

Title: 14/15 PSI - Statistical Mechanics - Lecture 1

Date: Oct 06, 2014 10:45 AM

URL: <http://pirsa.org/14100082>

Abstract:

Matter organizes itself in distinct phases.

1. Solid - liquid - gas.

2. Ferromagnet - paramagnet.

3. Metal - insulator.

position

distinct phases. Phases of matter are

distinguished by macroscopic properties

1. Solid has rigidity, while liquid does not.
2. Liquid has an interface, while gas does not.
3. FM has spontaneous magnetization, PM does not.

Fundamental idea: The presence of such macroscopic properties is
a consequence of ordering.

1. Rigidity of solid is a consequence
of ordering of atoms in a periodic crystal structure.



2.

macroscopic properties is

2. Ferromagnetism is a consequence
of ordering of atomic magnetic moments

ture.

In $d > 1$ Ising model exists in 2 phases: FM and PM.

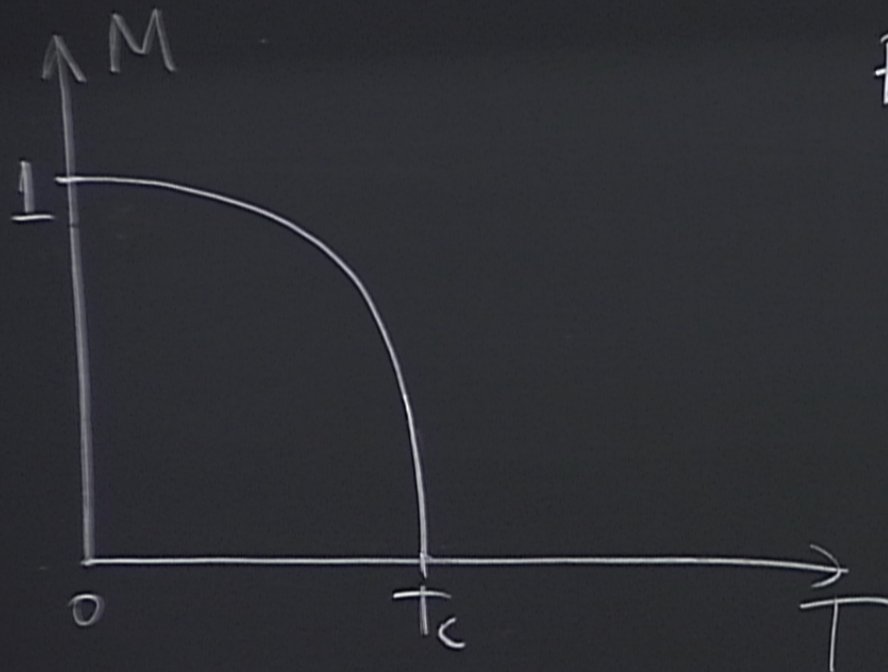
Distinguished by order parameter:

$M \neq 0$ in FM phase

$M = \langle \sigma_i \rangle$ - assuming translational symmetry.

$M = 0$ in PM phase

$$\langle \sigma_i \rangle = \frac{1}{Z} \sum_{\{\sigma_i\}} \sigma_i e^{-\frac{H}{T}}$$



For all i :

$\sigma_i \rightarrow -\sigma_i$ - symmetry
of Ising model.

In FM phase this symmetry is
violated spontaneously. - spontaneously
broken symmetry.

atoms is

paramagnetism is a consequence
of the presence of atomic magnetic moments.



$$H = -\frac{1}{2} \sum_{ij} J_{ij} \sigma_i \sigma_j - B \sum_i \sigma_i$$

$B > 0$ - prefers $\sigma_i = 1 \Rightarrow$ violates the $\sigma_i \rightarrow -\sigma_i$ symmetry.

N - # of lattice sites.

$$H = -\frac{1}{2} \sum_{ij} J_{ij} \sigma_i \sigma_j - BNM$$

$$\frac{P_{M<0}}{P_{M>0}} = \frac{e^{-\frac{NB|M|}{T}}}{e^{\frac{NB|M|}{T}}} = e^{-\frac{2NB|M|}{T}}, \quad \lim_{B \rightarrow 0} \lim_{N \rightarrow \infty} \frac{P_{M<0}}{P_{M>0}} = 0$$

Keep N fixed, while sending $B \rightarrow 0$

$$\lim_{N \rightarrow \infty} \lim_{B \rightarrow 0} \frac{P_{M<0}}{P_{M>0}} = 1 \Rightarrow M=0$$

$M \neq 0$

Phases and phase transitions are well-defined only in the limit $N \rightarrow \infty$ - thermodynamic limit.

$$H = -\frac{1}{2} \sum_{ij} J_{ij} \sigma_i \sigma_j - B \sum_i \sigma_i$$

$B > 0$ - prefers $\sigma_i = 1 \Rightarrow$ violates the $\sigma_i \rightarrow -\sigma_i$ symmetry.

N - # of lattice sites.

$$H = -\frac{1}{2} \sum_{ij} J_{ij} \sigma_i \sigma_j - BNM$$

$$Z = \sum_{\{\sigma_i\}} e^{-\frac{H}{T}}$$

$$\frac{P_{M < 0}}{P_{M > 0}} =$$

Keep N fixed

$$\lim_{N \rightarrow \infty} \lim_{B \rightarrow 0} \frac{P_{M < 0}}{P_{M > 0}}$$

- mean field theory (MFT)

$$M = (\sigma_i - M) \cdot (\sigma_j - M) + M(\sigma_i - M + \sigma_j - M) + M^2 \approx$$

not
null

$$\approx M(\sigma_i + \sigma_j) - 2M^2 + M^2 = M(\sigma_i + \sigma_j) - M^2$$

Simplest description of phase transition - mean field theory

$$\sigma_i = \sigma_i - \langle \sigma_i \rangle + \langle \sigma_i \rangle = \underbrace{(\sigma_i - M)}_{\substack{\uparrow \\ \text{assume that} \\ \text{this is small}}} + M = (\sigma_i - M) \cdot (\sigma_j - M)$$

$$\sigma_i \sigma_j = (\sigma_i - M + M) \cdot (\sigma_j - M + M) =$$

$$\approx M (\sigma_i + \sigma_j)$$

$$H = -\frac{1}{2} \sum_{ij} J_{ij} M (\sigma_i + \sigma_j) + \frac{1}{2} \sum_{ij} J_{ij} M^2$$

$$J = \sum_j J_{ij} \text{ - assume } J \text{ is finite}$$

$$H = -JM \sum_i \sigma_i + \frac{1}{2} NJM^2$$

$$\begin{aligned}
 Z &= \sum_{\{\sigma_i\}} e^{-\frac{H}{T}} = \sum_{\{\sigma_i\}} e^{\frac{JM}{T} \sum_i \sigma_i} e^{-\frac{NJM^2}{2T}} \\
 &= e^{-\frac{NJM^2}{2T}} \sum_{\sigma_i = \pm 1}^N e^{\frac{JM}{T} \sigma_i} = e^{-\frac{NJM^2}{2T}} \left[e^{\frac{JM}{T}} + e^{-\frac{JM}{T}} \right]^N \\
 &= e^{-\frac{NJM^2}{2T}} \left[2 \cosh\left(\frac{JM}{T}\right) \right]^N
 \end{aligned}$$

Helmholtz free energy:

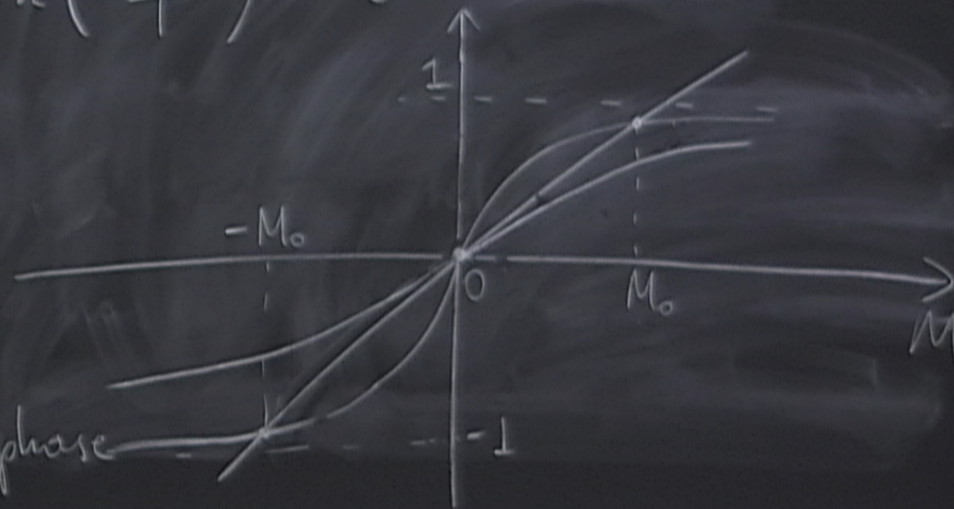
$$F = -T \ln Z = \frac{NJM^2}{2} - TN \ln \left[2 \cosh \left(\frac{JM}{T} \right) \right]$$

$$\frac{\partial F}{\partial M} = NJM - NJ \tanh\left(\frac{JM}{T}\right) = 0$$

$$M = \tanh\left(\frac{JM}{T}\right)$$

$$\tanh\left(\frac{JM}{T}\right) \approx \frac{JM}{T}$$

$\frac{J}{T} > 1$ - have multiple solutions - FM phase



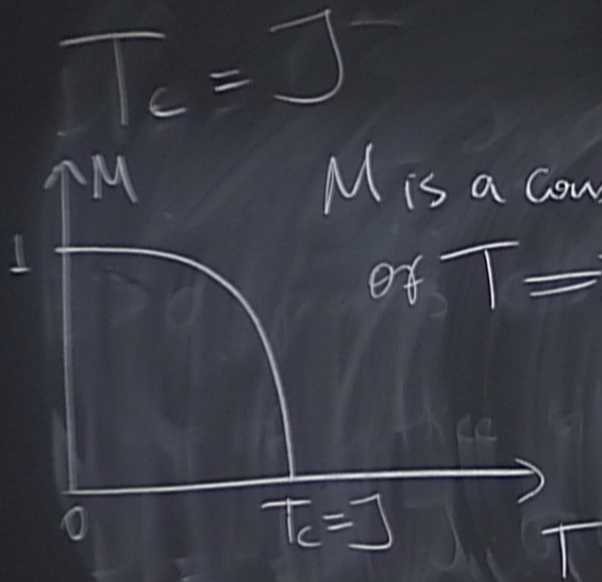
Helmholtz free energy:

$$F = -T \ln Z = \frac{NJM^2}{2} - TN \ln \left[2 \cosh \left(\frac{JM}{T} \right) \right]; \quad \frac{\partial F}{\partial M} = \dots$$

Minimize F as a function of M .

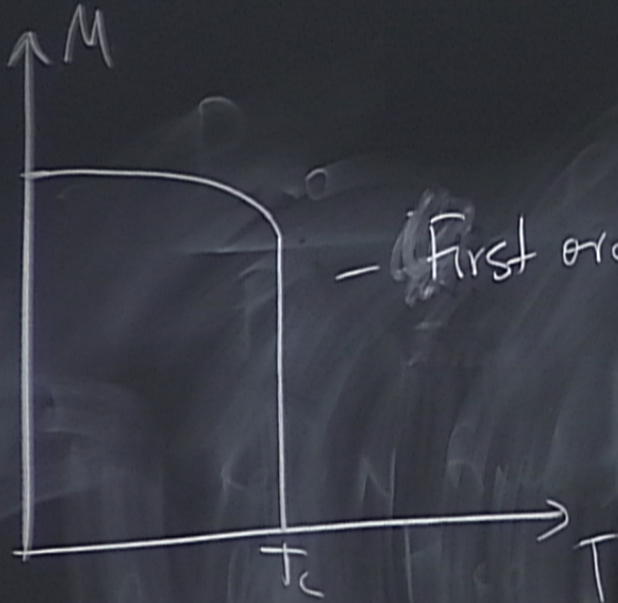
$\frac{J}{T} < 1$ - only $M=0$ is the solution - PM phase

$\frac{J}{T} > 1$ - have magnetization
 $\tanh \left(\frac{JM}{T} \right) \approx \left(\frac{J}{T} \right) M$



M is a continuous function
of $T \Rightarrow$ continuous (second order)
phase transition.





- First order (discontinuous) transition