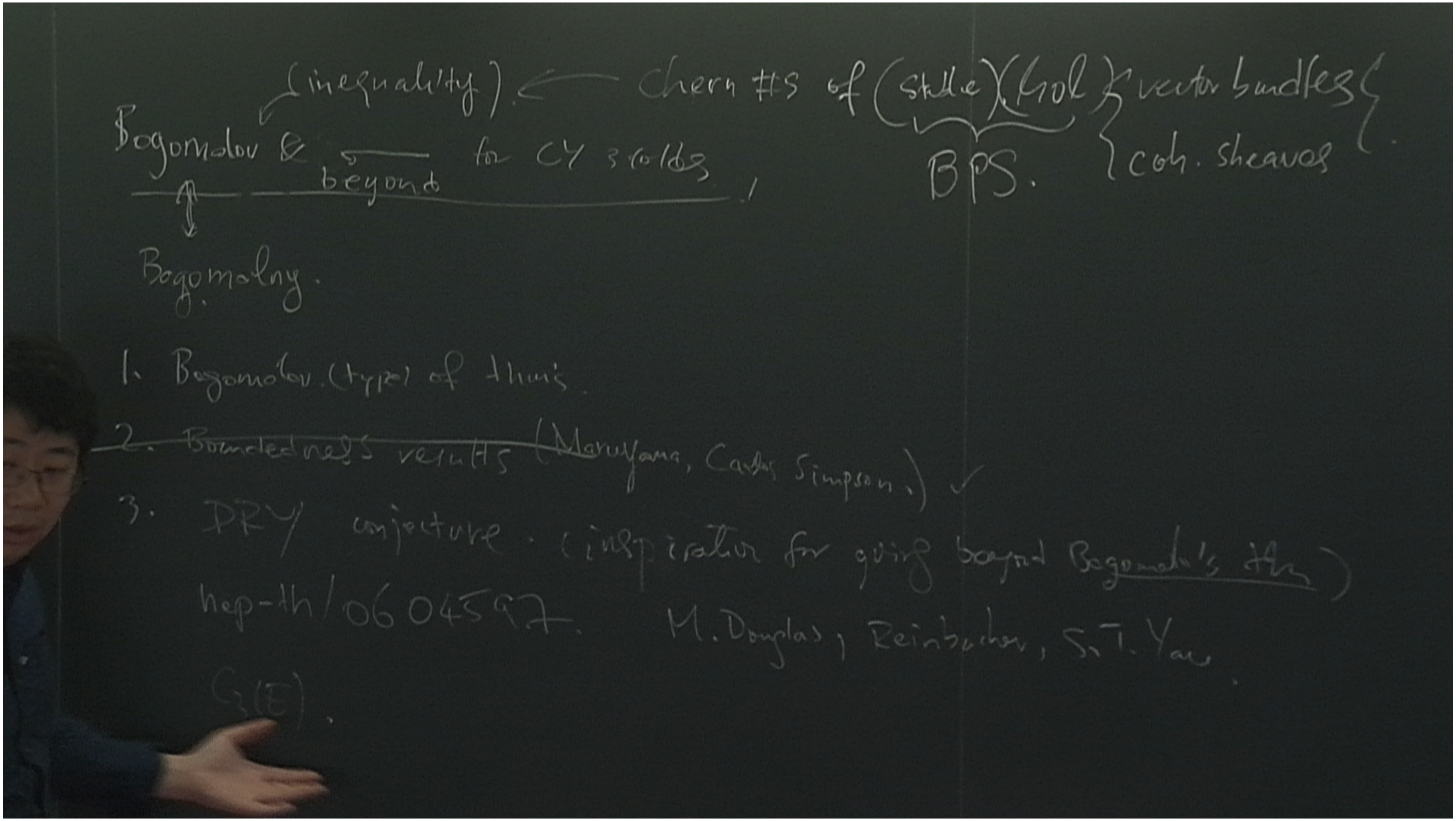
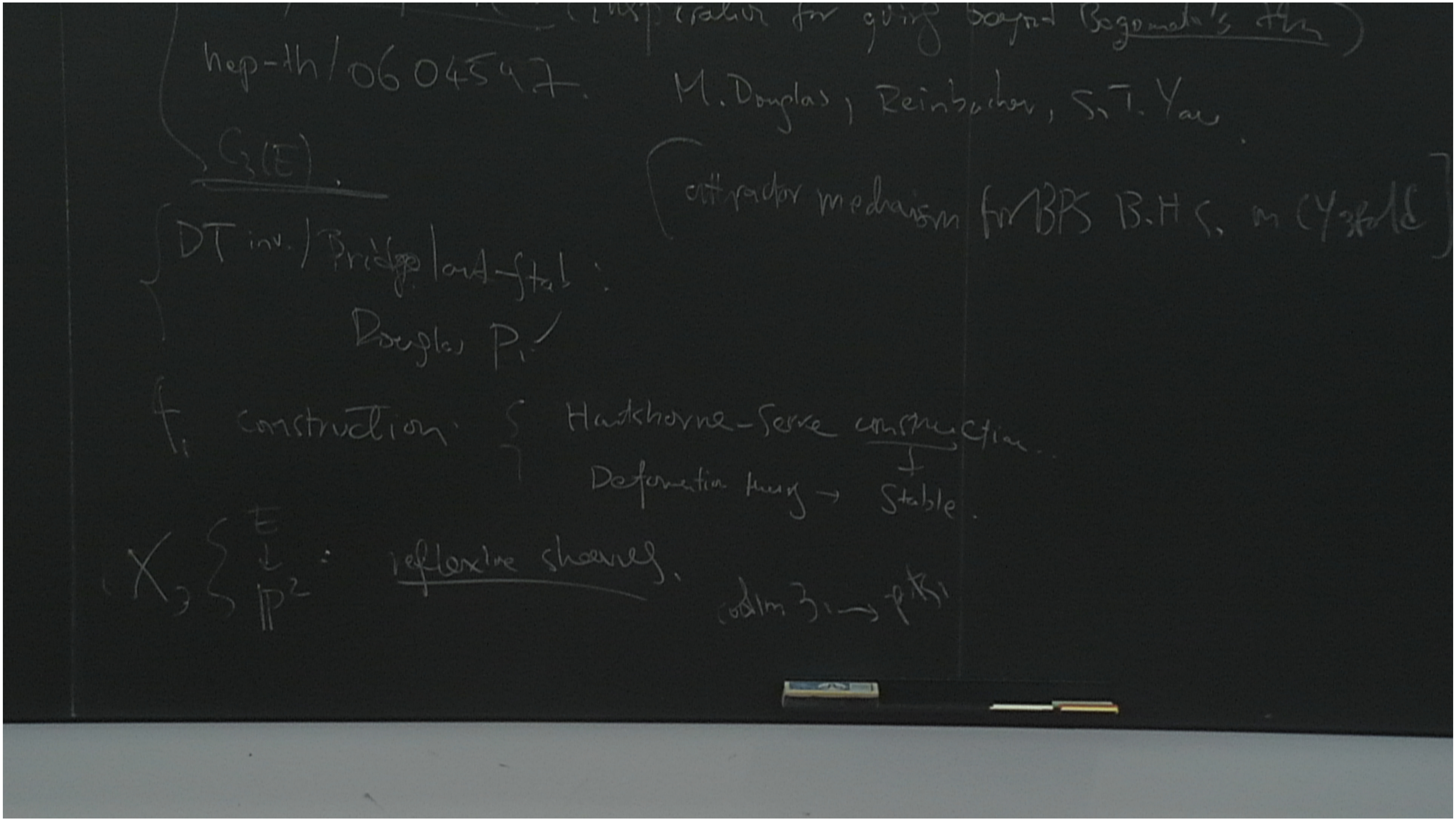
Title: Bogomolov and beyond for Calabi-Yau 3-folds

Date: Oct 08, 2014 02:00 PM

URL: http://pirsa.org/14100083

Abstract:


(inequality) ← Chern #s of (stable)(hol) vector bundles
coh. sheaves
Bogomolov & beyond for CY 3 folds
BPS.
Bogomolny.
1. Bogomolov (type) of thm's.
2. Boundedness results (Maruyama, Carlos Simpson.)
3. DRY conjecture. (inspiration for going beyond Bogomolov's thm)
hep-th/0604597. M. Douglas, Reinbacher, S.T. Yau.
c3(E).


hep-th/0604597. M. Douglas, Reinbacher, S.-T. Yau.
c3(E).
(attractor mechanism for BPS B.H's. in CY 3fold)
DT inv./Bridgeland-stab:
Douglas P.S.
f, construction: Hartshorne-Serre construction
Deformation theory → Stable.
X, E ↓ P2 : reflexive sheaves.
codim 3 ↦ pts

(inspiration for going beyond Bogomolov's thm)

hep-th/0604597. M. Douglas, Reinbacher, S. T. Yau.

$c_3(E)$.

(attractor mechanism for BPS B.H.s. in CY 3fold)

DT inv. / Bridgeland-stab:

Douglas P_1 ✓

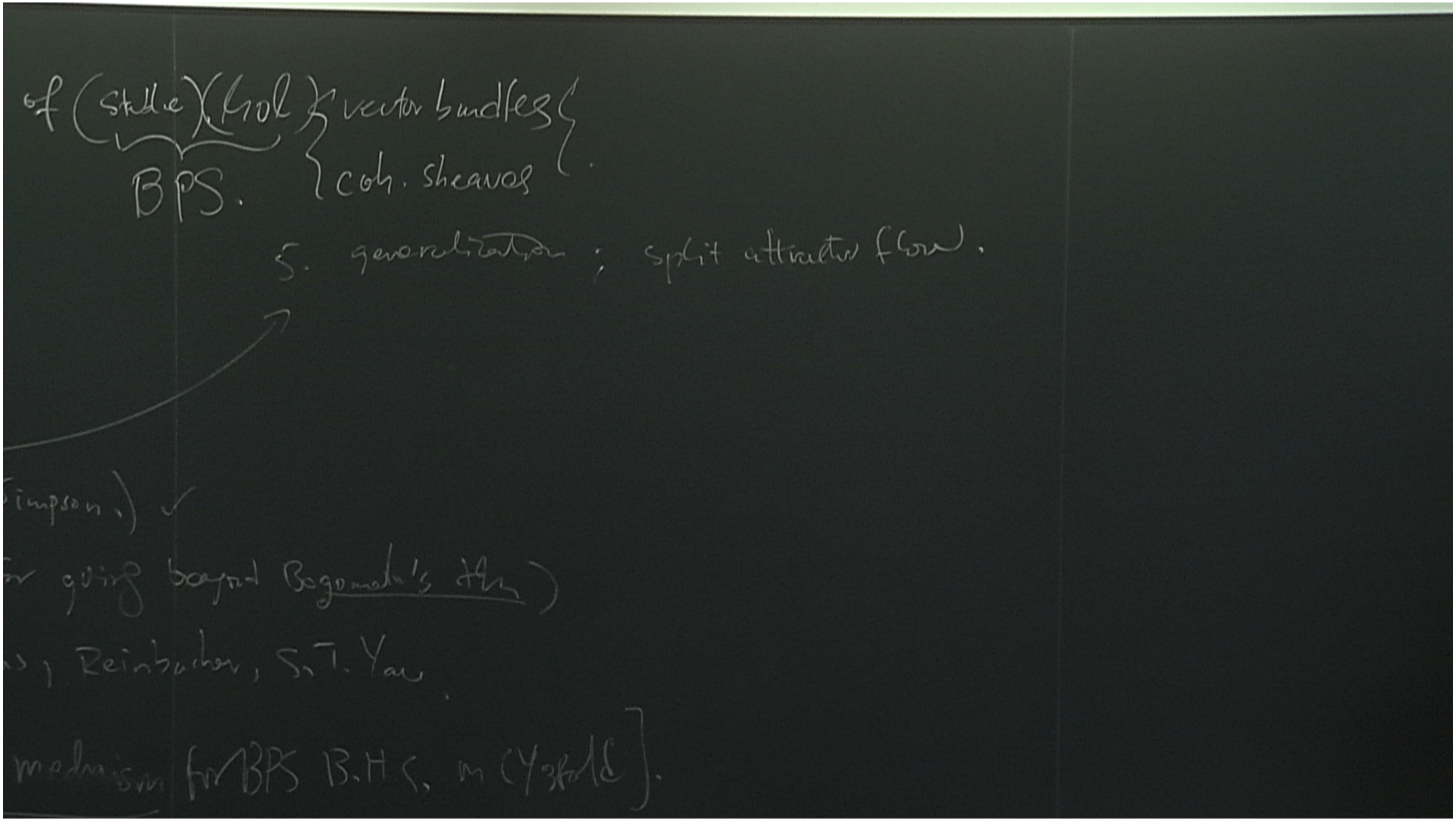
f_1 construction: { Hartshorne-Serre construction.

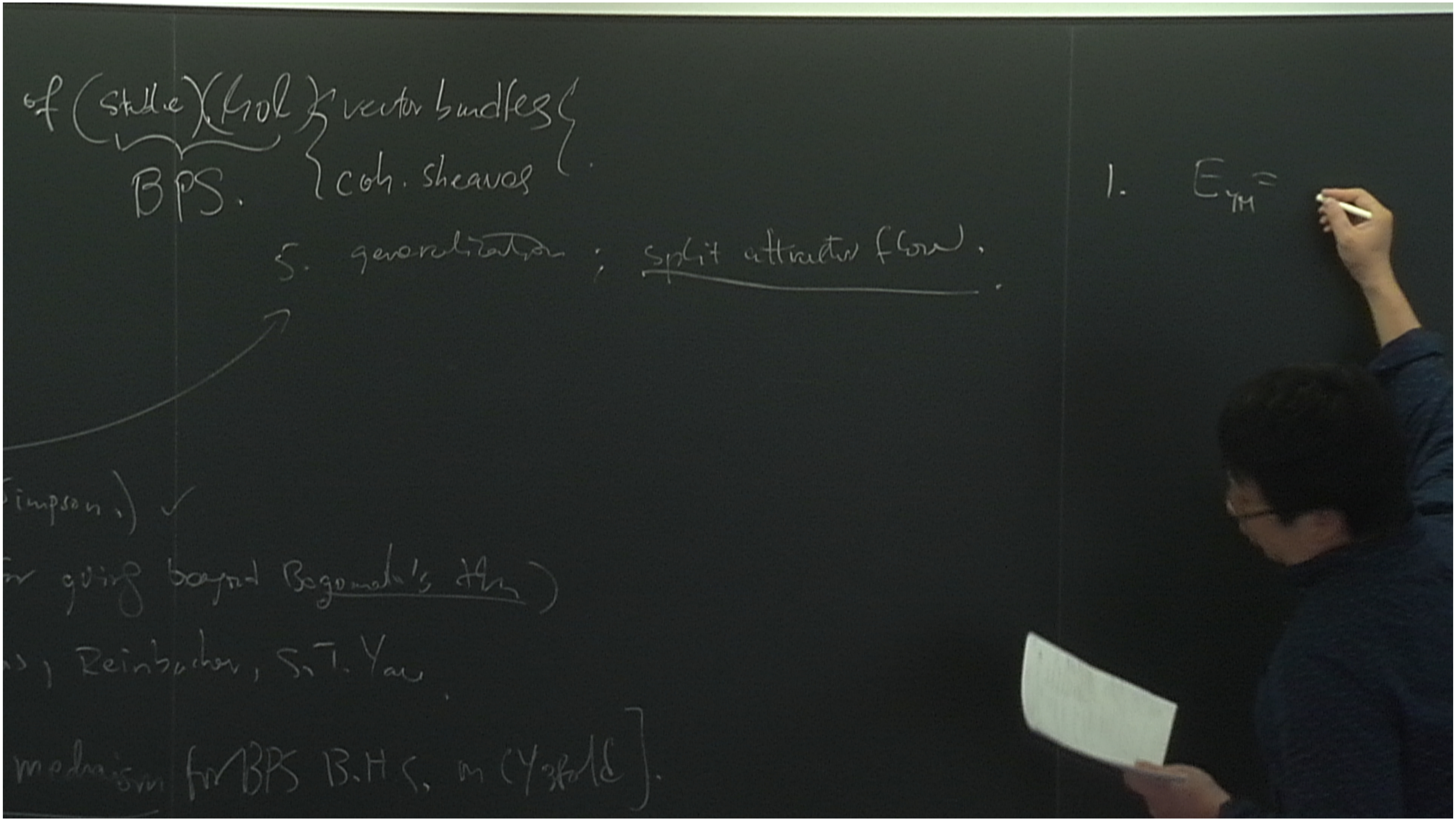
Deformation theory → Stable.

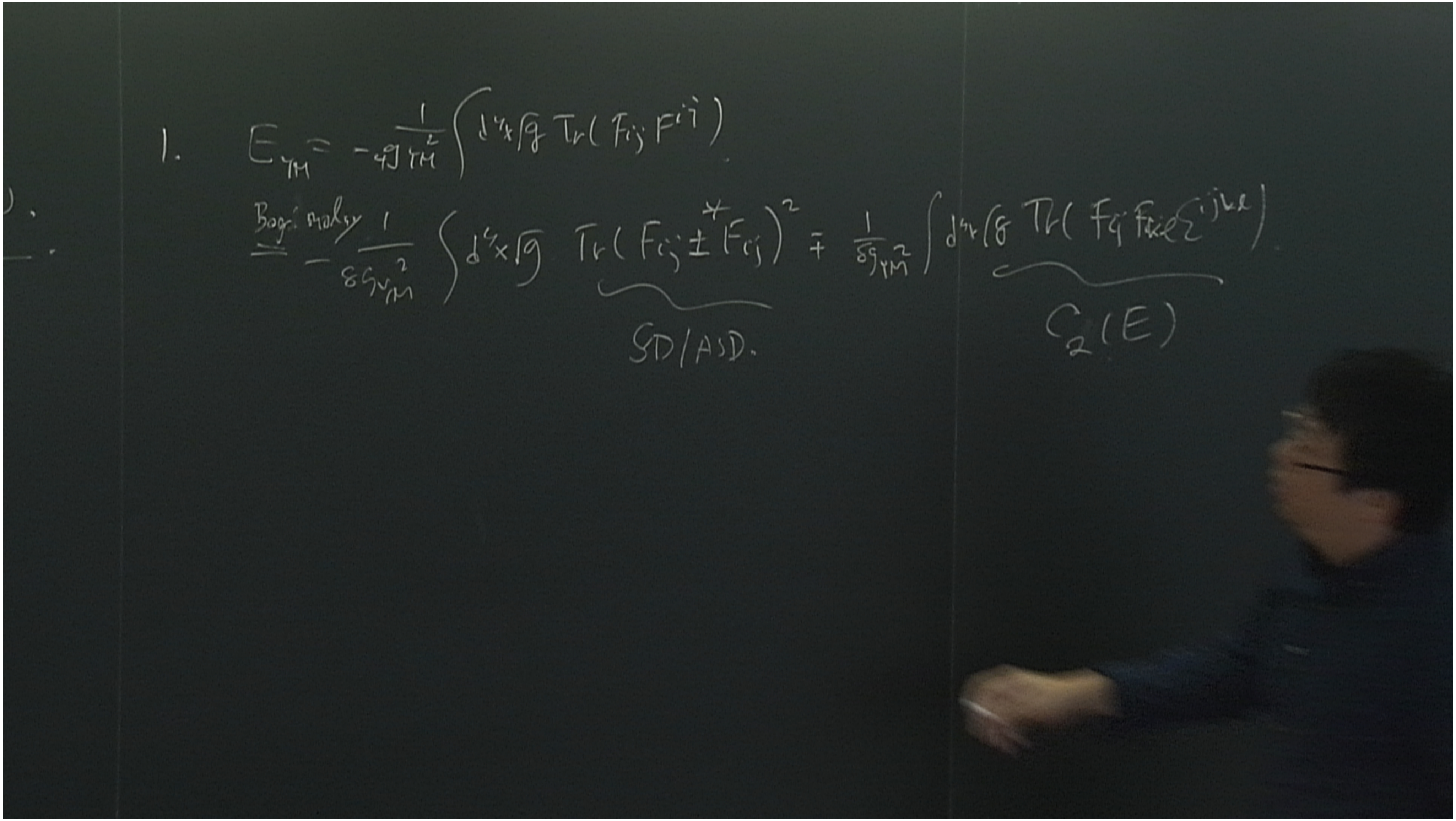
$X_3 \left\{ \begin{array}{c} E \\ \downarrow \\ \mathbb{P}^2 \end{array} \right.$: reflexive sheaves.

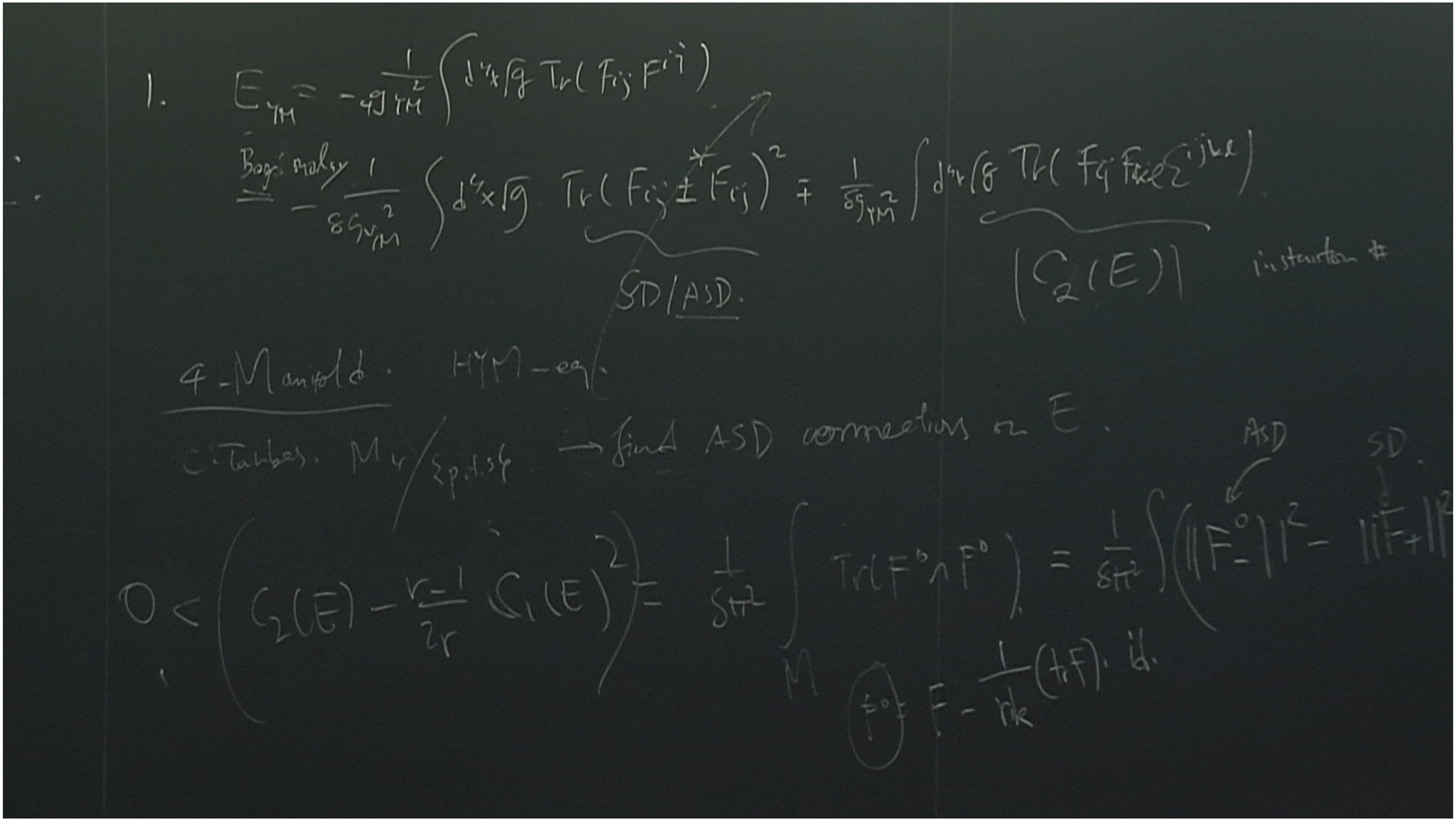
codim 3 → pts

Distler (heterotic string) $\mathcal{M}_{\text{stable obj}} \cong \mathcal{M}_{BPS}$ compact.

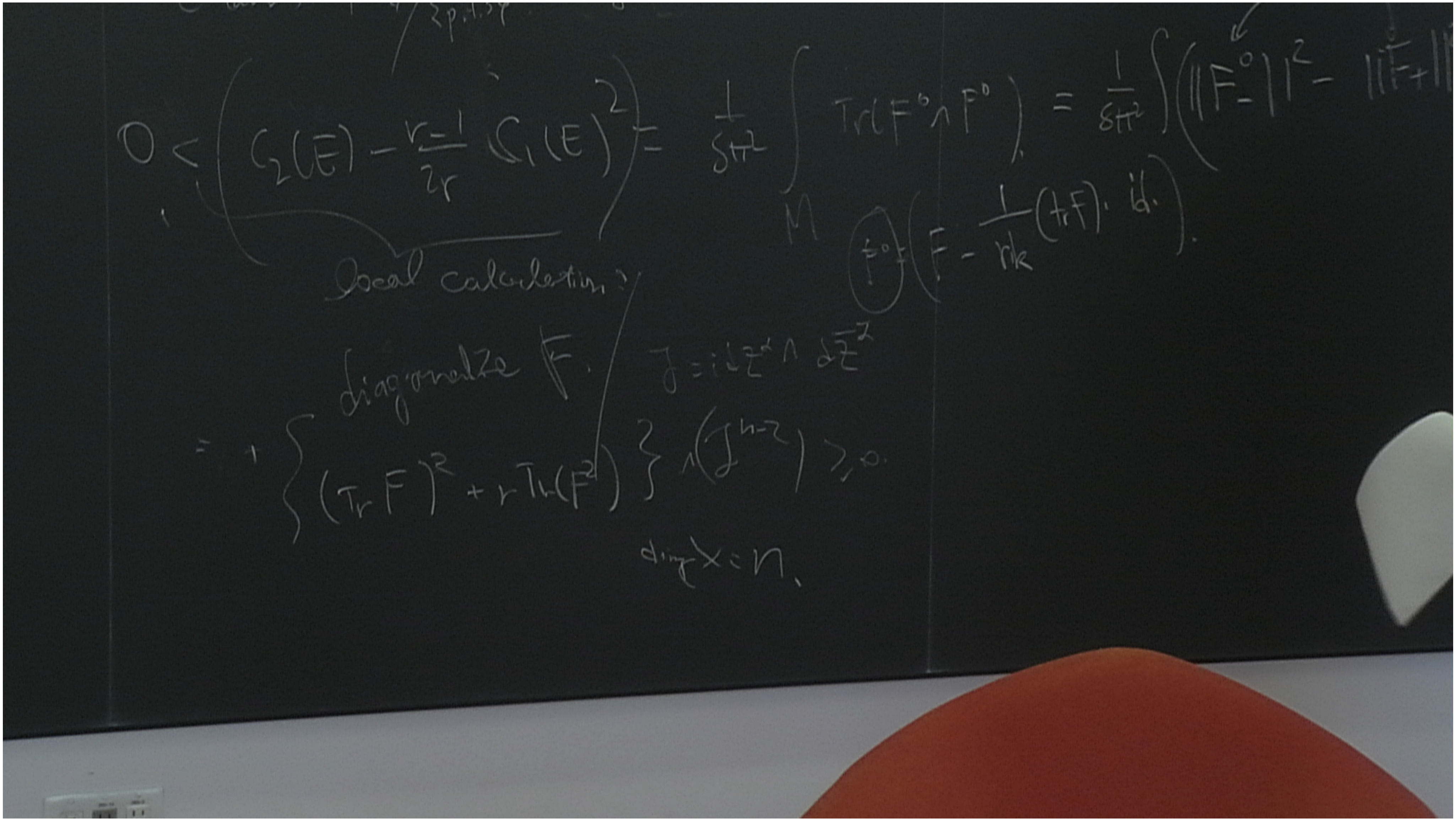

of (stable)(Hol) vector bundles
coh. sheaves
BPS.
5. generalization ; split attractor flow.
impson.)
going beyond Bogomolov's thm)
Reinbacher, S.T. Yau
mechanism for BPS B.H.s in CY3folds].

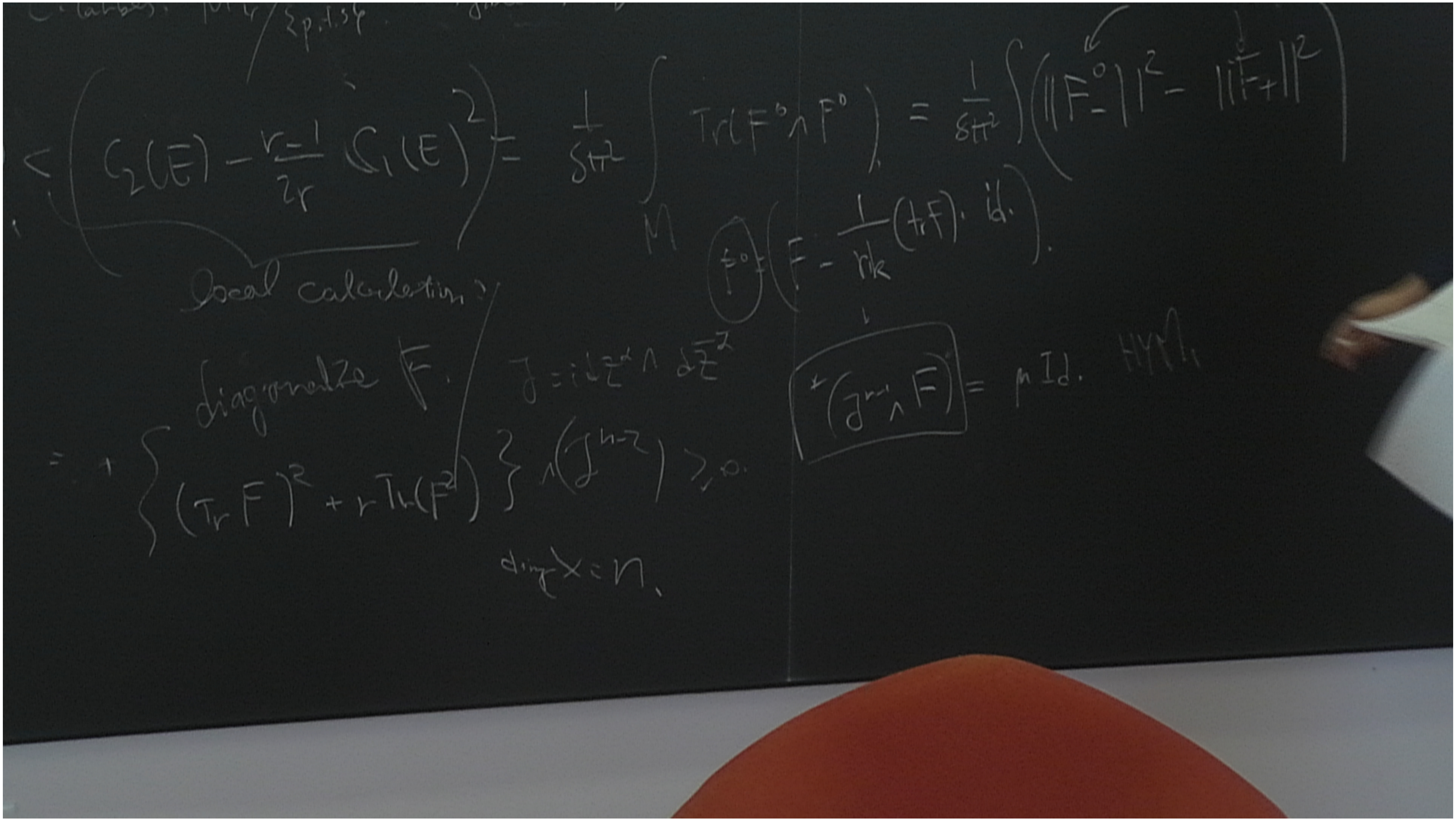

of (stable) (hol) {vector bundles / coh. sheaves}
BPS.
5. generalization ; split attractor flow.
Simpson.) ✓
for going beyond Bogomolov's thm)
Reinbacher, S.-T. Yau
mechanism for BPS B.H's. in CY3fold].
1. E_{YM} =

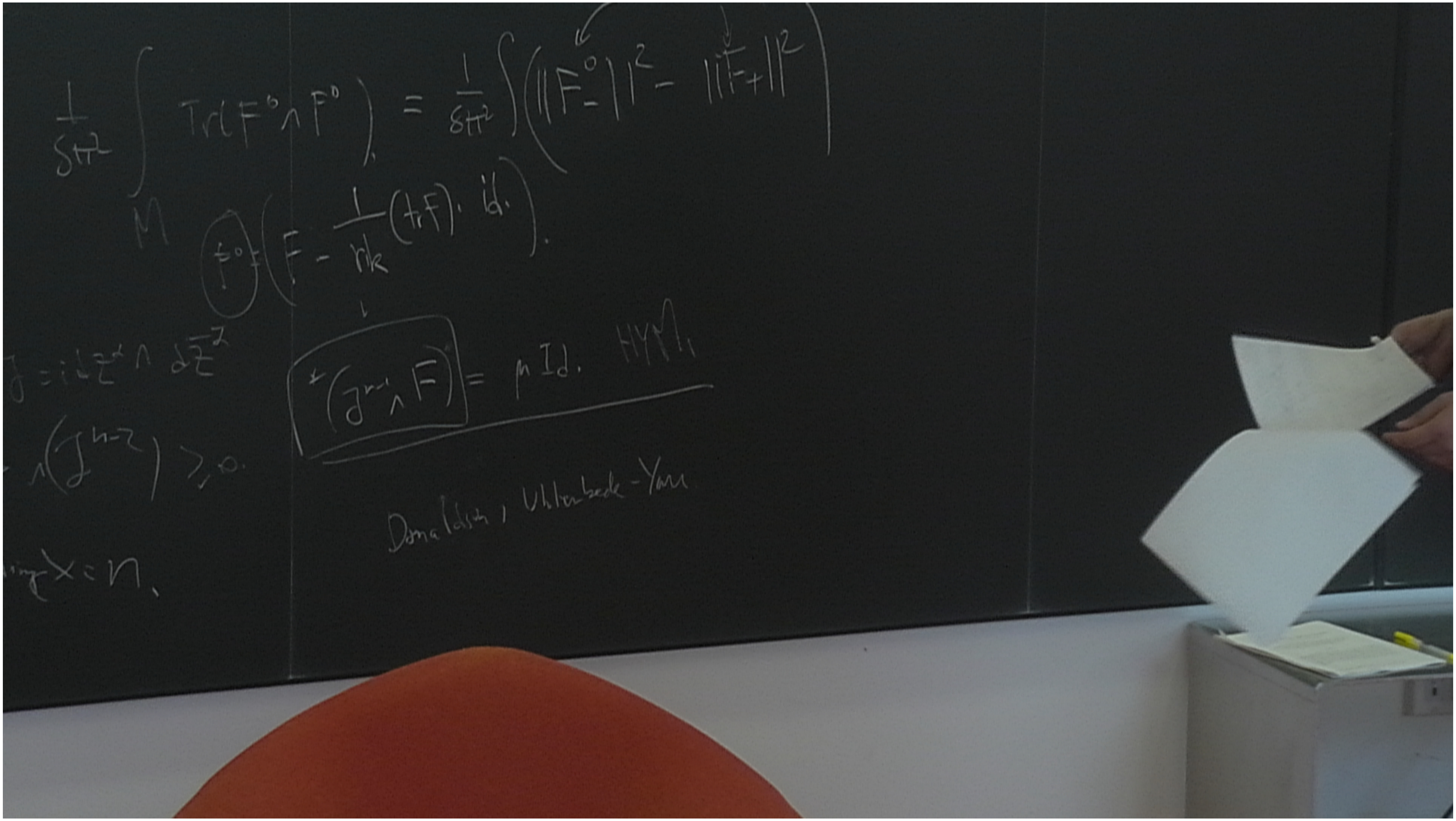

1. $E_{YM} = -\frac{1}{4g_{YM}^2}\int d^4x\sqrt{g}\,\mathrm{Tr}(F_{ij}F^{ij})$
Bogomolny $= -\frac{1}{8g_{YM}^2}\int d^4x\sqrt{g}\,\mathrm{Tr}(F_{ij} \pm {}^*F_{ij})^2 \mp \frac{1}{8g_{YM}^2}\int d^4x\sqrt{g}\,\mathrm{Tr}(F_{ij}F_{kl}\epsilon^{ijkl})$
SD/ASD.
$C_2(E)$

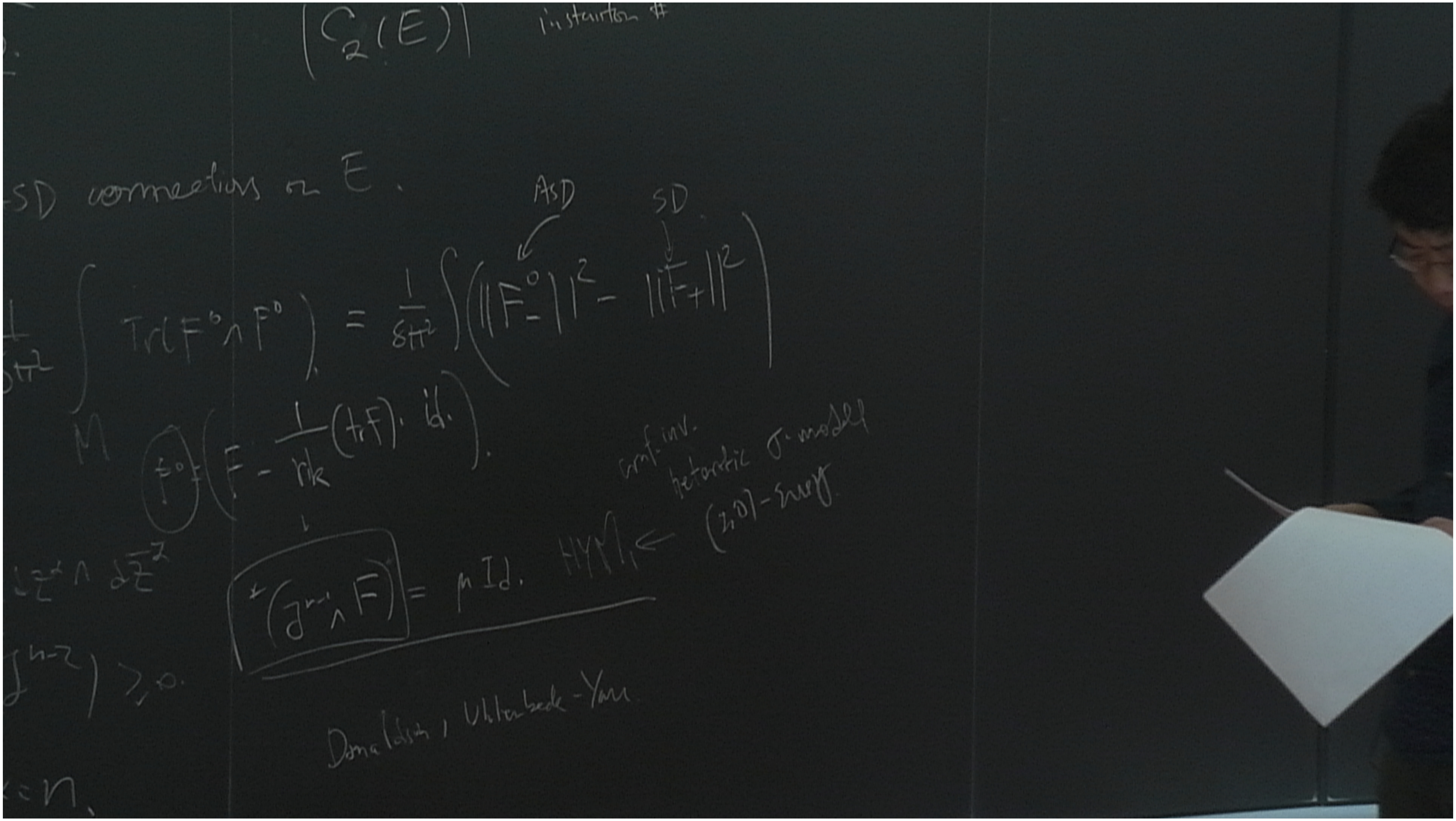


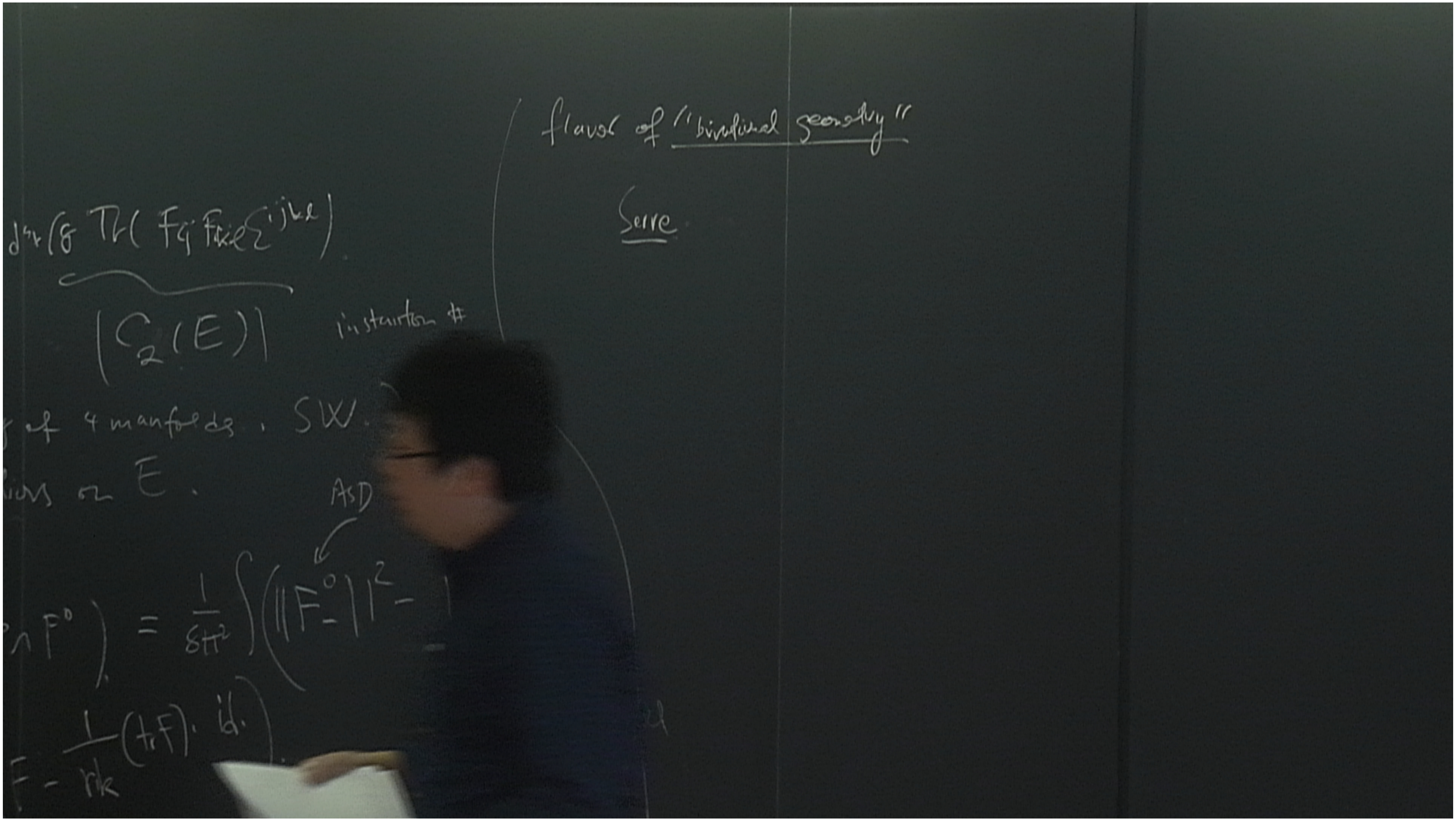
1. $E_{YM} = -\frac{1}{4g_{YM}^2}\int d^4x\sqrt{g}\,Tr(F_{ij}F^{ij})$
Bogomolny
$-\frac{1}{8g_{YM}^2}\int d^4x\sqrt{g}\,Tr(F_{ij} \pm {}^*F_{ij})^2 \mp \frac{1}{8g_{YM}^2}\int d^4x\sqrt{g}\,Tr(F_{ij}F_{kl}\epsilon^{ijkl})$
SD/ASD.
$|C_2(E)|$
instanton #
4-Manifolds. HYM-eq.
C. Taubes. $M_4/\{p_1,\ldots,p_n\}$ → find ASD connections on E.
ASD
SD.
$0 < \left(C_2(E) - \frac{r-1}{2r} C_1(E)^2\right) = \frac{1}{8\pi^2}\int_M Tr(F^0 \wedge F^0) = \frac{1}{8\pi^2}\int\left(\|F^0_-\|^2 - \|F^0_+\|^2\right)$
$F^0 = F - \frac{1}{rk}(trF)\cdot id.$

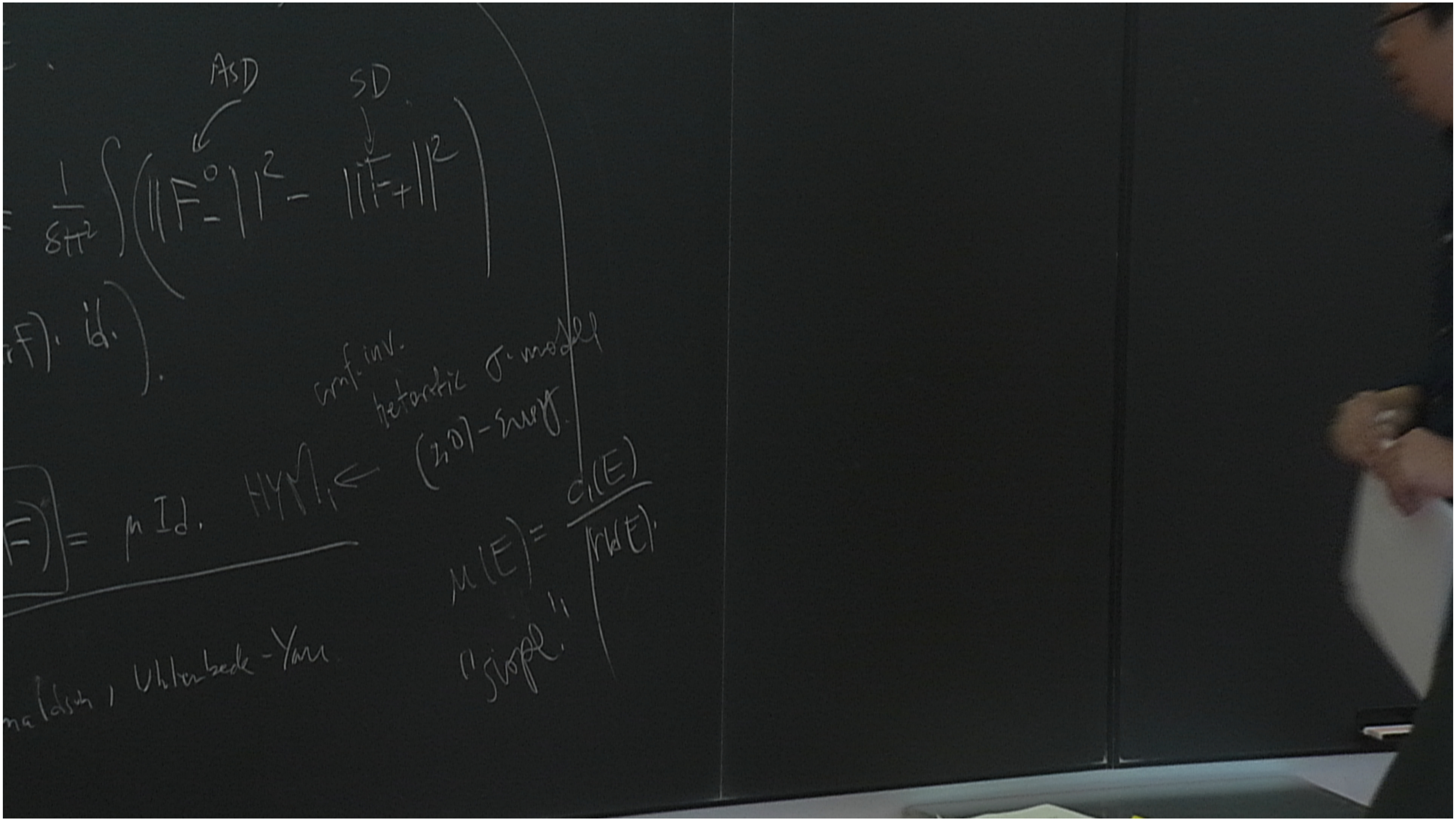

local calculation ?
diagonalize F.
dim X = n.

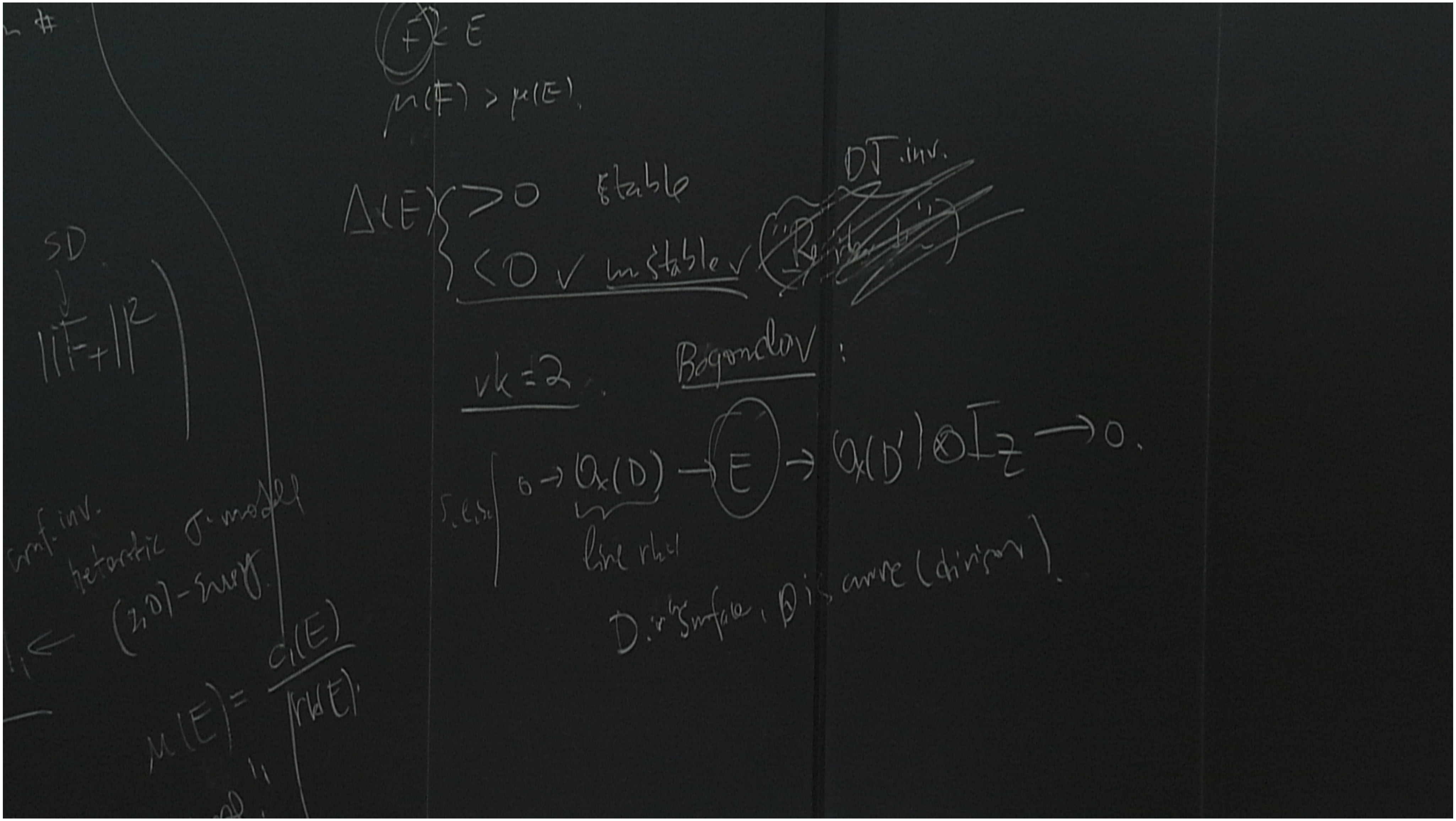

local calculation?
diagonalize F.
HYM
dim X = n.


HYM
Donaldson, Uhlenbeck-Yau

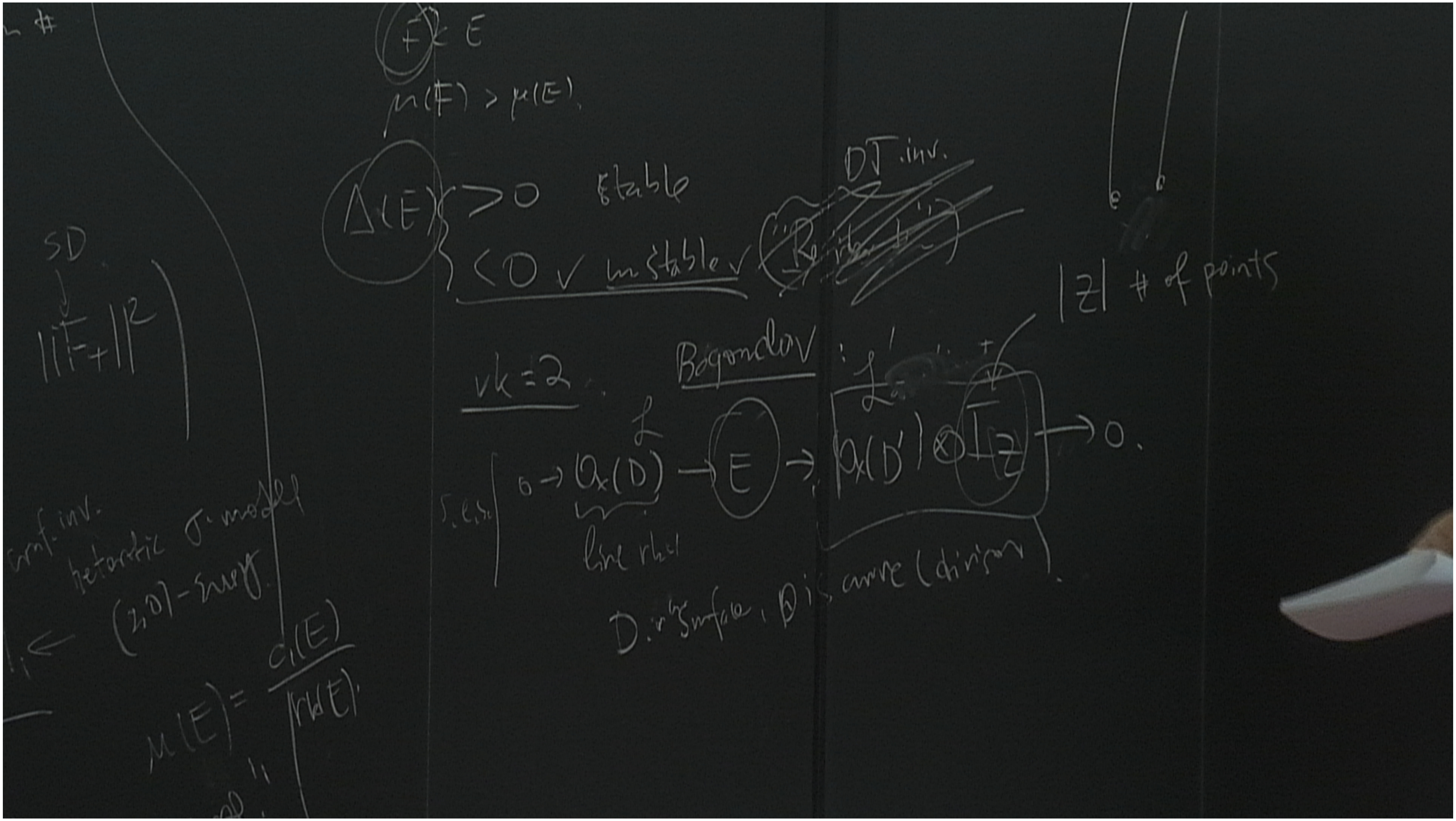

instanton #
SD connections on E.
ASD
SD
cnf. inv.
heterotic σ-model
(2,0)-Susy
HYM
= μ Id.
Donaldson, Uhlenbeck-Yau

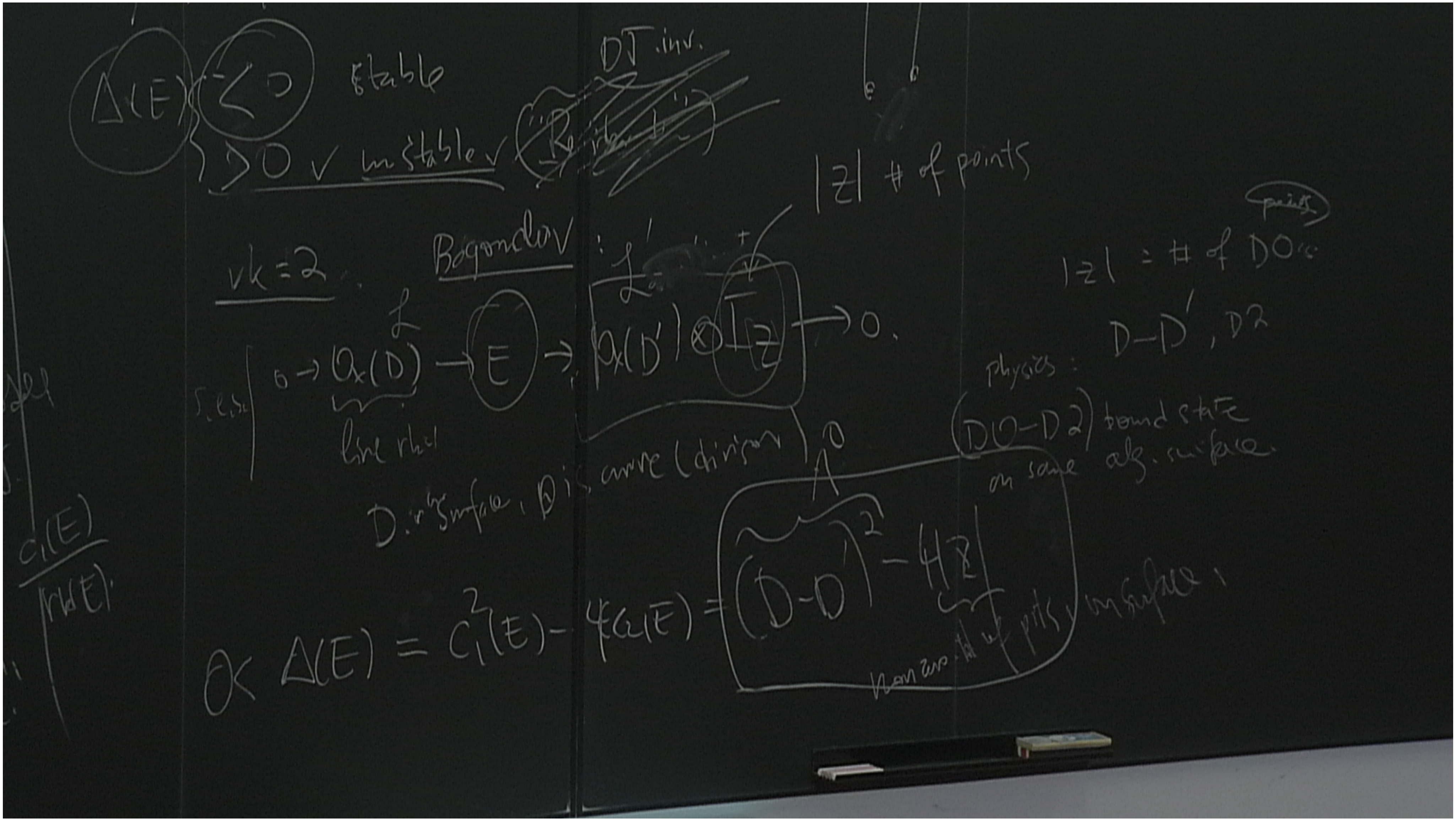

flavor of "birational geometry"
Serre
instanton #
ASD

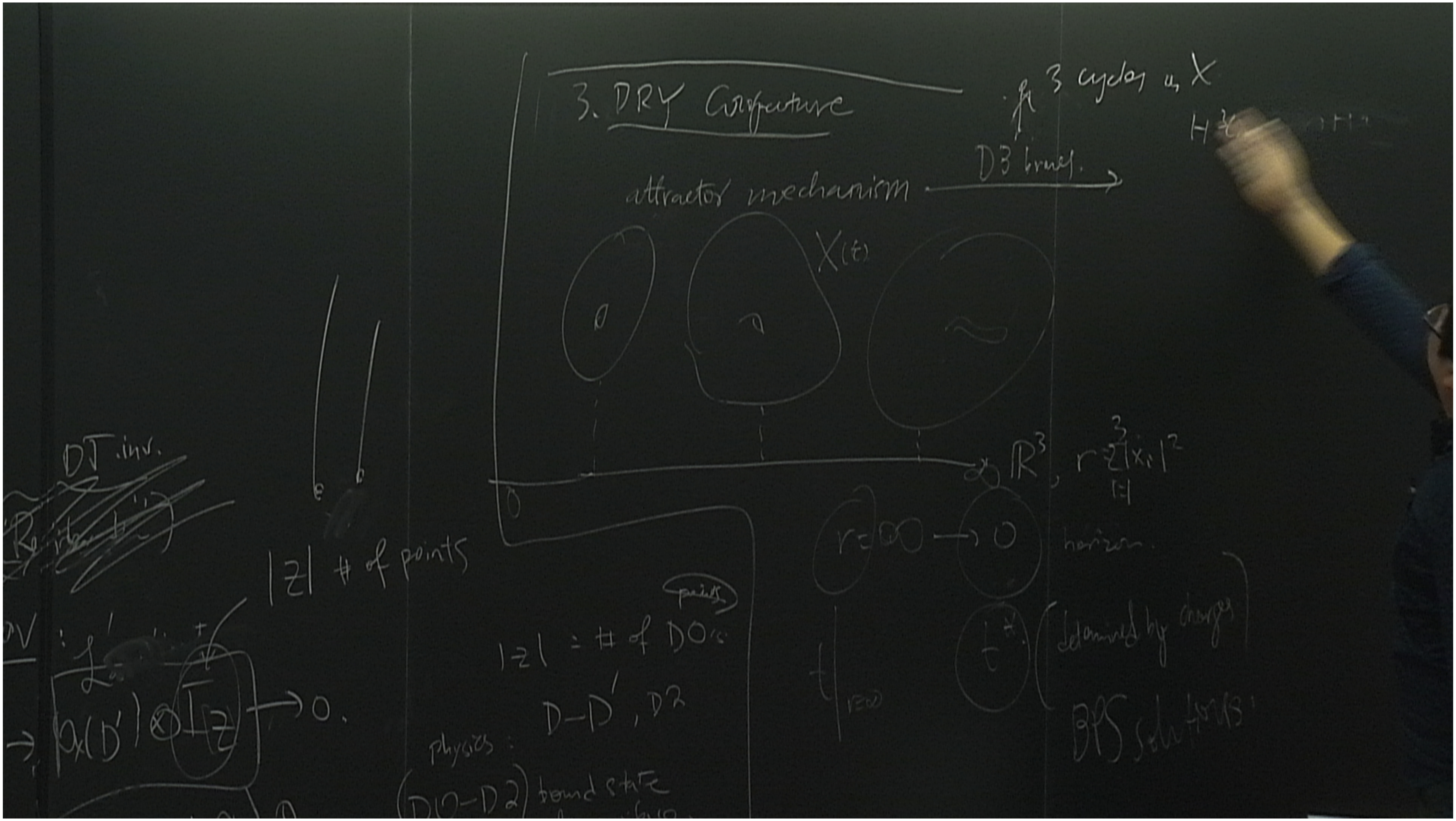

ASD
SD
HYM
(2,0)-susy
"slope"
Uhlenbeck-Yau

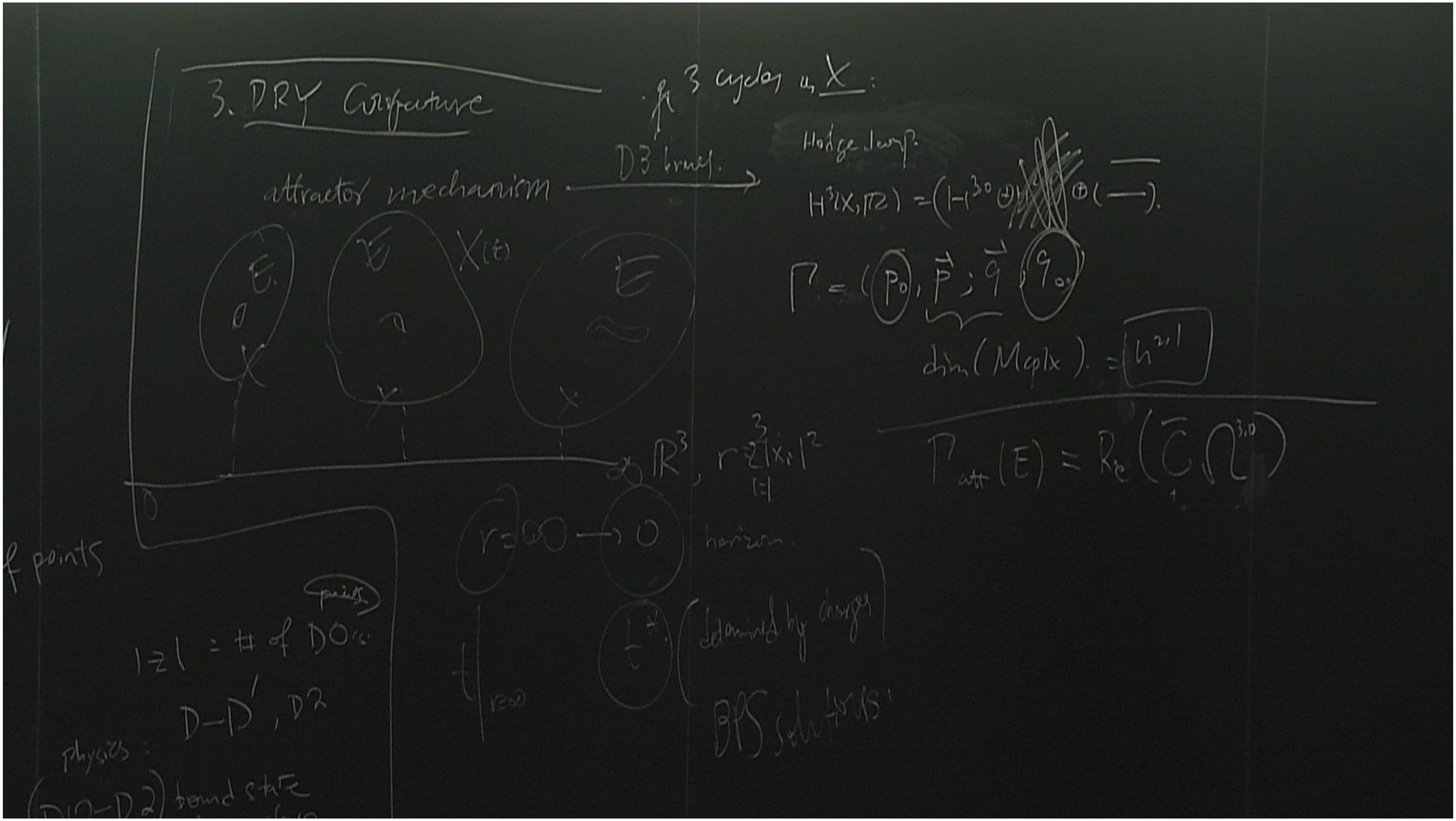


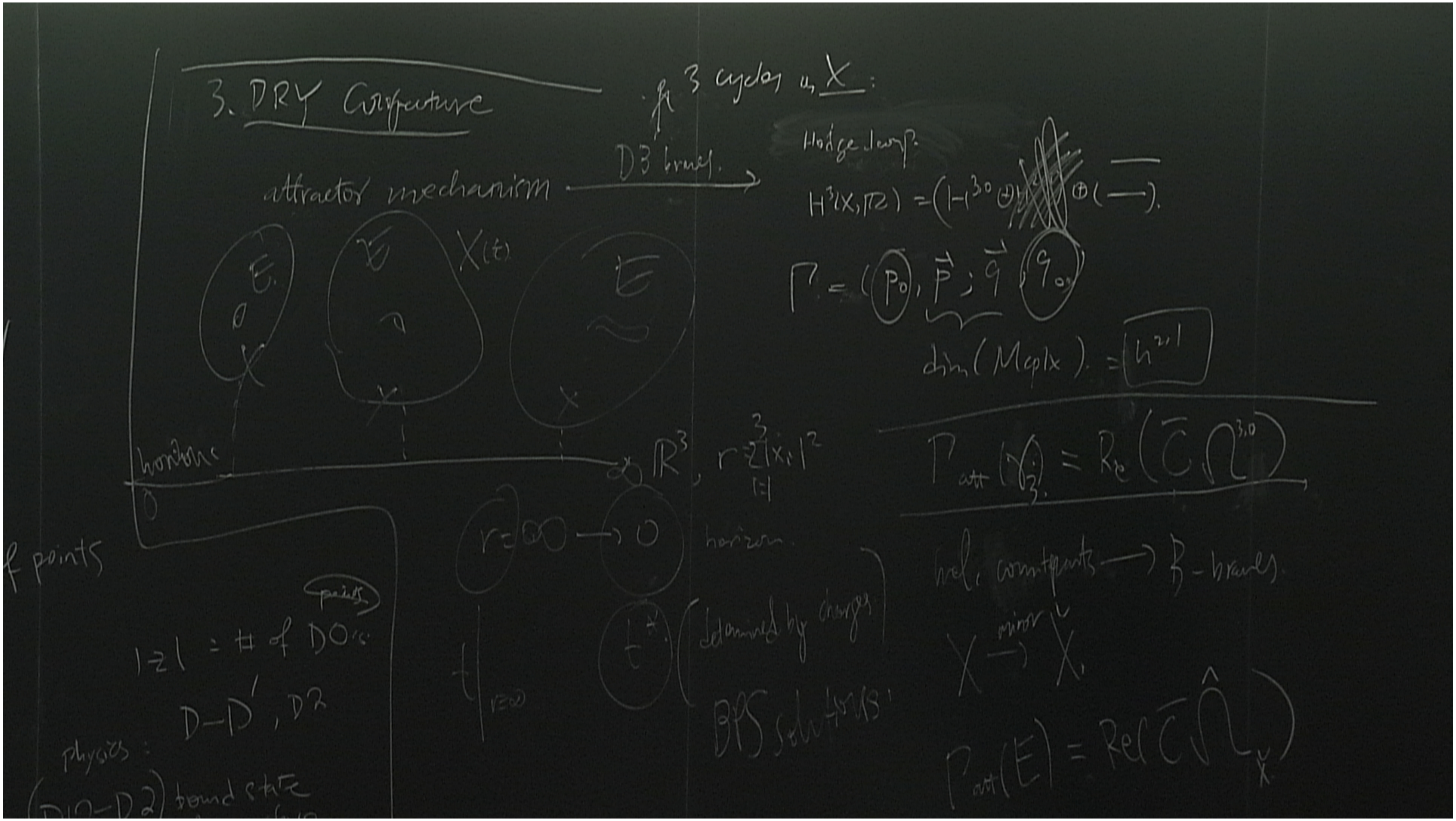
stable
unstable
rk = 2
Bogomolov
SD
conf. inv.

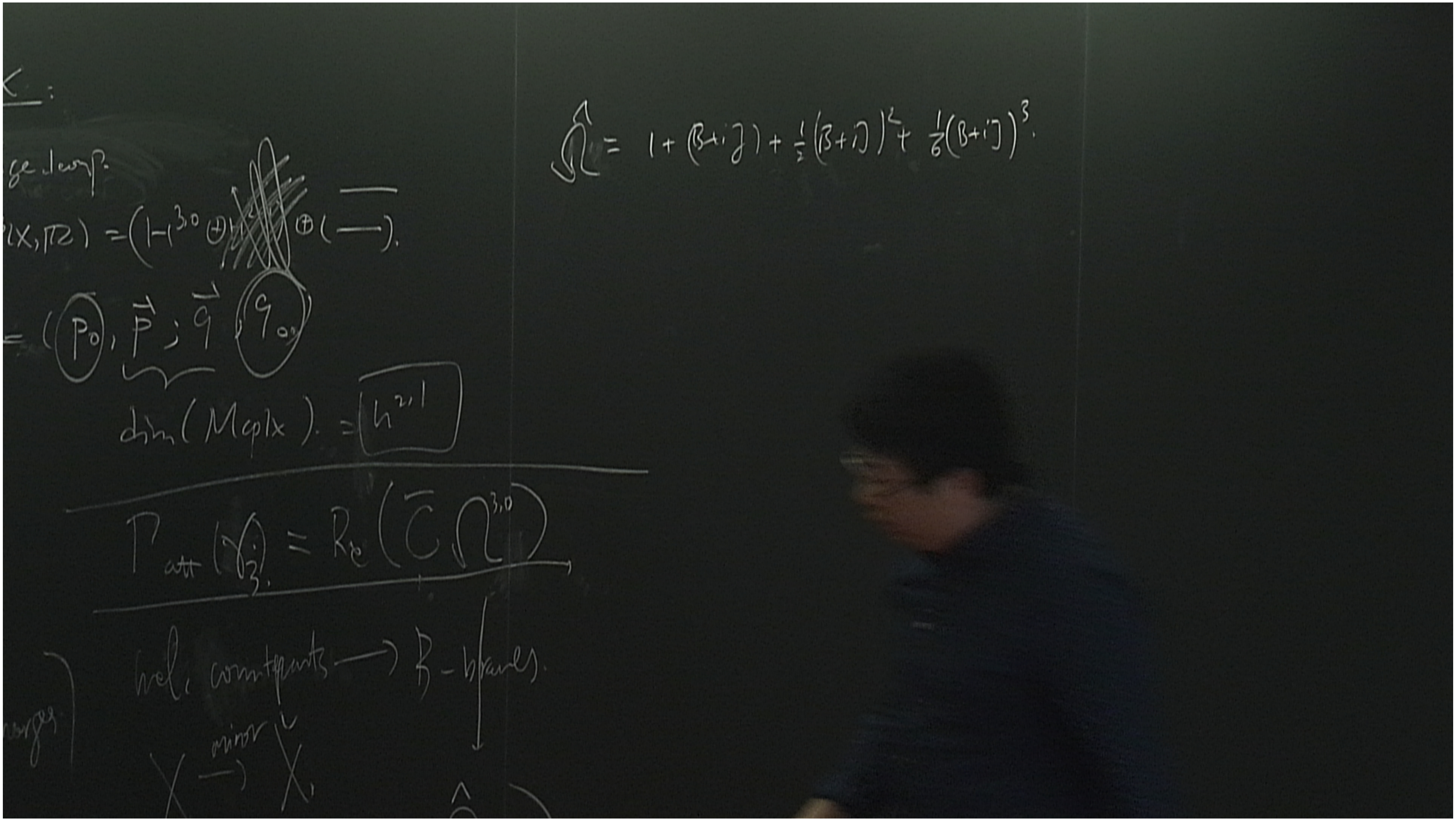

$\mu(F) > \mu(E)$
$\Delta(E) > 0$ stable
< 0 unstable
DT. inv.
$|Z|$ # of points
rk = 2
Bogomolov
$0 \to O_X(D) \to E \to O_X(D') \otimes I_Z \to 0$
line bundle
D : surface, D is curve (divisor)
SD
$\|F_+\|^2$
conf. inv.
heterotic σ-model
(2,0)-Energy
$\mu(E) = \frac{c_1(E)}{rk(E)}$

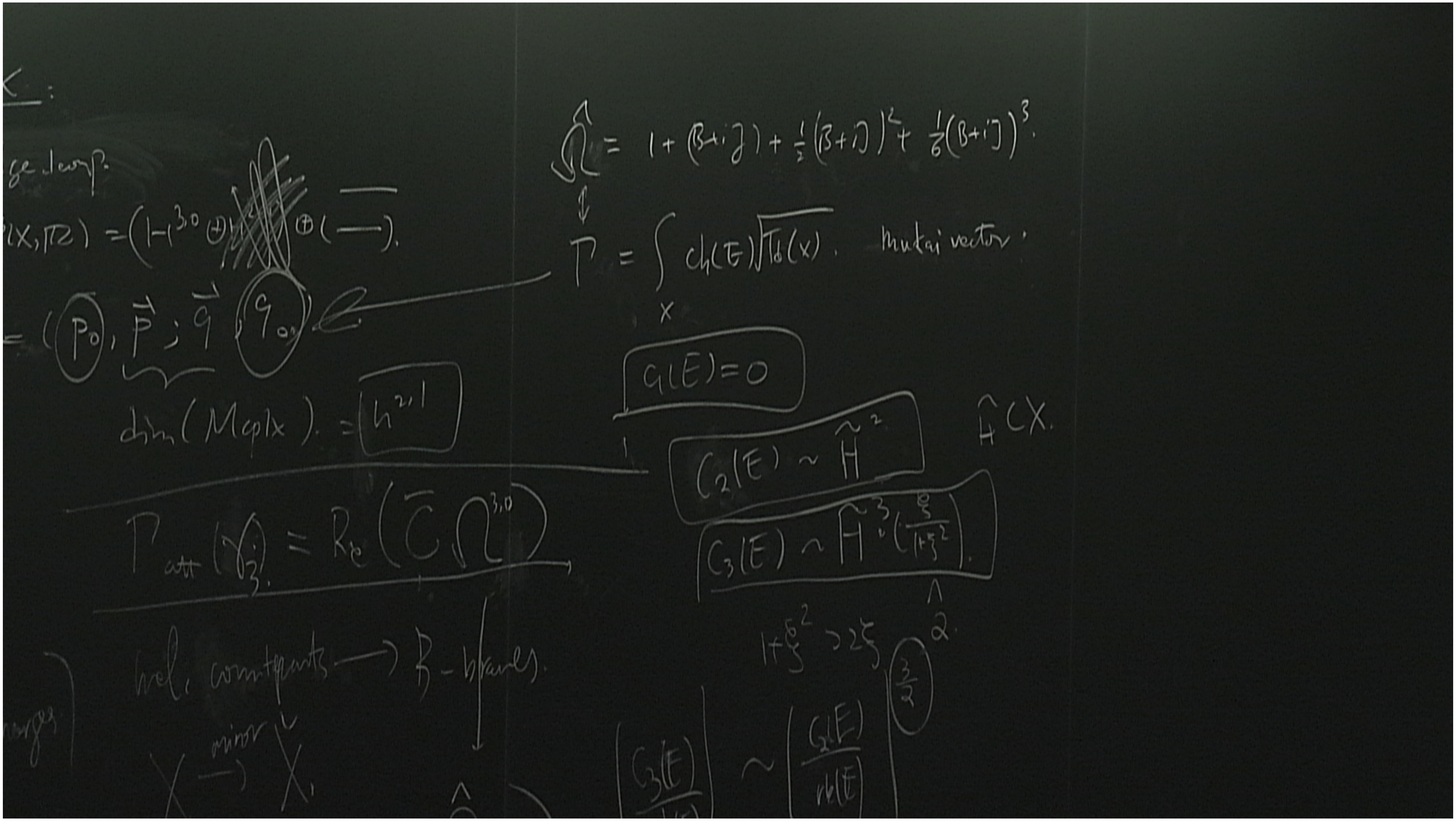

DT inv.
stable
unstable
|Z| # of points
Bogomolov
|Z| = # of D0's
D-D', D2
physics :
bound state
on same alg. surface.

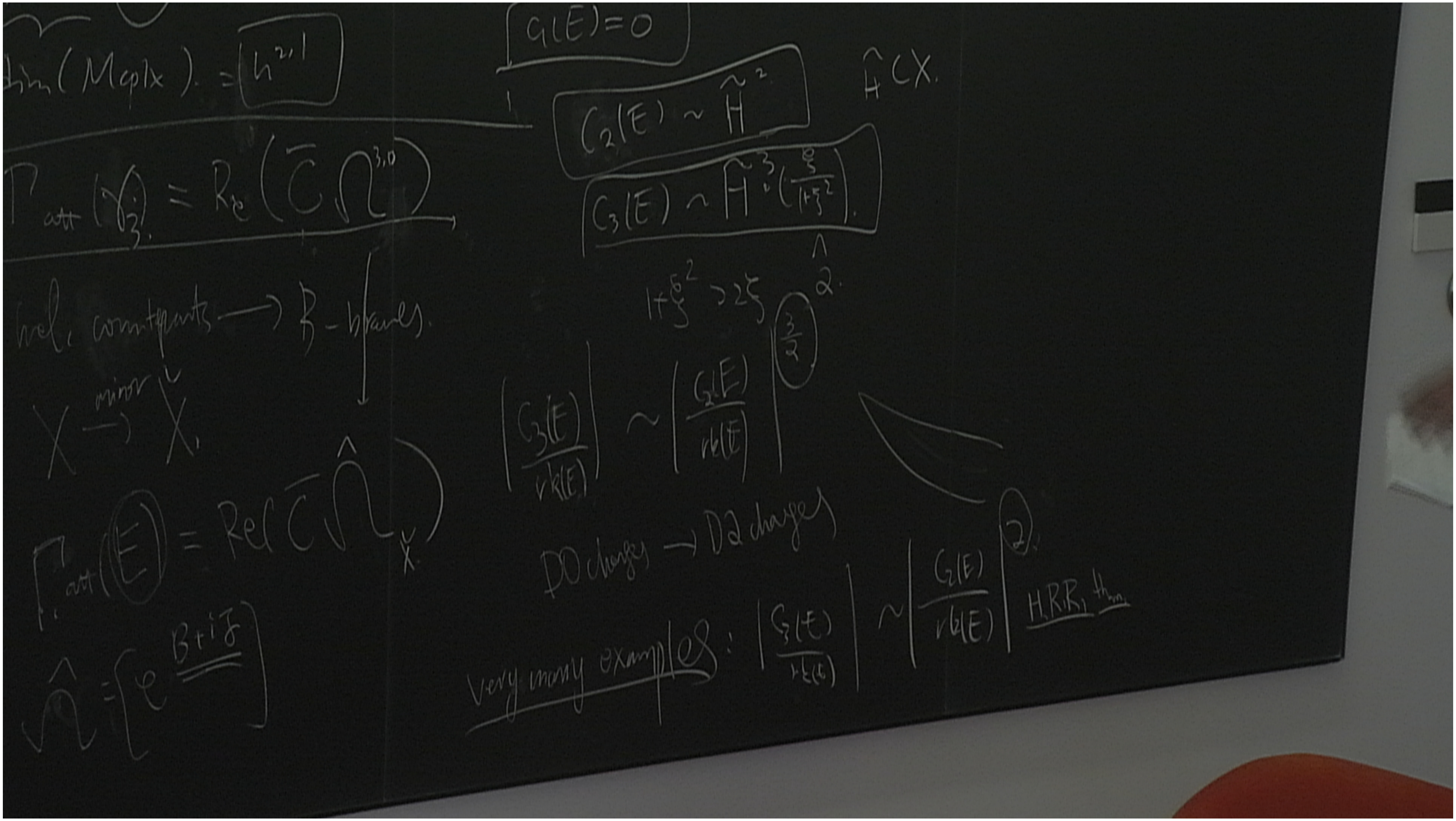

3. DRY Conjecture
attractor mechanism
D3 brane
3 cycles in X
X(t)
R^3
r=∞ → 0
horizon
determined by charges
BPS solutions
|Z| # of points
|Z| = # of D0's
D-D', D2
physics:
(D0-D2) bound state
DT. inv.


3. DRY Conjecture
3 cycles in X :
Hodge decomp.
D3 branes.
attractor mechanism
X(t)
dim (Mcplx) = h2,1
horizon.
determined by charges
BPS solutions :
|z| = # of D0's
D-D', D2
physics :
bound state
f points

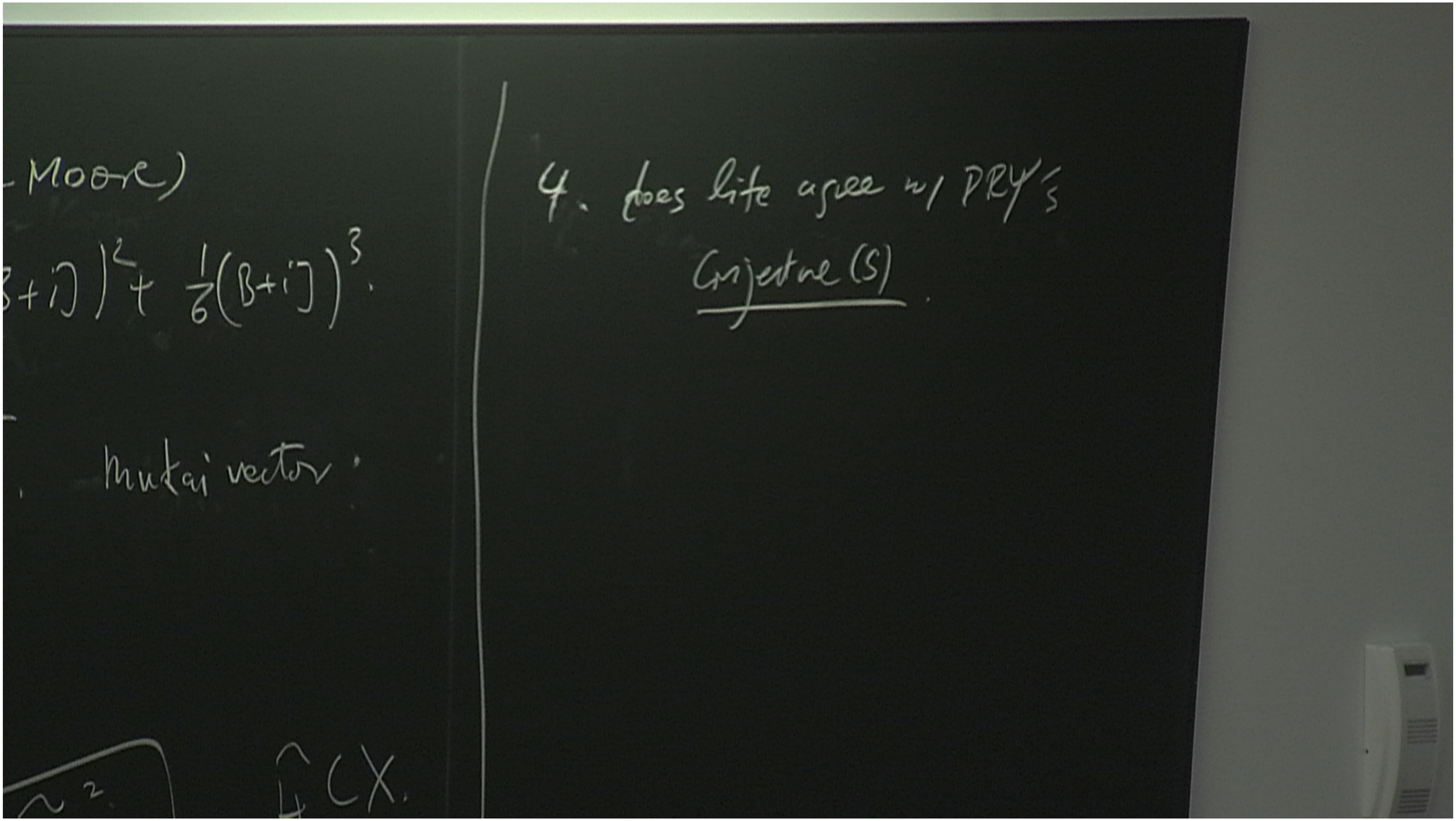

3. DRY Conjecture
attractor mechanism
D3 branes
3 cycles in X
Hodge decomp.
horizon
horizon
f points
# of D0's
physics
bound state
determined by charges
BPS solutions
dim (Mcplx) = h^{2,1}
mirror
-branes

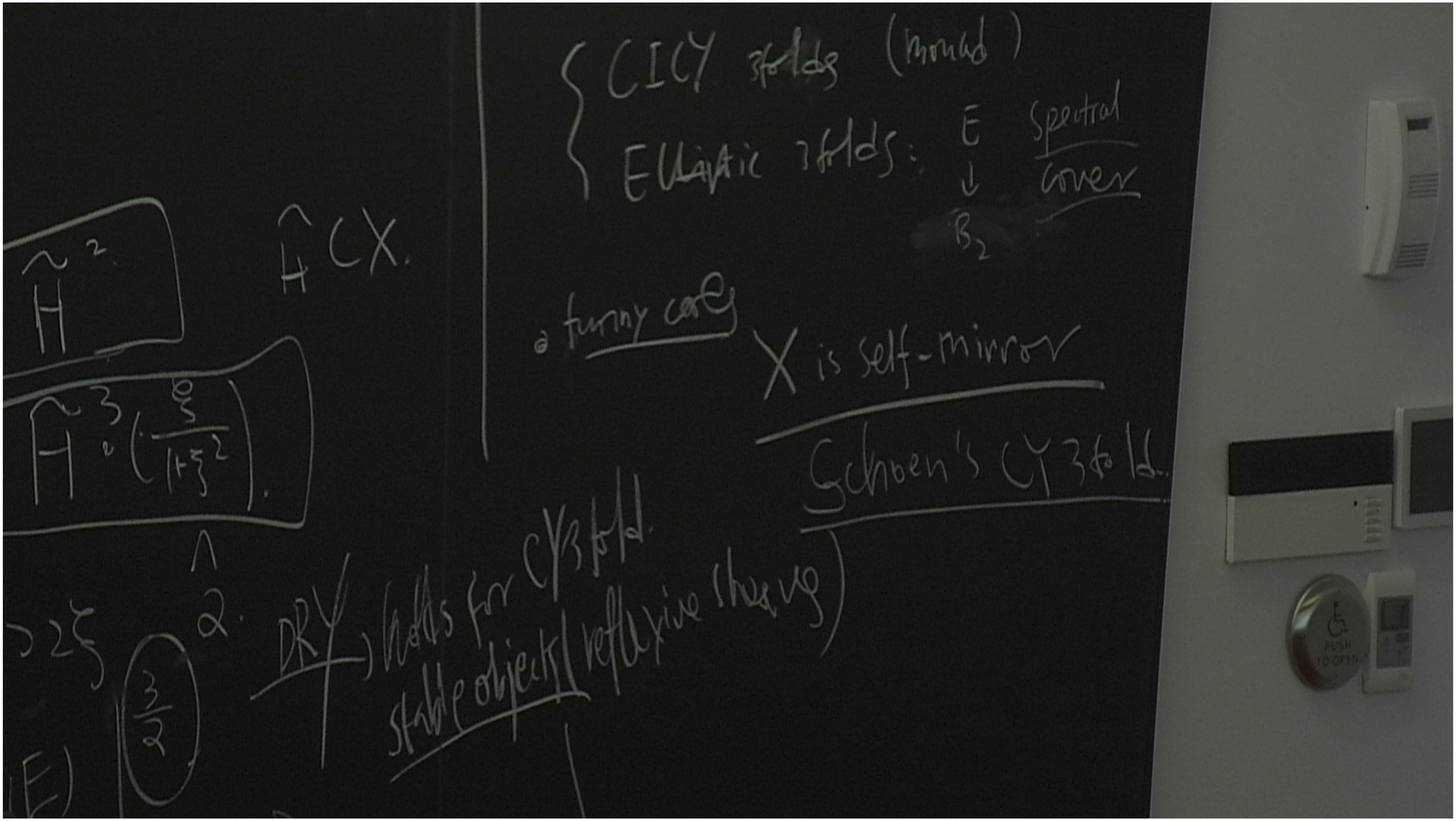

dim (Mcplx). = h^{2,1}
minor

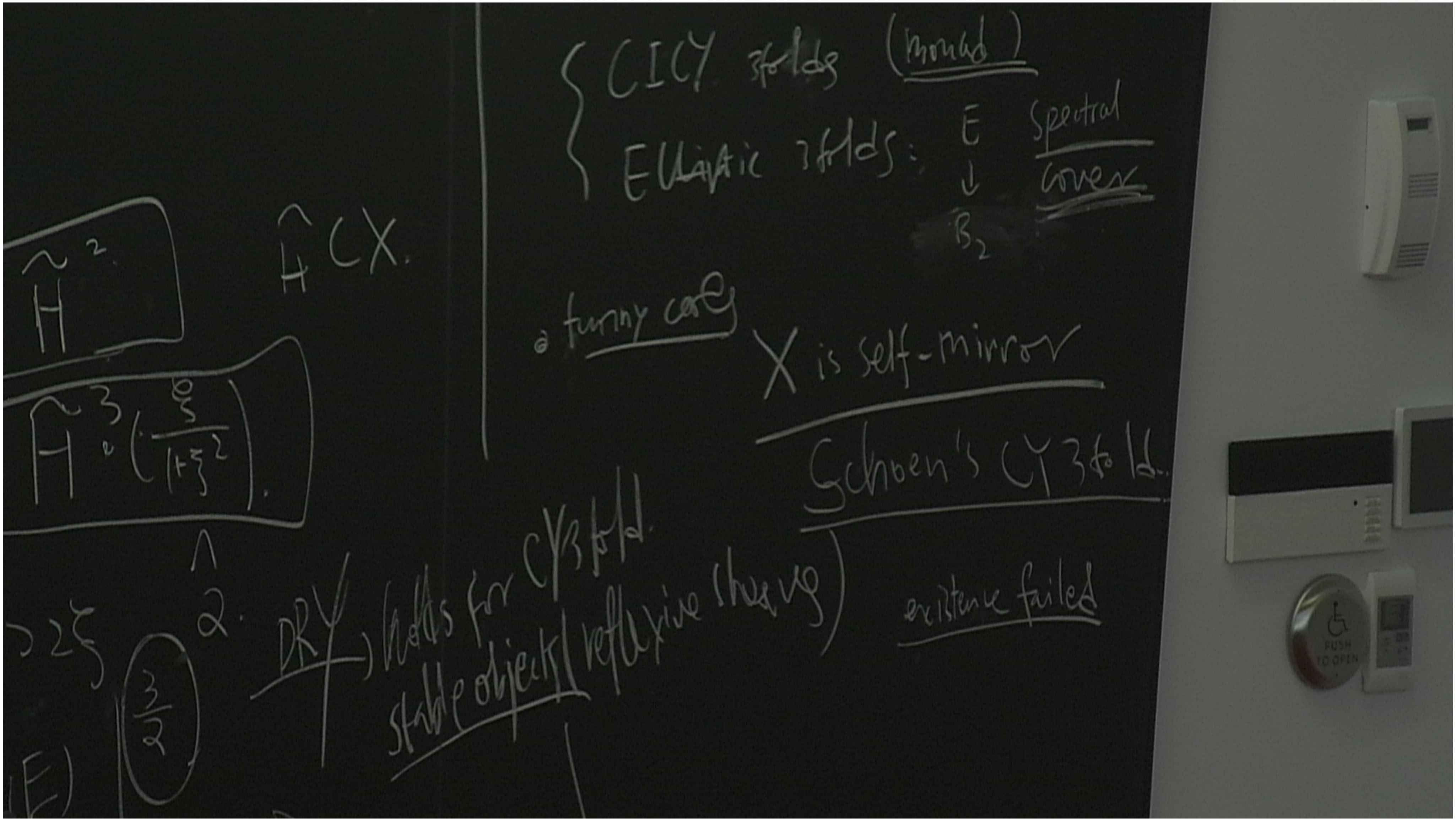

mukai vector
G(E)=0
dim(Mcplx) = h^{2,1}
B-branes
mirror

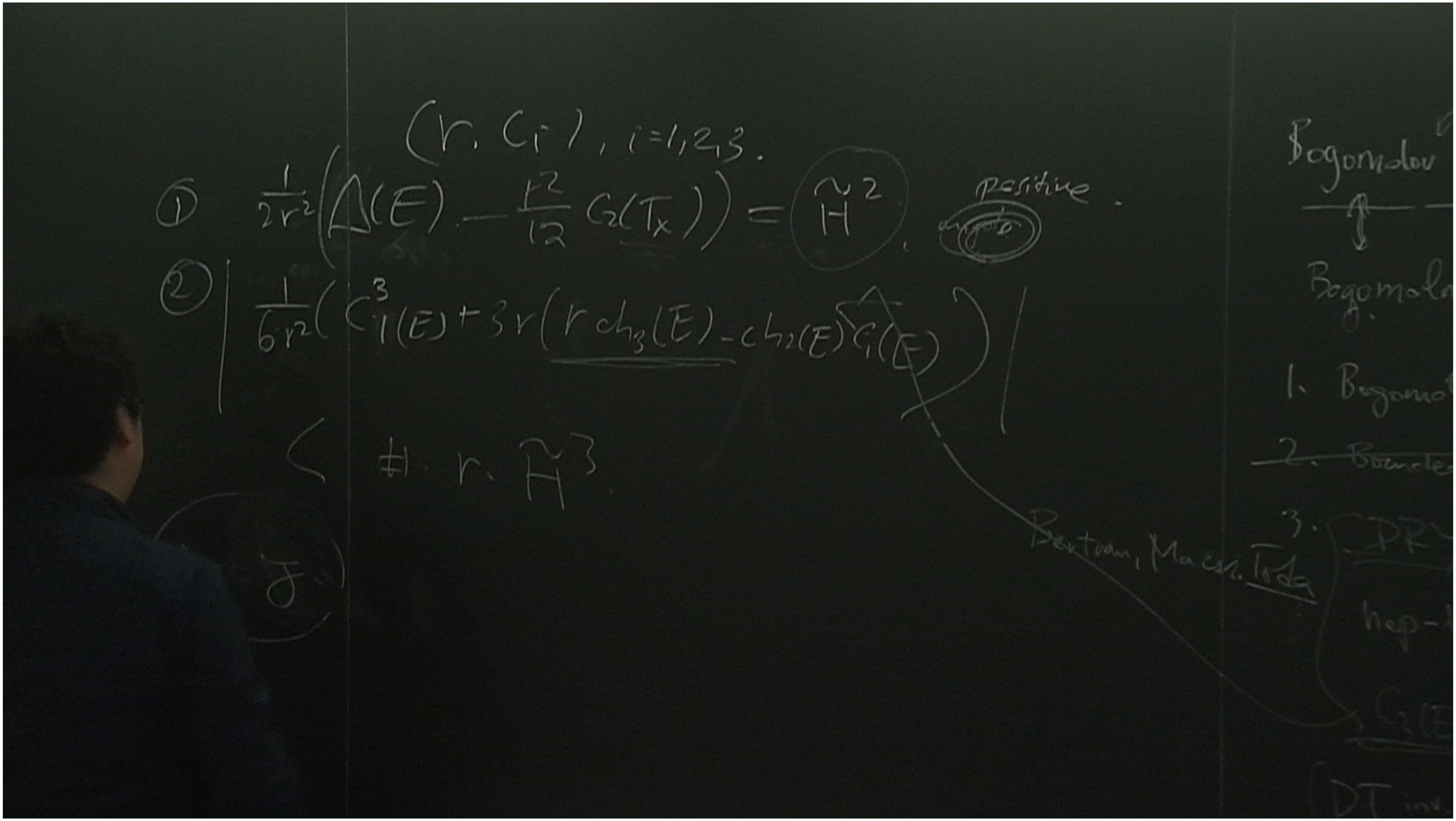


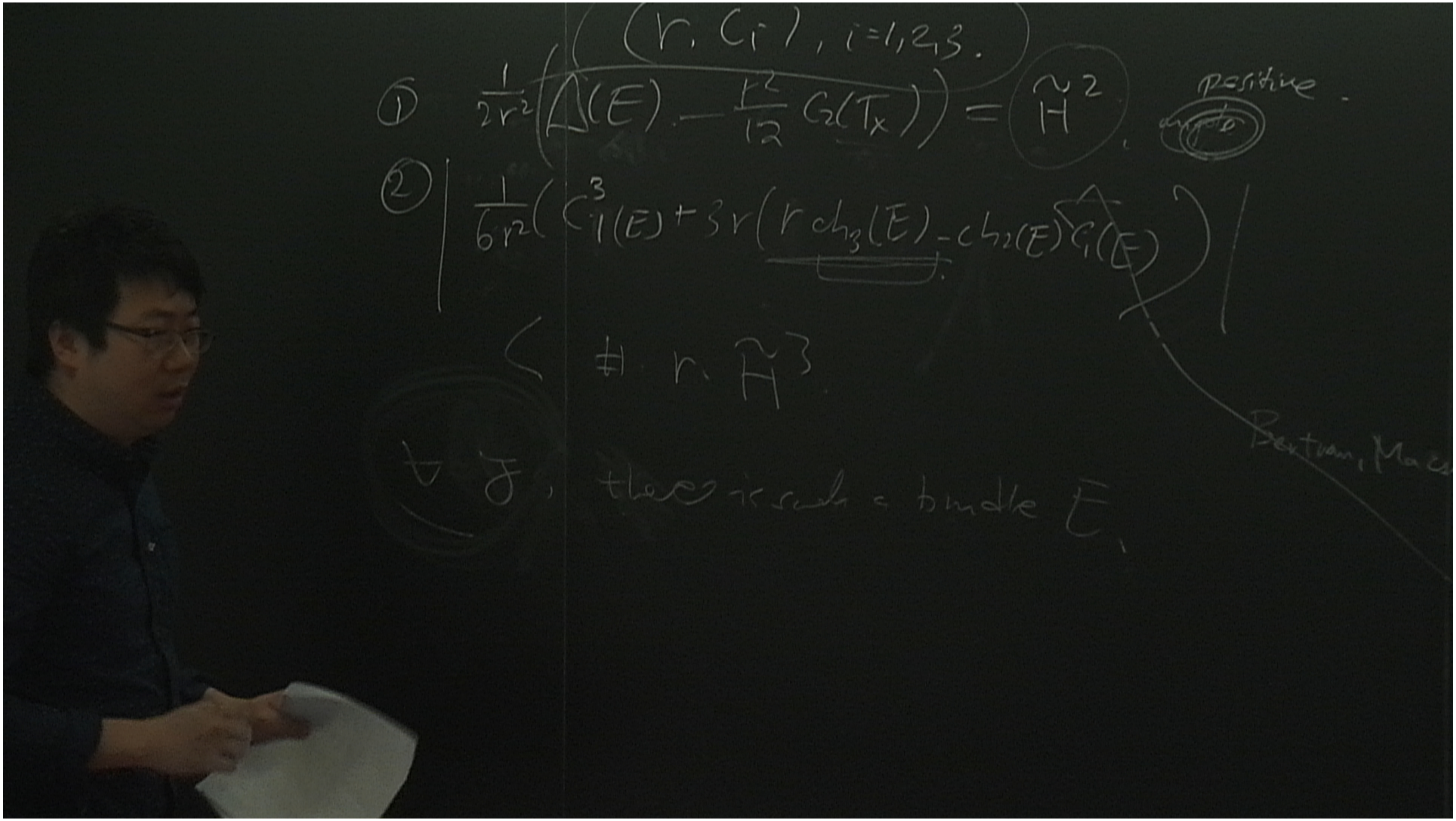
G(E)=0
dim(Mcplx) = h^{2,1}
Ĥ CX.
X —minor→ X̌
D0 charges → D2 charges
very many examples:
H.R.R. thm

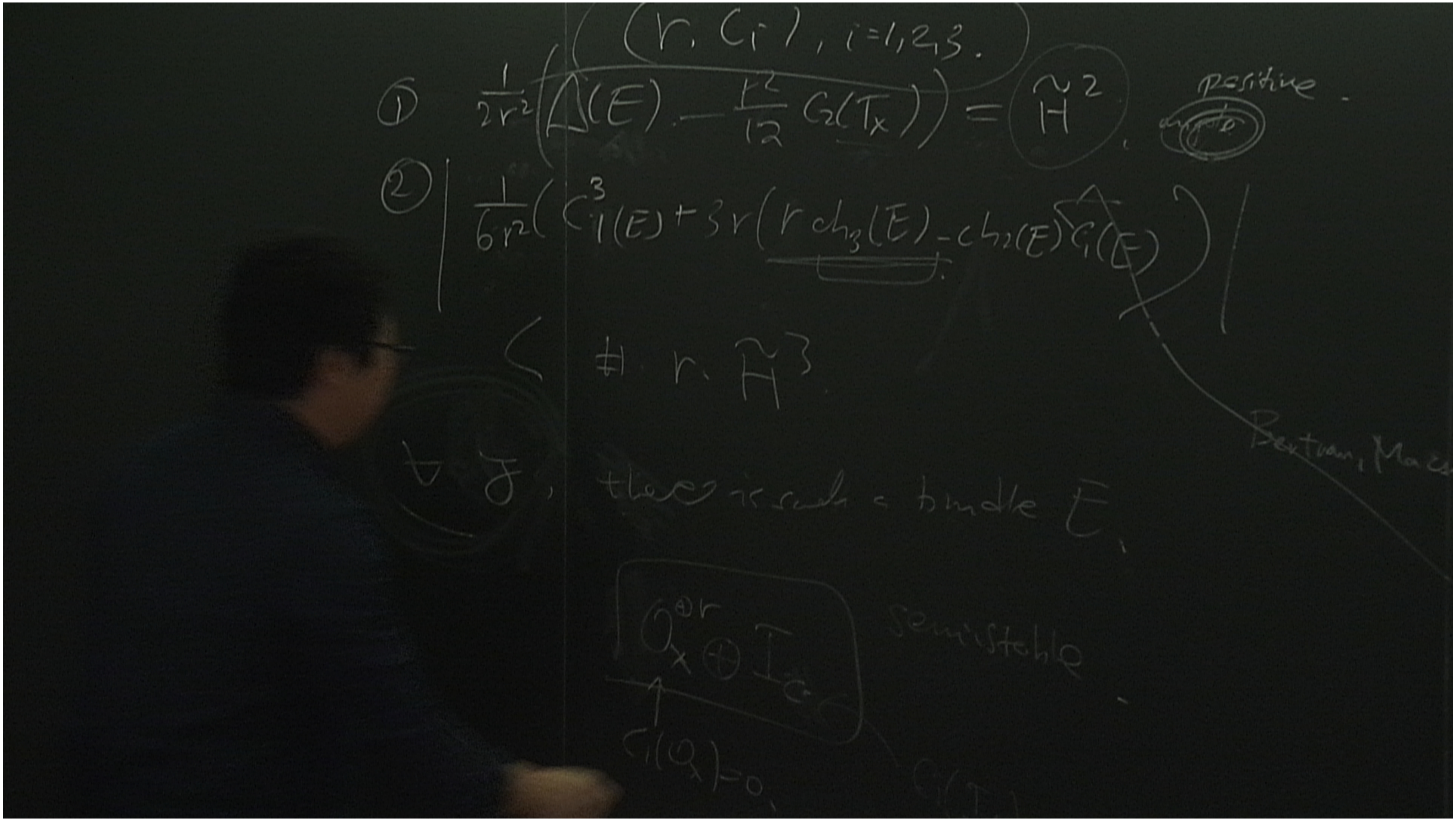

Moore)
4. does life agree w/ PRY's
Conjecture(s).
Mukai vector
CX.

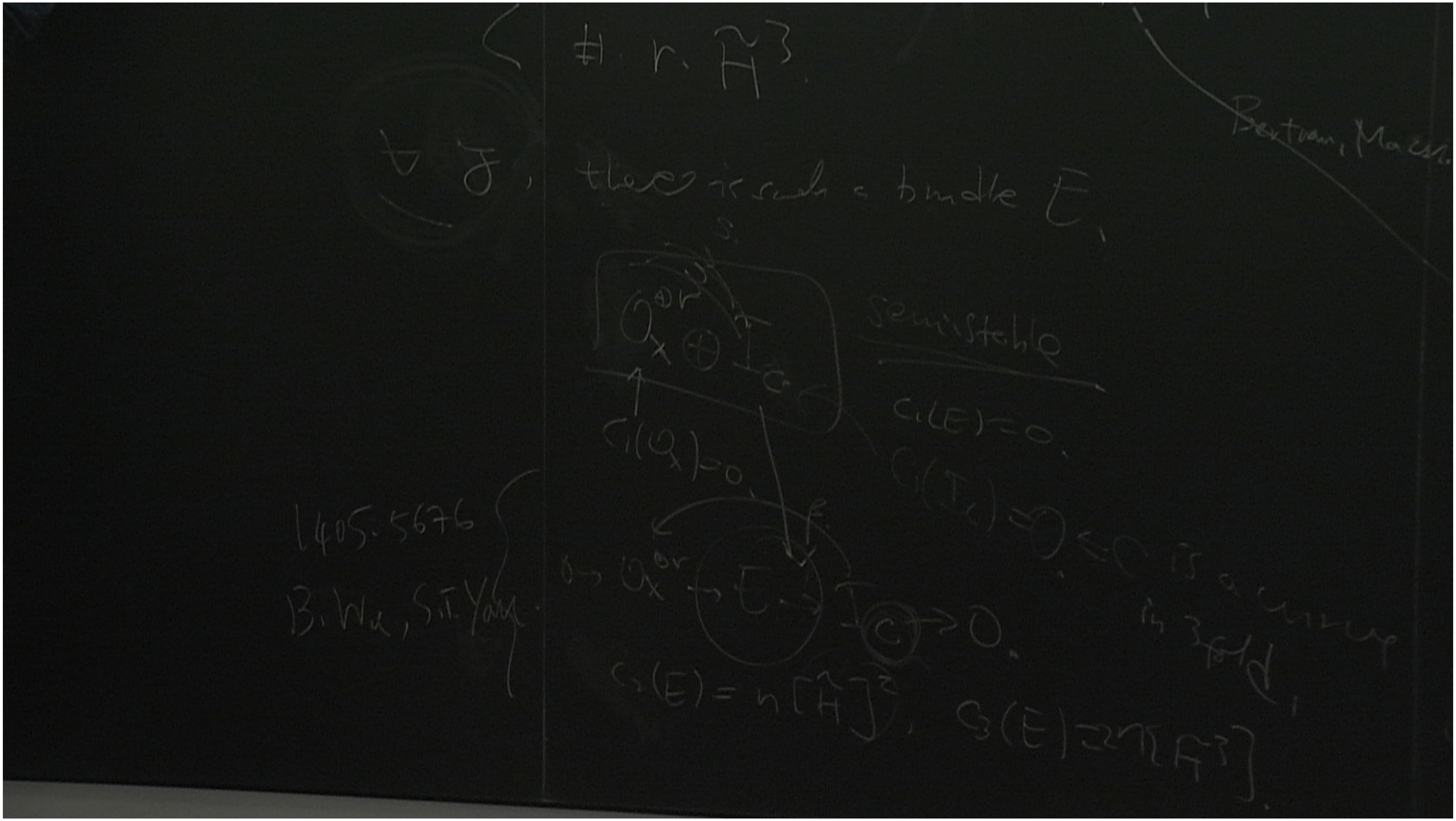

CICY 3folds (monad)
Elliptic 3folds:
spectral cover
funny categ
X is self-mirror
Schoen's CY 3fold
holds for CY3fold
stable objects
reflexive sheaves

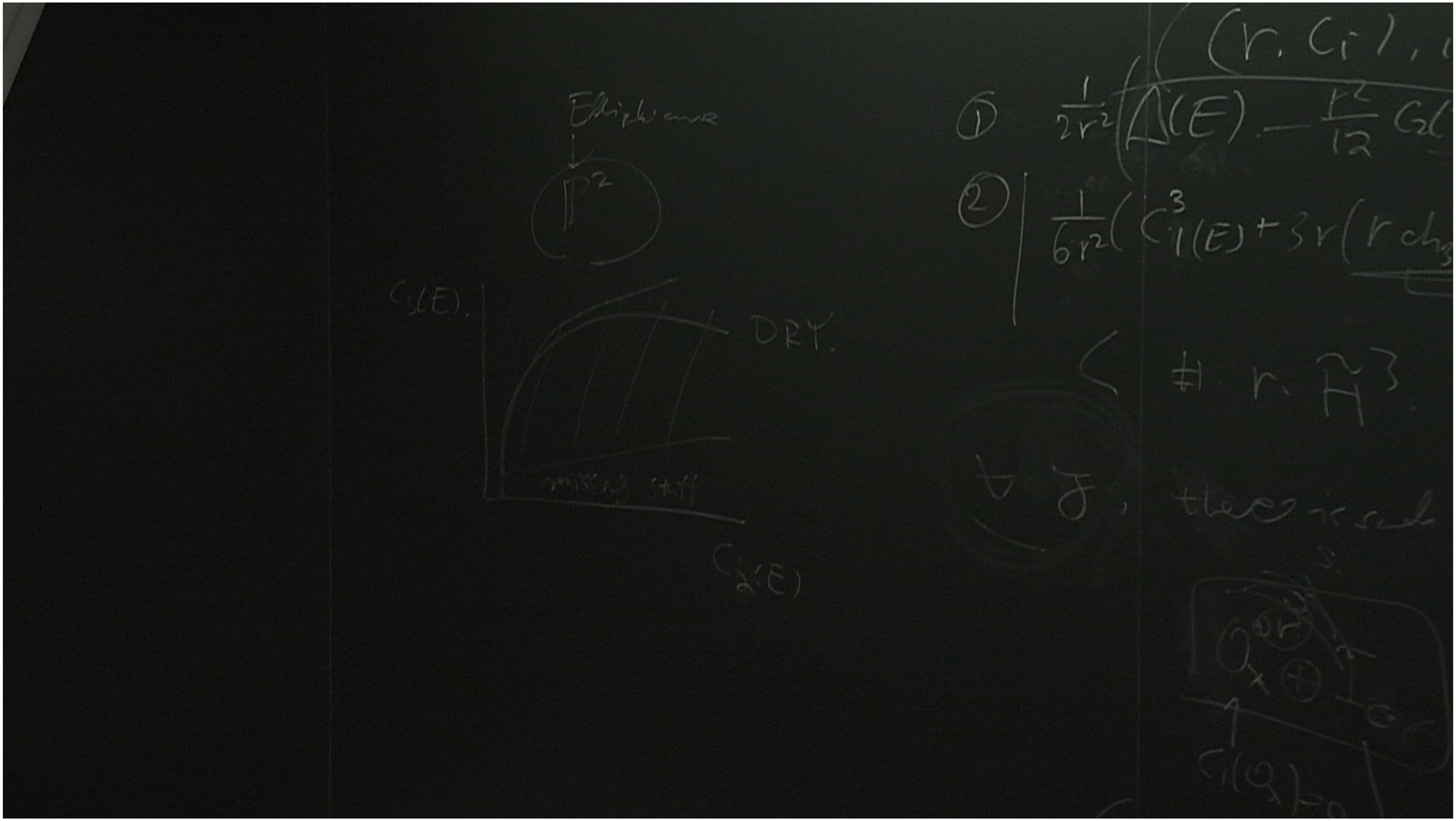

CICY 3folds (monad)
Elliptic 3folds
spectral cover
X is self-mirror
Schoen's CY 3fold
existence failed
holds for CY3fold
stable objects
DRY

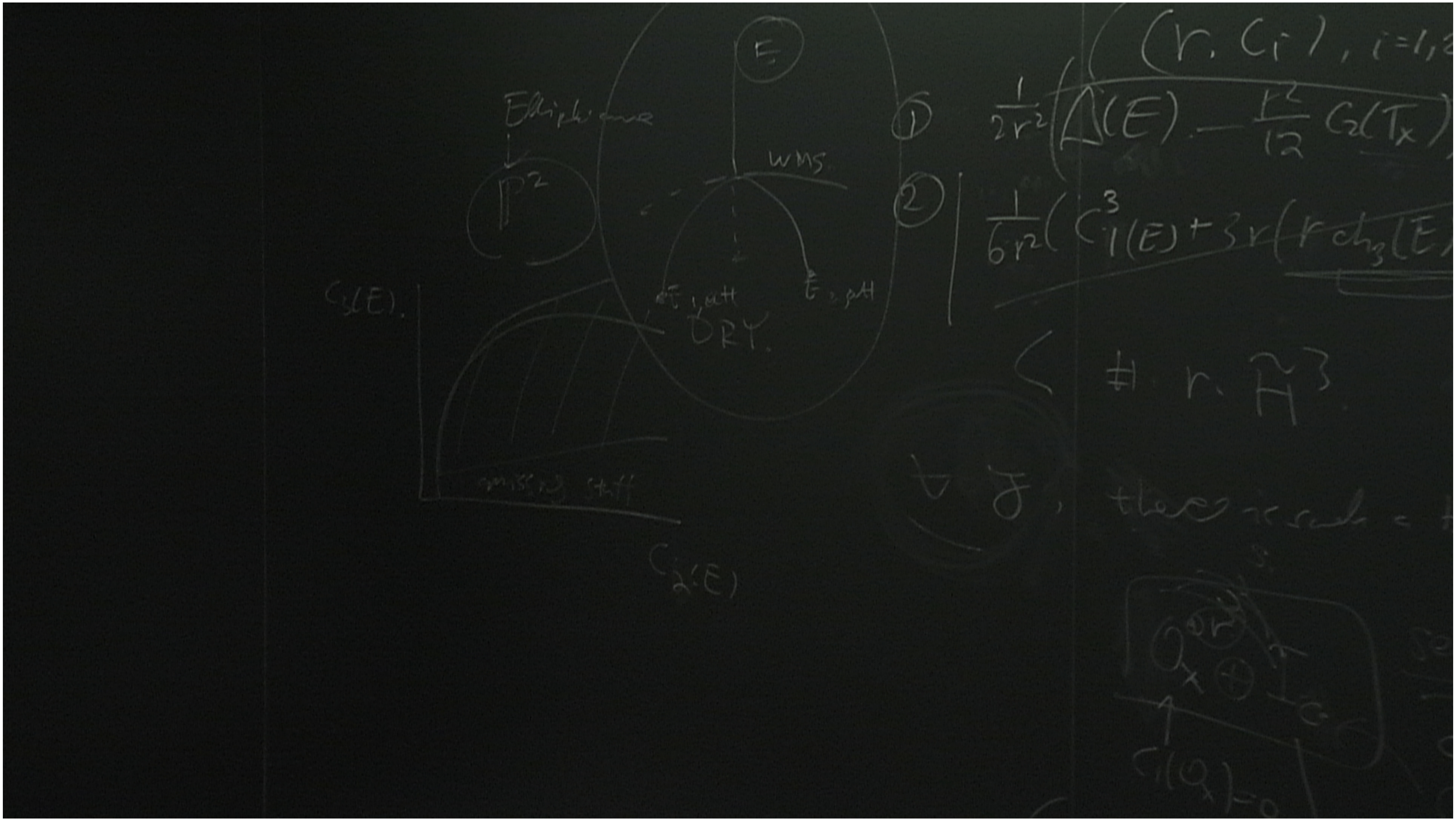

(r, c_i), i=1,2,3.
positive
Bogomolov
Bertram, Macri, Toda
hep-
DT inv

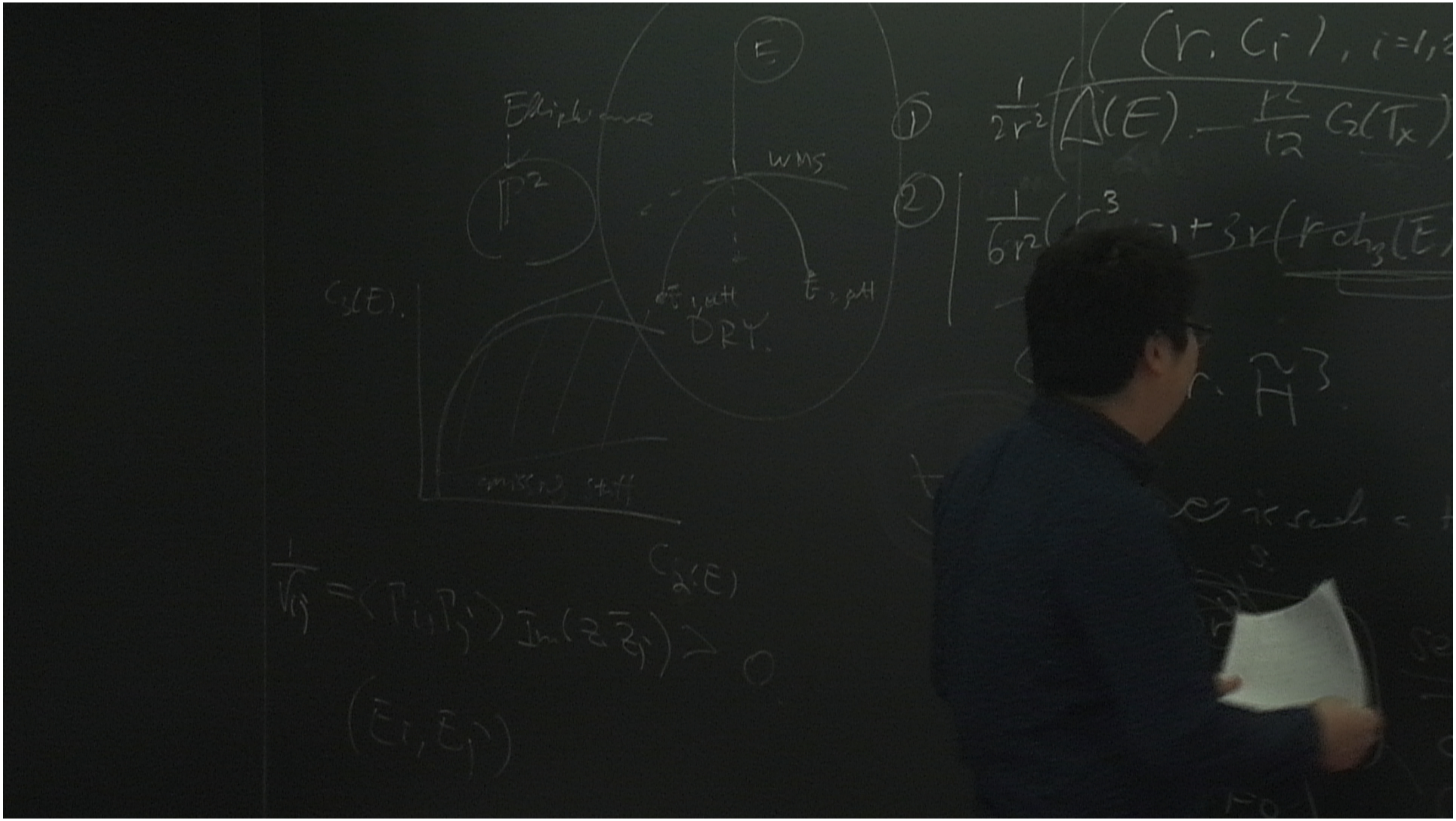

i=1,2,3.
positive

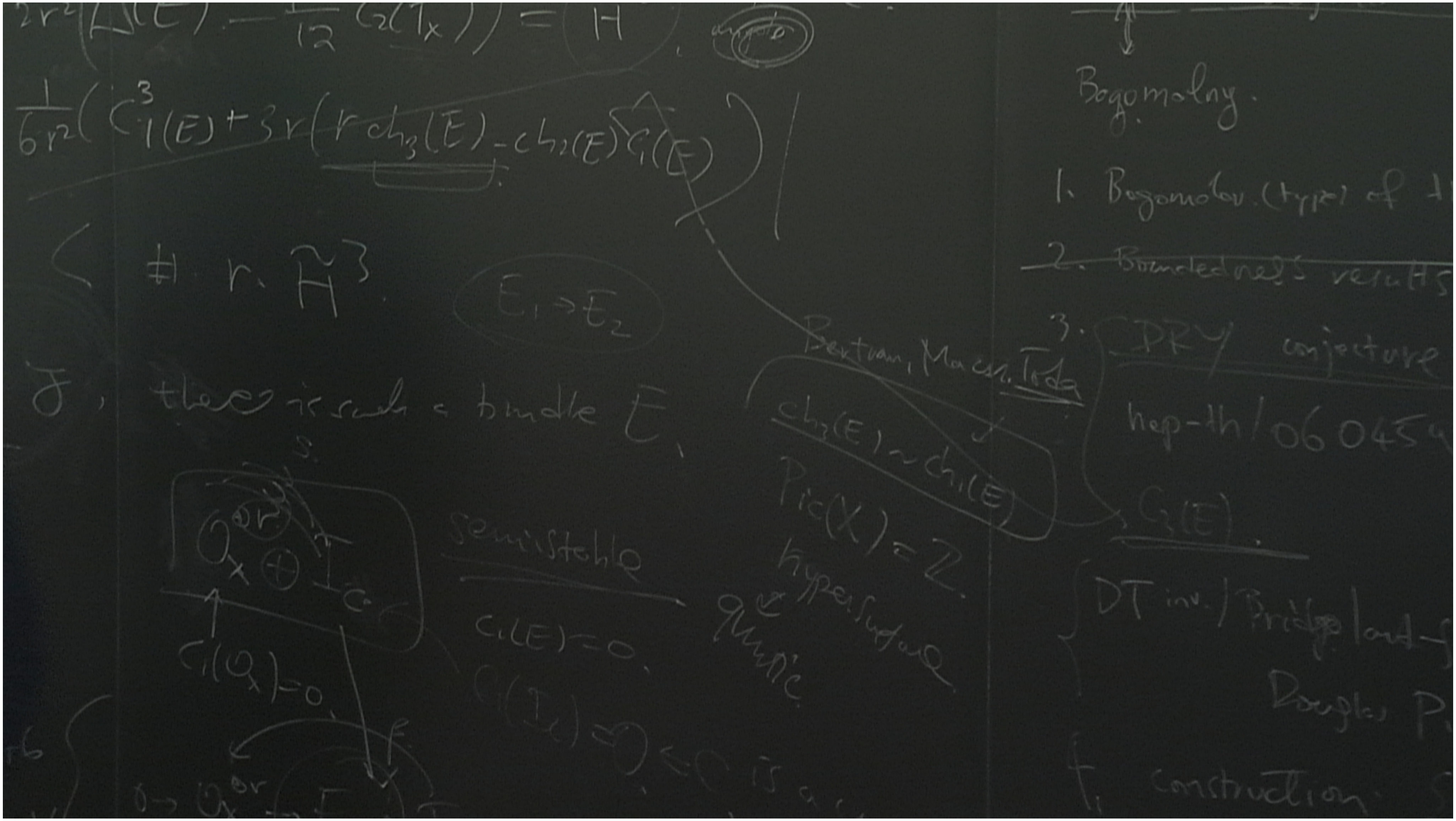

(r, c_i), i=1,2,3.
positive
semistable

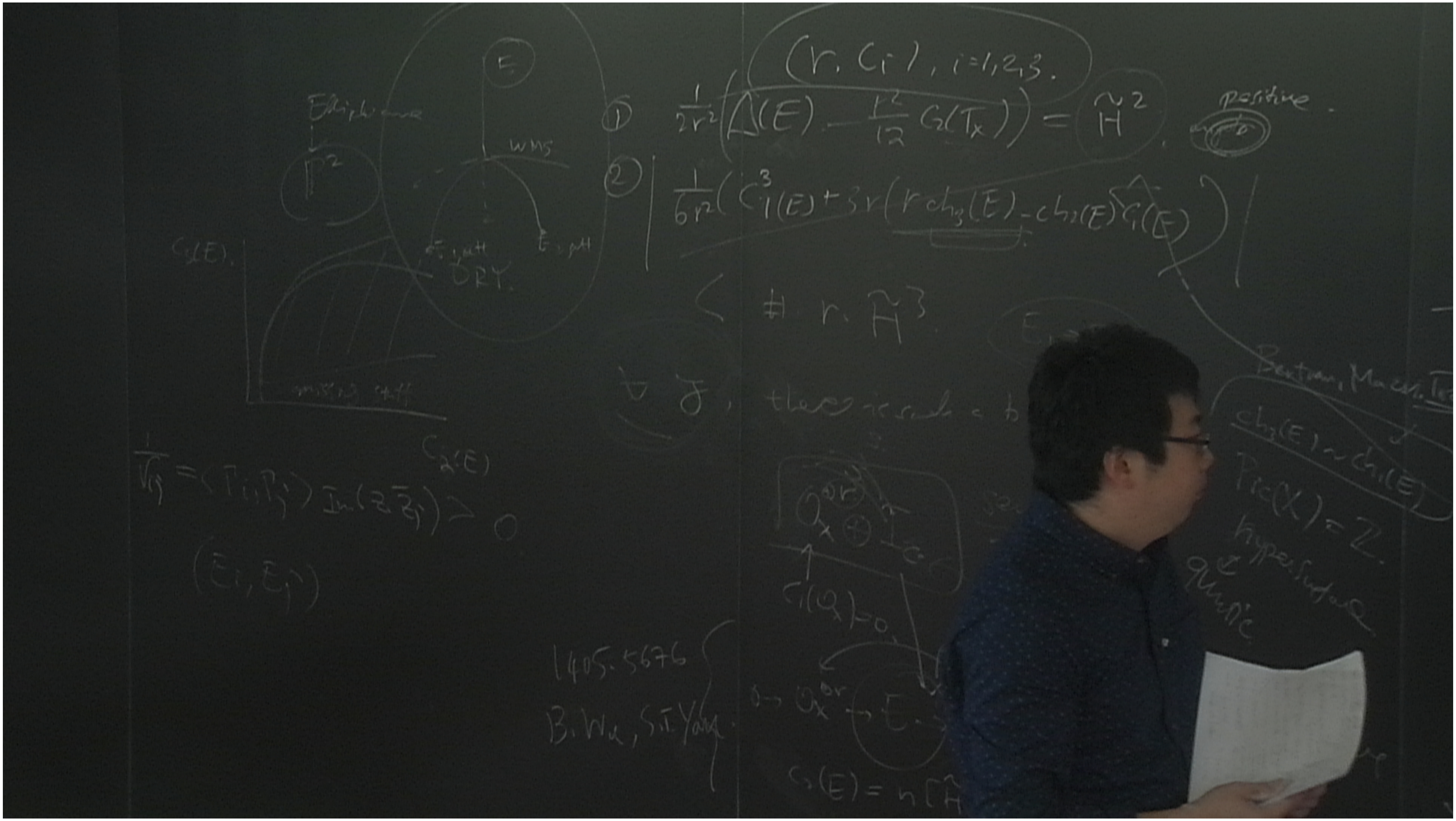

there is such a bundle E
semistable
1405.5676
B. Wu, S.T. Yau
is a curve
in 3fold


Elliptic curve
DRY.
missing stuff






Bogomolny.
1. Bogomolov (type) of
2. Boundedness results
3. CDRY conjecture
hep-th/060454
DT inv. / Bridgeland
Douglas
construction
Bertram, Macri, Toda
semistable
c1(E) = 0.
Pic(X) = Z.
hypersurface
quintic
E1 → E2


wms
BRT
missing stuff
$c_2(E)$
$(r, c_i), i=1,2,3.$
positive
1405.5676
B. Wu, S.T. Yau
$Pic(X) = \mathbb{Z}$
hypersurface
quintic