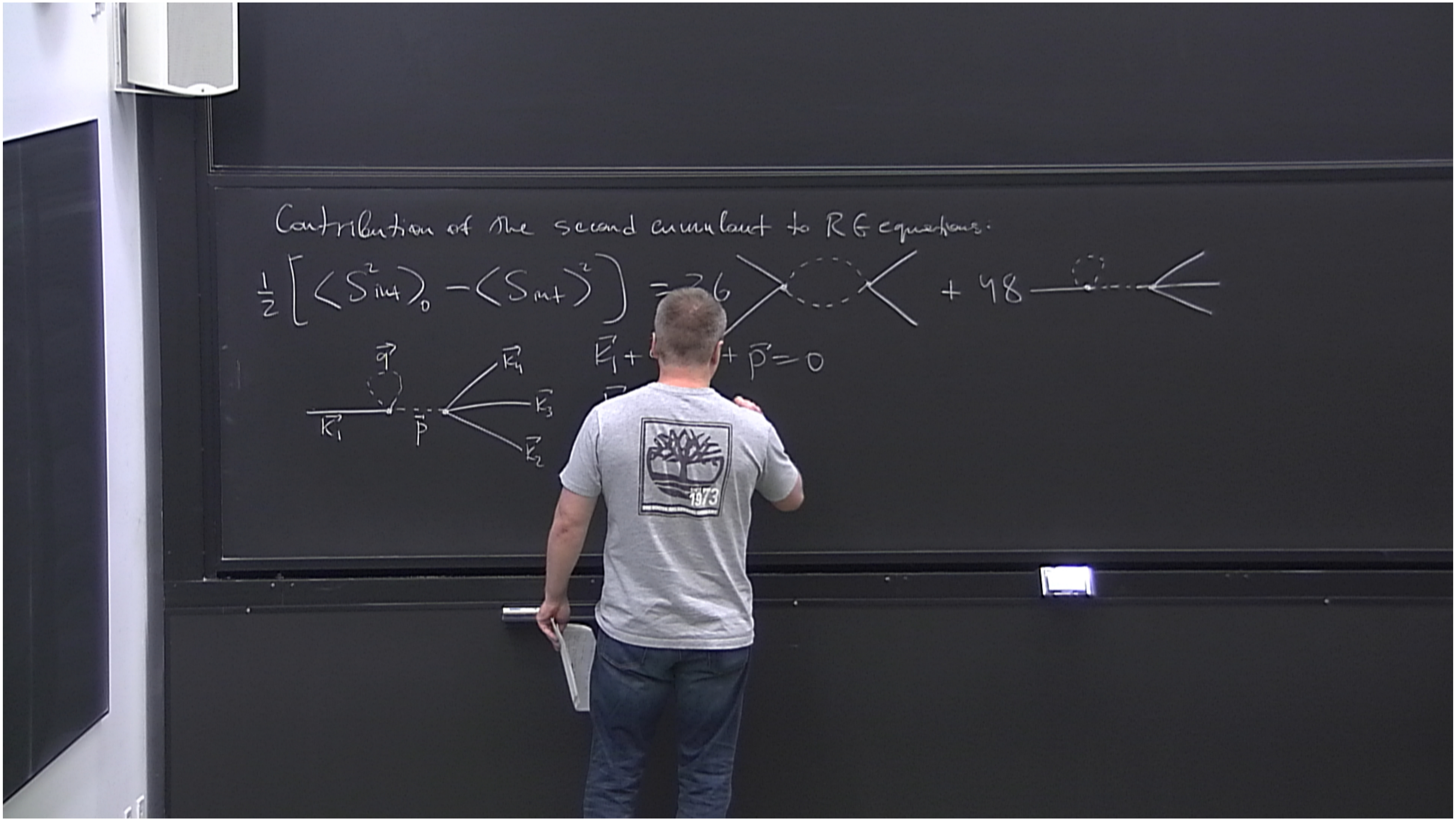
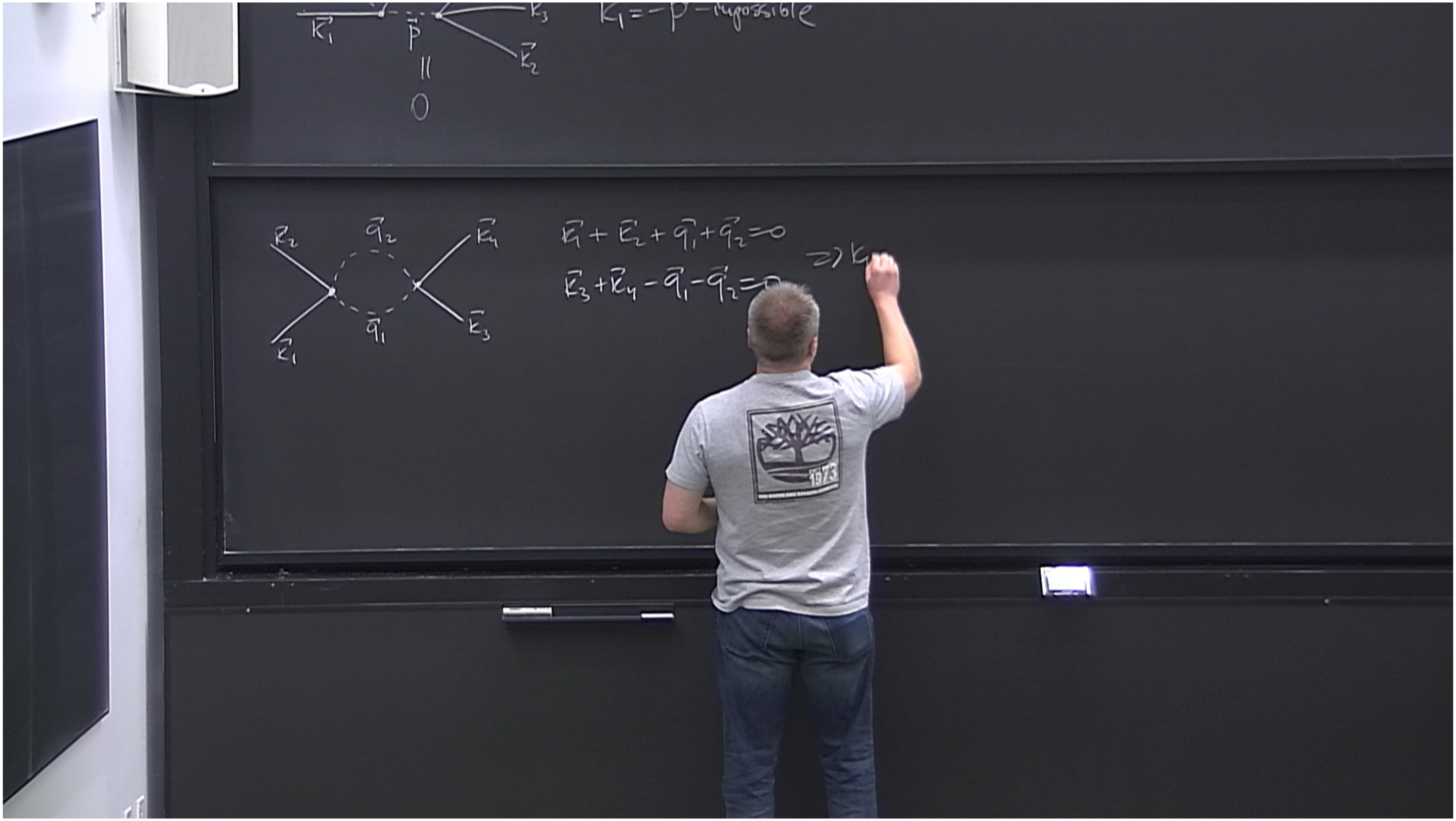
Title: 14/15 PSI - Statistical Mechanics - Lecture 9

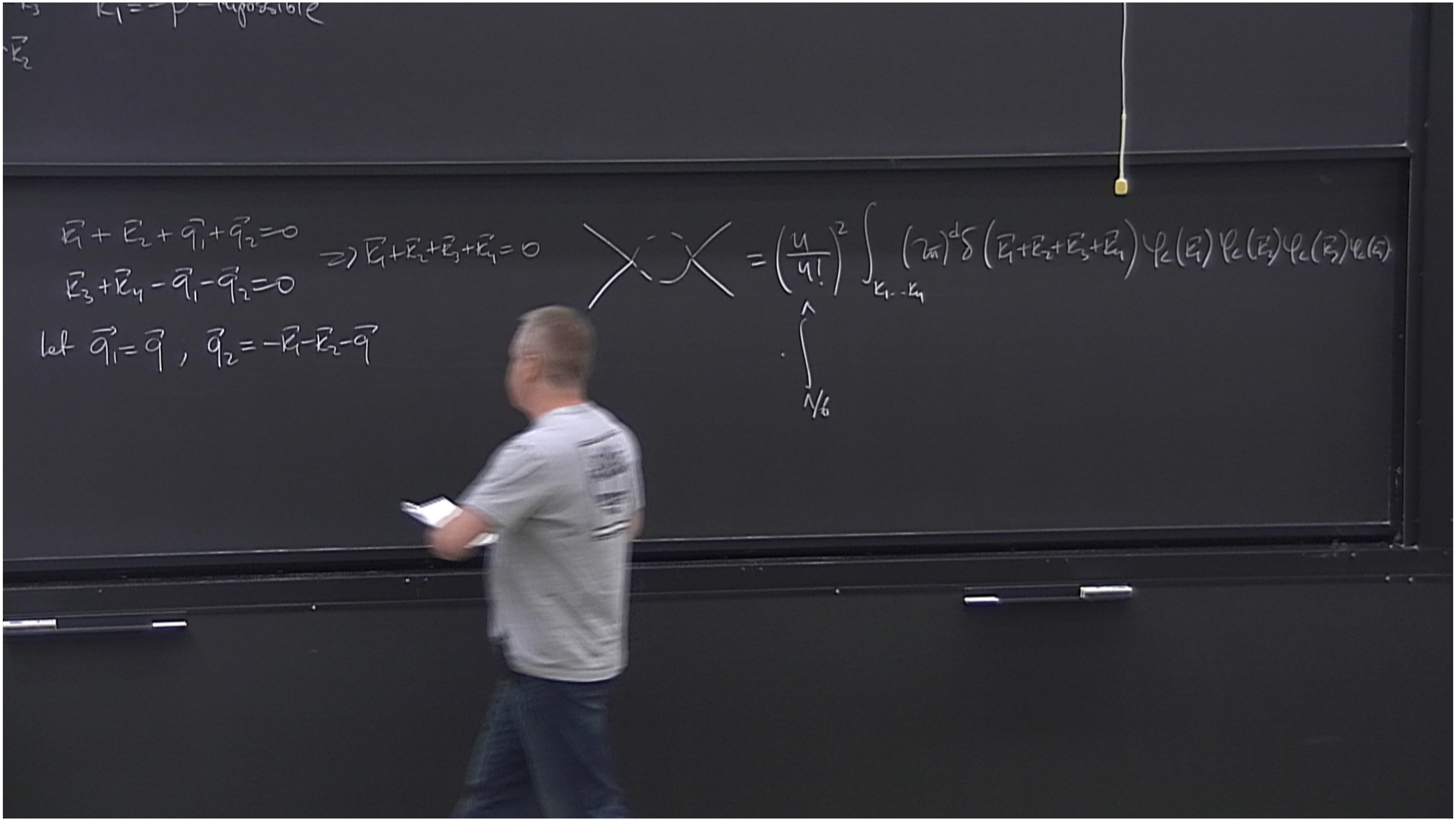
Date: Oct 17, 2014 10:45 AM

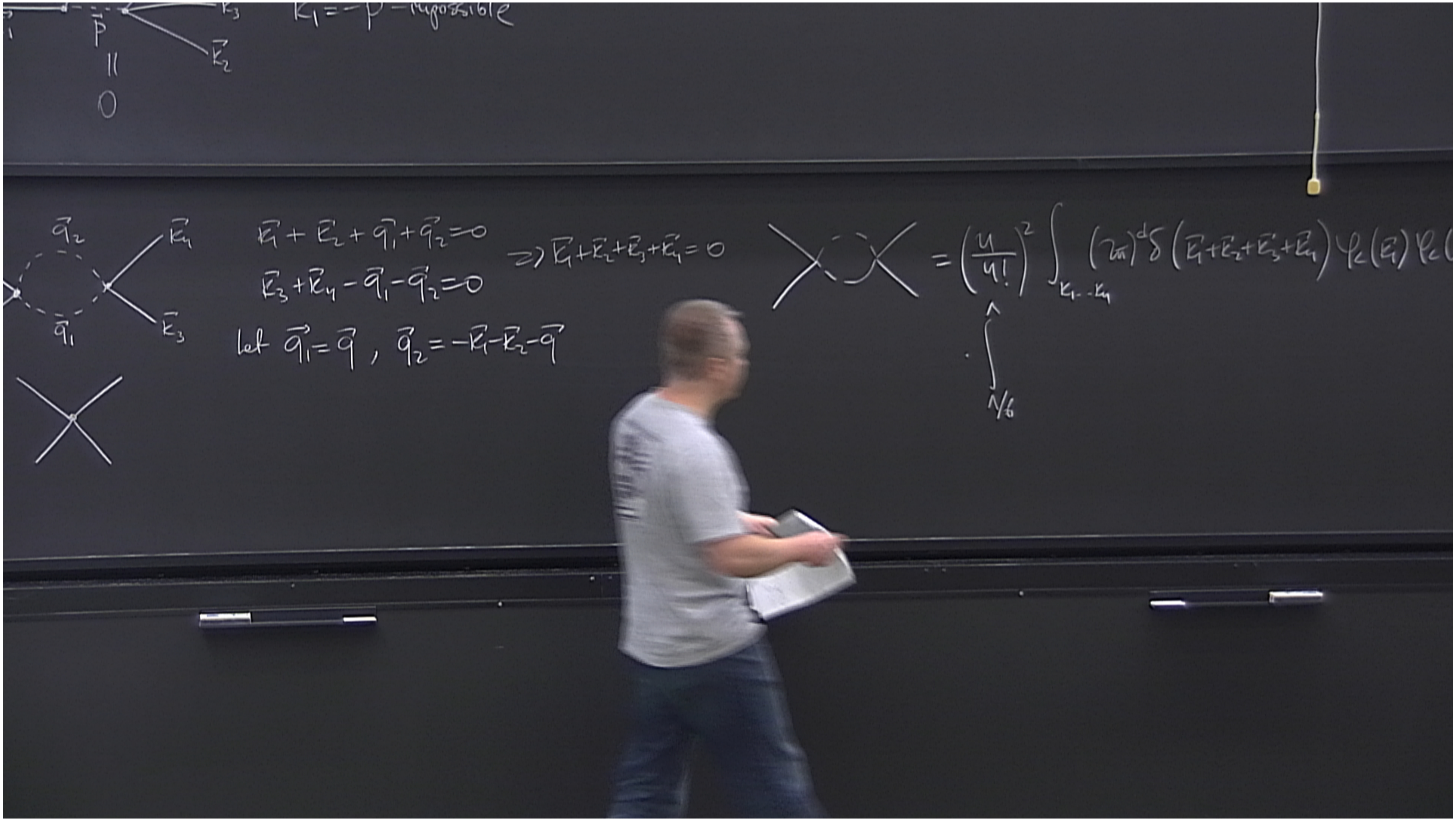
URL: http://pirsa.org/14100095

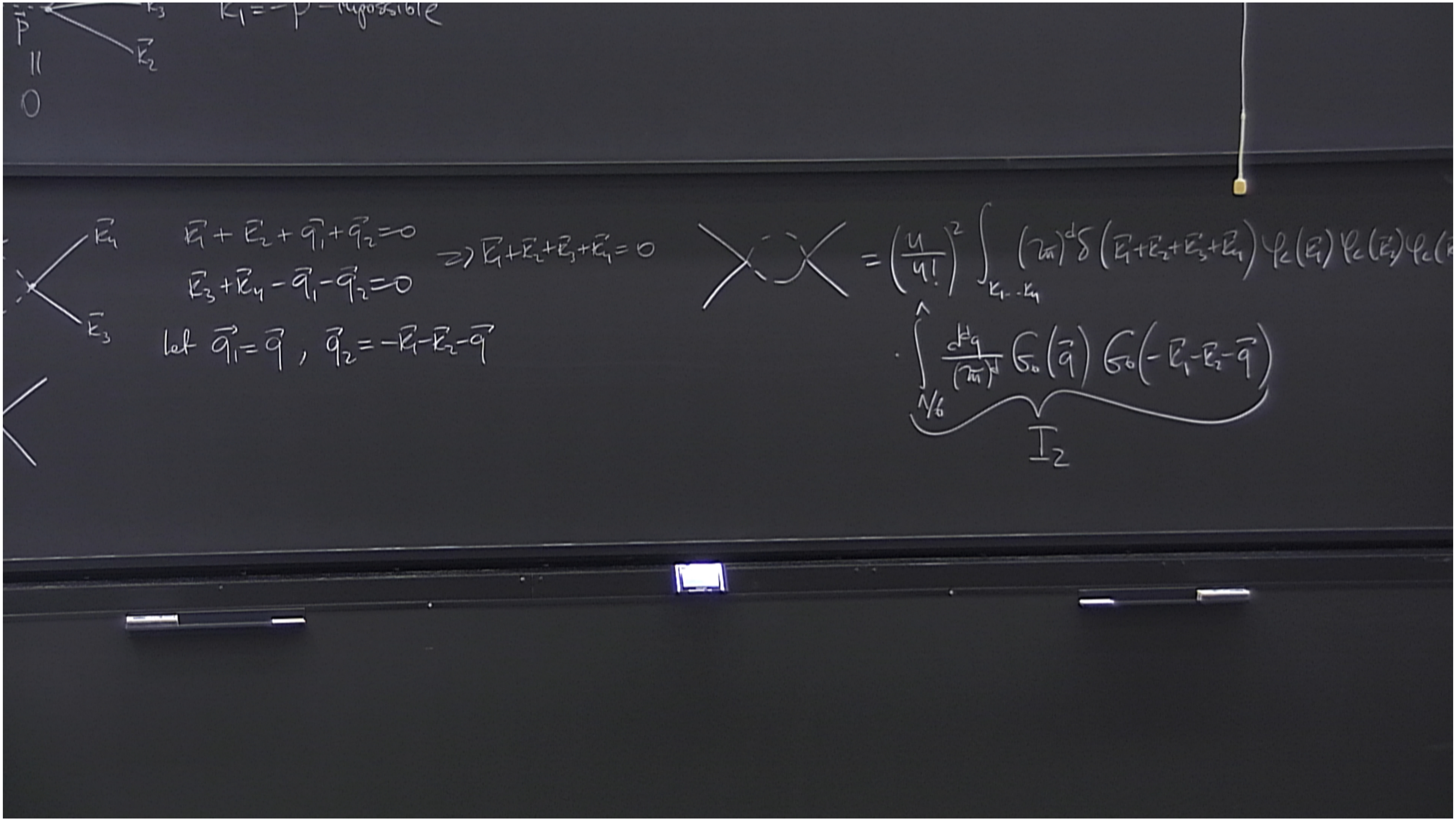
Abstract:

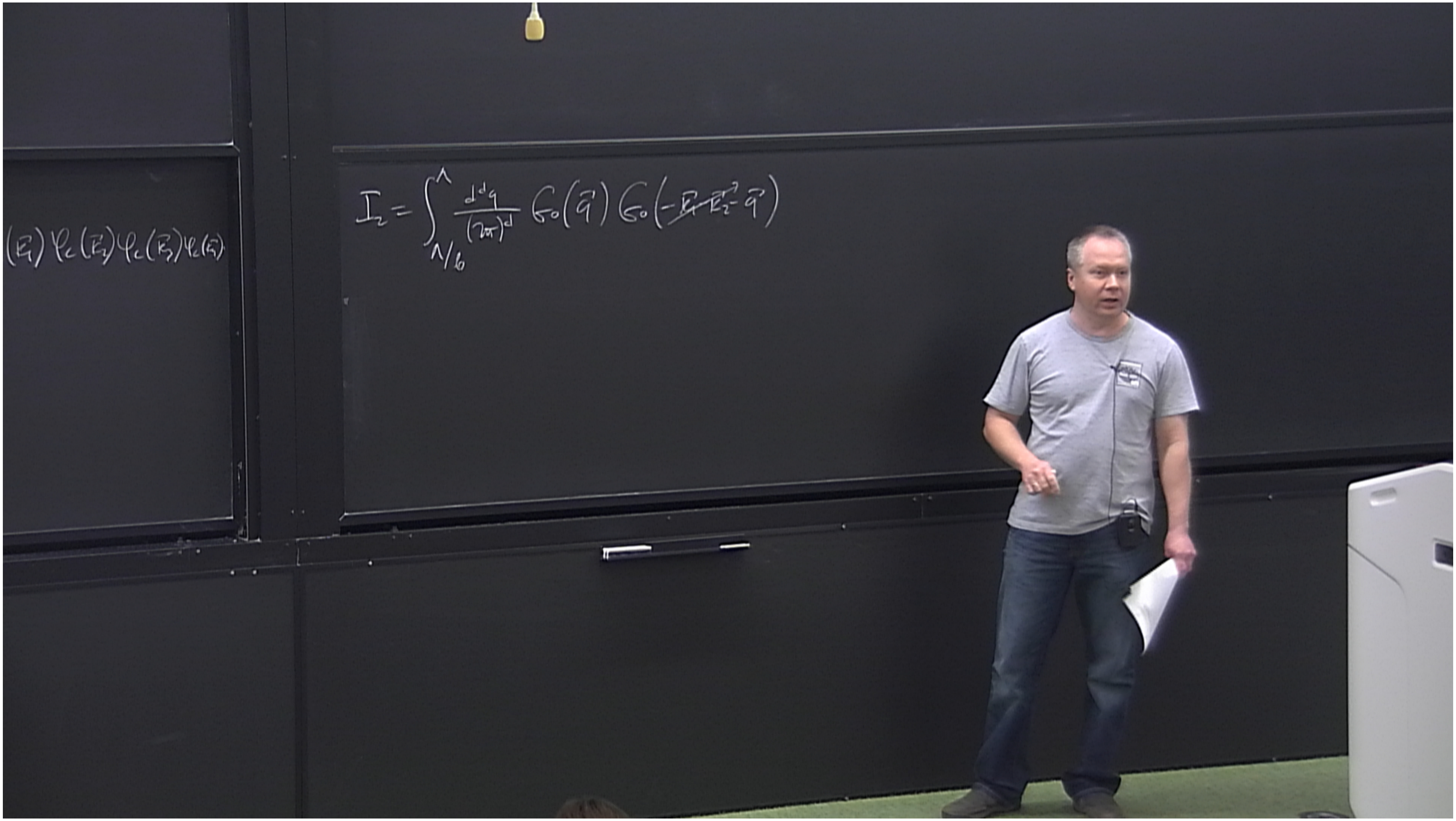

Contribution of the second cumulant to RG equations:
1/2 [⟨S²int⟩0 − ⟨Sint⟩²] = 36 + 48

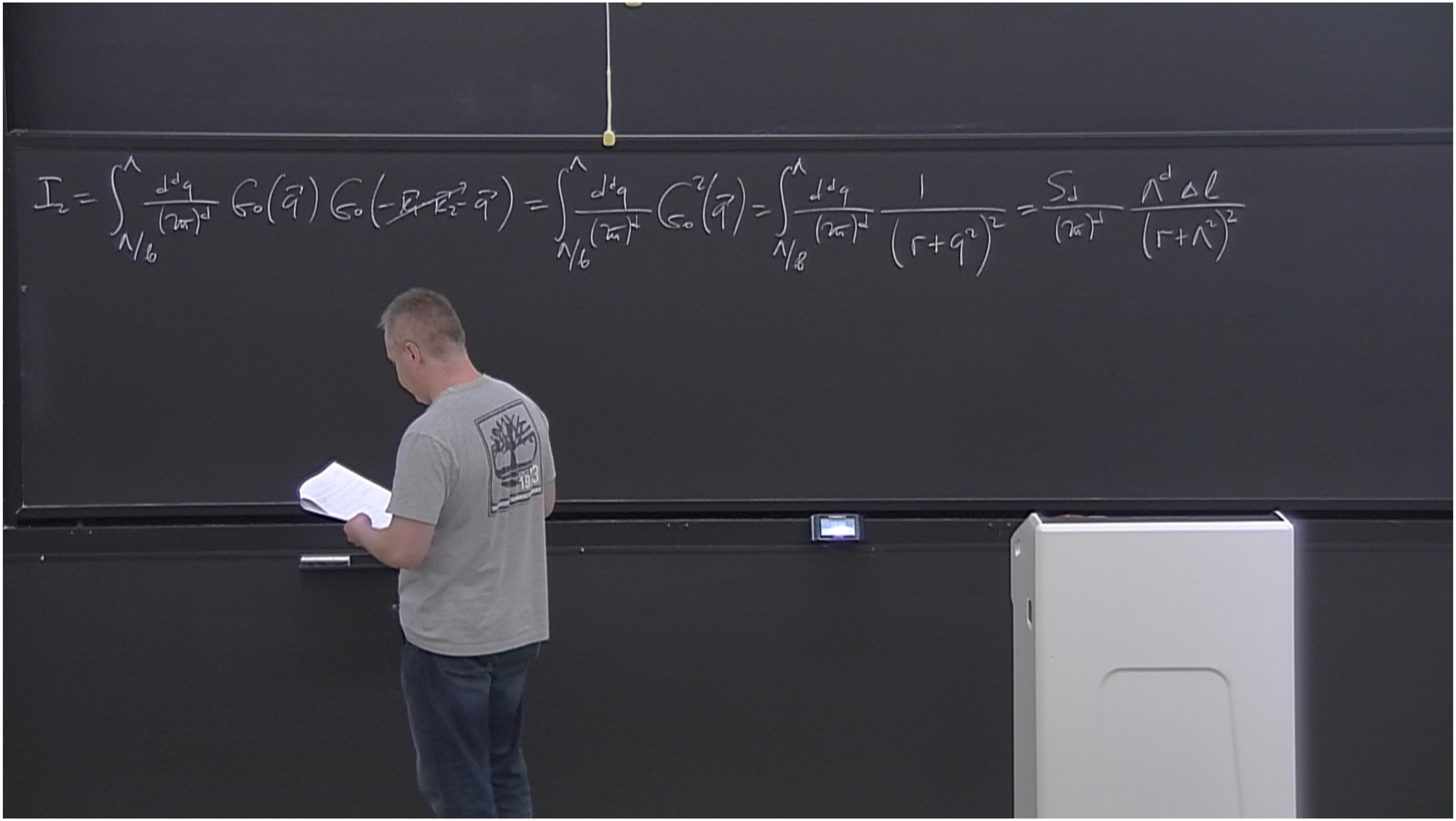

k1 = −p − impossible
k1 + k2 + q1 + q2 = 0
k3 + k4 − q1 − q2 = 0
1973

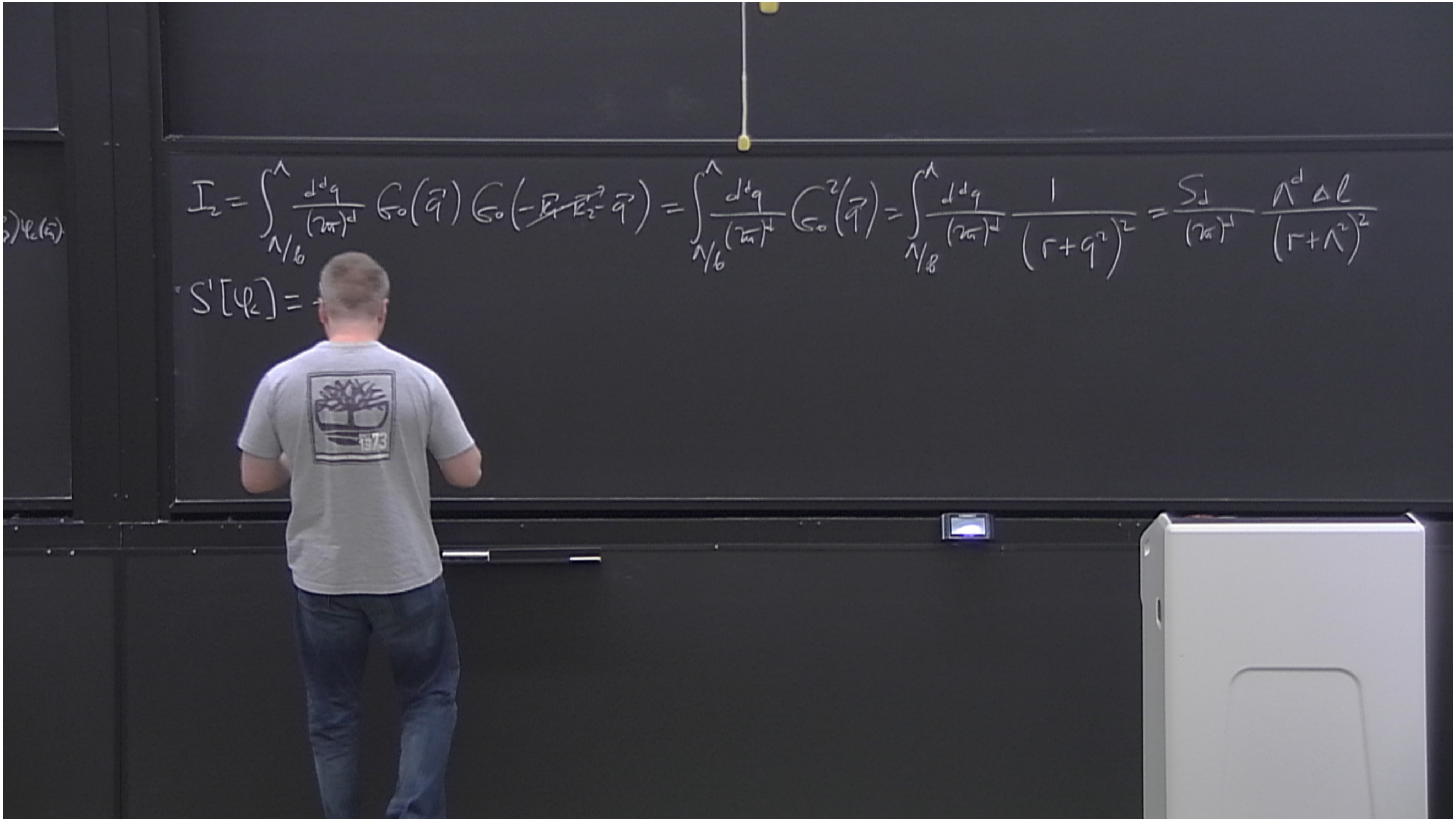

k1 + k2 + q1 + q2 = 0
k3 + k4 - q1 - q2 = 0
=> k1 + k2 + k3 + k4 = 0
let q1 = q , q2 = -k1 - k2 - q
= (u/4!)^2 ∫ k1..k4 (2π)^d δ(k1 + k2 + k3 + k4) φc(k1) φc(k2) φc(k3) φc(k4)
1/6

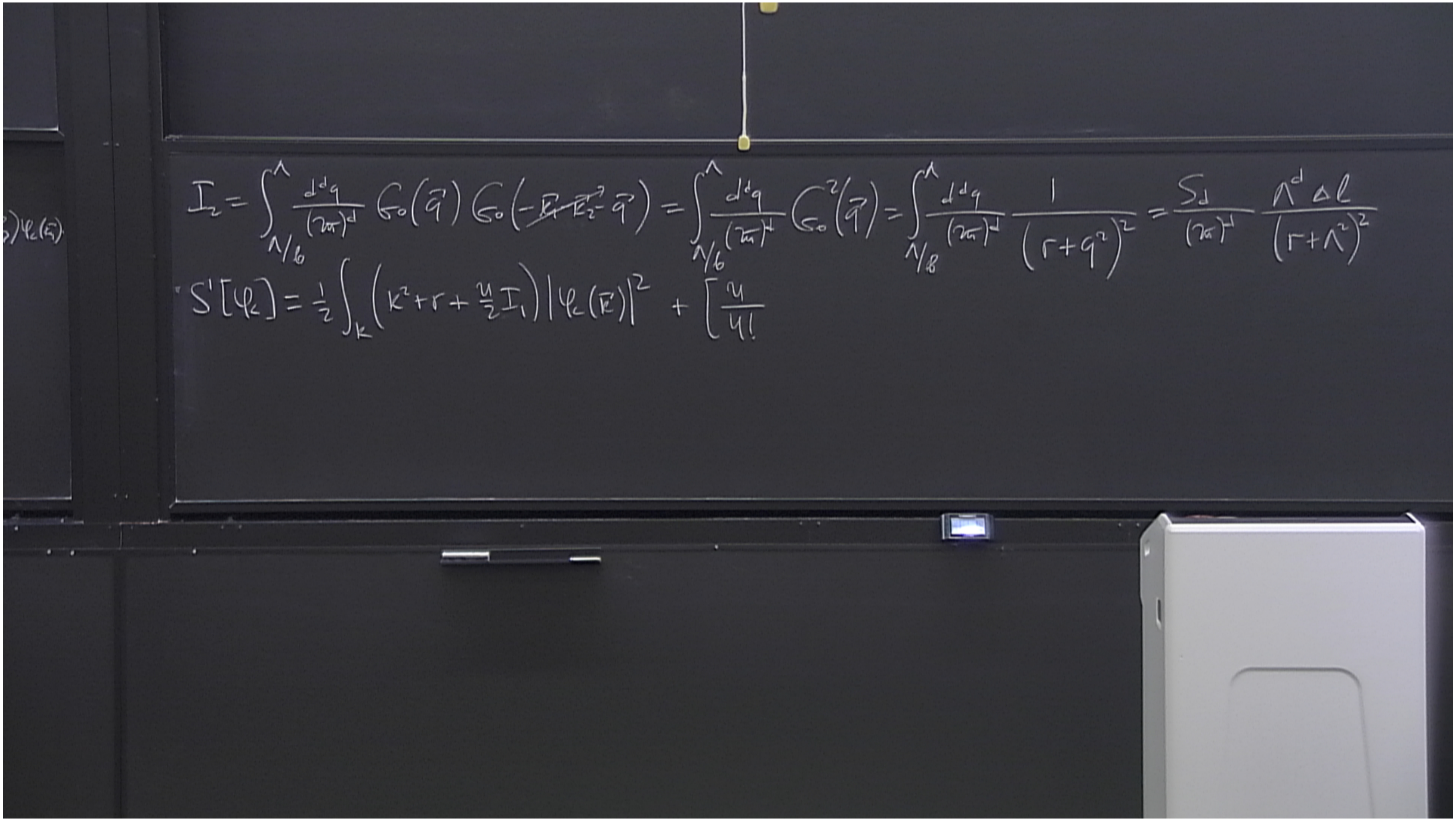

k1 + k2 + q1 + q2 = 0
k3 + k4 − q1 − q2 = 0
=> k1 + k2 + k3 + k4 = 0
let q1 = q , q2 = −k1 − k2 − q

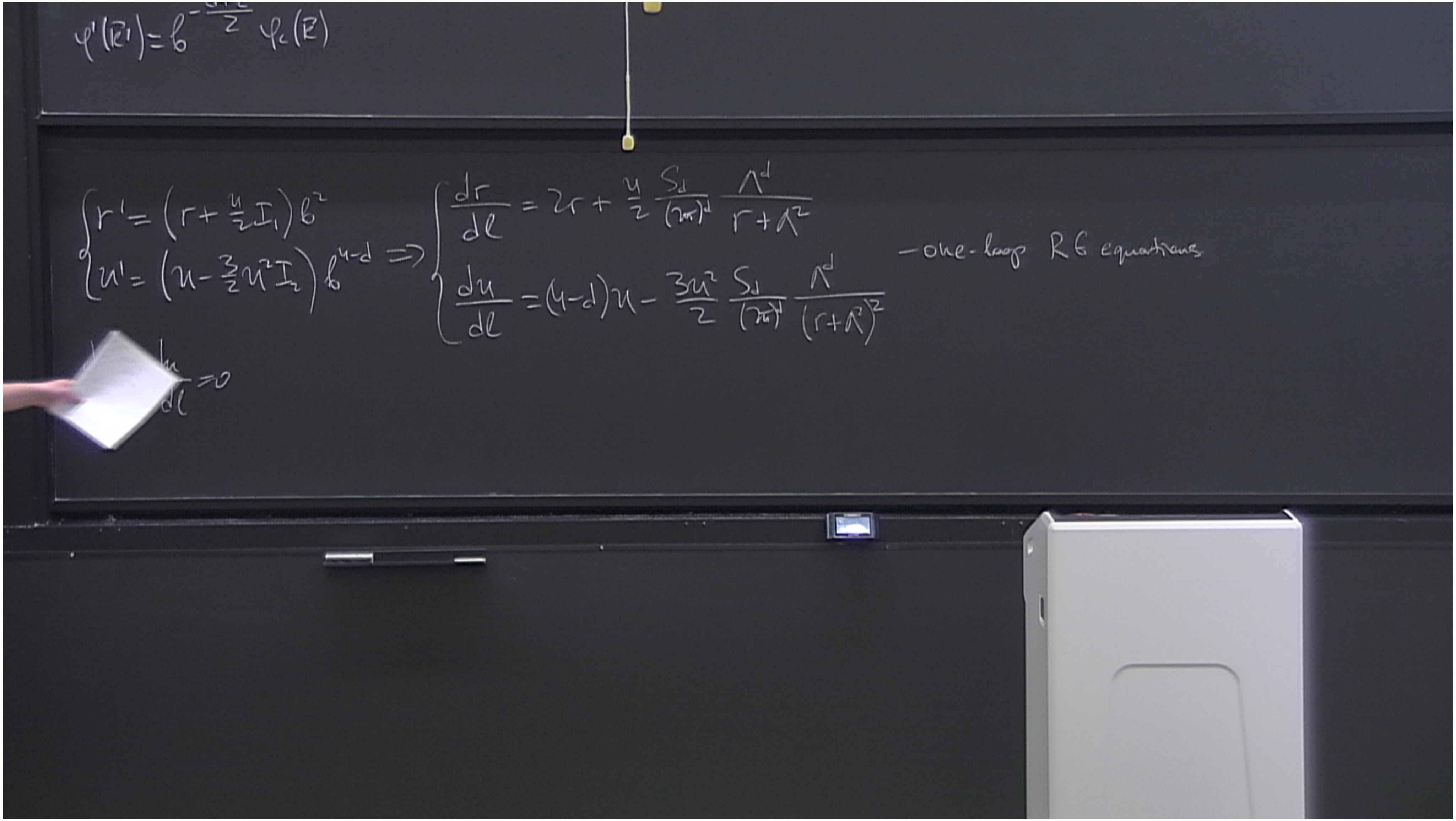

$\vec{k}_1 = -\vec{p}$ - impossible
$\vec{p} = 0$
$\vec{k}_1 + \vec{k}_2 + \vec{q}_1 + \vec{q}_2 = 0$
$\vec{k}_3 + \vec{k}_4 - \vec{q}_1 - \vec{q}_2 = 0$
$\Rightarrow \vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4 = 0$
let $\vec{q}_1 = \vec{q}$, $\vec{q}_2 = -\vec{k}_1 - \vec{k}_2 - \vec{q}$
$= \left(\frac{u}{4!}\right)^2 \int_{k_1 \dots k_4} (2\pi)^d \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4) \varphi_c(\vec{k}_1) \varphi_c(\vec{k}_2) \varphi_c($
$\cdot \int_{\Lambda/b}^{\Lambda} \frac{d^dq}{(2\pi)^d} G_0(\vec{q}) G_0(-\vec{k}_1 - \vec{k}_2 - \vec{q})$
I_2

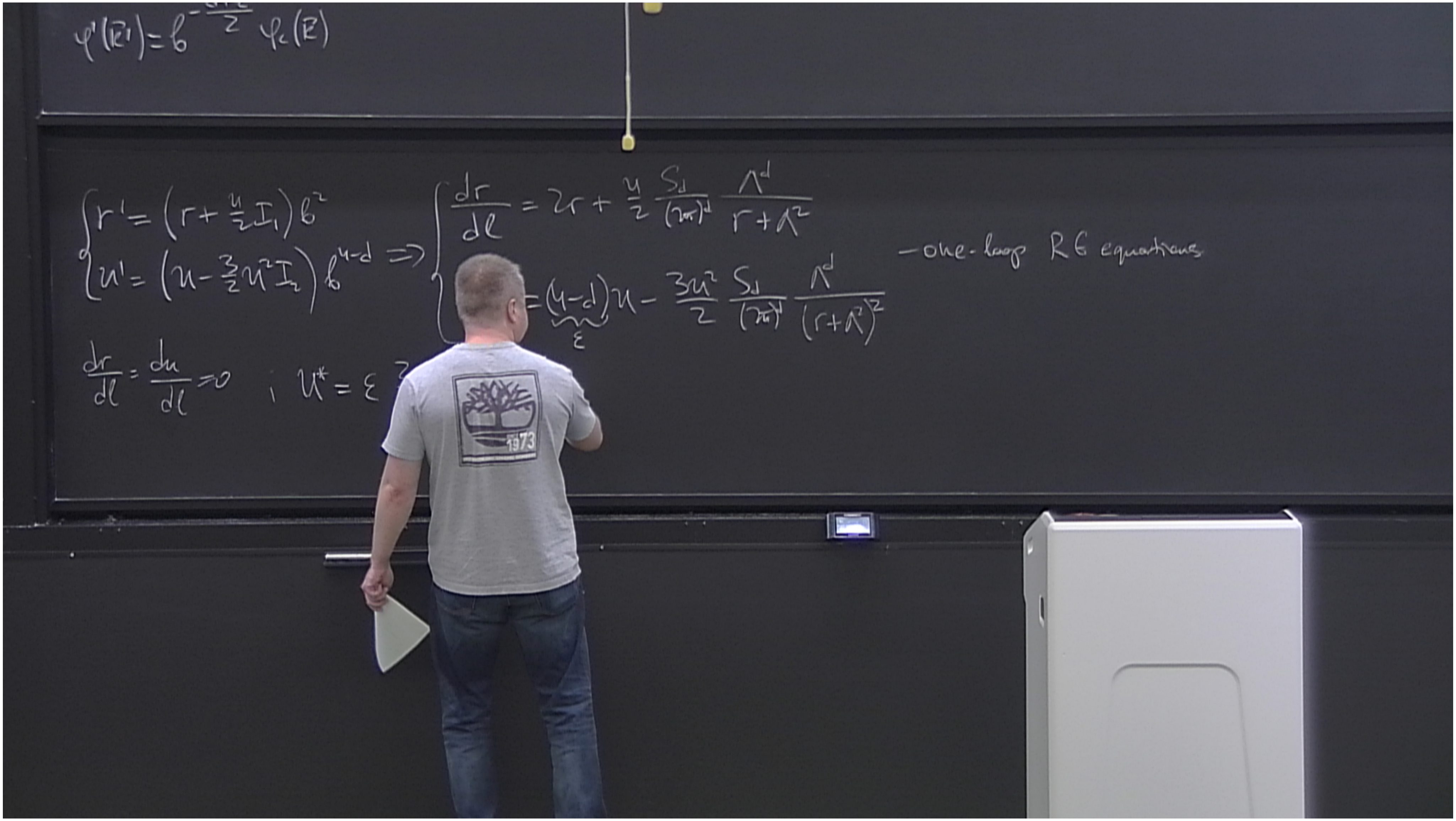

$I_2 = \int_{\Lambda/b}^{\Lambda} \frac{d^d q}{(2\pi)^d} G_0(\vec{q})\, G_0(-\vec{k}_2 - \vec{q})$


$I_2 = \int_{\Lambda/b}^{\Lambda} \frac{d^dq}{(2\pi)^d} G_0(\vec{q})\, G_0(-\vec{k_1}-\vec{k_2}-\vec{q}) = \int_{\Lambda/b}^{\Lambda} \frac{d^dq}{(2\pi)^d} G_0^2(\vec{q}) = \int_{\Lambda/b}^{\Lambda} \frac{d^dq}{(2\pi)^d} \frac{1}{(r+q^2)^2} = \frac{S_d}{(2\pi)^d} \frac{\Lambda^d \Delta l}{(r+\Lambda^2)^2}$

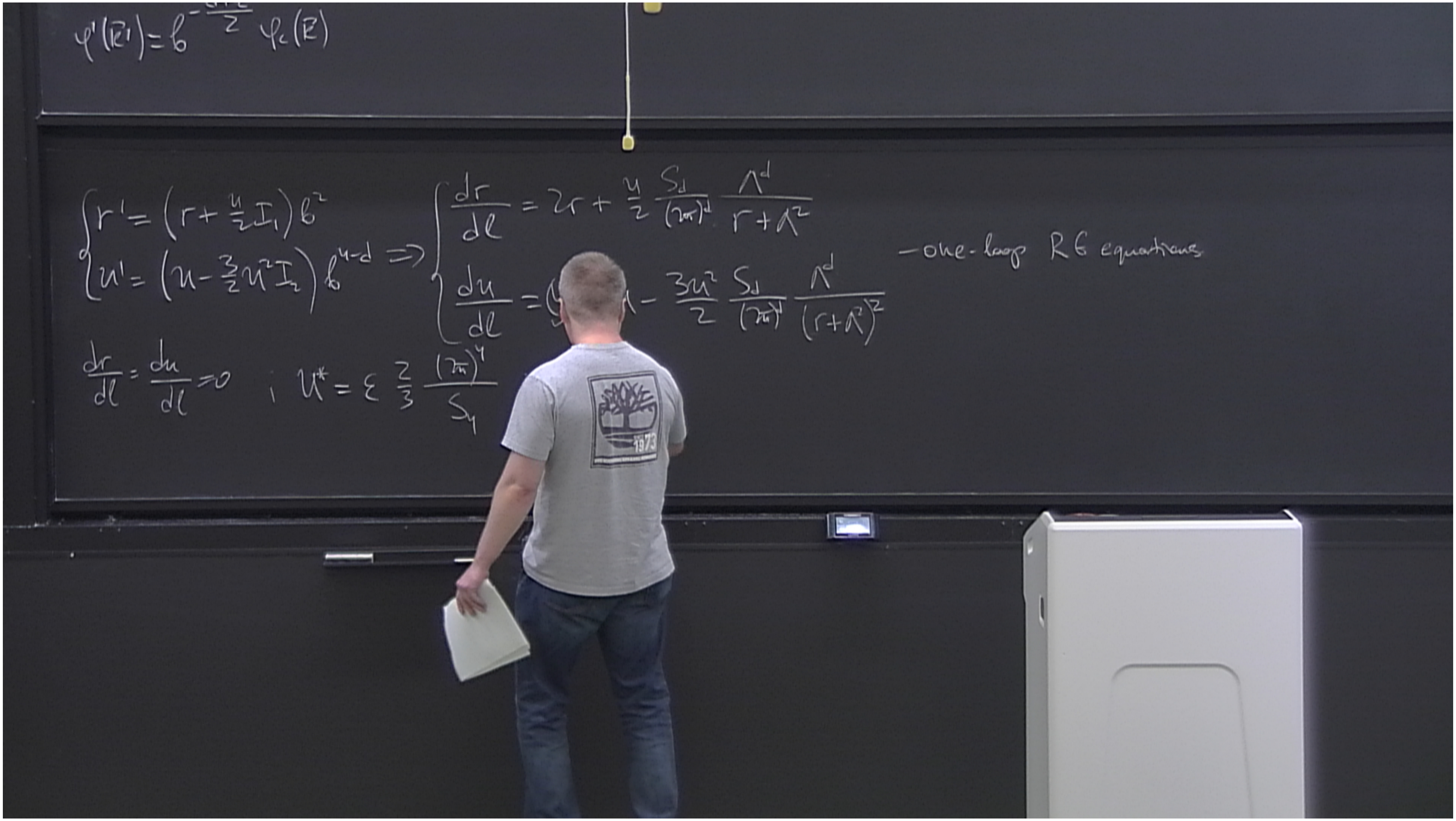

$I_2 = \int_{\Lambda/b}^{\Lambda} \frac{d^dq}{(2\pi)^d} G_0(\vec{q})\, G_0(-\vec{k}_1 - \vec{k}_2 - \vec{q}) = \int_{\Lambda/b}^{\Lambda} \frac{d^dq}{(2\pi)^d} G_0^2(\vec{q}) = \int_{\Lambda/b}^{\Lambda} \frac{d^dq}{(2\pi)^d} \frac{1}{(r+q^2)^2} = \frac{S_d}{(2\pi)^d} \frac{\Lambda^d \Delta \ell}{(r+\Lambda^2)^2}$
$S'[\varphi_<] =$

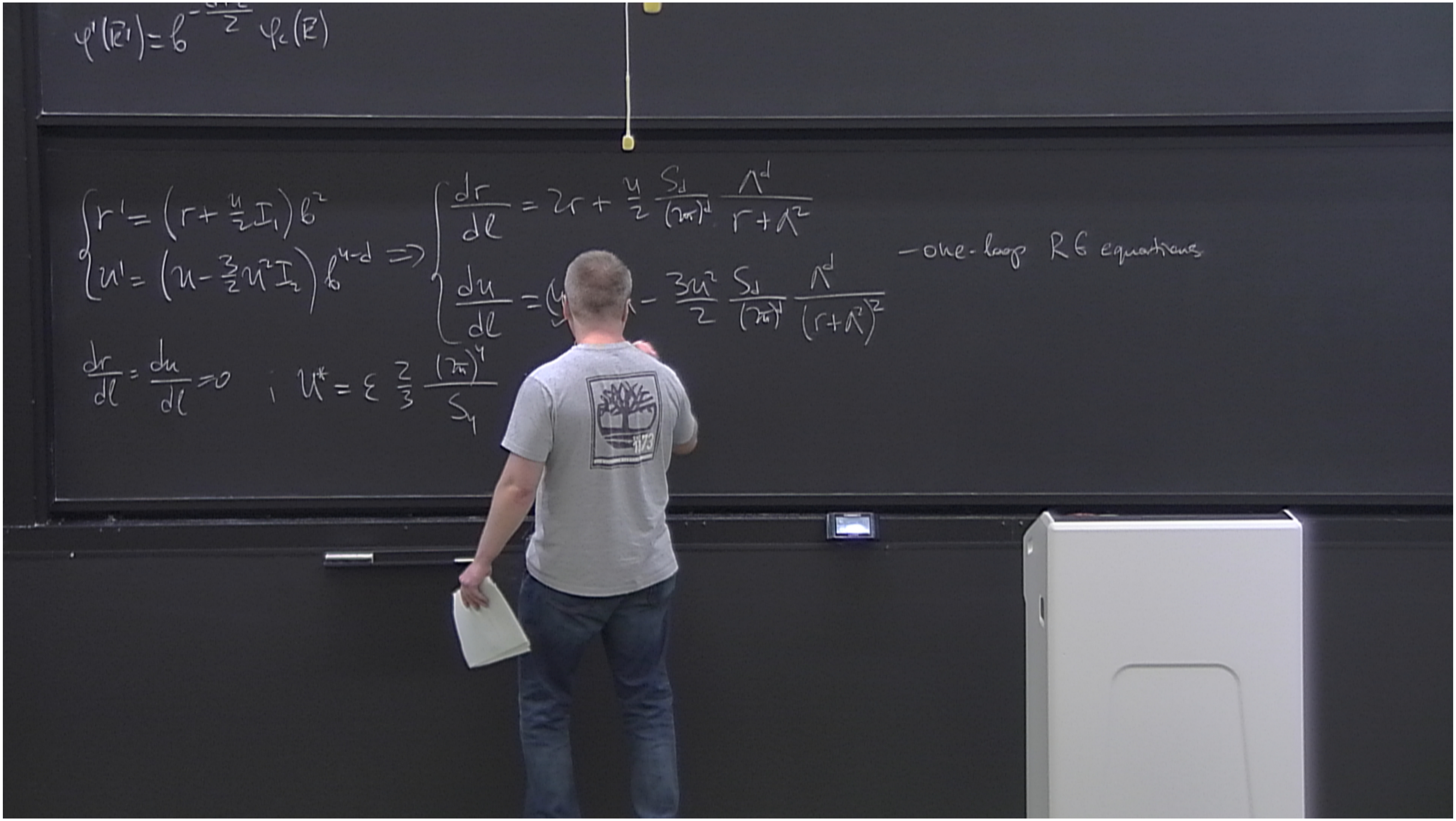

$I_2 = \int_{\Lambda/b}^{\Lambda} \frac{d^dq}{(2\pi)^d} G_0(\vec{q})\, G_0(-\vec{k}_1 - \vec{k}_2 - \vec{q}) = \int_{\Lambda/b}^{\Lambda} \frac{d^dq}{(2\pi)^d} G_0^2(\vec{q}) = \int_{\Lambda/b}^{\Lambda} \frac{d^dq}{(2\pi)^d} \frac{1}{(r+q^2)^2} = \frac{S_d}{(2\pi)^d} \frac{\Lambda^d \Delta \ell}{(r+\Lambda^2)^2}$
$S'[\varphi_<] = \frac{1}{2}\int_k \left(k^2 + r + \frac{u}{2} I_1\right) |\varphi_<(\vec{k})|^2 + \Big[\frac{u}{4!}$

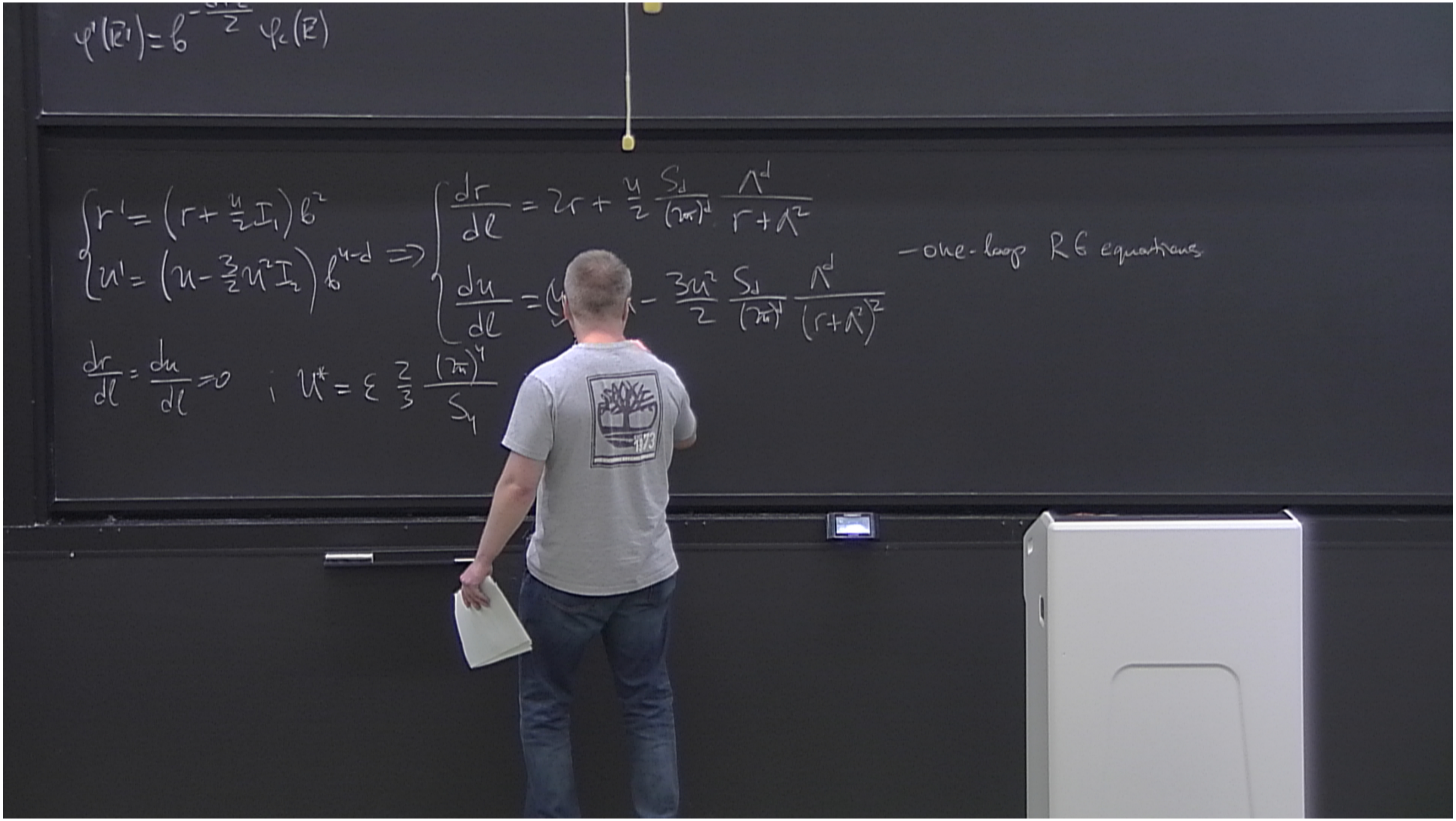

$\varphi'(\vec{k}') = b^{-\frac{d+2}{2}} \varphi_c(\vec{k})$
$r' = \left(r + \frac{u}{2} I_1\right) b^2$
$u' = \left(u - \frac{3}{2} u^2 I_2\right) b^{4-d}$
$\Rightarrow$
$\frac{dr}{dl} = 2r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2}$
$\frac{du}{dl} = (4-d)u - \frac{3u^2}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{(r+\Lambda^2)^2}$
— one-loop RG equations
$\frac{du}{dl} = 0$

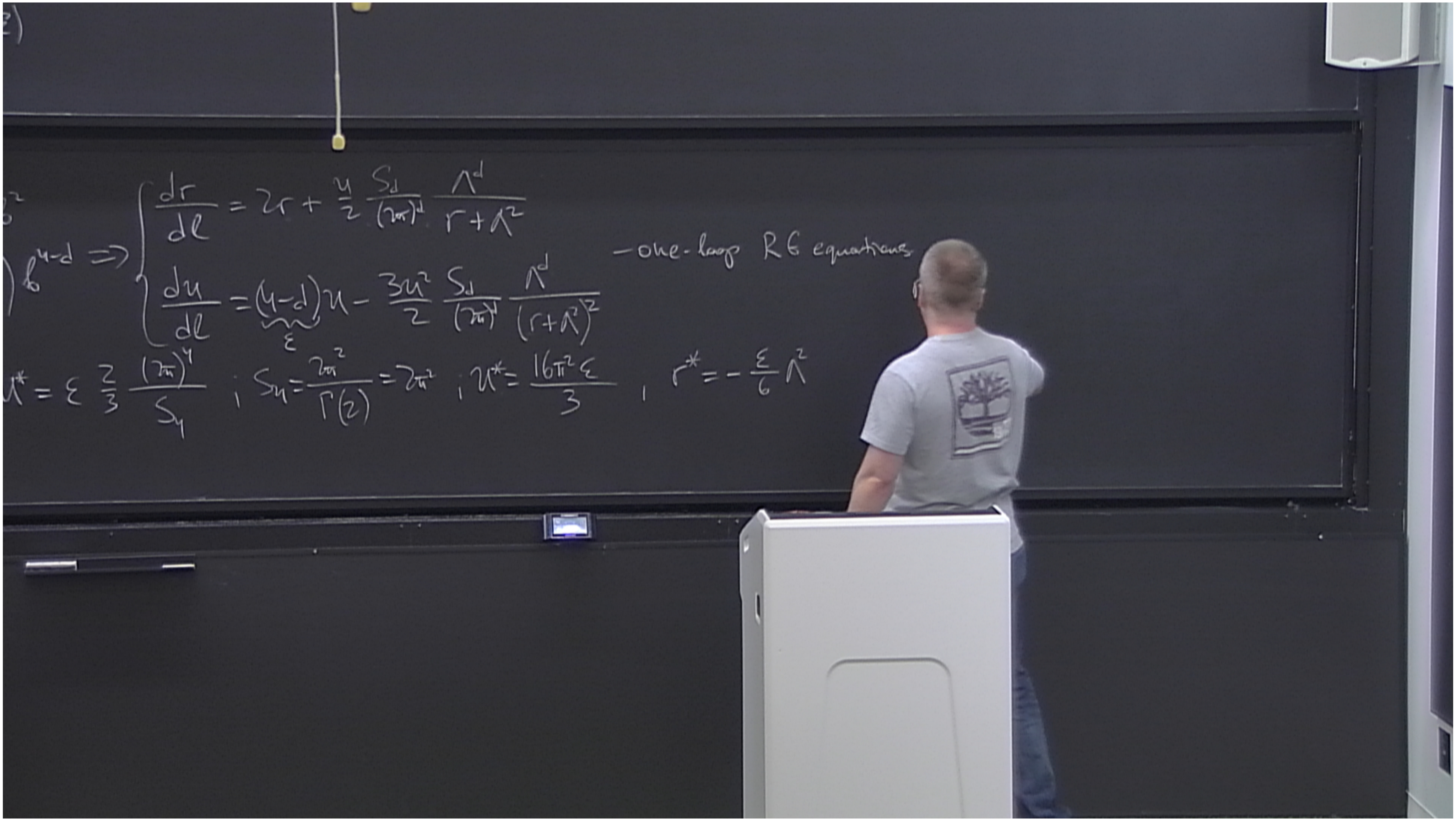


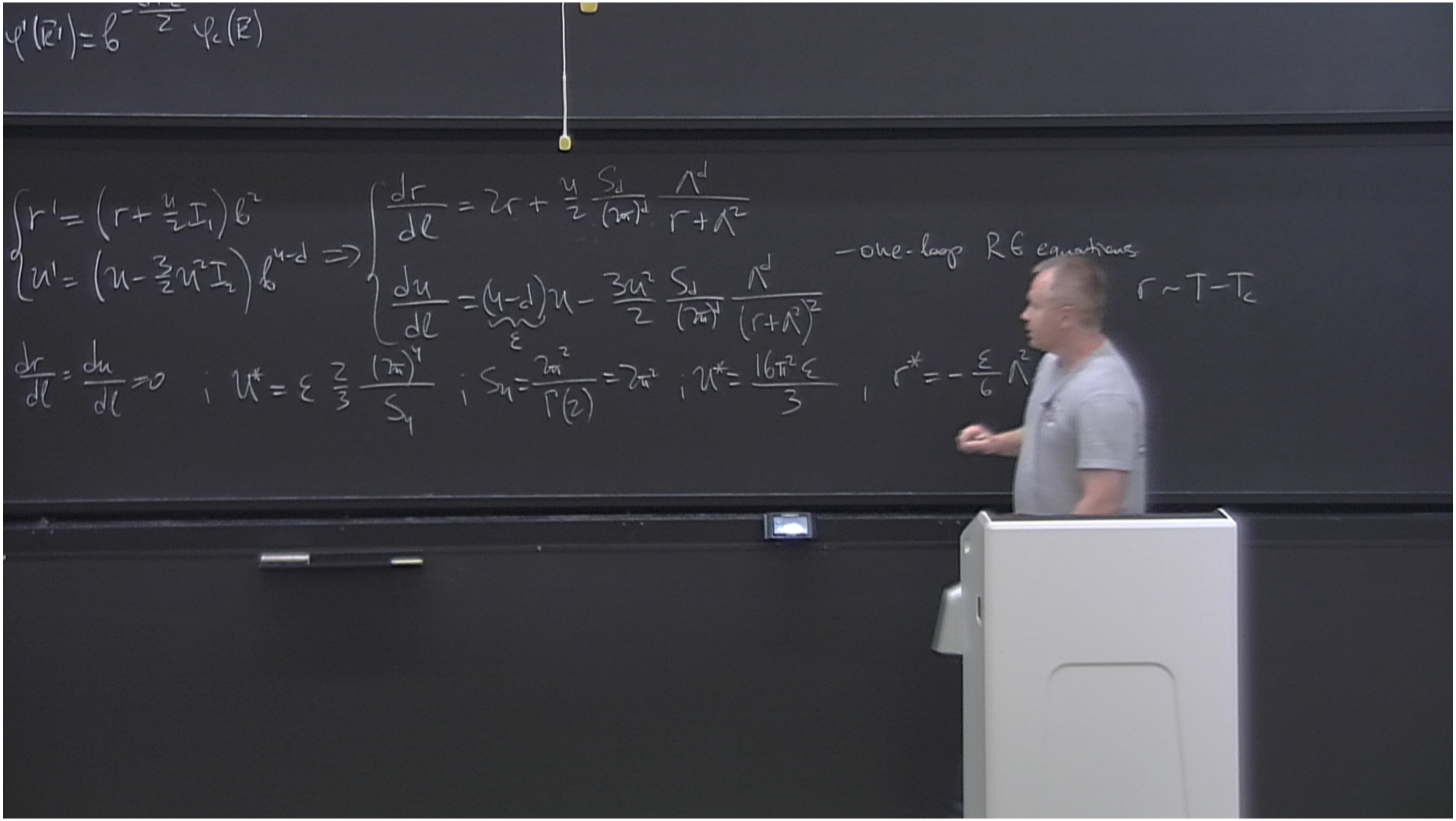
$\varphi'(k') = b^{-\frac{d+2}{2}} \varphi_<(k)$
$r' = (r + \frac{u}{2} I_1) b^2$
$u' = (u - \frac{3}{2} u^2 I_2) b^{4-d}$
$\frac{dr}{dl} = 2r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r+\Lambda^2}$
$= (4-d) u - \frac{3u^2}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{(r+\Lambda^2)^2}$
ε
— one-loop RG equations
$\frac{dr}{dl} = \frac{du}{dl} = 0$; $u^* = \varepsilon$

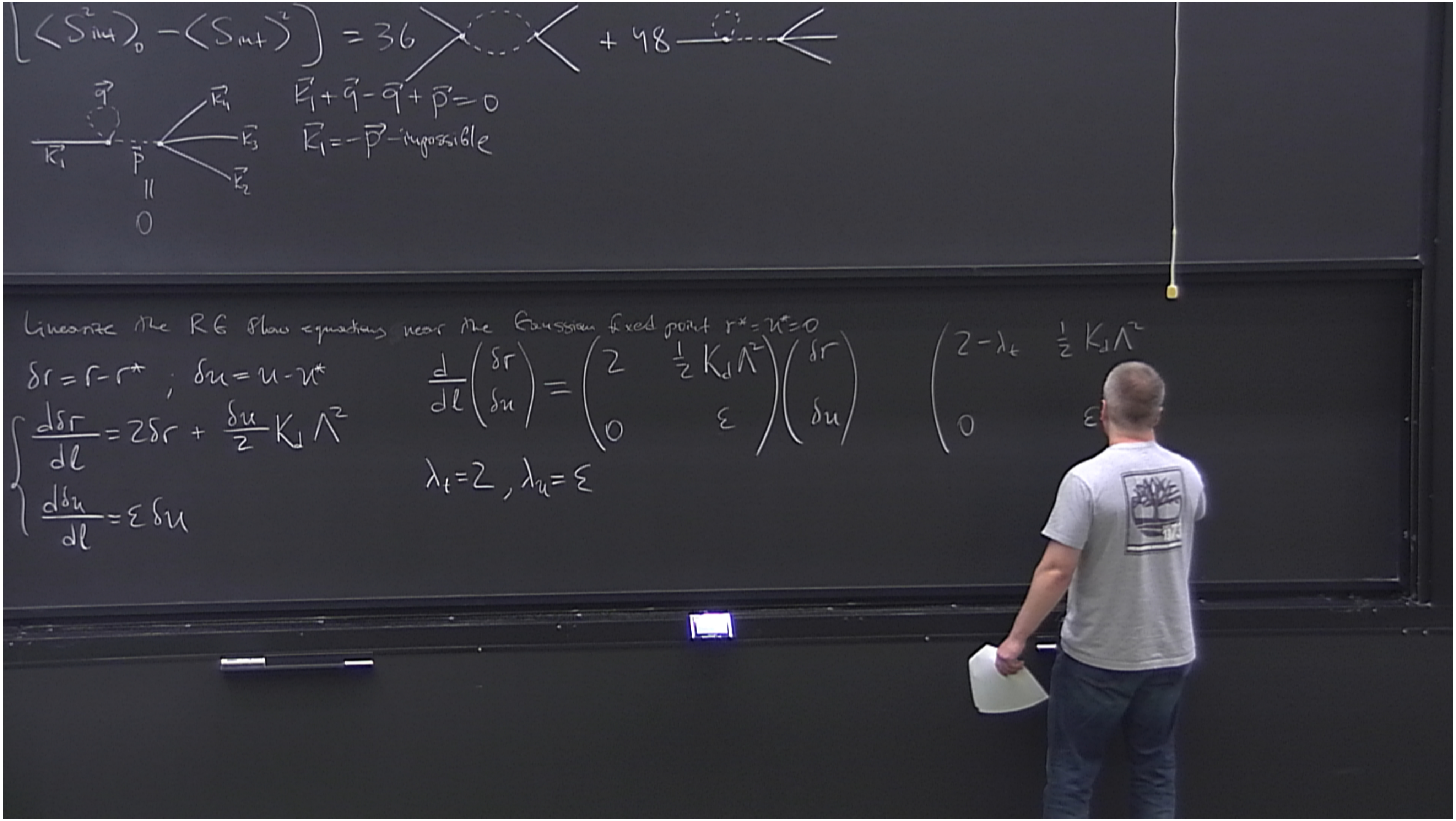

$\varphi'(k') = b^{-\frac{d+2}{2}} \varphi_<(k)$
$r' = \left(r + \frac{u}{2} I_1\right) b^2$
$u' = \left(u - \frac{3}{2} u^2 I_2\right) b^{4-d} \Rightarrow$
$\frac{dr}{dl} = 2r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r+\Lambda^2}$
$\frac{du}{dl} = \ldots u - \frac{3u^2}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{(r+\Lambda^2)^2}$
— one-loop RG equations
$\frac{dr}{dl} = \frac{du}{dl} = 0 \; ; \; u^* = \varepsilon \frac{2}{3} \frac{(2\pi)^4}{S_4}$


—one-loop RG equations

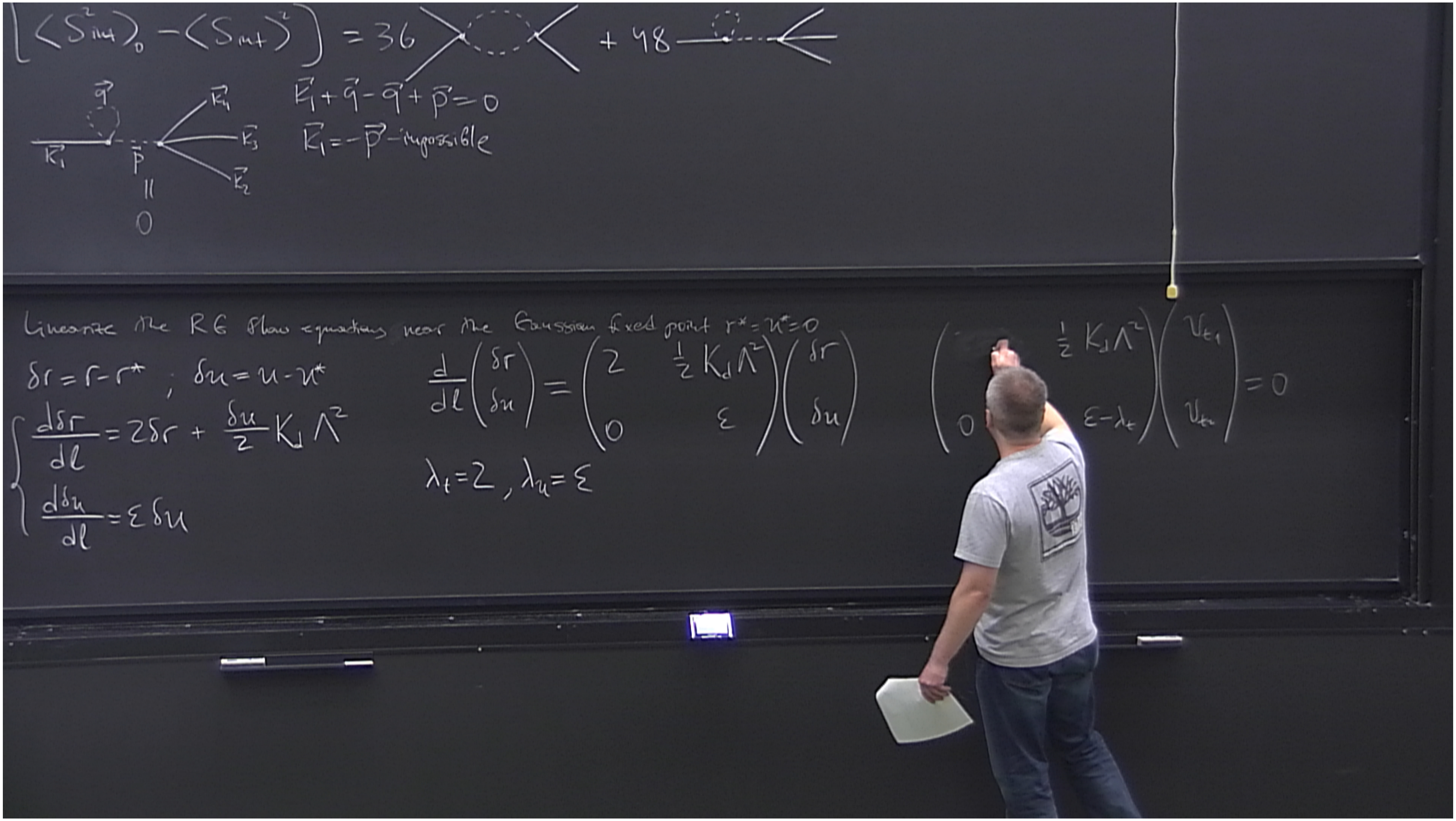

r' = (r + u/2 I_1) b^2
u' = (u - 3/2 u^2 I_2) b^{4-d}
dr/dl = 2r + u/2 S_d/(2π)^d Λ^d/(r+Λ^2)
du/dl
- 3u^2/2 S_d/(2π)^d Λ^d/(r+Λ^2)^2
— one-loop RG equations
dr/dl = du/dl = 0 ; u* = ε 2/3 (2π)^4/S_4

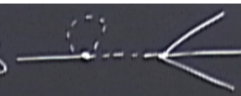

$\frac{dr}{dl} = 2r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r+\Lambda^2}$
$\frac{du}{dl} = (4-d)u - \frac{3u^2}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{(r+\Lambda^2)^2}$
ε
— one-loop RG equations
$u^* = \varepsilon \frac{2}{3} \frac{(2\pi)^4}{S_4}$; $S_4 = \frac{2\pi^2}{\Gamma(2)} = 2\pi^2$; $u^* = \frac{16\pi^2 \varepsilon}{3}$, $r^* = -\frac{\varepsilon}{6}\Lambda^2$

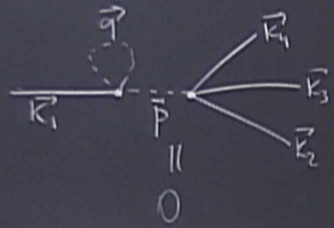

φ'(k') = b^{-...} φ_c(k)
r' = (r + u/2 I_1) b^2
u' = (u - 3/2 u^2 I_2) b^{4-d} =>
dr/dl = 2r + u/2 S_d/(2π)^d Λ^d/(r+Λ^2)
du/dl = (4-d)u - 3u^2/2 S_d/(2π)^d Λ^d/(r+Λ^2)^2
ε
- one-loop RG equations
r ~ T - T_c
dr/dl = du/dl = 0 ; u* = ε 2/3 (2π)^4/S_4 ; S_4 = 2π^2/Γ(2) = 2π^2 ; u* = 16π^2 ε/3 , r* = - ε/6 Λ^2



[⟨S²int⟩₀ − ⟨Sint⟩²] = 36 + 48
k₁ + q − q + p = 0
k₁ = −p − impossible
Linearize the RG flow equations near the Gaussian fixed point r* = u* = 0
δr = r − r* ; δu = u − u*
dδr/dl = 2δr + (δu/2) K_d Λ²
dδu/dl = ε δu
d/dl (δr, δu) = (2, ½ K_d Λ²; 0, ε)(δr, δu)
λ_t = 2 , λ_u = ε


$\left[\langle S_{int}^2\rangle_0 - \langle S_{int}\rangle^2\right] = 36 \;\; + 48$
$\vec{q}$ $\vec{k}_1$ $\vec{p}$ $\vec{k}_4$ $\vec{k}_3$ $\vec{k}_2$ $\parallel$ 0
$\vec{k}_1 + \vec{q} - \vec{q} + \vec{p} = 0$
$\vec{k}_1 = -\vec{p}$ - impossible
Linearize the RG flow equations near the Gaussian fixed point $r^* = u^* = 0$
$\delta r = r - r^*$; $\delta u = u - u^*$
$\frac{d\delta r}{dl} = 2\delta r + \frac{\delta u}{2} K_d \Lambda^2$
$\frac{d\delta u}{dl} = \varepsilon \delta u$
$\frac{d}{dl}\begin{pmatrix}\delta r\\ \delta u\end{pmatrix} = \begin{pmatrix}2 & \frac{1}{2}K_d\Lambda^2\\ 0 & \varepsilon\end{pmatrix}\begin{pmatrix}\delta r\\ \delta u\end{pmatrix}$
$\lambda_t = 2$, $\lambda_u = \varepsilon$
$\begin{pmatrix} & \frac{1}{2}K_d\Lambda^2\\ 0 & \varepsilon - \lambda_t\end{pmatrix}\begin{pmatrix}v_{t1}\\ v_{t2}\end{pmatrix} = 0$

$\left[\langle S_{int}^2\rangle_0 - \langle S_{int}\rangle^2\right] = 36$  $+\ 48$ 



$\vec{k}_1+\vec{q}-\vec{q}+\vec{p}=0$

$\vec{k}_1=-\vec{p}$ - impossible

Linearize the RG flow equations near the Gaussian fixed point $r^*=u^*=0$

$\delta r = r - r^*$; $\delta u = u - u^*$

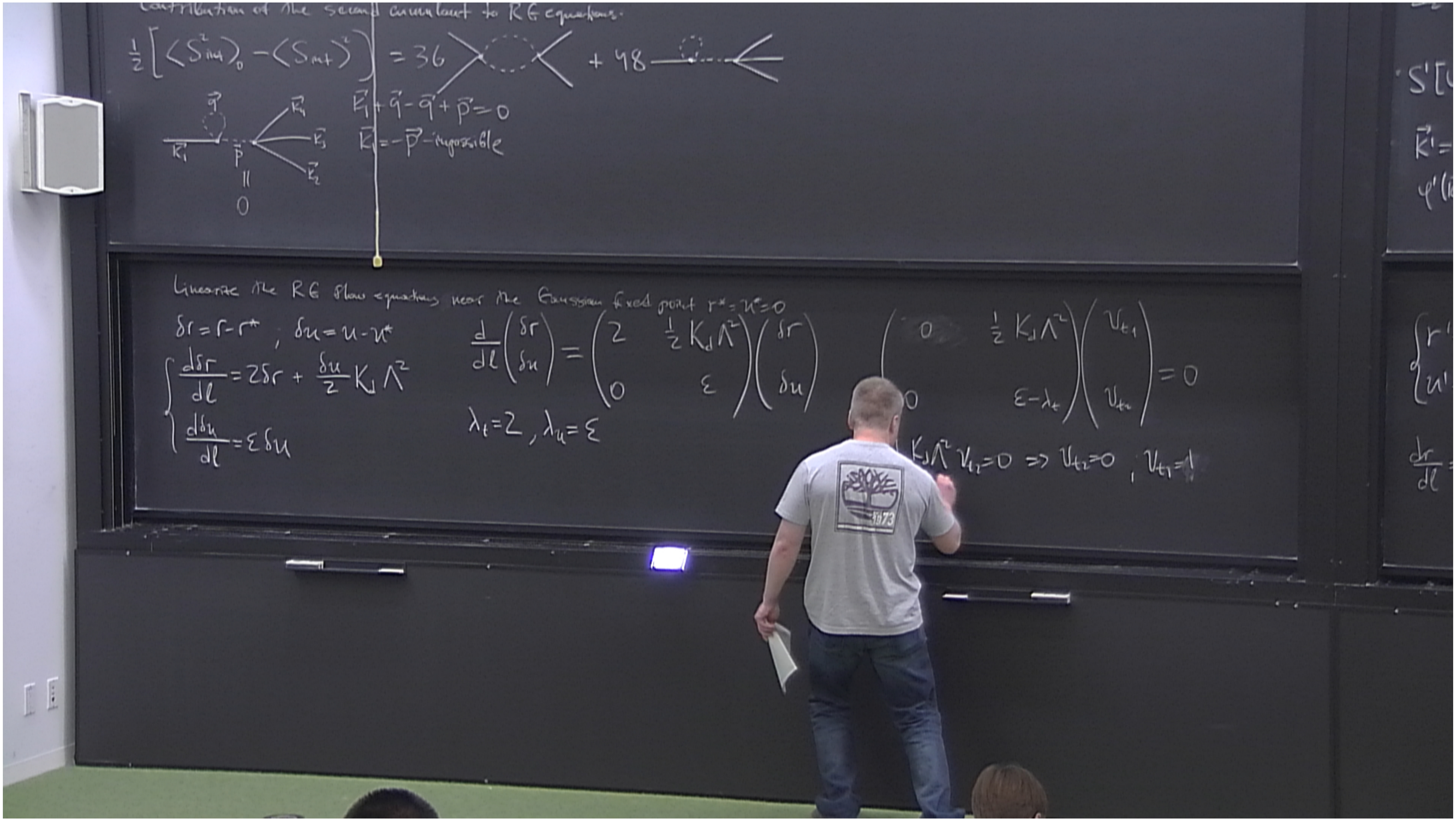
$$\begin{cases} \dfrac{d\delta r}{dl} = 2\delta r + \dfrac{\delta u}{2} K_d \Lambda^2 \\[2mm] \dfrac{d\delta u}{dl} = \varepsilon\, \delta u \end{cases}$$

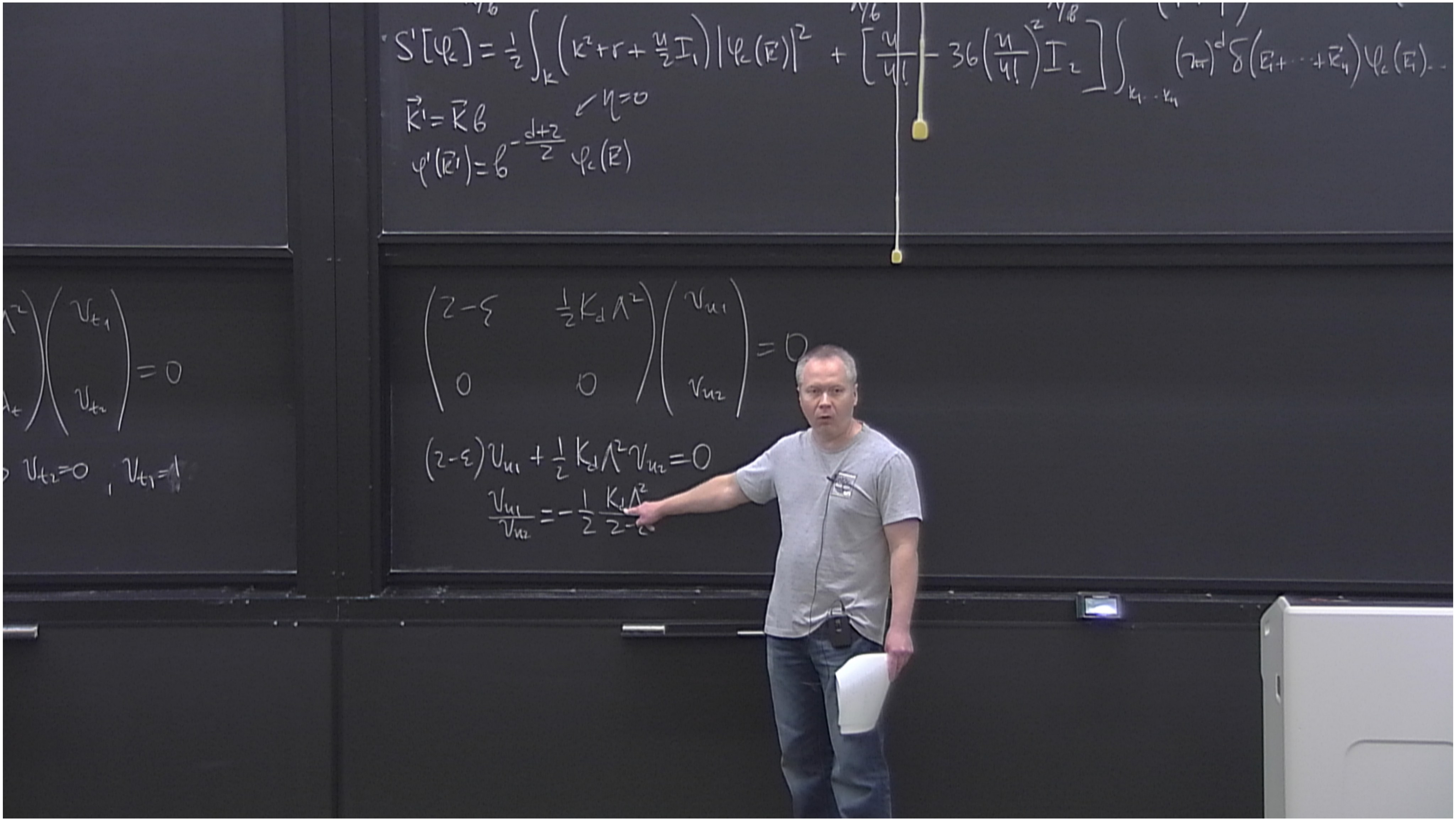
$$\frac{d}{dl}\begin{pmatrix} \delta r \\ \delta u \end{pmatrix} = \begin{pmatrix} 2 & \frac{1}{2}K_d\Lambda^2 \\ 0 & \varepsilon \end{pmatrix}\begin{pmatrix} \delta r \\ \delta u \end{pmatrix}$$

$\lambda_t = 2$, $\lambda_u = \varepsilon$

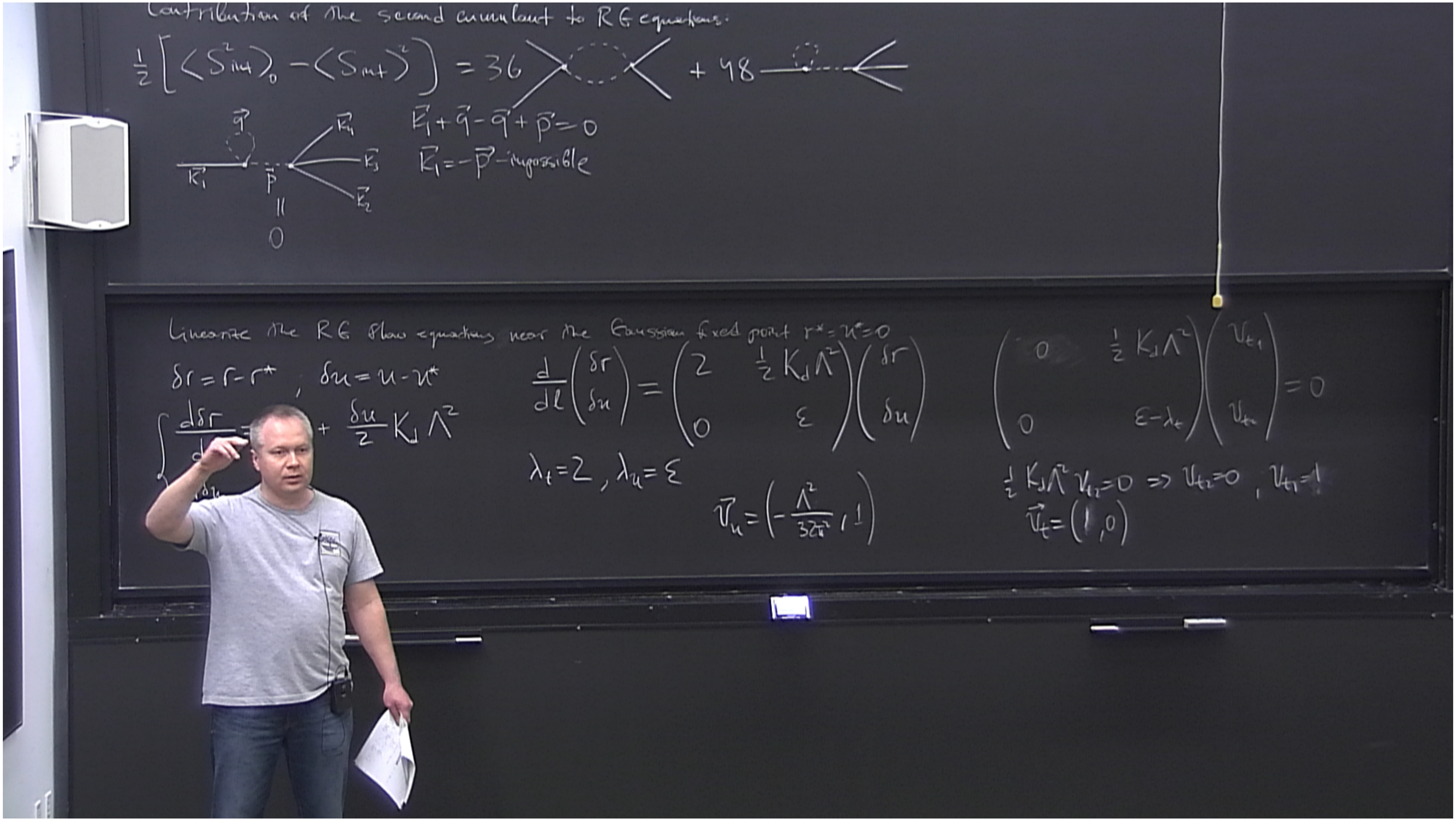
$$\begin{pmatrix} 0 & \frac{1}{2}K_d\Lambda^2 \\ 0 & \varepsilon-\lambda_t \end{pmatrix}\begin{pmatrix} v_{t_1} \\ v_{t_2} \end{pmatrix} = 0$$

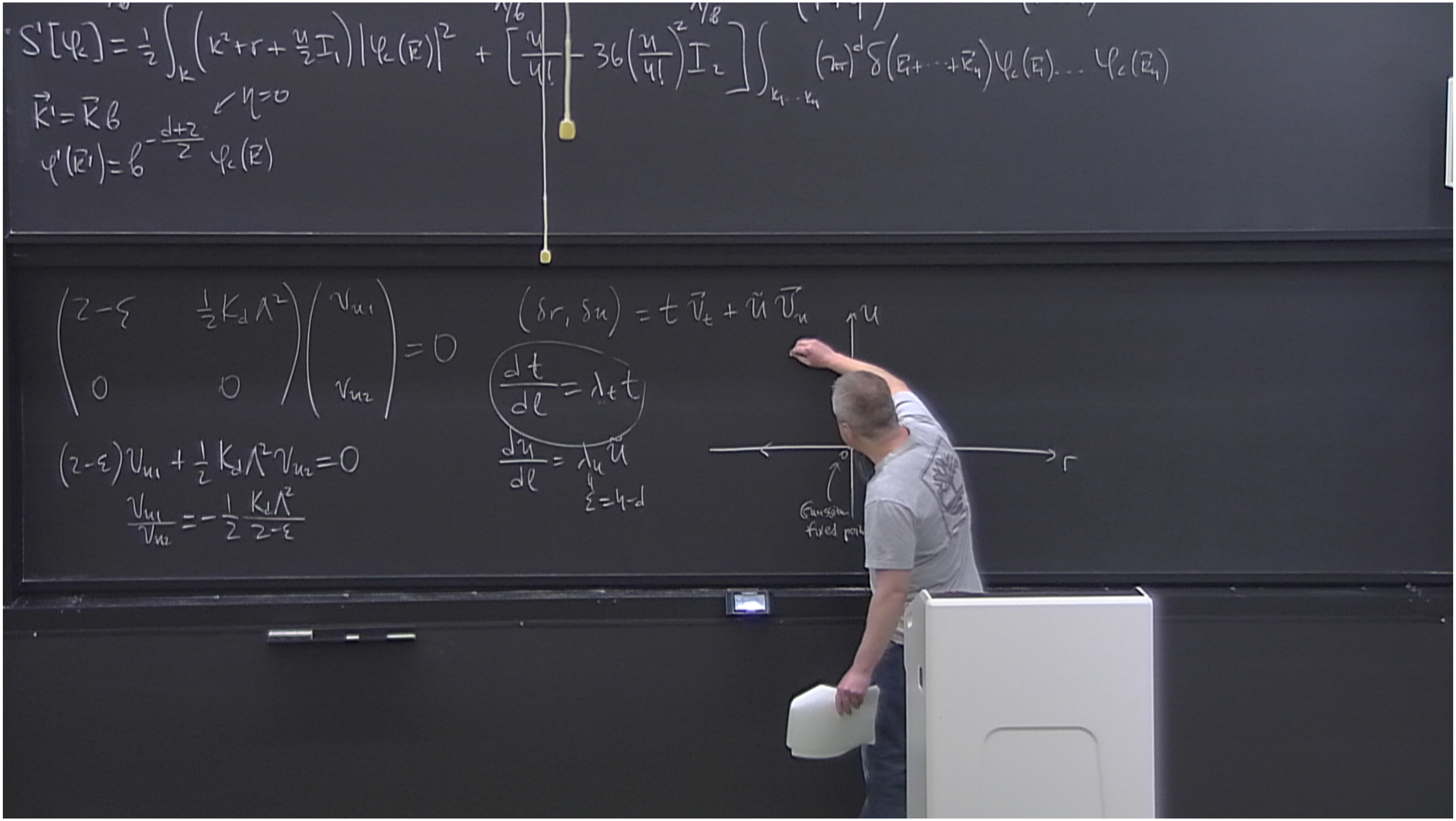
$\frac{1}{2}K_d\Lambda^2 v_{t_2} = 0 \Rightarrow v_{t_2}=0$, $v_{t_1}=1$

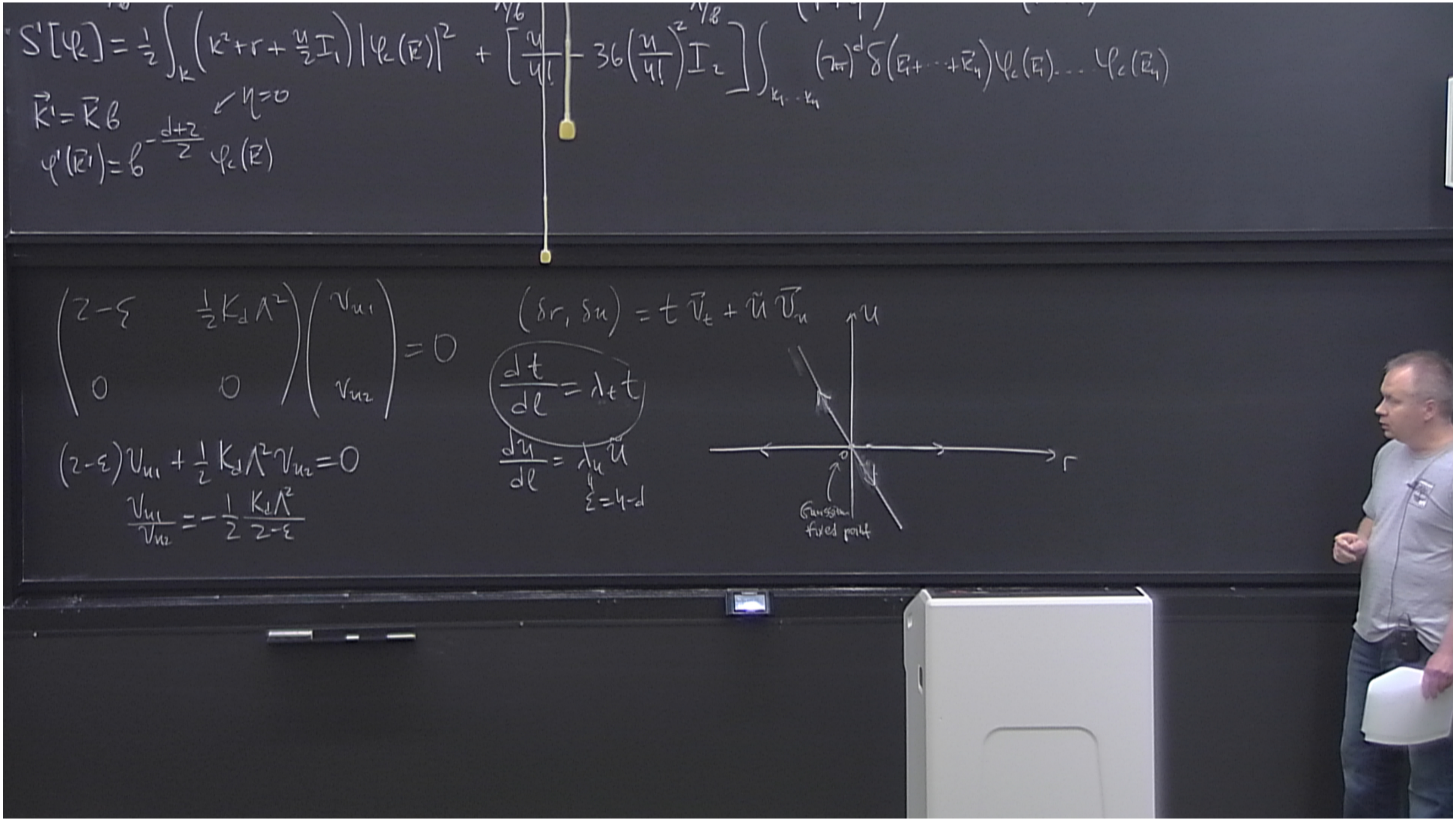

Contribution of the second cumulant to RG equations.
½[⟨S²int⟩₀ − ⟨Sint⟩²] = 36 + 48
k₁ + q − q + p = 0
k₁ = −p − impossible
Linearize the RG flow equations near the Gaussian fixed point r* = u* = 0
δr = r − r*, δu = u − u*
dδr/dl = 2δr + (δu/2) K_d Λ²
dδu/dl = ε δu
d/dl (δr, δu) = (2, ½ K_d Λ²; 0, ε)(δr, δu)
λ_t = 2, λ_u = ε
(0, ½ K_d Λ²; 0, ε − λ_t)(v_t1, v_t2) = 0
K_d Λ² v_t2 = 0 ⇒ v_t2 = 0, v_t1 = 1

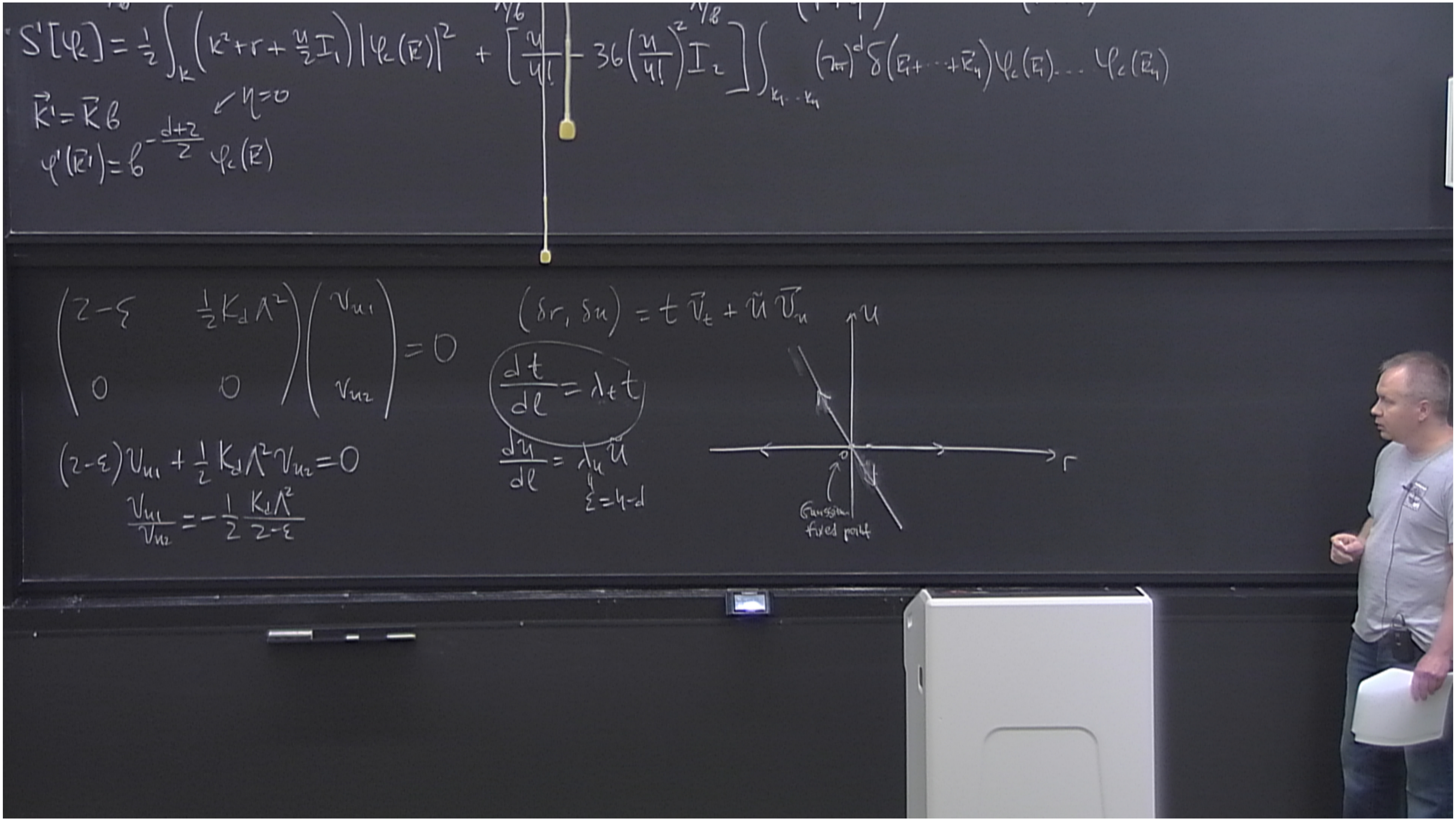


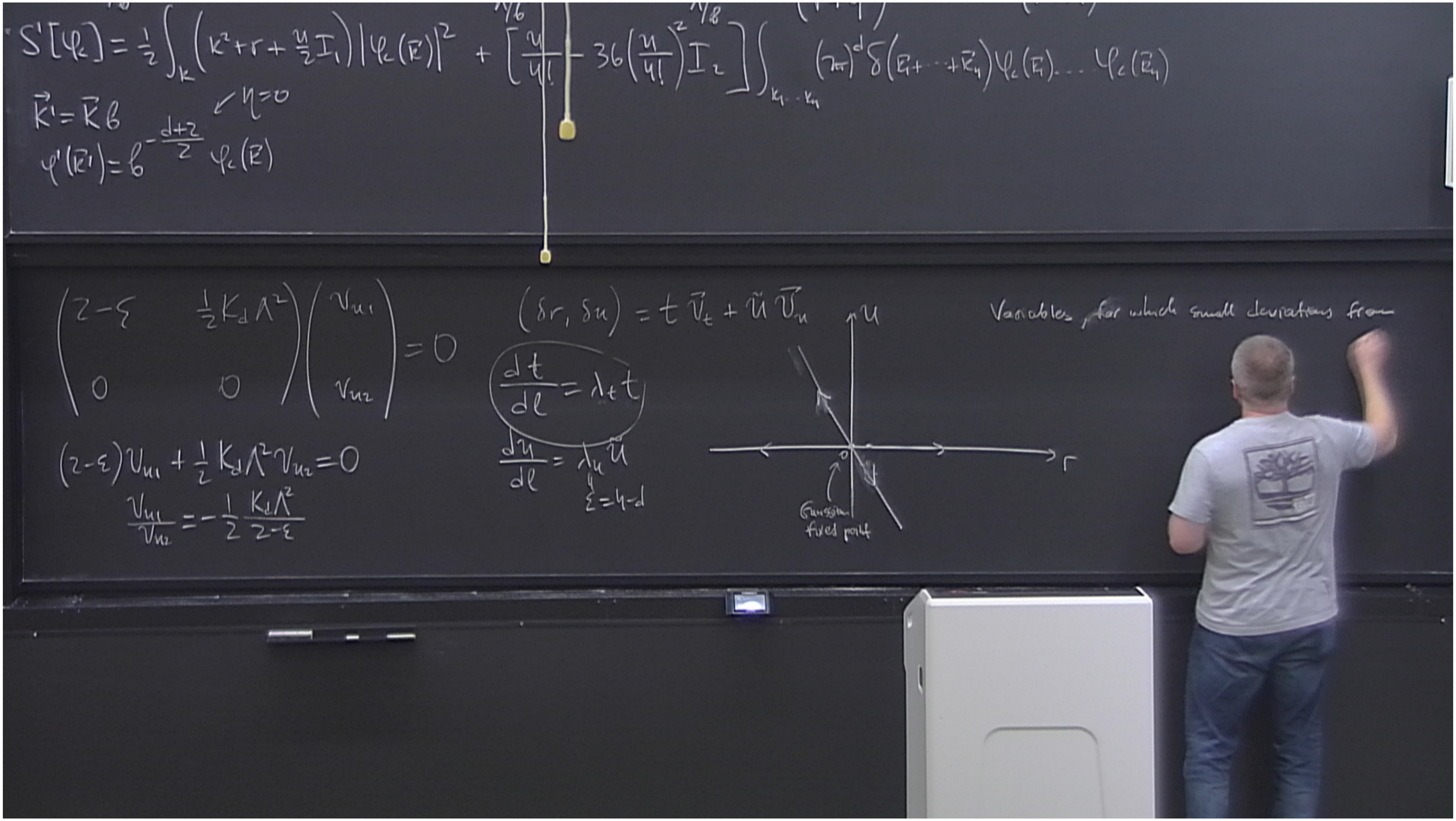
S'[φ<] = 1/2 ∫_k (k² + r + u/2 I₁) |φ<(k)|² + [u/4! − 36 (u/4!)² I₂] ∫_{k₁…k₄} (2π)^d δ(k₁ + … + k₄) φ<(k₁)…
k' = k b
η = 0
φ'(k') = b^{−(d+2)/2} φ<(k)
(2−ε ½ K_d Λ² ; 0 0)(v_{u1} ; v_{u2}) = 0
(v_{t1} ; v_{t2}) = 0
v_{t2} = 0 , v_{t1} = 1
(2−ε) v_{u1} + ½ K_d Λ² v_{u2} = 0
v_{u1}/v_{u2} = −½ K_d Λ²/(2−ε)

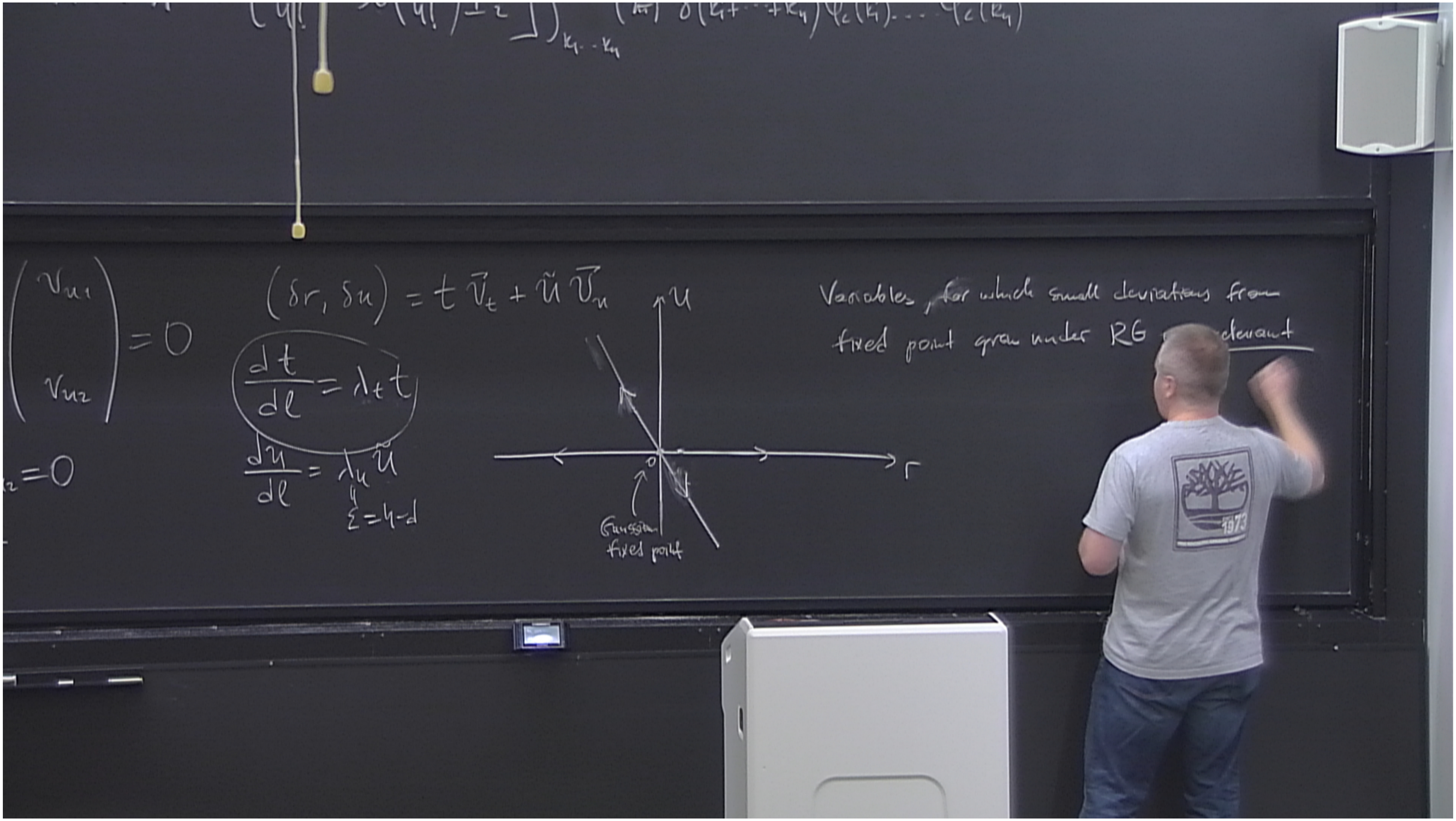

Contribution of the second cumulant to RG equations.
k1+q-q+p=0
k1=-p-impossible
Linearize the RG flow equations near the Gaussian fixed point r*=u*=0
δr=r-r*, δu=u-u*
λt=2, λu=ε

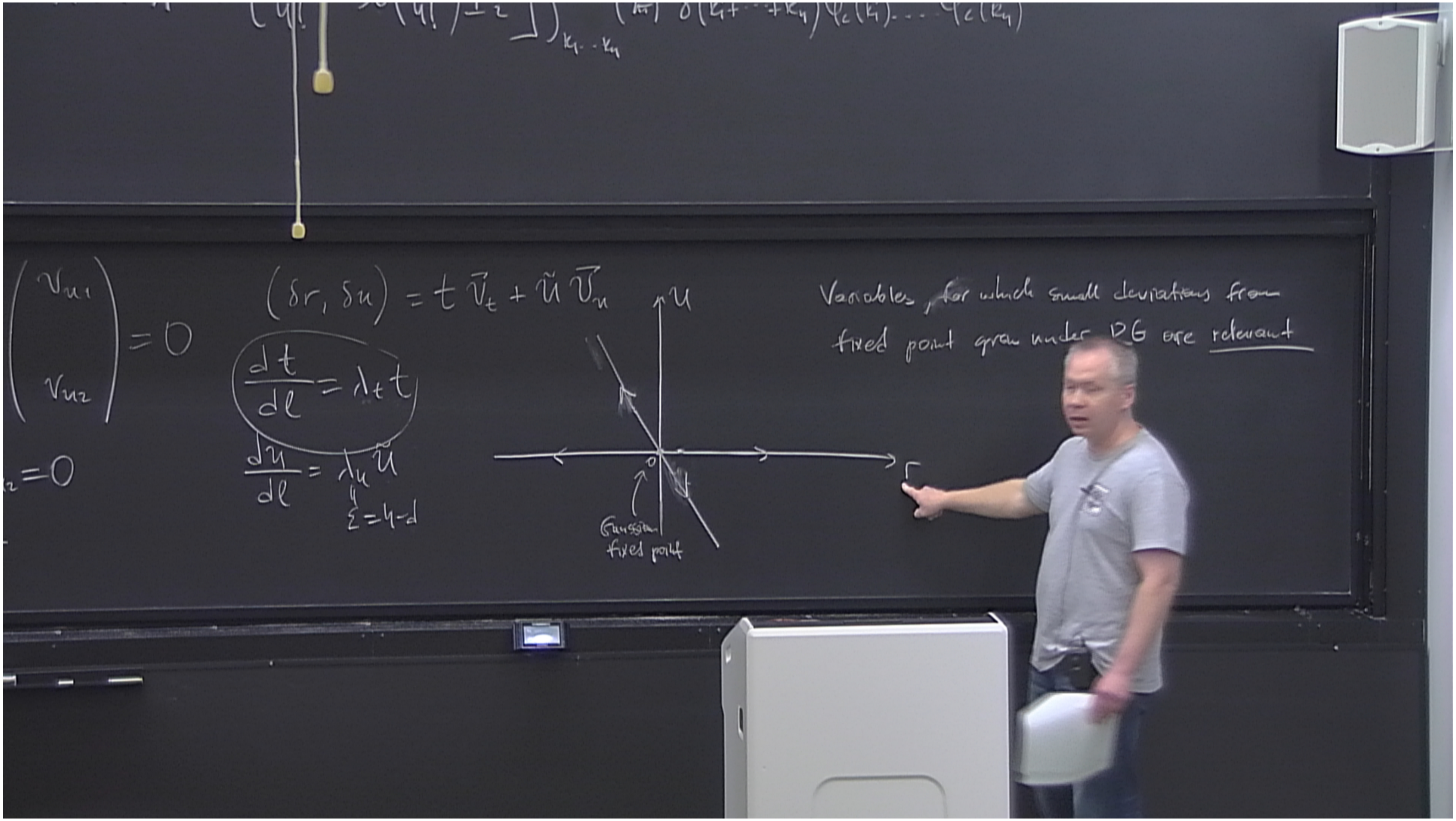

S'[φ_k] = ½∫ₖ (k² + r + u/2 I₁) |φ_k(k)|² + [u/4! − 36 (u/4!)² I₂] ∫ k₁…k₄ (2π)^d δ(k₁+…+k₄) φ_k(k₁)…φ_k(k₄)
k' = k b
η = 0
φ'(k') = b^(−(d+2)/2) φ_k(k)
(2−ε ½KdΛ² ; 0 0)(v_u1 ; v_u2) = 0
(2−ε) v_u1 + ½ KdΛ² v_u2 = 0
v_u1 / v_u2 = −½ KdΛ² / (2−ε)
(δr, δu) = t v_t + ũ v_u
dt/dl = λ_t t
dũ/dl = λ_u ũ
ε = 4−d
u
r
Gaussian fixed point


S'[φ<] = 1/2 ∫k (k²+r+ u/2 I1) |φ<(k)|² + [u/4! − 36 (u/4!)² I2] ∫k1...k4 (2π)^d δ(k1+...+k4) φ<(k1)...φ<(k4)
k' = k b
η=0
φ'(k') = b^-(d+2)/2 φ<(k)
(δr, δu) = t vt + ũ vu
dt/dl = λt t
du/dl = λu ũ
ε = 4−d
(2−ε) vu1 + 1/2 Kd Λ² vu2 = 0
vu1/vu2 = −1/2 Kd Λ²/(2−ε)
u
r
Gaussian fixed point

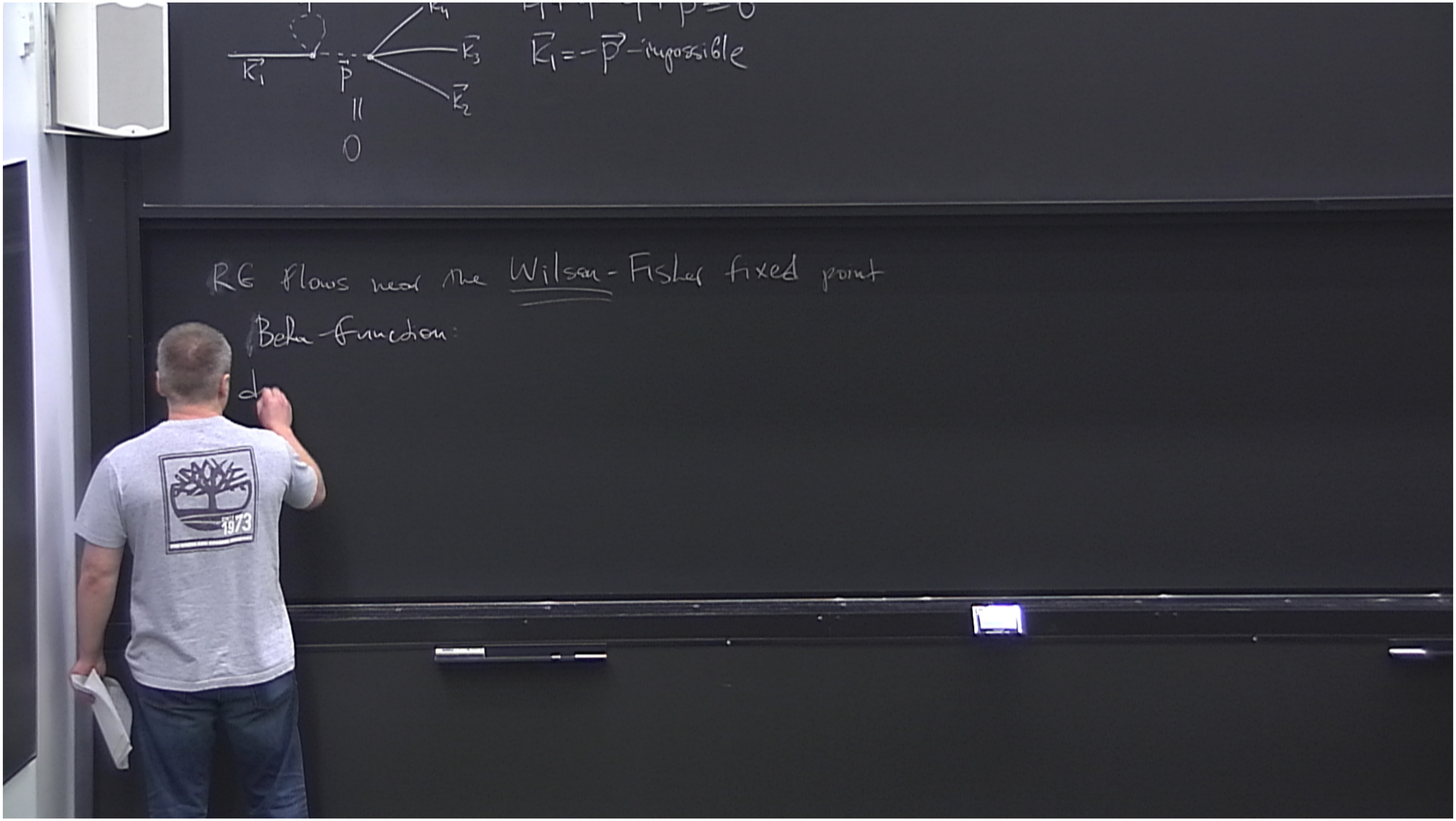

S'[φ<} = 1/2 ∫_k (k² + r + u/2 I₁) |φ<(k)|² + [u/4! − 36 (u/4!)² I₂] ∫_{k₁…k₄} (2π)^d δ(k₁ + … + k₄) φ<(k₁) … φ<(k₄)
k' = k b
η = 0
φ'(k') = b^{−(d+2)/2} φ<(k)
(2−ε ½ K_d Λ² ; 0 0) (v_{u1} ; v_{u2}) = 0
(2−ε) v_{u1} + ½ K_d Λ² v_{u2} = 0
v_{u1}/v_{u2} = −½ K_d Λ²/(2−ε)
(δr, δu) = t v_t + ũ v_u
dt/dl = λ_t t
dũ/dl = λ_u ũ
ε = 4−d
u
r
Gaussian fixed point

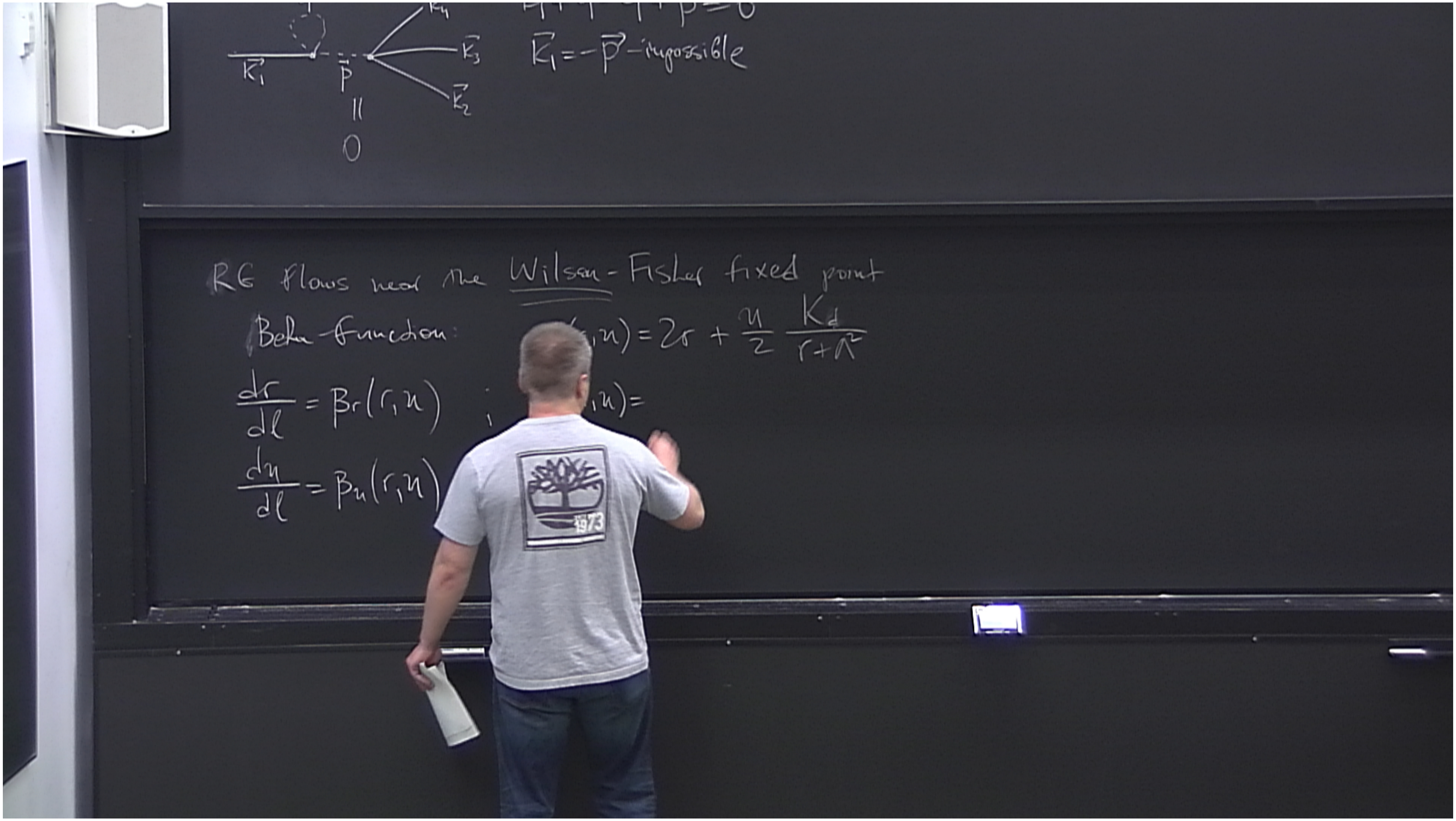

$S'[\varphi_<] = \frac{1}{2}\int_k \left(k^2 + r + \frac{u}{2} I_1\right)|\varphi_<(\vec{k})|^2 + \left[\frac{u}{4!} - 36\left(\frac{u}{4!}\right)^2 I_2\right]\int_{k_1 \ldots k_4} (2\pi)^d \delta(\vec{k}_1 + \cdots + \vec{k}_4)\varphi_<(\vec{k}_1)\ldots\varphi_<(\vec{k}_4)$
$\vec{k}' = \vec{k} b$
$\eta = 0$
$\varphi'(\vec{k}') = b^{-\frac{d+2}{2}} \varphi_<(\vec{k})$
$\begin{pmatrix} 2-\varepsilon & \frac{1}{2}K_d\Lambda^2 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} v_{u1} \\ v_{u2} \end{pmatrix} = 0$
$(2-\varepsilon)v_{u1} + \frac{1}{2}K_d\Lambda^2 v_{u2} = 0$
$\frac{v_{u1}}{v_{u2}} = -\frac{1}{2}\frac{K_d\Lambda^2}{2-\varepsilon}$
$(\delta r, \delta u) = t\vec{v}_t + \tilde{u}\vec{v}_u$
$\frac{dt}{dl} = \lambda_t t$
$\frac{d\tilde{u}}{dl} = \lambda_u \tilde{u}$
$\varepsilon = 4-d$
u
r
Gaussian fixed point
Variables, for which small deviations from

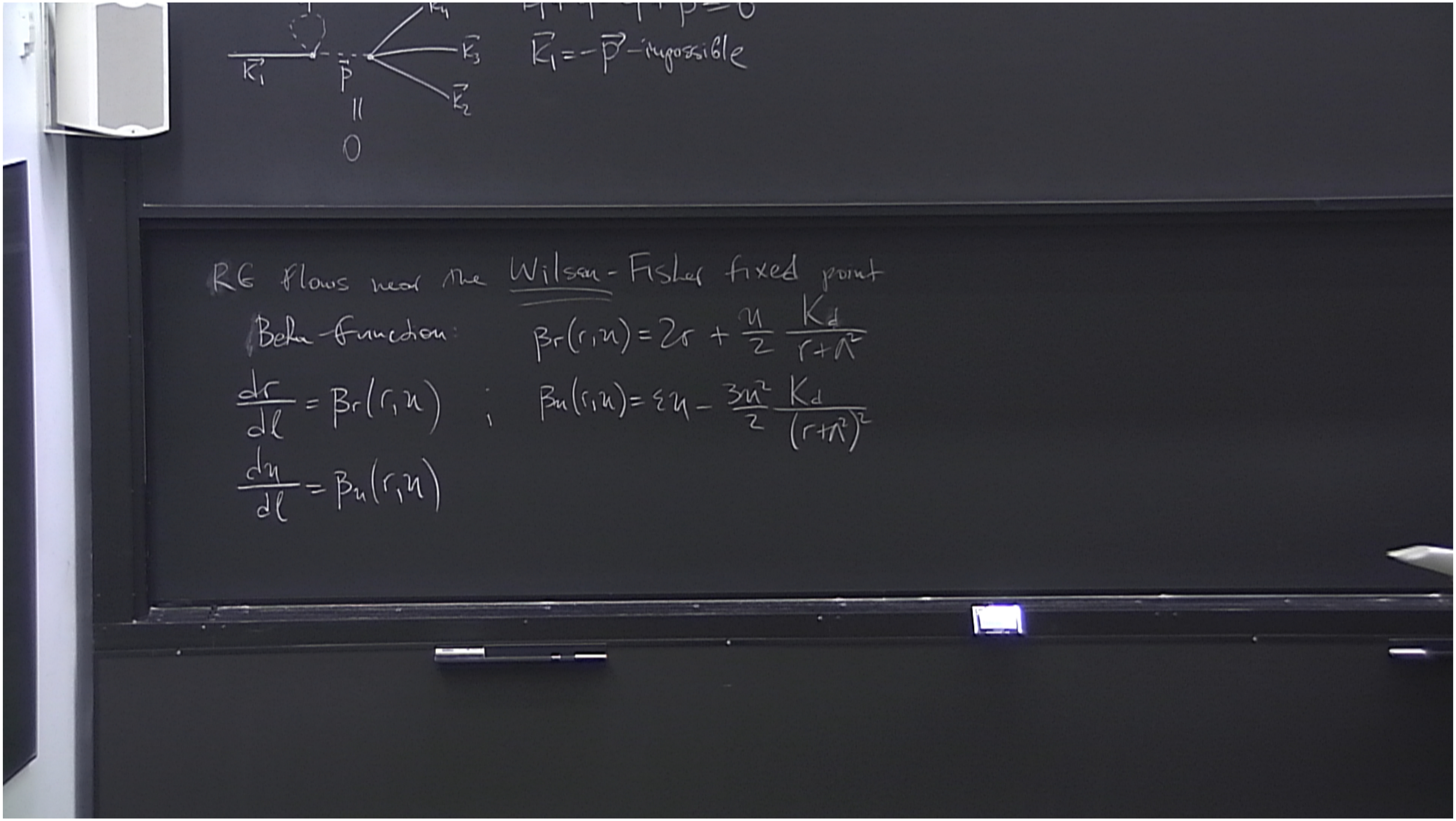

(δr, δu) = t V_t + ũ V_u
dt/dl = λ_t t
dũ/dl = λ_u ũ
ε = 4-d
u
r
Gaussian fixed point
Variables, for which small deviations from
fixed point grow under RG

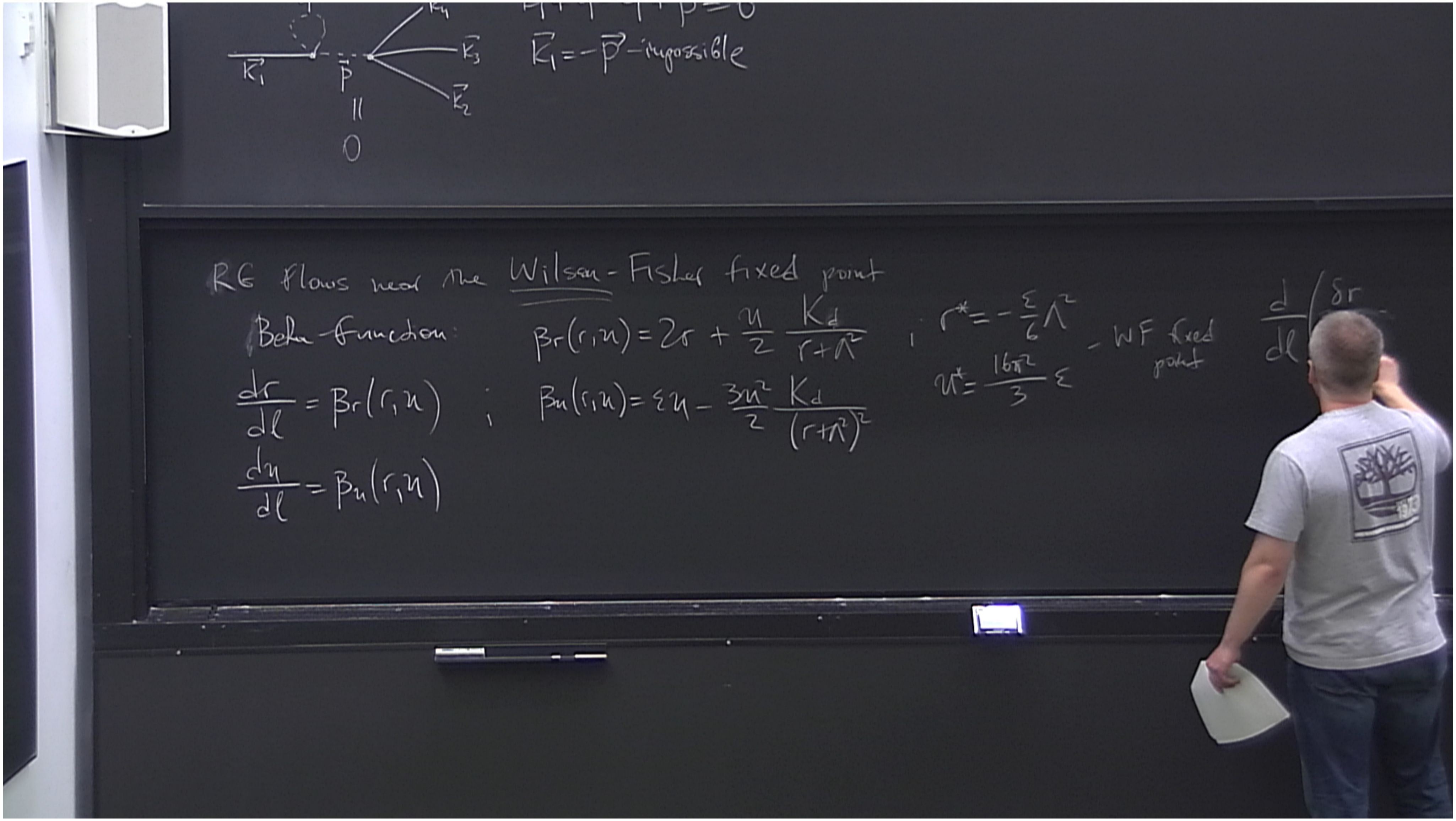


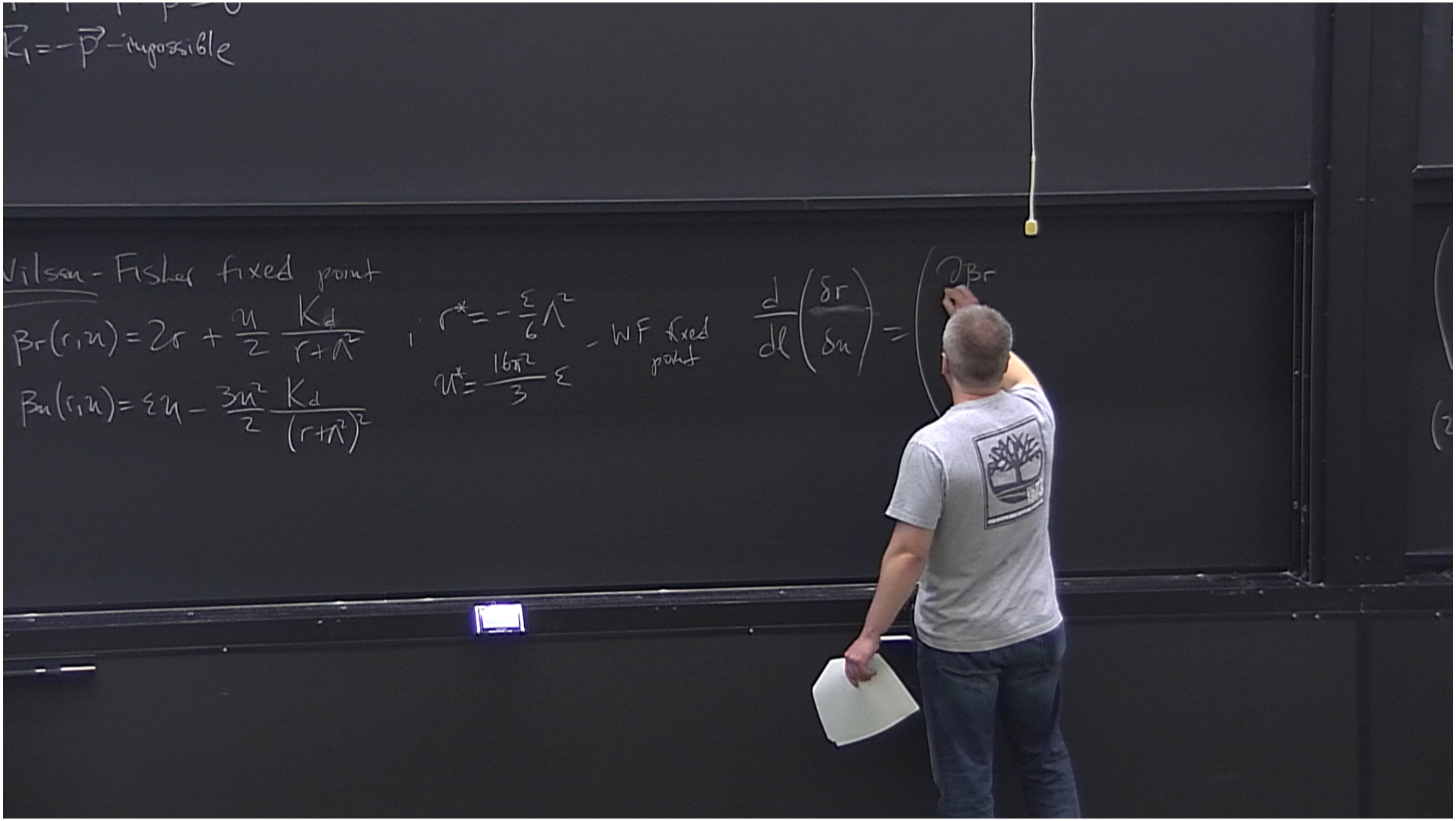
Variables, for which small deviations from fixed point grow under RG are relevant
Gaussian fixed point


$\vec{k}_1 = -\vec{p}$ - impossible
RG flows near the Wilson-Fisher fixed point
Beta-function:


k1 = -p - impossible
RG flows near the Wilson-Fisher fixed point
Beta-function:
(r,u) = 2r + u/2 Kd/(r+Λ2)
dr/dl = βr(r,u)
du/dl = βu(r,u)


$\vec{k}_1 = -\vec{p}$ - impossible
RG flows near the Wilson-Fisher fixed point
Beta-function:
$\beta_r(r,u) = 2r + \frac{u}{2}\frac{K_d}{r+\Lambda^2}$
$\frac{dr}{dl} = \beta_r(r,u)$; $\beta_u(r,u) = \varepsilon u - \frac{3u^2}{2}\frac{K_d}{(r+\Lambda^2)^2}$
$\frac{du}{dl} = \beta_u(r,u)$


$\vec{k}_1 = -\vec{p}$ - impossible
RG flows near the Wilson-Fisher fixed point
Beta-function:
$\beta_r(r,u) = 2r + \frac{u}{2}\frac{K_d}{r+\Lambda^2}$
$r^* = -\frac{\varepsilon}{6}\Lambda^2$
- WF fixed point
$u^* = \frac{16\pi^2}{3}\varepsilon$
$\frac{dr}{dl} = \beta_r(r,u)$
$\beta_u(r,u) = \varepsilon u - \frac{3u^2}{2}\frac{K_d}{(r+\Lambda^2)^2}$
$\frac{du}{dl} = \beta_u(r,u)$



$k_1 = -\vec{p}$ - impossible
Wilson-Fisher fixed point
$\beta_r(r,u) = 2r + \frac{u}{2}\frac{K_d}{r+\Lambda^2}$
$\beta_u(r,u) = \varepsilon u - \frac{3u^2}{2}\frac{K_d}{(r+\Lambda^2)^2}$
$r^* = -\frac{\varepsilon}{6}\Lambda^2$
$u^* = \frac{16\pi^2}{3}\varepsilon$
- WF fixed point
$\frac{d}{dl}\begin{pmatrix}\delta r \\ \delta u\end{pmatrix} = \Big(\partial\beta_r$

$\vec{k}_1 = -\vec{p}$ – impossible

Wilson-Fisher fixed point

$$\beta_r(r,u) = 2r + \frac{u}{2}\frac{K_d}{r+\Lambda^2}$$

$$\beta_u(r,u) = \varepsilon u - \frac{3u^2}{2}\frac{K_d}{(r+\Lambda^2)^2}$$

$$r^* = -\frac{\varepsilon}{6}\Lambda^2, \quad u^* = \frac{16\pi^2}{3}\varepsilon \quad \text{– WF fixed point}$$

$$\frac{d}{dl}\begin{pmatrix}\delta r\\ \delta u\end{pmatrix} = \begin{pmatrix}\frac{\partial\beta_r}{\partial r} & \frac{\partial\beta_r}{\partial u}\\ \frac{\partial\beta_u}{\partial r} & \frac{\partial\beta_u}{\partial u}\end{pmatrix}_{r^*,u^*}\begin{pmatrix}\delta r\\ \delta u\end{pmatrix}$$

$\vec{k}_1 = -\vec{p}$ – impossible

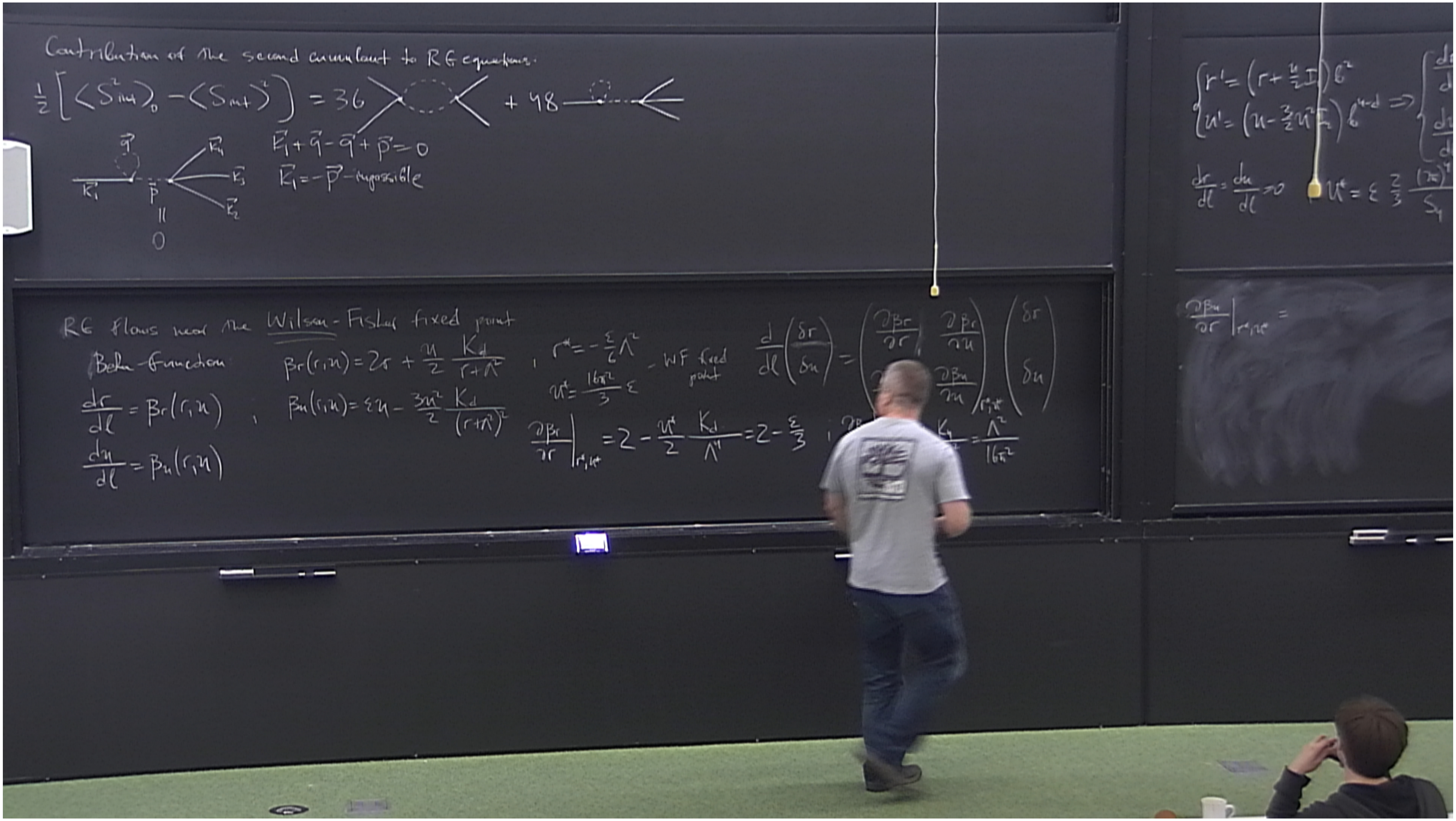
Wilson-Fisher fixed point

$$\beta_r(r,u) = 2r + \frac{u}{2}\frac{K_d}{r+\Lambda^2} \quad ; \quad r^* = -\frac{\varepsilon}{6}\Lambda^2 \quad \text{– WF fixed point}$$

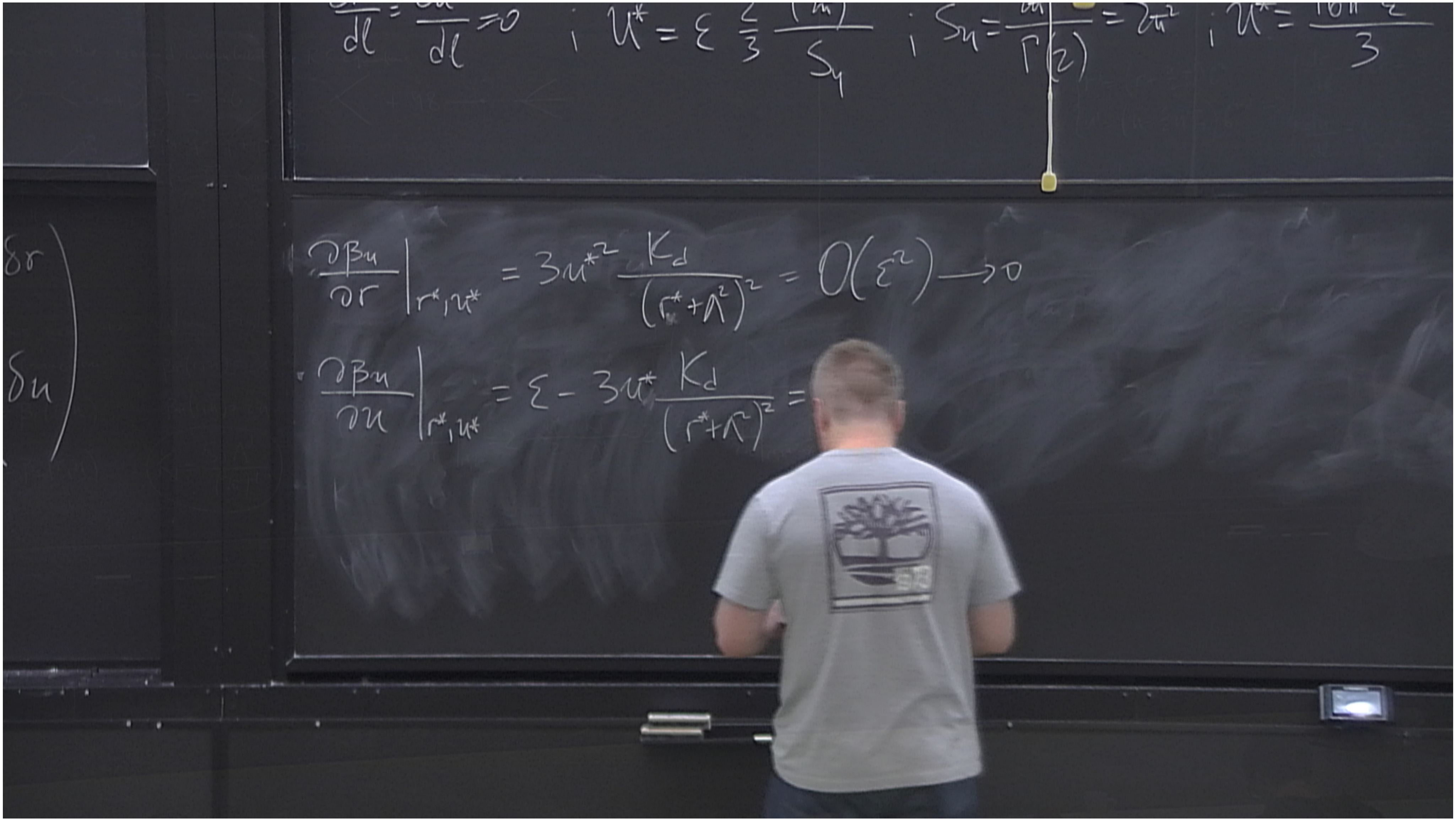
$$\beta_u(r,u) = \varepsilon u - \frac{3u^2}{2}\frac{K_d}{(r+\Lambda^2)^2} \qquad u^* = \frac{16\pi^2}{3}\varepsilon$$

$$\frac{d}{dl}\begin{pmatrix}\delta r\\ \delta u\end{pmatrix} = \begin{pmatrix}\frac{\partial\beta_r}{\partial r} & \frac{\partial\beta_r}{\partial u}\\ \frac{\partial\beta_u}{\partial r} & \frac{\partial\beta_u}{\partial u}\end{pmatrix}_{r^*,u^*}\begin{pmatrix}\delta r\\ \delta u\end{pmatrix}$$

$$\left.\frac{\partial\beta_r}{\partial r}\right|_{r^*,u^*} = 2 - \frac{u^*}{2}\frac{K_d}{\Lambda^4} = 2 - \frac{\varepsilon}{3} \quad ; \quad \left.\frac{\partial\beta_r}{\partial u}\right|_{r^*,u^*} = \frac{1}{2}\frac{K_4}{\Lambda^2} = \frac{\Lambda^2}{16\pi^2}$$



Contribution of the second cumulant to RG equations.
RG flows near the Wilson-Fisher fixed point
Beta-Function
WF fixed point
impossible



∂β_u/∂r |_{r*,u*} = 3u*² K_d/(r*+Λ²)² = O(ε²) → 0
∂β_u/∂u |_{r*,u*} = ε − 3u* K_d/(r*+Λ²)² =

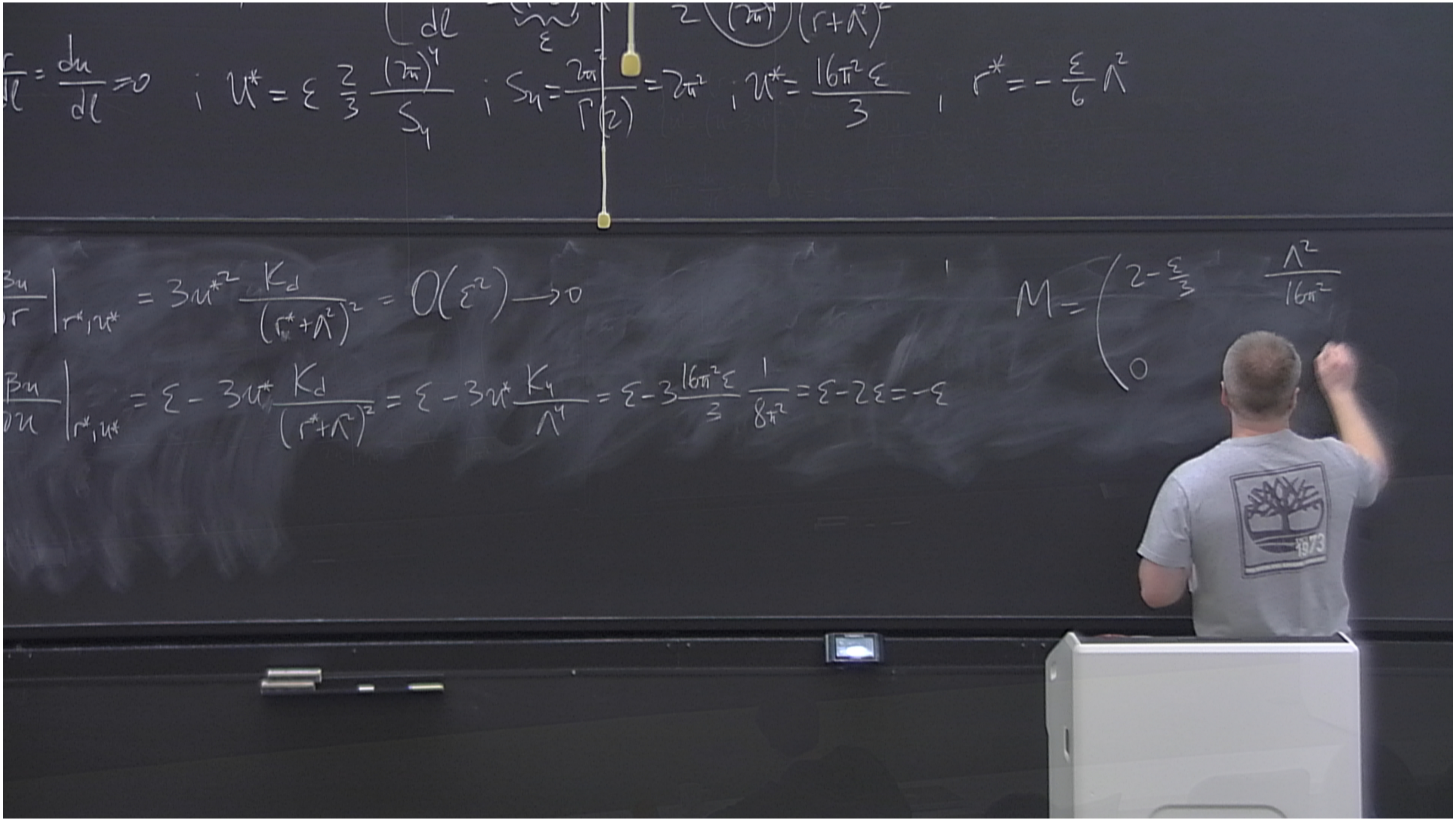
$$\frac{dr}{dl} = \frac{du}{dl} = 0 \;;\; u^* = \varepsilon\,\frac{2}{3}\,\frac{(\ldots)}{S_4} \;;\; S_4 = \frac{\ldots}{\Gamma(2)} = 2\pi^2 \;;\; u^* = \frac{16\pi^2\varepsilon}{3}$$

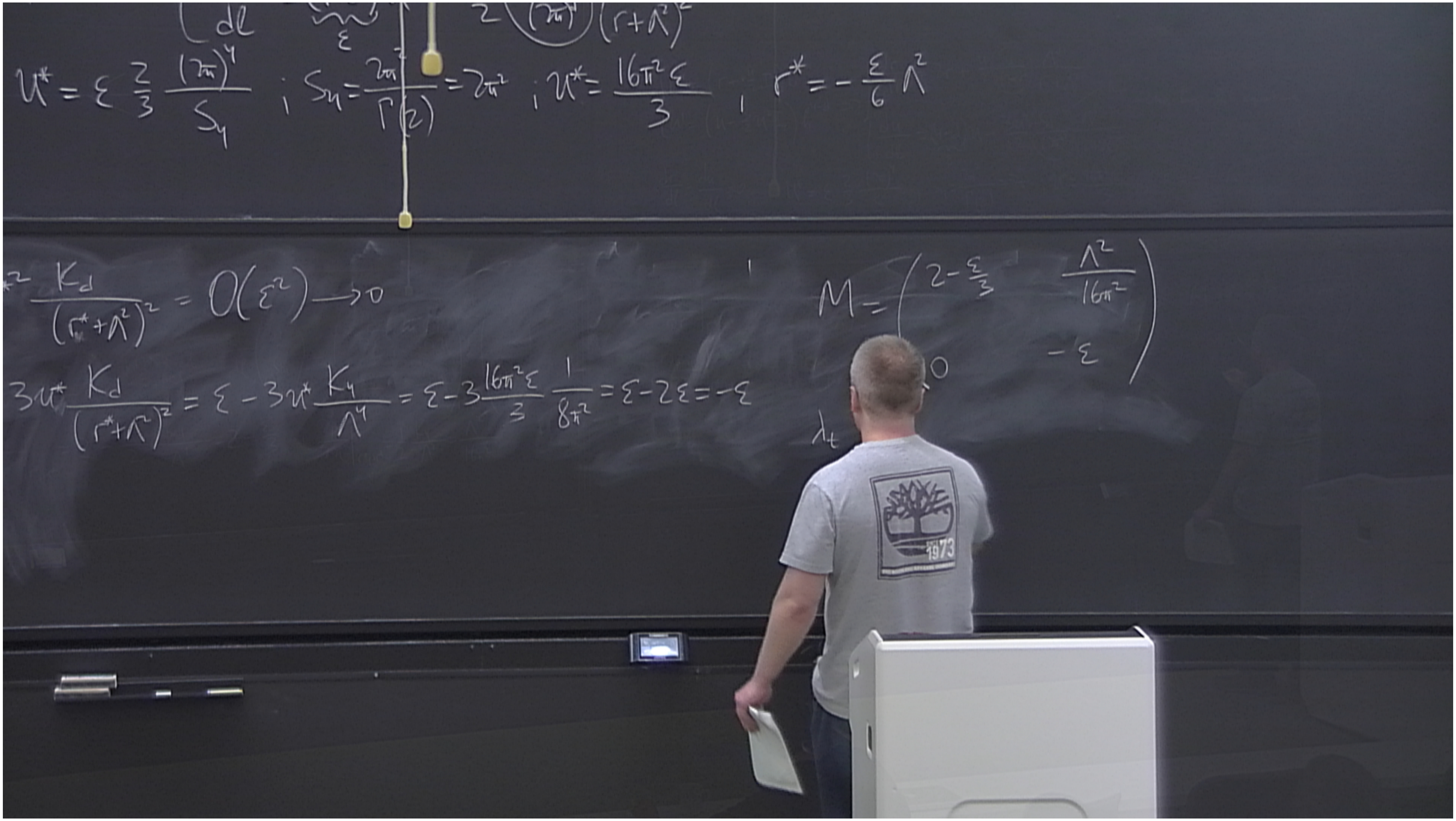
δr

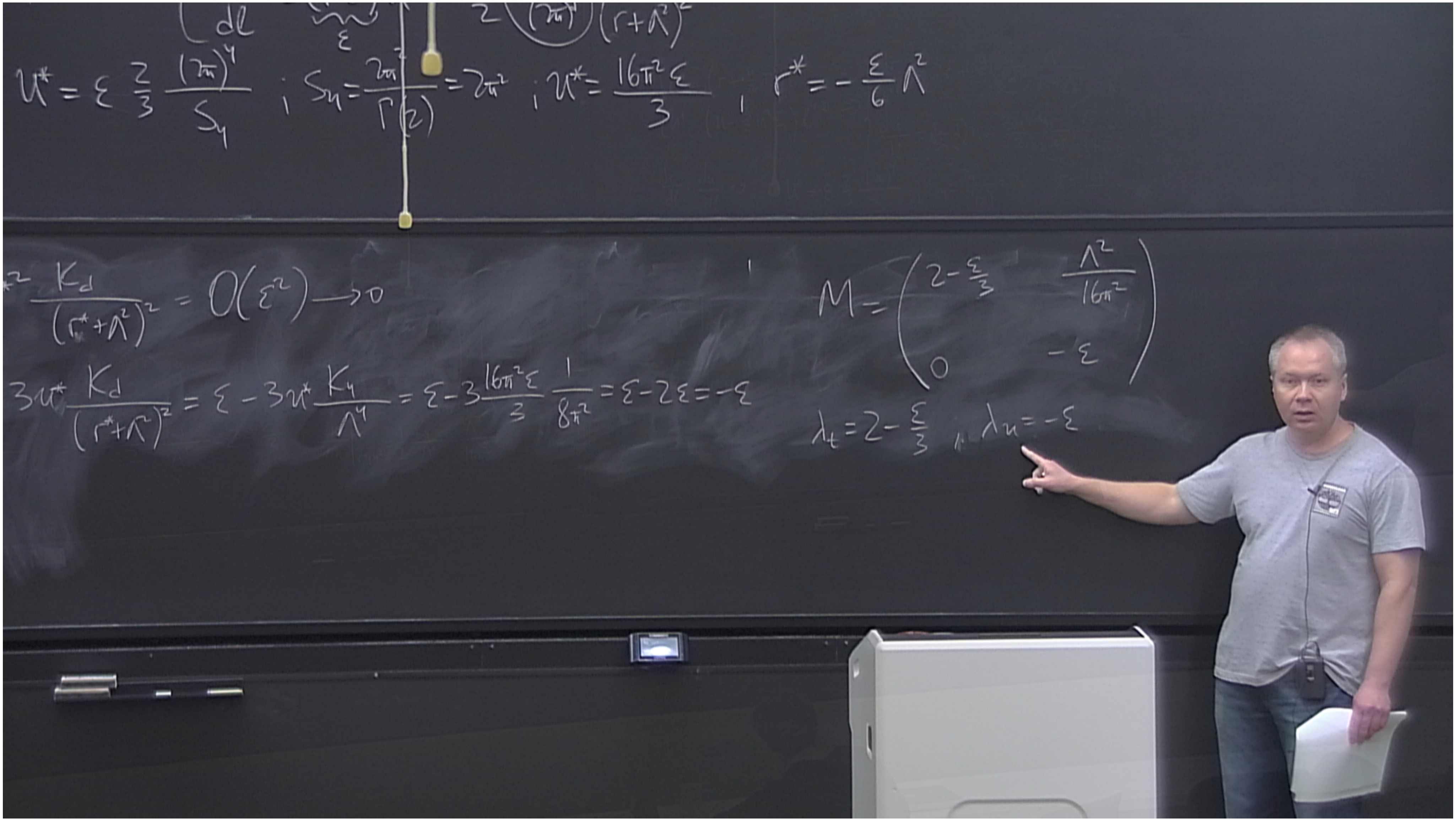
δu

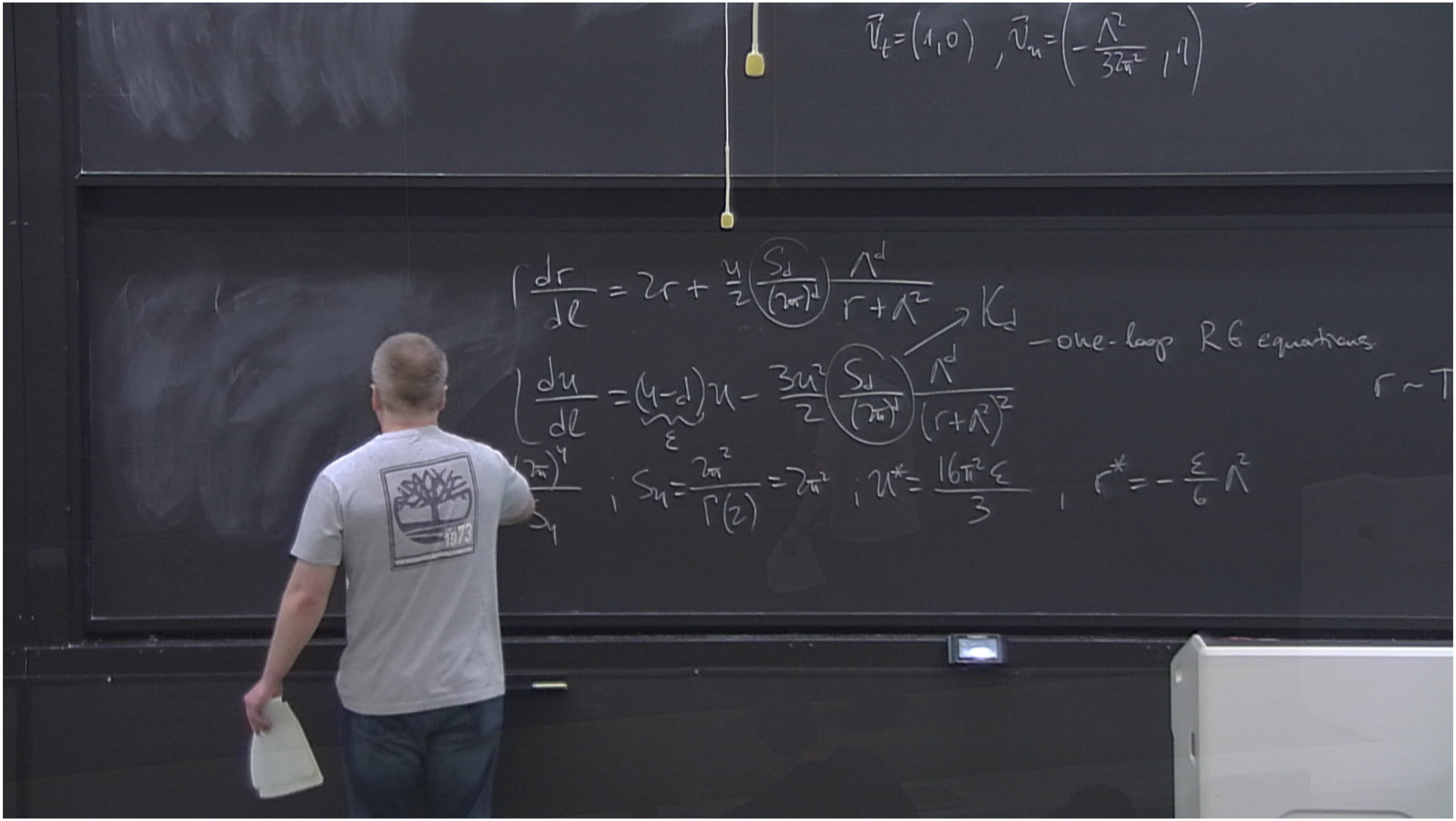
$$\left.\frac{\partial \beta_u}{\partial r}\right|_{r^*,u^*} = 3u^{*2}\,\frac{K_d}{(r^*+\Lambda^2)^2} = O(\varepsilon^2) \longrightarrow 0$$

$$\left.\frac{\partial \beta_u}{\partial u}\right|_{r^*,u^*} = \varepsilon - 3u^*\,\frac{K_d}{(r^*+\Lambda^2)^2} = \varepsilon - 3u^*\,\frac{K_4}{\Lambda^4} = \varepsilon - 3\,\frac{16\pi^2\varepsilon}{3}\,\frac{1}{8\pi^2} = \varepsilon - 2\varepsilon = -$$

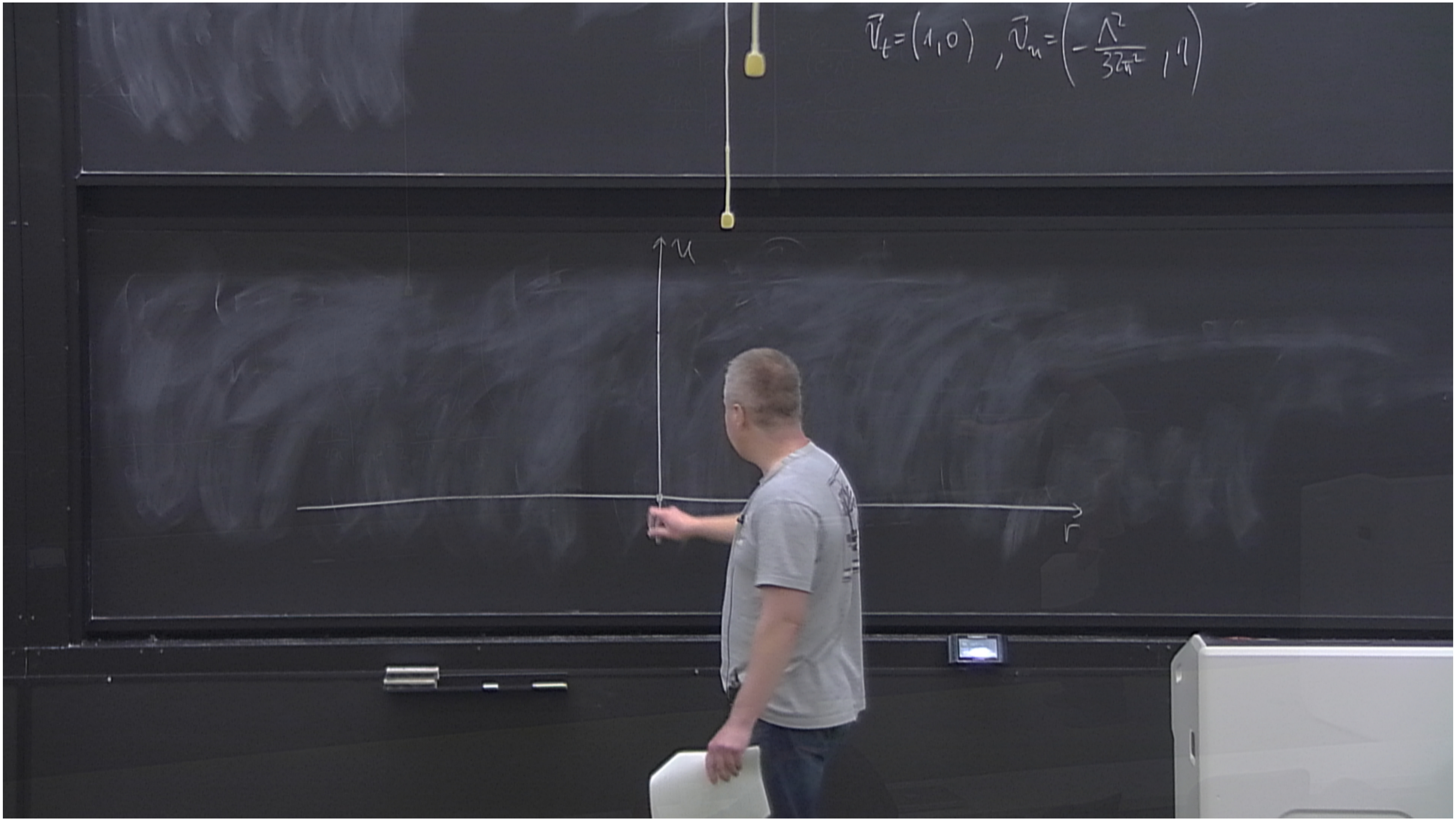

dr/dl = du/dl = 0 ; u* = ε (2/3) (2π)^4 / S_4 ; S_4 = 2π^2/Γ(2) = 2π^2 ; u* = 16π^2 ε / 3 , r* = −(ε/6) Λ^2
∂β_u/∂r |_{r*,u*} = 3u*^2 K_d/(r*+Λ^2)^2 = O(ε^2) → 0
∂β_u/∂u |_{r*,u*} = ε − 3u* K_d/(r*+Λ^2)^2 = ε − 3u* K_4/Λ^4 = ε − 3 (16π^2 ε/3)(1/8π^2) = ε − 2ε = −ε
M = (2 − ε/3   Λ^2/16π^2 ; 0)


$u^* = \varepsilon \frac{2}{3} \frac{(2\pi)^4}{S_4}$; $S_4 = \frac{2\pi^2}{\Gamma(2)} = 2\pi^2$; $u^* = \frac{16\pi^2 \varepsilon}{3}$, $r^* = -\frac{\varepsilon}{6}\Lambda^2$
$\frac{K_d}{(r^*+\Lambda^2)^2} = O(\varepsilon^2) \to 0$
$3u^* \frac{K_d}{(r^*+\Lambda^2)^2} = \varepsilon - 3u^* \frac{K_4}{\Lambda^4} = \varepsilon - 3\frac{16\pi^2\varepsilon}{3}\frac{1}{8\pi^2} = \varepsilon - 2\varepsilon = -\varepsilon$
$M = \begin{pmatrix} 2-\frac{\varepsilon}{3} & \frac{\Lambda^2}{16\pi^2} \\ 0 & -\varepsilon \end{pmatrix}$
λ_t

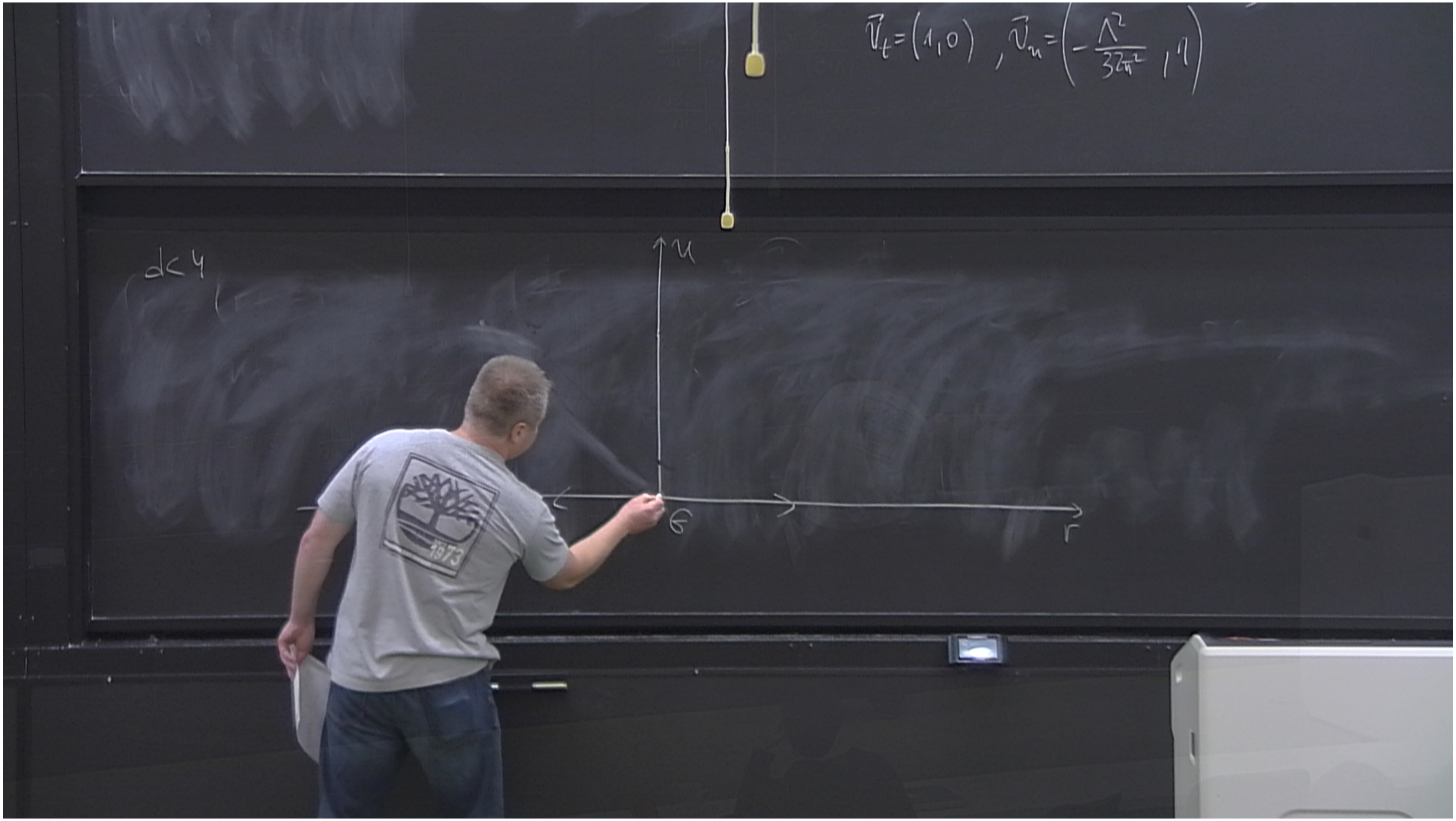

$u^* = \varepsilon \frac{2}{3} \frac{(2\pi)^4}{S_4}$; $S_4 = \frac{2\pi^2}{\Gamma(2)} = 2\pi^2$; $u^* = \frac{16\pi^2 \varepsilon}{3}$, $r^* = -\frac{\varepsilon}{6}\Lambda^2$
$\frac{K_d}{(r^* + \Lambda^2)^2} = O(\varepsilon^2) \to 0$
$3u^* \frac{K_d}{(r^* + \Lambda^2)^2} = \varepsilon - 3u^* \frac{K_4}{\Lambda^4} = \varepsilon - 3\frac{16\pi^2\varepsilon}{3}\frac{1}{8\pi^2} = \varepsilon - 2\varepsilon = -\varepsilon$
$M = \begin{pmatrix} 2-\frac{\varepsilon}{3} & \frac{\Lambda^2}{16\pi^2} \\ 0 & -\varepsilon \end{pmatrix}$
$\lambda_t = 2 - \frac{\varepsilon}{3}$, $\lambda_u = -\varepsilon$



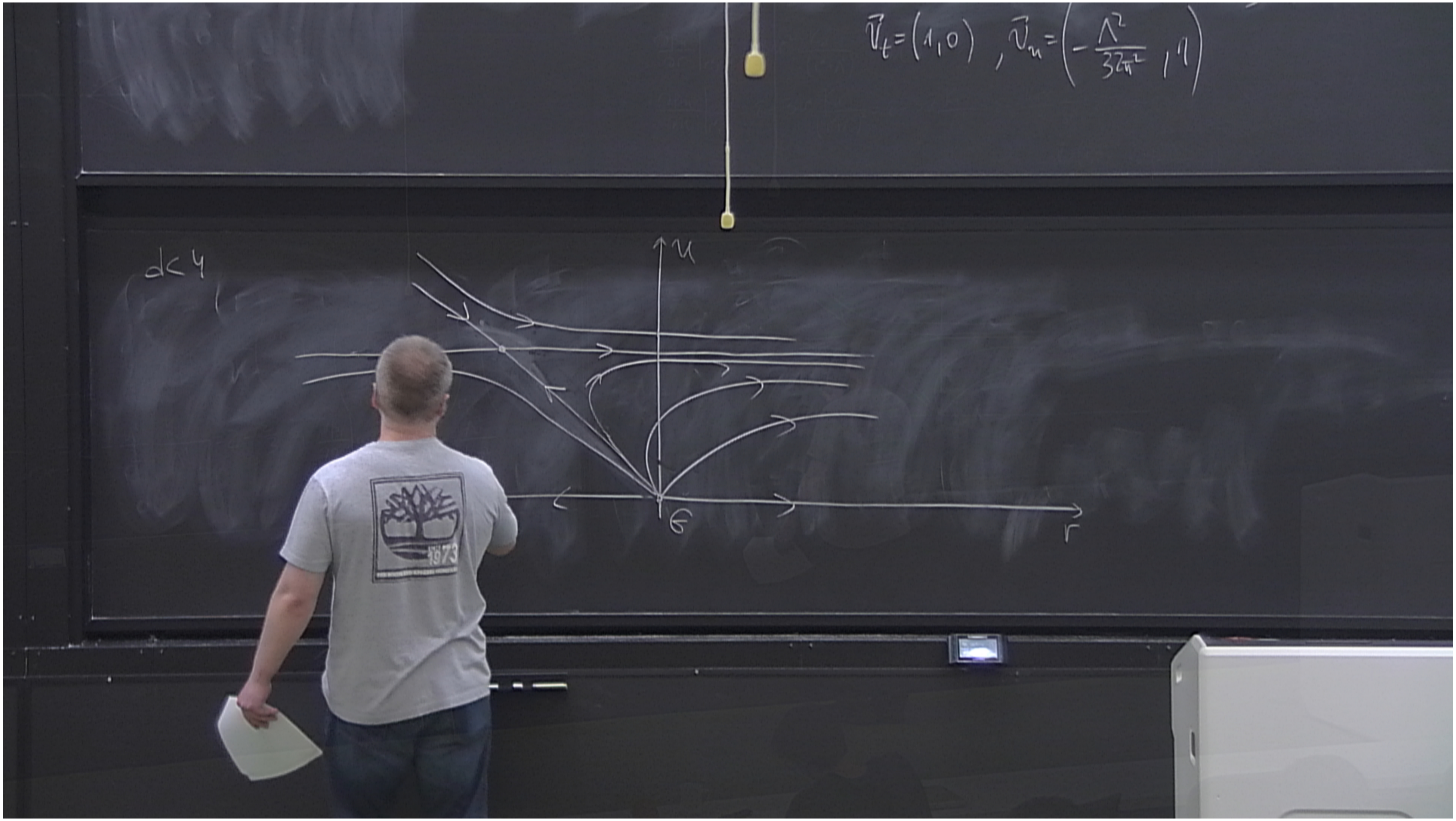
$\vec{v}_t = (1,0)$, $\vec{v}_u = \left(-\frac{\Lambda^2}{32\pi^2}, 1\right)$
$\frac{dr}{dl} = 2r + \frac{u}{2}\frac{S_d}{(2\pi)^d}\frac{\Lambda^d}{r+\Lambda^2}$
K_d
— one-loop RG equations
$\frac{du}{dl} = (4-d)u - \frac{3u^2}{2}\frac{S_d}{(2\pi)^d}\frac{\Lambda^d}{(r+\Lambda^2)^2}$
ε
$r \sim T$
$S_4 = \frac{2\pi^2}{\Gamma(2)} = 2\pi^2$; $u^* = \frac{16\pi^2\varepsilon}{3}$, $r^* = -\frac{\varepsilon}{6}\Lambda^2$



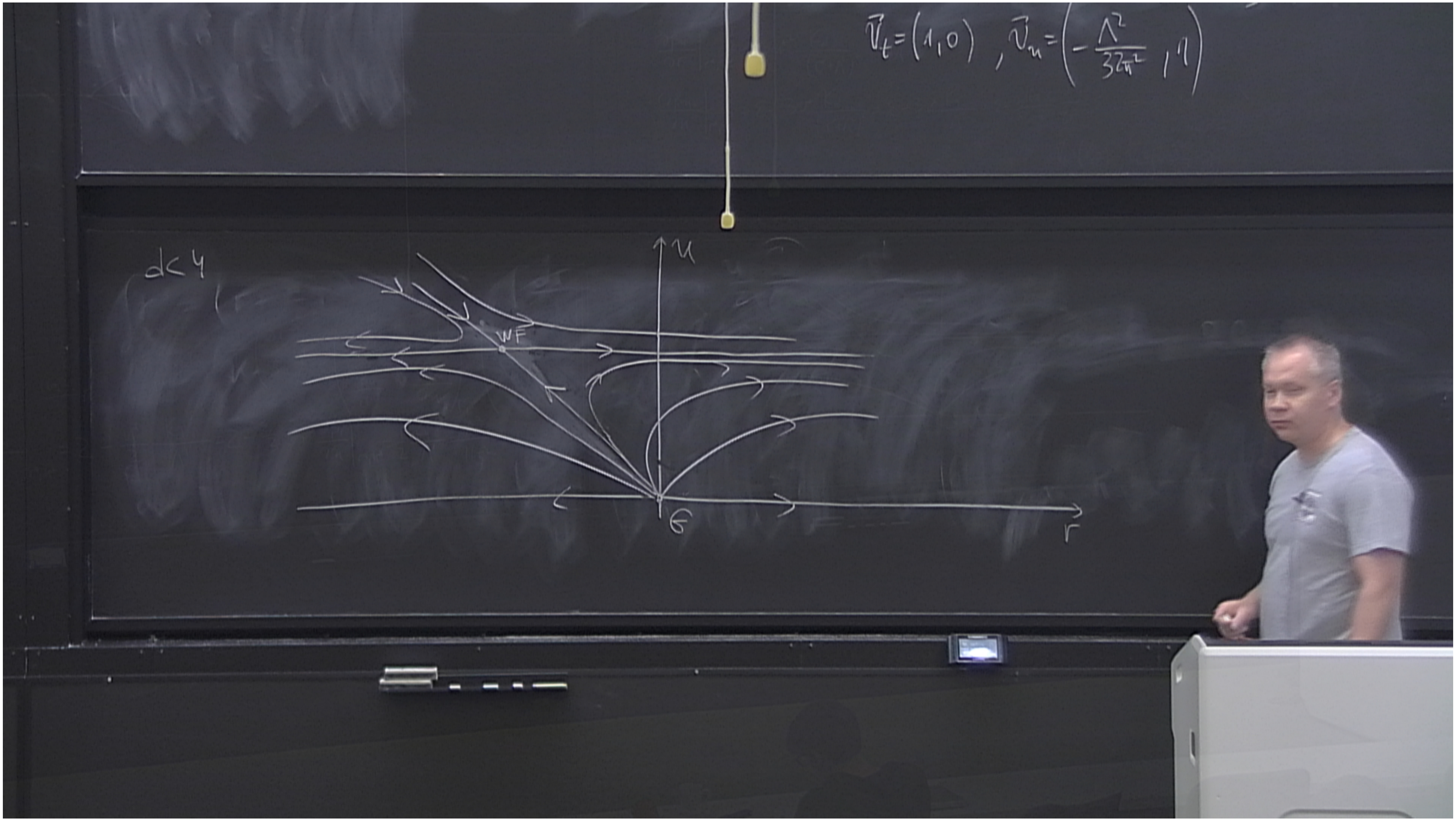
$\vec{v}_t = (1,0)$, $\vec{v}_u = \left(-\frac{\Lambda^2}{32\pi^2}, 1\right)$
u
r



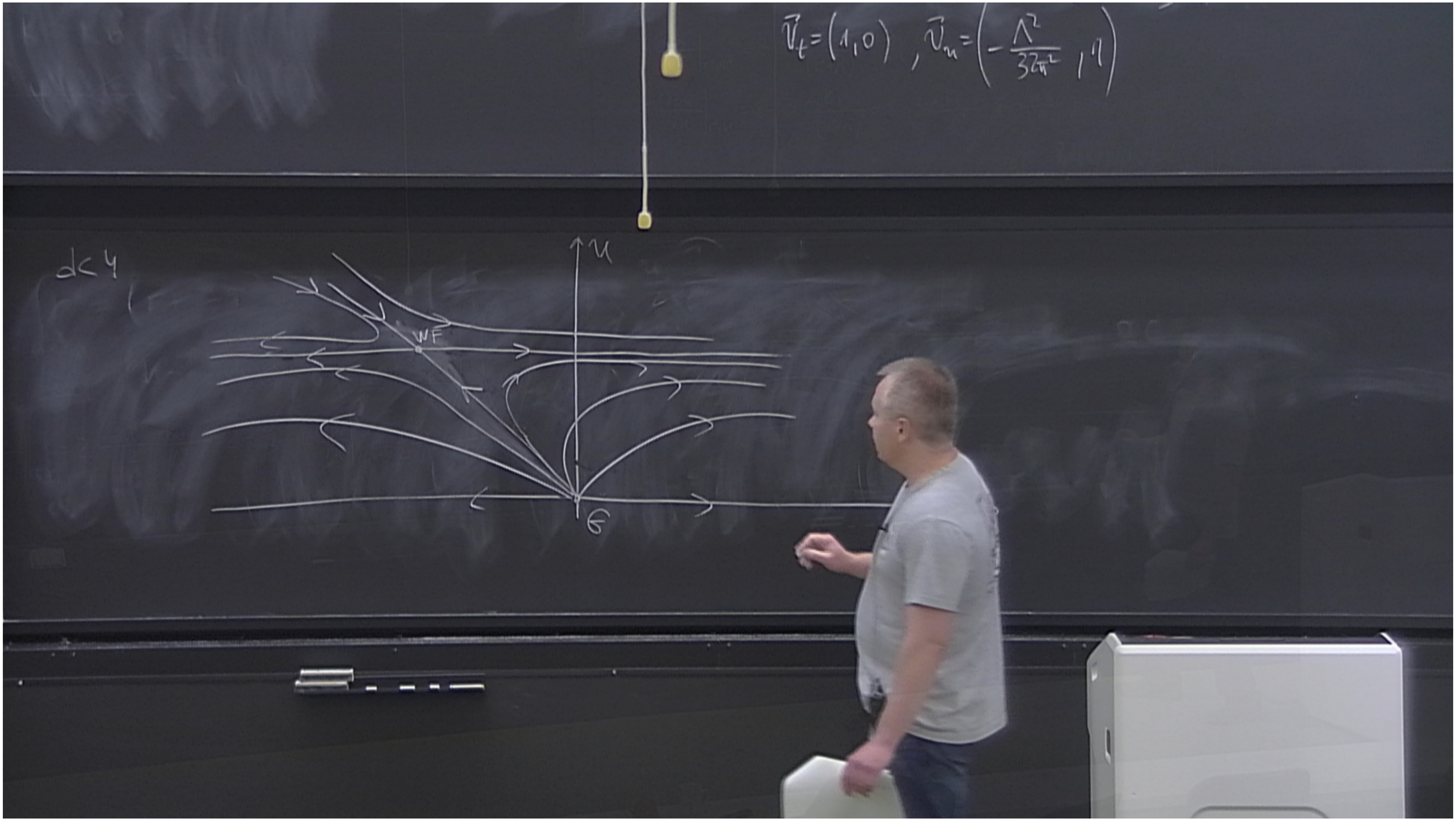
$\vec{V}_t = (1,0)$, $\vec{V}_u = \left(-\frac{\Lambda^2}{32\pi^2}, 1\right)$
d<4
u
G
r



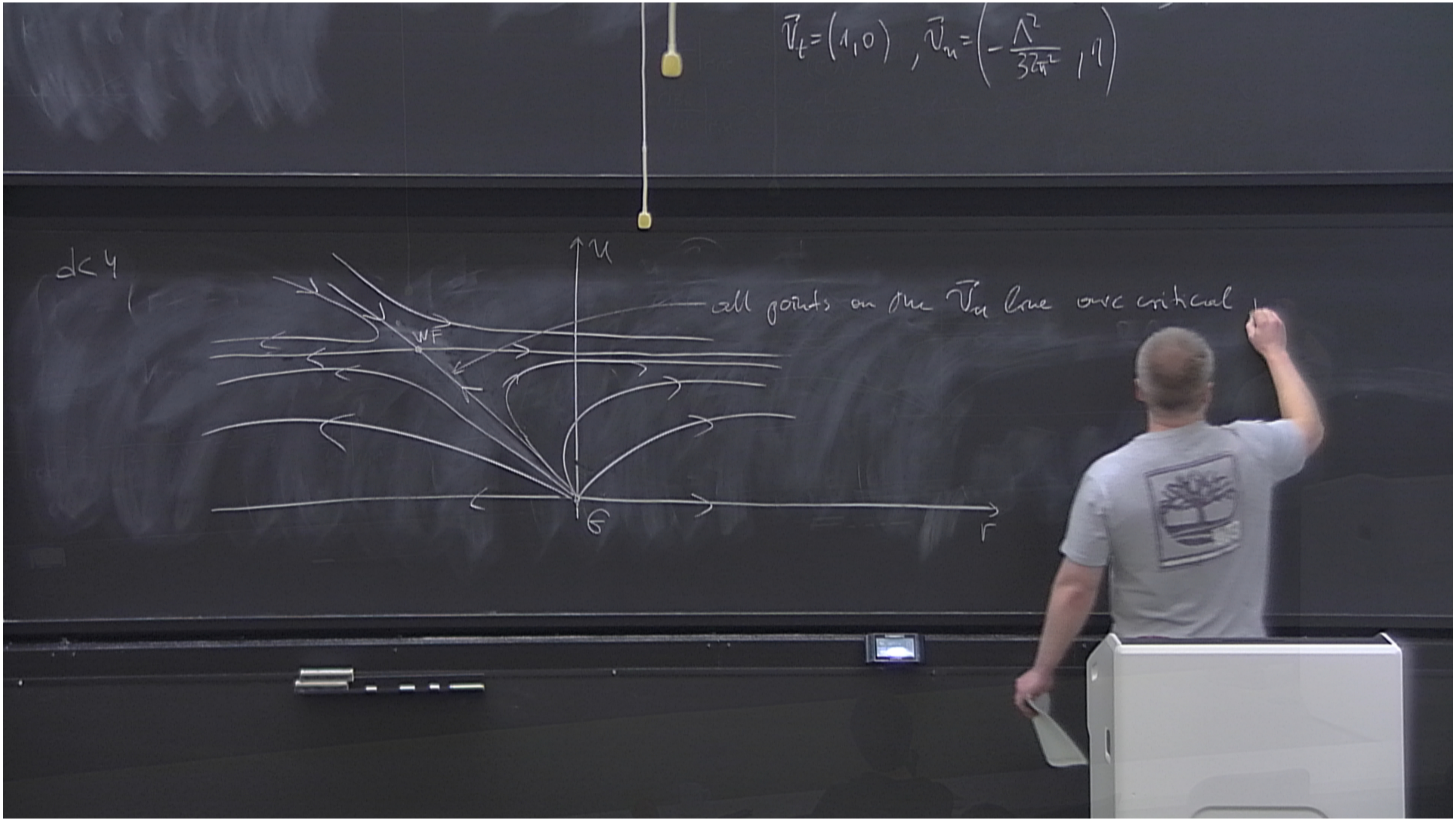
$\vec{v}_t = (1,0)$, $\vec{v}_u = \left(-\frac{\Lambda^2}{32\pi^2}, 1\right)$
d<4
u
G
r



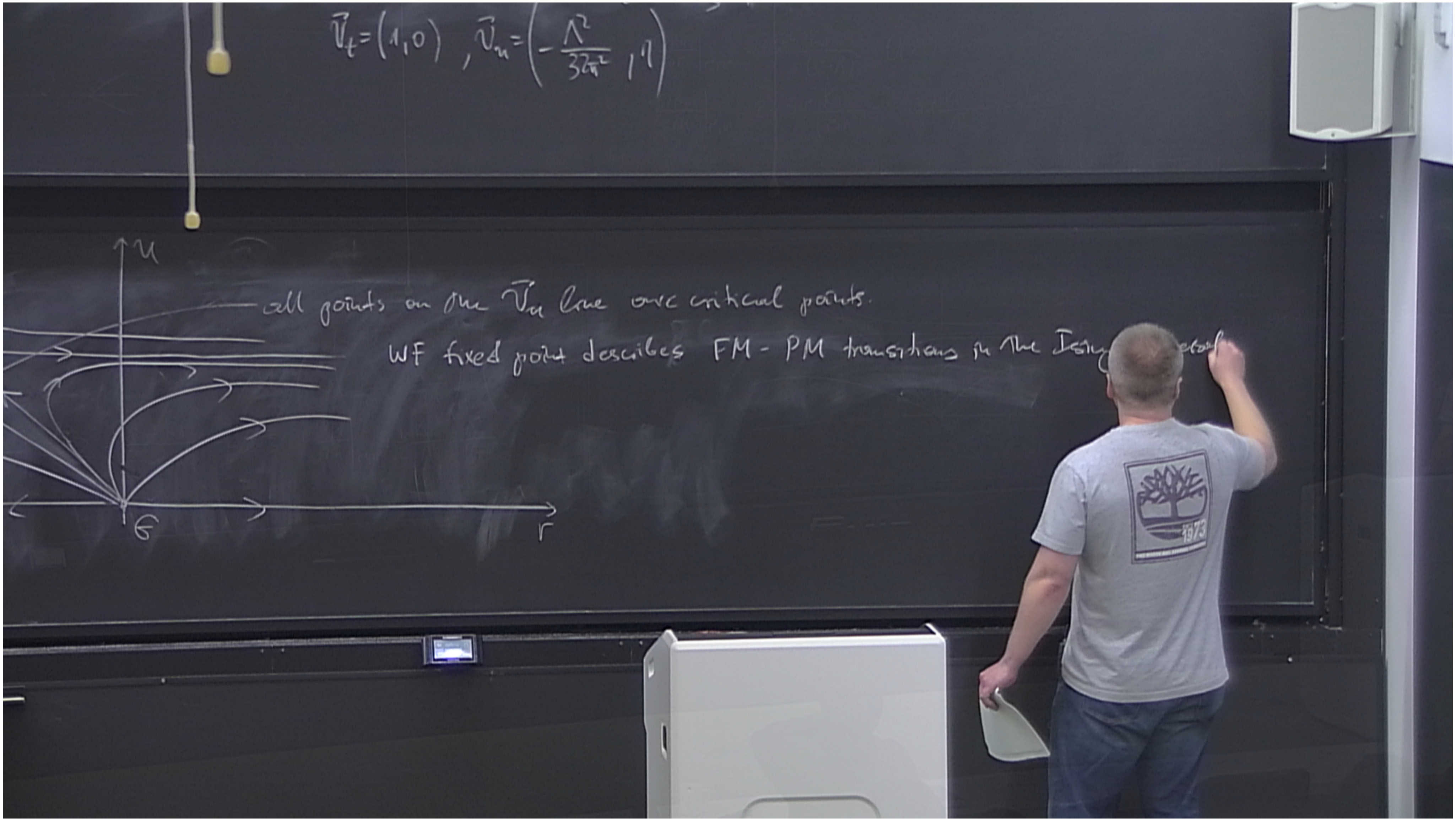
$\vec{v}_t = (1,0)$, $\vec{v}_u = \left(-\frac{\Lambda^2}{32\pi^2}, 1\right)$
d<4
u
WF
G
r

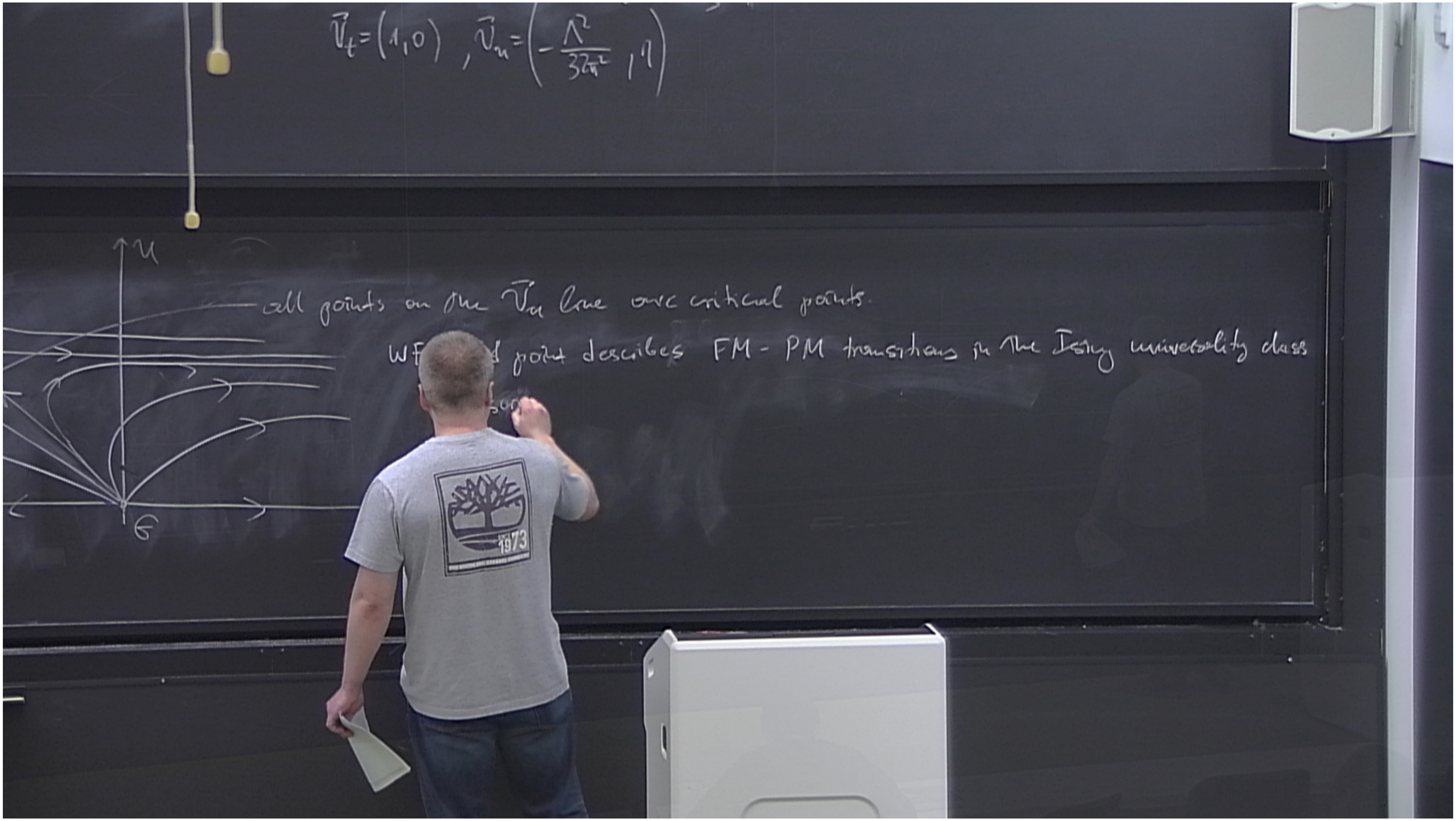


$\vec{v}_t = (1,0)$, $\vec{v}_u = \left(-\frac{\Lambda^2}{32\pi^2}, 1\right)$
d<4
u
WF
6

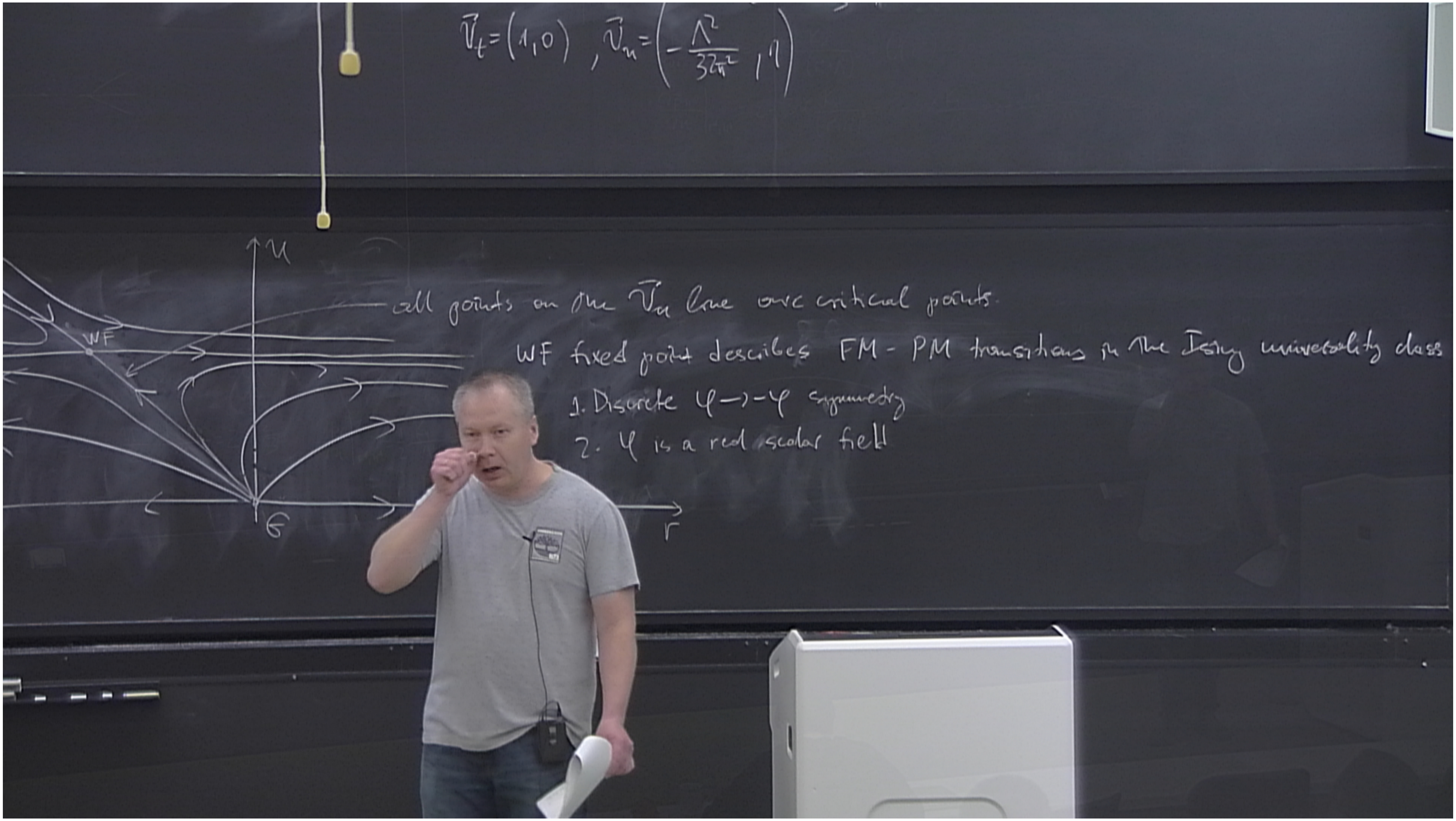


$\vec{v}_t = (1,0)$, $\vec{v}_u = \left(-\frac{\Lambda^2}{32\pi^2}, 1\right)$
d<4
u
WF
all points on the $\vec{v}_u$ line are critical
G
r

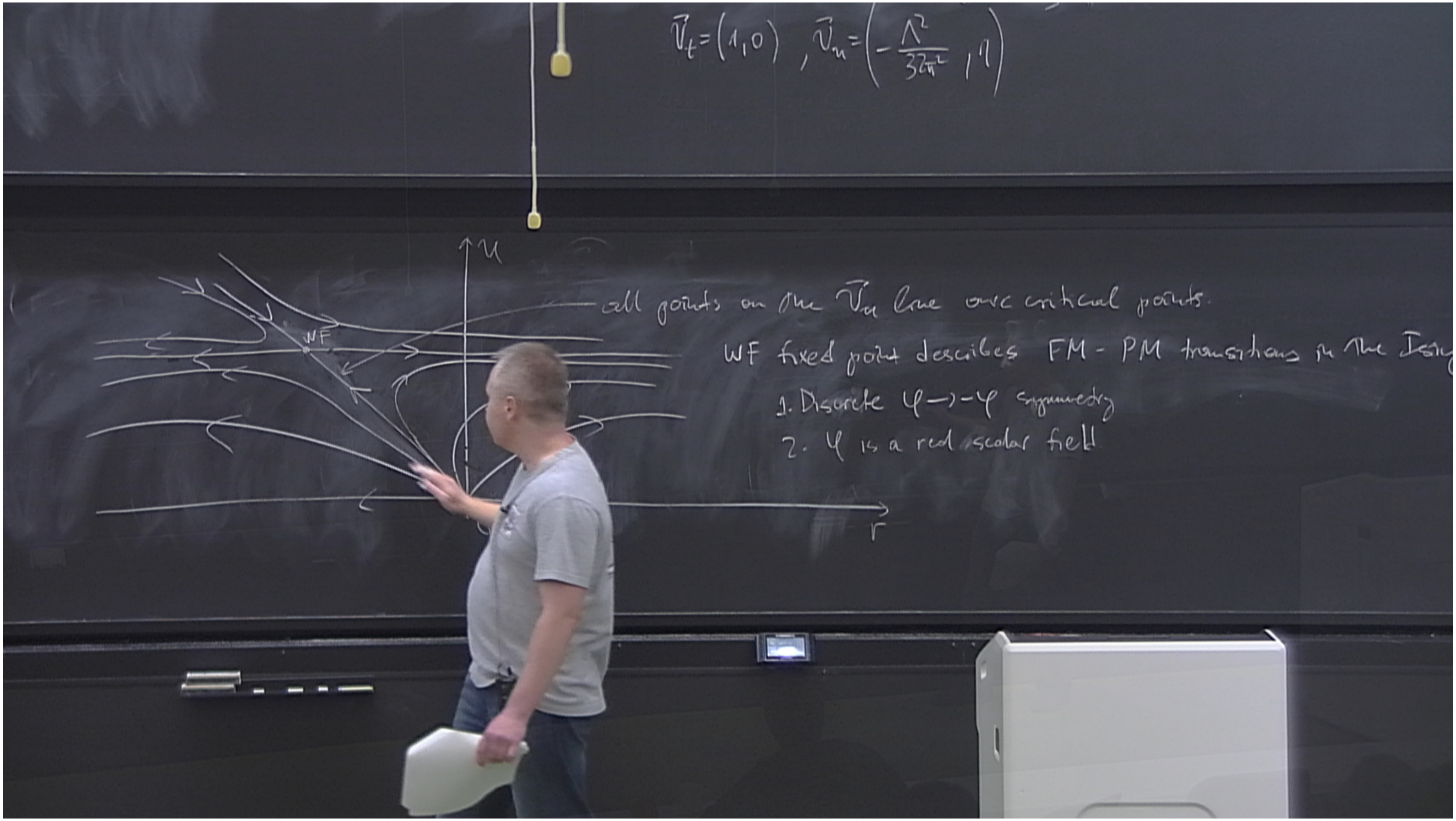

V_t = (1,0) , V_u = (− Λ²/32π² , 1)
all points on the V_u line are critical points.
WF fixed point describes FM - PM transitions in the Ising
u
r

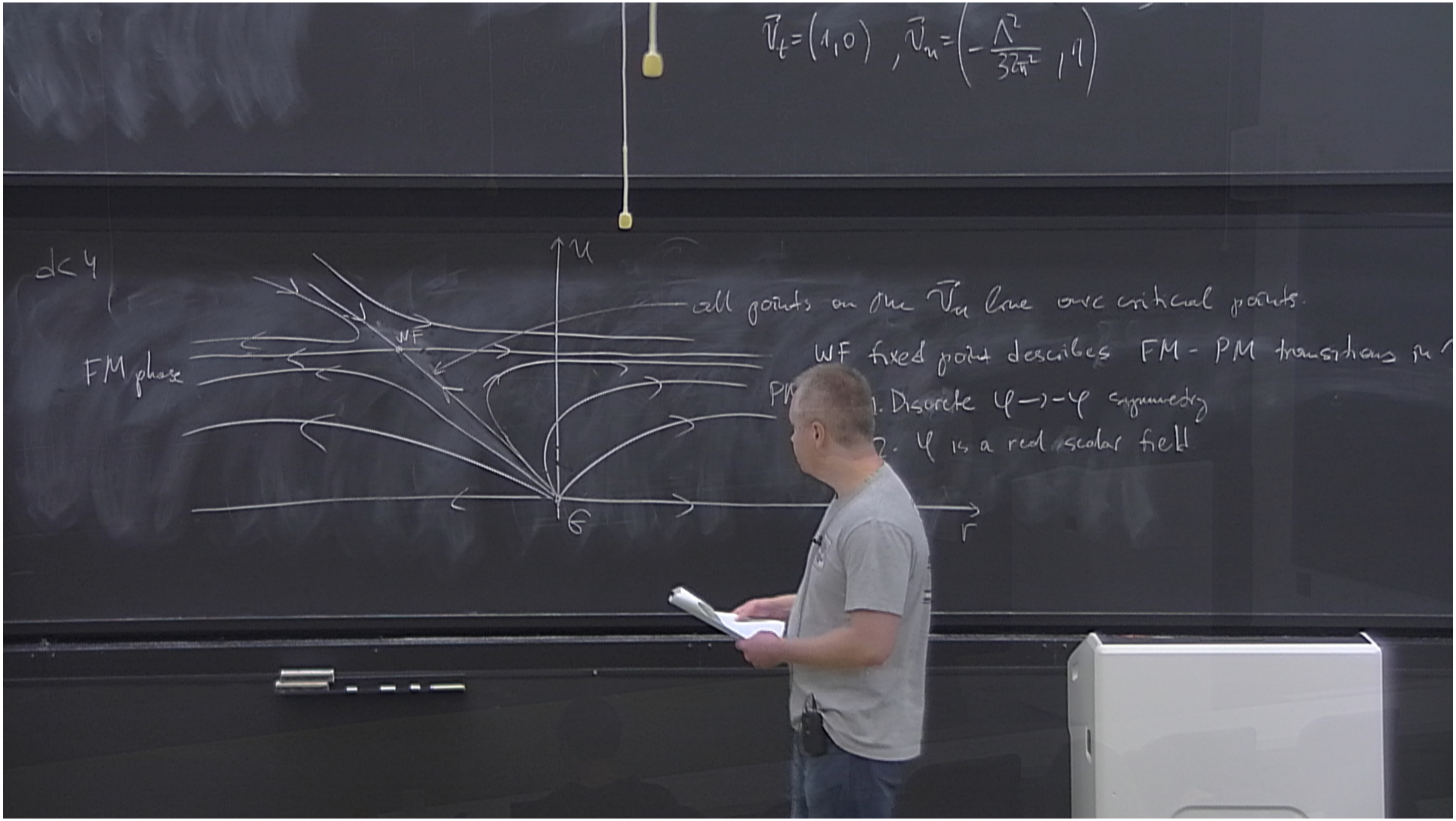


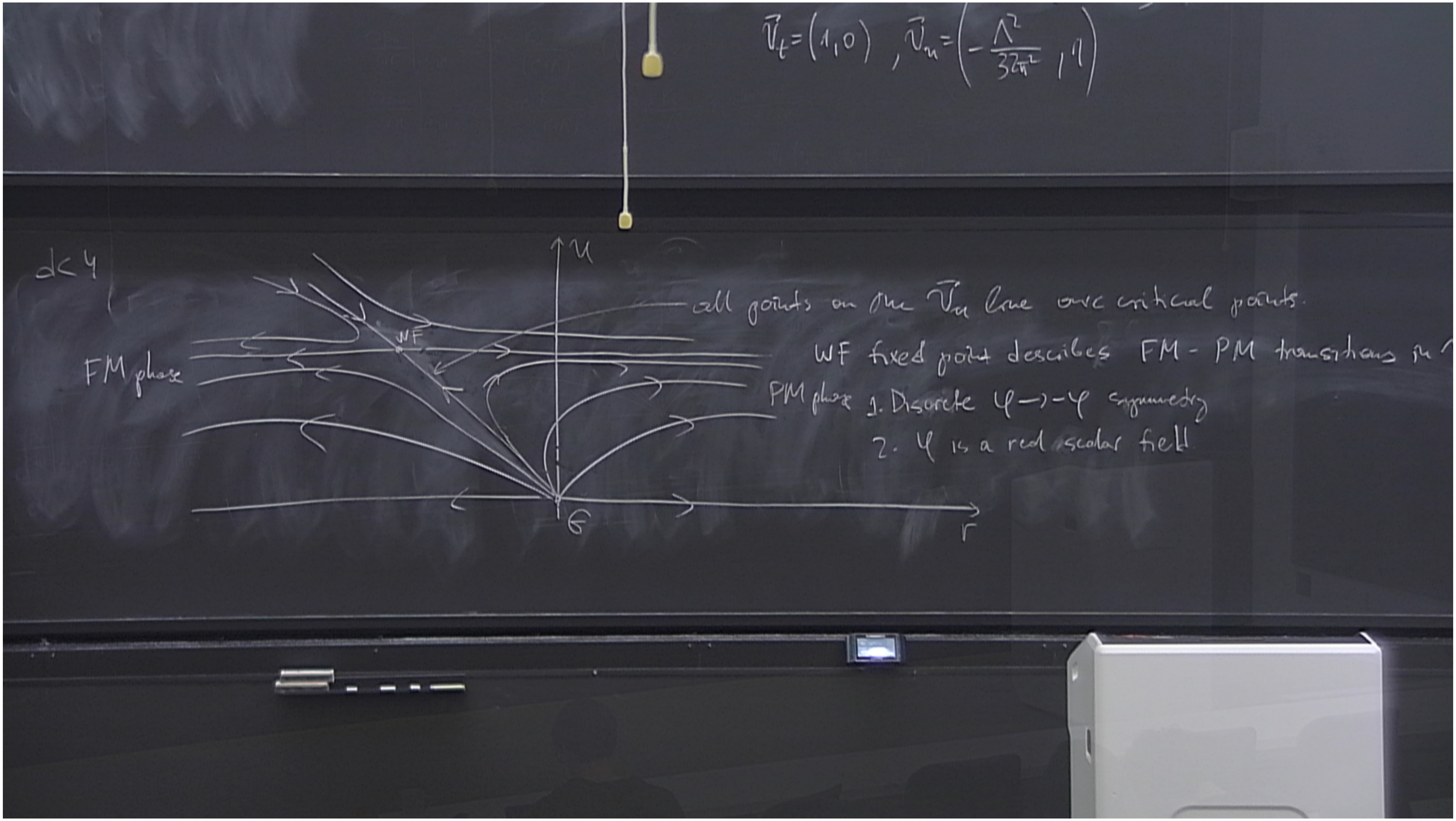
v_t = (1,0) , v_u = (- Λ^2 / 32π^2 , 1)
u
all points on the V_u line are critical points.
point describes FM - PM transitions in the Ising universality class
ε



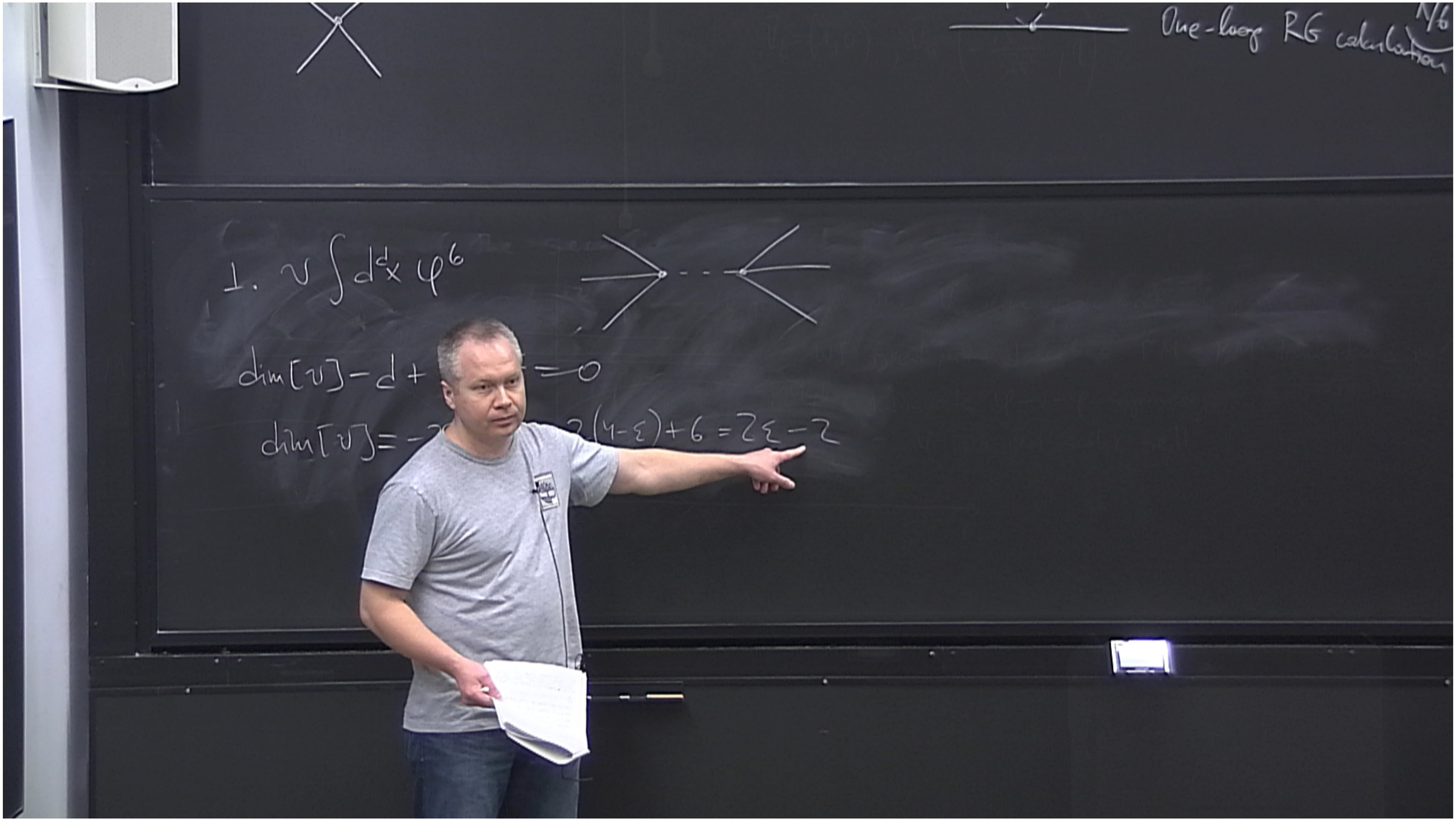
V_t = (1,0) , V_u = (- Λ^2 / 32π^2 , 1)
u
WF
all points on the V_u line are critical points.
WF fixed point describes FM - PM transitions in the Ising universality class
1. Discrete φ → -φ symmetry
2. φ is a real scalar field
r

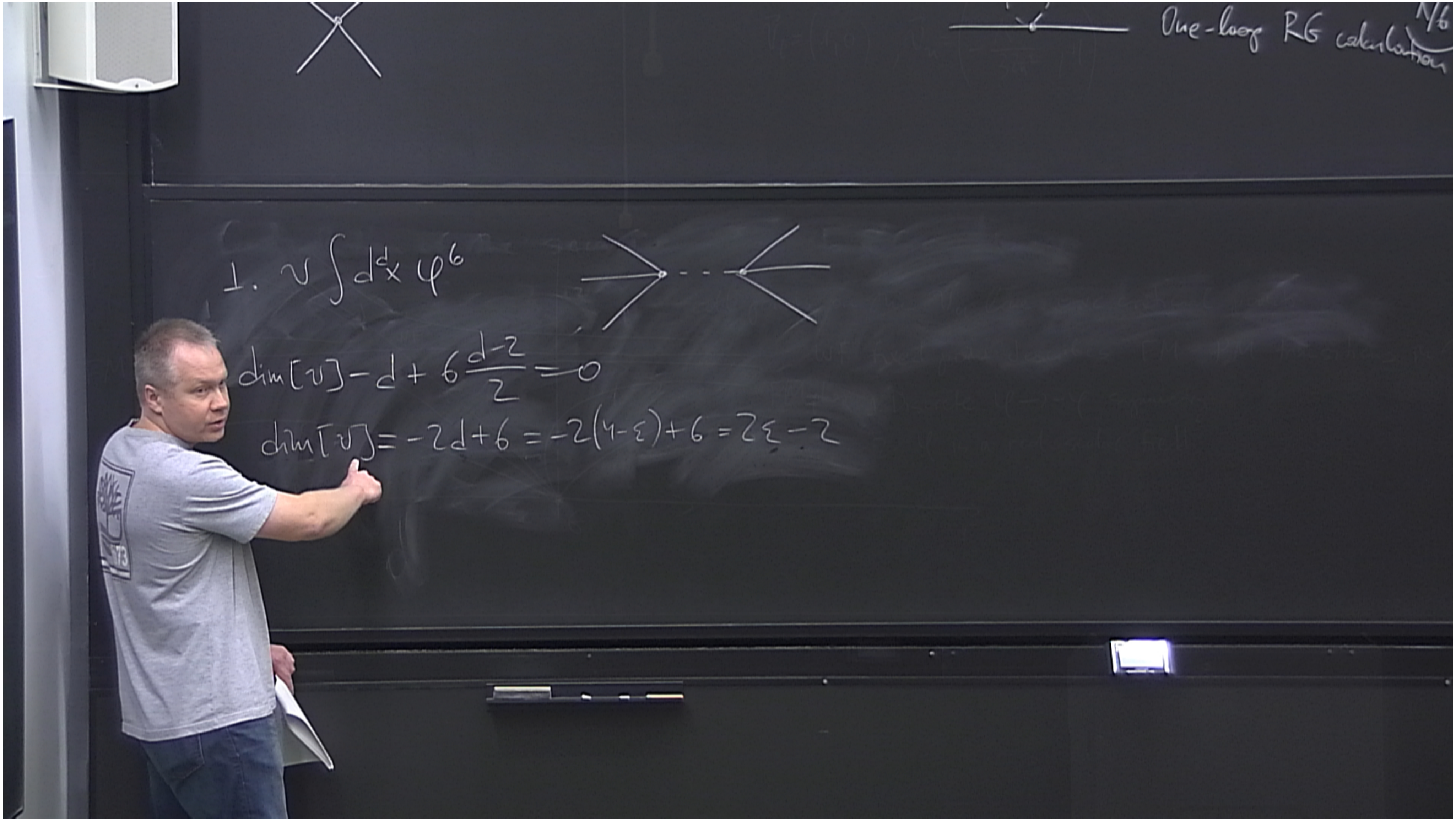

$\vec{v}_t = (1,0)$, $\vec{v}_u = \left(-\frac{\Lambda^2}{32\pi^2}, 1\right)$
u
r
WF
all points on the $\vec{v}_u$ line are critical points.
WF fixed point describes FM - PM transitions in the Ising
1. Discrete $\varphi \to -\varphi$ symmetry
2. φ is a real scalar field

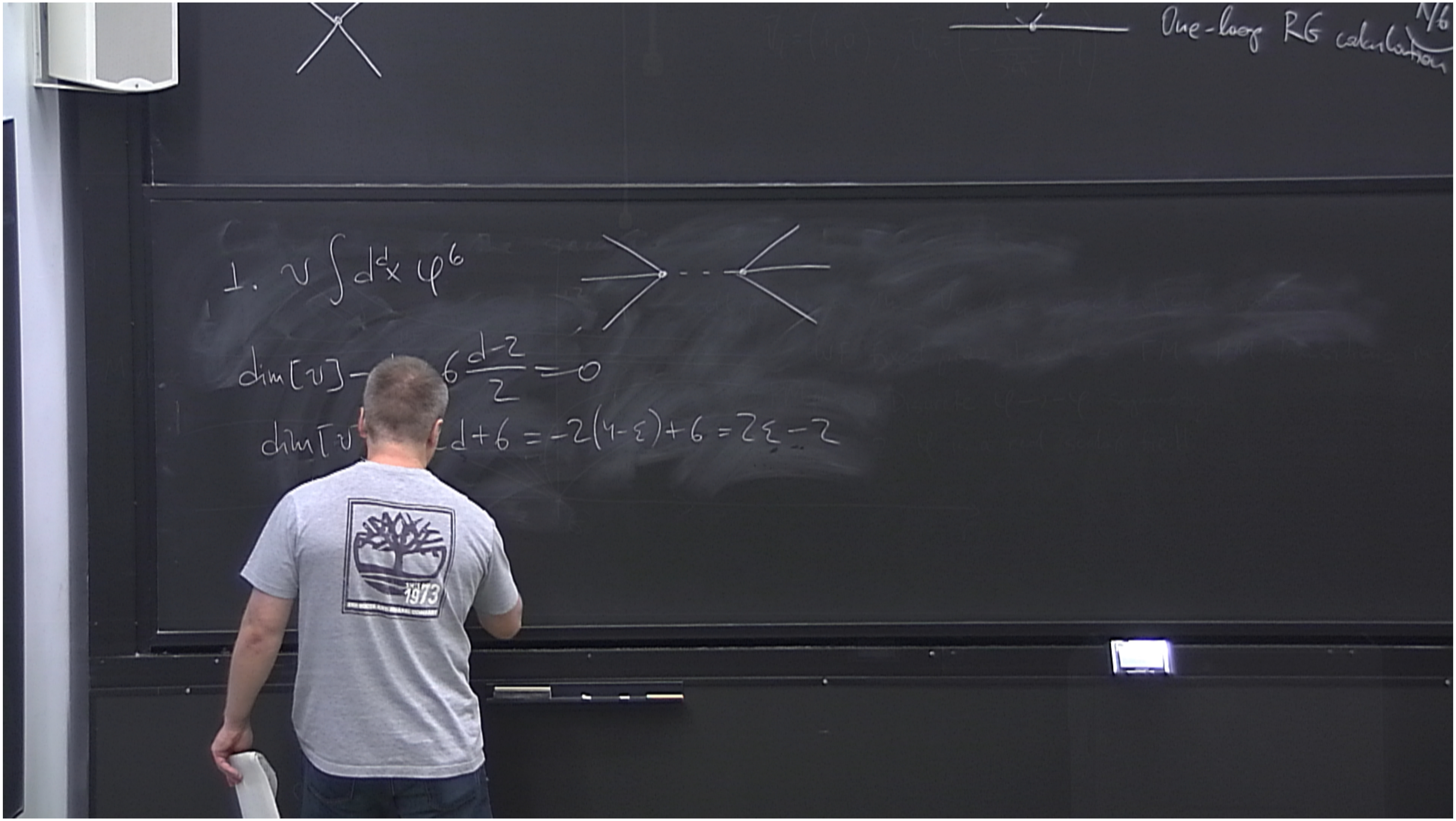

v_t = (1,0) , v_u = (- Λ^2/32π^2 , 1)
d<4
u
WF
FM phase
all points on the v_u line are critical points.
WF fixed point describes FM - PM transitions in
PM
1. Discrete φ → -φ symmetry
2. φ is a real scalar field
G
r

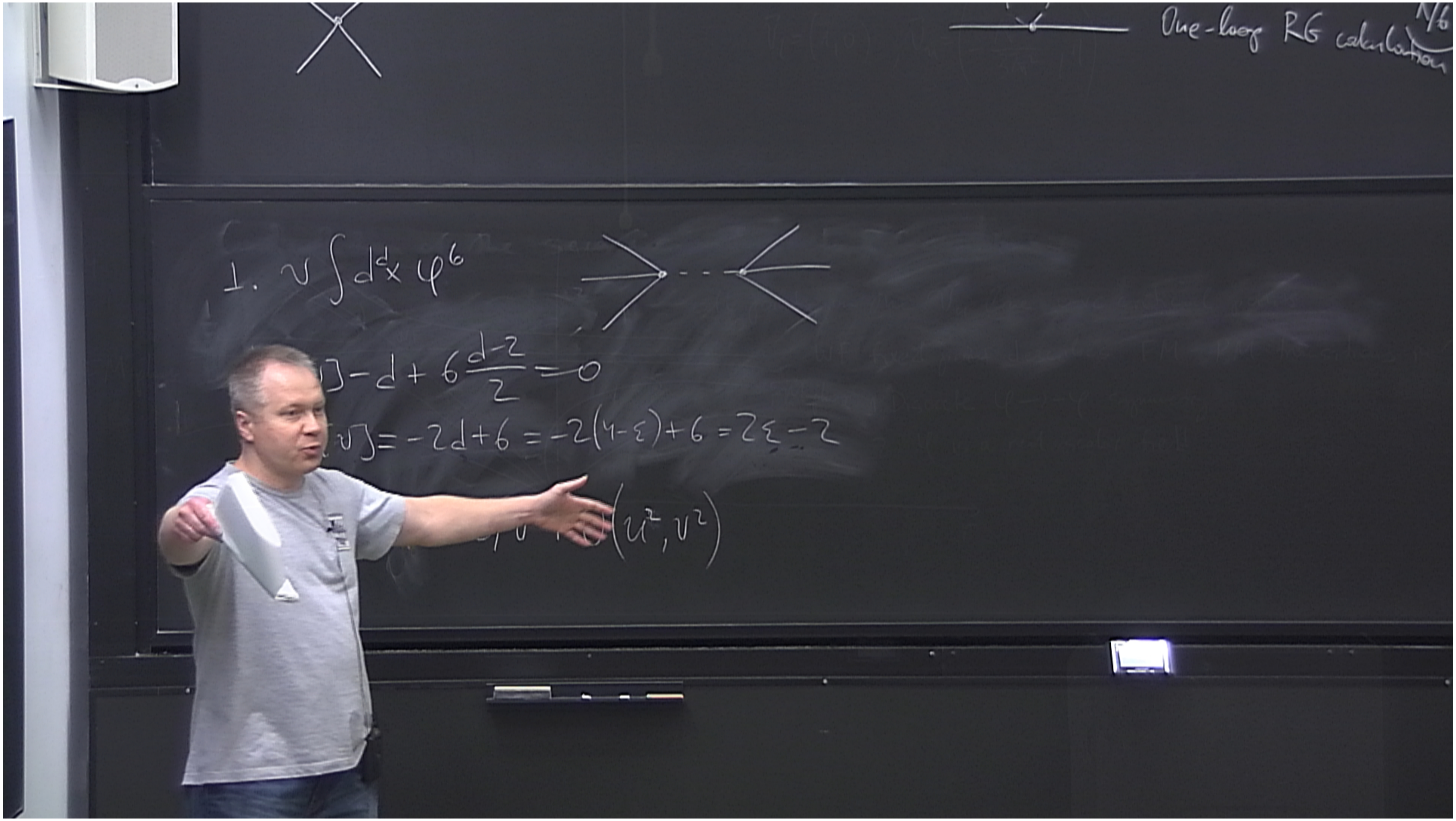


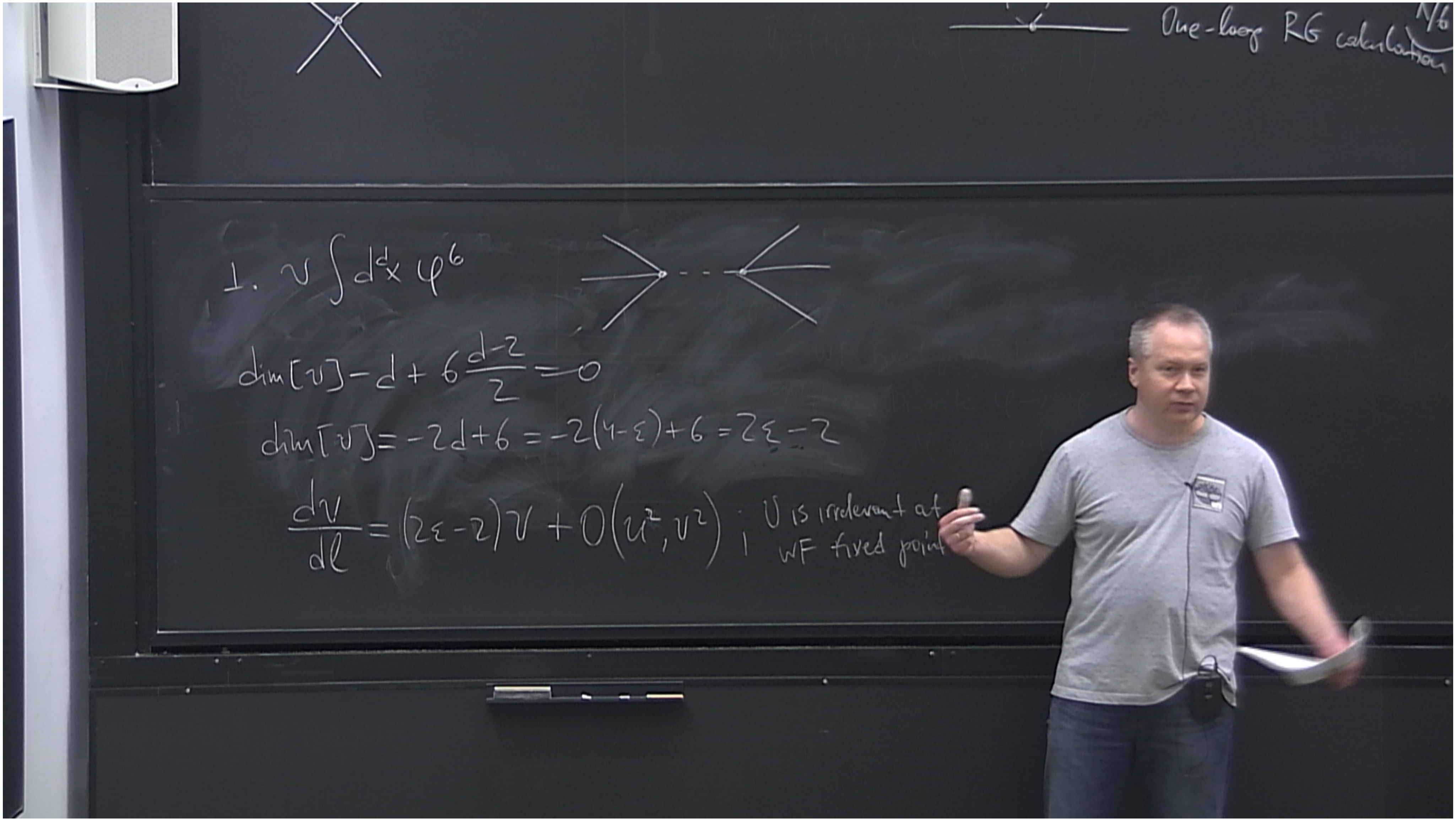
$\vec{V}_t = (1, 0)$, $\vec{V}_u = \left(-\frac{\Lambda^2}{32\pi^2}, 1\right)$
d<4
u
WF
FM phase
all points on the $\vec{V}_u$ line are critical points.
WF fixed point describes FM - PM transitions in
PM phase
1. Discrete $\varphi \to -\varphi$ symmetry
2. φ is a real scalar field
G
r

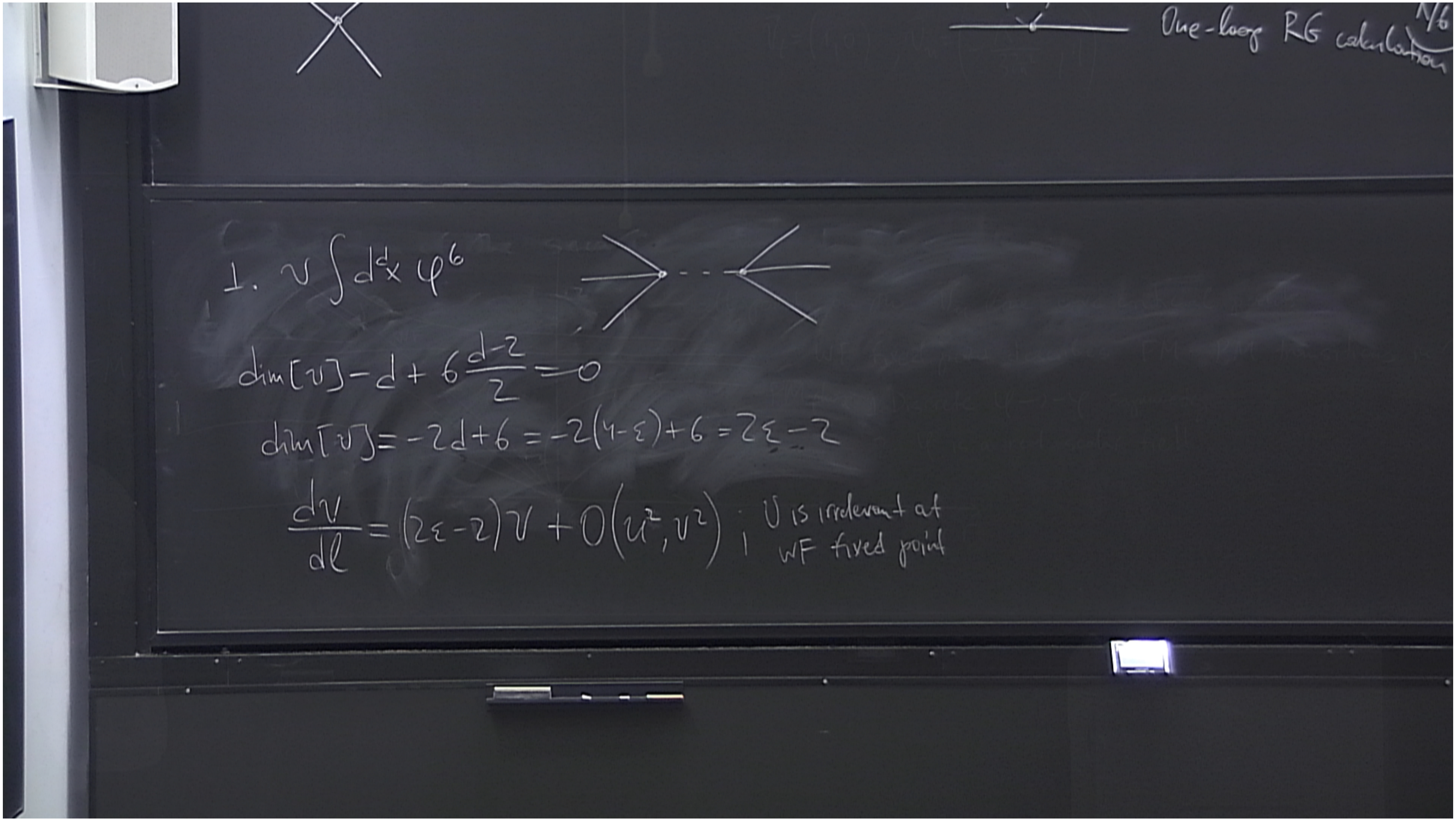

One-loop RG calculation
1. $v \int d^dx\, \varphi^6$
dim[v] − d +
= 0
dim[v] = −
2(4−ε)+6 = 2ε − 2

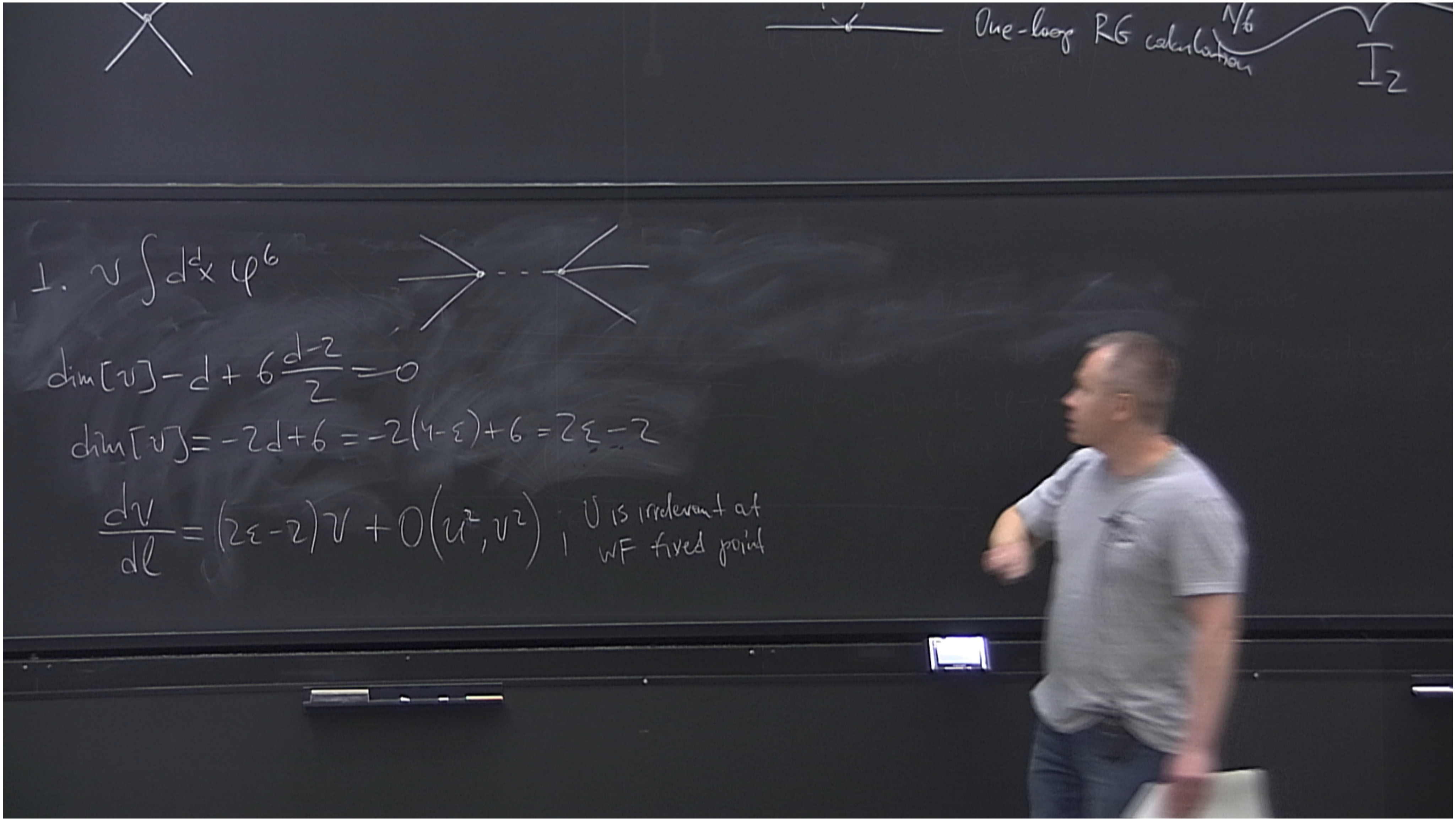

One-loop RG calculation
1. v ∫ d^d x φ^6
dim[v] − d + 6 (d−2)/2 = 0
dim[v] = −2d + 6 = −2(4−ε) + 6 = 2ε − 2

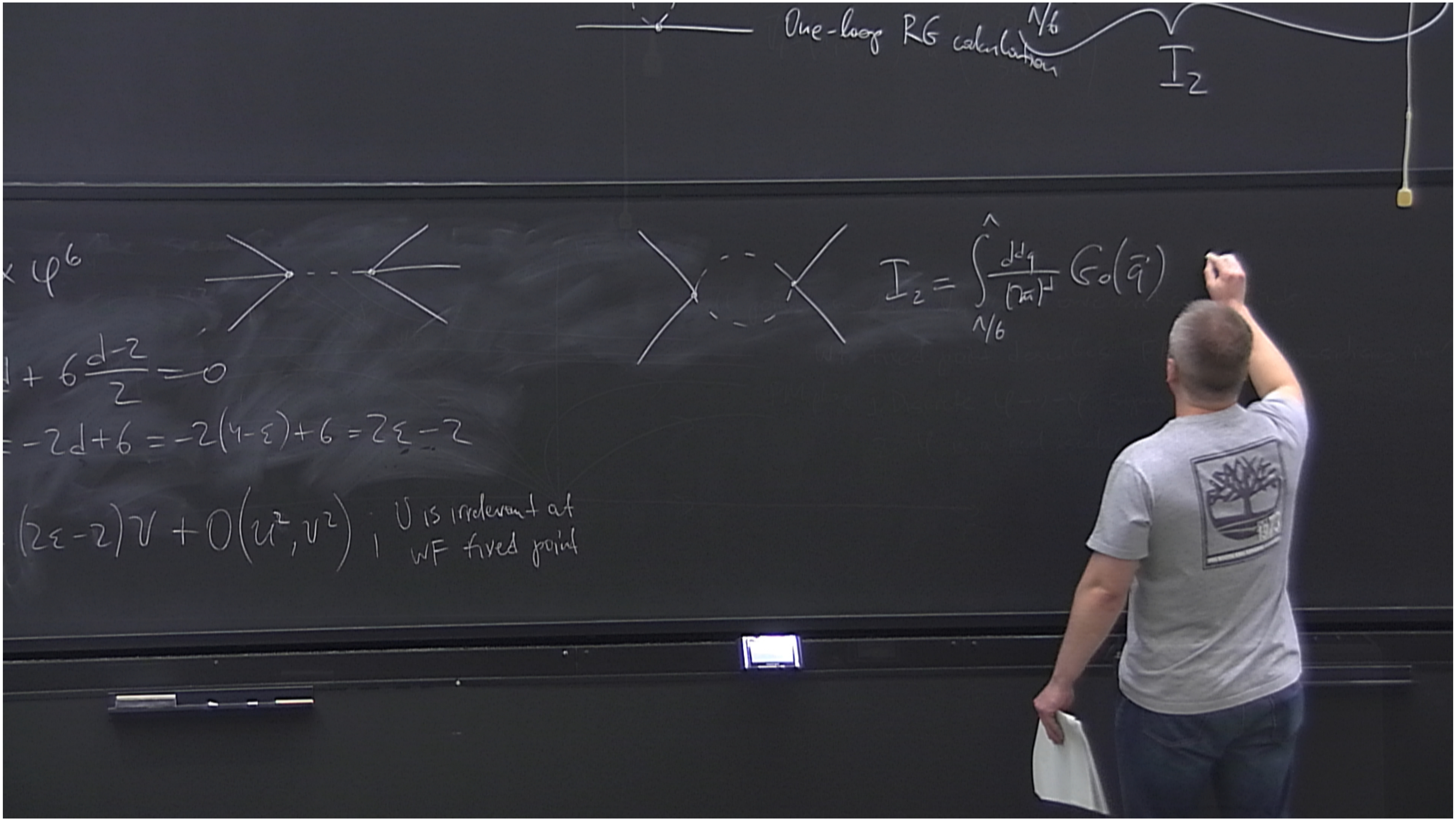

One-loop RG calculation
1. v ∫ d^d x φ^6
dim[v]
6 (d-2)/2 = 0
d+6 = -2(4-ε)+6 = 2ε-2

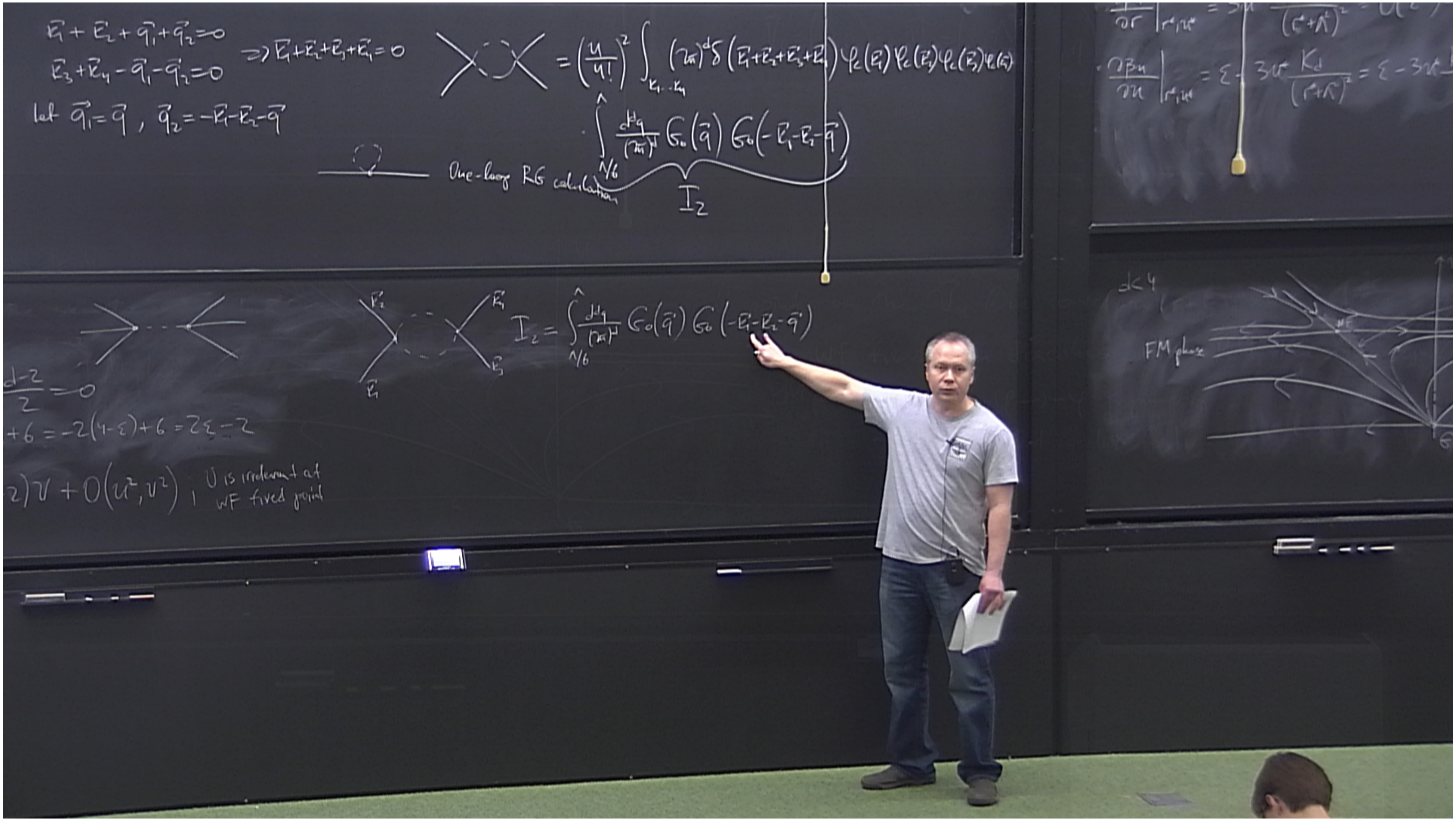

One-loop RG calculation
1. v ∫ d^d x φ^6
] − d + 6 (d−2)/2 = 0
v] = −2d + 6 = −2(4−ε) + 6 = 2ε − 2
(u², v²)

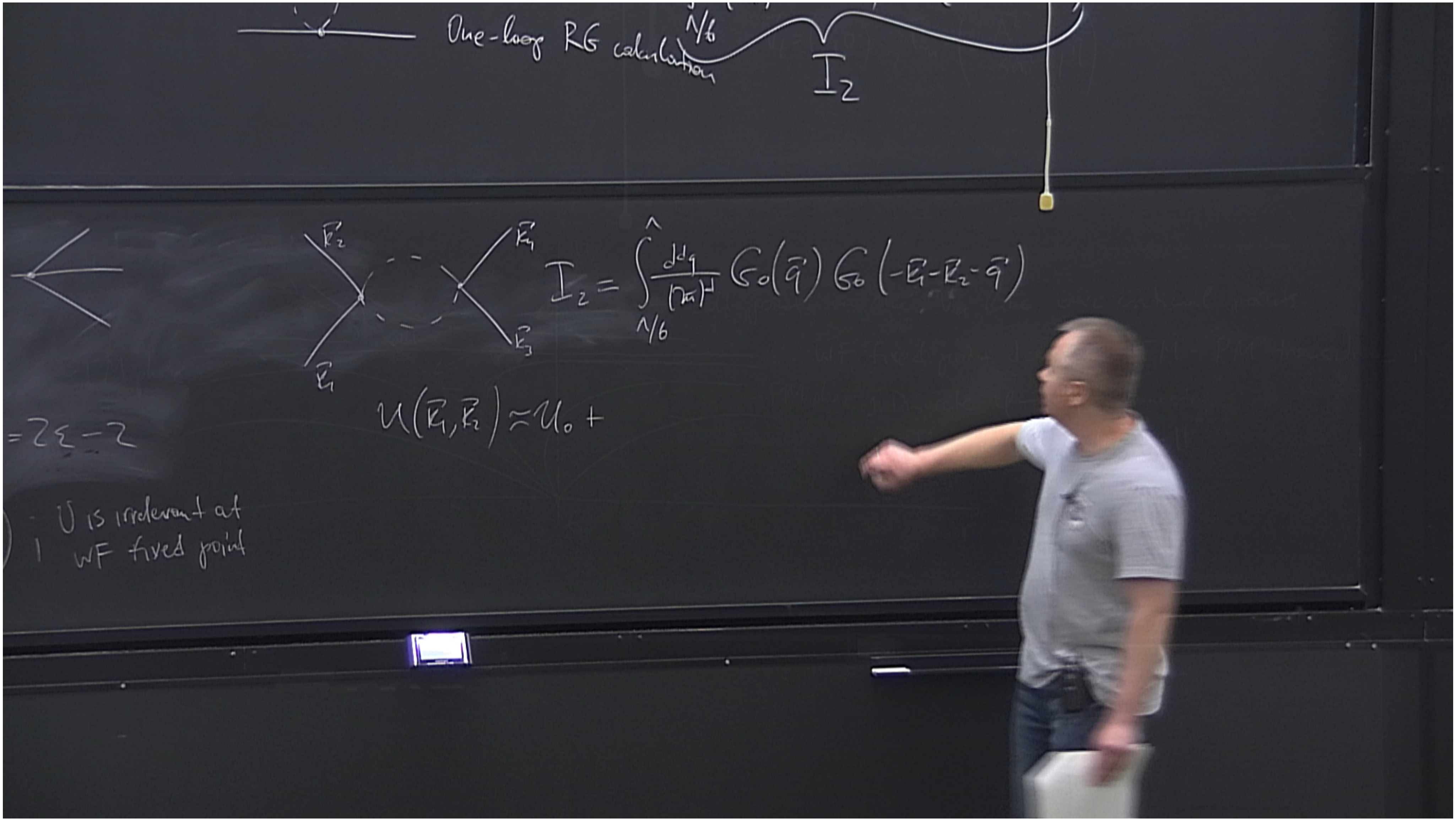

One-loop RG calculation
1. v ∫ d^d x φ^6
dim[v] − d + 6 (d−2)/2 = 0
dim[v] = −2d + 6 = −2(4−ε) + 6 = 2ε − 2
dv/dl = (2ε−2)v + O(u², v²) ; v is irrelevant at WF fixed point


One-loop RG calculation
1. v ∫ d^d x φ^6
dim[v] − d + 6 (d−2)/2 = 0
dim[v] = −2d + 6 = −2(4−ε) + 6 = 2ε − 2
dv/dl = (2ε−2)v + O(u², v²) ; v is irrelevant at WF fixed point


One-loop RG calculation
N/6
I2
1. v ∫ d^d x φ^6
dim[v] − d + 6 (d−2)/2 = 0
dim[v] = −2d+6 = −2(4−ε)+6 = 2ε−2
dv/dl = (2ε−2)v + O(u², v²) ;
v is irrelevant at
WF fixed point


One-loop RG calculation
Λ/b
I_2
I_2 = ∫ d^d q / (2π)^d G_0(q)
Λ/b
φ^6
+ 6 (d-2)/2 = 0
= -2d + 6 = -2(4-ε) + 6 = 2ε - 2
(2ε-2) v + O(u^2, v^2)
v is irrelevant at
WF fixed point


One-loop RG calculation
U is irrelevant at WF fixed point
FM phase


One-loop RG calculation
Λ/b
I_2
I_2 = ∫_{Λ/b}^{Λ} d^d q/(2π)^d G_0(q) G_0(-k_1-k_2-q)
U(k_1, k_2) ≈ U_0 +
= 2ε − 2
U is irrelevant at
WF fixed point

One-loop RG calculation

Λ/b

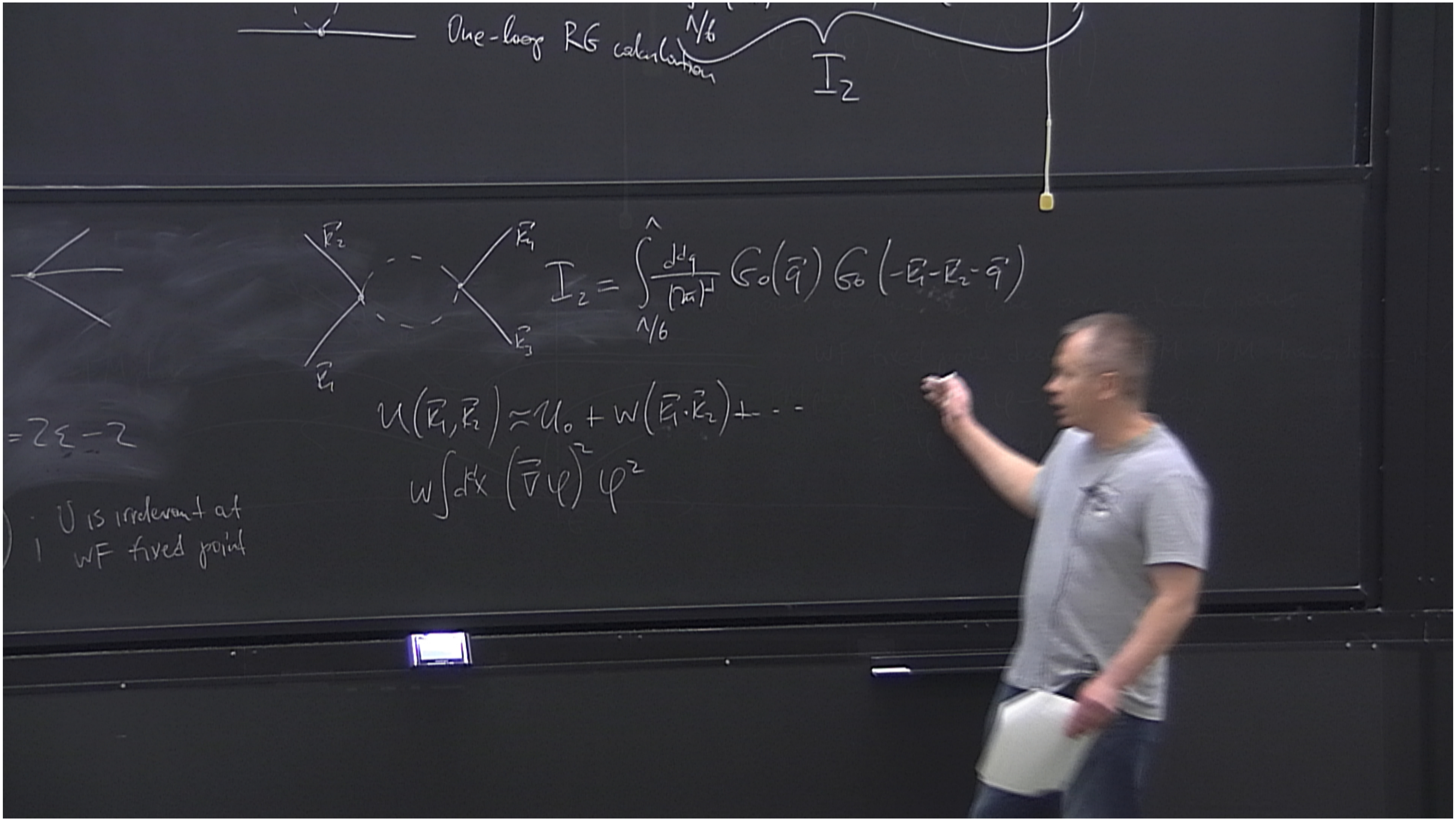
I_2

$$I_2 = \int_{\Lambda/b}^{\Lambda} \frac{d^dq}{(2\pi)^d}\, G_0(\vec q)\, G_0(-\vec k_1 - \vec k_2 - \vec q)$$

$$u(\vec k_1, \vec k_2) \simeq u_0 + w(\vec k_1 \cdot \vec k_2)$$

$= 2\varepsilon - 2$

U is irrelevant at WF fixed point



One-loop RG calculation
Λ/b
I2
I2 = ∫ d^dq/(2π)^d G0(q) G0(-k1-k2-q)
U(k1,k2) ≃ U0 + w(k1·k2) + ...
w ∫ d^dx (∇φ)^2 φ^2
U is irrelevant at
WF fixed point