Title: Three-Point Function in N=4 SYM and Spin Vertex

Date: Oct 21, 2014 02:00 PM

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Abstract: In this talk I will explain how to compute three-point functions of N=4 SYM theory in the planar limit for tree level and one-loop in perturbation theory. First I will recall how to formulate the problem of computing the three-point function of operators with determined R-charges in the language of integrable spin chains. In the su(2) sector, the tree-level three point function can be obtained in terms of determinants, whose large R-charge limit can be taken explicitly. Then I will report a systematic method to compute the su(2) three point function at higher loops. In particular, we are able to take the semi-classical limit, and we can compare our result with the calculation from string theory. In the Frolov-Tseytlin limit we find a perfect match at one-loop. Finally I will present a new formalism of computing three-point functions called the spin vertex formalism, which is the weak coupling counter-part of the string vertex in the string field theory. I will describe how to construct the spin vertex and discuss its important properties.


Correlation Functions in N=4 SYM
and Spin Vertex
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2014-10-21

Three-Point Function I

Tree Level

(Escobedo, Gromov, Sever, Vieira 2010-2011)

(Foda 2011)

Three-Point Function I

Tree Level

(Escobedo, Gromov, Sever, Vieira 2010-2011)

(Foda 2011)

Set-Up

(Escobedo, Gromov, Sever, Vieira 2010-2011)

Operators	Field Contents	Sectors
$\mathcal{O}_1$	$Z, \quad X$	$SU(2)_L$
$\mathcal{O}_2$	$\bar{Z}, \quad \bar{X}$	$SU(2)_L$
$\mathcal{O}_3$	$Z, \quad \bar{X}$	$SU(2)_R$

Eigenstates of one-loop dilatation operator.

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(Escobedo, Gromov, Sever, Vieira 2010-2011)

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Eigenstates of one-loop dilatation operator.

Structure Constant

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\rangle = \frac{L_i\delta_{ij}}{|x_{12}|^{2\Delta_i}}$$

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{L_1L_2L_3C_{123}(g)}{|x_{12}|^{\Delta_{12}}|x_{23}|^{\Delta_{23}}|x_{13}|^{\Delta_{13}}}$$

$$|x_{ij}| = |x_i - x_j|$$

$$\Delta_{ij} = \Delta_i + \Delta_j - \Delta_k \qquad i,j,k = 1,2,3$$

$$N_c C_{123}(g) = C_{123}^{(0)} + g^2 C_{123}^{(1)} + \cdots$$

Spin Chain Language

$$\mathcal{O}_1 \to |\mathbf{u}\rangle, \mathcal{O}_2 \to |\mathbf{v}\rangle, \mathcal{O}_3 \to |\mathbf{w}\rangle$$

$$\mathbf{u} = \{u_1, \cdots, u_{N_1}\}$$
$$\mathbf{v} = \{v_1, \cdots, v_{N_2}\}$$
$$\mathbf{w} = \{w_1, \cdots, w_{N_3}\}$$

$$C_{123}^{(0)} = \frac{\langle \mathbf{u}, \mathbf{v}, \mathbf{w} \rangle}{\sqrt{\langle \mathbf{u}|\mathbf{u}\rangle \langle \mathbf{v}|\mathbf{v}\rangle \langle \mathbf{w}|\mathbf{w}\rangle}}$$

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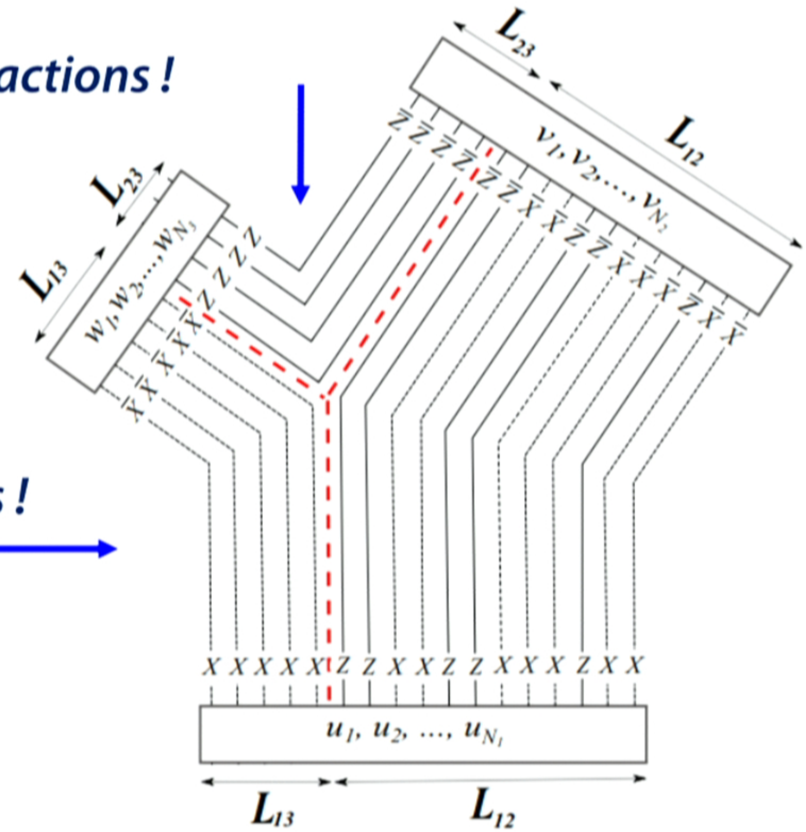
Planar Diagram

Trivial contractions !

Trivial contractions !



Planar Diagram

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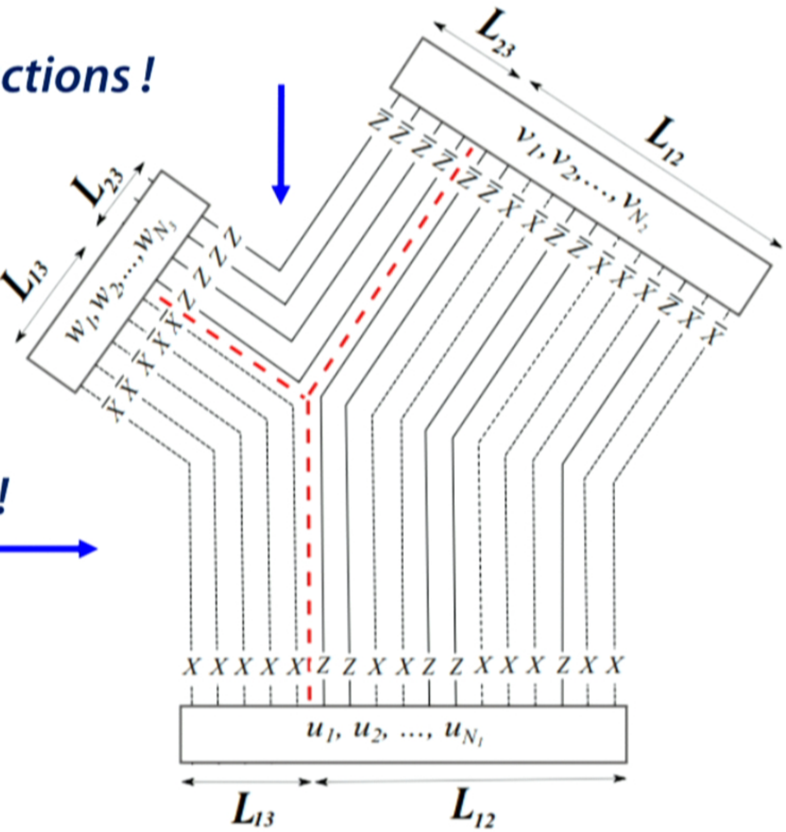

Tailoring

(Escobedo, Gromov, Sever, Vieira 2010-2011)

1. Cutting

$$|\mathbf{u}\rangle \to \sum_{\mathbf{u}'\cup\mathbf{u}''=\mathbf{u}} |\mathbf{u}'\rangle \otimes |\mathbf{u}''\rangle$$

$$|\mathbf{v}\rangle \to \sum_{\mathbf{v}'\cup\mathbf{v}''=\mathbf{v}} |\mathbf{v}'\rangle \otimes |\mathbf{v}''\rangle$$

$$|\mathbf{w}\rangle \to \sum_{\mathbf{w}'\cup\mathbf{w}''=\mathbf{w}} |\mathbf{w}'\rangle \otimes |\mathbf{w}''\rangle$$

2. Flipping

$$|\mathbf{u}'\rangle \otimes |\mathbf{u}''\rangle \to |\mathbf{u}'\rangle \otimes \langle \mathbf{u}''^*|$$

$$|\mathbf{v}'\rangle \otimes |\mathbf{v}''\rangle \to |\mathbf{v}'\rangle \otimes \langle \mathbf{v}''^*|$$

$$|\mathbf{w}'\rangle \otimes |\mathbf{w}''\rangle \to |\mathbf{w}'\rangle \otimes \langle \mathbf{w}''^*|$$

3. Sewing

$$C_{123}^{(0)} \sim \sum_{\text{partitions}} \frac{\langle \mathbf{u}''^*|\mathbf{v}'\rangle\langle \mathbf{v}''^*|\mathbf{w}'\rangle\langle \mathbf{w}''^*|\mathbf{u}'\rangle}{\sqrt{\langle \mathbf{u}|\mathbf{u}\rangle\langle \mathbf{v}|\mathbf{v}\rangle\langle \mathbf{w}|\mathbf{w}\rangle}}$$

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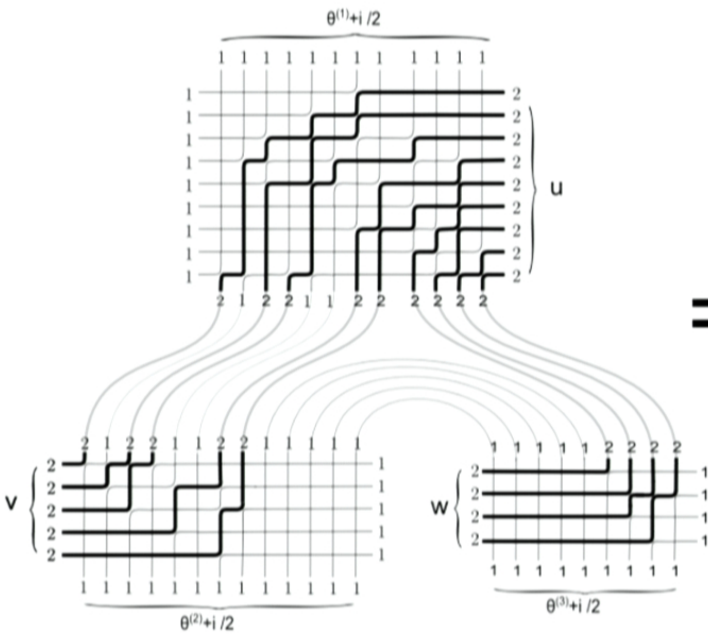
Freezing

(Foda 2011)

Cubic Vertex



$$= \langle \mathbf{v} \cup \mathbf{z} | \mathbf{u} \rangle_{\boldsymbol{\theta}^{(1)}} \langle \mathbf{w} | \mathbf{z} \rangle_{\boldsymbol{\theta}^{(3)}}$$

$$\mathbf{z} = \boldsymbol{\theta}^{(13)} - i/2$$

Freezing

Structure Constant

$$C_{123}^{(0)} = \frac{\langle \mathbf{v} \cup \mathbf{z} | \mathbf{u} \rangle_{\boldsymbol{\theta}^{(1)}} \langle \mathbf{w} | \mathbf{z} \rangle_{\boldsymbol{\theta}^{(3)}}}{\sqrt{\langle \mathbf{u}; \boldsymbol{\theta}^{(1)} | \mathbf{u}; \boldsymbol{\theta}^{(1)} \rangle \langle \mathbf{v}; \boldsymbol{\theta}^{(2)} | \mathbf{v}; \boldsymbol{\theta}^{(2)} \rangle \langle \mathbf{w}; \boldsymbol{\theta}^{(3)} | \mathbf{w}; \boldsymbol{\theta}^{(3)} \rangle}}$$

For tree level, take the homogeneous limit !

For higher loop, fix the values of impurities !

Freezing

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For tree level, take the homogeneous limit !

For higher loop, fix the values of impurities !

The semi-classical limit

(Gromov, Sever, Vieira 2011 ;
Kostov 2012 ; Kostov Matsuo 2012)

1. *L, N are large while N/L Finite*
2. *Compare with string theory*
3. *Bethe roots condense into cuts*
4. *Nice analytic results*

Inhomogeneous Case

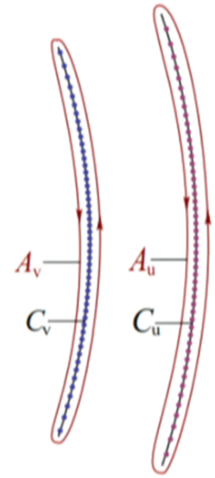
$$\log C_{123}^{(0)} = \oint_{A_{\mathbf{u}} \cup A_{\mathbf{v}}} \frac{\mathrm{d}u}{2\pi} \mathrm{Li}_2 \left(e^{ip_{\mathbf{u}} + ip_{\mathbf{v}} + \frac{i}{2} G_{\boldsymbol{\theta}^{(3)}}} \right)$$
$$+ \oint_{A_{\mathbf{w}}} \frac{\mathrm{d}u}{2\pi} \mathrm{Li}_2 \left(e^{ip_{\mathbf{w}} + \frac{i}{2} G_{\boldsymbol{\theta}^{(2)}} - \frac{i}{2} G_{\boldsymbol{\theta}^{(1)}}} \right)$$
$$- \oint_{A_{\mathbf{u}}} \frac{\mathrm{d}u}{4\pi} \mathrm{Li}_2 \left(e^{2ip_{\mathbf{u}}} \right)$$
$$- \oint_{A_{\mathbf{v}}} \frac{\mathrm{d}u}{4\pi} \mathrm{Li}_2 \left(e^{2ip_{\mathbf{v}}} \right)$$
$$- \oint_{A_{\mathbf{w}}} \frac{\mathrm{d}u}{4\pi} \mathrm{Li}_2 \left(e^{2ip_{\mathbf{w}}} \right)$$

Quasi-Momentum

$$p_{\mathbf{u}}(u) = G_{\mathbf{u}}(u) - \frac{1}{2} G_{\boldsymbol{\theta}}(u)$$

Homogeneous Limit

$$\log C_{123}^{(0)} = \oint_{A_{\mathbf{u}} \cup A_{\mathbf{v}}} \frac{du}{2\pi} \mathrm{Li}_2\left(e^{ip_{\mathbf{u}} + ip_{\mathbf{v}} + iL_3/2u}\right)$$
$$+ \oint_{A_{\mathbf{w}}} \frac{du}{2\pi} \mathrm{Li}_2\left(e^{ip_{\mathbf{w}} + i(L_2 - L_1)/2u}\right)$$
$$- \oint_{A_{\mathbf{u}}} \frac{du}{4\pi} \mathrm{Li}_2\left(e^{2ip_{\mathbf{u}}}\right)$$
$$- \oint_{A_{\mathbf{v}}} \frac{du}{4\pi} \mathrm{Li}_2\left(e^{2ip_{\mathbf{v}}}\right)$$
$$- \oint_{A_{\mathbf{w}}} \frac{du}{4\pi} \mathrm{Li}_2\left(e^{2ip_{\mathbf{w}}}\right)$$



Three-Point Function II

One Loop

(Gromov, Vieira 2012)

(Serban 2012)

(Y.J, Kostov, Loebbert, Serban 2014)

Long-Range Spin Chain

Hamiltonians

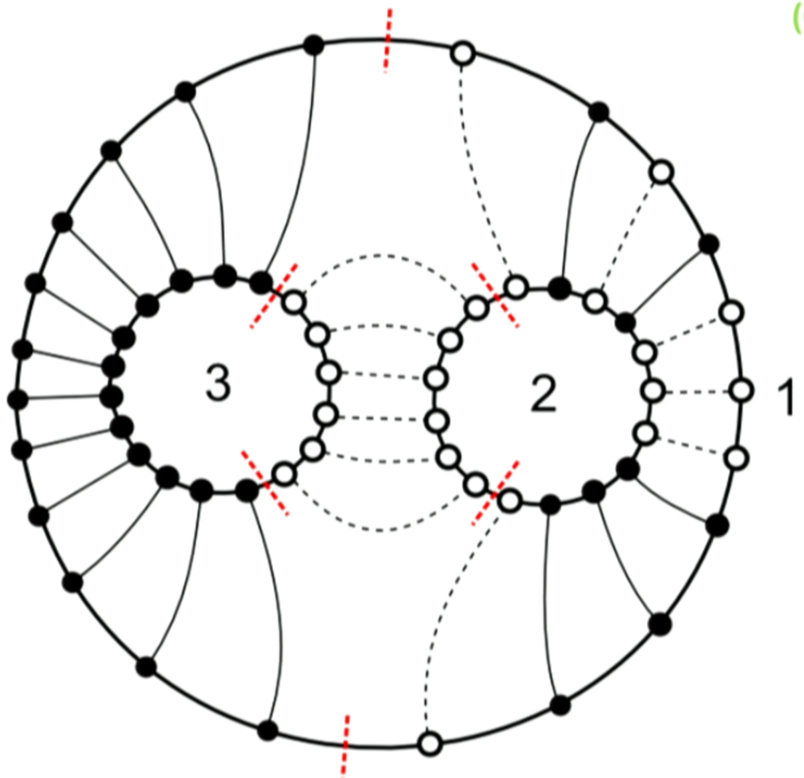
(Beisert, Dipper, Staudacher 2004)

$$H_2 = 2\sum_{k=1}^{L}(1 - \mathrm{P}_{k,k+1})$$

$$H_4 = 2\sum_{k=1}^{L}(4\mathrm{P}_{k,k+1} - \mathrm{P}_{k,k+2} - 3)$$

No systematic way to construct eigenstates !

One-Loop Insertion



(Okuyama and Tseng 2004)

$$\mathbb{I}_1 = \mathrm{H}^{(1)}_{L_{12}} + \mathrm{H}^{(1)}_{L_1}$$
$$\mathbb{I}_2 = \mathrm{H}^{(2)}_{L_{12}} + \mathrm{H}^{(2)}_{L_2}$$
$$\mathbb{I}_3 = \mathrm{H}^{(3)}_{L_{13}} + \mathrm{H}^{(3)}_{L_3}$$

$$\mathrm{H}_k = 1 - \mathrm{P}_{k,k+1}$$
$$[\mathrm{H}]_k = [\mathrm{H}_k, \mathrm{H}_{k+1}]$$

Unitarity Transformation

(Bargeer, Beisert, Loebbert 2010)

BDS ⟺ **Inhomogeneous XXX**

If $\theta_k^{\mathrm{BDS}} = 2g\sin\frac{2\pi k}{L}$

Monodromy Matrix

$$\mathcal{T}_{\mathrm{BDS}}(u) = S\mathcal{T}_{\mathrm{XXX}}(u;\boldsymbol{\theta}^{\mathrm{BDS}})S^{-1}$$

Bethe States

$$|\mathbf{u}\rangle_{\mathrm{BDS}} = S|\mathbf{u};\boldsymbol{\theta}^{\mathrm{BDS}}\rangle$$

The S-transformation preserves the RTT relation. This is called morphism property.

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S-Operator

$$\mathrm{S} = \mathrm{S}_{\mathcal{B}} \times \mathrm{S}_\theta^{-1}$$

Long-Range Deformation

$$\mathrm{S}_{\mathcal{B}} = \exp i\Phi$$

$$\Phi = 2g^2 \mathcal{B}[\mathrm{Q}_3]$$

$$\mathcal{B}[\mathrm{Q}_3] = \frac{i}{2} \sum_{k=1}^{L} k[\mathrm{H}]_{k-1}$$

Shift Impurities

$$\mathrm{S}_\theta^{-1} = \exp i\Theta$$

$$\Theta = \sum_{k=1}^{L} (\nu_k \mathrm{H}_k + \hat{\rho}_k [\mathrm{H}]_k)$$

$$\nu_k = -\sum_{j=1}^{k} \theta_j$$

$$\hat{\rho}_k = -\theta_k \nu_k - \sum_{j=1}^{k} \theta_j^2$$

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P-D Relations

First Order

(Gromov, Vieira 2012)

$$\mathrm{H}_k|\mathbf{u}\rangle = -\mathrm{D}_k|\mathbf{u}\rangle \qquad \mathrm{D}_k = i(\partial_k - \partial_{k+1})$$

$$\mathrm{H}_L|\mathbf{u}\rangle = -\mathrm{D}_L|\mathbf{u}\rangle + \mathrm{E}_2|\mathbf{u}\rangle \qquad \partial_k = \partial/\partial\theta_k$$

Second Order

$$[\mathrm{H}]_k|\mathbf{u}\rangle = \left(\frac{1}{2}(\mathrm{D}_k^2 - \mathrm{D}_{k+1}^2) + (\mathrm{D}_k - \mathrm{D}_{k+1})\right)|\mathbf{u}\rangle$$

$$[\mathrm{H}]_L|\mathbf{u}\rangle = \left(\frac{1}{2}(\mathrm{D}_L^2 - \mathrm{D}_1^2) + (\mathrm{D}_L - \mathrm{D}_1)\right)|\mathbf{u}\rangle - \left(i\mathrm{E}_3 - \frac{1}{2}\mathrm{E}_2^2 + \mathrm{E}_2(1 + \mathrm{D}_L)\right)|\mathbf{u}\rangle$$

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Final Result

$$C_{123}(g^2) = C_{123}^{\rm BDS}(g^2) + g^2\delta_{123}$$

$$C_{123}^{\rm BDS} = \left.\frac{\langle \mathbf{v}\cup\mathbf{z}|\mathbf{u}\rangle_{\boldsymbol{\theta}^{(1)}}\langle \mathbf{w}|\mathbf{z}\rangle_{\boldsymbol{\theta}^{(3)}}}{\sqrt{\langle \mathbf{u};\boldsymbol{\theta}^{(1)}|\mathbf{u};\boldsymbol{\theta}^{(1)}\rangle\langle \mathbf{v};\boldsymbol{\theta}^{(2)}|\mathbf{v};\boldsymbol{\theta}^{(2)}\rangle\langle \mathbf{w};\boldsymbol{\theta}^{(3)}|\mathbf{w};\boldsymbol{\theta}^{(3)}\rangle}}\right|_{\boldsymbol{\theta}^{\rm BDS}}$$

$$C_{123}^{\rm BDS}(g^2) \simeq C_{123}(1 + g^2\delta_{123}^{\rm BDS})$$

Semi-Classical Limit

$$g^2\delta_{123} \sim \frac{g^2}{L^2} = g'^2 \ll g^2\delta_{123}^{\rm BDS} \sim \frac{g^2}{L} = g'^2 L$$

Strong-Weak Comparison

Cubic Vertex

$$\langle \mathbf{u}, \mathbf{v}, \mathbf{w} \rangle \simeq \exp F_{123}^{\mathrm{BDS}}$$

Frolov-Tseytlin Limit

$$g' = \frac{g}{L} \ll 1$$

Due to order-of-limit problem, there is no guarantee that they should match !

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Weak Coupling

(Serban 2012)

(Y.J, Kostov, Loebbert, Serban 2014)

$$F_{123}^{\mathrm{BDS}} \simeq \oint_{\mathcal{C}_{\mathbf{u}}\cup\mathcal{C}_{\mathbf{v}}} \frac{\mathrm{d}u}{2\pi}\mathrm{Li}_2\left(e^{ip_{\mathbf{u}}+ip_{\mathbf{v}}-iq_{\mathbf{w}}}\right) + \oint_{\mathcal{C}_{\mathbf{w}}} \frac{\mathrm{d}u}{2\pi}\mathrm{Li}_2\left(e^{ip_{\mathbf{w}}+iq_{\mathbf{u}}-iq_{\mathbf{v}}}\right)$$

Strong Coupling

(Kazama, Komatsu 2013)

$$F_{123}^{\mathrm{KK}} \simeq \oint \frac{\mathrm{d}u}{2\pi}\mathrm{Li}_2\left(e^{ip_1+ip_2-ip_3}\right) + \oint \frac{\mathrm{d}u}{2\pi}\mathrm{Li}_2\left(e^{ip_3+ip_1-ip_2}\right) + \oint \frac{\mathrm{d}u}{2\pi}\mathrm{Li}_2\left(e^{ip_2+ip_3-ip_1}\right) + \oint \frac{\mathrm{d}u}{2\pi}\mathrm{Li}_2\left(e^{ip_1+ip_2+ip_3}\right)$$

Up to the order g'^2

✓ **Match:**

Integrand for the common terms

? **Unclear:**

Two extra terms from strong coupling side

? **Unclear:**

Integral contours

Spin Vertex Formalism

(Komastu, talk in Pohang workshop)

(Y.J, Kostov, Petrovskii, Serban
Work in progress...)

The String Vertex

Light-cone string field theory

(Spradlin and Volovich 2002)

(Dobashi, Shimada, Yoneya 2003)

(Dobashi and Yoneya 2004)

$$|H_3\rangle = \mathcal{P}\exp\left(-\frac{1}{2}\sum_{i=1}^{8}\sum_{r,s=1}^{3}\sum_{m,n}^{\infty} a_m^{(r)i\dagger}\tilde{N}_{mn}^{rs}a_n^{(s)i\dagger}\right)|0\rangle$$

BMN Correspondence

$$H_{123} = \langle 1|\langle 2|\langle 3|H_3\rangle \sim C_{123}$$

string theory *field theory*

The String Vertex

Light-cone string field theory

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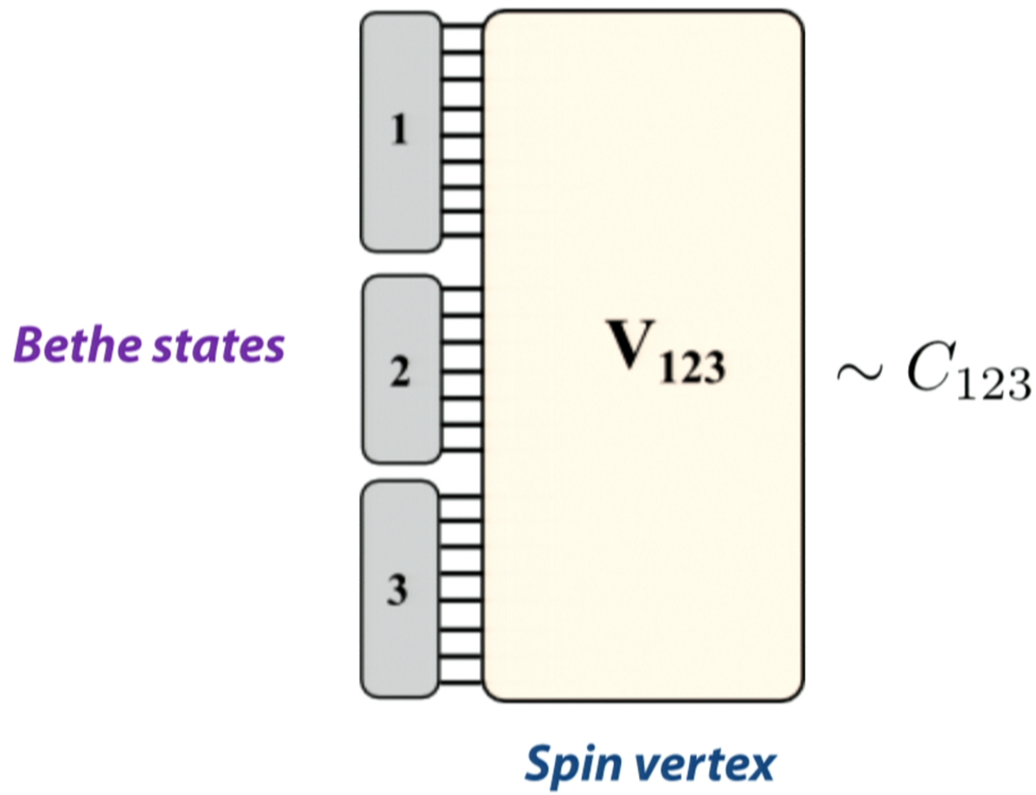
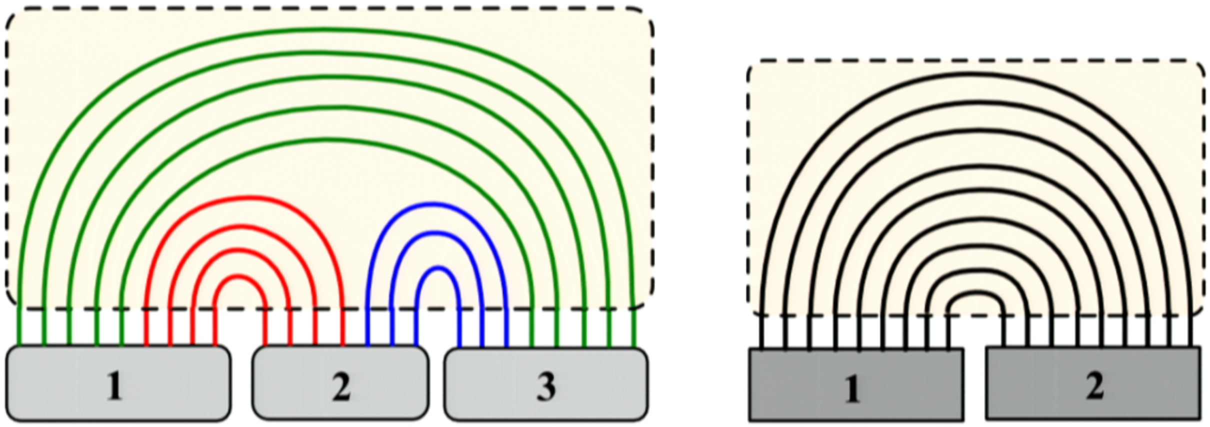
string theory *field theory*

The Spin Vertex



The Spin Vertex

Tree-level



$$|V_{123}\rangle = |V_{12}\rangle|V_{23}\rangle|V_{13}\rangle \qquad \langle \mathbf{u}|\langle \mathbf{v}|V_{12}\rangle = \langle \mathbf{u}|\mathbf{v}\rangle$$

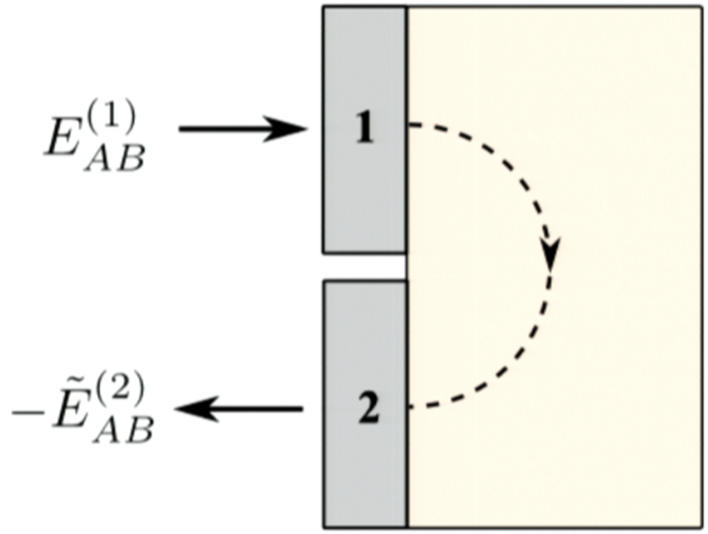
Wick Contraction + Planarity

$$|v_{12}\rangle = |Z\rangle \otimes |\bar{Z}\rangle + |\bar{Z}\rangle \otimes |Z\rangle + |X\rangle \otimes |\bar{X}\rangle + |\bar{X}\rangle \otimes |X\rangle$$

The Spin Vertex

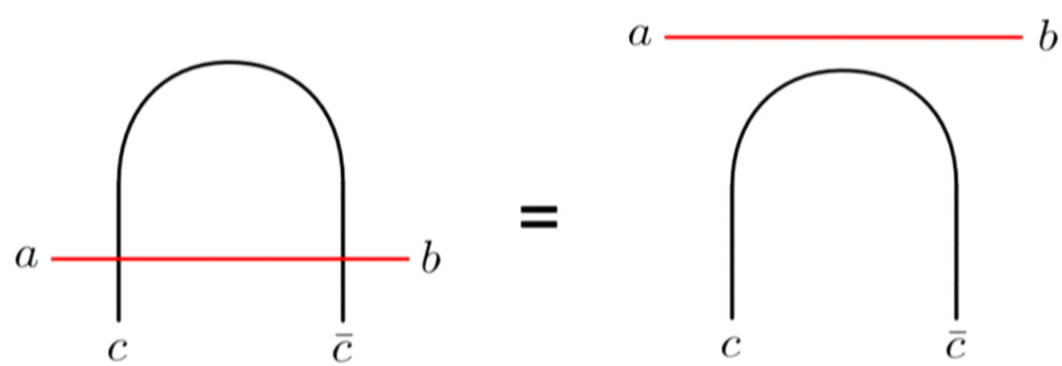
Reflection Property

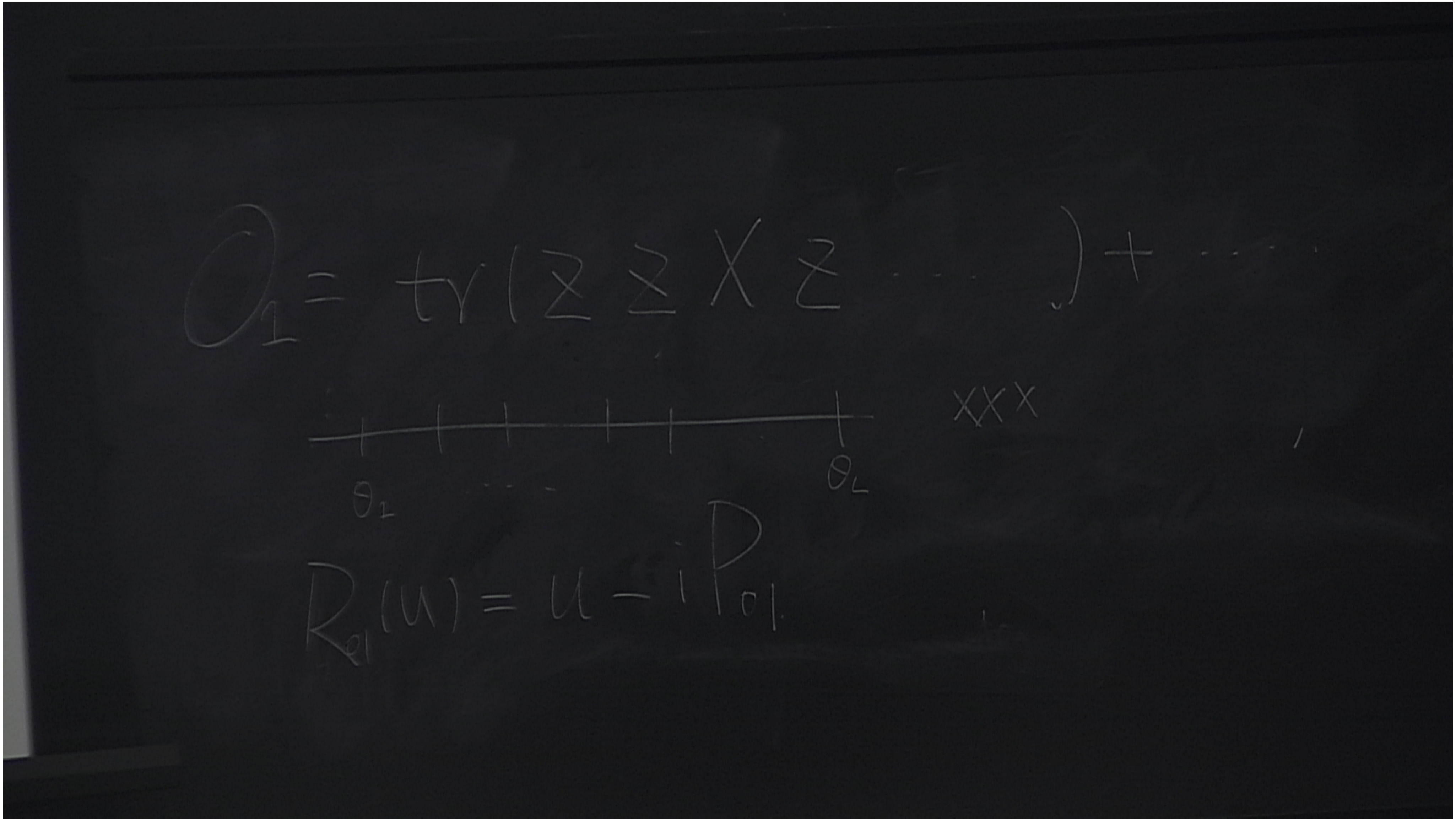
$$(E_{AB}^{(1)} + \tilde{E}_{AB}^{(2)})|V_{12}\rangle = 0$$



Monodromy Condition

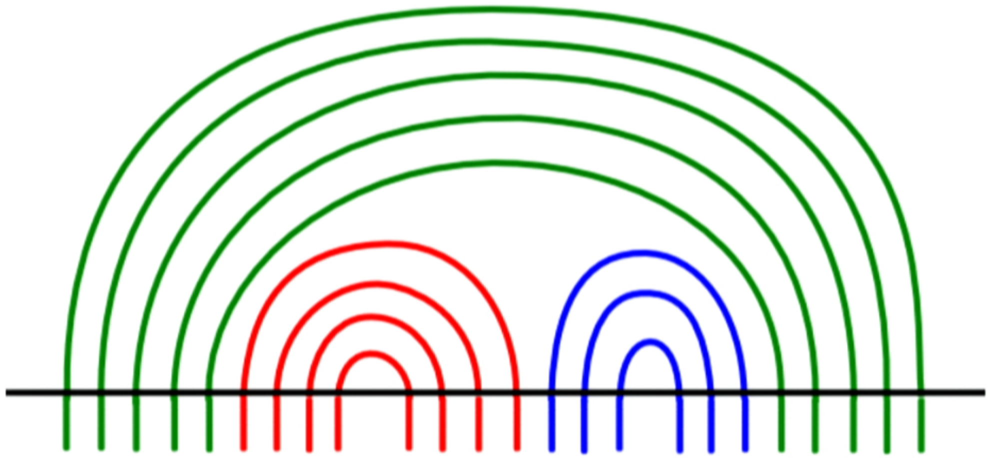
$$R_{01}(u)R_{0\bar{1}}(u)|V_{12}\rangle = |V_{12}\rangle$$



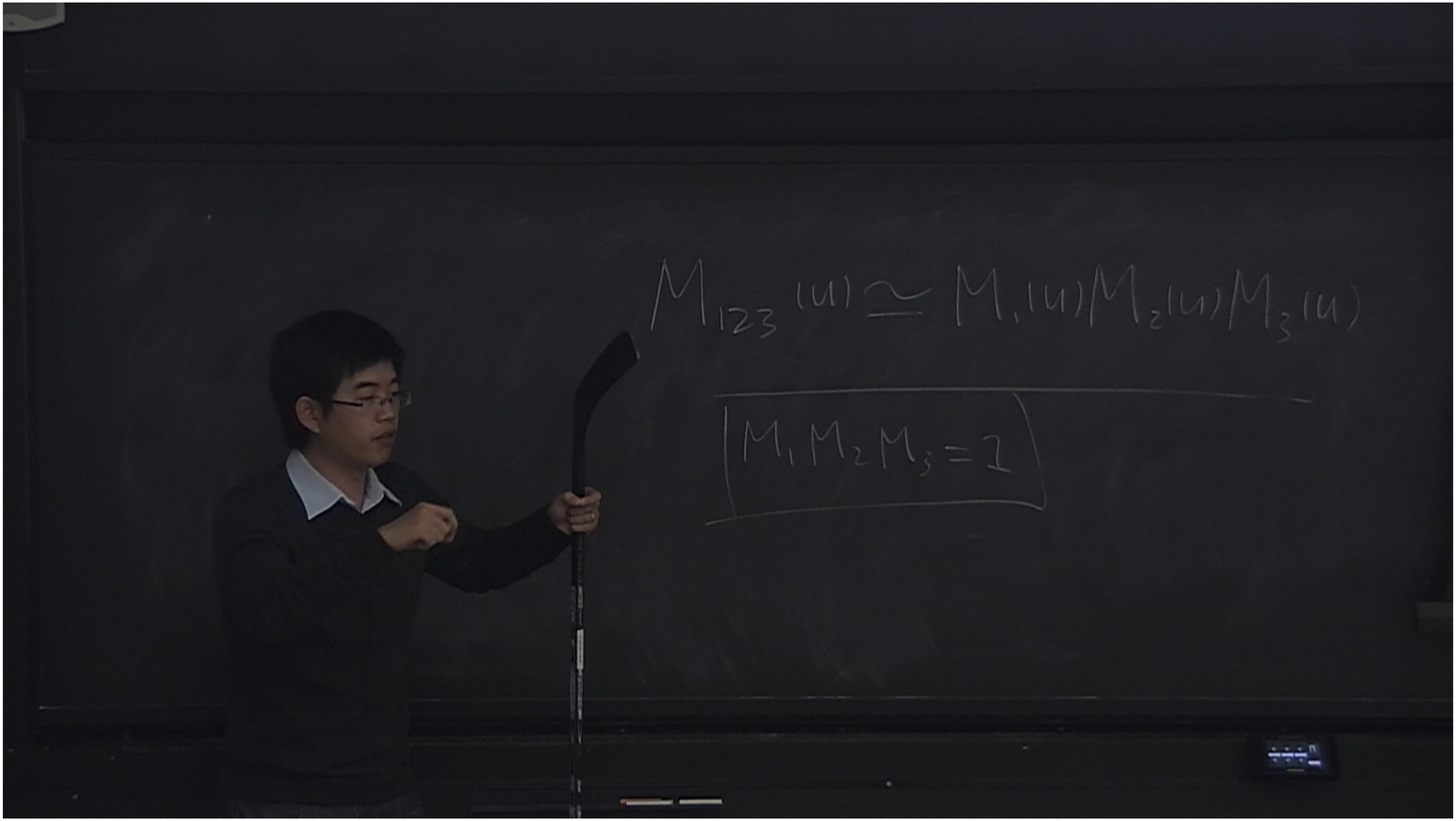

O1 = tr(Z Z X Z ......) + ...
θ1 ..... θL
XXX
R01(u) = u − i P01

Monodromy Condition

Yangian Invariant



$$M_{123}(u)|V_{123}\rangle = |V_{123}\rangle$$


$M_{123}(u) \simeq M_1(u) M_2(u) M_3(u)$
$M_1 M_2 M_3 = 1$

Monodromy Condition

Main Conjecture

$$M_{123}(u)|V_{123}\rangle = |V_{123}\rangle$$

The monodromy condition is true also at higher loops.

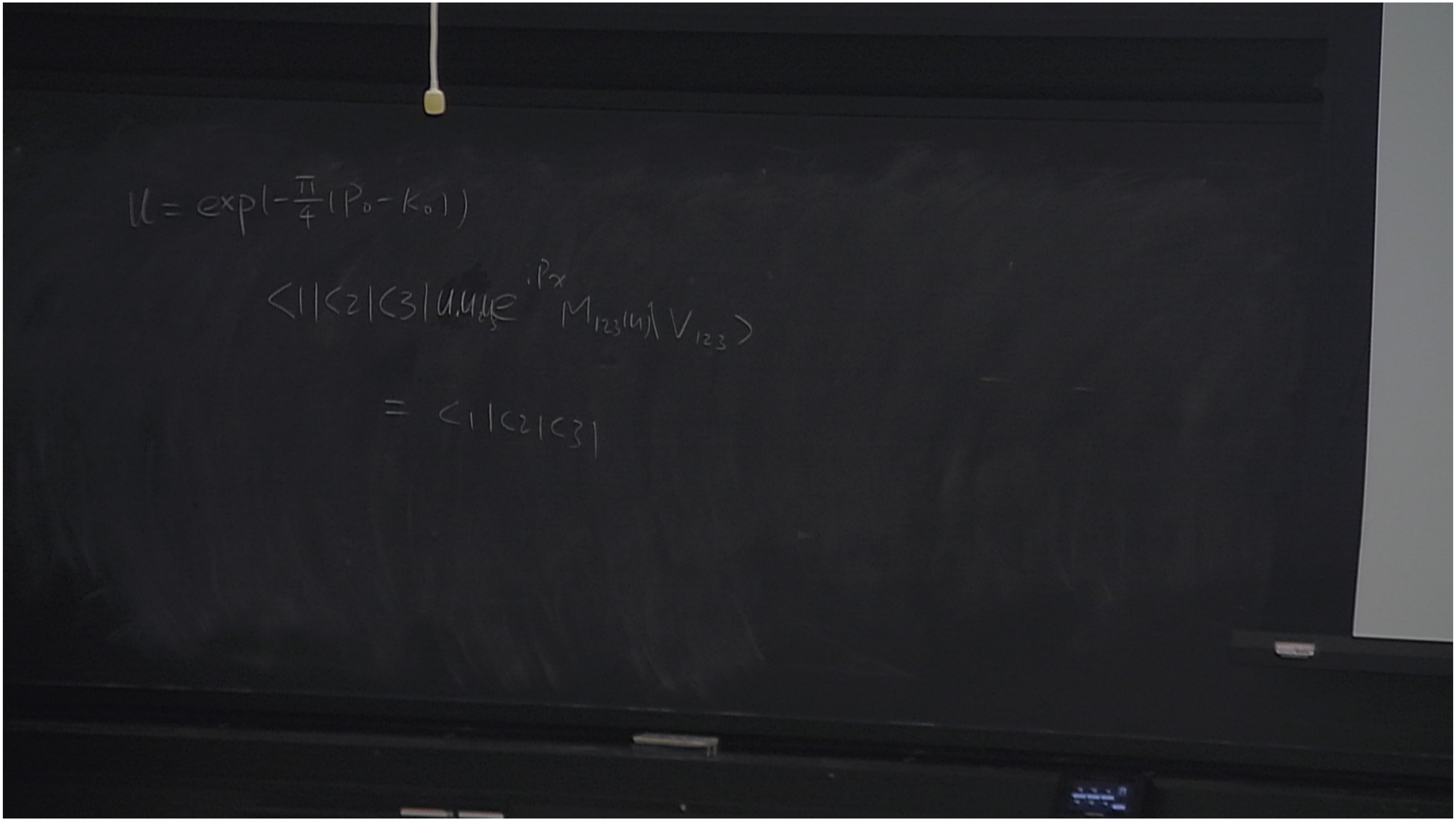
Next Step

Use the monodromy condition to fix one loop insertions without Feynmann diagram.

Outlook

- Use integrability to fix insertions
- Relations to scattering amplitudes
- Equations for three-point function
- All loop formalism


Thank you !


U = exp(-π/4 (P0 - K0))
<1|<2|<3| U1U2U3 e^iPx M123(u)|V123>
= <1|<2|<3|