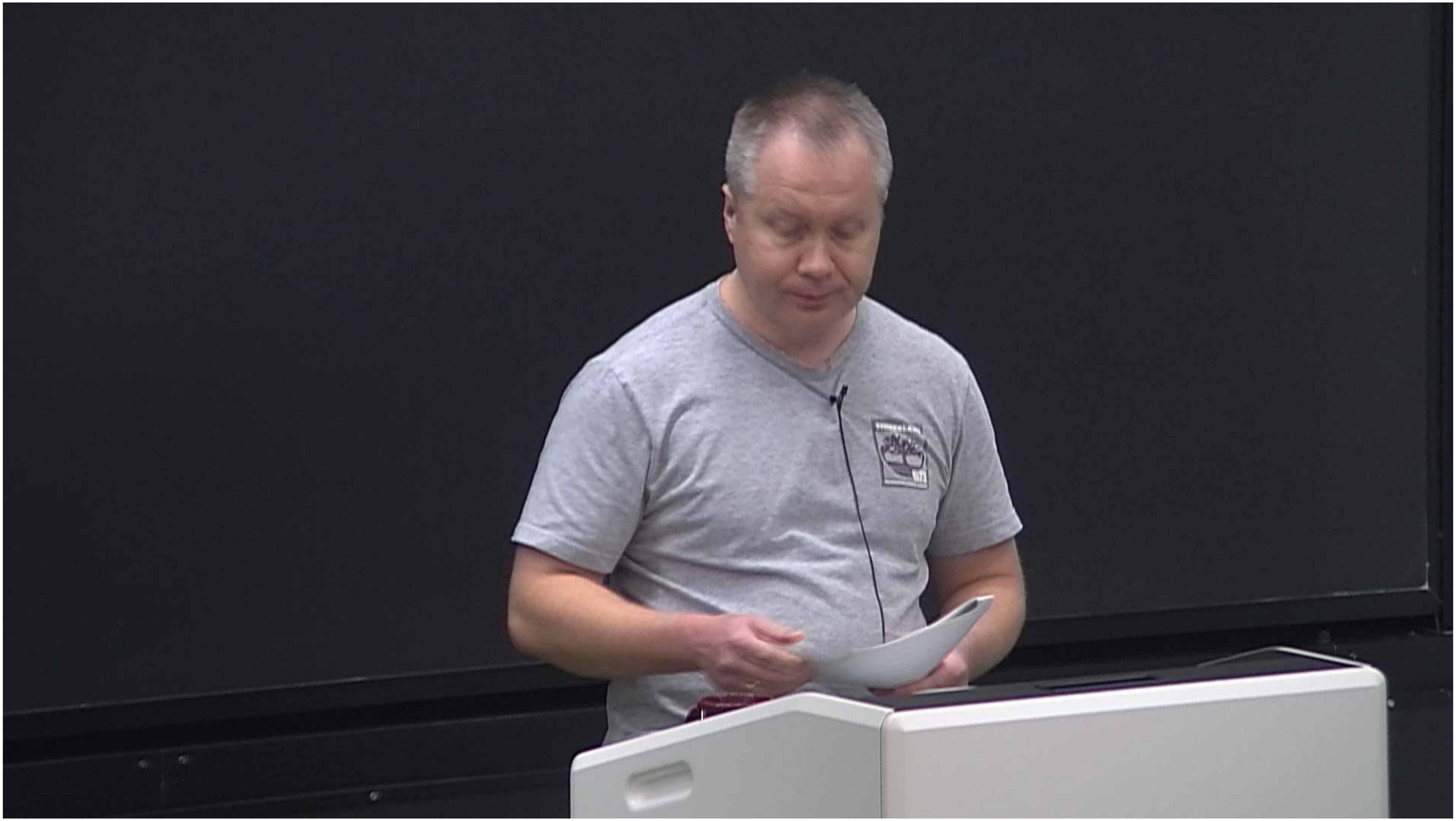
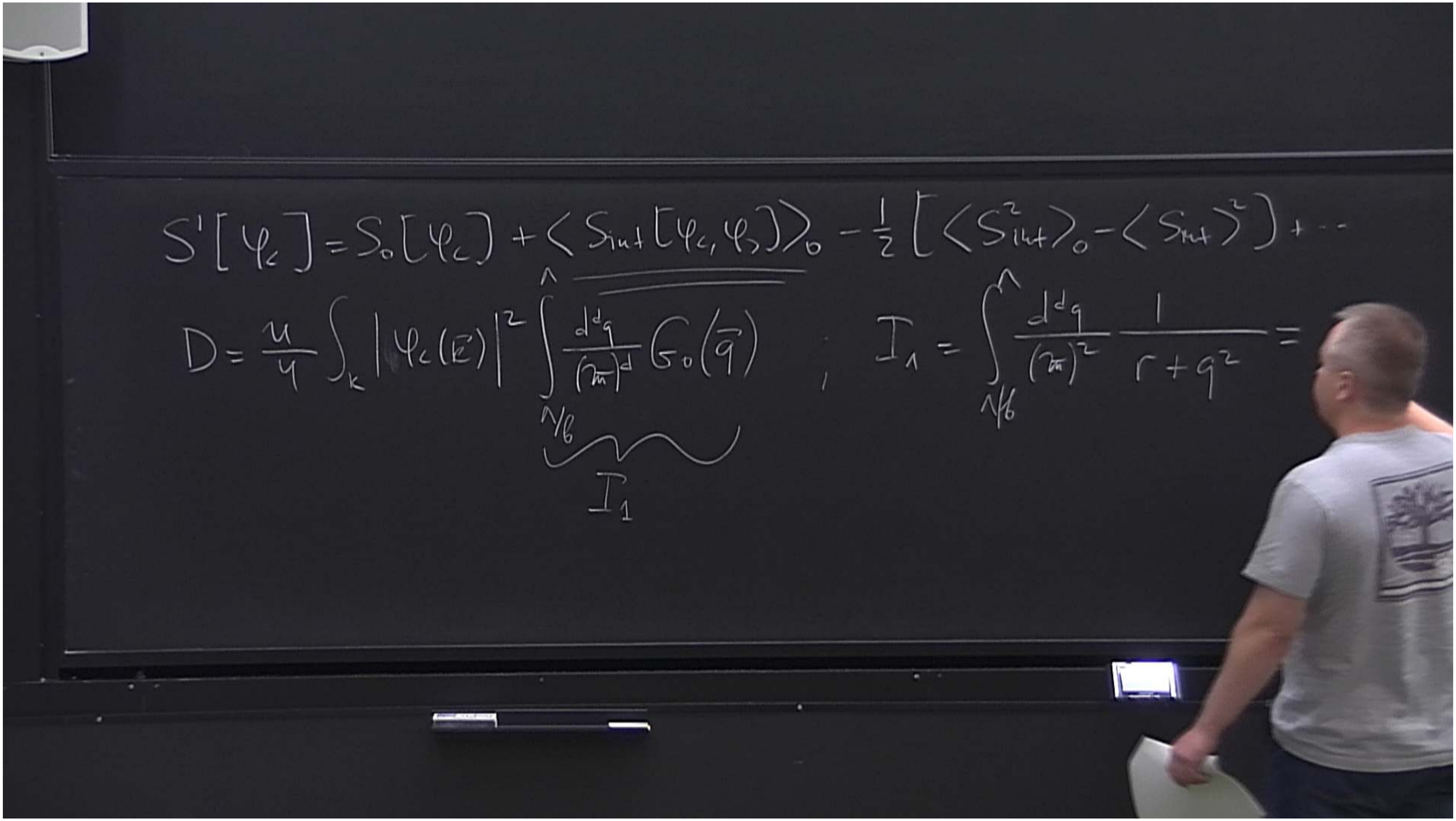
Title: 14/15 PSI - Statistical Mechanics - Lecture 8

Date: Oct 16, 2014 10:45 AM

URL: http://pirsa.org/14100094

Abstract:





$$S'[\varphi_<] = S_0[\varphi_<] + \underline{\underline{\langle S_{int}[\varphi_<, \varphi_>]\rangle_0}} - \frac{1}{2}\left[\langle S_{int}^2\rangle_0 - \langle S_{int}\rangle^2\right] + \cdots$$

$$D = \frac{u}{4}\int_k |\varphi_<(\vec{k})|^2 \underbrace{\int_{\Lambda/b}^{\Lambda} \frac{d^dq}{(2\pi)^d} G_0(\vec{q})}_{I_1} \quad ; \quad I_1 = \int_{\Lambda/b}^{\Lambda} \frac{d^dq}{(2\pi)^2}\frac{1}{r+q^2} =$$

$$S'[\varphi_<] = S_0[\varphi_<] + \underline{\underline{\langle S_{int}[\varphi_<, \varphi_>] \rangle_0}} - \frac{1}{2}\left[\langle S_{int}^2 \rangle_0 - \langle S_{int} \rangle^2\right] + \cdots$$

$$D = \frac{u}{4} \int_k |\varphi_<(\vec{k})|^2 \underbrace{\int_{\Lambda/b}^{\Lambda} \frac{d^d q}{(2\pi)^d} G_0(\vec{q})}_{I_1} \quad ; \quad I_1 = \int_{\Lambda/b}^{\Lambda} \frac{d^d q}{(2\pi)^d} \frac{1}{r+q^2} = \frac{S_d}{(2\pi)^d} \int_{\Lambda/b}^{\Lambda} dq \frac{q^{d-1}}{r+q^2}$$

$$S_d = \frac{2\pi^{d/2}}{\Gamma\left(\frac{d}{2}\right)}$$

$$S'[\varphi_<] = S_0[\varphi_<] + \underline{\underline{\langle S_{int}[\varphi_<, \varphi_>] \rangle_0}} - \frac{1}{2}\left[\langle S_{int}^2 \rangle_0 - \langle S_{int} \rangle^2\right] + \cdots$$

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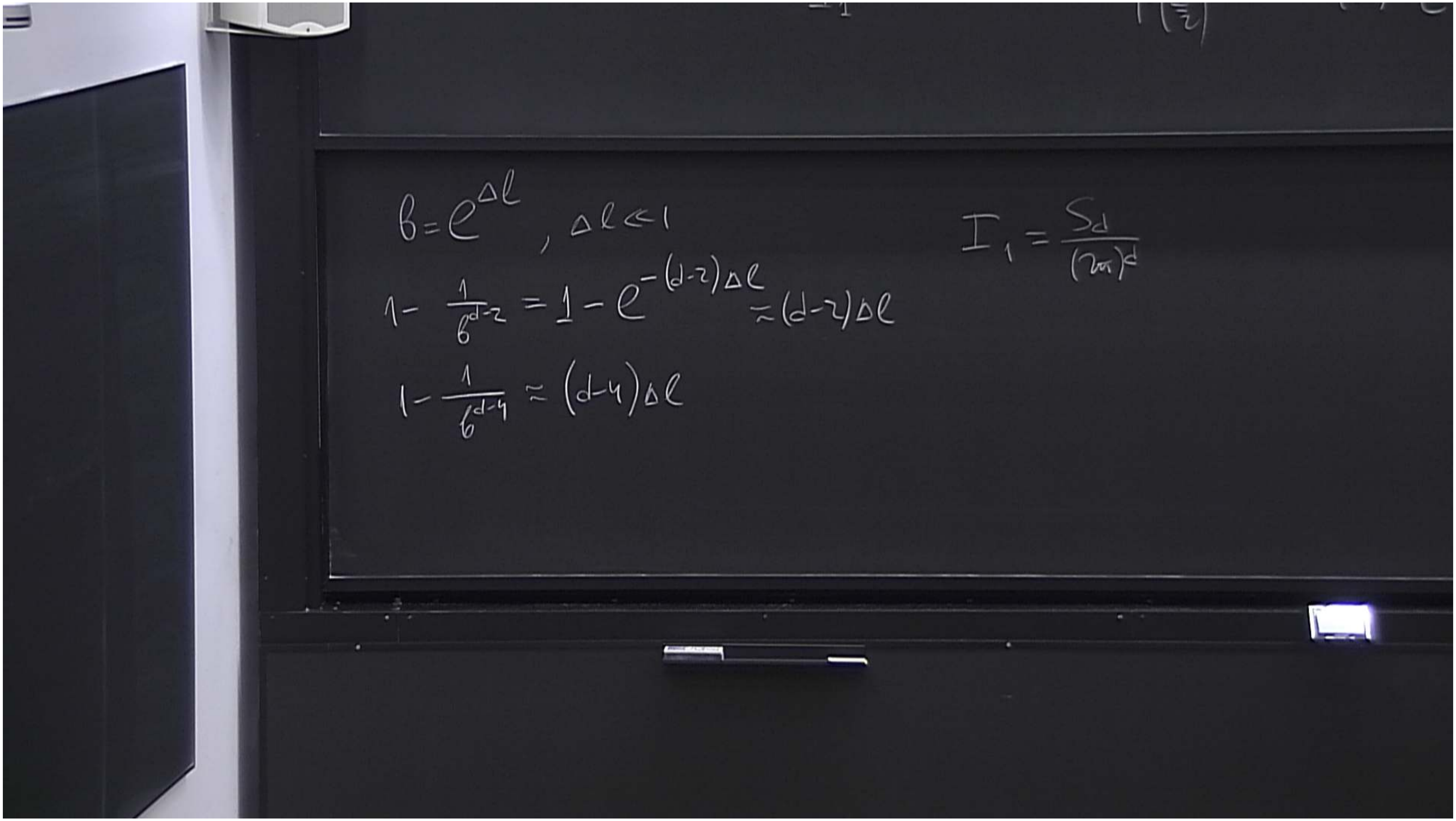
$$S_d = \frac{2\pi^{d/2}}{\Gamma\left(\frac{d}{2}\right)} \quad ; \quad = \frac{S_d}{(2\pi)^d}\left[\frac{1}{d-2}\left(\Lambda^{d-2} - \left(\frac{\Lambda}{b}\right)^{d-2}\right) - \frac{r}{d-4}\left(\Lambda^{d-4} - \left(\frac{\Lambda}{b}\right)^{d-4}\right) + \cdots\right]$$

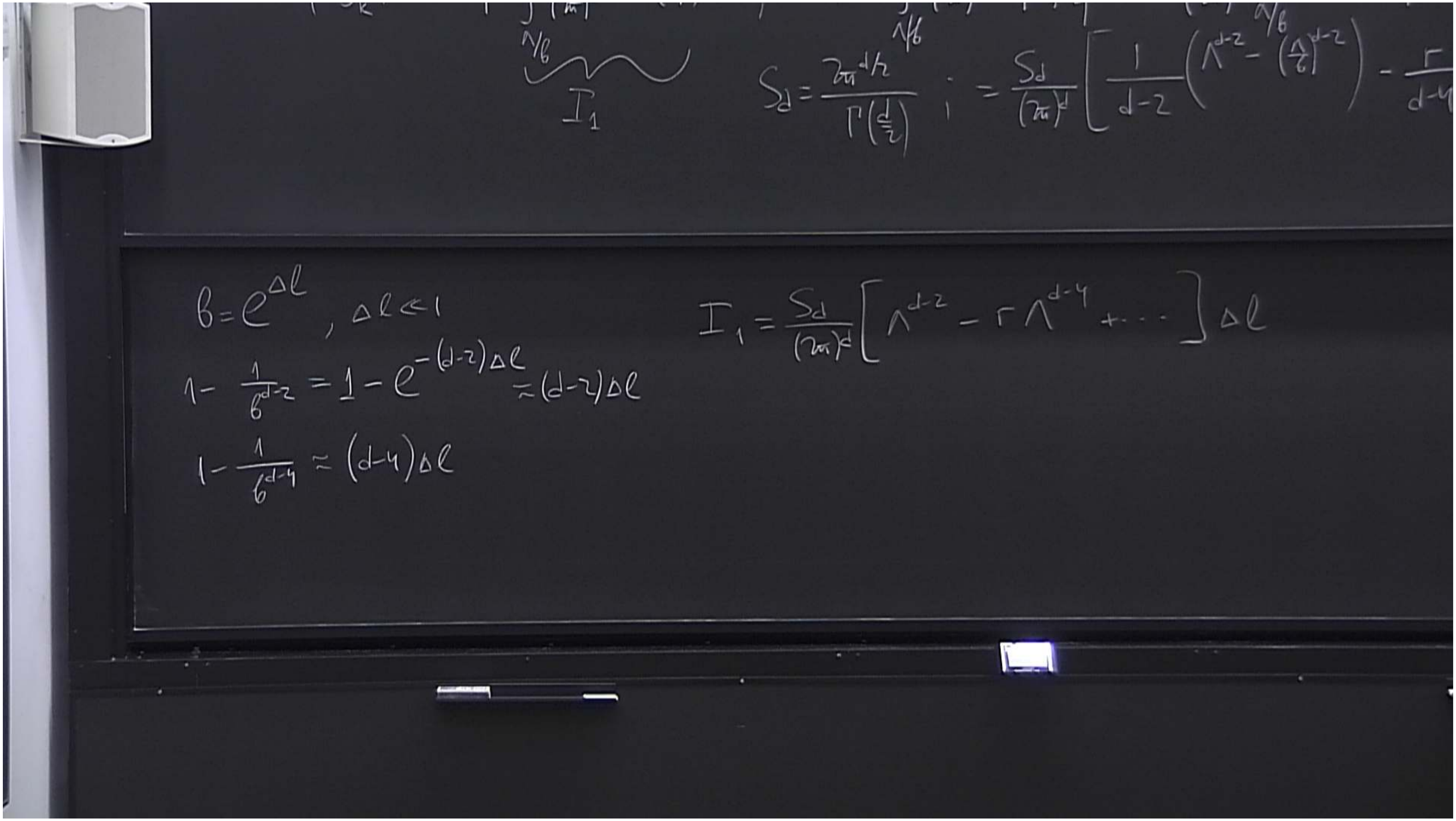


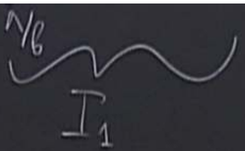
$b = e^{\Delta \ell}$, $\Delta \ell \ll 1$

$1 - \frac{1}{b^{d-2}} = 1 - e^{-(d-2)\Delta \ell} \simeq (d-2)\Delta \ell$

$1 - \frac{1}{b^{d-4}} \simeq (d-4)\Delta \ell$

$I_1 = \frac{S_d}{(2\pi)^d}$





$S_d = \frac{2\pi^{d/2}}{\Gamma\left(\frac{d}{2}\right)}$; $= \frac{S_d}{(2\pi)^d}\left[\frac{1}{d-2}\left(\Lambda^{d-2} - \left(\frac{\Lambda}{b}\right)^{d-2}\right) - \frac{r}{d-4}\left(\Lambda^{d-4} - \right.\right.$

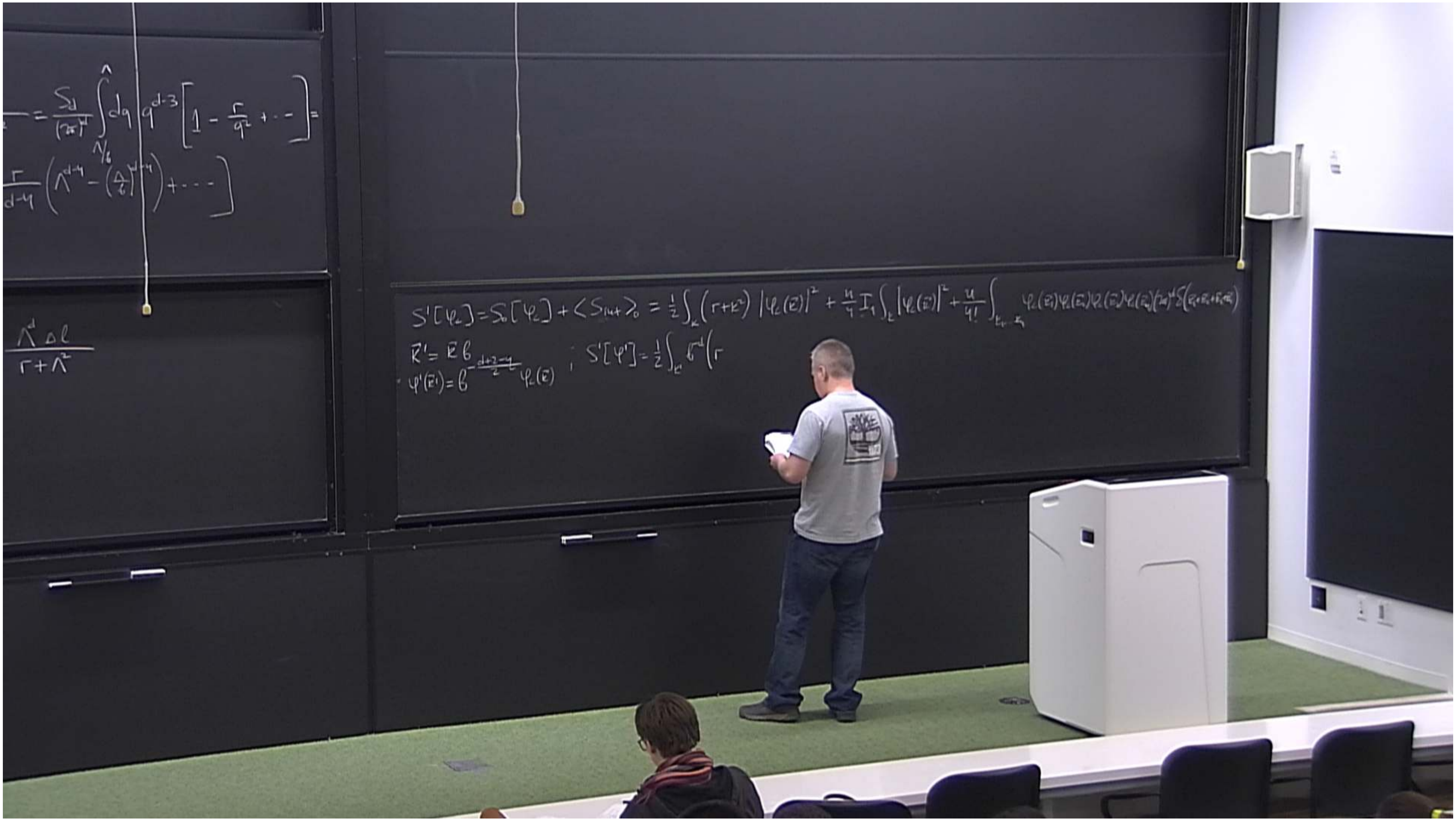
$b = e^{\Delta l}$, $\Delta l \ll 1$

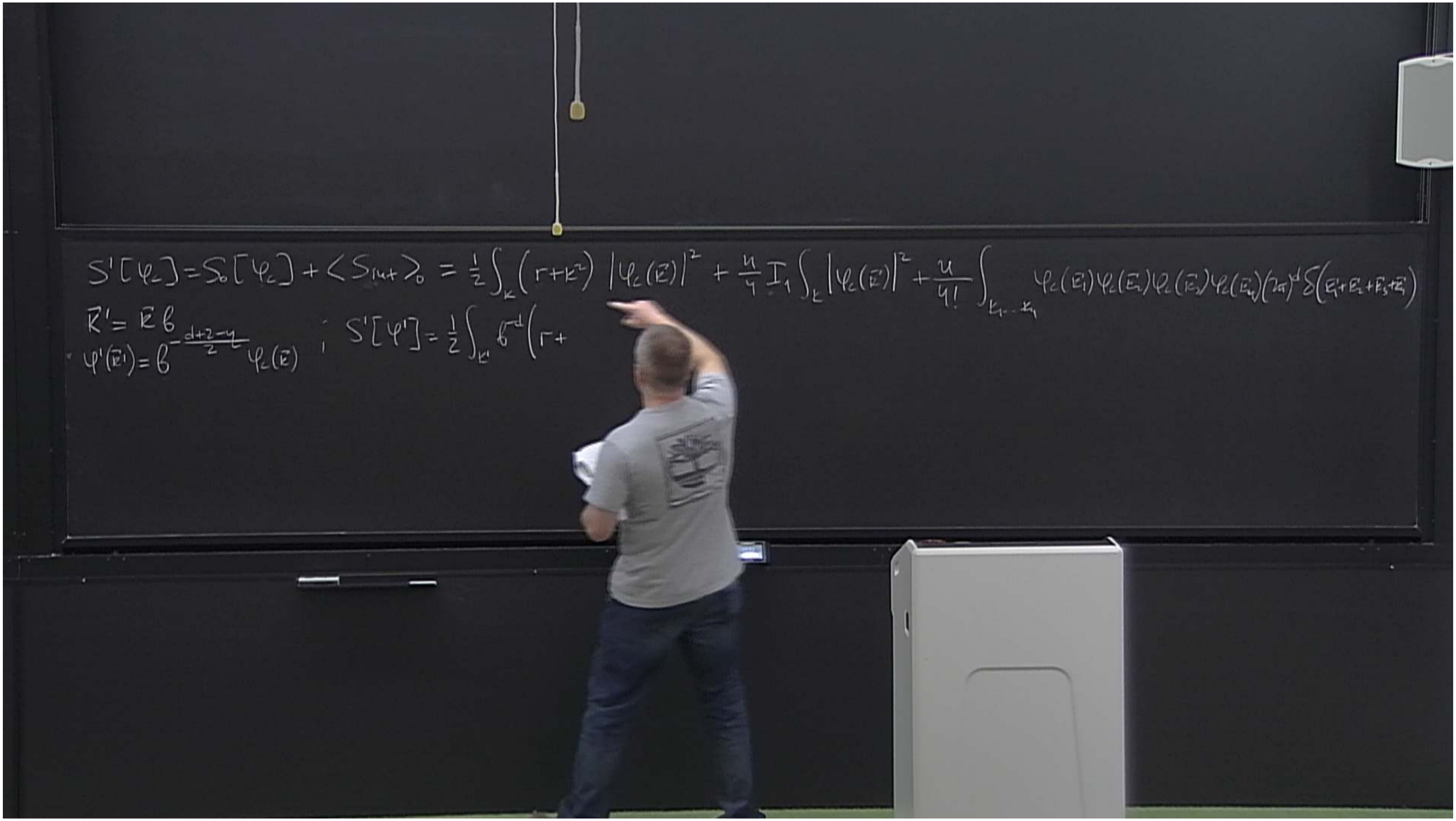
$1 - \frac{1}{b^{d-2}} = 1 - e^{-(d-2)\Delta l} \approx (d-2)\Delta l$

$1 - \frac{1}{b^{d-4}} \approx (d-4)\Delta l$

$I_1 = \frac{S_d}{(2\pi)^d}\left[\Lambda^{d-2} - r\Lambda^{d-4} + \dots\right]\Delta l = \frac{S_d}{(2\pi)^d}\frac{\Lambda^d \Delta l}{r + \Lambda^2}$

$\langle D \rangle_0 = I_1 \frac{u}{4}\int_k |\varphi_<(k)|^2$

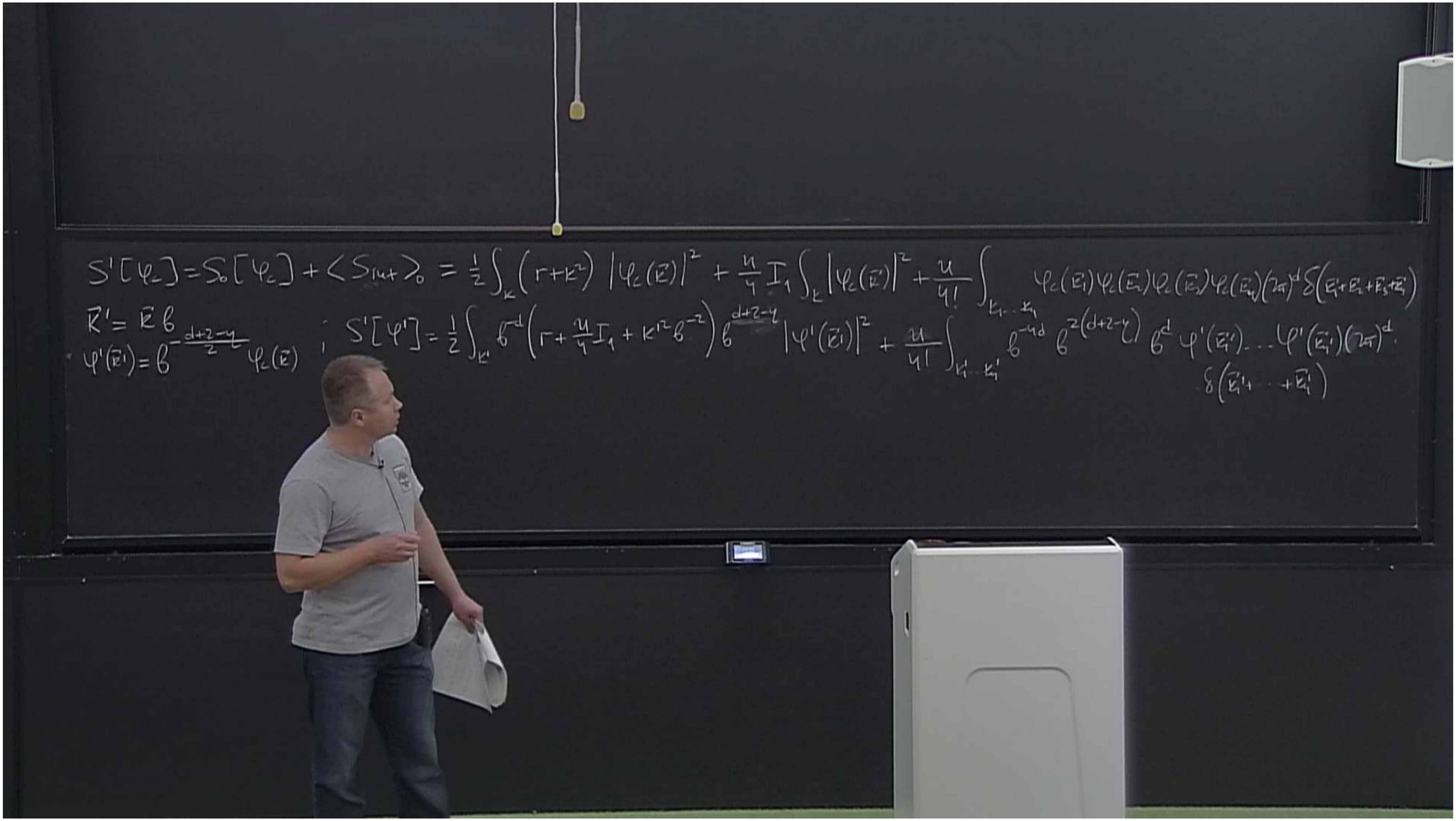




$$S'[\varphi_<] = S_0[\varphi_<] + \langle S_{int} \rangle_0 = \frac{1}{2}\int_k (r+k^2)\,|\varphi_<(\vec{k})|^2 + \frac{u}{4} I_1 \int_k |\varphi_<(\vec{k})|^2 + \frac{u}{4!}\int_{k_1 \ldots k_4} \varphi_<(\vec{k}_1)\varphi_<(\vec{k}_2)\varphi_<(\vec{k}_3)\varphi_<(\vec{k}_4)(2\pi)^d \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)$$

$$\vec{k}' = \vec{k}\, b$$

$$\varphi'(\vec{k}') = b^{-\frac{d+2-\eta}{2}} \varphi_<(\vec{k}) \quad ; \quad S'[\varphi'] = \frac{1}{2}\int_{k'} b^{-d}\Big(r +$$

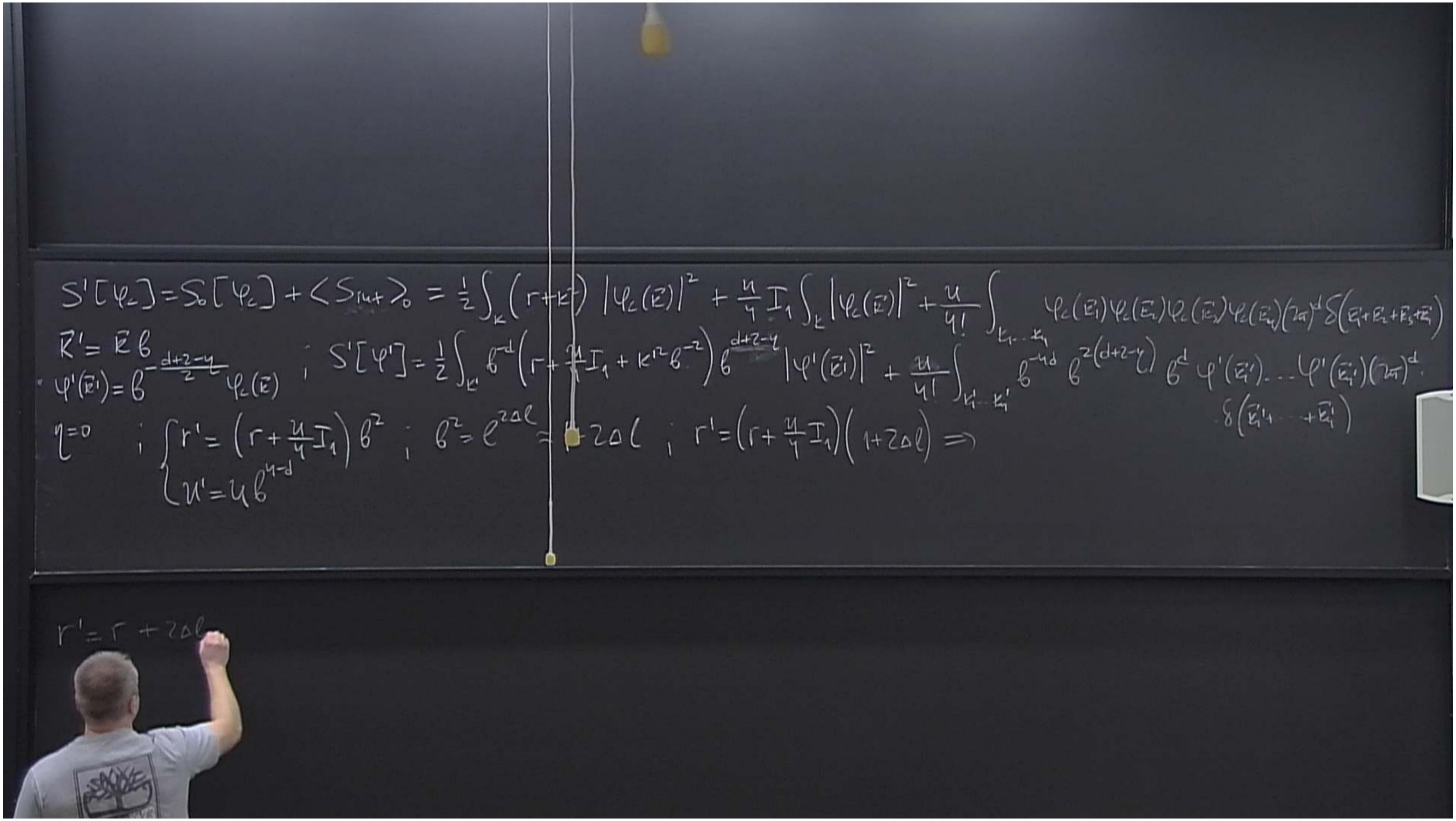


$$S'[\varphi_<] = S_0[\varphi_<] + \langle S_{int} \rangle_0 = \frac{1}{2}\int_k (r+k^2)\,|\varphi_<(\vec{k})|^2 + \frac{u}{4} I_1 \int_k |\varphi_<(\vec{k})|^2 + \frac{u}{4!}\int_{k_1 \dots k_4} \varphi_<(\vec{k}_1)\varphi_<(\vec{k}_2)\varphi_<(\vec{k}_3)\varphi_<(\vec{k}_4)(2\pi)^d \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)$$

$$\vec{k}' = \vec{k}\, b$$

$$\varphi'(\vec{k}') = b^{-\frac{d+2-\eta}{2}} \varphi_<(\vec{k})$$

$$; \quad S'[\varphi'] = \frac{1}{2}\int_{k'} b^{-d}\left(r + \frac{u}{4} I_1 + k'^2 b^{-2}\right) b^{d+2-\eta} |\varphi'(\vec{k}')|^2 + \frac{u}{4!}\int_{k'_1 \dots k'_4} b^{-4d}\, b^{2(d+2-\eta)}\, b^{d}\, \varphi'(\vec{k}'_1)\dots\varphi'(\vec{k}'_4)(2\pi)^d \cdot \delta(\vec{k}'_1 + \dots + \vec{k}'_4)$$



$$S'[\varphi_<] = S_0[\varphi_<] + \langle S_{int}\rangle_0 = \frac{1}{2}\int_k (r+k^2)\,|\varphi_<(\vec{k})|^2 + \frac{u}{4} I_1 \int_k |\varphi_<(\vec{k})|^2 + \frac{u}{4!}\int_{k_1\ldots k_4} \varphi_<(\vec{k}_1)\varphi_<(\vec{k}_2)\varphi_<(\vec{k}_3)\varphi_<(\vec{k}_4)(2\pi)^d\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)$$

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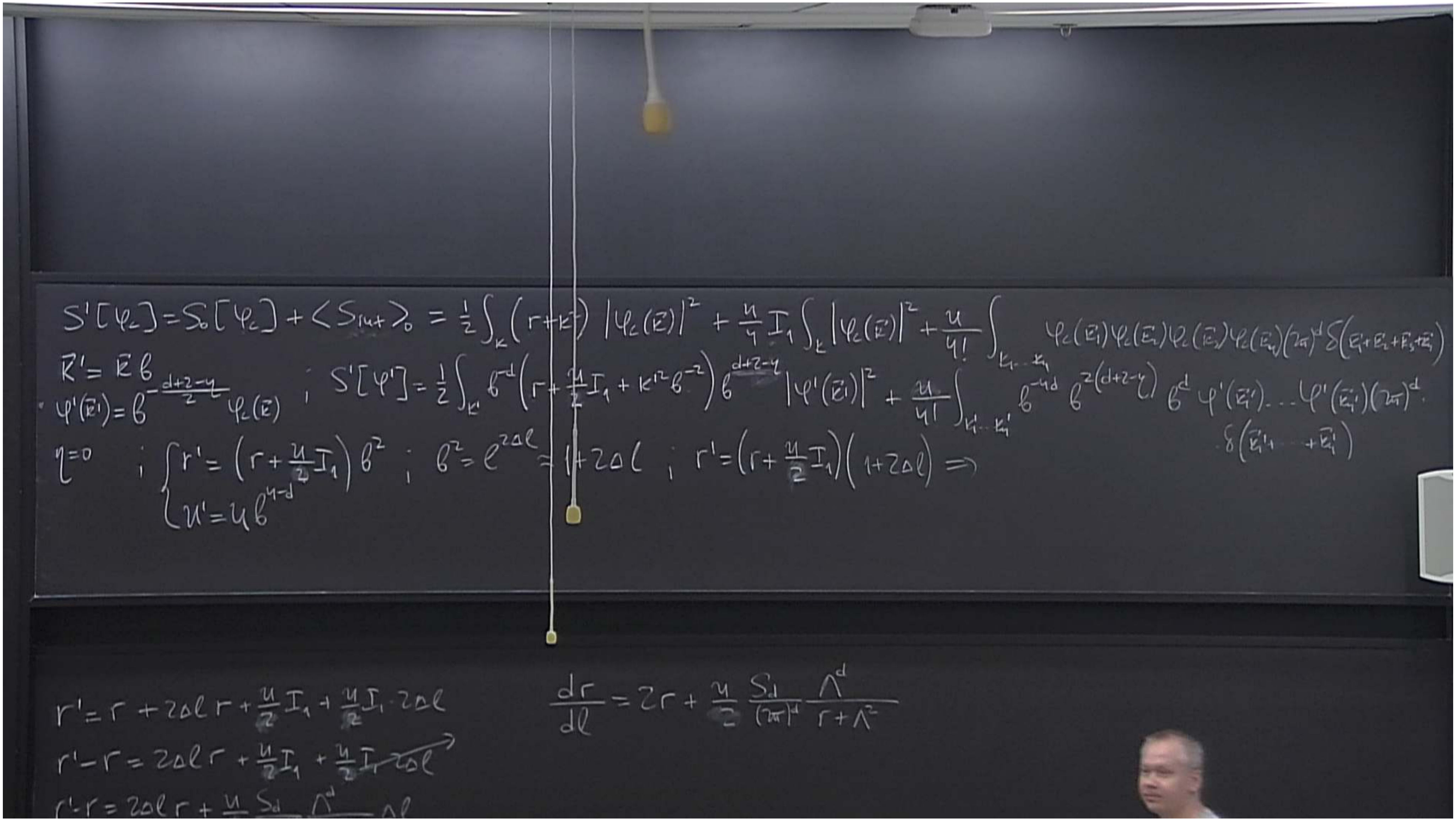
$$\eta = 0 \quad ; \quad \begin{cases} r' = \left(r + \frac{u}{4} I_1\right) b^2 \\ u' = u\, b^{4-d} \end{cases} \quad ; \quad b^2 = e^{2\Delta l} \approx 1 + 2\Delta l \quad ; \quad r' = \left(r + \frac{u}{4} I_1\right)(1 + 2\Delta l) \Longrightarrow$$

$$r' = r + 2\Delta l$$



$$S'[\varphi_c] = S_0[\varphi_c] + \langle S_{int} \rangle_0 = \frac{1}{2}\int_k (r+k^2)\,|\varphi_c(\vec{k})|^2 + \frac{u}{4} I_1 \int_k |\varphi_c(\vec{k})|^2 + \frac{u}{4!}\int_{k_1 \dots k_4} \varphi_c(\vec{k}_1)\varphi_c(\vec{k}_2)\varphi_c(\vec{k}_3)\varphi_c(\vec{k}_4)(2\pi)^d \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)$$

$$\vec{k}' = \vec{k}\, b$$

$$\varphi'(\vec{k}') = b^{-\frac{d+2-\eta}{2}} \varphi_c(\vec{k}) \; ; \; S'[\varphi'] = \frac{1}{2}\int_{k'} b^{-d}\left(r + \frac{u}{2} I_1 + k'^2 b^{-2}\right) b^{d+2-\eta} |\varphi'(\vec{k}')|^2 + \frac{u}{4!}\int_{k_1' \dots k_4'} b^{-4d}\, b^{2(d+2-\eta)}\, b^d\, \varphi'(\vec{k}_1') \dots \varphi'(\vec{k}_4')(2\pi)^d \cdot \delta(\vec{k}_1' + \dots + \vec{k}_4')$$

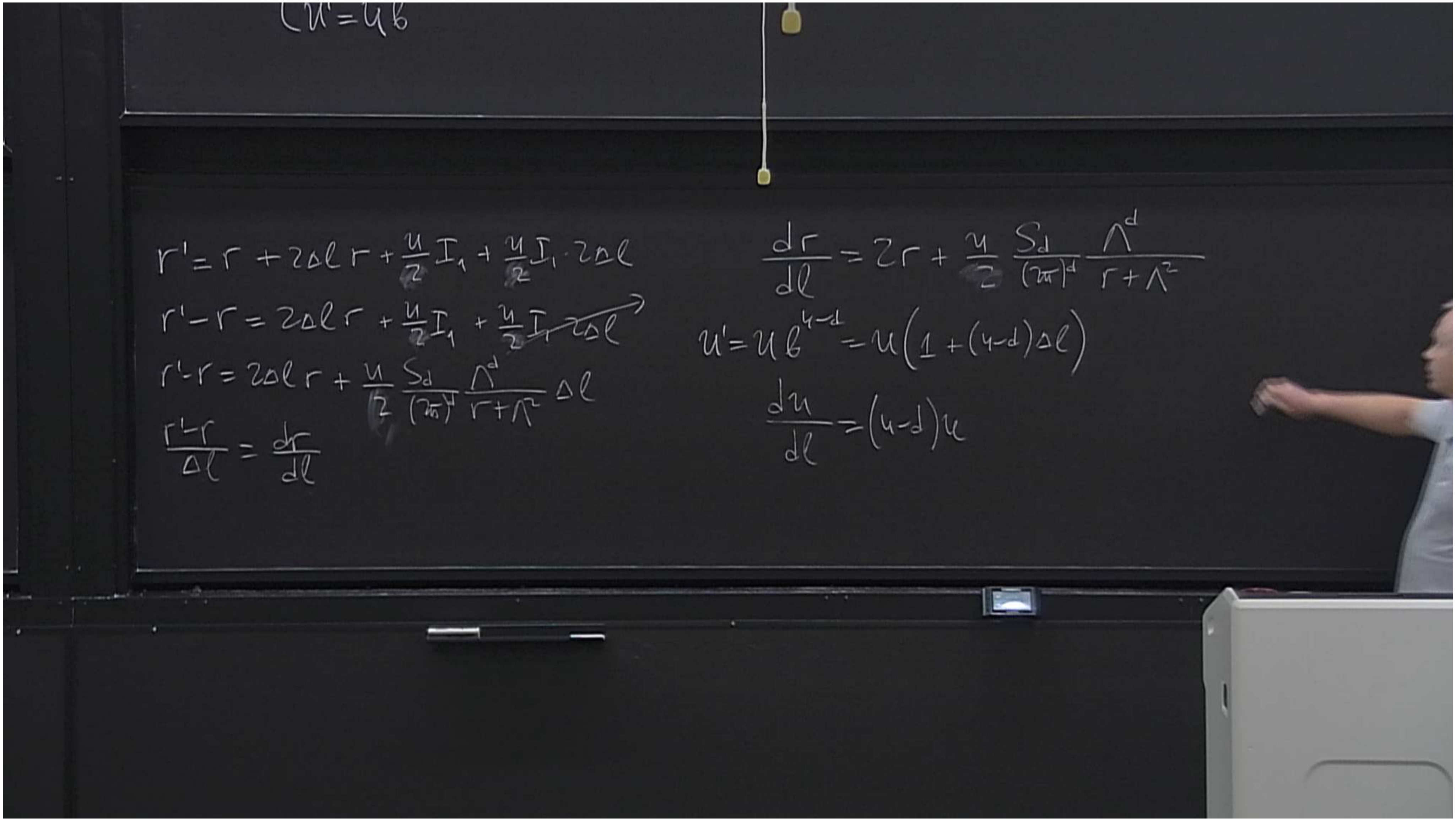
$$\eta = 0 \; ; \; \begin{cases} r' = \left(r + \frac{u}{2} I_1\right) b^2 \\ u' = u\, b^{4-d} \end{cases} ; \; b^2 = e^{2\Delta l} \approx 1 + 2\Delta l \; ; \; r' = \left(r + \frac{u}{2} I_1\right)\left(1 + 2\Delta l\right) \Rightarrow$$

$$r' = r + 2\Delta l\, r + \frac{u}{2} I_1 + \frac{u}{2} I_1 \cdot 2\Delta l$$

$$r' - r = 2\Delta l\, r + \frac{u}{2} I_1 + \frac{u}{2} \cancel{I_1 2\Delta l}$$

$$r' - r = 2\Delta l\, r + \frac{u}{} \frac{S_d}{} \frac{\Lambda^d}{} \Delta l$$

$$\frac{dr}{dl} = 2r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2}$$



$(u' = u b$

$$r' = r + 2\Delta l\, r + \frac{u}{2} I_1 + \frac{u}{2} I_1 \cdot 2\Delta l$$

$$r' - r = 2\Delta l\, r + \frac{u}{2} I_1 + \cancel{\frac{u}{2} I_1\, 2\Delta l}$$

$$r' - r = 2\Delta l\, r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2} \Delta l$$

$$\frac{r' - r}{\Delta l} = \frac{dr}{dl}$$

$$\frac{dr}{dl} = 2r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2}$$

$$u' = u b^{4-d} = u\left(1 + (4-d)\Delta l\right)$$

$$\frac{du}{dl} = (4-d) u$$

$(u' = u b^{4-d}$

$$r' = r + 2\Delta\ell\, r + \frac{u}{2} I_1 + \frac{u}{2} I_1 \cdot 2\Delta\ell$$

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$$\frac{r' - r}{\Delta\ell} = \frac{dr}{d\ell}$$

$$\frac{dr}{d\ell} = \boxed{2} r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2}$$

$$u' = u b^{4-d} = u\left(1 + (4-d)\Delta\ell\right)$$

$$\frac{du}{d\ell} = \boxed{(4-d)} u$$

$\dim[r] = 2$

$\dim[u] = 4-d$

$u' = u b^{4-d}$

$$r' = r + 2\Delta l\, r + \frac{u}{2} I_1 + \frac{u}{2} I_1 \cdot 2\Delta l$$

$$r' - r = 2\Delta l\, r + \frac{u}{2} I_1 + \frac{u}{2} \cancel{I_1 2\Delta l} \rightarrow$$

$$r' - r = 2\Delta l\, r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2} \Delta l$$

$$\frac{r' - r}{\Delta l} = \frac{dr}{dl}$$

$$\frac{dr}{dl} = \enclose{circle}{2} r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2}$$

$$u' = u b^{4-d} = u\left(1 + (4-d)\Delta l\right)$$

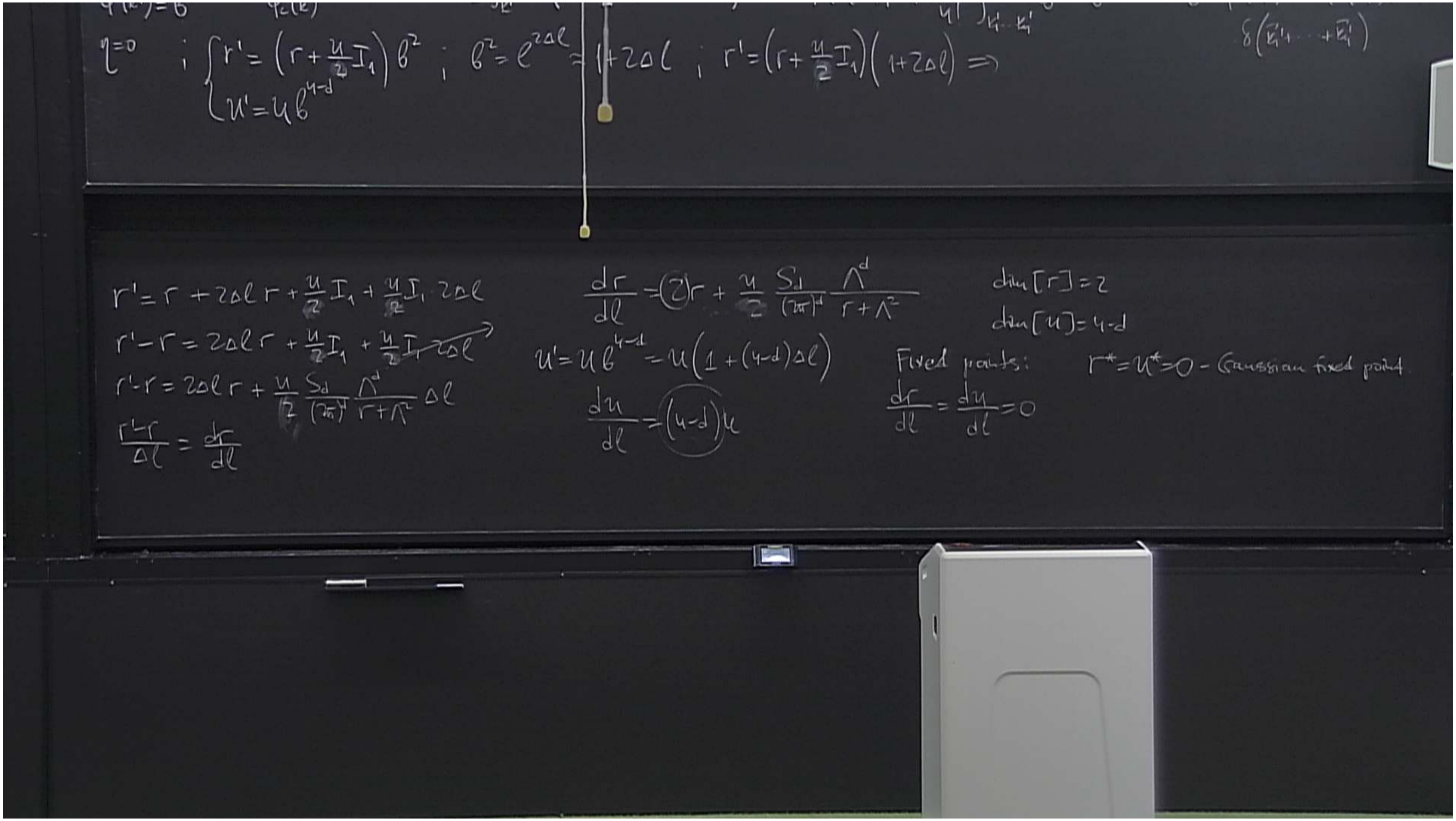
$$\frac{du}{dl} = \enclose{circle}{(4-d)} u$$

$\dim[r] = 2$

$\dim[u] = 4-d$

Fixed points:

$$\frac{dr}{dl} = \frac{du}{dl} = 0$$



$$\delta(\vec{k}'_1 \cdots + \vec{k}'_4)$$

$$\eta = 0 \; ; \; \begin{cases} r' = \left(r + \frac{u}{2} I_1\right) b^2 \\ u' = u\, b^{4-d} \end{cases} ; \; b^2 = e^{2\Delta\ell} \approx 1 + 2\Delta\ell \; ; \; r' = \left(r + \frac{u}{2} I_1\right)\left(1 + 2\Delta\ell\right) \Rightarrow$$

$$r' = r + 2\Delta\ell\, r + \frac{u}{2} I_1 + \frac{u}{2} I_1 \, 2\Delta\ell$$

$$r' - r = 2\Delta\ell\, r + \frac{u}{2} I_1 + \frac{u}{2} \cancel{I_1 \, 2\Delta\ell}$$

$$r' - r = 2\Delta\ell\, r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2} \Delta\ell$$

$$\frac{r' - r}{\Delta\ell} = \frac{dr}{d\ell}$$

$$\frac{dr}{d\ell} = \boxed{2} r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r + \Lambda^2}$$

$$u' = u\, b^{4-d} = u\left(1 + (4-d)\Delta\ell\right)$$

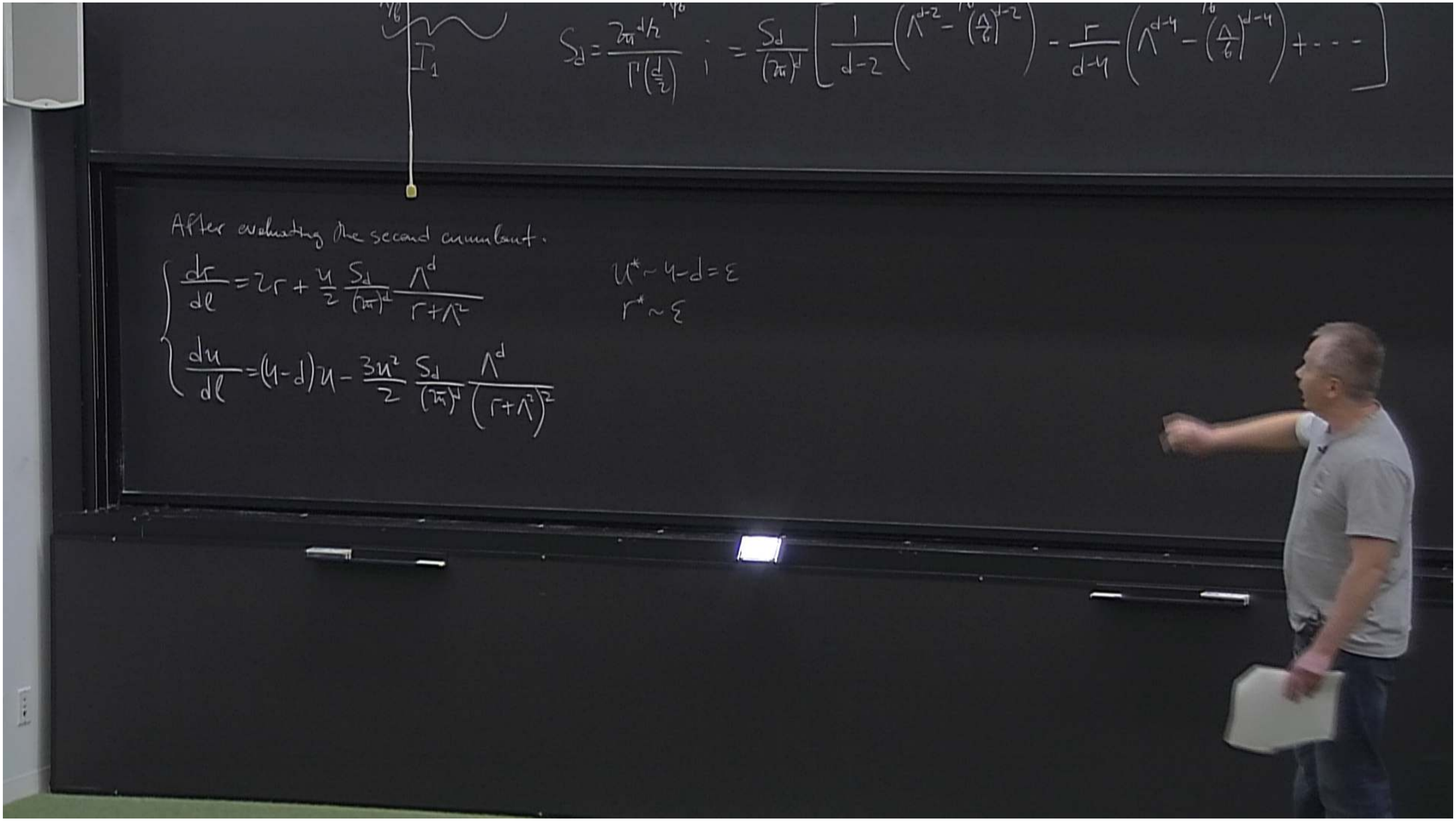
$$\frac{du}{d\ell} = \boxed{(4-d)} u$$

$$\dim[r] = 2$$

$$\dim[u] = 4-d$$

Fixed points: $r^* = u^* = 0$ - Gaussian fixed point

$$\frac{dr}{d\ell} = \frac{du}{d\ell} = 0$$



I_1

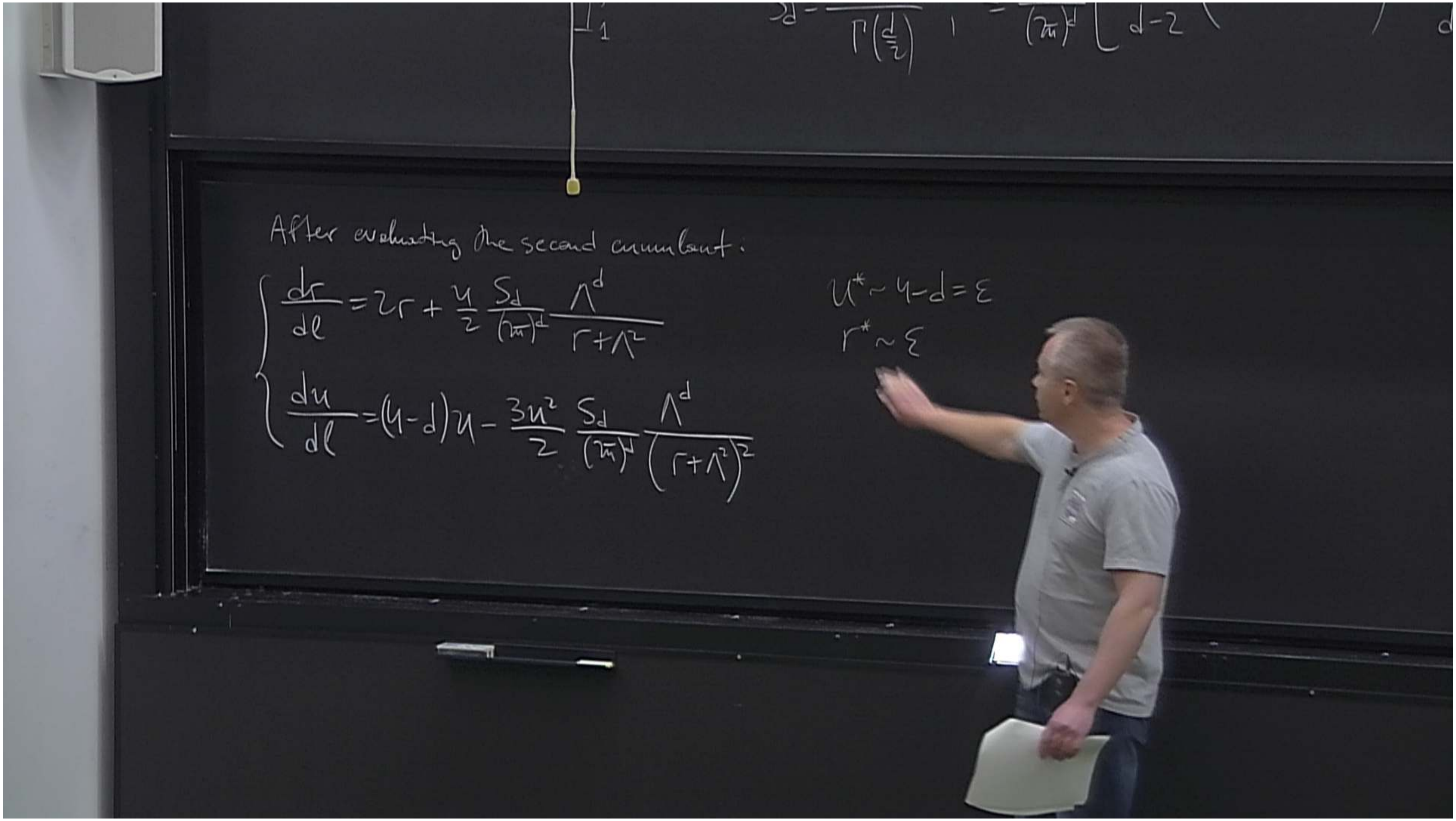
$$S_d = \frac{2\pi^{d/2}}{\Gamma\left(\frac{d}{2}\right)} \quad ; \quad = \frac{S_d}{(2\pi)^d}\left[\frac{1}{d-2}\left(\Lambda^{d-2} - \left(\frac{\Lambda}{b}\right)^{d-2}\right) - \frac{r}{d-4}\left(\Lambda^{d-4} - \left(\frac{\Lambda}{b}\right)^{d-4}\right) + \cdots\right]$$

After evaluating the second cumulant.

$$\begin{cases} \dfrac{dr}{dl} = 2r + \dfrac{u}{2}\dfrac{S_d}{(2\pi)^d}\dfrac{\Lambda^d}{r+\Lambda^2} \\ \dfrac{du}{dl} = (4-d)u - \dfrac{3u^2}{2}\dfrac{S_d}{(2\pi)^d}\dfrac{\Lambda^d}{\left(r+\Lambda^2\right)^2} \end{cases}$$

$u^* \sim 4-d = \varepsilon$

$r^* \sim \varepsilon$



$S_d = \frac{}{\Gamma(\frac{d}{2})}$ $\frac{}{(2\pi)^d}\Big[d-2$

After evaluating the second cumulant:

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$u^* \sim 4-d = \varepsilon$

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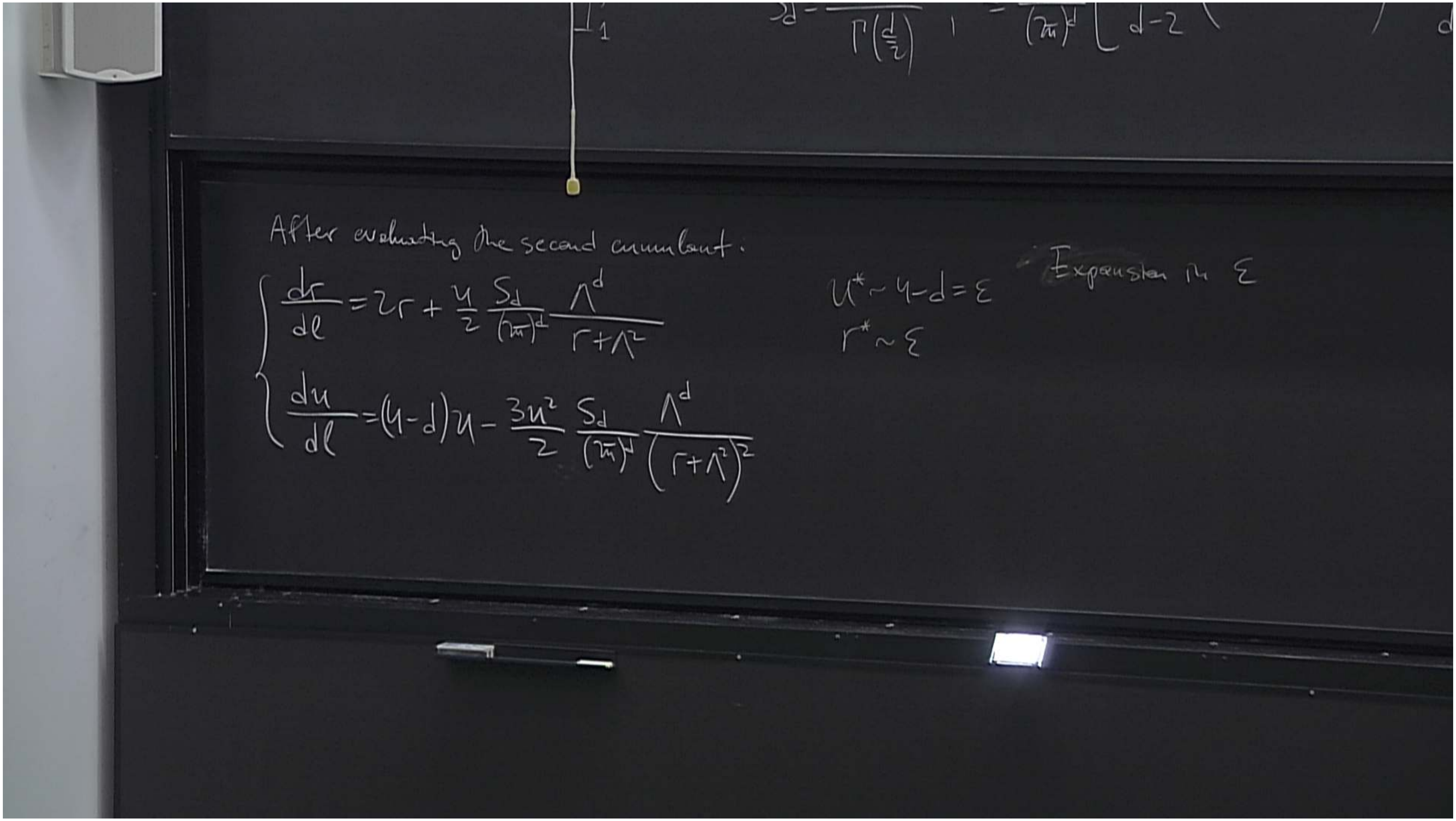

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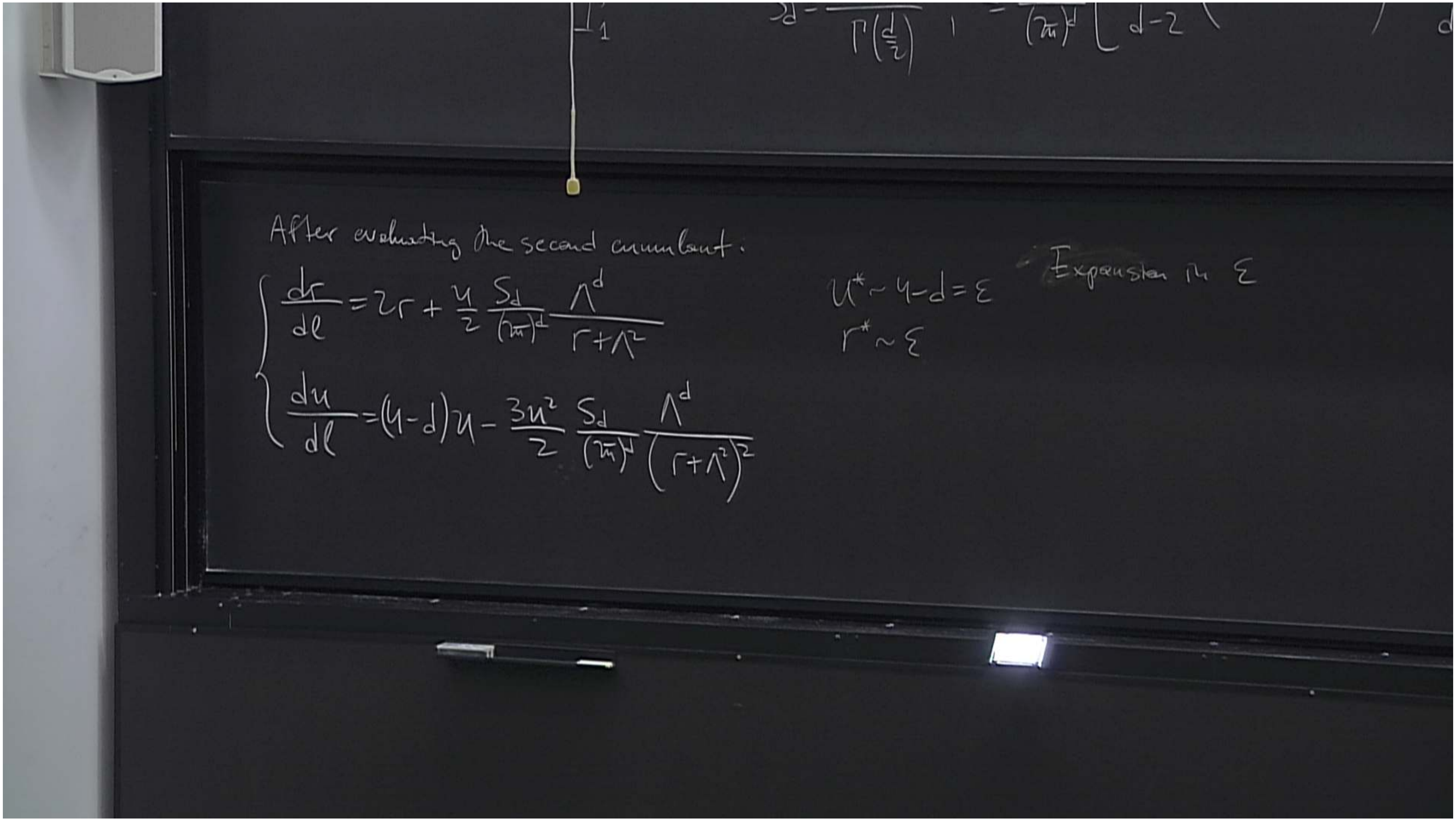
$u^* \sim 4-d = \varepsilon$

$r^* \sim \varepsilon$

Expansion in ε



Γ_1 $\quad S_d = \frac{\quad}{\Gamma\left(\frac{d}{2}\right)} \quad = \frac{\quad}{(2\pi)^d}\Big[d-2$

After evaluating the second cumulant:

$$\begin{cases} \dfrac{dr}{dl} = 2r + \dfrac{u}{2}\dfrac{S_d}{(2\pi)^d}\dfrac{\Lambda^d}{r+\Lambda^2} \\ \dfrac{du}{dl} = (4-d)u - \dfrac{3u^2}{2}\dfrac{S_d}{(2\pi)^d}\dfrac{\Lambda^d}{\left(r+\Lambda^2\right)^2} \end{cases}$$

$u^* \sim 4-d = \varepsilon$

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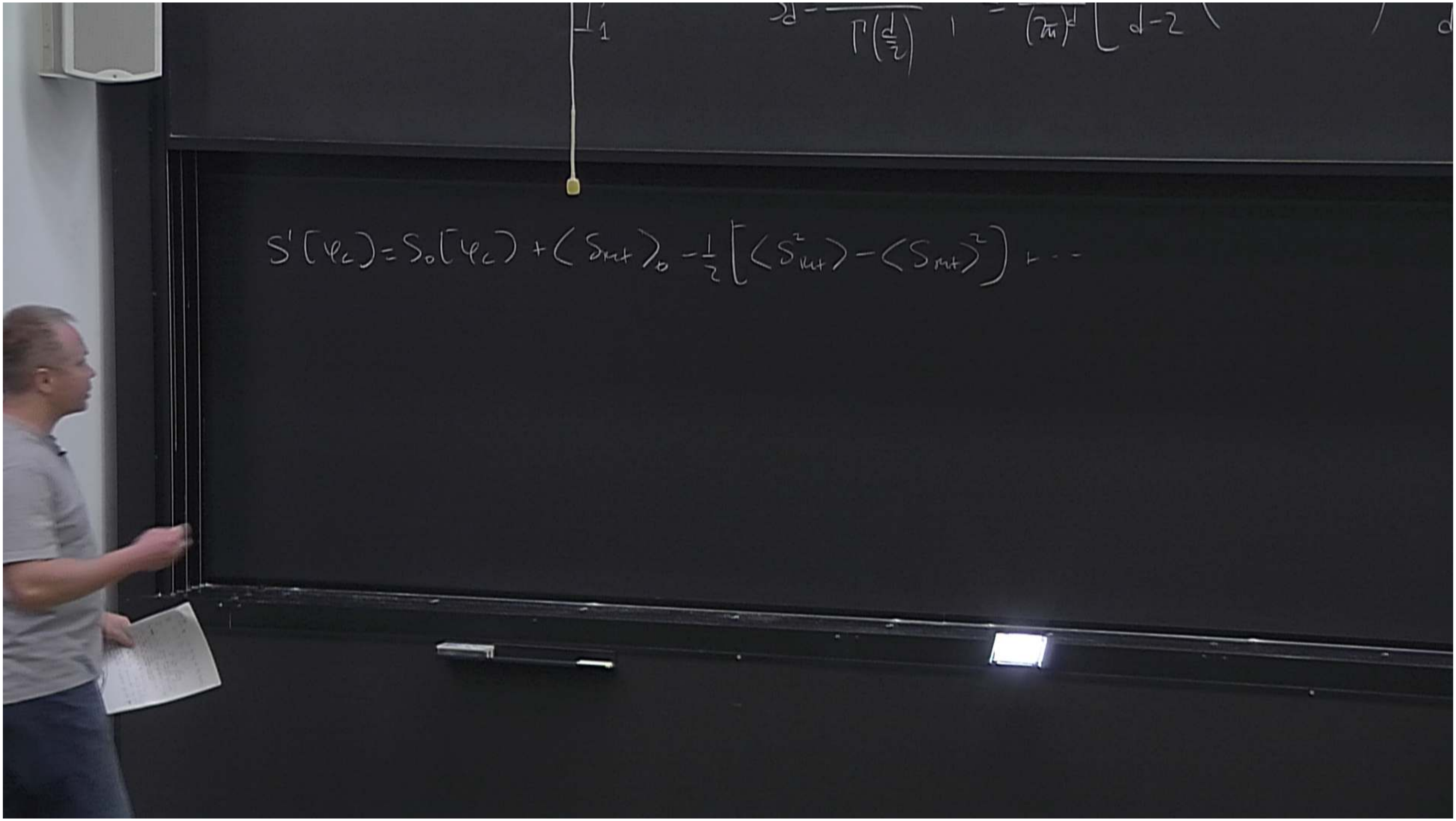
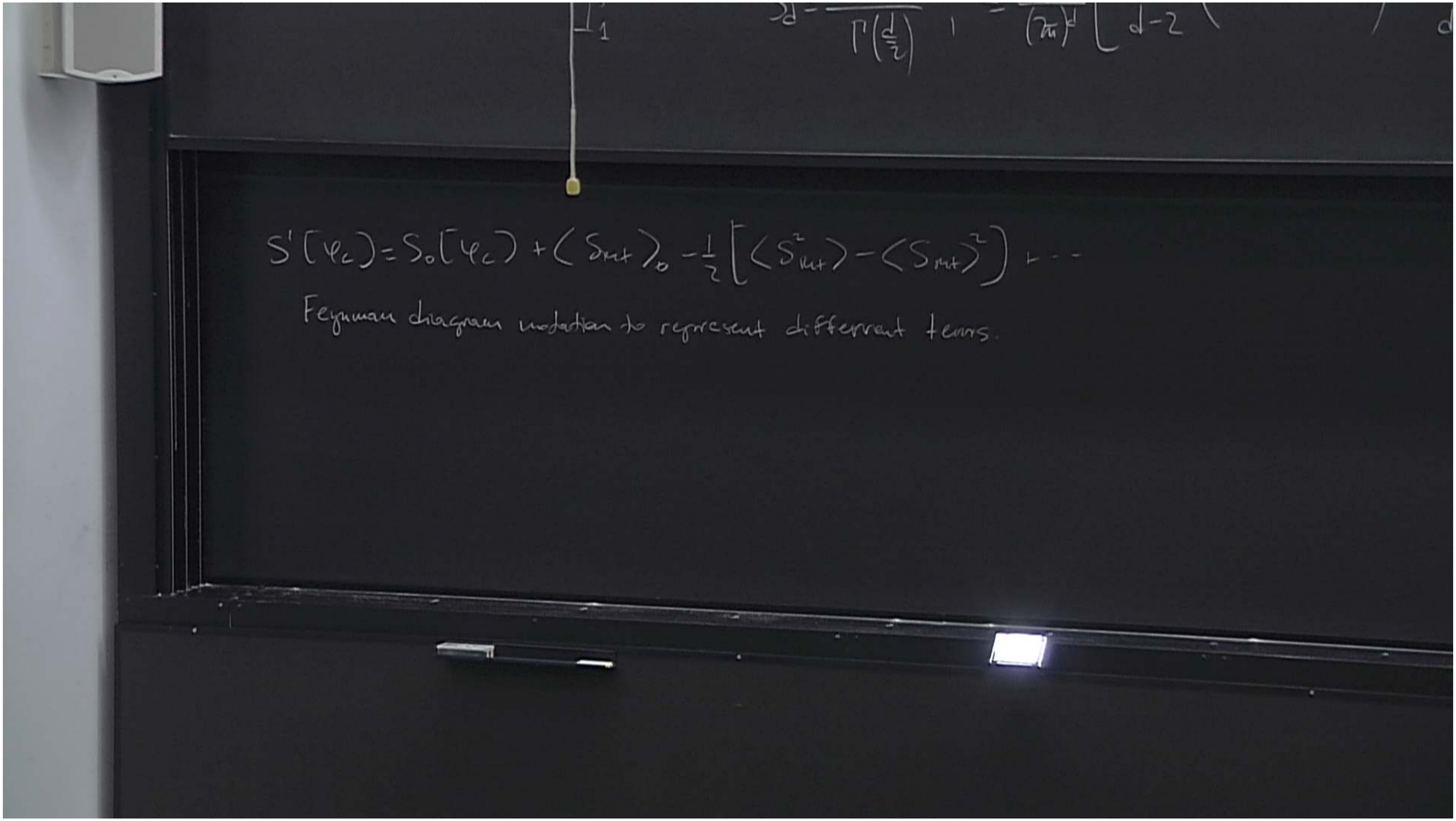
Expansion in ε

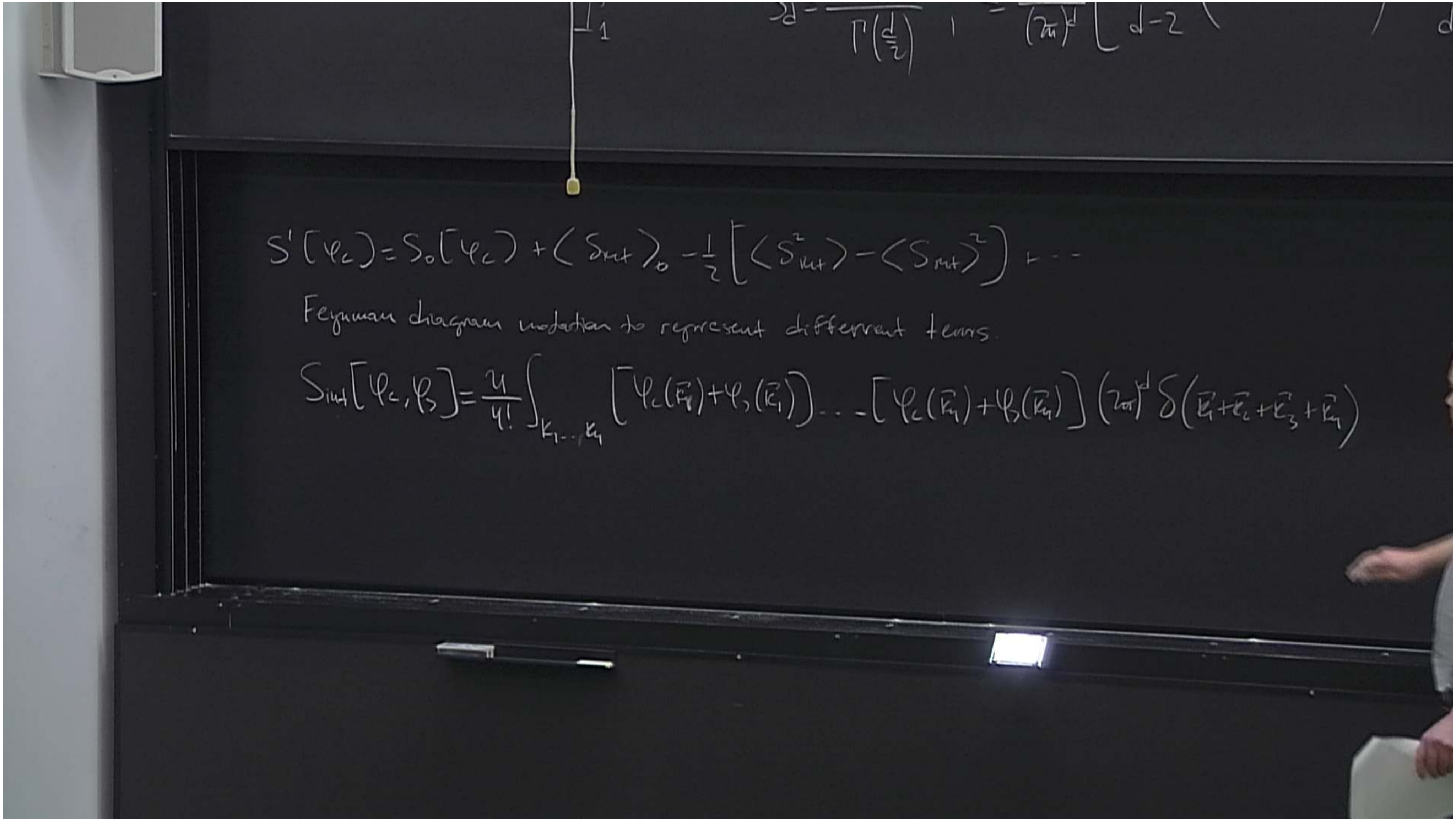


$$S'[\varphi_c] = S_0[\varphi_c] + \langle S_{int} \rangle_0 - \frac{1}{2}\left[\langle S_{int}^2 \rangle - \langle S_{int} \rangle^2 \right] + \cdots$$



$$S'[\varphi_c] = S_0[\varphi_c] + \langle S_{int}\rangle_0 - \frac{1}{2}\left[\langle S_{int}^2\rangle - \langle S_{int}\rangle^2\right) + \cdots$$

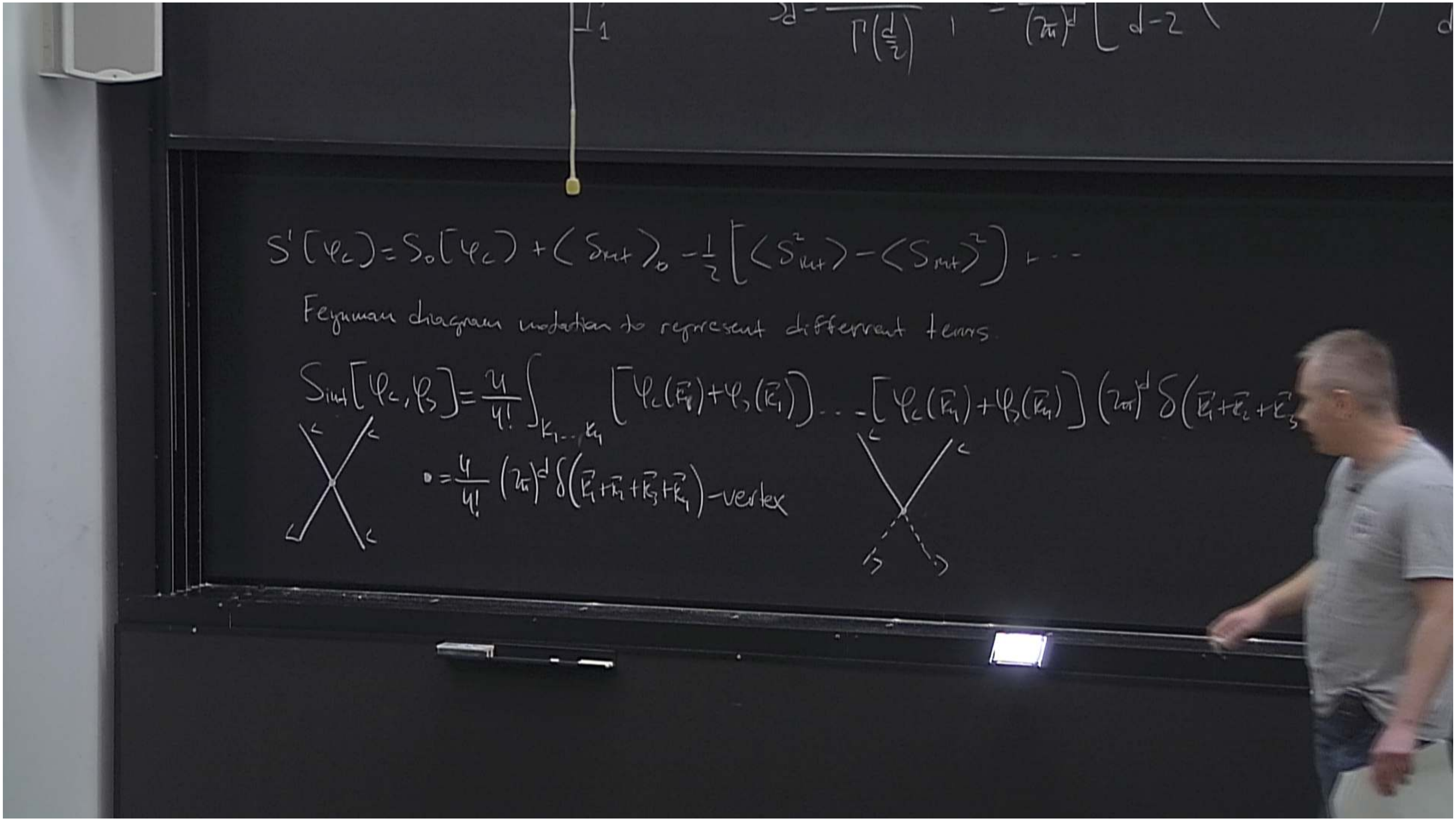
Feynman diagram notation to represent differrent terms.



$$S'[\varphi_c] = S_0[\varphi_c] + \langle S_{int}\rangle_0 - \frac{1}{2}\left[\langle S_{int}^2\rangle - \langle S_{int}\rangle^2\right) + \cdots$$

Feynman diagram notation to represent differrent terms.

$$S_{int}[\varphi_c,\varphi_>] = \frac{u}{4!}\int_{k_1\ldots k_4}\left[\varphi_c(\vec{k}_1)+\varphi_>(\vec{k}_1)\right)\cdots\left[\varphi_c(\vec{k}_4)+\varphi_>(\vec{k}_4)\right](2\pi)^d\,\delta\left(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4\right)$$

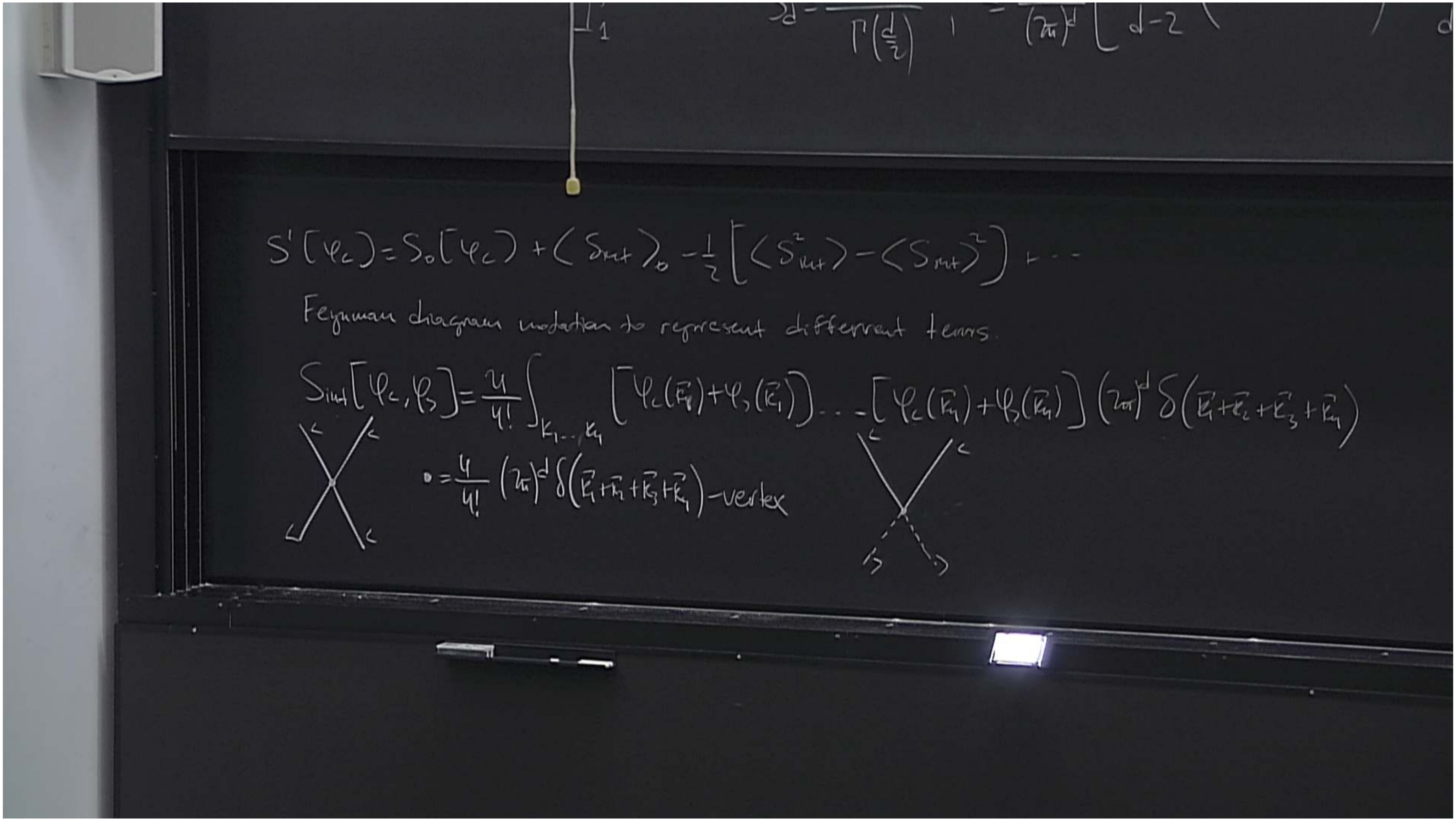


$$S'[\varphi_<] = S_0[\varphi_<] + \langle S_{int} \rangle_0 - \frac{1}{2}\left[\langle S^2_{int}\rangle - \langle S_{int}\rangle^2\right] + \cdots$$

Feynman diagram notation to represent different terms.

$$S_{int}[\varphi_<, \varphi_>] = \frac{u}{4!}\int_{k_1 \ldots k_4} \left[\varphi_<(\vec{k}_1) + \varphi_>(\vec{k}_1)\right] \ldots \left[\varphi_<(\vec{k}_4) + \varphi_>(\vec{k}_4)\right] (2\pi)^d \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3$$

$$\bullet = \frac{u}{4!}(2\pi)^d \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4) - \text{vertex}$$

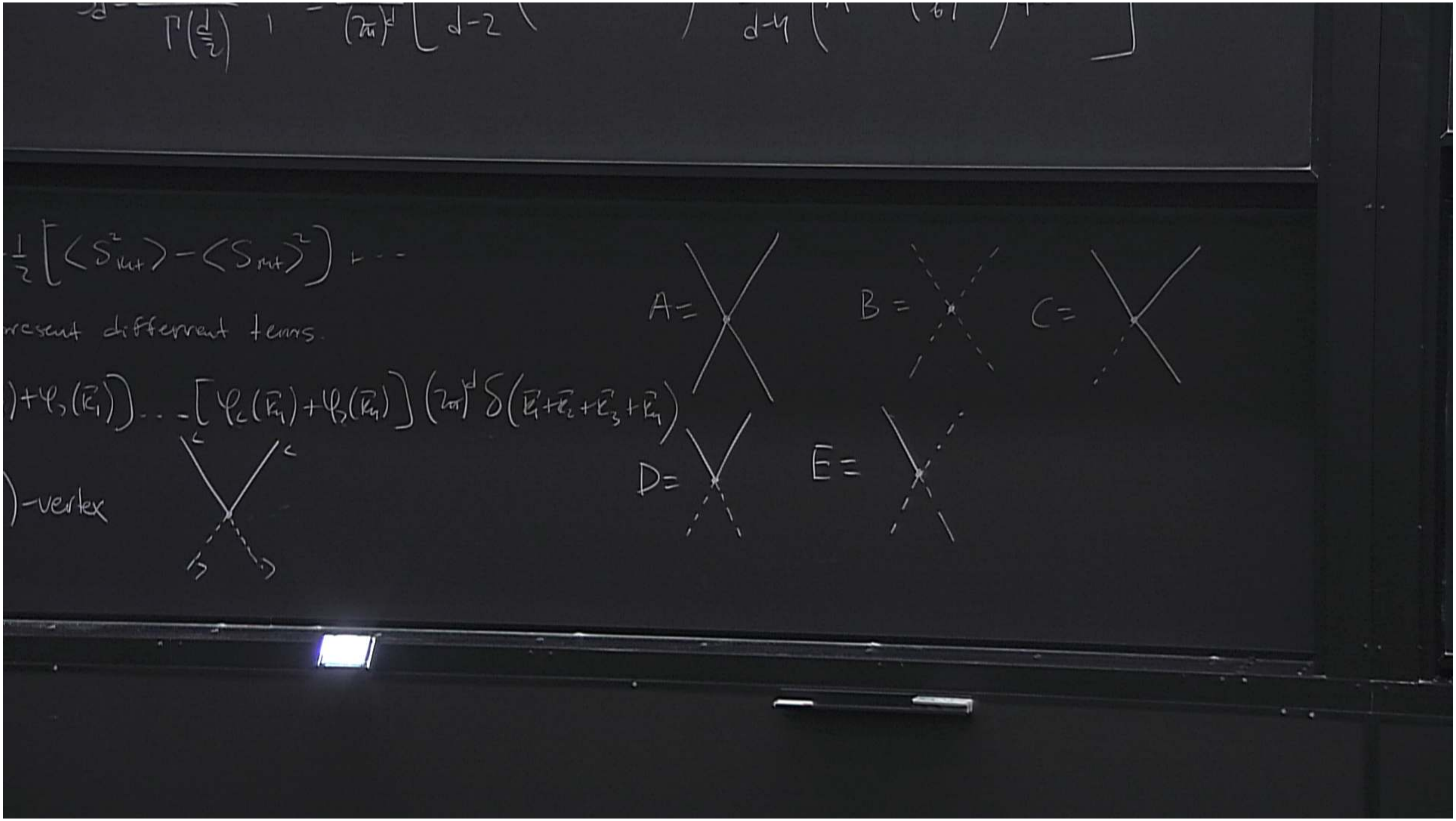


$$S'[\varphi_<] = S_0[\varphi_<] + \langle S_{int}\rangle_0 - \frac{1}{2}\Big[\langle S^2_{int}\rangle - \langle S_{int}\rangle^2\Big] + \cdots$$

Feynman diagram notation to represent differrent terms.

$$S_{int}[\varphi_<, \varphi_>] = \frac{u}{4!}\int_{k_1 \ldots k_4}\Big[\varphi_<(\vec{k}_1) + \varphi_>(\vec{k}_1)\Big] \ldots \Big[\varphi_<(\vec{k}_4) + \varphi_>(\vec{k}_4)\Big](2\pi)^d\,\delta\big(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4\big)$$

$$\bullet = \frac{u}{4!}(2\pi)^d\,\delta\big(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4\big) - \text{vertex}$$



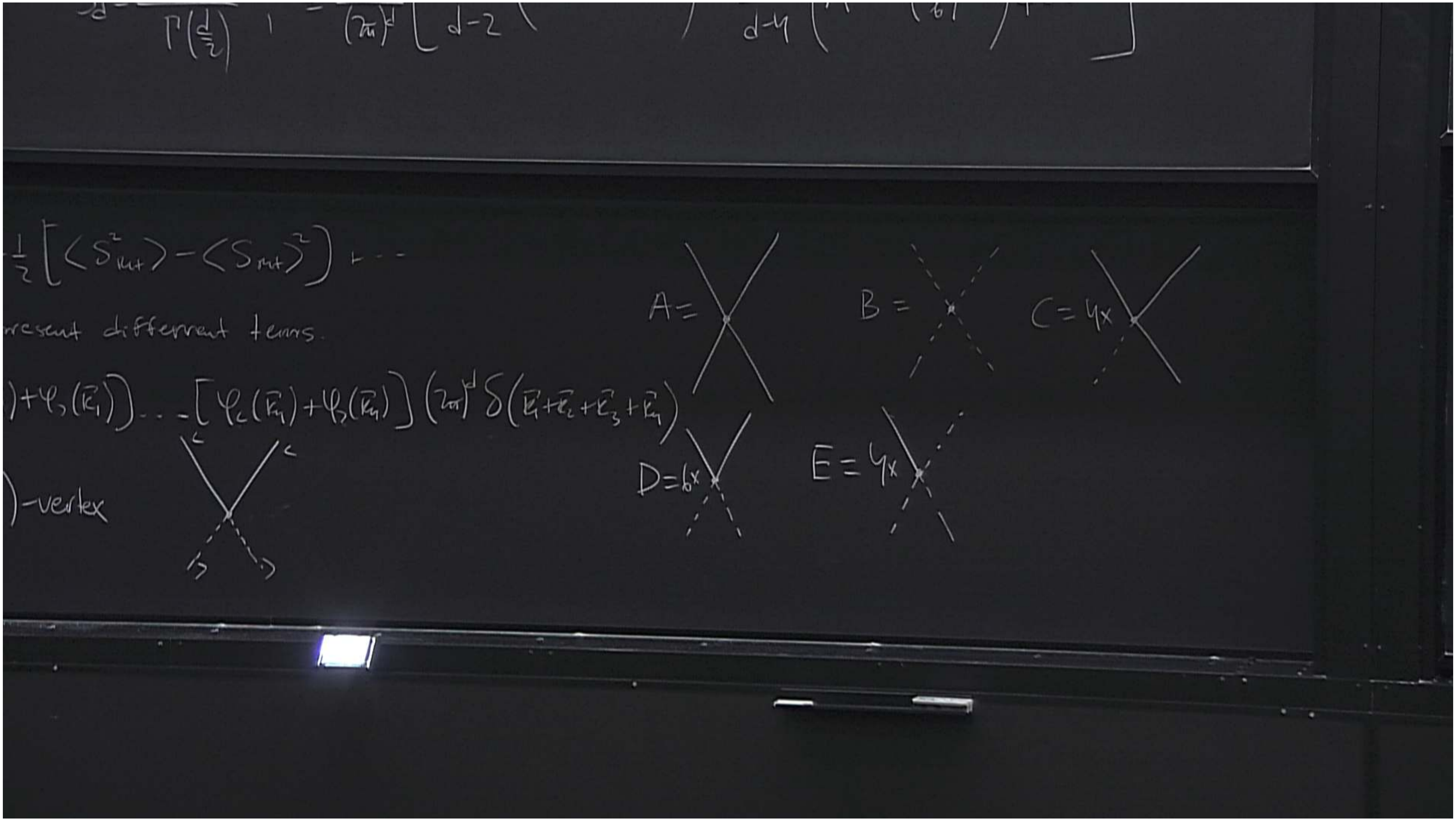
$\Gamma(\frac{d}{2})$ $(2\pi)^d\Big[\, d-2$ $d-4$

$\frac{1}{2}\left[\langle S_{int}^2\rangle - \langle S_{int}\rangle^2\right] + \cdots$

resent differrent terms.

$)+\varphi_>(\vec{k}_1)\big]\cdots\big[\varphi_<(\vec{k}_4)+\varphi_>(\vec{k}_4)\big]\,(2\pi)^d\,\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)$

)-vertex



$\Gamma\left(\frac{d}{2}\right)$ $(2\pi)^d \Big[\ d-2$ $d-4$

$\frac{1}{2}\Big[\langle S^2_{int}\rangle - \langle S_{int}\rangle^2\Big) + \cdots$

resent differrent terms.

$) + \varphi_>(\vec{k}_1)\Big]\cdots\Big[\varphi_<(\vec{k}_4) + \varphi_>(\vec{k}_4)\Big](2\pi)^d\,\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)$

)-vertex

A =

B =

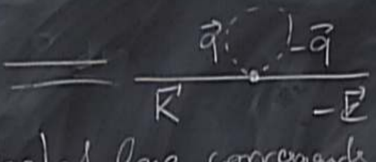
C = 4x

D = 6x

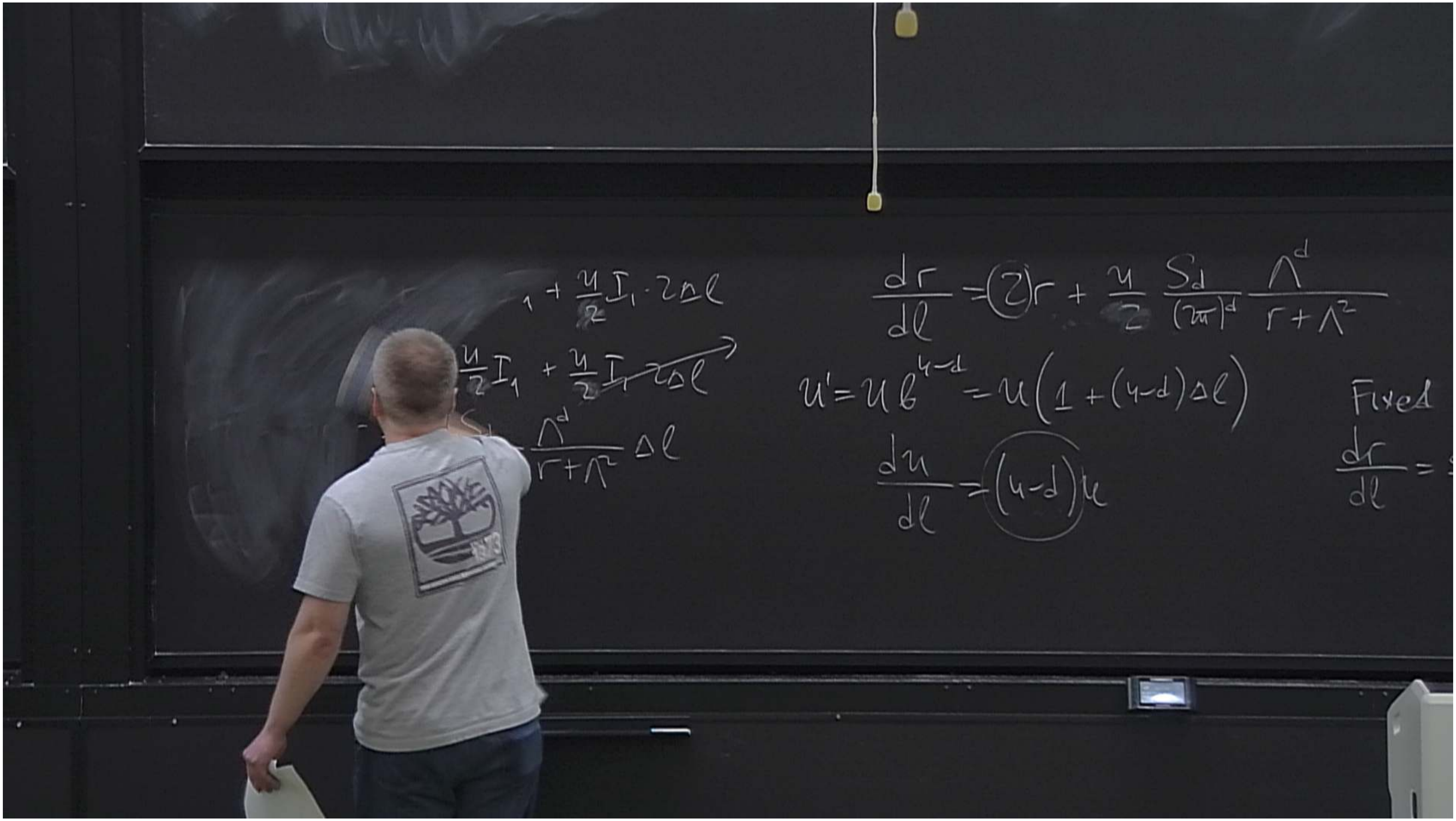
E = 4x

Denote averaging over fast field by connecting the dashed lines

$\langle D \rangle$



Each dashed line corresponds to $\int_{\Lambda/b}^{\Lambda} \frac{d^dq}{(2\pi)} G_0(\vec{q})$



$$1 + \frac{u}{2} I_1 \cdot 2\Delta l$$

$$\frac{u}{2} I_1 + \frac{u}{2} \cancel{I_1 \, 2\Delta l}$$

$$S_d \; \frac{\Lambda^d}{r+\Lambda^2} \Delta l$$

$$\frac{dr}{dl} = \textcircled{2} r + \frac{u}{2} \frac{S_d}{(2\pi)^d} \frac{\Lambda^d}{r+\Lambda^2}$$

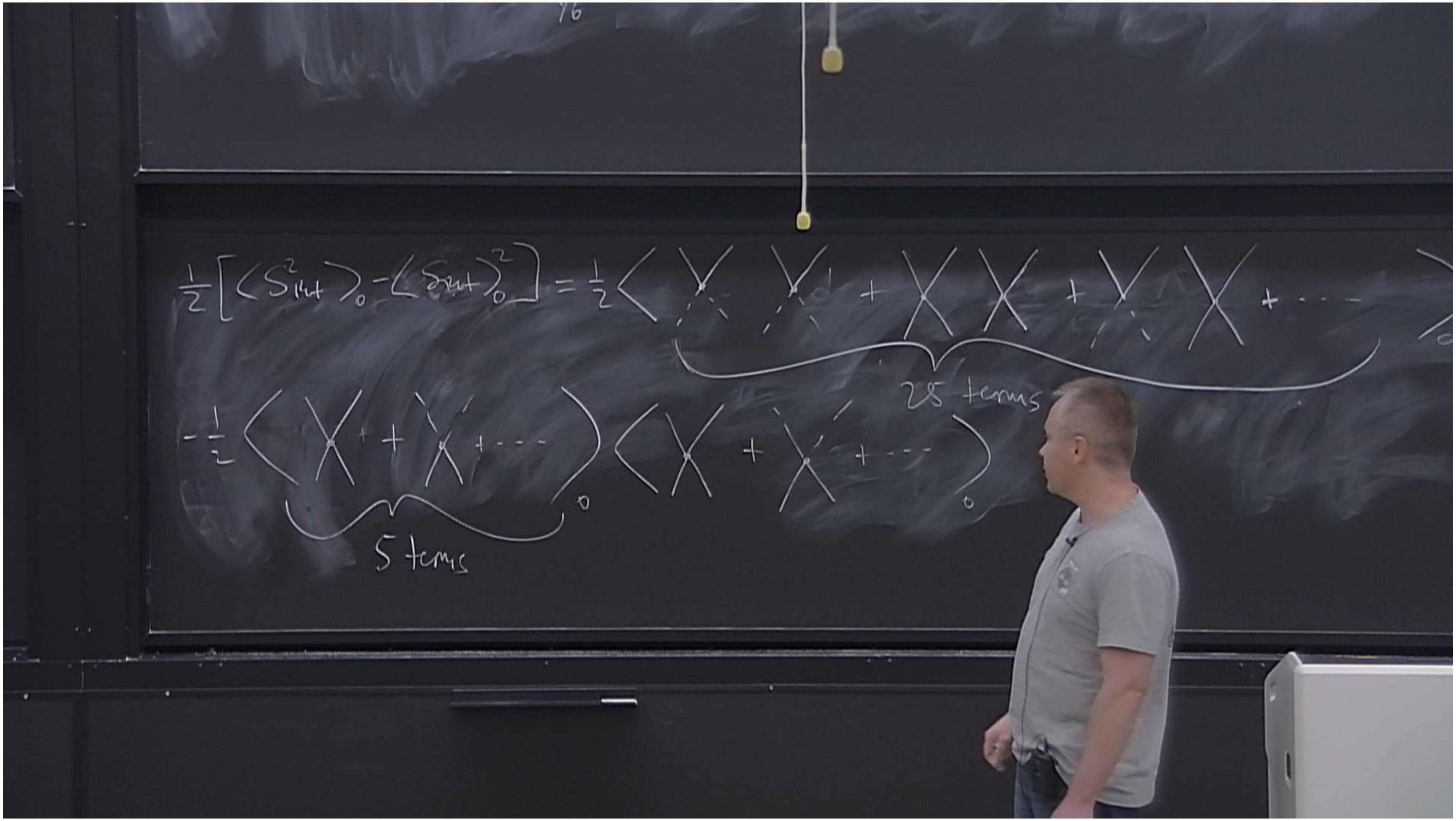
$$u' = u b^{4-d} = u\left(1 + (4-d)\Delta l\right)$$

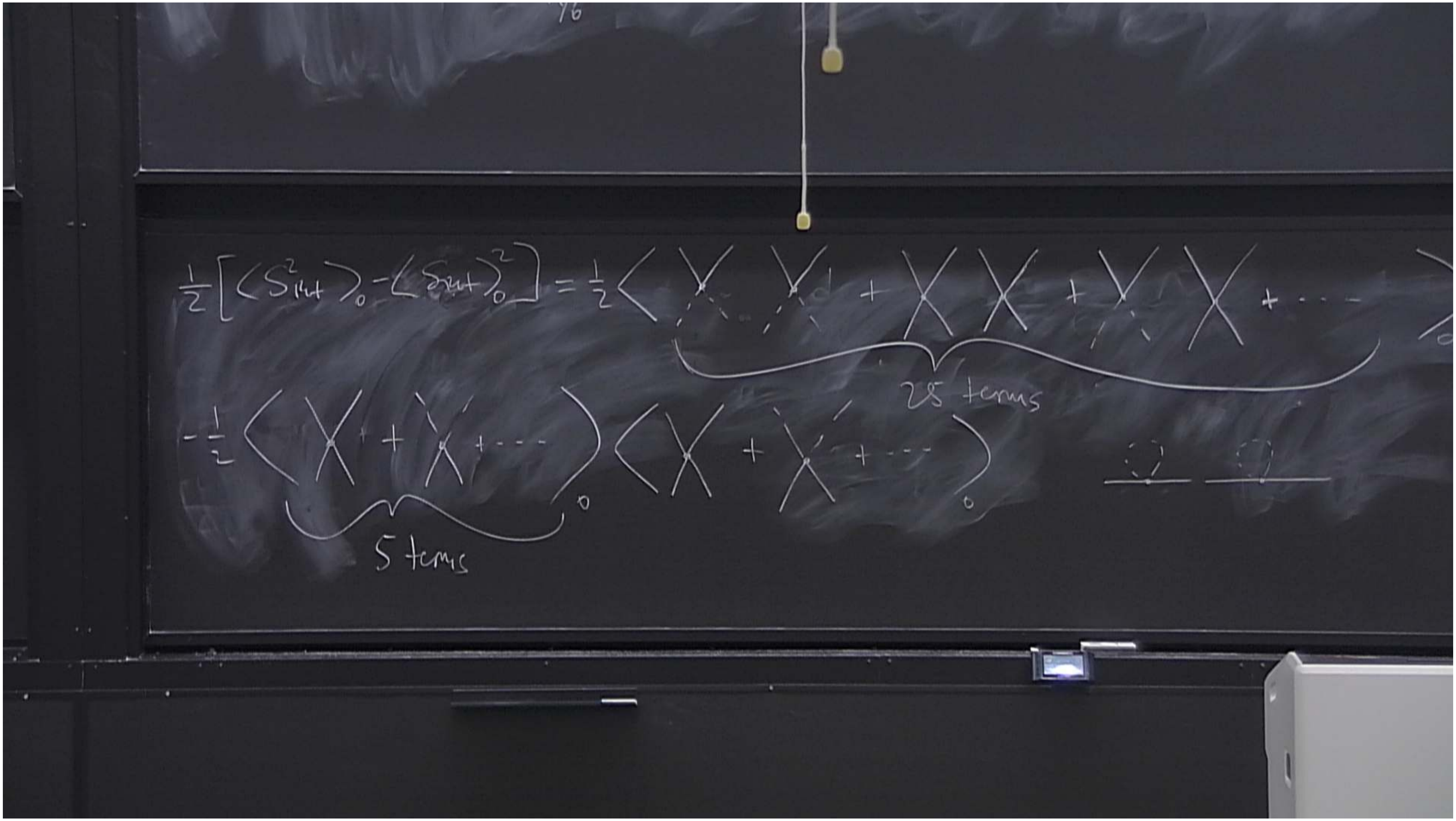
$$\frac{du}{dl} = \boxed{(4-d)} u$$

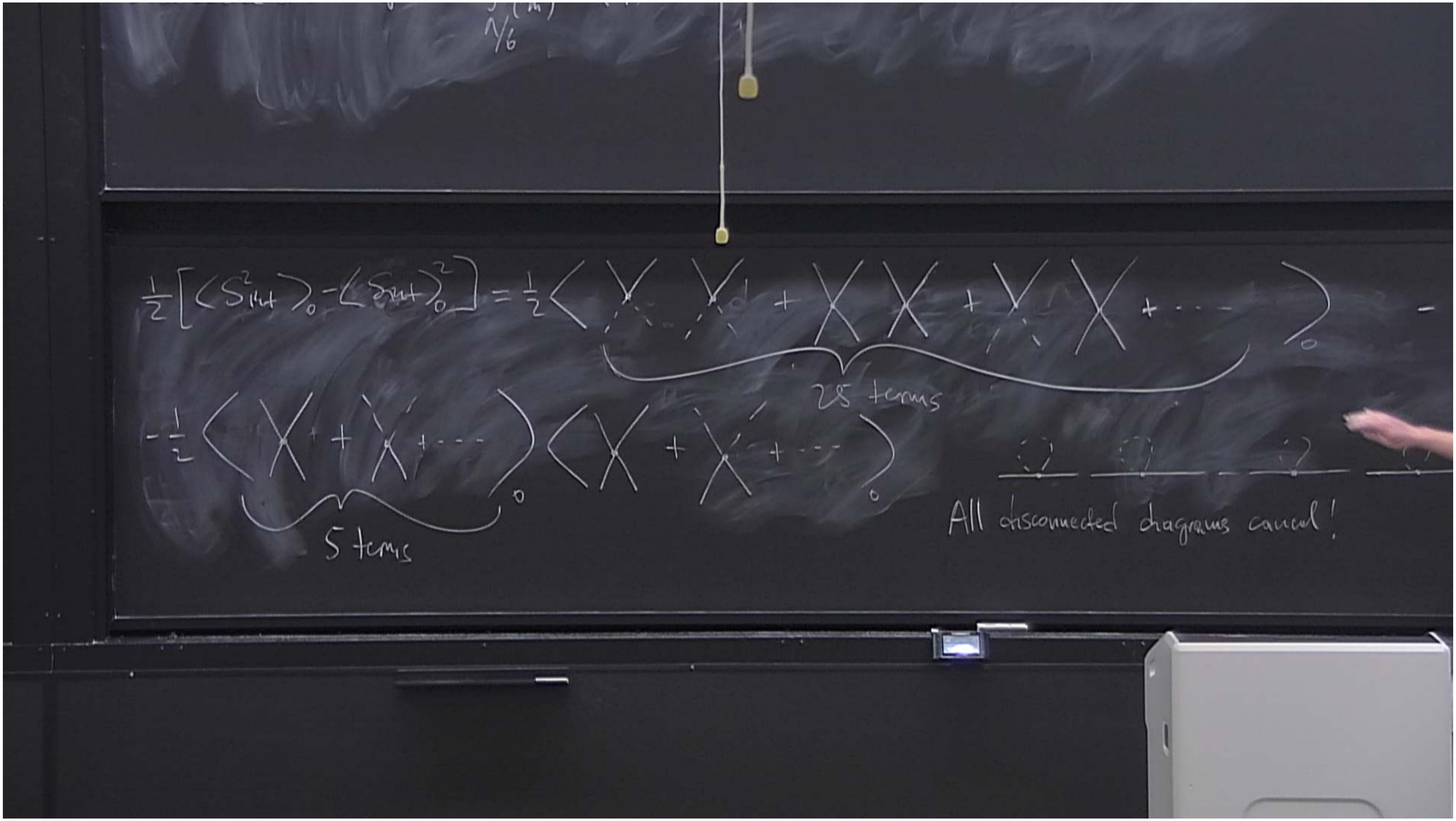
Fixed

$$\frac{dr}{dl} =$$

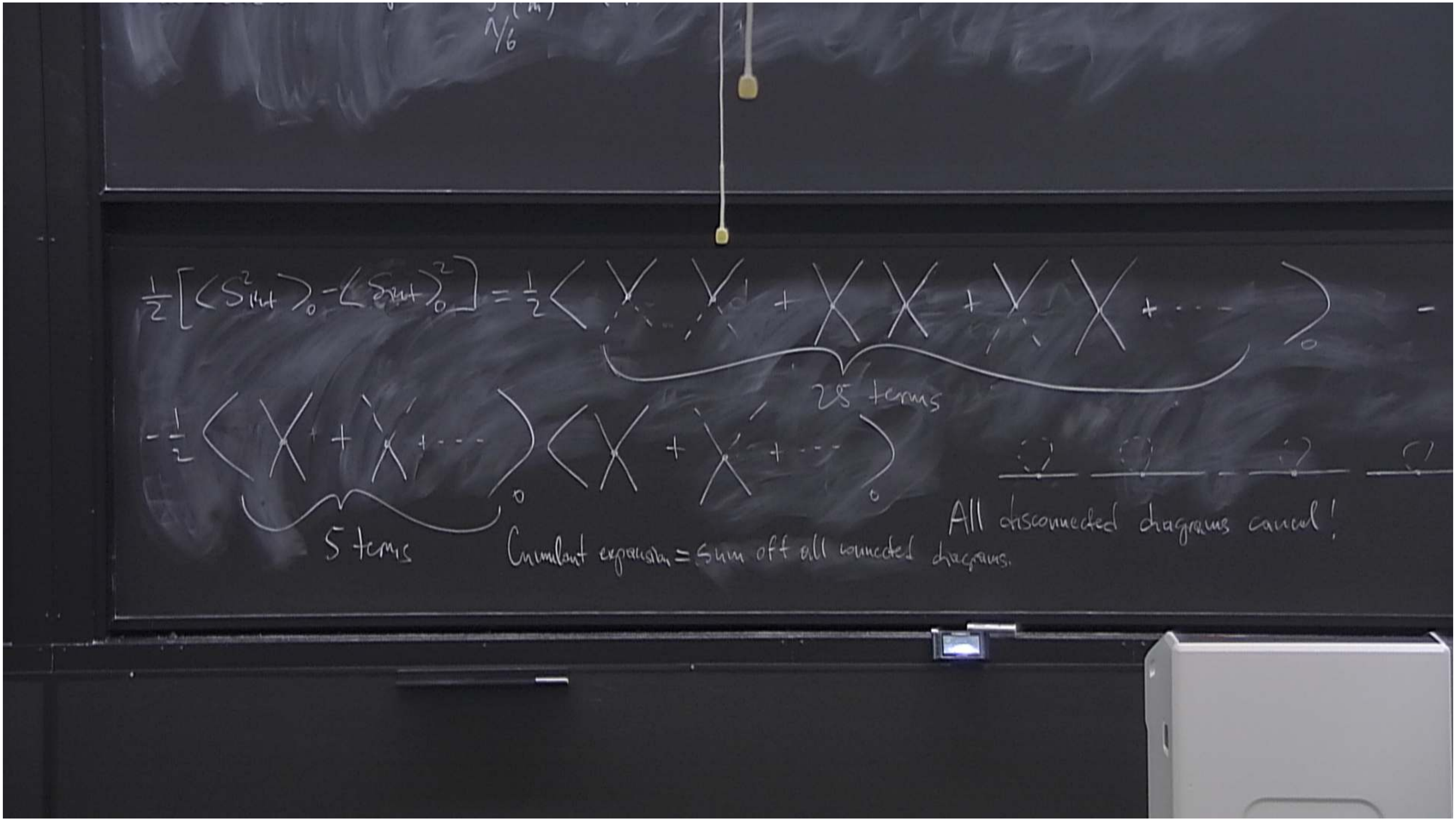
$$\frac{1}{2}\left[\langle S_{int}^2\rangle_0 - \langle S_{int}\rangle_0^2\right] = \frac{1}{2}\langle$$


$\frac{1}{2}\left[\langle S_{int}^2\rangle_0 - \langle S_{int}\rangle_0^2\right] = \frac{1}{2}\langle \; + \; + \; + \cdots \rangle_0$
25 terms
$-\frac{1}{2}\langle \; + \; + \cdots \rangle_0 \langle \; + \; + \cdots \rangle_0$
5 terms


96
$\frac{1}{2}\left[\langle S_{int}^2\rangle_0-\langle S_{int}\rangle_0^2\right]=\frac{1}{2}\langle$
$+$
$+$
$+ \cdots$
$\rangle_0$
25 terms
$-\frac{1}{2}\langle$
$+$
$+ \cdots$
$\rangle_0\langle$
$+$
$+ \cdots$
$\rangle_0$
5 terms



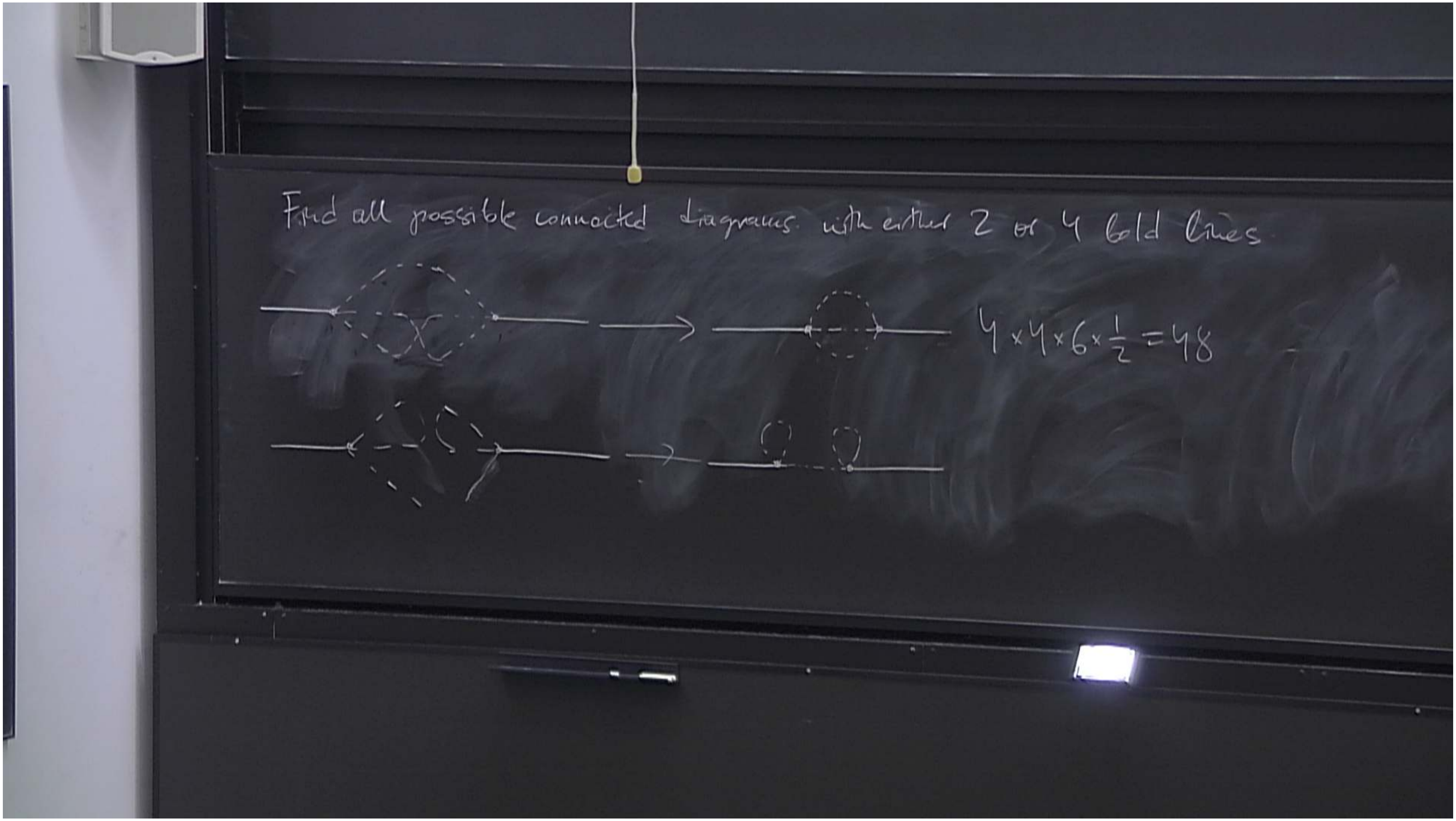
All disconnected diagrams cancel!



$\frac{1}{2}\left[\langle S_{int}^2\rangle_0 - \langle S_{int}\rangle_0^2\right] = \frac{1}{2}\langle \cdots \rangle_0 - \frac{1}{2}\langle \cdots \rangle_0 \langle \cdots \rangle_0$

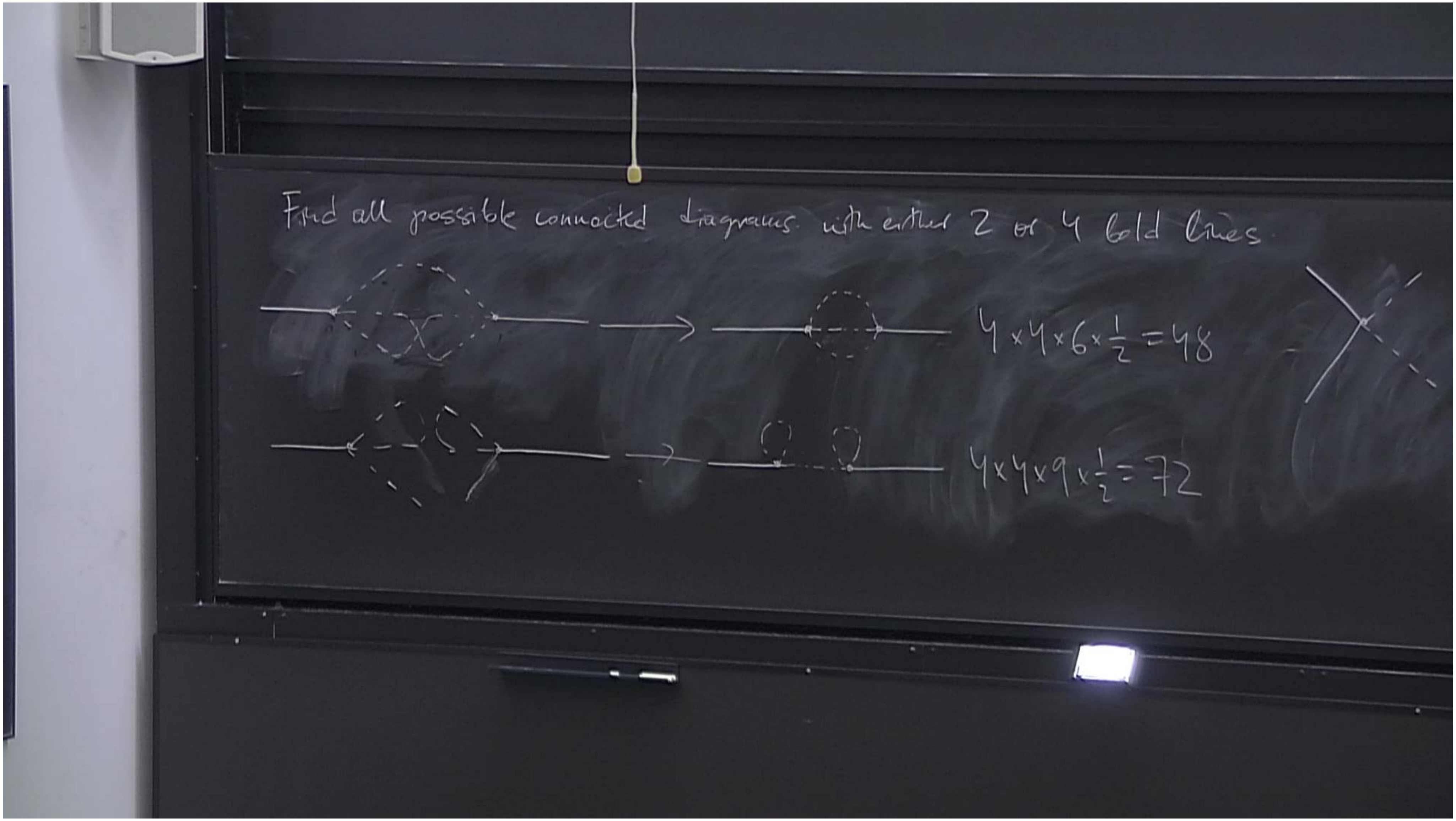
Cumulant expansion = Sum off all connected diagrams.

All disconnected diagrams cancel!



Find all possible connected diagrams with either 2 or 4 bold lines

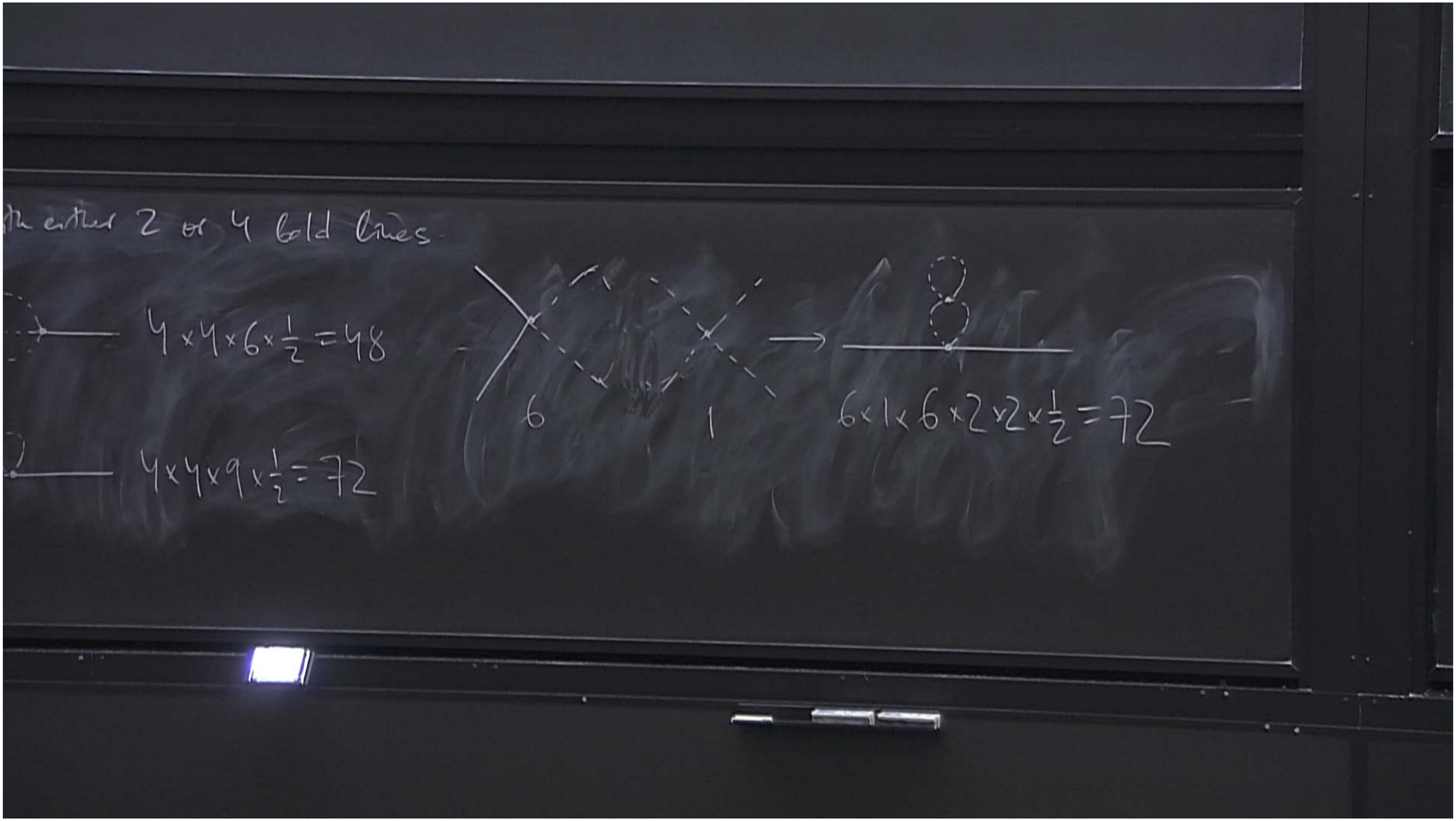
$4 \times 4 \times 6 \times \frac{1}{2} = 48$



Find all possible connected diagrams with either 2 or 4 bold lines.

$4 \times 4 \times 6 \times \frac{1}{2} = 48$

$4 \times 4 \times 9 \times \frac{1}{2} = 72$

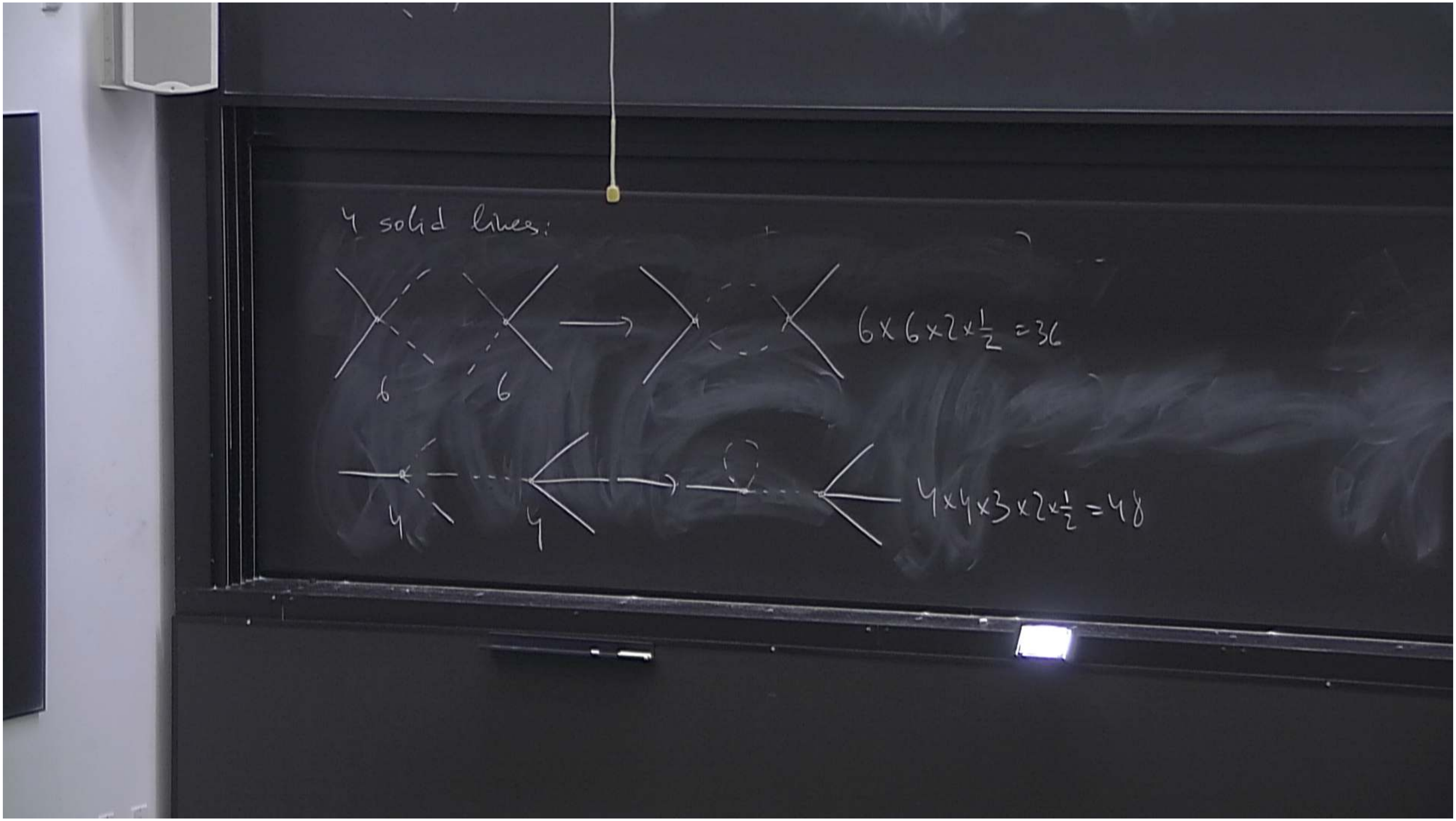


th either 2 or 4 bold lines.

$4 \times 4 \times 6 \times \frac{1}{2} = 48$

$4 \times 4 \times 9 \times \frac{1}{2} = 72$

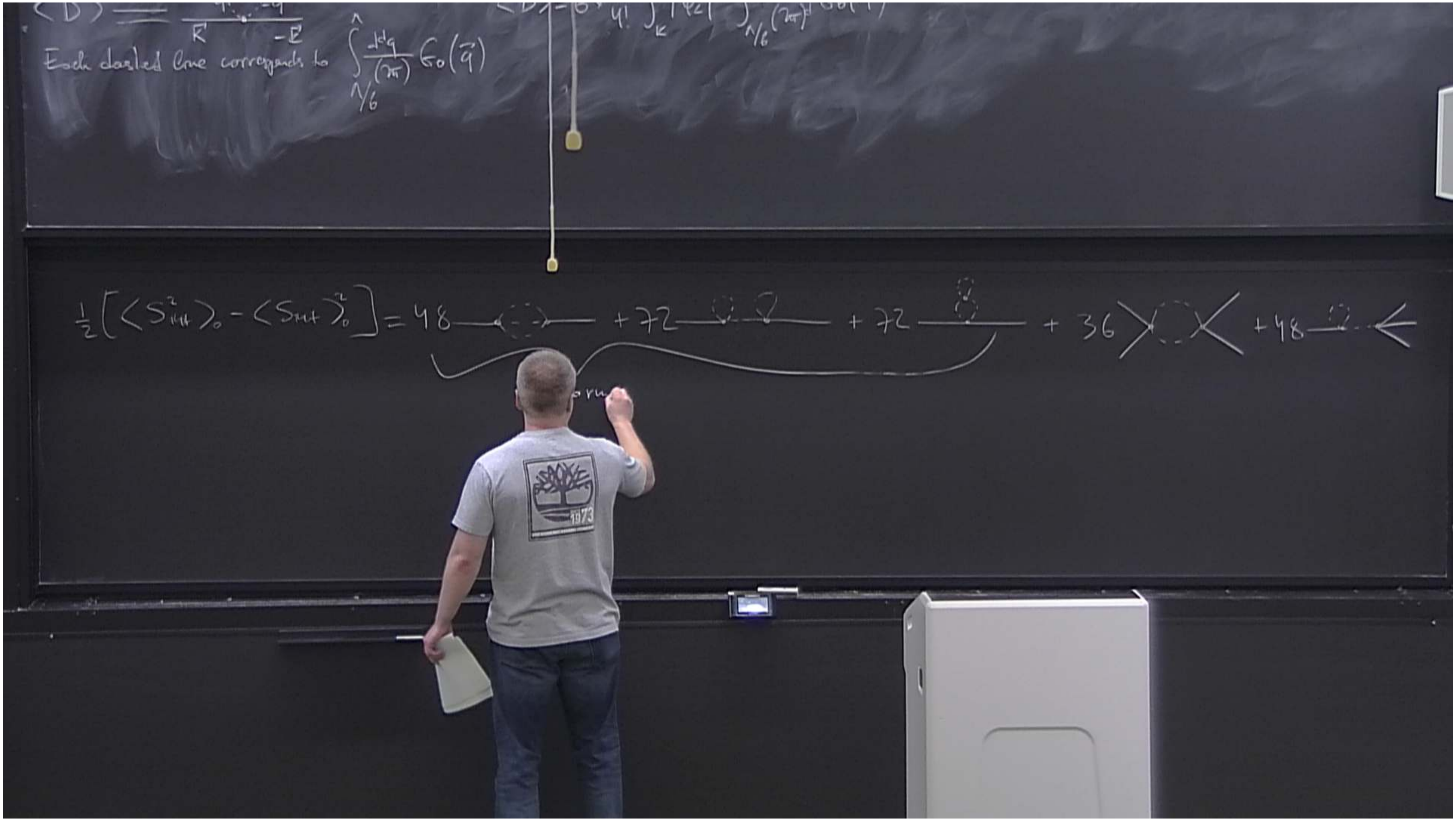
$6 \times 1 \times 6 \times 2 \times 2 \times \frac{1}{2} = 72$



4 solid lines:

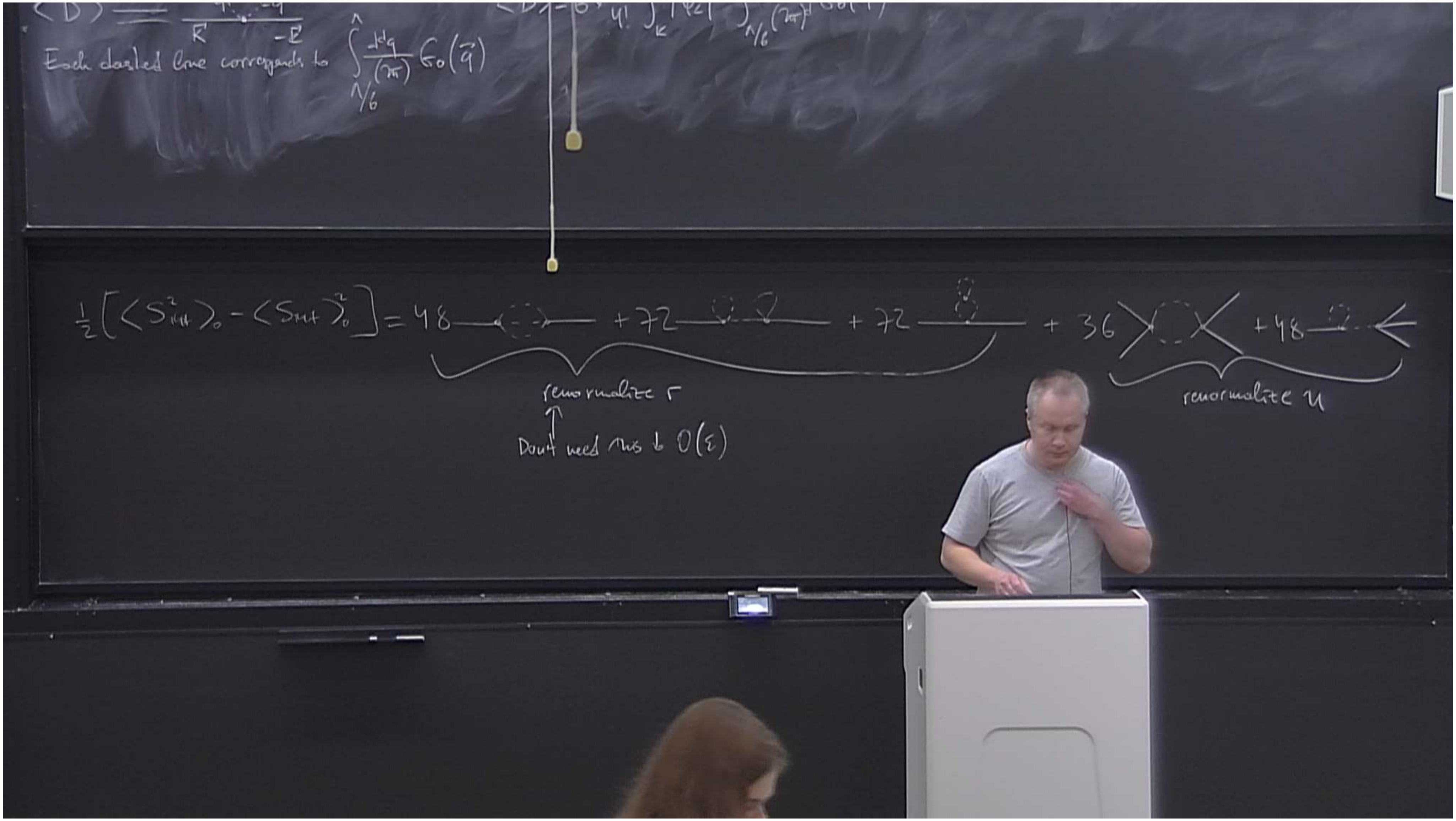
$6 \times 6 \times 2 \times \frac{1}{2} = 36$

$4 \times 4 \times 3 \times 2 \times \frac{1}{2} = 48$



Each dashed line corresponds to $\int_{\Lambda/6}^{\Lambda} \frac{d^dq}{(2\pi)^d} G_0(\vec{q})$

$\frac{1}{2}\left[\langle S_{int}^2\rangle_0 - \langle S_{int}\rangle_0^2\right] = 48 \qquad + 72 \qquad + 72 \qquad + 36 \qquad + 48$



Each dashed line corresponds to $\int_{\Lambda/b}^{\Lambda} \frac{d^dq}{(2\pi)^d} G_0(\vec{q})$

$\frac{1}{2}\left[\langle S_{int}^2\rangle_0 - \langle S_{int}\rangle_0^2\right] = 48 \quad + 72 \quad + 72 \quad + 36 \quad + 48$

renormalize r

Don't need this to $O(\varepsilon)$

renormalize u