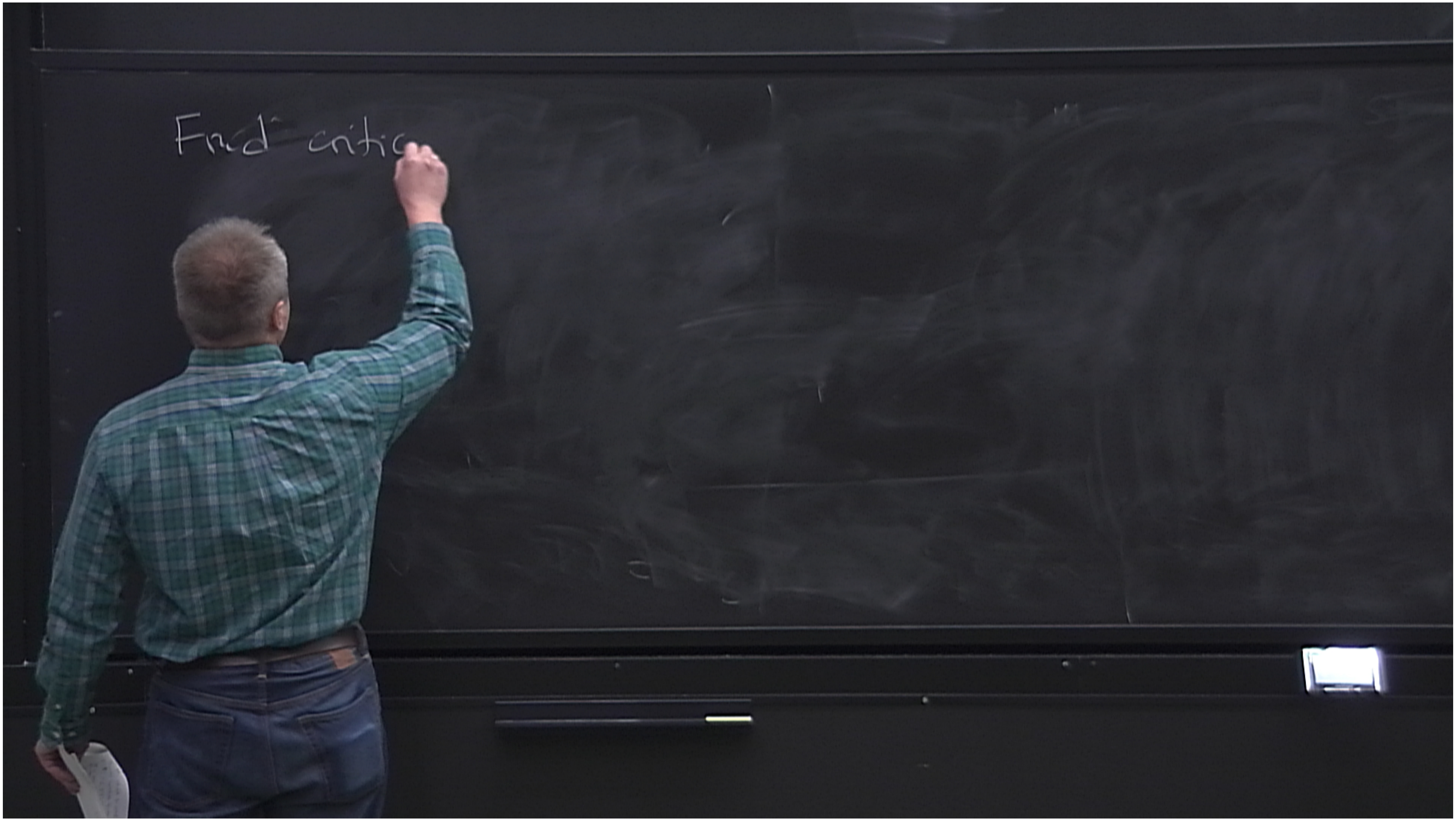
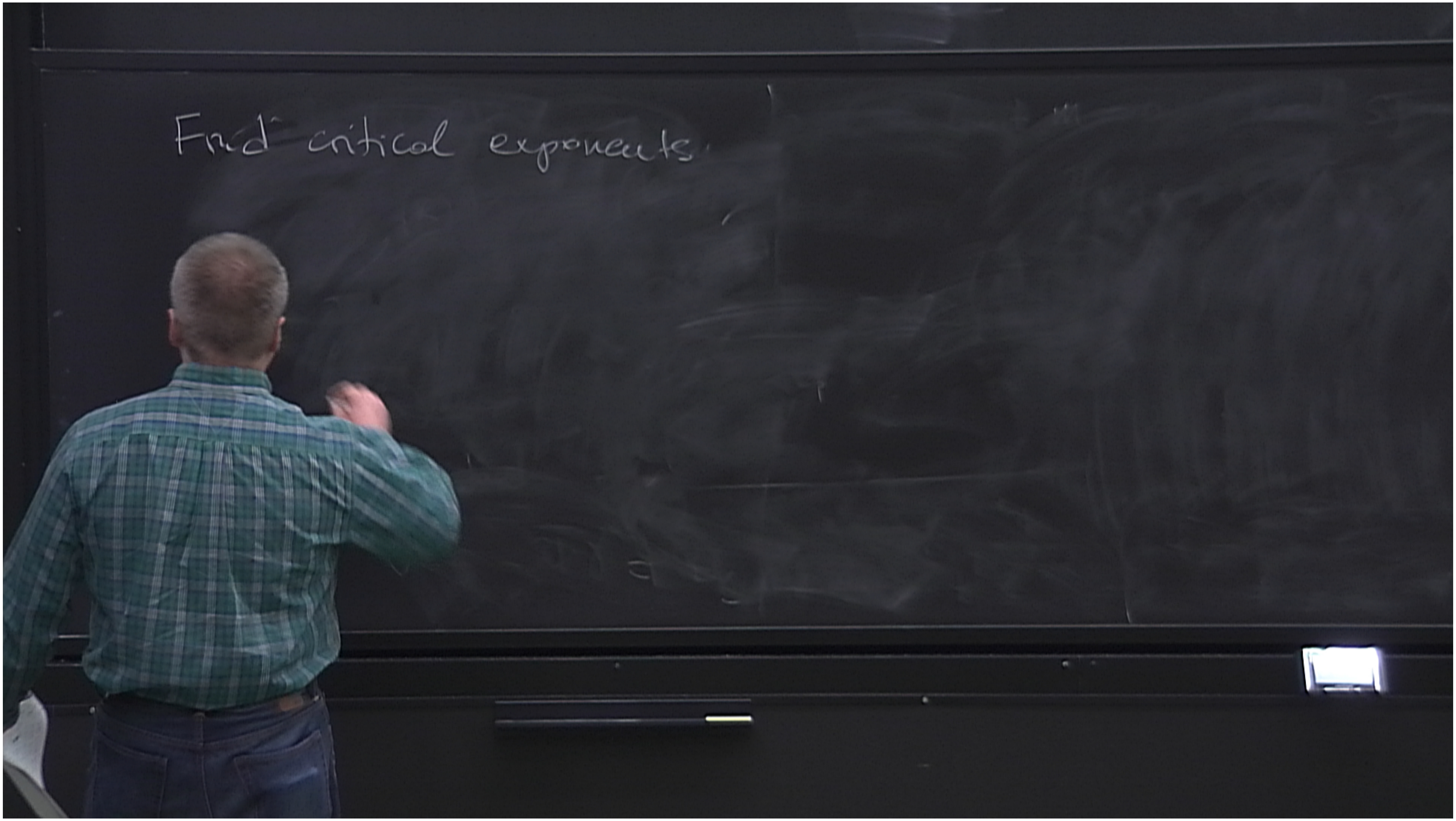
Title: 14/15 PSI - Statistical Mechanics - Lecture 10

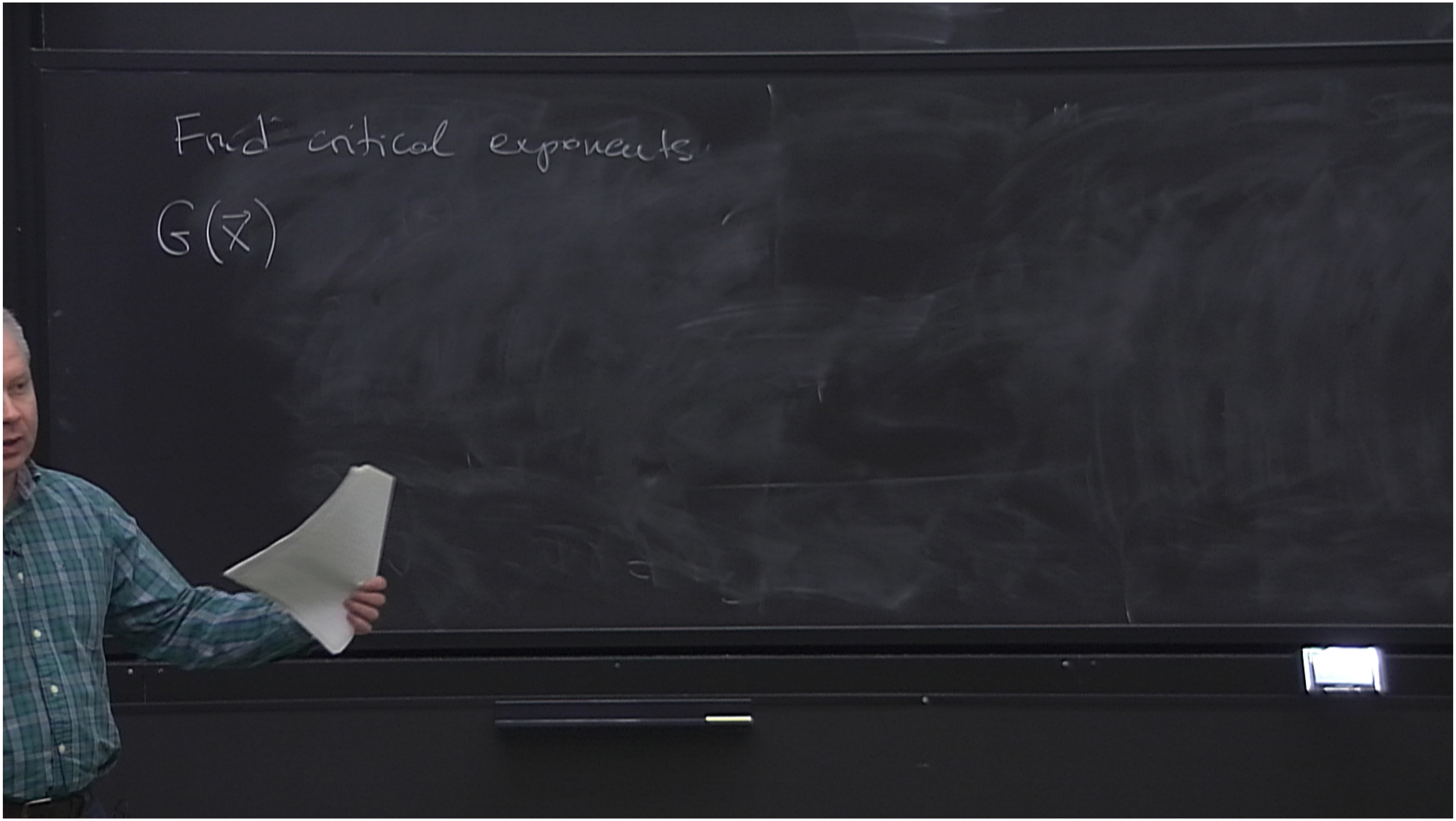
Date: Oct 20, 2014 10:45 AM

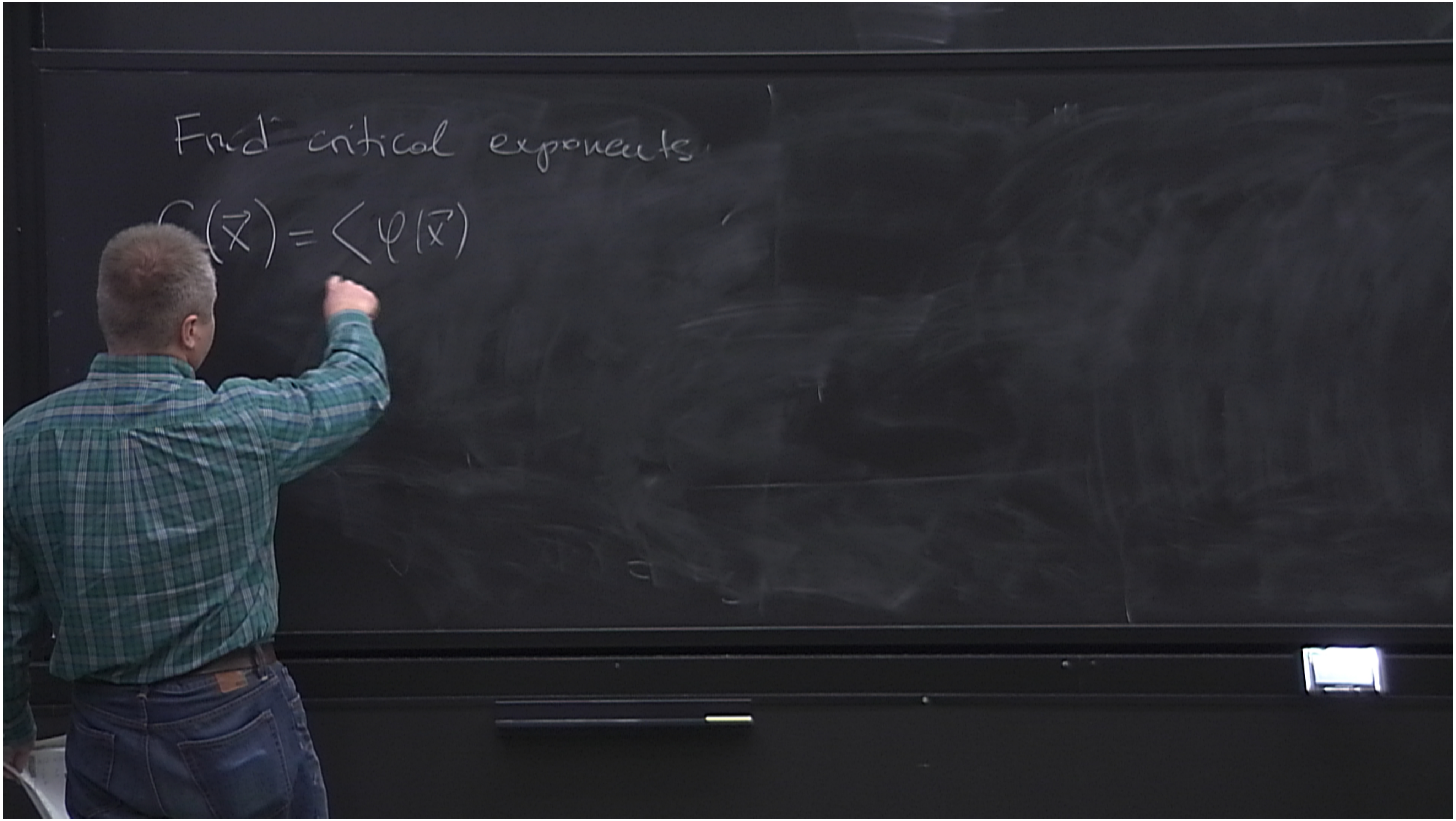
URL: http://pirsa.org/14100098

Abstract:




Find critical exponents.


Find critical exponents:
G(x⃗)


Find critical exponents
G(x⃗) = < φ(x⃗)

Find critical exponents.

$$G(\vec{x}) = \langle \varphi(\vec{x})\, \varphi(0) \rangle - \langle \varphi(\vec{x})\, \varphi(0) \rangle$$

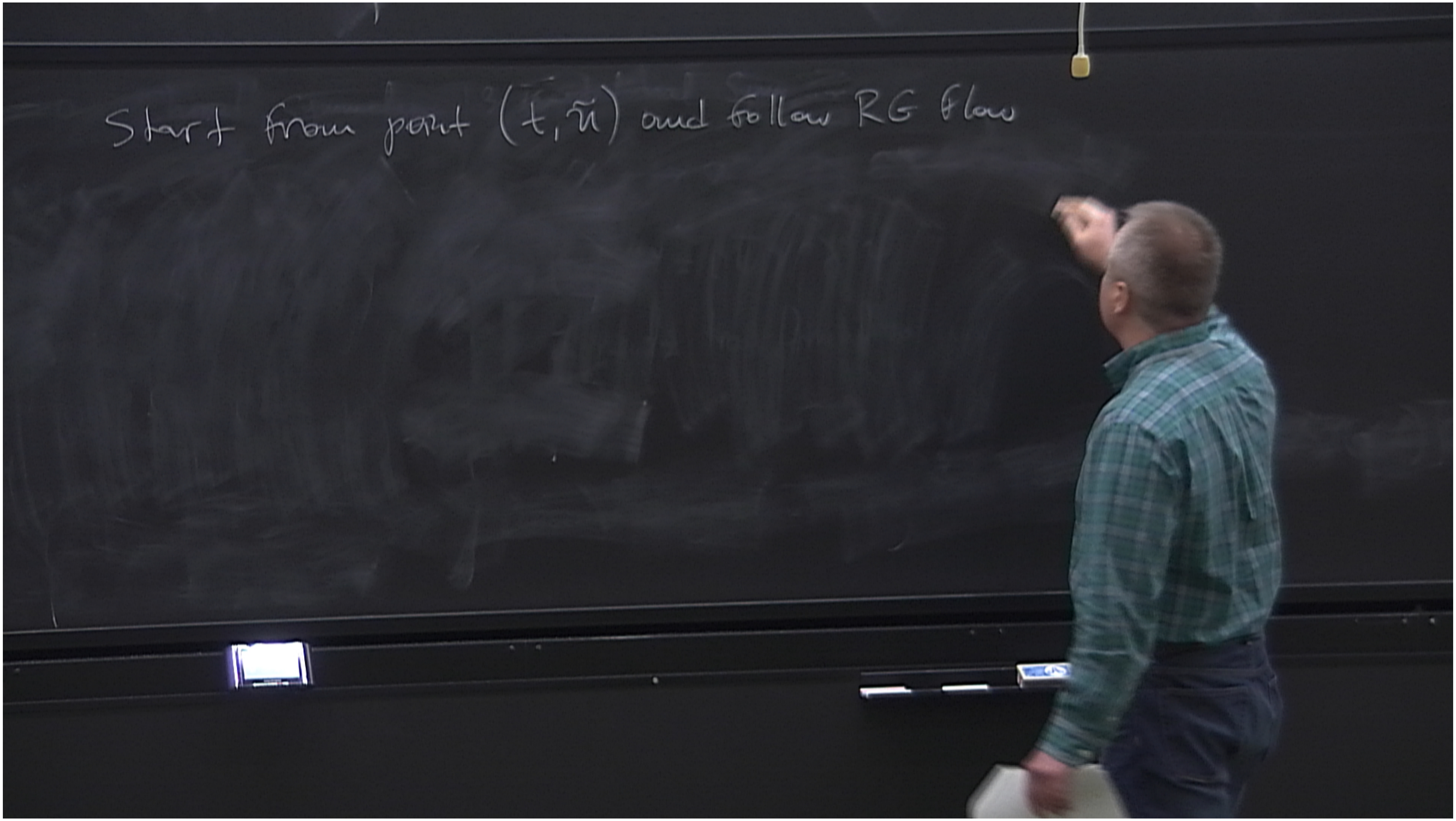
Find critical exponents

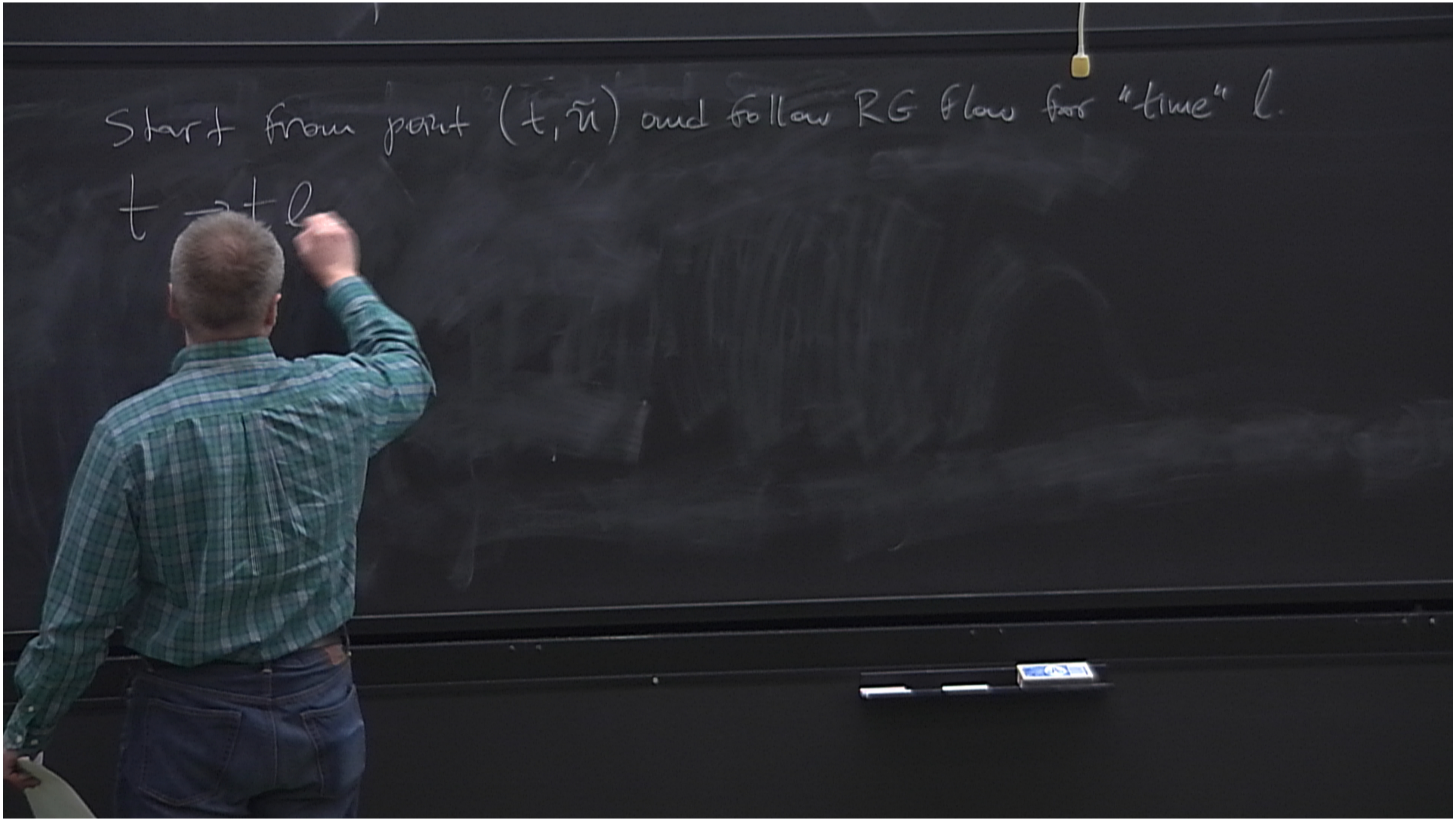
Start from

$$G(\vec{x}) = \langle \varphi(\vec{x})\varphi(0)\rangle - \langle \varphi(\vec{x})\rangle\langle \varphi(0)\rangle$$

(r, u) ; $t, \tilde{u}$

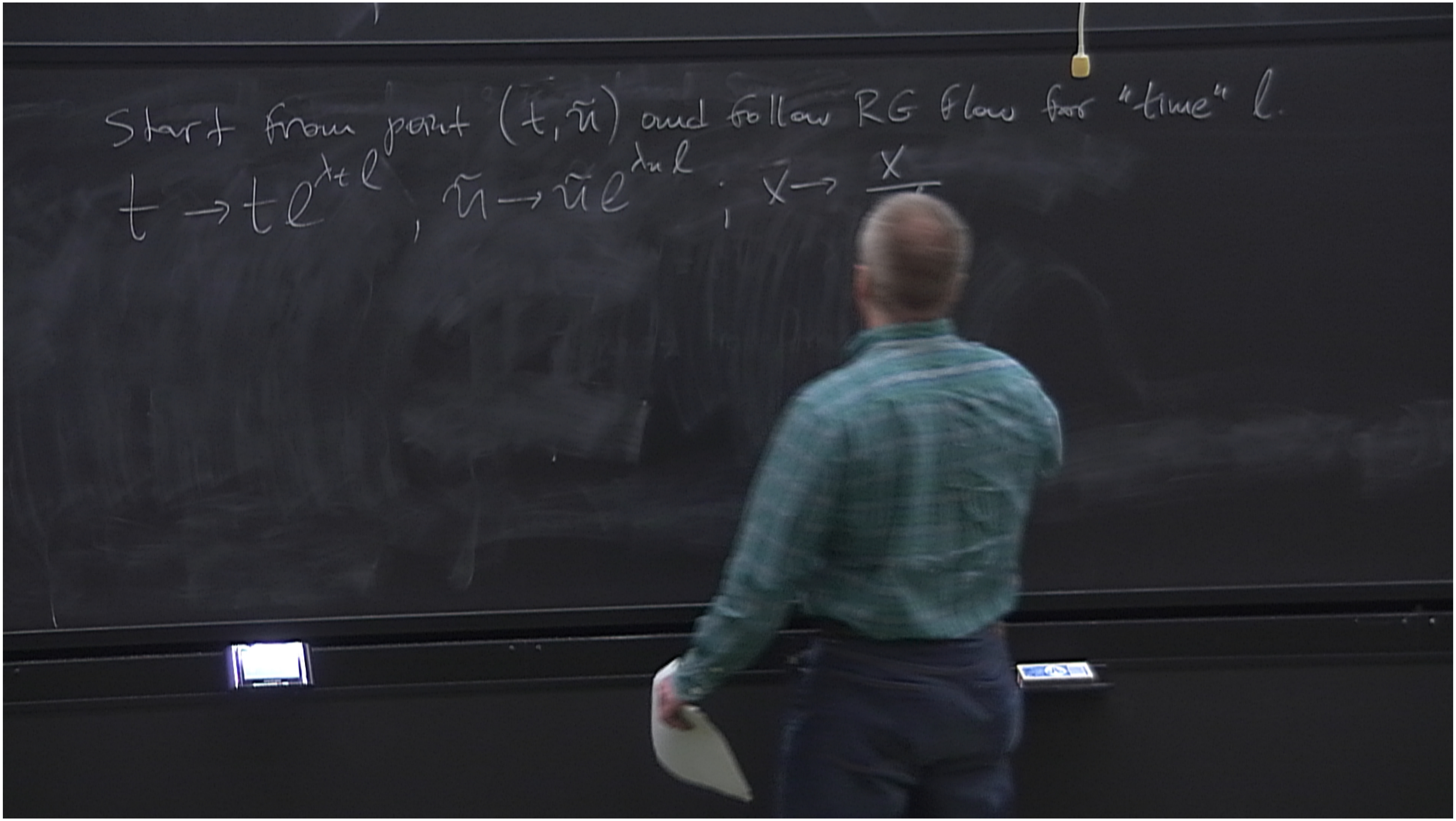
$$\frac{dt}{dl} = \lambda_t t \quad ; \quad \frac{d\tilde{u}}{dl} = \lambda_u \tilde{u}$$


Start from point $(\bar{t}, \tilde{u})$ and follow RG flow


Start from point (t̃, ũ) and follow RG flow for "time" l.
t → t l

Start from point $(t, \tilde{u})$ and follow RG flow for "time" l.

$$t \to t\, e^{\lambda_t l}, \quad \tilde{u} \to \tilde{u}\, e^{\lambda_u l}$$

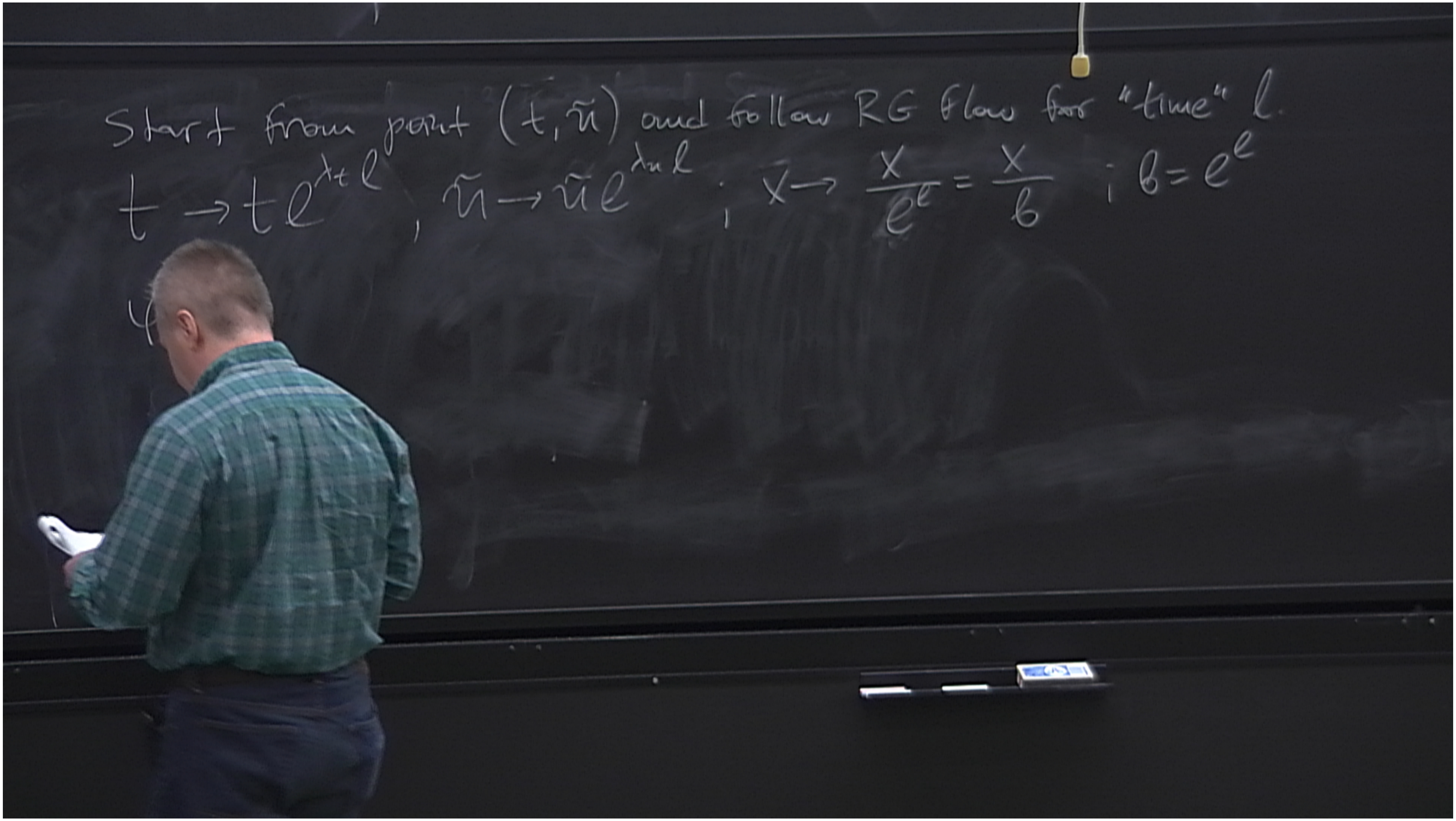

Start from point $(\tilde{t}, \tilde{u})$ and follow RG flow for "time" l.
$t \to t\, e^{\lambda_t l}$, $\tilde{u} \to \tilde{u}\, e^{\lambda_u l}$; $x \to \frac{x}{}$

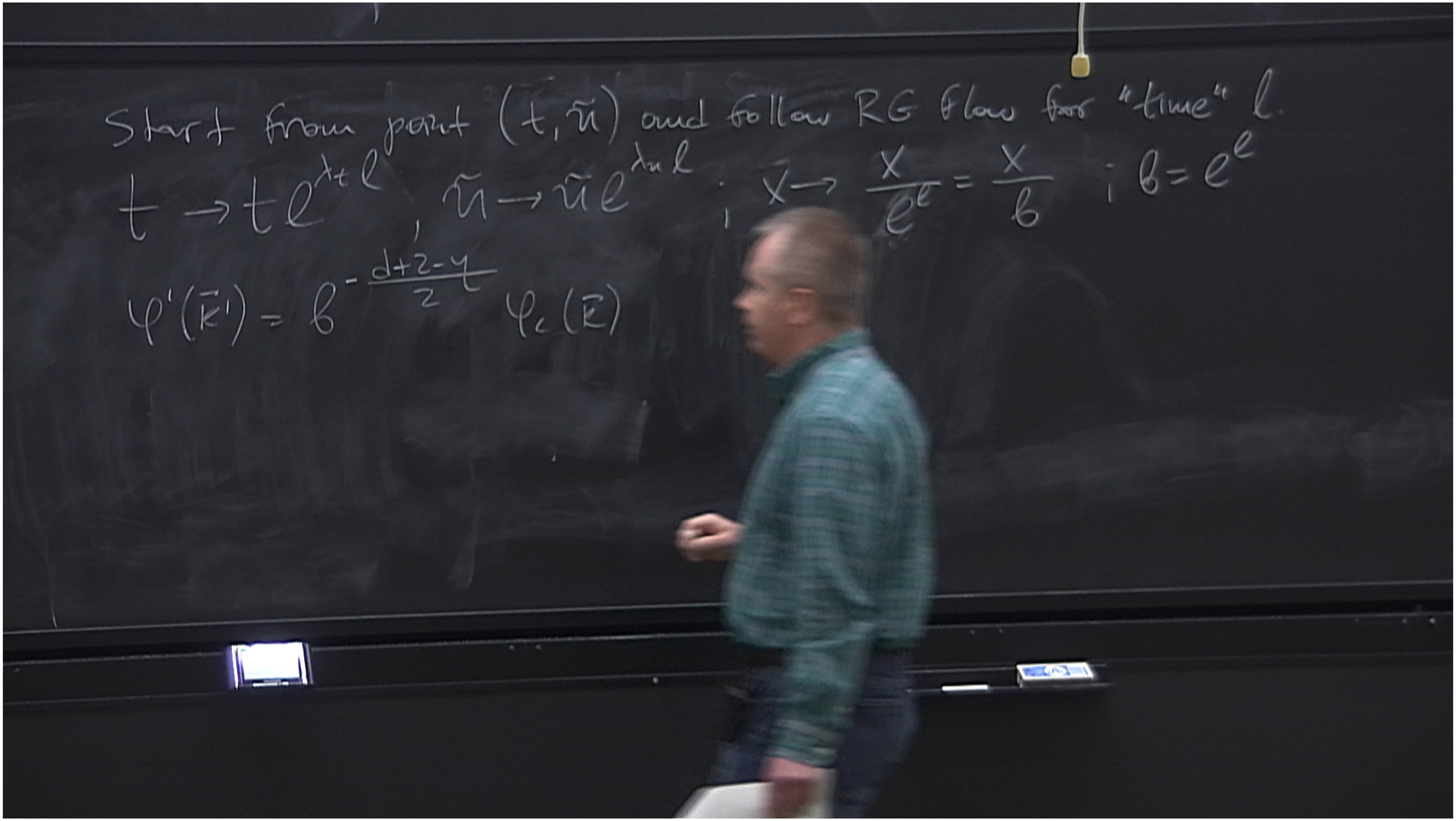
Start from point $(\bar{t}, \tilde{u})$ and follow RG flow for "time" l.

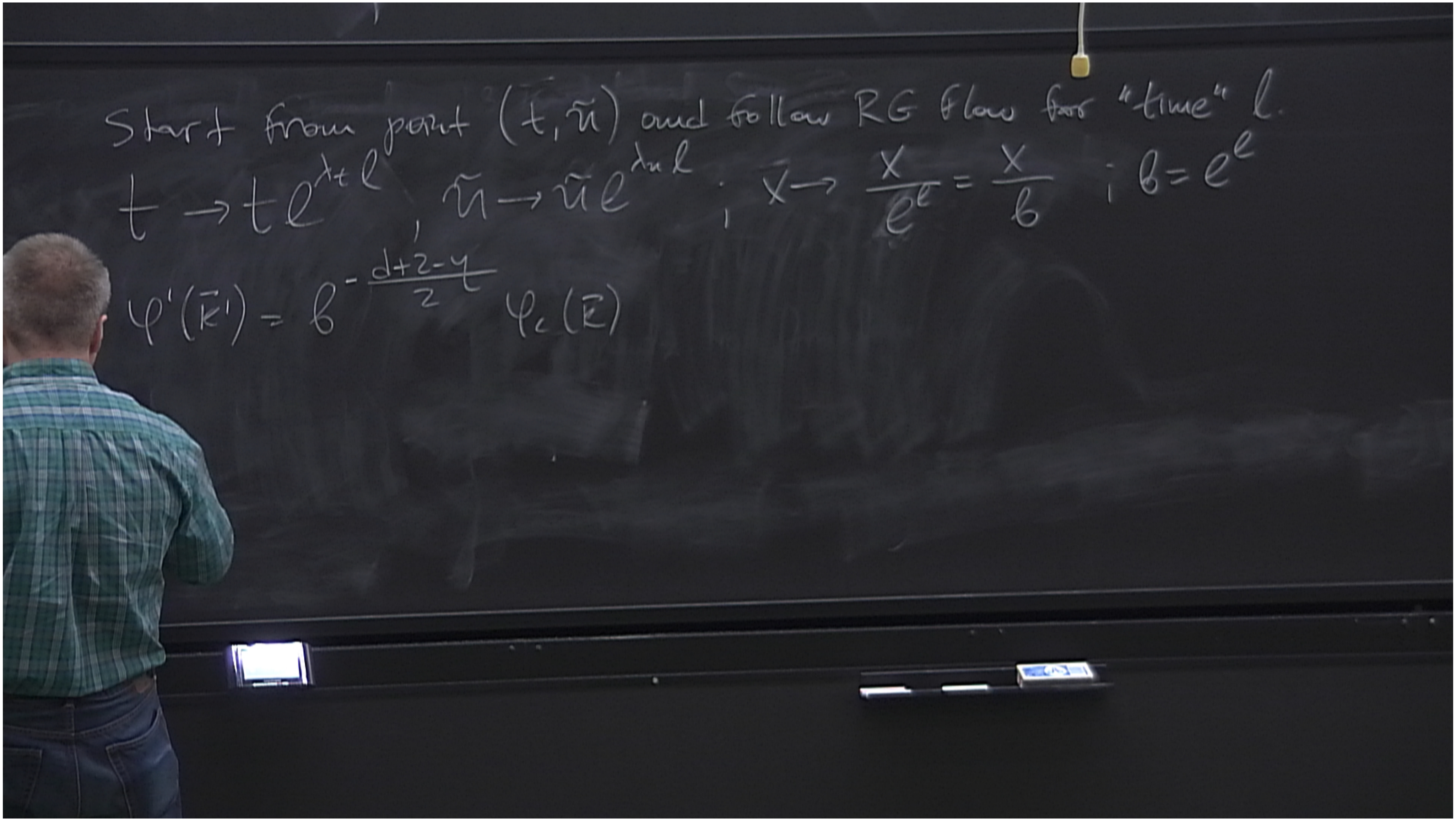
$t \to t e^{\lambda_t l}$, $\tilde{u} \to \tilde{u} e^{\lambda_u l}$; $\bar{x} \to \frac{x}{e^{l}}$

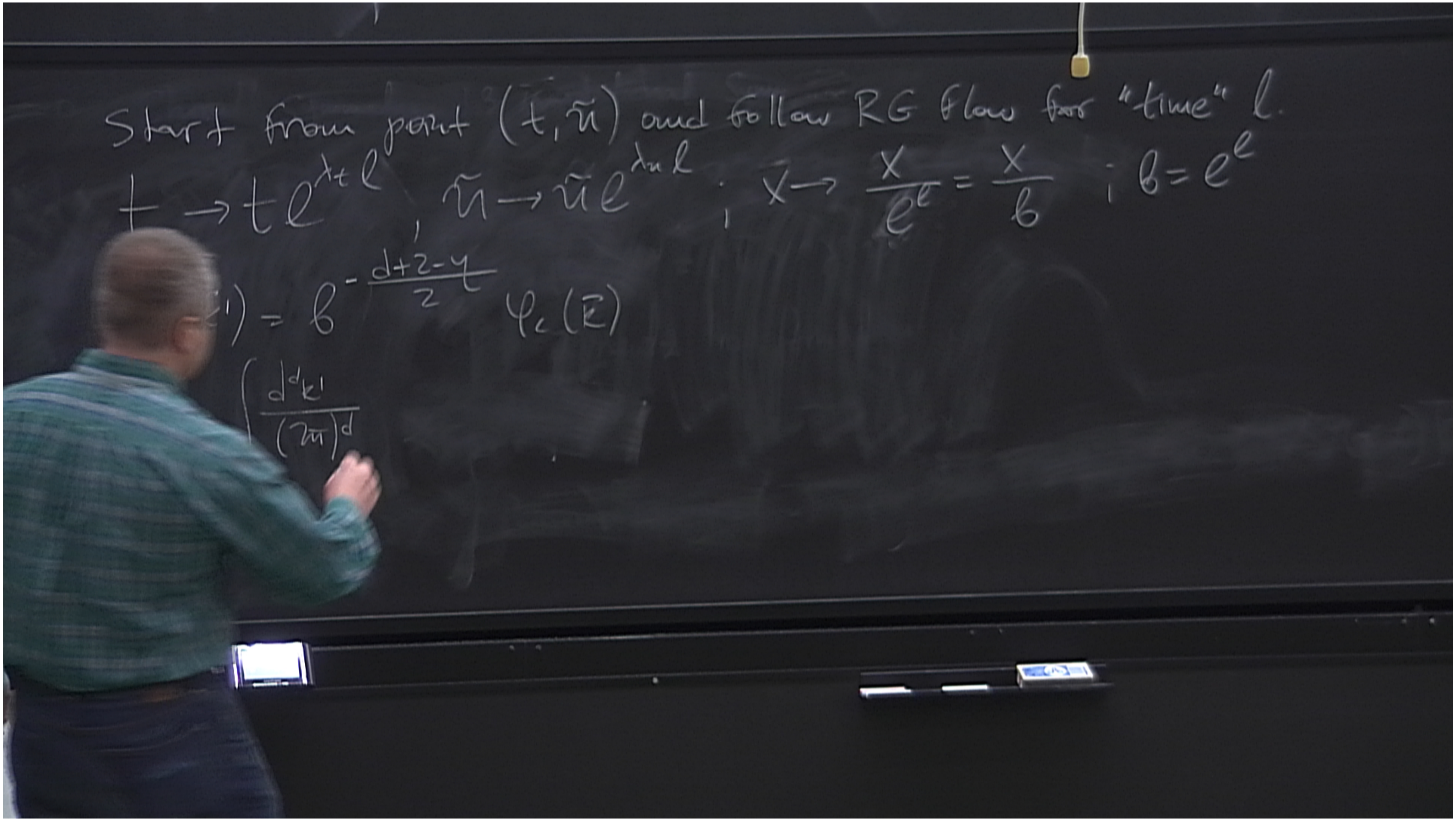
Start from point $(t, \tilde{u})$ and follow RG flow for "time" l.

$$t \to t e^{\lambda_t l}, \quad \tilde{u} \to \tilde{u} e^{\lambda_u l} \; ; \quad x \to \frac{x}{e^l} = \frac{x}{b} \; ; \quad b = e^l$$


Start from point $(t, \tilde{u})$ and follow RG flow for "time" l.
$t \to t e^{\lambda_t l}$, $\tilde{u} \to \tilde{u} e^{\lambda_u l}$; $x \to \frac{x}{e^l} = \frac{x}{b}$; $b = e^l$


Start from point $(\tilde{t}, \tilde{u})$ and follow RG flow for "time" ℓ.
$t \to t e^{\lambda_t \ell}$, $\tilde{u} \to \tilde{u} e^{\lambda_u \ell}$; $\bar{x} \to \frac{x}{e^{\ell}} = \frac{x}{b}$; $b = e^{\ell}$
$\varphi'(\bar{k}') = b^{-\frac{d+2-\eta}{2}} \varphi_c(\bar{k})$


Start from point $(\tilde{t}, \tilde{u})$ and follow RG flow for "time" l.
$t \to t e^{\lambda_t l}$, $\tilde{u} \to \tilde{u} e^{\lambda_u l}$; $\bar{x} \to \frac{x}{e^l} = \frac{x}{b}$; $b = e^l$
$\varphi'(\bar{k}') = b^{-\frac{d+2-\eta}{2}} \varphi_c(\bar{k})$

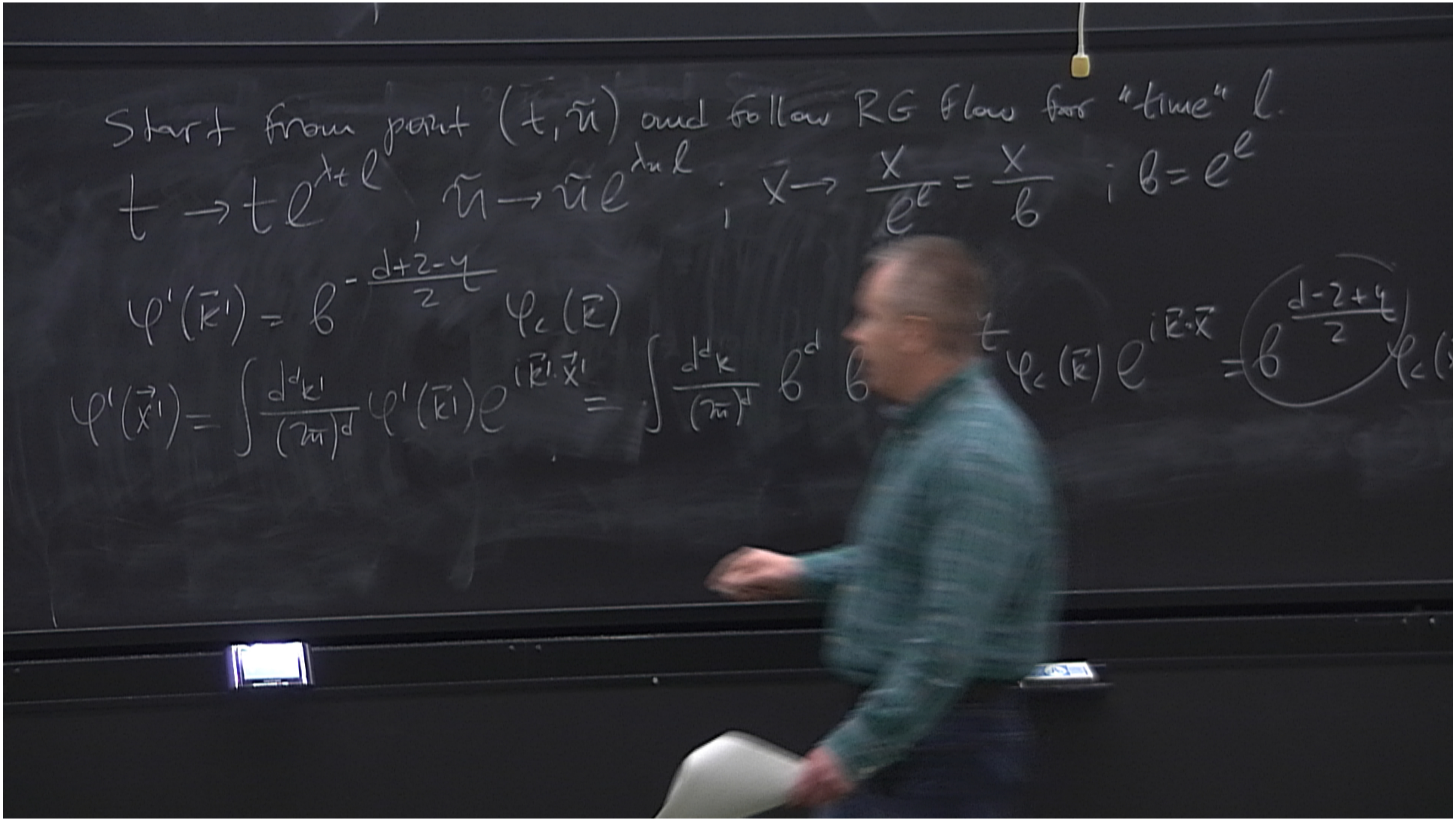

Start from point $(t, \tilde{u})$ and follow RG flow for "time" l.
$t \to t e^{\lambda_t l}$, $\tilde{u} \to \tilde{u} e^{\lambda_u l}$; $x \to \frac{x}{e^l} = \frac{x}{b}$; $b = e^l$
$) = b^{-\frac{d+2-\eta}{2}}$ $\varphi_c(k)$
$\int \frac{d^dk'}{(2\pi)^d}$

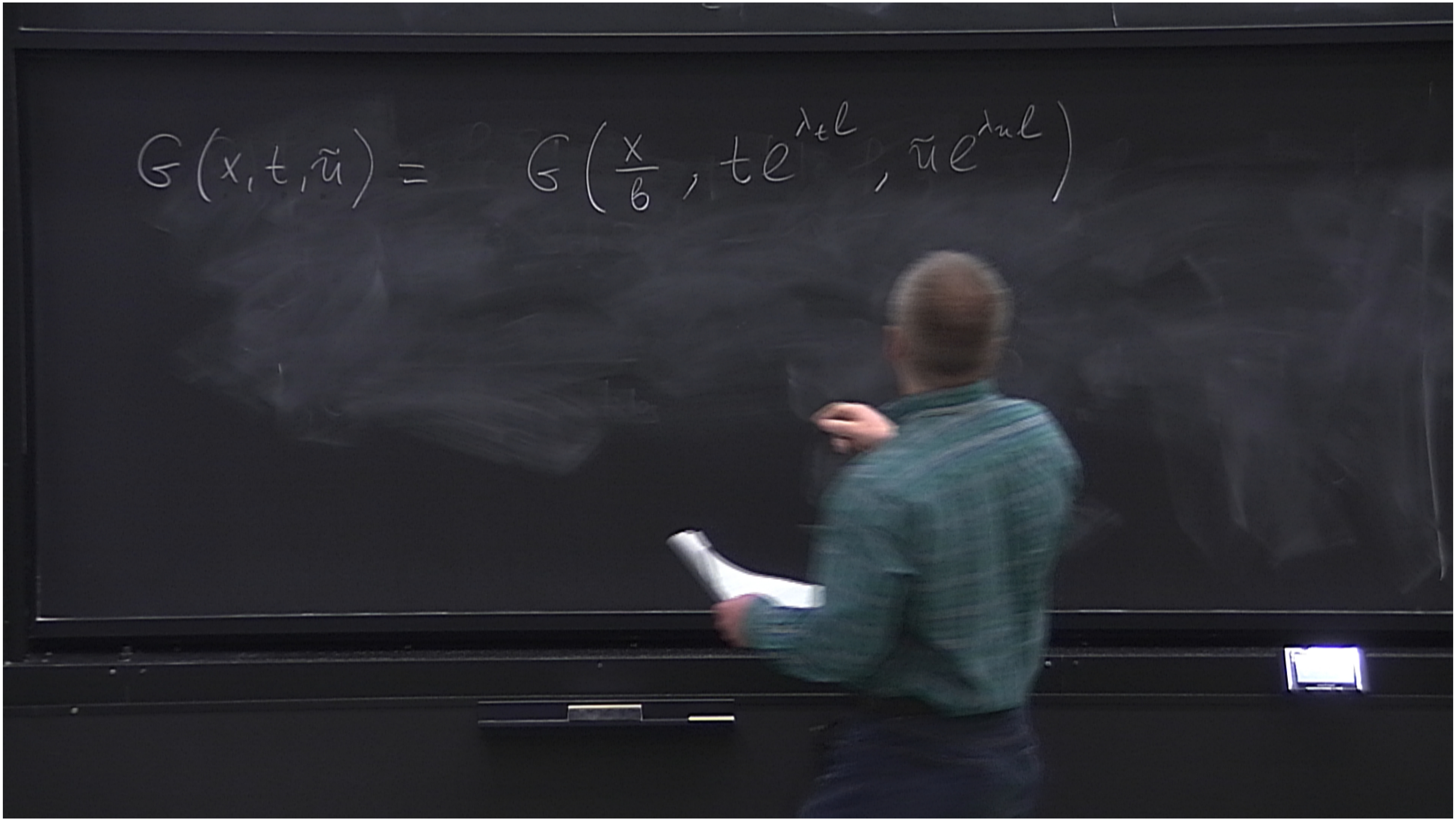
Start from point $(t, \tilde{u})$ and follow RG flow for "time" l.

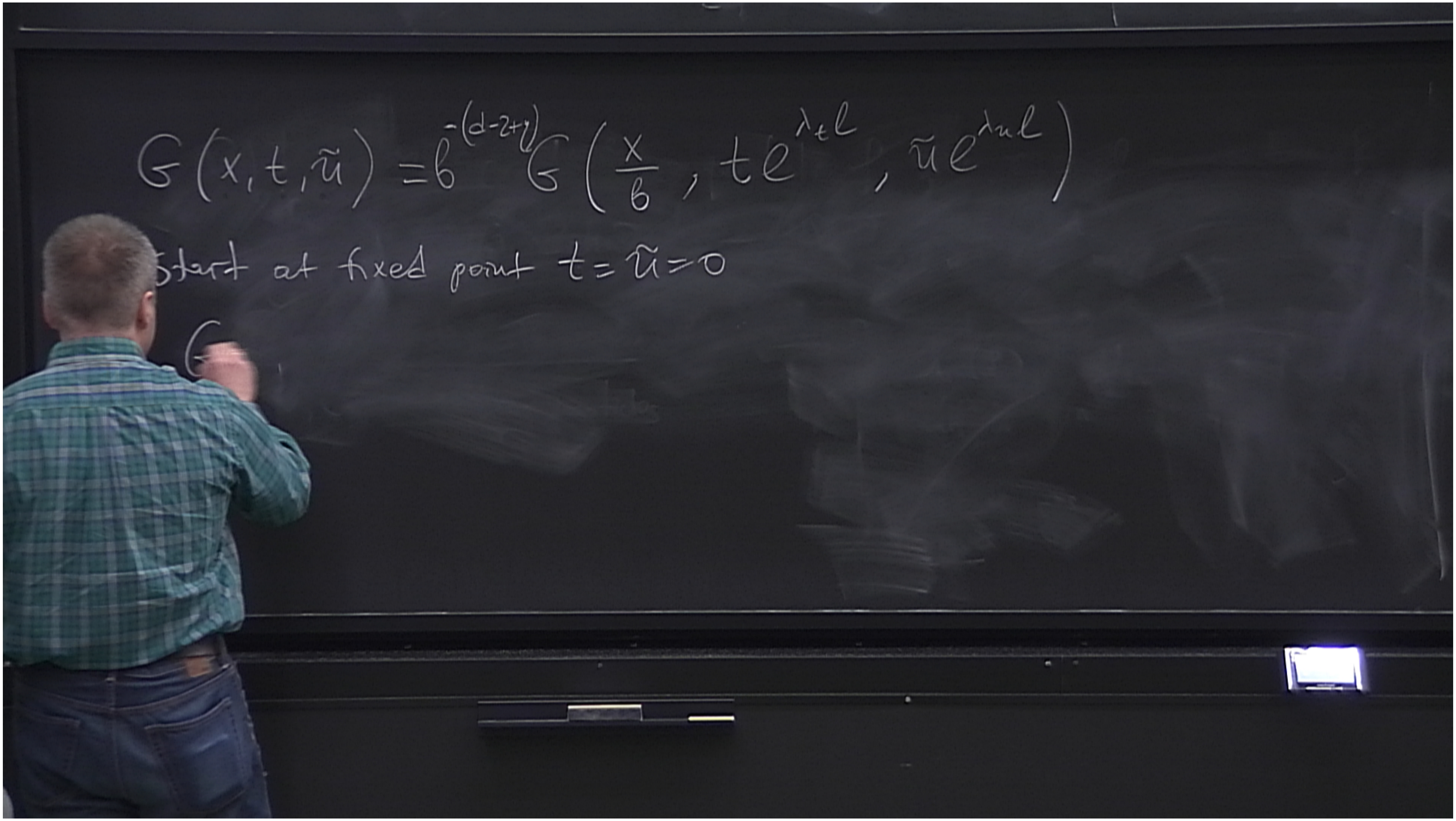
$$t \to t e^{\lambda_t l}, \quad \tilde{u} \to \tilde{u} e^{\lambda_u l}; \quad \vec{x} \to \frac{x}{e^l} = \frac{x}{b} \quad ; \quad b = e^l$$

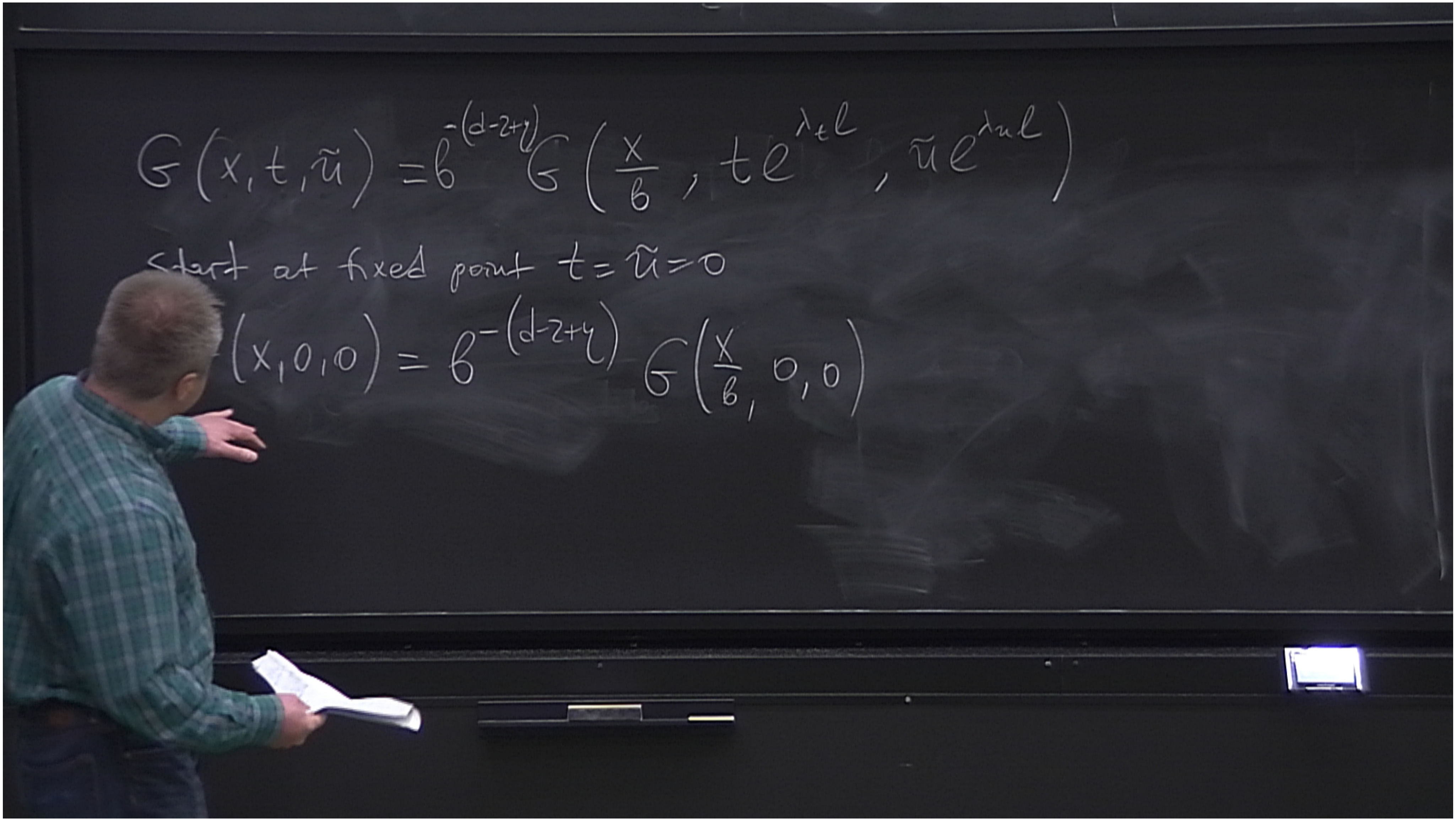
$$\varphi'(\vec{k}') = b^{-\frac{d+2-\eta}{2}} \varphi_c(\vec{k})$$

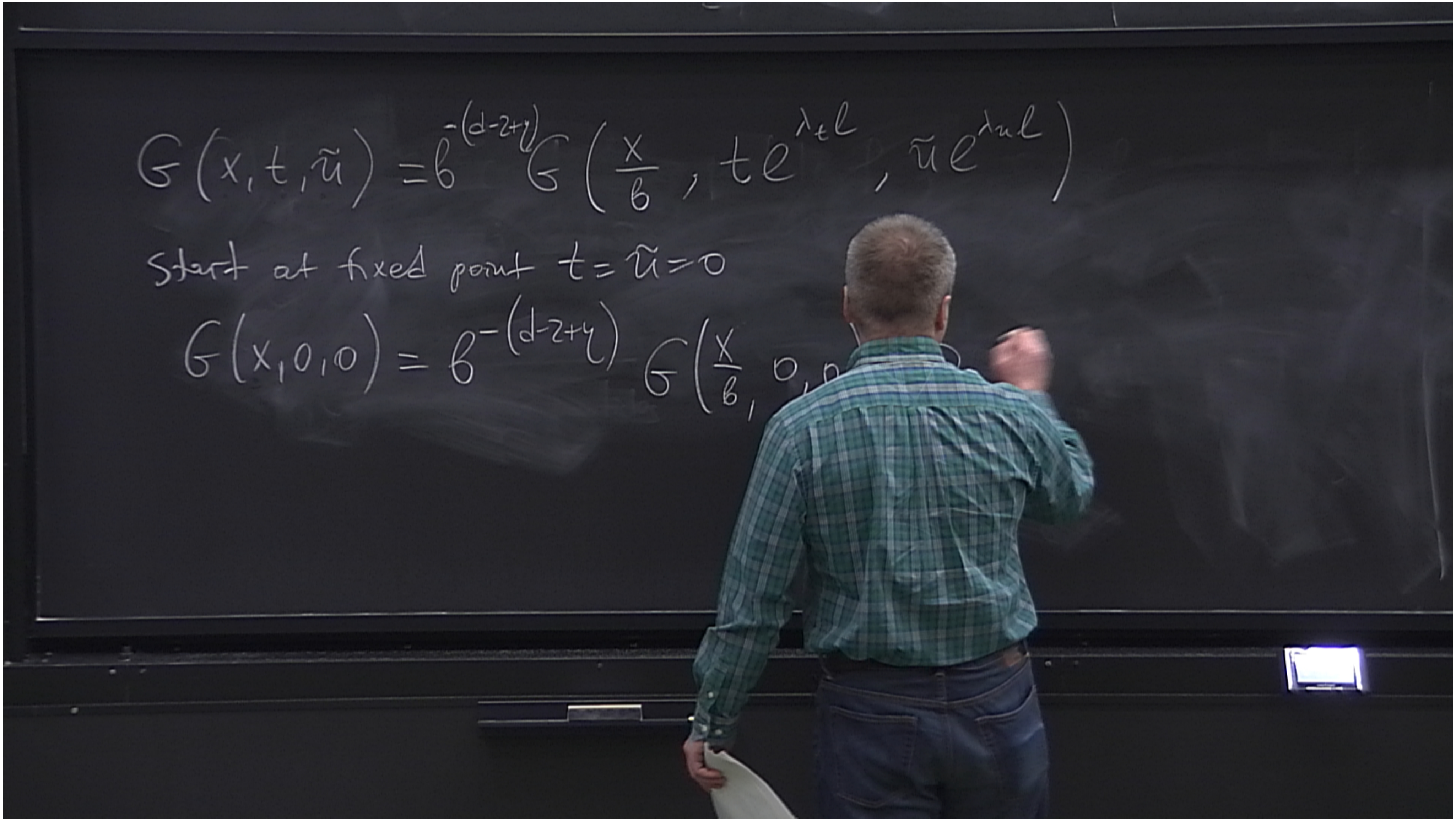
$$\varphi'(\vec{x}') = \int \frac{d^d k'}{(2\pi)^d} \varphi'(\vec{k}') e^{i\vec{k}' \cdot \vec{x}'} = \int \frac{d^d k}{(2\pi)^d} b^d \, b^{-\frac{d+2-\eta}{2}} \varphi_c(\vec{k}) e^{i\vec{k}\cdot\vec{x}}$$


Start from point (t, ũ) and follow RG flow for "time" l.
t → t e^{λ_t l}, ũ → ũ e^{λ_u l}; x̄ → x/e^l = x/b; b = e^l
φ'(k̄') = b^{-(d+2-η)/2} φ_<(k̄)
φ'(x̄') = ∫ d^d k'/(2π)^d φ'(k̄') e^{i k̄'·x̄'} = ∫ d^d k/(2π)^d b^d b φ_<(k̄) e^{i k̄·x̄} = b^{(d-2+η)/2} φ_<(


$G(x, t, \tilde{u}) = G\left(\frac{x}{b}, t e^{\lambda_t \ell}, \tilde{u} e^{\lambda_u \ell}\right)$


$G(x,t,\tilde{u}) = b^{-(d-2+\eta)} G\left(\frac{x}{b}, t e^{\lambda_t l}, \tilde{u} e^{\lambda_u l}\right)$
Start at fixed point $t = \tilde{u} = 0$

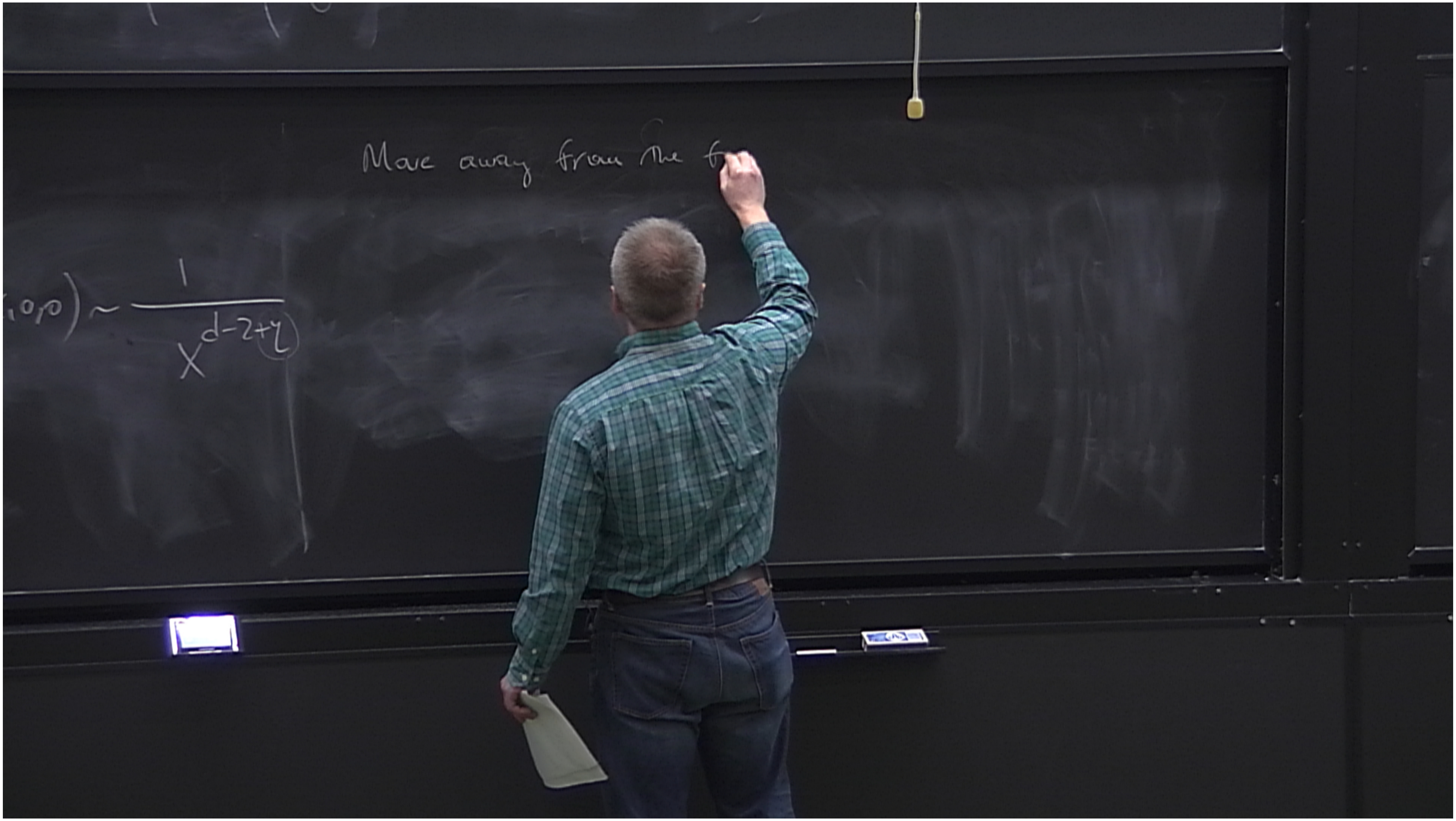

$G(x,t,\tilde{u}) = b^{-(d-2+\eta)} G\left(\frac{x}{b}, t e^{\lambda_t l}, \tilde{u} e^{\lambda_u l}\right)$
Start at fixed point $t = \tilde{u} = 0$
$G(x,0,0) = b^{-(d-2+\eta)} G\left(\frac{x}{b}, 0, 0\right)$

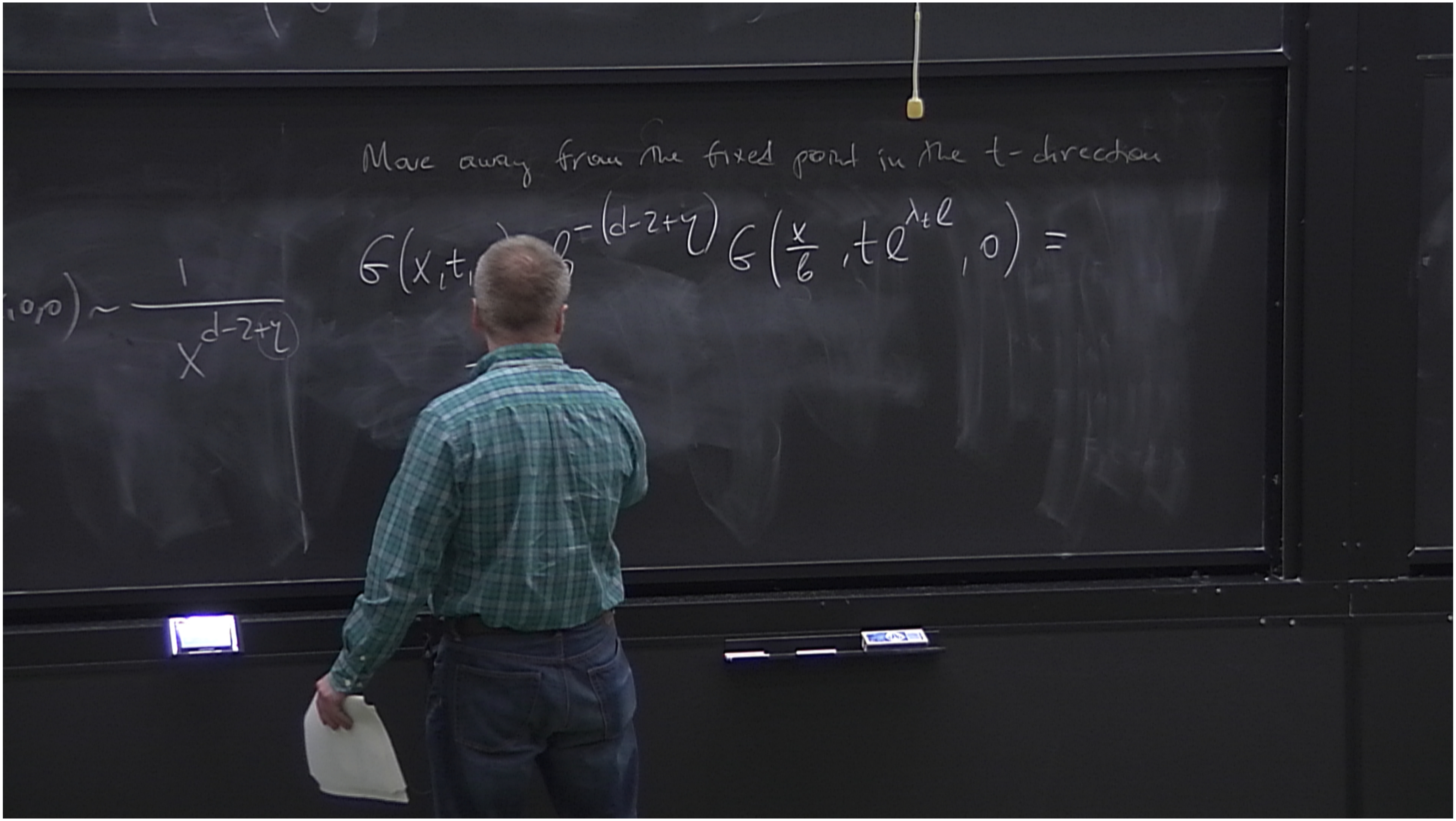

$G(x,t,\tilde{u}) = b^{-(d-2+\eta)} G\left(\frac{x}{b}, t e^{\lambda_t \ell}, \tilde{u} e^{\lambda_u \ell}\right)$
Start at fixed point $t = \tilde{u} = 0$
$G(x,0,0) = b^{-(d-2+\eta)} G\left(\frac{x}{b}, 0, 0\right.$

$$G(x, t, \tilde{u}) = b^{-(d-2+\eta)} G\left(\frac{x}{b}, t e^{\lambda_t l}, \tilde{u} e^{\lambda_u l}\right)$$

Start at fixed point $t = \tilde{u} = 0$

$$G(x, 0, 0) = b^{-(d-2+\eta)} G\left(\frac{x}{b}, 0, 0\right) \Rightarrow \qquad G(x, 0, 0) \sim \frac{1}{x^{d-2+\eta}}$$


Move away from the f
$\sim \frac{1}{x^{d-2+\eta}}$

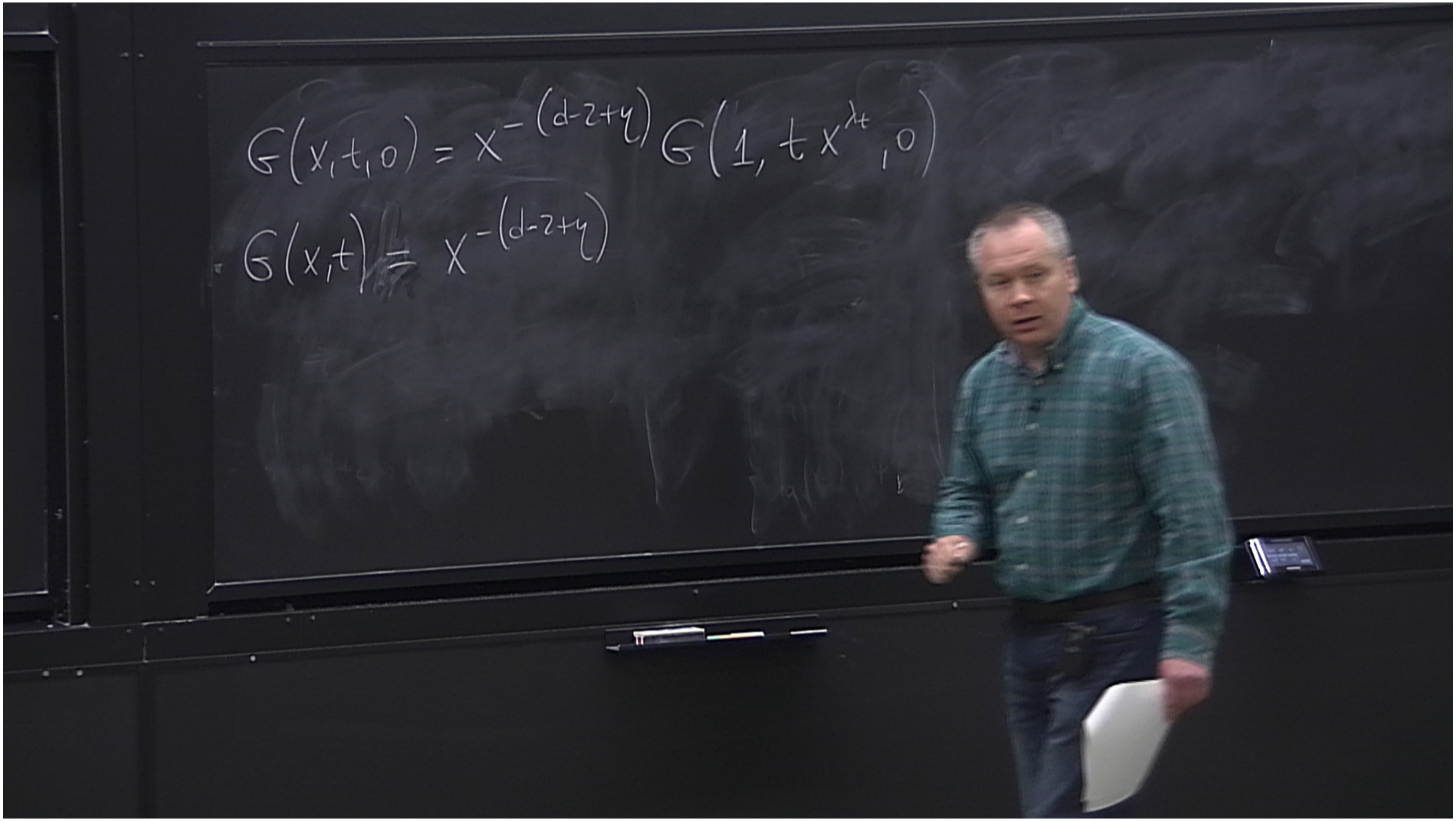

Move away from the fixed point in the t-direction

$$,0,0) \sim \frac{1}{x^{d-2+\eta}}$$

Move away from the fixed point in the t-direction

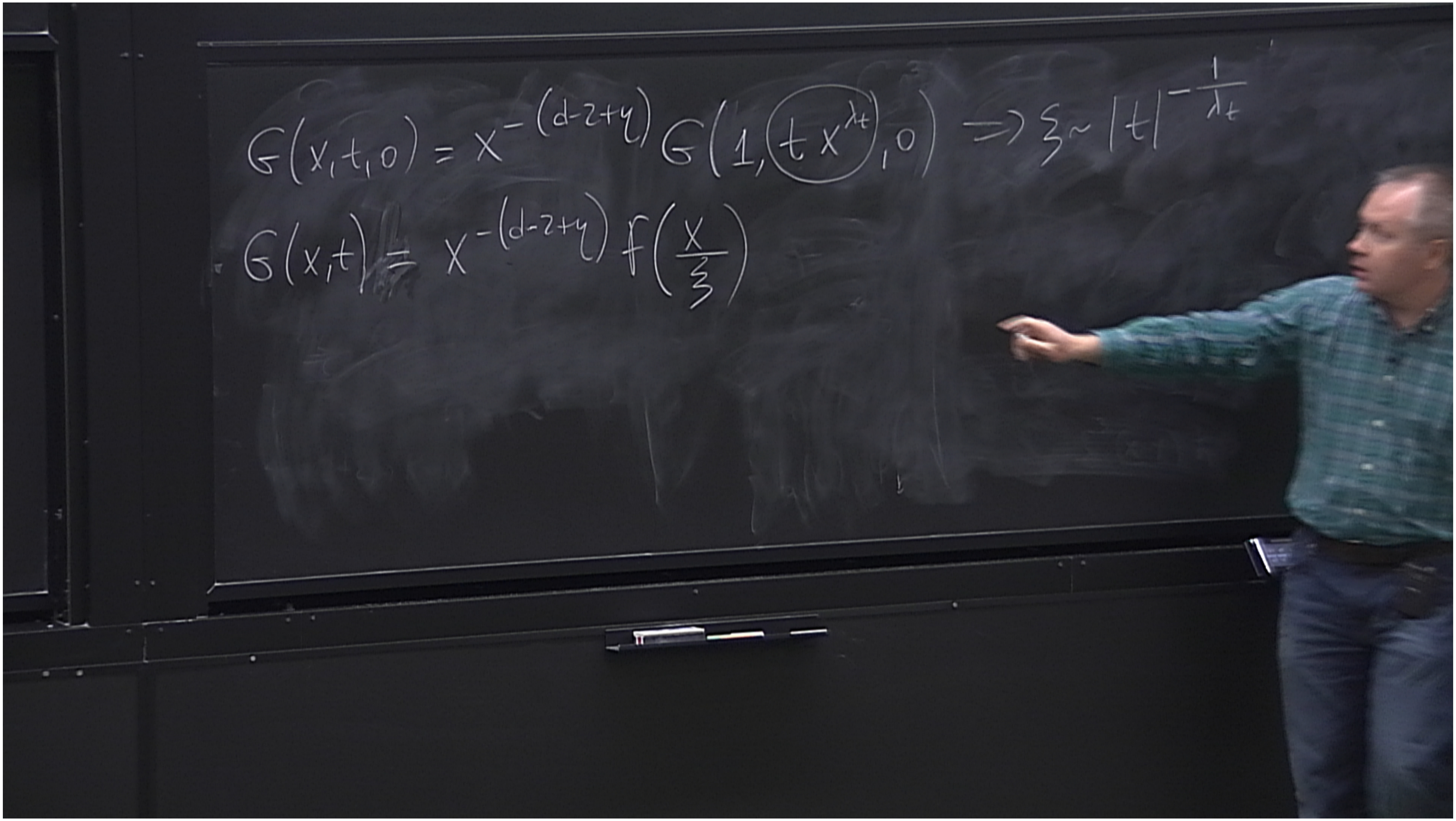
$$G(x,t,0) = b^{-(d-2+\eta)}\, G\left(\frac{x}{b}, t e^{\lambda_t \ell}, 0\right) =$$

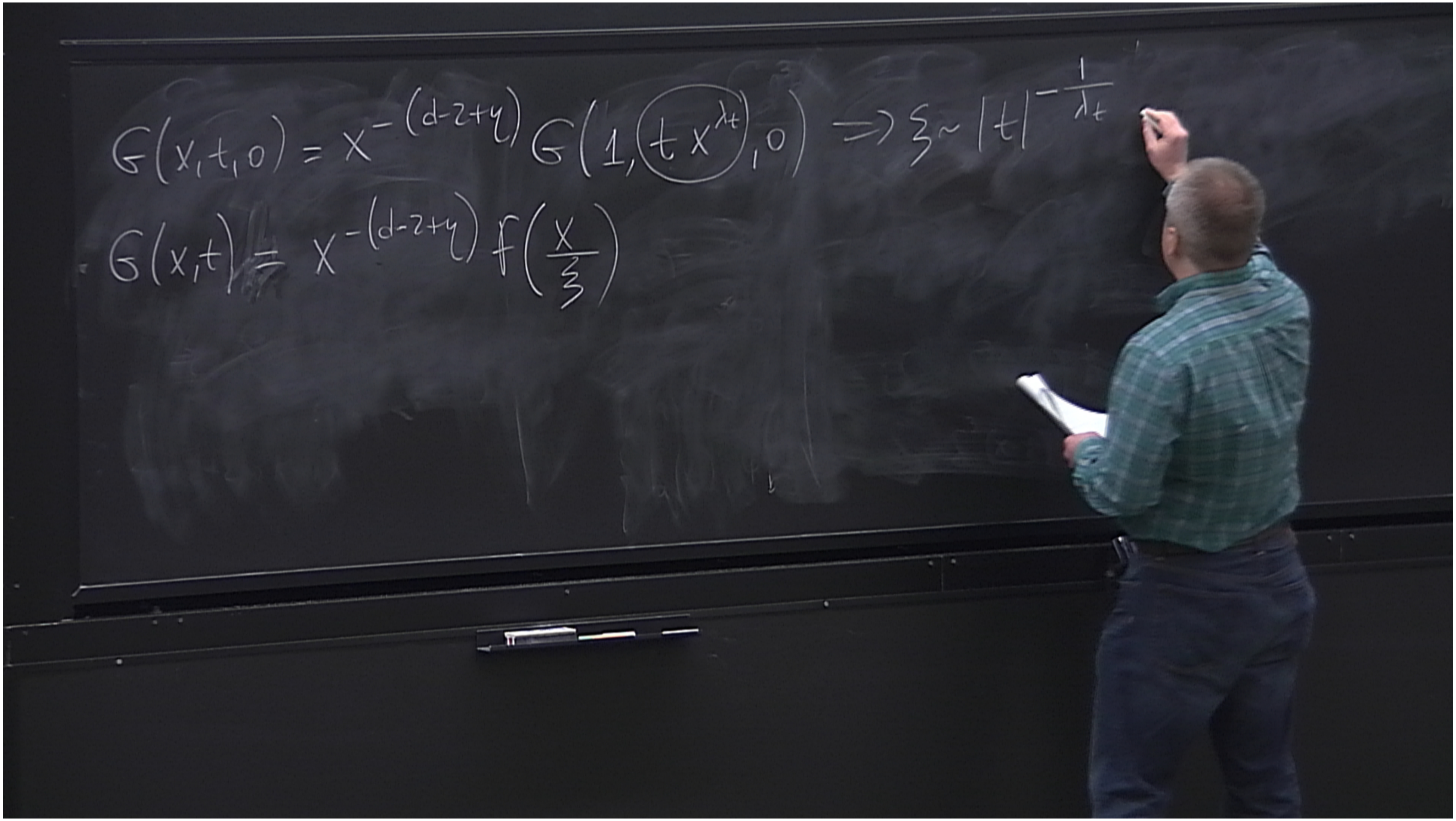
$$= b^{-(d-2+\eta)}\, G\left(\frac{x}{b}, t b^{\lambda_t}, 0\right), \quad b = e^{\ell}$$

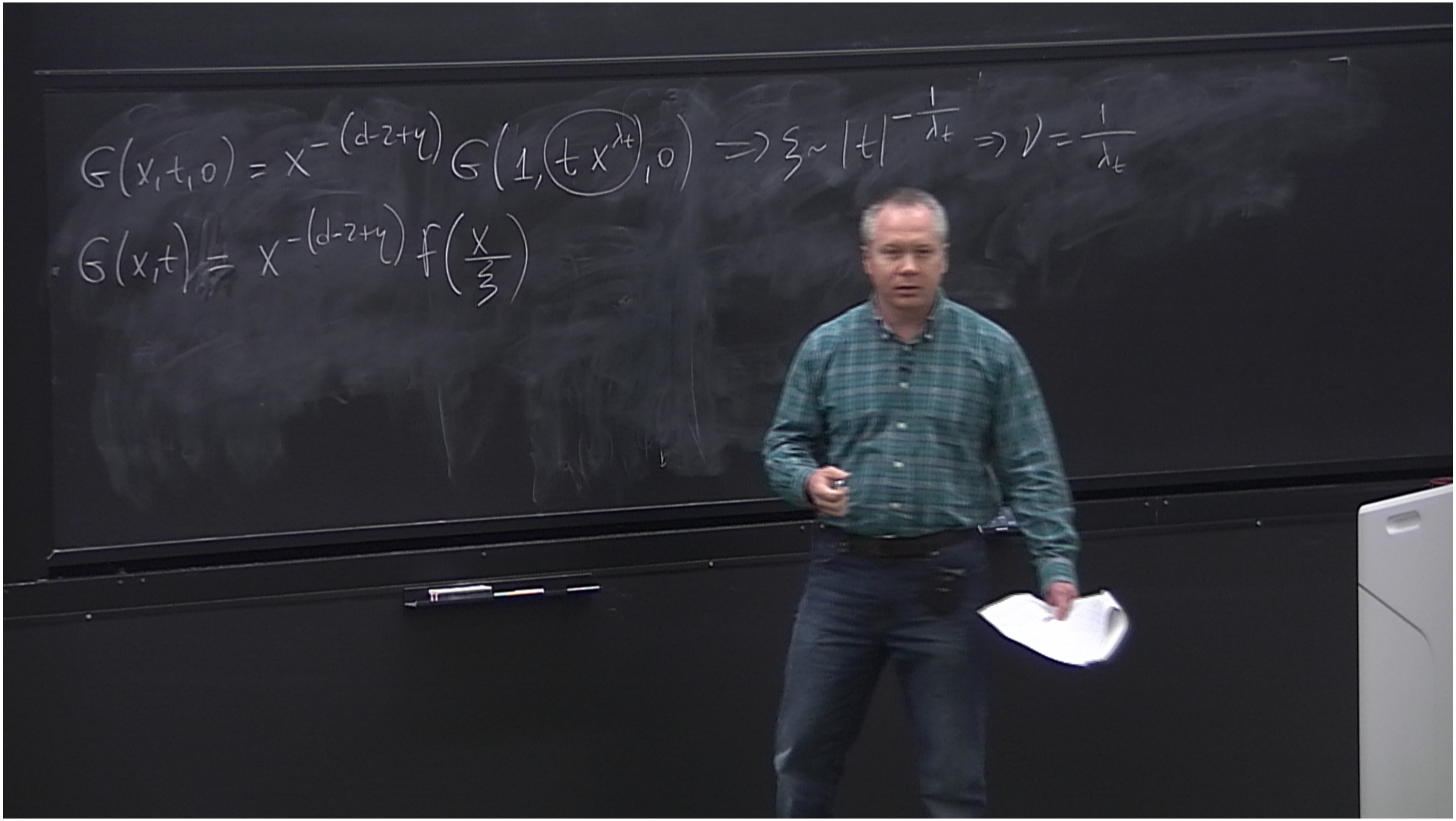

$G(x,t,0) = x^{-(d-2+\eta)} G(1, t x^{\lambda_t}, 0)$
$G(x,t) = x^{-(d-2+\eta)}$

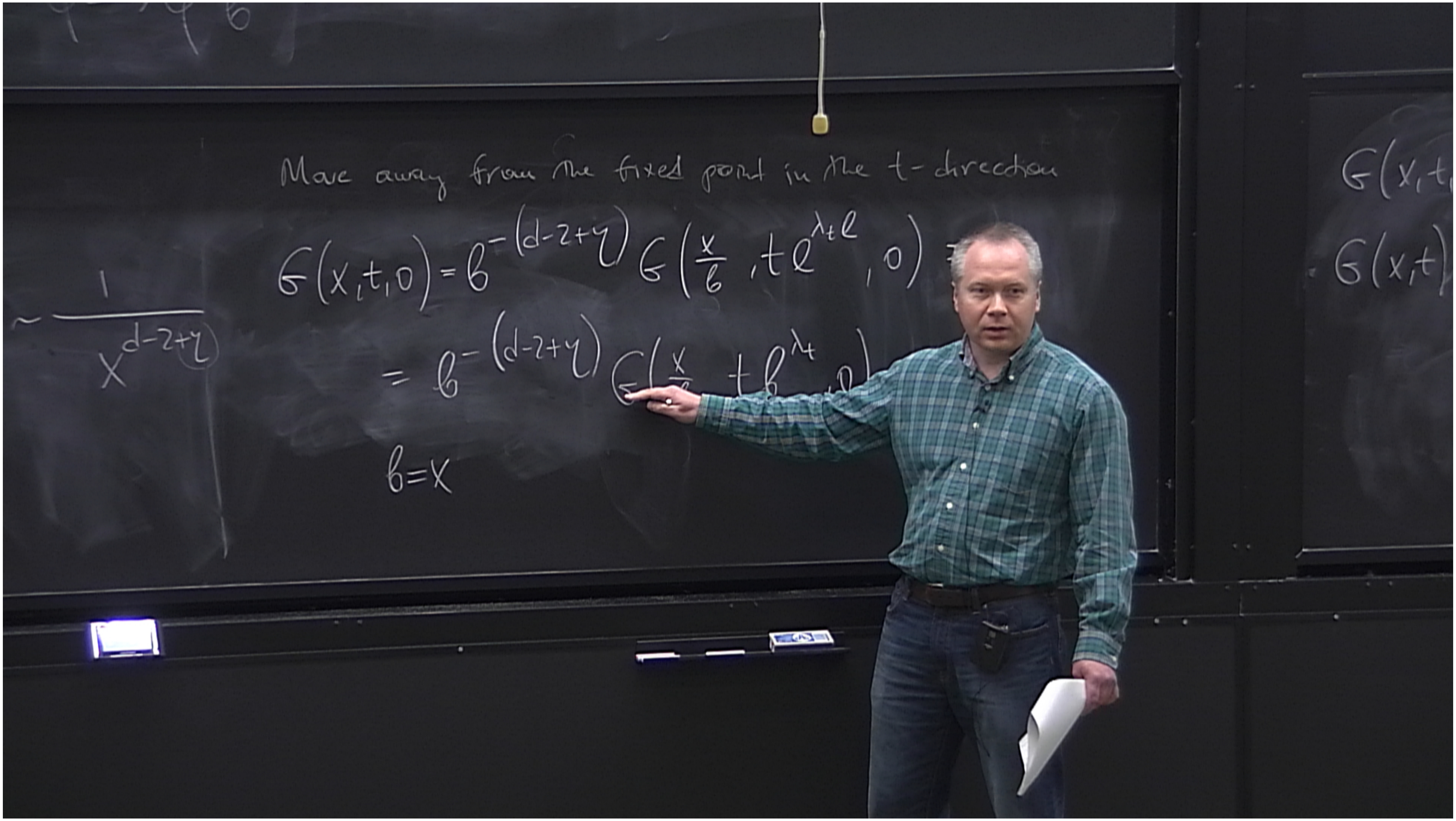
$$G(x,t,0) = x^{-(d-2+\eta)}\, G(1, t\, x^{\lambda_t}, 0)$$

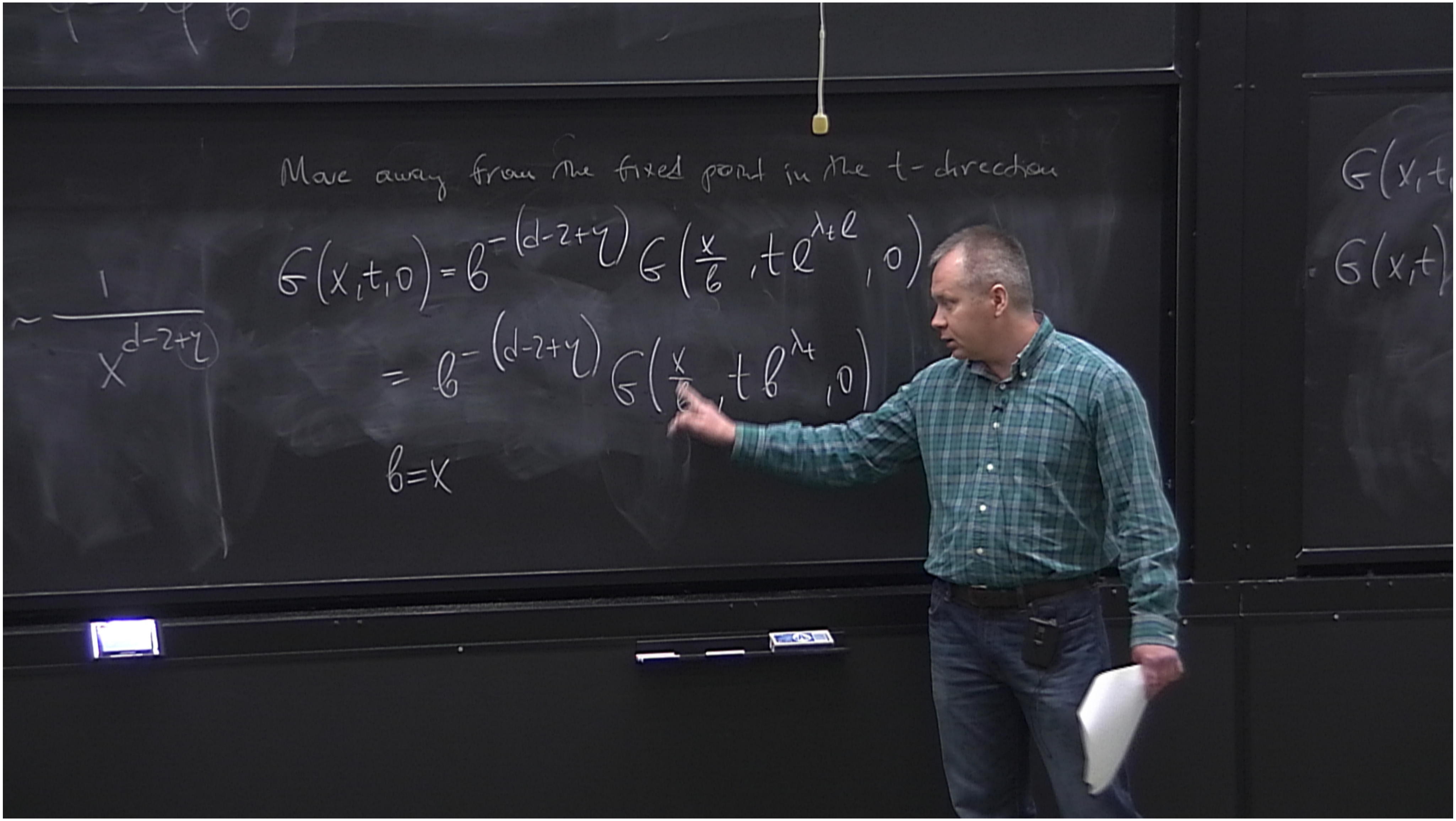
$$G(x,t) = x^{-(d-2+\eta)}\, f\left(\frac{x}{\xi}\right)$$

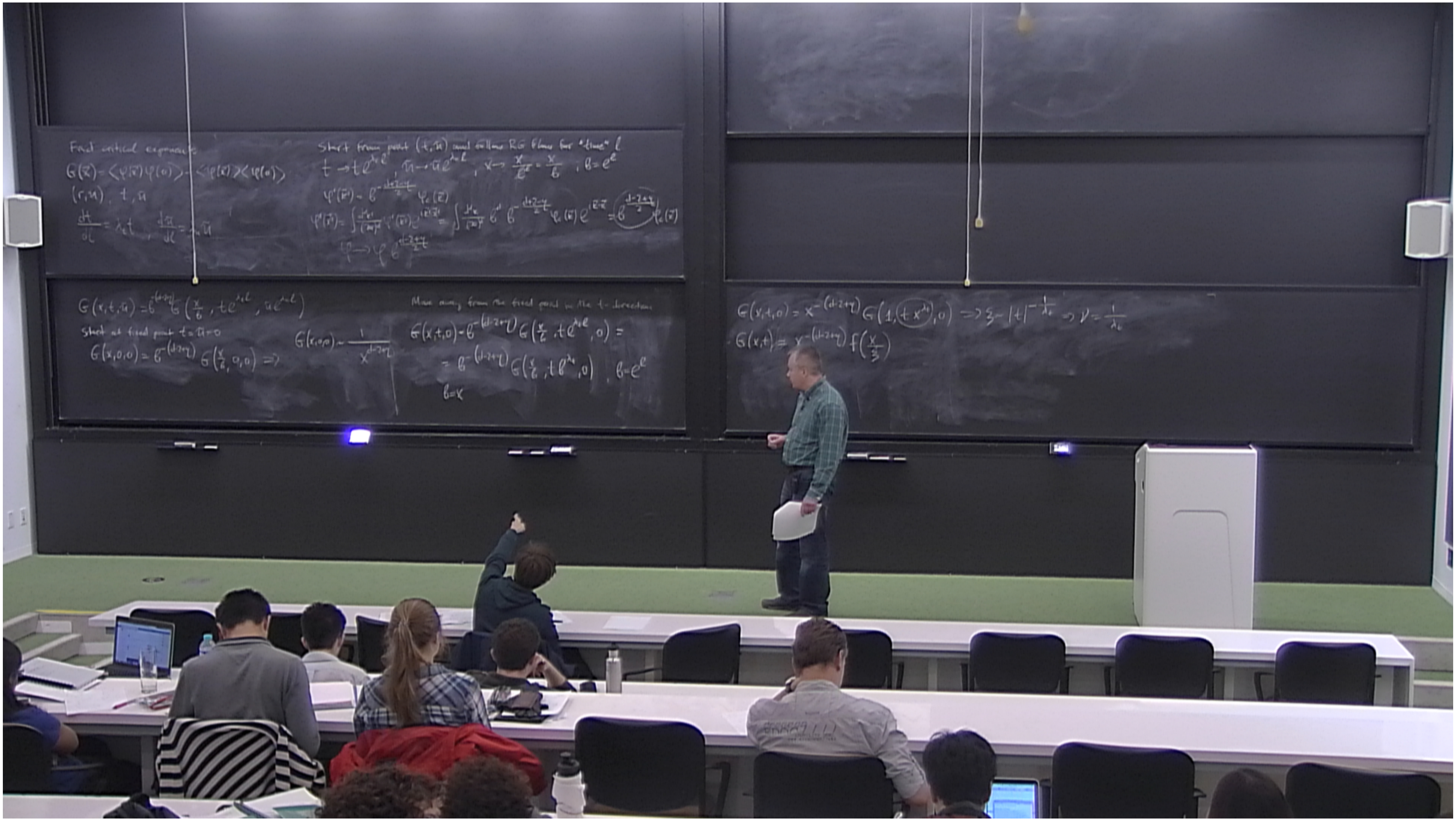

$G(x,t,0) = x^{-(d-2+\eta)} G(1, t x^{\lambda_t}, 0) \Rightarrow \xi \sim |t|^{-\frac{1}{\lambda_t}}$
$G(x,t) = x^{-(d-2+\eta)} f\left(\frac{x}{\xi}\right)$

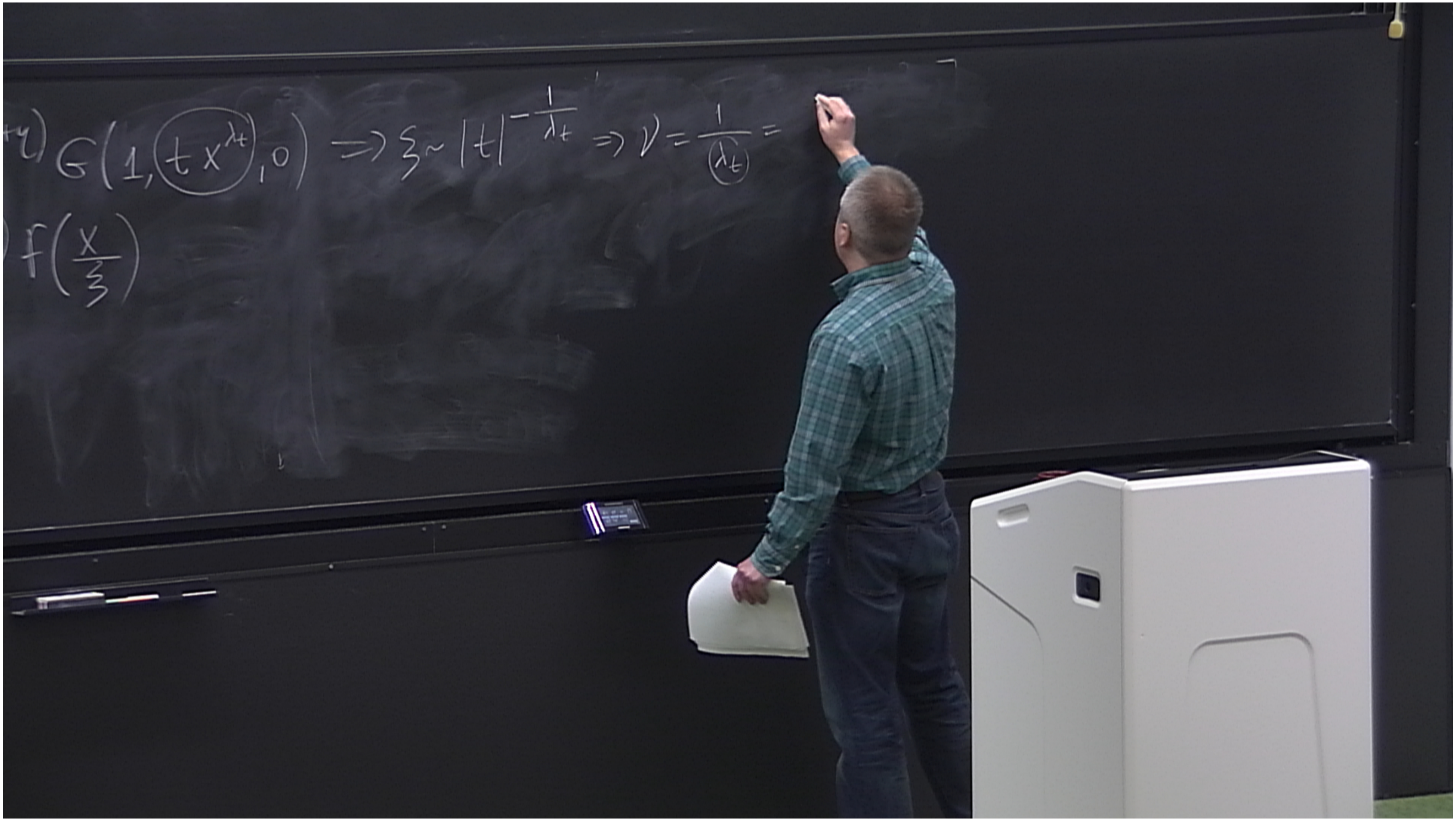

$G(x,t,0) = x^{-(d-2+\eta)} G(1, t x^{\lambda_t}, 0) \Rightarrow \xi \sim |t|^{-\frac{1}{\lambda_t}}$
$G(x,t) = x^{-(d-2+\eta)} f\left(\frac{x}{\xi}\right)$

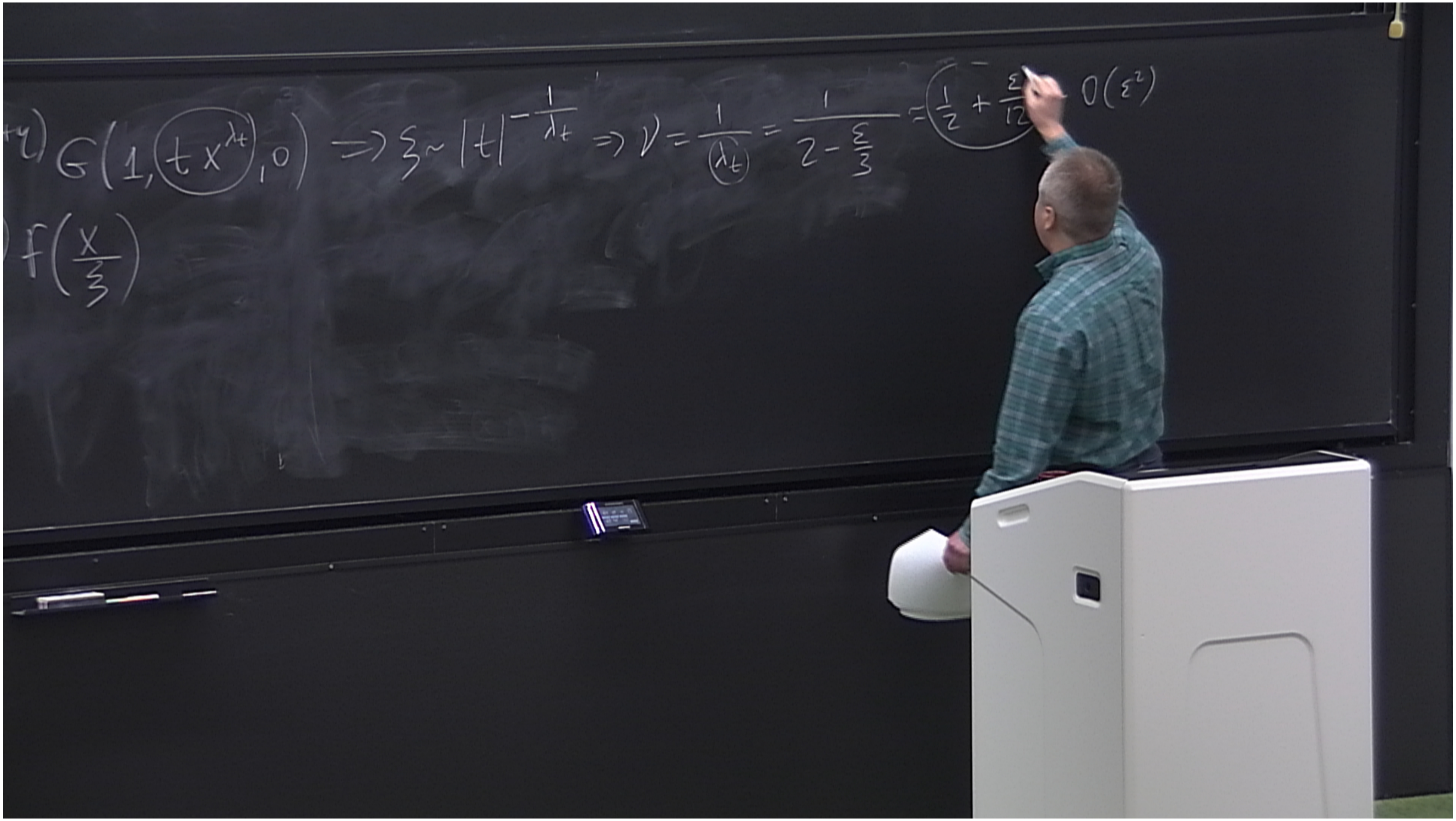

$G(x,t,0) = x^{-(d-2+\eta)} G(1, t x^{\lambda_t}, 0) \Rightarrow \xi \sim |t|^{-\frac{1}{\lambda_t}} \Rightarrow \nu = \frac{1}{\lambda_t}$
$G(x,t) = x^{-(d-2+\eta)} f\left(\frac{x}{\xi}\right)$

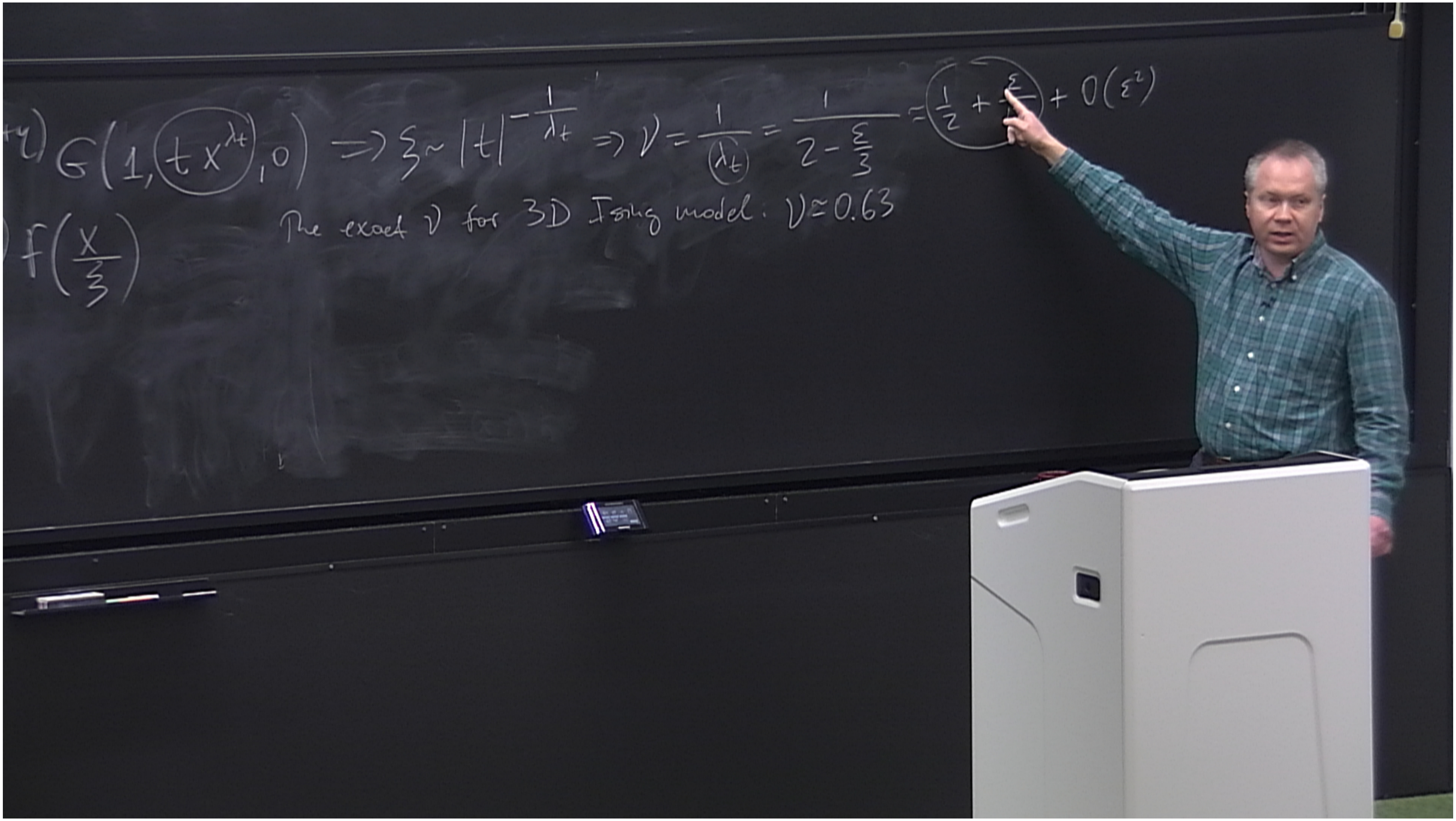

Move away from the fixed point in the t-direction
G(x,t,0) = b^-(d-2+η) G(x/b, t b^λt l, 0)
= b^-(d-2+η) G(x/b, t b^λt, 0)
b=x
~ 1/x^(d-2+η)

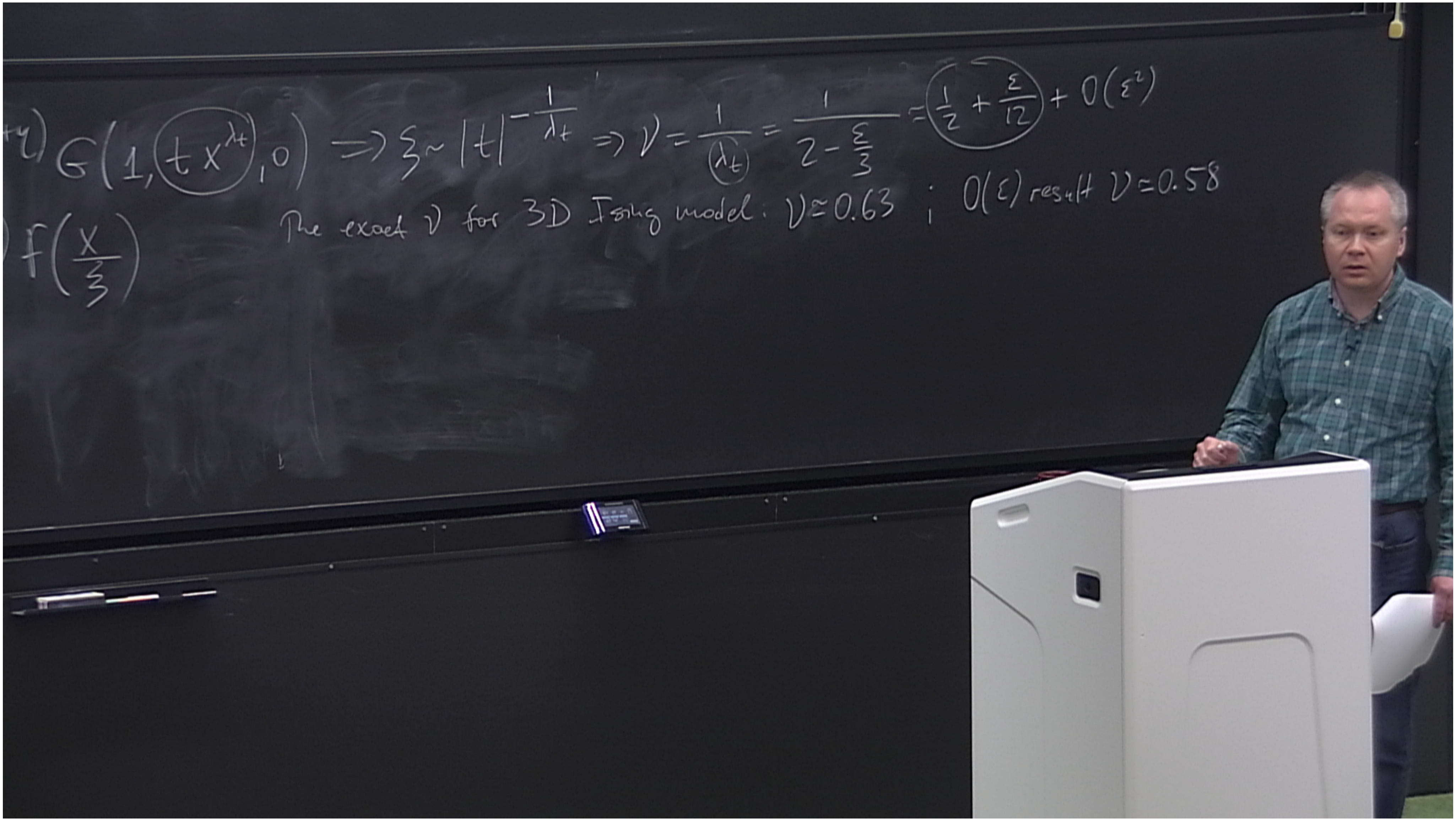

Move away from the fixed point in the t-direction
G(x,t,0) = b^-(d-2+η) G(x/b, t e^λt l, 0)
= b^-(d-2+η) G(x/b, t b^λt, 0)
b=x
~ 1/x^(d-2+η)

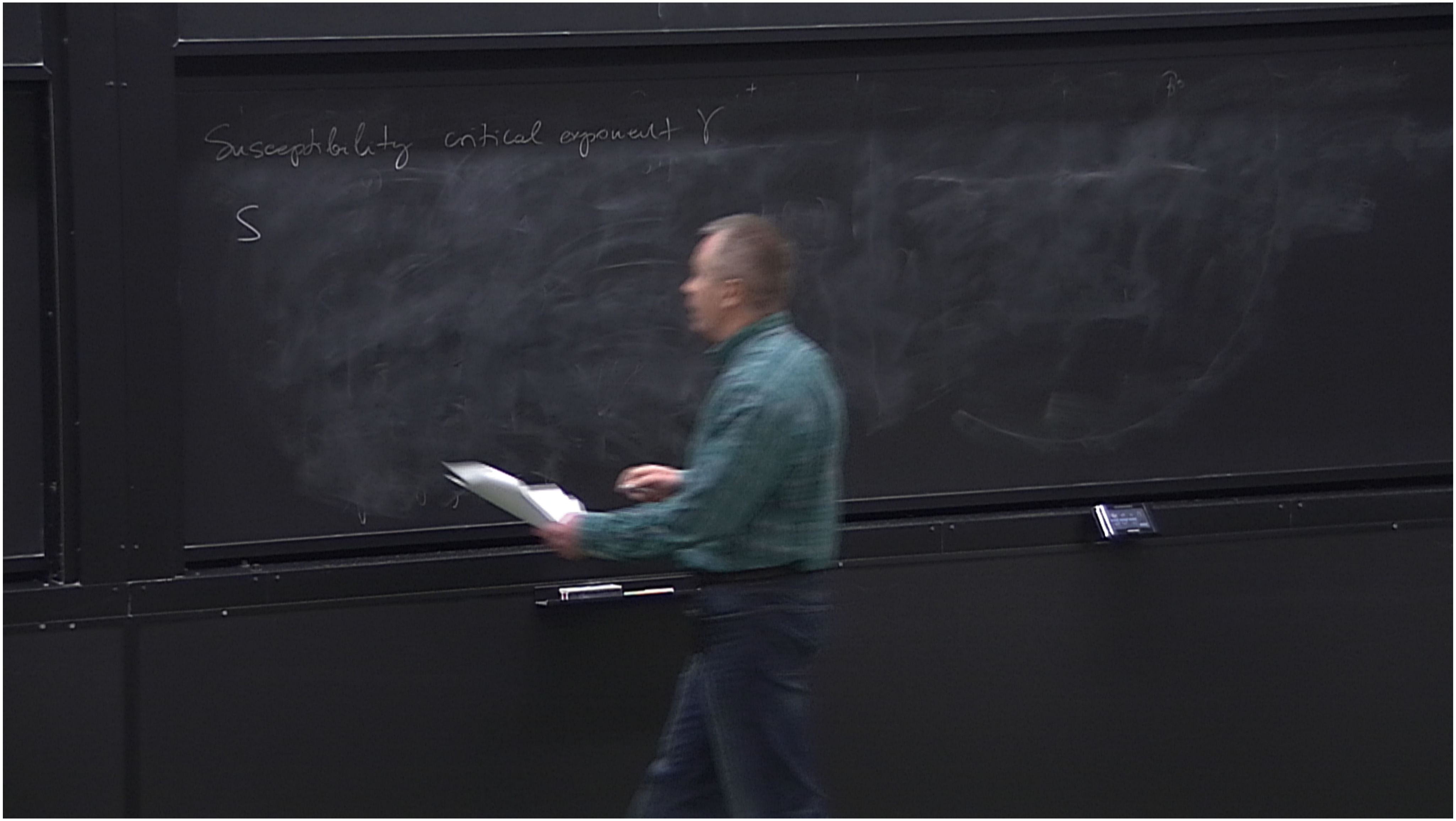

Find critical exponents
Start from point (t, u) and follow RG flow for "time" l
Start at fixed point t=u=0
Move away from the fixed point in the t-direction

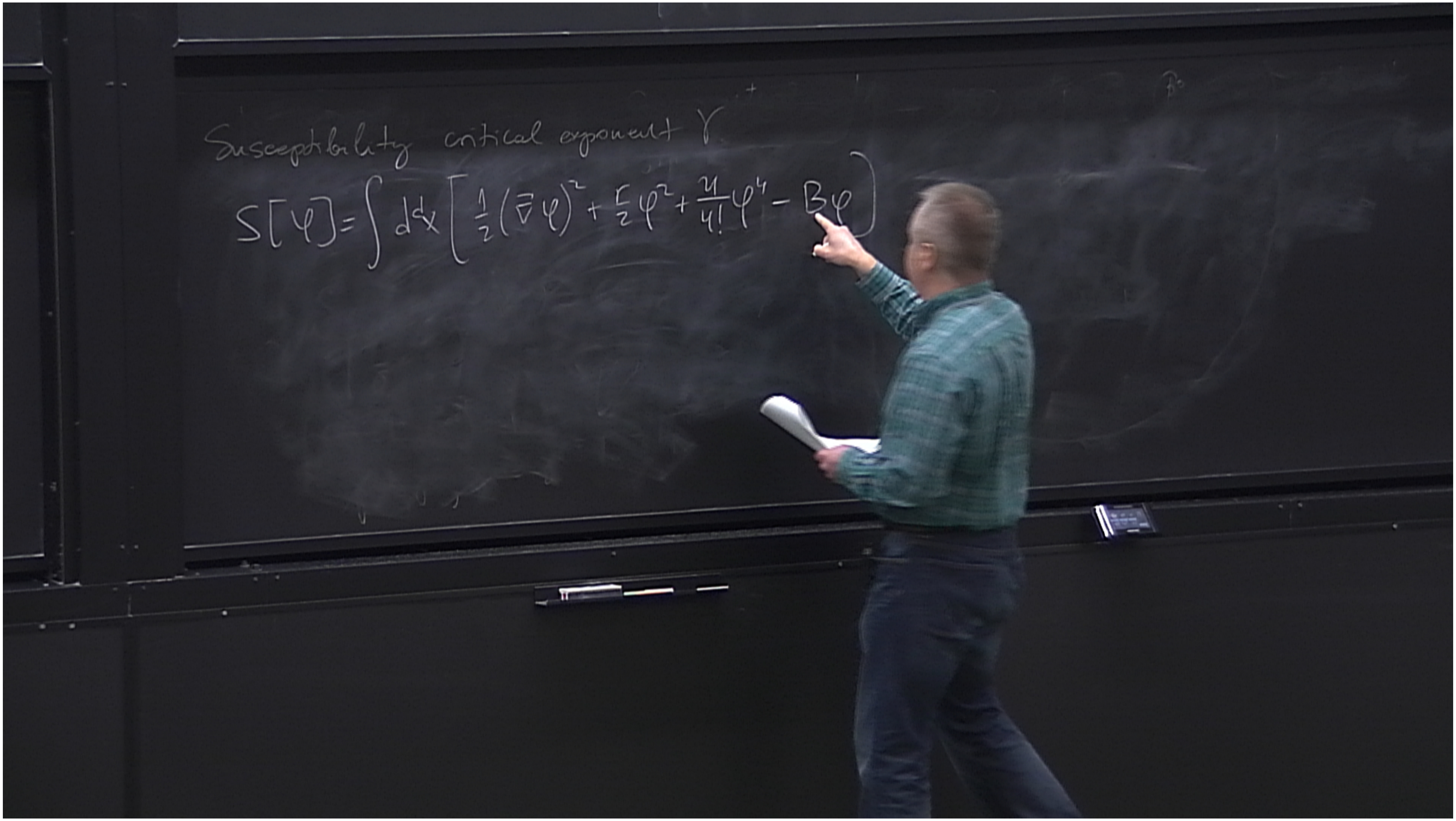

$G(1, t x^{\lambda_t}, 0) \Rightarrow \xi \sim |t|^{-\frac{1}{\lambda_t}} \Rightarrow \nu = \frac{1}{\lambda_t} =$
$f\left(\frac{x}{\xi}\right)$

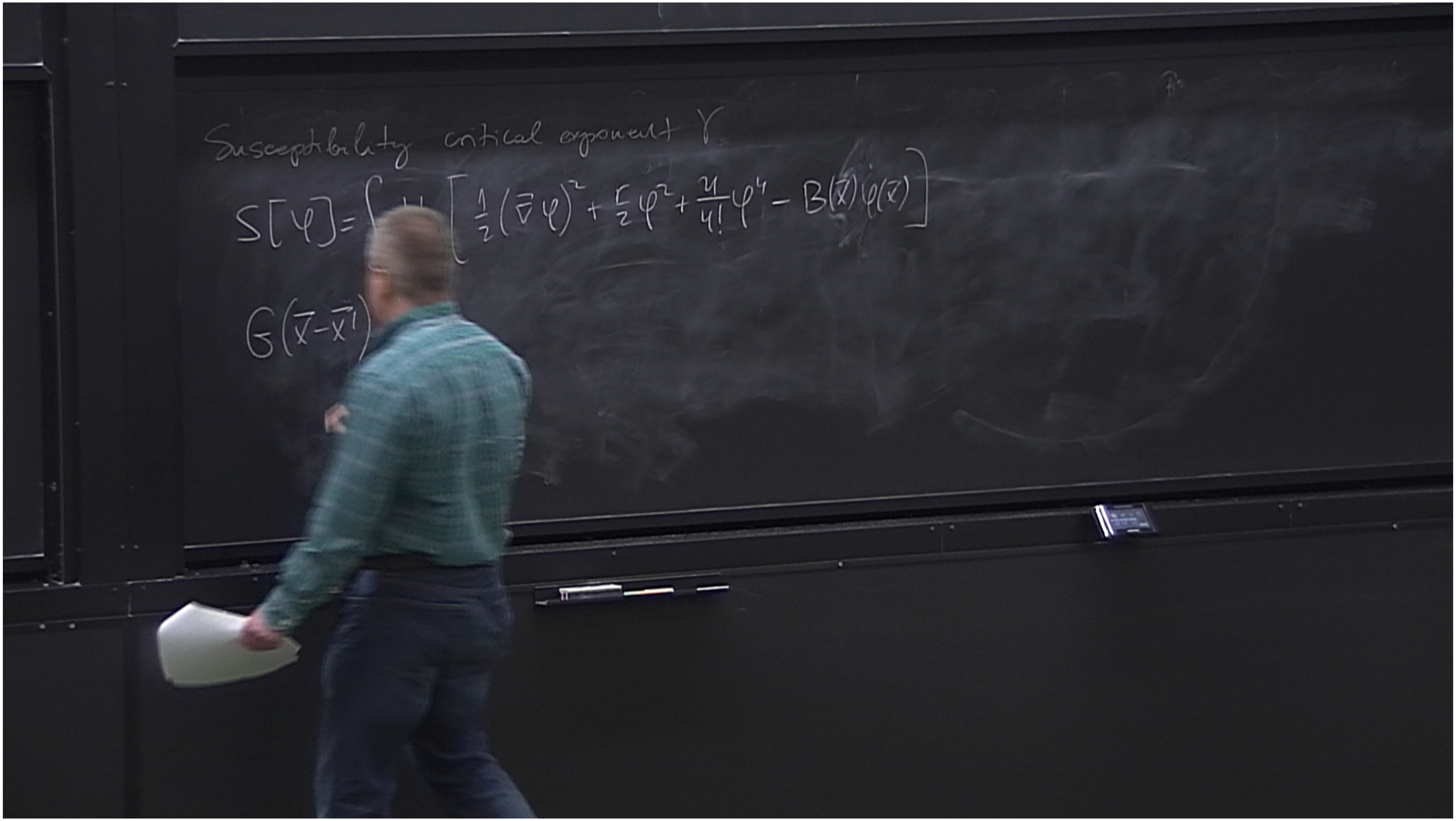

$G(1, t x^{\lambda_t}, 0) \Rightarrow \xi \sim |t|^{-\frac{1}{\lambda_t}} \Rightarrow \nu = \frac{1}{\lambda_t} = \frac{1}{2 - \frac{\varepsilon}{3}} \approx \frac{1}{2} + \frac{\varepsilon}{12} + O(\varepsilon^2)$
$f\left(\frac{x}{\xi}\right)$

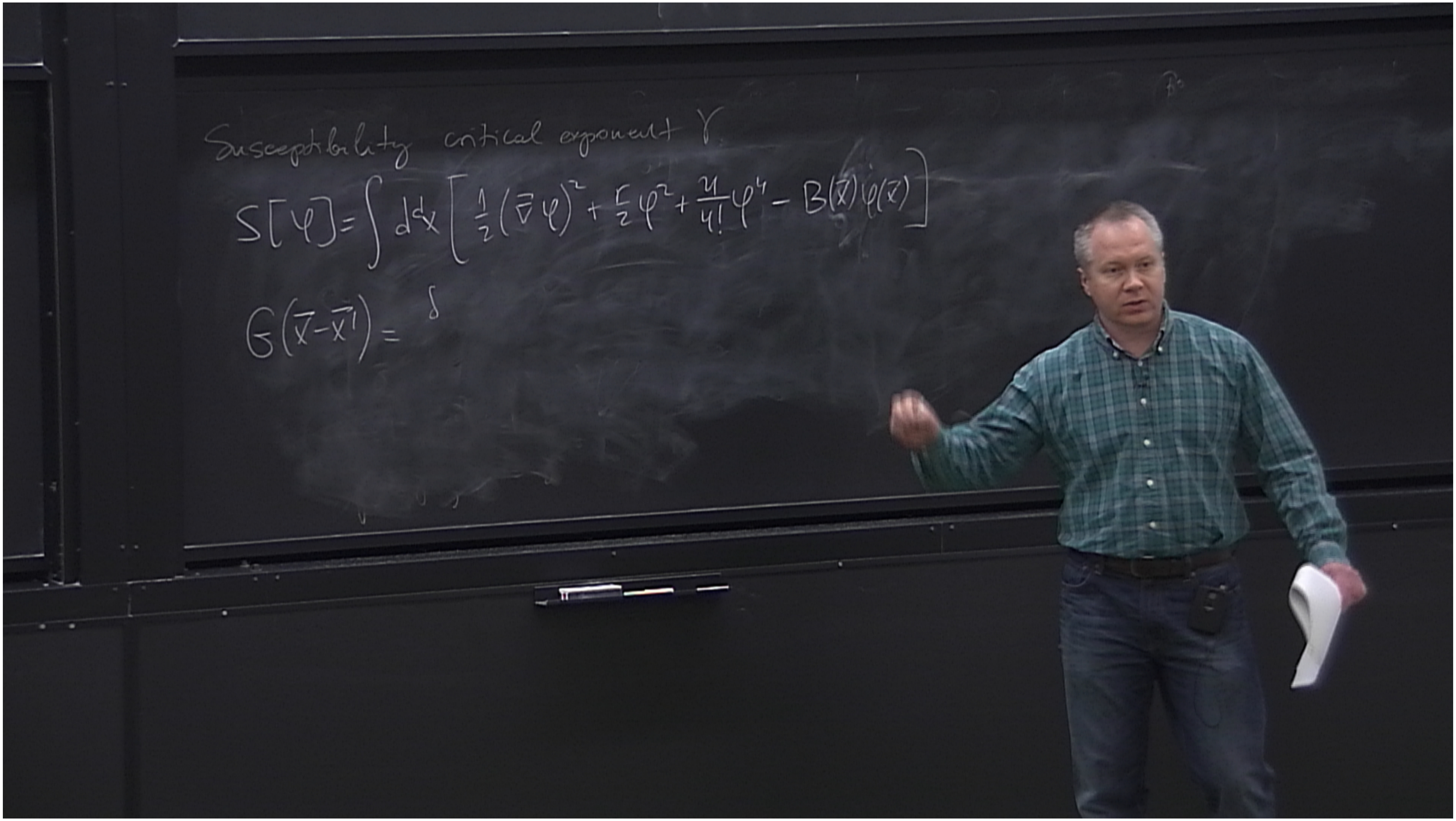

G(1, t x^{λ_t}, 0) => ξ ~ |t|^{-1/λ_t} => ν = 1/λ_t = 1/(2 - ε/3) = 1/2 + ε/12 + O(ε^2)
f(x/ξ)
The exact ν for 3D Ising model: ν ≃ 0.63

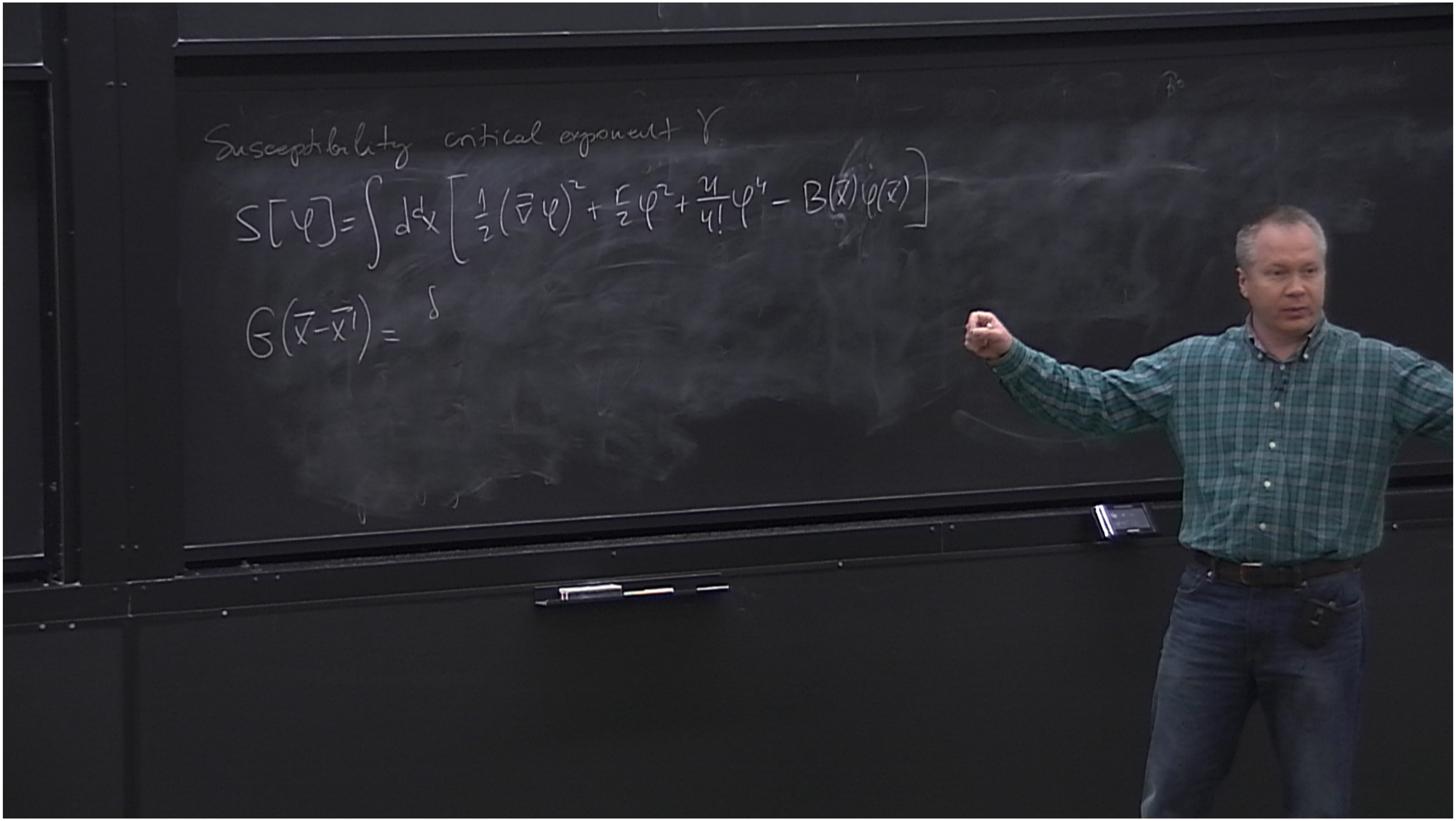

$G(1, t x^{\lambda_t}, 0) \Rightarrow \xi \sim |t|^{-\frac{1}{\lambda_t}} \Rightarrow \nu = \frac{1}{\lambda_t} = \frac{1}{2-\frac{\varepsilon}{3}} \simeq \frac{1}{2} + \frac{\varepsilon}{12} + O(\varepsilon^2)$
$f\left(\frac{x}{\xi}\right)$
The exact ν for 3D Ising model: $\nu \simeq 0.63$; $O(\varepsilon)$ result $\nu \simeq 0.58$

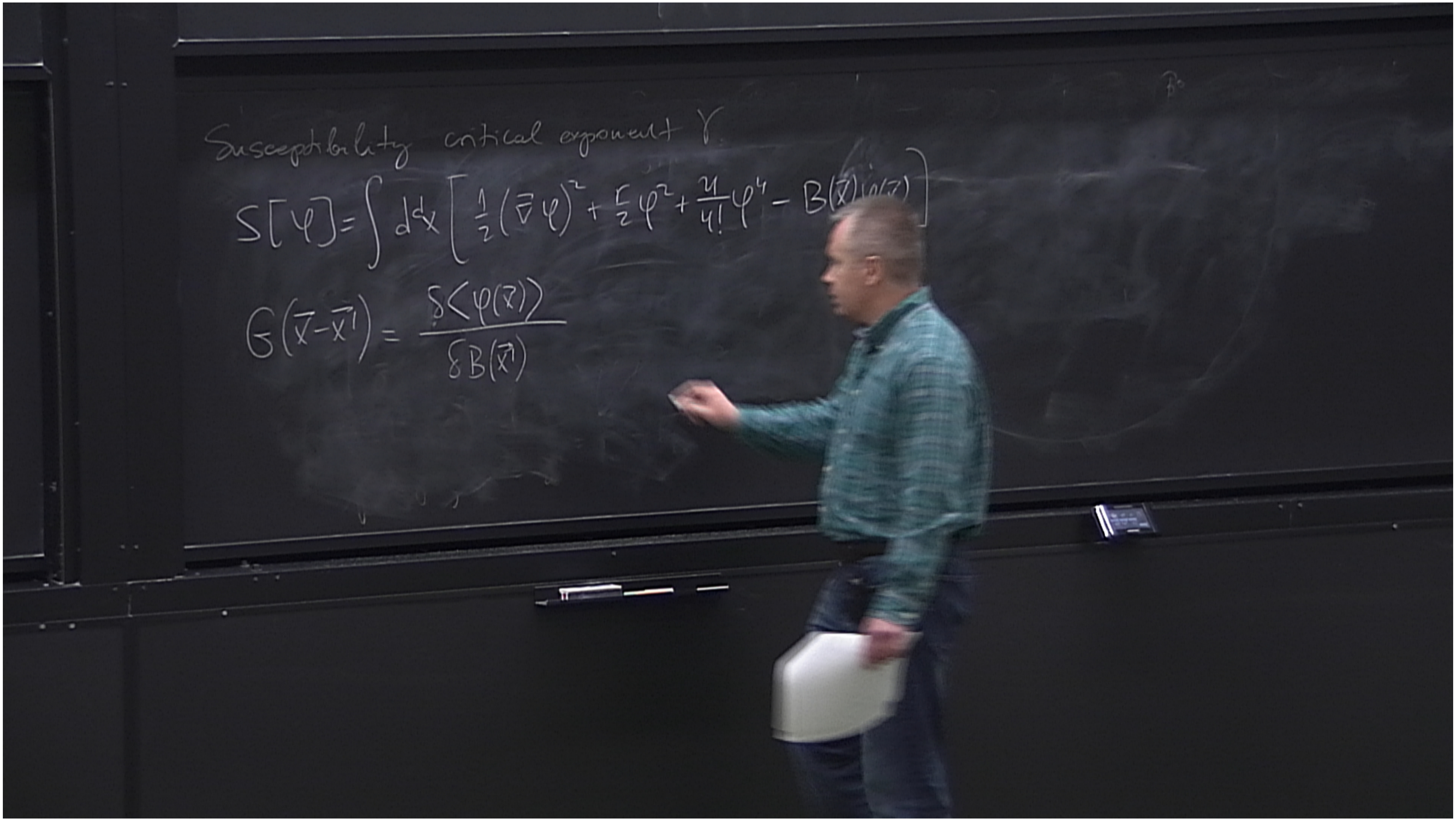

Susceptibility critical exponent γ
S


Susceptibility critical exponent γ
$S[\varphi] = \int d^dx \left[\frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 - B\varphi \right)$


Susceptibility critical exponent γ
S[φ] = ∫ [½(∇φ)² + r/2 φ² + u/4! φ⁴ − B(x)φ(x)]
G(x − x')


Susceptibility critical exponent γ
S[φ] = ∫ d^d x [1/2 (∇φ)^2 + r/2 φ^2 + u/4! φ^4 − B(x) φ(x)]
G(x − x') = δ


Susceptibility critical exponent γ
$S[\varphi] = \int d^dx \left[\frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 - B(\vec{x})\varphi(\vec{x}) \right]$
$G(\vec{x}-\vec{x}') = \delta$

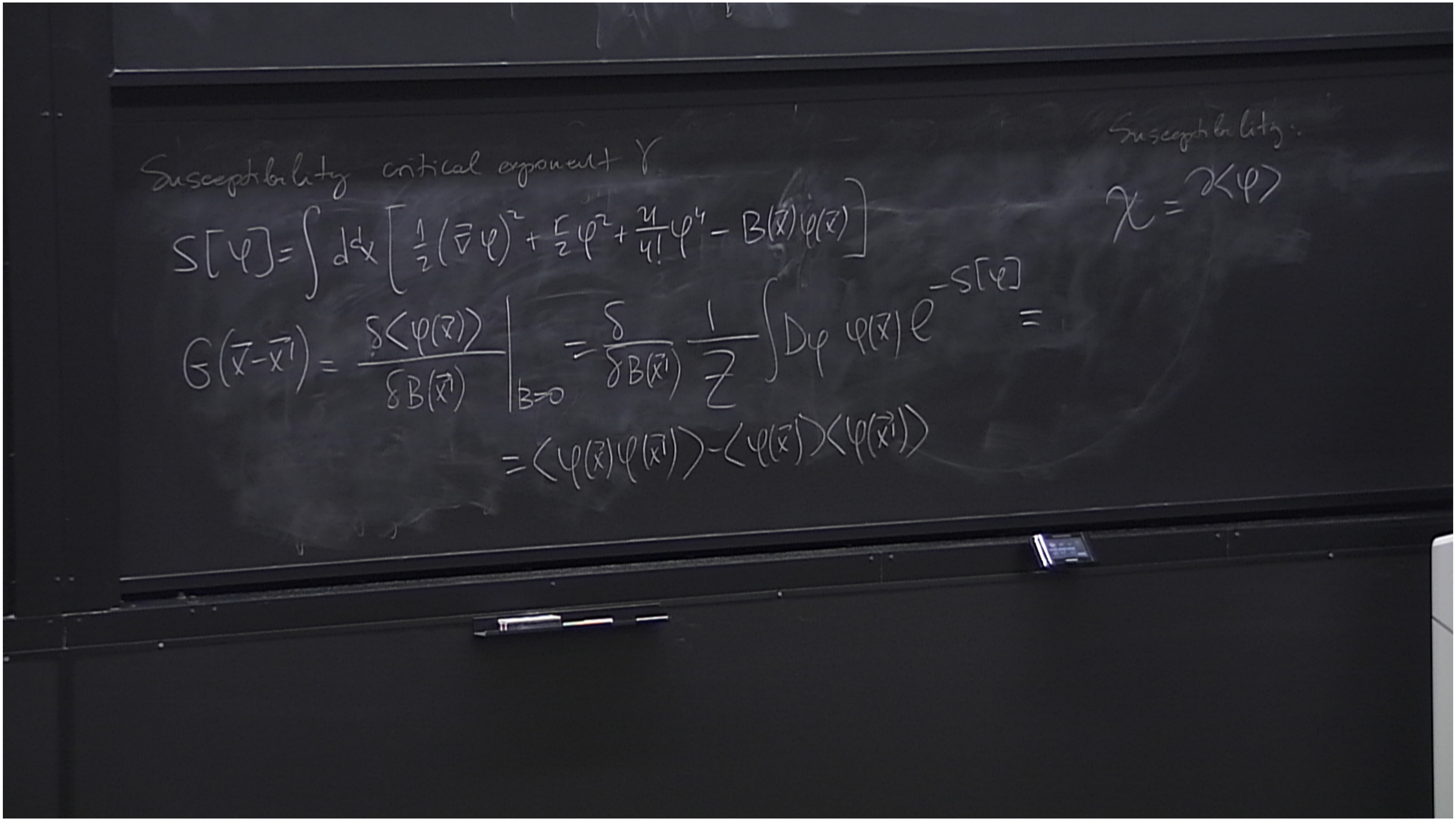

Susceptibility critical exponent γ
$S[\varphi]=\int d^dx\left[\frac{1}{2}(\vec{\nabla}\varphi)^2+\frac{r}{2}\varphi^2+\frac{u}{4!}\varphi^4-B(\vec{x})\varphi(\vec{x})\right]$
$G(\vec{x}-\vec{x}')=\frac{\delta\langle\varphi(\vec{x})\rangle}{\delta B(\vec{x}')}$

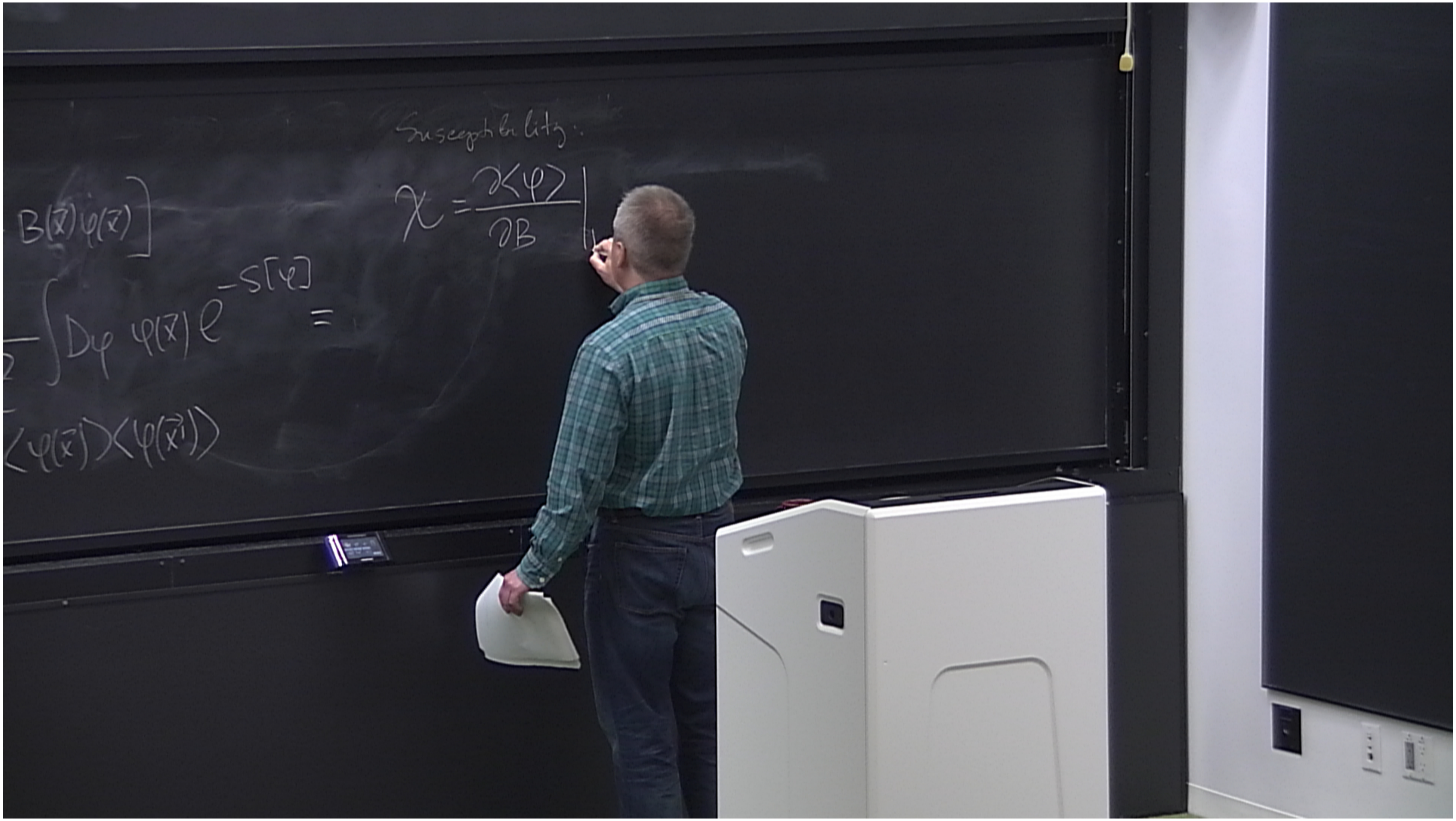
Susceptibility critical exponent γ

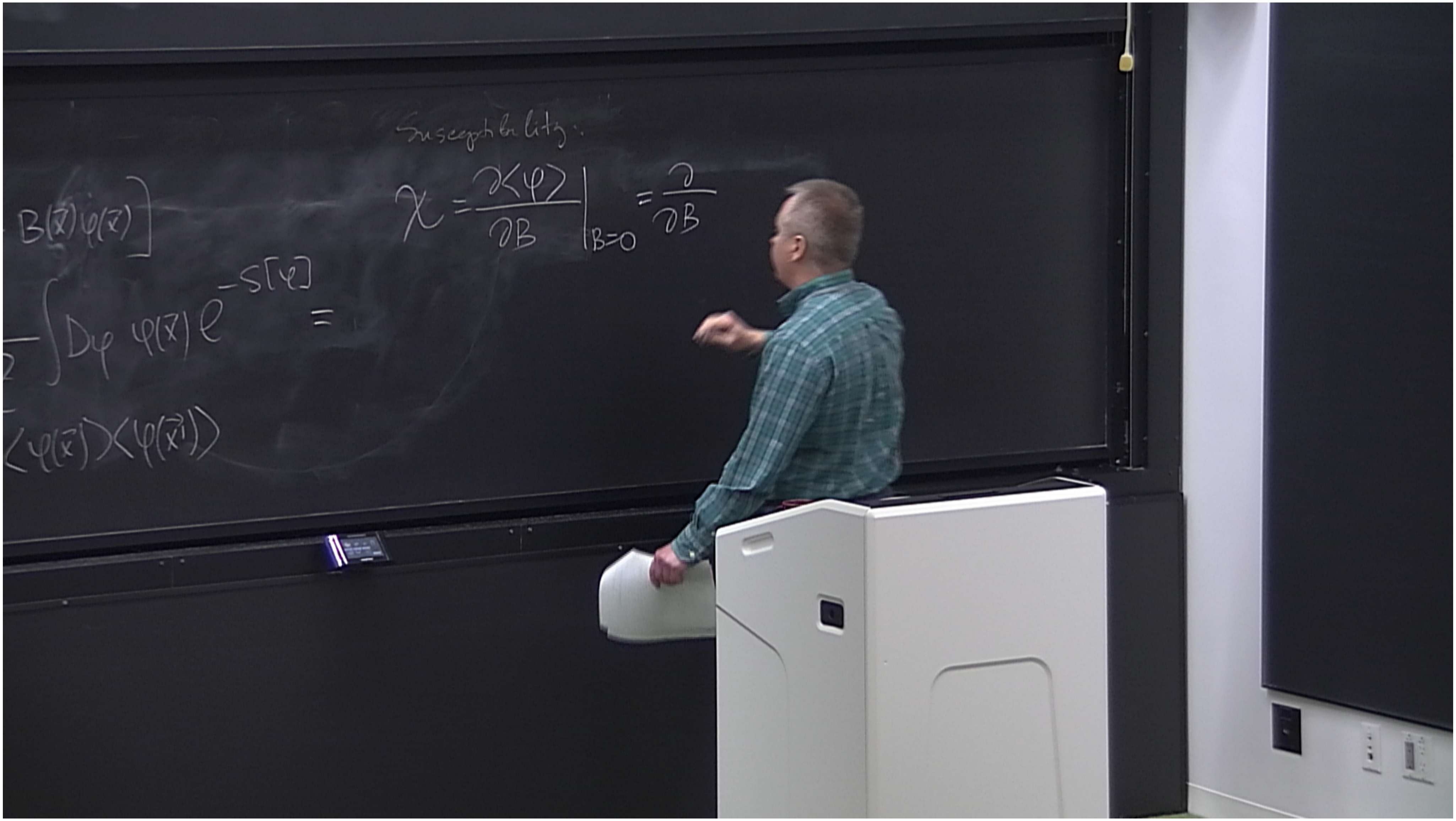
$$S[\varphi] = \int d^d x \left[\frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 - B(\vec{x})\varphi(\vec{x}) \right]$$

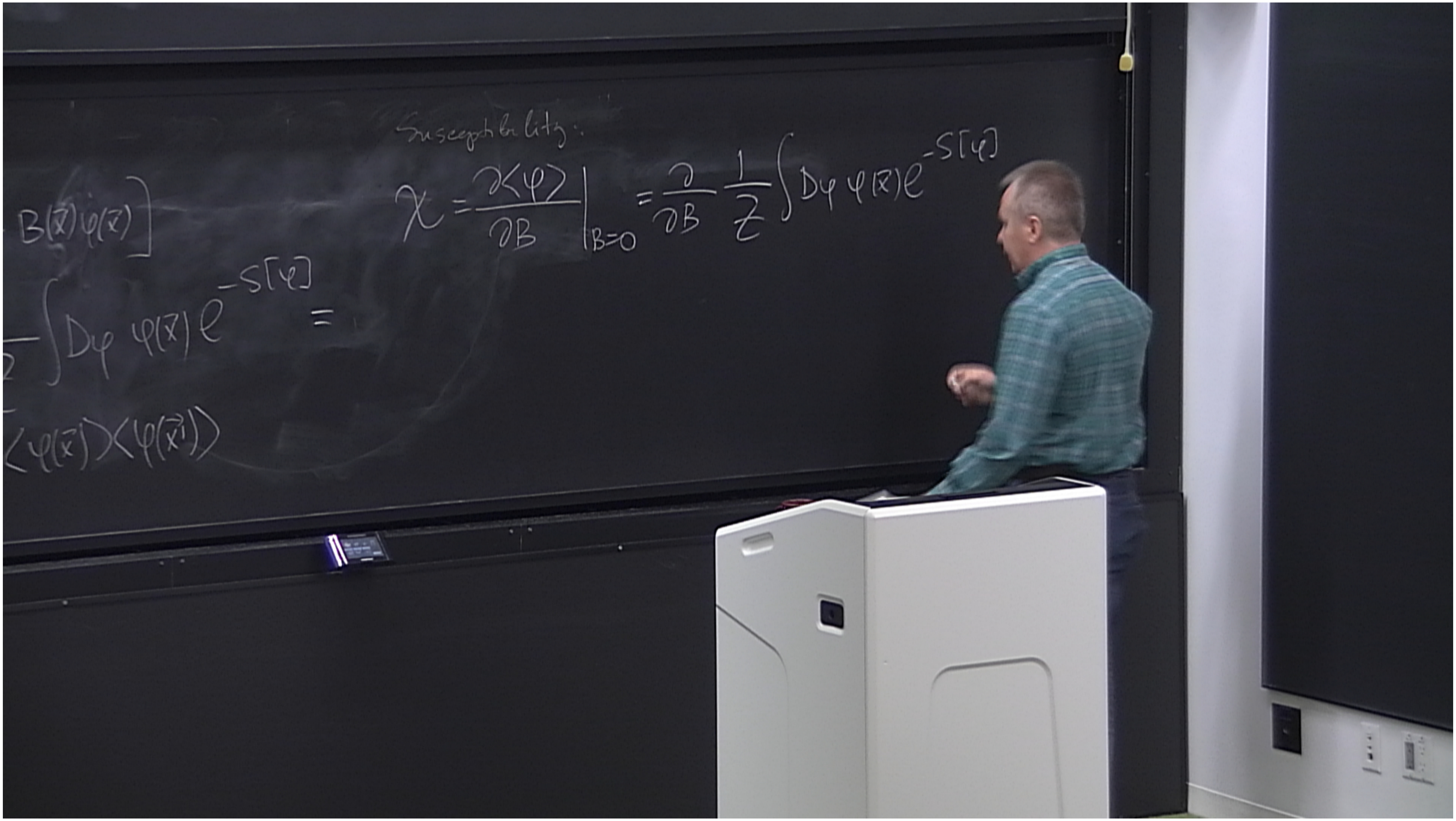
$$G(\vec{x}-\vec{x}') = \left.\frac{\delta\langle\varphi(\vec{x})\rangle}{\delta B(\vec{x}')}\right|_{B=0} = \frac{\delta}{\delta B(\vec{x}')} \frac{1}{Z} \int D\varphi\, \varphi(\vec{x})\, e^{-S[\varphi]} =$$

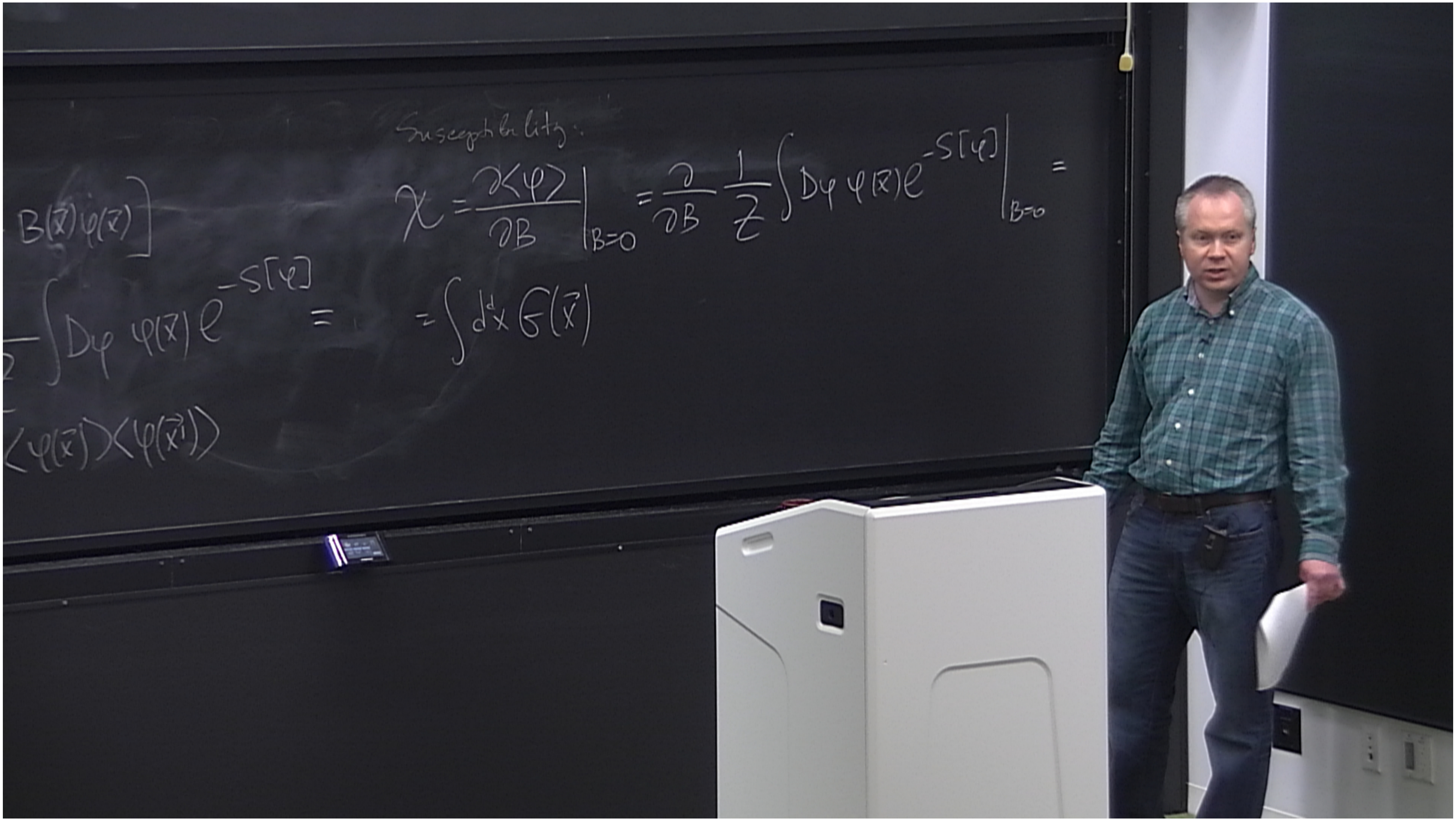
$$= \langle\varphi(\vec{x})\varphi(\vec{x}')\rangle - \langle\varphi(\vec{x})\rangle\langle\varphi(\vec{x}')\rangle$$

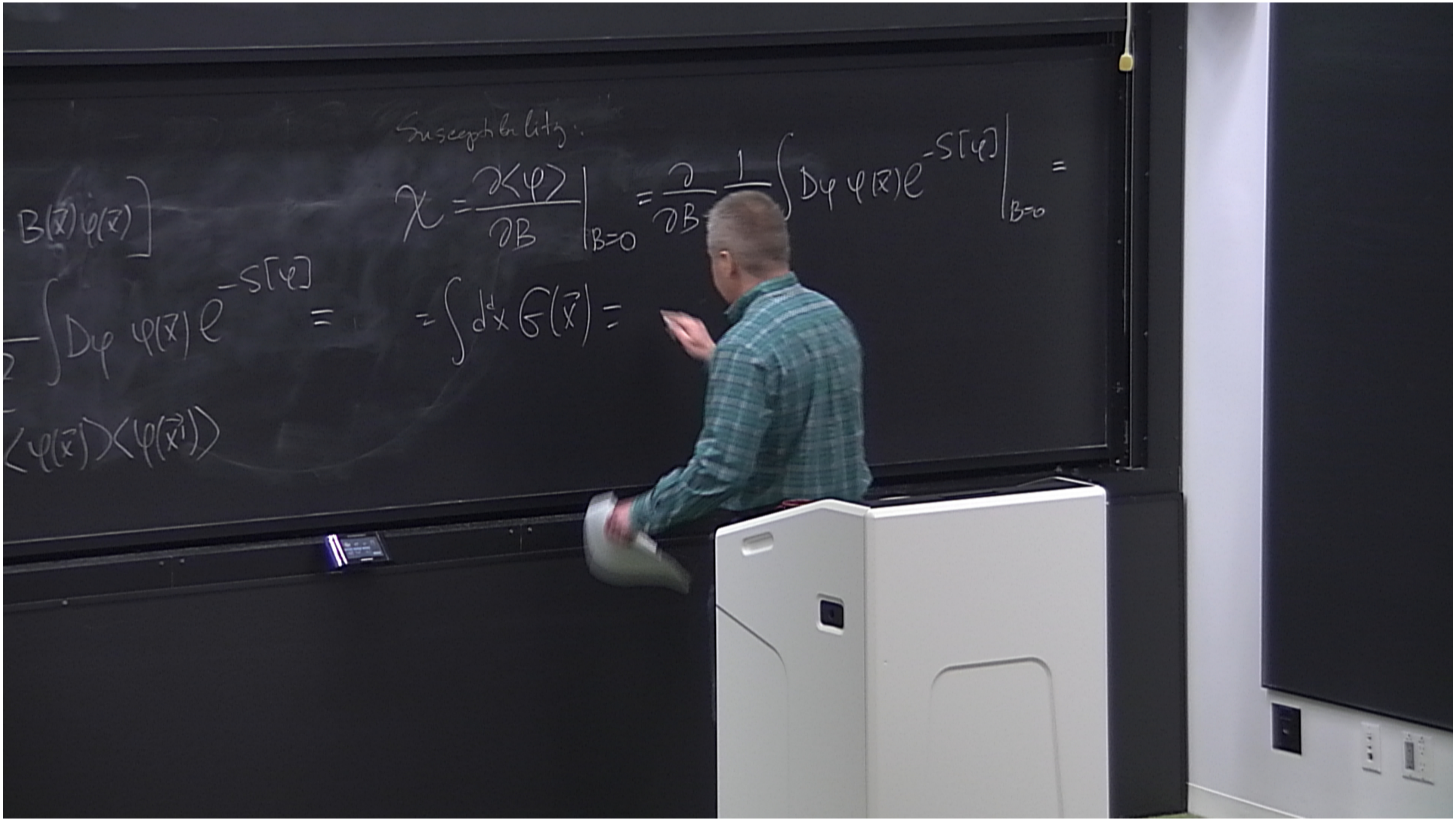

Susceptibility critical exponent γ
$S[\varphi] = \int d^dx \left[\frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 - B(\vec{x})\varphi(\vec{x}) \right]$
$G(\vec{x}-\vec{x}') = \left.\frac{\delta\langle\varphi(\vec{x})\rangle}{\delta B(\vec{x}')}\right|_{B=0} = \frac{\delta}{\delta B(\vec{x}')}\frac{1}{Z}\int D\varphi\, \varphi(\vec{x})\, e^{-S[\varphi]} =$
$= \langle\varphi(\vec{x})\varphi(\vec{x}')\rangle - \langle\varphi(\vec{x})\rangle\langle\varphi(\vec{x}')\rangle$
Susceptibility:
$\chi = \partial\langle\varphi\rangle$

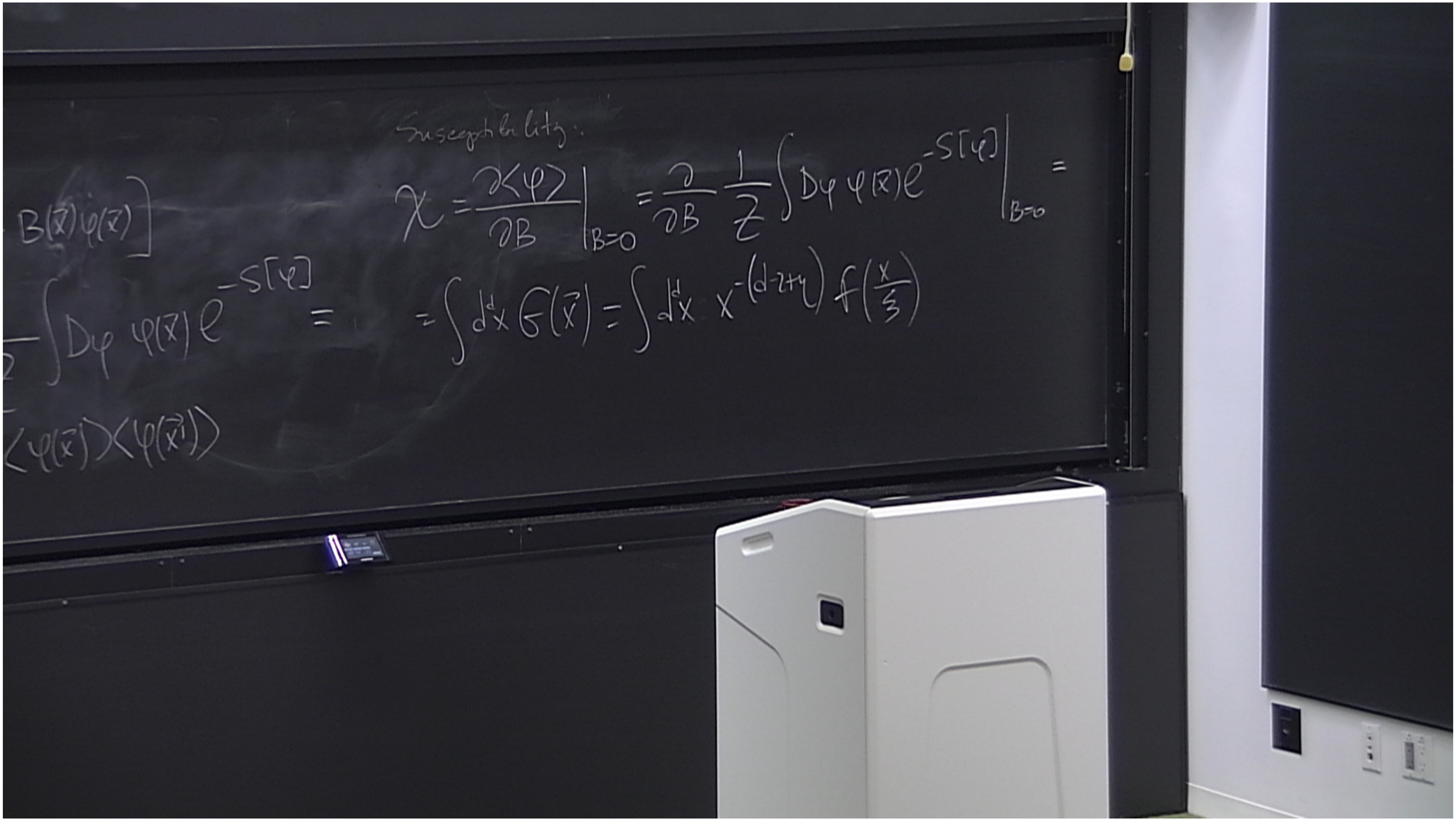

Susceptibility:
$\chi = \frac{\partial\langle\varphi\rangle}{\partial B}\Big|$
$B(\vec{x})\,\varphi(\vec{x})]$
$\int D\varphi\ \varphi(\vec{x})\, e^{-S[\varphi]} =$
$\langle\varphi(\vec{x})\rangle\langle\varphi(\vec{x}')\rangle$

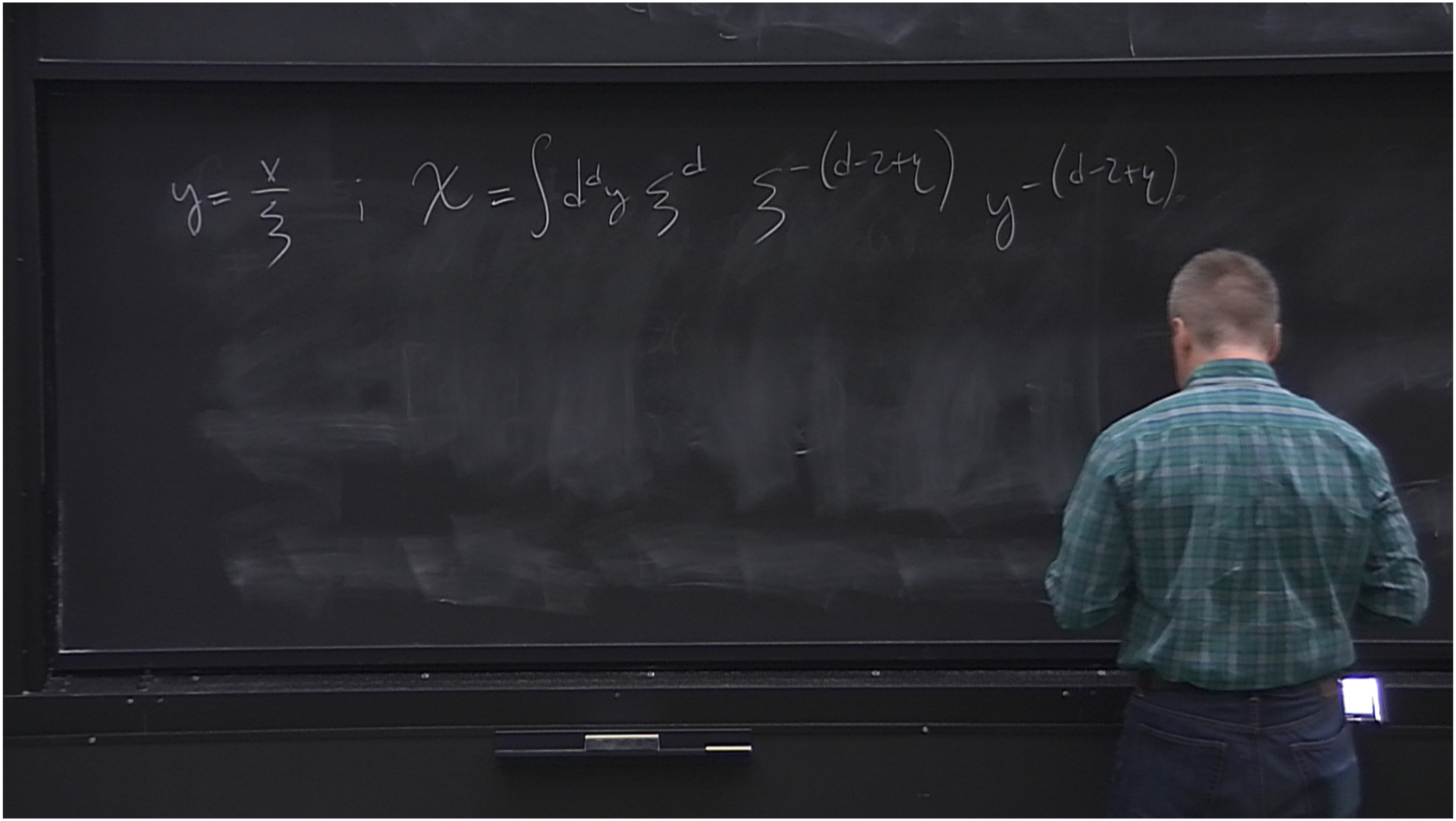

Susceptibility:
$\chi = \frac{\partial\langle\varphi\rangle}{\partial B}\Big|_{B=0} = \frac{\partial}{\partial B}$
$B(\vec{x})\varphi(\vec{x})]$
$\int D\varphi\, \varphi(\vec{x})\, e^{-S[\varphi]} =$
$\langle\varphi(\vec{x})\rangle\langle\varphi(\vec{x}')\rangle$

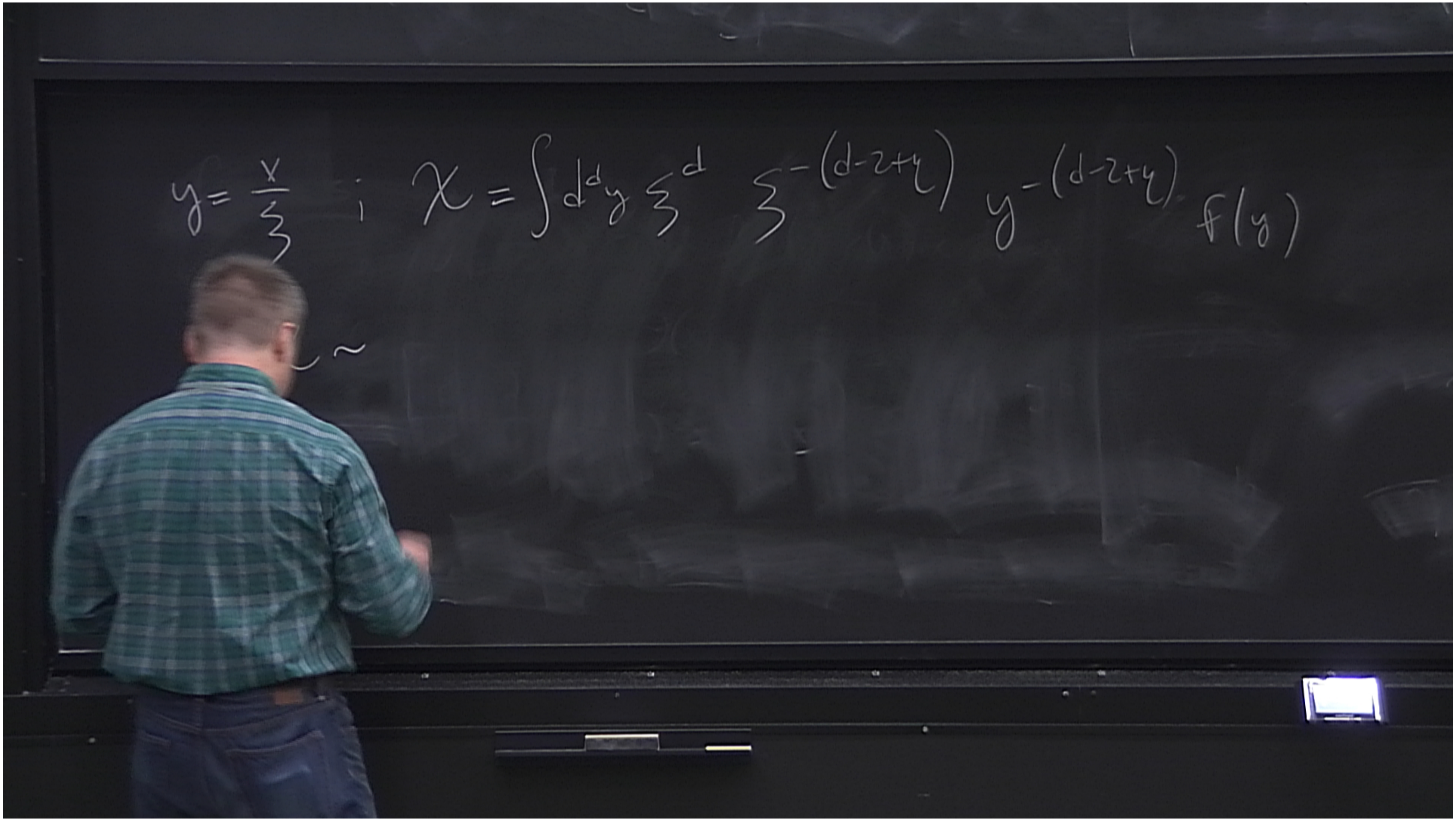

Susceptibility:
χ = ∂⟨φ⟩/∂B |B=0 = ∂/∂B 1/Z ∫ Dφ φ(x) e^−S[φ]
B(x) φ(x)]
∫ Dφ φ(x) e^−S[φ] =
⟨φ(x)⟩⟨φ(x')⟩


Susceptibility:
$\chi = \frac{\partial\langle\varphi\rangle}{\partial B}\Big|_{B=0} = \frac{\partial}{\partial B}\frac{1}{Z}\int D\varphi\,\varphi(\vec{x})\,e^{-S[\varphi]}\Big|_{B=0} =$
$= \int d^dx\, G(\vec{x})$
$B(\vec{x})\varphi(\vec{x})]$
$\int D\varphi\,\varphi(\vec{x})\,e^{-S[\varphi]} =$
$\langle\varphi(\vec{x})\rangle\langle\varphi(\vec{x}')\rangle$


Susceptibility:
$\chi = \frac{\partial \langle \varphi \rangle}{\partial B}\Big|_{B=0} = \frac{\partial}{\partial B} \frac{1}{\ } \int D\varphi\, \varphi(\vec{x})\, e^{-S[\varphi]} \Big|_{B=0} =$
$= \int d^d x\, G(\vec{x}) =$
$B(\vec{x})\varphi(\vec{x})]$
$\int D\varphi\, \varphi(\vec{x})\, e^{-S[\varphi]} =$
$\langle \varphi(\vec{x}) \rangle \langle \varphi(\vec{x}') \rangle$


Susceptibility:
$\chi = \frac{\partial\langle\varphi\rangle}{\partial B}\Big|_{B=0} = \frac{\partial}{\partial B}\frac{1}{Z}\int D\varphi\, \varphi(\vec{x})\, e^{-S[\varphi]}\Big|_{B=0} =$
$= \int d^d x\, G(\vec{x}) = \int d^d x\, x^{-(d-2+\eta)} f\left(\frac{x}{\xi}\right)$
$B(\vec{x})\varphi(\vec{x})]$
$\int D\varphi\, \varphi(\vec{x})\, e^{-S[\varphi]} =$
$\langle\varphi(\vec{x})\rangle\langle\varphi(\vec{x}')\rangle$

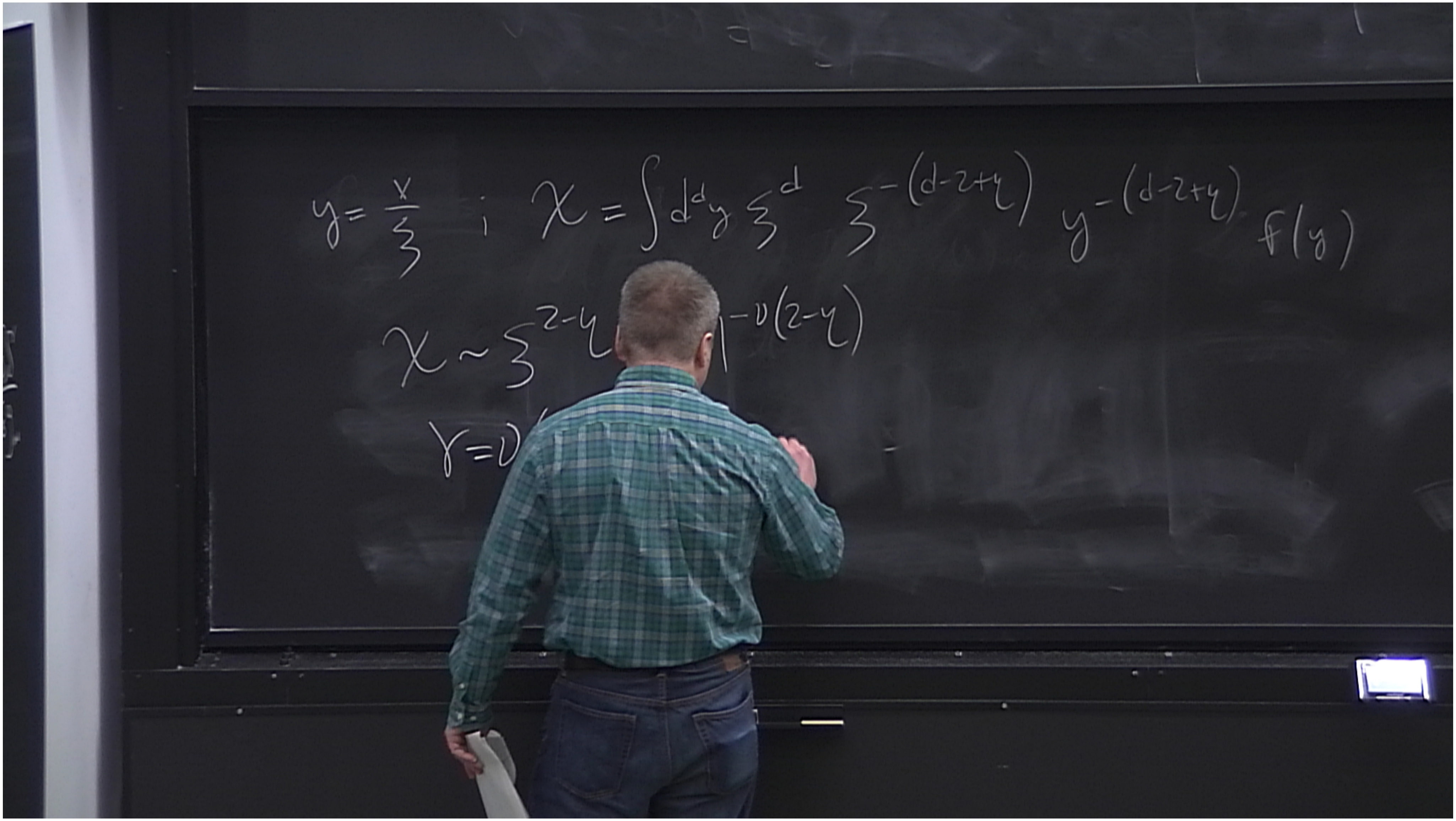

$y = \frac{x}{\xi}$; $x = \int d^d y \, \xi^d \, \xi^{-(d-2+\eta)} \, y^{-(d-2+\eta)}$


$y = \frac{x}{\xi}$; $\chi = \int d^d y \, \xi^d \, \xi^{-(d-2+\eta)} \, y^{-(d-2+\eta)} \, f(y)$

$$y = \frac{x}{\xi} \; ; \quad \chi = \int d^d y \, \xi^d \, \xi^{-(d-2+\eta)} \, y^{-(d-2+\eta)} \, f(y)$$

$$\chi \sim \xi^{2-\eta} \sim |t|^{-\nu(2-\eta)}$$

$$\gamma = \nu(2-\eta)$$



$y = \frac{x}{\xi}$; $\chi = \int d^d y \, \xi^d \, \xi^{-(d-2+\eta)} \, y^{-(d-2+\eta)} \, f(y)$
$\chi \sim \xi^{2-\eta}$
$|^{-\nu(2-\eta)}$
$\gamma = \nu($

$$y = \frac{x}{\xi} \; ; \; \chi = \int d^d y \, \xi^d \, \xi^{-(d-2+\eta)} \, y^{-(d-2+\eta)} \, f(y)$$

$$\chi \sim \xi^{2-\eta} \sim |t|^{-\nu(2-\eta)}$$

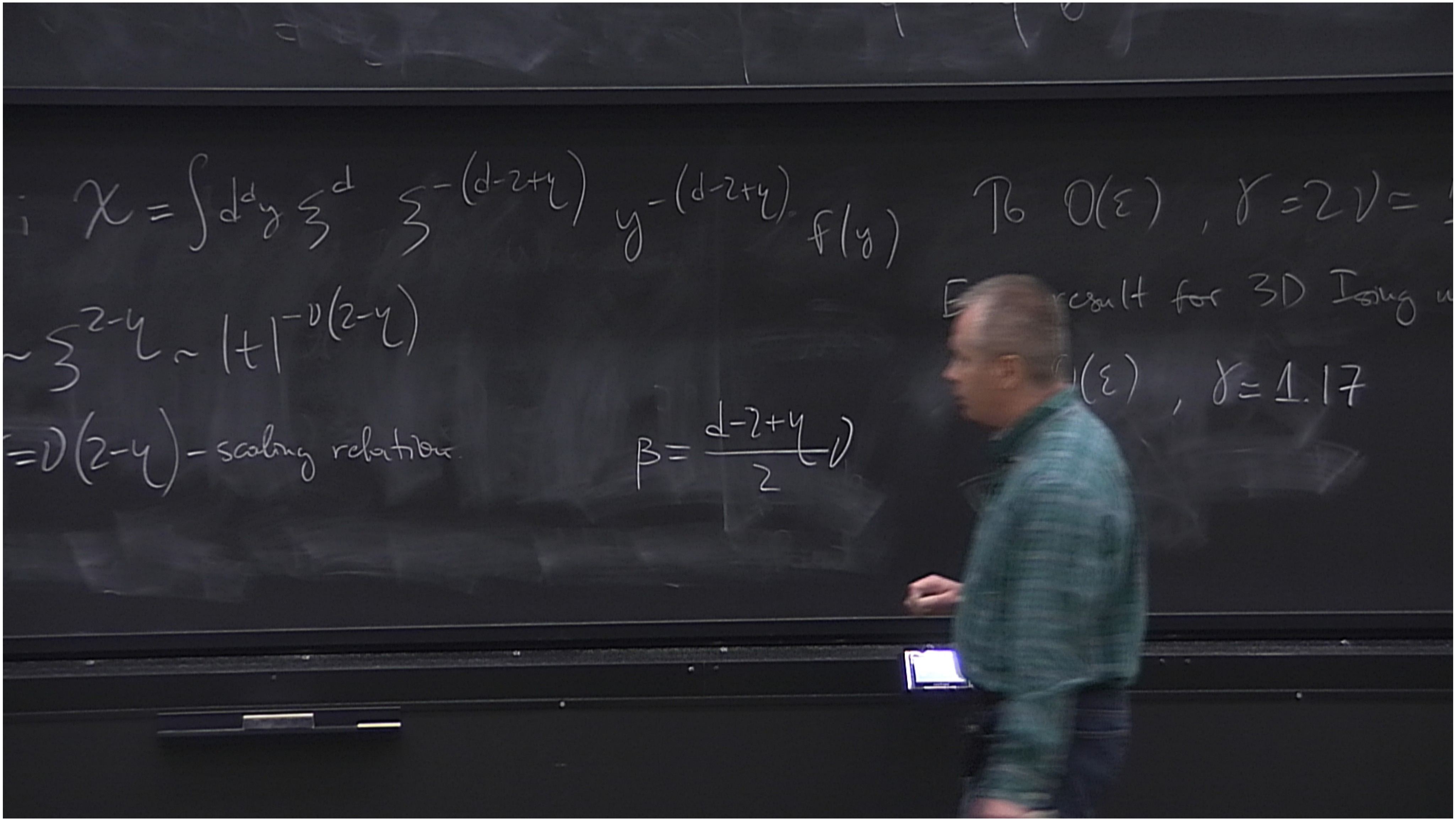
$\gamma = \nu(2-\eta)$ – scaling relation

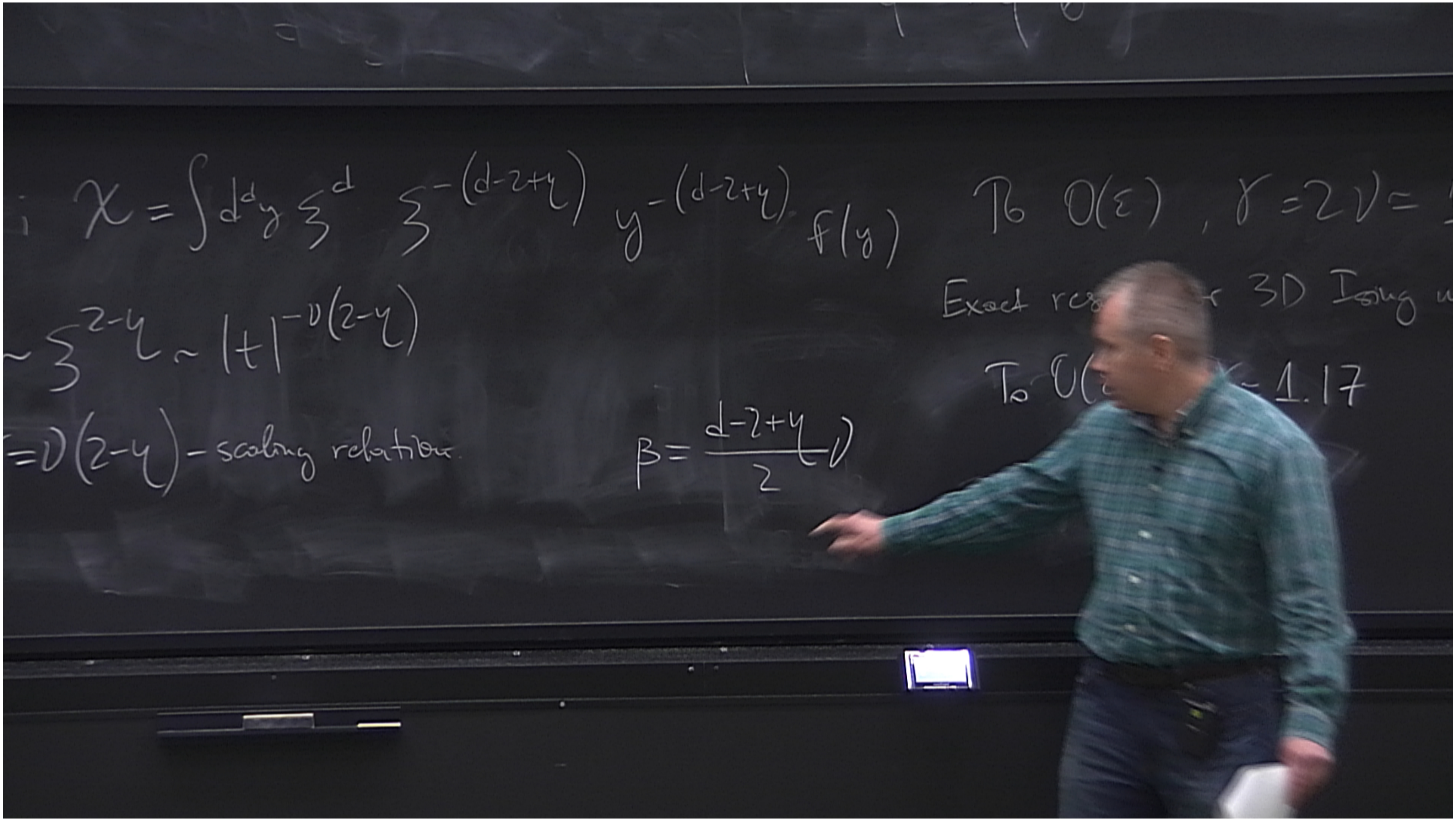
$f(y)$

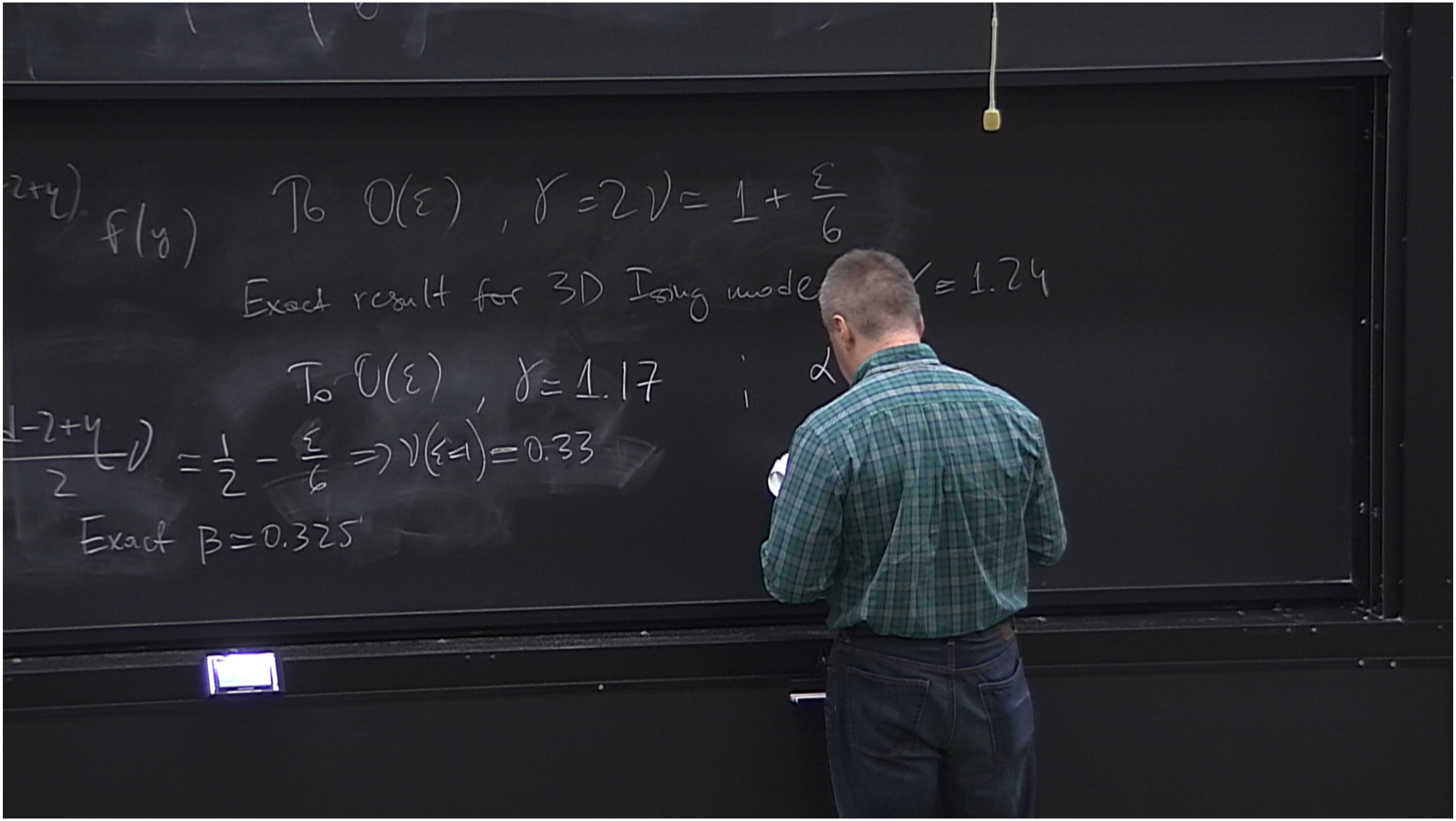
To $O(\varepsilon)$, $\gamma = 2\nu = 1 + \frac{\varepsilon}{6}$

Exact result for 3D Ising model : $\gamma \simeq 1.24$

To $O(\varepsilon)$, $\gamma \simeq 1.17$


χ = ∫ d^d y ξ^d ξ^-(d-2+η) y^-(d-2+η) f(y)
~ ξ^(2-η) ~ |t|^-ν(2-η)
= ν(2-η) - scaling relation
β = (d-2+η)/2 ν
To O(ε), γ = 2ν =
result for 3D Ising
(ε), γ = 1.17


$\chi = \int d^d y \, \xi^d \, \xi^{-(d-2+\eta)} \, y^{-(d-2+\eta)} f(y)$
$\sim \xi^{2-\eta} \sim |t|^{-\nu(2-\eta)}$
$= \nu(2-\eta)$ - scaling relation
$\beta = \frac{d-2+\eta}{2}\nu$
To $O(\varepsilon)$, $\gamma = 2\nu =$


f(y)
To $O(\varepsilon)$, $\gamma = 2\nu = 1 + \frac{\varepsilon}{6}$
Exact result for 3D Ising mode
$\simeq 1.24$
To $O(\varepsilon)$, $\gamma \simeq 1.17$; α
$\frac{d-2+\eta}{2}\nu = \frac{1}{2} - \frac{\varepsilon}{6} \Rightarrow \nu(\varepsilon=1) = 0.33$
Exact $\beta \simeq 0.325$

$2+\eta)$ $f(y)$

To $O(\varepsilon)$, $\gamma = 2\nu = 1 + \frac{\varepsilon}{6}$

Exact result for 3D Ising model : $\gamma \simeq 1.24$

To $O(\varepsilon)$, $\gamma \simeq 1.17$; $\alpha = 2 - d\nu$

$\frac{d-2+\eta}{2}\nu = \frac{1}{2} - \frac{\varepsilon}{6} \Rightarrow \nu(\varepsilon=1) = 0.33$

Exact $\beta = 0.325$

$2+y)$ $f(y)$

To $O(\varepsilon)$, $\gamma = 2\nu = 1 + \frac{\varepsilon}{6}$

Exact result for 3D Ising model: $\gamma \simeq 1.24$

To $O(\varepsilon)$, $\gamma \simeq 1.17$; $\alpha = 2 - d\nu = \frac{\varepsilon}{6}$

$\frac{d-2+y}{2}\nu = \frac{1}{2} - \frac{\varepsilon}{6} \Rightarrow \nu(\varepsilon=1) = 0.33$

Exact $\beta \simeq 0.325$

$2+\eta)$ $f(y)$

To $O(\varepsilon)$, $\gamma = 2\nu = 1 + \frac{\varepsilon}{6}$

Exact result for 3D Ising model: $\gamma \simeq 1.24$

To $O(\varepsilon)$, $\gamma \simeq 1.17$; $\alpha = 2 - d\nu = \frac{\varepsilon}{6}$

$\frac{d-2+\eta}{2}\nu = \frac{1}{2} - \frac{\varepsilon}{6} \Rightarrow \nu(\varepsilon=1) \simeq 0.33$

$\alpha = 0.17$, exact $\alpha \simeq 0.11$

Exact $\beta \simeq 0.325$

Susceptibility critical exponent γ

$$S[\varphi] = \int d^d x \left[\frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 - B(\vec{x})\varphi(\vec{x}) \right]$$

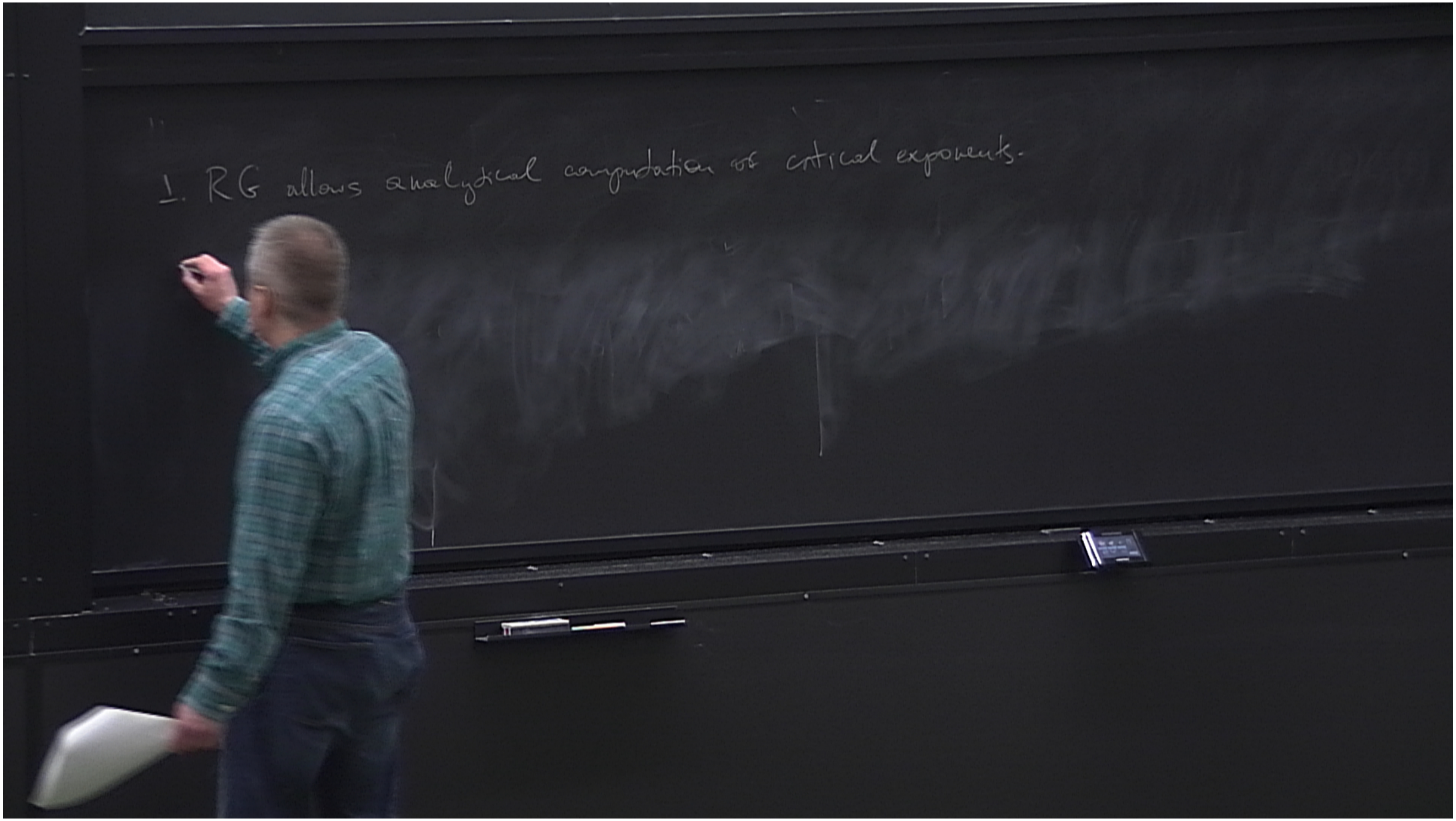
$$G(\vec{x}-\vec{x}') = \left.\frac{\delta\langle\varphi(\vec{x})\rangle}{\delta B(\vec{x}')}\right|_{B=0} = \frac{\delta}{\delta B(\vec{x}')} \frac{1}{Z}\int D\varphi\, \varphi(\vec{x})\, e^{-S[\varphi]} =$$

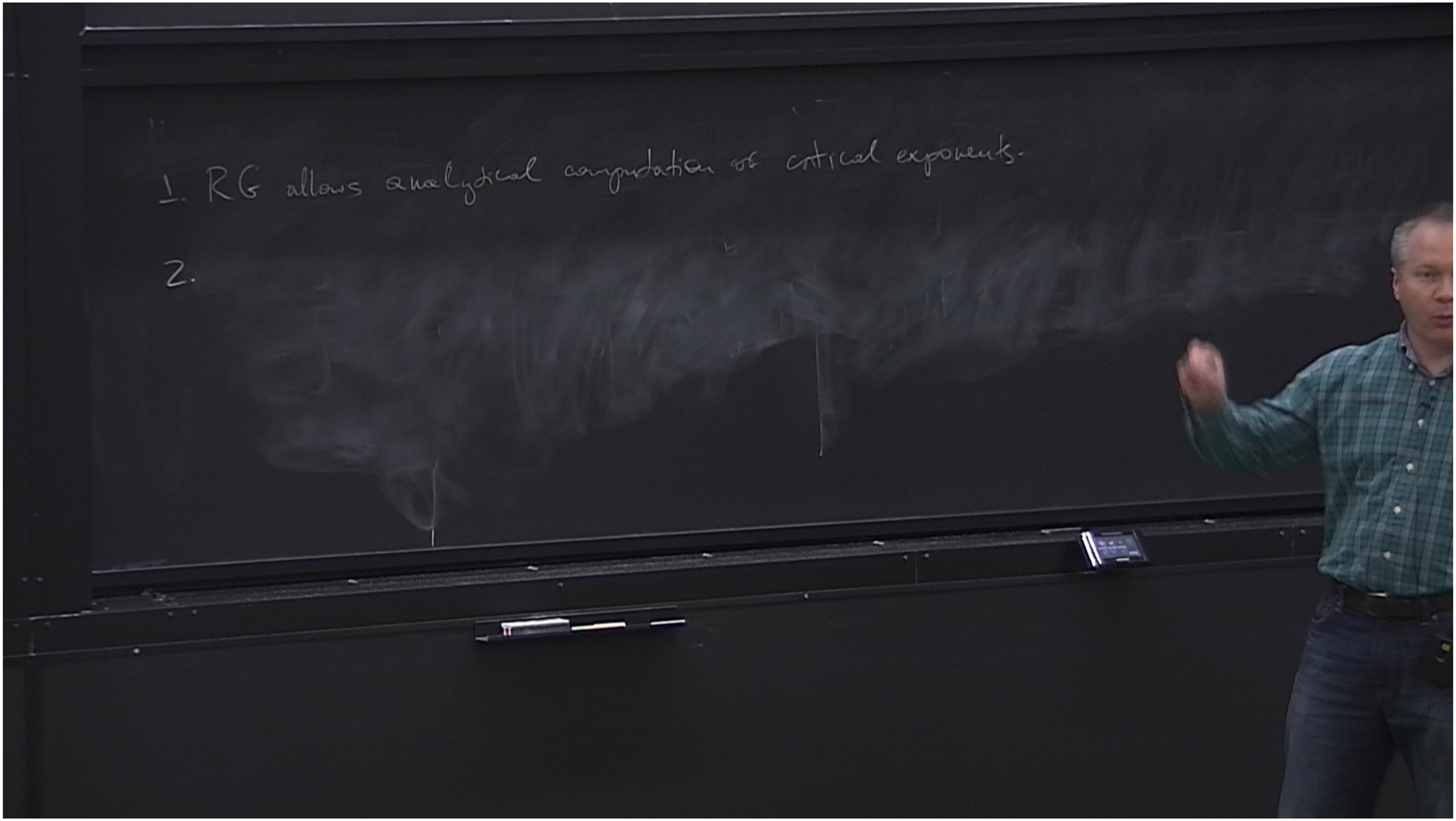
$$= \langle\varphi(\vec{x})\varphi(\vec{x}')\rangle - \langle\varphi(\vec{x})\rangle\langle\varphi(\vec{x}')\rangle$$

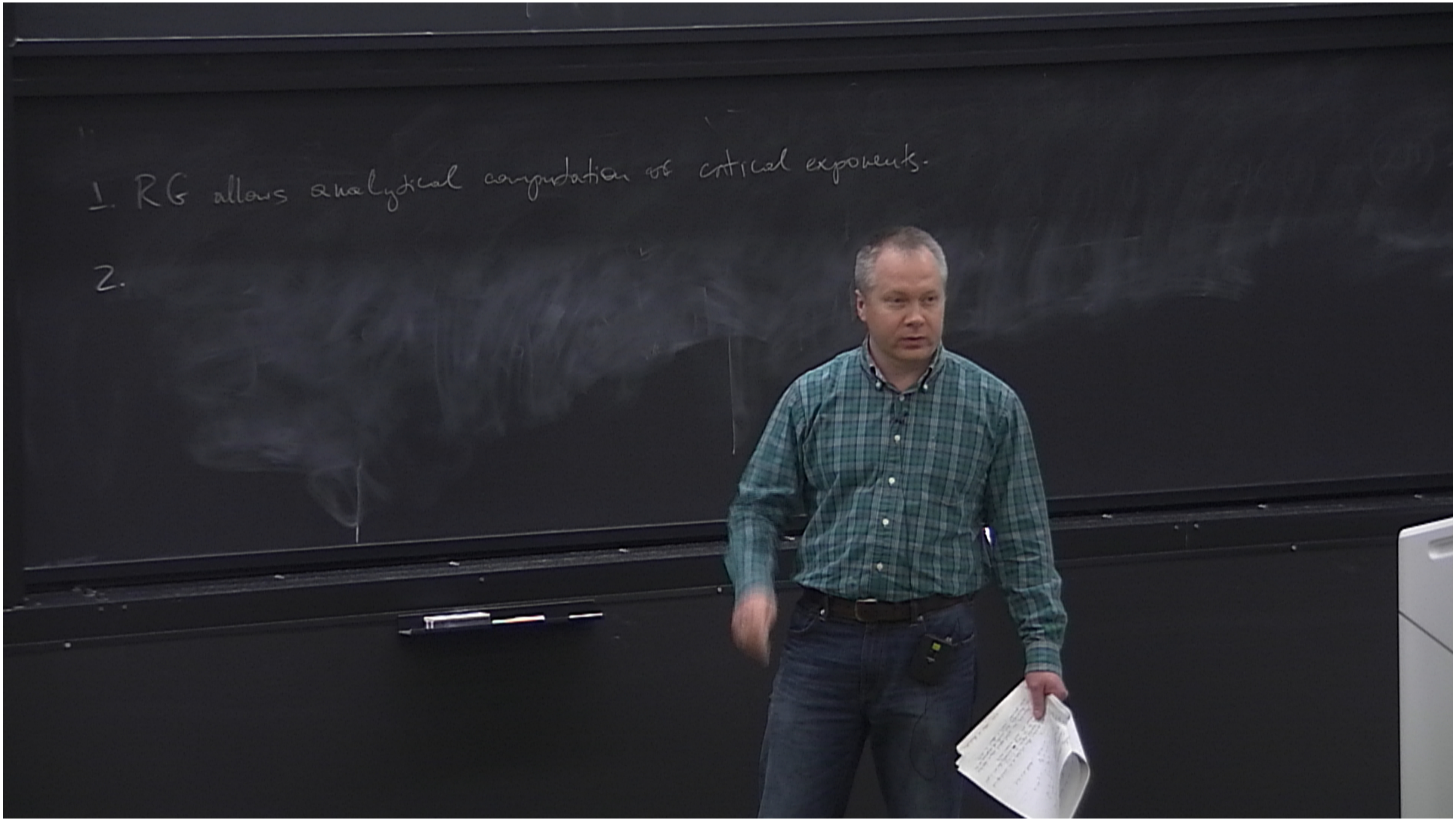
Susceptibility

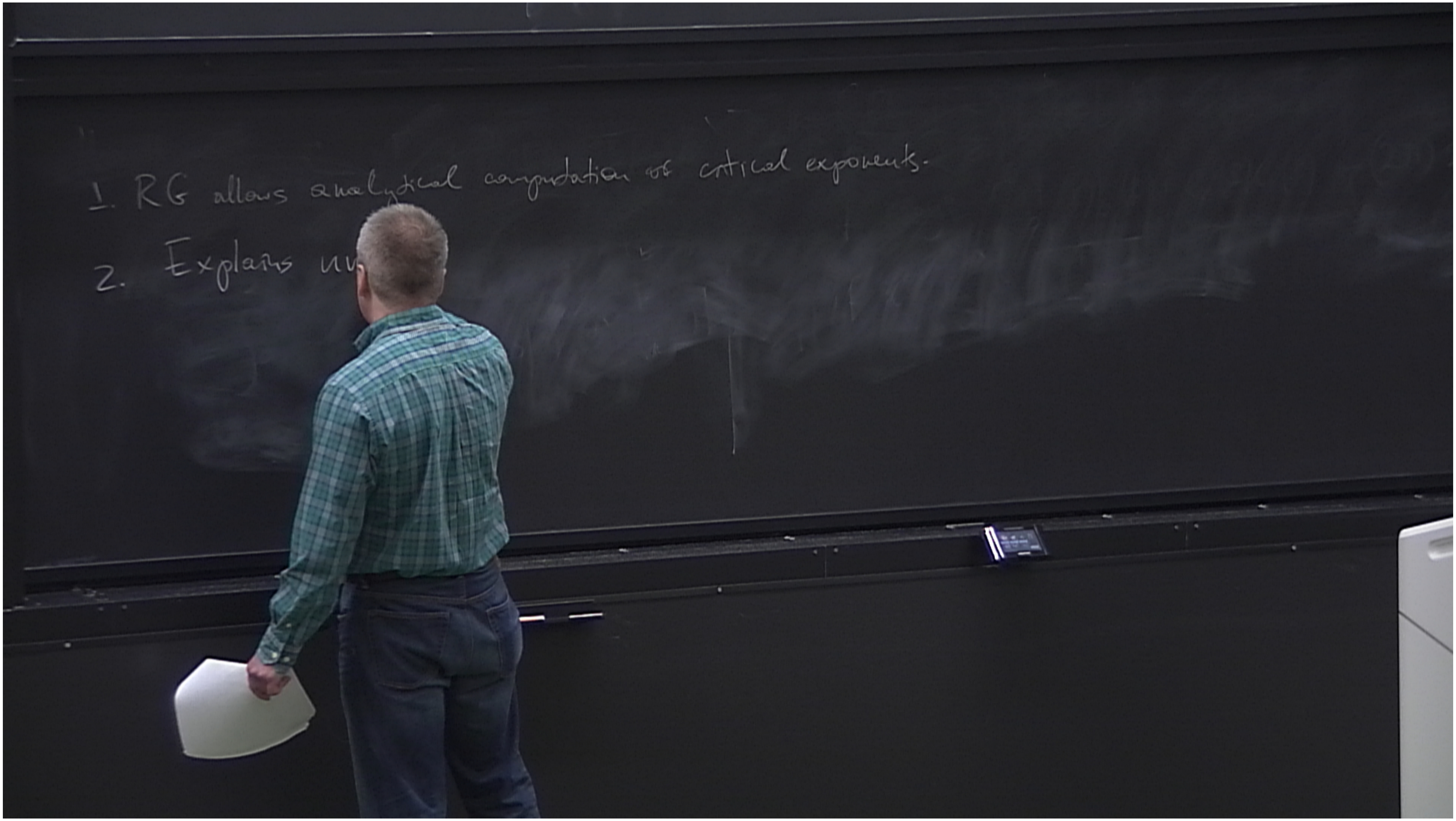
$$\chi = \frac{\partial\langle\varphi\rangle}{\partial B}$$

$$= \int d^d x\, G(\vec{x}$$


1. RG allows analytical computation of critical exponents.


1. RG allows analytical computation of critical exponents.
2.

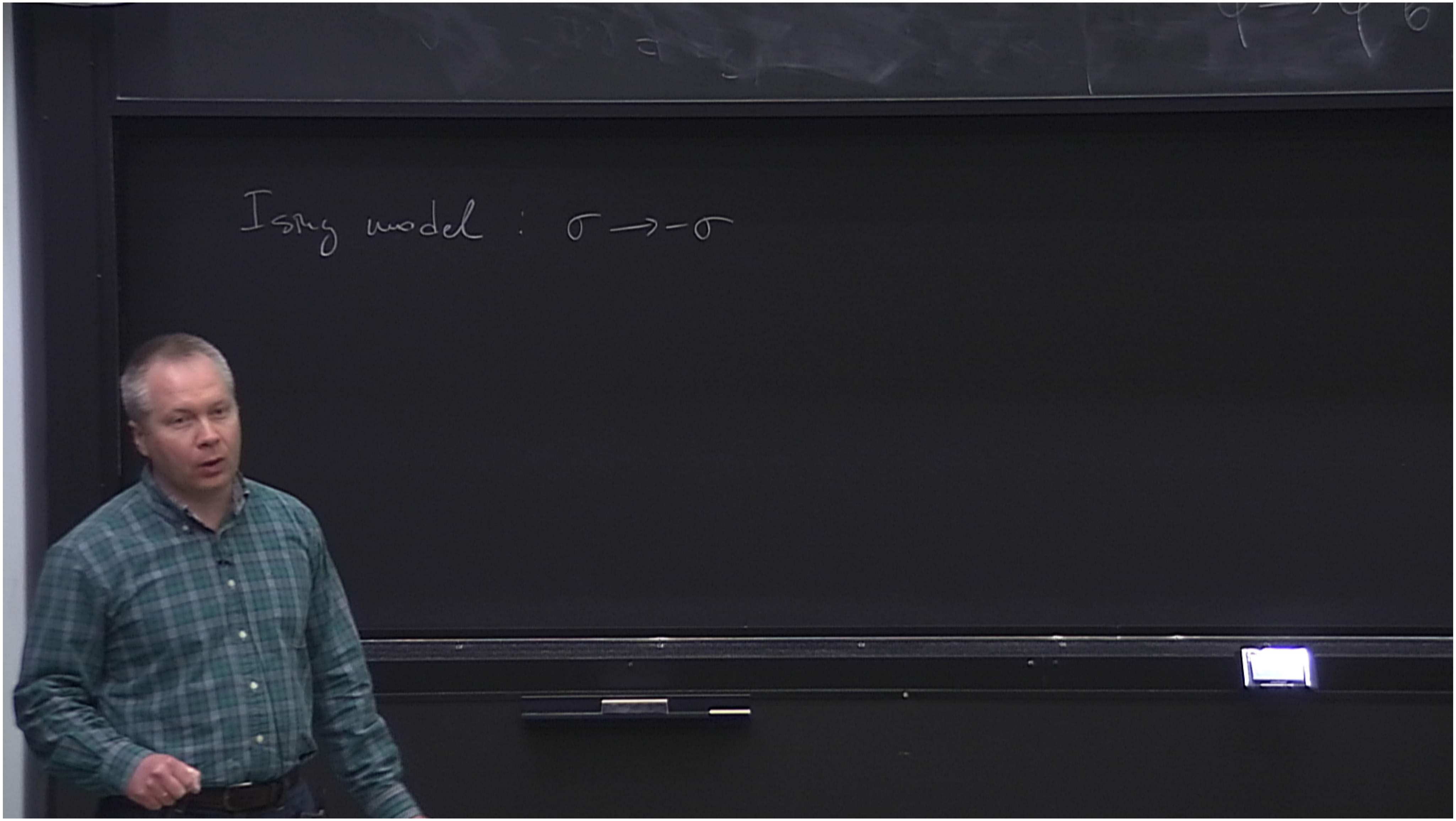

1. RG allows analytical computation of critical exponents.
2.

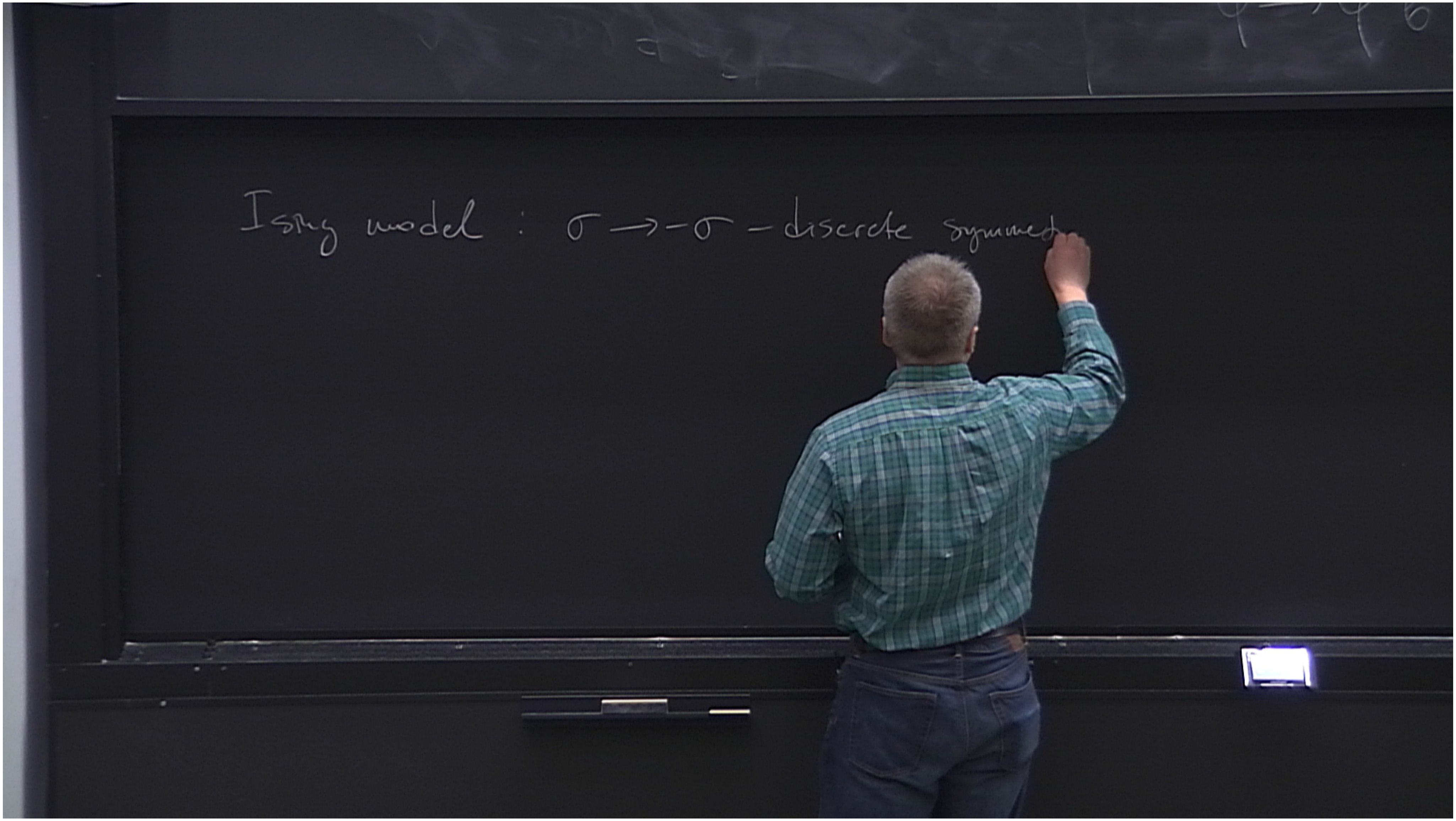

1. RG allows analytical computation of critical exponents.
2. Explains

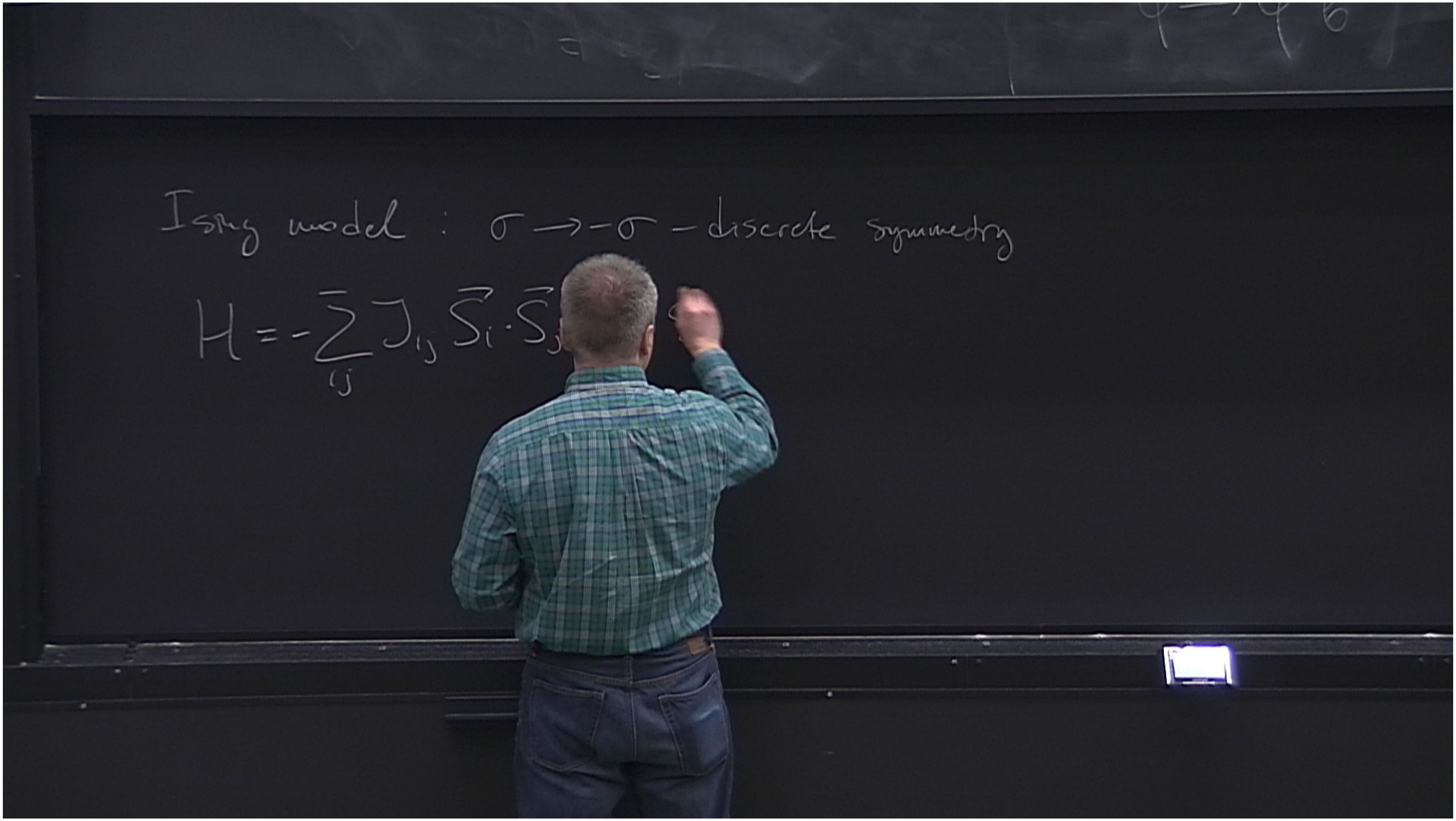
1. RG allows analytical computation of critical exponents.

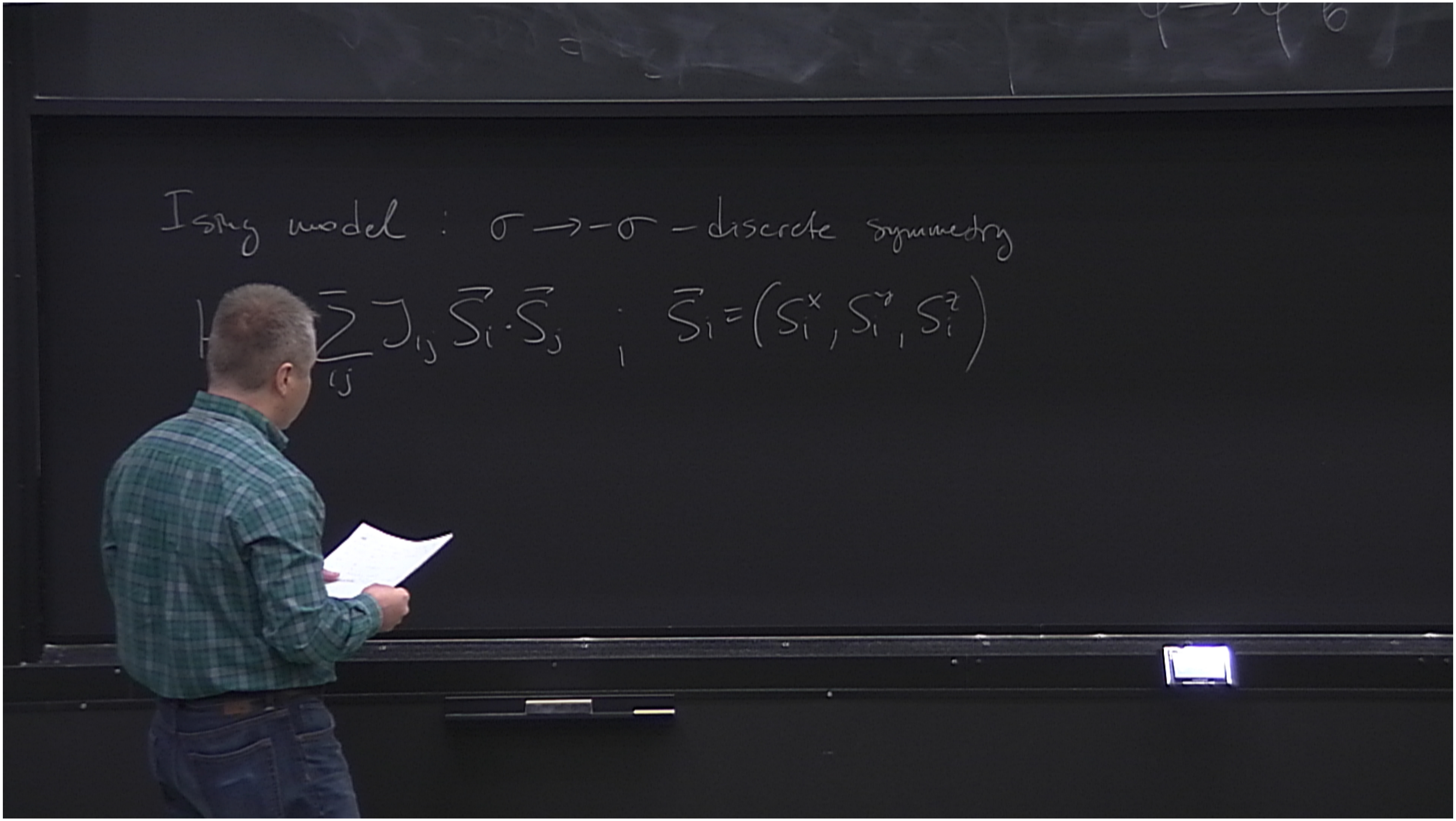
2. Explains universality of continuous phase transitions.

1. RG allows analytical computation of critical exponents.
2. Explains universality of continuous phase transitions.
3. Provides conceptual foundation for the way we model condensed matter system.


Ising model : σ → −σ

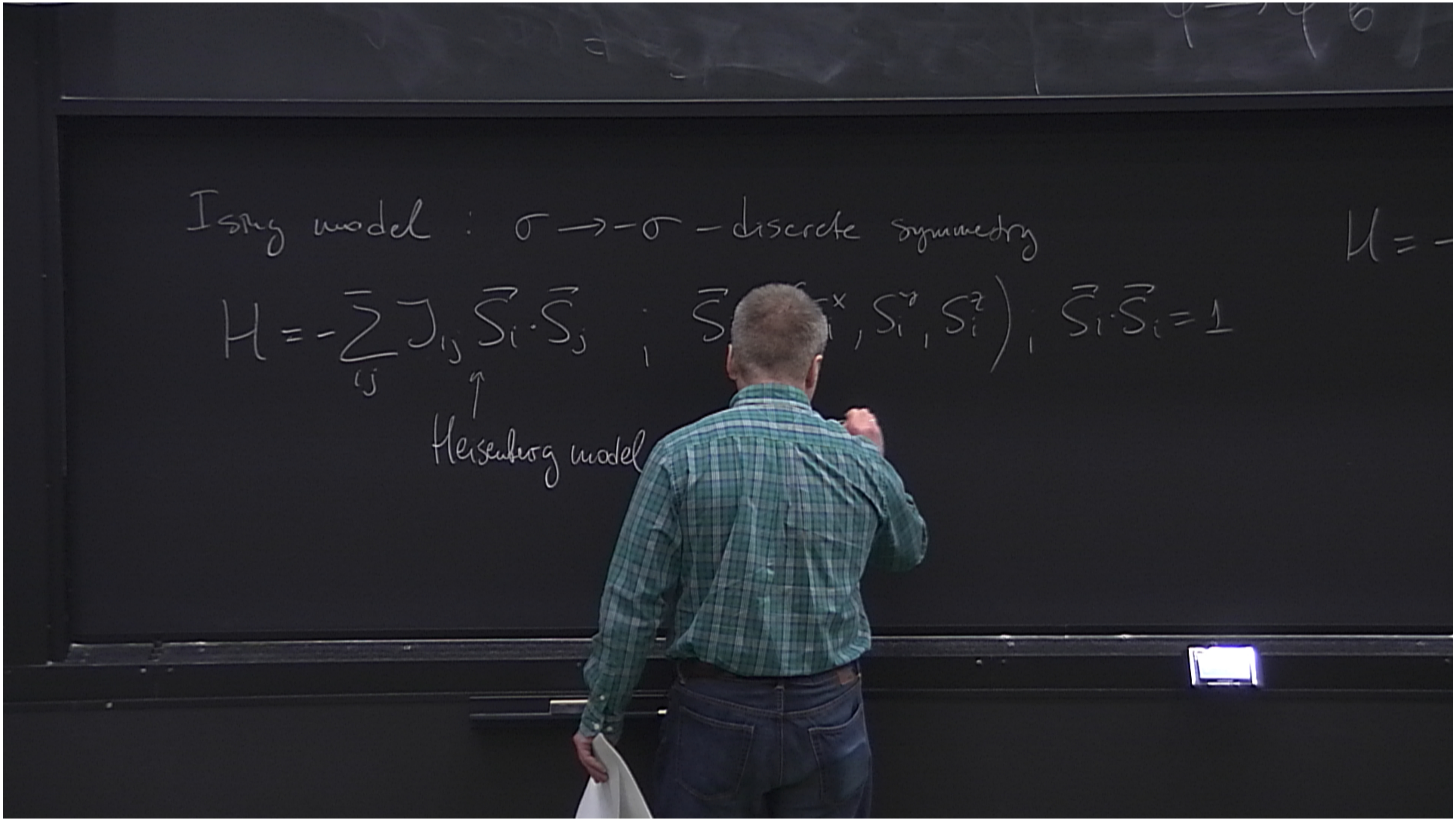

Ising model : σ → −σ − discrete symmet

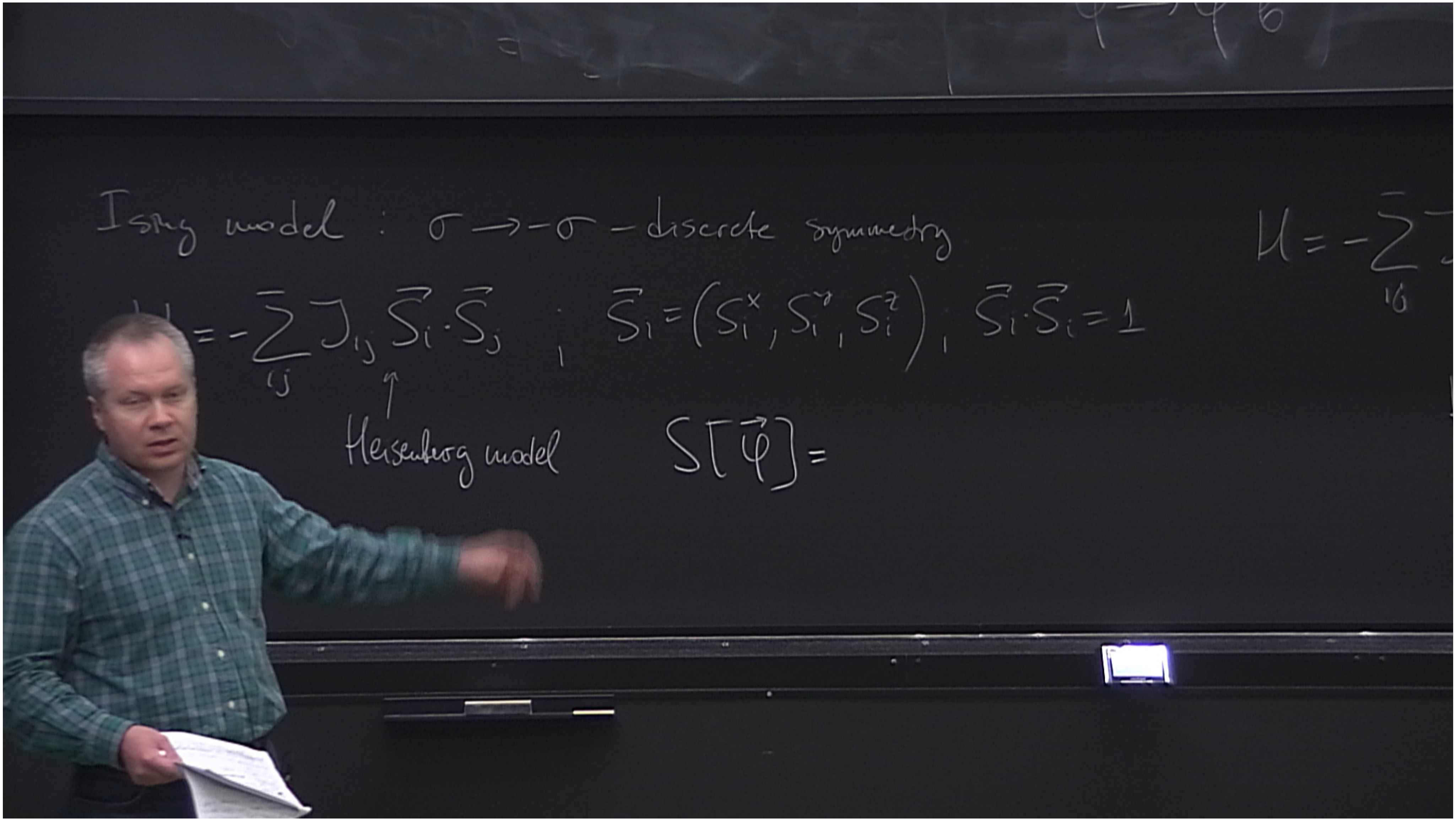

Ising model : σ → -σ - discrete symmetry
H = -Σ_ij J_ij S_i · S_j


Ising model : $\sigma \to -\sigma$ – discrete symmetry
$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$; $\vec{S}_i = (S_i^x, S_i^y, S_i^z)$

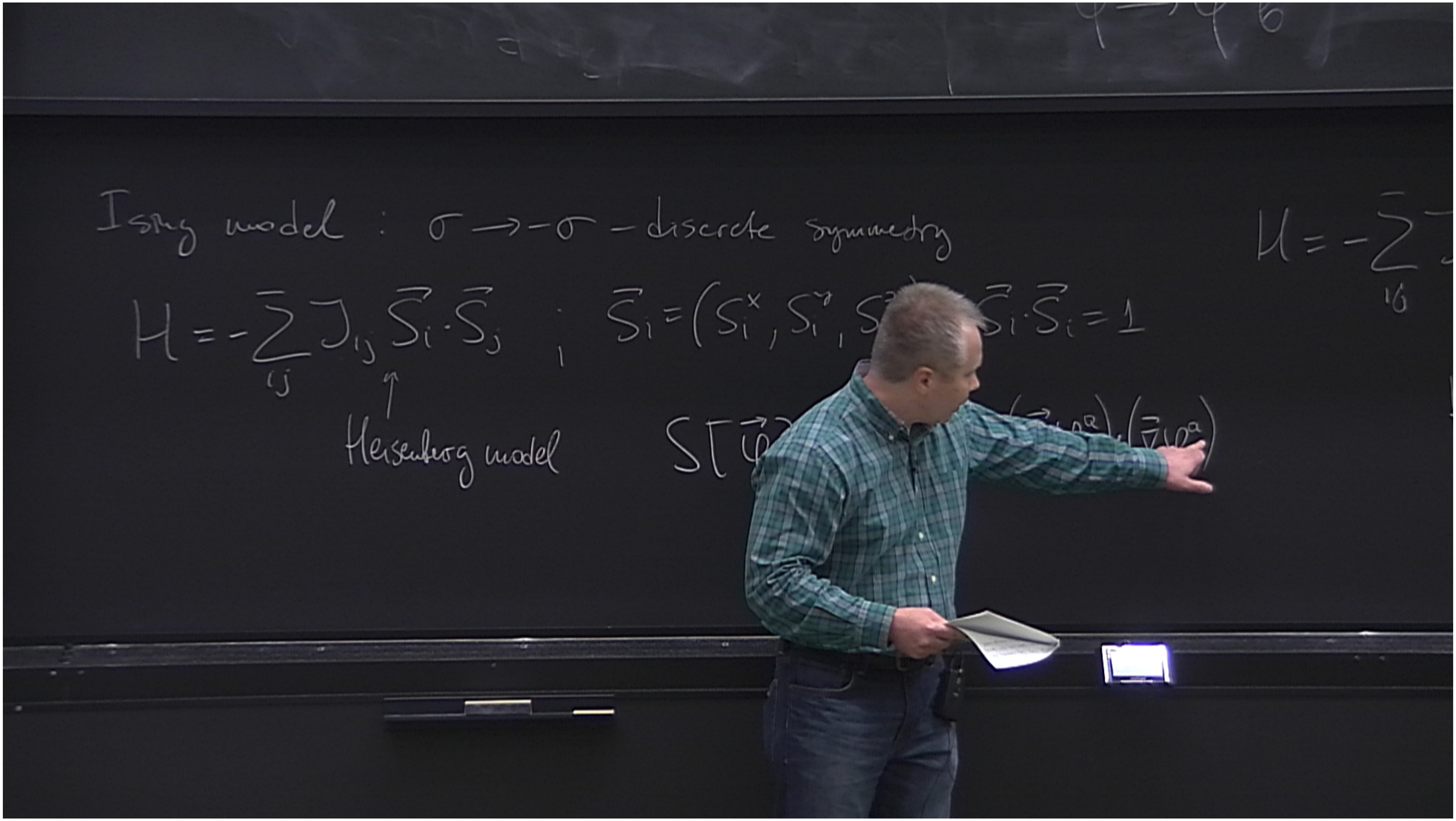
Ising model : $\sigma \to -\sigma$ – discrete symmetry

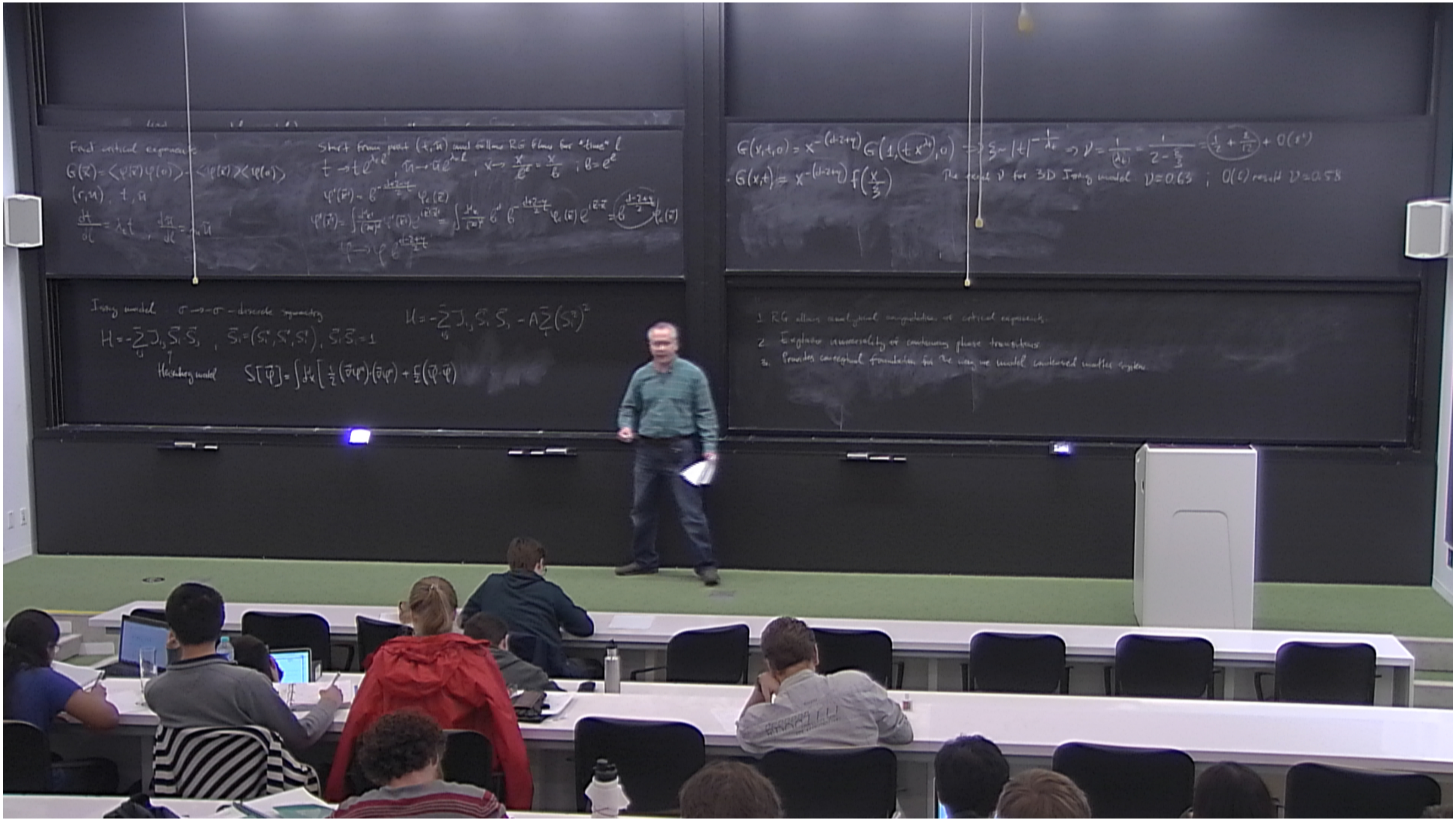
$$H = -\sum_{ij} J_{ij}\, \vec{S}_i \cdot \vec{S}_j \; ; \; \vec{S}_i = (S_i^x, S_i^y, S_i^z) \; ; \; \vec{S}_i \cdot \vec{S}_i = 1$$


Ising model : σ → -σ - discrete symmetry
H = -∑ J_ij S_i · S_j ; S_i = (S_i^x, S_i^y, S_i^z) ; S_i · S_i = 1
ij
Heisenberg model



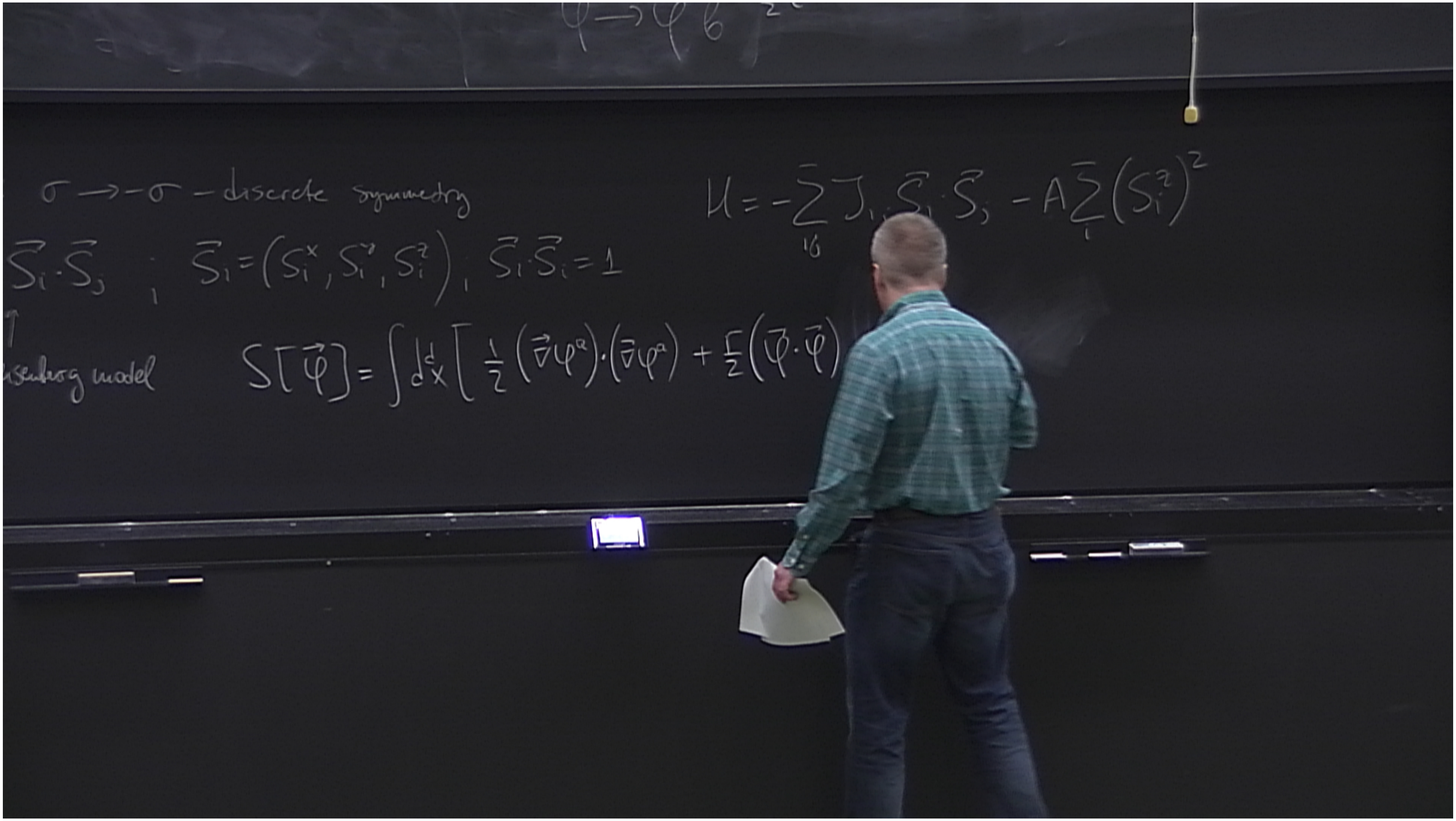
Ising model : $\sigma \to -\sigma$ – discrete symmetry
$H = -\sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$; $\vec{S}_i = (S_i^x, S_i^y, S_i^z)$; $\vec{S}_i \cdot \vec{S}_i = 1$
Heisenberg model
$S[\vec{\varphi}] =$


Ising model : σ → -σ - discrete symmetry
Heisenberg model



$$\vec{S}_i = \left(S_i^x, S_i^y, S_i^z\right); \quad \vec{S}_i \cdot \vec{S}_i = 1$$

$$S[\vec{\varphi}] = \int d^dx \left[\frac{1}{2} \left(\vec{\nabla}\varphi^a\right) \cdot \left(\vec{\nabla}\varphi^a\right) + \frac{r}{2} \left(\vec{\varphi} \cdot \vec{\varphi}\right) \right.$$


σ → −σ − discrete symmetry
$U = -\sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j - A \sum_i (S_i^z)^2$
$\vec{S}_i \cdot \vec{S}_j$; $\vec{S}_i = (S_i^x, S_i^y, S_i^z)$; $\vec{S}_i \cdot \vec{S}_i = 1$
Heisenberg model
$S[\vec{\varphi}] = \int d^dx \left[\frac{1}{2} (\vec{\nabla}\varphi^a)\cdot(\vec{\nabla}\varphi^a) + \frac{r}{2}(\vec{\varphi}\cdot\vec{\varphi})\right.$

$\varphi \to \varphi\, b$

$\sigma \to -\sigma$ – discrete symmetry

$$U = -\sum_{ij} J_{ij}\, \vec{S}_i \cdot \vec{S}_j - A \sum_i \left(S_i^z\right)^2$$

$\vec{S}_i \cdot \vec{S}_j$; $\vec{S}_i = \left(S_i^x, S_i^y, S_i^z\right)$; $\vec{S}_i \cdot \vec{S}_i = 1$

↑

isenberg model

$$S[\vec{\varphi}] = \int d^dx \Big[\frac{1}{2} \left(\vec{\nabla}\varphi^a\right) \cdot \left(\vec{\nabla}\varphi^a\right) + \frac{r}{2} \left(\vec{\varphi} \cdot \vec{\varphi}\right) + \frac{u}{4} \left(\vec{\varphi} \cdot \vec{\varphi}\right)^2 + \dots$$

$\sigma \to -\sigma$ - discrete symmetry

$\vec{S}_i \cdot \vec{S}_j$; $\vec{S}_i = (S_i^x, S_i^y, S_i^z)$; $\vec{S}_i \cdot \vec{S}_i = 1$

$$H = -\sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j - A \sum_i (S_i^z)^2$$

Heisenberg model

$$S[\vec{\varphi}] = \int d^d x \left[\frac{1}{2} (\vec{\nabla} \varphi^a) \cdot (\vec{\nabla} \varphi^a) + \frac{r}{2} (\vec{\varphi} \cdot \vec{\varphi}) + \frac{u}{4} (\vec{\varphi} \cdot \vec{\varphi})^2 + \dots \right]$$

$d = 4$ - upper critical dimension, can set up ε-expansion.

...metry

$$U = -\sum_{ij} J_{ij}\, \vec{S}_i \cdot \vec{S}_j - A \sum_i \left(S_i^z\right)^2$$

S_i^z), $\vec{S}_i \cdot \vec{S}_i = 1$

$$\int d^dx \left[\frac{1}{2} \left(\vec{\nabla}\varphi^a\right)\cdot\left(\vec{\nabla}\varphi^a\right) + \frac{r}{2}\left(\vec{\varphi}\cdot\vec{\varphi}\right) + \frac{u}{4}\left(\vec{\varphi}\cdot\vec{\varphi}\right)^2 + \dots \right]$$

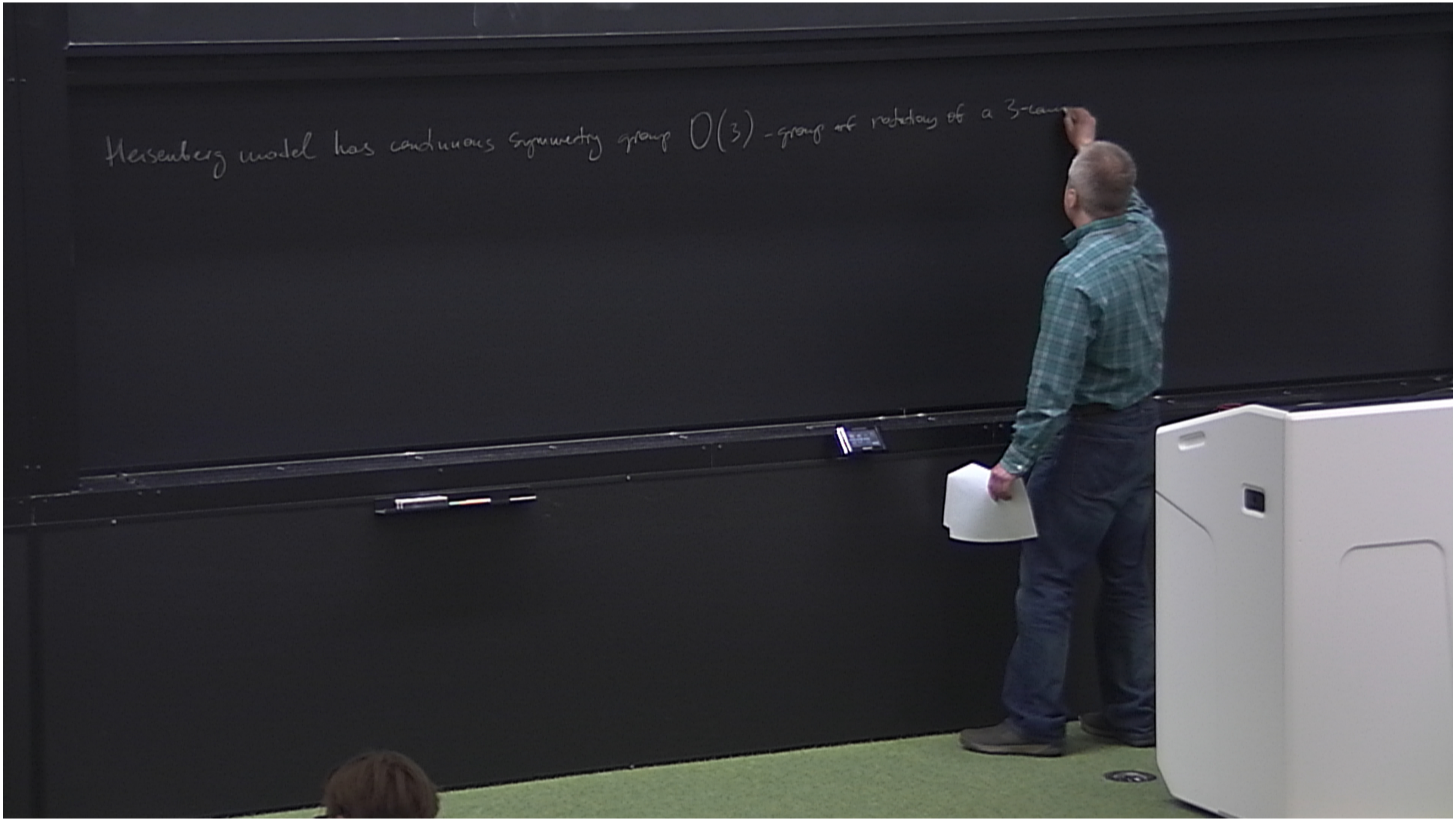
$d=4$ - upper critical dimension, can set up ε-expansion.

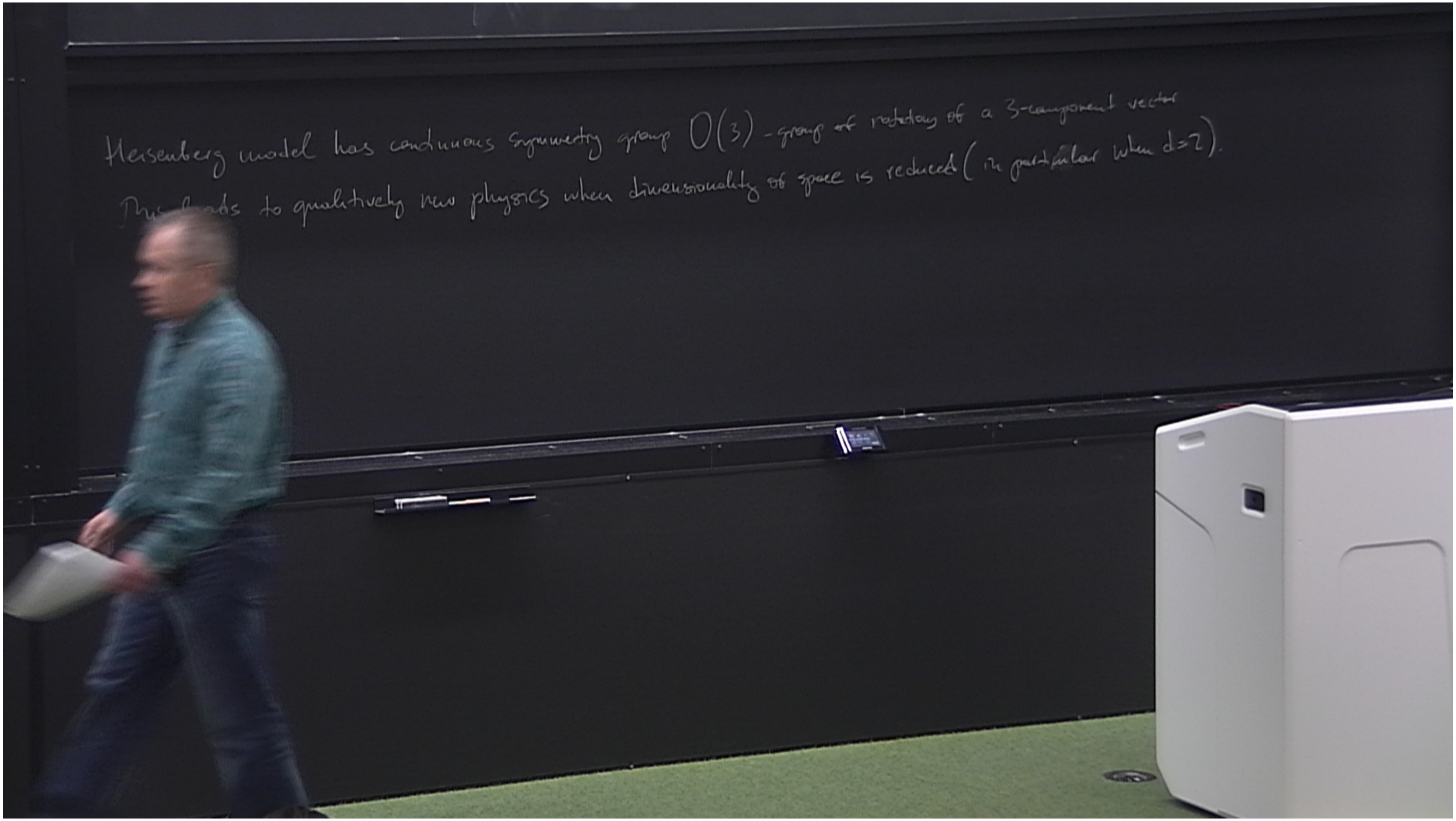
New feature compared to Ising model is dependence on the number of components of $\vec{\varphi}$

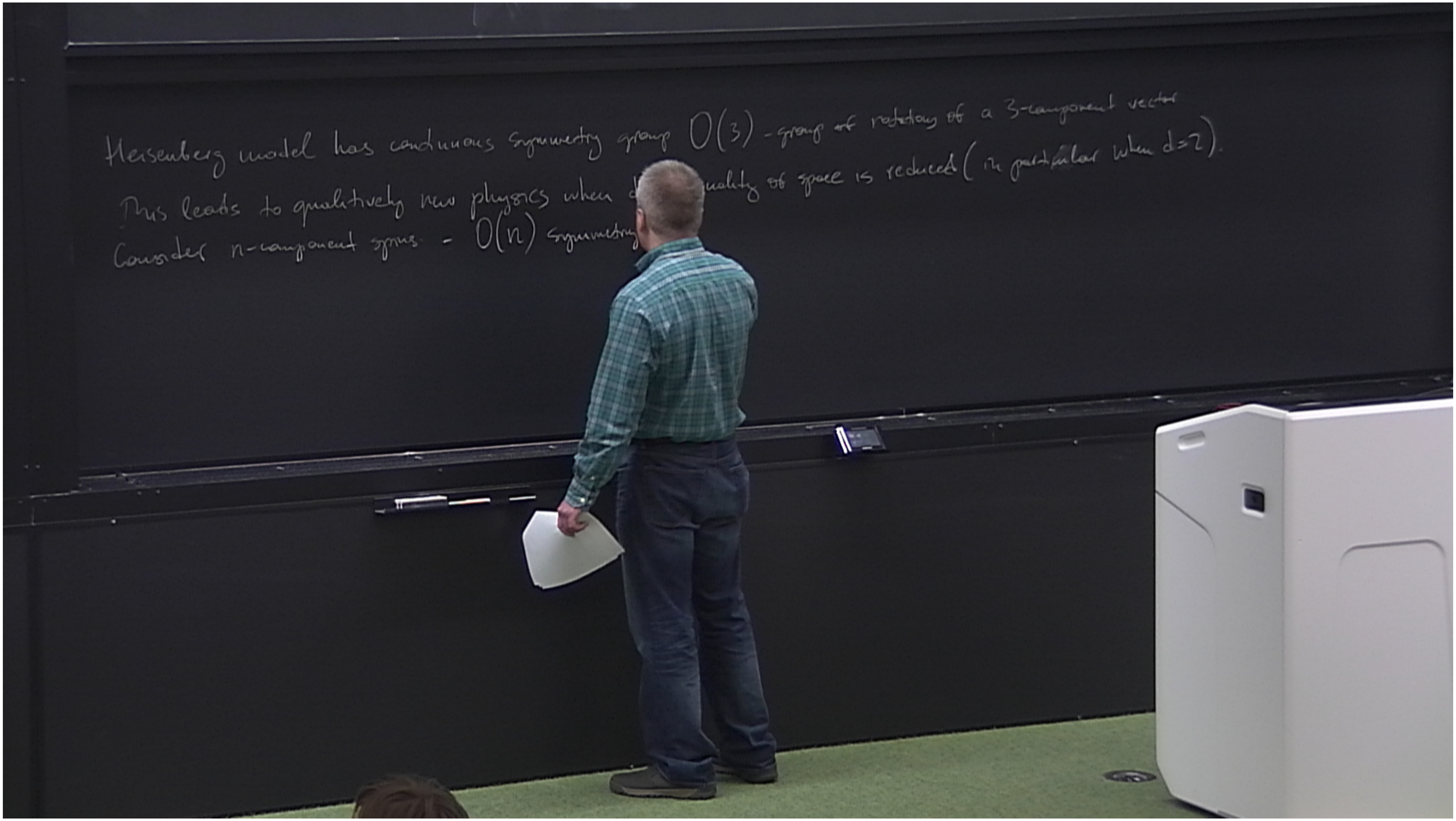
1. RG allows a

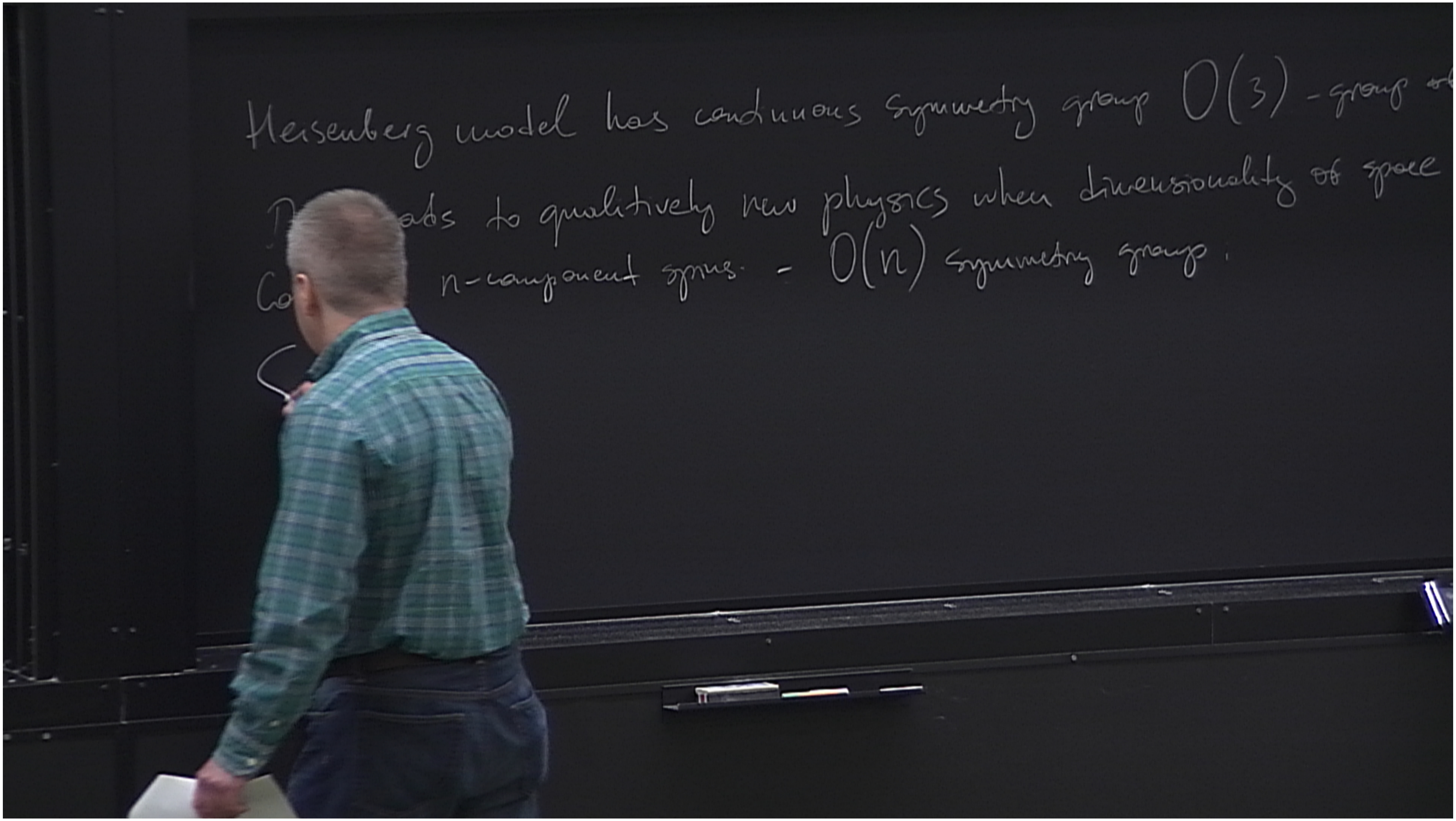
2. Explains

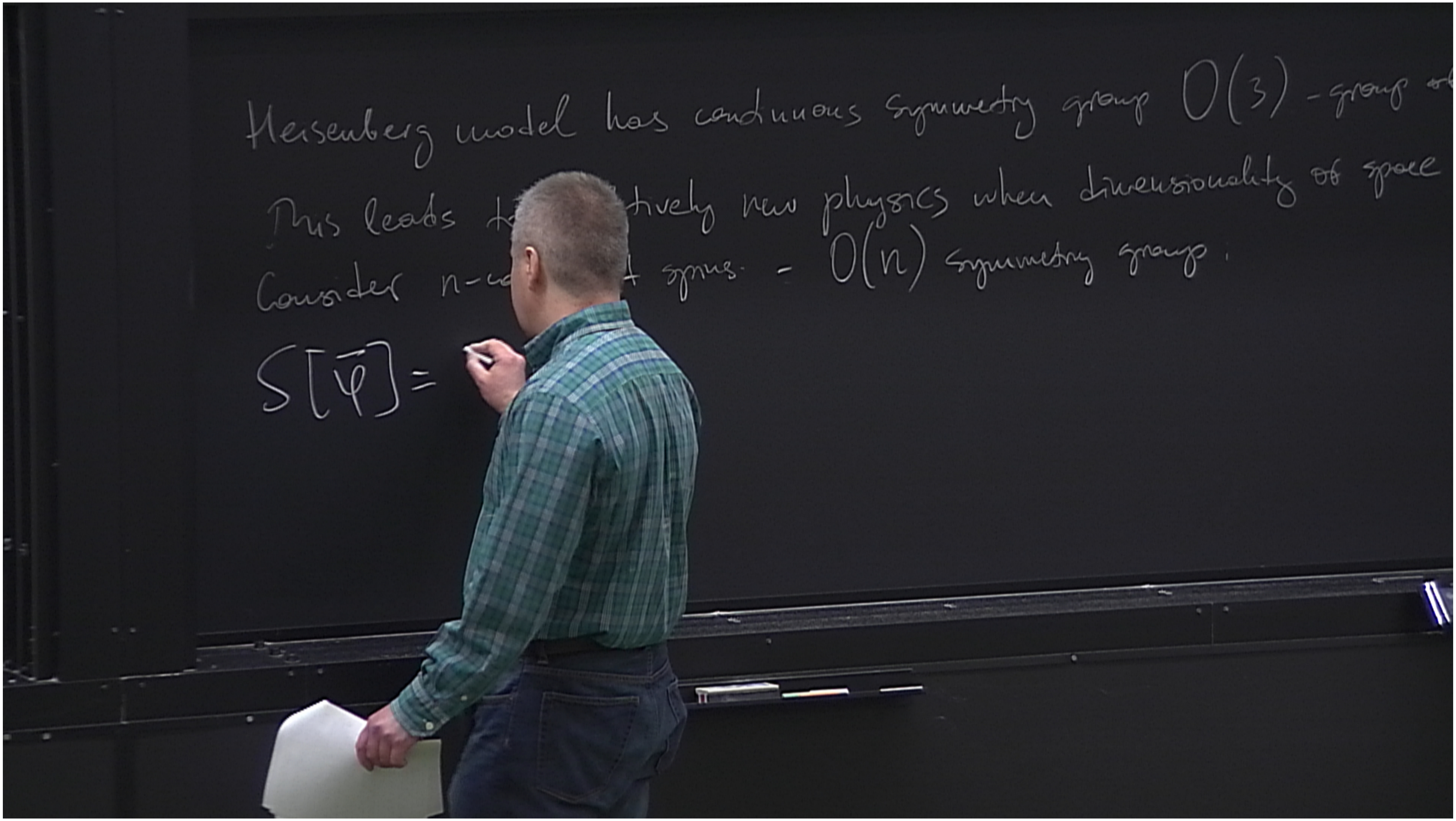
3. Provides con


Heisenberg model has continuous symmetry group O(3) - group of rotations of a 3-


Heisenberg model has continuous symmetry group O(3) - group of rotations of a 3-component vector
to qualitatively new physics when dimensionality of space is reduced (in particular when d=2).


Heisenberg model has continuous symmetry group O(3) - group of rotations of a 3-component vector
This leads to qualitatively new physics when
uality of space is reduced (in particular when d=2).
Consider n-component spins - O(n) symmetry


Heisenberg model has continuous symmetry group O(3) - group of
leads to qualitatively new physics when dimensionality of space
n-component spins. - O(n) symmetry group.

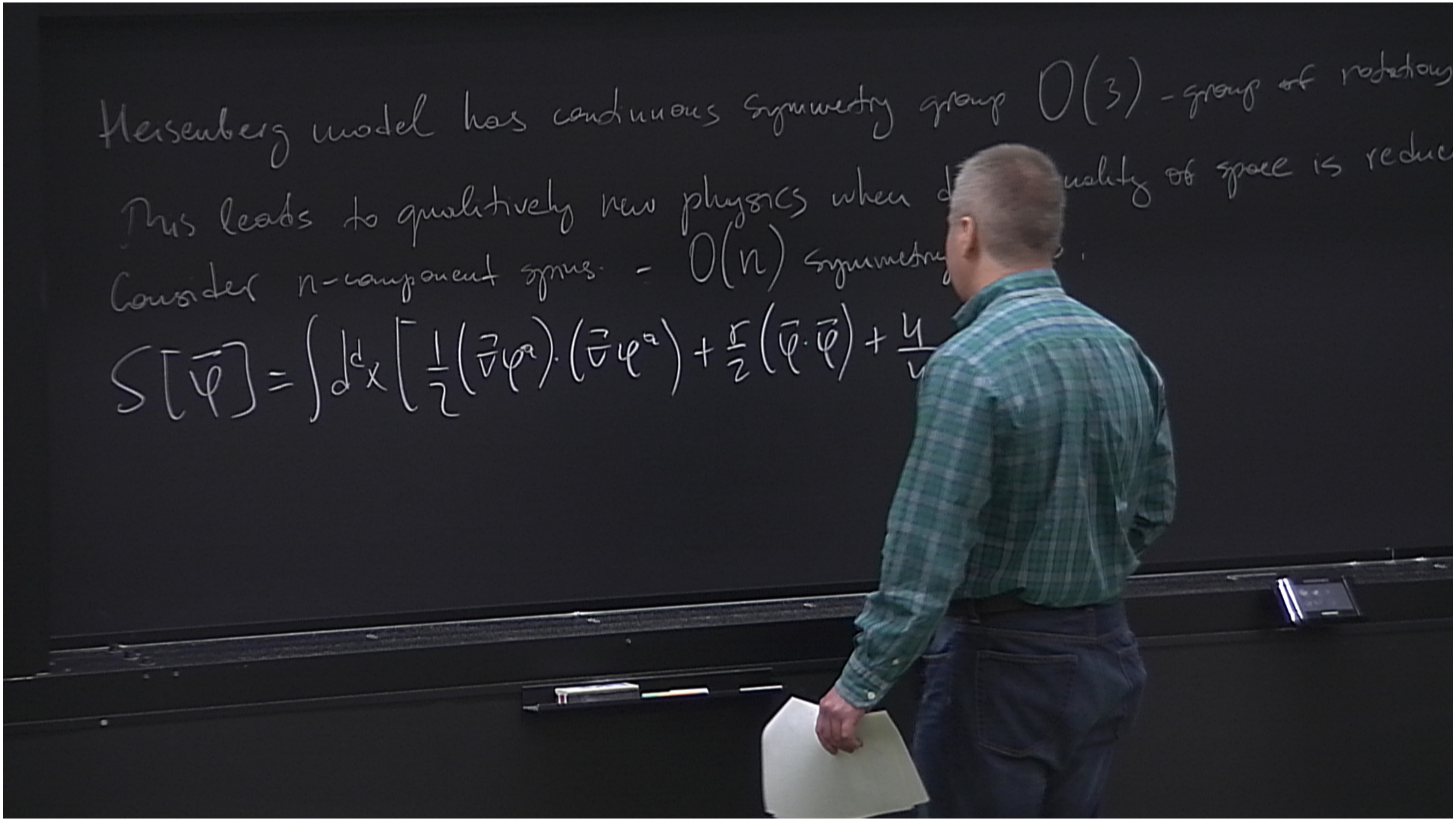

Heisenberg model has continuous symmetry group O(3) - group of
This leads t tively new physics when dimensionality of space
Consider n-co spins - O(n) symmetry group.
S[φ̄] =

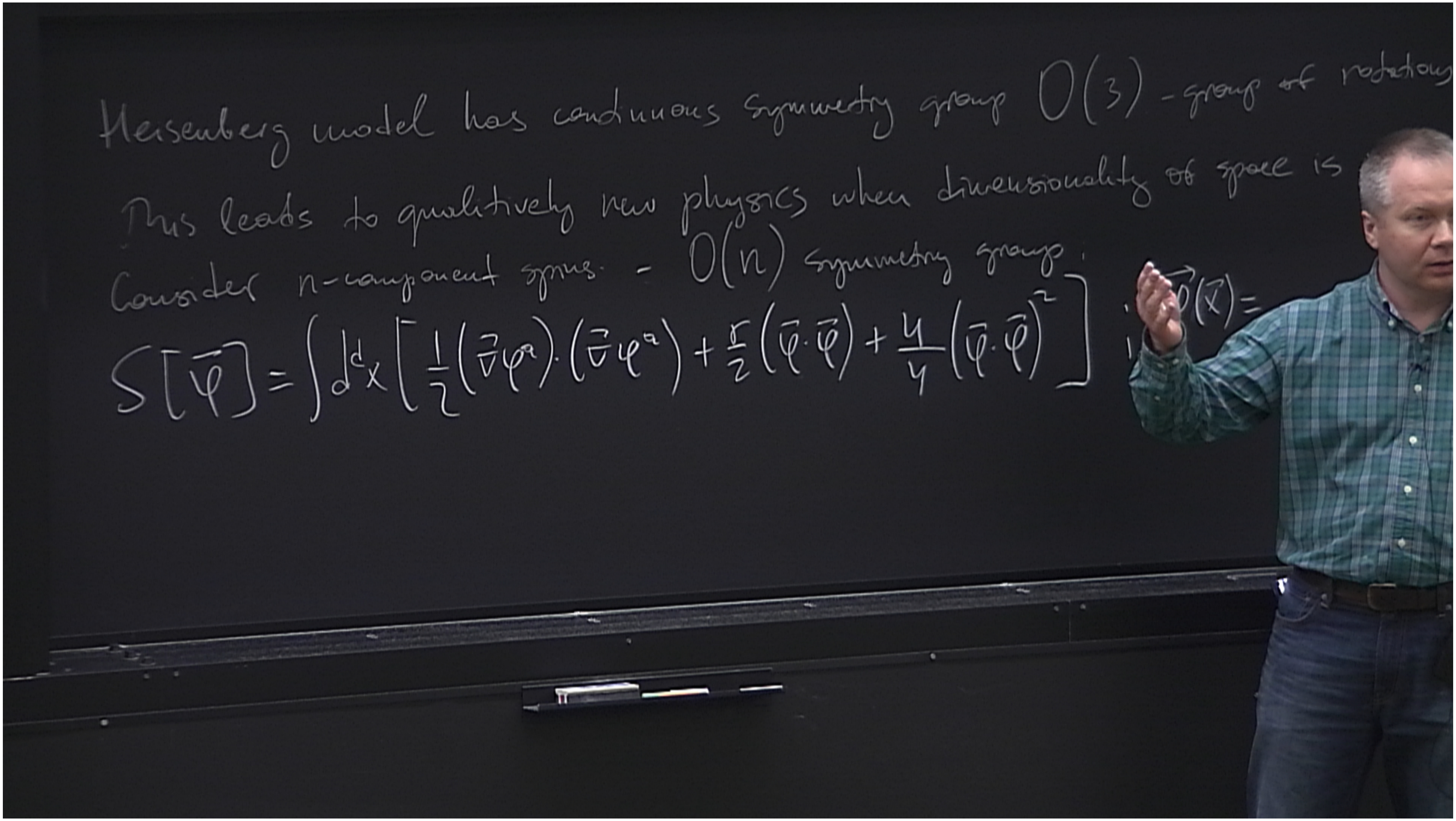
Heisenberg model has continuous symmetry group $O(3)$ - group of

This leads to qualitatively new physics when dimensionality of space

Consider n-component spins. - $O(n)$ symmetry group.

$$S[\vec{\varphi}] = \int d^dx \Big[\frac{1}{2} (\vec{\nabla}\varphi^a)\cdot(\vec{\nabla}\varphi^a)$$


Heisenberg model has continuous symmetry group $O(3)$ - group of rotations
This leads to qualitatively new physics when d
uality of space is reduc
Consider n-component spins - $O(n)$ symmetry
$S[\vec{\varphi}] = \int d^dx \left[\frac{1}{2}(\vec{\nabla}\varphi^a)\cdot(\vec{\nabla}\varphi^a) + \frac{r}{2}(\vec{\varphi}\cdot\vec{\varphi}) + \frac{u}{4}\right.$

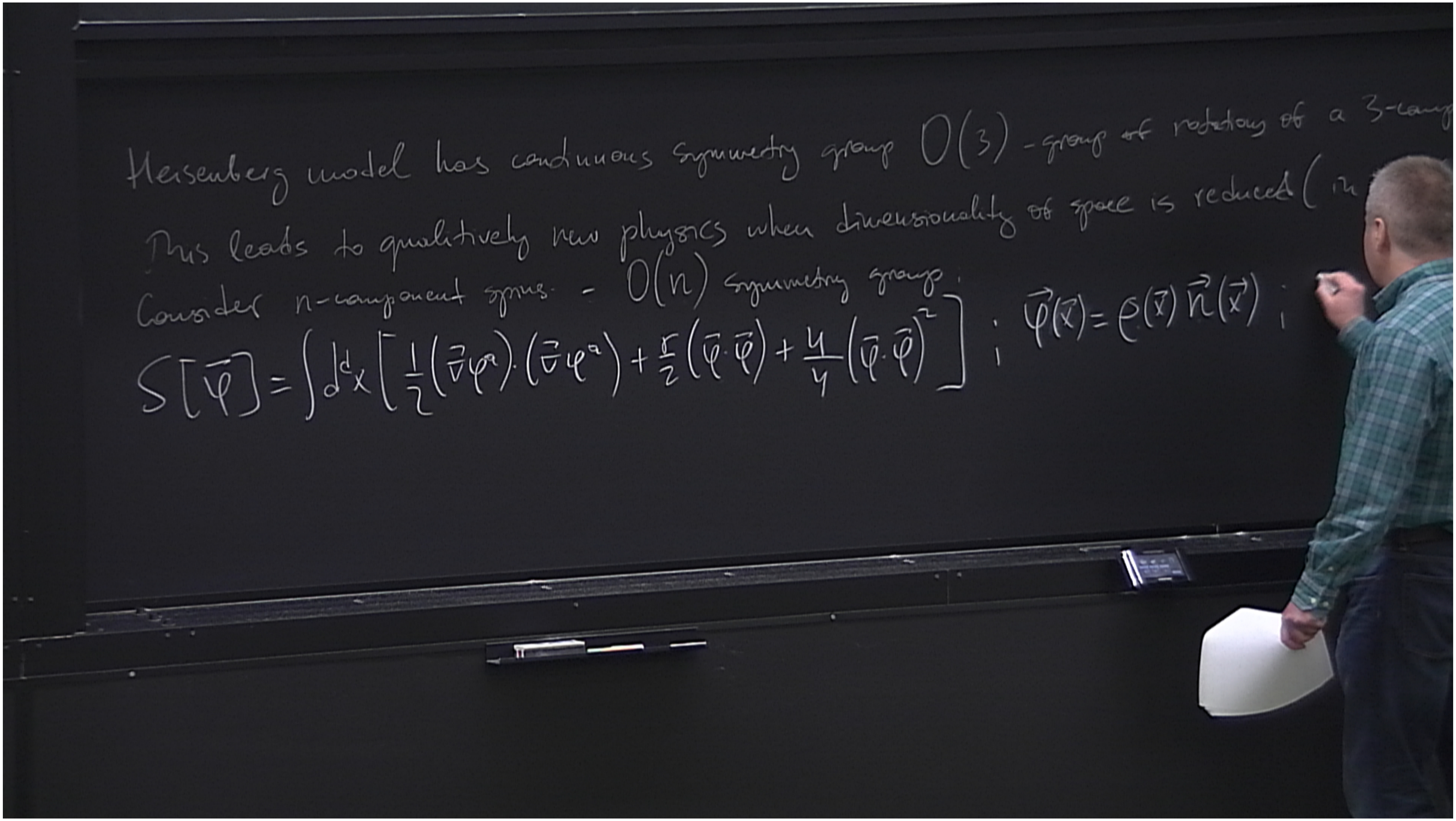

Heisenberg model has continuous symmetry group O(3) - group of rotations
This leads to qualitatively new physics when dimensionality of space is
Consider n-component spins. - O(n) symmetry group.
$S[\vec{\varphi}] = \int d^d x \left[\frac{1}{2}(\vec{\nabla}\varphi^a)\cdot(\vec{\nabla}\varphi^a) + \frac{r}{2}(\vec{\varphi}\cdot\vec{\varphi}) + \frac{u}{4}(\vec{\varphi}\cdot\vec{\varphi})^2 \right]$
$\vec{\varphi}(\vec{x}) =$

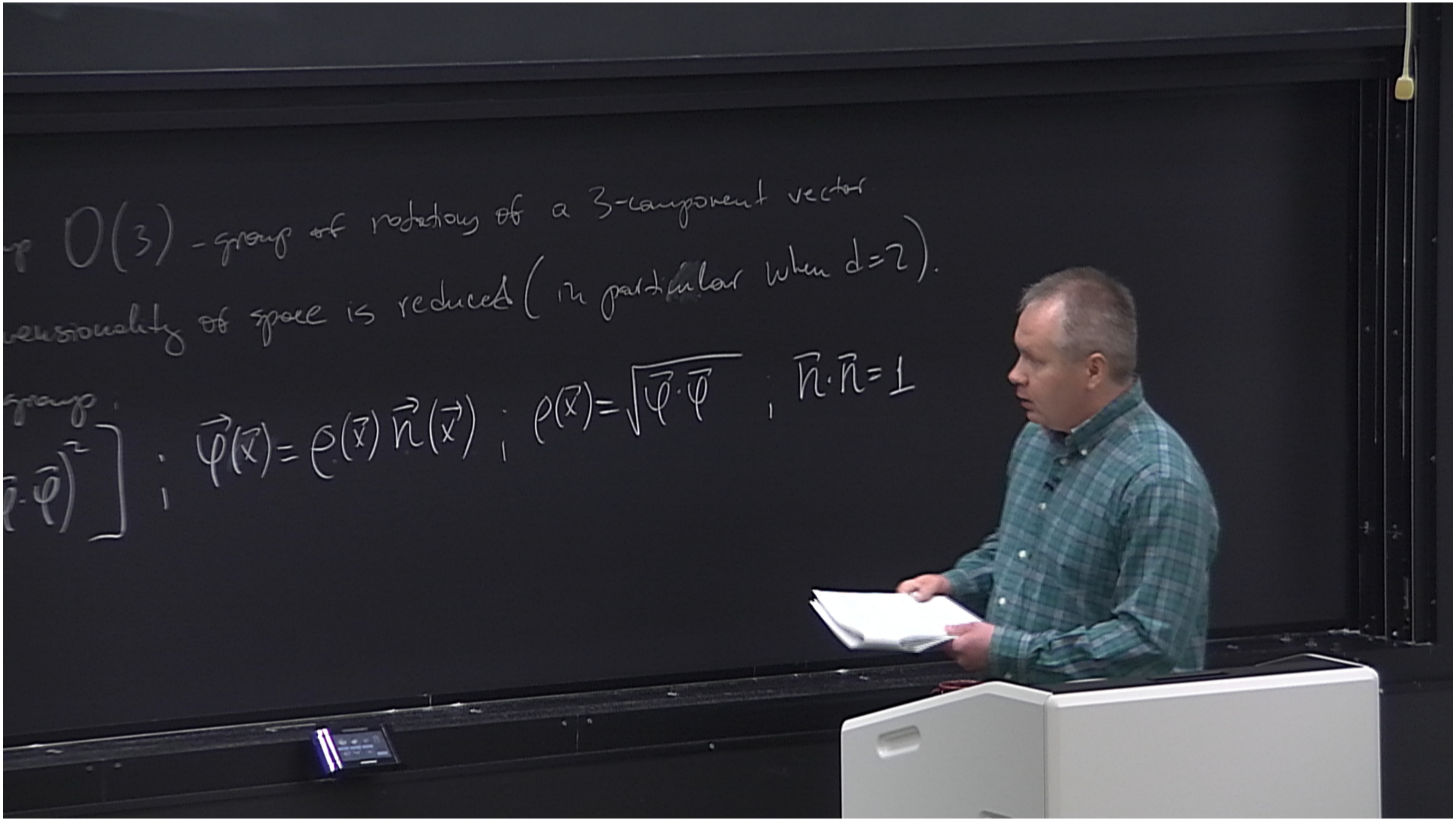
Heisenberg model has continuous symmetry group $O(3)$ - group of rotations

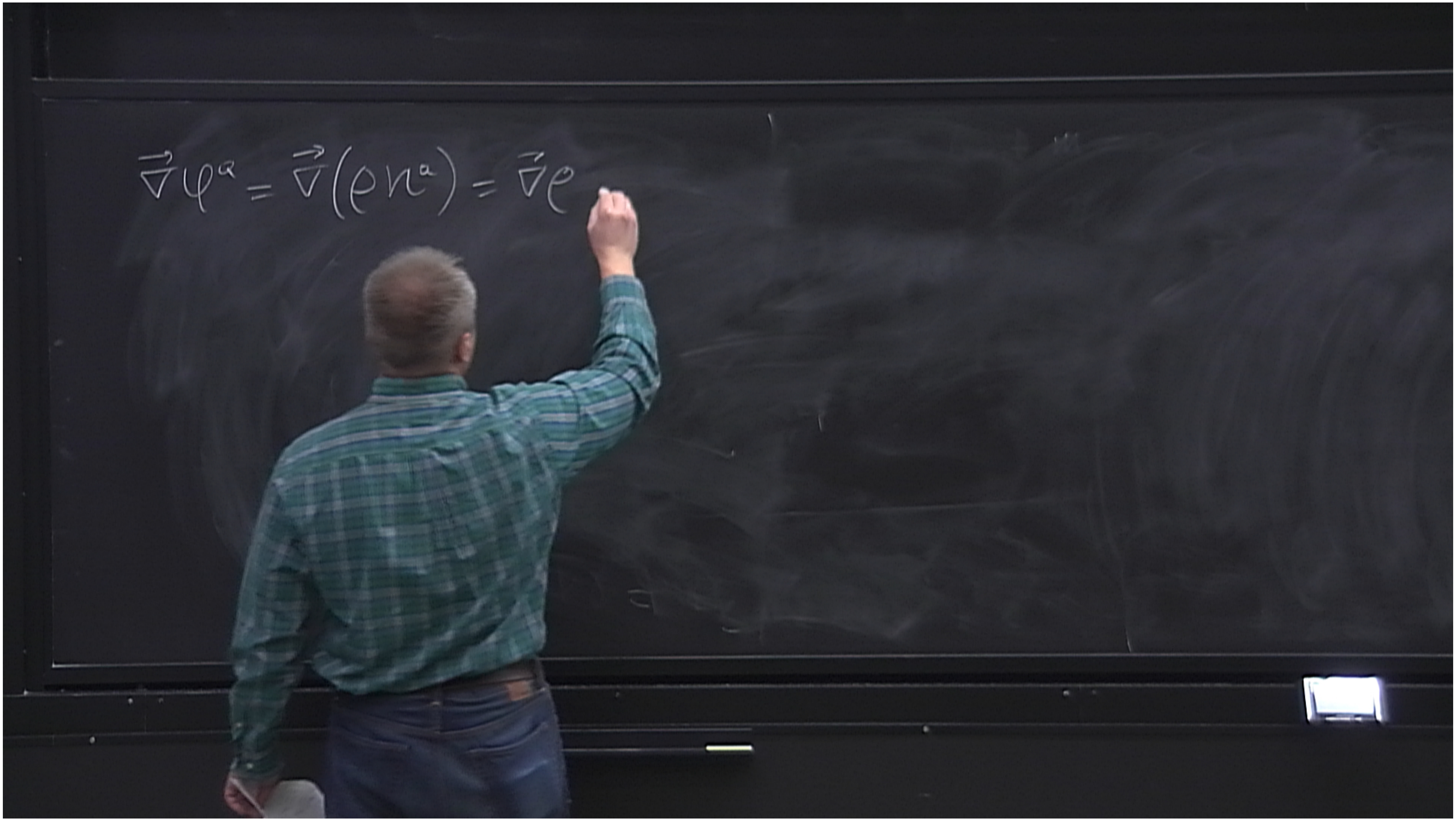
This leads to qualitatively new physics when dimensionality of space is reduc

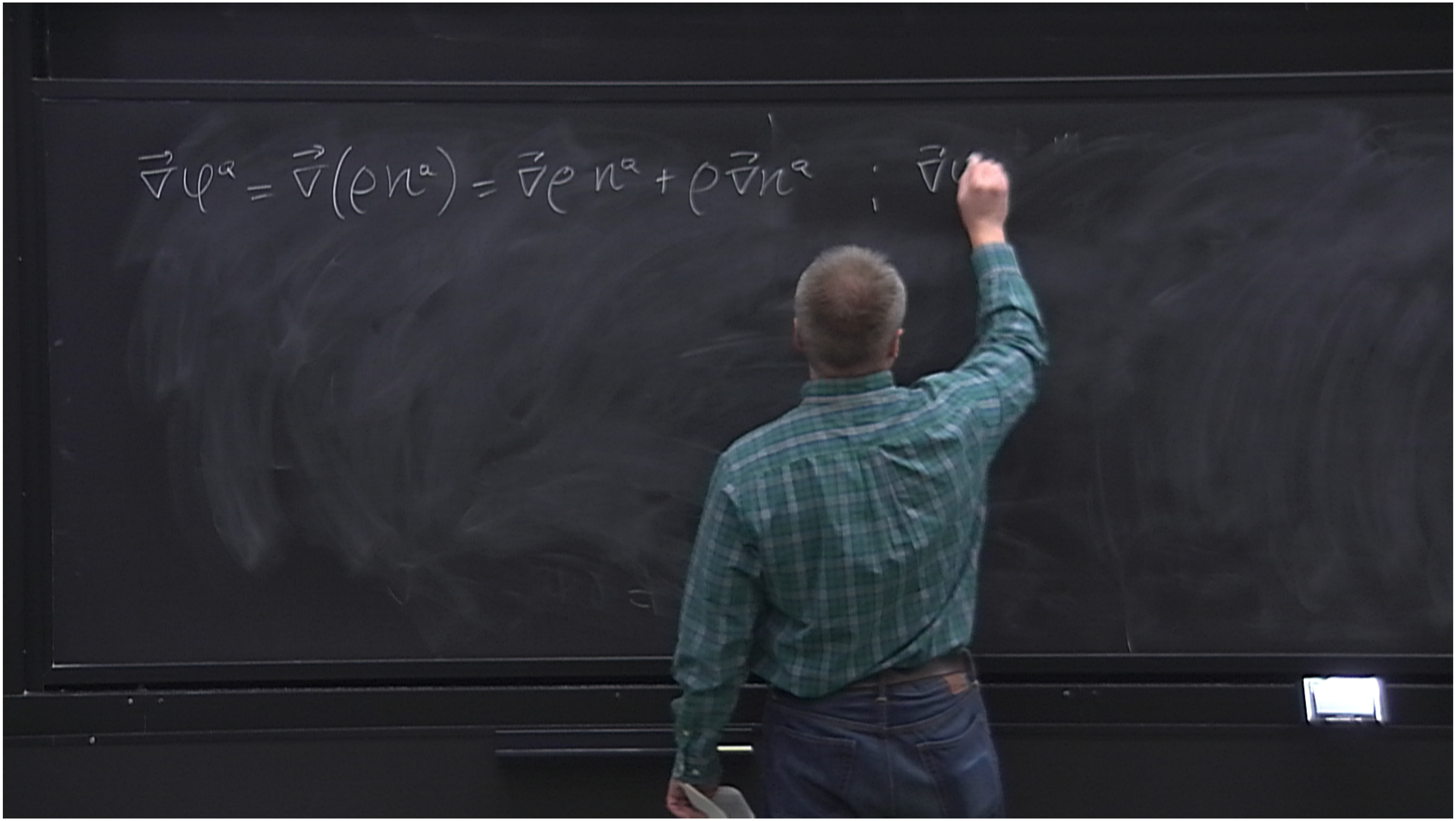
Consider n-component spins. - $O(n)$ symmetry group.

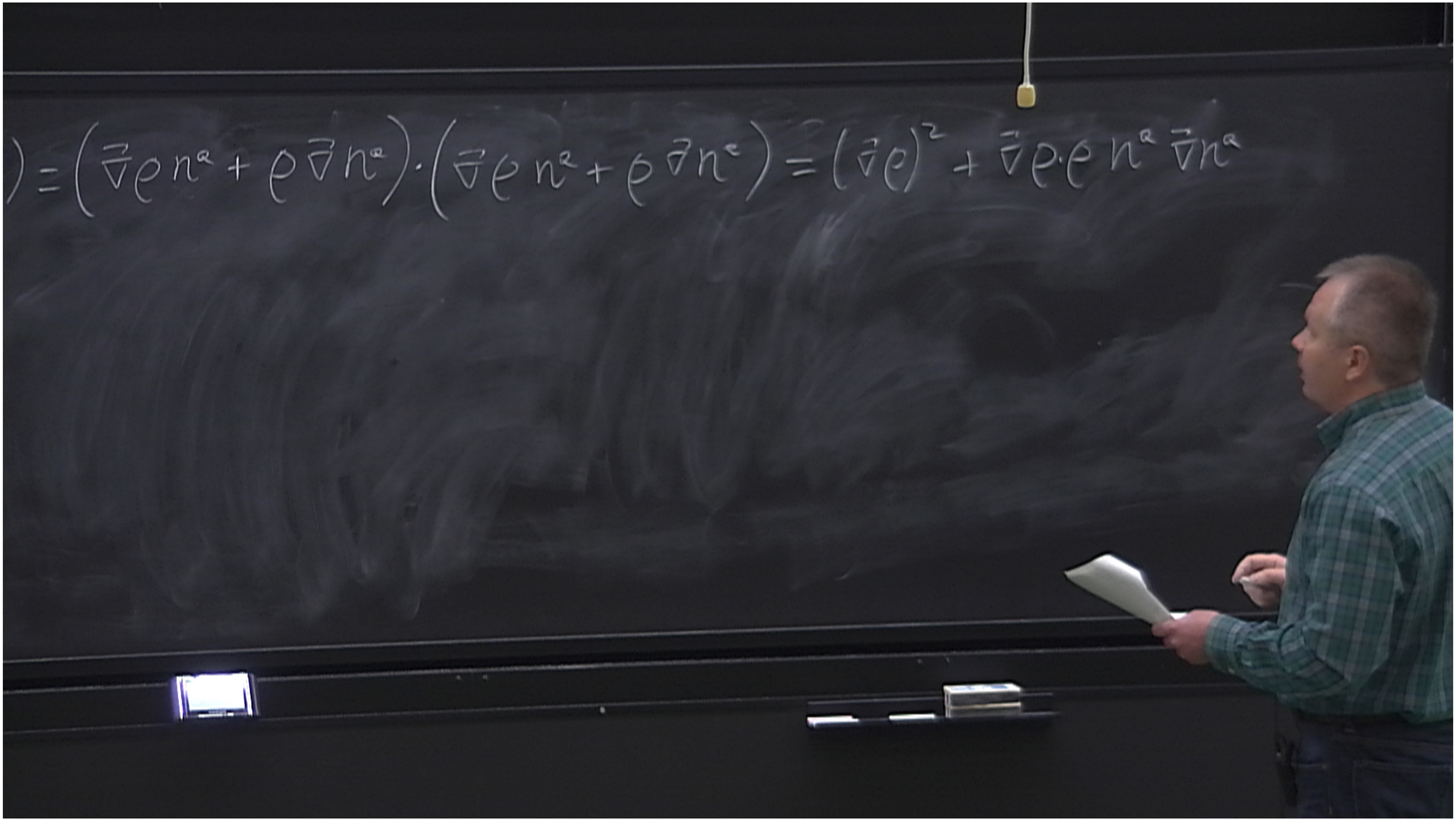
$$S[\vec{\varphi}] = \int d^dx \left[\frac{1}{2}(\vec{\nabla}\varphi^a)\cdot(\vec{\nabla}\varphi^a) + \frac{r}{2}(\vec{\varphi}\cdot\vec{\varphi}) + \frac{u}{4}(\vec{\varphi}\cdot\vec{\varphi})^2 \right] \; ; \; \vec{\varphi}(\vec{x}) = \rho(\vec{x})\,\vec{n}(\vec{x})$$


Heisenberg model has continuous symmetry group O(3) - group of rotations of a 3-comp
This leads to qualitatively new physics when dimensionality of space is reduced (in
Consider n-component spins - O(n) symmetry group.
$S[\vec{\varphi}] = \int d^dx \left[\frac{1}{2}(\vec{\nabla}\varphi^a)\cdot(\vec{\nabla}\varphi^a) + \frac{r}{2}(\vec{\varphi}\cdot\vec{\varphi}) + \frac{u}{4}(\vec{\varphi}\cdot\vec{\varphi})^2 \right]$; $\vec{\varphi}(\vec{x}) = \rho(\vec{x})\,\vec{n}(\vec{x})$;

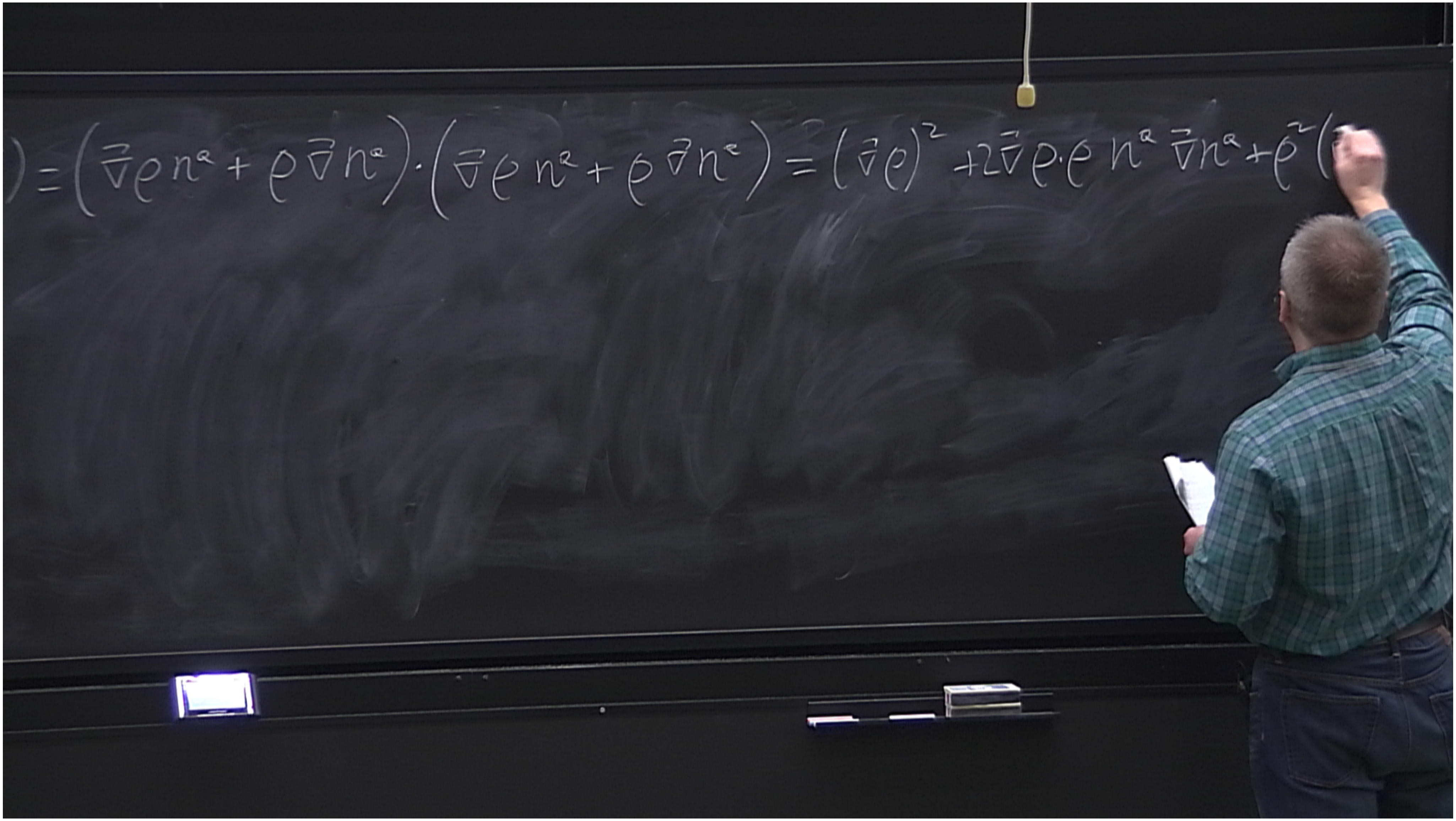

O(3) - group of rotations of a 3-component vector
ensionality of space is reduced (in particular when d=2).
group:
φ(x) = ρ(x) n(x) ; ρ(x) = √(φ·φ) ; n·n = 1

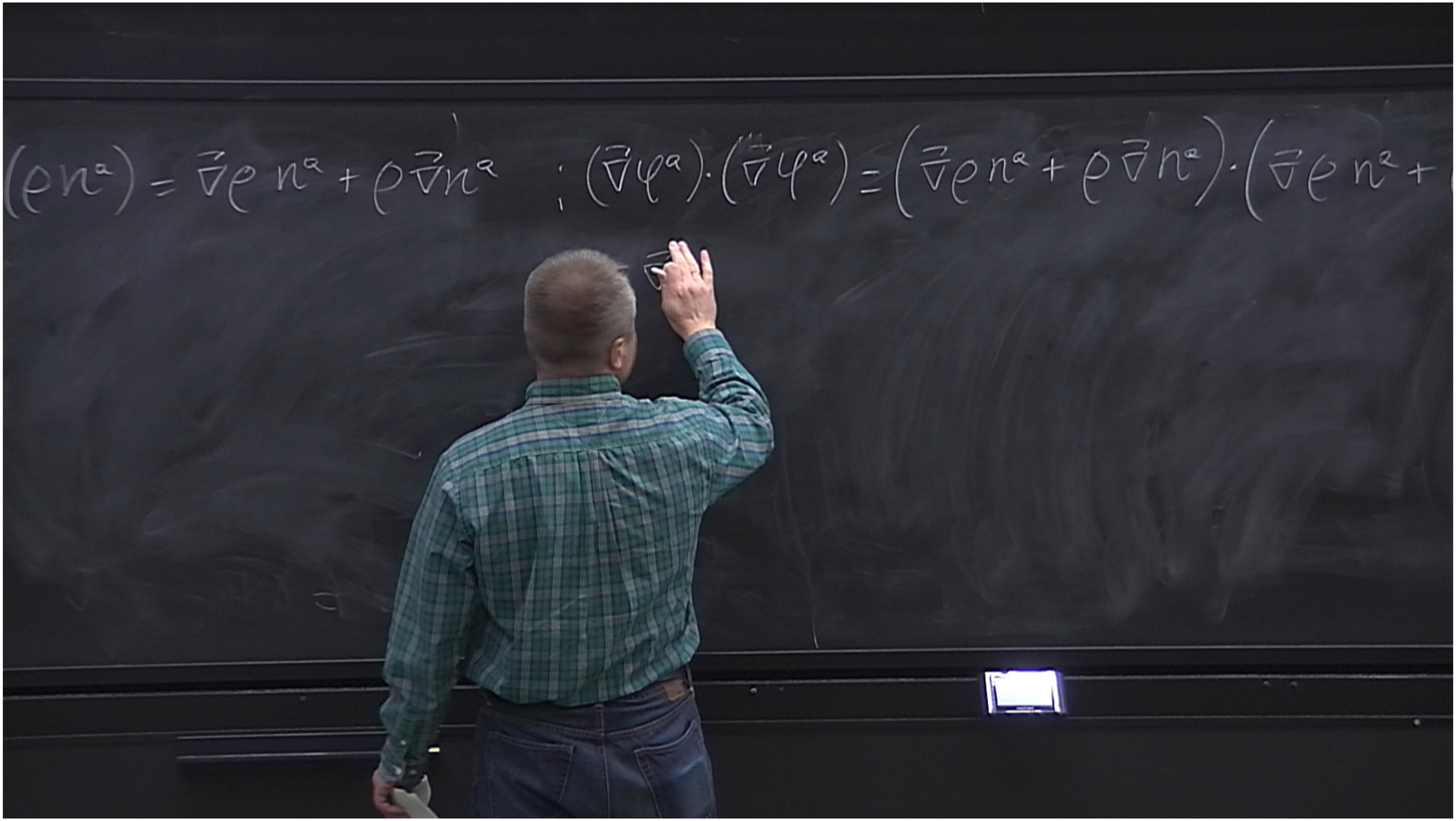

$\vec{\nabla}\varphi^a = \vec{\nabla}(\rho n^a) = \vec{\nabla}\rho$


$\vec{\nabla}\varphi^a = \vec{\nabla}(\rho n^a) = \vec{\nabla}\rho\, n^a + \rho\vec{\nabla} n^a$;



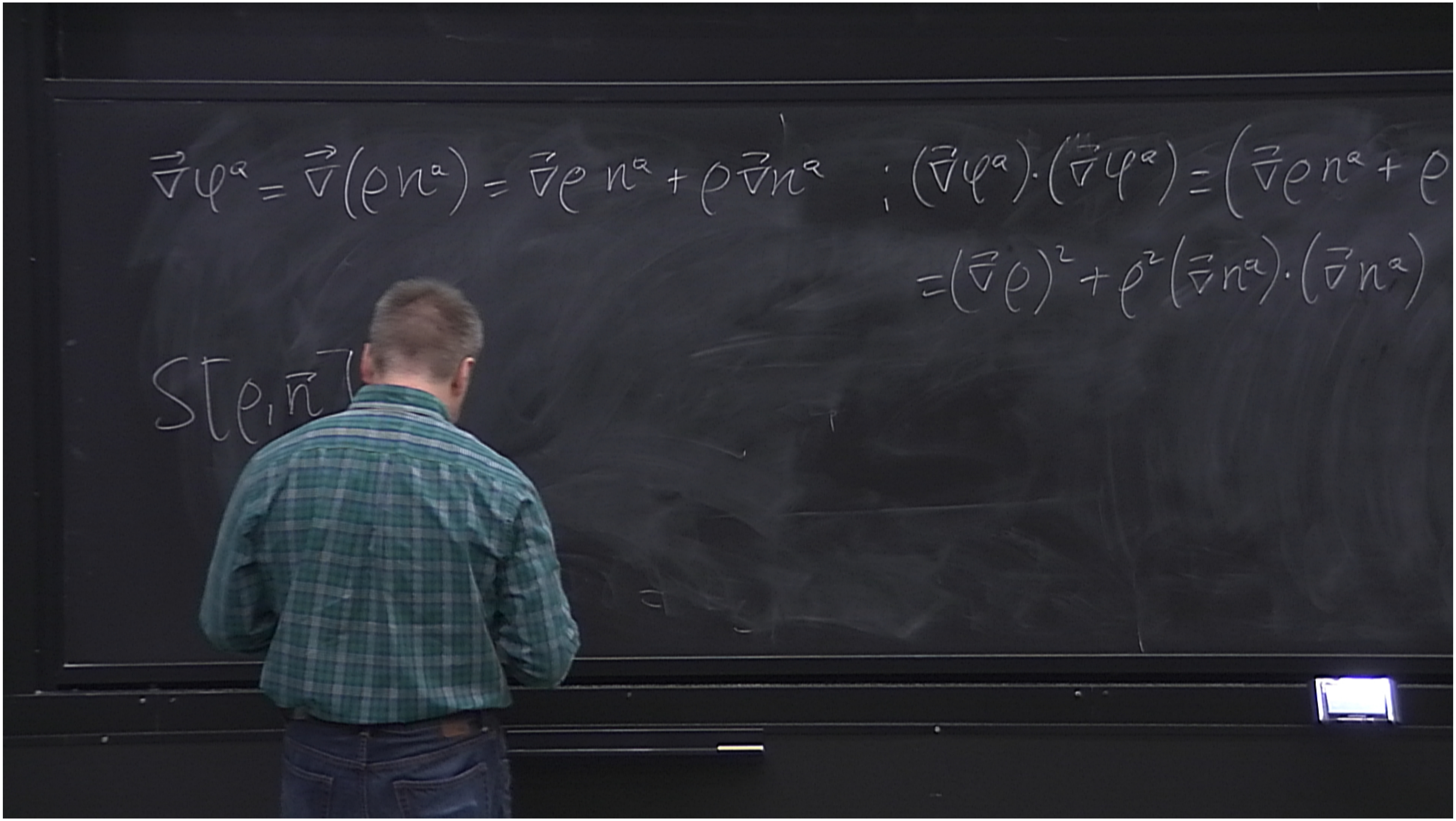
$) = (\vec\nabla\rho\, n^a + \rho\, \vec\nabla n^a)\cdot(\vec\nabla\rho\, n^a + \rho\, \vec\nabla n^a) = (\vec\nabla\rho)^2 + \vec\nabla\rho\cdot\rho\, n^a\, \vec\nabla n^a$


$) = (\vec{\nabla}\rho\, n^a + \rho\,\vec{\nabla} n^a)\cdot(\vec{\nabla}\rho\, n^a + \rho\,\vec{\nabla} n^a) = (\vec{\nabla}\rho)^2 + 2\vec{\nabla}\rho\cdot\rho\, n^a\,\vec{\nabla} n^a + \rho^2($


$(\rho n^a) = \vec{\nabla}\rho\, n^a + \rho \vec{\nabla} n^a \quad ; (\vec{\nabla}\varphi^a)\cdot(\vec{\nabla}\varphi^a) = (\vec{\nabla}\rho\, n^a + \rho\vec{\nabla} n^a)\cdot(\vec{\nabla}\rho\, n^a +$

$$(\rho n^a) = \vec{\nabla}\rho\, n^a + \rho\vec{\nabla}n^a \quad ; \quad (\vec{\nabla}\varphi^a)\cdot(\vec{\nabla}\varphi^a) = (\vec{\nabla}\rho\, n^a + \rho\vec{\nabla}n^a)\cdot(\vec{\nabla}\rho\, n^a +$$

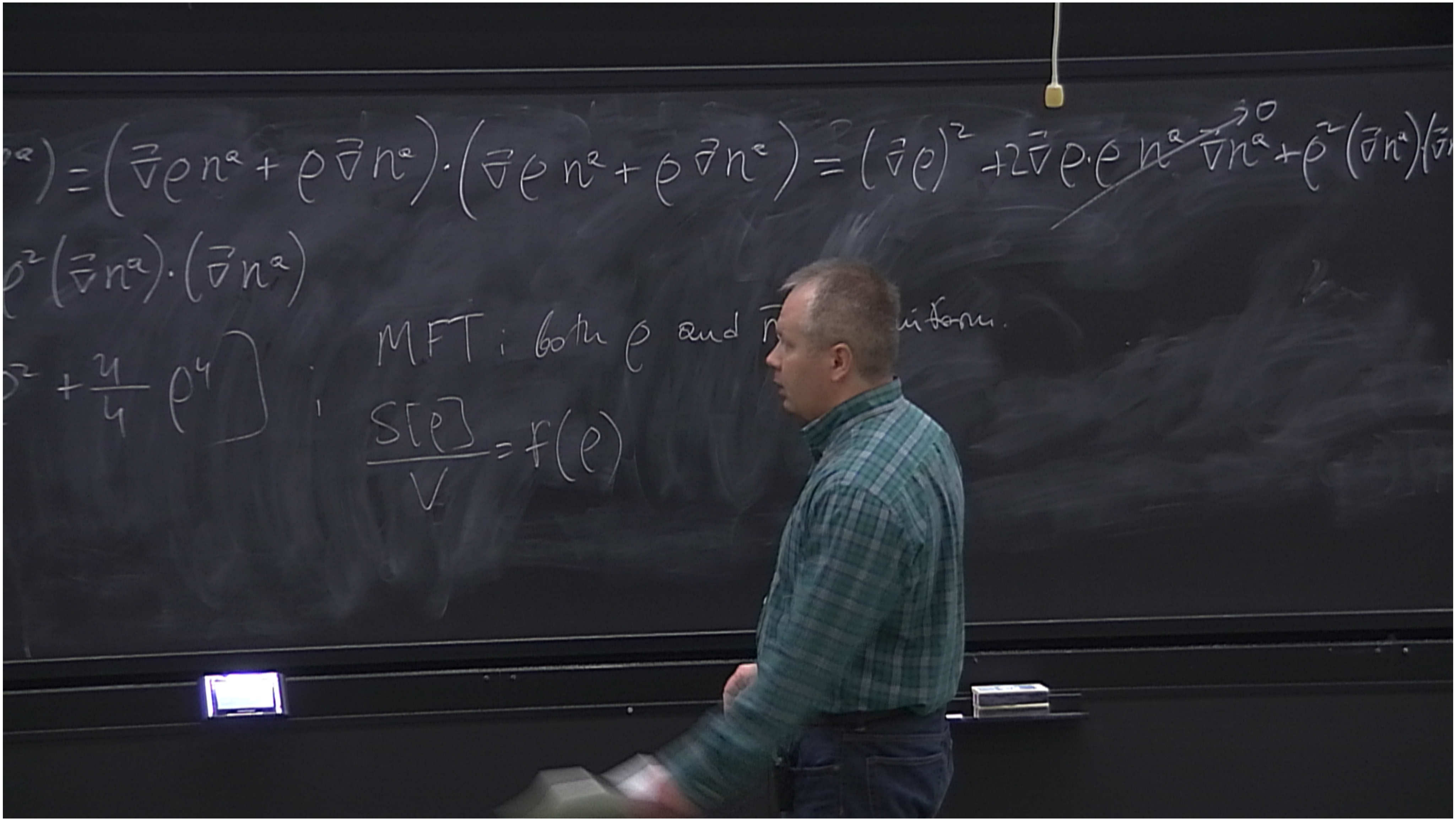
$$= (\vec{\nabla}\rho)^2 + \rho^2(\vec{\nabla}n^a)\cdot(\vec{\nabla}n^a)$$

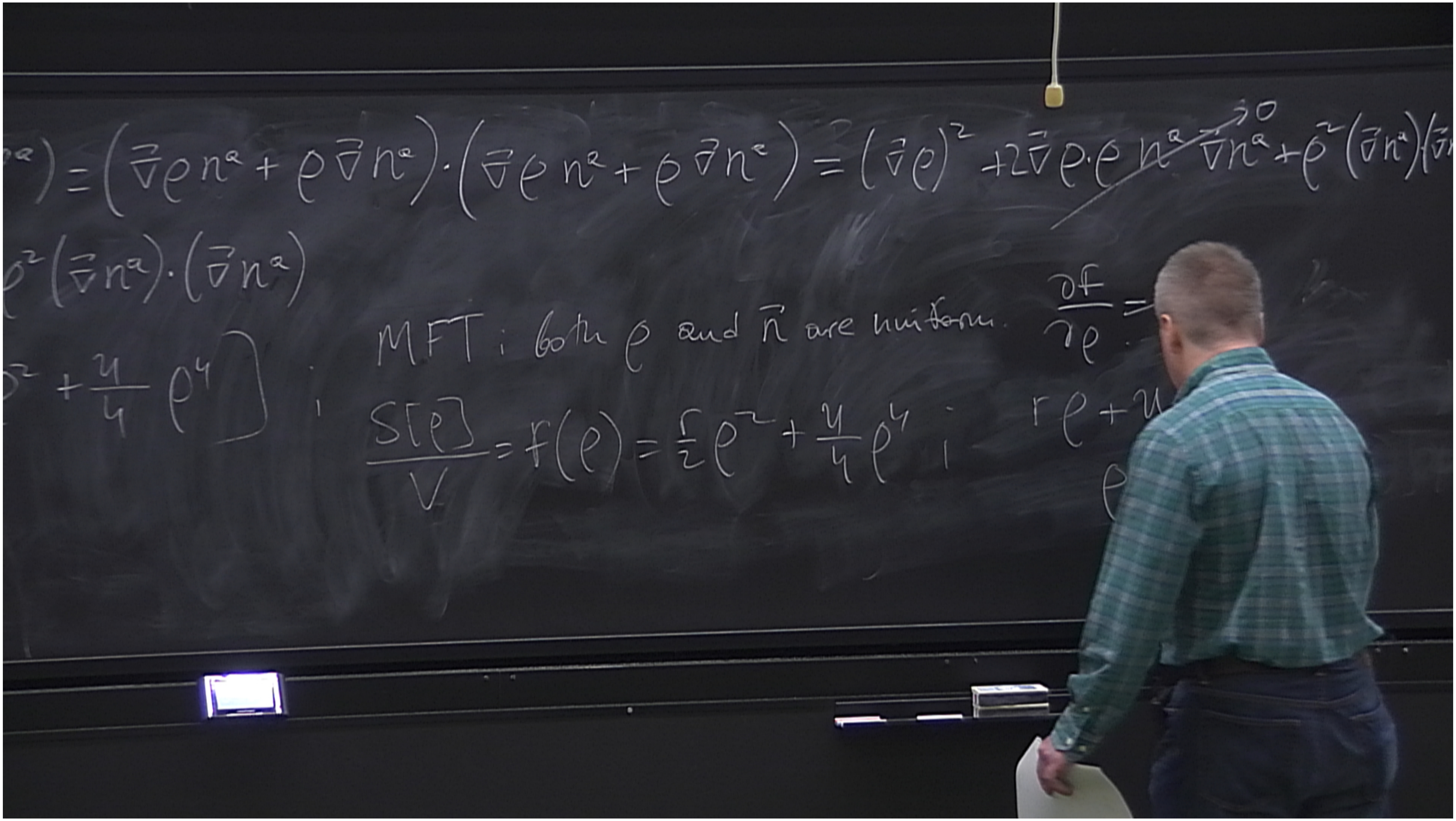

$\vec{\nabla}\varphi^a = \vec{\nabla}(\rho n^a) = \vec{\nabla}\rho\, n^a + \rho\vec{\nabla}n^a$; $(\vec{\nabla}\varphi^a)\cdot(\vec{\nabla}\varphi^a) = (\vec{\nabla}\rho\, n^a + \rho$
$= (\vec{\nabla}\rho)^2 + \rho^2(\vec{\nabla}n^a)\cdot(\vec{\nabla}n^a)$
$S[\rho, \vec{n}]$

$$\vec{\nabla}\varphi^a = \vec{\nabla}(\rho n^a) = \vec{\nabla}\rho\, n^a + \rho\vec{\nabla}n^a \quad ; \quad (\vec{\nabla}\varphi^a)\cdot(\vec{\nabla}\varphi^a) = (\vec{\nabla}\rho\, n^a + \rho$$

$$= (\vec{\nabla}\rho)^2 + \rho^2(\vec{\nabla}n^a)\cdot(\vec{\nabla}n^a)$$

$$S[\rho, \vec{n}] = \int d^dx \left[\frac{1}{2}\rho^2 (\vec{\nabla}n^a)\cdot(\vec{\nabla}n^a) + \frac{1}{2}(\vec{\nabla}\rho)^2 + \frac{r}{2}\rho^2 + \frac{u}{4}\rho^4 \right]$$


MFT: both ρ and
S[ρ]/V = f(ρ)


MFT: both ρ and n̄ are uniform.
S[ρ]/V = f(ρ) = r/2 ρ² + u/4 ρ⁴ ;
∂f/∂ρ =

$$\ldots n^a + \rho\vec\nabla n^a)\cdot(\vec\nabla\rho\, n^a + \rho\vec\nabla n^a) = (\vec\nabla\rho)^2 + 2\vec\nabla\rho\cdot\rho\,\underbrace{\cancel{n^a\,\vec\nabla n^a}}_{\to 0} + \rho^2(\vec\nabla n^a)\cdot(\vec\nabla n^a)$$

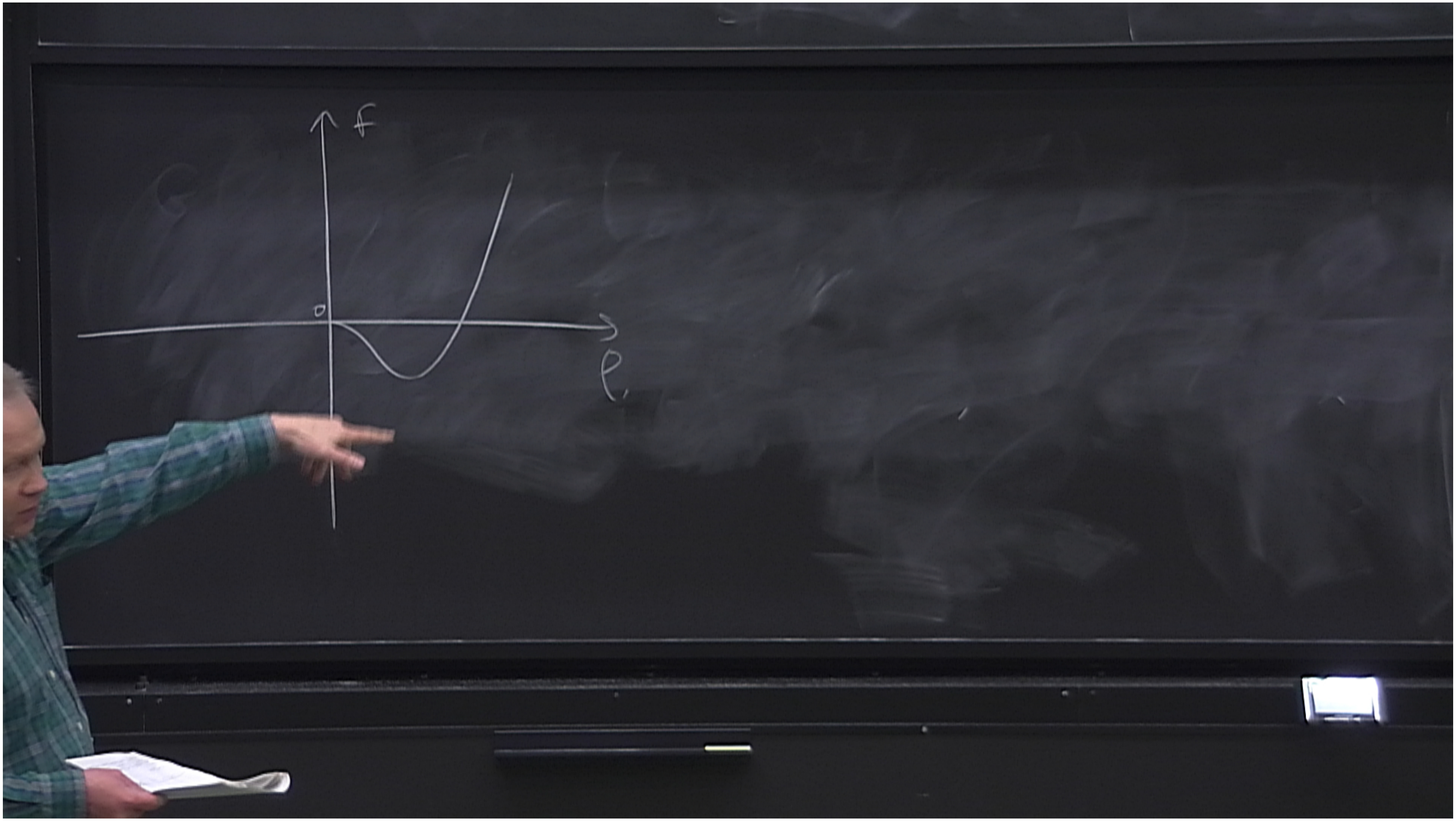
$\ldots\vec\nabla n^a)$

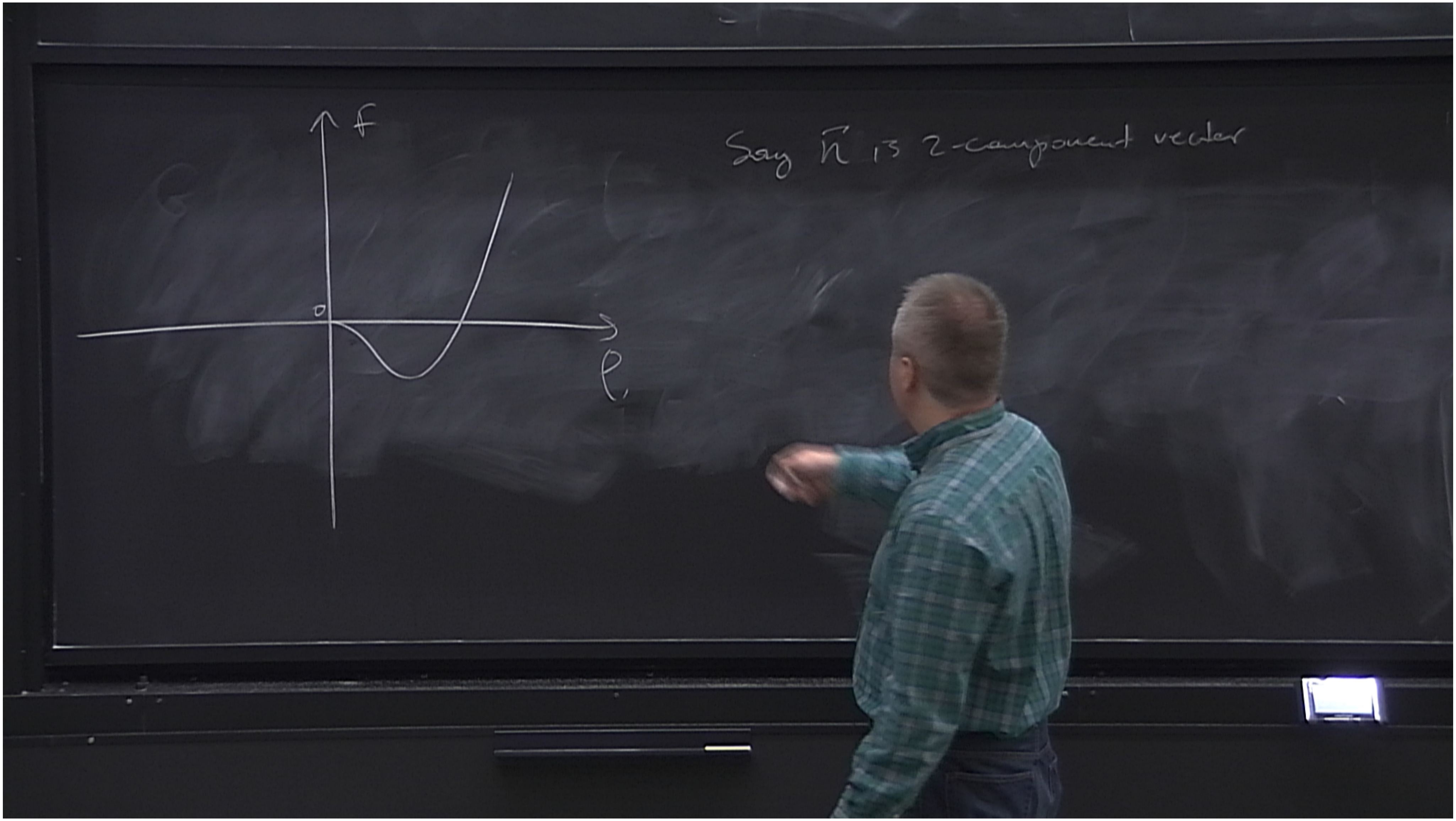
MFT: both ρ and $\vec n$ are uniform. $\frac{\partial f}{\partial \rho} = 0$

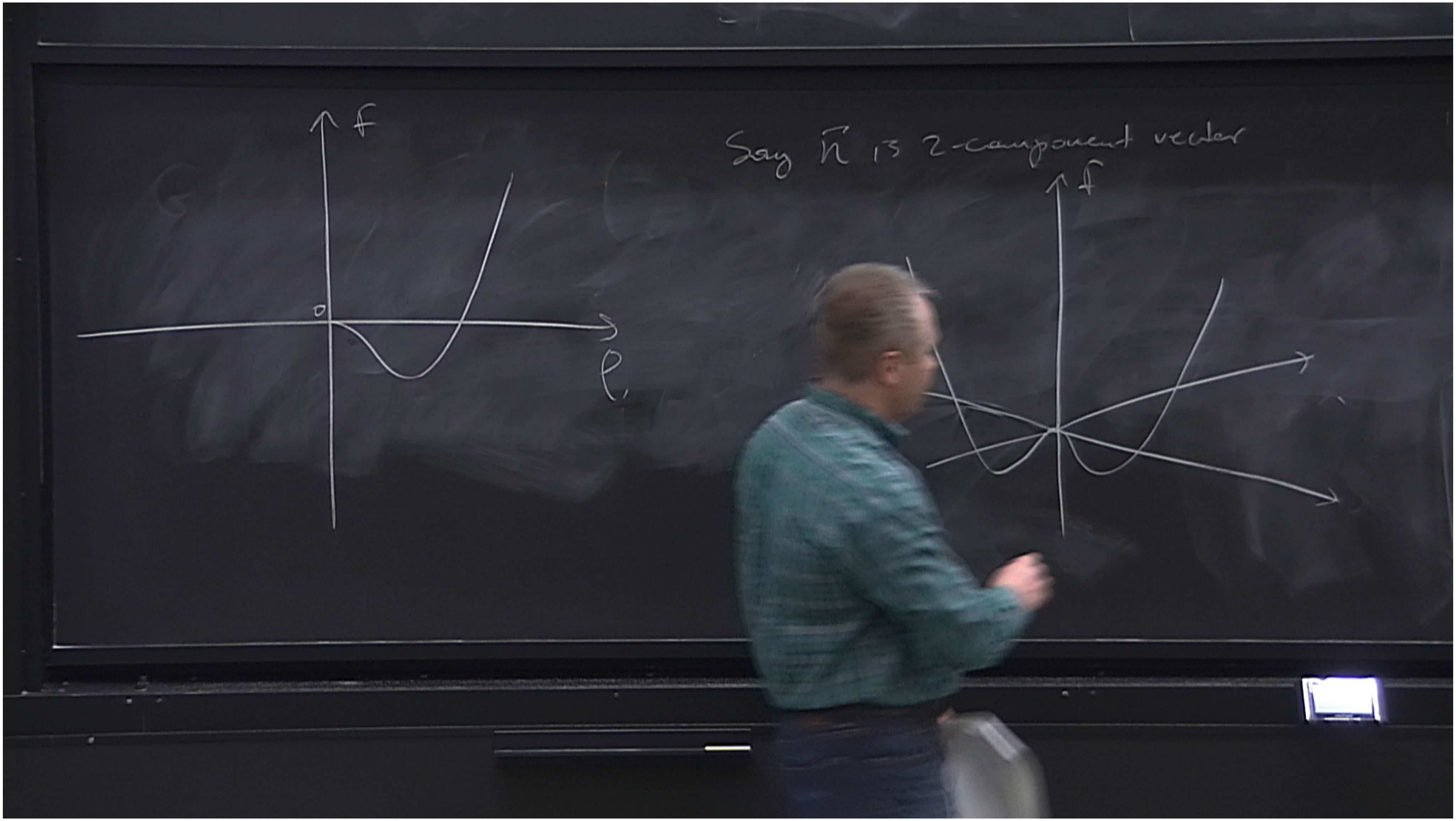
$$\frac{S[\rho]}{V} = f(\rho) = \frac{r}{2}\rho^2 + \frac{u}{4}\rho^4 ;$$

$$r\rho + u\rho^3 = 0$$

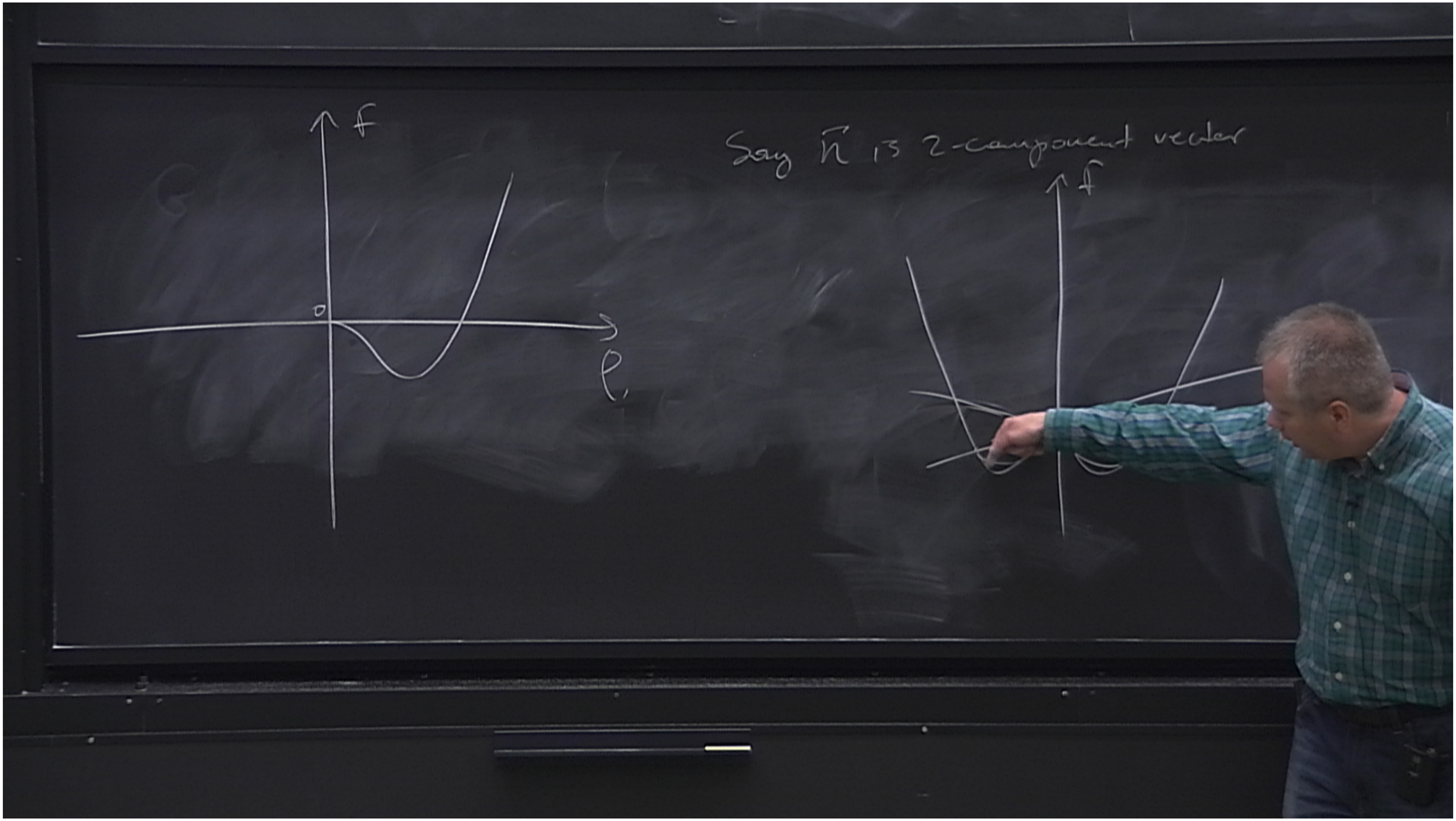
$$\rho = \sqrt{-\frac{r}{u}}$$


f
0
e


f
0
ρ
Say $\vec{n}$ is 2-component vector



f
0
ρ
Say $\vec{n}$ is 2-component vector
f


f
0
ρ
Say $\vec{n}$ is 2-component vector
f