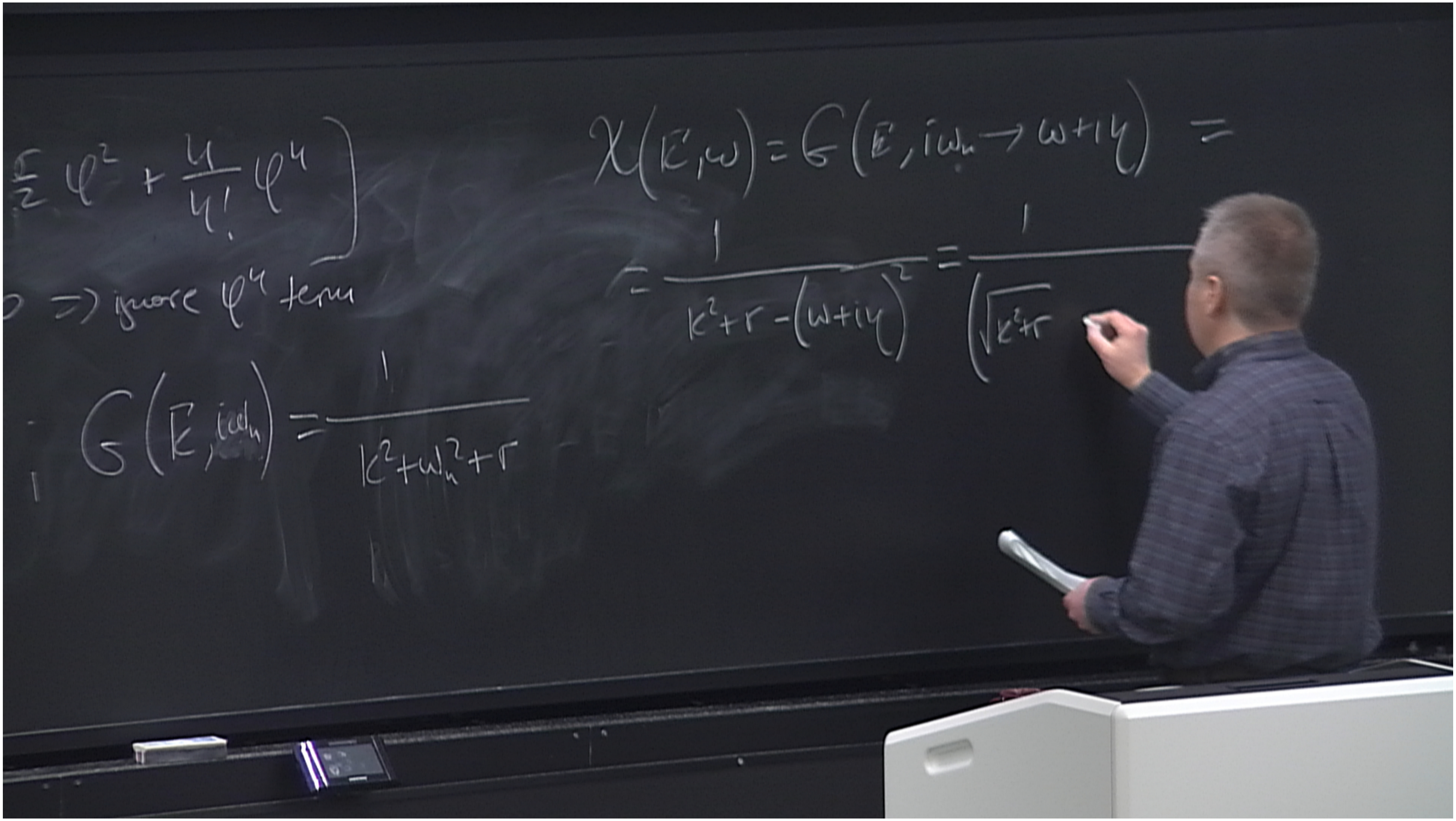

$S[\varphi] = \int_0^\beta d\tau \int d^d x \left[ \frac{1}{2}(\partial_\tau \varphi)^2 + \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 \right]$
$G(\vec{x},t) = \langle \varphi(\vec{x},t)\,\varphi(0,0) \rangle$
$T \gg T_c$

$$S[\varphi] = \int_0^\beta d\tau \int d^dx \left[ \frac{1}{2}(\partial_\tau \varphi)^2 + \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 \right]$$

$$G(\vec{x}, t) = \langle \varphi(\vec{x}, t)\, \varphi(0,0) \rangle$$

$T \gg T_c \Rightarrow r > 0 \Rightarrow$ ignore $\varphi^4$ term

$$S[\varphi] = \frac{1}{V\beta} \sum_{\vec{k}, \omega_n} \frac{1}{2}\left(k^2 + \omega_n^2 + r\right) \left|\varphi(\vec{k}, \omega_n)\right|^2 ; \quad G(\vec{k}, i\omega_n)$$


χ(k, ω) = G(k, iωn → ω+iη) =
r/2 φ² + u/4! φ⁴
=> ignore φ⁴ term
= 1 / (k²+r − (ω+iη)²) = 1 / (√(k²+r
G(k, iωn) = 1 / (k²+ωn²+r)

$$\frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 \Big]$$

$\Rightarrow$ ignore $\varphi^4$ term

$$G(\vec{k}, i\omega_n) = \frac{1}{k^2+\omega_n^2+r}$$

$$\chi(\vec{k},\omega) = G(\vec{k}, i\omega_n \to \omega + i\eta) =$$

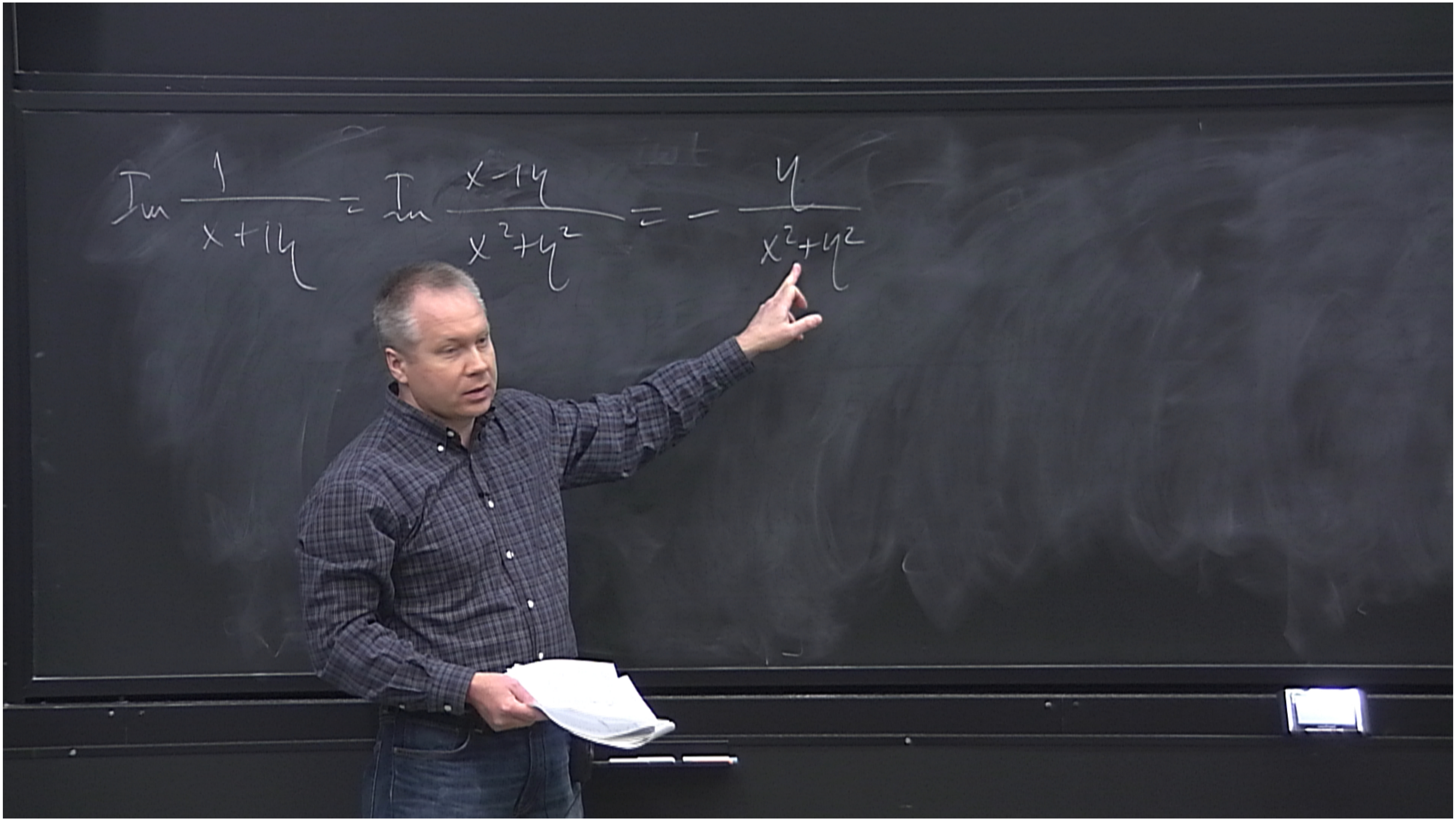
$$= \frac{1}{k^2+r-(\omega+i\eta)^2} = \frac{1}{\left(\sqrt{k^2+r}-\omega-i\eta\right)\left(\sqrt{k^2+r}+\omega+i\eta\right)}$$

$$\frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 \Big]$$

$\Rightarrow$ ignore $\varphi^4$ term

$$G(\vec{k}, i\omega_n) = \frac{1}{k^2+\omega_n^2+r}$$

$$\chi(\vec{k},\omega) = G(\vec{k}, i\omega_n \to \omega + i\eta) = \frac{1}{k^2+r-(\omega+i\eta)^2} = \frac{1}{\left(\sqrt{k^2+r}-\omega-i\eta\right)\left(\sqrt{k^2+r}+\omega+i\eta\right)}$$


$\mathrm{Im}\,\frac{1}{x+i\eta} = \mathrm{Im}\,\frac{x-i\eta}{x^2+\eta^2} = -\frac{\eta}{x^2+\eta^2}$

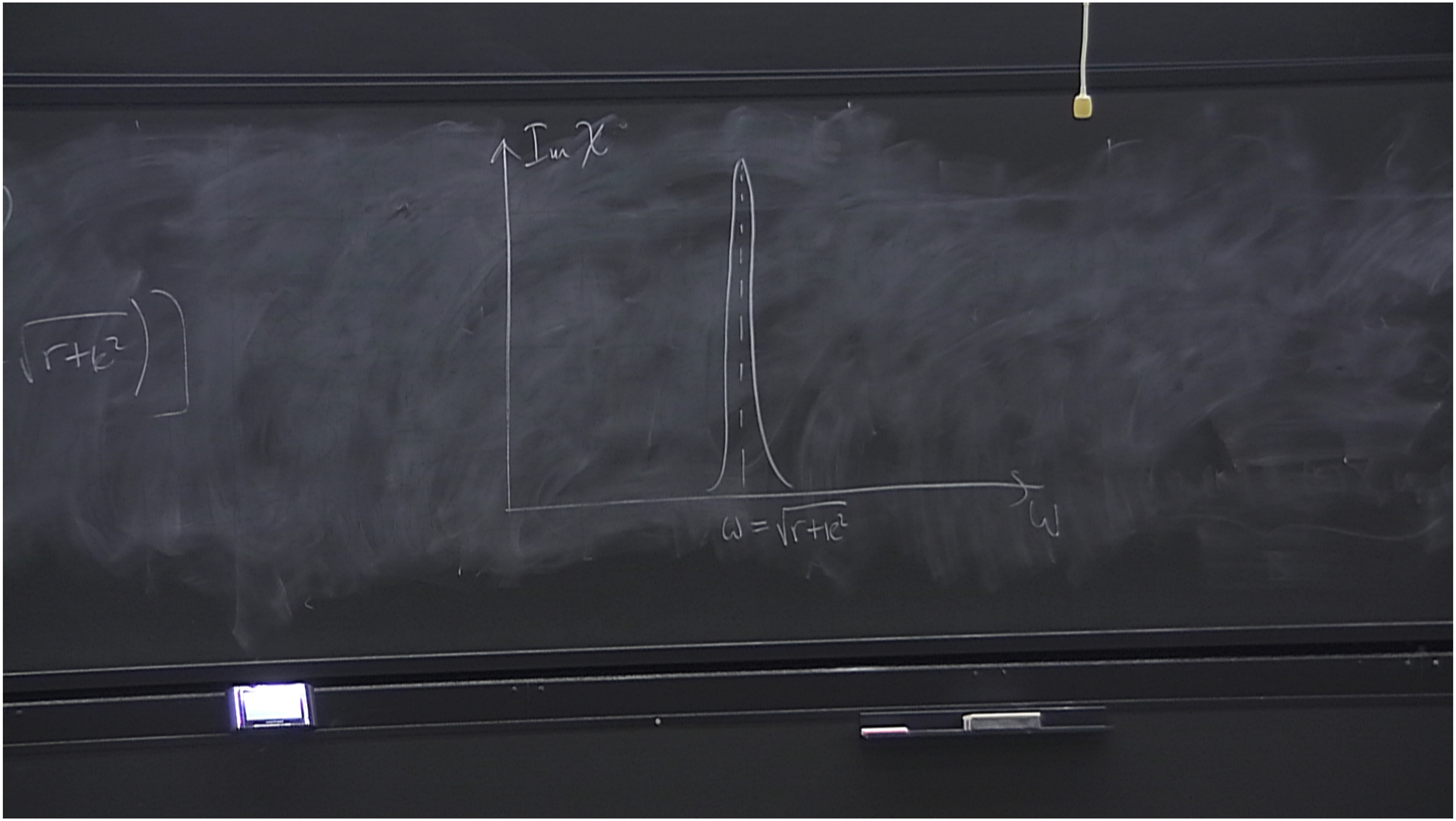
$$\mathrm{Im}\,\frac{1}{x+i\eta}=\mathrm{Im}\,\frac{x-i\eta}{x^2+\eta^2}=-\frac{\eta}{x^2+\eta^2}=-\pi\,\delta(x)$$

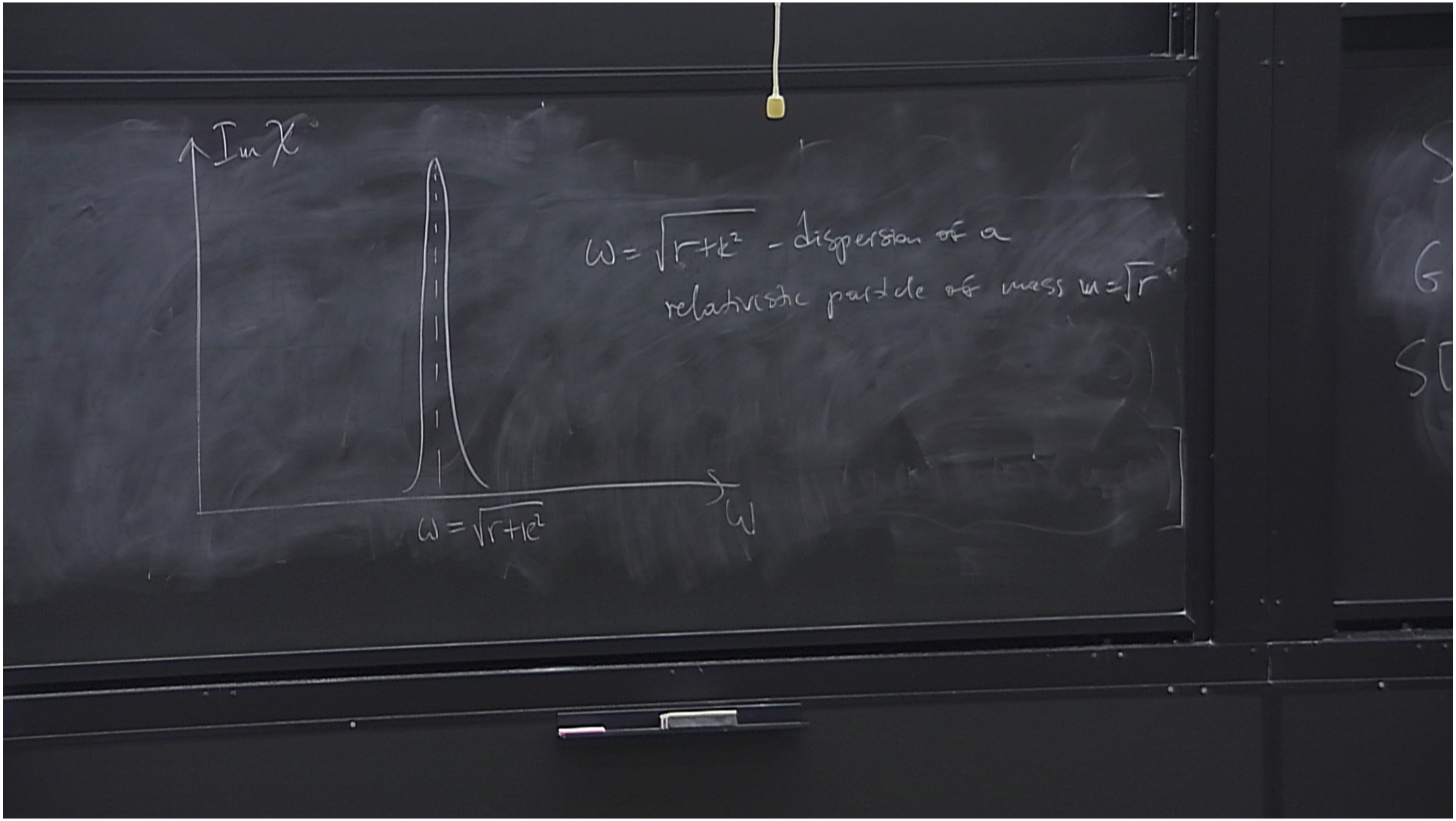
$$\mathrm{Im}\,\frac{1}{x+i\eta} = \mathrm{Im}\,\frac{x-i\eta}{x^2+\eta^2} = -\frac{\eta}{x^2+\eta^2} = -\pi\,\delta(x)$$

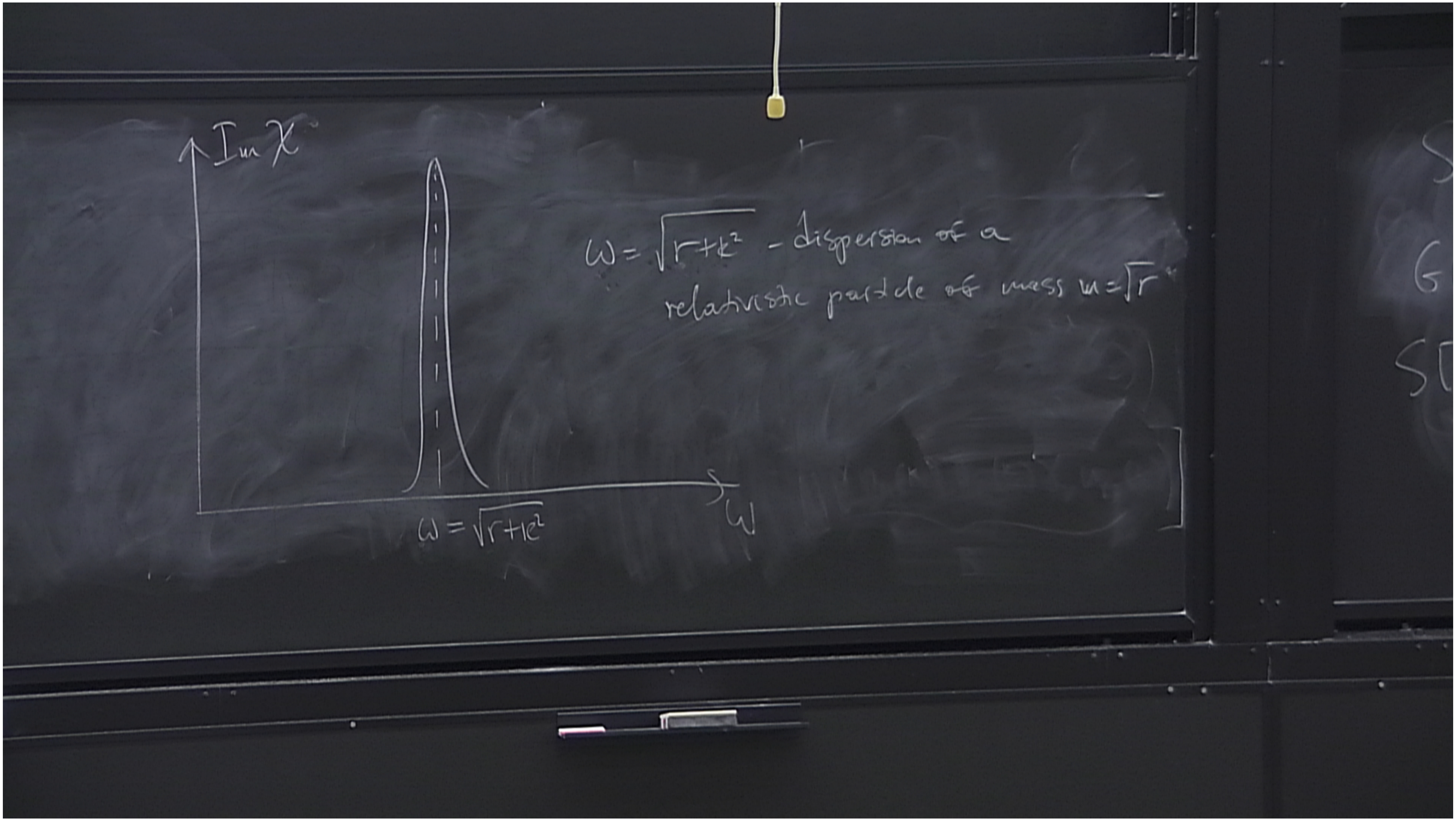
$$\mathrm{Im}\,\chi(k,\omega) = \frac{\pi}{2\sqrt{r+k^2}}\left[\delta\left(\omega-\sqrt{r+k^2}\right)-\delta\left(\omega+\sqrt{r+k^2}\right)\right]$$

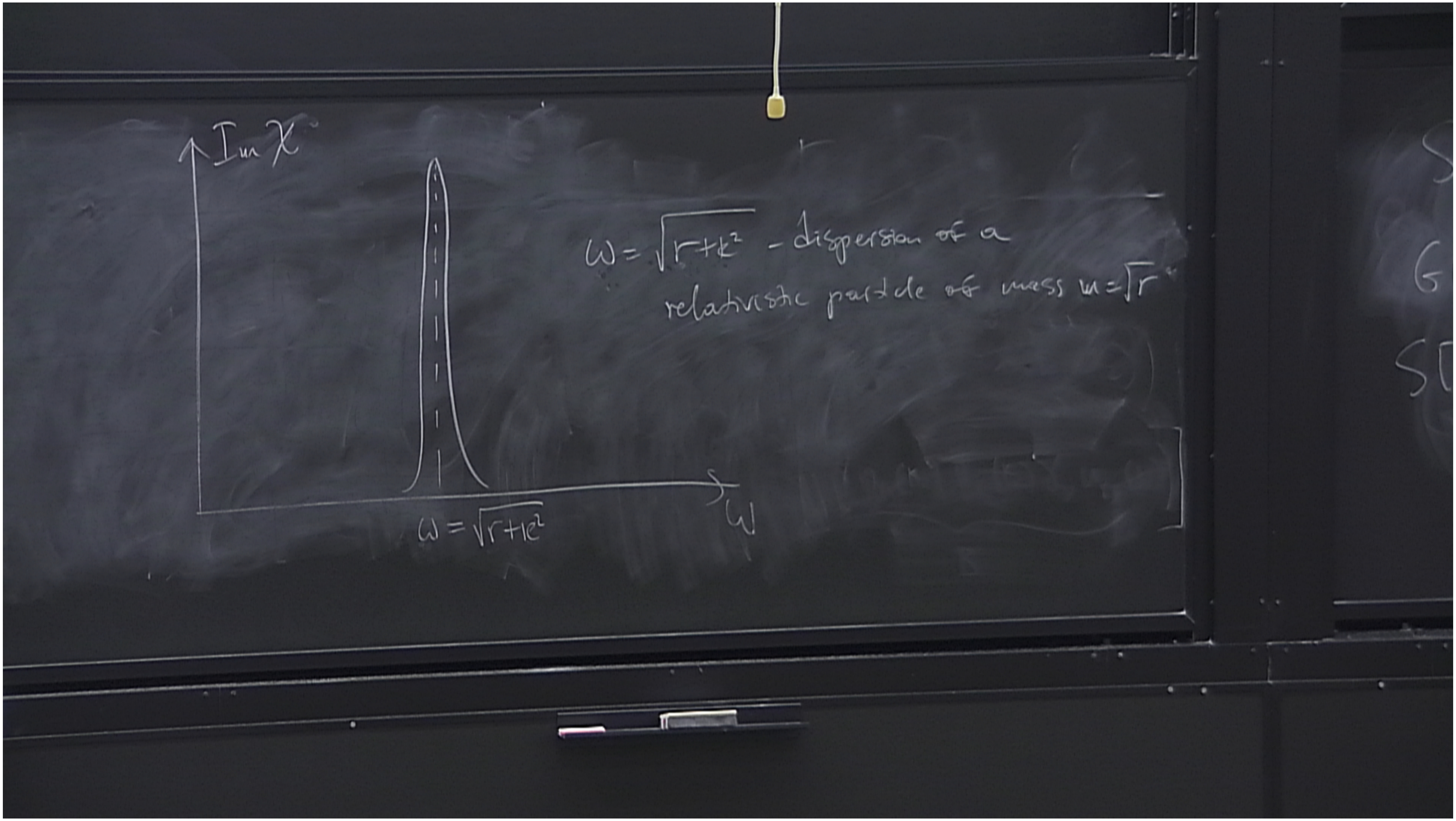
$$\operatorname{Im} \frac{1}{x+i\eta} = \operatorname{Im} \frac{x-i\eta}{x^2+\eta^2} = -\frac{\eta}{x^2+\eta^2} = -\pi\delta(x)$$

$$\operatorname{Im} \chi(k,\omega) = \frac{\pi}{2\sqrt{r+k^2}} \left[ \delta\left(\omega - \sqrt{r+k^2}\right) - \delta\left(\omega + \sqrt{r+k^2}\right) \right]$$

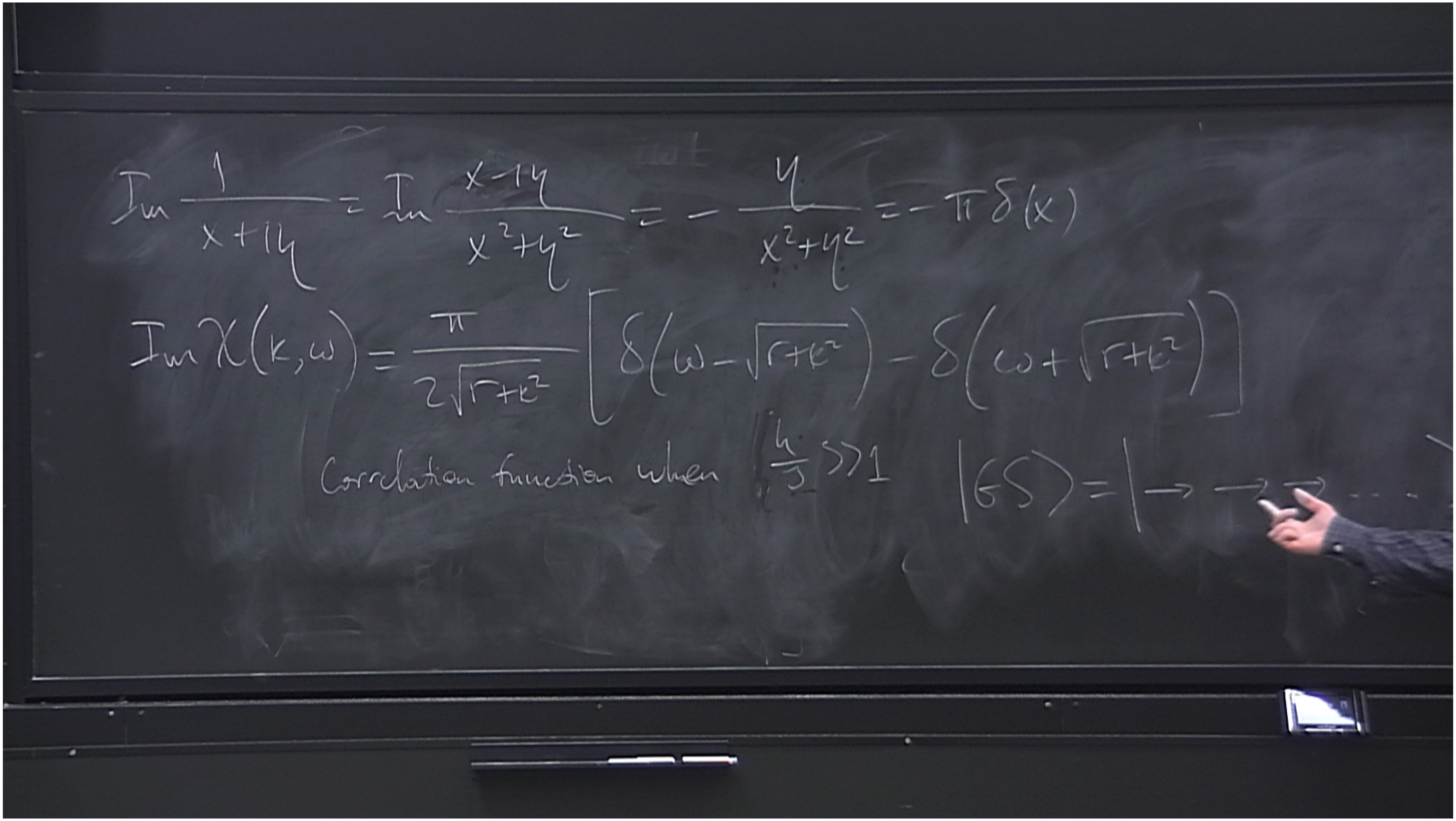

√r+k²
Im χ
ω
ω = √r+k²

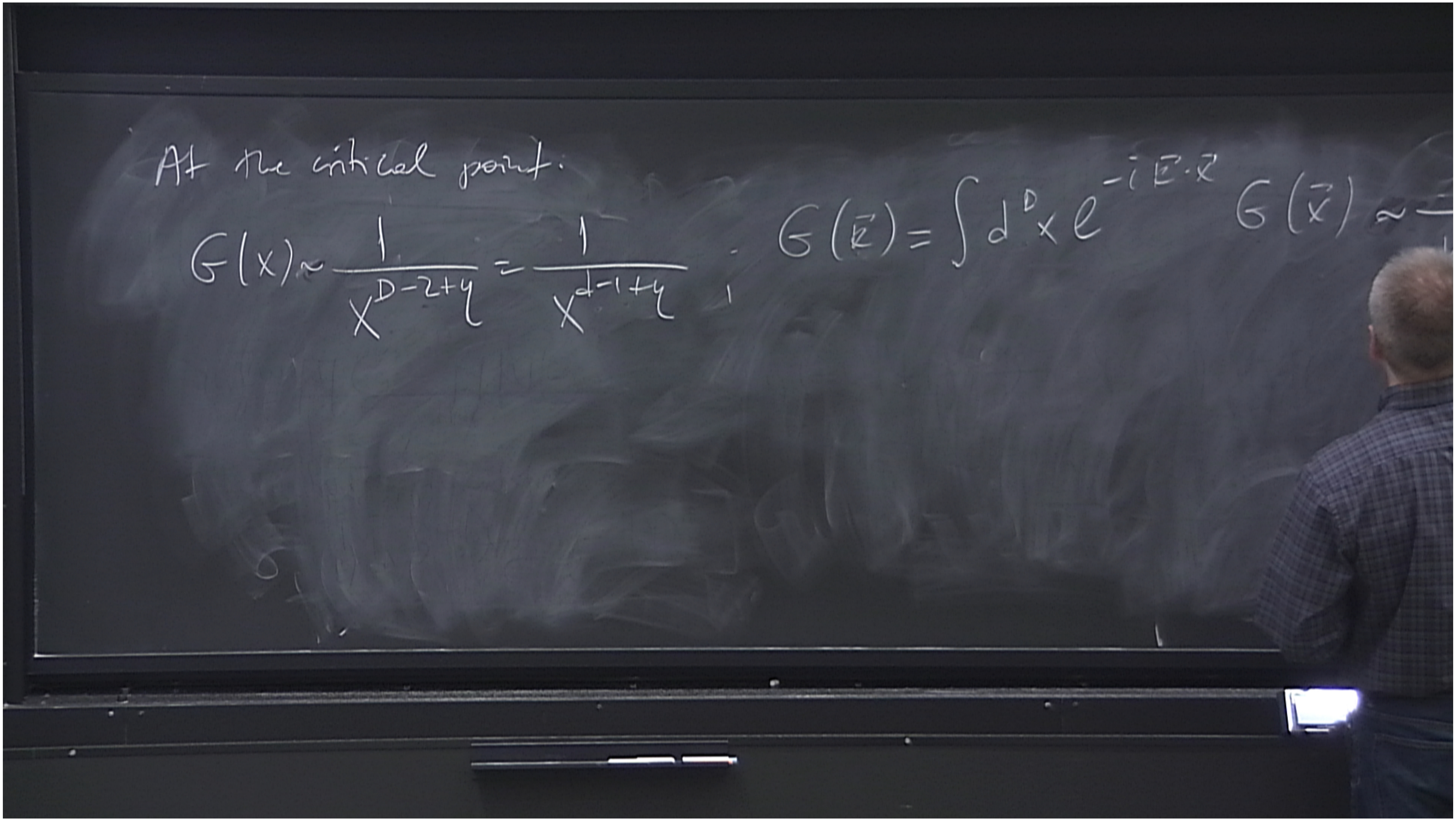

Im χ
ω = √(r+k²) - dispersion of a
relativistic particle of mass m = √r
ω = √(r+k²)
ω


Im χ
ω = √(r+k²) - dispersion of a
relativistic particle of mass m = √r
ω = √(r+k²)
ω



$\mathrm{Im}\,\chi$
$\omega = \sqrt{r+k^2}$ - dispersion of a
relativistic particle of mass $m = \sqrt{r}$
$\omega = \sqrt{r+k^2}$
$\omega$


$\mathrm{Im}\frac{1}{x+i\eta} = \mathrm{Im}\frac{x-i\eta}{x^2+\eta^2} = -\frac{\eta}{x^2+\eta^2} = -\pi\delta(x)$
$\mathrm{Im}\,\chi(k,\omega) = \frac{\pi}{2\sqrt{r+k^2}}\left[\delta\left(\omega-\sqrt{r+k^2}\right)-\delta\left(\omega+\sqrt{r+k^2}\right)\right]$
Correlation function when $\frac{h}{J} \gg 1$
$|GS\rangle = |\rightarrow\rightarrow\rightarrow\cdots$


At the critical point:
$G(x) \sim \frac{1}{x^{D-2+\eta}} = \frac{1}{x^{d-1+\eta}}$ ;
$G(\vec{k}) = \int d^D x \, e^{-i \vec{k} \cdot \vec{x}}$
$G(\vec{x}) \sim$

At the critical point:

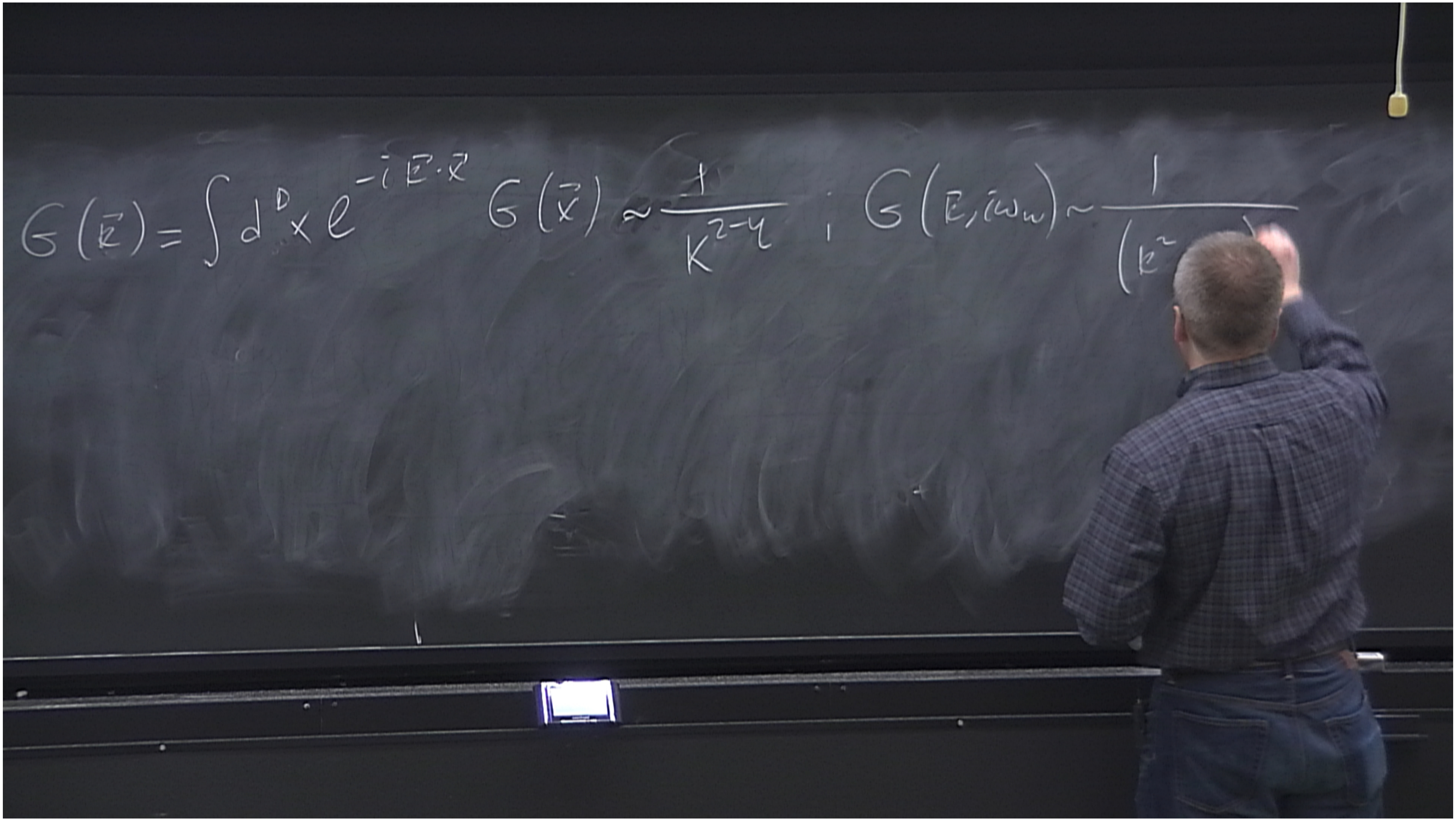
$$G(x) \sim \frac{1}{x^{D-2+\eta}} = \frac{1}{x^{d-1+\eta}}$$

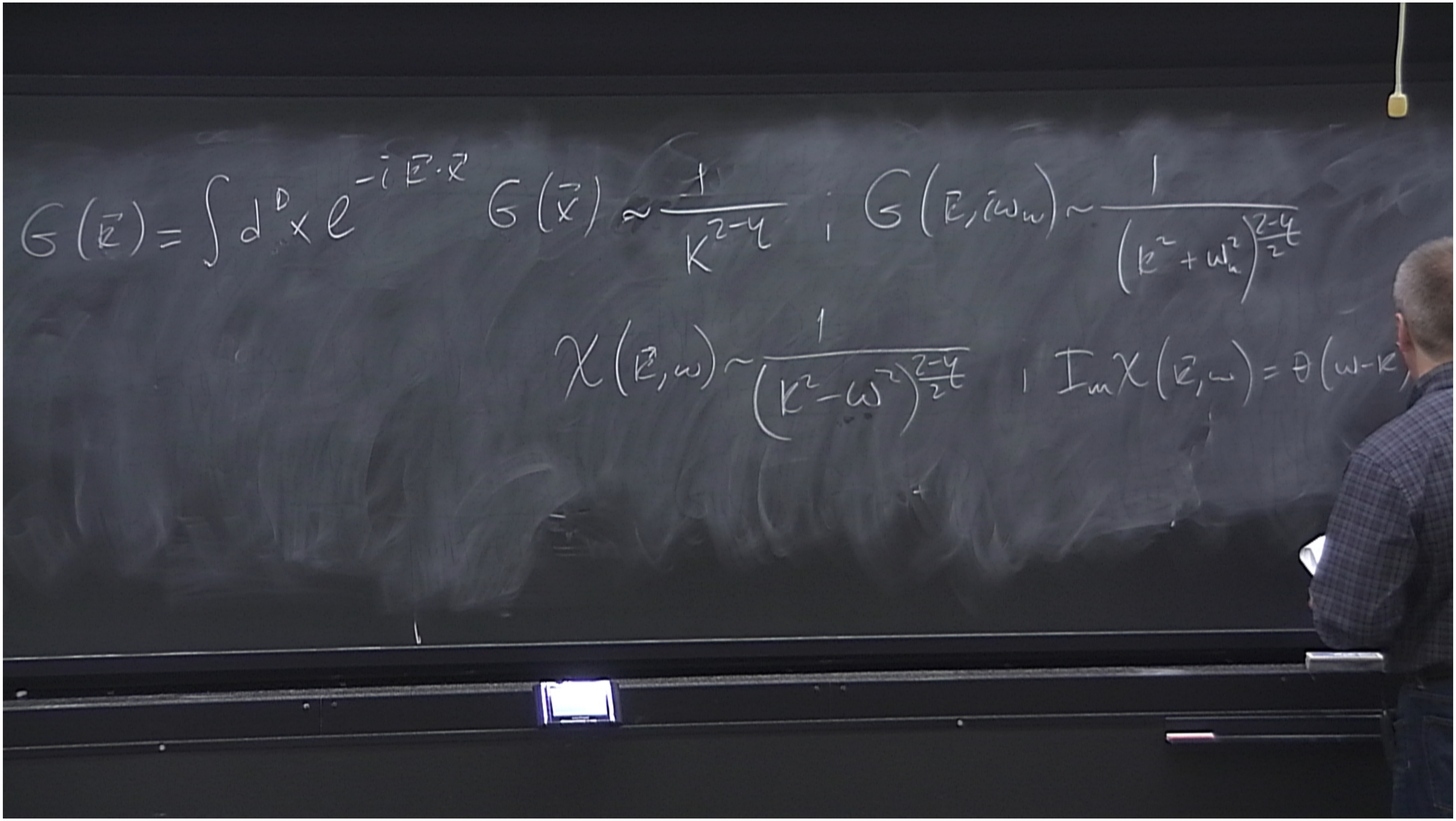
$$G(\vec{k}) = \int d^D x \, e^{-i\vec{k}\cdot\vec{x}} \, G(\vec{x}) \sim$$

At the critical point:

$$G(x) \sim \frac{1}{x^{D-2+\eta}} = \frac{1}{x^{d-1+\eta}}$$

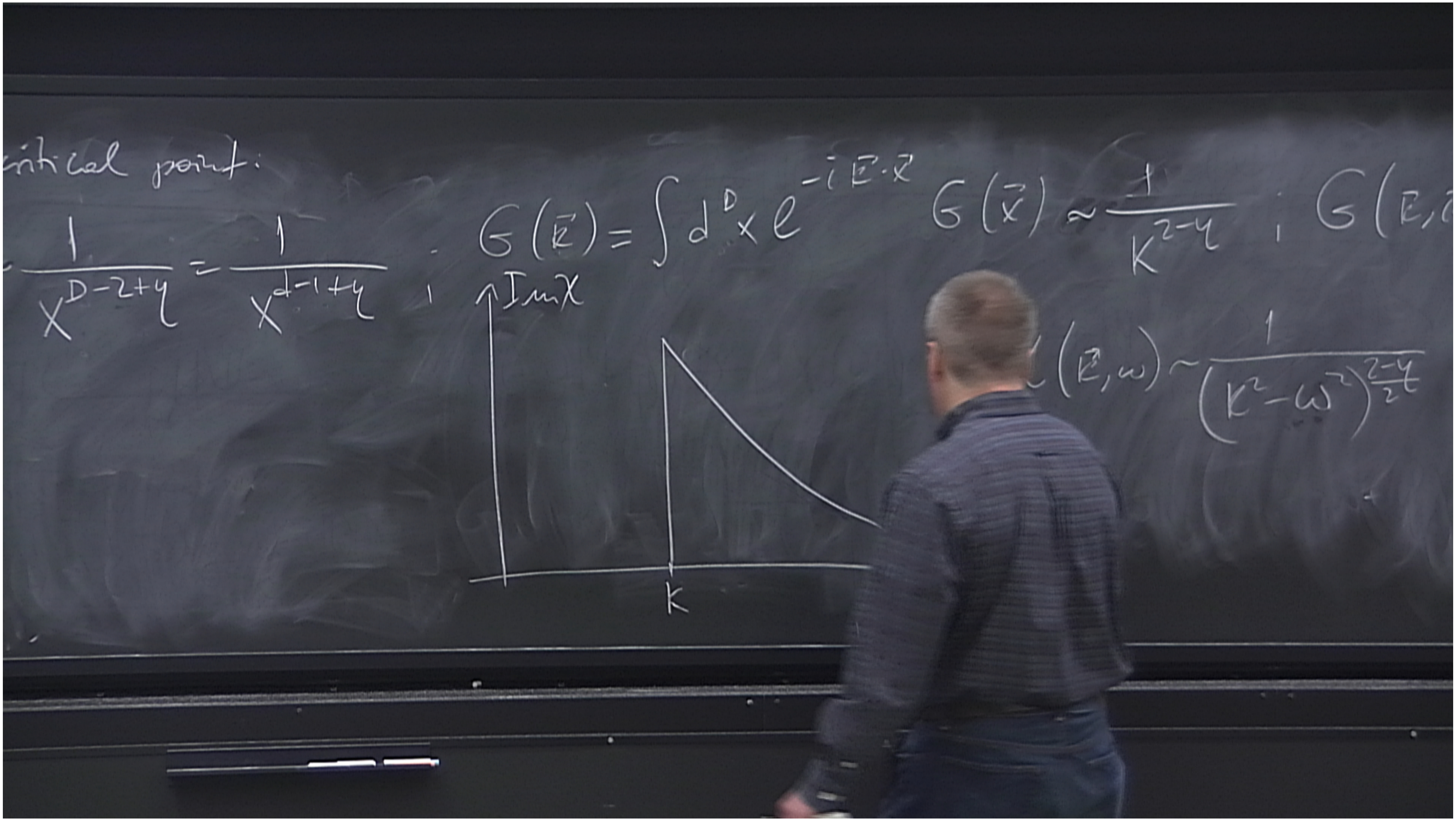
$$G(\vec{k}) = \int d^D x\, e^{-i\vec{k}\cdot\vec{x}}\, G(\vec{x}) \sim \frac{1}{k^{2-\eta}}$$

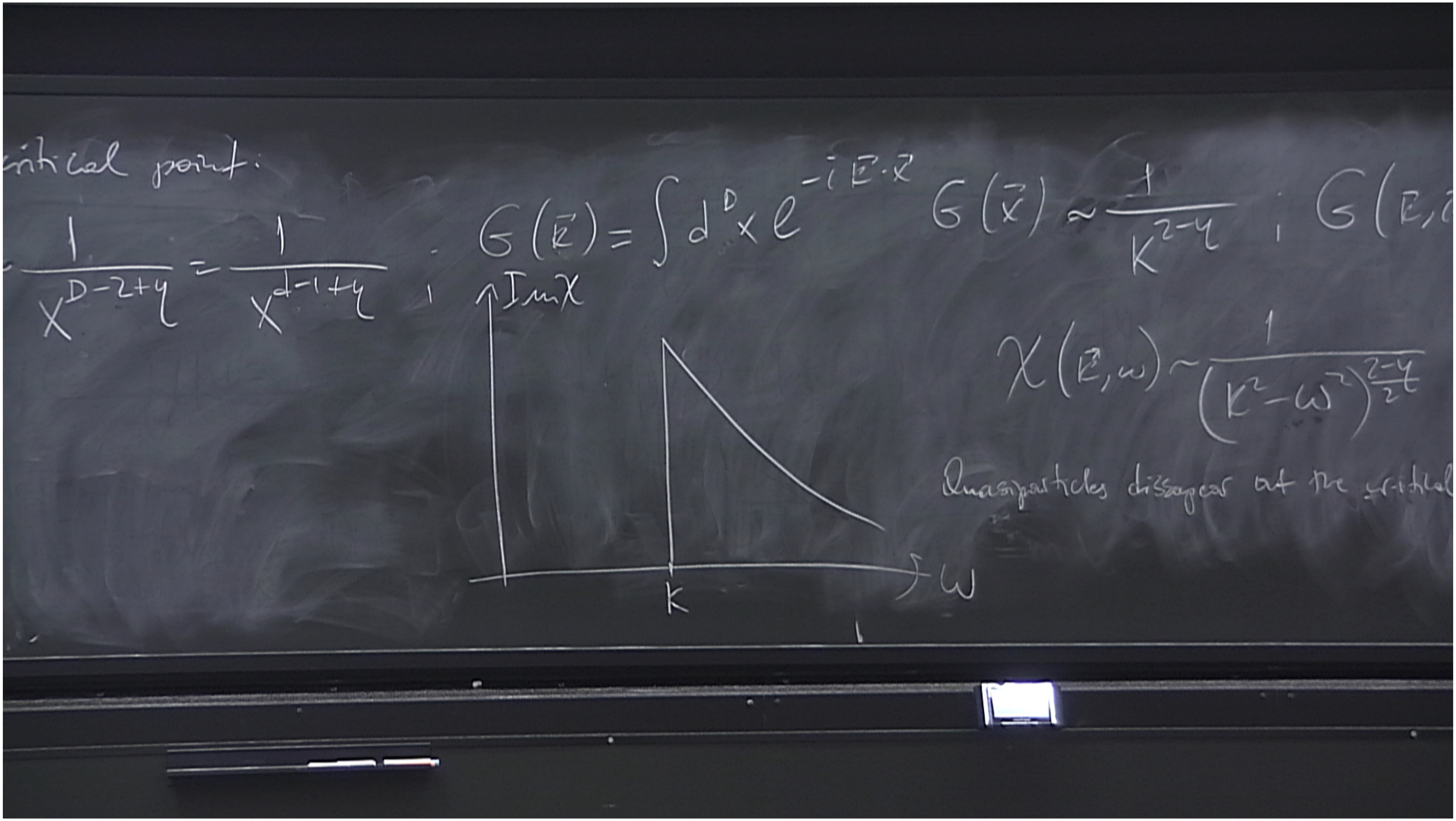

G(k) = ∫ d^D x e^{-ik·x} G(x) ~ 1/K^{2-η} ; G(k, iω_n) ~ 1/(k^2


$G(\vec{k}) = \int d^D x \, e^{-i\vec{k}\cdot\vec{x}}$
$G(\vec{x}) \sim \frac{1}{K^{2-\eta}}$ ; $G(\vec{k}, i\omega_n) \sim \frac{1}{(k^2+\omega_n^2)^{\frac{2-\eta}{2}}}$
$\chi(\vec{k}, \omega) \sim \frac{1}{(k^2-\omega^2)^{\frac{2-\eta}{2}}}$ ; $\mathrm{Im}\,\chi(\vec{k}, \omega) = \theta(\omega - k)$

$$G(\vec{x}) \sim \frac{1}{K^{2-\eta}} \; ; \; G(k, i\omega_n) \sim \frac{1}{\left(k^2+\omega_n^2\right)^{\frac{2-\eta}{2}}}$$

$$\chi(\vec{k},\omega) \sim \frac{1}{\left(K^2-\omega^2\right)^{\frac{2-\eta}{2}}} \; ; \; \mathrm{Im}\,\chi(k,\omega) = \theta(\omega-k)\,\mathrm{Im}\left[\frac{e^{-i\pi\left(\frac{2-\eta}{2}\right)}}{\left(\omega^2-k^2\right)^{\frac{2-\eta}{2}}}\right]$$


ritical point:
$\frac{1}{x^{D-2+\eta}} = \frac{1}{x^{d-1+\eta}}$
$G(\vec{k}) = \int d^D x \, e^{-i\vec{k}\cdot\vec{x}}$
$G(\vec{x}) \sim \frac{1}{K^{2-\eta}}$
$(\vec{k},\omega) \sim \frac{1}{(K^2-\omega^2)^{\frac{2-\eta}{2}}}$
Im χ
k


ritical point:
$\frac{1}{x^{D-2+\eta}} = \frac{1}{x^{d-1+\eta}}$
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$G(\vec{x}) \sim \frac{1}{k^{2-\eta}}$
$\chi(\vec{k},\omega) \sim \frac{1}{(k^2-\omega^2)^{\frac{2-\eta}{2}}}$
Im $\chi$
k
$\omega$
Quasiparticles disappear at the critical