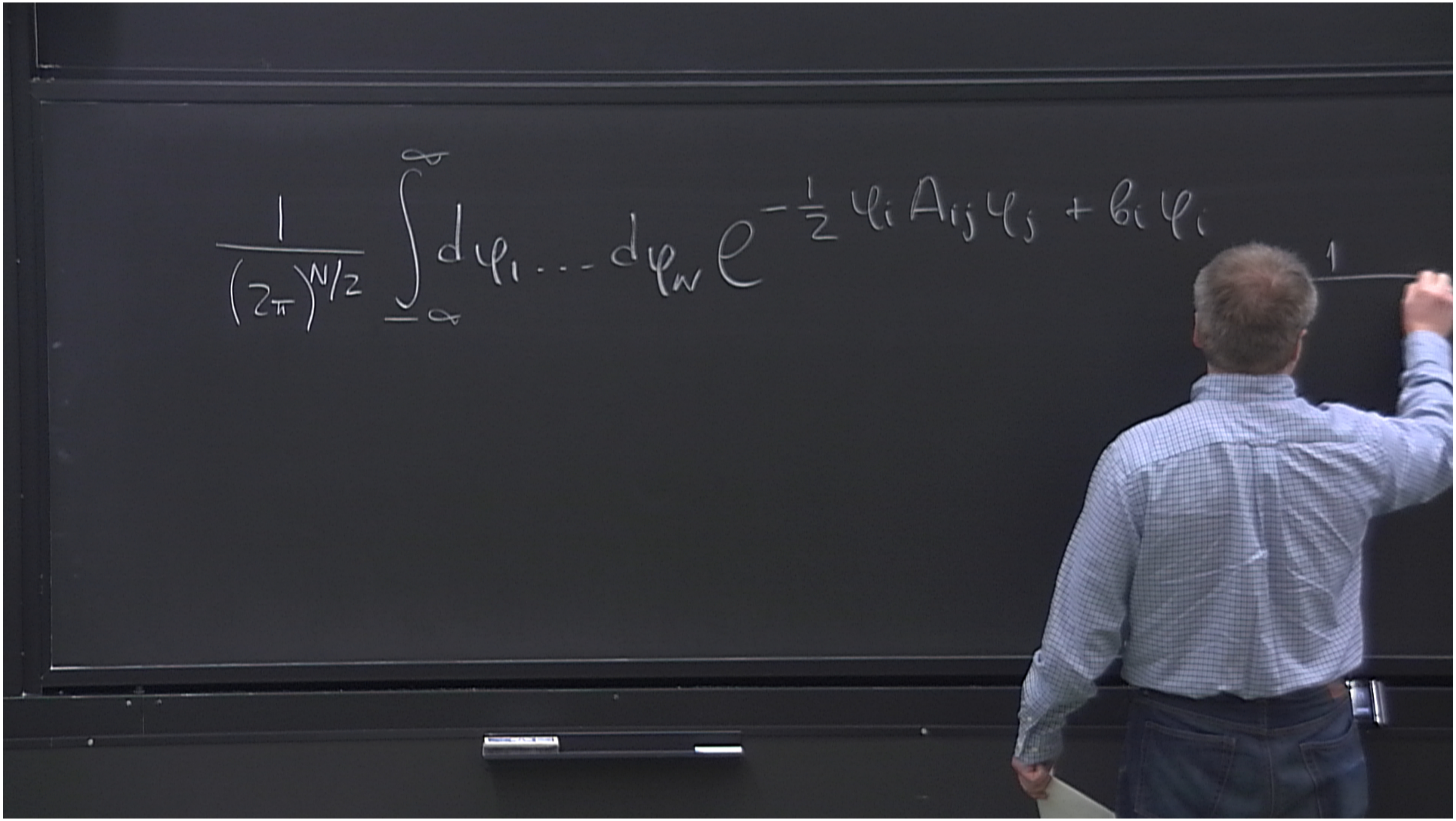
Title: 14/15 PSI - Statistical Mechanics - Lecture 3

Date: Oct 08, 2014 10:45 AM

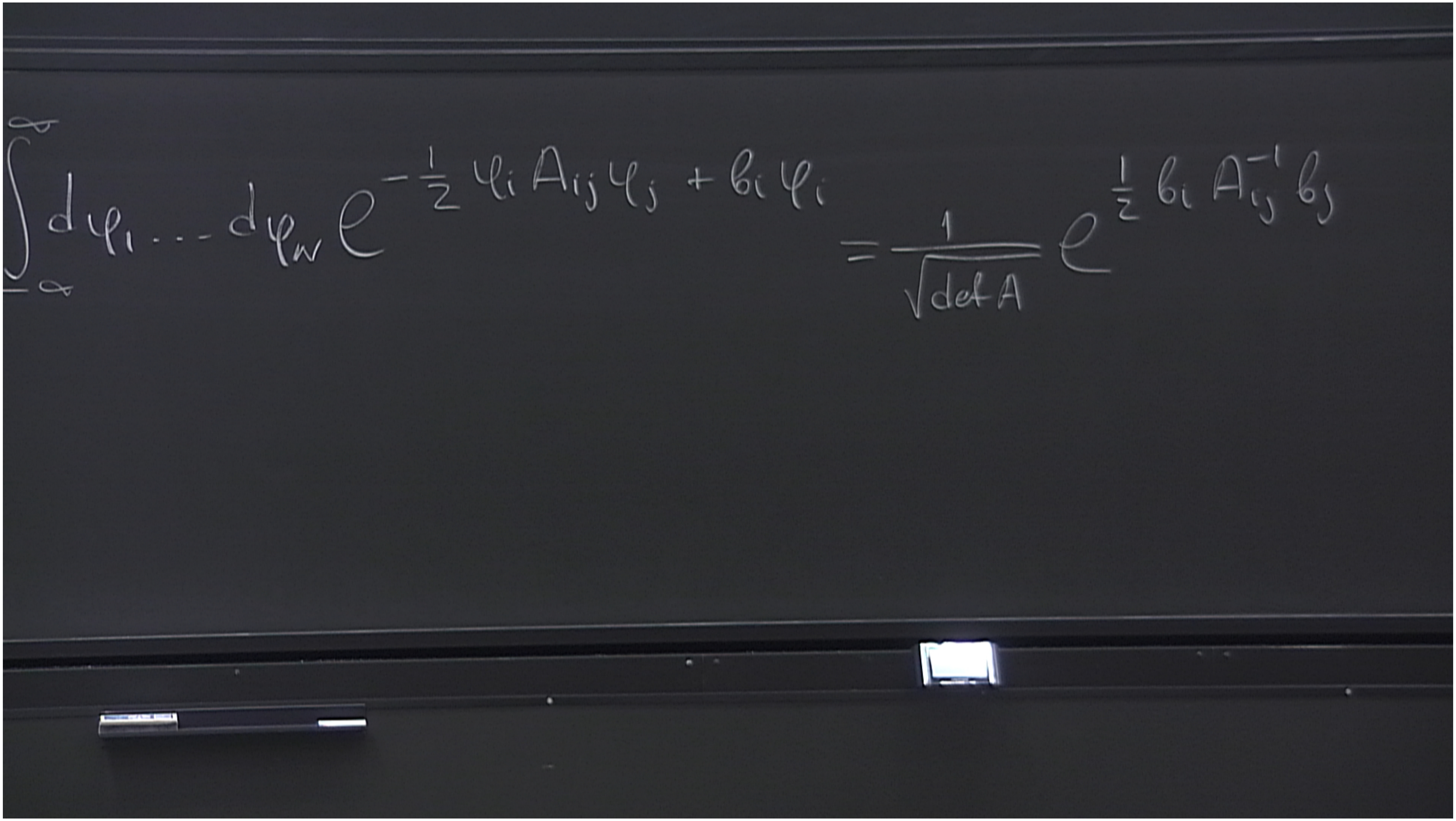
URL: http://pirsa.org/14100086

Abstract:

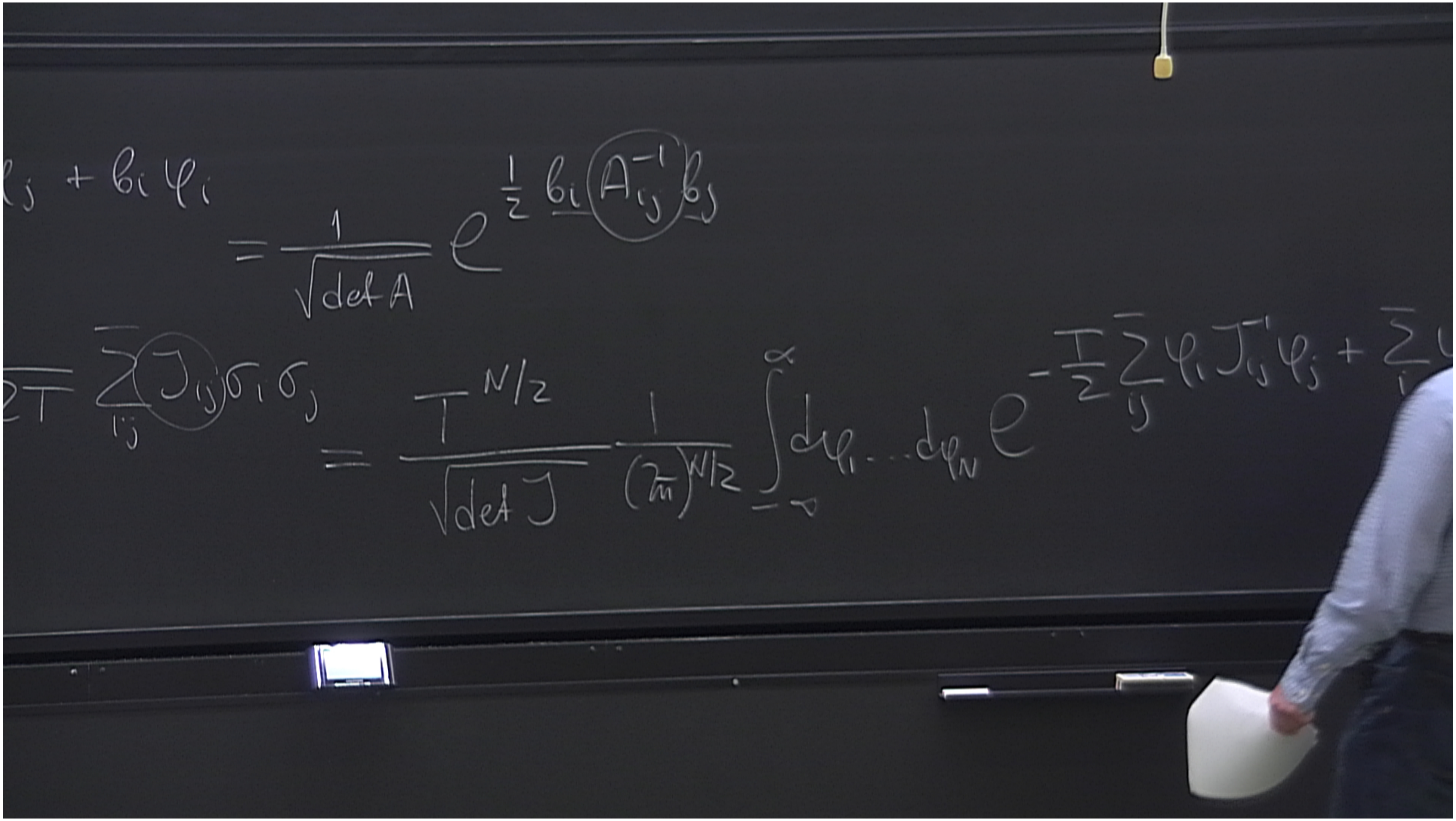


$$\frac{1}{(2\pi)^{N/2}}\int_{-\infty}^{\infty} d\varphi_1 \ldots d\varphi_N\, e^{-\frac{1}{2}\varphi_i A_{ij}\varphi_j + b_i\varphi_i}$$

$$\frac{1}{\quad}$$



$$\int_{-\infty}^{\infty} d\varphi_1 \dots d\varphi_N \, e^{-\frac{1}{2}\varphi_i A_{ij}\varphi_j + b_i\varphi_i} = \frac{1}{\sqrt{\det A}} \, e^{\frac{1}{2} b_i A^{-1}_{ij} b_j}$$

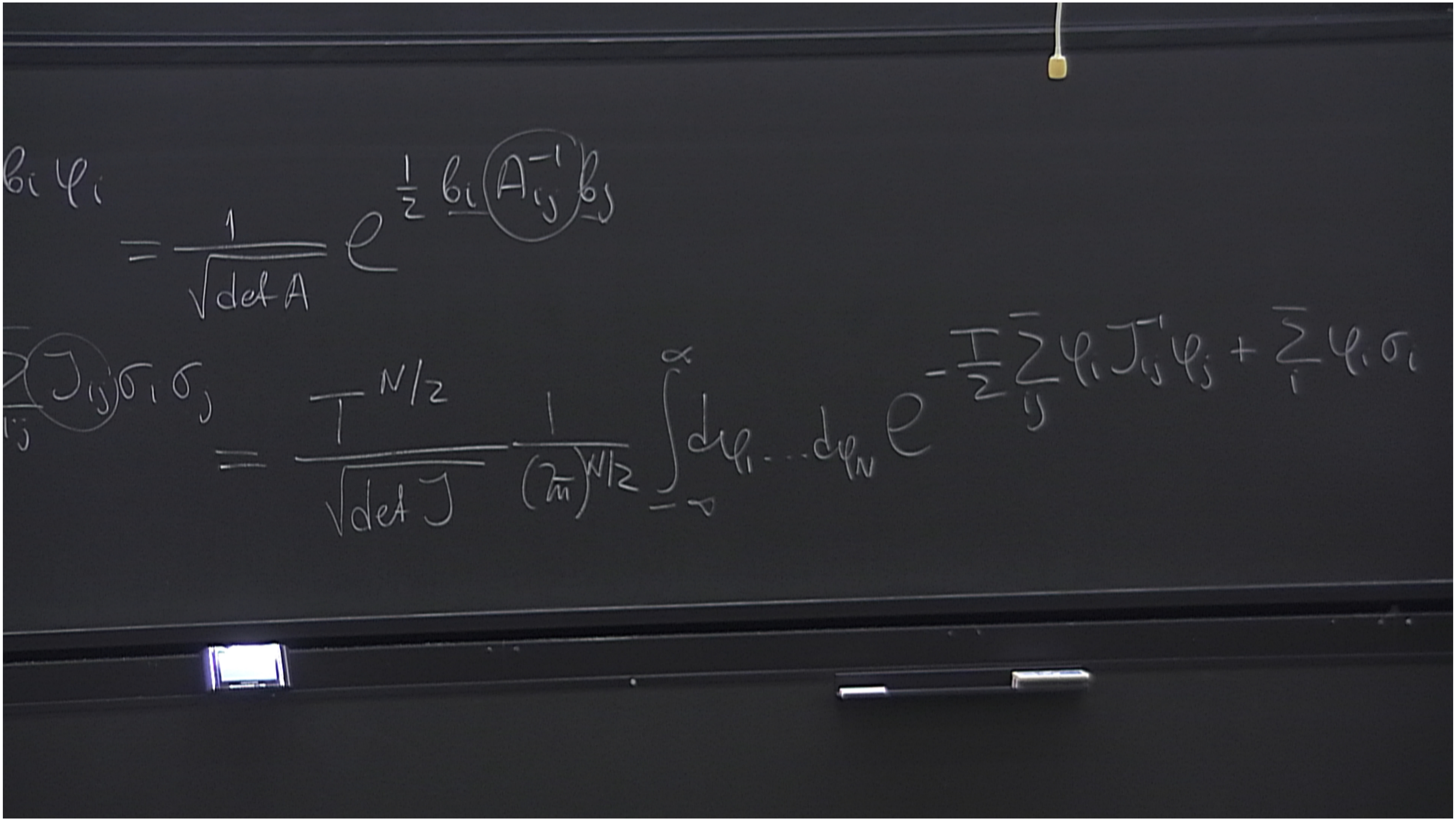


$$\dots \varphi_j + b_i \varphi_i$$

$$= \frac{1}{\sqrt{\det A}} e^{\frac{1}{2} \underline{b_i} \boxed{A^{-1}_{ij}} \underline{b_j}}$$

$$\dots 2T \sum_{ij} \boxed{J_{ij}} \sigma_i \sigma_j$$

$$= \frac{T^{N/2}}{\sqrt{\det J}} \frac{1}{(2\pi)^{N/2}} \int_{-\infty}^{\infty} d\varphi_1 \dots d\varphi_N \, e^{-\frac{T}{2} \sum_{ij} \varphi_i J^{-1}_{ij} \varphi_j + \sum_i \dots}$$



$$b_i \varphi_i$$

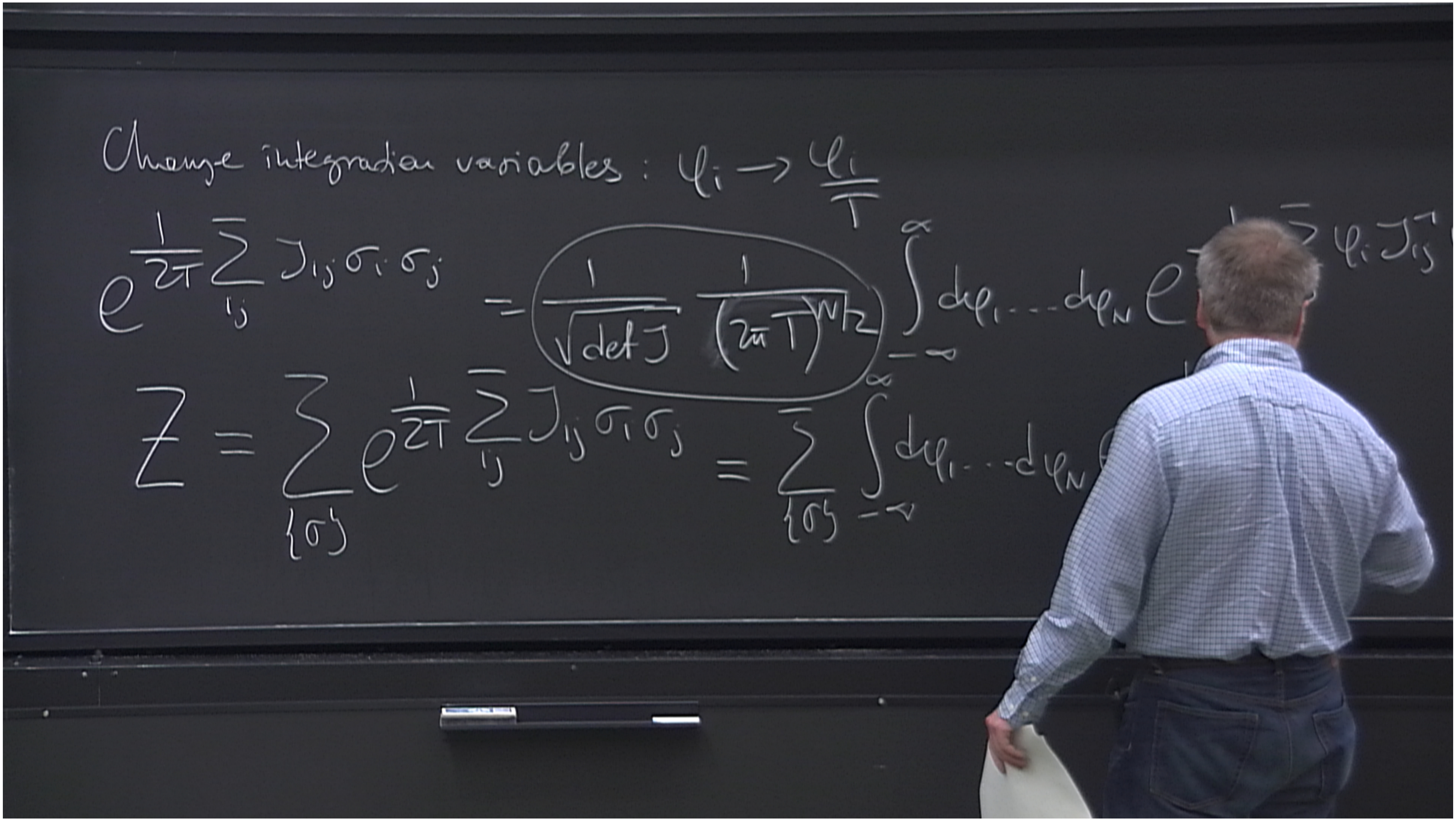
$$= \frac{1}{\sqrt{\det A}} e^{\frac{1}{2} \underline{b_i} \boxed{A^{-1}_{ij}} \underline{b_j}}$$

$$\sum_{ij} \boxed{J_{ij}} \sigma_i \sigma_j$$

$$= \frac{T^{N/2}}{\sqrt{\det J}} \frac{1}{(2\pi)^{N/2}} \int_{-\infty}^{\infty} d\varphi_1 \dots d\varphi_N \, e^{-\frac{T}{2} \sum_{ij} \varphi_i J^{-1}_{ij} \varphi_j + \sum_i \varphi_i \sigma_i}$$

riables : $\varphi_i \to \frac{\varphi_i}{T}$

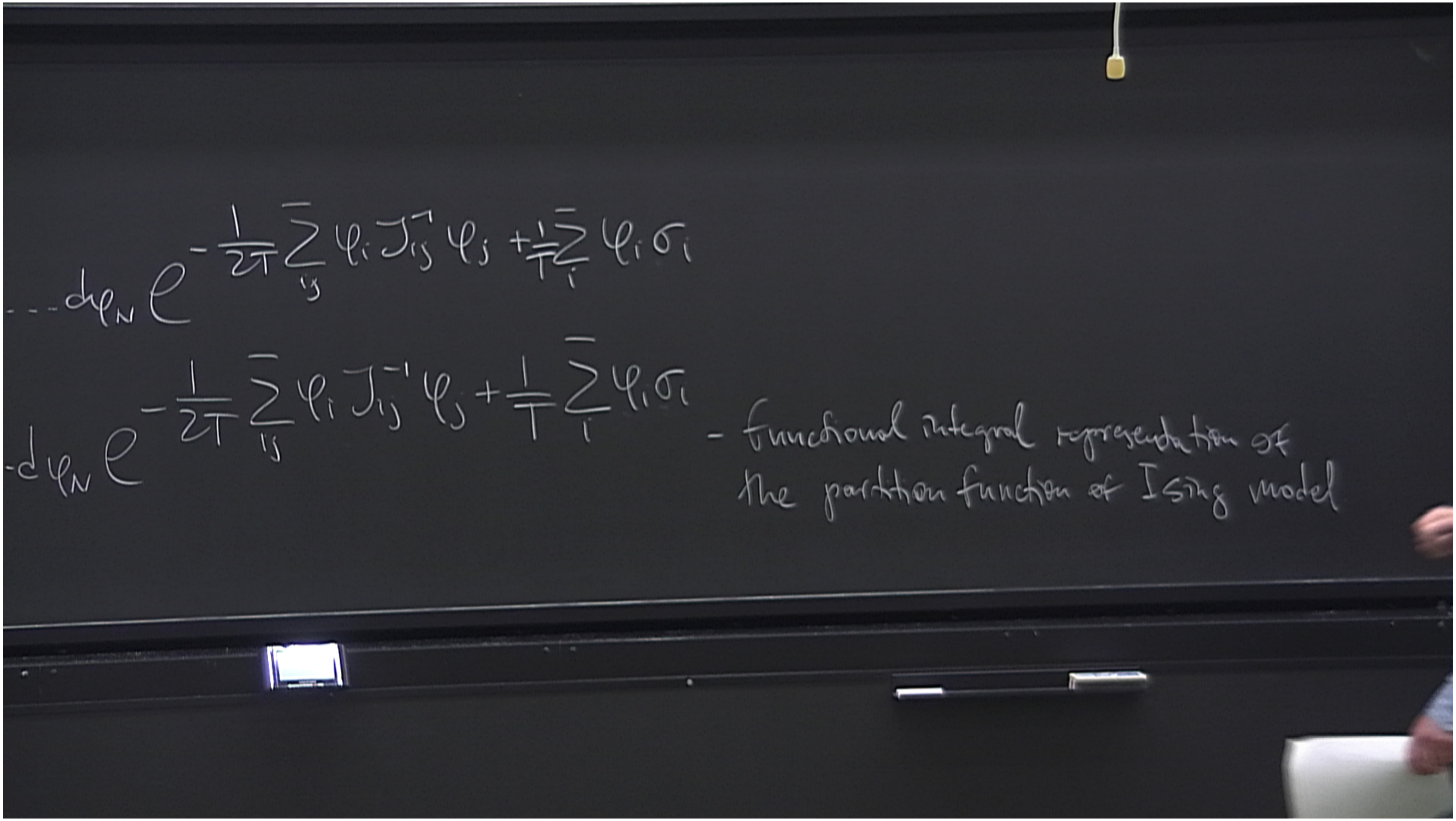
$$= \frac{1}{\sqrt{\det J}} \frac{1}{(2\pi T)^{N/2}} \int_{-\infty}^{\infty} d\varphi_1 \cdots d\varphi_N \, e^{-\frac{1}{2T}\sum_{ij} \varphi_i J_{ij}^{-1} \varphi_j + \frac{1}{T}\sum_i \varphi_i \sigma_i}$$



Change integration variables: $\varphi_i \to \frac{\varphi_i}{T}$

$$e^{\frac{1}{2T}\sum_{ij} J_{ij}\sigma_i\sigma_j} = \boxed{\frac{1}{\sqrt{\det J}}\,\frac{1}{(2\pi T)^{N/2}}}\int_{-\infty}^{\infty} d\varphi_1\ldots d\varphi_N\, e^{-\frac{1}{\,}\sum \varphi_i J^{-1}_{ij}}$$

$$Z = \sum_{\{\sigma\}} e^{\frac{1}{2T}\sum_{ij} J_{ij}\sigma_i\sigma_j} = \sum_{\{\sigma\}}\int_{-\infty}^{\infty} d\varphi_1\ldots d\varphi_N\, e$$

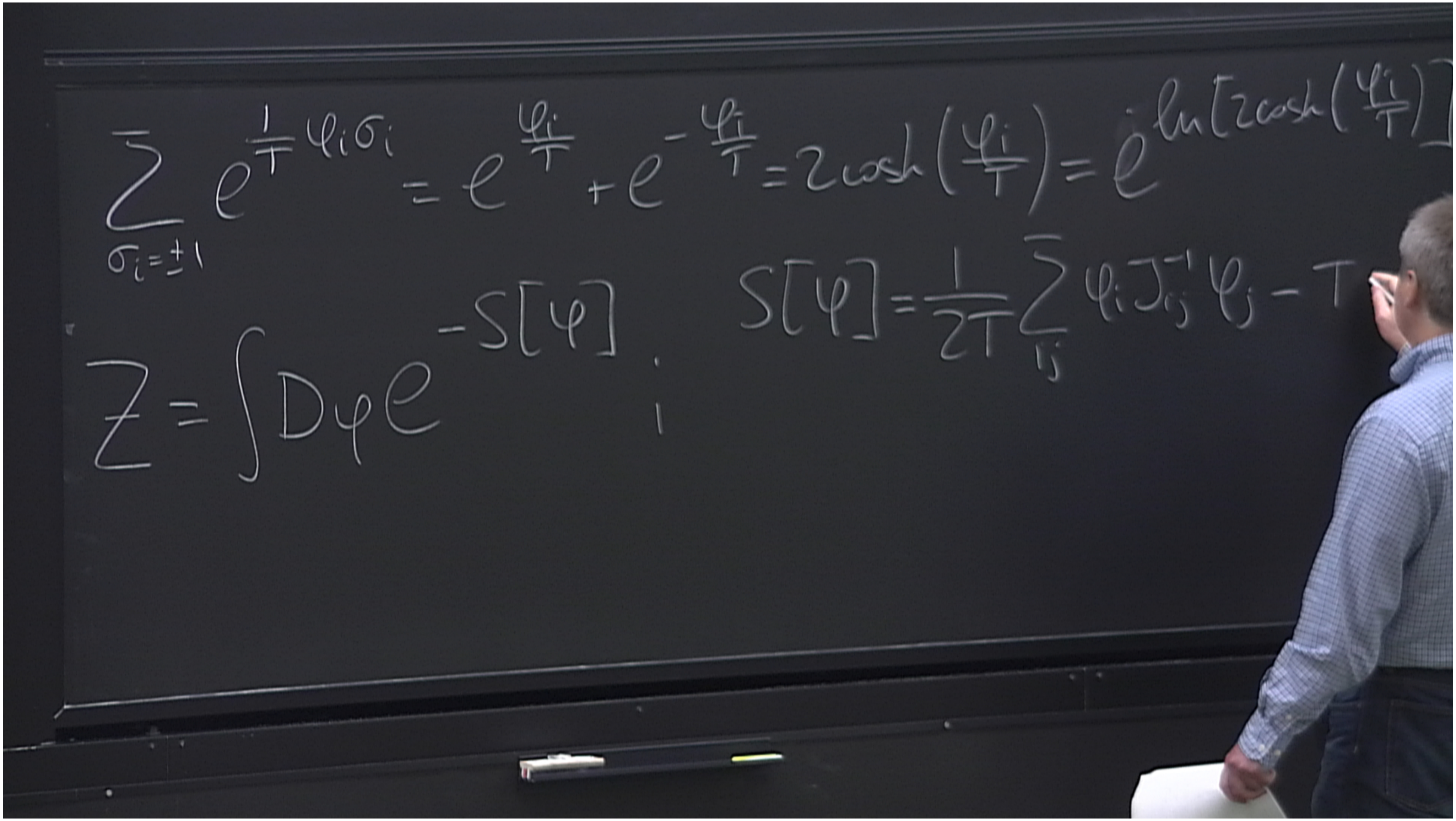


$$\ldots d\varphi_N \, e^{-\frac{1}{2T}\sum_{ij}\varphi_i J^{-1}_{ij}\varphi_j + \frac{1}{T}\sum_i \varphi_i \sigma_i}$$

$$- d\varphi_N \, e^{-\frac{1}{2T}\sum_{ij}\varphi_i J^{-1}_{ij}\varphi_j + \frac{1}{T}\sum_i \varphi_i \sigma_i}$$

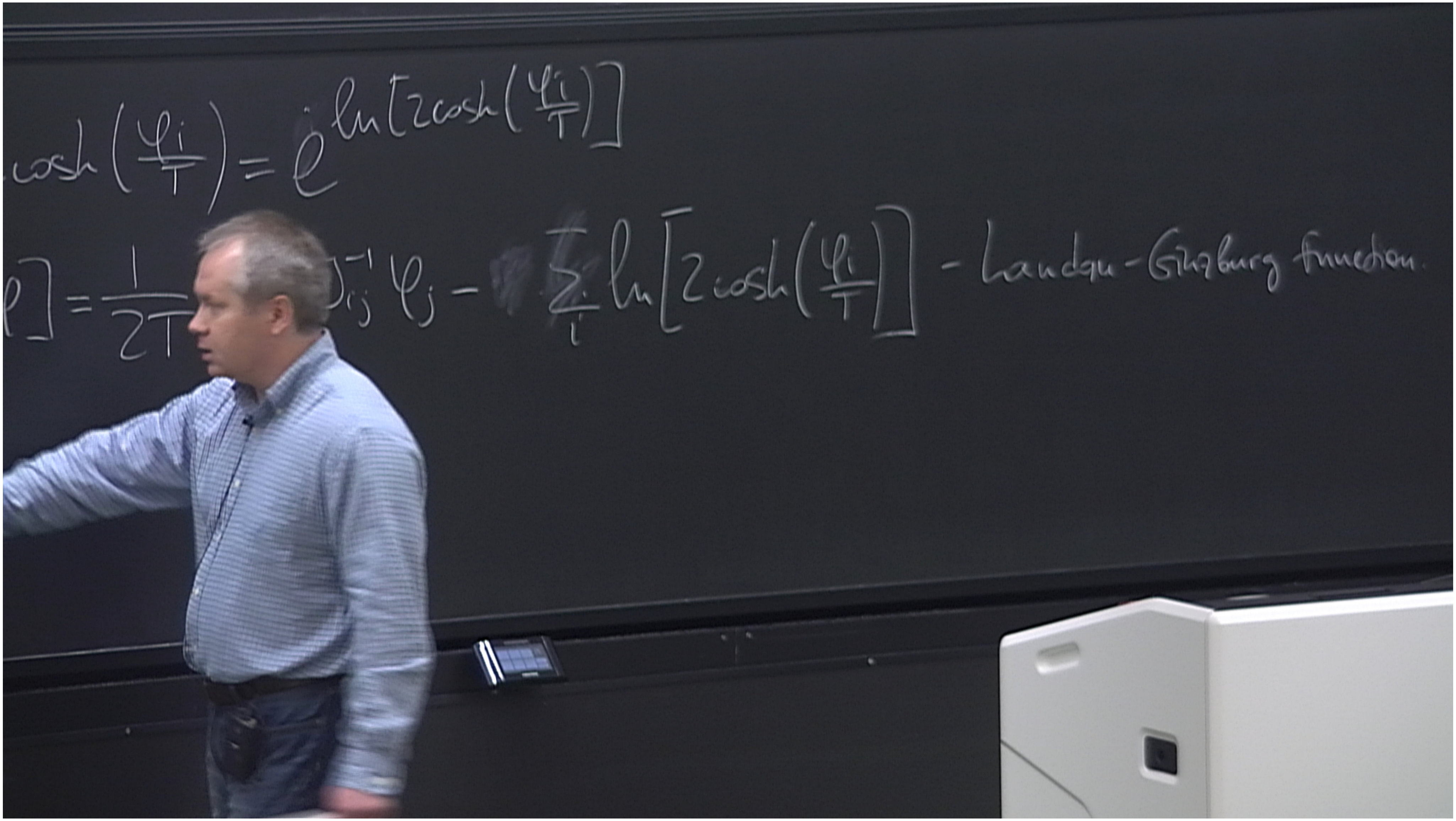
- functional integral representation of the partition function of Ising model

$$\sum_{\sigma_i=\pm 1} e^{\frac{1}{T}\varphi_i\sigma_i} = e^{\frac{\varphi_i}{T}} + e^{-\frac{\varphi_i}{T}} = 2\cosh\left(\frac{\varphi_i}{T}\right) = e^{\ln[2\cosh}$$



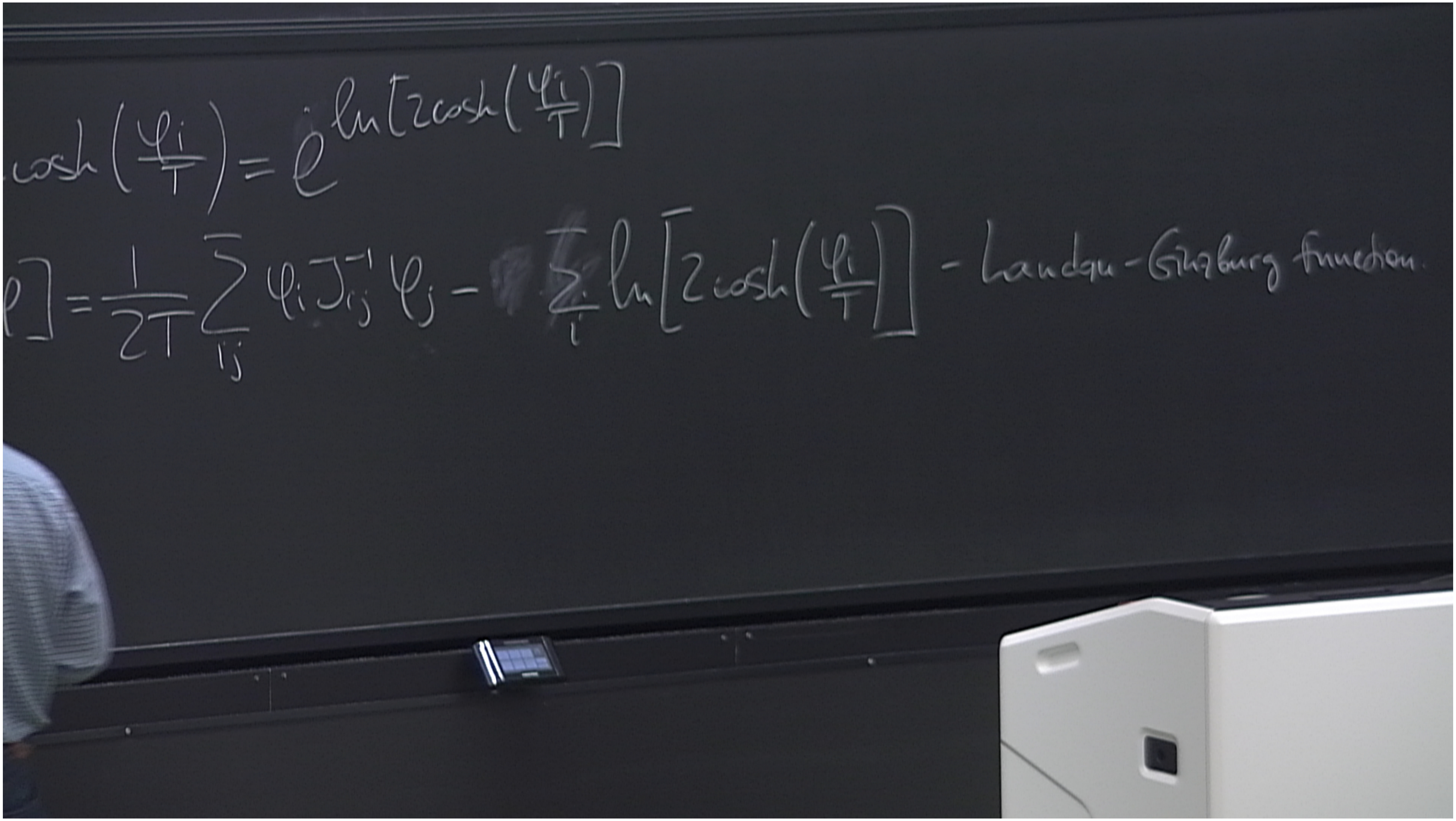
$$= e^{\frac{\varphi_i}{T}} + e^{-\frac{\varphi_i}{T}} = 2\cosh\left(\frac{\varphi_i}{T}\right) = e^{\ln\left[2\cosh\left(\frac{\varphi_i}{T}\right)\right]}$$

$$e^{-S[\varphi]} \; ; \quad S[\varphi] = \frac{1}{2T}\sum_{ij} \varphi_i J^{-1}_{ij} \varphi_j - \sum_i \ln\left[2\cosh\left(\frac{\varphi_i}{T}\right)\right] -$$



$$\cosh\left(\frac{\varphi_i}{T}\right) = e^{\ln\left[2\cosh\left(\frac{\varphi_i}{T}\right)\right]}$$

$$\varphi\big] = \frac{1}{2T} \quad {}^{-1}_{ij}\,\varphi_j - \sum_i \ln\left[2\cosh\left(\frac{\varphi_i}{T}\right)\right]$$ - Landau-Ginzburg function.



$$\cosh\left(\frac{\varphi_i}{T}\right) = e^{\ln\left[2\cosh\left(\frac{\varphi_i}{T}\right)\right]}$$

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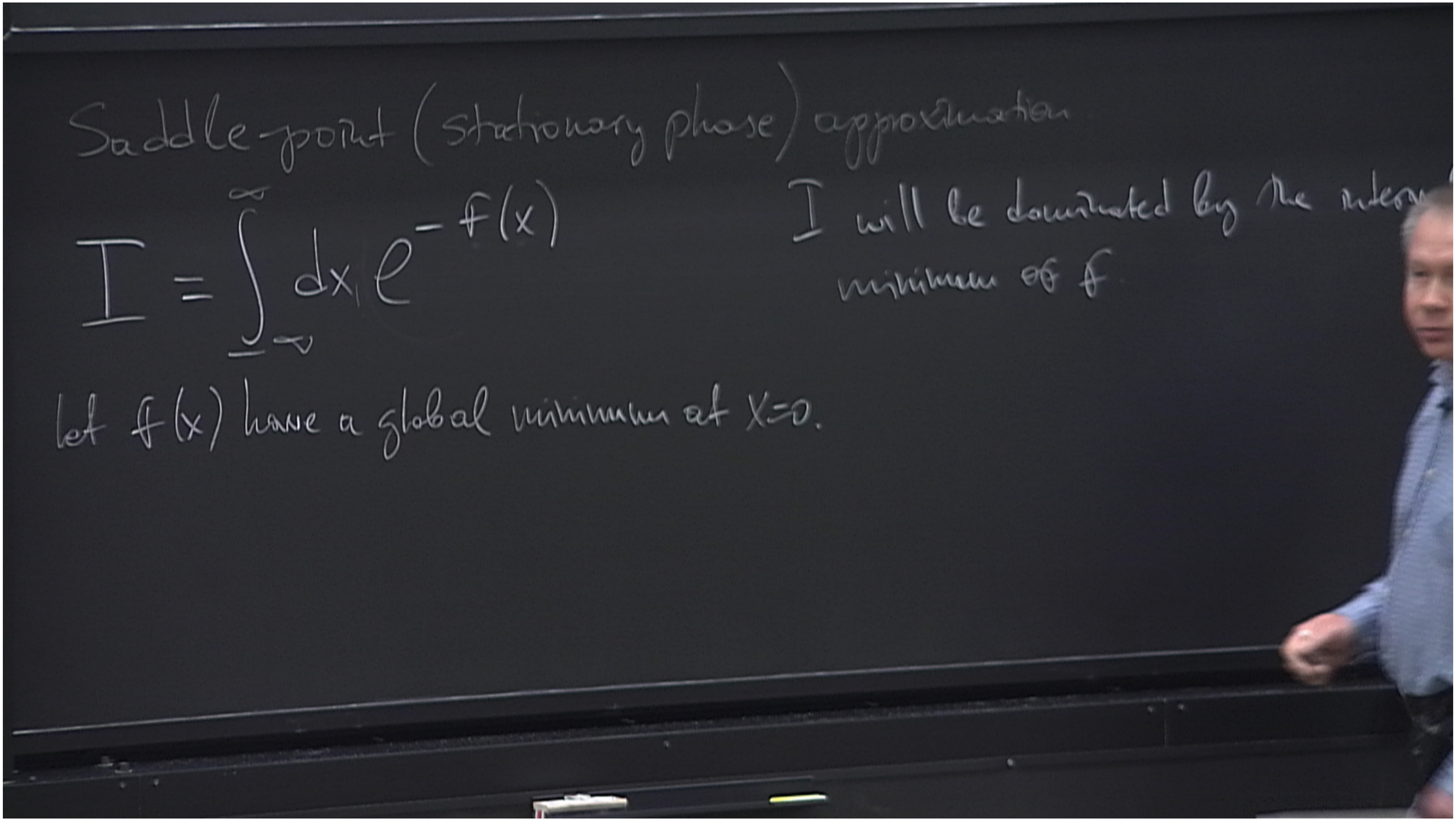


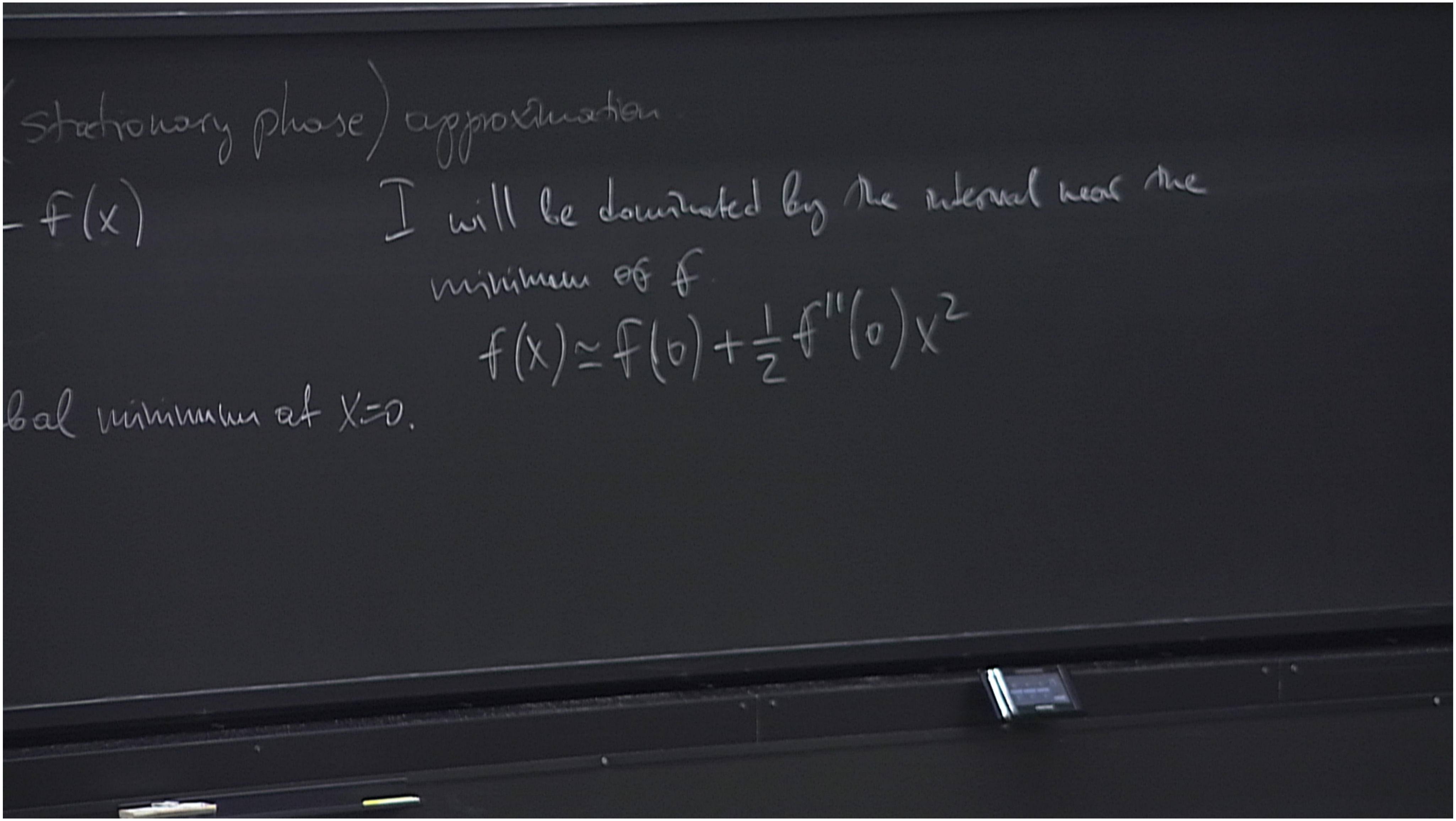
Saddle-point (stationary phase) approximation

$$I = \int_{-\infty}^{\infty} dx\, e^{-f(x)}$$

I will be dominated by the inter

minimum of f.

let f(x) have a global minimum at x=0.



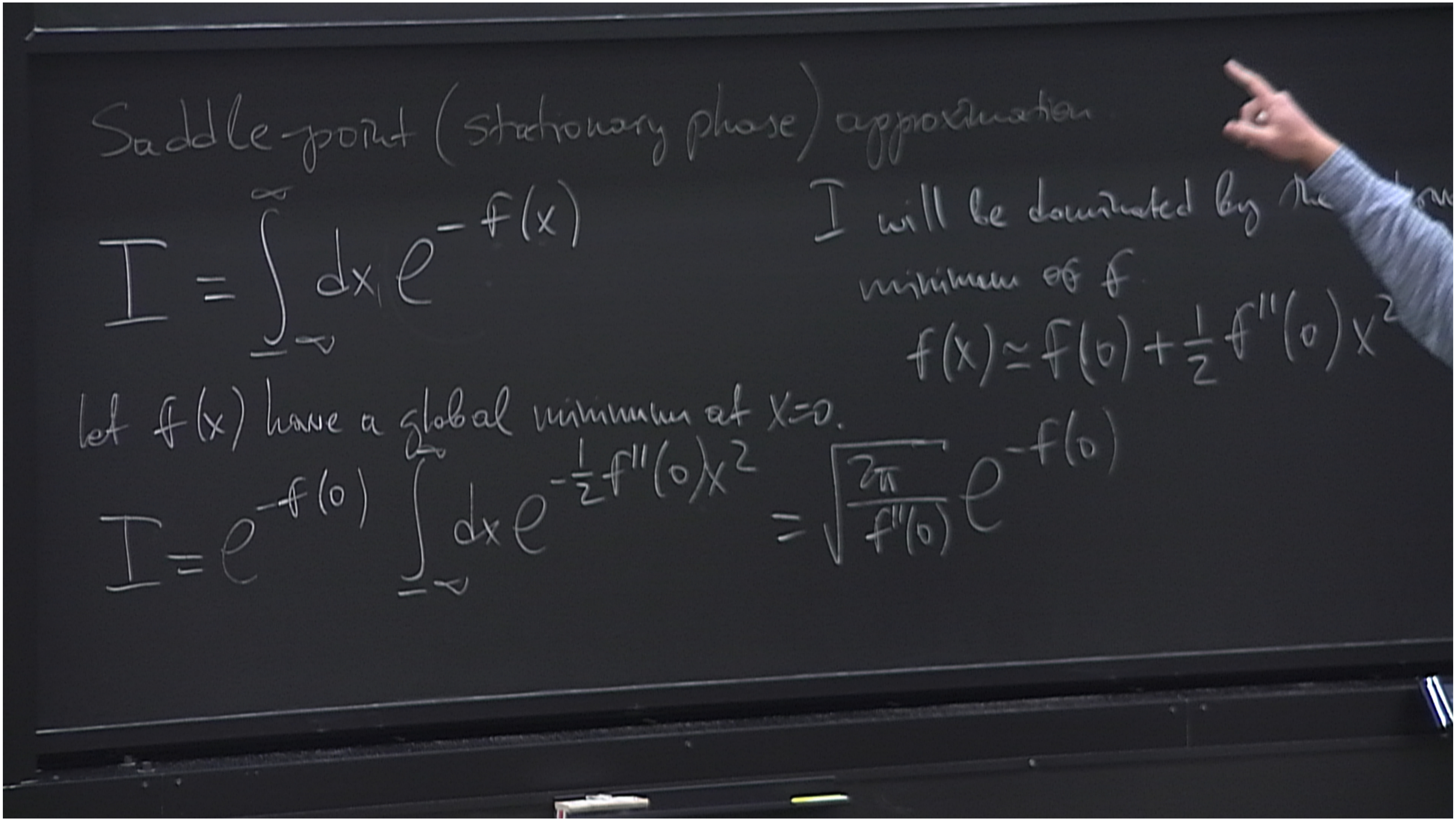
(stationary phase) approximation

$-f(x)$

I will be dominated by the interval near the minimum of f

$$f(x) \simeq f(0) + \frac{1}{2} f''(0) x^2$$

bal minimum at $x=0$.



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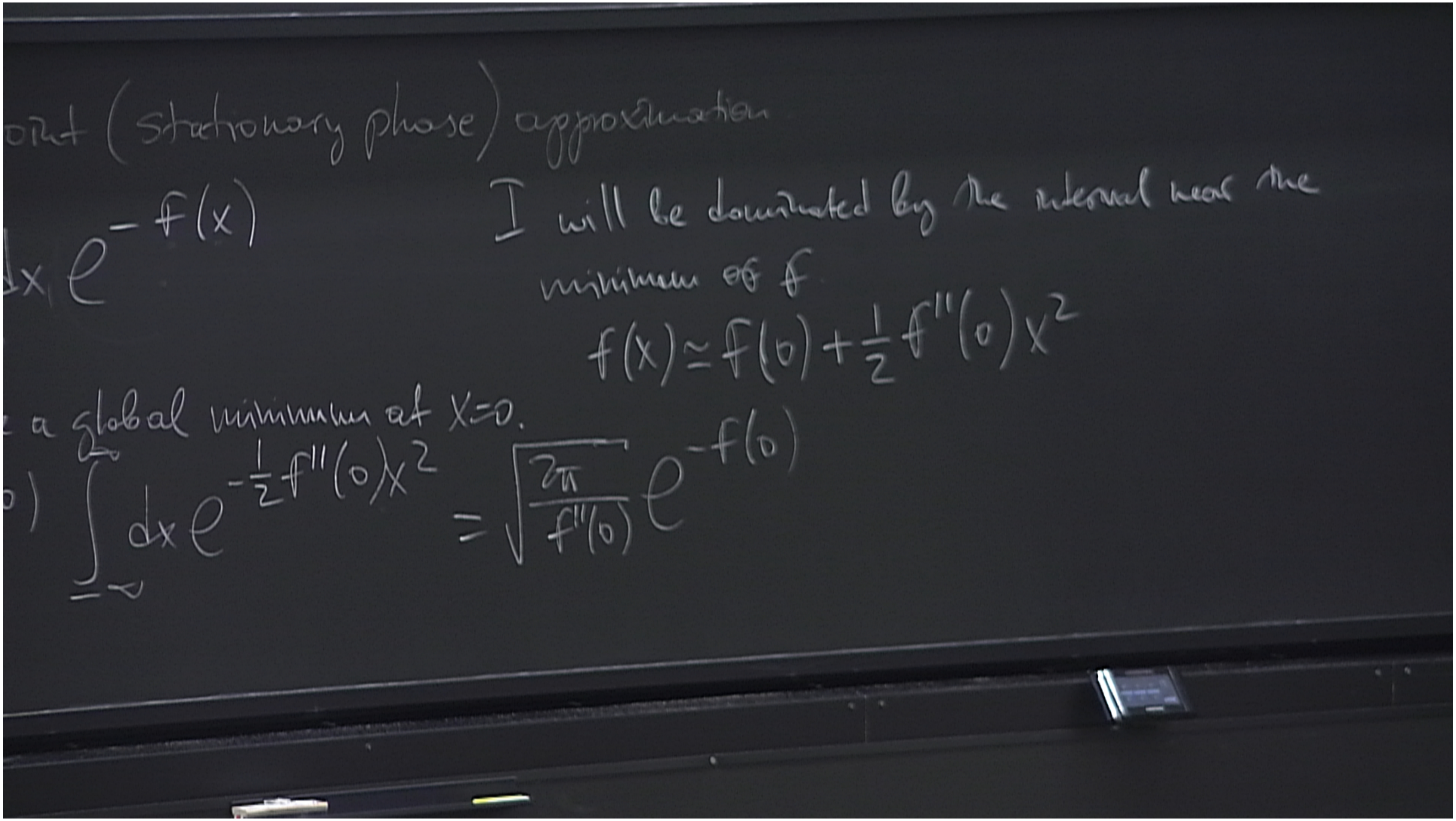
let f(x) have a global minimum at x=0.

$$I = e^{-f(0)} \int_{-\infty}^{\infty} dx\, e^{-\frac{1}{2} f''(0) x^2} = \sqrt{\frac{2\pi}{f''(0)}}\, e^{-f(0)}$$



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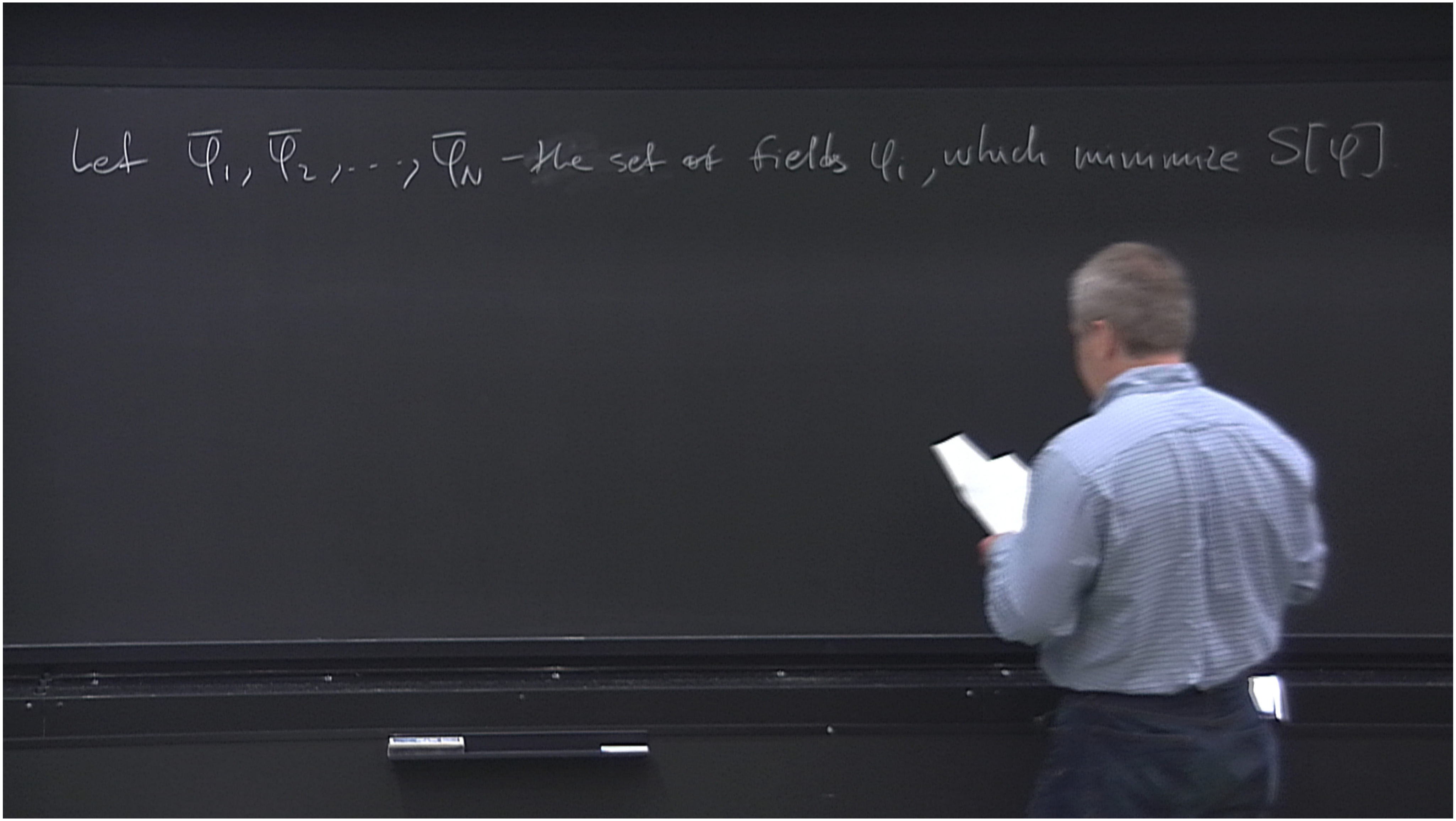
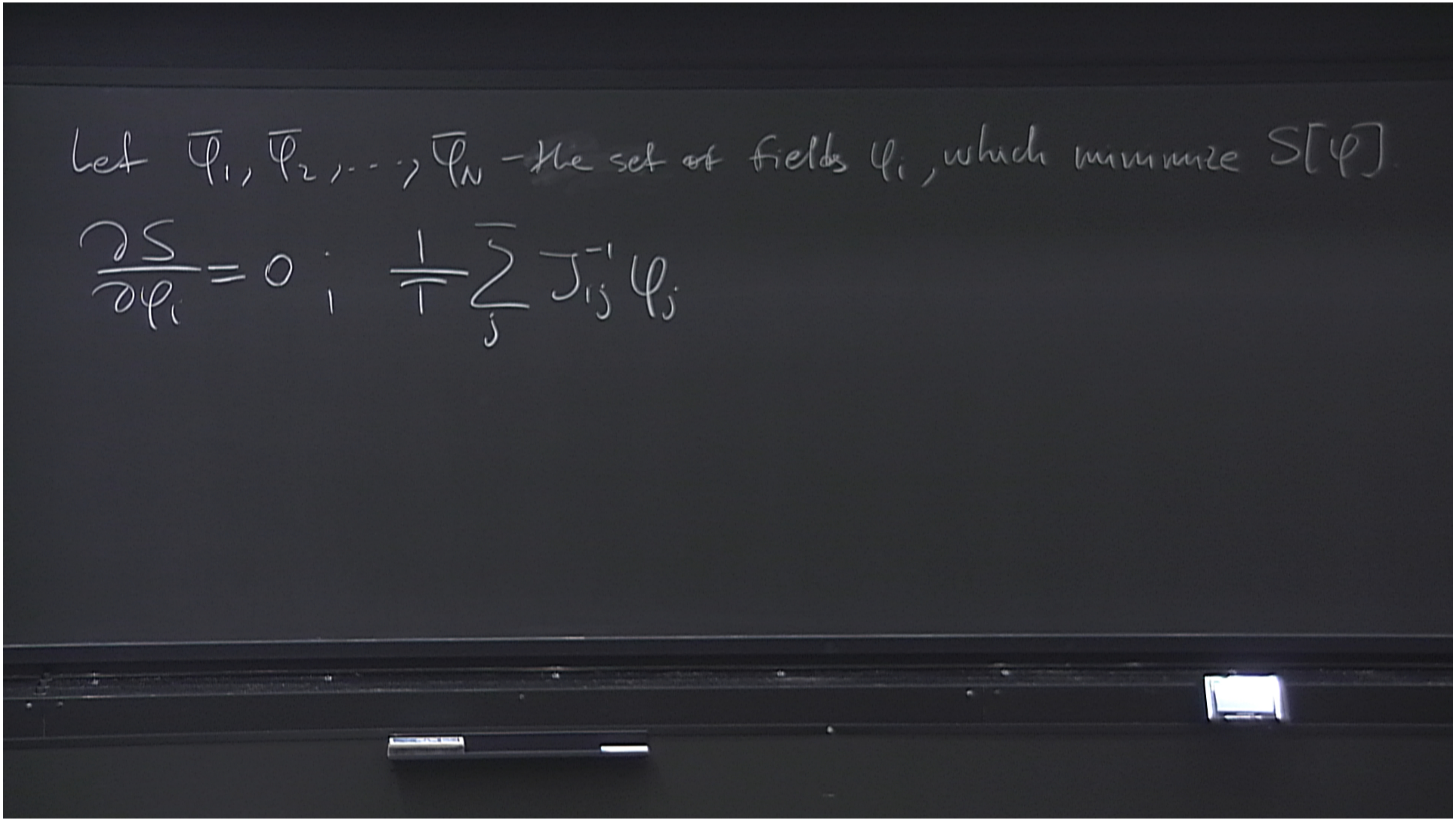
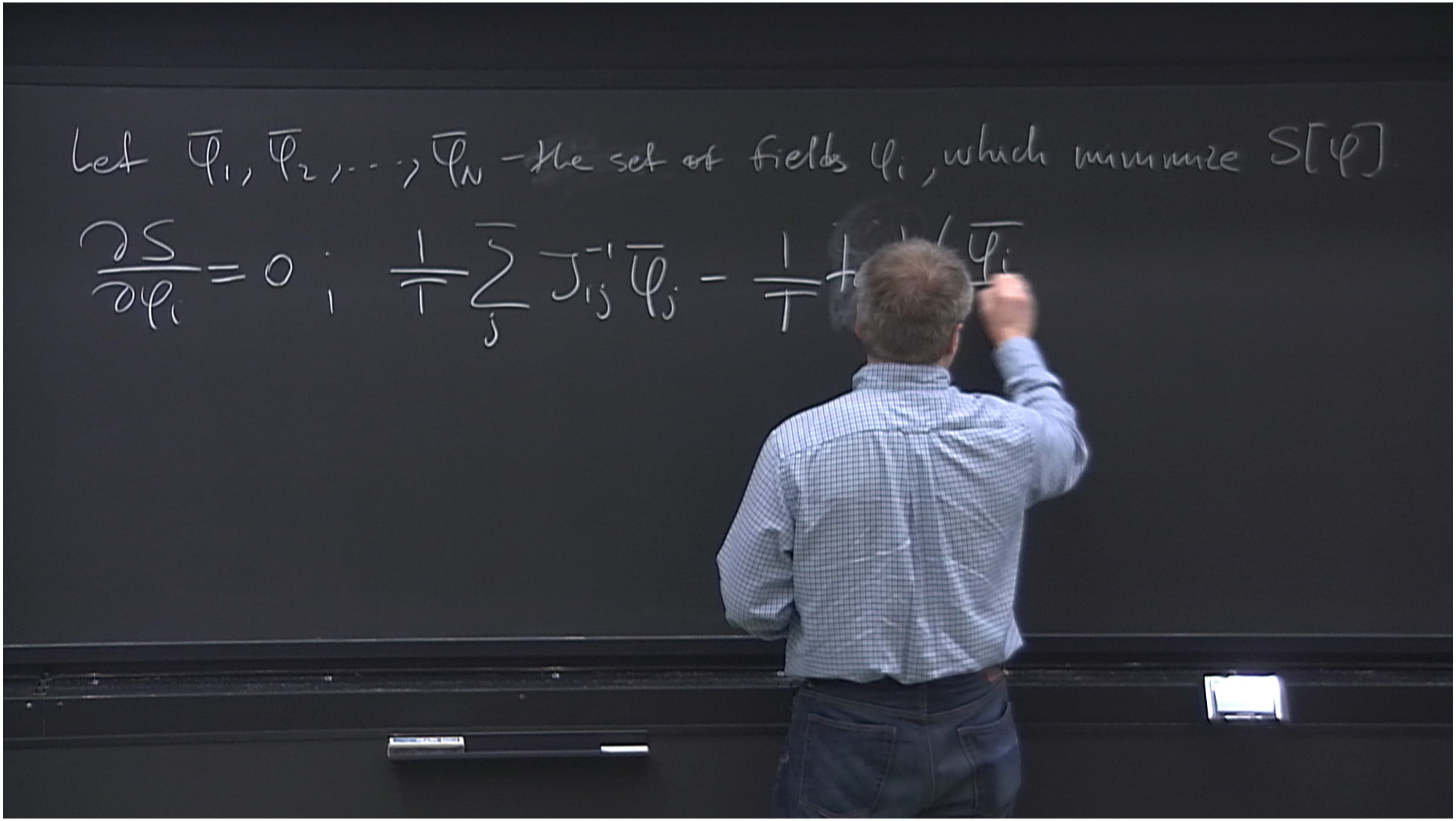


Let $\overline{\varphi}_1, \overline{\varphi}_2, \dots, \overline{\varphi}_N$ – the set of fields $\varphi_i$, which minimize $S[\varphi]$



Let $\bar{\varphi}_1, \bar{\varphi}_2, \dots, \bar{\varphi}_N$ – the set of fields $\varphi_i$, which minimize $S[\varphi]$.

$$\frac{\partial S}{\partial \varphi_i} = 0 \; ; \quad \frac{1}{T} \sum_j J^{-1}_{ij} \varphi_j$$



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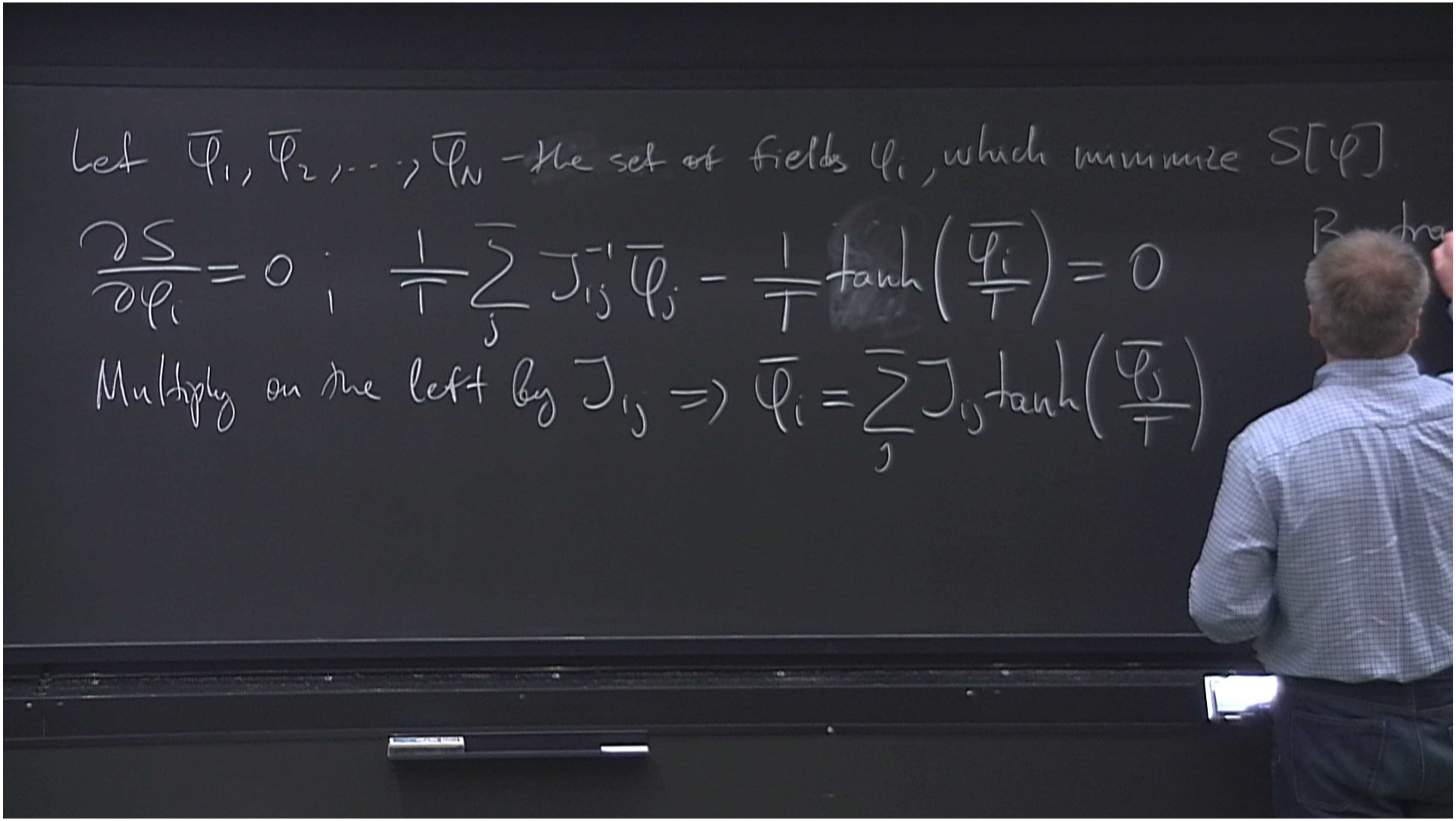
$$\frac{\partial S}{\partial \varphi_i} = 0 \; ; \quad \frac{1}{T} \sum_j J^{-1}_{ij} \bar{\varphi}_j - \frac{1}{T}$$ [illegible] $\bar{\varphi}_i$



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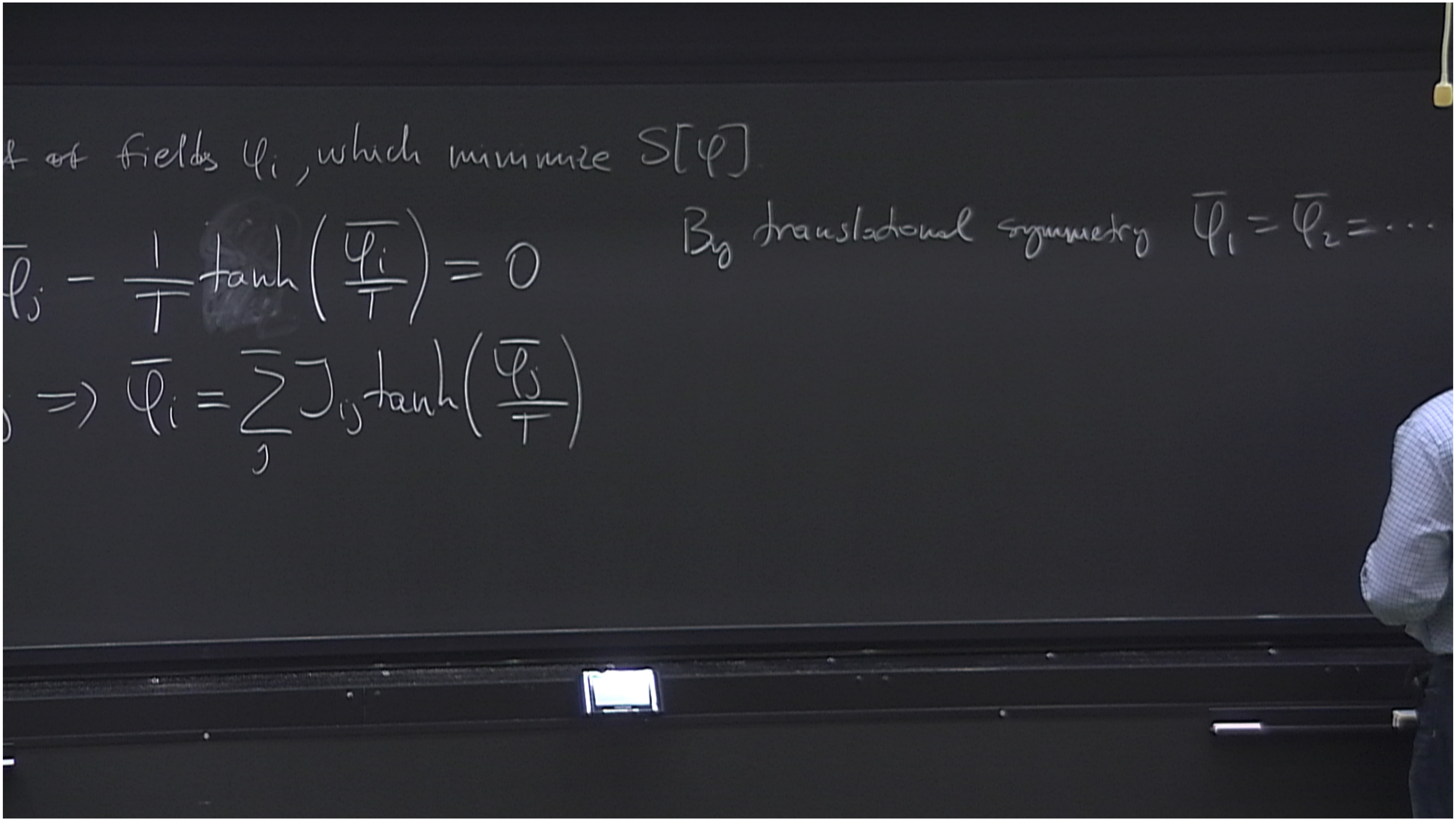
$$\frac{\partial S}{\partial \varphi_i} = 0 \; ; \quad \frac{1}{T}\sum_j J^{-1}_{ij}\bar{\varphi}_j - \frac{1}{T}\tanh\left(\frac{\bar{\varphi}_i}{T}\right) = 0$$

Multiply on the left by $J_{ij}$ $\Rightarrow$ $\bar{\varphi}_i = \sum_j J_{ij}\tanh\left(\frac{\bar{\varphi}_j}{T}\right)$

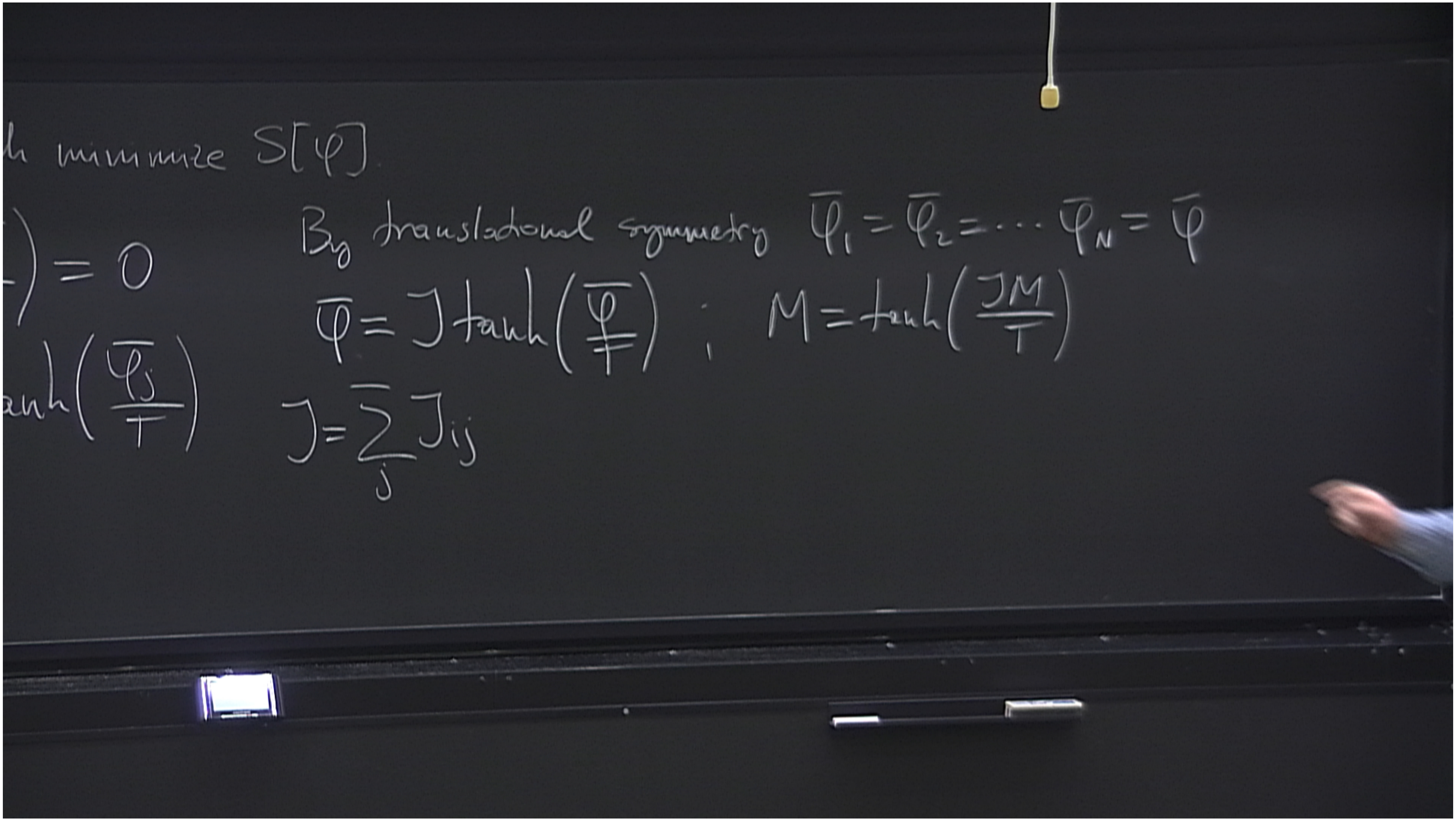


t of fields $\varphi_i$, which minimize $S[\varphi]$

$$\bar{\varphi}_j - \frac{1}{T}\tanh\left(\frac{\bar{\varphi}_i}{T}\right) = 0$$

By translational symmetry $\bar{\varphi}_1 = \bar{\varphi}_2 = \dots$

$$j \Rightarrow \bar{\varphi}_i = \sum_j J_{ij}\tanh\left(\frac{\bar{\varphi}_j}{T}\right)$$



h minimize $S[\varphi]$

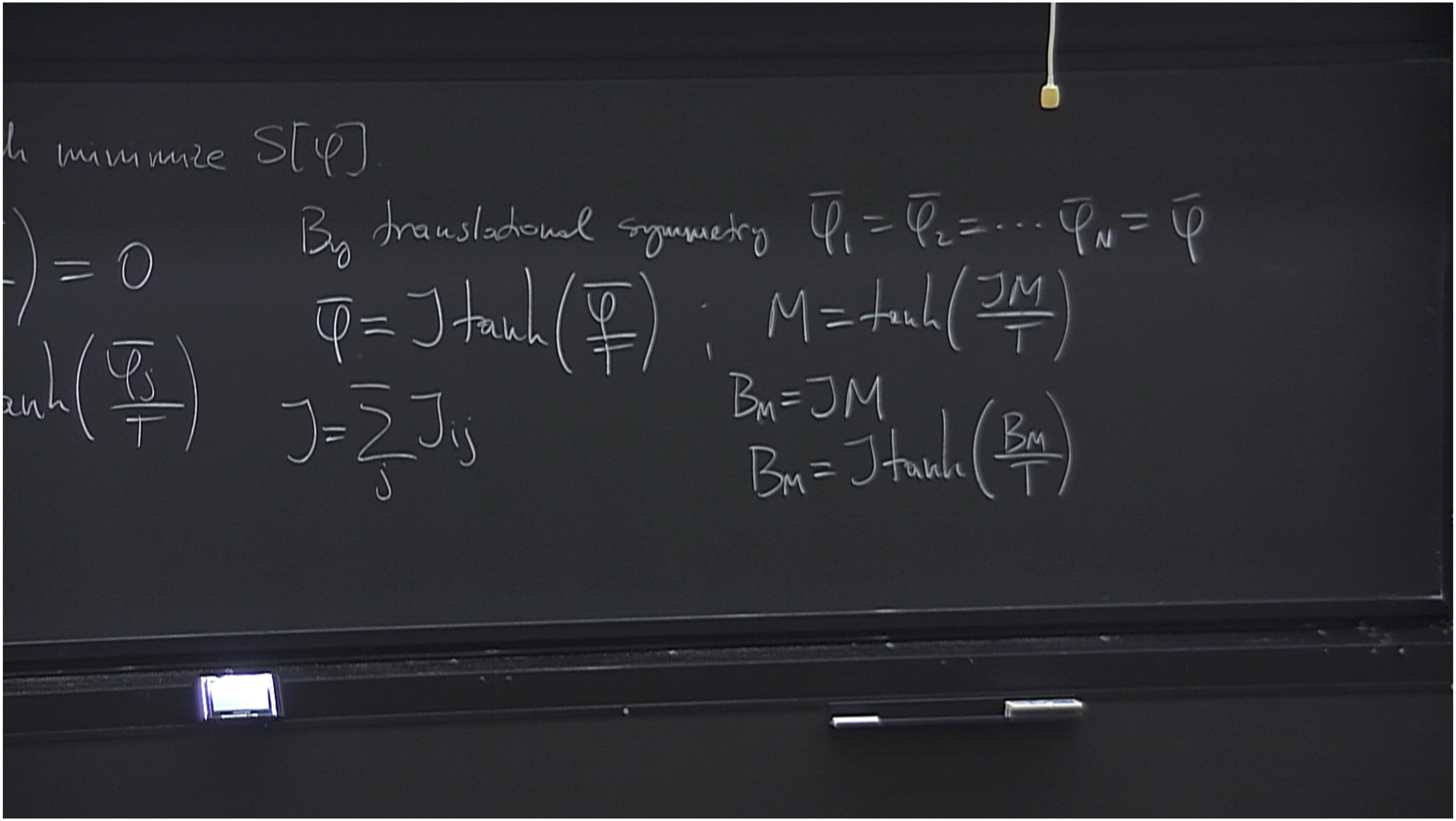
$) = 0$

By translational symmetry $\bar{\varphi}_1 = \bar{\varphi}_2 = \cdots \varphi_N = \bar{\varphi}$

$\bar{\varphi} = J \tanh\left(\frac{\bar{\varphi}}{T}\right)$ ; $M = \tanh\left(\frac{JM}{T}\right)$

$\mathrm{anh}\left(\frac{\bar{\varphi}_j}{T}\right)$

$J = \overline{\sum_j} J_{ij}$



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$B_M = JM$

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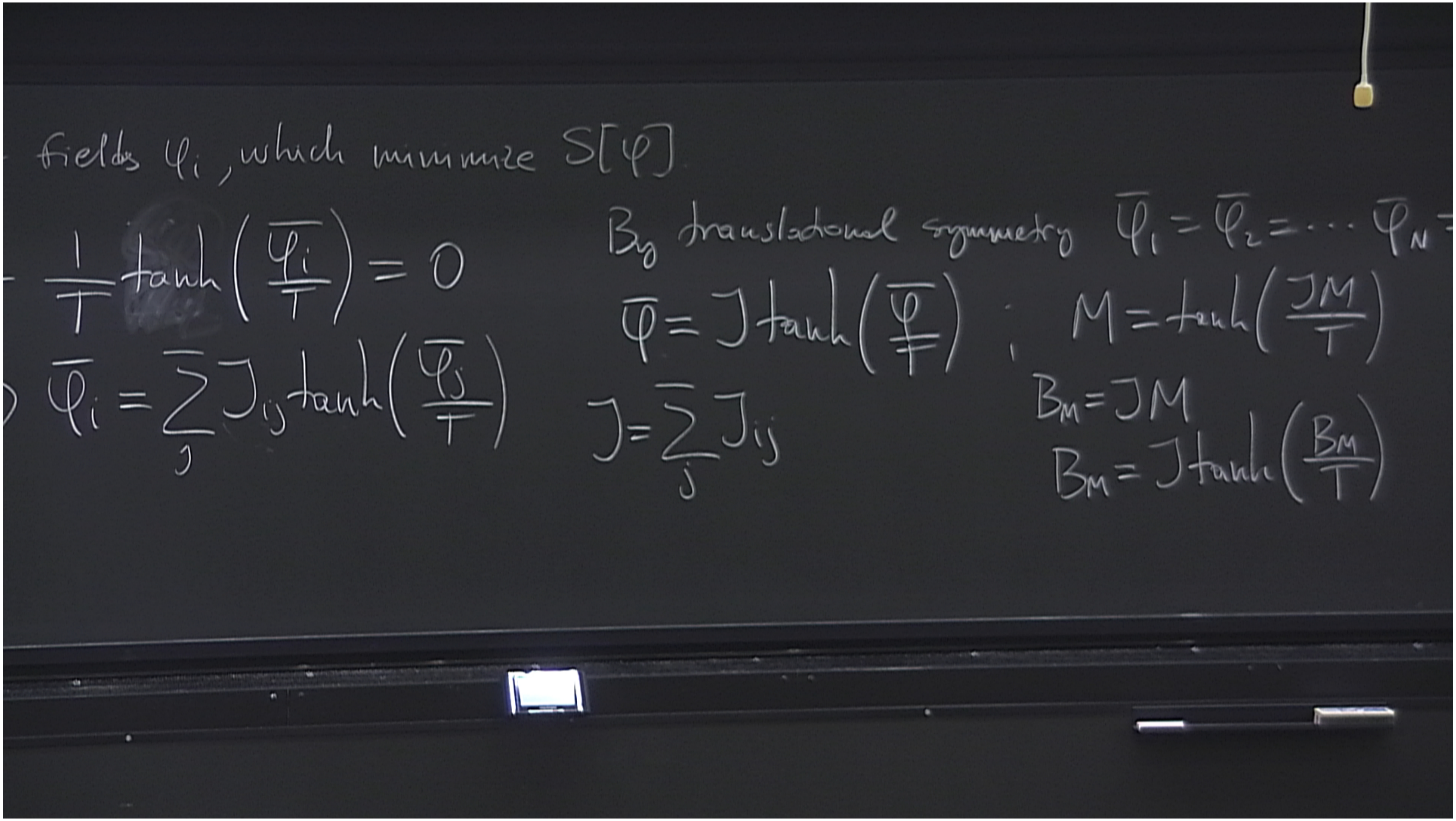

fields $\varphi_i$, which minimize $S[\varphi]$

$$\frac{1}{T}\tanh\left(\frac{\bar{\varphi}_i}{T}\right) = 0$$

$$\bar{\varphi}_i = \sum_j J_{ij}\tanh\left(\frac{\bar{\varphi}_j}{T}\right)$$

By translational symmetry $\bar{\varphi}_1 = \bar{\varphi}_2 = \cdots \varphi_N =$

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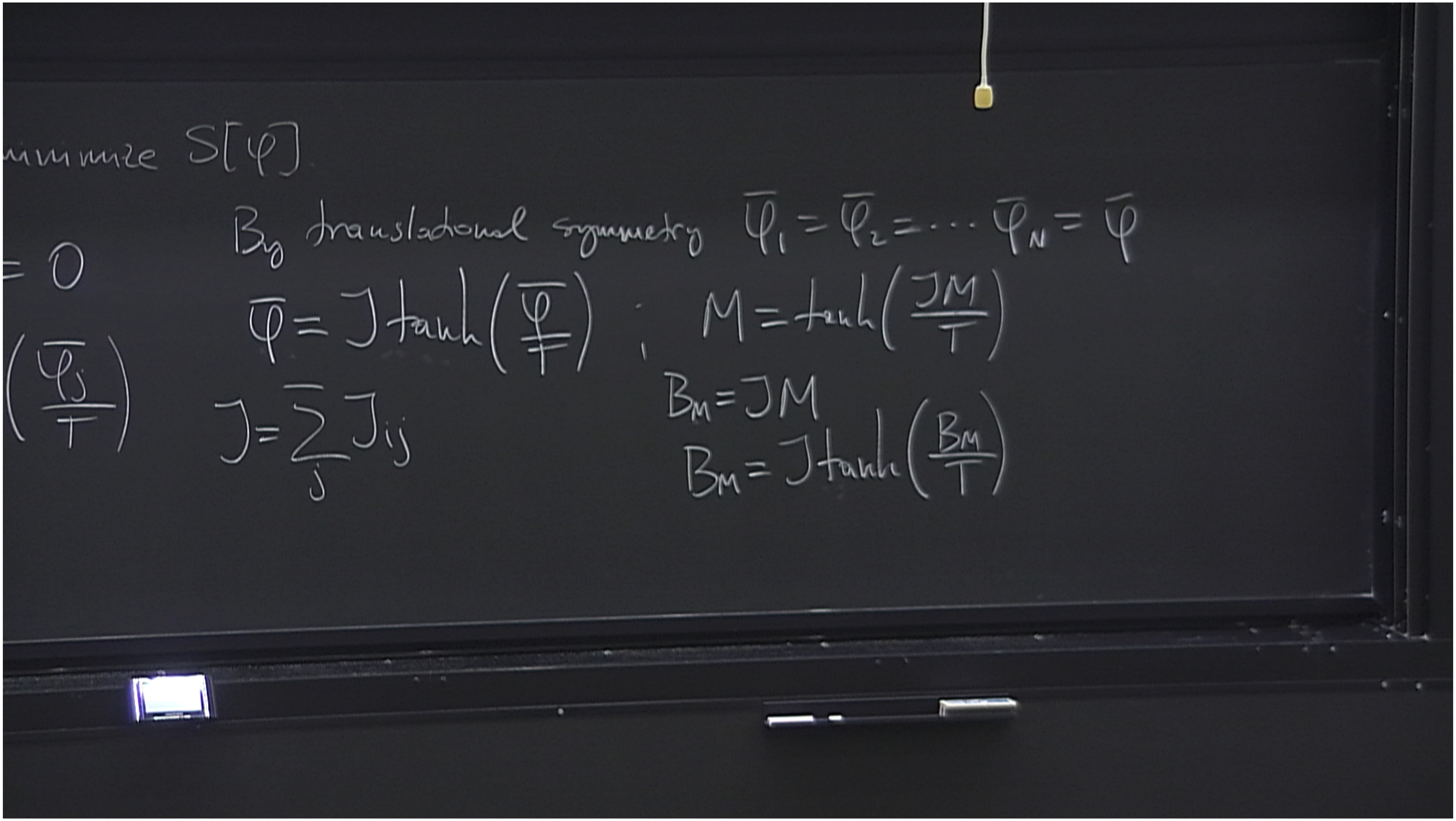

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$= 0$

$\left(\frac{\bar{\varphi}_j}{T}\right)$

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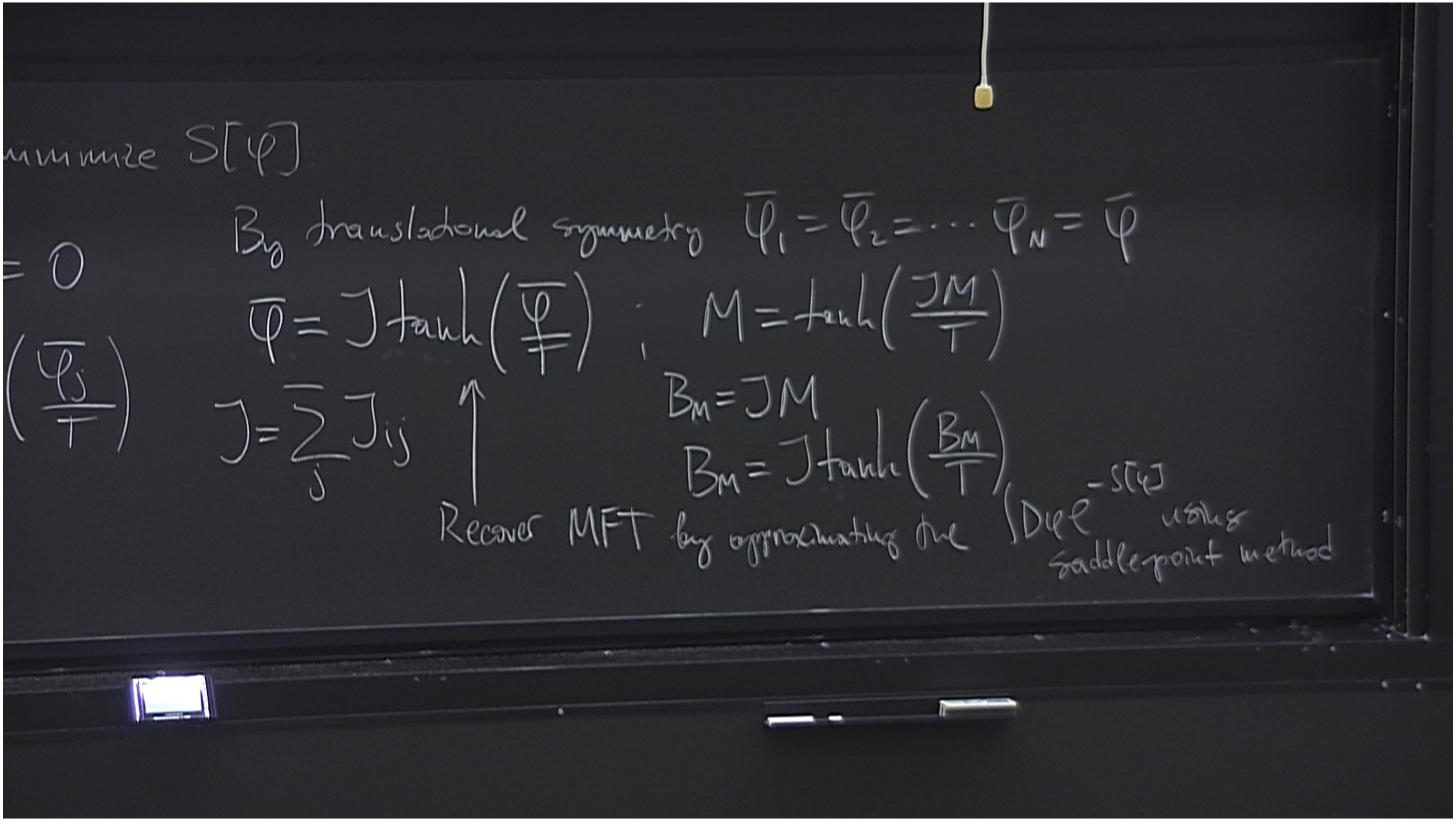

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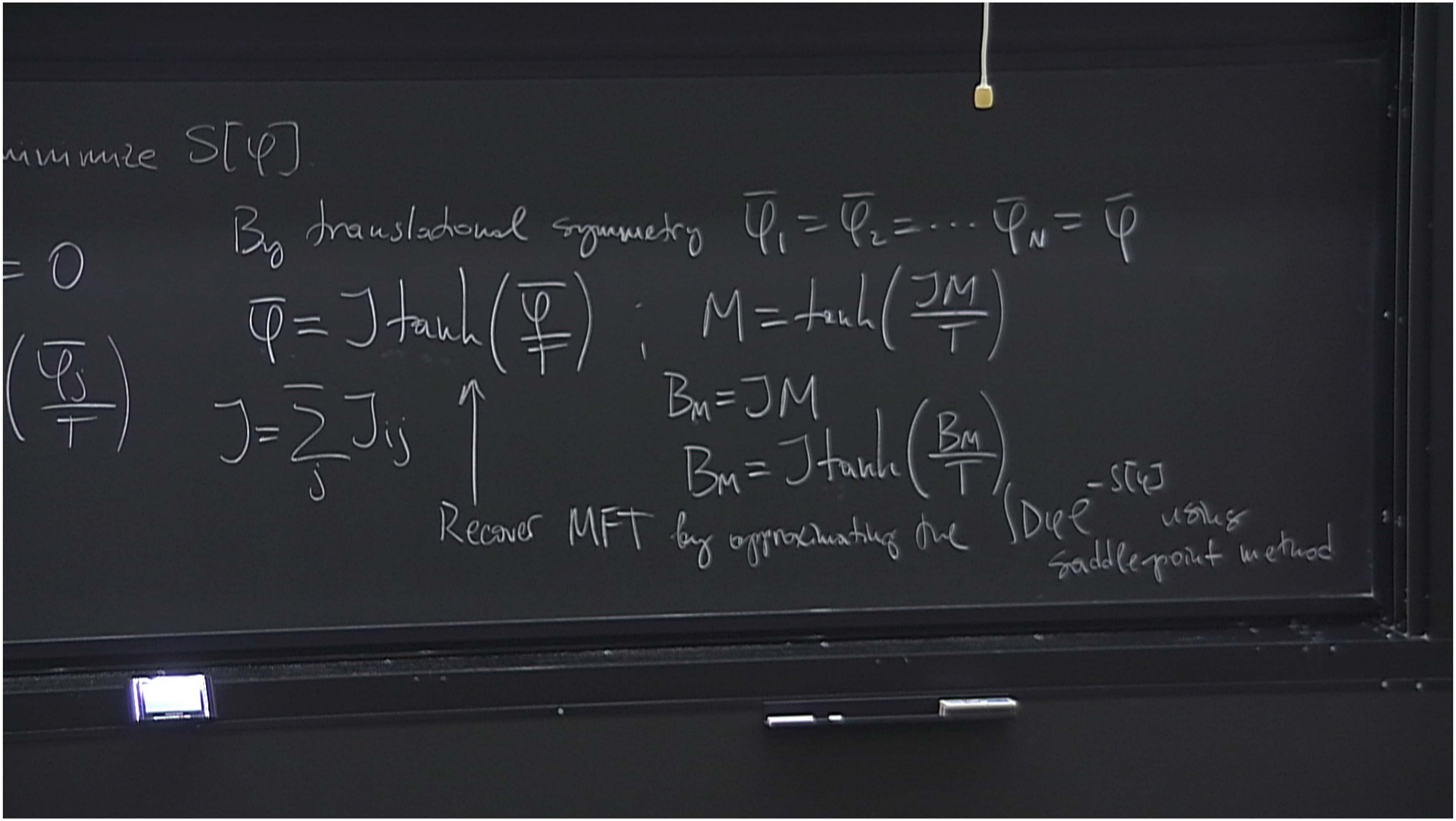
$\uparrow$ Recover MFT by approximating the $\int D\varphi \, e^{-S[\varphi]}$ using saddlepoint method



inimize $S[\varphi]$

$= 0$

By translational symmetry $\bar{\varphi}_1 = \bar{\varphi}_2 = \cdots \bar{\varphi}_N = \bar{\varphi}$

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$J = \sum_j J_{ij}$ ↑

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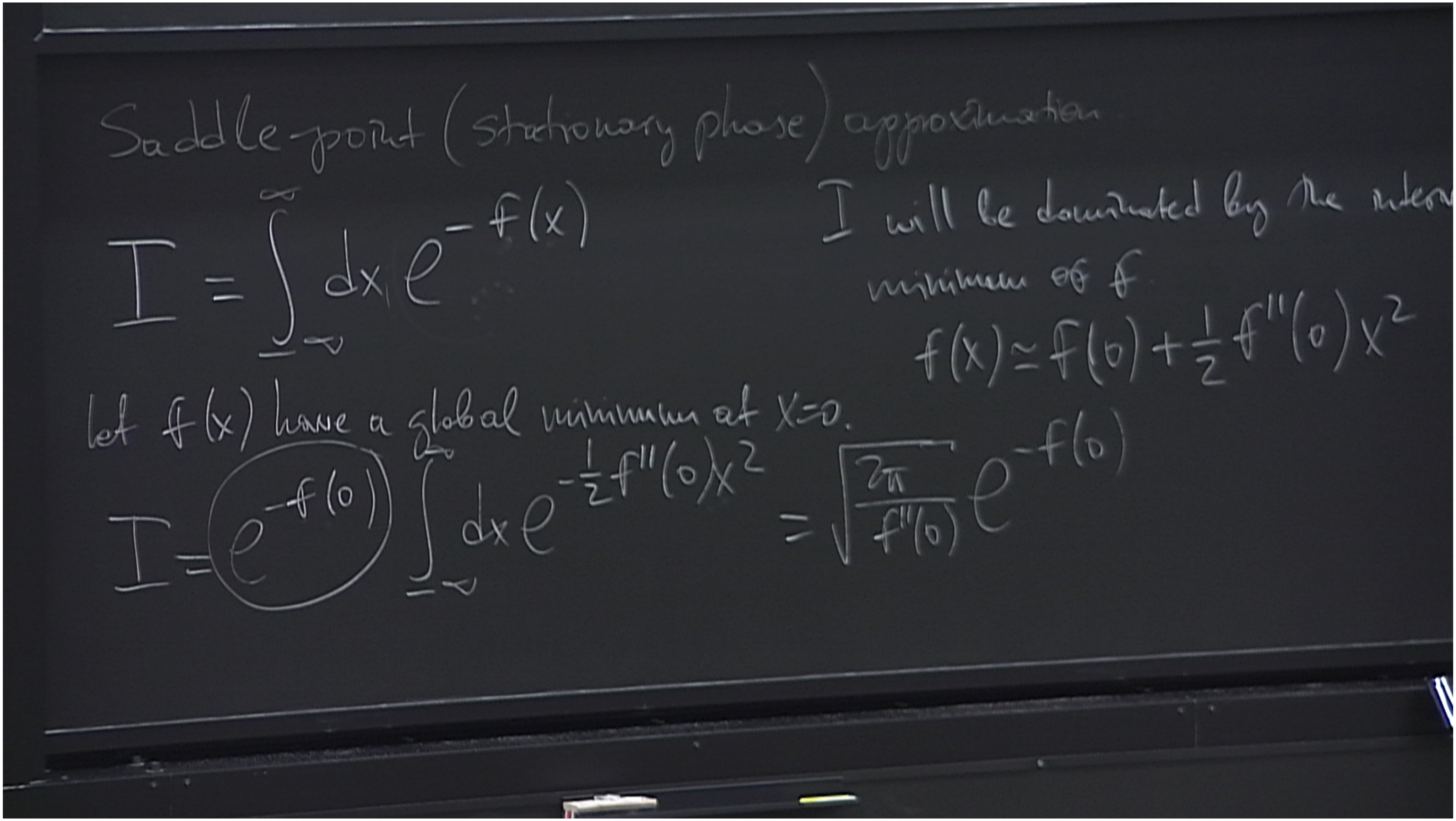
Recover MFT by approximating the $\int D\varphi \, e^{-S[\varphi]}$ using saddlepoint method



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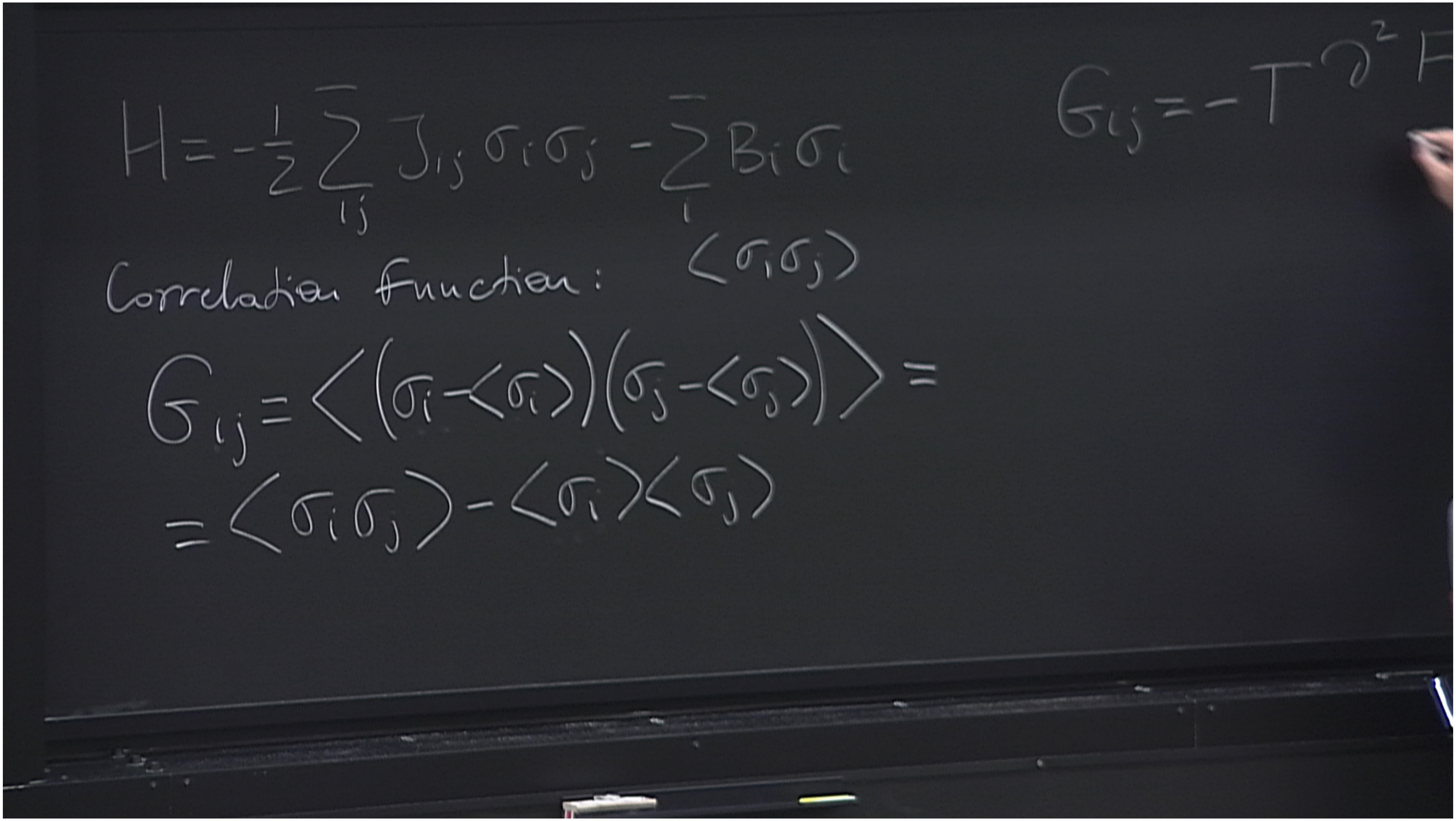


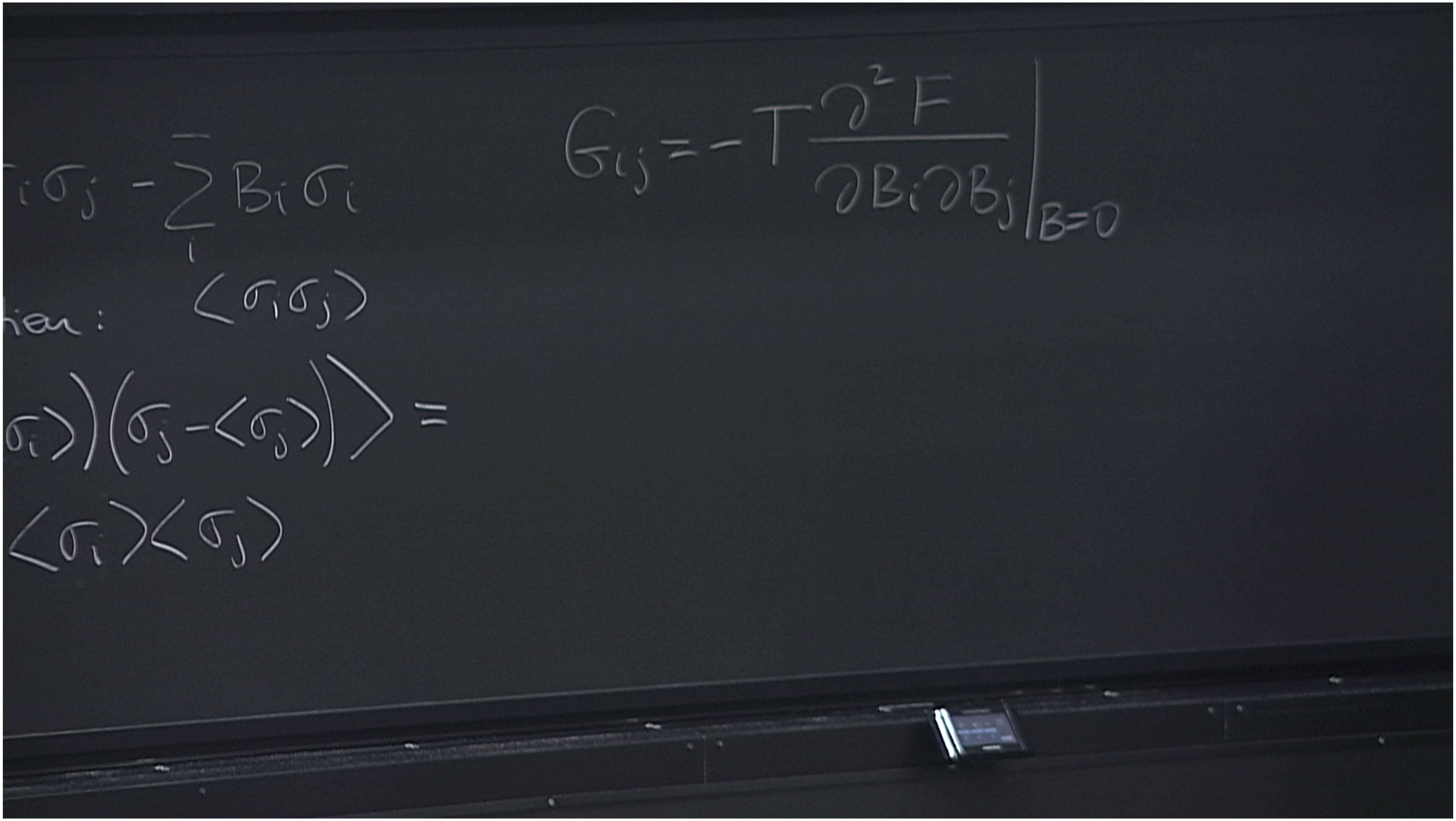
$$H = -\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j - \sum_i B_i\sigma_i$$

$$G_{ij} = -T\partial^2 F$$

Correlation function: $\langle\sigma_i\sigma_j\rangle$

$$G_{ij} = \Big\langle\big(\sigma_i - \langle\sigma_i\rangle\big)\big(\sigma_j - \langle\sigma_j\rangle\big)\Big\rangle =$$

$$= \langle\sigma_i\sigma_j\rangle - \langle\sigma_i\rangle\langle\sigma_j\rangle$$



$_i\sigma_j - \sum_i B_i \sigma_i$

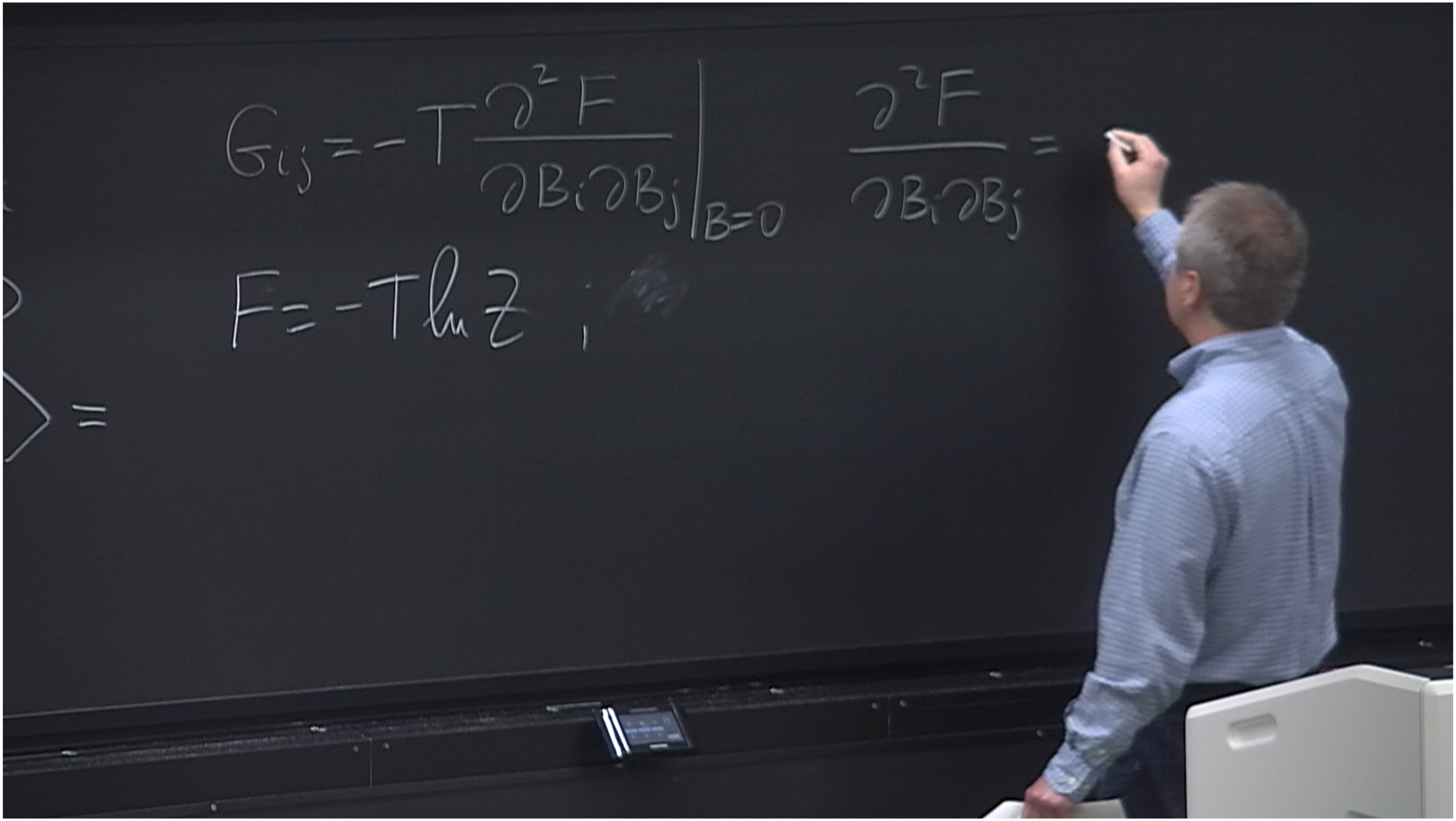
ion :

$\langle \sigma_i \sigma_j \rangle$

$\sigma_i \rangle)(\sigma_j - \langle \sigma_j \rangle)\rangle =$

$\langle \sigma_i \rangle \langle \sigma_j \rangle$

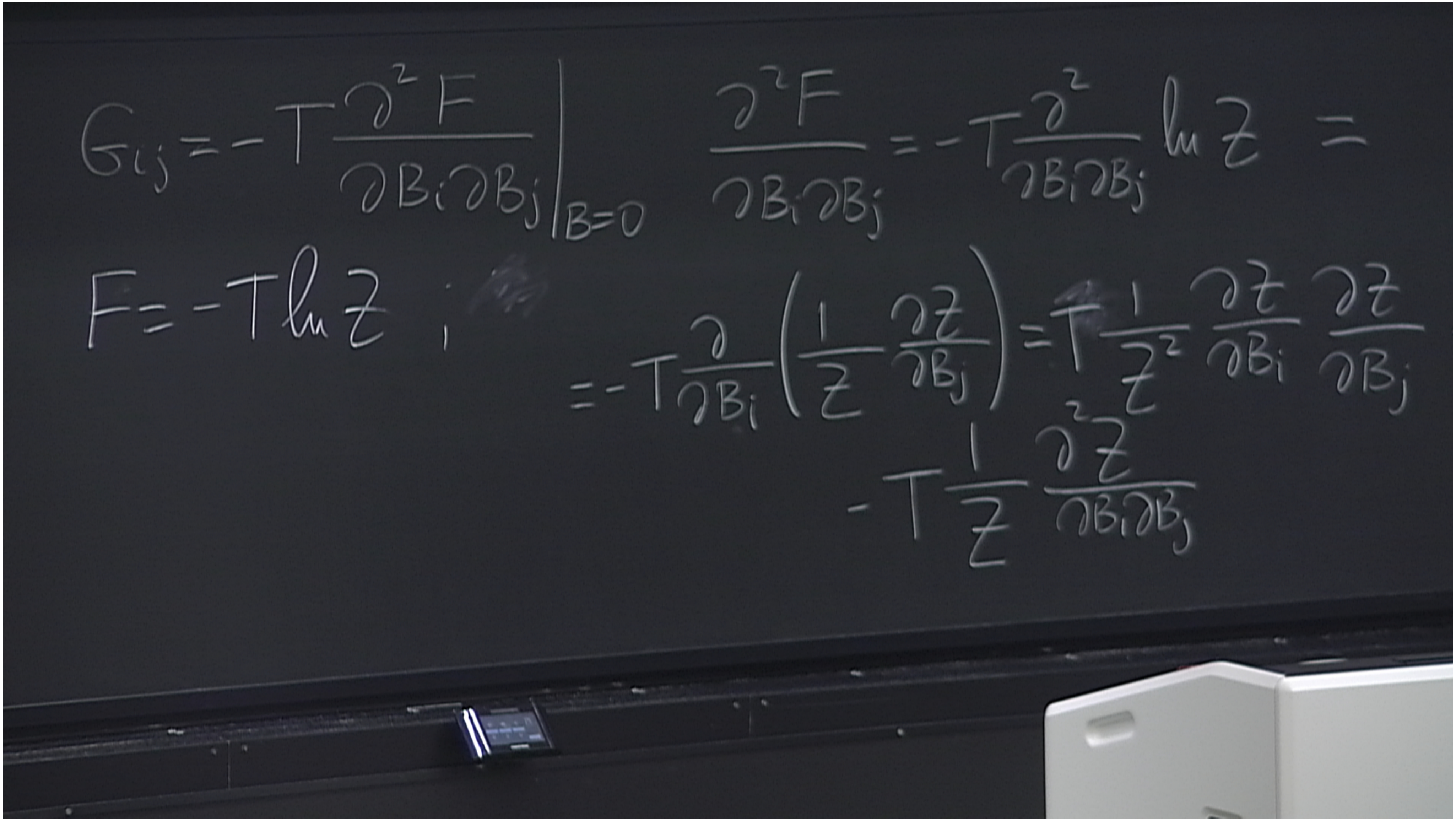
$$G_{ij} = -T \frac{\partial^2 F}{\partial B_i \partial B_j}\Big|_{B=0}$$



$$G_{ij} = -T \frac{\partial^2 F}{\partial B_i \partial B_j}\Bigg|_{B=0} \qquad \frac{\partial^2 F}{\partial B_i \partial B_j} =$$

$$F = -T \ln Z \;;$$

$$\Big\rangle =$$

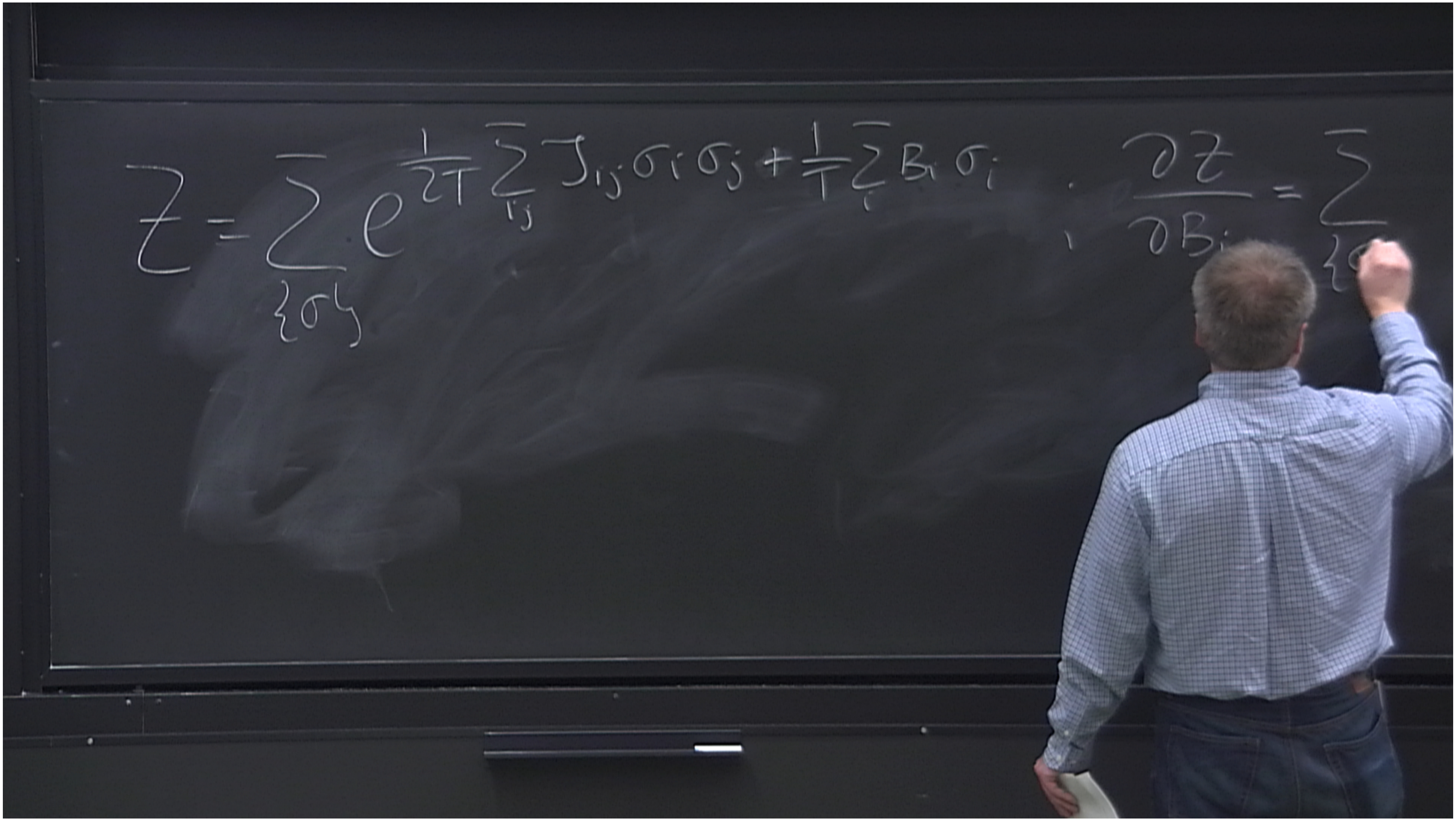


$$G_{ij} = -T \frac{\partial^2 F}{\partial B_i \partial B_j}\bigg|_{B=0} \qquad \frac{\partial^2 F}{\partial B_i \partial B_j} = -T \frac{\partial^2}{\partial B_i \partial B_j} \ln Z =$$

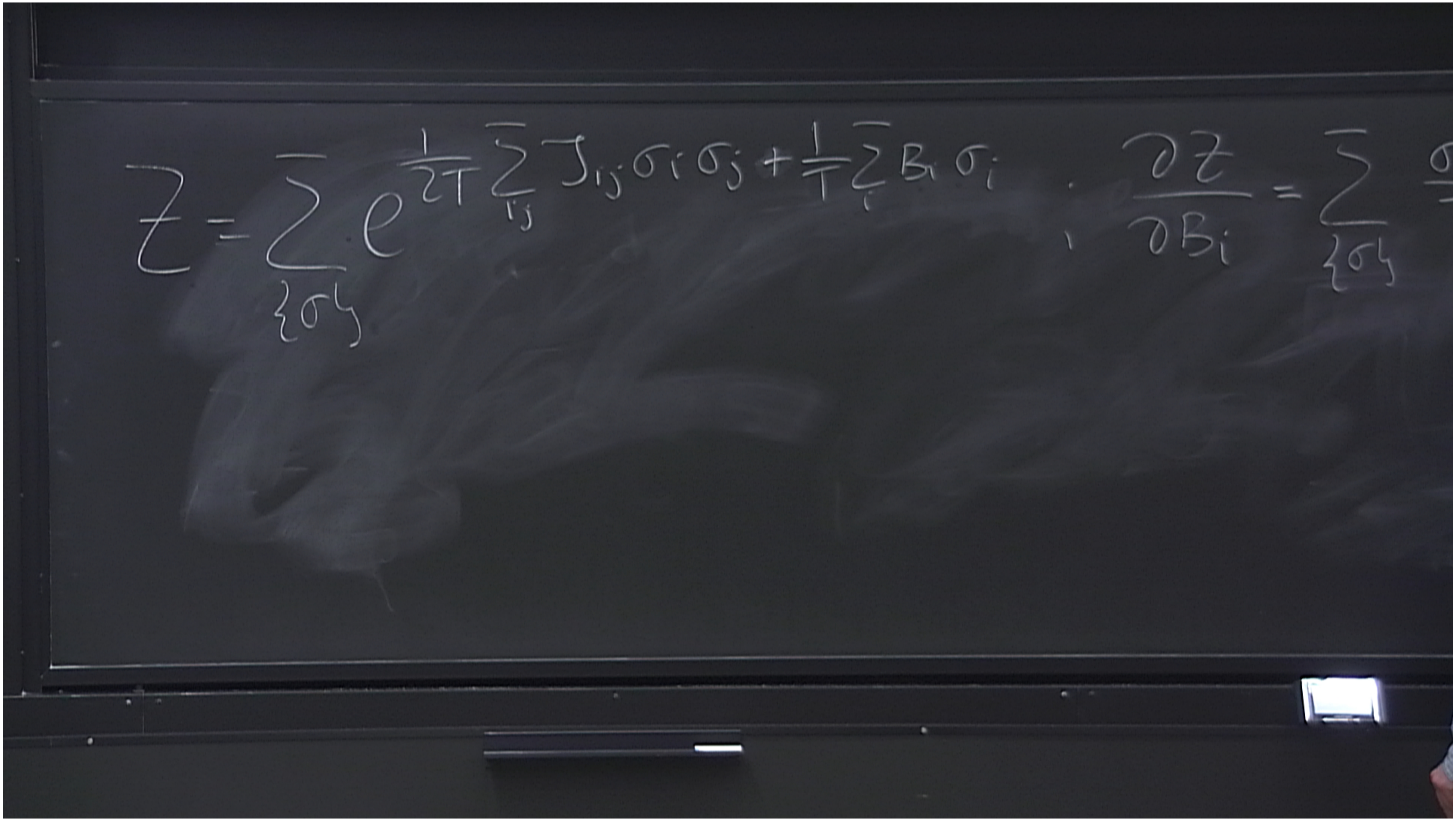
$$F = -T \ln Z \;;$$

$$= -T \frac{\partial}{\partial B_i}\left(\frac{1}{Z}\frac{\partial Z}{\partial B_j}\right) = T \frac{1}{Z^2}\frac{\partial Z}{\partial B_i}\frac{\partial Z}{\partial B_j}$$

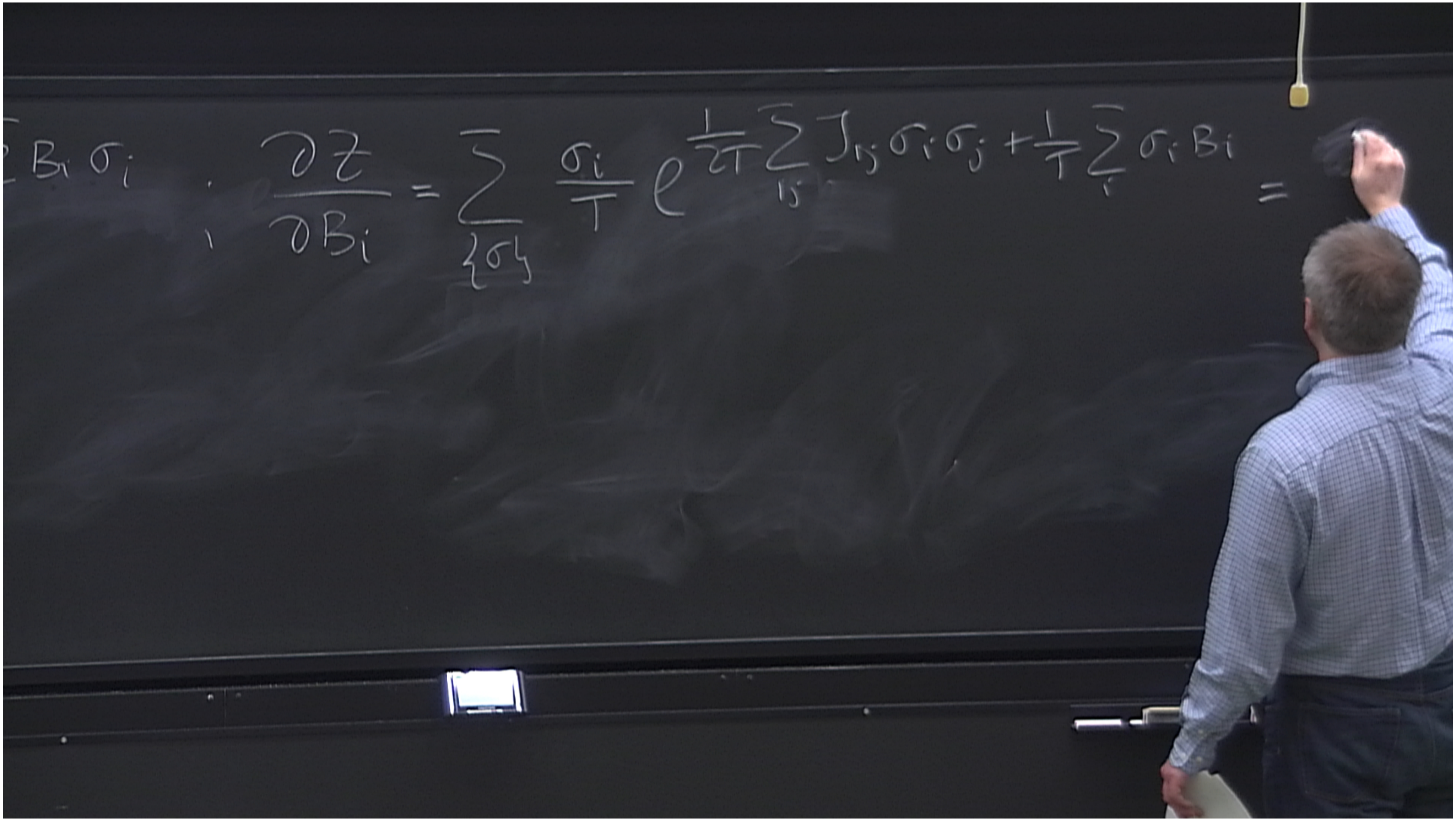
$$-T \frac{1}{Z}\frac{\partial^2 Z}{\partial B_i \partial B_j}$$



$$Z = \sum_{\{\sigma\}} e^{\frac{1}{2T}\sum_{ij} J_{ij}\sigma_i\sigma_j + \frac{1}{T}\sum_i B_i\sigma_i} \; ; \quad \frac{\partial Z}{\partial B_i} = \sum_{\{\sigma}$$



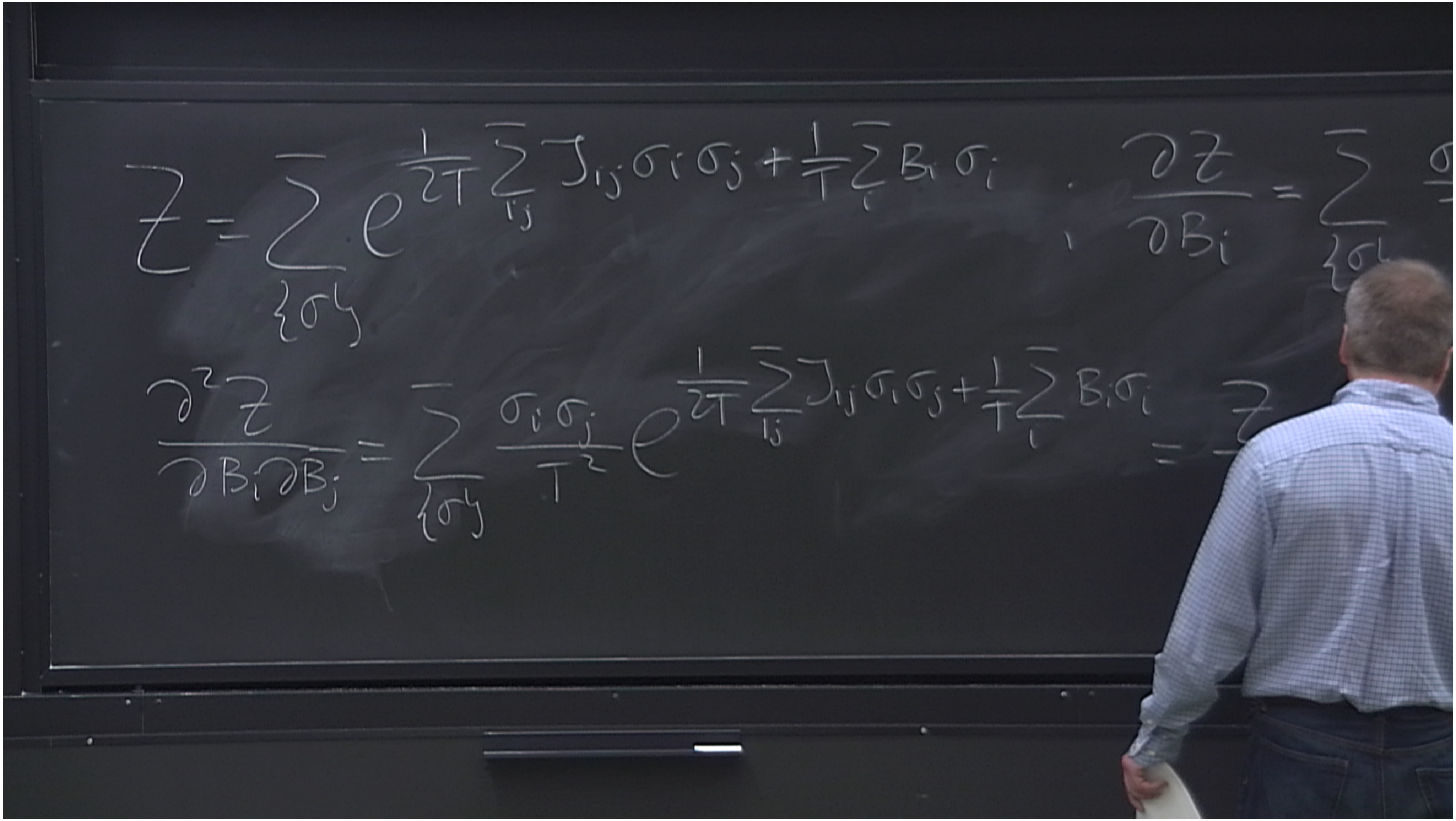
$$Z = \sum_{\{\sigma\}} e^{\frac{1}{2T}\sum_{ij} J_{ij}\sigma_i\sigma_j + \frac{1}{T}\sum_{i} B_i\sigma_i} \quad ; \quad \frac{\partial Z}{\partial B_i} = \sum_{\{\sigma\}}$$



$B_i \sigma_i$

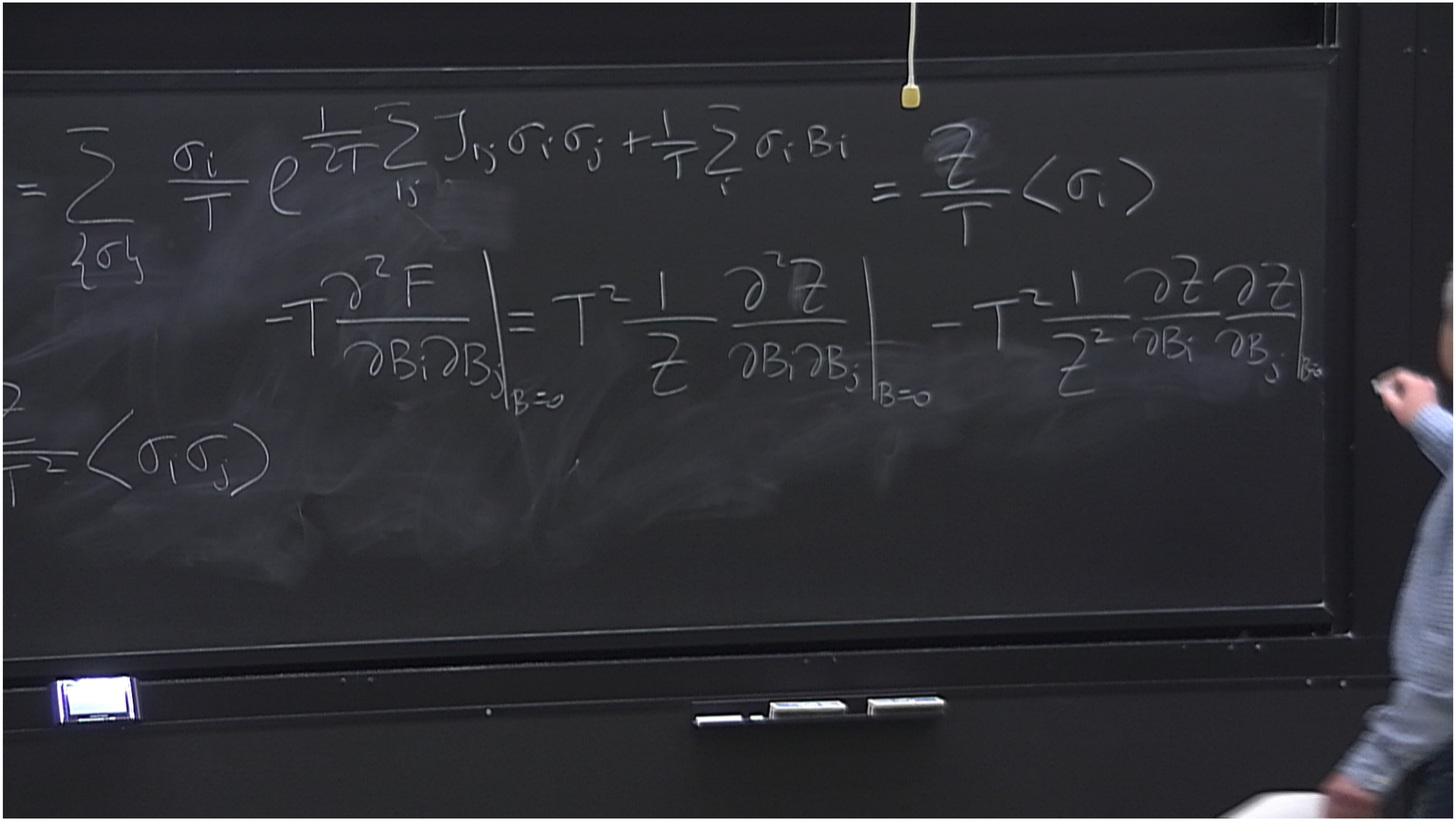
$$\frac{\partial Z}{\partial B_i} = \sum_{\{\sigma\}} \frac{\sigma_i}{T} e^{\frac{1}{2T}\sum_{ij} J_{ij}\sigma_i\sigma_j + \frac{1}{T}\sum_i \sigma_i B_i} =$$

$$\sum_{\{\sigma\}} \frac{\sigma_i}{T} e^{\frac{1}{2T}\sum_{ij} J_{ij}\sigma_i\sigma_j + \frac{1}{T}\sum_i \sigma_i B_i} = \frac{Z}{T}\langle\sigma_i\rangle$$



$$Z = \sum_{\{\sigma\}} e^{\frac{1}{2T}\sum_{ij} J_{ij}\sigma_i\sigma_j + \frac{1}{T}\sum_i B_i\sigma_i} \quad , \quad \frac{\partial Z}{\partial B_i} = \sum_{\{\sigma\}}$$

$$\frac{\partial^2 Z}{\partial B_i \partial B_j} = \sum_{\{\sigma\}} \frac{\sigma_i\sigma_j}{T^2} e^{\frac{1}{2T}\sum_{ij} J_{ij}\sigma_i\sigma_j + \frac{1}{T}\sum_i B_i\sigma_i} = \frac{Z}{}$$



$$= \sum_{\{\sigma\}} \frac{\sigma_i}{T} e^{\frac{1}{2T}\sum_{ij} J_{ij}\sigma_i\sigma_j + \frac{1}{T}\sum_i \sigma_i B_i} = \frac{Z}{T}\langle \sigma_i \rangle$$

$$-T\frac{\partial^2 F}{\partial B_i \partial B_j}\Big|_{B=0} = T^2 \frac{1}{Z}\frac{\partial^2 Z}{\partial B_i \partial B_j}\Big|_{B=0} - T^2 \frac{1}{Z^2}\frac{\partial Z}{\partial B_i}\frac{\partial Z}{\partial B_j}\Big|_{B=0}$$

$$\frac{Z}{T^2}\langle \sigma_i \sigma_j \rangle$$

Evaluate $G_{ij}$ using the functional integral representation

$$Z = \int D\varphi \, e^{-S[\varphi]}$$

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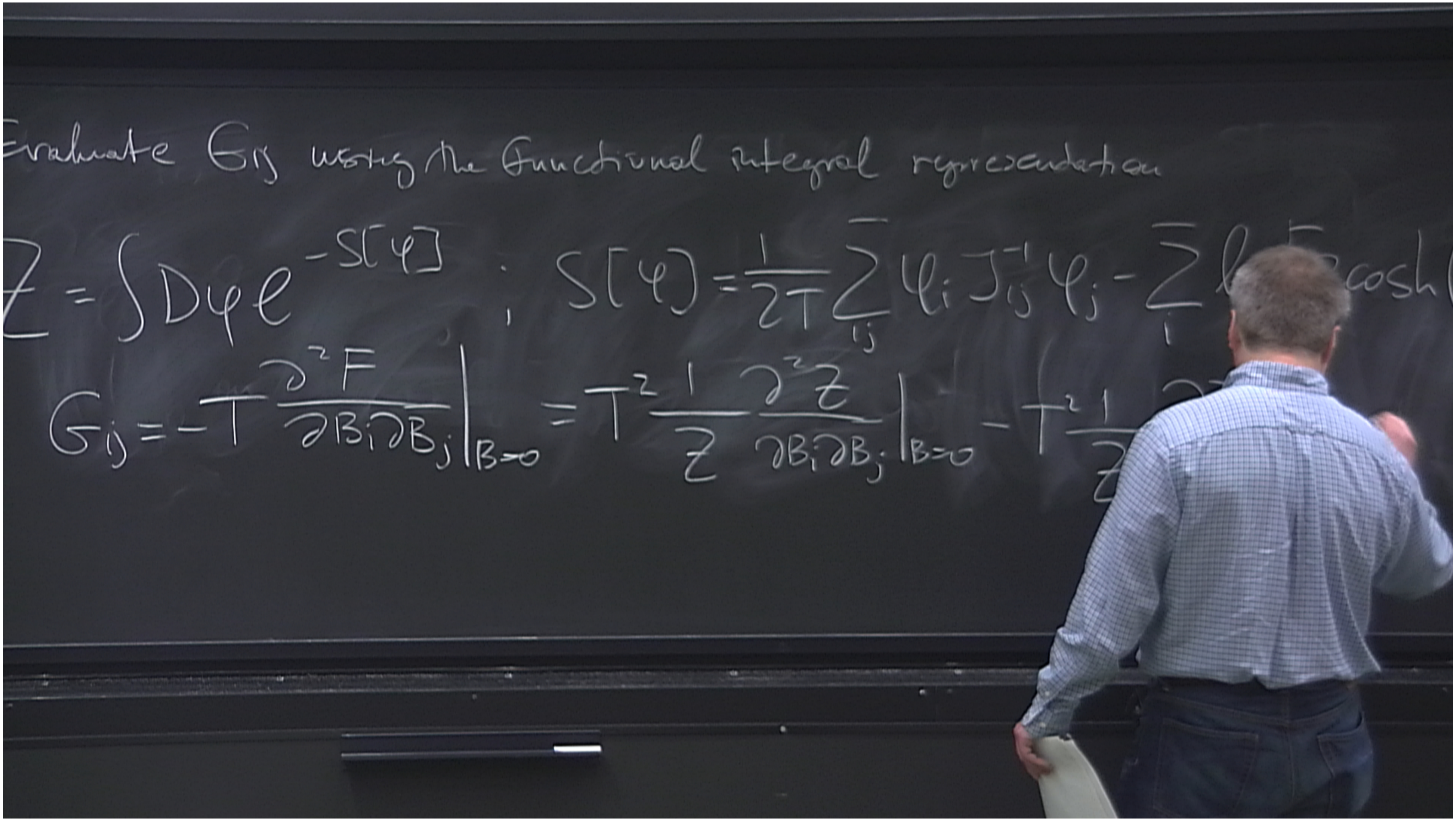

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tation

$$\varphi_j - \sum_i \ln\left[2\cosh\left(\frac{\varphi_i + B_i}{T}\right)\right] ,$$

$$^{-2}\,\frac{1}{Z^2}\,\frac{\partial Z}{\partial B_i}\Big|_{B=0}\,\frac{\partial Z}{\partial B_j}\Big|_{B=0} ,$$

Change integration variables:

$$\varphi_i \longrightarrow \varphi_i - B_i$$

$$S[\varphi] = \frac{1}{2T}\sum_{ij}(\varphi_i - B_i)\,J^{-1}_{ij}\,(\varphi_j - B_j) - \sum_i \ln\left[2\cosh\left(\frac{\varphi_i}{T}\right)\right]$$

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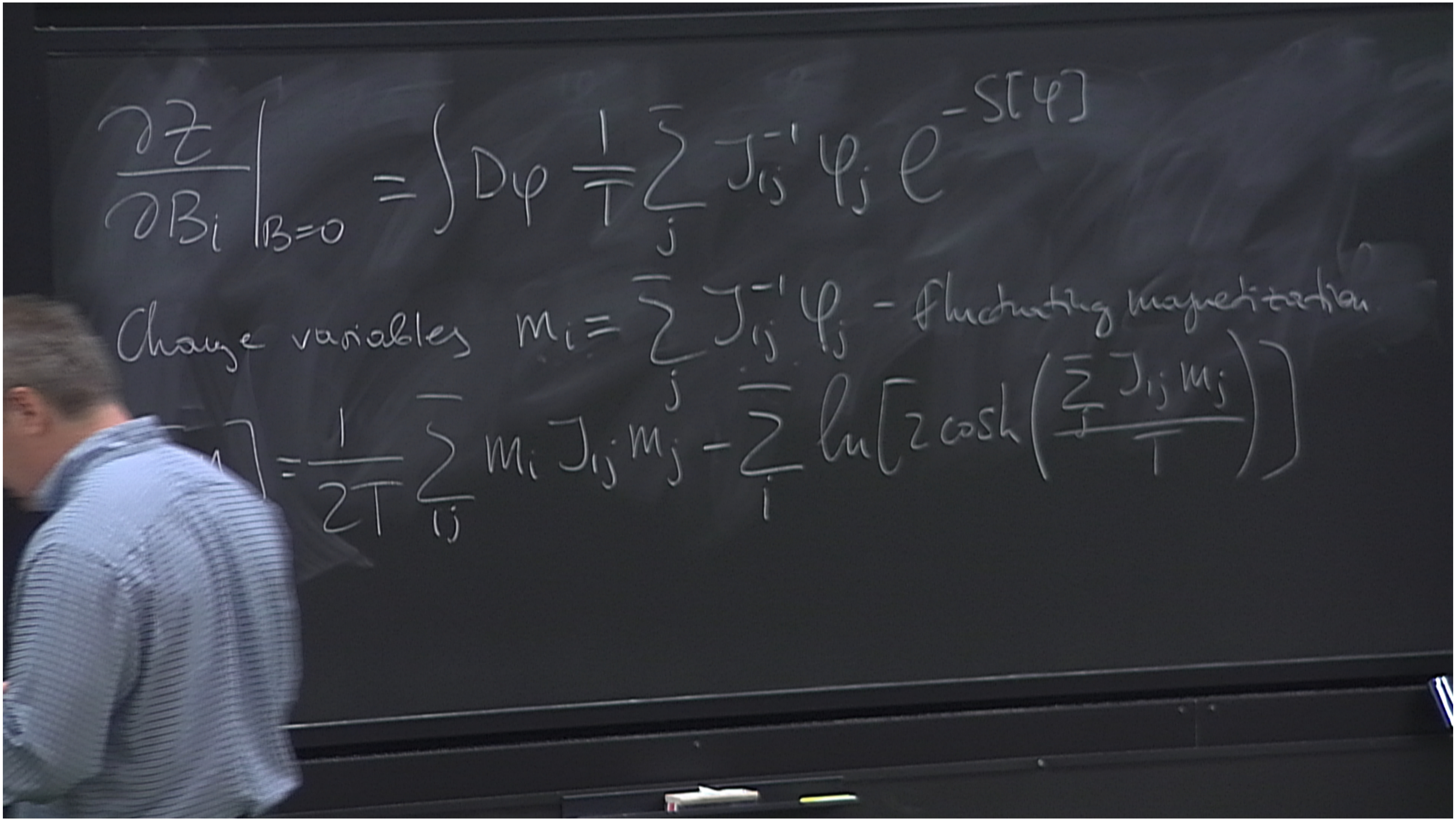
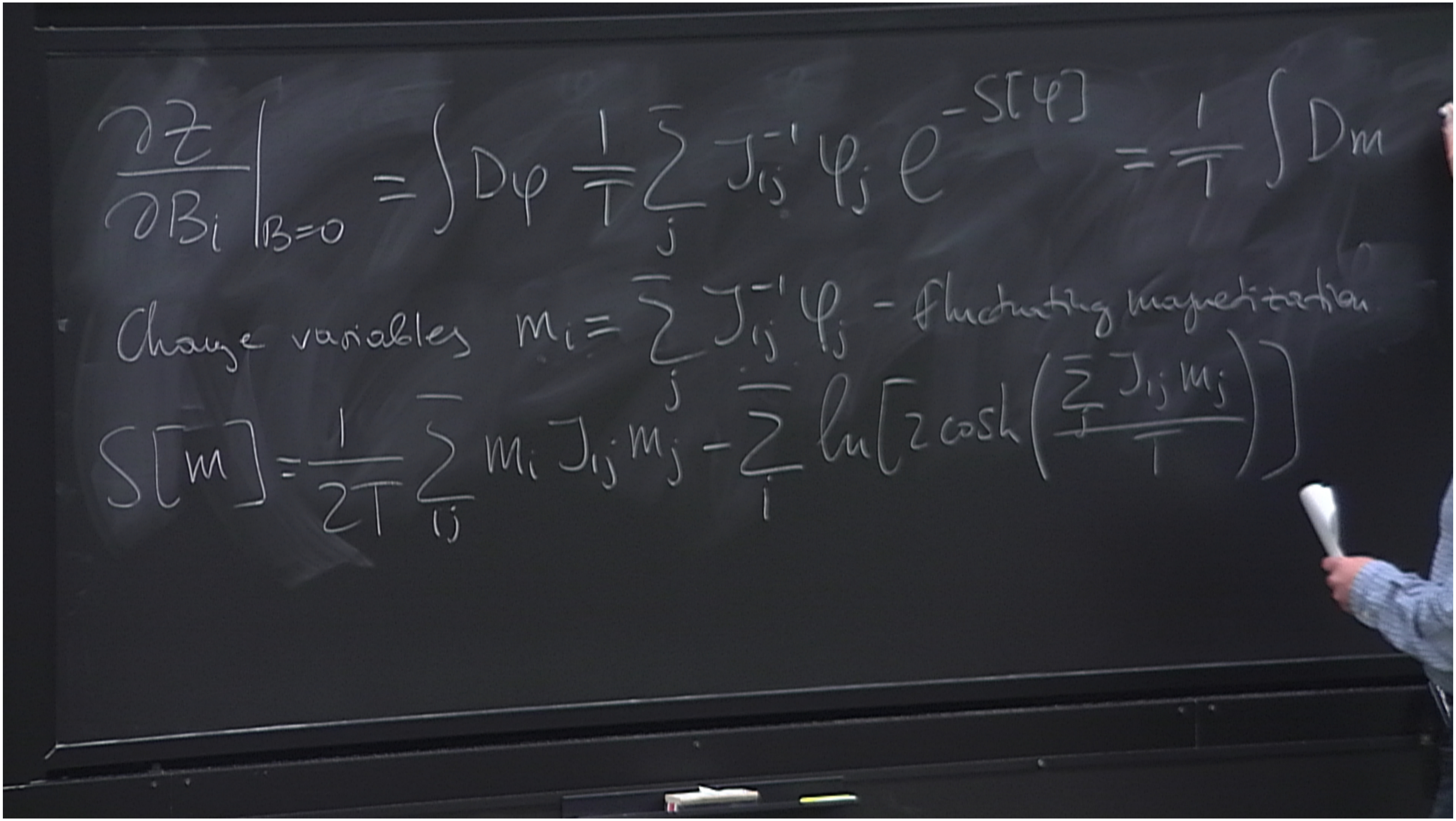


$$\left.\frac{\partial Z}{\partial B_i}\right|_{B=0} = \int D\varphi \, \frac{1}{T}\sum_j J^{-1}_{ij}\varphi_j \, e^{-S[\varphi]}$$

Change variables $m_i = \sum_j J^{-1}_{ij}\varphi_j$ - fluctuating magnetization

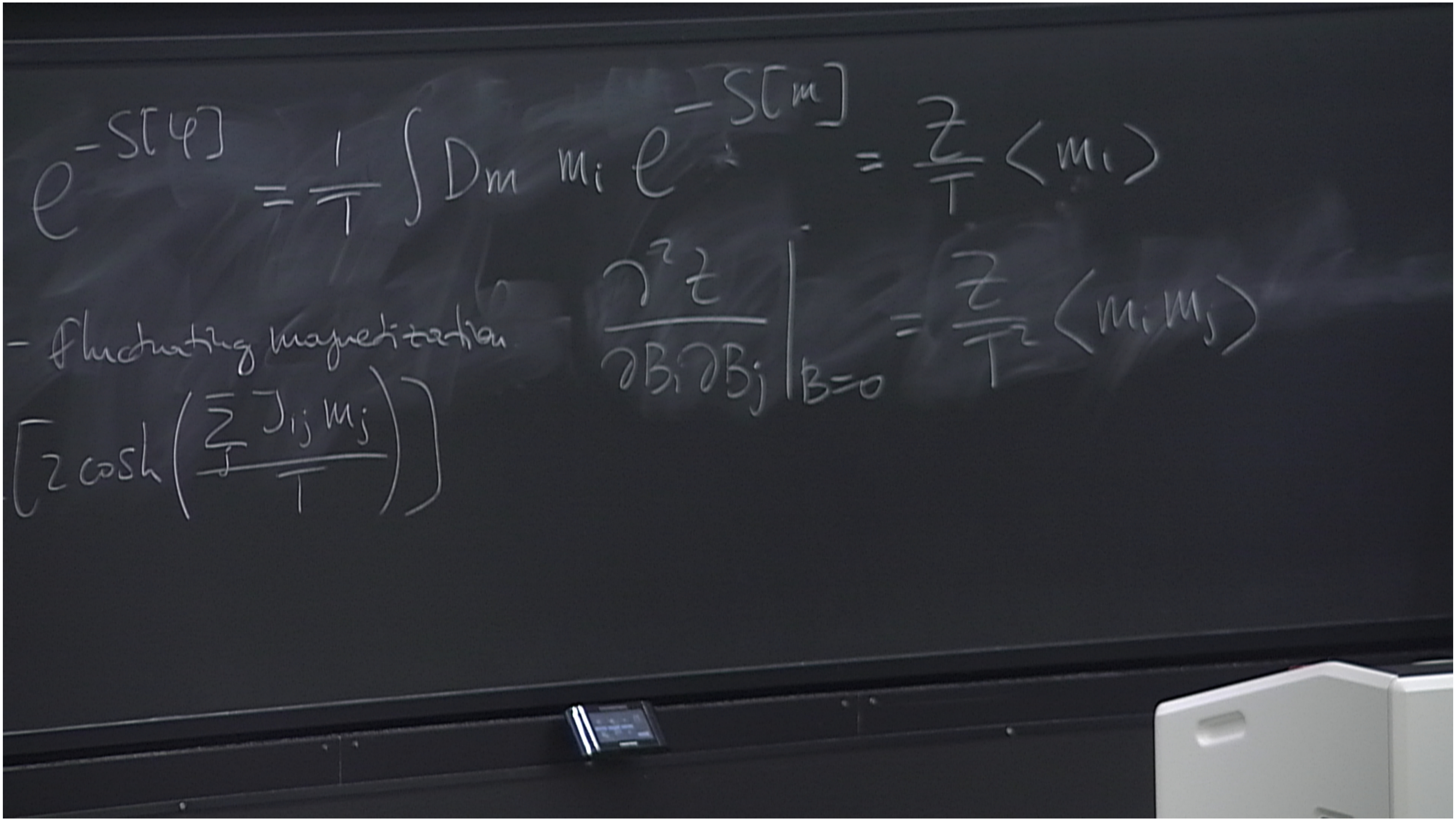
$$] = \frac{1}{2T}\sum_{ij} m_i J_{ij} m_j - \sum_i \ln\left[2\cosh\left(\frac{\sum_j J_{ij} m_j}{T}\right)\right]$$



$$\left.\frac{\partial Z}{\partial B_i}\right|_{B=0} = \int D\varphi \, \frac{1}{T} \sum_j J^{-1}_{ij} \varphi_j \, e^{-S[\varphi]} = \frac{1}{T} \int Dm$$

Change variables $m_i = \sum_j J^{-1}_{ij} \varphi_j$ - fluctuating magnetization

$$S[m] = \frac{1}{2T} \sum_{ij} m_i J_{ij} m_j - \sum_i \ln\left[2\cosh\left(\frac{\sum_j J_{ij} m_j}{T}\right)\right]$$

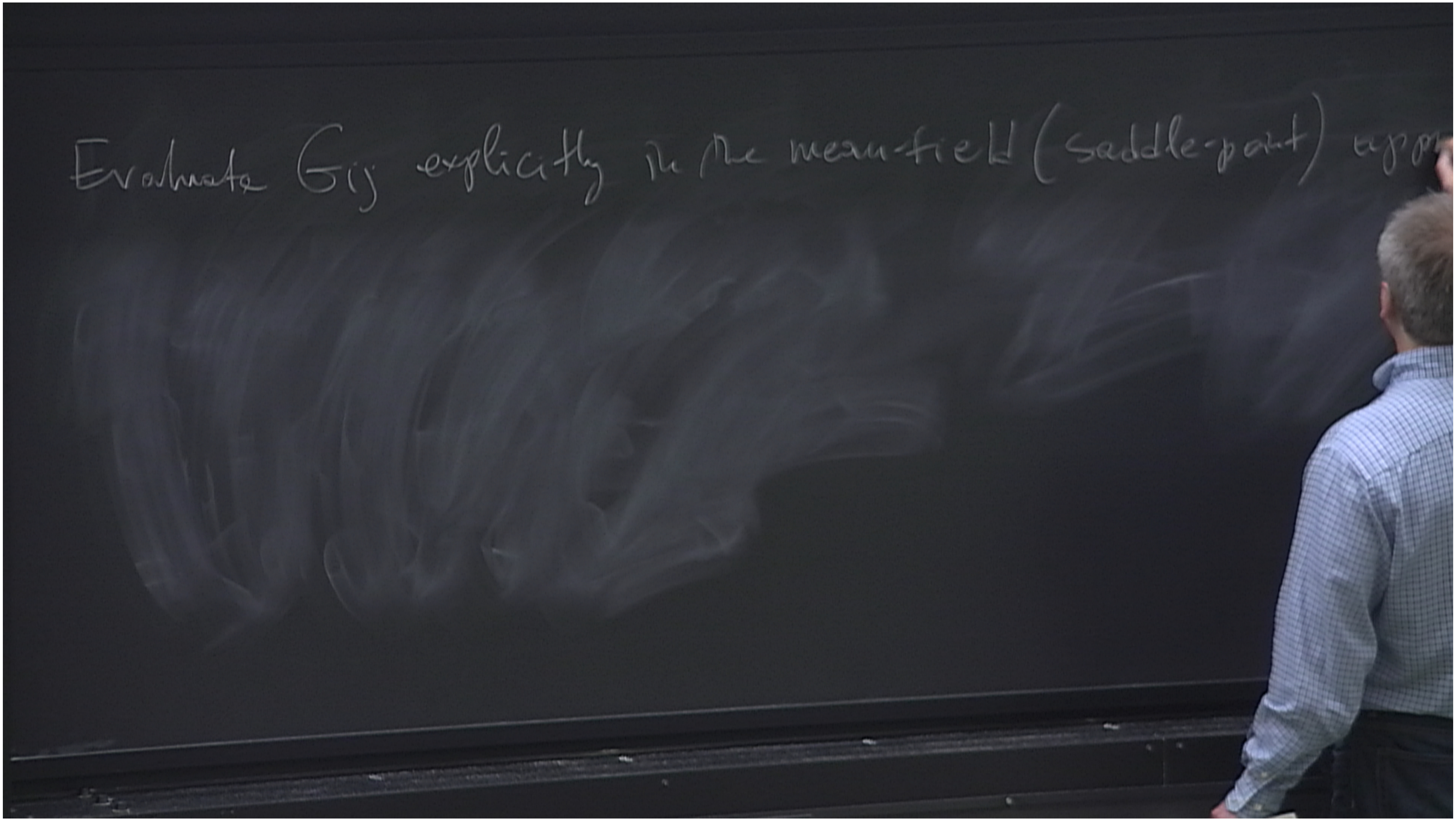


$$e^{-S[\varphi]} = \frac{1}{T}\int Dm \; m_i \, e^{-S[m]} = \frac{Z}{T}\langle m_i \rangle$$

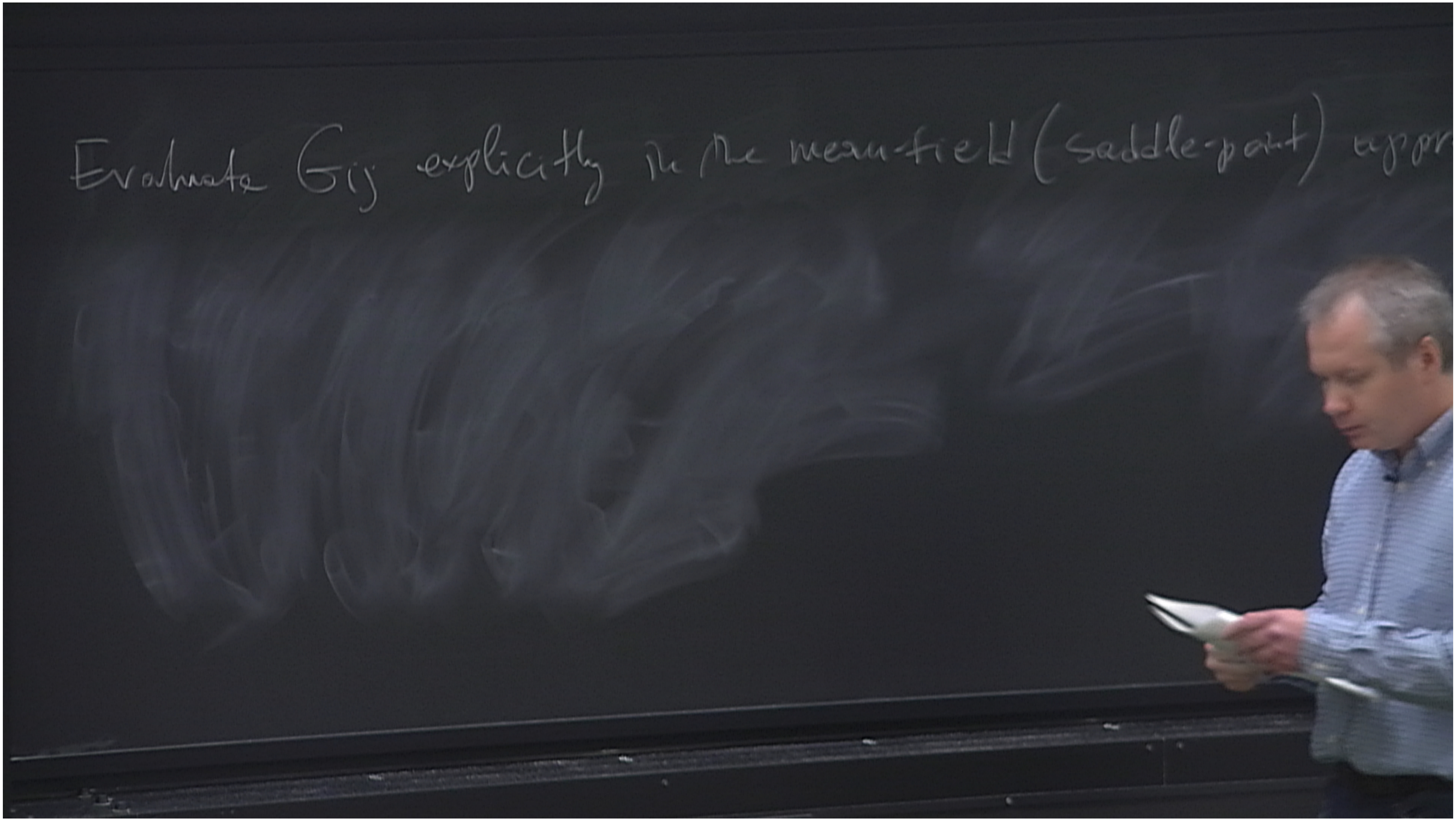
- fluctuating magnetization

$$\frac{\partial^2 Z}{\partial B_i \partial B_j}\Big|_{B=0} = \frac{Z}{T^2}\langle m_i m_j \rangle$$

$$\left[2\cosh\left(\frac{\sum_j J_{ij} m_j}{T}\right)\right]$$



Evaluate $G_{ij}$ explicitly in the mean-field (saddle-point) appr



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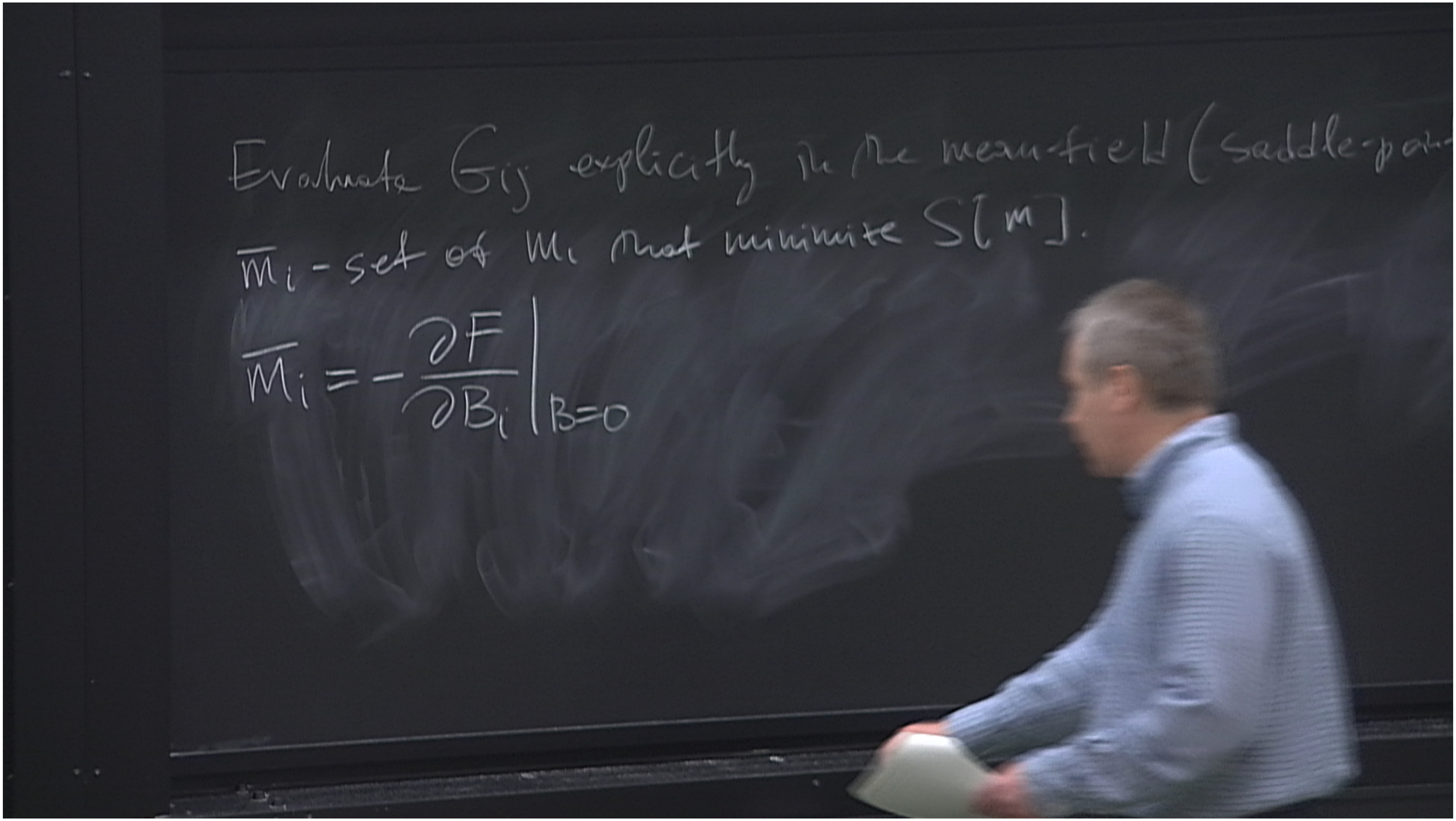
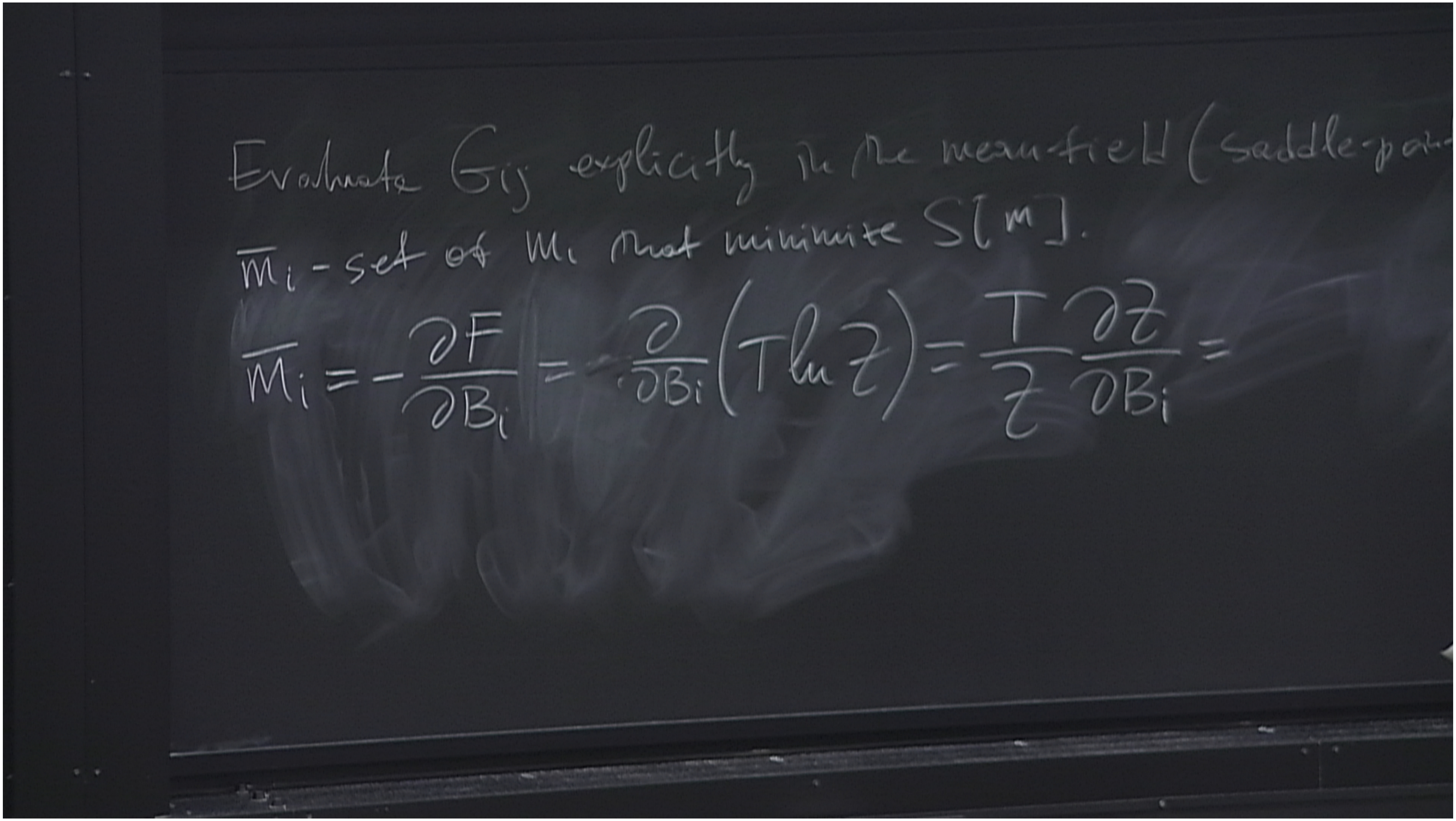


Evaluate $G_{ij}$ explicitly in the mean-field (saddle-poi

$\overline{m}_i$ - set of $m_i$ that minimize $S[m]$.

$$\overline{m_i} = -\frac{\partial F}{\partial B_i}\Big|_{B=0}$$



Evaluate $G_{ij}$ explicitly in the mean-field (saddle-poi

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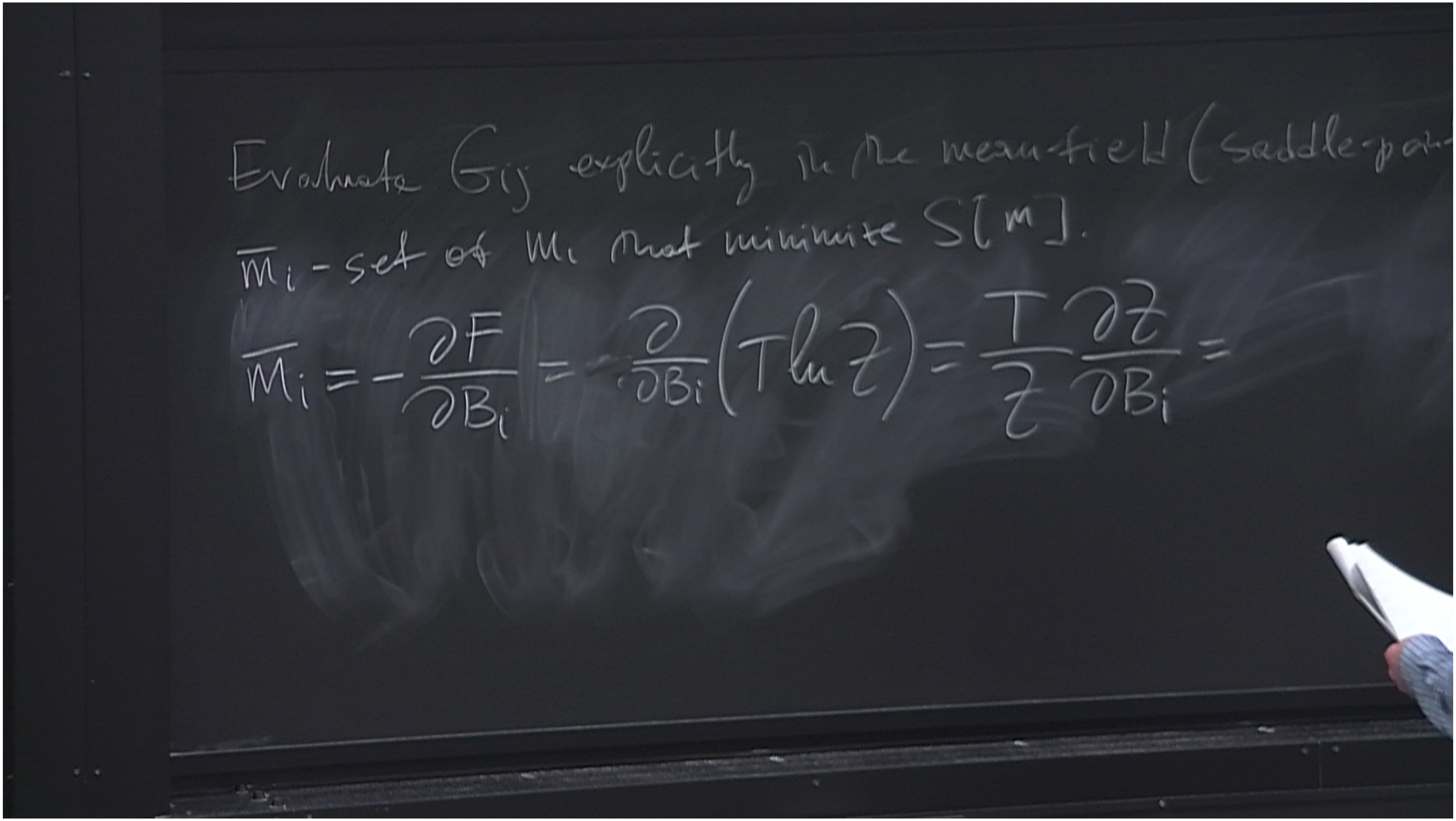
$$\bar{m}_i = -\frac{\partial F}{\partial B_i} = -\frac{\partial}{\partial B_i}\left(T \ln Z\right) = \frac{T}{Z}\frac{\partial Z}{\partial B_i} =$$



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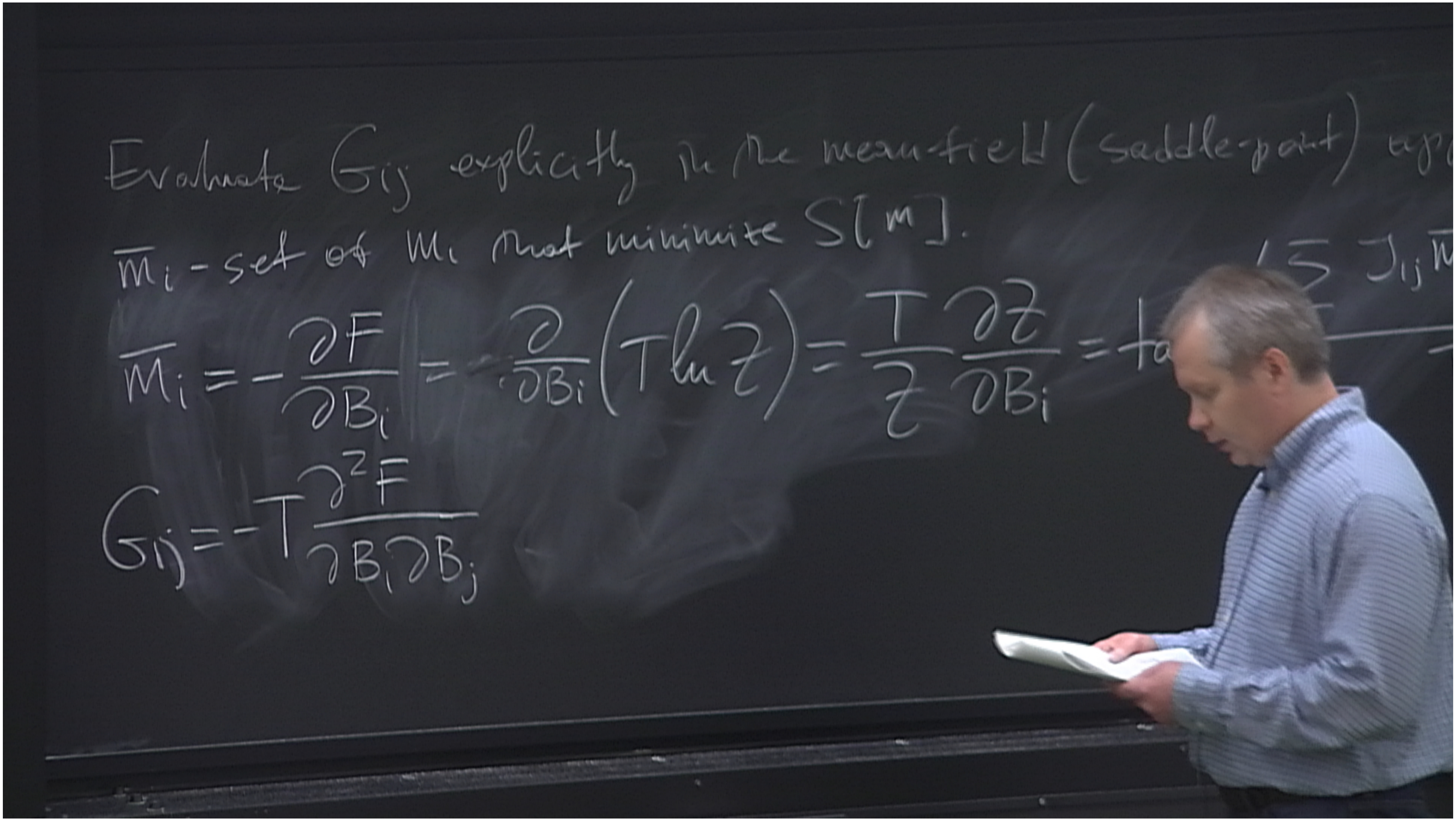

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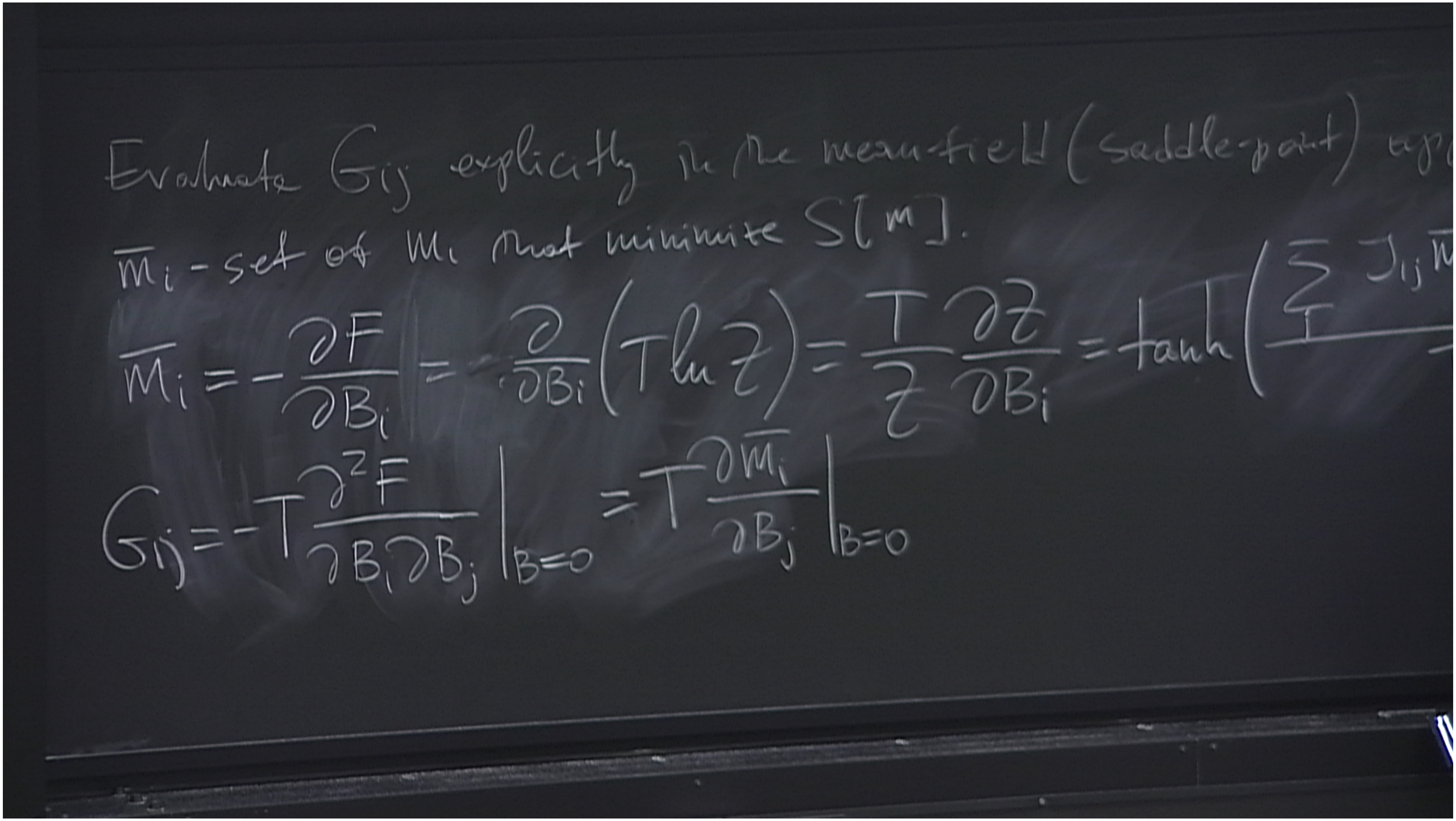
$$\left(\sum J_{ij}\overline{m}\right.$$

$$G_{ij} = -T\frac{\partial^2 F}{\partial B_i \partial B_j}$$



Evaluate $G_{ij}$ explicitly in the mean-field (saddle-point) app

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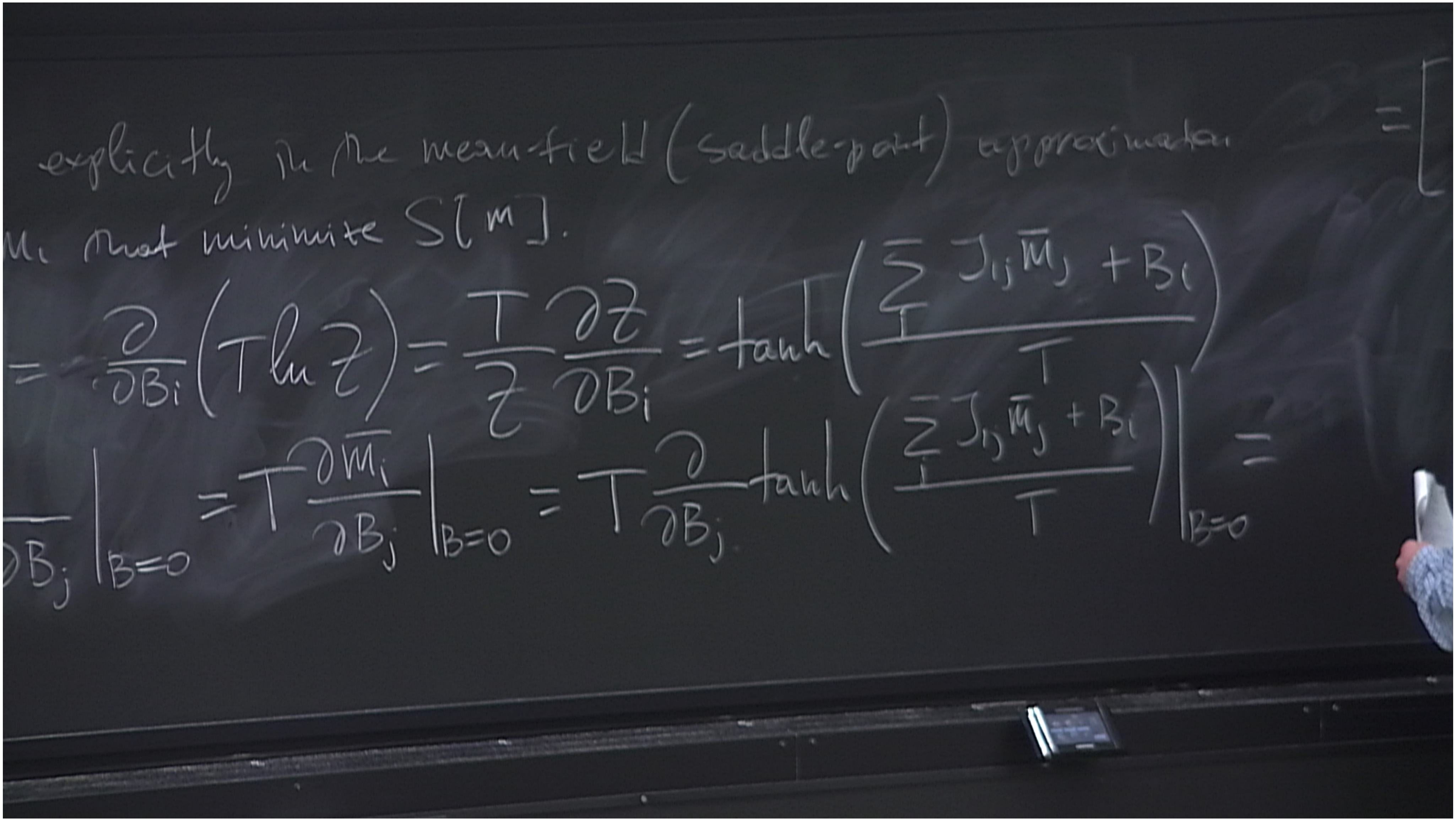
$$\bar{m}_i = -\frac{\partial F}{\partial B_i} = \frac{\partial}{\partial B_i}\left(T \ln Z\right) = \frac{T}{Z}\frac{\partial Z}{\partial B_i} = \tanh\left(\frac{\sum_j J_{ij}\bar{m}}{}\right.$$

$$G_{ij} = -T\frac{\partial^2 F}{\partial B_i \partial B_j}\Big|_{B=0} = T\frac{\partial \bar{m}_i}{\partial B_j}\Big|_{B=0}$$

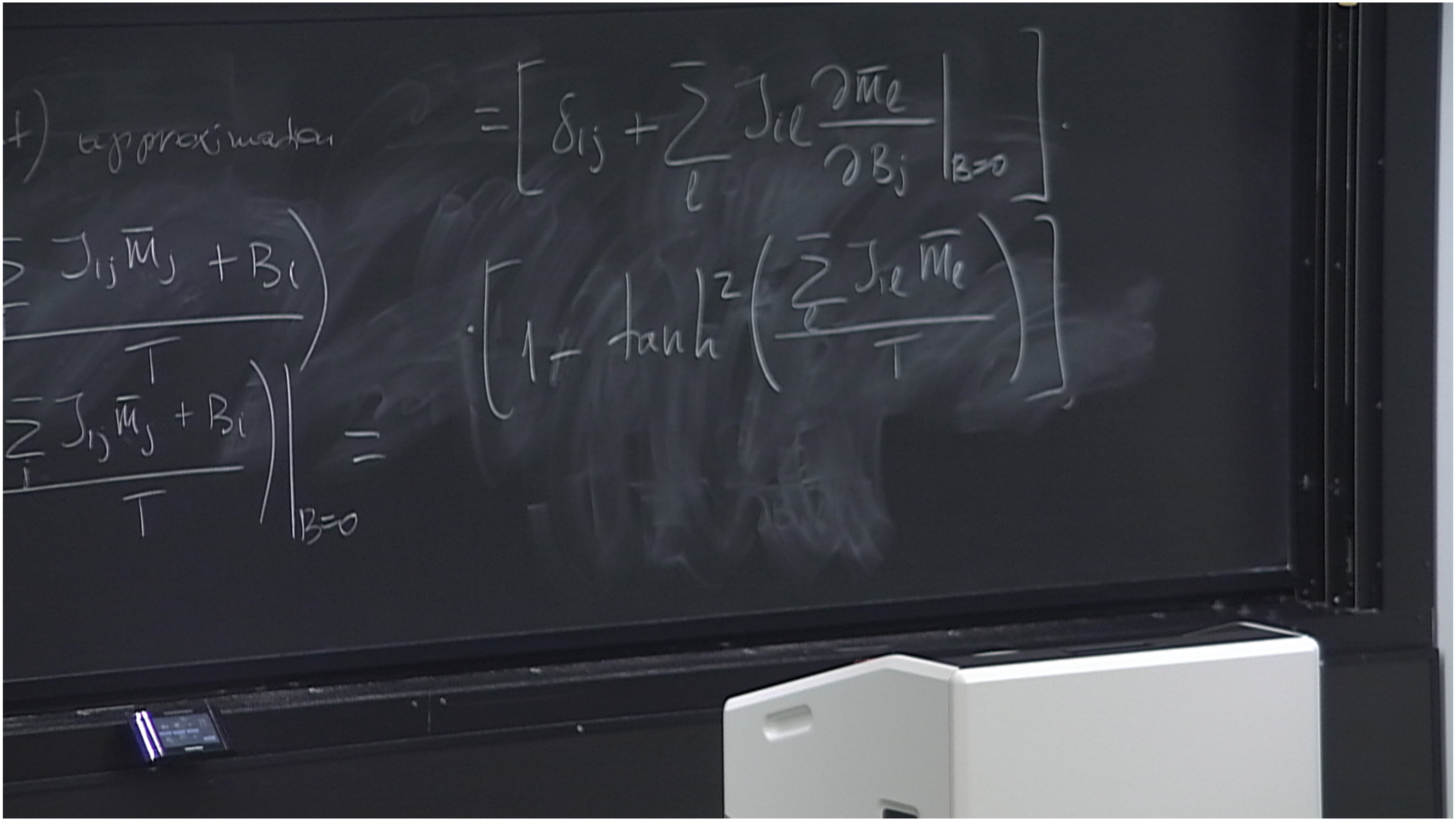


explicitly in the mean-field (saddle-point) approximation $= [$

$m_i$ that minimize $S[m]$.

$$= \frac{\partial}{\partial B_i}\left(T \ln Z\right) = \frac{T}{Z}\frac{\partial Z}{\partial B_i} = \tanh\left(\frac{\sum_j J_{ij}\bar{m}_j + B_i}{T}\right)$$

$$\partial B_j \Big|_{B=0} = T\frac{\partial \bar{m}_i}{\partial B_j}\Big|_{B=0} = T\frac{\partial}{\partial B_j}\tanh\left(\frac{\sum_j J_{ij}\bar{m}_j + B_i}{T}\right)\Big|_{B=0} =$$

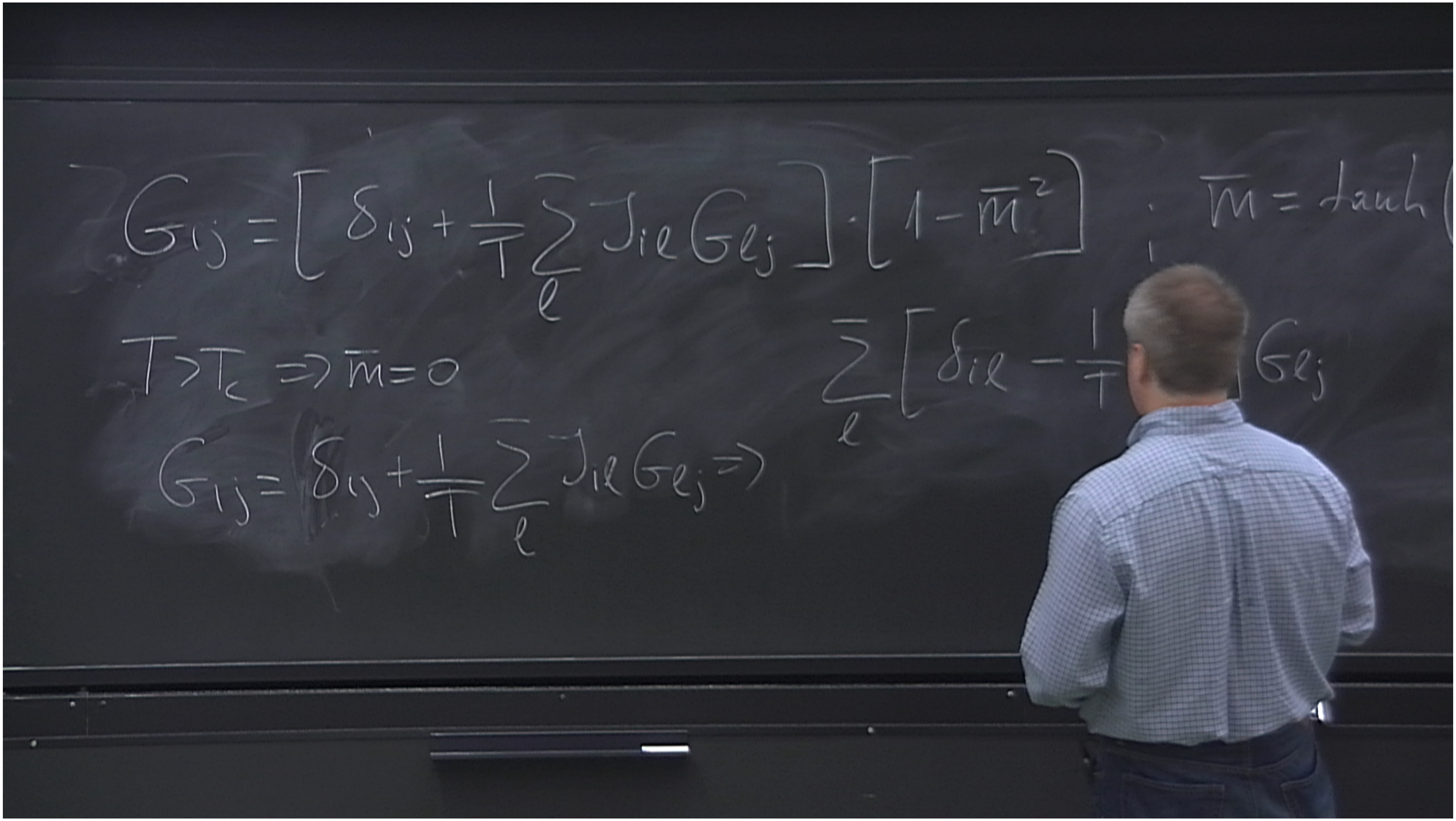


t) approximation

$$\sum J_{ij}\bar{m}_j + B_i \over T \Big)$$

$$\sum_j J_{ij}\bar{m}_j + B_i \over T \Big)\Big|_{B=0} = \left[\delta_{ij} + \sum_l J_{il}\frac{\partial \bar{m}_l}{\partial B_j}\Big|_{B=0}\right] \cdot \left[1 - \tanh^2\left(\frac{\sum_l J_{il}\bar{m}_l}{T}\right)\right],$$

$$\Big] \cdot \left[ 1 - \bar{m}^2 \right] \; ; \; \bar{m} = \tanh\left( \frac{J \bar{m}}{T} \right) \; ; \; J = \sum_j J_{ij}$$



$$G_{ij} = \left[\delta_{ij} + \frac{1}{T}\sum_{\ell} J_{i\ell}G_{\ell j}\right]\cdot\left[1-\bar{m}^2\right] \;;\; \bar{m} = \tanh($$

$$T > T_c \Rightarrow \bar{m} = 0$$

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$$\sum_{\ell}\left[\delta_{i\ell} - \frac{1}{T} \quad \right] G_{\ell j}$$

$$G_{ij} = \left[\delta_{ij} + \frac{1}{T}\sum_{\ell} J_{i\ell}\, G_{\ell j}\right]\cdot\left[1-\bar{m}^2\right] \quad ; \quad \bar{m} = \tan$$

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