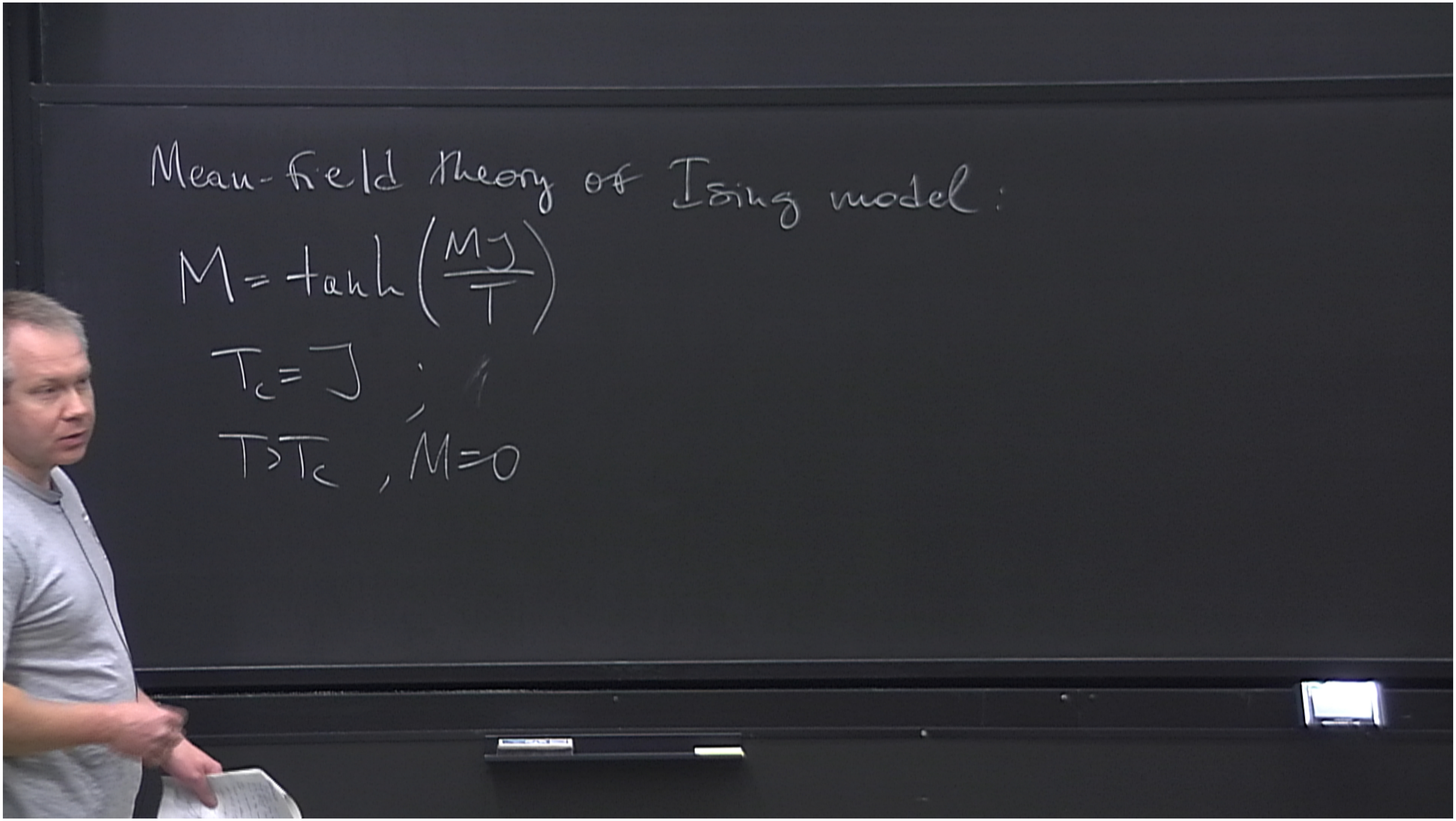
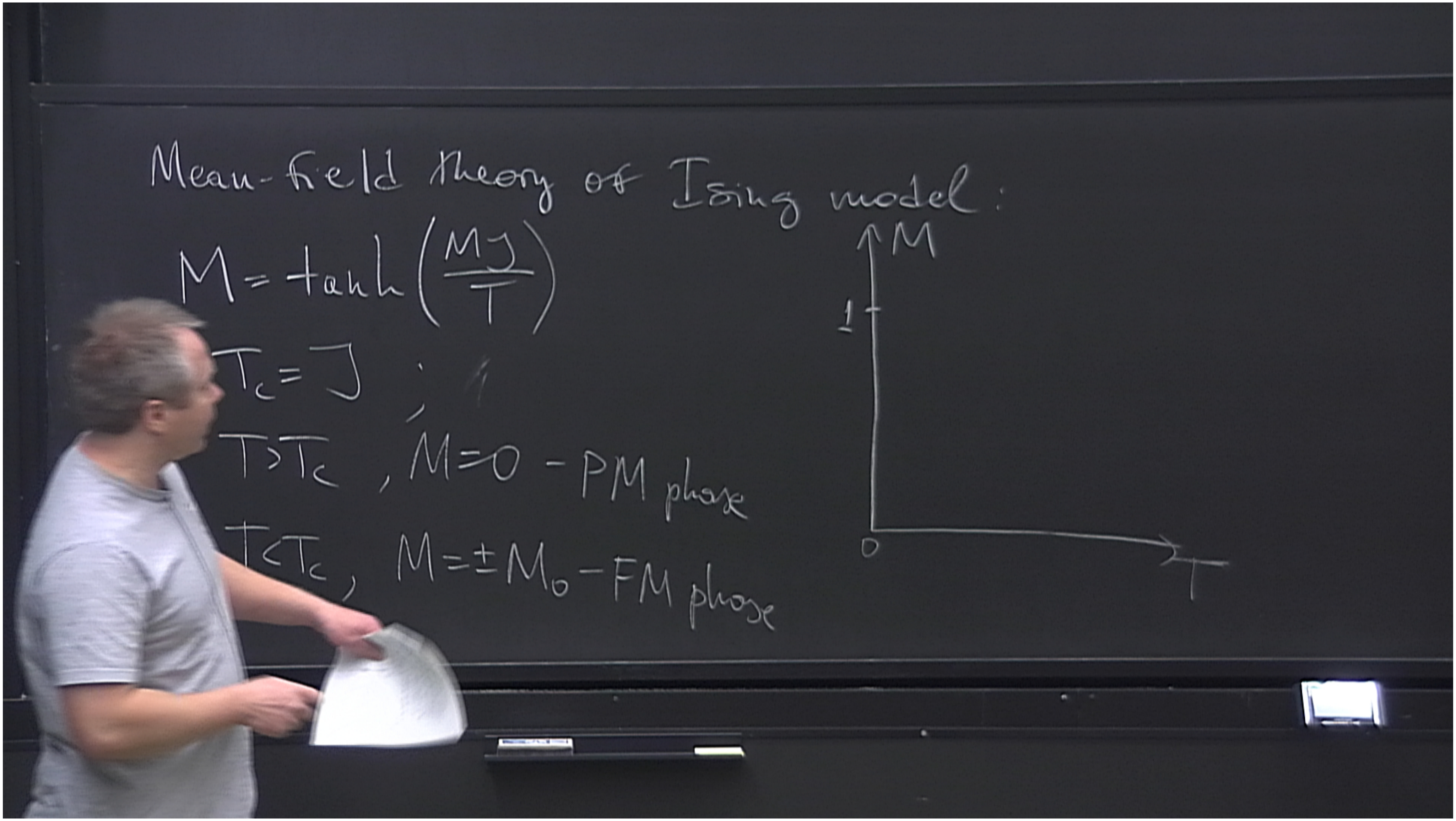
Title: 14/15 PSI - Statistical Mechanics - Lecture 2

Date: Oct 07, 2014 10:45 AM

URL: http://pirsa.org/14100085

Abstract:


Mean-field theory of Ising model:
$M = \tanh\left(\frac{MJ}{T}\right)$
$T_c = J$;
$T > T_c$, $M = 0$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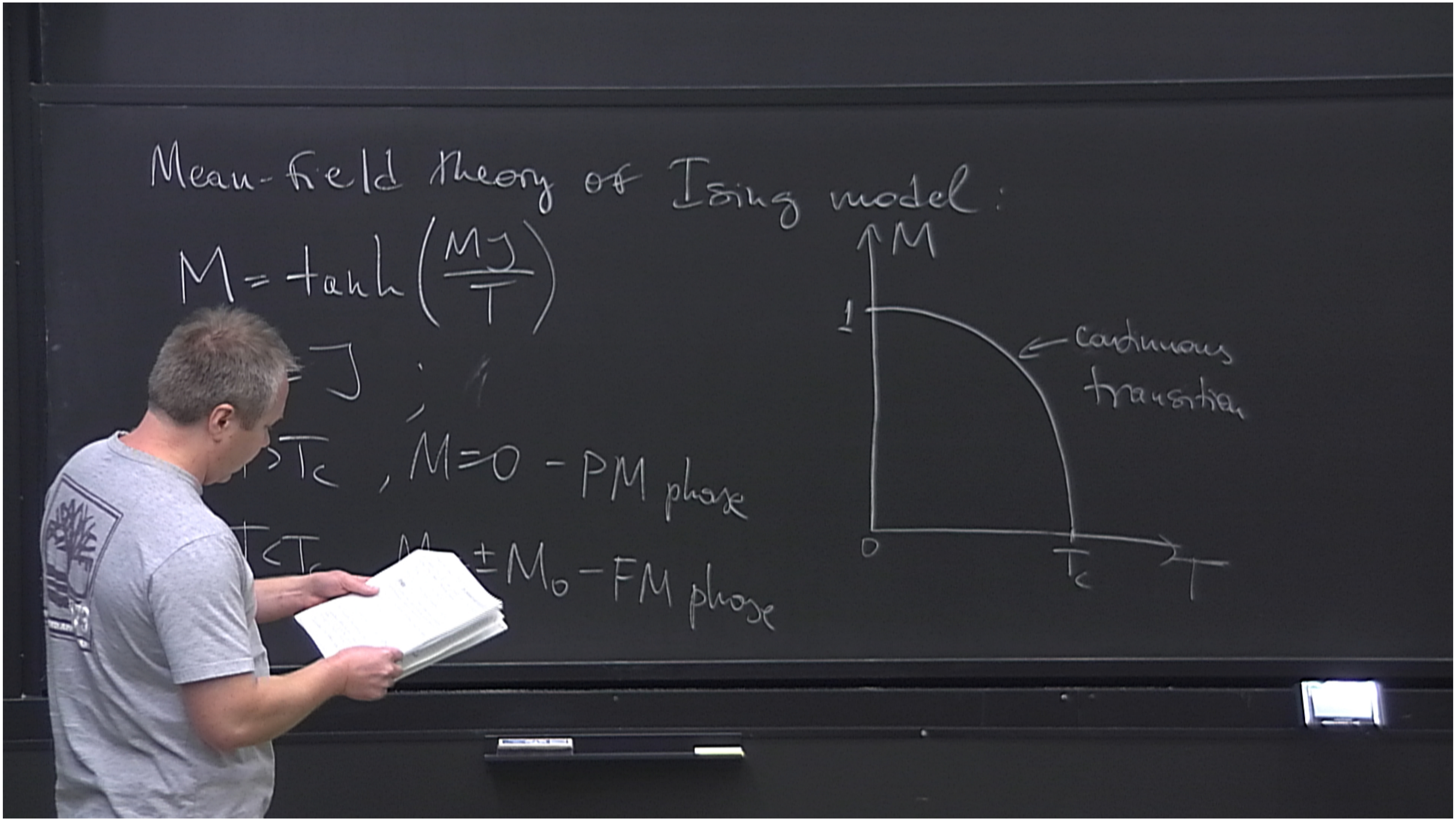
$M = \tanh\left(\frac{MJ}{T}\right)$
$T_c = J$;
$T > T_c$, $M = 0$ - PM phase
$T < T_c$, $M = \pm M_0$ - FM phase
M
1
0
T


Mean-field theory of Ising model:
$M = \tanh\left(\frac{MJ}{T}\right)$
$>T_c$, $M=0$ - PM phase
$\pm M_0$ - FM phase
M
1
continuous transition
0
T_c
T

Since M vanishes at T_c continuously, can assume that M is small when T is close to T_c.

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Can expand $F(M)$ in Taylor series.

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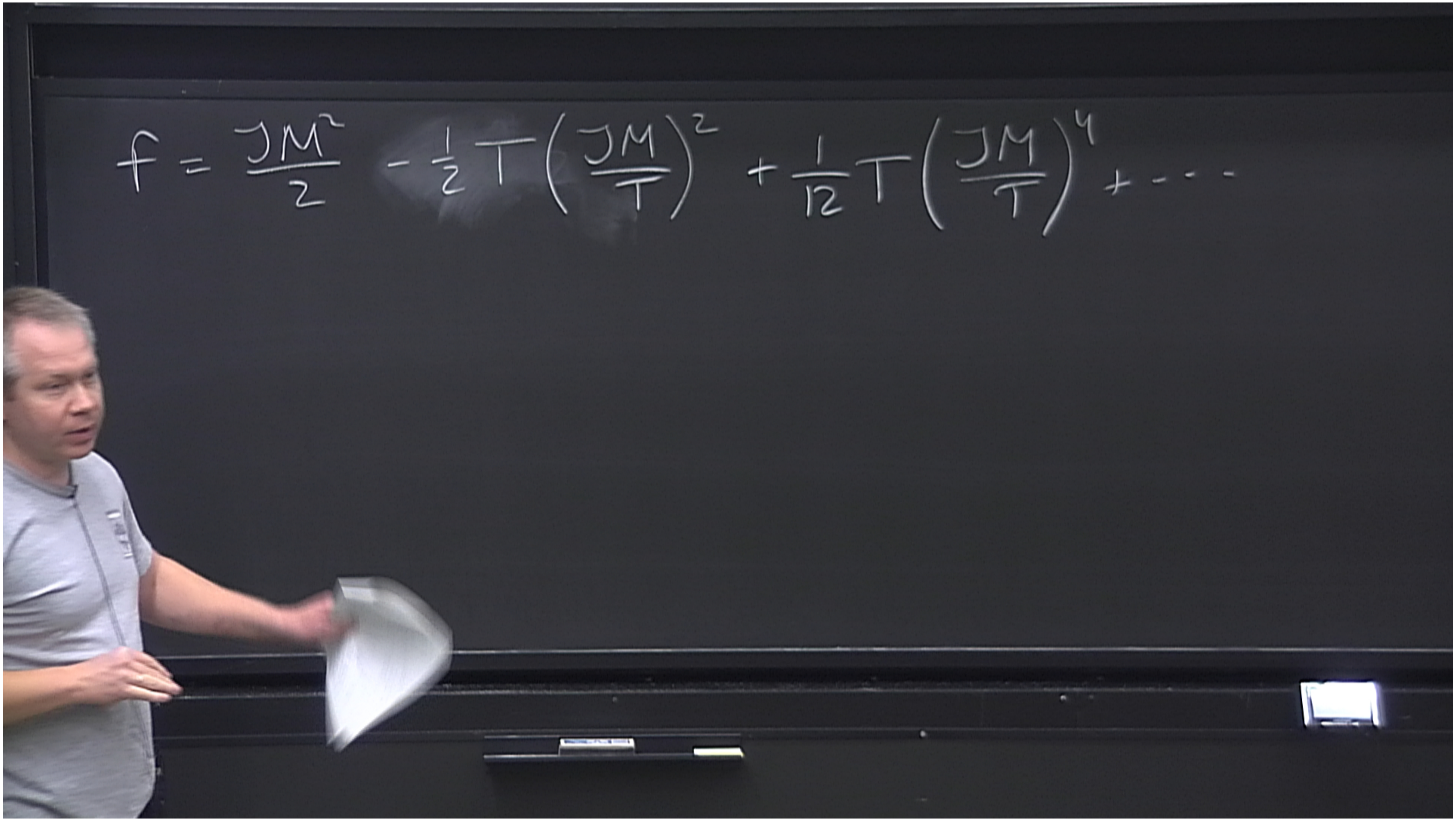
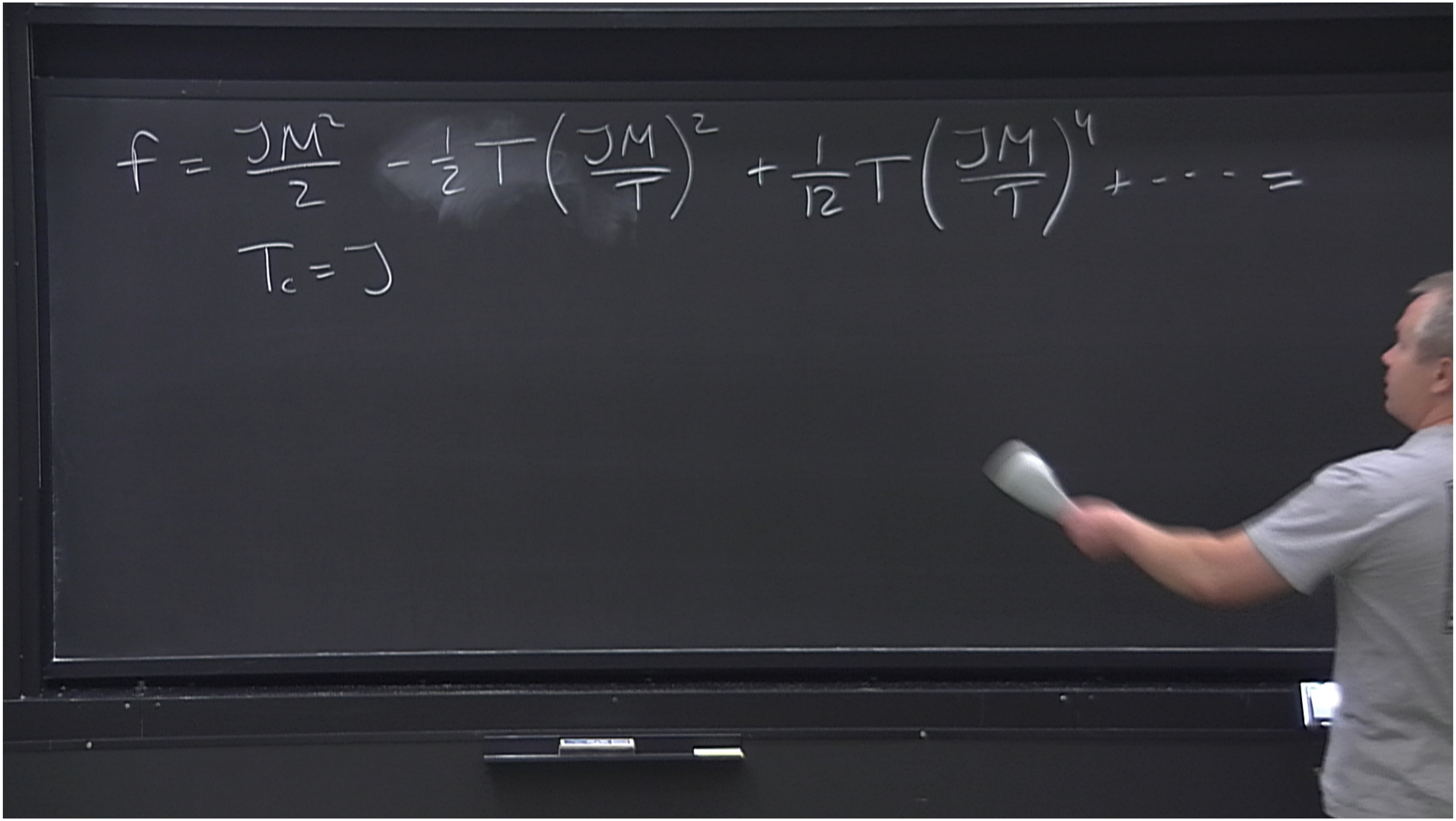
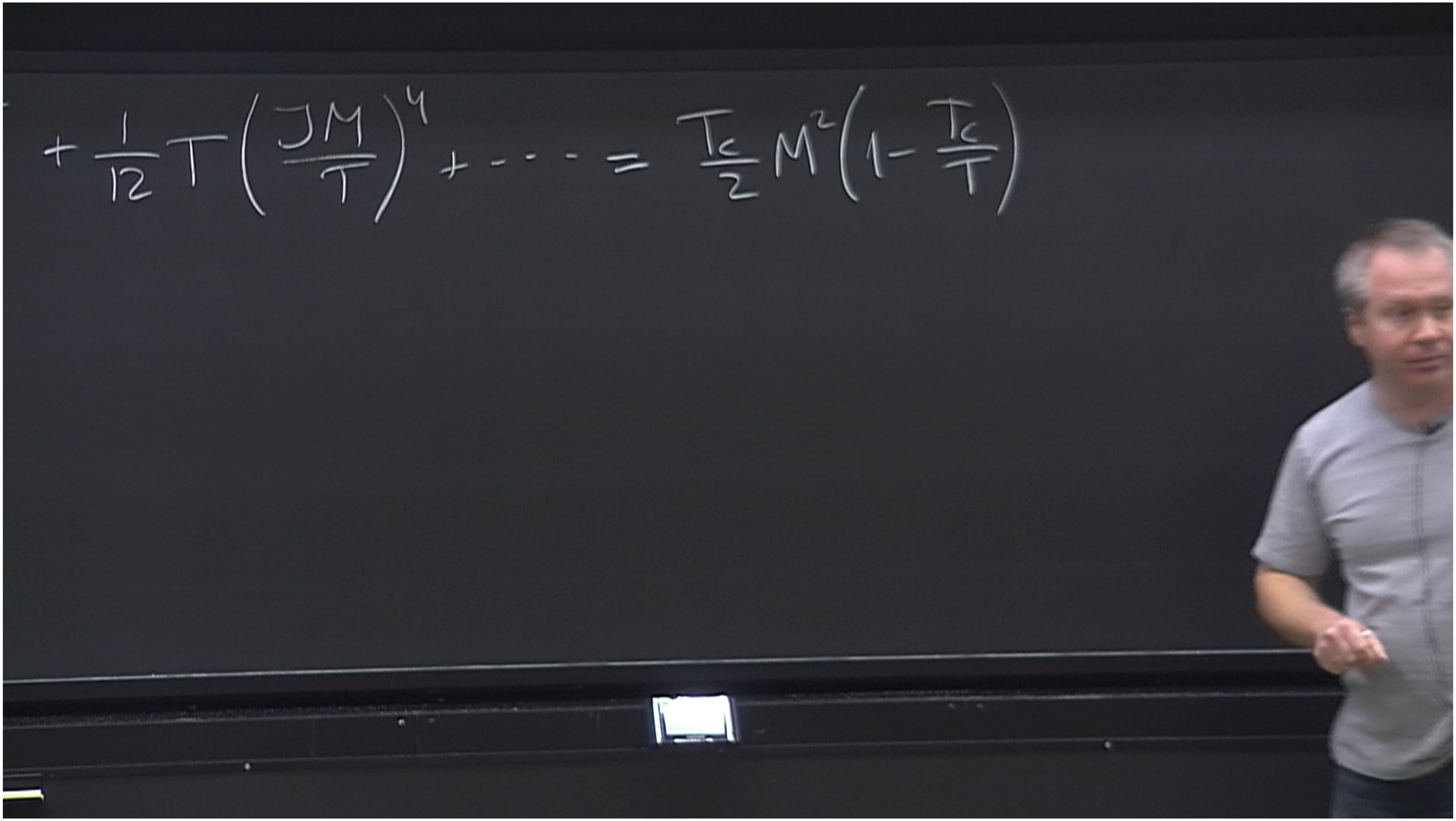
Can expand $F(M)$ in Taylor series.

$$f = \frac{F}{N} = \frac{JM^2}{2} - T \ln\left[2\cosh\left(\frac{JM}{T}\right)\right]$$

$$f = \frac{JM^2}{2} - \frac{1}{2} T \left(\frac{JM}{T} \right)^2$$

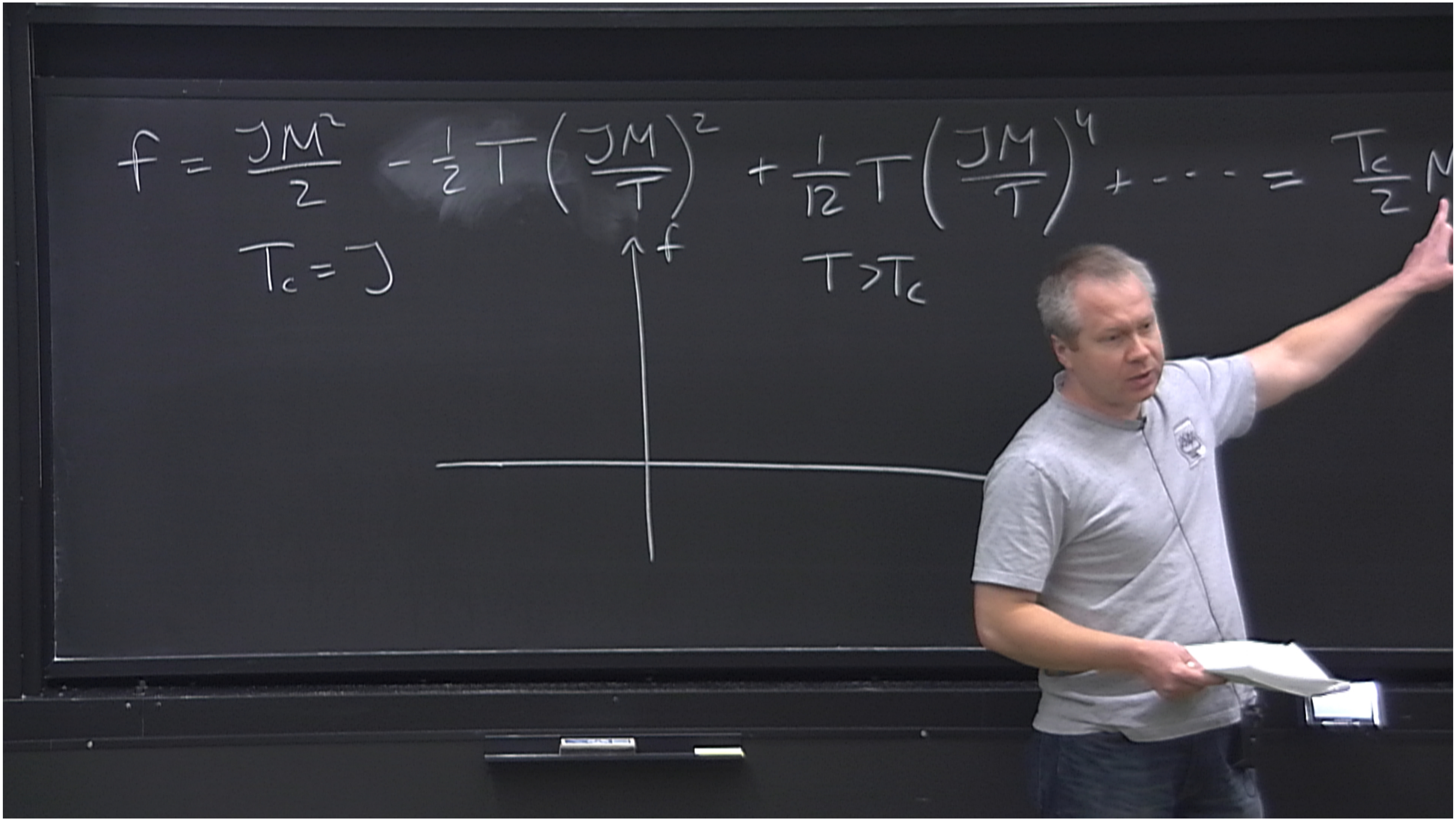

$f = \frac{JM^2}{2} - \frac{1}{2} T \left(\frac{JM}{T}\right)^2 + \frac{1}{12} T \left(\frac{JM}{T}\right)^4 + \cdots$

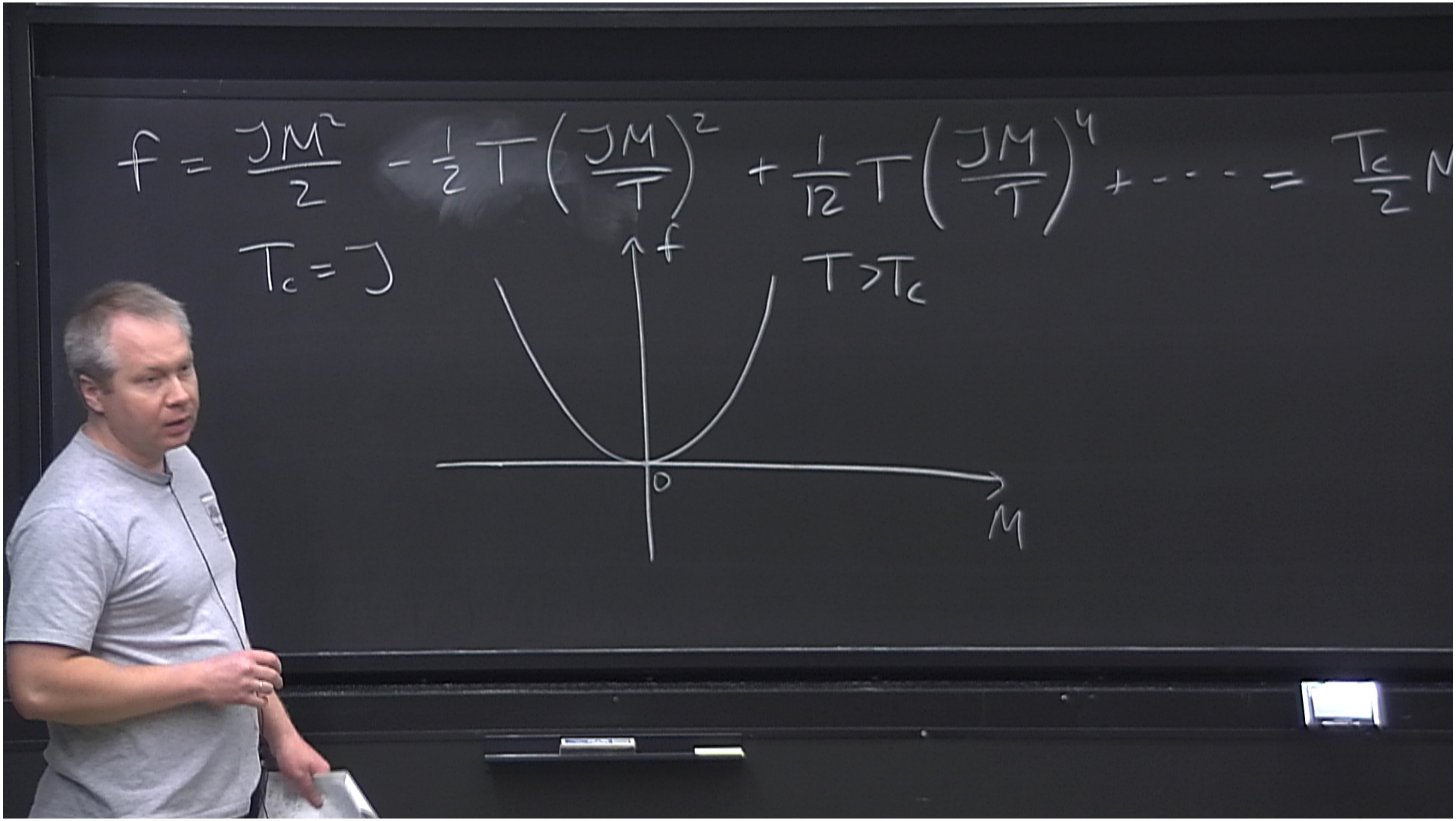

$f = \frac{JM^2}{2} - \frac{1}{2} T \left(\frac{JM}{T}\right)^2 + \frac{1}{12} T \left(\frac{JM}{T}\right)^4 + \cdots =$
$T_c = J$


$+\frac{1}{12}T\left(\frac{JM}{T}\right)^4+\cdots=\frac{T_c}{2}M^2\left(1-\frac{T_c}{T}\right)$

$$= \frac{T_c}{2} M^2 \left(1 - \frac{T_c}{T}\right) + \frac{T_c^4}{12 T^3} M^4$$

$$= \frac{T_c}{2} M^2 \left(1 - \frac{T_c}{T}\right) + \underbrace{\frac{T_c^4}{12T^3}}_{>0} M^4$$


$f = \frac{JM^2}{2} - \frac{1}{2}T\left(\frac{JM}{T}\right)^2 + \frac{1}{12}T\left(\frac{JM}{T}\right)^4 + \cdots = \frac{T_c}{2}M$
$T_c = J$
f
$T > T_c$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$T > T_c$
0
M

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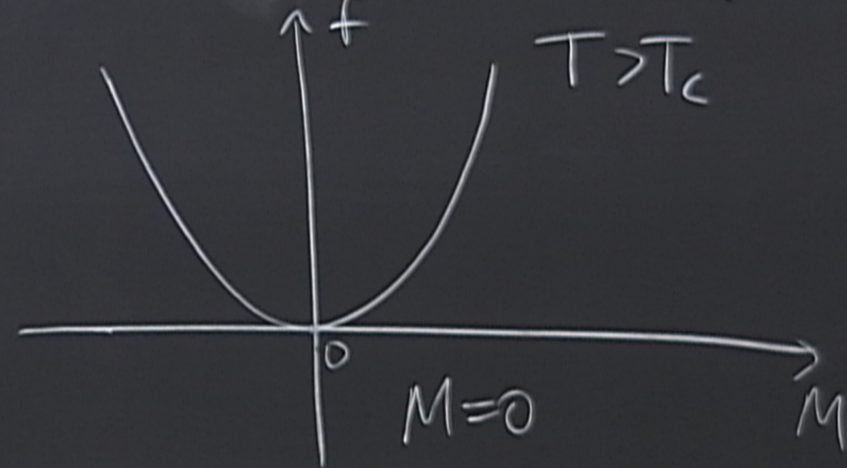
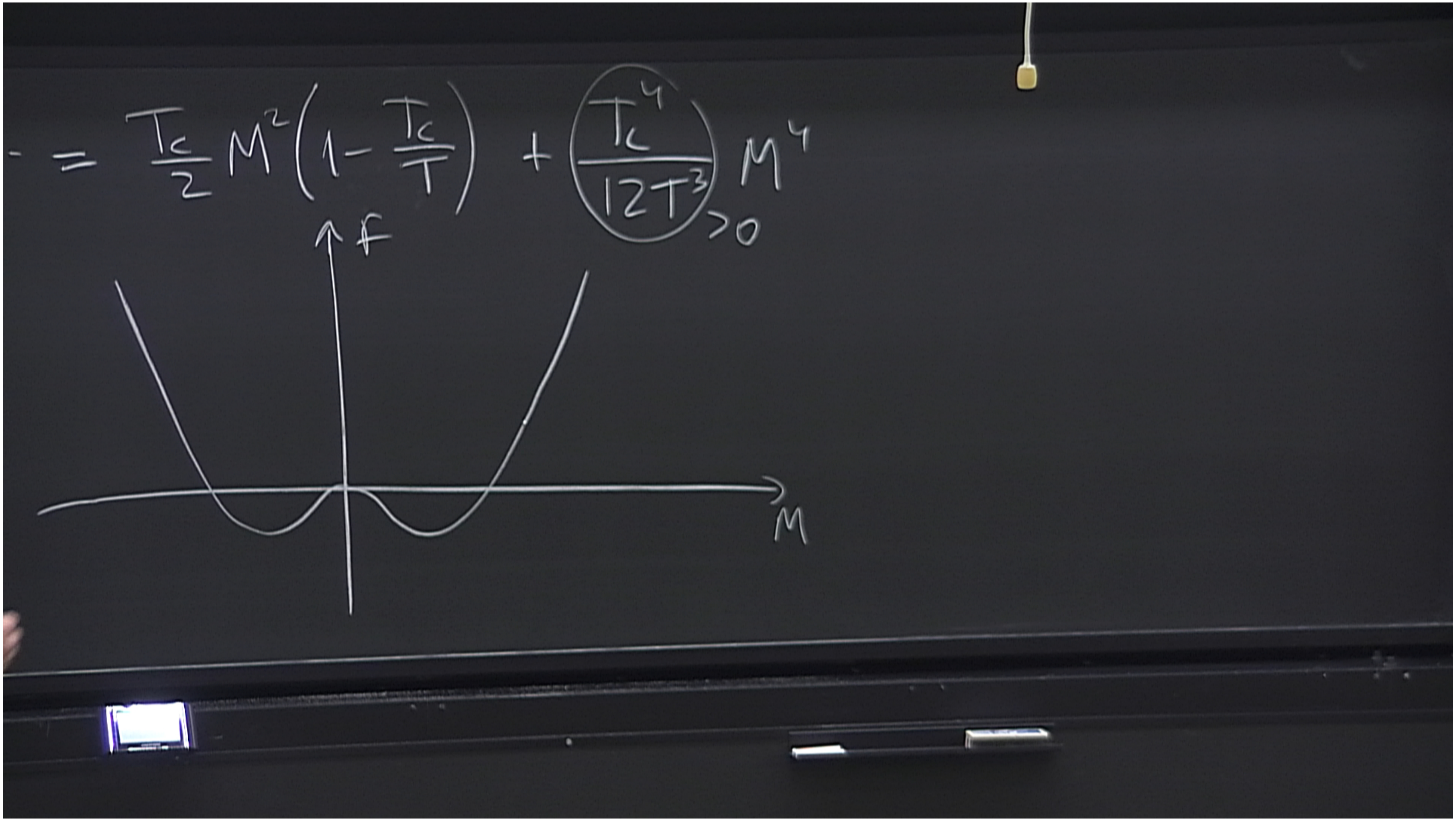
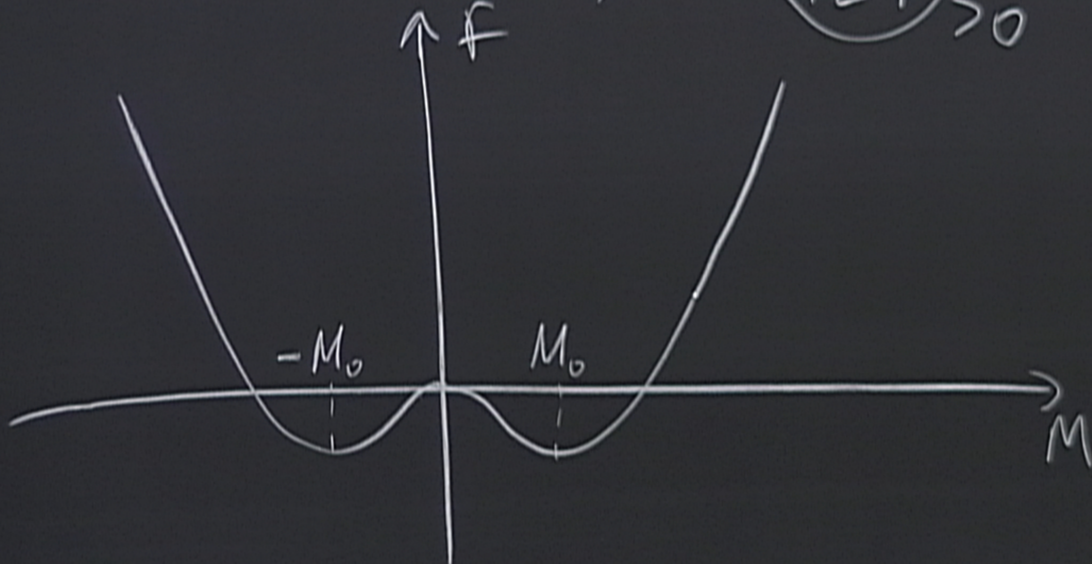
$$T_c = J$$




$$- = \frac{T_c}{2} M^2 \left(1 - \frac{T_c}{T}\right) + \frac{T_c^4}{12T^3} M^4$$
> 0
f
M

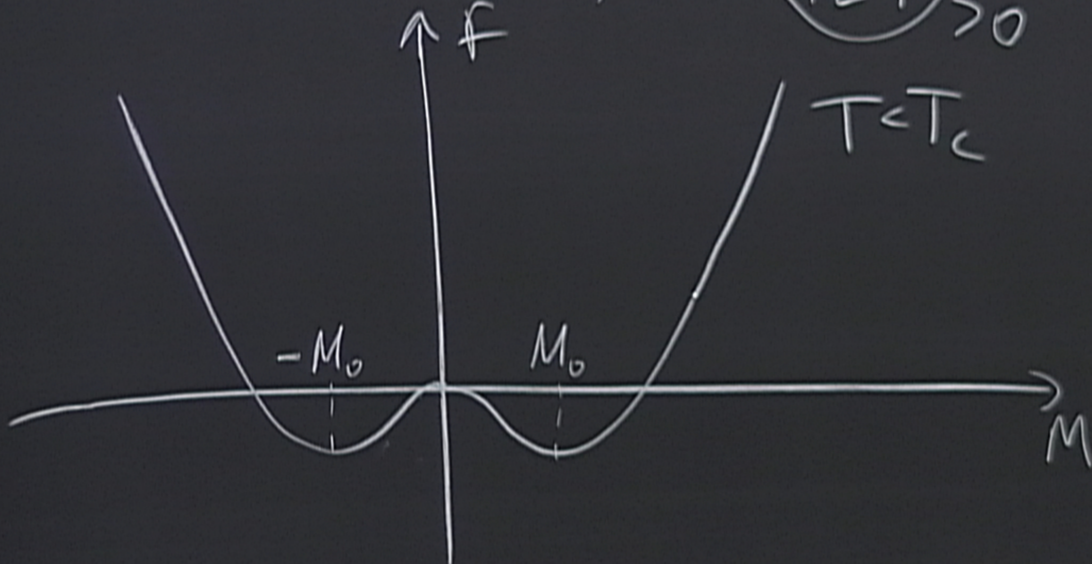
$$\cdot = \frac{T_c}{2} M^2 \left(1 - \frac{T_c}{T}\right) + \frac{T_c^4}{12T^3} M^4$$

>0



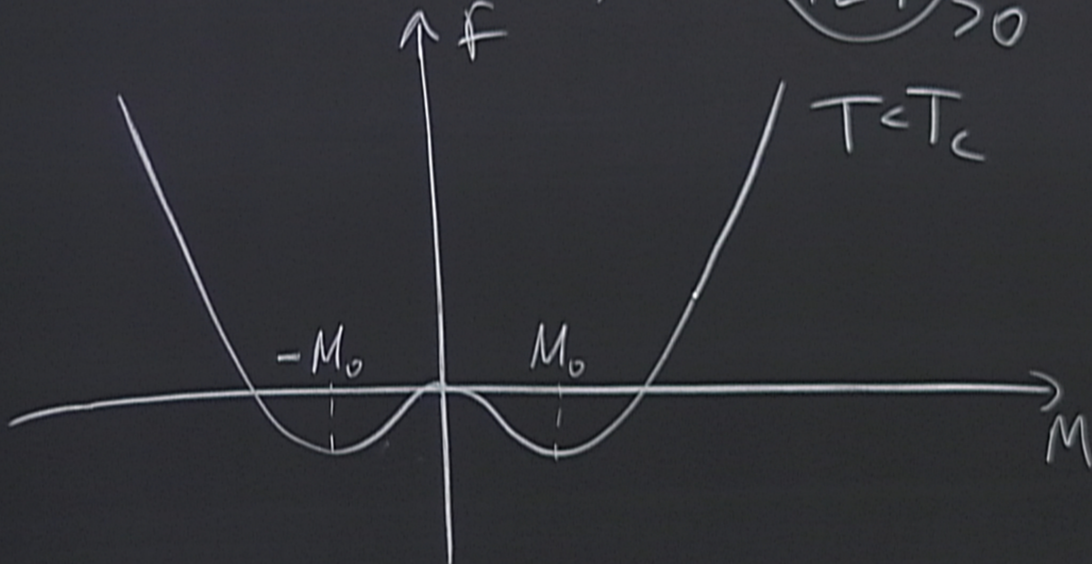
$$= \frac{T_c}{2} M^2 \left(1 - \frac{T_c}{T}\right) + \frac{T_c^4}{12 T^3} M^4$$

> 0



$$\ldots = \frac{T_c}{2} M^2 \left(1 - \frac{T_c}{T}\right) + \underbrace{\frac{T_c^4}{12T^3}}_{>0} M^4$$

$T < T_c$



$$\frac{\partial f}{\partial M} = T_c\left(1 - \frac{T_c}{T}\right)M + \frac{T_c^4}{3T^3}M^3 = 0$$

$$M^2 = \frac{3T^3}{T_c^3}\left(\frac{T_c}{T} - 1\right)$$

$$\frac{\partial f}{\partial M} = T_c\left(1 - \frac{T_c}{T}\right)M + \frac{T_c^4}{3T^3}M^3 = 0$$

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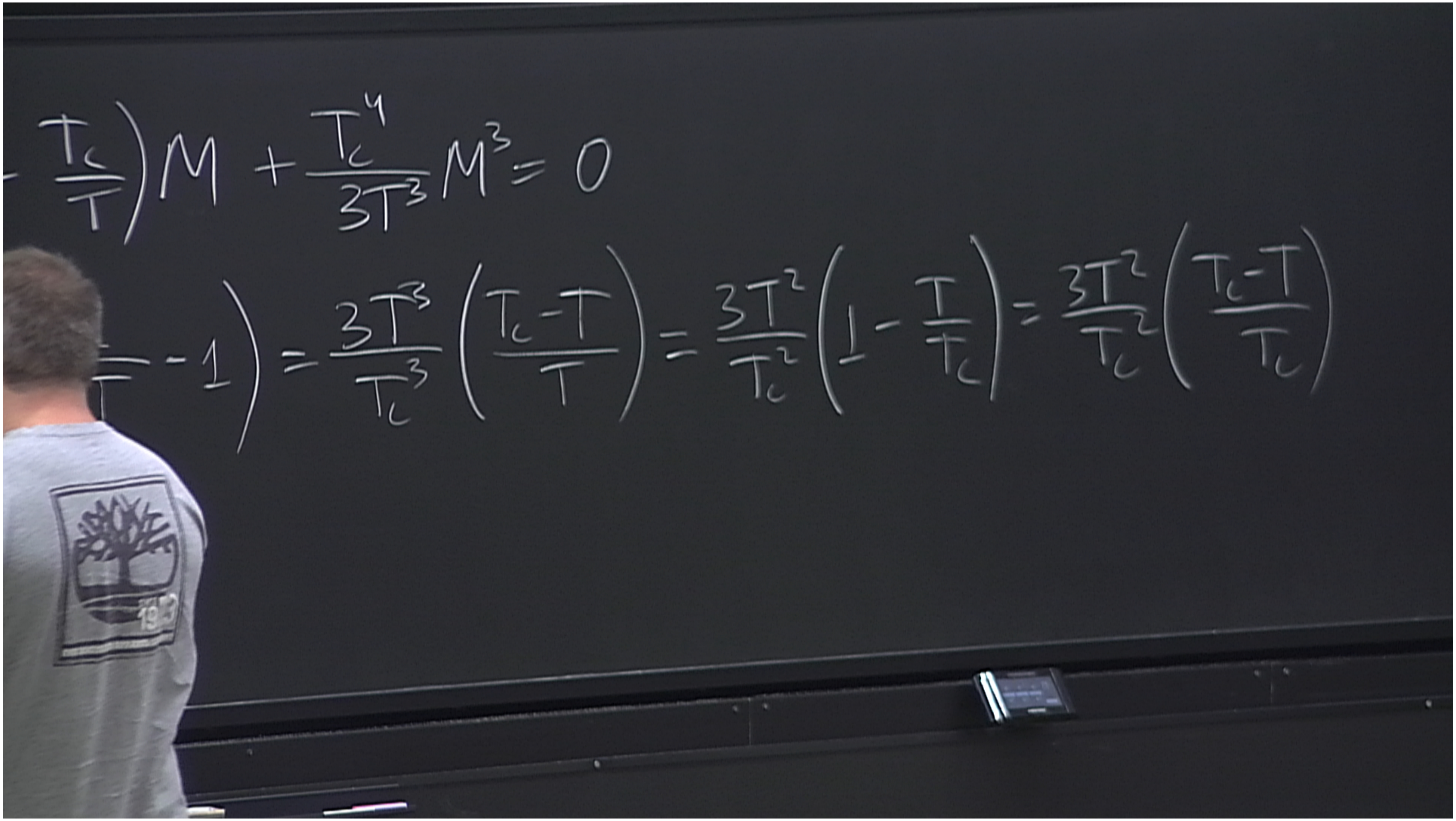
$-\frac{T_c}{T}\Big) M + \frac{T_c^4}{3T^3} M^3 = 0$
$-1\Big) = \frac{3T^3}{T_c^3}\left(\frac{T_c - T}{T}\right) = \frac{3T^2}{T_c^2}\left(1 - \frac{T}{T_c}\right) = \frac{3T^2}{T_c^2}\left(\frac{T_c - T}{T_c}\right)$

$$\frac{\partial f}{\partial M} = T_c\left(1-\frac{T_c}{T}\right)M + \frac{T_c^4}{3T^3}M^3 = 0$$

$$M^2 = \frac{3T^3}{T_c^3}\left(\frac{T_c}{T}-1\right) = \frac{3T^3}{T_c^3}\left(\frac{T_c-T}{T}\right) = \frac{3T^2}{T_c^2}\left(1-\frac{T}{T_c}\right)$$

$$M = \pm\frac{T}{T_c}\sqrt{3\left(\frac{T_c-T}{T_c}\right)} \simeq \pm\sqrt{3\left(\frac{T_c-T}{T_c}\right)}$$

$$\frac{\partial f}{\partial M} = T_c\left(1 - \frac{T_c}{T}\right)M + \frac{T_c^4}{3T^3}M^3 = 0$$

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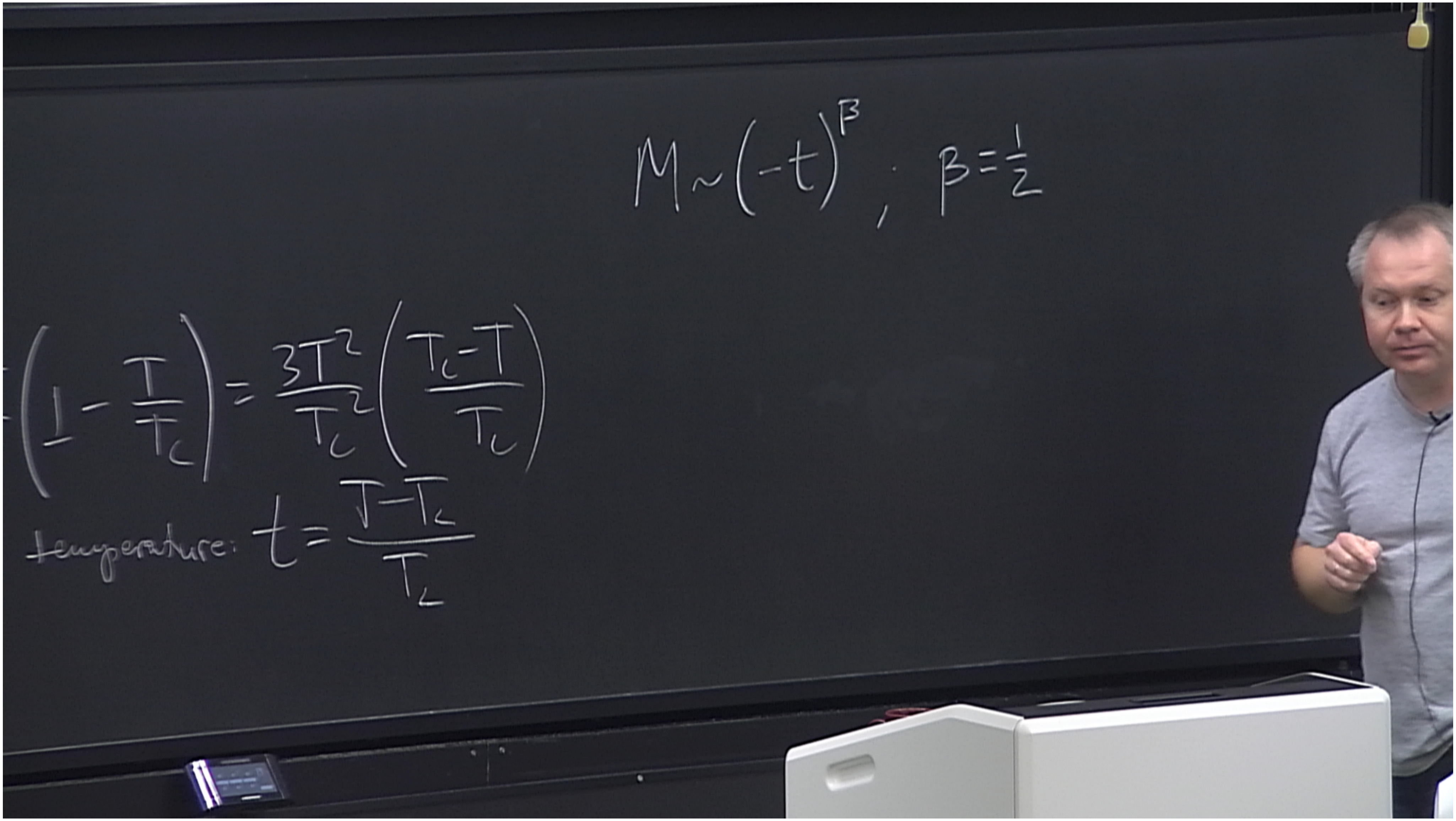
Reduced temperature:

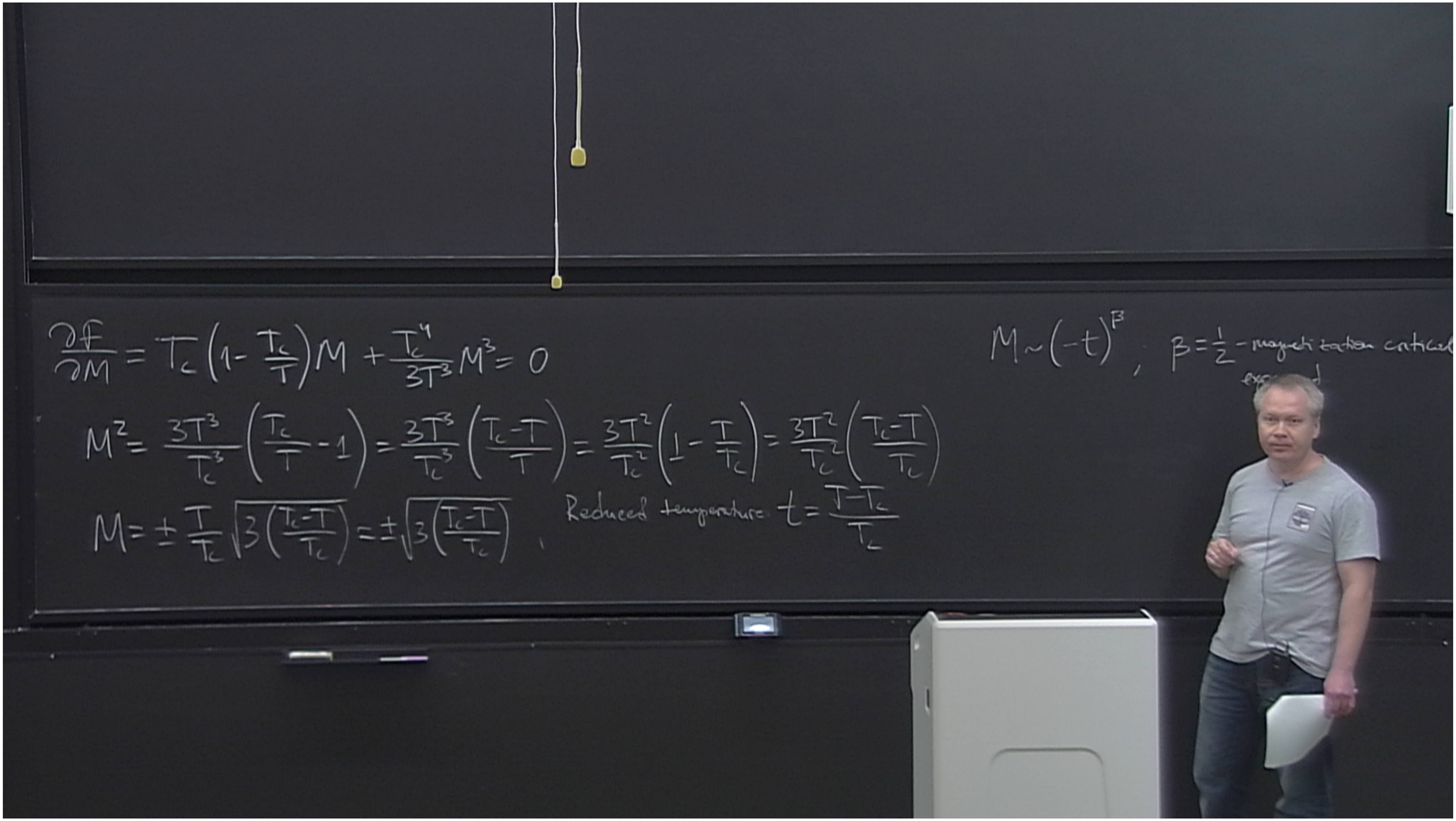
$$) M + \frac{T_c^4}{3T^3} M^3 = 0$$

$$-1 \Big) = \frac{3T^3}{T_c^3}\left(\frac{T_c - T}{T}\right) = \frac{3T^2}{T_c^2}\left(1 - \frac{T}{T_c}\right) = \frac{3T^2}{T_c^2}\left(\frac{T_c - T}{T_c}\right)$$

$$\left(\frac{-T}{T_c}\right) = \pm\sqrt{3\left(\frac{T_c - T}{T_c}\right)}$$

Reduced temperature: $t = \frac{T - T_c}{T_c}$


M ~ (-t)^β ; β = 1/2
(1 - T/T_c) = 3T^2/T_c^2 ((T_c - T)/T_c)
temperature: t = (T - T_c)/T_c


$\frac{\partial f}{\partial M} = T_c\left(1 - \frac{T_c}{T}\right)M + \frac{T_c^4}{3T^3}M^3 = 0$
$M^2 = \frac{3T^3}{T_c^3}\left(\frac{T_c}{T} - 1\right) = \frac{3T^3}{T_c^3}\left(\frac{T_c - T}{T}\right) = \frac{3T^2}{T_c^2}\left(1 - \frac{T}{T_c}\right) = \frac{3T^2}{T_c^2}\left(\frac{T_c - T}{T_c}\right)$
$M = \pm\frac{T}{T_c}\sqrt{3\left(\frac{T_c - T}{T_c}\right)} = \pm\sqrt{3\left(\frac{T_c - T}{T_c}\right)}$
Reduced temperature: $t = \frac{T - T_c}{T_c}$
$M \sim (-t)^\beta$, $\beta = \frac{1}{2}$ - magnetization critical

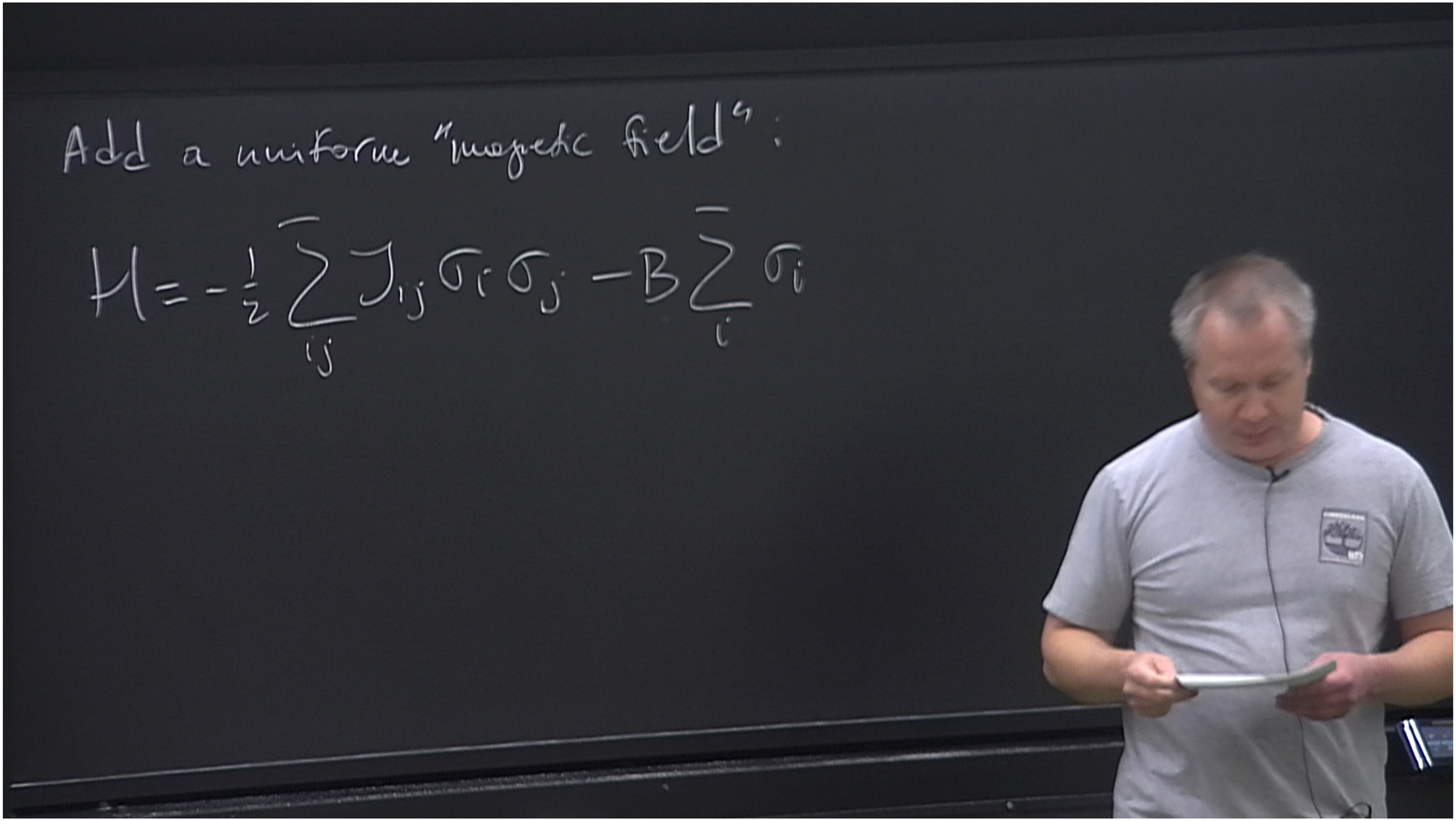
$$M \sim (-t)^{\beta} \; ; \quad \beta = \frac{1}{2}$$ - magnetization critical exponent

universal power law dependence.

$$\left(1 - \frac{T}{T_c}\right) = \frac{3T^2}{T_c^2}\left(\frac{T_c - T}{T_c}\right)$$

temperature: $t = \dfrac{T - T_c}{T_c}$

Add a uniform "magnetic field"


Add a uniform "magnetic field":
$H = -\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j - B\sum_i \sigma_i$

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$$H = -\frac{1}{2}\sum_{ij} J_{ij}\,\sigma_i \sigma_j - B\sum_i \sigma_i$$

$$H = -MJ\sum_i \sigma_i - B\sum_i \sigma_i + \frac{1}{2}NJM^2 =$$

$$= -(MJ+B)\sum_i \sigma_i + \frac{1}{2}NJM^2$$

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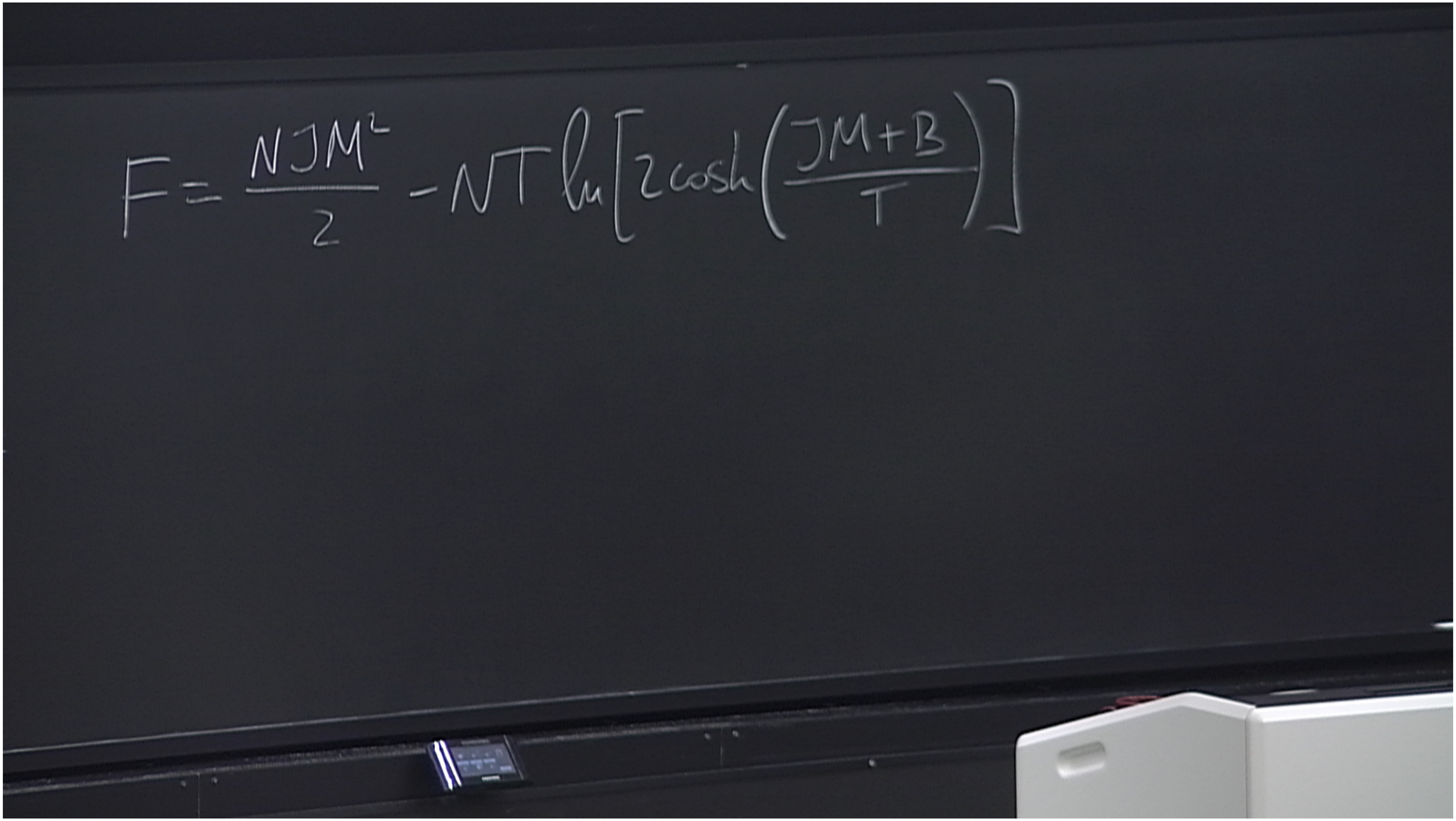
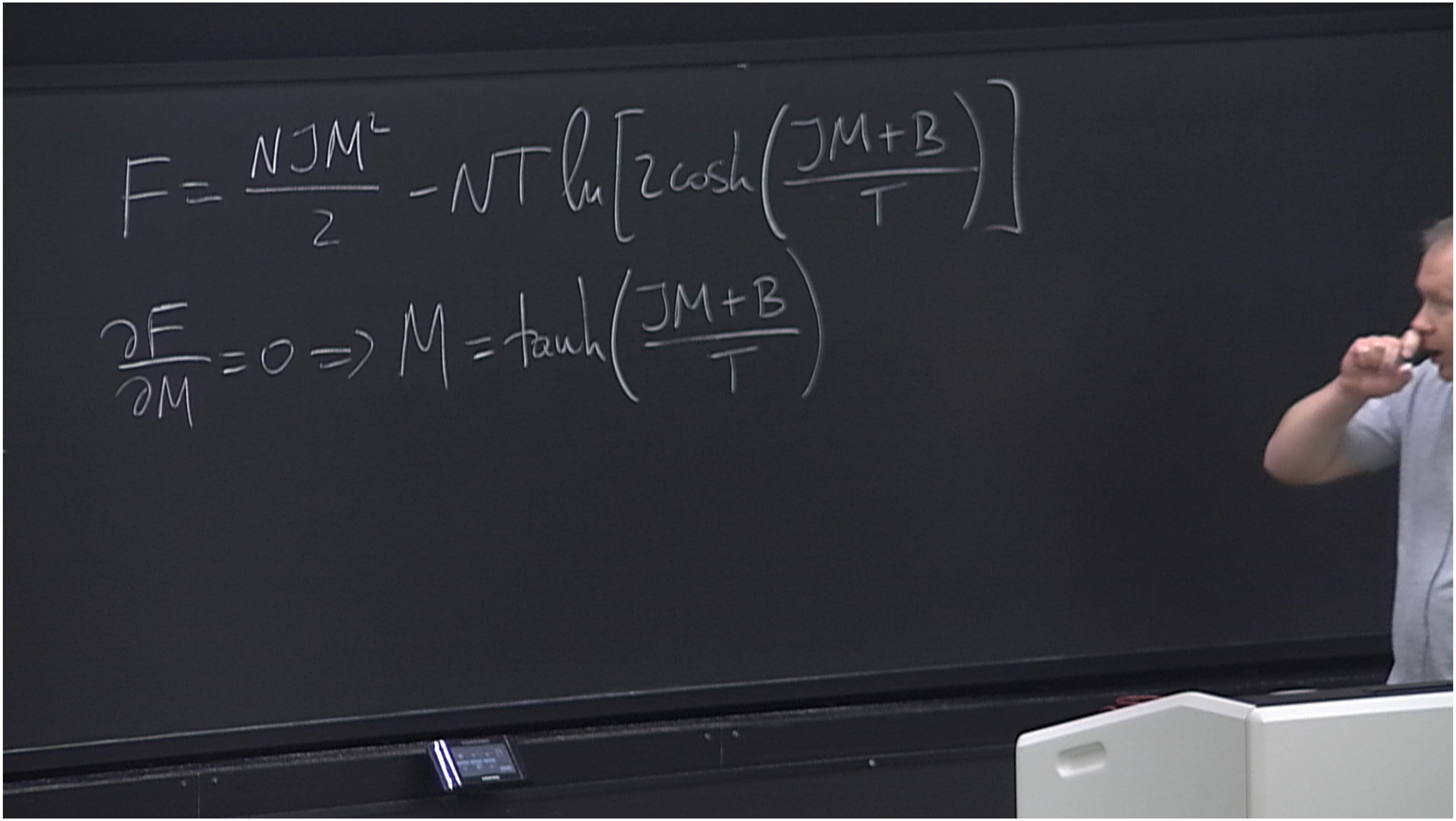
$$= -(MJ + B)\sum_i \sigma_i + \frac{1}{2}NJM^2$$

$B_M = MJ$ - molecular field.


$F = \frac{NJM^2}{2} - NT \ln\left[2\cosh\left(\frac{JM+B}{T}\right)\right]$


$F = \frac{NJM^2}{2} - NT \ln\left[2\cosh\left(\frac{JM+B}{T}\right)\right]$
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Magnetic susceptibility: $\chi = \frac{\partial M}{\partial B}$

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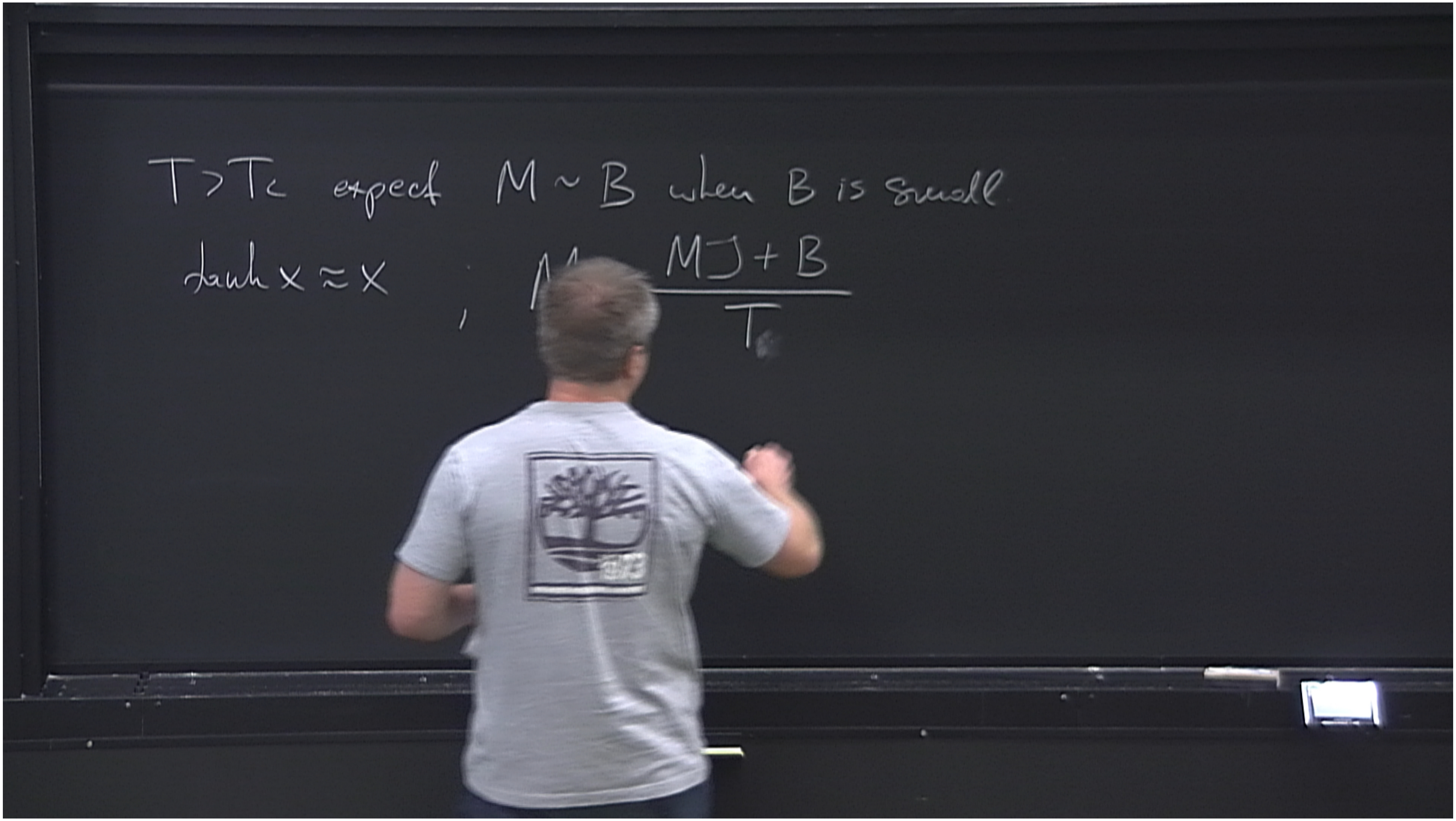
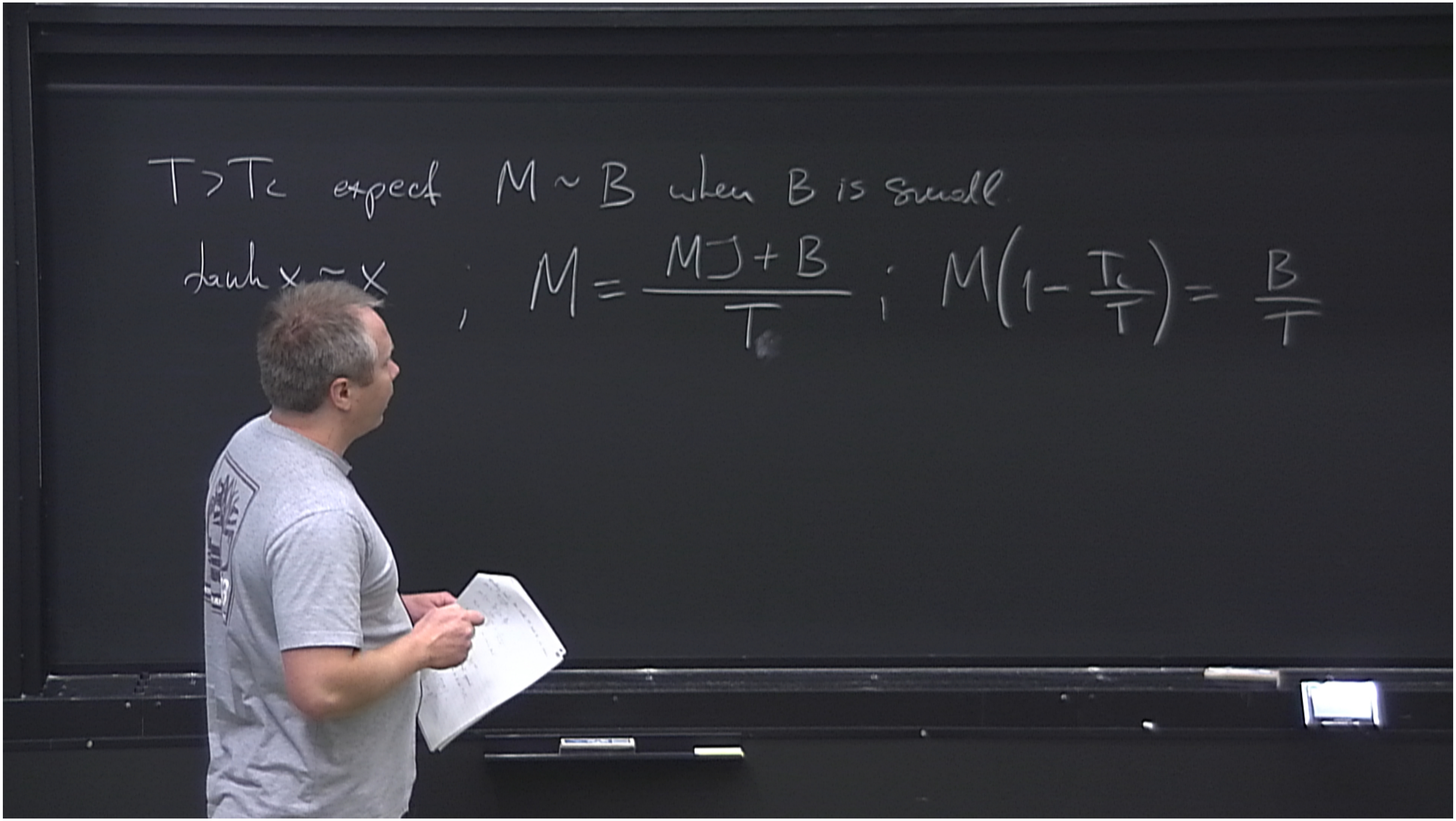
Magnetic susceptibility: $\chi = \frac{\partial M}{\partial B}$, $T > T_c$

$T > T_c$ expect

$T > T_c$ expect $M \sim B$ when B is small


T > Tc expect M ~ B when B is small
tanh x ≈ x
MJ + B
T


T>Tc expect M ~ B when B is small
tanh x ~ x ; M = (MJ + B)/T ; M(1 - Tc/T) = B/T

$$\frac{B}{T} ;$$

$$\chi = \frac{\partial M}{\partial B} = \frac{M}{B} = \frac{1}{T}\frac{1}{1-\frac{T_c}{T}} = \frac{1}{T-T_c}$$

$$\frac{B}{T} \;;\quad \chi = \frac{\partial M}{\partial B} = \frac{M}{B} = \frac{1}{T}\,\frac{1}{1-\frac{T_c}{T}} = \frac{1}{T-T_c} \sim t^{-\gamma}$$

$$\frac{B}{T} \; ; \quad \chi = \frac{\partial M}{\partial B} = \frac{M}{B} = \frac{1}{T} \frac{1}{1 - \frac{T_c}{T}} = \frac{1}{T - T_c} \sim t^{-\gamma}$$

$\gamma = 1$ - susceptibility critical exponent.

$$\frac{B}{T} \; ; \quad \chi = \frac{\partial M}{\partial B} = \frac{M}{B} = \frac{1}{T}\,\frac{1}{1-\frac{T_c}{T}} = \frac{1}{T-T_c} \sim t^{-\gamma}$$

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$$T < T_c \; ; \quad \chi = \frac{1}{2}\,\frac{1}{T_c - T}$$

$$\frac{B}{T} \; ; \quad \chi = \frac{\partial M}{\partial B} = \frac{M}{B} = \frac{1}{T}\frac{1}{1-\frac{T_c}{T}} = \frac{1}{T-T_c} \sim t^{-\gamma}$$

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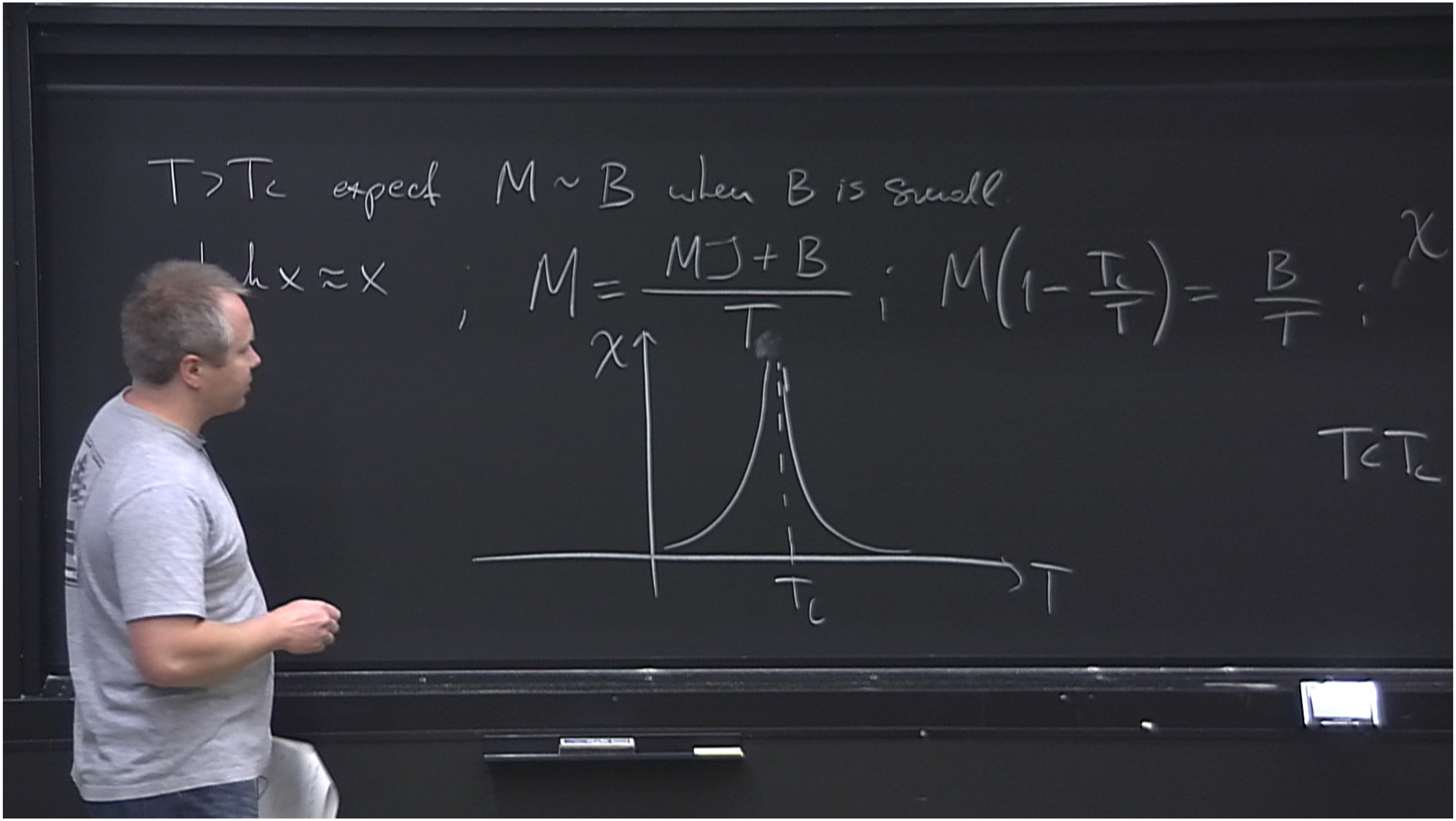

T > Tc expect M ~ B when B is small
x ≈ x ; M = MJ + B / T ; M(1 − Tc/T) = B/T ;
χ
χ
Tc
T
T<Tc

Specific heat :

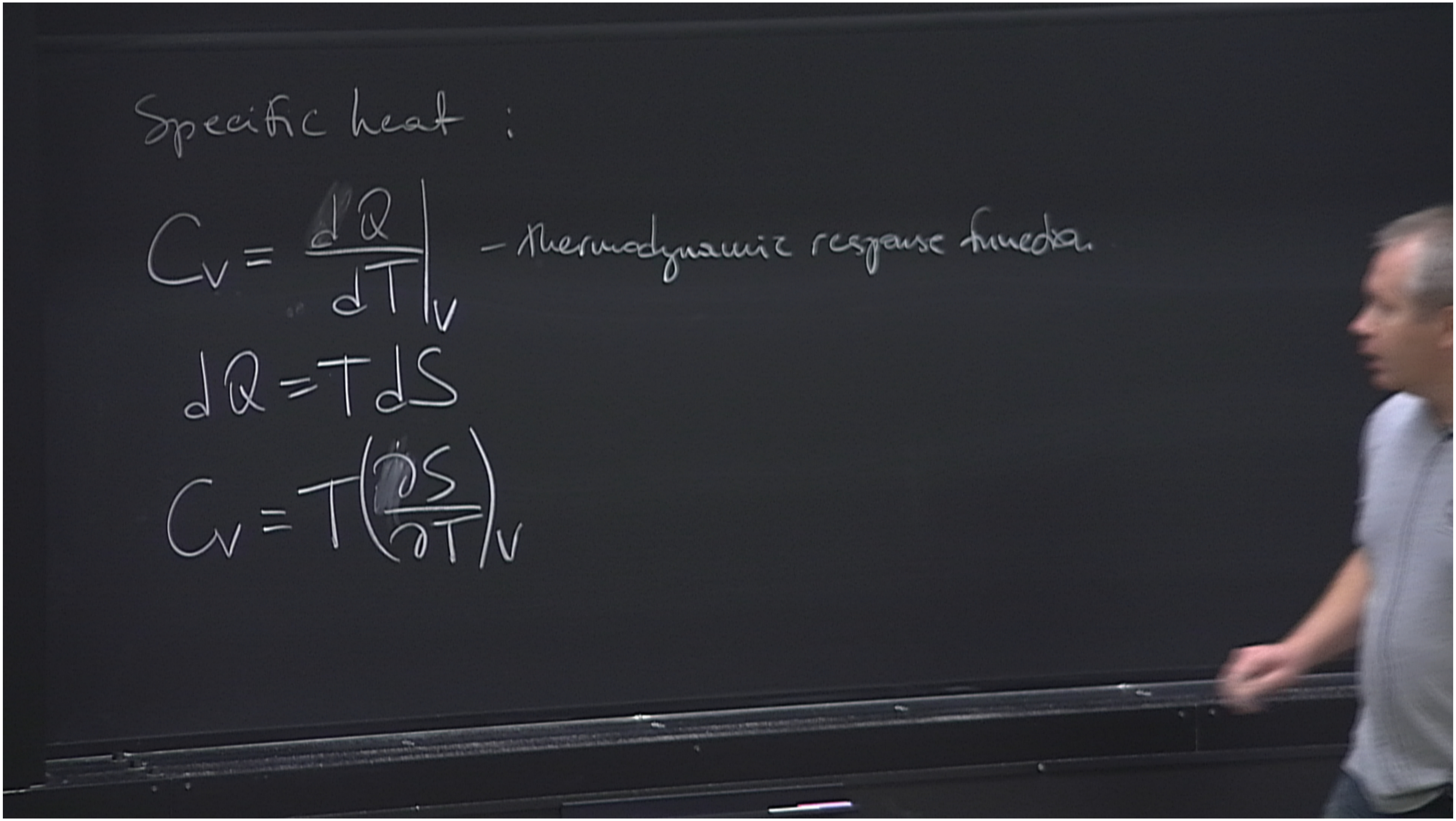
$$C_V = \frac{dQ}{dT}\Big|_V$$

$$dQ = T\,dS$$

Specific heat :

$$C_V = \frac{dQ}{dT}\Big|_V$$ – thermodynamic response function

$$dQ = T\,dS$$


Specific heat :
$C_V = \frac{đQ}{dT}\Big|_V$ – thermodynamic response function
$đQ = T\,dS$
$C_V = T\left(\frac{\partial S}{\partial T}\right)_V$

Specific heat :

$$C_V = \left.\frac{đQ}{dT}\right|_V \quad \text{- thermodynamic response function.}$$

$$đQ = T\,dS \qquad ; \quad S = -\left(\frac{\partial F}{\partial T}\right)_V$$

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V$$

Specific heat :

$$C_V = \left.\frac{dQ}{dT}\right|_V$$ – thermodynamic response function

$$dQ = T\,dS \quad ; \quad S = -\left(\frac{\partial F}{\partial T}\right)_V$$

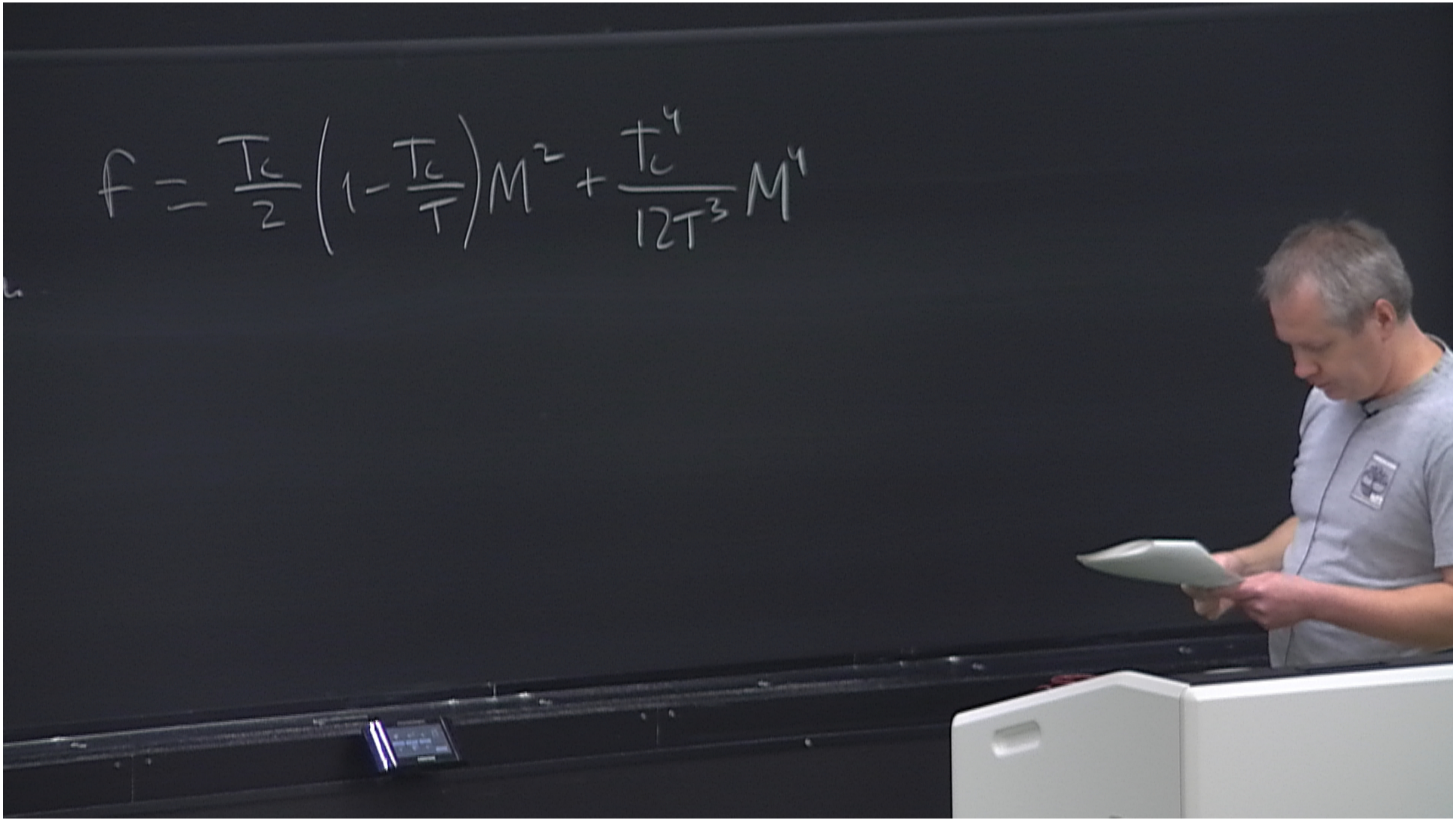
$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V \quad ; \quad C_V = -T\left(\frac{\partial^2 F}{\partial T^2}\right)_V$$

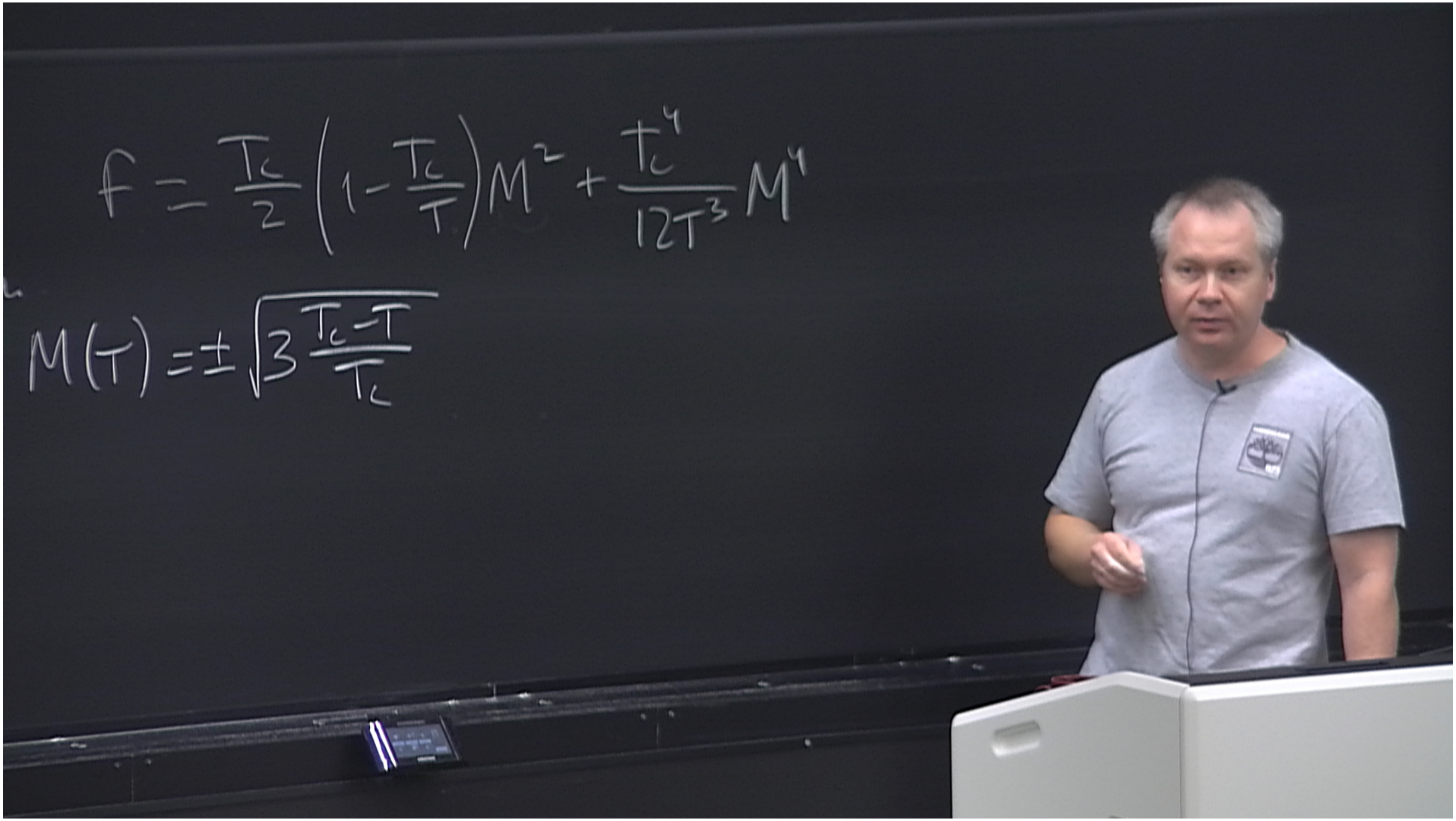
Specific heat :

$$C_V = \frac{đQ}{dT}\Big|_V \quad \text{- thermodynamic response function}$$

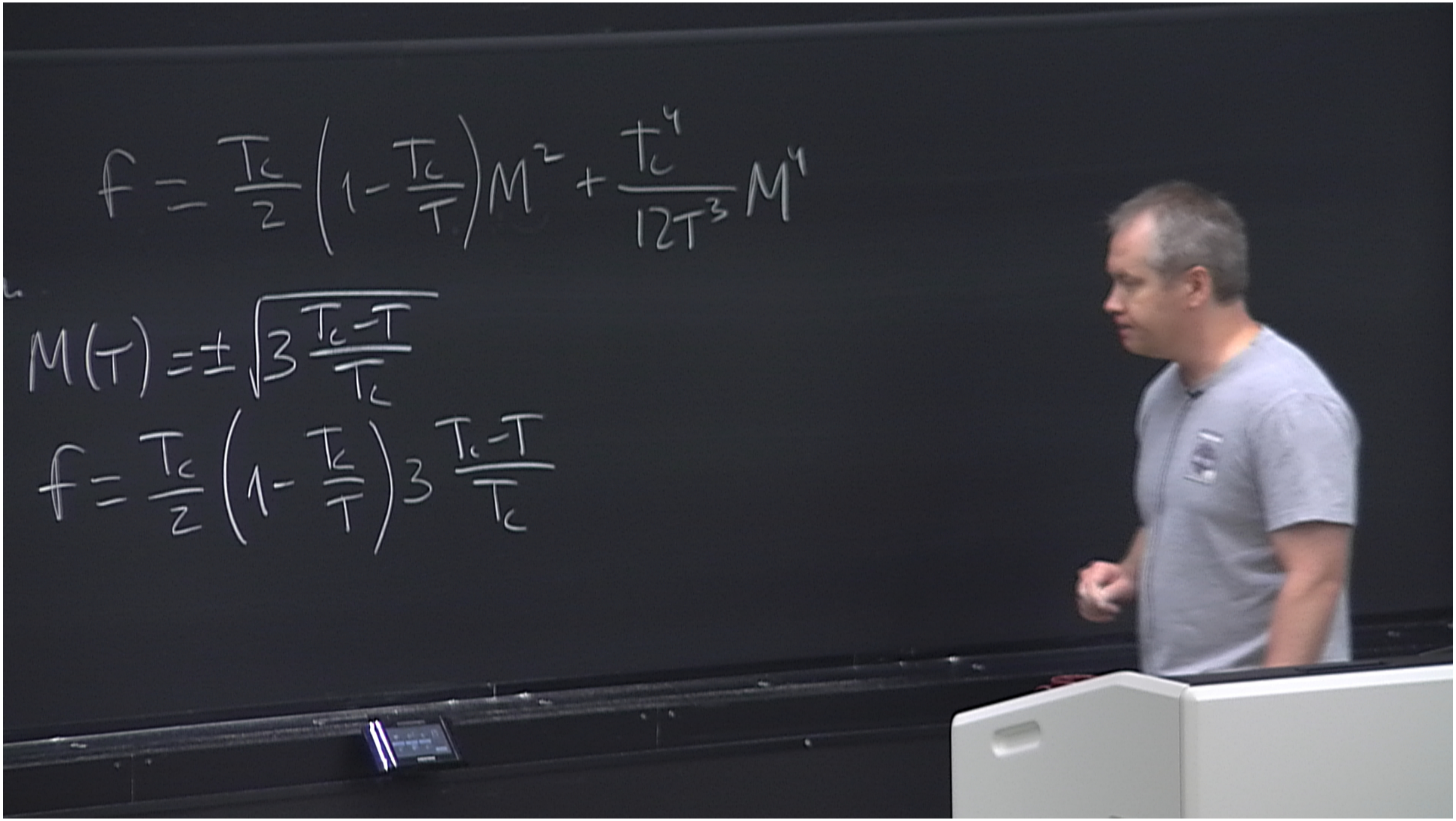
$$đQ = T\,dS \quad ; \quad S = -\left(\frac{\partial F}{\partial T}\right)_V$$

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V \quad ; \quad C_V = -\frac{T}{N}\left(\frac{\partial^2 F}{\partial T^2}\right)_V$$


$f = \frac{T_c}{2}\left(1-\frac{T_c}{T}\right)M^2 + \frac{T_c^4}{12T^3}M^4$



$f = \frac{T_c}{2}\left(1 - \frac{T_c}{T}\right)M^2 + \frac{T_c^4}{12T^3}M^4$
$M(T) = \pm\sqrt{3\frac{T_c - T}{T_c}}$


$f = \frac{T_c}{2}\left(1-\frac{T_c}{T}\right)M^2 + \frac{T_c^4}{12T^3}M^4$
$M(T) = \pm\sqrt{3\frac{T_c - T}{T_c}}$
$f = \frac{T_c}{2}\left(1-\frac{T_c}{T}\right)3\frac{T_c - T}{T_c}$

$$f = \frac{T_c}{2}\left(1 - \frac{T_c}{T}\right)M^2 + \frac{T_c^4}{12T^3}M^4$$

$$M(T) = \pm\sqrt{3\,\frac{T_c - T}{T_c}}$$

$$f = \frac{T_c}{2}\left(1 - \frac{T_c}{T}\right)3\,\frac{T_c - T}{T_c} + \frac{T_c^4}{12T^3}\,9\left(\frac{T_c - T}{T_c}\right)^2$$

$$f = \frac{T_c}{2}\left(1 - \frac{T_c}{T}\right) M^2 + \frac{T_c^4}{12 T^3} M^4$$

$$M(T) = \pm \sqrt{3 \frac{T_c - T}{T_c}}$$

$$f = \frac{T_c}{2}\left(1 - \frac{T_c}{T}\right) 3 \frac{T_c - T}{T_c} + \frac{T_c^4}{12 T^3} 9 \left(\frac{T_c - T}{T_c}\right)^2 = -\frac{3}{2T_c}\left(\frac{T_c - T}{T_c}\right)^2$$

$$f = \frac{T_c}{2}\left(1-\frac{T_c}{T}\right)M^2 + \frac{T_c^4}{12T^3}M^4$$

$$M(T) = \pm\sqrt{3\,\frac{T_c-T}{T_c}}$$

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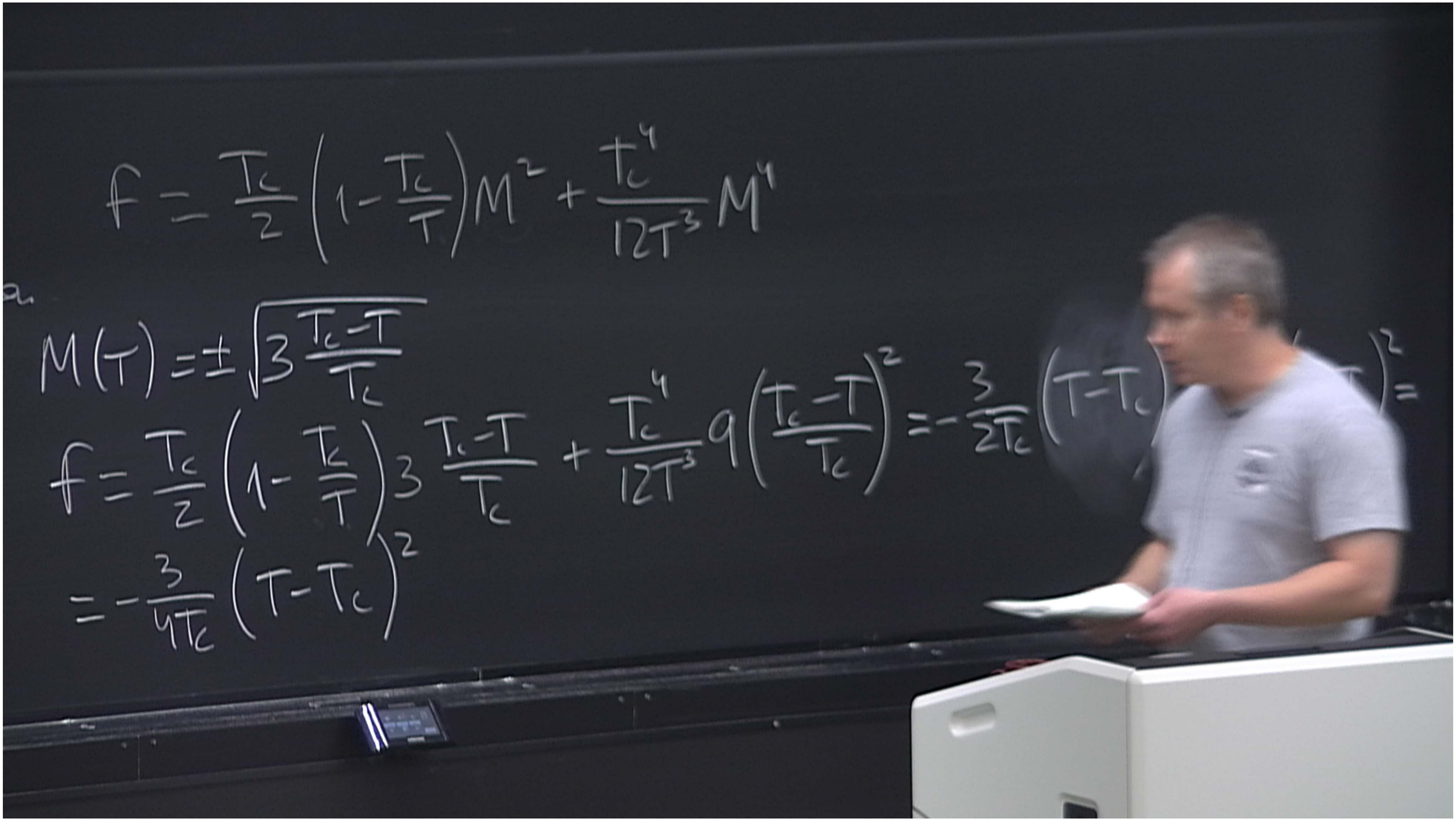
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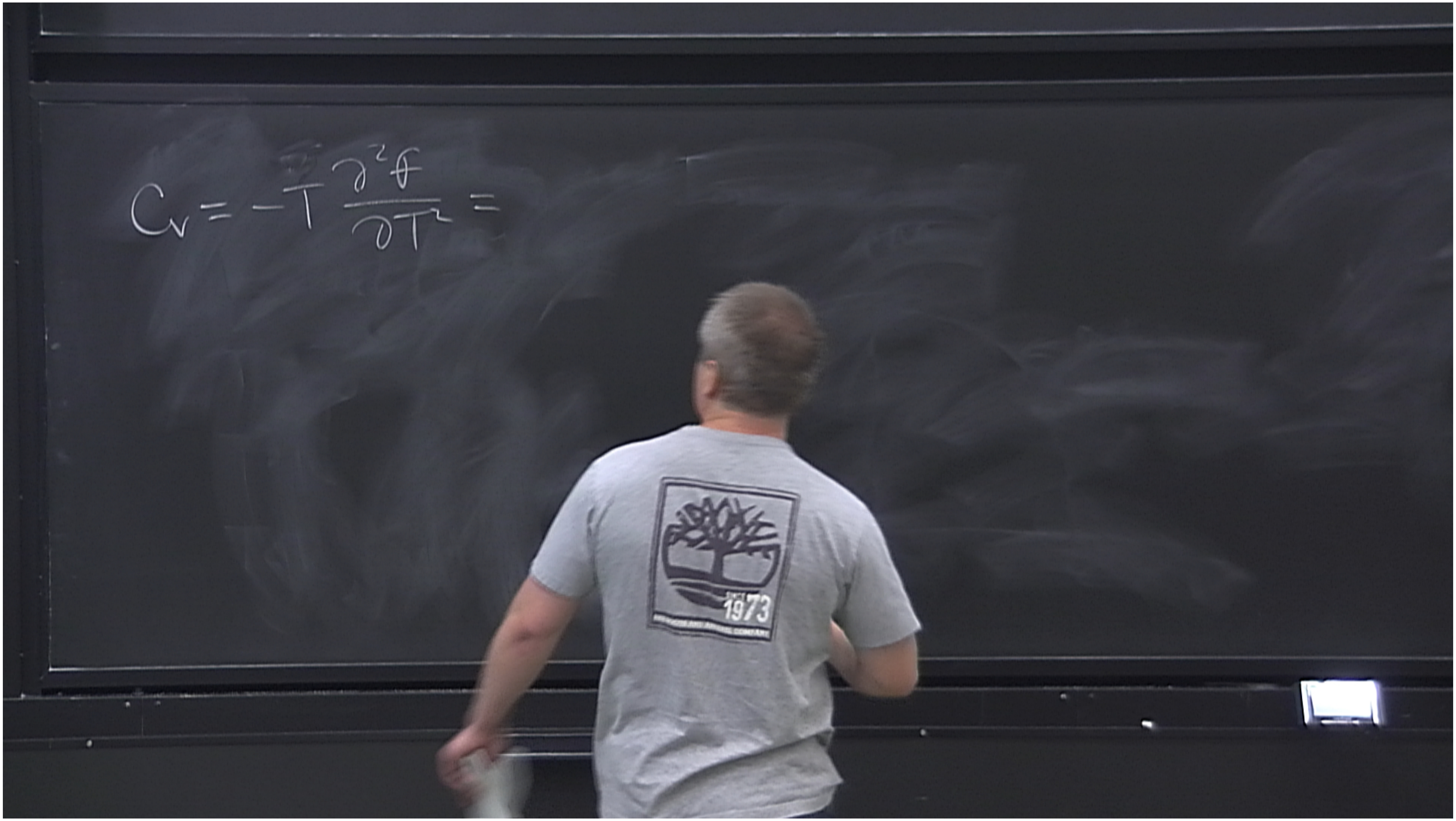
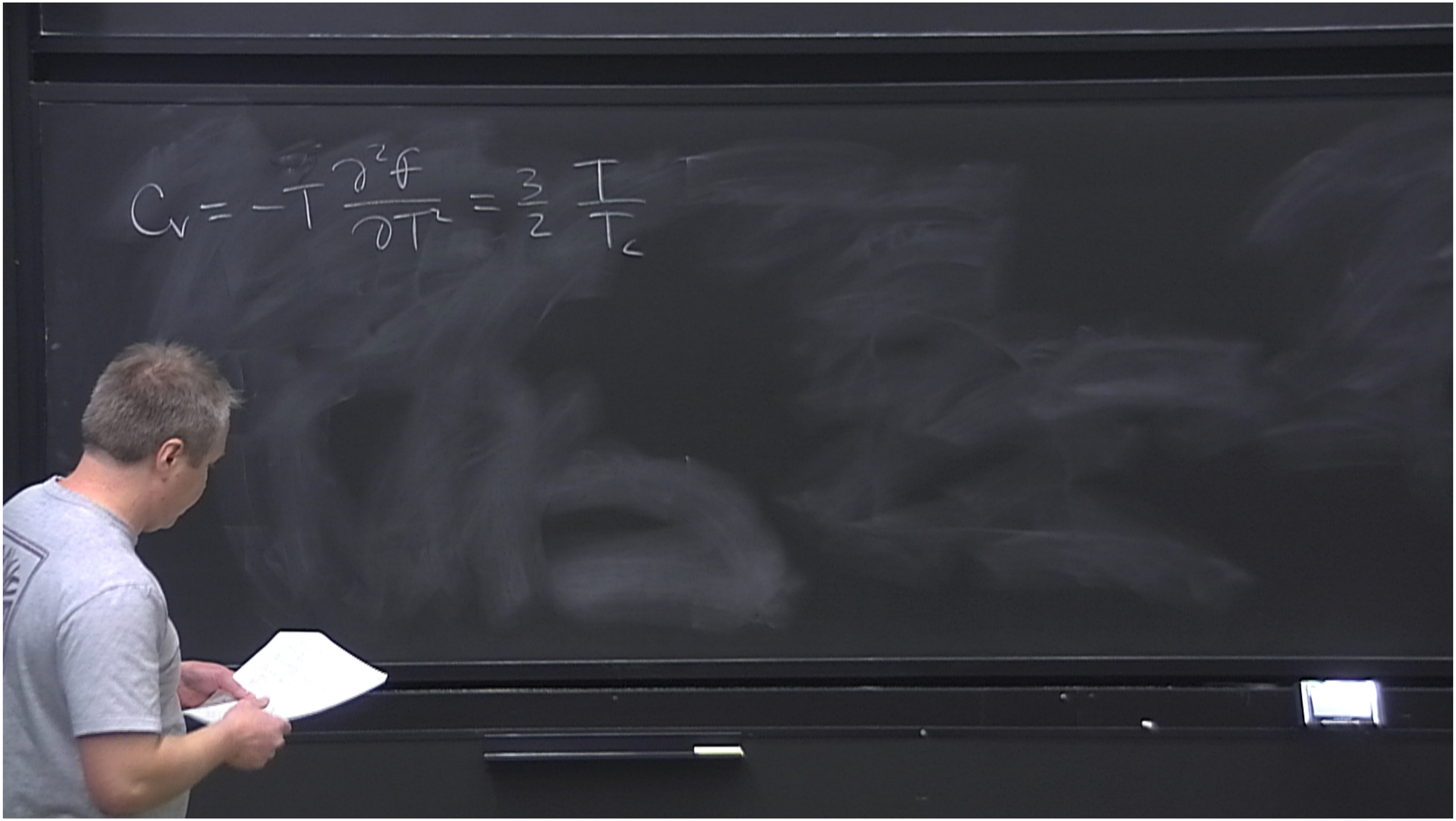

$C_v = -T \frac{\partial^2 F}{\partial T^2} =$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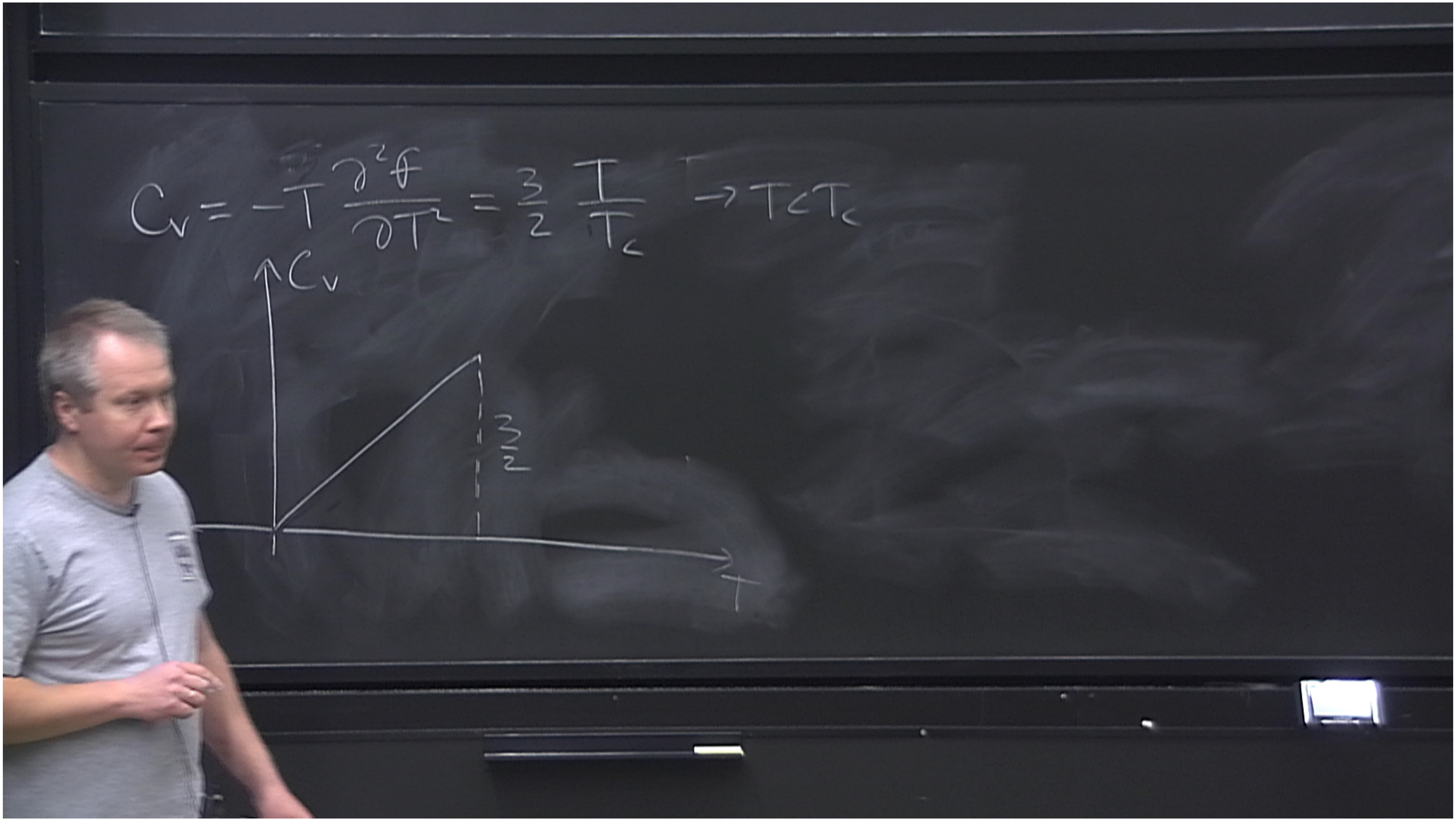
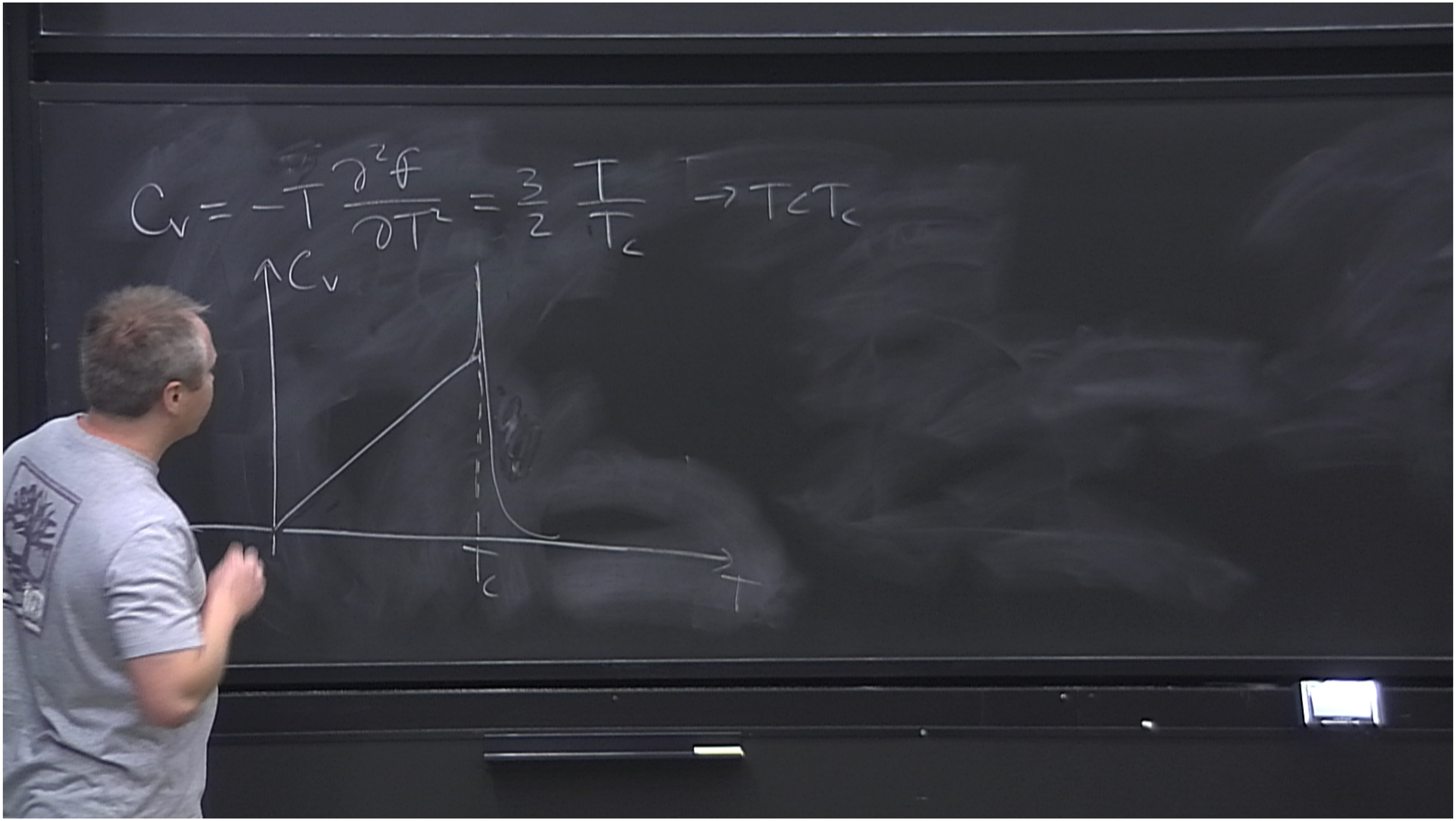

$C_v = -T \frac{\partial^2 f}{\partial T^2} = \frac{3}{2} \frac{T}{T_c} \rightarrow T < T_c$
C_v
$\frac{3}{2}$
T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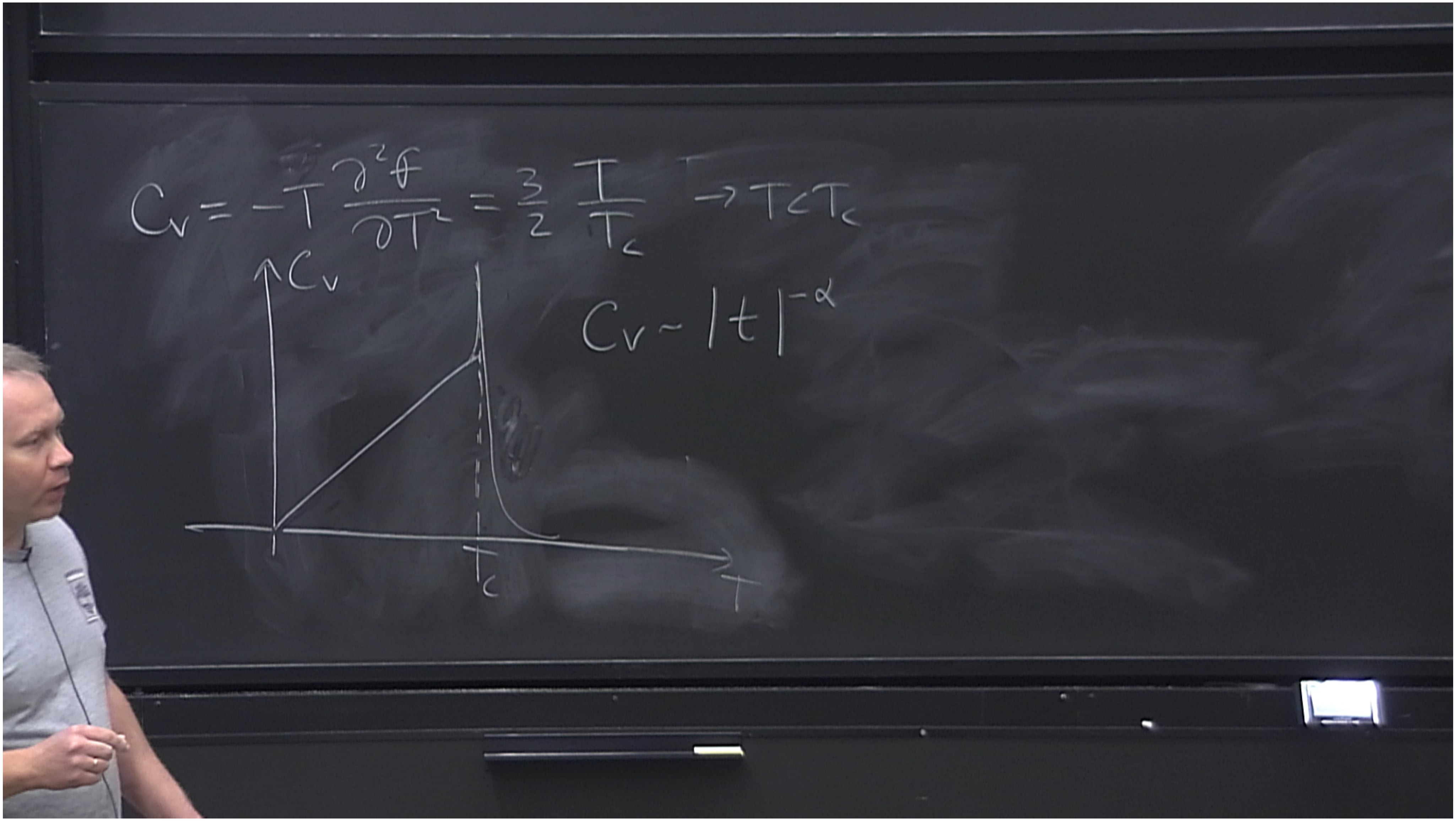
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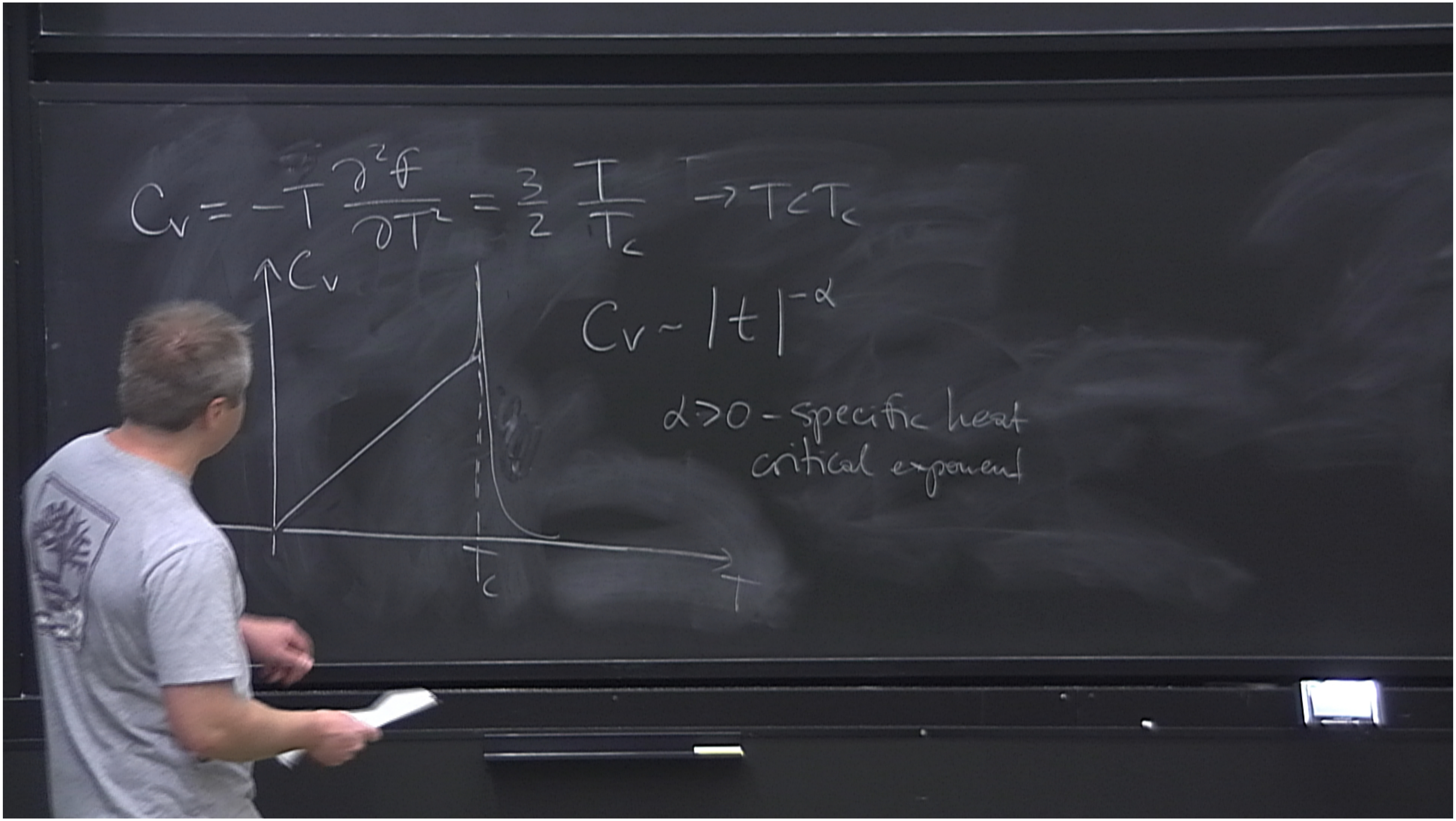
T_c
T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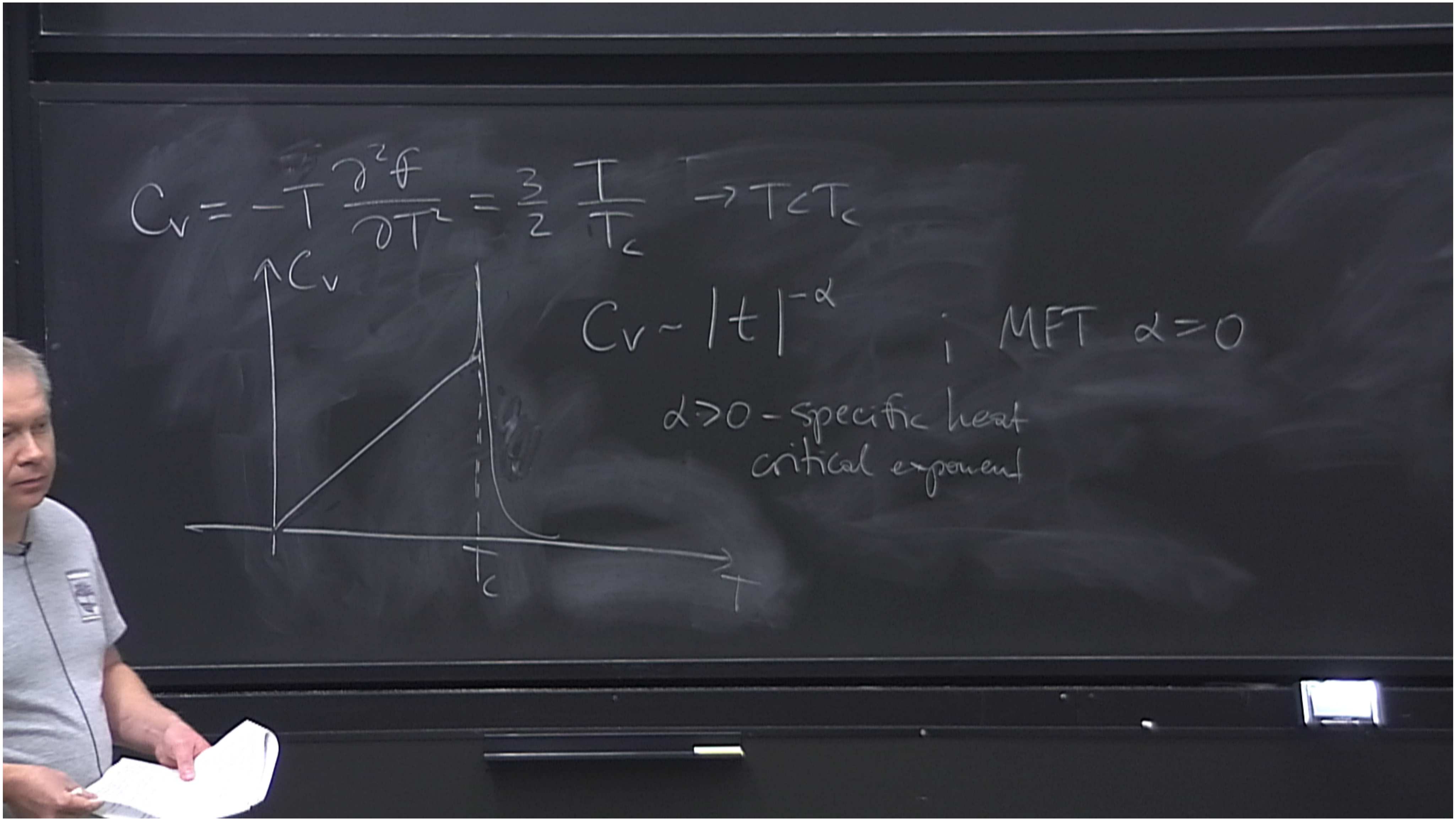
critical exponent
T_c
T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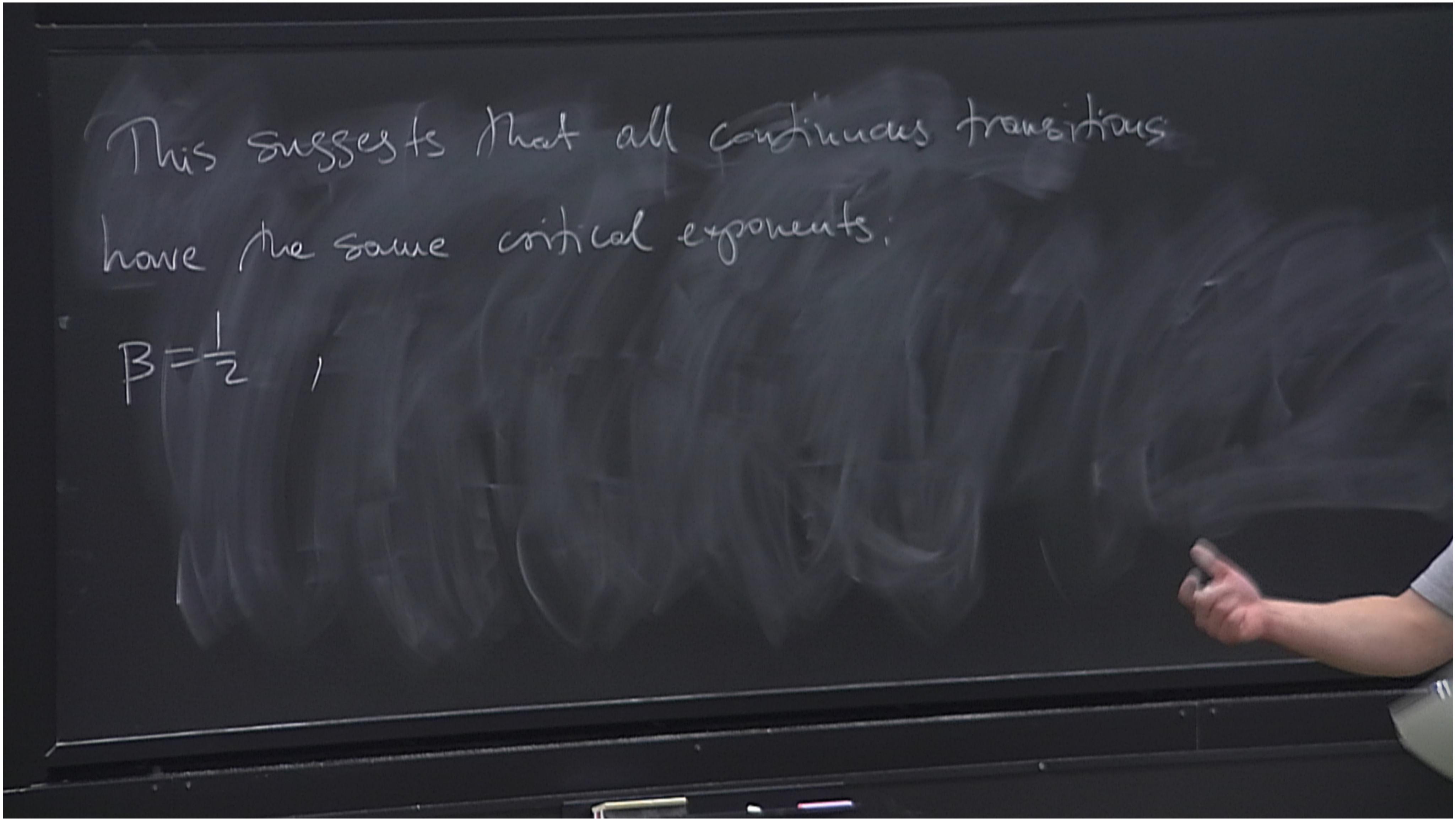
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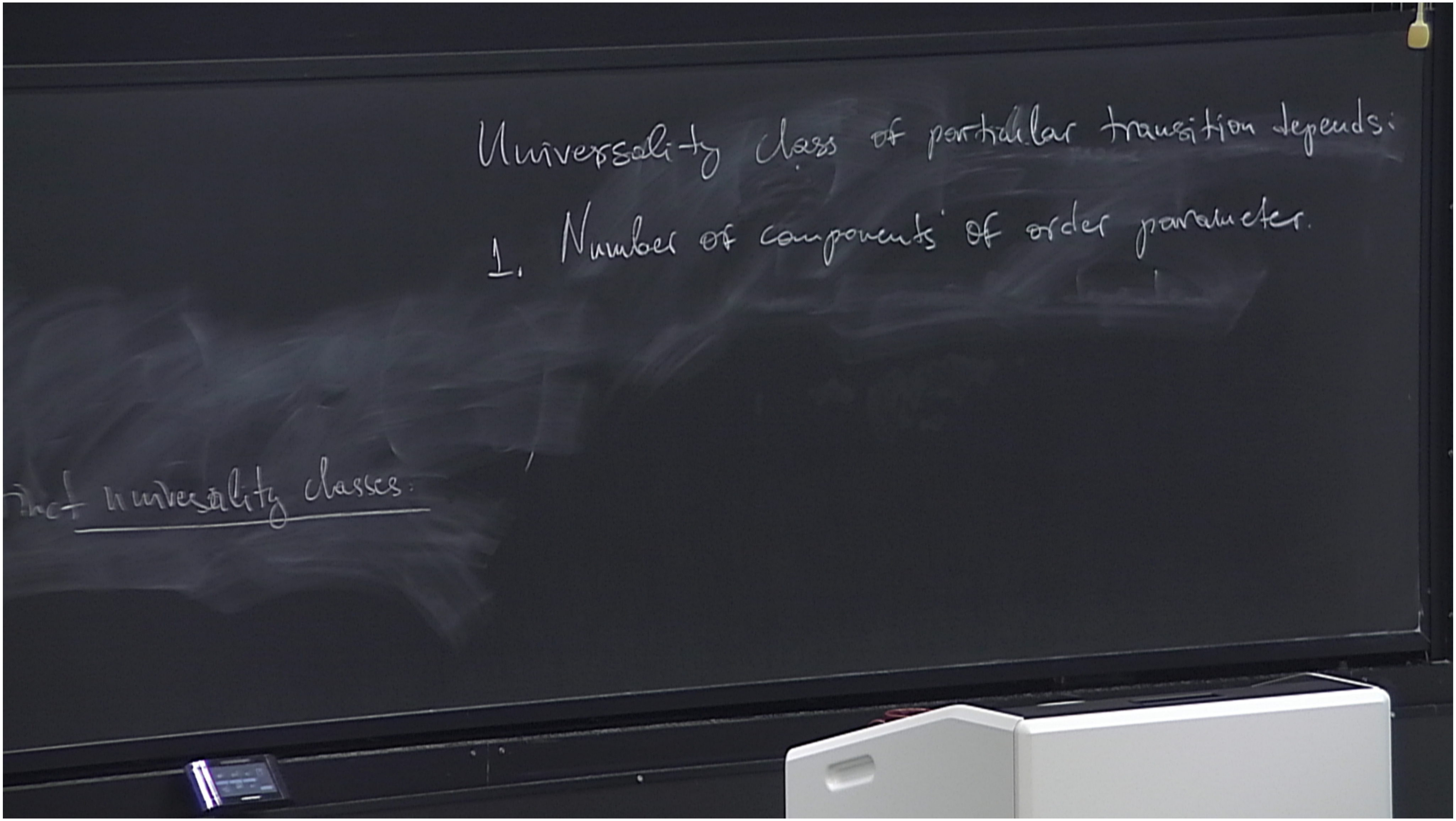
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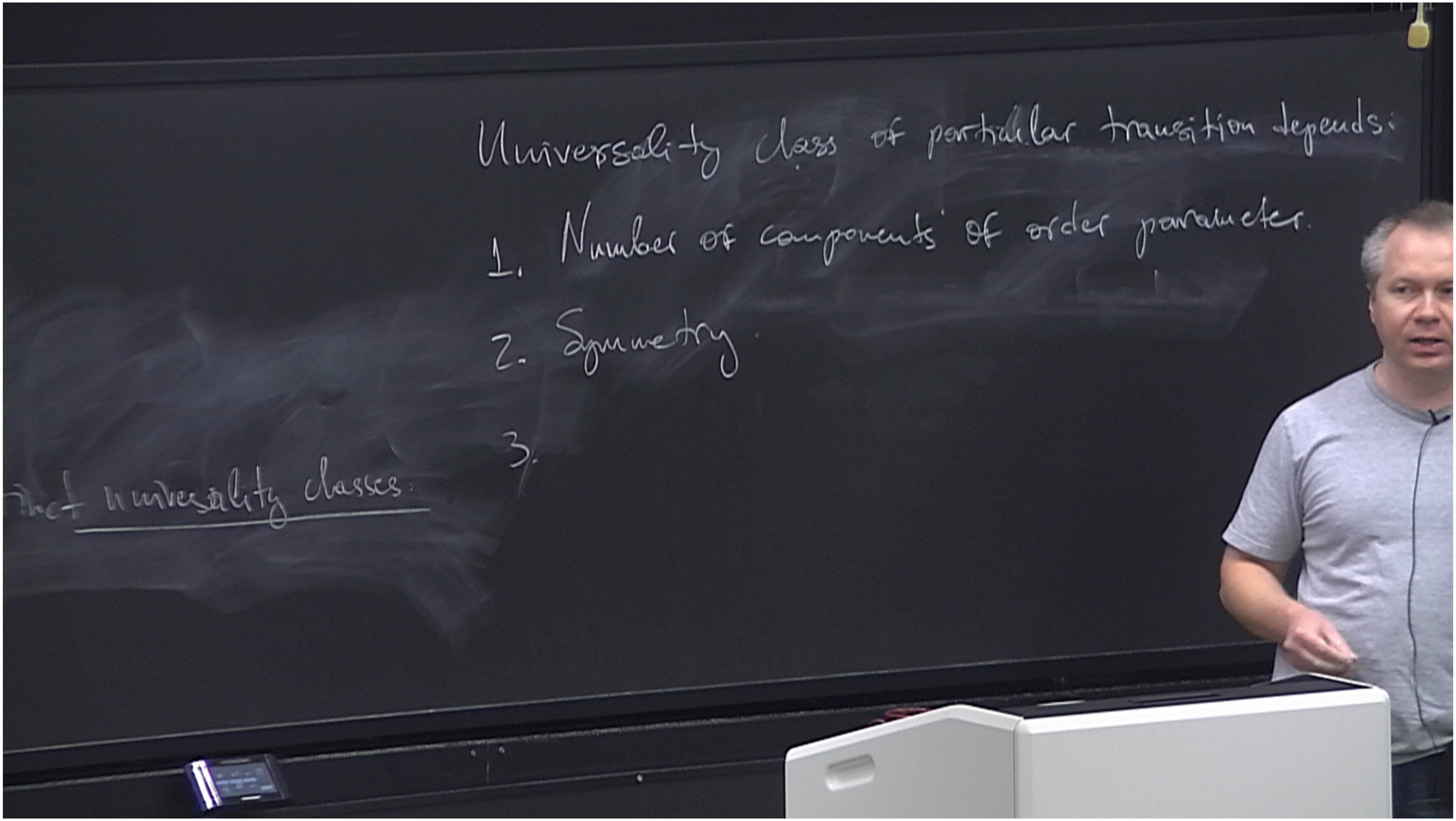
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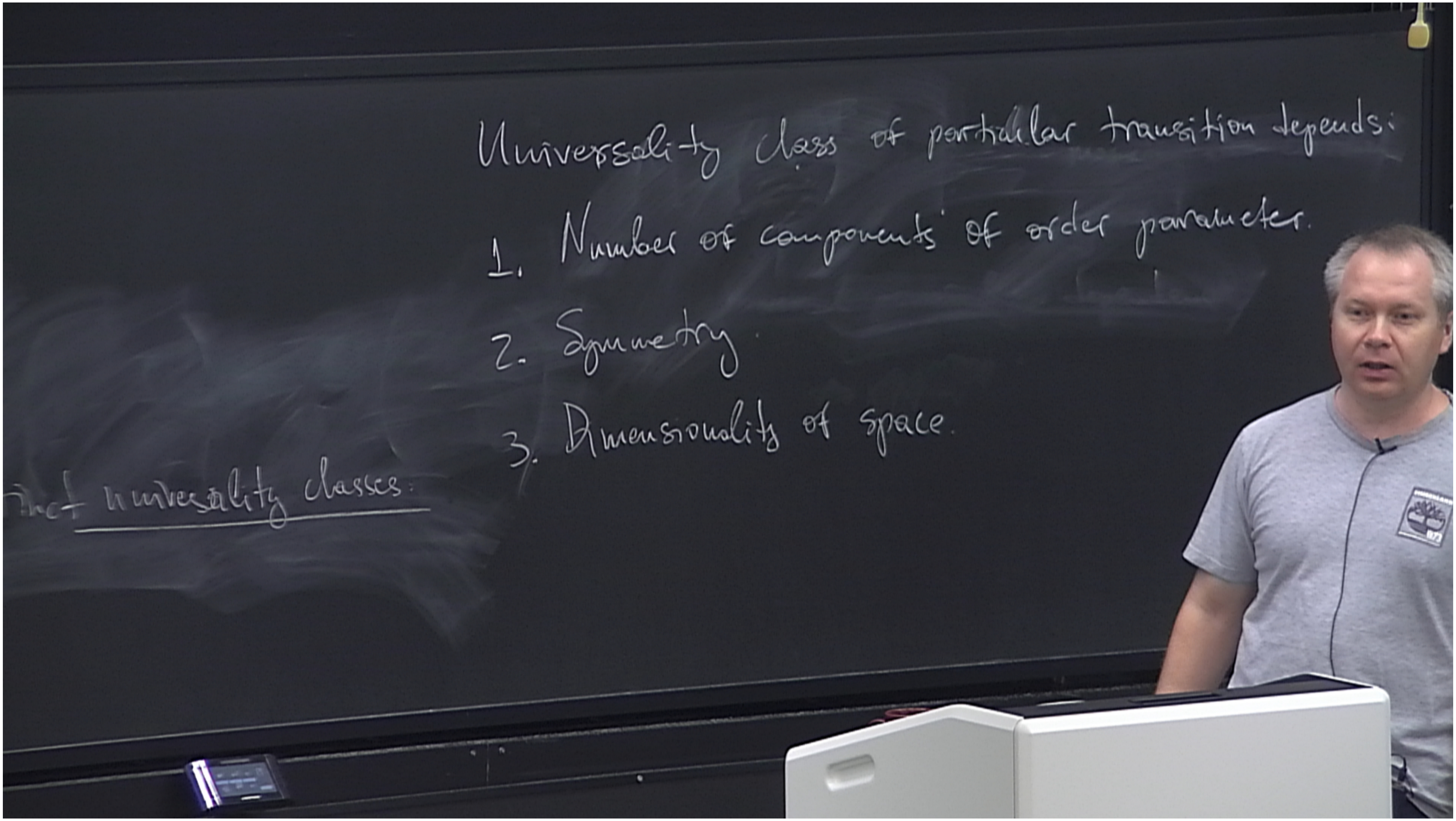
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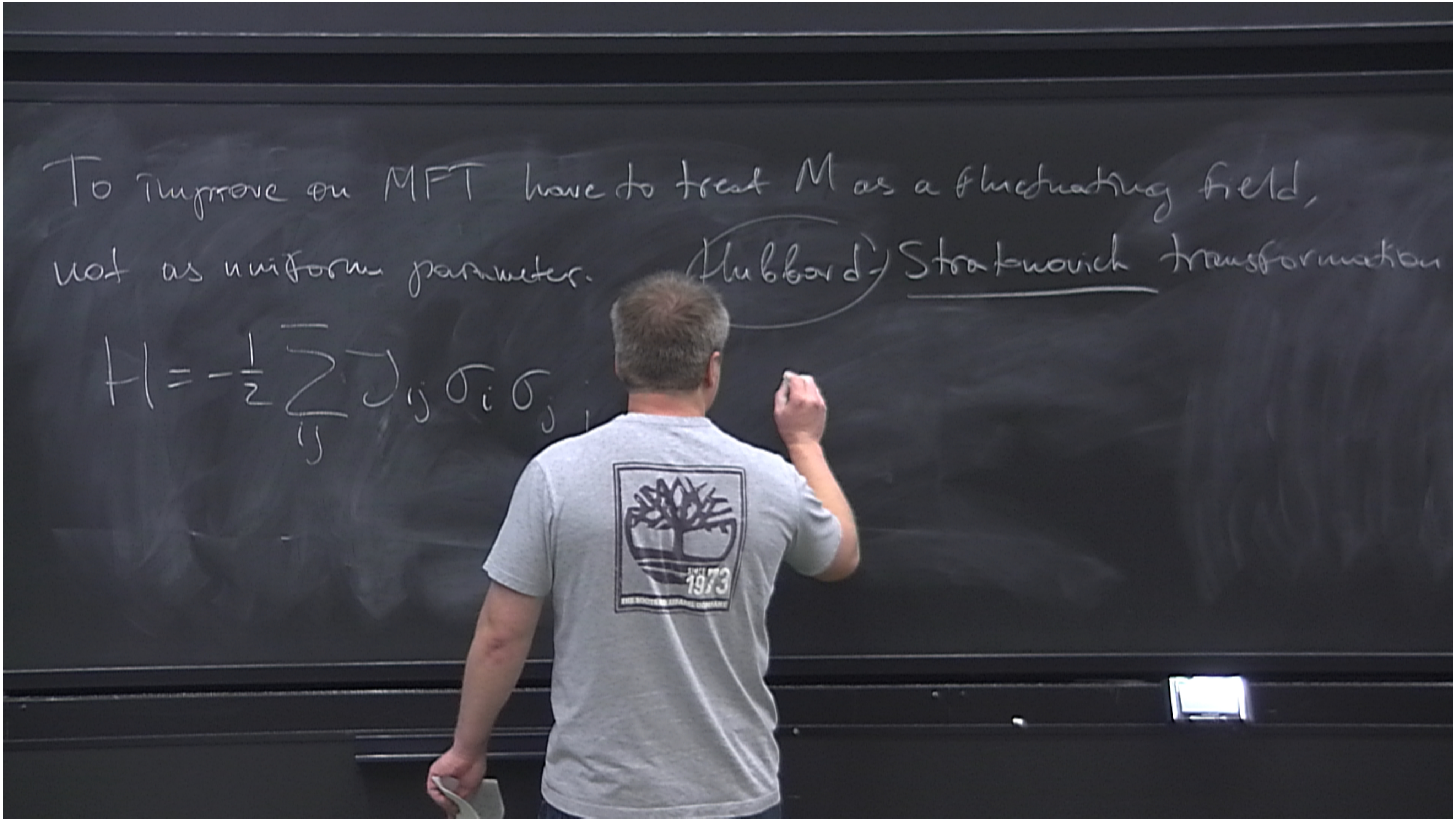
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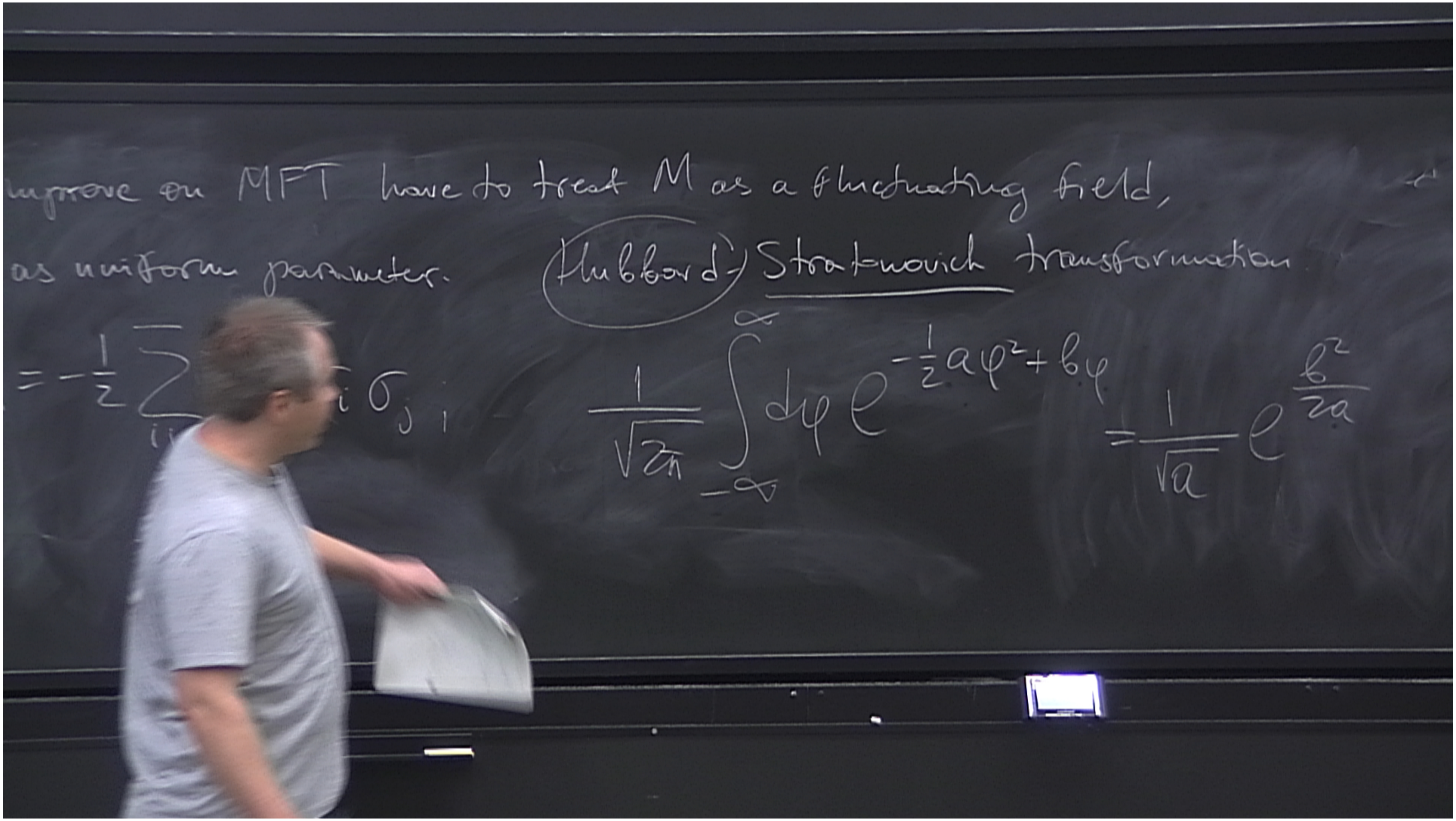
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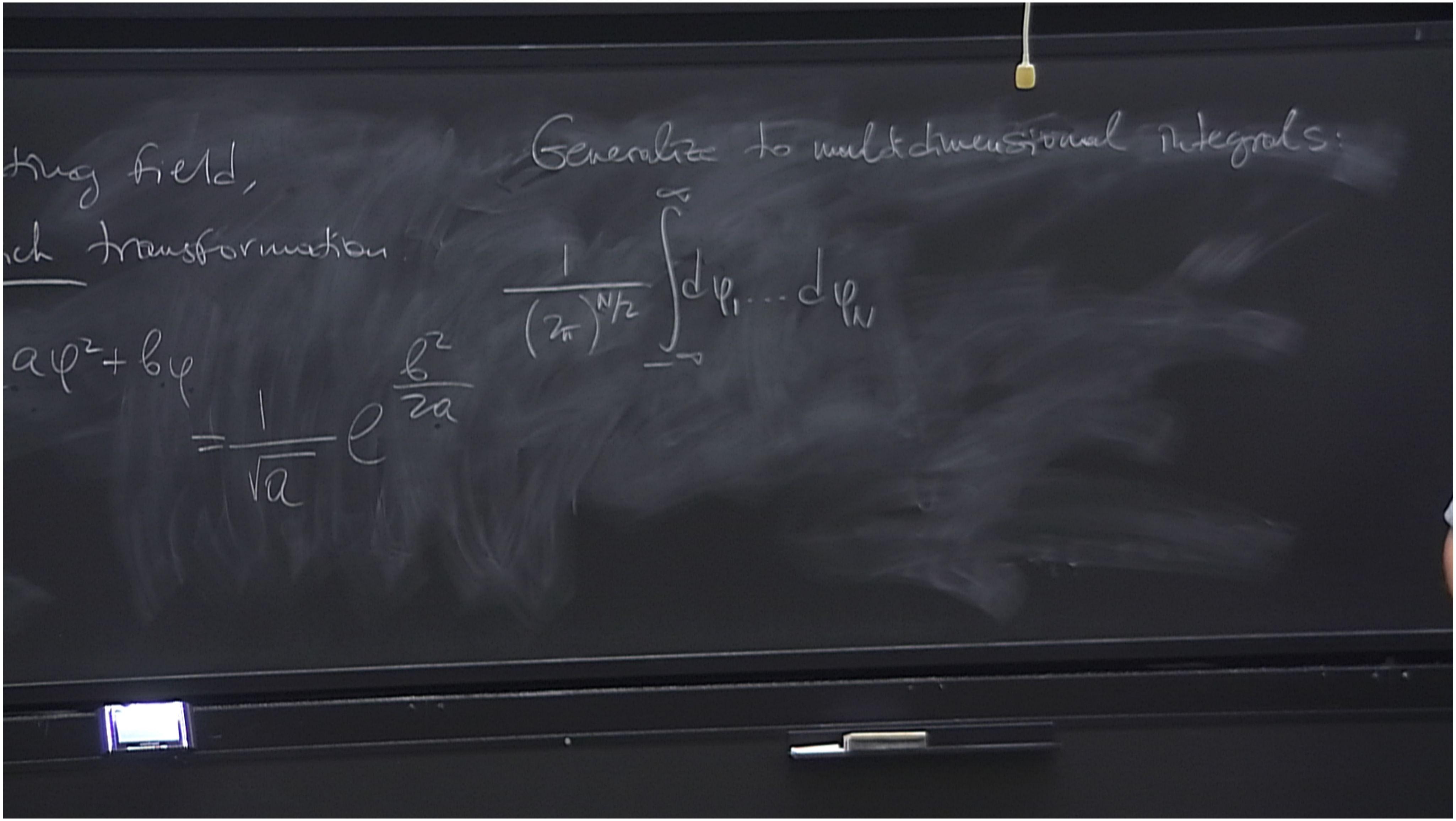
Generalize to multidimensional integrals:
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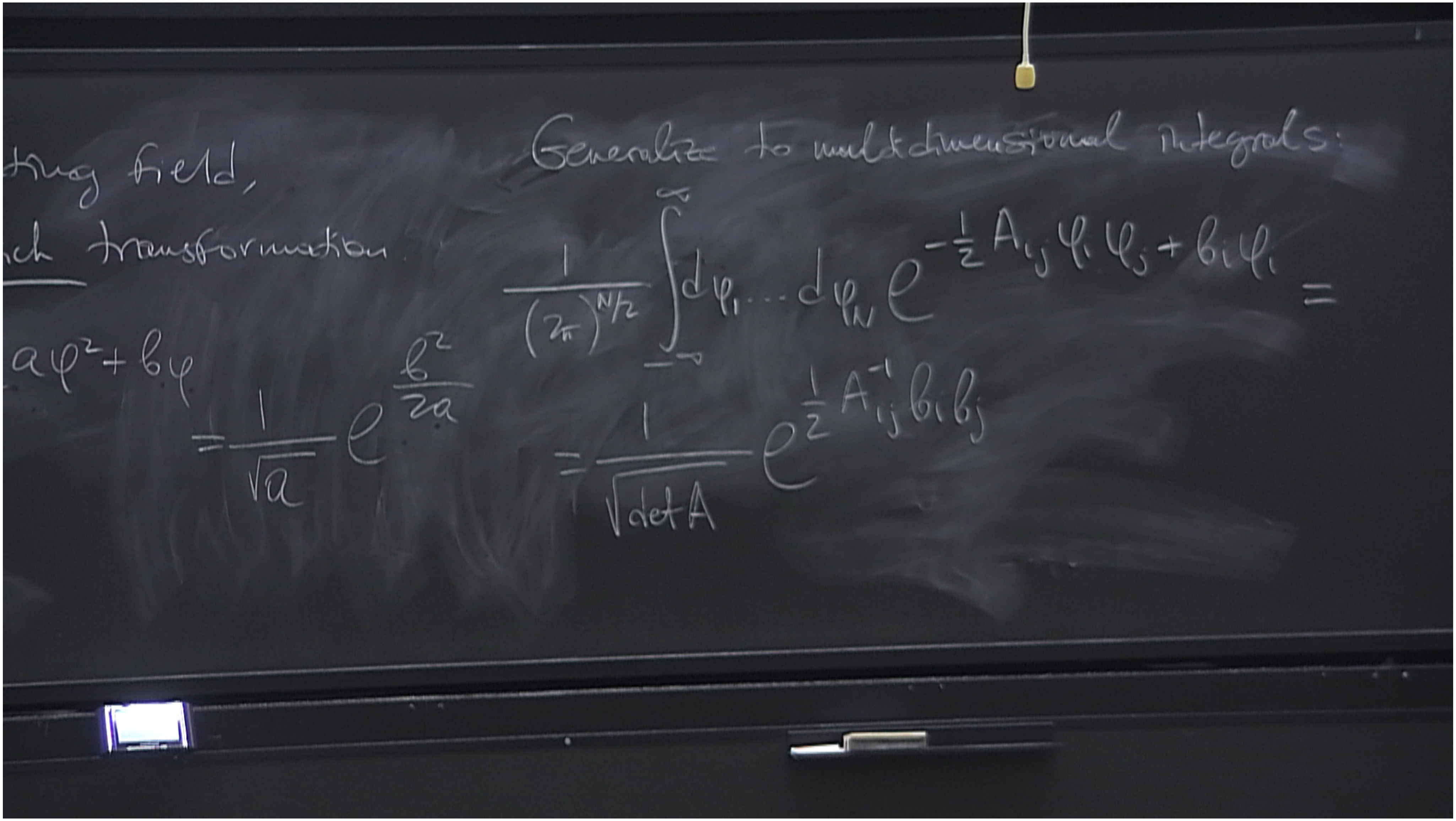
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