Title: Flux Compactifications Grow Lumps

Date: Oct 07, 2014 11:00 AM

URL: http://pirsa.org/14100081

Abstract: The simplest flux compactifications are highly symmetricâ€”a q-form flux is wrapped uniformly around an extra-dimensional q-sphere. I will discuss a family of solutions that break the internal SO(q+1) symmetry of these solutions down to SO(q)Ã—Z_2, and show that often at least one of them has lower vacuum energy, larger entropy, and is more stable than the symmetric solution. I will describe the phase diagram of lumpy solutions and provide an interpretation in terms of an effective potential. Finally, I will provide evidence that the perturbatively stable vacua have a non-perturbative instability to spontaneously sprout lumps; generically this new decay is exponentially faster than all other known decays of the model.

Flux Compactifications Grow Lumps

Claire Zukowski
U.C. Berkeley

Alex Dahlen and CZ [1404.5979]
Kurt Hinterbichler, Janna Levin and CZ [1310.6353]

Perimeter cosmology seminar, October 7, 2014

Outline

- The Toy Model
- Symmetric Solutions
    - Properties
    - Stability
- Warped Solutions
- Effective Potential Description
- Non-Perturbative Decays
- Future Work

The Toy Model Freund, Rubin (1980)

$(p \geq 3,\ q \geq 2)$

$$S = \int d^D x \sqrt{-g} \left[\frac{1}{2} M_D^{D-2} \mathcal{R} - \frac{1}{2q!} F_q^2 - \Lambda_D \right]$$

- q-dimensional internal manifold wrapped by a q-form flux F_q
- $p = D - q$ extended dimensions

The Toy Model: Motivations

"Simplest model of stabilized extra dimensions"
(Repulsive flux + attractive curvature can form stable minimum)

• One of several toy models often invoked to gain intuition about the much more complicated string landscape

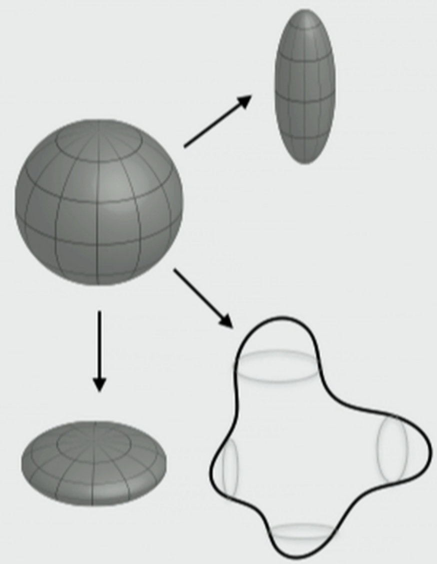
• We should fully study this simple model, then try to understand if its physical implications carry over to the more general case

• An extremely simple physical system (around for > 50 years!), with some rich and tractable corners of phase space that remain to be understood!

The Toy Model

The take-home message:

Even this "simplest" model is actually far from simple. There are symmetry-breaking effects whereby a symmetrically shaped internal manifold generically develops lumps, either perturbatively or non-perturbatively.



Symmetric Solution

$$S = \int d^D x \sqrt{-g} \left[\frac{1}{2} M_D^{D-2} \mathcal{R} - \frac{1}{2q!} F_q^2 - \Lambda_D \right]$$

Direct product total space $\Rightarrow$ both components are Einstein spaces

$$\boxed{\mathcal{M}_p \times S^q}$$

"Freund-Rubin model"

Symmetric Solution

Metric: $$ds^2 = L^2 ds_p^2 + R^2 d\Omega_q^2$$

Effective c.c.: $$\frac{\Lambda_{\text{eff}}}{M_p^{p-2}} = \frac{(p-1)(p-2)}{2L^2}$$

Conserved flux: $$n \equiv \int_{S^q} F_q = \rho \, \text{Vol}_{S^q}$$

flux number
(quantized)

flux density

Symmetric Solution: Effective Potential

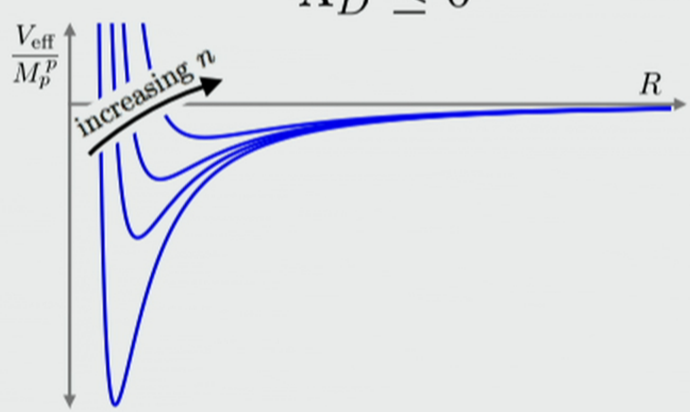
Treat sphere as fixed shape and vary radius as field R:

$$\frac{V_{\text{eff}}(R)}{M_p^p} \sim \left(\frac{1}{R}\right)^{2q/(p-2)} \left[\frac{n^2}{R^{2q}} - \frac{1}{R^2} + \Lambda_D\right]$$

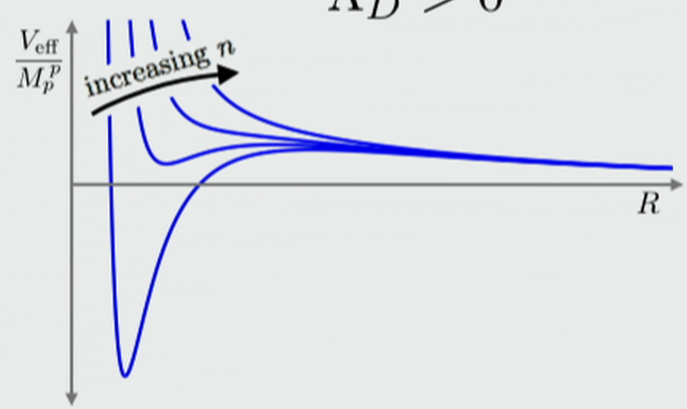
conversion from D- to p-dimensional Planck mass

flux (repulsive) **curvature** (attractive) **c.c.** (repulsive)

$\Lambda_D \leq 0$



$\Lambda_D > 0$



DeWolfe, Freedman, Gubser, Horowitz, Mitra (2002)
Bousso, DeWolfe, Myers (2003)
Hinterbichler, Levin, CZ (2013)

Symmetric Solution: Stability

Metric: $g_{MN} + h_{MN}$

Flux: $\underbrace{A_{\alpha_1 \ldots \alpha_{q-1}}}_{\text{Background}} + \underbrace{\delta A_{\alpha_1 \ldots \alpha_{q-1}}}_{\text{Fluctuation}}$

The procedure:

- Expand fluctuations in complete basis of forms
- Expand action to quadratic order
- Integrate over internal manifold
- Diagonalize, then read off spectrum and stability

Shape mode sector: Expand in spherical harmonics $Y_{lm}(\theta, \phi)$

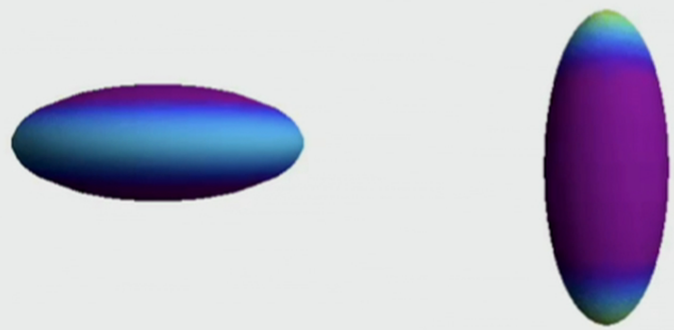
Symmetric Solution: Stability

h_ℓ and a_ℓ mix! $\rightarrow$ physical shape mode is linear combination $\psi_{\pm}^{(\ell)}$

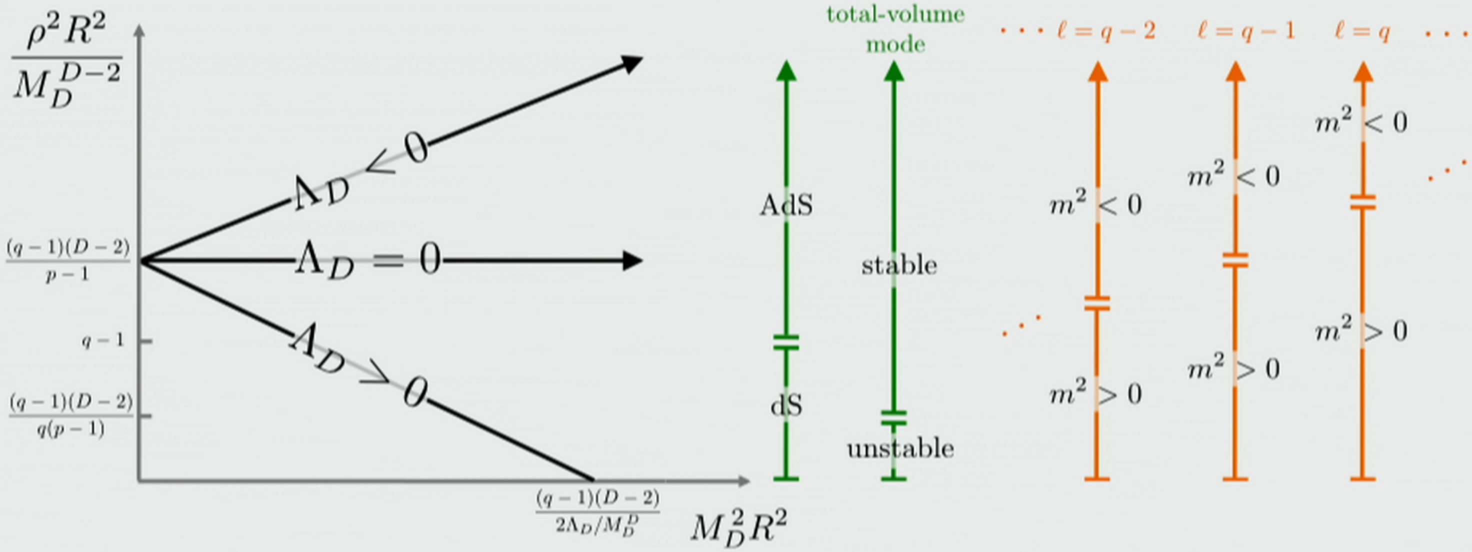
$\psi_{-}^{(\ell)}$ is the "danger mode" that sometimes has negative mass squared

$\Rightarrow$ **shape mode instability**

For $\psi_{-}^{(\ell)}$, flux density larger where radius is larger
(opposite for $\psi_{+}^{(\ell)}$)



Stability



Danger mode has negative mass squared iff:

$$\rho^2 R^2 = \frac{1}{p-1}\left[(q-1)(D-2) - 2\Lambda_D R^2\right] > \frac{D-2}{2(p-1)(q-2)}\Big[\ell(\ell+q-1) - 2(q-1)\Big]$$

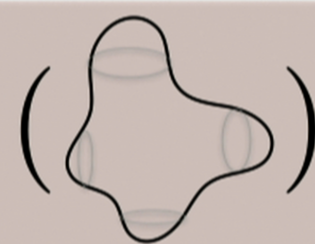
Warped Solutions

Kinoshita (2007)
Kinoshita, Mukohyama (2012)

$$S = \int d^D x \sqrt{-g} \left[\frac{1}{2} M_D^{D-2} \mathcal{R} - \frac{1}{2q!} F_q^2 - \Lambda_D \right]$$

A new set of lumpy, warped solutions can be found numerically for $q \geq 3$!

$$\mathcal{M}_p \times_{\text{warped}}$$



Let's explore their physics...

Warped Solutions

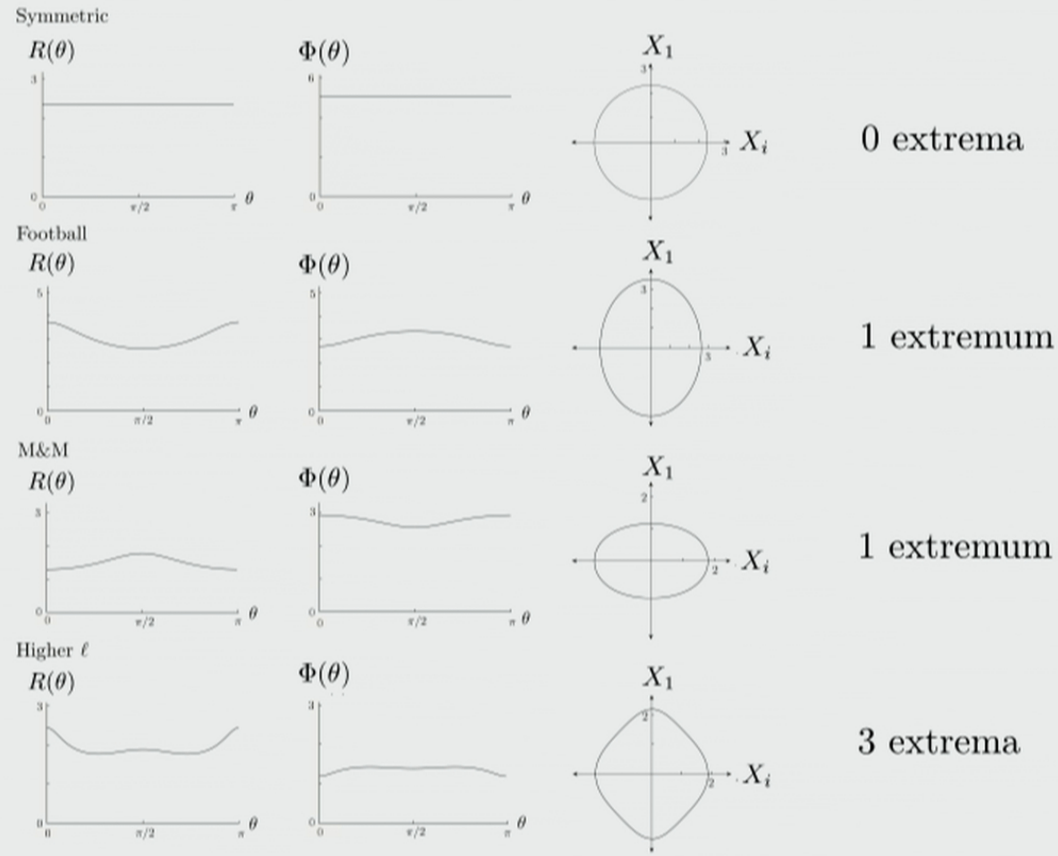
Metric: $$ds^2 = \Phi(\theta)^2 ds_p^2 + R(\theta)^2 d\Omega_q^2$$

Flux ansatz: $$F_{\alpha_1 \cdots \alpha_q} = Q\Phi^{-p}(\theta)\epsilon_{\alpha_1 \cdots \alpha_q}$$

To find these solutions, shoot numerically from the equator varying Q, Λ_D and imposing regularity at poles, evenness about equator

$$\mathrm{SO}(q+1) \to \mathrm{SO}(q) \times \mathbb{Z}_2$$

Warped Solutions: Sample Solutions



Warped Solutions: Game Plan

- Consider the number of extrema as a classifier.

- For simplicity, we restrict to the 1 extremum solutions.

- To study these solutions, we use numerical techniques to map out their phase space. Later, we will return to motivate this picture using an effective potential description.

- First we parametrize the solutions using a convenient order parameter, the ellipticity.

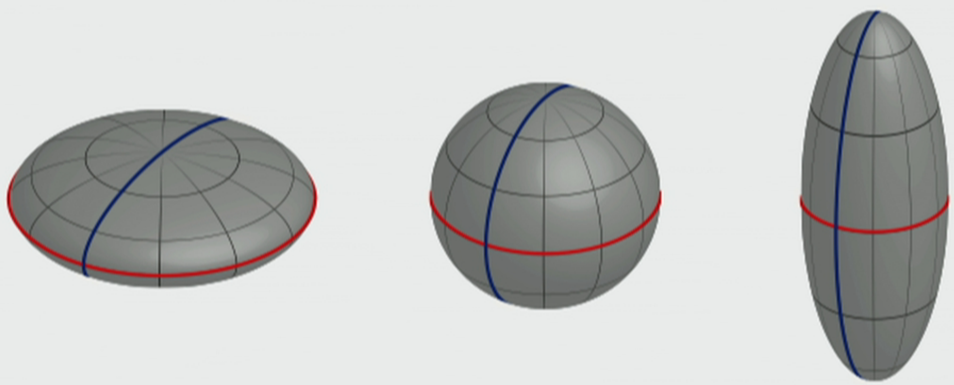
Warped Solutions: Ellipsoidal Case

(1 extremum)

Order parameter: Ellipticity

$$\varepsilon \equiv \frac{2 \times (\text{distance from north to south pole})}{\text{distance around equator}} = \frac{2\int_0^\pi R(\theta)d\theta}{2\pi R(\theta = \pi/2)}$$

M&M | Symmetric | Football



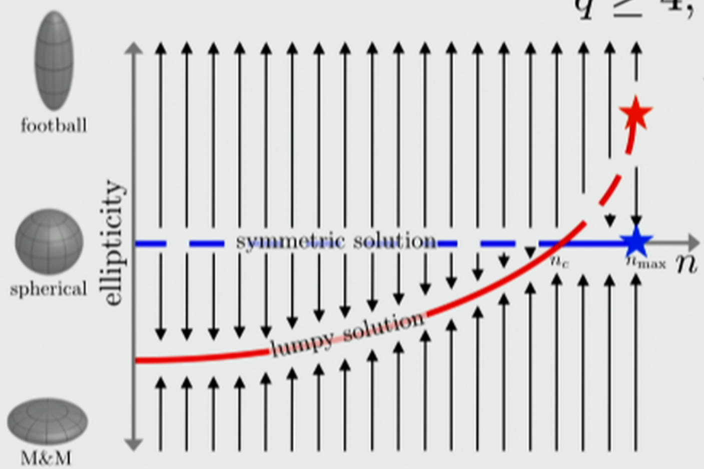
$\varepsilon < 1$ | $\varepsilon = 1$ | $\varepsilon > 1$

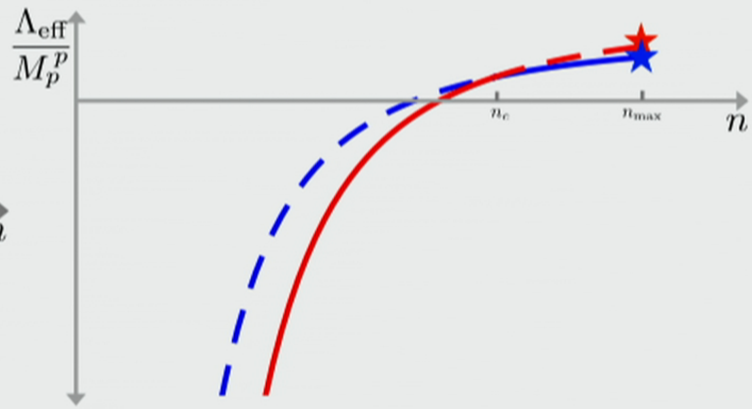
Warped Solutions: Ellipsoidal Case

(1 extremum)

Cartoon Distillation:

$q \geq 4, \Lambda_D > 0$

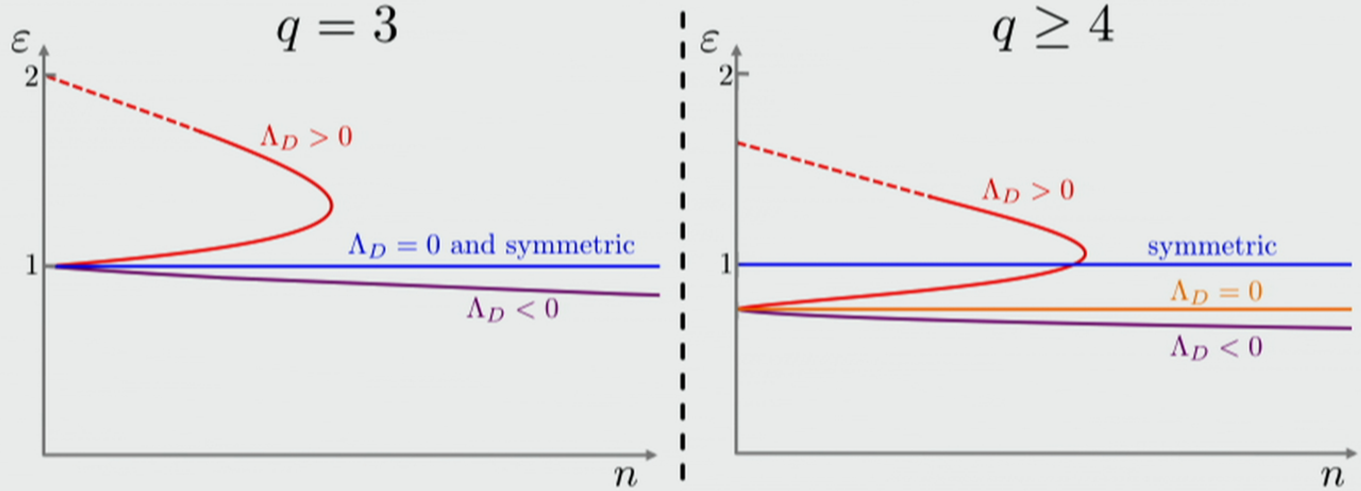




Warped Solutions: Ellipsoidal Case

(1 extremum)

Full phase diagram:



Scaling properties of EOM:

$$\Lambda_D \to \alpha^{-2}\Lambda_D\,, \qquad Q \to \beta^{p-1}Q \quad (\Lambda_D = 0)$$

Warped Solutions: Ellipsoidal Case

(1 extremum)

The underlying principles:

- When the $\ell = 2$ 'danger mode' has a negative mass squared, the ellipsoidal solution is M&M shaped and energetically favored. When the mode has a positive mass squared, the ellipsoidal solution is football-shaped and energetically disfavored.

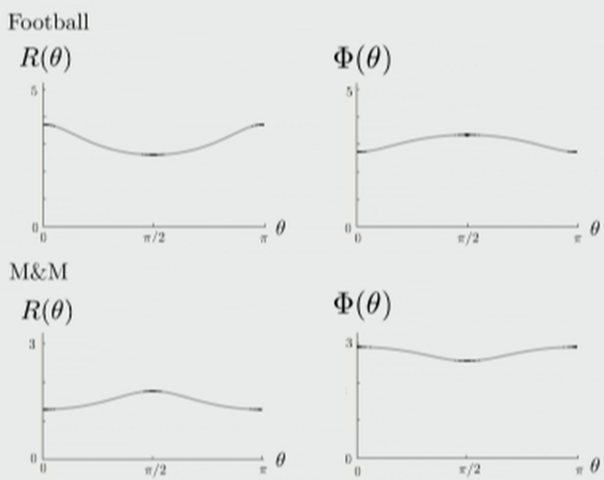
- The $n \to 0$ behavior is independent of Λ_D

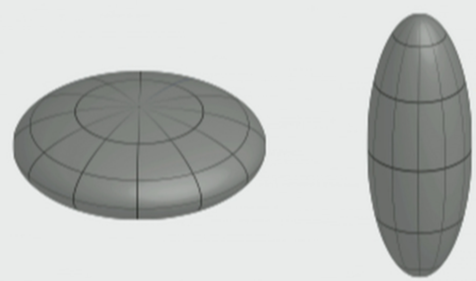
- Each Freund-Rubin solution is accompanied by a single ellipsoidal solution with the same flux n

Warped Solutions: Ellipsoidal Case

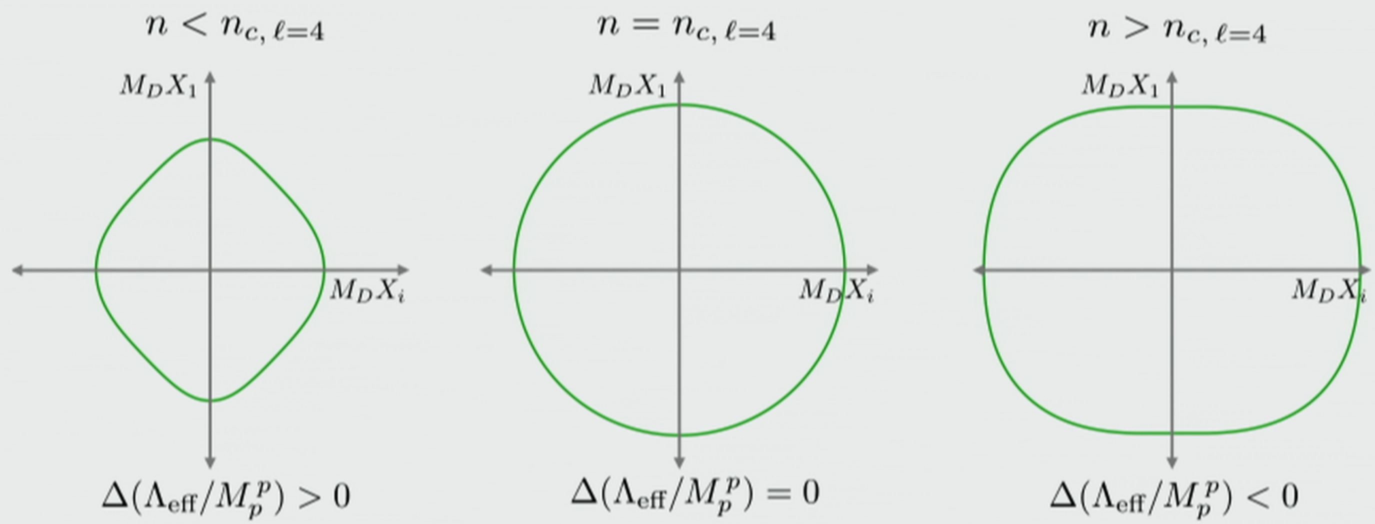
(1 extremum)

1 extremum solution $\Leftrightarrow$ $\ell = 2$ instability of the symmetric solution





Warped Solutions: Higher-ℓ



Same principles hold!

Conjecture:

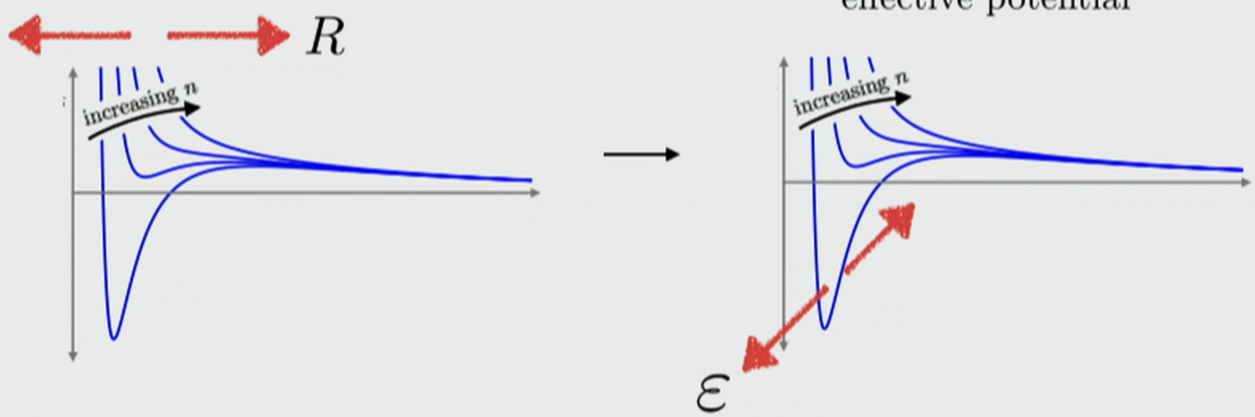
$\ell - 1$ extremum solution	$\Leftrightarrow$	ℓ-mode instability of the symmetric solution

Effective Potential Derivation

Now that we've mapped out the phase space of some warped solutions, we will argue for a neat effective potential picture that encapsulates the physics.

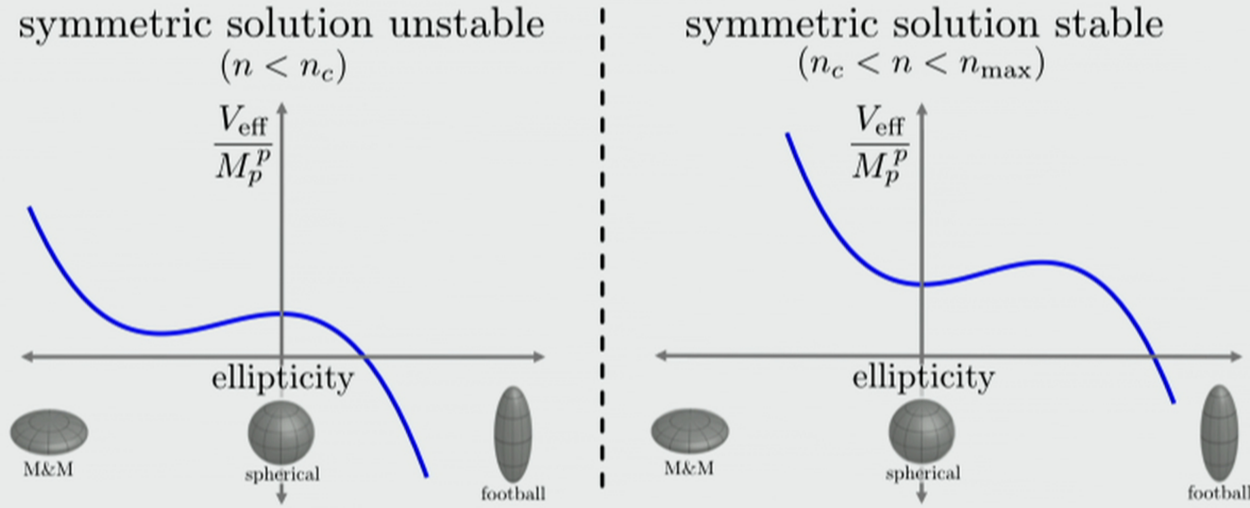
Volume-direction effective potential

(Perpendicular) Shape-direction effective potential



Warped Solutions: Effective Potential

Effective potential in shape direction:



Critical value of n where symmetric flips from positive to negative mass squared + fixed asymptotics

$\Rightarrow$ New (for us, lumpy) solutions are "ejected"

Warped Solutions: Effective Potential

Asymptotics:

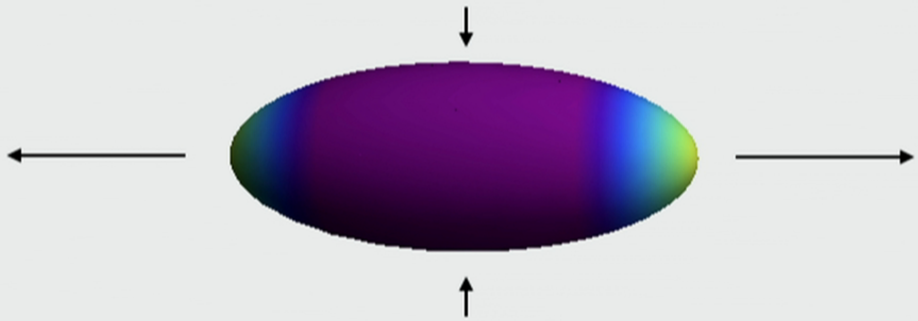
- $V \to +\infty$ (M&M) and $V \to -\infty$ (football)

- Turns around in either or both directions

  If so, should have found evidence of additional solutions

- Finite asymptote in either or both directions

  Long flat section with no symmetry to protect it, we find unlikely



Warped Solutions: Effective Potential

Asymptotics:

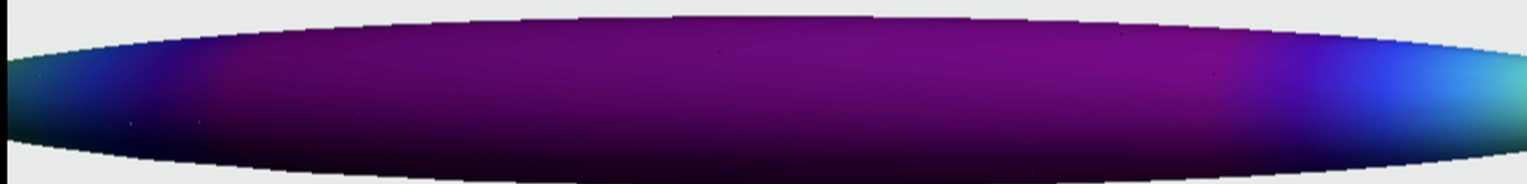
- $V \to +\infty$ (M&M) and $V \to -\infty$ (football)

- Turns around in either or both directions

  If so, should have found evidence of additional solutions

- Finite asymptote in either or both directions

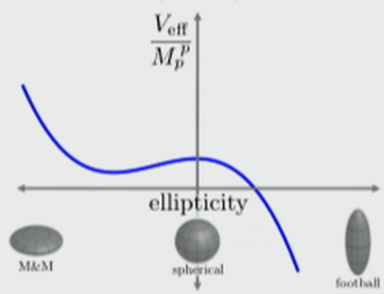
  Long flat section with no symmetry to protect it, we find unlikely



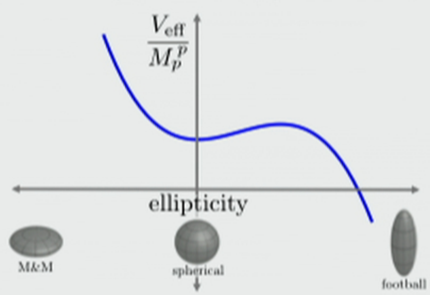
Warped Solutions: Effective Potential

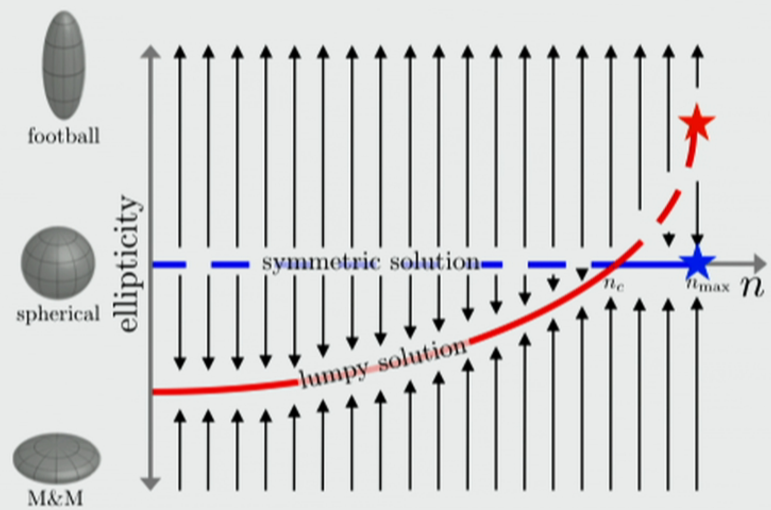
Summary: Consistent effective potential description that captures all the physics of our phase diagram

symmetric solution unstable
$(n < n_c)$



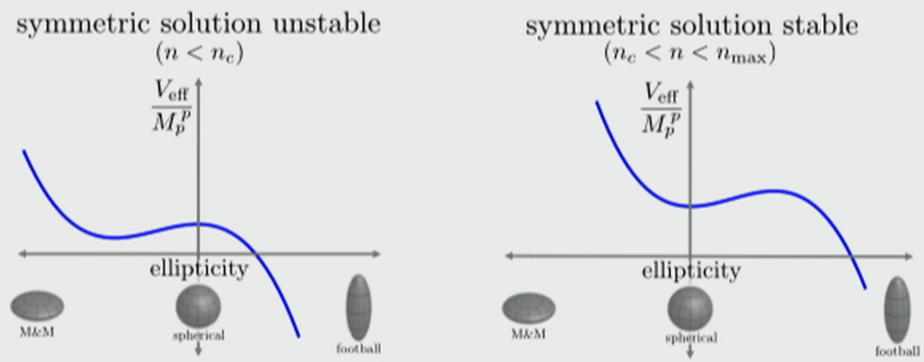
symmetric solution stable
$(n_c < n < n_{\rm max})$





Warped Solutions: Effective Potential

The classical stability of the symmetric solution determines the form of the lumpy solutions: their shape, stability and whether they are dominant or subdominant.

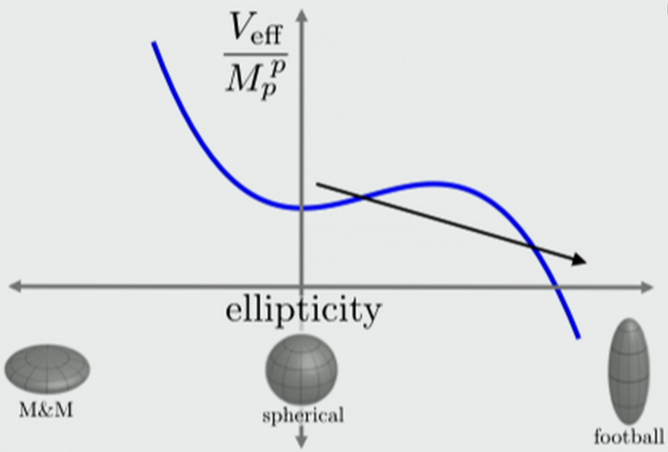


Non-Perturbative Instabilities

New non-perturbative instability: Tunnel to infinite football!

Typical mass squareds in shape direction are a couple orders of magnitude smaller than in total-volume direction.

symmetric solution stable
$(n_c < n < n_{\max})$



$\Rightarrow$ Shape-mode tunneling is fastest decay (vs. e.g. decompactification).

$\underline{p = q = 4, \;\; \Lambda_D > 0}$:

- 97% of FR perturbatively unstable
- 2.97% decay to footballs

Future Work

- Study spontaneous symmetry breaking along different axes at each point in space-time, and whether this leads to topologically nontrivial configurations
- Study the evolution in this shape direction potential further from sphericality
- Generalize to some more realistic models
- Do the full stability calculation around the warped product solutions.

Conclusions

- There exist additional lumpy solutions to the Freund-Rubin equations that can be more stable than the symmetric branch.

- There is a symmetry-breaking effect where the internal sphere generically rolls to these new solutions.

- Even when the symmetric solution is stable, we found new non-perturbative decays to lumpiness, including to the infinite football-shaped direction. These are the fastest-yet-known decays for this model.

- We derived a phase diagram and argued all physics is neatly encapsulated by a shape-direction effective potential.