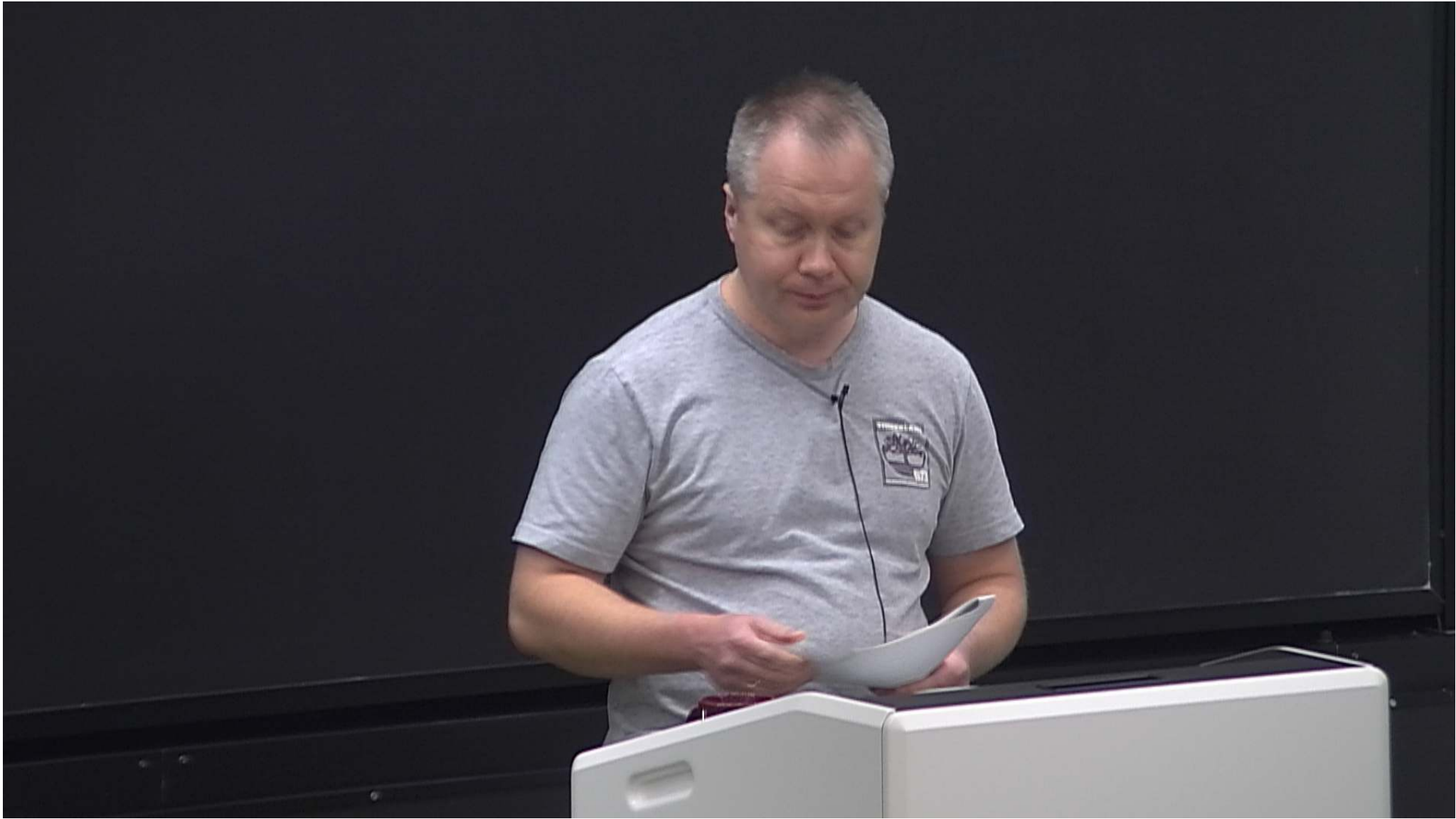


Title: 14/15 PSI - Statistical Mechanics - Lecture 8

Date: Oct 16, 2014 10:45 AM

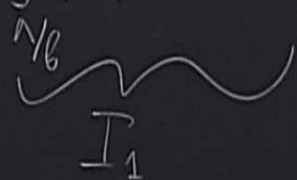
URL: <http://pirsa.org/14100094>

Abstract:



$$S'[\psi_c] = S_0[\psi_c] + \langle S_{int}[\psi_c, \psi_s] \rangle_0 - \frac{1}{2} [\langle S_{int}^2 \rangle_0 - \langle S_{int} \rangle_0^2] + \dots$$

$$D = \frac{\mu}{\gamma} \int_k |\psi_c(k)|^2 \int_{\Lambda_B} \frac{d^d q}{(2\pi)^d} G_0(\vec{q}) \quad ; \quad I_1 = \int_{\Lambda_B} \frac{d^d q}{(2\pi)^d} \frac{1}{r + q^2} =$$



$$S'[\psi_c] = S_0[\psi_c] + \langle S_{int}[\psi_c, \psi_b] \rangle_0 - \frac{1}{2} [\langle S_{int}^2 \rangle_0 - \langle S_{int} \rangle_0^2] + \dots$$

$$D = \frac{u}{4} \int_k |\psi_c(E)|^2 \int_{\mathbb{R}^d} \frac{d^d q}{(2\pi)^d} G_0(\vec{q})$$

$\underbrace{\int_{\mathbb{R}^d} \frac{d^d q}{(2\pi)^d}}_{I_1}$

$$I_1 = \int_{\mathbb{R}^d} \frac{d^d q}{(2\pi)^d} \frac{1}{r+q^2} = \frac{S_d}{(2\pi)^d} \int_0^\infty dq \frac{q^{d-1}}{r+q^2}$$

$$S_d = \frac{2\pi^{d/2}}{\Gamma(\frac{d}{2})}$$

$$S'[\psi_c] = S_0[\psi_c] + \langle S_{int}[\psi_c, \psi_s] \rangle_0 - \frac{1}{2} [\langle S_{int}^2 \rangle_0 - \langle S_{int} \rangle_0^2] + \dots$$

$$D = \frac{u}{4} \int_k |\psi_c(E)|^2 \int \frac{d^d q}{(2\pi)^d} G_0(\vec{q})$$

$\underbrace{\int \frac{d^d q}{(2\pi)^d}}_{I_1}$

$$I_1 = \int \frac{d^d q}{(2\pi)^d} \frac{1}{r+q^2} = \frac{S_d}{(2\pi)^d} \int d\Omega \frac{q^{d-1}}{r+q^2} = \frac{S_d}{(2\pi)^d} \int d\Omega q^{d-3} \left[1 - \frac{r}{q^2} + \dots \right] =$$

$$S_d = \frac{2\pi^{d/2}}{\Gamma(\frac{d}{2})} \quad ; \quad = \frac{S_d}{(2\pi)^d} \left[\frac{1}{d-2} \left(\frac{\Lambda^{d-2}}{6} - \left(\frac{\Lambda}{6}\right)^{d-2} \right) - \frac{r}{d-4} \left(\frac{\Lambda^{d-4}}{6} - \left(\frac{\Lambda}{6}\right)^{d-4} \right) + \dots \right]$$

$$b = e^{\Delta l}, \quad \Delta l \ll 1$$

$$1 - \frac{1}{b^{d-2}} = 1 - e^{-(d-2)\Delta l} \approx (d-2)\Delta l$$

$$1 - \frac{1}{b^{d-4}} \approx (d-4)\Delta l$$

$$I_1 = \frac{S_d}{(2\pi)^d}$$

$$I_1$$

$$S_d = \frac{2\pi^{d/2}}{\Gamma(d/2)} = \frac{S_d}{(2\pi)^d} \left[\frac{1}{d-2} \left(\Lambda^{d-2} - \left(\frac{\Lambda}{b}\right)^{d-2} \right) - \frac{r}{d-4} \right]$$

$$b = e^{\Delta l}, \Delta l \ll 1$$

$$1 - \frac{1}{b^{d-2}} = 1 - e^{-(d-2)\Delta l} \approx (d-2)\Delta l$$

$$1 - \frac{1}{b^{d-4}} \approx (d-4)\Delta l$$

$$I_1 = \frac{S_d}{(2\pi)^d} \left[\Lambda^{d-2} - r \Lambda^{d-4} + \dots \right] \Delta l$$

$$I_1$$

$$S_d = \frac{2\pi^{d/2}}{\Gamma(d/2)} = \frac{S_d}{(2\pi)^d} \left[\frac{1}{d-2} \left(\Lambda^{d-2} - \left(\frac{\Lambda}{b}\right)^{d-2} \right) - \frac{\Gamma}{d-4} \left(\Lambda^{d-4} - \left(\frac{\Lambda}{b}\right)^{d-4} \right) \right]$$

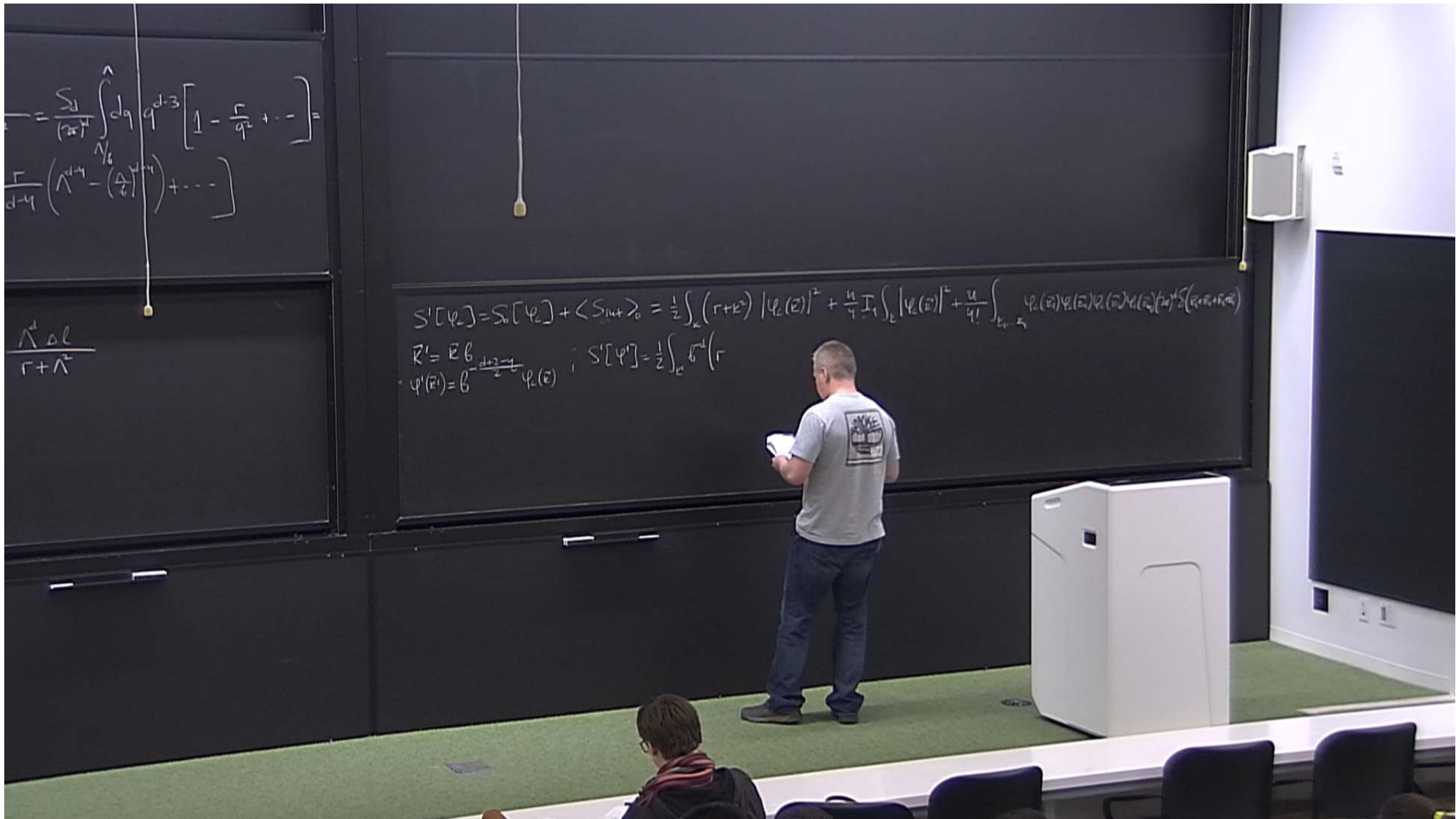
$$b = e^{\Delta l}, \quad \Delta l \ll 1$$

$$1 - \frac{1}{b^{d-2}} = 1 - e^{-(d-2)\Delta l} \approx (d-2)\Delta l$$

$$1 - \frac{1}{b^{d-4}} \approx (d-4)\Delta l$$

$$I_1 = \frac{S_d}{(2\pi)^d} \left[\Lambda^{d-2} - \Gamma \Lambda^{d-4} + \dots \right] \Delta l = \frac{S_d}{(2\pi)^d} \frac{\Lambda^d \Delta l}{\Gamma + \Lambda^2}$$

$$\langle D \rangle_0 = I_1 \frac{N}{4} \int_k |\psi_k(E)|^2$$



$$S'[\psi_c] = S_0[\psi_c] + \langle S_{int} \rangle_0 = \frac{1}{2} \int_{\mathbb{R}^d} (\Gamma + k^2) |\psi_c(\mathbb{R})|^2 + \frac{u}{4} \mathbb{I}_1 \int_{\mathbb{R}^d} |\psi_c(\mathbb{R})|^2 + \frac{u}{4!} \int_{\mathbb{R}^d} \psi_c(\mathbb{R}_1) \psi_c(\mathbb{R}_2) \psi_c(\mathbb{R}_3) \psi_c(\mathbb{R}_4) (\gamma \pi)^d \delta(\mathbb{R}_1 + \mathbb{R}_2 + \mathbb{R}_3 + \mathbb{R}_4)$$

$$\tilde{R}' = \tilde{R} \beta^{-\frac{d+2-\eta}{2}} \quad ; \quad S'[\psi'] = \frac{1}{2} \int_{\mathbb{R}^d} \tilde{R}' |\psi'(\mathbb{R})|^2$$

$$\psi'(\mathbb{R}) = \beta^{-\frac{d+2-\eta}{2}} \psi_c(\mathbb{R})$$



$$S'[\psi_c] = S_0[\psi_c] + \langle S_{int} \rangle_0 = \frac{1}{2} \int_k (r + k^2) |\psi_c(\vec{k})|^2 + \frac{\mu}{4} I_1 \int_k |\psi_c(\vec{k})|^2 + \frac{\mu}{4!} \int_{k_1 \dots k_4} \psi_c(\vec{k}_1) \psi_c(\vec{k}_2) \psi_c(\vec{k}_3) \psi_c(\vec{k}_4) (2\pi)^d \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4)$$

$$\vec{k}' = \vec{k} \beta^{-1} \quad ; \quad S'[\psi'] = \frac{1}{2} \int_{k'} \beta^{-d} \left(r + \frac{\mu}{4} I_1 + k'^2 \beta^{-2} \right) \beta^{\frac{d+2-\epsilon}{2}} |\psi'(\vec{k}')|^2 + \frac{\mu}{4!} \int_{k'_1 \dots k'_4} \beta^{-4d} \beta^{2(d+2-\epsilon)} \beta^d \psi'(\vec{k}'_1) \dots \psi'(\vec{k}'_4) (2\pi)^d \delta(\vec{k}'_1 + \dots + \vec{k}'_4)$$

$$\psi'(\vec{k}') = \beta^{-\frac{d+2-\epsilon}{2}} \psi_c(\vec{k})$$



$$S'[\psi_c] = S_0[\psi_c] + \langle S_{int} \rangle_0 = \frac{1}{2} \int_{\mathbb{R}^d} (r + k^2) |\psi_c(\vec{k})|^2 + \frac{\mu}{4} I_1 \int_{\mathbb{R}^d} |\psi_c(\vec{k})|^2 + \frac{\mu}{4!} \int_{\mathbb{R}^d} \psi_c(\vec{k}_1) \psi_c(\vec{k}_2) \psi_c(\vec{k}_3) \psi_c(\vec{k}_4) (2\pi)^d \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4)$$

$$\vec{k}' = \vec{k} \beta \quad ; \quad S'[\psi'] = \frac{1}{2} \int_{\mathbb{R}^d} \beta^{-d} \left(r + \frac{\mu}{4} I_1 + k'^2 \beta^{-2} \right) \beta^{\frac{d+2-\eta}{2}} |\psi'(\vec{k}')|^2 + \frac{\mu}{4!} \int_{\mathbb{R}^d} \beta^{-4d} \beta^{2(d+2-\eta)} \beta^d \psi'(\vec{k}'_1) \dots \psi'(\vec{k}'_4) (2\pi)^d \delta(\vec{k}'_1 + \dots + \vec{k}'_4)$$

$$\eta = 0 \quad ; \quad \int r' = \left(r + \frac{\mu}{4} I_1 \right) \beta^2 \quad ; \quad \beta^2 = e^{2\Delta l} = 1 + 2\Delta l \quad ; \quad r' = \left(r + \frac{\mu}{4} I_1 \right) (1 + 2\Delta l) \Rightarrow$$

$$\left\{ \begin{array}{l} \mu' = \mu \beta^{4-d} \end{array} \right.$$

$$r' = r + 2\Delta l$$

$$S'[\psi_c] = S_0[\psi_c] + \langle S_{int} \rangle_0 = \frac{1}{2} \int_{\mathbb{R}^d} (r+k^2) |\psi_c(\vec{z})|^2 + \frac{u}{4} I_1 \int_{\mathbb{R}^d} |\psi_c(\vec{z})|^2 + \frac{u}{4!} \int_{\mathbb{R}^d} \psi_c(\vec{z}_1) \psi_c(\vec{z}_2) \psi_c(\vec{z}_3) \psi_c(\vec{z}_4) (2\pi)^d \delta(\vec{z}_1 + \vec{z}_2 + \vec{z}_3 + \vec{z}_4)$$

$$\vec{r}' = \vec{r} \beta$$

$$\psi'(\vec{z}') = \beta^{-\frac{d+2-\eta}{2}} \psi_c(\vec{z}) ; S'[\psi'] = \frac{1}{2} \int_{\mathbb{R}^d} \beta^{-d} \left(r + \frac{u}{2} I_1 + k'^2 \beta^{-2} \right) \beta^{\frac{d+2-\eta}{2}} |\psi'(\vec{z}')|^2 + \frac{u}{4!} \int_{\mathbb{R}^d} \beta^{-4d} \beta^{2(d+2-\eta)} \beta^d \psi'(\vec{z}'_1) \dots \psi'(\vec{z}'_4) (2\pi)^d \delta(\vec{z}'_1 + \dots + \vec{z}'_4)$$

$$\eta=0 ; \int r' = \left(r + \frac{u}{2} I_1 \right) \beta^2 ; \beta^2 = e^{2\Delta l} = 1 + 2\Delta l ; r' = \left(r + \frac{u}{2} I_1 \right) (1 + 2\Delta l) \Rightarrow$$

$$\left\{ \begin{array}{l} u' = u \beta^{4-d} \end{array} \right.$$

$$r' = r + 2\Delta l r + \frac{u}{2} I_1 + \frac{u}{2} I_1 \cdot 2\Delta l$$

$$r' - r = 2\Delta l r + \frac{u}{2} I_1 + \frac{u}{2} I_1 \cdot 2\Delta l$$

$$r' - r = 2\Delta l r + \frac{u}{2} S_d \frac{\Lambda^d}{r + \Lambda^2} \Delta l$$

$$\frac{dr}{dl} = 2r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2}$$

$$u' = ub$$

$$r' = r + 2\Delta l r + \frac{u}{2} I_1 + \frac{u}{2} I_1 \cdot 2\Delta l$$

$$r' - r = 2\Delta l r + \frac{u}{2} I_1 + \frac{u}{2} I_1 \cdot 2\Delta l$$

$$r' - r = 2\Delta l r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2} \Delta l$$

$$\frac{r' - r}{\Delta l} = \frac{dr}{dl}$$

$$\frac{dr}{dl} = 2r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2}$$

$$u' = ub^{4-d} = u(1 + (4-d)\Delta l)$$

$$\frac{du}{dl} = (4-d)u$$

$$u' = u b^{4-d}$$

$$r' = r + 2\Delta l r + \frac{u}{2} I_1 + \frac{u}{2} I_1 \cdot 2\Delta l$$

$$r' - r = 2\Delta l r + \frac{u}{2} I_1 + \frac{u}{2} I_1 \cdot 2\Delta l \rightarrow$$

$$r' - r = 2\Delta l r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2} \Delta l$$

$$\frac{r' - r}{\Delta l} = \frac{dr}{dl}$$

$$\frac{dr}{dl} = (2)r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2}$$

$$u' = u b^{4-d} = u(1 + (4-d)\Delta l)$$

$$\frac{du}{dl} = (4-d)u$$

$$\dim[r] = 2$$

$$\dim[u] = 4-d$$

$$u' = u b^{4-d}$$

$$r' = r + 2\Delta l r + \frac{u}{2} I_1 + \frac{u}{2} I_1 - 2\Delta l$$

$$r' - r = 2\Delta l r + \frac{u}{2} I_1 + \frac{u}{2} I_1 - 2\Delta l \rightarrow$$

$$r' - r = 2\Delta l r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2} \Delta l$$

$$\frac{r' - r}{\Delta l} = \frac{dr}{dl}$$

$$\frac{dr}{dl} = (2) r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2}$$

$$u' = u b^{4-d} = u (1 + (4-d)\Delta l)$$

$$\frac{du}{dl} = (4-d)u$$

$$\dim[r] = 2$$

$$\dim[u] = 4-d$$

Fixed points:

$$\frac{dr}{dl} = \frac{du}{dl} = 0$$

$$q(r) = b \quad q_2(l) \quad q_1 \quad q_1 - q_1' \quad 8(\bar{E}_1 + \bar{E}_1')$$

$$q=0 \quad ; \quad r' = \left(r + \frac{u}{2} I_1\right) b^2 \quad ; \quad b^2 = e^{2\alpha l} = 1 + 2\alpha l \quad ; \quad r' = \left(r + \frac{u}{2} I_1\right) (1 + 2\alpha l) \Rightarrow$$

$$\begin{cases} u' = u b^{4-d} \end{cases}$$

$$r' = r + 2\alpha l r + \frac{u}{2} I_1 + \frac{u}{2} I_1 2\alpha l$$

$$r' - r = 2\alpha l r + \frac{u}{2} I_1 + \frac{u}{2} I_1 2\alpha l$$

$$r' - r = 2\alpha l r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2} \Delta l$$

$$\frac{r' - r}{\Delta l} = \frac{dr}{dl}$$

$$\frac{dr}{dl} = (2)r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2}$$

$$u' = u b^{4-d} = u(1 + (4-d)\alpha l)$$

$$\frac{du}{dl} = (4-d)u$$

$$\dim[r] = 2$$

$$\dim[u] = 4-d$$

Fixed points: $r^* = u^* = 0$ - Gaussian fixed point.

$$\frac{dr}{dl} = \frac{du}{dl} = 0$$

$$S_d = \frac{2\pi^{d/2} h^{d/2}}{\Gamma(\frac{d}{2})} ; = \frac{S_d}{(2\pi)^d} \left[\frac{1}{d-2} \left(\Lambda^{d-2} - \left(\frac{\Lambda}{6}\right)^{d-2} \right) - \frac{r}{d-4} \left(\Lambda^{d-4} - \left(\frac{\Lambda}{6}\right)^{d-4} \right) + \dots \right]$$

After evaluating the second cumulant.

$$\left\{ \frac{dr}{dl} = 2r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2} \right.$$

$$u^* \sim 4-d = \epsilon$$

$$r^* \sim \epsilon$$

$$\left\{ \frac{du}{dl} = (4-d)u - \frac{3u^2}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{(r + \Lambda^2)^2} \right.$$



After evaluating the second cumulant.

$$\begin{cases} \frac{dr}{dt} = 2r + \frac{\mu}{2} \frac{S_d}{(\bar{m})^d} \frac{\Lambda^d}{r + \Lambda^2} \\ \frac{du}{dt} = (4-d)u - \frac{3u^2}{2} \frac{S_d}{(\bar{m})^d} \frac{\Lambda^d}{(r + \Lambda^2)^2} \end{cases}$$

$$u^* \sim 4-d = \varepsilon$$

$$r^* \sim \varepsilon$$

After evaluating the second cumulant.

$$\left\{ \frac{dr}{dl} = 2r + \frac{u}{2} \frac{S_d}{(\pi)^d} \frac{\Lambda^d}{r + \Lambda^2} \right.$$

$$\left\{ \frac{du}{dl} = (4-d)u - \frac{3u^2}{2} \frac{S_d}{(\pi)^d} \frac{\Lambda^d}{(r + \Lambda^2)^2} \right.$$

$$u^* \sim 4-d = \epsilon$$

$$r^* \sim \epsilon$$

Expansion in ϵ

After evaluating the second cumulant.

$$\left\{ \begin{aligned} \frac{dr}{dl} &= 2r + \frac{u}{2} \frac{S_d}{(\bar{m})^d} \frac{\Lambda^d}{r + \Lambda^2} \\ \frac{du}{dl} &= (4-d)u - \frac{3u^2}{2} \frac{S_d}{(\bar{m})^d} \frac{\Lambda^d}{(r + \Lambda^2)^2} \end{aligned} \right.$$

$$u^* \sim 4-d = \varepsilon$$

$$r^* \sim \varepsilon$$

Expansion in ε

$$S'(\varphi_L) = S_0(\varphi_L) + \langle S_{int} \rangle_0 - \frac{1}{2} \left[\langle S_{int}^2 \rangle - \langle S_{int} \rangle^2 \right] + \dots$$

$$S'[\varphi_c] = S_0[\varphi_c] + \langle S_{int} \rangle_0 - \frac{1}{2} \left[\langle S_{int}^2 \rangle - \langle S_{int} \rangle^2 \right] + \dots$$

Feynman diagram notation to represent different terms.

$$S'[\varphi_c] = S_0[\varphi_c] + \langle S_{int} \rangle_0 - \frac{1}{2} \left[\langle S_{int}^2 \rangle - \langle S_{int} \rangle^2 \right] + \dots$$

Feynman diagram notation to represent different terms.

$$S_{int}[\varphi_c, \varphi_s] = \frac{\eta}{4!} \int_{k_1, \dots, k_4} \left[\varphi_c(\vec{k}_1) + \varphi_s(\vec{k}_1) \right] \dots \left[\varphi_c(\vec{k}_4) + \varphi_s(\vec{k}_4) \right] (2\pi)^d \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4)$$

$$S'[\psi_c] = S_0[\psi_c] + \langle S_{int} \rangle_0 - \frac{1}{2} \left[\langle S_{int}^2 \rangle_0 - \langle S_{int} \rangle_0^2 \right] + \dots$$

Feynman diagram notation to represent different terms.

$$S_{int}[\psi_c, \psi_s] = \frac{y}{4!} \int_{k_1, \dots, k_4} [\psi_c(\vec{k}_1) + \psi_s(\vec{k}_1)] \dots [\psi_c(\vec{k}_4) + \psi_s(\vec{k}_4)] (2\pi)^d \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4)$$



$$= \frac{y}{4!} (2\pi)^d \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4) \text{-vertex}$$



$$S'[\varphi_c] = S_0[\varphi_c] + \langle S_{int} \rangle_0 - \frac{1}{2} \left[\langle S_{int}^2 \rangle_0 - \langle S_{int} \rangle_0^2 \right] + \dots$$

Feynman diagram notation to represent different terms.

$$S_{int}[\varphi_c, \varphi_s] = \frac{y}{4!} \int_{k_1, \dots, k_4} [\varphi_c(\vec{k}_1) + \varphi_s(\vec{k}_1)] \dots [\varphi_c(\vec{k}_4) + \varphi_s(\vec{k}_4)] (2\pi)^d \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4)$$



$$= \frac{y}{4!} (2\pi)^d \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4) \text{-vertex}$$



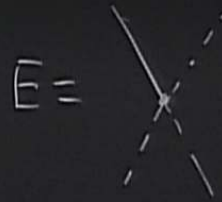
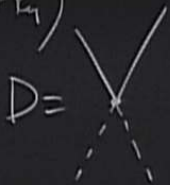
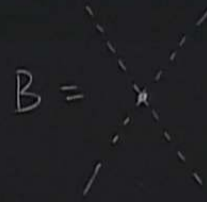
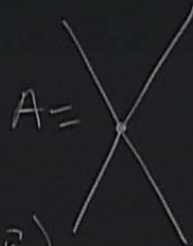
$$\Gamma\left(\frac{d}{2}\right) \frac{1}{(2\pi)^d} \int d^d x \dots$$

$$\frac{1}{2} \left[\langle S_{int}^2 \rangle - \langle S_{int} \rangle^2 \right] + \dots$$

represent different terms.

$$\dots + \psi_2(\vec{r}_1) \dots \left[\psi_c(\vec{r}_1) + \psi_b(\vec{r}_1) \right] (2\pi)^d \delta(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4)$$

)-vertex



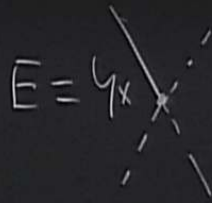
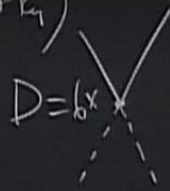
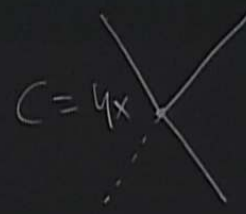
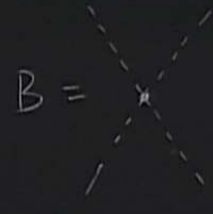
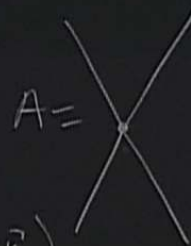
$$P\left(\frac{d}{2}\right) - (2\pi)^d \left[d-2 \right] \quad d-4 \quad (6)$$

$$\frac{1}{2} \left[\langle S_{int}^2 \rangle - \langle S_{int} \rangle^2 \right] + \dots$$

represent different terms.

$$\psi_1(\vec{r}_1) \dots \left[\psi_2(\vec{r}_1) + \psi_3(\vec{r}_1) \right] (2\pi)^d \delta(\vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4)$$

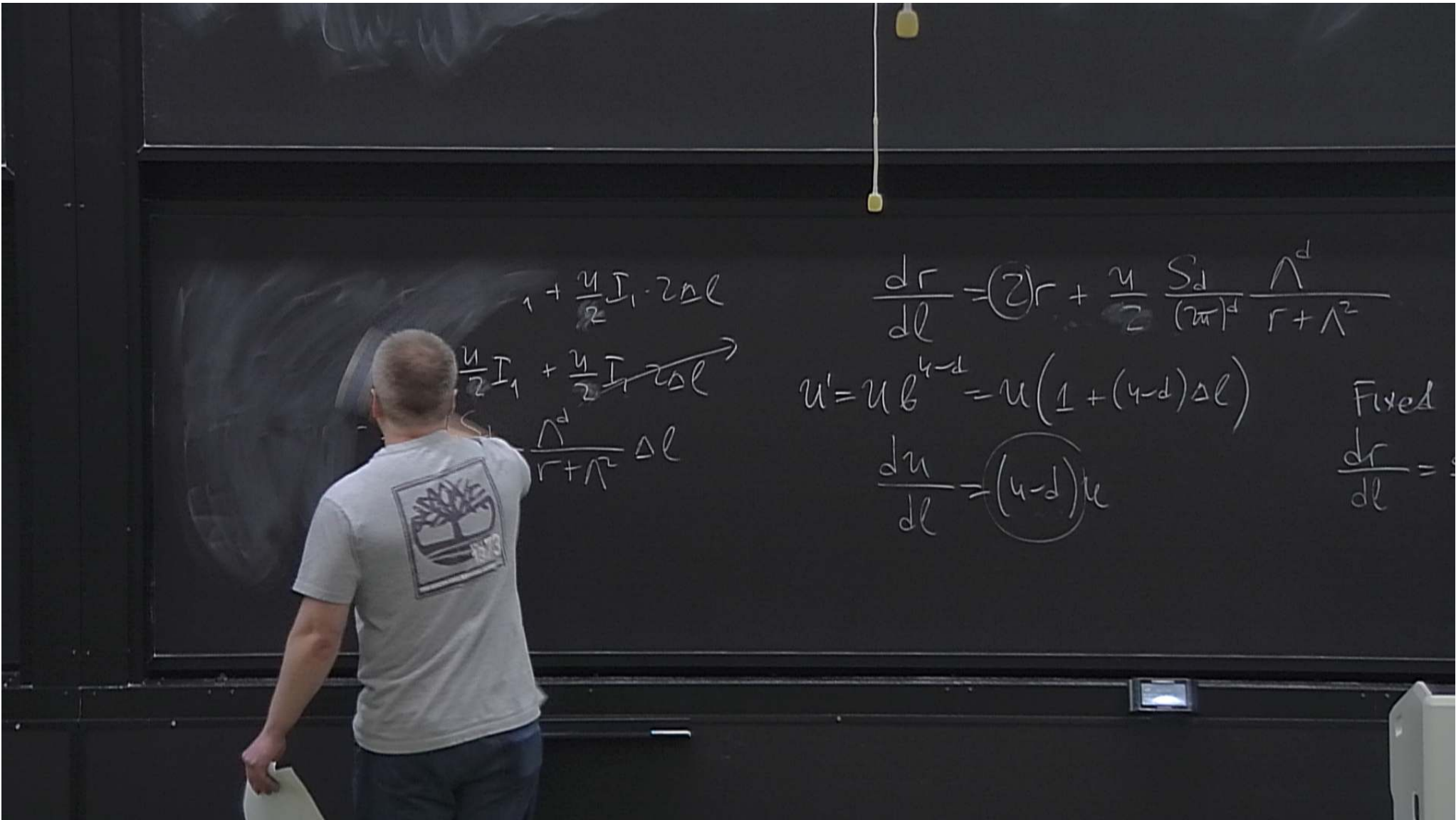
vertex



Define averaging over fast field by connecting the dashed lines

$$\langle D \rangle = \frac{\vec{q} \cdot (-\vec{q})}{R} \int_{-\pi/6}^{\pi/6} \frac{d\vec{q}}{(2\pi)^3} G_0(\vec{q})$$

Each dashed line corresponds to



$$1 + \frac{y}{2} I_1 \cdot 2 \Delta l$$

$$\frac{y}{2} I_1 + \frac{y}{2} I_1 \cdot 2 \Delta l \rightarrow$$

$$\frac{\Lambda^d}{r + \Lambda^2} \Delta l$$

$$\frac{dr}{dl} = (2)r + \frac{y}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2}$$

$$u' = u b^{y-d} = u (1 + (y-d) \Delta l)$$

Fixed

$$\frac{du}{dl} = (y-d)u$$

$$\frac{dr}{dl} =$$

$$\frac{1}{2} \left[\langle S_{114}^2 \rangle_0 - \langle S_{114} \rangle_0^2 \right] = \frac{1}{2} \langle \text{---} \times \text{---} + \text{---} \times \text{---} + \text{---} \times \text{---} + \dots \rangle$$

76

$$\frac{1}{2} [\langle S_{144}^2 \rangle_0 - \langle S_{244} \rangle_0^2] = \frac{1}{2} \langle \underbrace{\begin{matrix} \diagup \diagdown & \diagdown \diagup & \diagup \diagdown & \diagdown \diagup & \dots \\ \diagdown \diagup & \diagup \diagdown & \diagdown \diagup & \diagup \diagdown & \dots \end{matrix}}_{25 \text{ terms}} \rangle_0$$

$$= \frac{1}{2} \langle \underbrace{\begin{matrix} \diagup & \diagdown & \dots \\ \diagdown & \diagup & \dots \end{matrix}}_{5 \text{ terms}} \rangle_0 \langle \underbrace{\begin{matrix} \diagup & \diagdown & \dots \\ \diagdown & \diagup & \dots \end{matrix}}_{5 \text{ terms}} \rangle_0$$



$$\frac{1}{2} [\langle S_{144}^2 \rangle_0 - \langle S_{224} \rangle_0^2] = \frac{1}{2} \langle \underbrace{\dots + \dots + \dots}_{25 \text{ terms}} \rangle_0$$

$$= \frac{1}{2} \langle \underbrace{\dots}_{5 \text{ terms}} \rangle_0 \langle \underbrace{\dots}_{5 \text{ terms}} \rangle_0$$

$$\frac{1}{2} \left[\langle S_{144}^2 \rangle_0 - \langle S_{144} \rangle_0^2 \right] = \frac{1}{2} \left\langle \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \\ \vdots \end{array} + \begin{array}{c} \diagup \quad \diagup \\ \diagdown \quad \diagdown \\ \vdots \end{array} + \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagdown \\ \vdots \end{array} + \dots \right\rangle_0$$

25 terms

$$-\frac{1}{2} \left\langle \begin{array}{c} \diagup \\ \diagdown \\ \vdots \end{array} + \begin{array}{c} \diagdown \\ \diagup \\ \vdots \end{array} + \dots \right\rangle_0 \left\langle \begin{array}{c} \diagup \\ \diagdown \\ \vdots \end{array} + \begin{array}{c} \diagdown \\ \diagup \\ \vdots \end{array} + \dots \right\rangle_0$$

5 terms

All disconnected diagrams cancel!

$$\frac{1}{2} \left[\langle S_{int}^2 \rangle_0 - \langle S_{int} \rangle_0^2 \right] = \frac{1}{2} \left(\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \dots \right)$$

$$= \frac{1}{2} \left(\text{diagram 4} + \text{diagram 5} + \dots \right) \left(\text{diagram 6} + \text{diagram 7} + \dots \right)$$

5 terms 25 terms

Cumulant expansion = sum of all connected diagrams. All disconnected diagrams cancel!

Find all possible connected diagrams with either 2 or 4 bold lines



$$4 \times 4 \times 6 \times \frac{1}{2} = 48$$

Find all possible connected diagrams with either 2 or 4 bold lines.



the either 2 or 4 bold lines

$$4 \times 4 \times 6 \times \frac{1}{2} = 48$$

$$4 \times 4 \times 9 \times \frac{1}{2} = 72$$



$$8 \times 6 \times 1 \times 6 \times 2 \times 2 \times \frac{1}{2} = 72$$

4 solid lines:



$\langle D \rangle = \frac{1}{K} \frac{-E}{\int_{-1/6}^{1/6} G_0(\vec{q}) \frac{d^3q}{(2\pi)^3}}$
 Each dashed line corresponds to

$$\frac{1}{2} [\langle S_{111}^2 \rangle_0 - \langle S_{111} \rangle_0^2] = 48 \left(\leftarrow \right) + 72 \left(\circ \circ \right) + 72 \left(\circ \right) + 36 \left(\times \times \right) + 48 \left(\leftarrow \right)$$



$\langle D \rangle = \frac{1}{K} \frac{-E}{\int \frac{d^3q}{(2\pi)^3} G_0(\vec{q})}$
 Each dashed line corresponds to $\int \frac{d^3q}{(2\pi)^3} G_0(\vec{q})$

$$\frac{1}{2} [\langle S_{tt}^2 \rangle_0 - \langle S_{tt} \rangle_0^2] = 48 \text{ (dashed circle)} + 72 \text{ (dashed circle)} + 72 \text{ (dashed circle)} + 36 \text{ (crossed circle)} + 48 \text{ (dashed circle)}$$

renormalize r renormalize u

Don't need this $\downarrow O(\epsilon)$

