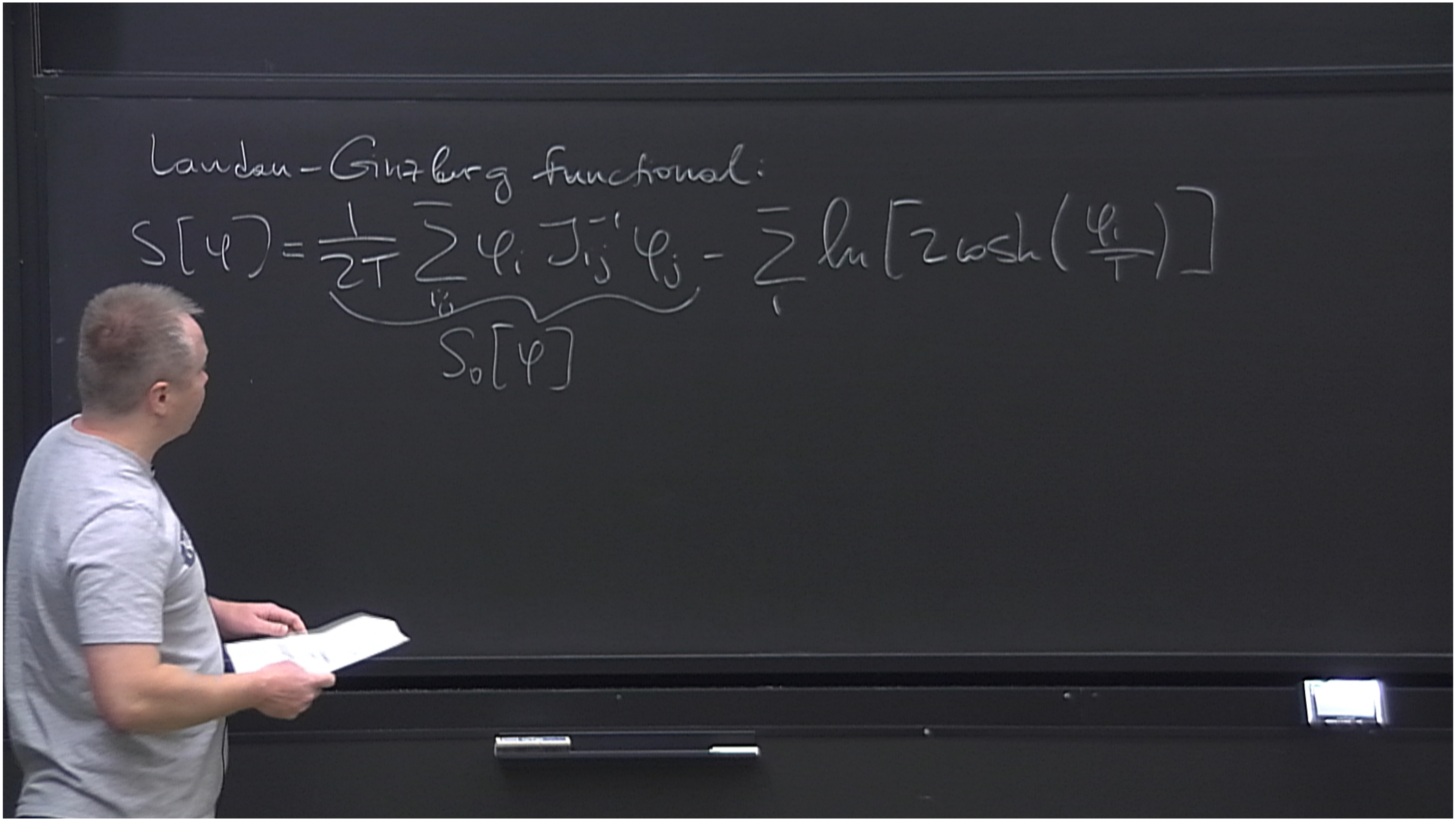
Title: 14/15 PSI - Statistical Mechanics - Lecture 6

Date: Oct 14, 2014 10:45 AM

URL: http://pirsa.org/14100092

Abstract:


Landau-Ginzburg functional:
$S[\varphi] = \underbrace{\frac{1}{2T}\sum_{ij}\varphi_i J^{-1}_{ij}\varphi_j}_{S_0[\varphi]} - \sum_i \ln\left[2\cosh\left(\frac{\varphi_i}{T}\right)\right]$

Landau-Ginzburg functional:

$$S[\varphi] = \frac{1}{2T}\sum_{ij} \varphi_i J^{-1}_{ij} \varphi_j - \sum_i \ln\left[2\cosh\left(\frac{\varphi_i}{T}\right)\right] \simeq \frac{1}{2T}\sum_{ij}$$

$$= \frac{1}{2T} \sum_{ij} \varphi_i J^{-1}_{ij} \varphi_j - \frac{1}{2} \sum_i \left( \frac{\varphi_i}{T} \right)^2 + \frac{1}{12} \sum_i \left( \frac{\varphi_i}{T} \right)^4 + \cdots$$

Landau-Ginzburg functional:

$$S[\varphi] = \frac{1}{2T}\sum_{ij} \varphi_i J^{-1}_{ij} \varphi_j - \sum_i \ln\left[2\cosh\left(\frac{\varphi_i}{T}\right)\right] \simeq \underbrace{\frac{1}{2T}\sum_{ij} \varphi_i J^{-1}_{ij} \varphi_j - \frac{1}{2}\sum_i \left(\frac{\varphi_i}{T}\right)^2}_{S_0[\varphi]} + \frac{1}{12}\sum_i \left(\frac{\varphi_i}{T}\right)^4 + \cdots$$

Landau-Ginzburg functional:

$$S[\varphi] = \frac{1}{2T}\sum_{ij} \varphi_i J_{ij}^{-1} \varphi_j - \sum_i \ln\left[2\cosh\left(\frac{\varphi_i}{T}\right)\right] \simeq \frac{1}{2T}\sum_{ij}$$

$$\varphi_i = \frac{1}{N}\sum_q \varphi(\vec{q})\, e^{i\vec{q}\cdot\vec{r}_i}$$

$$J_{ij} = \frac{1}{N}\sum_{ij} J(\vec{q})\, e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)} \quad ; \quad S_0[\varphi] = \frac{1}{2T^2N}\sum_q \varphi(-\vec{q})$$

$$\underbrace{\frac{1}{2T}\sum_{ij}\varphi_i J^{-1}_{ij}\varphi_j - \frac{1}{2}\sum_i\left(\frac{\varphi_i}{T}\right)^2}_{S_0[\varphi]} + \frac{1}{12}\sum_i\left(\frac{\varphi_i}{T}\right)^4 + \cdots$$

$$\varphi(-\vec{q})\left[\frac{T}{J(\vec{q})} - 1\right]\varphi(\vec{q})$$

$$= \underbrace{\frac{1}{2T}\sum_{ij} \varphi_i J^{-1}_{ij} \varphi_j - \frac{1}{2}\sum_i \left(\frac{\varphi_i}{T}\right)^2}_{S_0[\varphi]} + \frac{1}{12}\sum_i \left(\frac{\varphi_i}{T}\right)^4 + \cdots$$

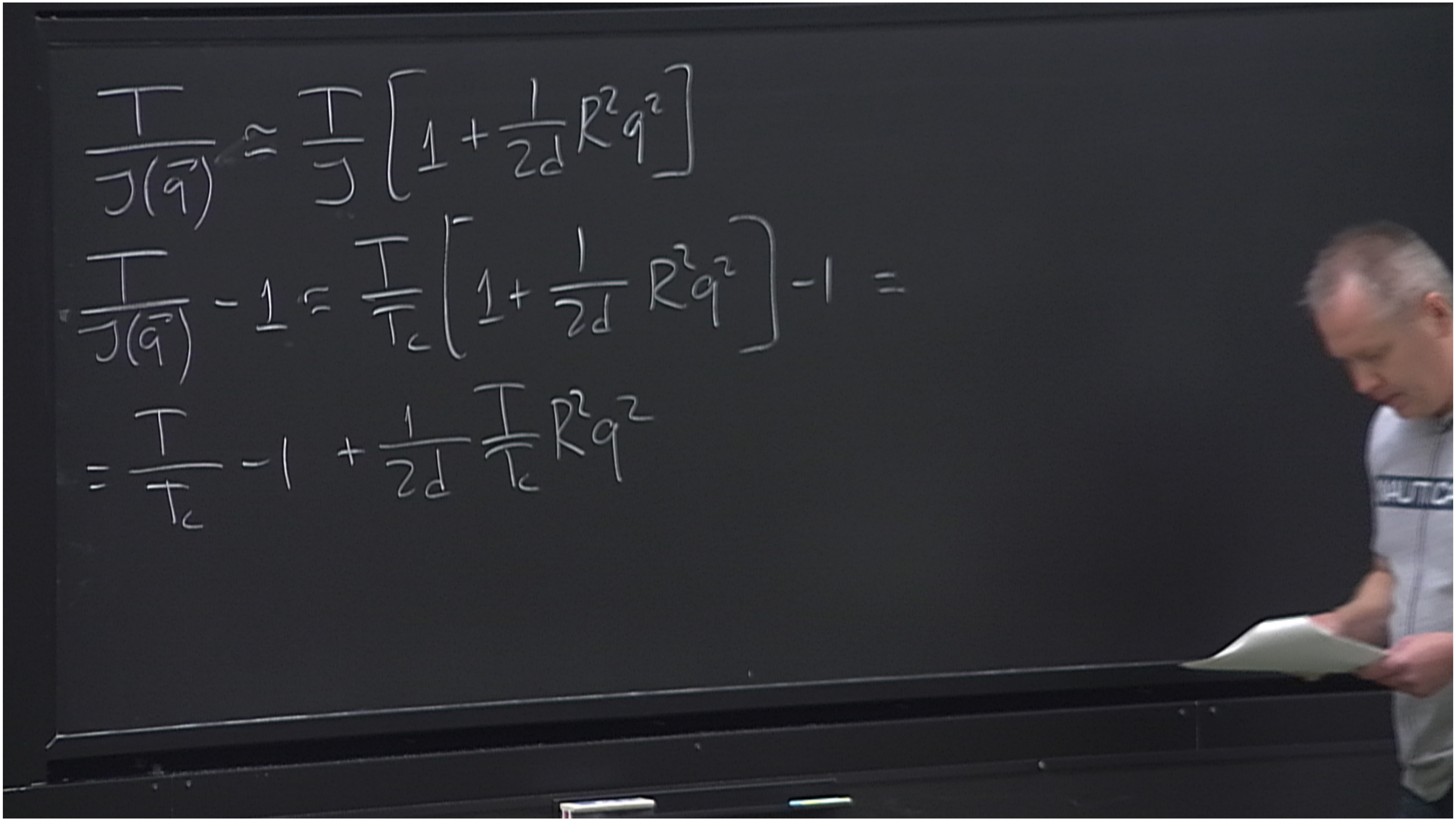
$$\varphi(-\vec{q})\left[\frac{T}{J(\vec{q})} - 1\right]\varphi(\vec{q}) \quad ; \quad J(\vec{q}) \simeq J - \frac{1}{2} K q^2 = J\left[1 - \frac{K}{2J} q^2\right]$$

$$= \underbrace{\frac{1}{2T}\sum_{ij}\varphi_i J^{-1}_{ij}\varphi_j - \frac{1}{2}\sum_i\left(\frac{\varphi_i}{T}\right)^2}_{S_0[\varphi]} + \frac{1}{12}\sum_i\left(\frac{\varphi_i}{T}\right)^4 + \dots$$

$$\varphi(-\vec{q})\left[\frac{T}{J(\vec{q})} - 1\right]\varphi(\vec{q}) \quad ; \quad J(\vec{q}) \simeq J - \frac{1}{2}Kq^2 = J\left[1 - \frac{K}{2J}q^2\right] =$$

$$= J\left[1 - \frac{1}{2d}R^2 q^2\right]$$

$$\frac{T}{J(\vec{q})} \simeq \frac{T}{J}\left[1 + \frac{1}{2d}R^2 q^2\right]$$


$\frac{T}{J(\vec{q})} \simeq \frac{T}{J}\left[1 + \frac{1}{2d}R^2 q^2\right]$
$\frac{T}{J(\vec{q})} - 1 \simeq \frac{T}{T_c}\left[1 + \frac{1}{2d}R^2 q^2\right] - 1 =$
$= \frac{T}{T_c} - 1 + \frac{1}{2d}\frac{T}{T_c}R^2 q^2$

$$\frac{T}{J(\vec{q})} \simeq \frac{T}{J}\left[1+\frac{1}{2d}R^2q^2\right]$$

$$\frac{T}{J(\vec{q})} - 1 \simeq \frac{T}{T_c}\left[1+\frac{1}{2d}R^2q^2\right]-1 =$$

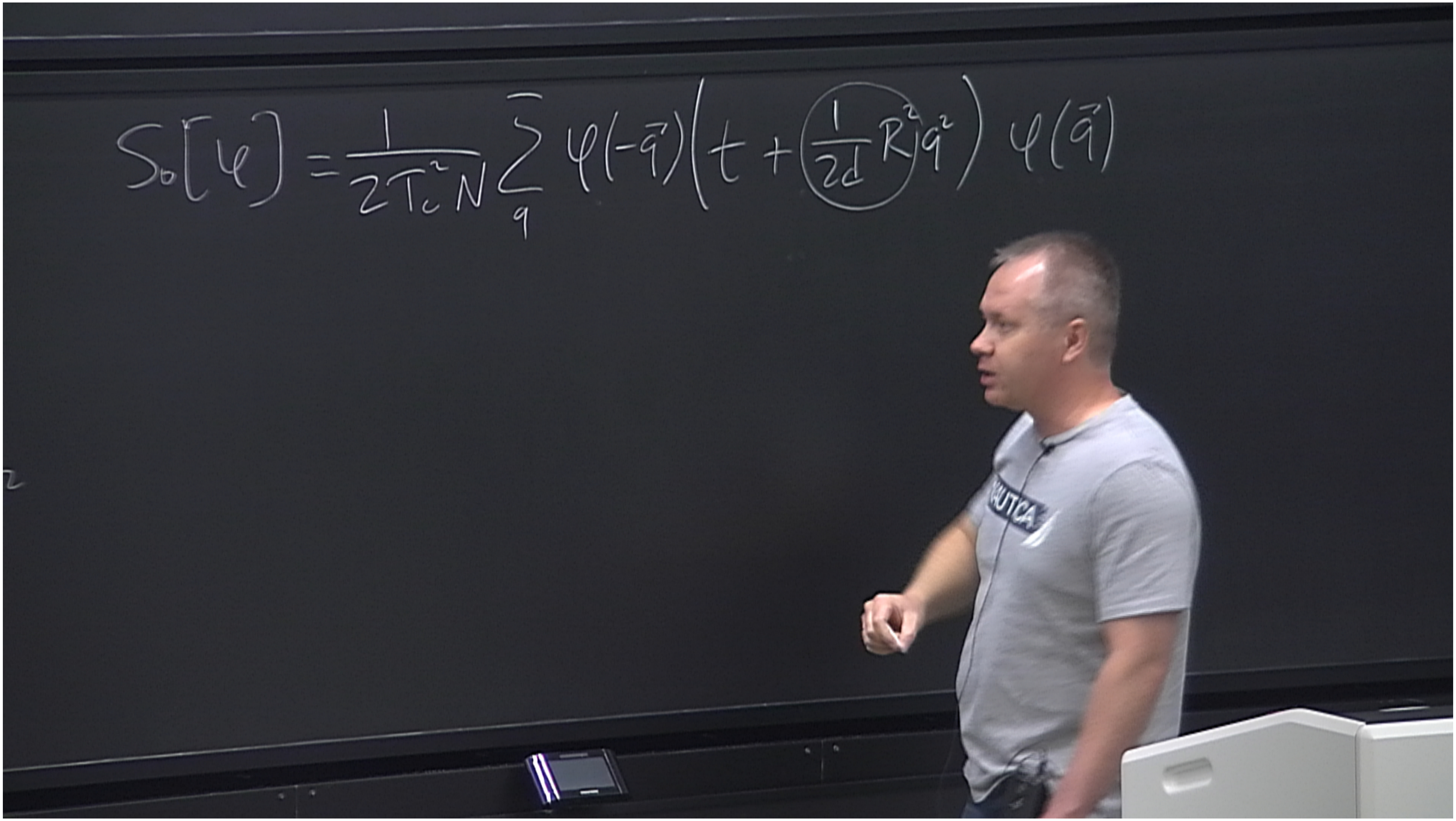
$$= \frac{T}{T_c} - 1 + \frac{1}{2d}\frac{T}{T_c}R^2q^2$$

$$\frac{T}{J(\vec{q})} \simeq \frac{T}{J}\left[1 + \frac{1}{2d}R^2q^2\right]$$

$$\frac{T}{J(\vec{q})} - 1 \simeq \frac{T}{T_c}\left[1 + \frac{1}{2d}R^2q^2\right] - 1 =$$

$$= \underbrace{\left(\frac{T}{T_c} - 1\right)}_{t} + \frac{1}{2d}\frac{T}{T_c}R^2q^2 \simeq t + \frac{1}{2d}R^2q^2$$

$$S_0[\varphi] = \frac{1}{2T_c^2 N}\sum_{q} \varphi(-\vec{q})\left(t + \frac{1}{2d}R^2 q^2\right)\varphi(\vec{q})$$


$S_0[\varphi] = \frac{1}{2T_c^2 N} \sum_q \varphi(-\vec{q}) \left( t + \frac{1}{2d} R^2 q^2 \right) \varphi(\vec{q})$

$$S_0[\varphi] = \frac{1}{2T_c^2 N}\sum_q \varphi(-\vec{q})\left(t + \frac{1}{2d}R^2 q^2\right)\varphi(\vec{q})$$

Introduce $\tilde{\varphi}(\vec{q}) = \dfrac{R a^{d/2}}{\sqrt{2d}\;T_c}$

$$S_0[\tilde{\varphi}] = \frac{1}{2Na^d}\sum_q \tilde{\varphi}(-\vec{q})\left(\frac{2dt}{R^2} + q^2\right)\varphi(\vec{q})$$

$$S_0[\varphi] = \frac{1}{2T_c^2 N}\sum_q \varphi(-\vec{q})\left(t + \frac{1}{2d}R^2 q^2\right)\varphi(\vec{q})$$

$$\frac{1}{V}\sum_q$$

Introduce $\tilde{\varphi}(\vec{q}) = \dfrac{R a^{d/2}}{\sqrt{2d}\, T_c}$

$$S_0[\tilde{\varphi}] = \frac{1}{2Na^d}\sum_q \tilde{\varphi}(-\vec{q})\left(\underbrace{\frac{2dt}{R^2}}_{r} + q^2\right)\varphi(\vec{q})$$

$r>0,\ T>T_c$ ; $r<0,\ T<T_c$

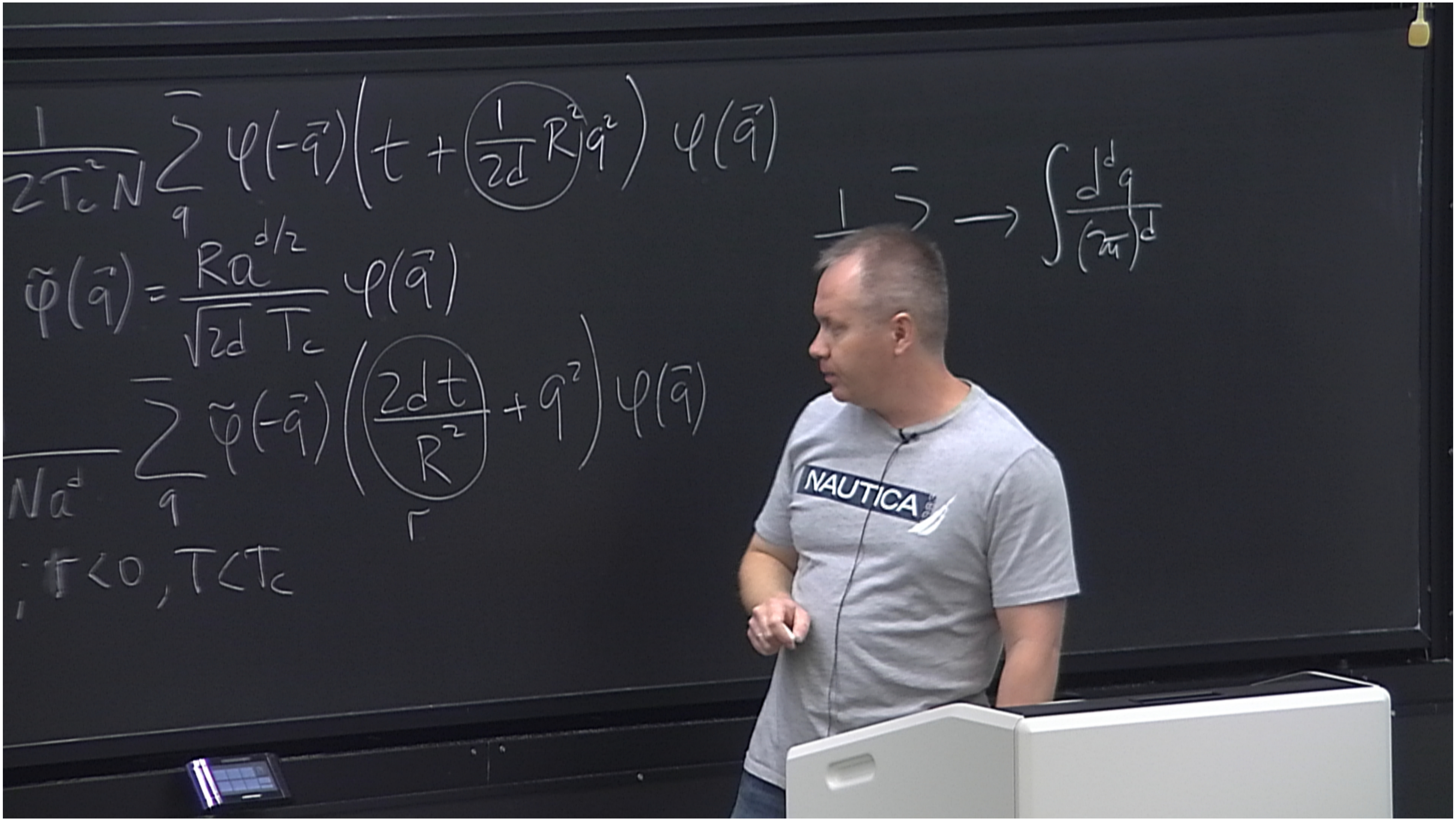
$$\frac{1}{2T_c^2 N}\sum_{q} \varphi(-\vec{q})\left(t + \frac{1}{2d}R^2 q^2\right)\varphi(\vec{q})$$

$$\tilde{\varphi}(\vec{q}) = \frac{R a^{d/2}}{\sqrt{2d}\, T_c}$$

$$\frac{\quad}{N a^d}\sum_{q} \tilde{\varphi}(-\vec{q})\left(\underbrace{\frac{2dt}{R^2}}_{r} + q^2\right)\varphi(\vec{q})$$

$; \; t<0, \; T<T_c$

$$\frac{1}{V}\sum_{q} \longrightarrow \int \frac{d^d q}{(2\pi)^d}$$


$\frac{1}{2T_c^2 N} \sum_q \varphi(-\vec{q})\left(t + \frac{1}{2d} R^2 q^2\right) \varphi(\vec{q})$
$\tilde{\varphi}(\vec{q}) = \frac{R a^{d/2}}{\sqrt{2d}\, T_c} \varphi(\vec{q})$
$\sum_q \tilde{\varphi}(-\vec{q})\left(\frac{2dt}{R^2} + q^2\right) \varphi(\vec{q})$
$N a^d$
$r$
$t < 0,\ T < T_c$
$\sum \longrightarrow \int \frac{d^d q}{(2\pi)^d}$
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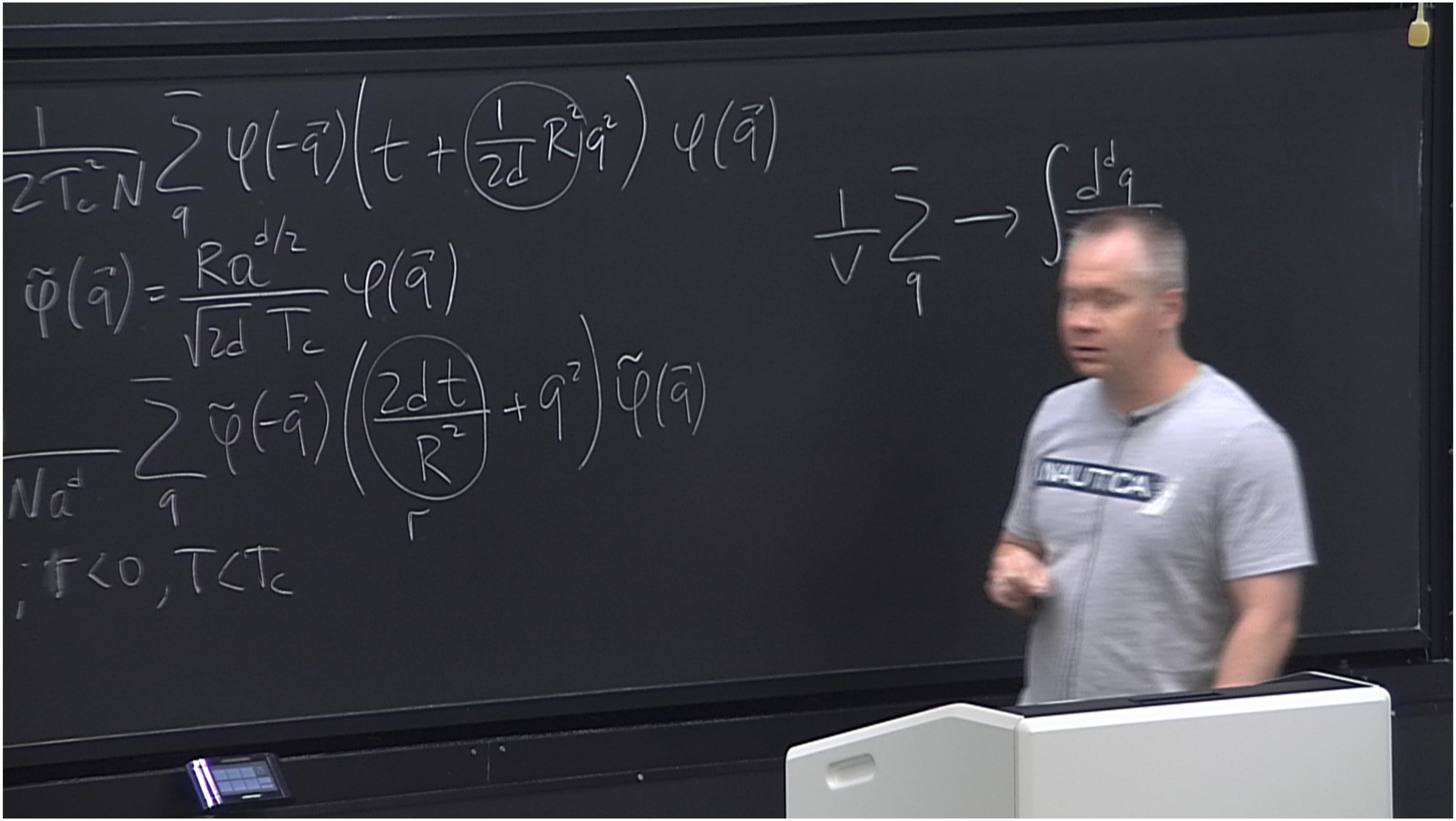
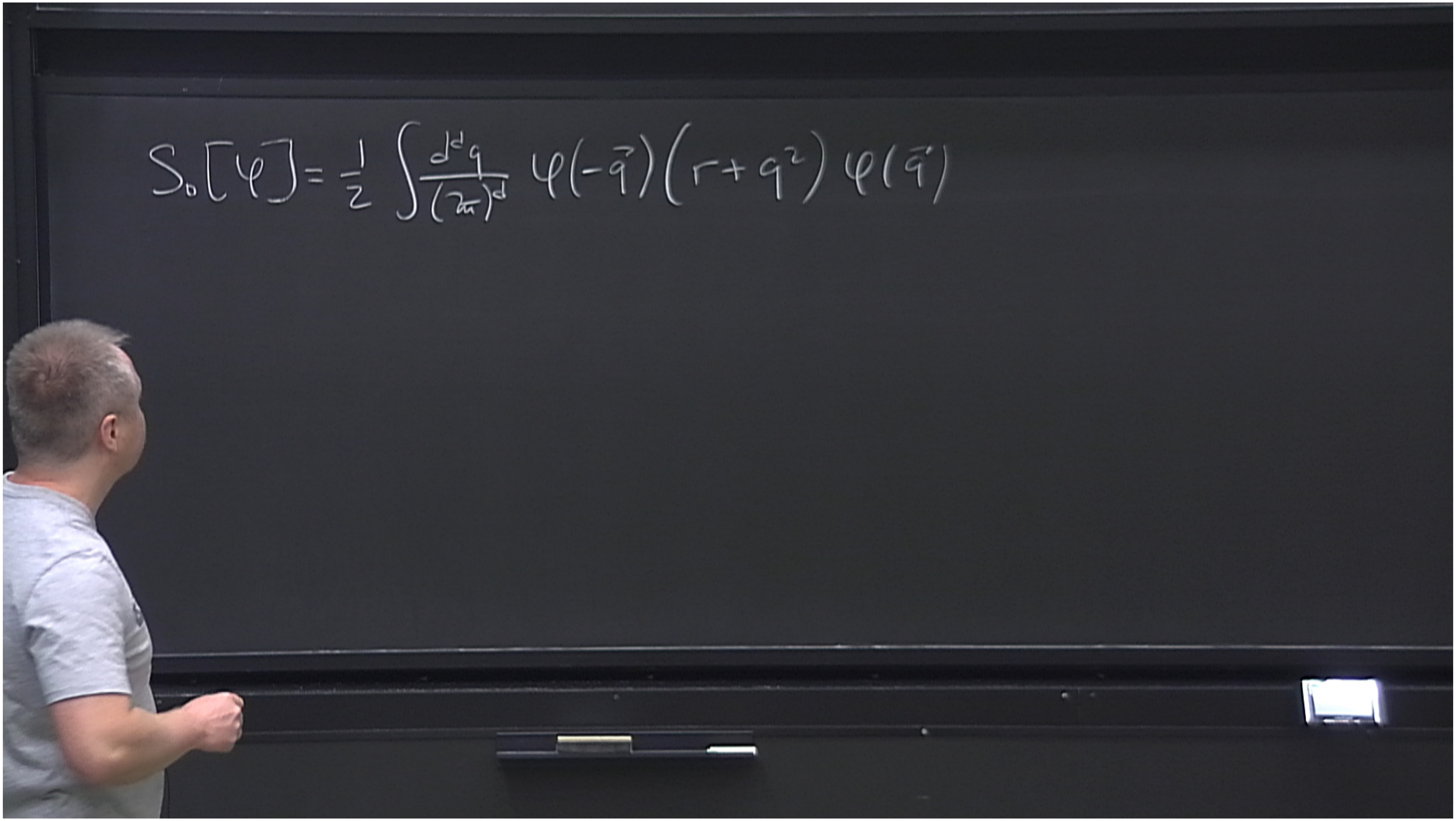
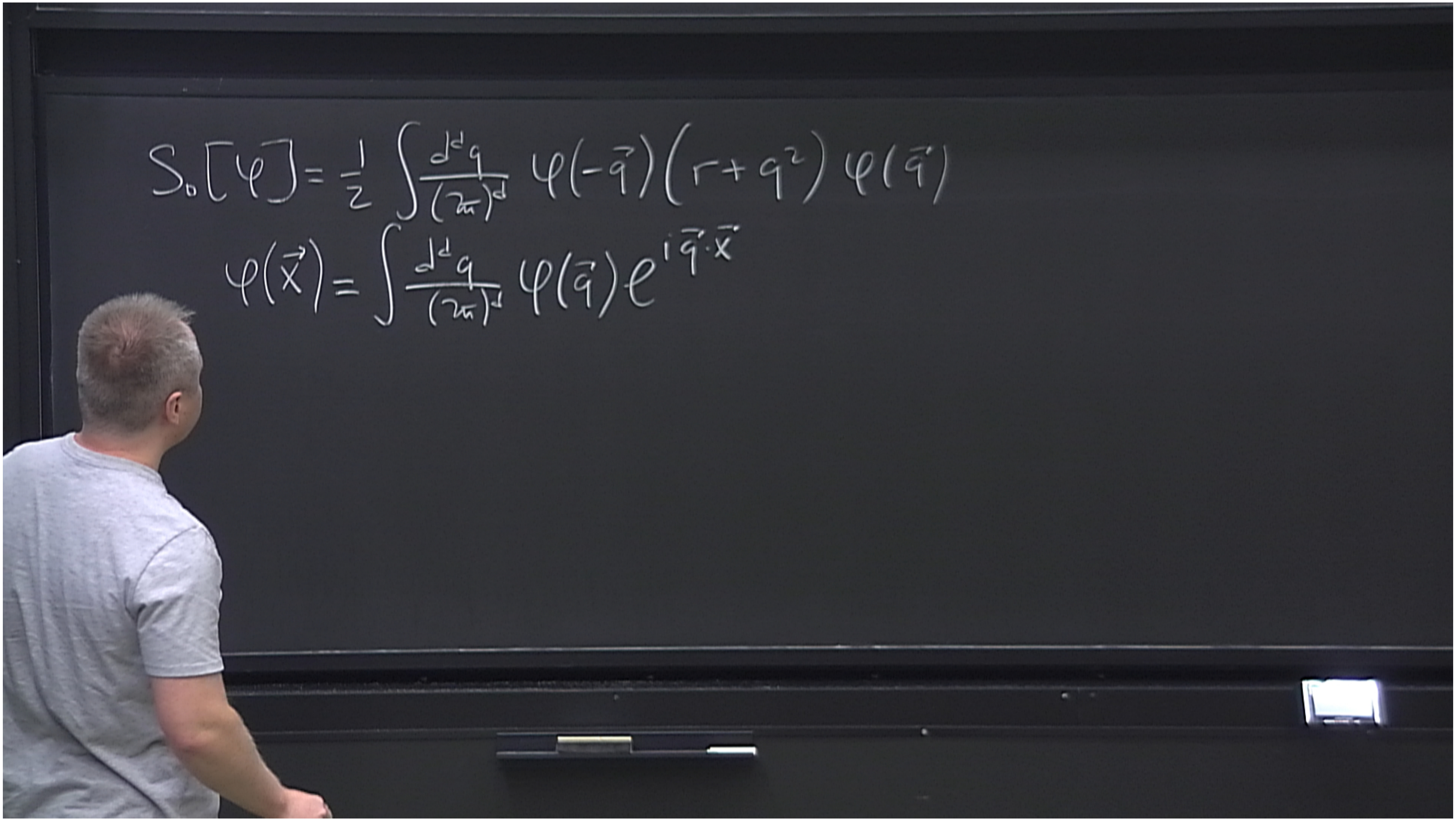
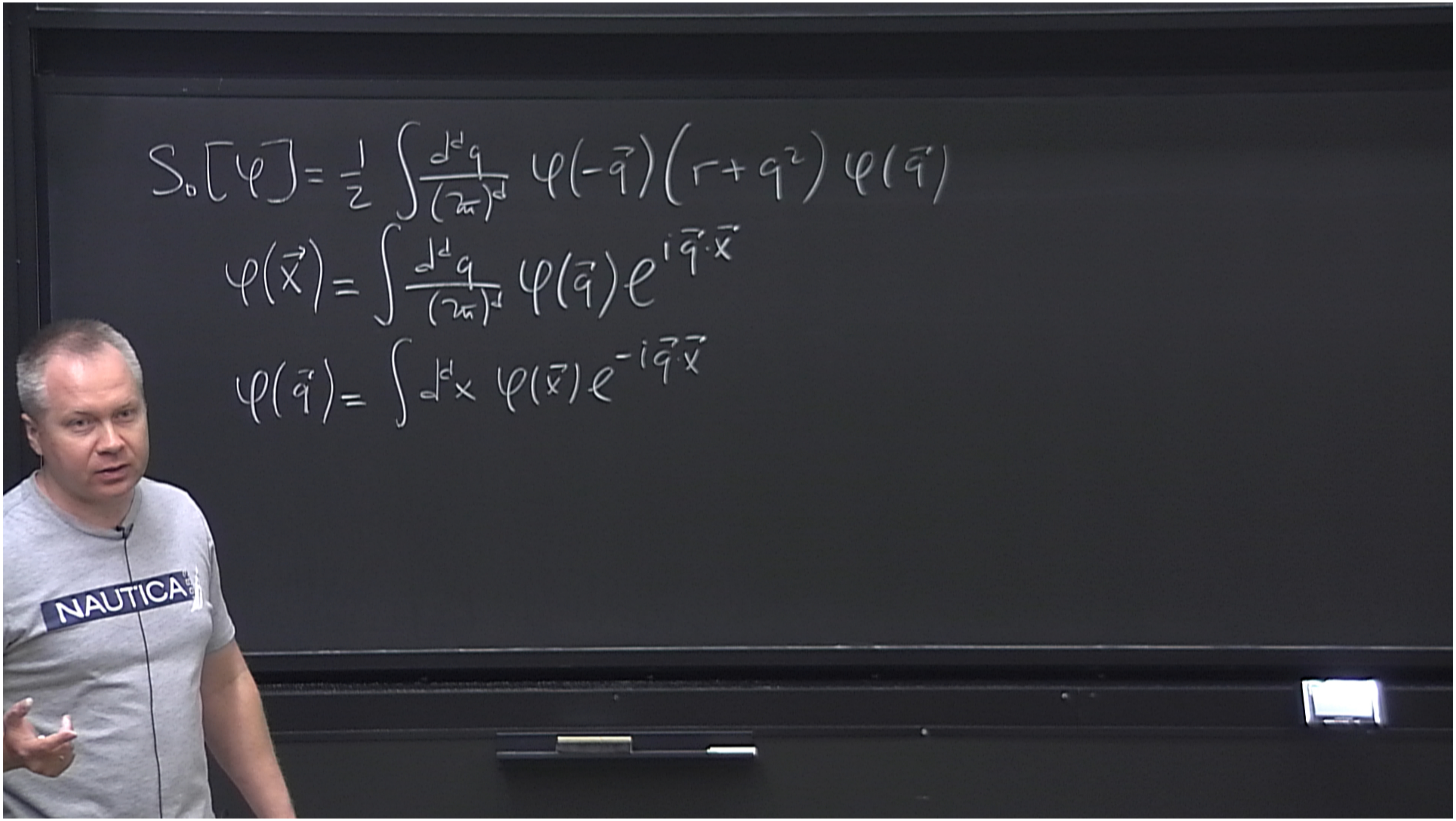

$\frac{1}{2T_c^2 N}\sum_q \varphi(-\vec{q})\left(t + \frac{1}{2d}R^2 q^2\right)\varphi(\vec{q})$
$\tilde{\varphi}(\vec{q}) = \frac{Ra^{d/2}}{\sqrt{2d}\,T_c}\varphi(\vec{q})$
$\frac{}{Na^d}\sum_q \tilde{\varphi}(-\vec{q})\left(\frac{2dt}{R^2} + q^2\right)\tilde{\varphi}(\vec{q})$
$r$
$t<0,\ T<T_c$
$\frac{1}{V}\sum_q \longrightarrow \int d^d q$


$S_0[\varphi] = \frac{1}{2}\int \frac{d^dq}{(2\pi)^d}\,\varphi(-\vec{q})\,(r+q^2)\,\varphi(\vec{q})$


$S_0[\varphi] = \frac{1}{2} \int \frac{d^d q}{(2\pi)^d} \varphi(-\vec{q}) \left( r + q^2 \right) \varphi(\vec{q})$
$\varphi(\vec{x}) = \int \frac{d^d q}{(2\pi)^d} \varphi(\vec{q}) e^{i \vec{q} \cdot \vec{x}}$


$S_0[\varphi] = \frac{1}{2}\int \frac{d^dq}{(2\pi)^d}\, \varphi(-\vec{q})(r+q^2)\varphi(\vec{q})$
$\varphi(\vec{x}) = \int \frac{d^dq}{(2\pi)^d}\, \varphi(\vec{q}) e^{i\vec{q}\cdot\vec{x}}$
$\varphi(\vec{q}) = \int d^dx\, \varphi(\vec{x}) e^{-i\vec{q}\cdot\vec{x}}$
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$$\Rightarrow \quad S_0[\varphi] = \frac{1}{2}\int d^dx \left[ (\vec{\nabla}\varphi)^2 + r\varphi^2 \right]$$

$$S_0[\varphi] = \frac{1}{2T_c^2 N}\sum_q \varphi(-\vec{q})\left(t + \frac{1}{2d}R^2 q^2\right)\varphi(\vec{q})$$

$$\frac{1}{V}\sum_q$$

Introduce $\varphi(\vec{q}) = \left(\frac{R a^{d/2}}{\sqrt{2d}\,T_c}\right)^{-1}\tilde{\varphi}(\vec{q})$

$=$

$$S_0[\tilde{\varphi}] = \frac{1}{2Na^d}\sum_q \tilde{\varphi}(-\vec{q})\left(\underbrace{\frac{2dt}{R^2}}_{r} + q^2\right)\tilde{\varphi}(\vec{q})$$

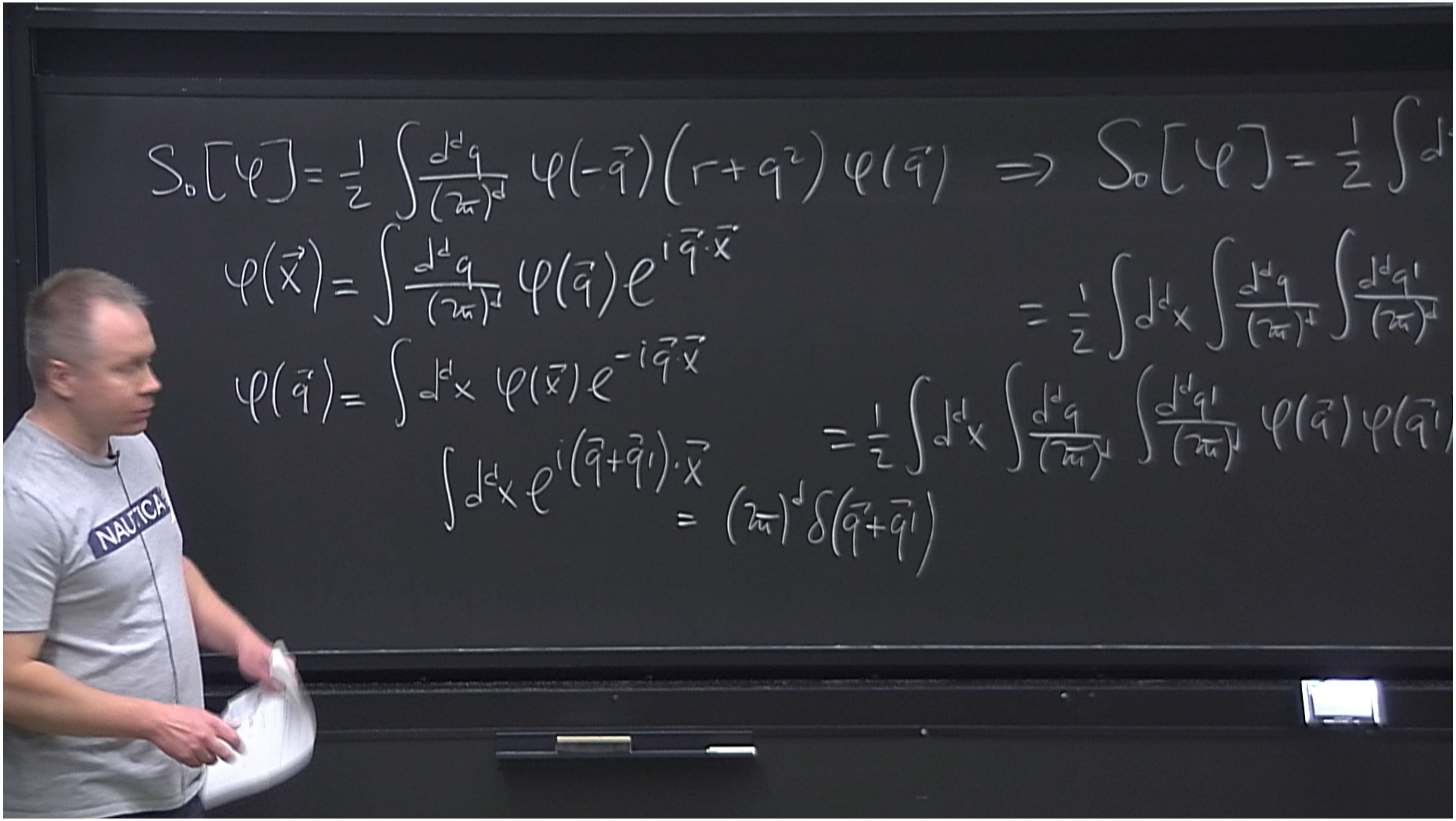
$R^2 q^2$

$r > 0,\ T > T_c$ ; $r < 0,\ T < T_c$

$$q^2)\,\varphi(\vec{q}) \Rightarrow S_0[\varphi] = \frac{1}{2}\int d^dx \left[ (\vec{\nabla}\varphi)^2 + r\varphi^2 \right] =$$

$$= \frac{1}{2}\int d^dx \int \frac{d^dq}{(2\pi)^d} \int \frac{d^dq'}{(2\pi)^d}\, \varphi(\vec{q})\,\varphi(\vec{q}\,')\, \vec{\nabla} e^{i\vec{q}\cdot\vec{x}} \cdot \vec{\nabla} e^{i\vec{q}\,'\cdot\vec{x}} =$$

$$= \frac{1}{2}\int d^dx \int \frac{d^dq}{(2\pi)^d} \int \frac{d^dq'}{(2\pi)^d}\, \varphi(\vec{q})\,\varphi(\vec{q}\,')\,(-1)\,\vec{q}\cdot\vec{q}\,'\, e^{i(\vec{q}+\vec{q}\,')\cdot\vec{x}}$$


$S_0[\varphi] = \frac{1}{2}\int \frac{d^dq}{(2\pi)^d}\,\varphi(-\vec{q})(r+q^2)\varphi(\vec{q}) \Rightarrow S_0[\varphi] = \frac{1}{2}\int d$
$\varphi(\vec{x}) = \int \frac{d^dq}{(2\pi)^d}\,\varphi(\vec{q})\,e^{i\vec{q}\cdot\vec{x}}$
$= \frac{1}{2}\int d^dx \int \frac{d^dq}{(2\pi)^d}\int \frac{d^dq'}{(2\pi)^d}$
$\varphi(\vec{q}) = \int d^dx\,\varphi(\vec{x})\,e^{-i\vec{q}\cdot\vec{x}}$
$= \frac{1}{2}\int d^dx \int \frac{d^dq}{(2\pi)^d}\int \frac{d^dq'}{(2\pi)^d}\,\varphi(\vec{q})\,\varphi(\vec{q}'$
$\int d^dx\,e^{i(\vec{q}+\vec{q}')\cdot\vec{x}} = (2\pi)^d\,\delta(\vec{q}+\vec{q}')$
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$$) = \frac{1}{2}\int d^dx \left[ (\vec{\nabla}\varphi)^2 + r\varphi^2 \right] =$$

$$\frac{d^dq}{(2\pi)^d}\int \frac{d^dq'}{(2\pi)^d}\, \varphi(\vec{q})\,\varphi(\vec{q}\,')\, \vec{\nabla} e^{i\vec{q}\cdot\vec{x}} \cdot \vec{\nabla} e^{i\vec{q}\,'\cdot\vec{x}} =$$

$$\varphi(\vec{q})\,\varphi(\vec{q}\,')\,(-1)\,\vec{q}\cdot\vec{q}\,'\, e^{i(\vec{q}+\vec{q}\,')\cdot\vec{x}} = \frac{1}{2}\int \frac{d^dq}{(2\pi)^d}\, \varphi(\vec{q})\,\varphi(-\vec{q})\, q^2$$

$$) = \frac{1}{2}\int d^dx \left[ (\vec{\nabla}\varphi)^2 + r\varphi^2 \right] =$$

$$\frac{d^dq}{(2\pi)^d}\int \frac{d^dq'}{(2\pi)^d}\,\varphi(\vec{q})\,\varphi(\vec{q}\,')\,\vec{\nabla}e^{i\vec{q}\cdot\vec{x}}\cdot\vec{\nabla}e^{i\vec{q}\,'\cdot\vec{x}} =$$

$$\varphi(\vec{q})\,\varphi(\vec{q}\,')\,(-1)\,\vec{q}\cdot\vec{q}\,'\,e^{i(\vec{q}+\vec{q}\,')\cdot\vec{x}} = \frac{1}{2}\int\frac{d^dq}{(2\pi)^d}\,\varphi(\vec{q})\,\varphi(-\vec{q})\,q^2$$

$$\frac{1}{a^d}\sum_i \longrightarrow \int d^dx$$

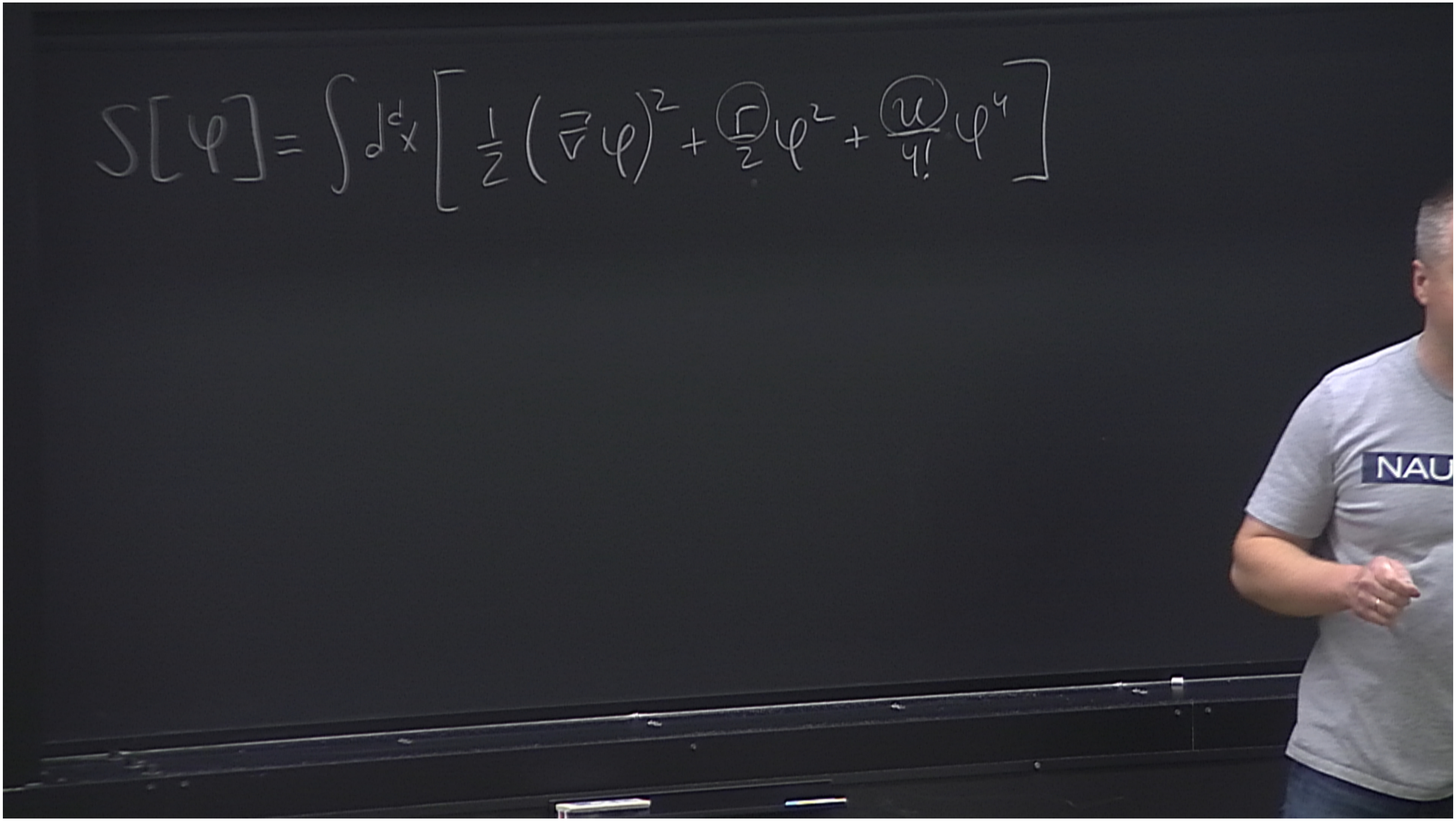
$$) = \frac{1}{2}\int d^d x \left[ (\vec{\nabla}\varphi)^2 + r\varphi^2 \right] =$$

$$\frac{d^d q}{(2\pi)^d} \int \frac{d^d q'}{(2\pi)^d}\, \varphi(\vec{q})\,\varphi(\vec{q}\,')\, \vec{\nabla} e^{i\vec{q}\cdot\vec{x}} \cdot \vec{\nabla} e^{i\vec{q}\,'\cdot\vec{x}} =$$

$$\varphi(\vec{q})\,\varphi(\vec{q}\,')\,(-1)\,\vec{q}\cdot\vec{q}\,'\, e^{i(\vec{q}+\vec{q}\,')\cdot\vec{x}} = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d}\, \varphi(\vec{q})\,\varphi(-\vec{q})\, q^2$$

$$\frac{1}{a^d}\sum_i \longrightarrow \int d^d x$$

$$S[\varphi] = \int d^d x \left[ \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 \right]$$


$$S[\varphi] = \int d^d x \left[ \frac{1}{2}\left(\vec{\nabla}\varphi\right)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 \right]$$
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$$S[\varphi] = \int d^d x \left[ \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 \right]$$

$r$ changes sign at $T = T_c$ and $u > 0$

$$S[\varphi] = \int d^d x \left[ \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 \right]$$

$r$ changes sign at $T = T_c$ and $u > 0$.

$S[\varphi]$ is an even functional of $\varphi$ follows from $\sigma \to -\sigma$.

$$S[\varphi] = \int d^d x \left[ \frac{1}{2}\left(\vec{\nabla}\varphi\right)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 \right]$$

$r$ changes sign at $T = T_c$ and $u > 0$.

$S[\varphi]$ is an even functional of $\varphi$ follows from $\sigma \to -\sigma$.

Even powers of $\vec{\nabla}$ - follows from inversion symmetry.

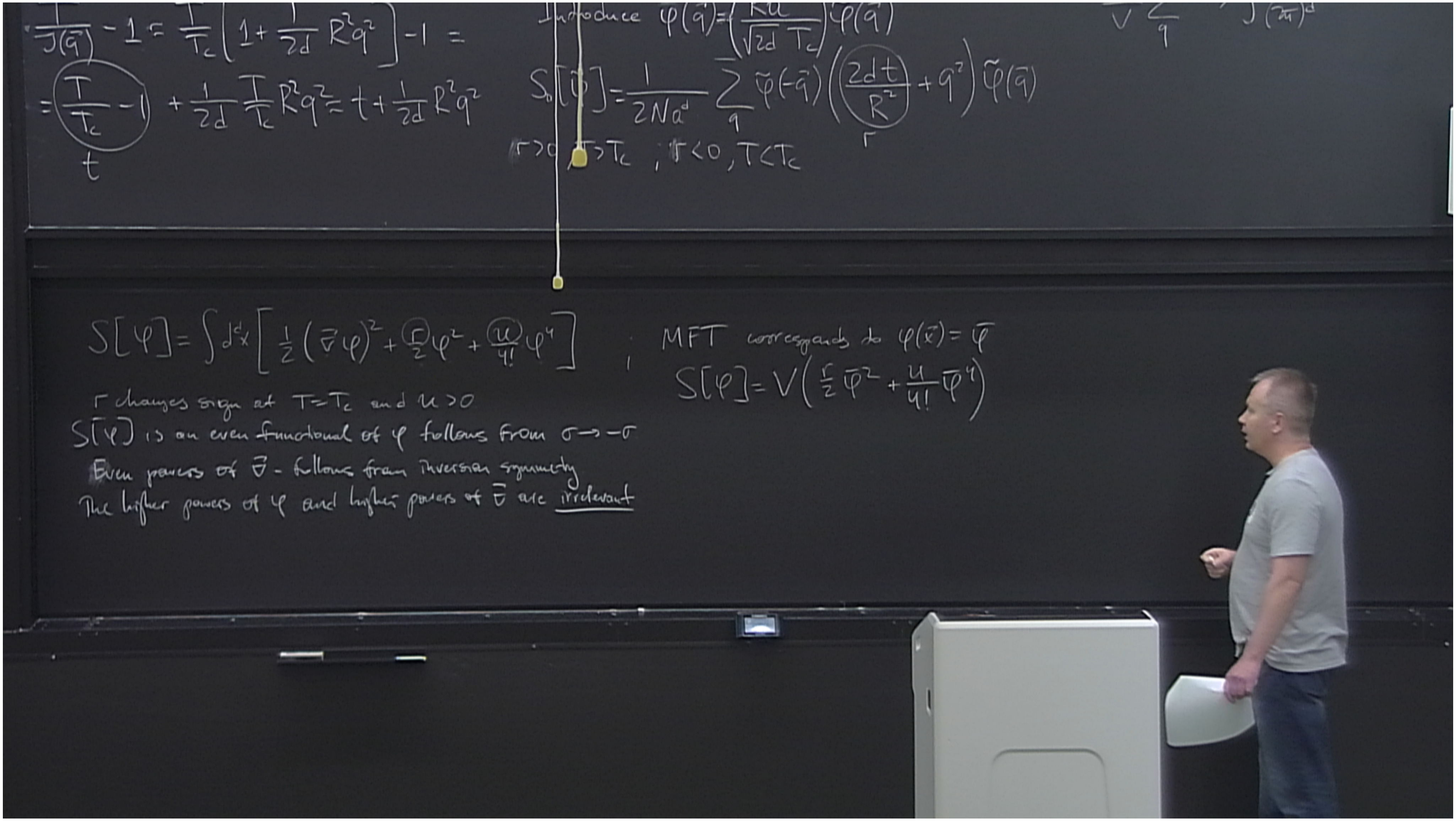
$$S[\varphi] = \int d^dx \left[ \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 \right]$$

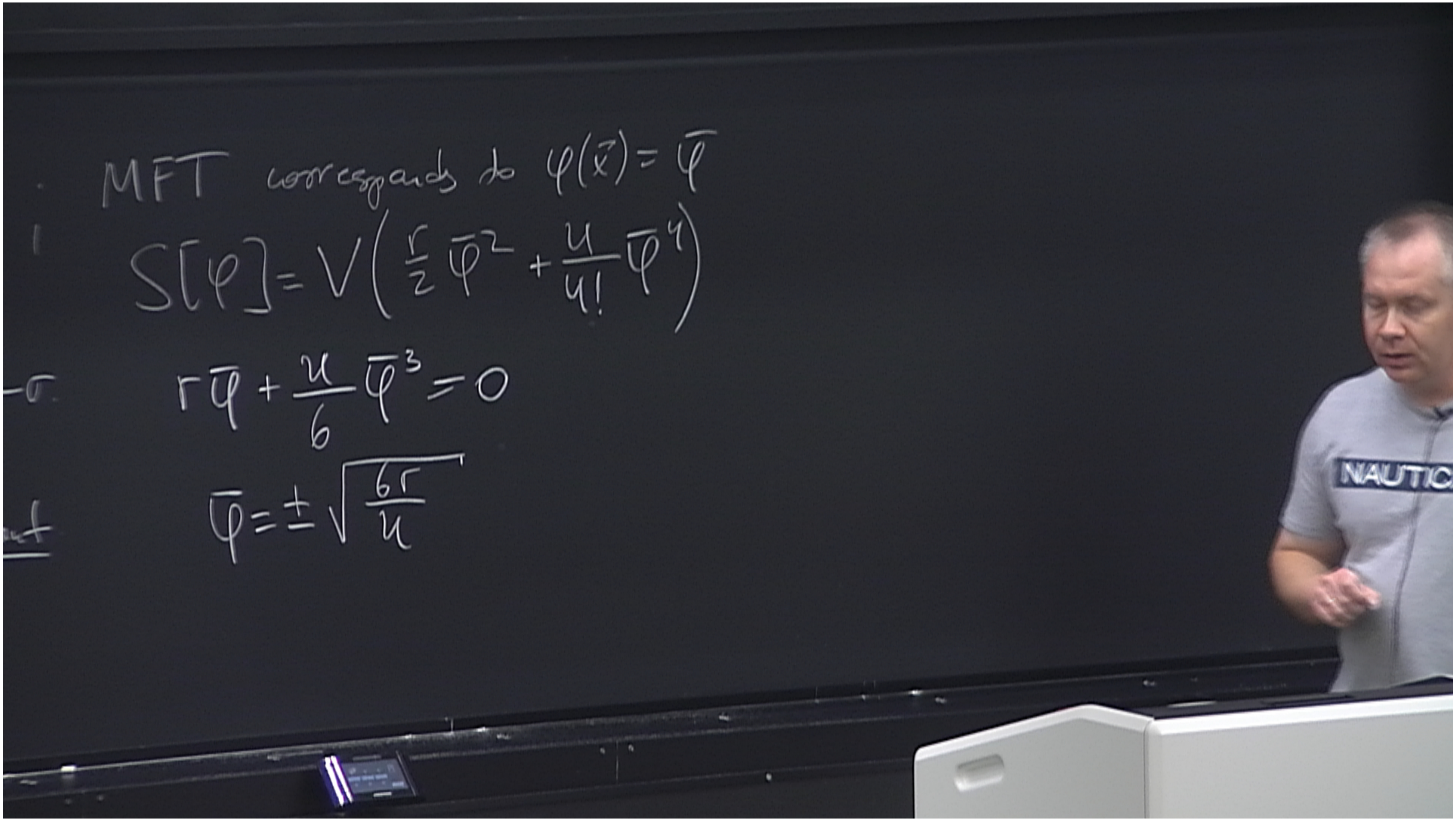
$r$ changes sign at $T = T_c$ and $u > 0$

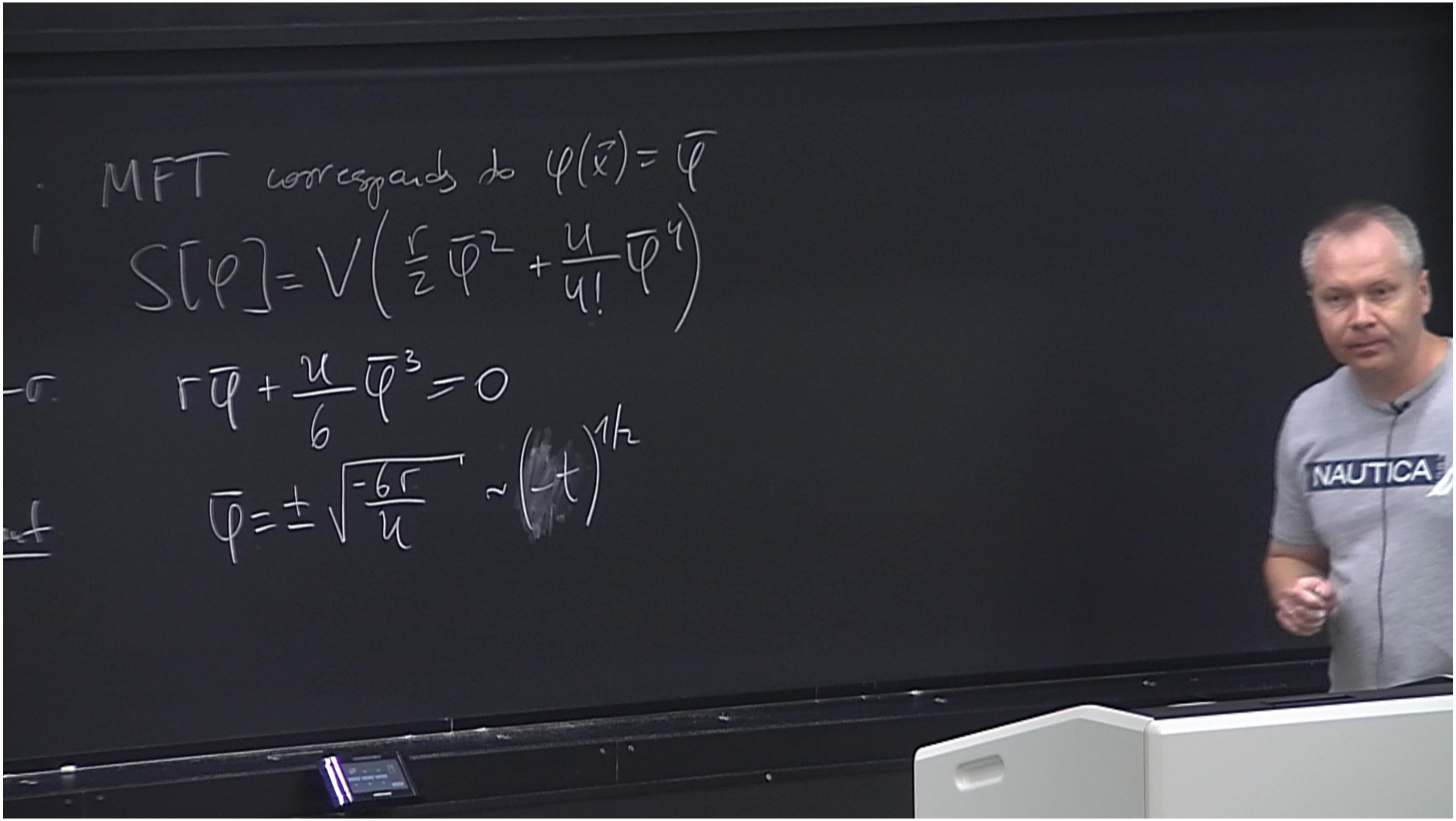
$S[\varphi]$ is an even functional of $\varphi$ follows from $\sigma \to -\sigma$.

Even powers of $\vec{\nabla}$ - follows from inversion symmetry.

The higher powers of $\varphi$ and higher powers of $\vec{\nabla}$ are irrelevant


$\frac{1}{J(\vec q)} - 1 = \frac{T}{T_c}\left[1+\frac{1}{2d}R^2q^2\right]-1 =$
$= \left(\frac{T}{T_c}-1\right) + \frac{1}{2d}\frac{T}{T_c}R^2q^2 \simeq t + \frac{1}{2d}R^2q^2$
$t$
Introduce $\varphi(\vec q) = \left(\frac{Ra}{\sqrt{2d\,T_c}}\right)\tilde\varphi(\vec q)$
$S_0[\tilde\varphi] = \frac{1}{2Na^d}\sum_q \tilde\varphi(-\vec q)\left(\frac{2dt}{R^2}+q^2\right)\tilde\varphi(\vec q)$
$r$
$r>0,\ T>T_c\ ;\ r<0,\ T<T_c$
$S[\varphi] = \int d^dx\left[\frac{1}{2}(\vec\nabla\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4\right]$ ;
MFT corresponds to $\varphi(\vec x)=\bar\varphi$
$S[\varphi] = V\left(\frac{r}{2}\bar\varphi^2 + \frac{u}{4!}\bar\varphi^4\right)$
$r$ changes sign at $T=T_c$ and $u>0$
$S[\varphi]$ is an even functional of $\varphi$ follows from $\sigma\to-\sigma$
Even powers of $\vec\nabla$ - follows from inversion symmetry
The higher powers of $\varphi$ and higher powers of $\vec\nabla$ are irrelevant


MFT corresponds to $\varphi(\vec{x}) = \bar{\varphi}$
$S[\varphi] = V\left(\frac{r}{2}\bar{\varphi}^2 + \frac{u}{4!}\bar{\varphi}^4\right)$
$r\bar{\varphi} + \frac{u}{6}\bar{\varphi}^3 = 0$
$\bar{\varphi} = \pm\sqrt{\frac{6r}{u}}$


MFT corresponds to $\varphi(\vec{x}) = \bar{\varphi}$
$S[\varphi] = V\left(\frac{r}{2}\bar{\varphi}^2 + \frac{u}{4!}\bar{\varphi}^4\right)$
$r\bar{\varphi} + \frac{u}{6}\bar{\varphi}^3 = 0$
$\bar{\varphi} = \pm\sqrt{\frac{-6r}{u}} \sim (-t)^{1/2}$
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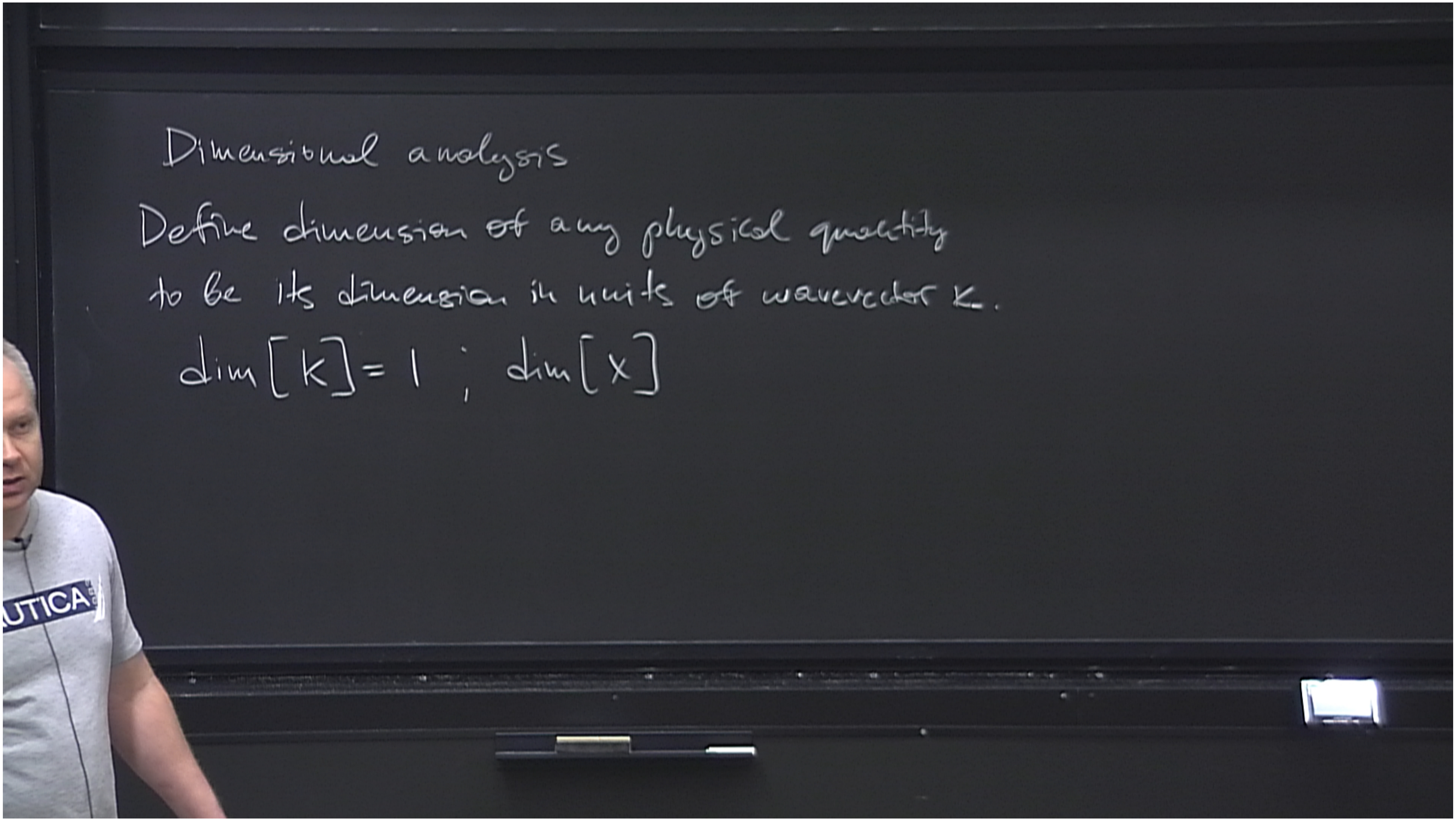
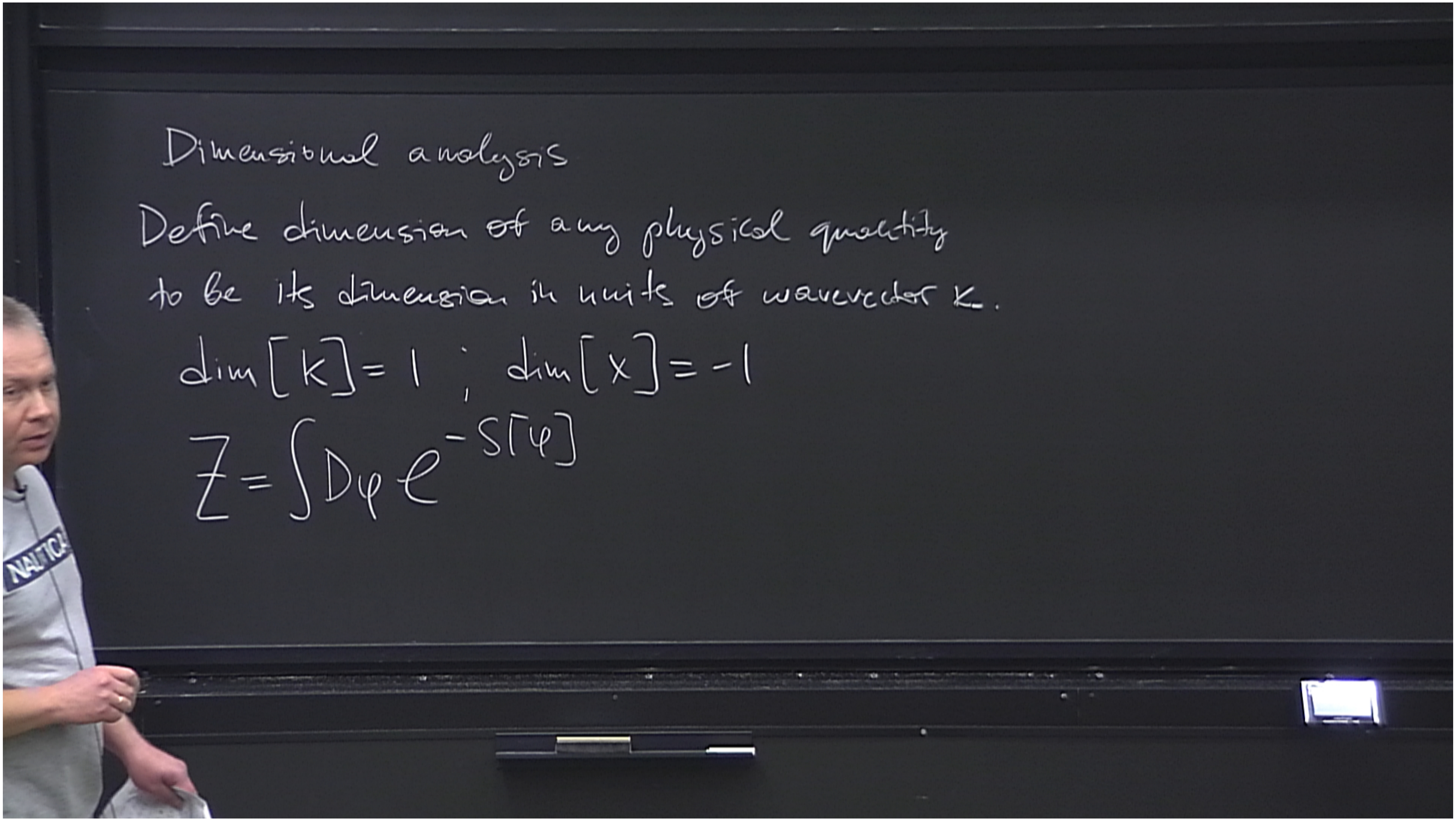
Dimensional analysis


Dimensional analysis
Define dimension of any physical quantity
to be its dimension in units of wavevector k.
dim[k] = 1 ; dim[x]


Dimensional analysis
Define dimension of any physical quantity
to be its dimension in units of wavevector k.
$\dim[k] = 1$ ; $\dim[x] = -1$
$Z = \int D\varphi \, e^{-S[\varphi]}$

$S[\varphi]$ - dimensionless

$\dim[S] = 0$

Dimensional analysis

Define dimension of any physical quantity to be its dimension in units of wavevector $k$.

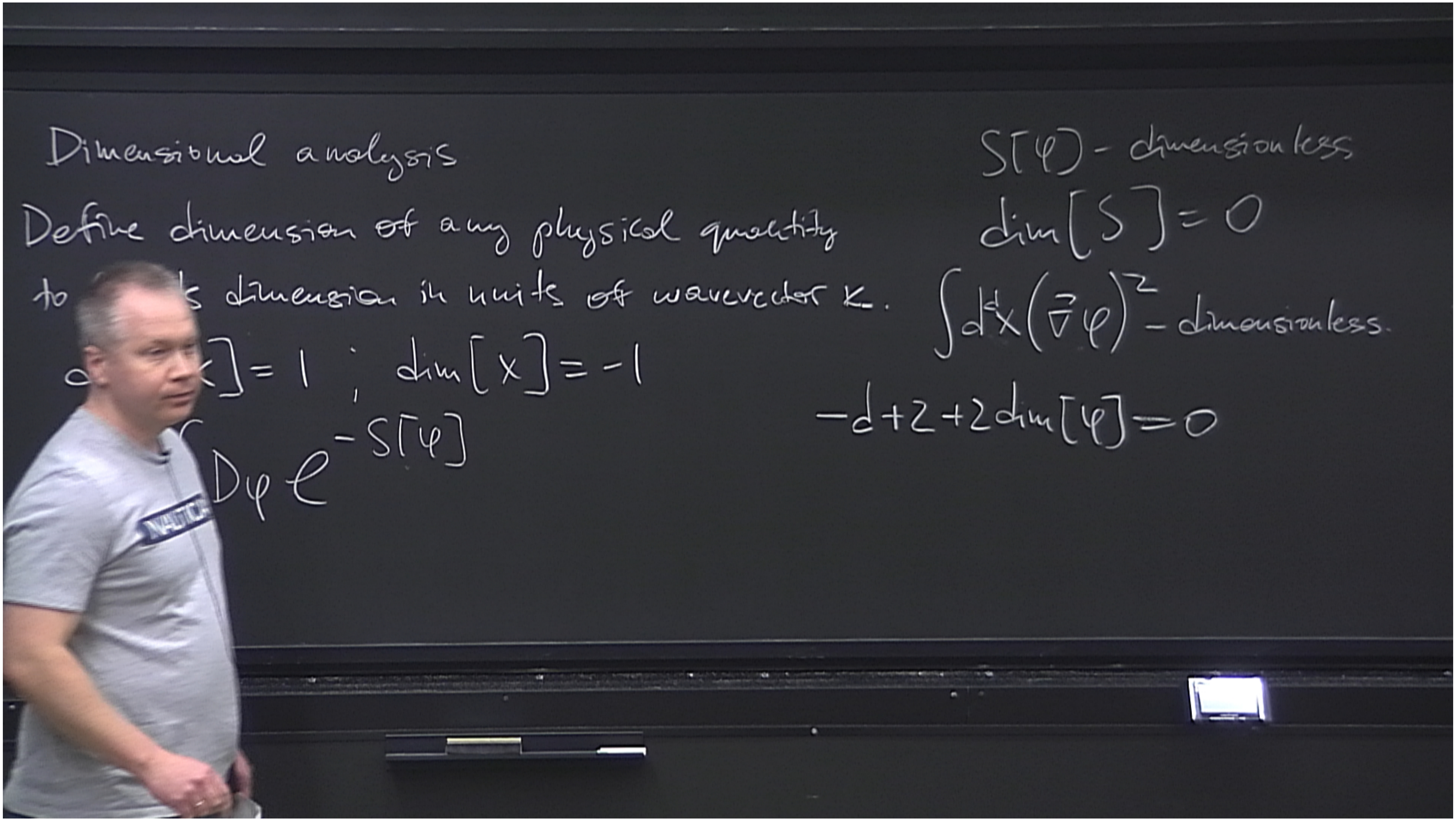
$\dim[k] = 1$ ; $\dim[x] = -1$

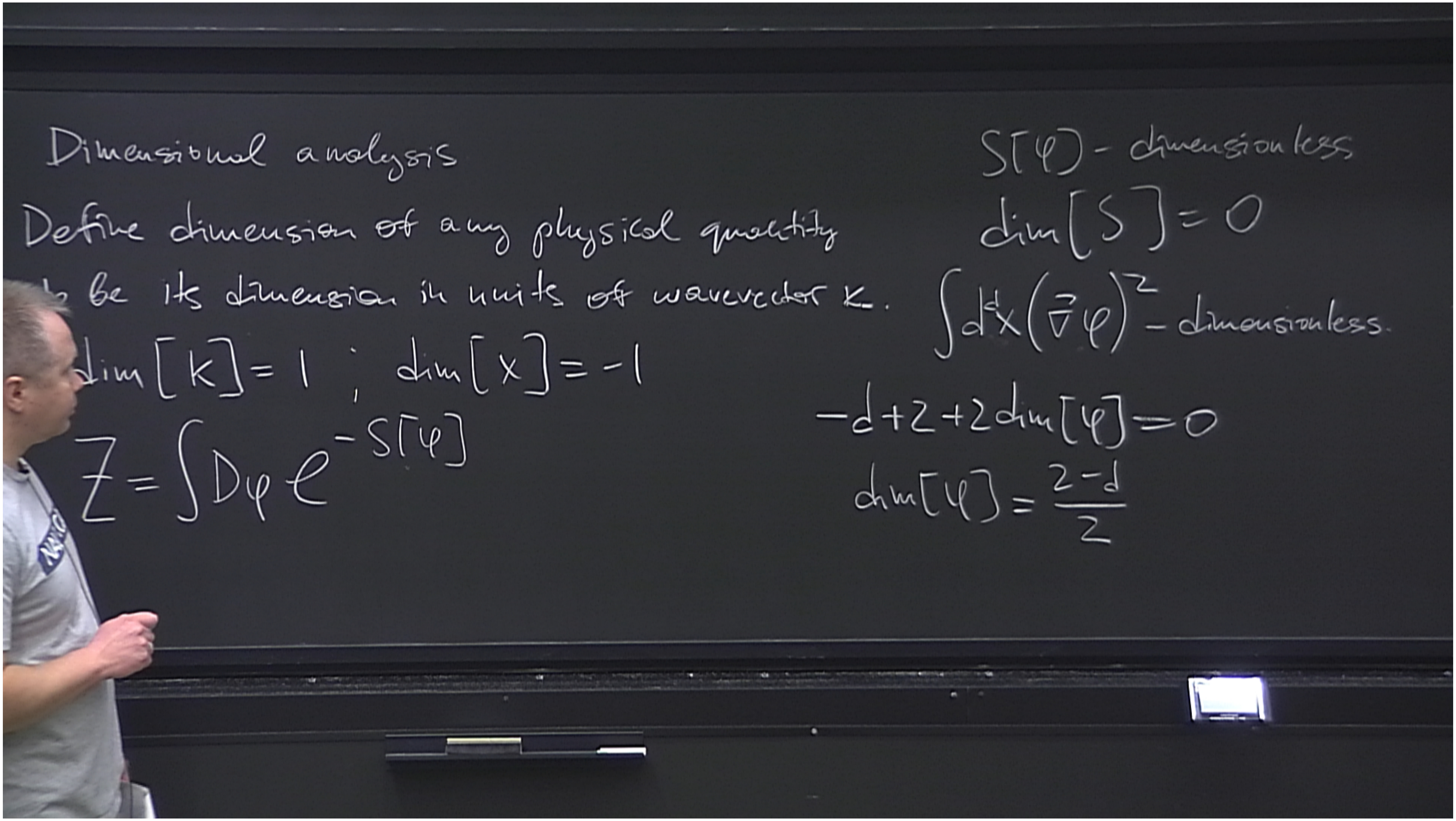
$$Z = \int D\varphi \, e^{-S[\varphi]}$$

$S[\varphi]$ - dimensionless

$\dim[S] = 0$

$\int d^d x (\vec{\nabla}\varphi)^2$ - dimensionless.


Dimensional analysis
Define dimension of any physical quantity
to
s dimension in units of wavevector k.
$] = 1$ ; $\dim[x] = -1$
$D\varphi\, e^{-S[\varphi]}$
$S[\varphi]$ - dimensionless
$\dim[S] = 0$
$\int d^d x (\vec{\nabla}\varphi)^2$ - dimensionless.
$-d+2+2\dim[\varphi]=0$


Dimensional analysis
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$S[\varphi]$ - dimensionless
$\dim[S] = 0$
$\int d^dx (\vec{\nabla}\varphi)^2$ - dimensionless.
$-d + 2 + 2\dim[\varphi] = 0$
$\dim[\varphi] = \frac{2-d}{2}$

## Dimensional analysis

Define dimension of any physical quantity

... be its dimension in units of wavevector $k$.

$$\ldots[k] = 1 \; ; \; \mathrm{dim}[x] = -1$$

$$\ldots = \int D\varphi \, e^{-S[\varphi]}$$

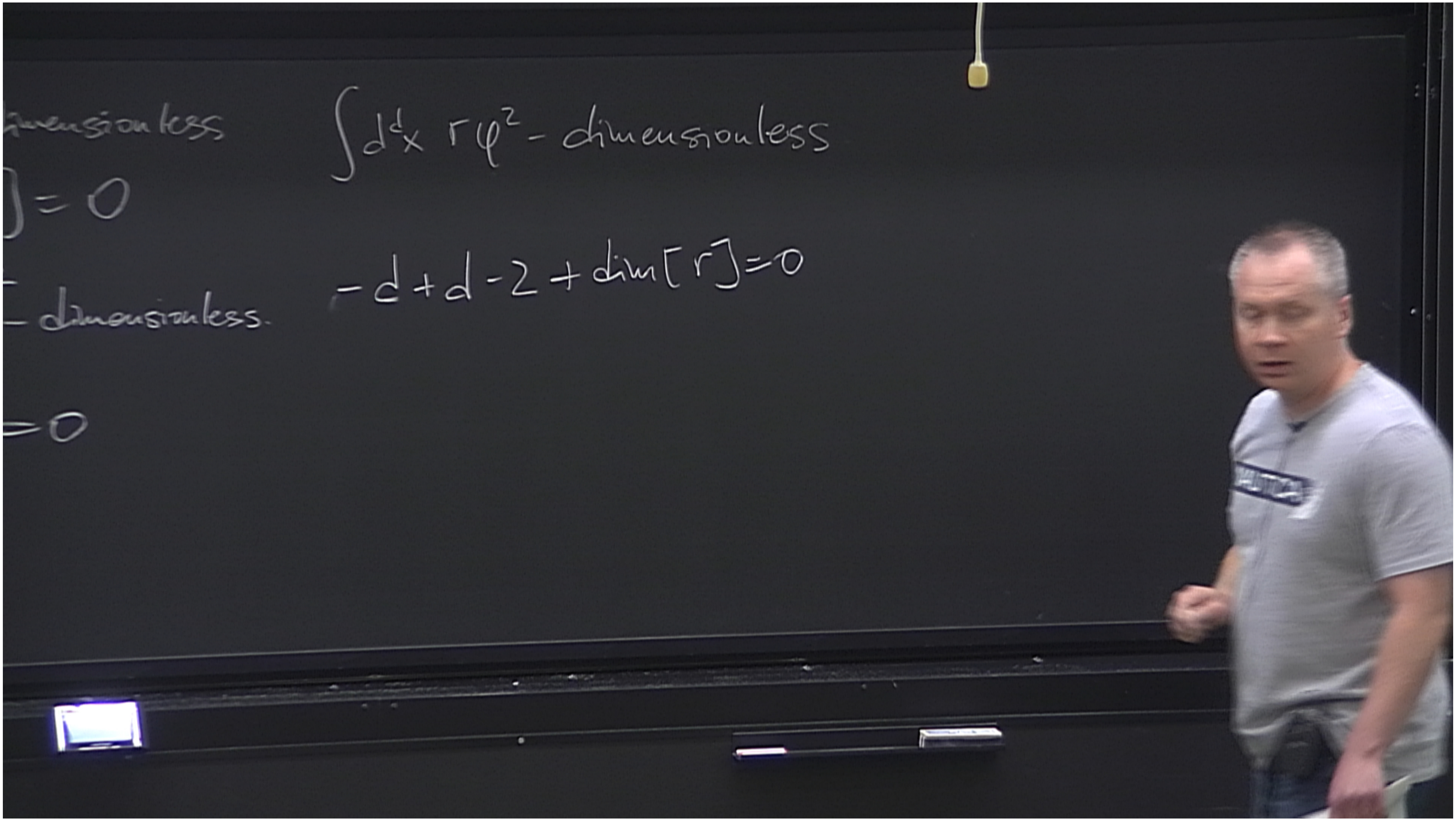
$S[\varphi]$ - dimensionless

$$\mathrm{dim}[S] = 0$$

$\int d^dx \left(\vec{\nabla}\varphi\right)^2$ - dimensionless.

$$-d + 2 + 2\,\mathrm{dim}[\varphi] = 0$$

$$\mathrm{dim}[\varphi] = \frac{d-2}{2}$$


imensionless
] = 0
dimensionless.
= 0
$\int d^d x \, r \varphi^2$ - dimensionless
$-d + d - 2 + \dim[r] = 0$

...mensionless

$...] = 0$

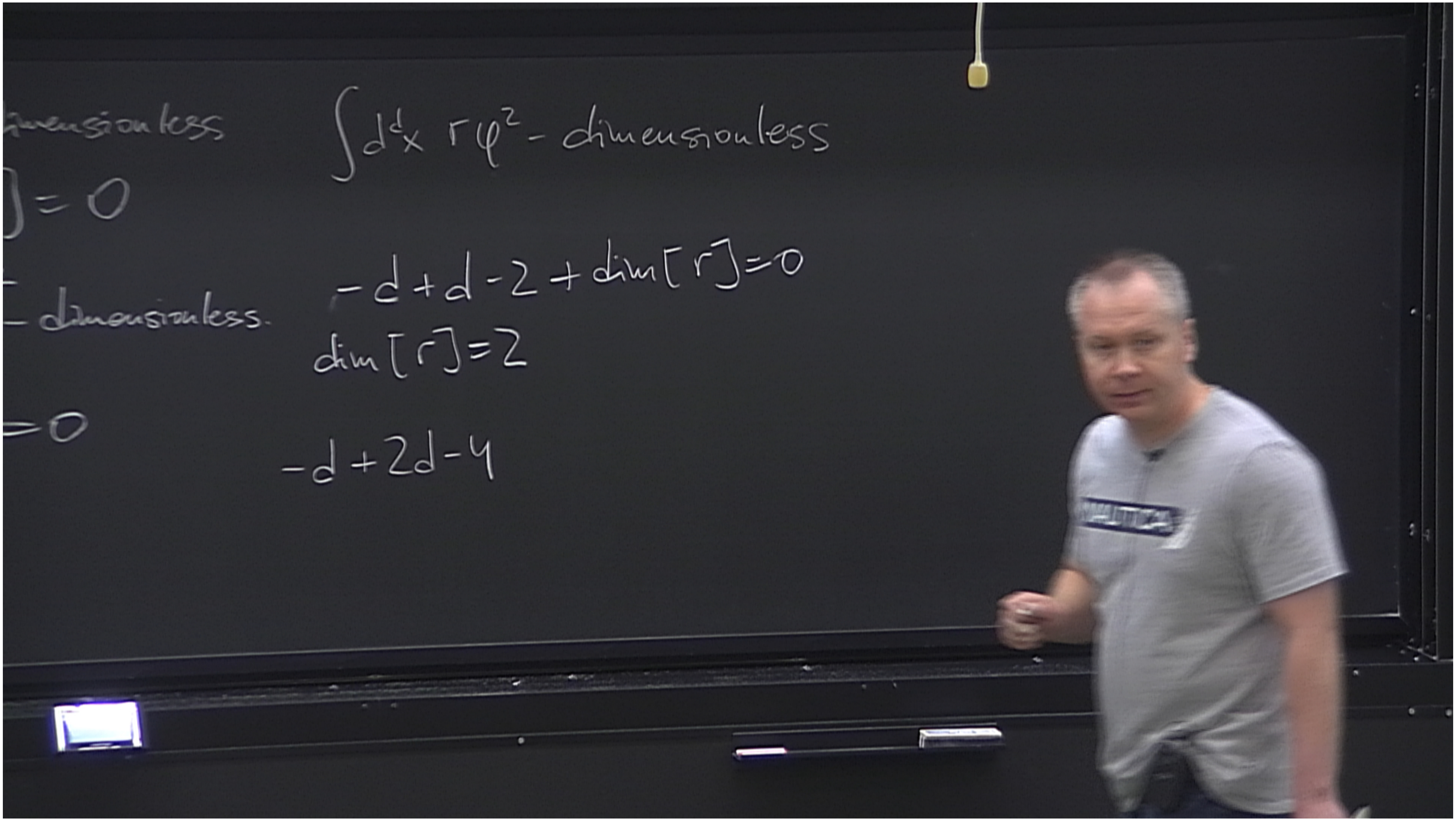
... dimensionless.

$... = 0$

$\int d^d x \, r \varphi^2$ - dimensionless

$-d + d - 2 + \dim[r] = 0$

$\dim[r] = 2$


dimensionless
$] = 0$
dimensionless.
$= 0$
$\int d^d x \, r \varphi^2$ - dimensionless
$-d + d - 2 + \dim[r] = 0$
$\dim[r] = 2$
$-d + 2d - 4$

imensionless

$] = 0$

– dimensionless.

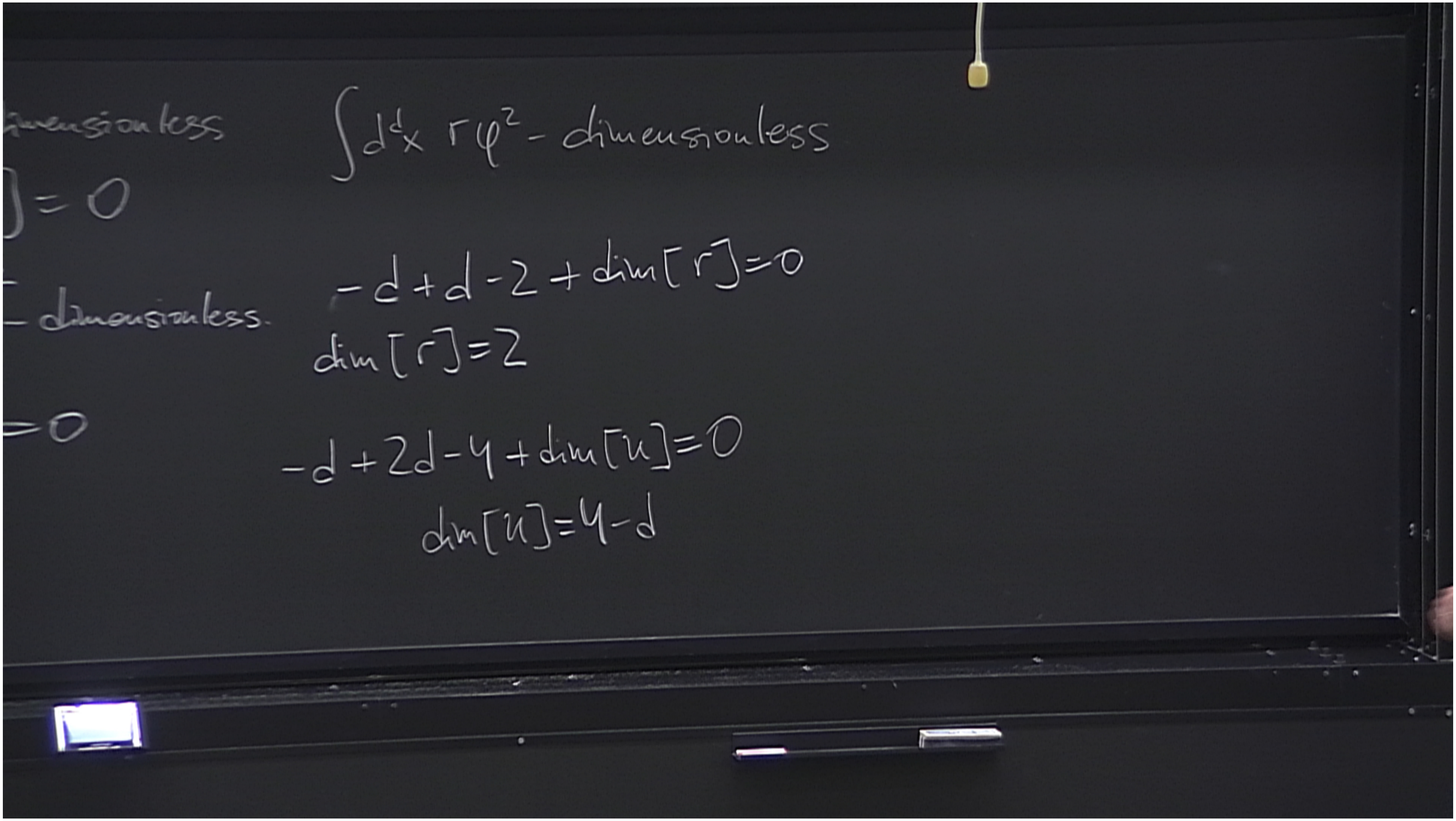
$= 0$

$\int d^dx \, r\varphi^2$ - dimensionless

$$-d + d - 2 + \dim[r] = 0$$

$$\dim[r] = 2$$

$$-d + 2d - 4 + \dim[u] = 0$$


$\int d^dx \; r\varphi^2$ - dimensionless
$] = 0$
$-d+d-2+\dim[r]=0$
- dimensionless.
$\dim[r]=2$
$=0$
$-d+2d-4+\dim[u]=0$
$\dim[u]=4-d$

...imensionless

$] = 0$

... dimensionless.

$= 0$

$\int d^dx \, r\varphi^2$ - dimensionless

$-d + d - 2 + \dim[r] = 0$

$\dim[r] = 2$

$-d + 2d - 4 + \dim[u] = 0$

$\dim[u] = 4 - d$

Positive dimension - relevant.

...mensionless

$] = 0$

... dimensionless.

$= 0$

$\int d^dx \, r\varphi^2$ - dimensionless

$-d + d - 2 + \dim[r] = 0$

$\dim[r] = 2$

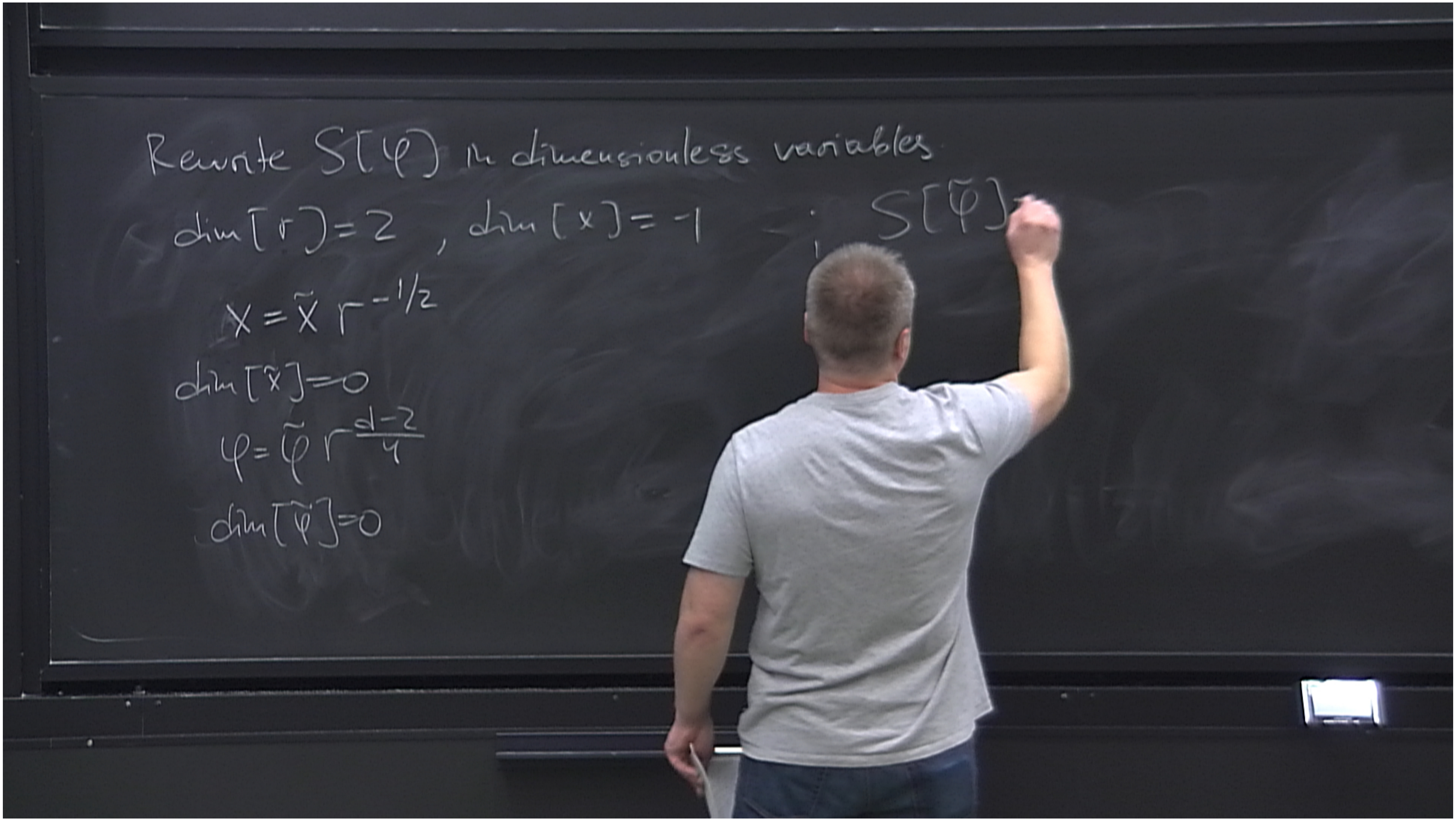
$-d + 2d - 4 + \dim[u] = 0$

$\dim[u] = 4 - d$

Positive dimension – relevant.

Negative dimension – irrelevant.

Rewrite $S[\varphi]$ in dimensionless variables


Rewrite $S[\varphi]$ in dimensionless variables
$\dim[r]=2$ , $\dim[x]=-1$ ; $S[\tilde{\varphi}]$
$x=\tilde{x}\, r^{-1/2}$
$\dim[\tilde{x}]=0$
$\varphi=\tilde{\varphi}\, r^{\frac{d-2}{4}}$
$\dim[\tilde{\varphi}]=0$

variables

$$S[\tilde{\varphi}] = \int d^d\tilde{x} \left[ \frac{1}{2}\left(\tilde{\nabla}\tilde{\varphi}\right)^2 + \frac{1}{2}\tilde{\varphi}^2 + \frac{u}{4!}\, r^{\frac{d-4}{2}}\, \tilde{\varphi}^4 \right]$$

variables

$$S[\tilde{\varphi}] = \int d^d\tilde{x} \left[ \frac{1}{2}\left(\tilde{\nabla}\tilde{\varphi}\right)^2 + \frac{1}{2}\tilde{\varphi}^2 + \underbrace{\frac{u}{4!} r^{\frac{d-4}{2}}}_{\tilde{u}} \tilde{\varphi}^4 \right]$$

variables

$$S[\tilde{\varphi}] = \int d^d\tilde{x} \left[ \frac{1}{2}(\tilde{\nabla}\tilde{\varphi})^2 + \frac{1}{2}\tilde{\varphi}^2 + \underbrace{\frac{u}{4!} r^{\frac{d-4}{2}}}_{\tilde{u}} \tilde{\varphi}^4 \right]$$

$r \sim t$ ; $\tilde{u} \sim u\, t^{\frac{d-4}{2}}$

variables

$$S[\tilde{\varphi}] = \int d^d\tilde{x} \left[ \frac{1}{2}\left(\tilde{\nabla}\tilde{\varphi}\right)^2 + \frac{1}{2}\tilde{\varphi}^2 + \frac{u}{4!} r^{\frac{d-4}{2}} \tilde{\varphi}^4 \right]$$

$r \sim t$ ; $\tilde{u} \sim u\, t^{\frac{d-4}{2}} \Rightarrow \tilde{u}$ diverges as $t \xrightarrow{\tilde{u}} 0$

sionless variables

$= -1$ ; $S[\tilde{\varphi}] = \int d^d\tilde{x} \left[ \frac{1}{2}\left(\tilde{\nabla}\tilde{\varphi}\right)^2 + \frac{1}{2}\tilde{\varphi}^2 + \frac{u}{4!} r^{\frac{d-4}{2}} \tilde{\varphi}^4 \right]$

$r \sim t$ ; $\tilde{u} \sim u\, t^{\frac{d-4}{2}} \Rightarrow \tilde{u}$ diverges as $t \xrightarrow{\tilde{u}} 0$

Previous lecture: $S[\varphi] \simeq S[\bar{\varphi}] + \frac{1}{2}\frac{\partial^2 S}{\partial \varphi^2}(\delta\varphi)^2 + \ldots$ – does not work sionless variables

$= -1$ ; $S[\tilde{\varphi}] = \int d^d\tilde{x} \left[ \frac{1}{2}(\tilde{\nabla}\tilde{\varphi})^2 + \frac{1}{2}\tilde{\varphi}^2 + \frac{u}{4!} r^{\frac{d-4}{2}} \tilde{\varphi}^4 \right]$

$r \sim t$ ; $\tilde{u} \sim u\, t^{\frac{d-4}{2}} \Rightarrow \tilde{u}$ diverges as $t \xrightarrow{\tilde{u}} 0$

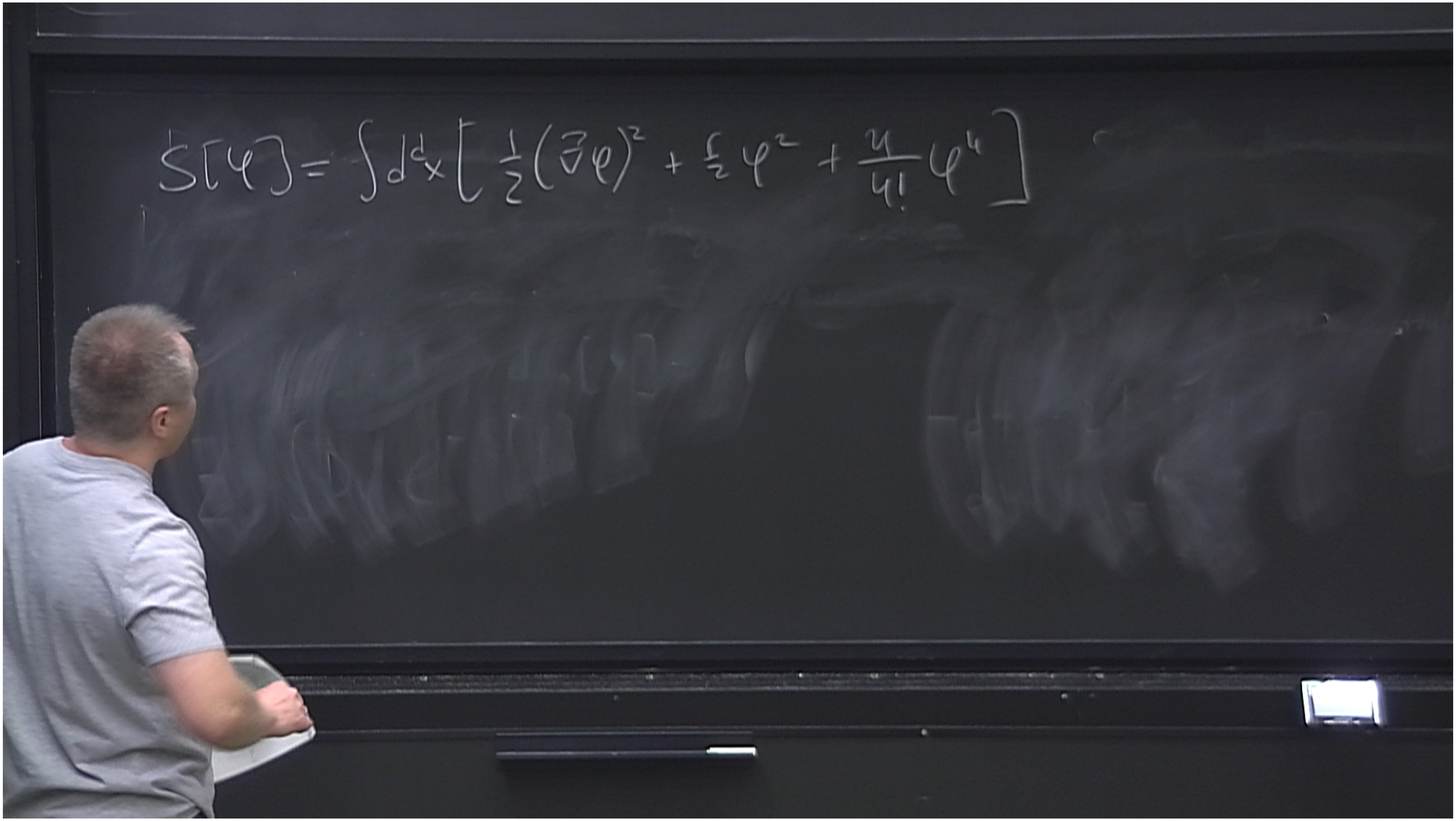
Previous lecture: $S[\varphi] \cong S[\bar{\varphi}] + \frac{1}{2}\frac{\partial^2 S}{\partial \varphi^2}(\delta\varphi)^2 + \dots$ - does not work

$Z = \int D\varphi\, e^{-S[\varphi]}$

$$\tilde{\varphi})^2 + \frac{1}{2}\tilde{\varphi}^2 + \underbrace{\frac{u}{4!} r^{\frac{d-4}{2}}}_{\tilde{u}} \tilde{\varphi}^4 \Big]$$ – we can not do perturbation theory in $u\varphi^4$ term either

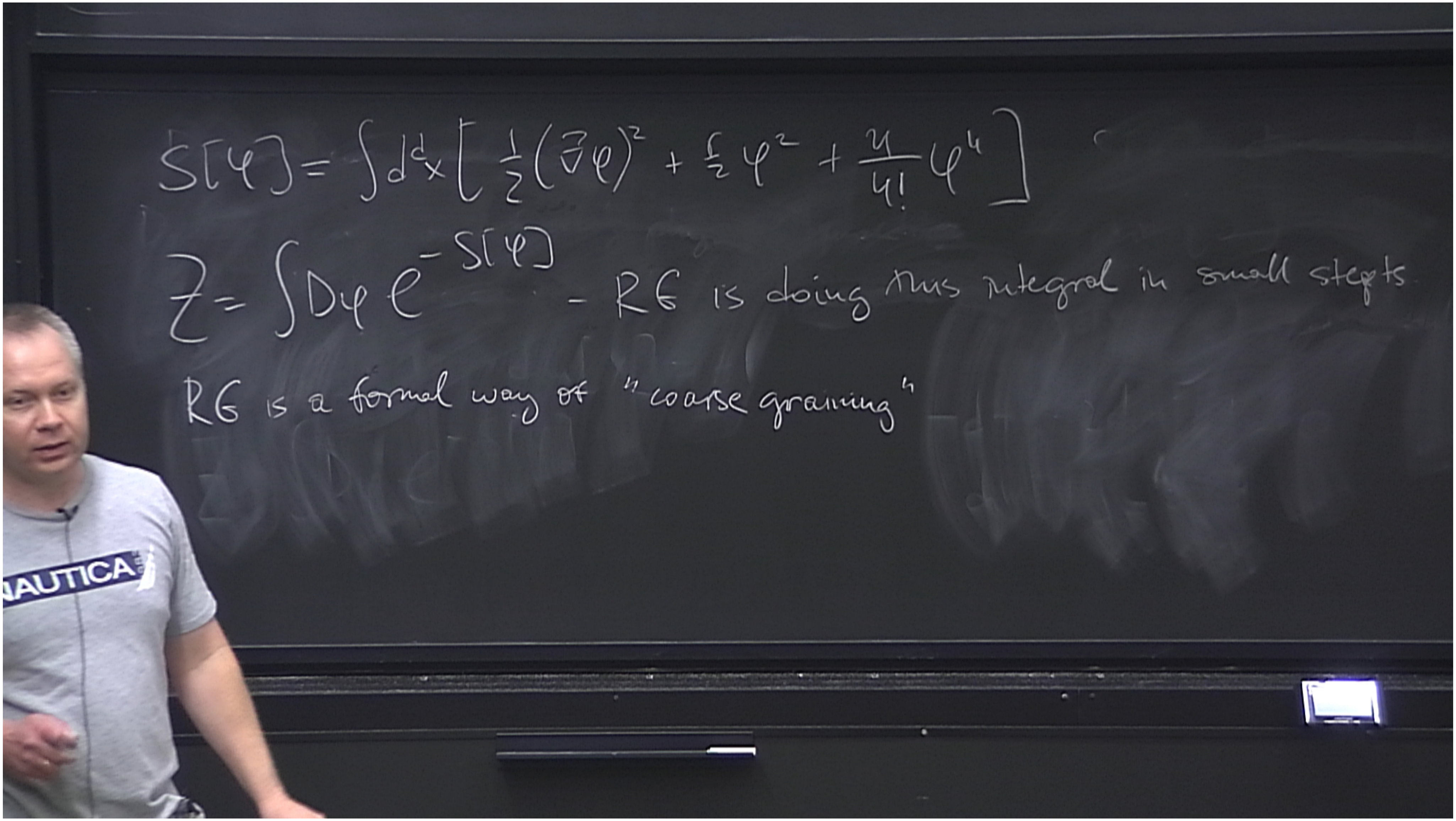
$\Rightarrow \tilde{u}$ diverges as $t \to 0$

$$\Big] + \frac{1}{2}\frac{\partial^2 S}{\partial \varphi^2}(\delta\varphi)^2 + \dots$$ – does not work


$S[\varphi] = \int d^d x \left[ \frac{1}{2} (\vec{\nabla} \varphi)^2 + \frac{r}{2} \varphi^2 + \frac{u}{4!} \varphi^4 \right]$

$$S[\varphi] = \int d^dx \left[ \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 \right]$$

$$Z = \int D\varphi \, e^{-S[\varphi]}$$ - RG is doing this integral in small steps


$S[\varphi] = \int d^d x \left[ \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 \right]$
$Z = \int D\varphi \, e^{-S[\varphi]}$ - RG is doing this integral in small steps
RG is a formal way of "coarse graining"
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$S[\varphi] = \int d^dx \left[ \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 \right]$
$Z = \int D\varphi \, e^{-S[\varphi]}$ – RG is doing this integral in small steps
RG is a formal
"coarse graining":
$\varphi(\vec{x}) = \int_0^{\Lambda} \frac{d^dk}{(2\pi)^d} \varphi(\vec{k}) e^{i\vec{k}\cdot\vec{x}}$
$\Lambda$
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$$S[\varphi] = \int d^dx \left[ \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 \right]$$

$$Z = \int D\varphi \, e^{-S[\varphi]}$$

– RG is doing this integral in small steps

RG is a formal way of "coarse graining":

$$\varphi(\vec{x}) = \int_0^{\Lambda} \frac{d^dk}{(2\pi)^d} \varphi(\vec{k}) e^{i\vec{k}\cdot\vec{x}}$$

$$\Lambda \sim \frac{\pi}{a}$$

ll steps

$$\ldots \frac{\ldots k}{\ldots)^d} \varphi(\vec{k}) e^{i\vec{k}\cdot\vec{x}} = \varphi_<(\vec{x}) + \varphi_>(\vec{x})$$

$$\varphi_<(\vec{x}) = \int_0^{\Lambda/b} \frac{d^dk}{(2\pi)^d} \varphi(\vec{k}) e^{i\vec{k}\cdot\vec{x}}$$

$$\varphi_>(\vec{x}) = \int_{\Lambda/b}^{\Lambda} \frac{d^dk}{(2\pi)^d} \varphi(\vec{k}) e^{i\vec{k}\cdot\vec{x}}$$

ll steps

$$\varphi_<(\vec{x}) = \int_0^{\Lambda/b} \frac{d^dk}{(2\pi)^d}\,\varphi(\vec{k})\,e^{i\vec{k}\cdot\vec{x}}$$

$$\varphi_>(\vec{x}) = \int_{\Lambda/b}^{\Lambda} \frac{d^dk}{(2\pi)^d}\,\varphi(\vec{k})\,e^{i\vec{k}\cdot\vec{x}}$$

$$\ldots\frac{k}{\ldots)^d}\,\varphi(\vec{k})\,e^{i\vec{k}\cdot\vec{x}} = \varphi_<(\vec{x}) + \varphi_>(\vec{x})$$

$b > 1$

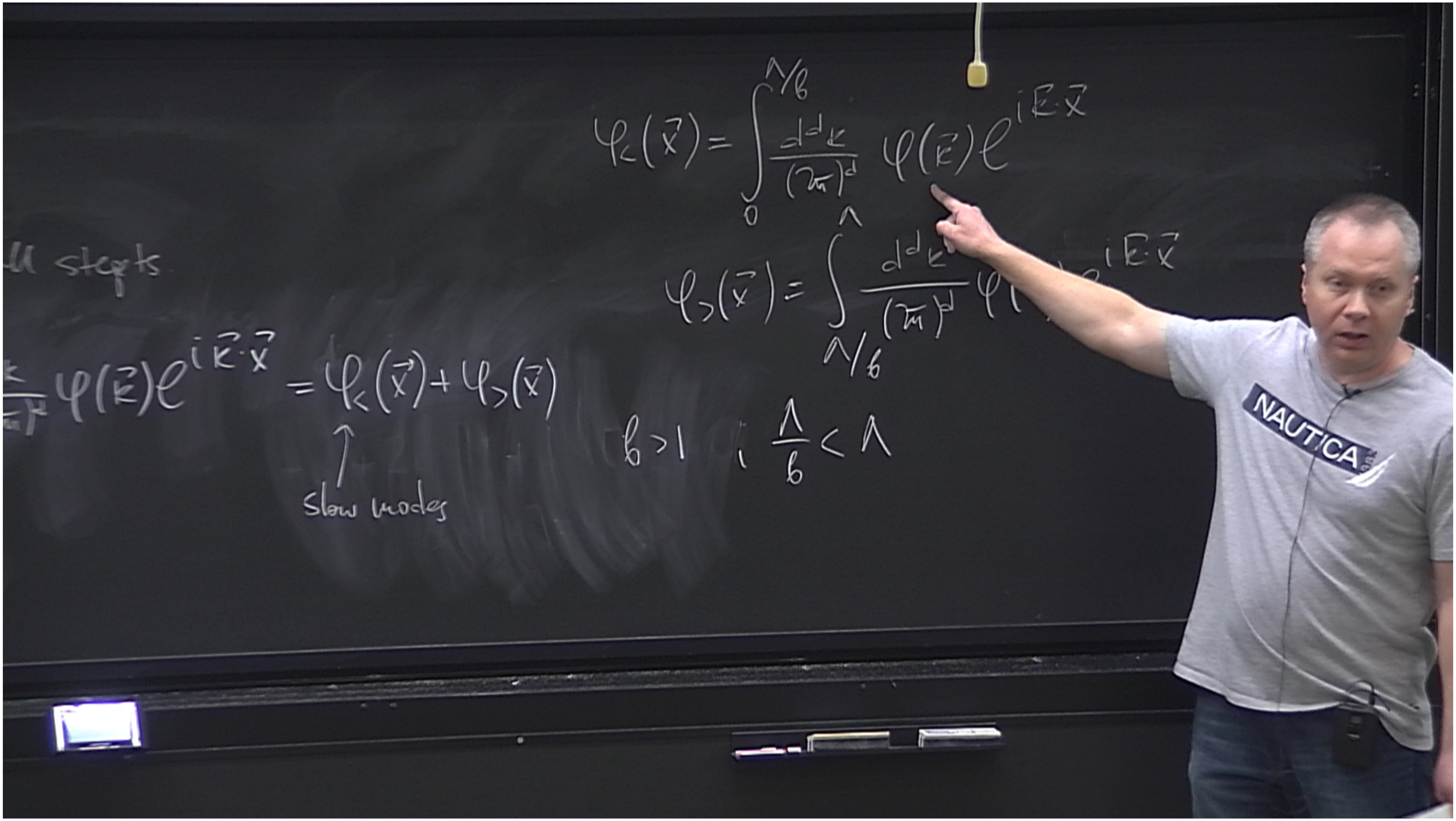
$$\varphi_<(\vec{x}) = \int_0^{\Lambda/b} \frac{d^dk}{(2\pi)^d}\, \varphi(\vec{k})\, e^{i\vec{k}\cdot\vec{x}}$$

ll steps.

$$\varphi_>(\vec{x}) = \int_{\Lambda/b}^{\Lambda} \frac{d^dk}{(2\pi)^d}\, \varphi(\vec{k})\, e^{i\vec{k}\cdot\vec{x}}$$

$$\frac{k}{(2\pi)^d}\varphi(\vec{k})e^{i\vec{k}\cdot\vec{x}} = \varphi_<(\vec{x}) + \varphi_>(\vec{x})$$

$$b>1 \quad ; \quad \frac{\Lambda}{b} < \Lambda$$


ll steps.
$\varphi_<(\vec{x}) = \int_0^{\Lambda/b} \frac{d^dk}{(2\pi)^d}\, \varphi(\vec{k})\, e^{i\vec{k}\cdot\vec{x}}$
$\varphi_>(\vec{x}) = \int_{\Lambda/b}^{\Lambda} \frac{d^dk}{(2\pi)^d}\, \varphi(\vec{k})\, e^{i\vec{k}\cdot\vec{x}}$
$= \varphi_<(\vec{x}) + \varphi_>(\vec{x})$
slow modes
$b > 1$ ; $\frac{\Lambda}{b} < \Lambda$

$$\varphi_<(\vec{x}) = \int_0^{\Lambda/b} \frac{d^dk}{(2\pi)^d}\, \varphi(\vec{k})\, e^{i\vec{k}\cdot\vec{x}}$$

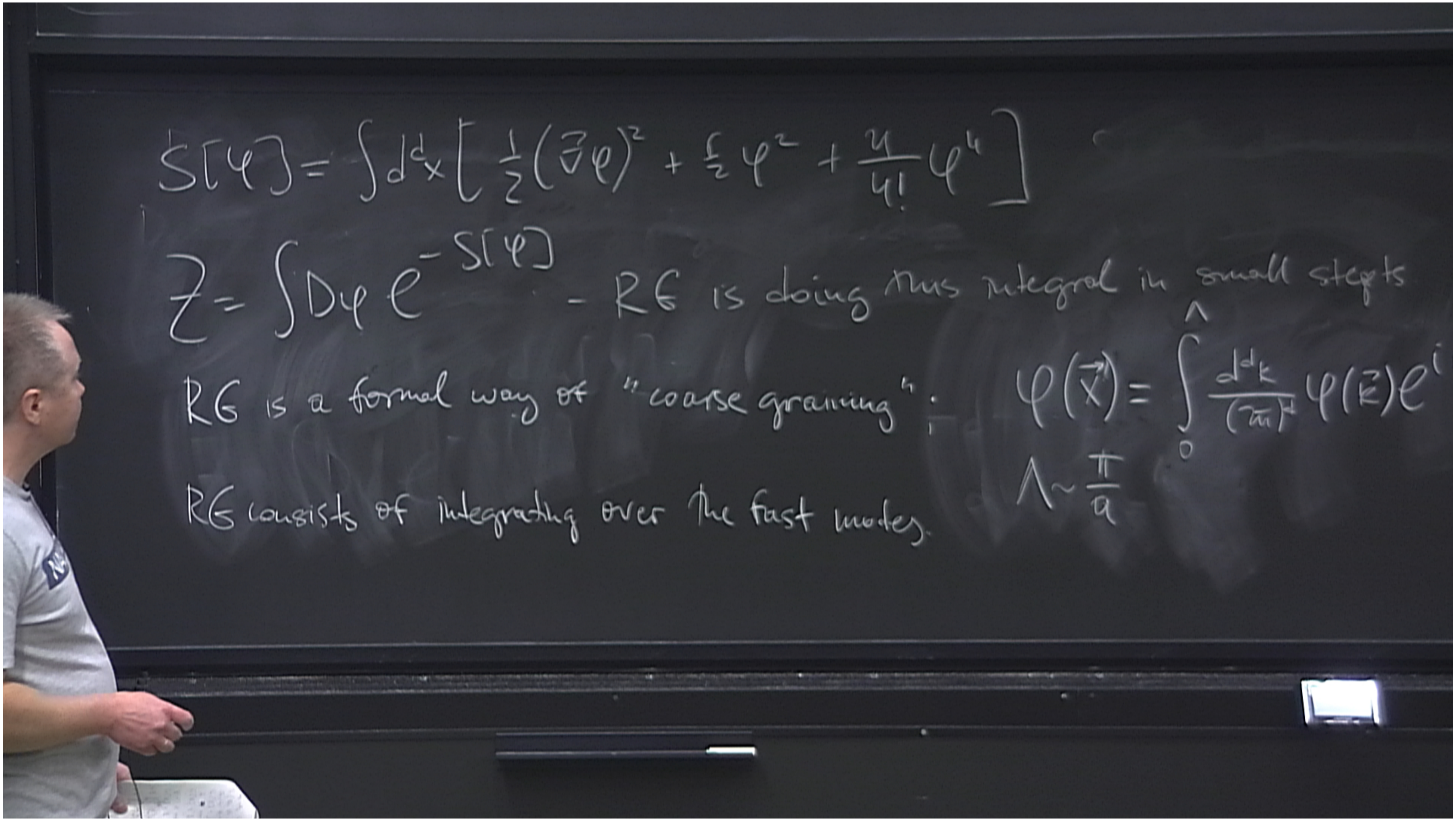
$$\varphi_>(\vec{x}) = \int_{\Lambda/b}^{\Lambda} \frac{d^dk}{(2\pi)^d}\, \varphi(\vec{k})\, e^{i\vec{k}\cdot\vec{x}}$$

...ll steps

$$\ldots \frac{\ldots k}{(2\pi)^d}\, \varphi(\vec{k})\, e^{i\vec{k}\cdot\vec{x}} = \varphi_<(\vec{x}) + \varphi_>(\vec{x})$$

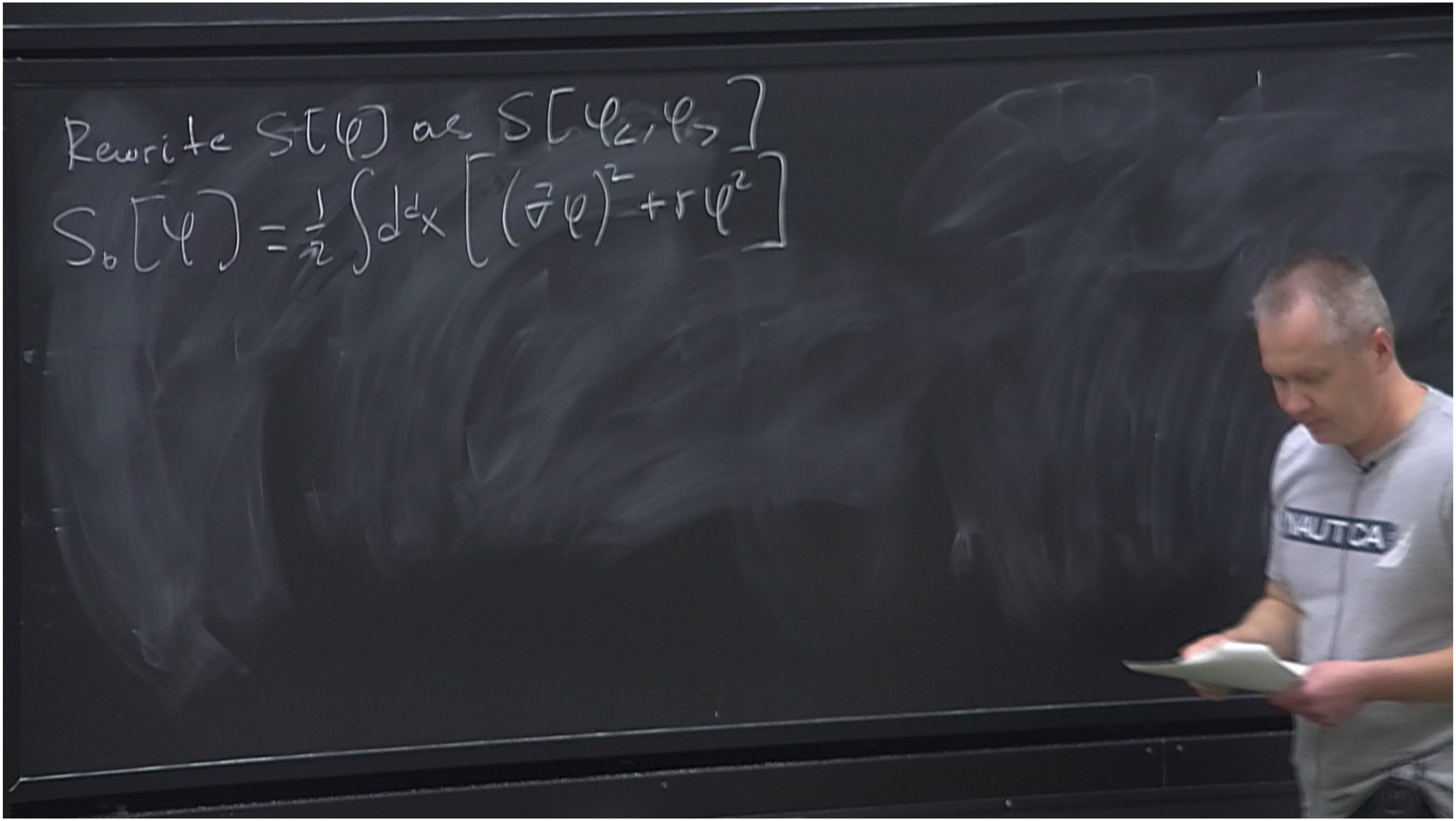
slow modes fast modes

$$b > 1 \;;\; \frac{\Lambda}{b} < \Lambda$$



$S[\varphi] = \int d^dx \left[ \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{u}{4!}\varphi^4 \right]$
$Z = \int D\varphi\, e^{-S[\varphi]}$ - RG is doing this integral in small steps
RG is a formal way of "coarse graining":
$\varphi(\vec{x}) = \int_0^\Lambda \frac{d^dk}{(2\pi)^d} \varphi(\vec{k}) e^{i}$
$\Lambda \sim \frac{\pi}{a}$
RG consists of integrating over the fast modes.

Rewrite $S[\varphi]$ as $S[\varphi_<, \varphi_>]$


Rewrite $S[\varphi]$ as $S[\varphi_<, \varphi_>]$
$S_0[\varphi] = \frac{1}{2}\int d^dx \left[ (\vec{\nabla}\varphi)^2 + r\varphi^2 \right]$

Rewrite $S[\varphi]$ as $S[\varphi_<, \varphi_>]$

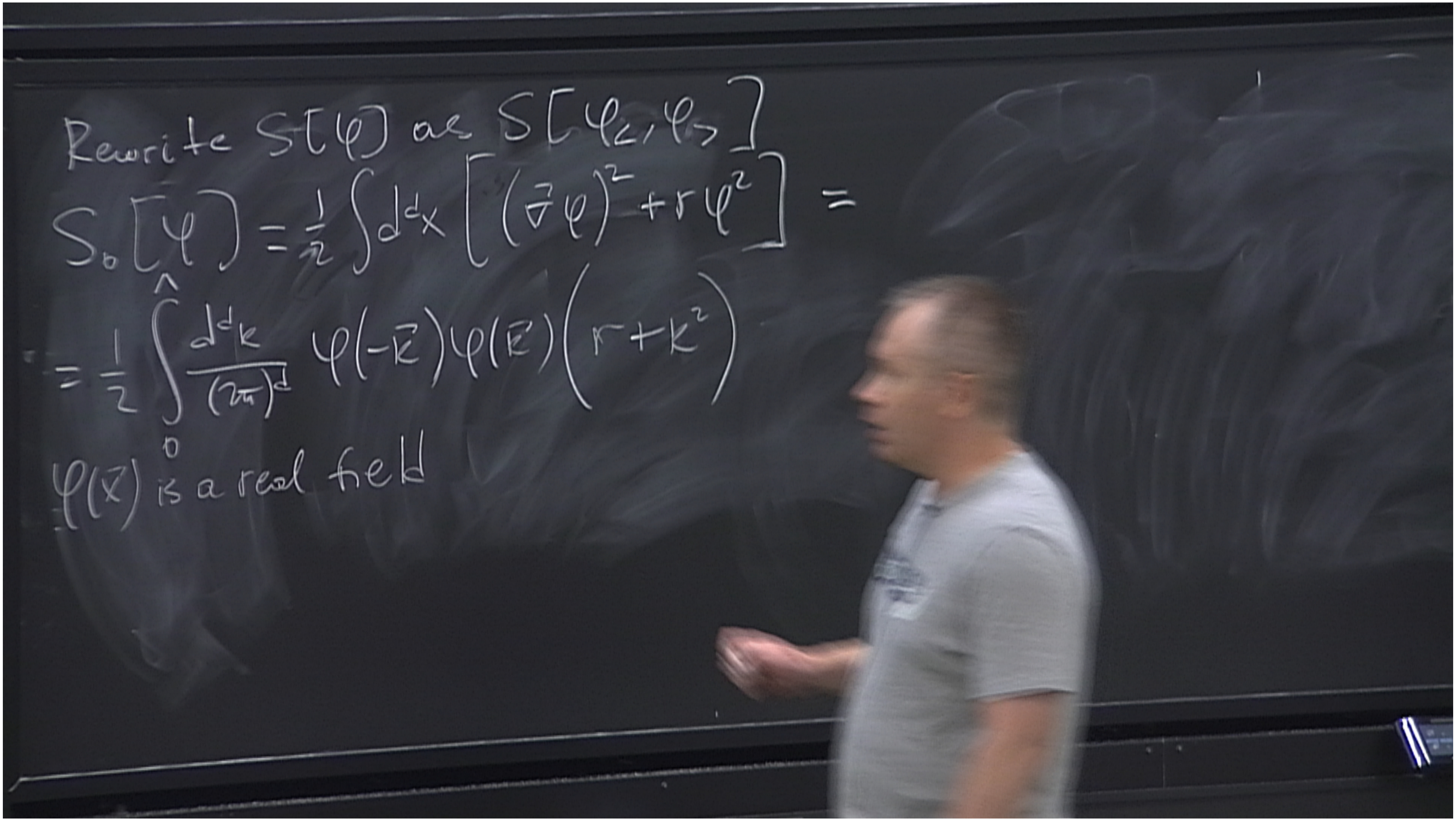
$$S_0[\varphi] = \frac{1}{2}\int d^dx \left[(\vec{\nabla}\varphi)^2 + r\varphi^2\right] =$$

$$= \frac{1}{2}\int_0^{\Lambda} \frac{d^dk}{(2\pi)^d}$$

Rewrite $S[\varphi]$ as $S[\varphi_<, \varphi_>]$

$$S_0[\varphi] = \frac{1}{2}\int d^dx \left[ (\vec{\nabla}\varphi)^2 + r\varphi^2 \right] =$$

$$= \frac{1}{2}\int_0^{\Lambda} \frac{d^dk}{(2\pi)^d} \varphi(-\vec{k})\varphi(\vec{k})\left(r + k^2\right)$$



Rewrite $S[\varphi]$ as $S[\varphi_<, \varphi_>]$
$S_0[\varphi] = \frac{1}{2}\int d^dx \left[(\vec{\nabla}\varphi)^2 + r\varphi^2\right] =$
$= \frac{1}{2}\int_0^{\Lambda} \frac{d^dk}{(2\pi)^d}\, \varphi(-\vec{k})\varphi(\vec{k})\left(r + k^2\right)$
$\varphi(\vec{x})$ is a real field

Rewrite $S[\varphi]$ as $S[\varphi_<, \varphi_>]$

$$S_0[\varphi] = \frac{1}{2}\int d^dx \left[ (\vec{\nabla}\varphi)^2 + r\varphi^2 \right] =$$

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$\varphi(\vec{x})$ is a real field

$$\varphi^*(\vec{x}) = \int_0^{\Lambda} \frac{d^dk}{(2\pi)^d}\, \varphi^*(\vec{k})\, e^{-i\vec{k}\cdot\vec{x}}$$

Rewrite $S[\varphi]$ as $S[\varphi_<, \varphi_>]$

$$S_0[\varphi] = \frac{1}{2}\int d^d x \left[ (\vec{\nabla}\varphi)^2 + r\varphi^2 \right] =$$

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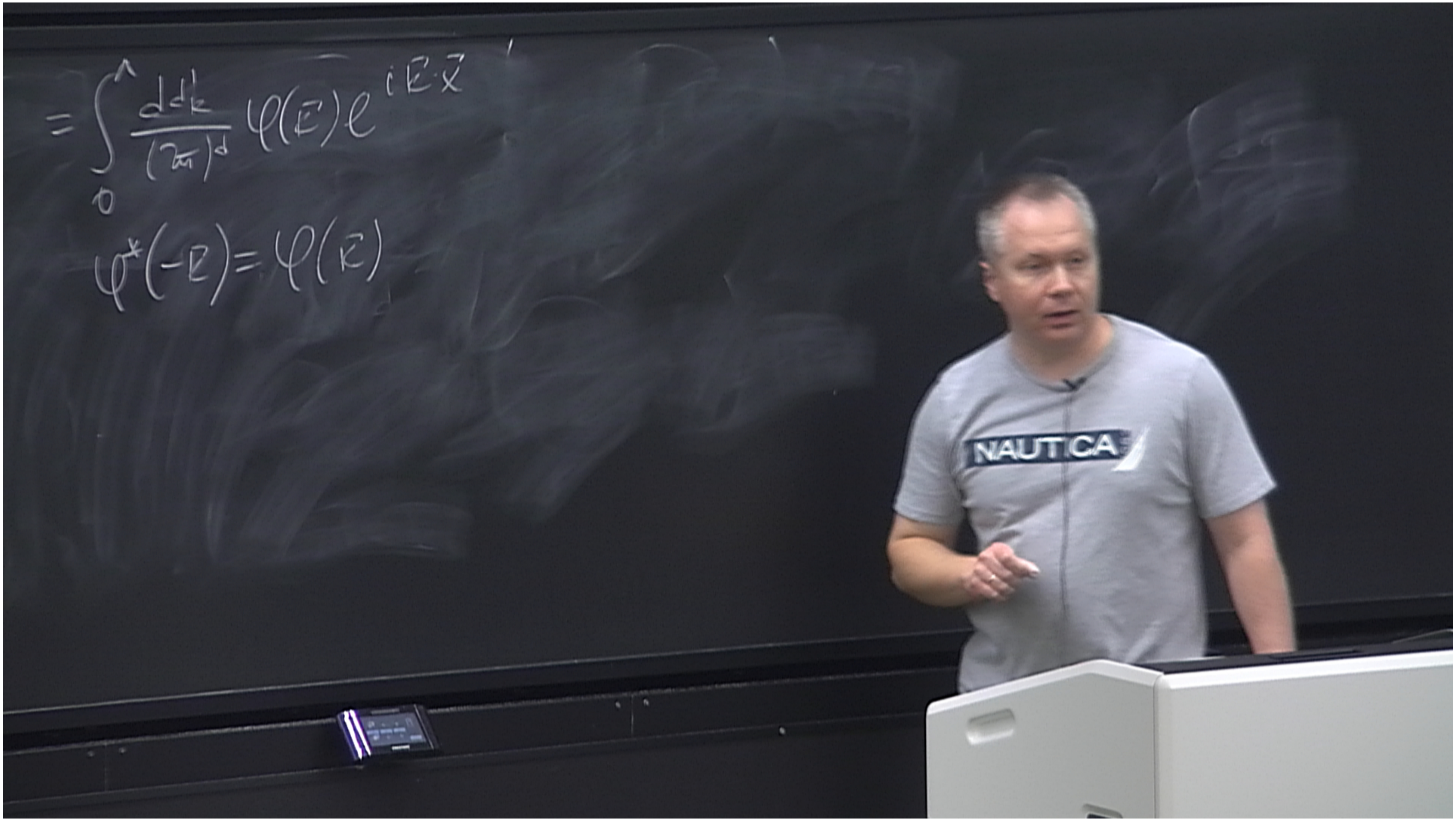
$\varphi(\vec{x})$ is a real field

$$\varphi^*(\vec{x}) = \int_0^{\Lambda} \frac{d^d k}{(2\pi)^d} \varphi^*(\vec{k}) e^{-i\vec{k}\cdot\vec{x}} = \int_0^{\Lambda} \frac{d^d k}{(2\pi)^d} \varphi^*(-\vec{k}) e^{i\vec{k}\cdot\vec{x}}$$


$= \int_0^{\Lambda} \frac{d^dk}{(2\pi)^d} \varphi(\vec{k}) e^{i\vec{k}\cdot\vec{x}}$
$\varphi^*(-\vec{k}) = \varphi(\vec{k})$
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Rewrite $S[\varphi]$ as $S[\varphi_<, \varphi_>]$

$$S_0[\varphi] = \frac{1}{2}\int d^dx \left[ (\vec{\nabla}\varphi)^2 + r\varphi^2 \right] =$$

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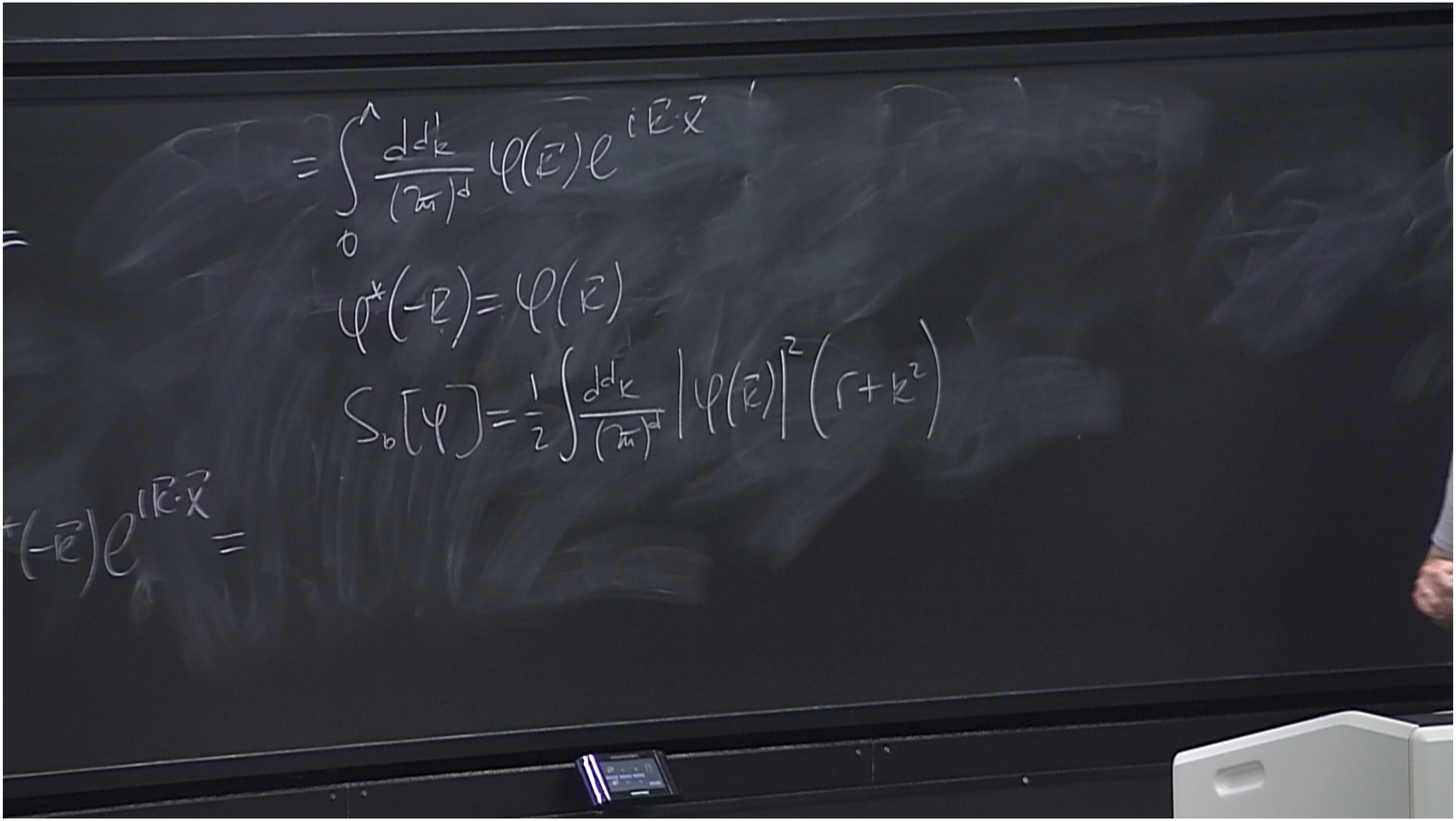
$\varphi(\vec{x})$ is a real field

$$\varphi^*(\vec{x}) = \int_0^{\Lambda} \frac{d^dk}{(2\pi)^d}\, \varphi^*(\vec{k})\, e^{-i\vec{k}\cdot\vec{x}} = \int_0^{\Lambda} \frac{d^dk}{(2\pi)^d}\, \varphi^*(-\vec{k})\, e^{i\vec{k}\cdot\vec{x}} =$$

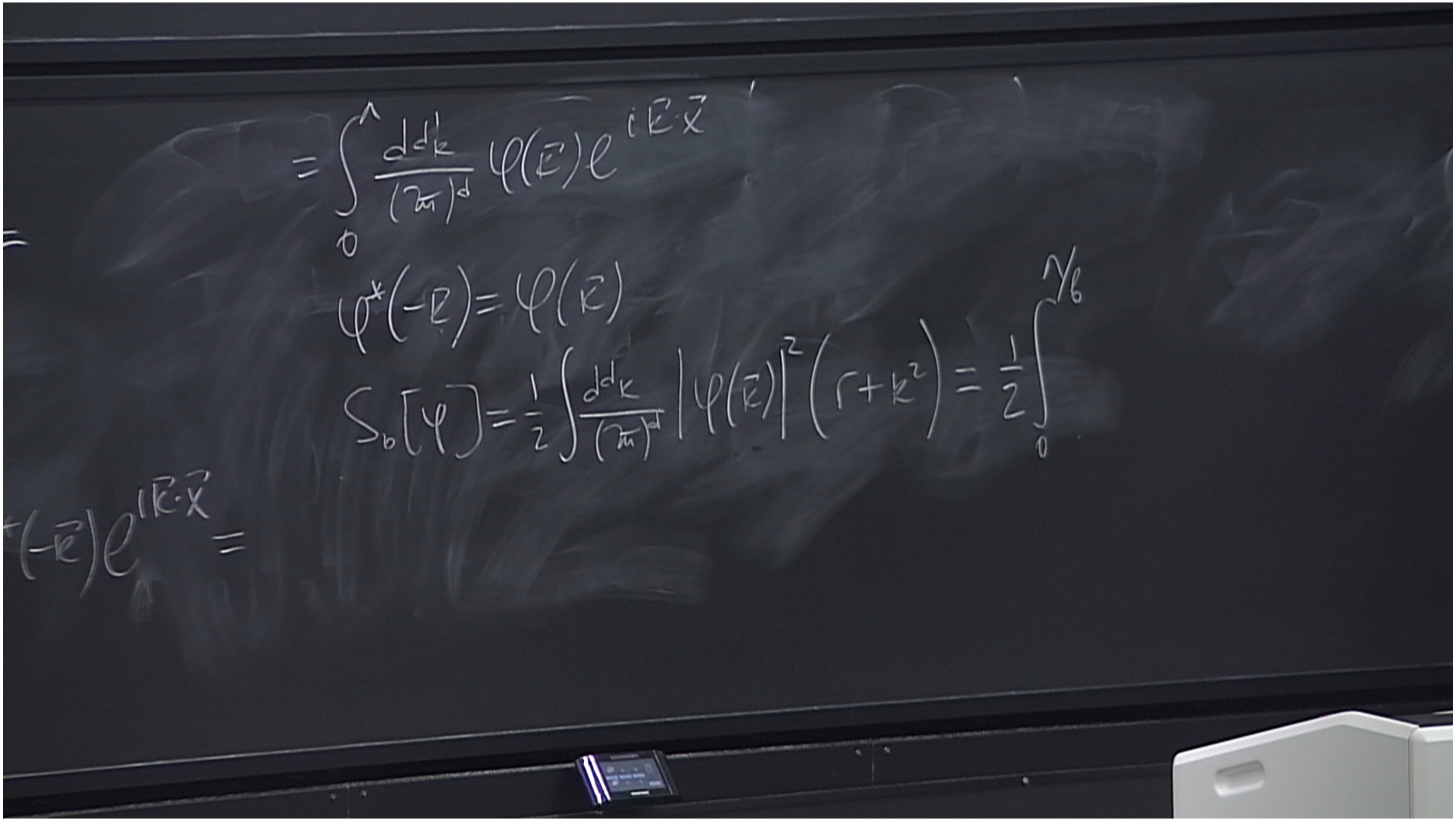
$$= \int_0^{\Lambda} \frac{d^dk}{(2\pi)^d}\, \varphi(\vec{k})\, e$$

$$\varphi^*(-\vec{k}) \ldots \varphi(\vec{k})$$

$$S_0[\varphi$$



$= \int_0^\Lambda \frac{d^d k}{(2\pi)^d} \varphi(\vec{k}) e^{i\vec{k}\cdot\vec{x}}$
$\varphi^*(-\vec{k}) = \varphi(\vec{k})$
$S_0[\varphi] = \frac{1}{2}\int \frac{d^d k}{(2\pi)^d} |\varphi(\vec{k})|^2 (r + k^2)$
$(-\vec{k}) e^{i\vec{k}\cdot\vec{x}} =$

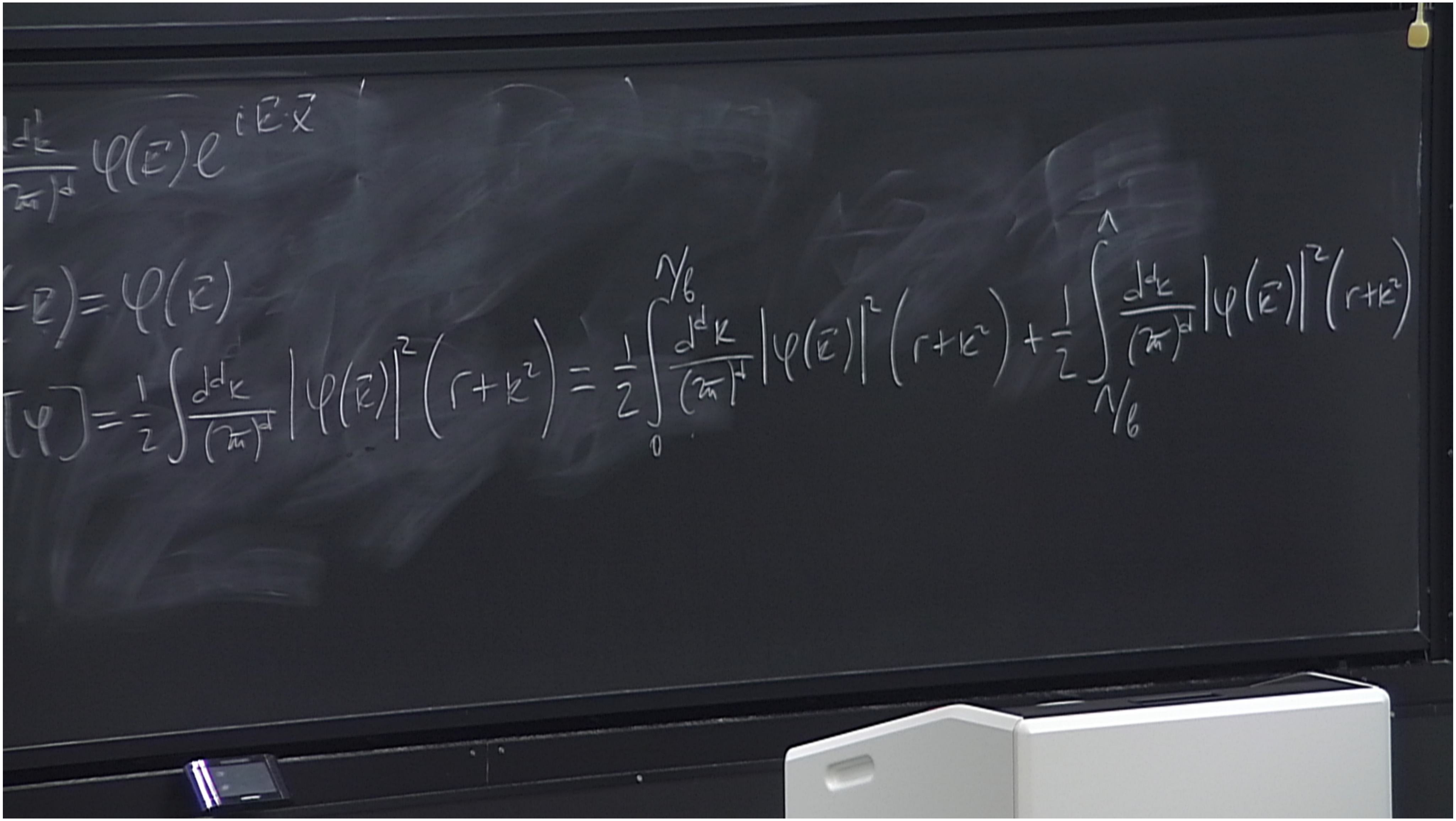

$= \int_0^\Lambda \frac{d^dk}{(2\pi)^d} \varphi(\vec{k}) e^{i\vec{k}\cdot\vec{x}}$
$\varphi^*(-\vec{k}) = \varphi(\vec{k})$
$S_0[\varphi] = \frac{1}{2}\int \frac{d^dk}{(2\pi)^d} |\varphi(\vec{k})|^2 (r+k^2) = \frac{1}{2}\int_0^{\Lambda/b}$
$(-\vec{k}) e^{i\vec{k}\cdot\vec{x}} =$

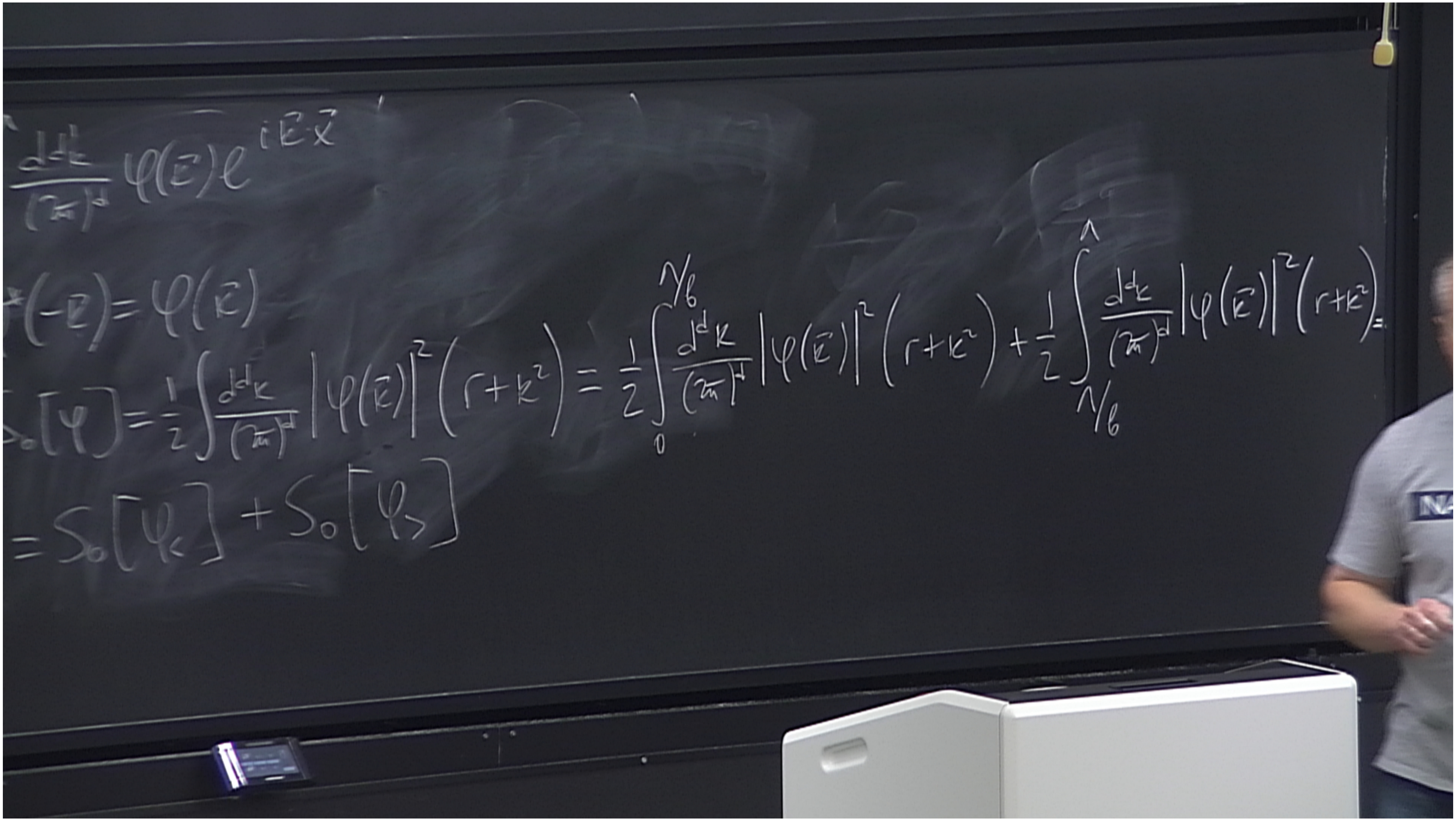
$$= \int_0^{\Lambda} \frac{d^dk}{(2\pi)^d}\, \varphi(\vec{k})\, e^{i\vec{k}\cdot\vec{x}}$$

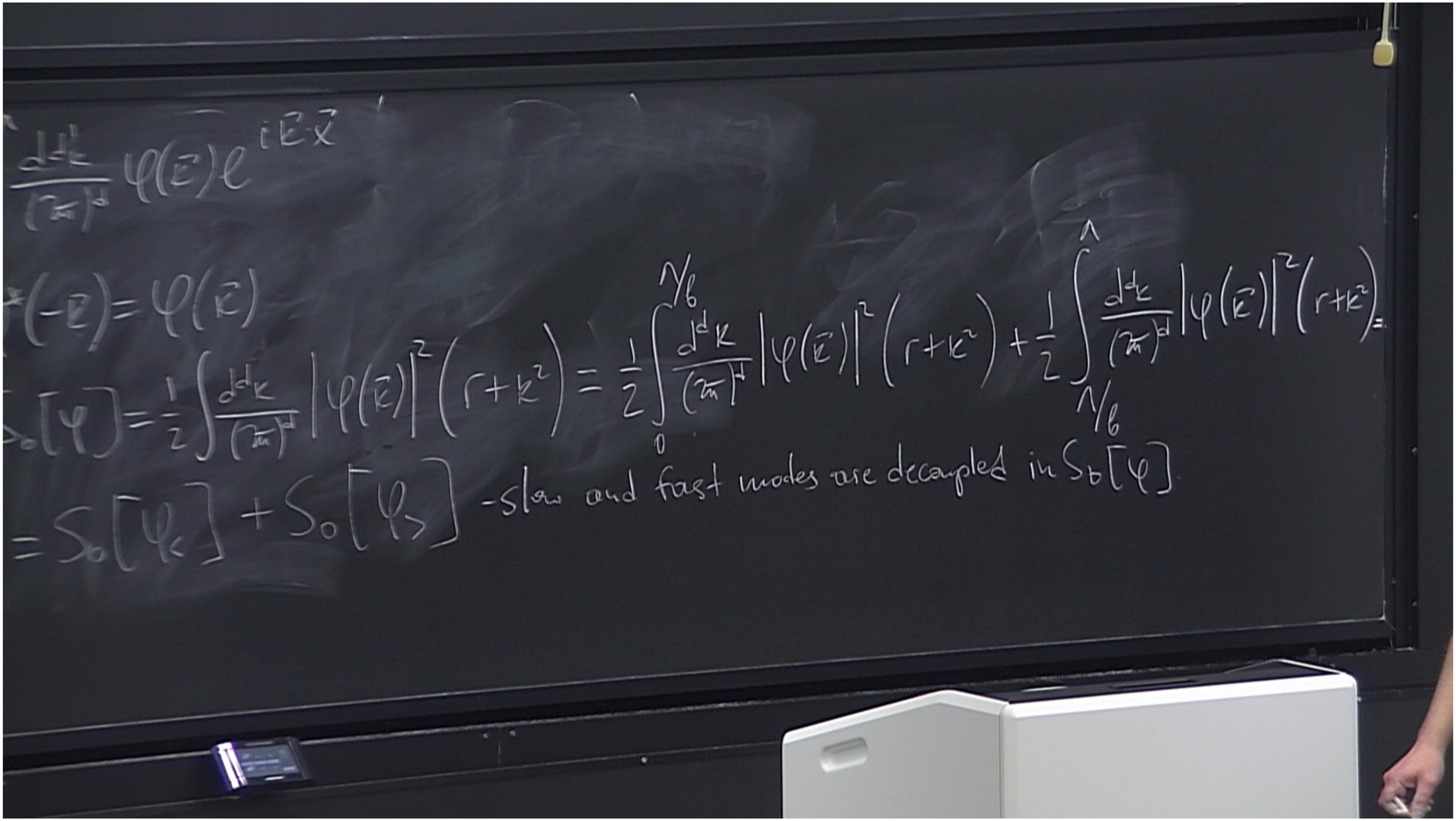
$$\varphi^*(-\vec{k}) = \varphi(\vec{k})$$

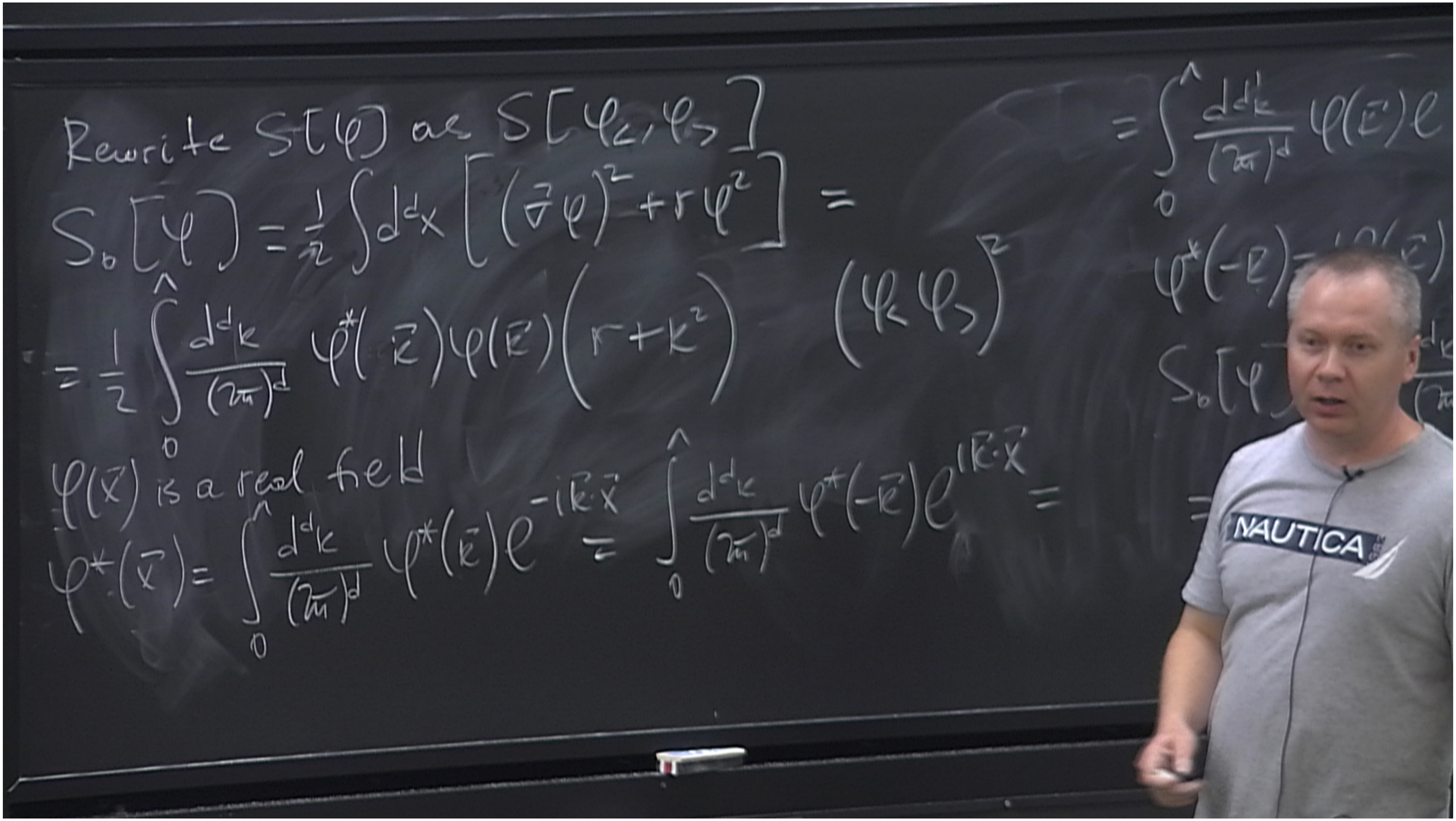
$$S_0[\varphi] = \frac{1}{2}\int \frac{d^dk}{(2\pi)^d}\, |\varphi(\vec{k})|^2 (r+k^2) = \frac{1}{2}\int_0^{\Lambda/b} \frac{d^dk}{(2\pi)^d}\, |\varphi(\vec{k})|^2 (r+k^2)$$

$$(-\vec{k})\, e^{i\vec{k}\cdot\vec{x}} =$$


$\frac{d^dk}{(2\pi)^d}\varphi(\vec{k})e^{i\vec{k}\cdot\vec{x}}$
$(-\vec{k})=\varphi(\vec{k})$
$[\varphi]=\frac{1}{2}\int\frac{d^dk}{(2\pi)^d}|\varphi(\vec{k})|^2(r+k^2)=\frac{1}{2}\int_0^{\Lambda/b}\frac{d^dk}{(2\pi)^d}|\varphi(\vec{k})|^2(r+k^2)+\frac{1}{2}\int_{\Lambda/b}^{\Lambda}\frac{d^dk}{(2\pi)^d}|\varphi(\vec{k})|^2(r+k^2)$


$\frac{d^dk}{(2\pi)^d}\varphi(\vec{k})e^{i\vec{k}\cdot\vec{x}}$
$(-\vec{k})=\varphi(\vec{k})$
$S_0[\varphi]=\frac{1}{2}\int\frac{d^dk}{(2\pi)^d}|\varphi(\vec{k})|^2(r+k^2)=\frac{1}{2}\int_0^{\Lambda/b}\frac{d^dk}{(2\pi)^d}|\varphi(\vec{k})|^2(r+k^2)+\frac{1}{2}\int_{\Lambda/b}^{\Lambda}\frac{d^dk}{(2\pi)^d}|\varphi(\vec{k})|^2(r+k^2)=$
$=S_0[\varphi_<]+S_0[\varphi_>]$


$\frac{d^dk}{(2\pi)^d}\varphi(\vec{k})e^{i\vec{k}\cdot\vec{x}}$
$\varphi^*(-\vec{k})=\varphi(\vec{k})$
$S_0[\varphi]=\frac{1}{2}\int\frac{d^dk}{(2\pi)^d}|\varphi(\vec{k})|^2(r+k^2)=\frac{1}{2}\int_0^{\Lambda/b}\frac{d^dk}{(2\pi)^d}|\varphi(\vec{k})|^2(r+k^2)+\frac{1}{2}\int_{\Lambda/b}^{\Lambda}\frac{d^dk}{(2\pi)^d}|\varphi(\vec{k})|^2(r+k^2)=$
$=S_0[\varphi_<]+S_0[\varphi_>]$ - slow and fast modes are decoupled in $S_0[\varphi]$.


Rewrite $S[\varphi]$ as $S[\varphi_<, \varphi_>]$
$S_0[\varphi] = \frac{1}{2}\int d^dx \left[(\vec\nabla\varphi)^2 + r\varphi^2\right] =$
$= \frac{1}{2}\int_0^\Lambda \frac{d^dk}{(2\pi)^d}\, \varphi^*(-\vec k)\varphi(\vec k)\left(r + k^2\right)$
$(\varphi_< \varphi_>)^2$
$\varphi(\vec x)$ is a real field
$\varphi^*(\vec x) = \int_0^\Lambda \frac{d^dk}{(2\pi)^d}\, \varphi^*(\vec k)\, e^{-i\vec k\cdot\vec x} = \int_0^\Lambda \frac{d^dk}{(2\pi)^d}\, \varphi^*(-\vec k)\, e^{i\vec k\cdot\vec x} =$
$= \int_0^\Lambda \frac{d^dk}{(2\pi)^d}\, \varphi(\vec k)\, e$
$\varphi^*(-\vec k) = \varphi(\vec k)$
$S_0[\varphi]$
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Rewrite $S[\varphi]$ as $S[\varphi_<, \varphi_>]$

$$S_0[\varphi_<] = \frac{1}{2}\int d^dx \left[(\vec{\nabla}\varphi_<)^2 + r\varphi_<^2\right] =$$

$$= \frac{1}{2}\int_0^{\Lambda} \frac{d^dk}{(2\pi)^d}\, \varphi^*(\vec{k})\varphi(\vec{k})\left(r + k^2\right)$$

$$(\varphi_< \varphi_>)^2$$

$\varphi(\vec{x})$ is a real field

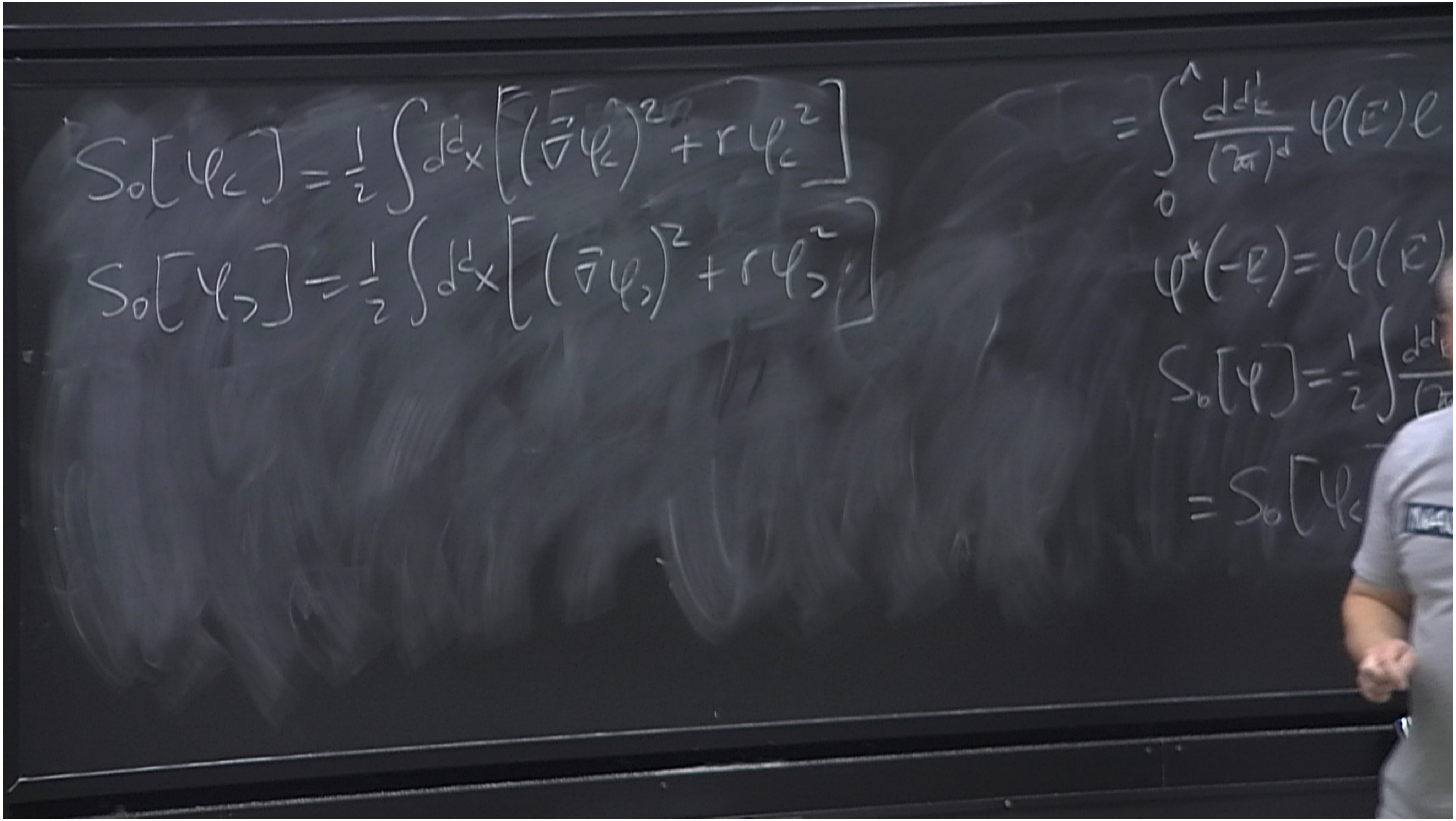
$$\varphi^*(\vec{x}) = \int_0^{\Lambda} \frac{d^dk}{(2\pi)^d}\, \varphi^*(\vec{k})\, e^{-i\vec{k}\cdot\vec{x}} = \int_0^{\Lambda} \frac{d^dk}{(2\pi)^d}\, \varphi^*(-\vec{k})\, e^{i\vec{k}\cdot\vec{x}} =$$

$$= \int_0^{\Lambda} \frac{d^dk}{(2\pi)^d}\, \varphi(\vec{k})\, e$$

$$\varphi^*(-\vec{k}) = \varphi(\vec{k})$$

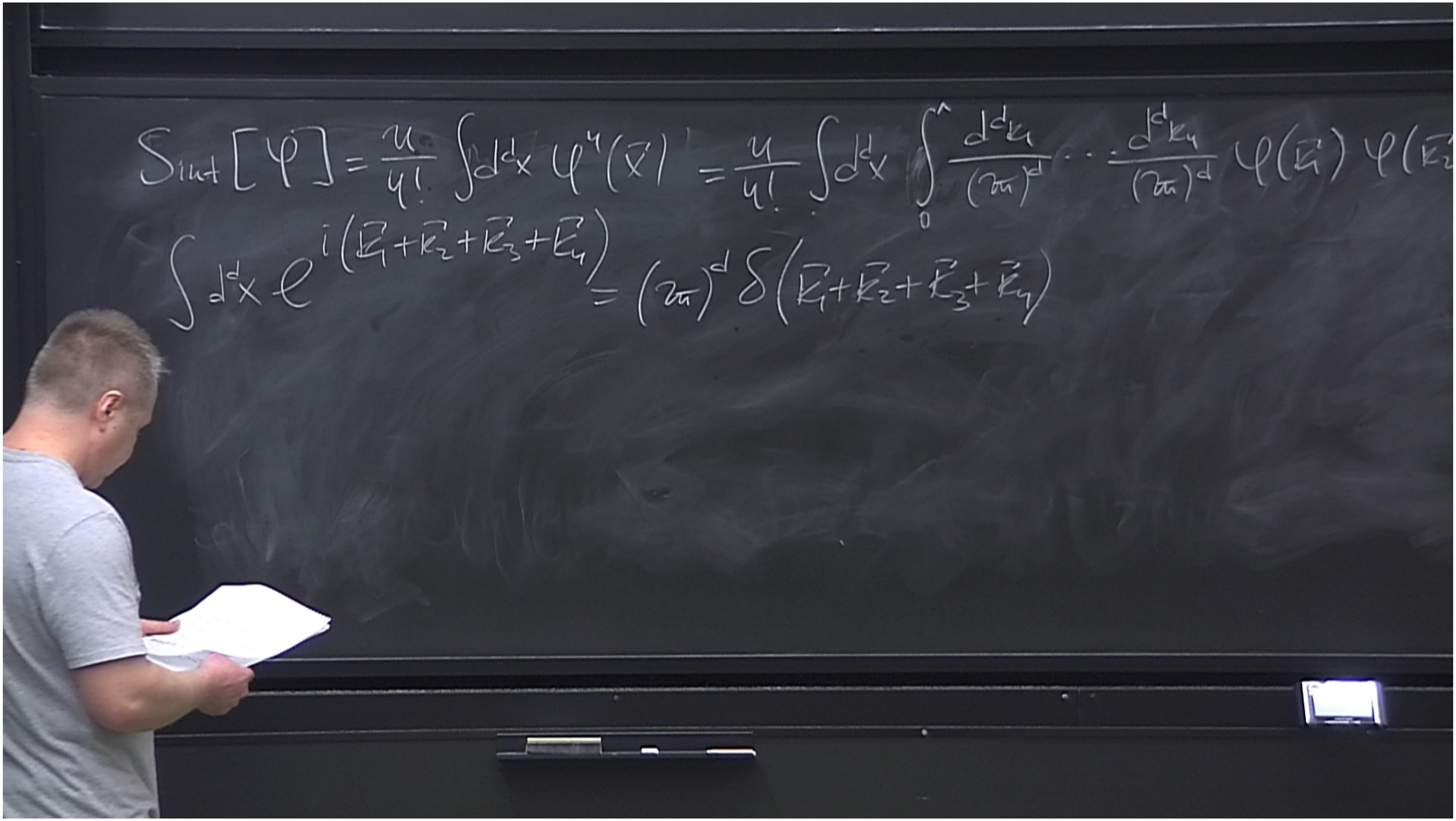
$$S_0[\varphi] = \frac{1}{2}\int \frac{d^dk}{(2\pi}$$

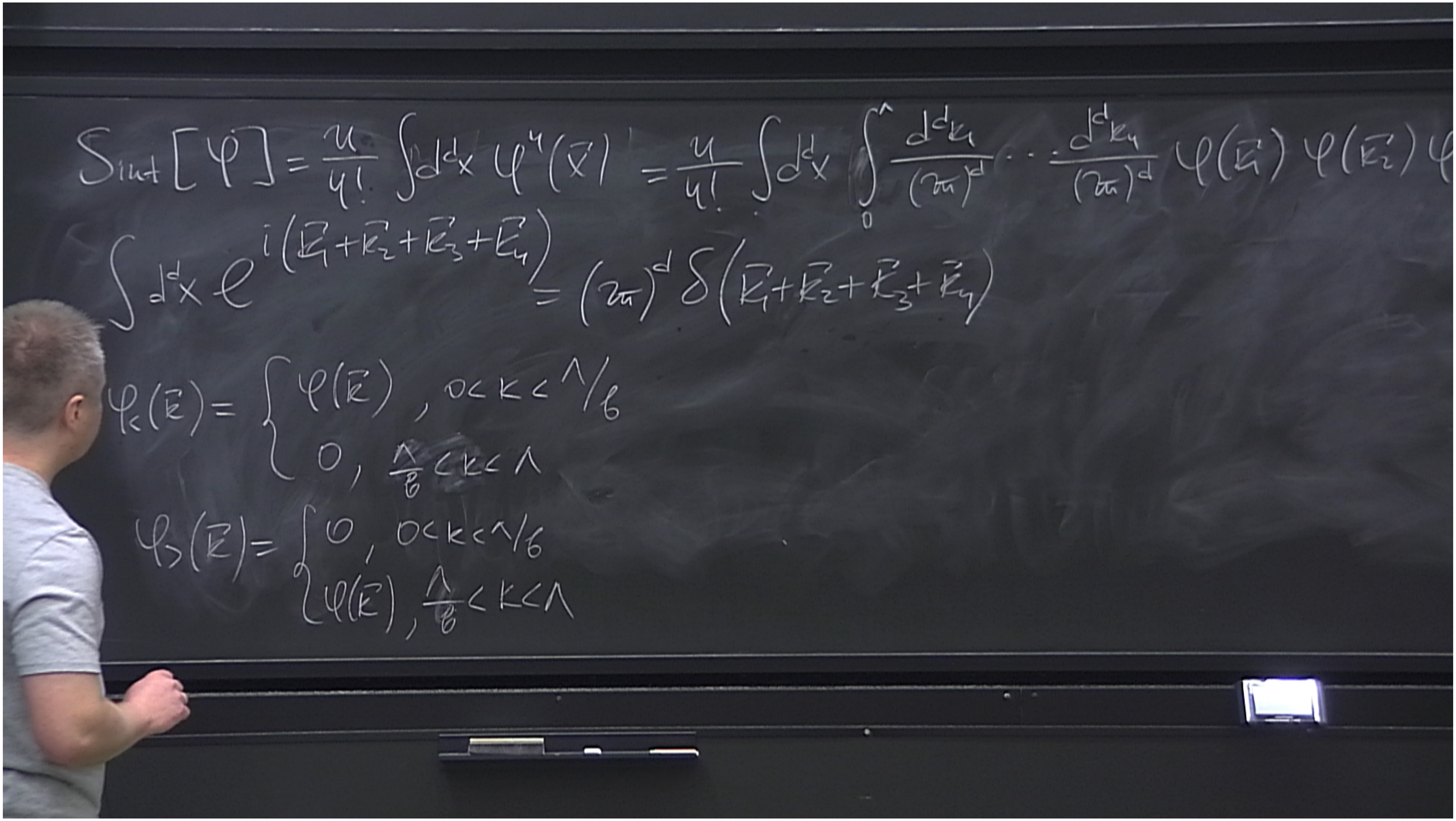
$$= S_0[\varphi_<] +$$


$S_0[\varphi_<] = \frac{1}{2}\int d^dx \left[ (\vec\nabla \varphi_<)^2 + r\varphi_<^2 \right]$
$S_0[\varphi_>] = \frac{1}{2}\int d^dx \left[ (\vec\nabla \varphi_>)^2 + r\varphi_>^2 \right]$
$= \int_0^\Lambda \frac{d^dk}{(2\pi)^d} \varphi(\vec k) e$
$\varphi^*(-\vec k) = \varphi(\vec k)$
$S_0[\varphi] = \frac{1}{2}\int$
$= S_0[\varphi_<$

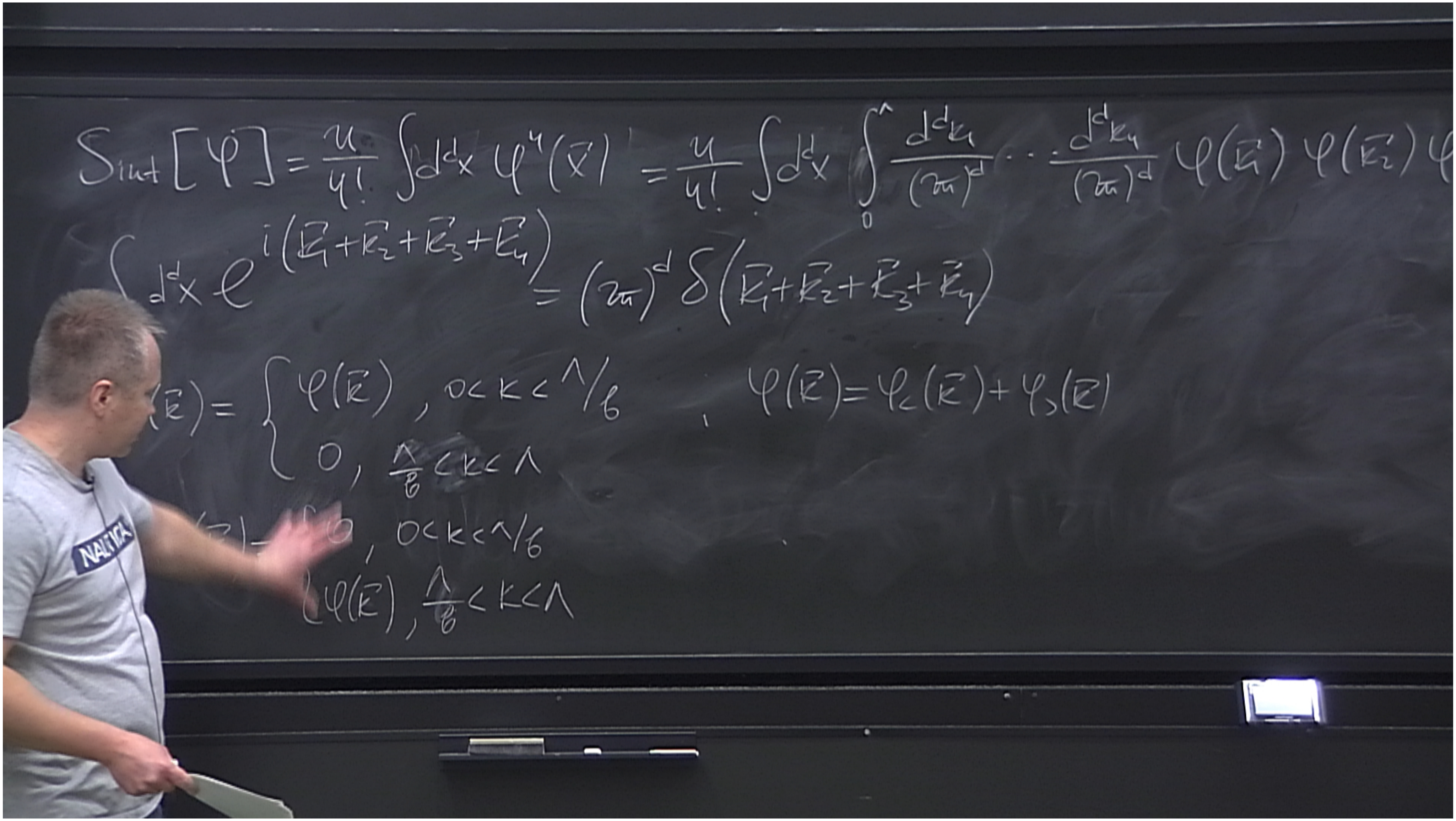
$$S_{int}[\varphi] = \frac{u}{4!}\int d^d x\, \varphi^4(\vec{x}) = \frac{u}{4!}\int d^d x \int_0^\Lambda \frac{d^d k_1}{(2\pi)^d} \cdots \frac{d^d k_4}{(2\pi)^d}$$

$$x \int_0^{\Lambda} \frac{d^d k_1}{(2\pi)^d} \cdots \frac{d^d k_4}{(2\pi)^d}\, \varphi(\vec{k}_1)\,\varphi(\vec{k}_2)\,\varphi(\vec{k}_3)\,\varphi(\vec{k}_4)\, e^{i(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)\cdot\vec{x}}$$


$S_{int}[\varphi] = \frac{u}{4!}\int d^dx\, \varphi^4(\vec{x}) = \frac{u}{4!}\int d^dx \int_0^\Lambda \frac{d^dk_1}{(2\pi)^d}\cdots\frac{d^dk_4}{(2\pi)^d}\, \varphi(\vec{k}_1)\, \varphi(\vec{k}_2$
$\int d^dx\, e^{i(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)} = (2\pi)^d\, \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)$



$S_{int}[\varphi] = \frac{u}{4!}\int d^dx\, \varphi^4(\vec{x}) = \frac{u}{4!}\int d^dx \int_0^\Lambda \frac{d^dk_1}{(2\pi)^d} \cdots \frac{d^dk_4}{(2\pi)^d}\, \varphi(\vec{k}_1)\, \varphi(\vec{k}_2)\, \varphi$
$\int d^dx\, e^{i(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)} = (2\pi)^d\, \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)$
$\varphi_<(\vec{k}) = \begin{cases} \varphi(\vec{k}), & 0<k<\Lambda/b \\ 0, & \frac{\Lambda}{b}<k<\Lambda \end{cases}$
$\varphi_>(\vec{k}) = \begin{cases} 0, & 0<k<\Lambda/b \\ \varphi(\vec{k}), & \frac{\Lambda}{b}<k<\Lambda \end{cases}$


$S_{int}[\varphi] = \frac{u}{4!}\int d^dx\, \varphi^4(\vec{x}) = \frac{u}{4!}\int d^dx \int_0^\Lambda \frac{d^dk_1}{(2\pi)^d} \cdots \frac{d^dk_4}{(2\pi)^d}\, \varphi(\vec{k}_1)\,\varphi(\vec{k}_2)\,\varphi$
$\int d^dx\, e^{i(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)} = (2\pi)^d\,\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)$
$(\vec{k}) = \begin{cases} \varphi(\vec{k}), & 0<k<\Lambda/b \\ 0, & \frac{\Lambda}{b}<k<\Lambda \end{cases}$
$\varphi(\vec{k}) = \varphi_<(\vec{k}) + \varphi_>(\vec{k})$
$0, \quad 0<k<\Lambda/b$
$\varphi(\vec{k}), \quad \frac{\Lambda}{b}<k<\Lambda$

$$S_{int}[\varphi] = \frac{u}{4!}\int d^dx\, \varphi^4(\vec{x}) = \frac{u}{4!}\int d^dx \int_0^\Lambda \frac{d^dk_1}{(2\pi)^d}\cdots\frac{d^dk_4}{(2\pi)^d}\,\varphi(\vec{k}_1)\,\varphi(\vec{k}_2)\,\varphi(\vec{k}_3$$

$$\int d^dx\, e^{i(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)} = (2\pi)^d\,\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)$$

$$= \frac{u}{4!}\int_0^\Lambda \frac{d^dk_1}{(2\pi)^d}\cdots\frac{d^dk_4}{(2\pi)}$$

$$\varphi_<(\vec{k}) = \begin{cases} \varphi(\vec{k}), & 0<k<\Lambda/b \\ 0, & \frac{\Lambda}{b}<k<\Lambda \end{cases}, \qquad \varphi(\vec{k}) = \varphi_<(\vec{k}) + \varphi_>(\vec{k})$$

$$\varphi_>(\vec{k}) = \begin{cases} 0, & 0<k<\Lambda/b \\ \varphi(\vec{k}), & \frac{\Lambda}{b}<k<\Lambda \end{cases}$$

$$\cdots \varphi(\vec{k}_1)\,\varphi(\vec{k}_2)\,\varphi(\vec{k}_3)\,\varphi(\vec{k}_4)\, e^{i(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)\cdot\vec{x}} =$$

$$= \frac{u}{4!}\int_0^{\Lambda} \frac{d^d k_1}{(2\pi)^d}\cdots\frac{d^d k_4}{(2\pi)^d}\left[\varphi_<(\vec{k}_1)+\varphi_>(\vec{k}_1)\right]\cdot\left[\varphi_<(\vec{k}_2)+\varphi_>(\vec{k}_2)\right]\cdot\left[\varphi_<(\vec{k}_3)+\varphi_>(\vec{k}_3)\right]\cdot$$

$$\cdot\left[\varphi_<(\vec{k}_4)+\varphi_>(\vec{k}_4)\right]\cdot(2\pi)^d\,\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)$$

$$\ldots \frac{}{)^d}\,\varphi(\vec{k}_1)\,\varphi(\vec{k}_2)\,\varphi(\vec{k}_3)\,\varphi(\vec{k}_4)\,e^{i(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)\cdot\vec{x}} =$$

$$= \frac{u}{4!}\int_0^{\Lambda}\frac{d^dk_1}{(2\pi)^d}\cdots\frac{d^dk_4}{(2\pi)^d}\left[\varphi_<(\vec{k}_1)+\varphi_>(\vec{k}_1)\right]\cdot\left[\varphi_<(\vec{k}_2)+\varphi_>(\vec{k}_2)\right]\cdot\left[\varphi_<(\vec{k}_3)+\varphi_>(\vec{k}_3)\right]\cdot$$

$$\cdot\left[\varphi_<(\vec{k}_4)+\varphi_>(\vec{k}_4)\right]\cdot(2\pi)^d\,\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)$$

– slow and fast modes are coupled

$$S_{int}[\varphi] = \frac{u}{4!}\int d^dx\, \varphi^4(\vec{x}) = \frac{u}{4!}\int d^dx \int_0^\Lambda \frac{d^dk_1}{(2\pi)^d}\cdots\frac{d^dk_4}{(2\pi)^d}\,\varphi(\vec{k}_1)\,\varphi(\vec{k}_2)\,\varphi(\vec{k}_3$$

$$\int d^dx\, e^{i(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)} = (2\pi)^d\,\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)$$

$$= \frac{u}{4!}\int_0^\Lambda \frac{d^dk_1}{(2\pi)^d}\cdots\frac{d^dk_4}{(2\pi)}$$

$$\varphi_<(\vec{k}) = \begin{cases}\varphi(\vec{k}), & 0<k<\Lambda/b \\ 0, & \frac{\Lambda}{b}<k<\Lambda\end{cases}, \qquad \varphi(\vec{k}) = \varphi_<(\vec{k}) + \varphi_>(\vec{k})$$

$$\varphi_>(\vec{k}) = \begin{cases}0, & 0<k<\Lambda/b \\ \varphi(\vec{k}), & \frac{\Lambda}{b}<k<\Lambda\end{cases}$$

$$S_{int}[\varphi_<, \varphi_>]$$

$$Z = \int D\varphi \, e^{-S[\varphi]} = \int D\varphi_> D\varphi_< \, e^{-S[\varphi_<, \varphi_>]}$$