Title: Schroedinger method as field theoretical model to describe structure formation

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Abstract: We investigate large-scale structure formation of collisionless dark matter in the phase space description based on the Vlasov equation whose nonlinearity is induced solely by gravitational interaction according to the Poisson equation. Determining the time-evolution of density and velocity demands solving the full Vlasov hierarchy for the cumulants of the distribution function. In the presence of long-range interaction no consistent truncation is known apart from the dust model which is incapable of describing the formation of bound structures due to the inability to generate higher cumulants like velocity dispersion. Our goal is to find a simple ansatz for the phase space distribution function that approximates the full Vlasov distribution function and can serve as theoretical N-body double to replace the dust model. We present the Schroedinger method which is based on the coarse-grained Wigner probability distribution obtained from a wave function fulfilling the Schroedinger-Poisson equation as sought-after model. We show that its evolution equation approximates the Vlasov equation in a controlled way, cures the shell-crossing singularities of the dust model and is able to describe multi-streaming which is crucial for halo formation. This feature has already been employed in cosmological simulations of large-scale structure formation by Widrow & Kaiser (1993). We explain how the coarse-grained Wigner ansatz allows to calculate higher cumulants like velocity dispersion analytically from density and velocity in a self-consistent manner. On this basis we show that instead of solving the Vlasov-Poisson system one can use the SchrÃ¶dinger method and solve the SchrÃ¶dinger-Poission equation to directly determine density and velocity and all higher cumulants. As a first application we study the coarse-grained dust model, which is a limiting case of the SchrÃ¶dinger method, within Eulerian and Lagrangian perturbation theory.

Schrödinger method
as field theoretical model for
Structure formation

Cora Uhlemann

Arnold Sommerfeld Center, LMU
& Excellence Cluster Universe

Advisor: Stefan Hofmann

in collaboration with

group of Jochen Weller
University Observatory, LMU

Outline

1. Structure formation
2. Analytical description cold dark matter
   a. dust model
   b. Schrödinger method
3. Correlation functions
   a. coarse-grained dust power spectra
   b. halo correlation incl. redshift space distortions
4. Summary

Cosmological Structure Formation

-13.8 billion years: nearly uniform initial state

today: rich structures in cosmic web

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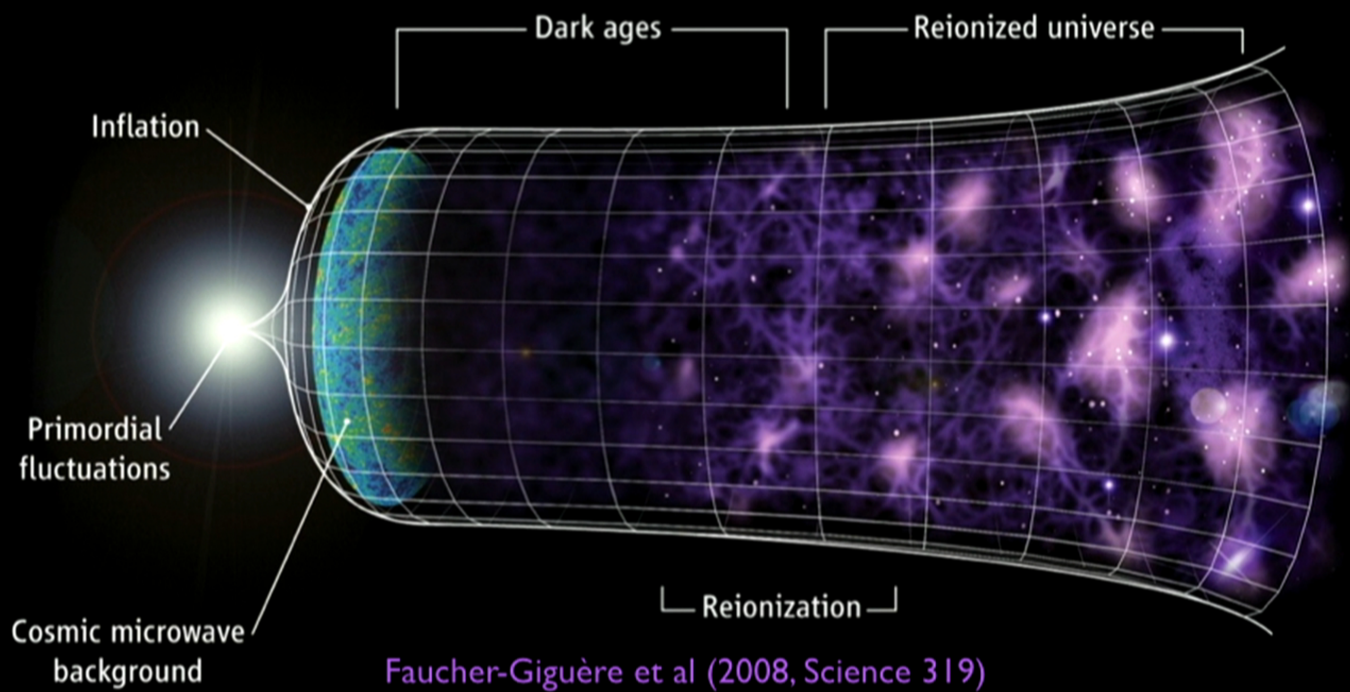

Faucher-Giguère et al (2008, Science 319)

today: rich structures in cosmic web

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Inflation

- established `boring` initial conditions
  - quantum fluctuations get amplified
  - primordial plasma cools → recombination → **CMB**

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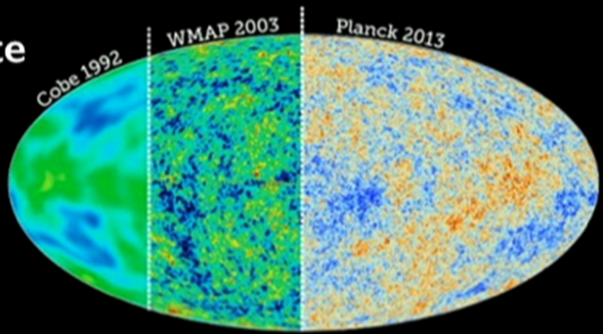

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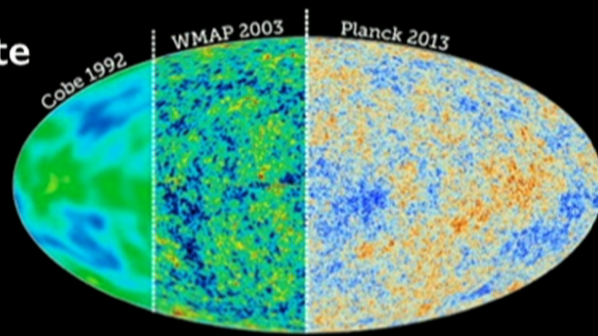

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- hierarchical
- tiny over-densities act as seeds
  - congregation via gravitational instability
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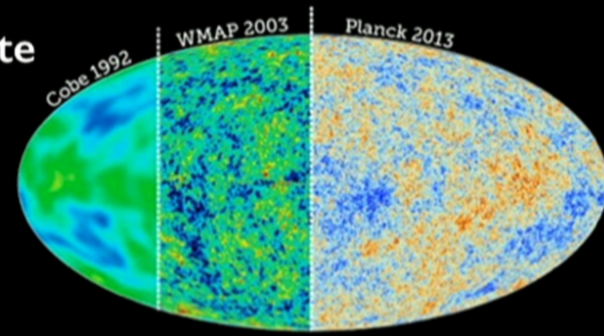

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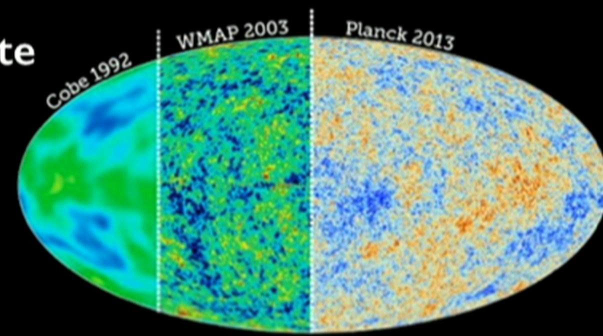

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- linear regime
  ✓ analytically understood
- nonlinear stage
  ?! N-body simulations inevitable

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Kravtsov & Klypin (simulations @NCSA)

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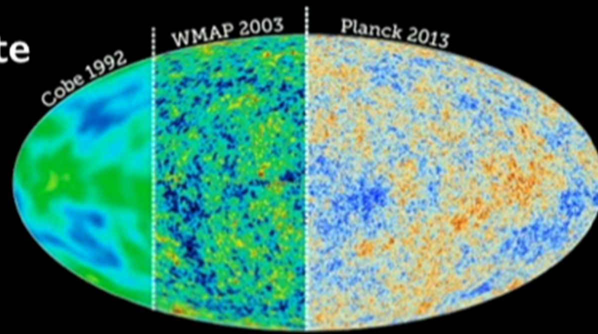

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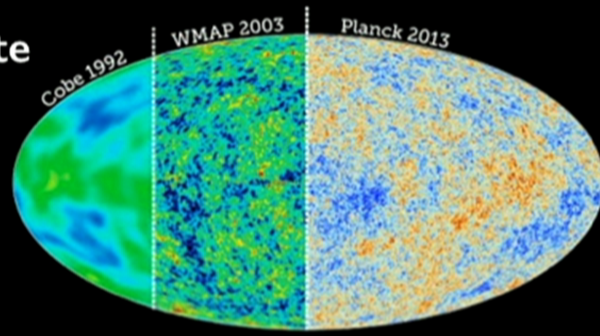

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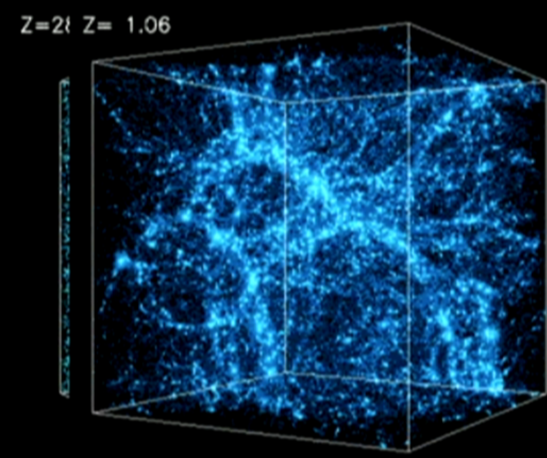

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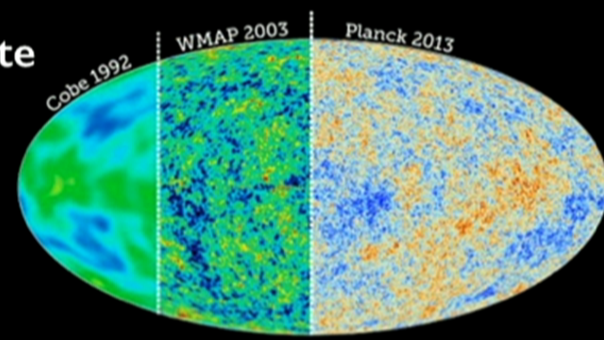

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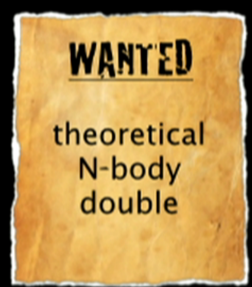

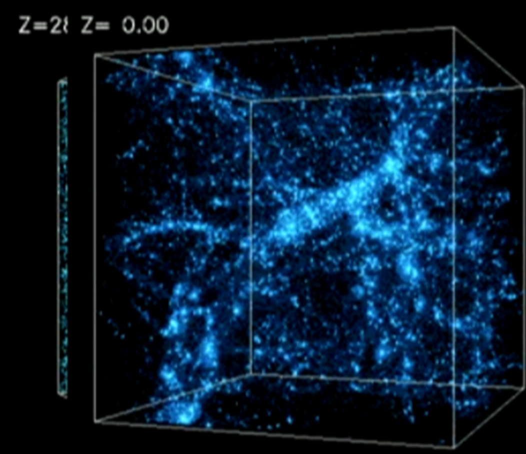

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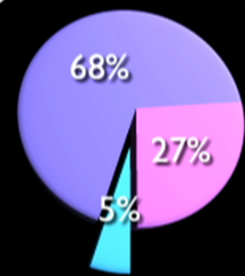
Cosmological Standard Model

ΛCDM



Dark Energy

cosmological constant

Cold Dark Matter

collisionless

cold (initially)

baryons

standard model
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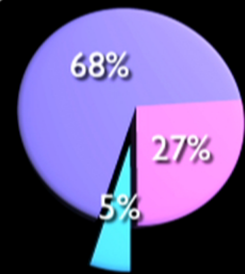

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Large Scale Structure (LSS)

dark matter halos

galaxy surveys

constrain parameters

halo density profiles

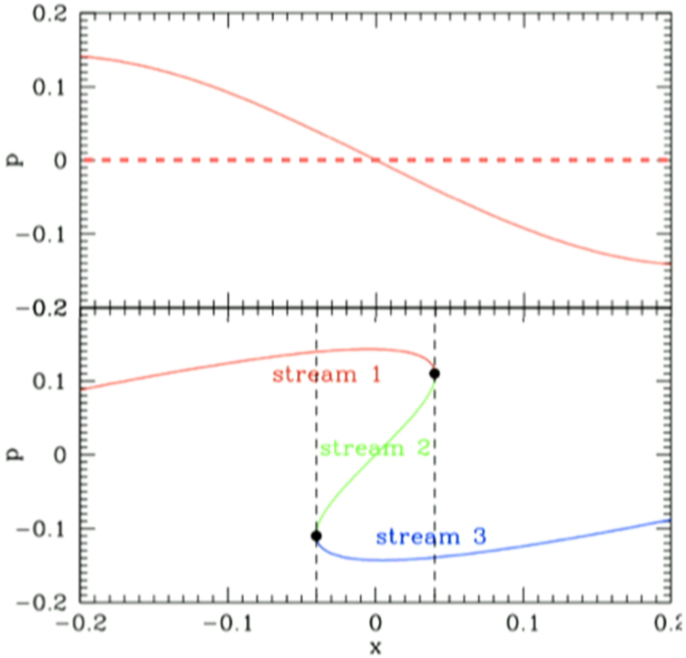
understand universality

Describing Cold Dark Matter

phase space distribution function f(t,x,p)

- describes number density & distribution of momenta *p*

theoretical expectation



Pueblas & Scoccimarro (2009, PRD 80)

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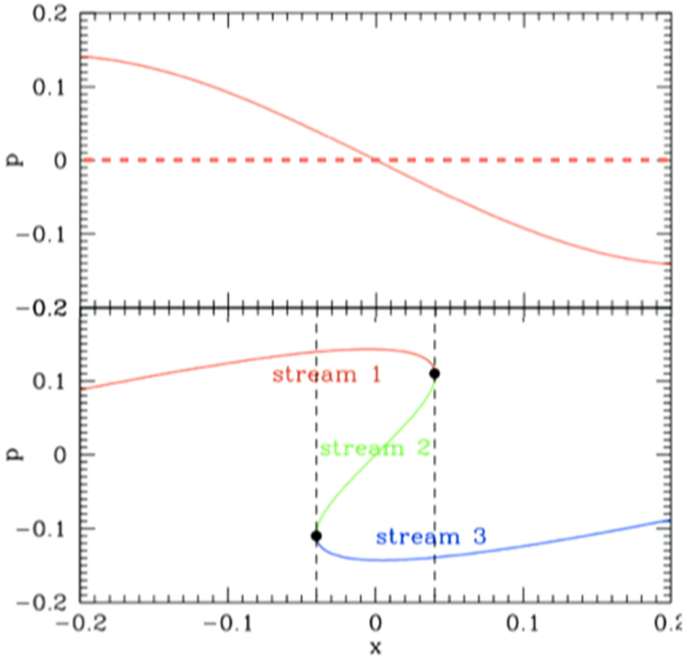

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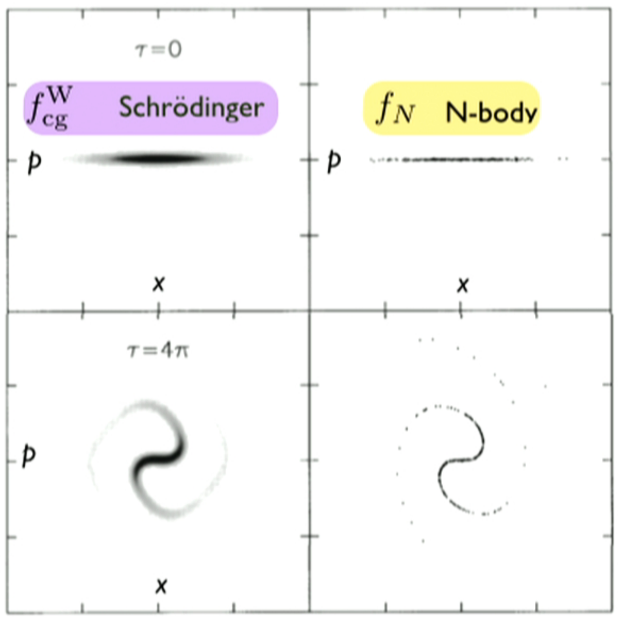
numerical realization



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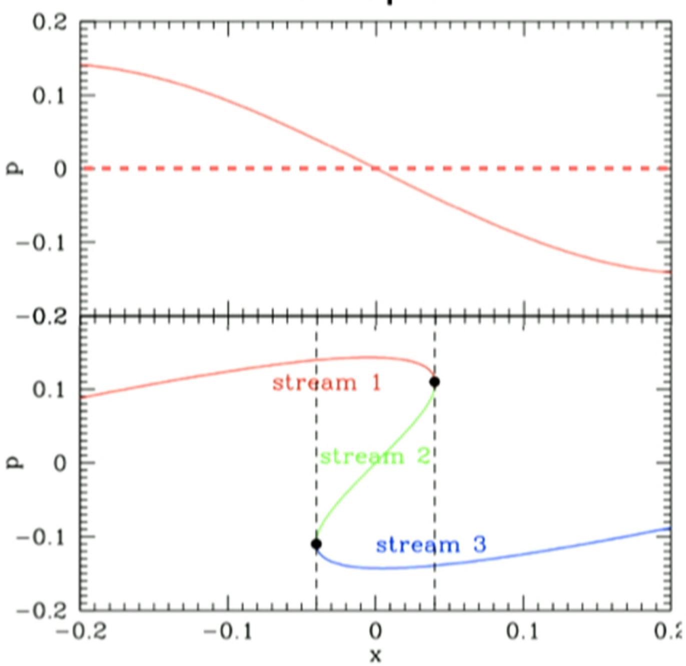

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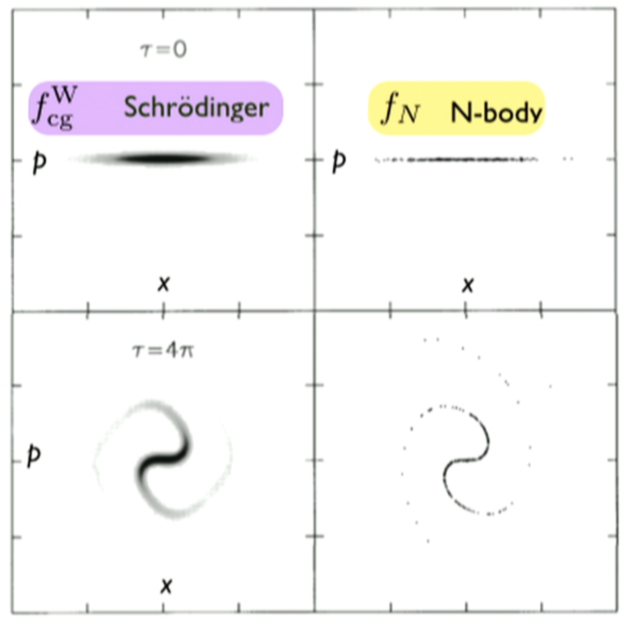

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Describing Cold Dark Matter

phase space distribution function f(t,x,p)

- **N-body**: non-relativistic, only gravitationally
- **continuous**: ensemble average, no collisions

$$f_N = \sum_i \delta_D(\boldsymbol{x} - \boldsymbol{x}_i)\delta_D(\boldsymbol{p} - \boldsymbol{p}_i)$$

$f_N \rightarrow f$

Vlasov - Poisson equation

$$\partial_\tau f(\boldsymbol{x}, \boldsymbol{p}, \tau) = -\frac{\boldsymbol{p}}{am}\boldsymbol{\nabla}_x f + am\boldsymbol{\nabla}_x V \boldsymbol{\nabla}_p f$$

gravitational potential

$$\Delta V(\boldsymbol{x}, \tau) = \frac{4\pi G m}{a}\left(n(\boldsymbol{x}, \tau) - \langle n \rangle\right)$$

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partial

nonlinear

integro

number density

$$n = \int \mathrm{d}^3 p \ f$$

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Hierarchy of Moments $M^{(n)}(\boldsymbol{x}) = \int d^3p\; p_{i_1}\ldots p_{i_n} f$

- density n(x): $M^{(0)} = n(\boldsymbol{x})$, velocity **v**(x): $M^{(1)} = n\boldsymbol{v}(\boldsymbol{x})$
- velocity dispersion $\boldsymbol{\sigma}$(x): $M^{(2)} = n\,(\boldsymbol{v}\boldsymbol{v} + \boldsymbol{\sigma})\,(\boldsymbol{x}), \ldots$

cumulant

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$$\partial_t M^{(n)} = -\frac{1}{a^2 m}\nabla \cdot M^{(n+1)} - m\nabla V \cdot M^{(n-1)}$$

infinite coupled hierarchy

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infinite coupled hierarchy

Dust model

dust model

- only consistent truncation of hierarchy
- pressureless fluid: density and velocity

$$f_{\rm d}(\boldsymbol{x}, \boldsymbol{p}, \tau) = n(\boldsymbol{x}, \tau)\delta_D^{(3)}(\boldsymbol{p} - \boldsymbol{\nabla}\phi(\boldsymbol{x}, \tau))$$

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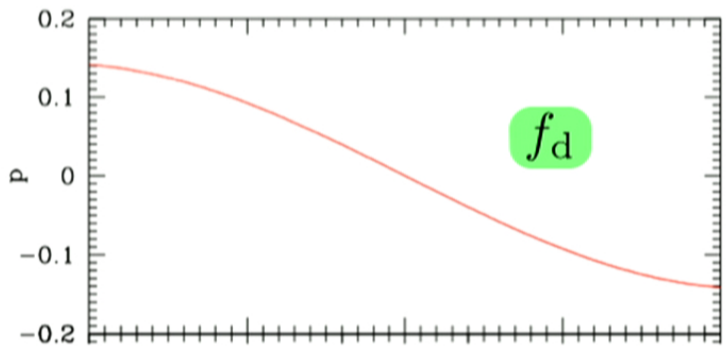
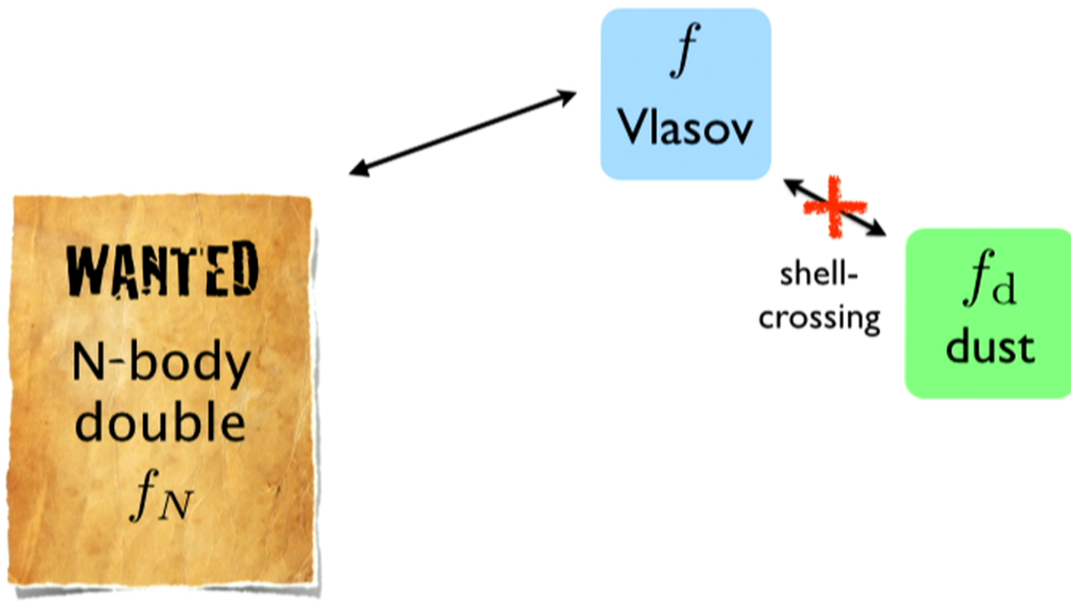
Euler $\partial_\tau \phi = -\frac{1}{2am}(\boldsymbol{\nabla}\phi)^2 - amV$



Schrödinger method at a glance



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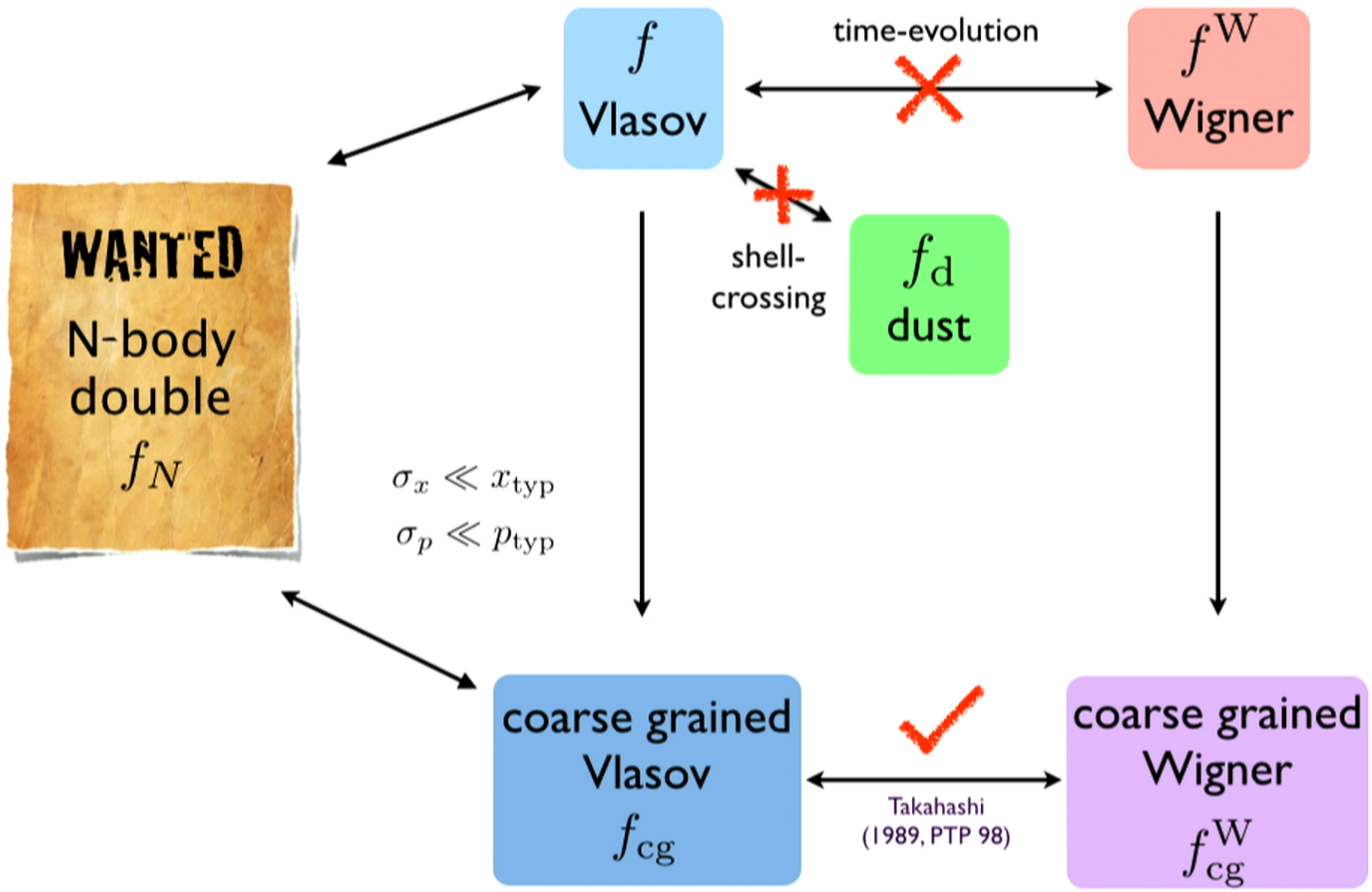

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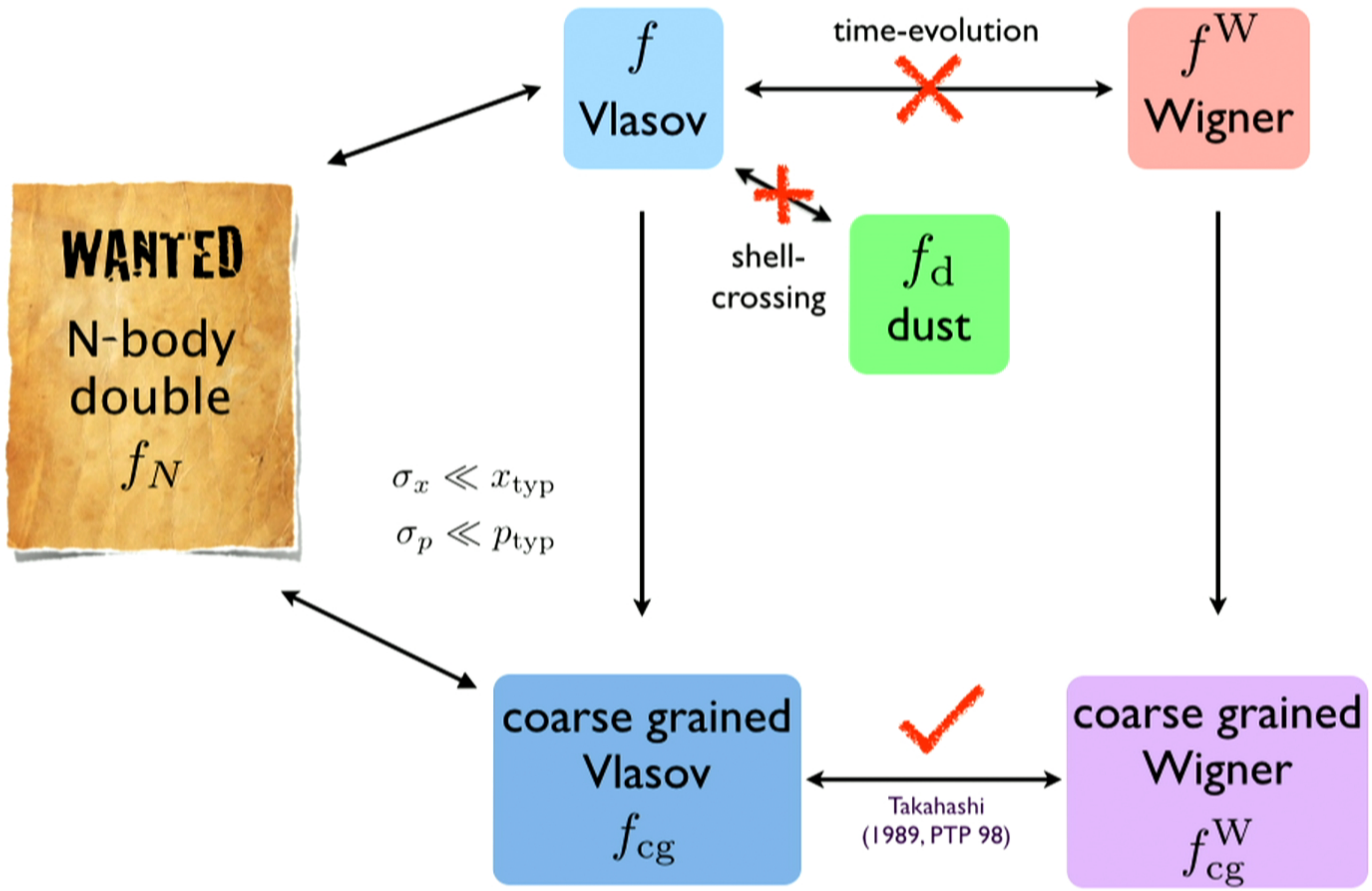

Schrödinger method

Schrödinger method

- Coarse-grained Wigner function, constructed from self-gravitating field

$$\psi = \sqrt{n} \exp\left(\frac{i}{\hbar}\phi\right)$$

Schrödinger - Poisson equation

$$i\hbar\partial_t\psi = \left[-\frac{\hbar^2}{2a^2m}\Delta + mV\right]\psi$$

$$\Delta V = \frac{4\pi G\rho_0}{a}(|\psi|^2 - 1)$$

degrees of freedom

- 2: amplitude n & phase ϕ

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parameters

- coarse-graining σ_x, σ_p
  - fundamental resolution $\sigma_x\sigma_p \gtrsim \hbar/2$
- Schrödinger scale $\hbar$
  - degree of restriction
  - dust as special case

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Euler $\partial_\tau\phi = -\frac{1}{2am}(\boldsymbol{\nabla}\phi)^2 - amV + \frac{\hbar^2}{2am}\left(\frac{\Delta\sqrt{n}}{\sqrt{n}}\right)$

quantum potential

Features of Schrödinger Method

phase space density

Multi-streaming

❌ dust model: fails at shell-crossing

✅ Schrödinger method: **beyond shell-crossing**

blue S contours: Schrödinger method
red dotted Z line: Zeldovich solution (dust model)

Schrödinger method

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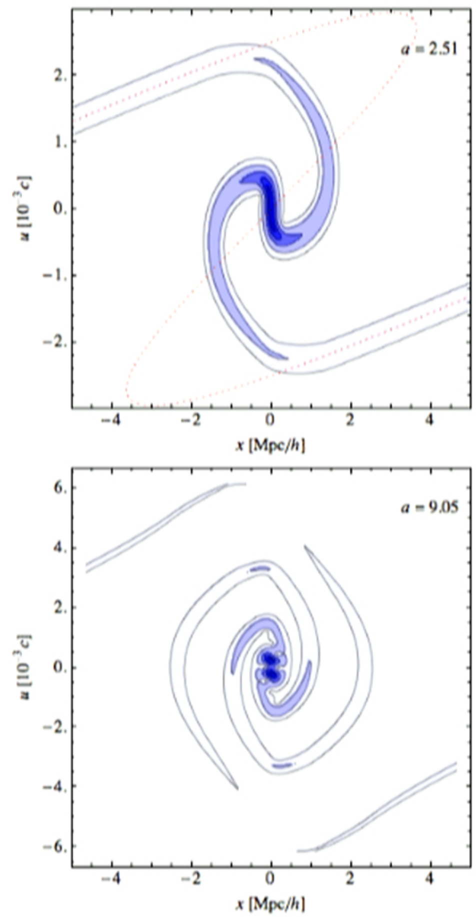
Virialization

❌ even in extended models: no **virialization**

✅ Schrödinger method: **bound structures** - halo formation

phase space density



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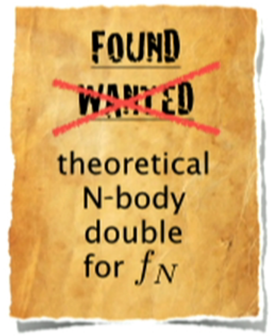
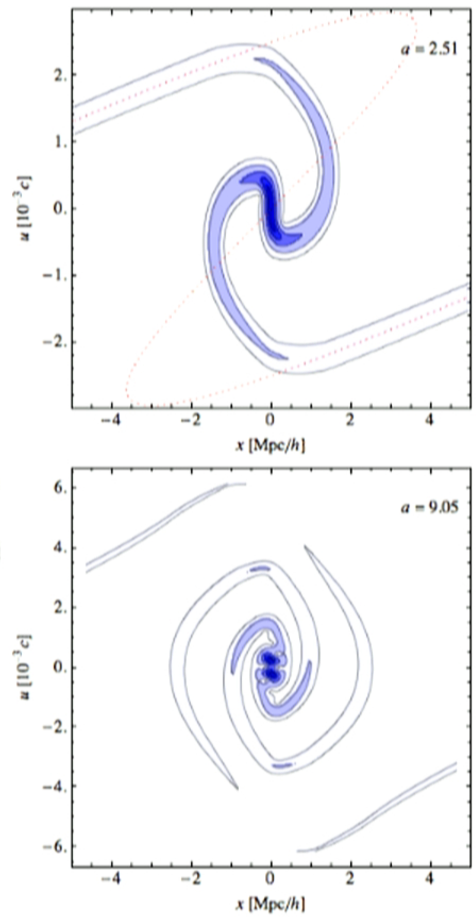


Virialization

❌ even in extended models: no **virialization**

✅ Schrödinger method: **bound structures** - halo formation

phase space density



Features of Schrödinger Method

Schrödinger method

- Coarse-grained Wigner function, constructed from self-gravitating field

$$\psi = \sqrt{n} \exp\left(\frac{i}{\hbar}\phi\right)$$

$$f_{cg}^W(\boldsymbol{x},\boldsymbol{p}) = \int \frac{d^3x' d^3p'}{(\pi\sigma_x\sigma_p)^3} \exp\left[-\frac{(\boldsymbol{x}-\boldsymbol{x}')^2}{2\sigma_x^2} - \frac{(\boldsymbol{p}-\boldsymbol{p}')^2}{2\sigma_p^2}\right] \int \frac{d^3\tilde{x}}{(2\pi\hbar)^3} \exp\left[2\frac{i}{\hbar}\boldsymbol{p}'\cdot\tilde{\boldsymbol{x}}\right] \psi(\boldsymbol{x}'-\tilde{\boldsymbol{x}})\bar{\psi}(\boldsymbol{x}'+\tilde{\boldsymbol{x}})$$

special p-dependence
allows to calculate
cumulants analytically

Cumulants

- lowest two: macroscopic density & velocity

$$\bar{n}(\boldsymbol{x}) = \exp\left[\tfrac{1}{2}\sigma_x^2\Delta\right] n(\boldsymbol{x}) \qquad \bar{\boldsymbol{v}}(\boldsymbol{x}) = \frac{1}{am\bar{n}(\boldsymbol{x})} \exp\left[\tfrac{1}{2}\sigma_x^2\Delta\right] (n\boldsymbol{\nabla}\phi)(\boldsymbol{x})$$

- higher cumulants given self-consistently
  evolution equations fulfilled automatically

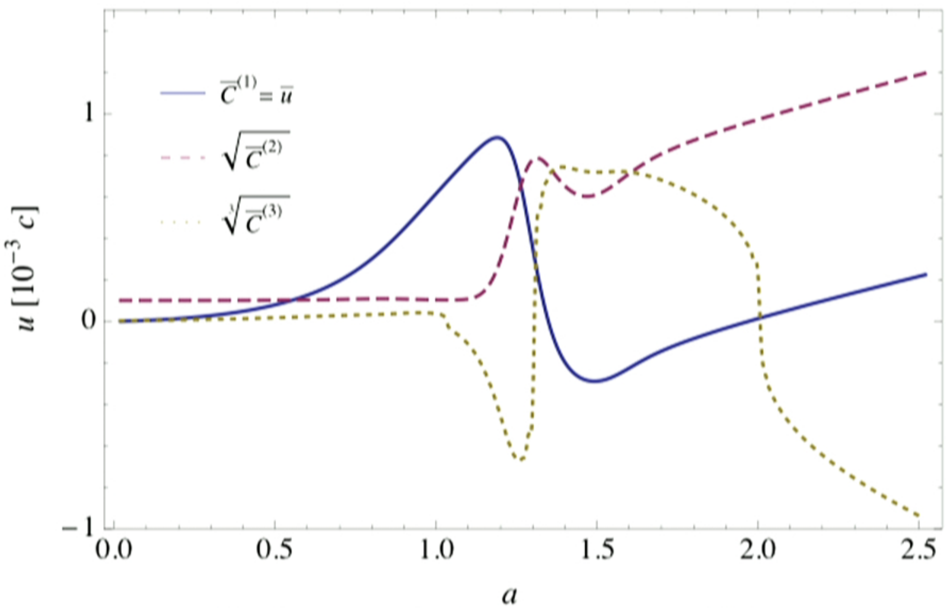
closure of hierarchy

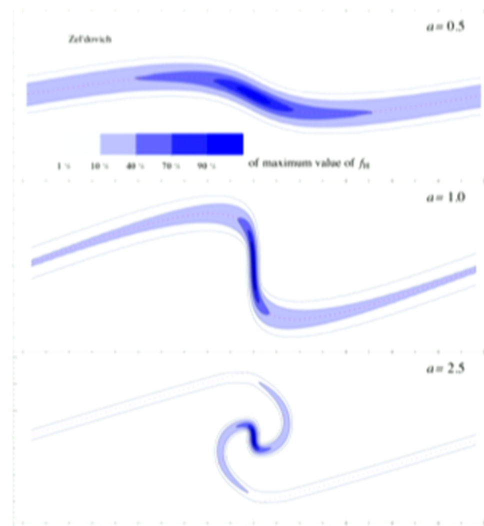
CU, Kopp & Haugg (2014, PRD 90, 023517)

Features of Schrödinger Method

Multi-streaming

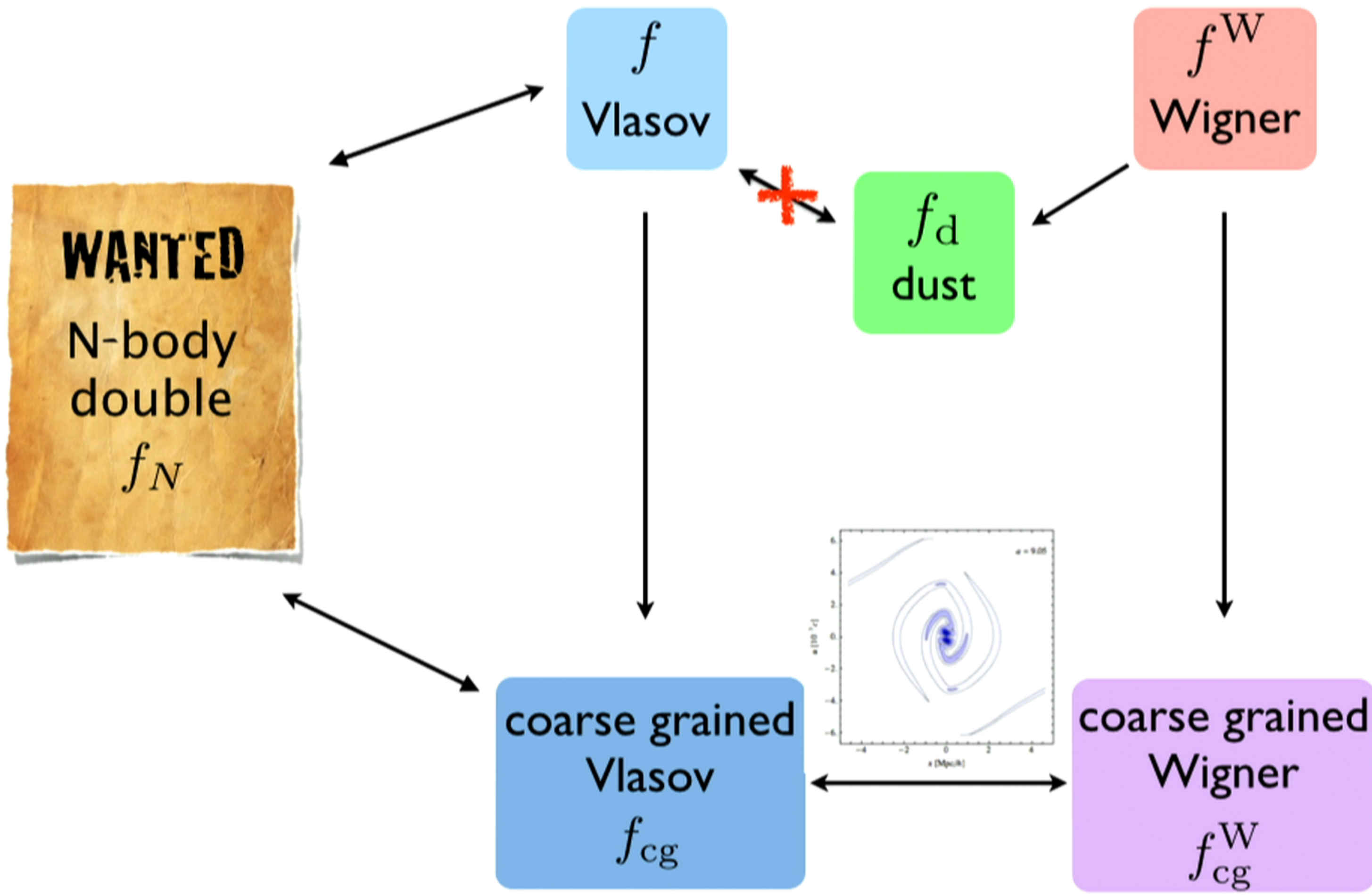
- higher cumulants encode **multi-streaming** effects
- during shell-crossing: **higher moments sourced dynamically**



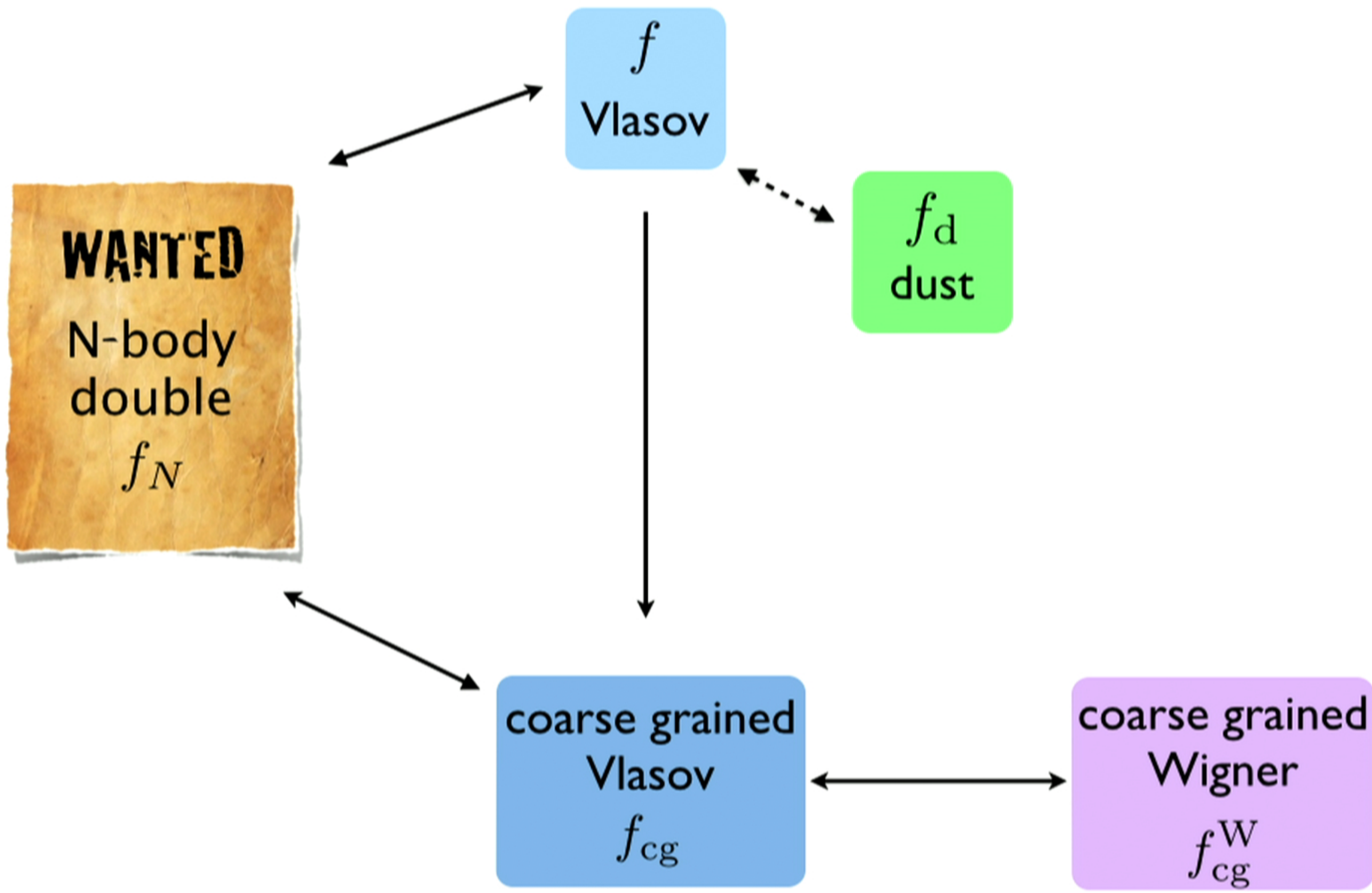


Schrödinger method: cumulants at x = −0.5 Mpc:
all equally important after shell crossing

Schrödinger method at a glance



Application: Perturbation theory



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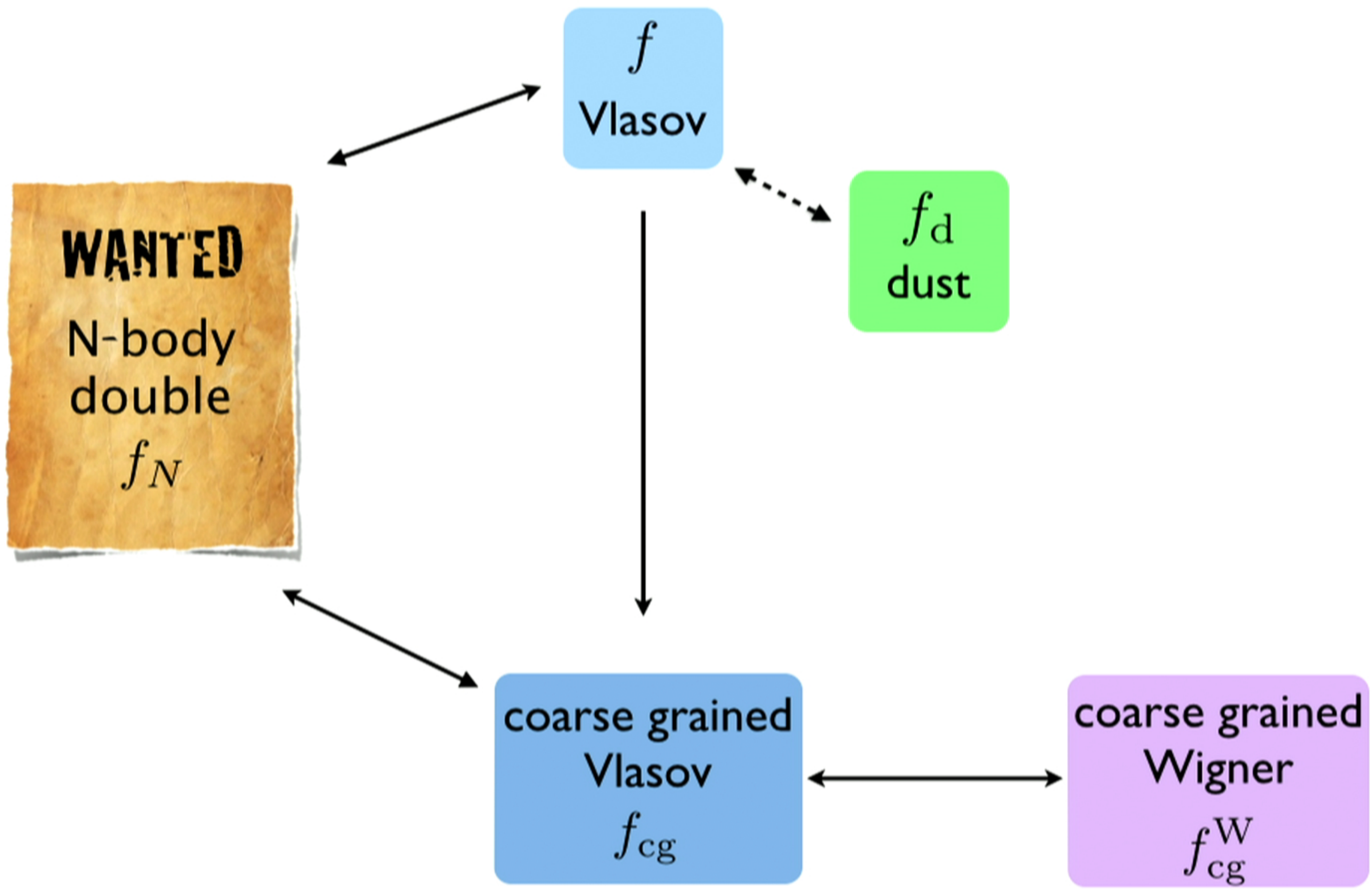

Eulerian Perturbation Theory

Dust model

- express fluid equations in terms of $\delta = n - 1$ and $\theta = \boldsymbol{\nabla} \cdot \boldsymbol{v} \propto \Delta\phi$ (no vorticity)
- perturbative expansion: separation ansatz (fastest growing mode)

$$\delta(\tau, \boldsymbol{k}) = \sum_{n=1}^{\infty} a^n(\tau)\ \delta_n(\boldsymbol{k}) \qquad \theta(\tau, \boldsymbol{k}) = \mathcal{H}(\tau) \sum_{n=1}^{\infty} a^n(\tau)\ \theta_n(\boldsymbol{k})$$

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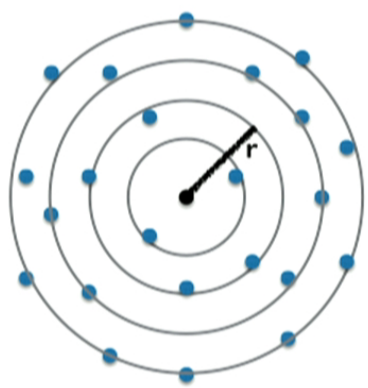
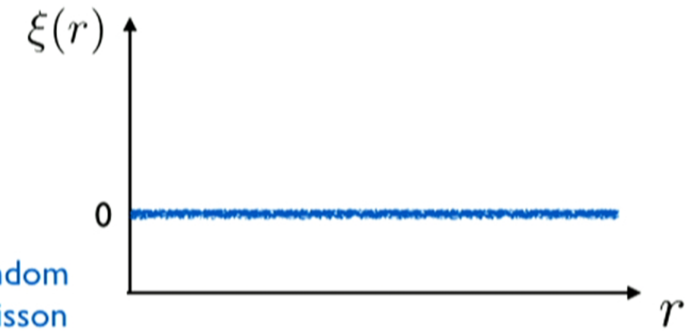
Correlation function

- 2-point correlation: excess probability of finding 2 objects separated by **r**

$$dP = n[1 + \xi(r)]dV \qquad \text{homogeneity \& isotropy: } \xi(\boldsymbol{r}) = \xi(r)$$





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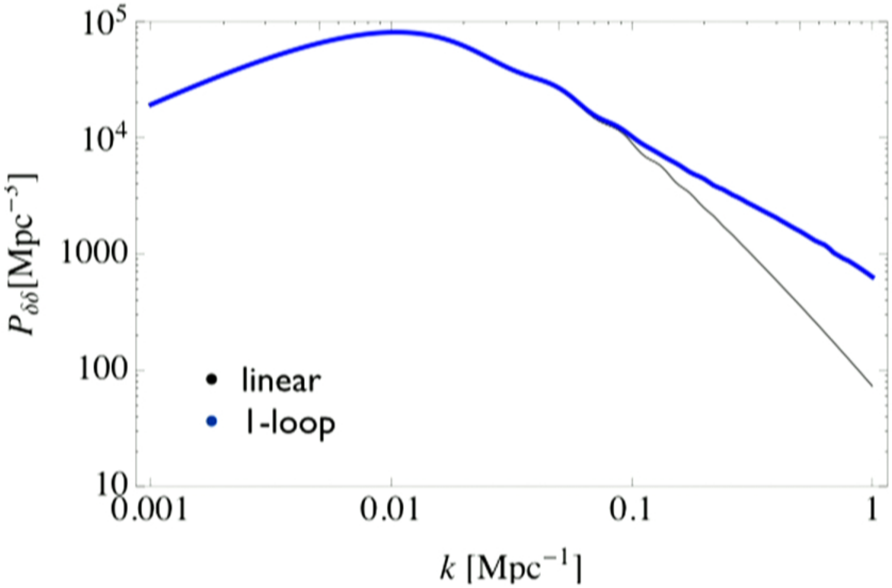
Density power spectrum



correlation function

=

FT of power spectrum

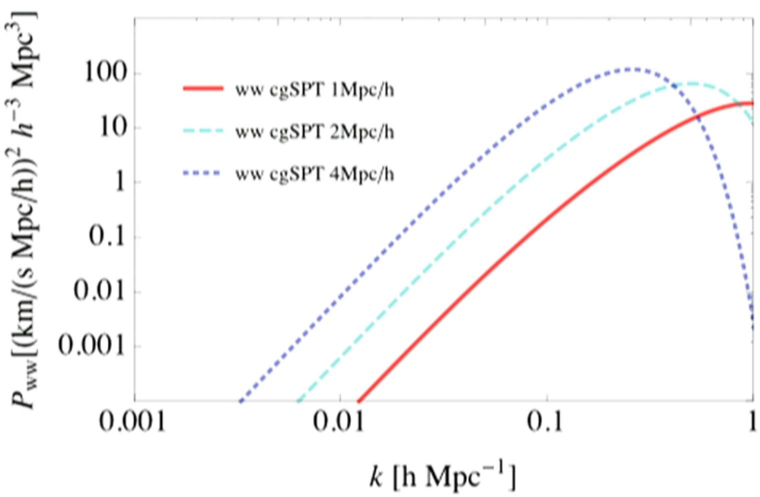
$$\xi(r) = \frac{1}{2\pi^2} \int dk\ k^2 P(k) \frac{\sin(kr)}{kr}$$

Eulerian Perturbation Theory

Coarse grained dust model

- same procedure, but mass-weighted velocity $\bar{v} := \frac{\overline{nv}}{\bar{n}}$
- large scale vorticity $\bar{w} := \nabla \times \bar{v} \neq 0$!

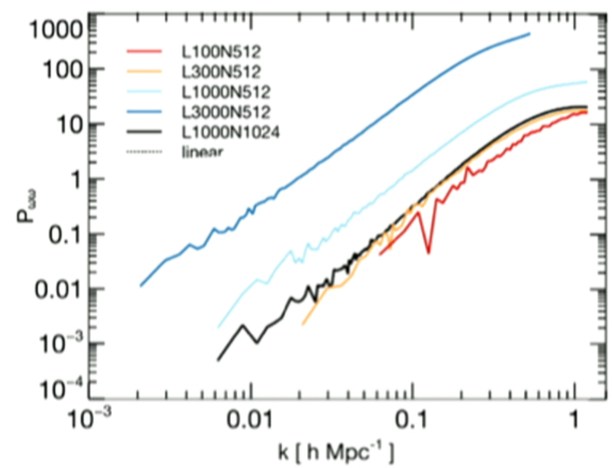
Vorticity power spectrum $P_{ww}(k)$



CU & Kopp
arXiv: 1407.4810

corresponding N-body data

Hahn, Angulo & Abel (2014, arXiv:1404.2280)



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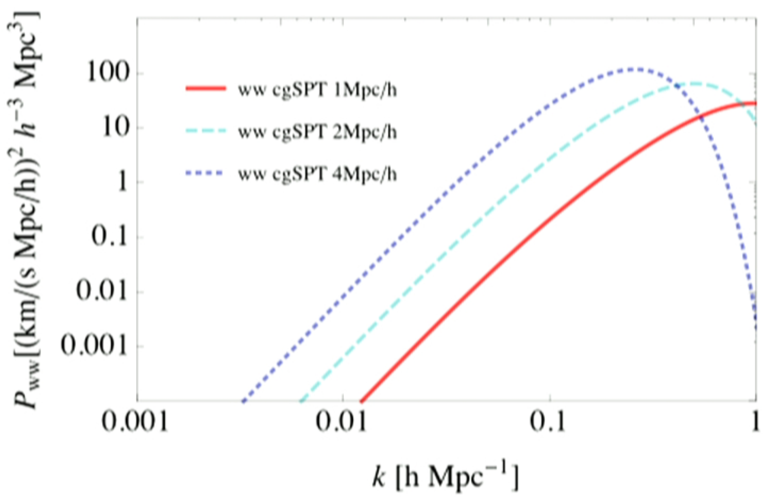

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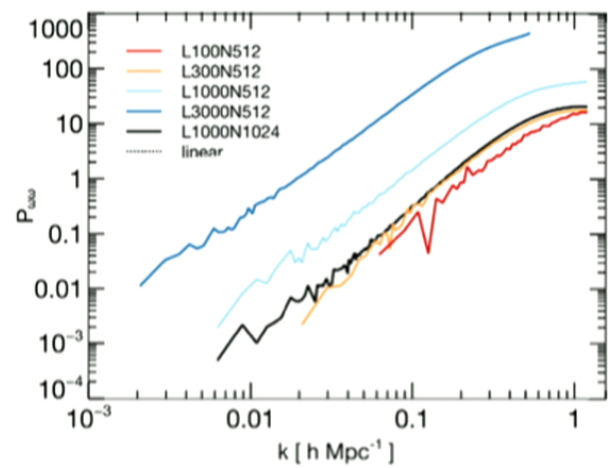

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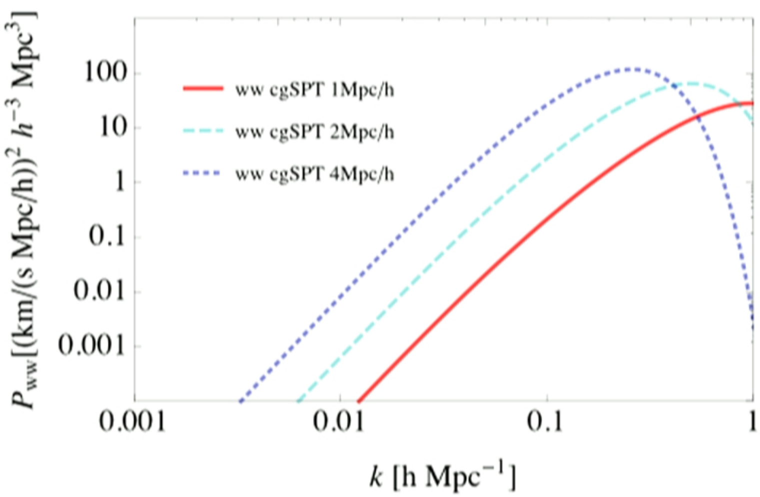

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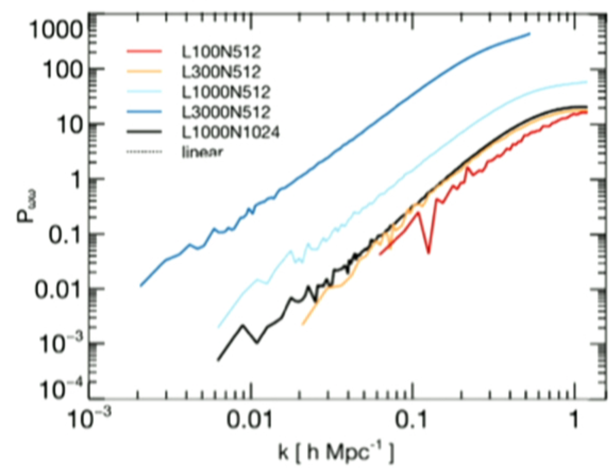

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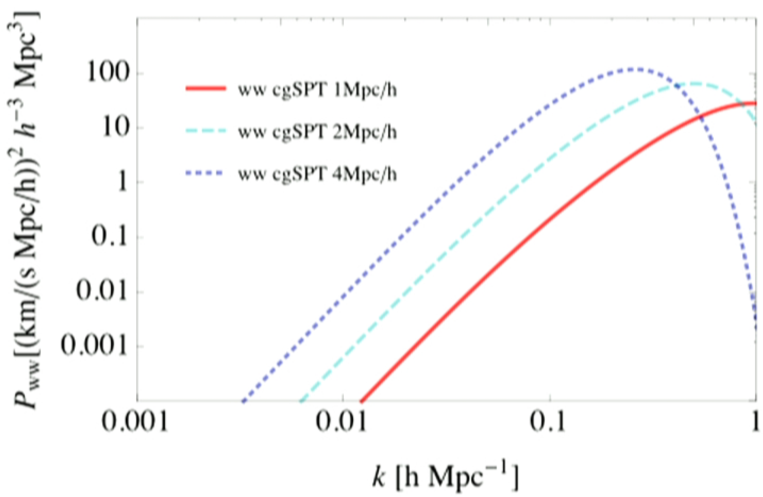

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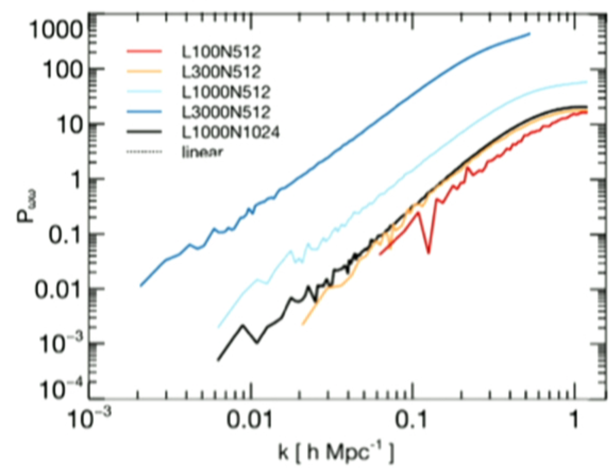


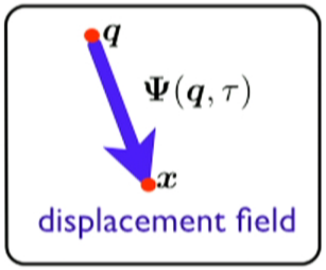
Halo correlation in redshift space with the Coarse-grained dust model

Halo correlation function

Lagrangian perturbation theory

- central quantity: $\boldsymbol{\Psi}(\boldsymbol{q}, \tau)$, perturbative expansion
- relation to density: mass conservation $[1+\delta(\boldsymbol{x})]d^3x = d^3q$

Rampf & Buchert (2012, JCAP 6)



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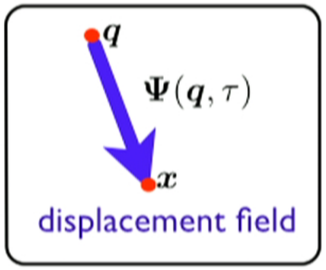
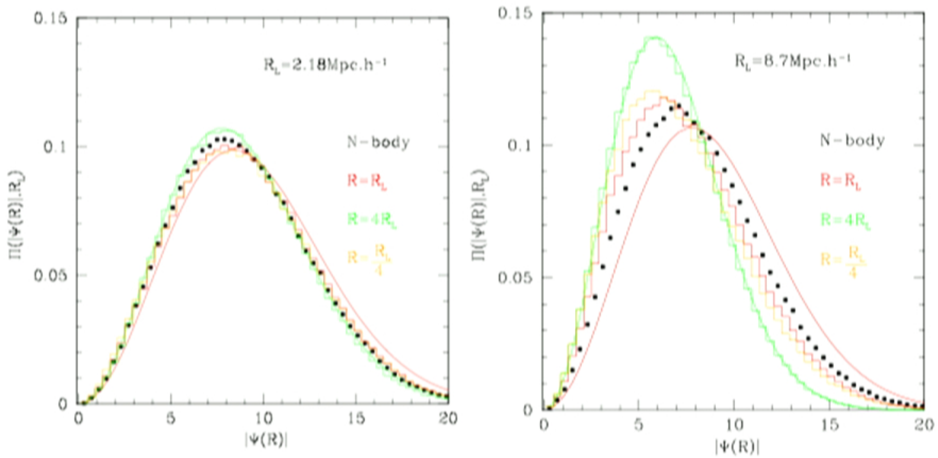


(Post-)Zel'dovich approximation

- **Zel'dovich**: 1st order, relation non-pert. Zel'dovich (1970, A&A 5, 84)
  - physically motivated resummation of SPT
- **truncated**: smoothed input power spectrum Coles, Melott, Shandarin (1993, MNRAS 260)
  - improves agreement with N-body



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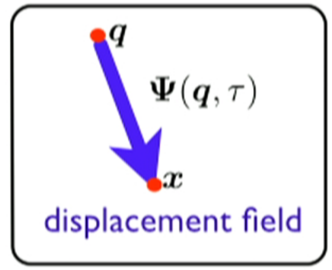

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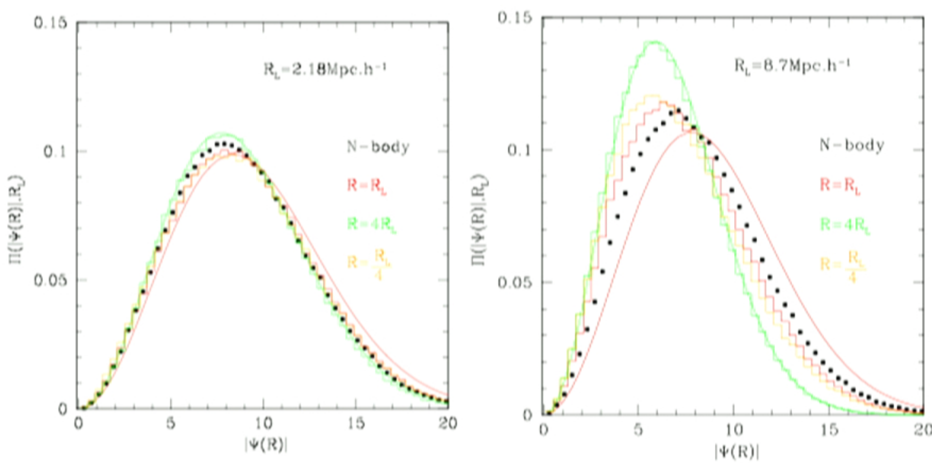
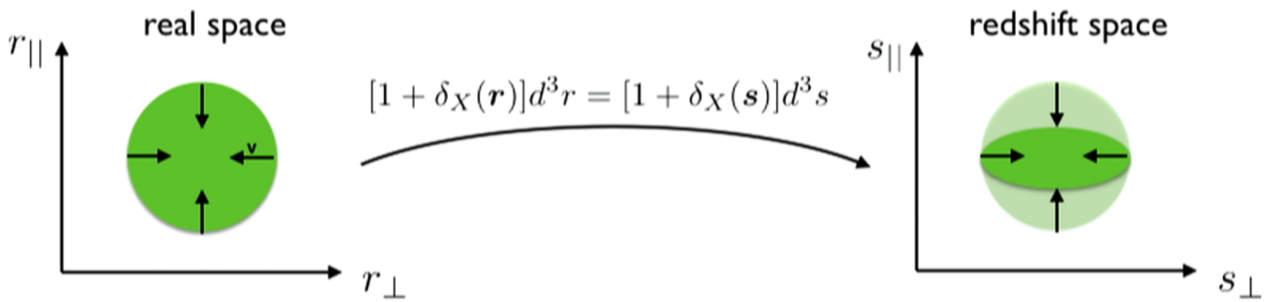


Redshift space distortions

Redshift space distortions

- observations in redshift space affected by velocities $s_{||} = r_{||} + v/\mathcal{H} \quad s_\perp = r_\perp$



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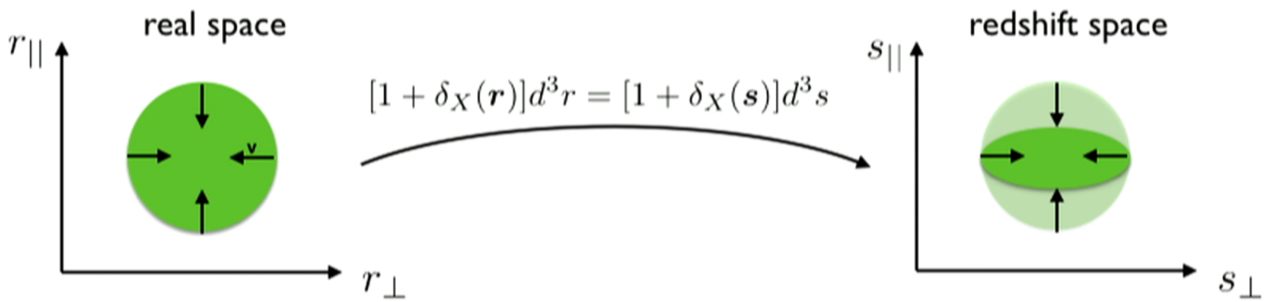


Gaussian streaming model

- joint probability distribution: density & velocity

Redshift space correlation function

real space correlation

Gaussian velocity distribution

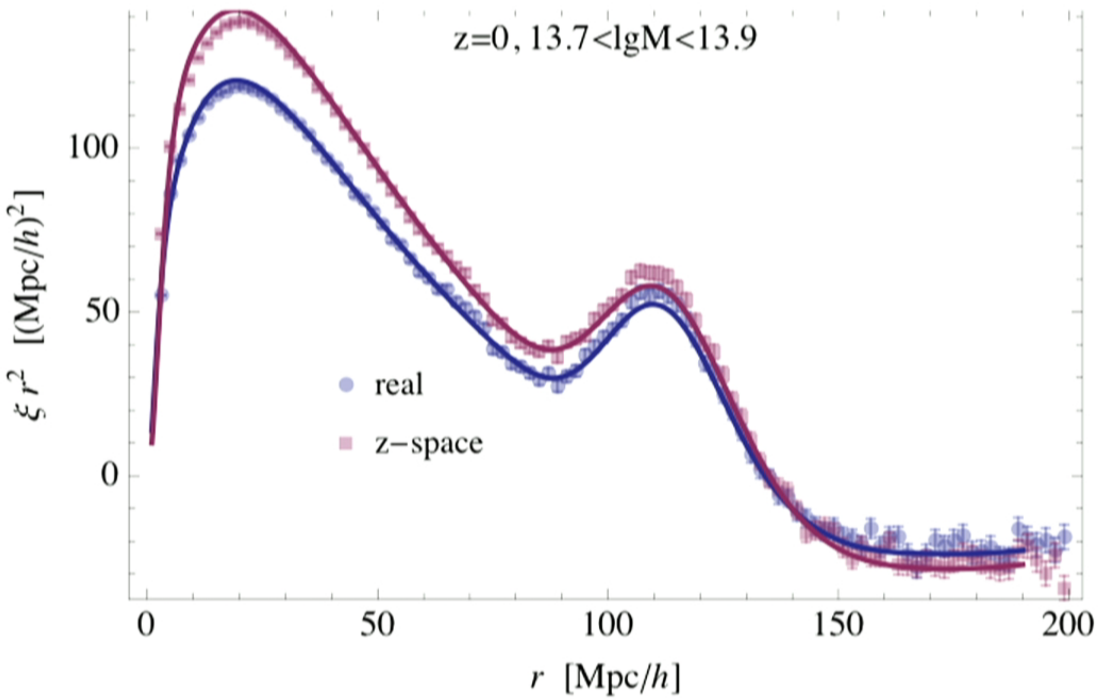
$$1+\xi_X(s_{||}, s_\perp, t) = \int_{-\infty}^{\infty} \frac{dr_{||}}{\sqrt{2\pi}\sigma_{12}(r, r_{||}, t)} (1+\xi_X(r,t)) \exp\left[-\frac{(s_{||} - r_{||} - v_{12}(r,t) r_{||}/r)^2}{2\sigma_{12}^2(r, r_{||}, t)}\right]$$

pairwise velocity: mean & variance

Redshift space distortions

Gaussian streaming model with CLPT

- real space ξ(r) & redshift space monopole $\xi_0(s)$
- well described by Convolution LPT   Carlson et al. (2012, MNRAS 429)



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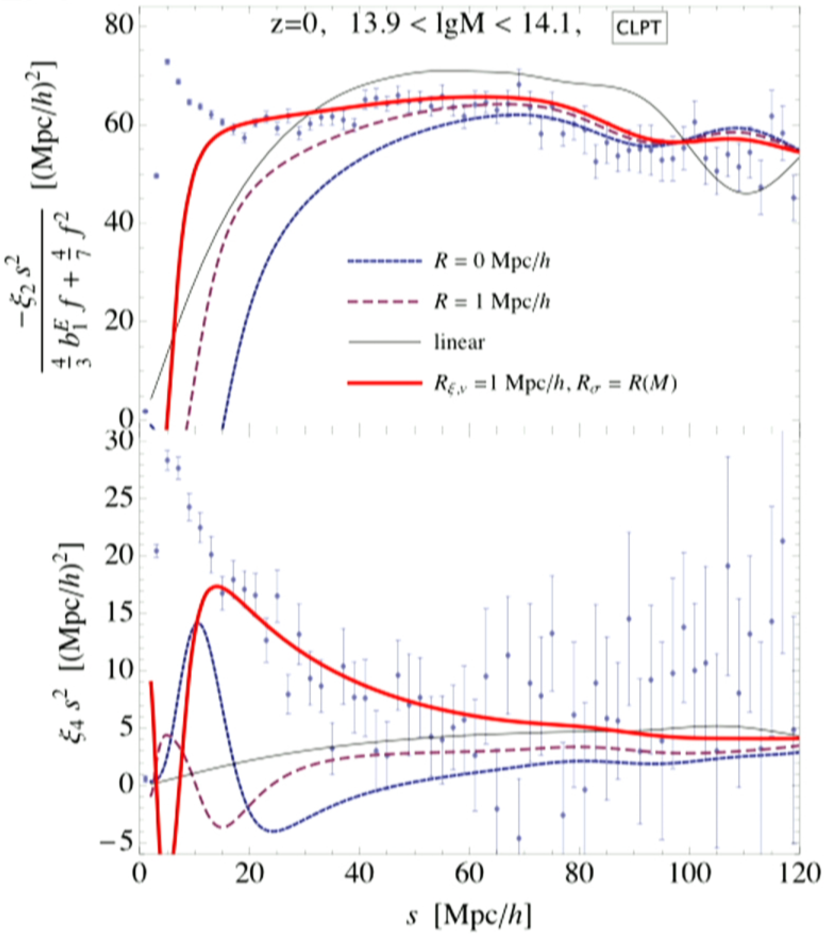

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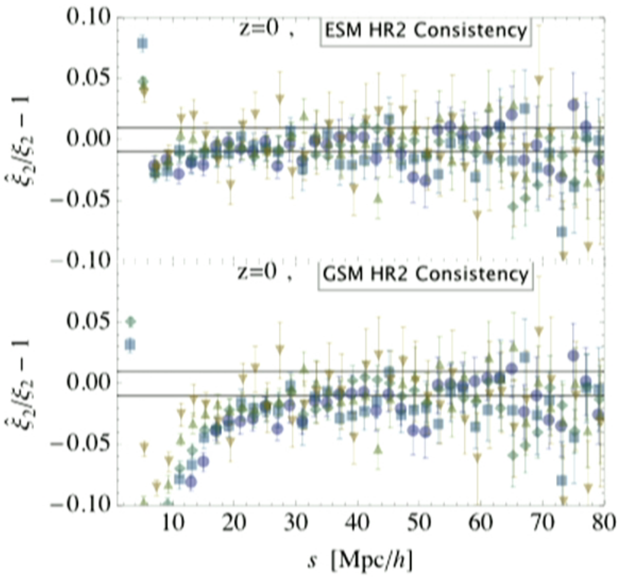
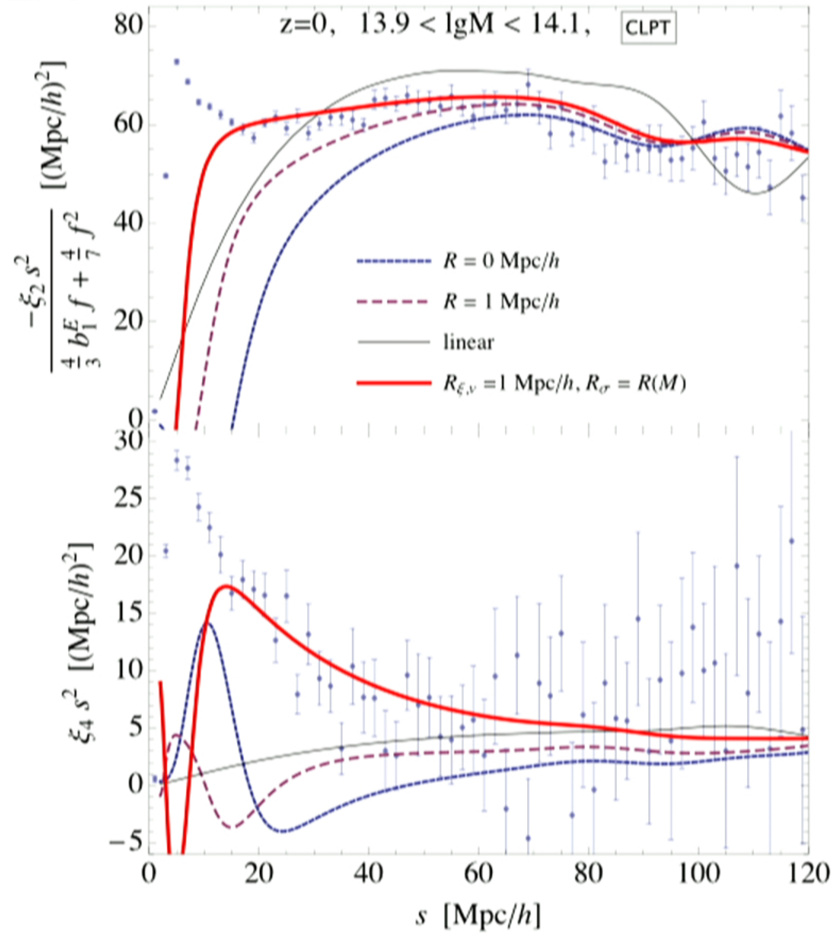
Edgeworth streaming model

incl. non-Gaussian corrections





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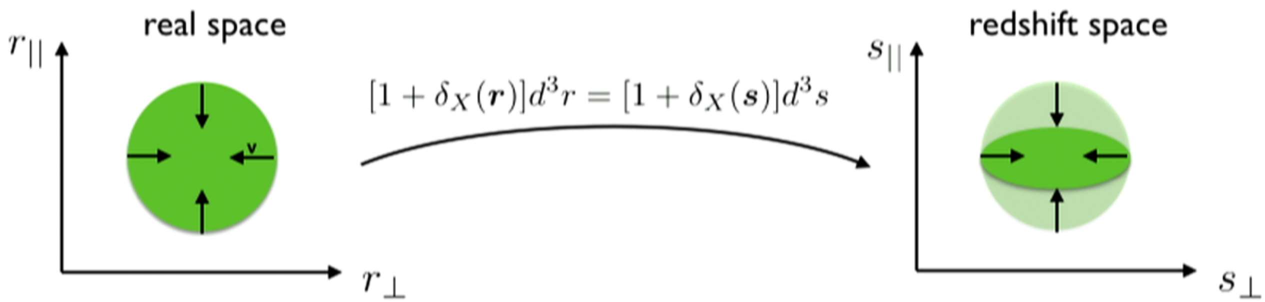

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Conclusion & Prospects

Schrödinger method

- models CDM using self-gravitating scalar field
- analytical tool for structure formation
  - multi-streaming & virialization

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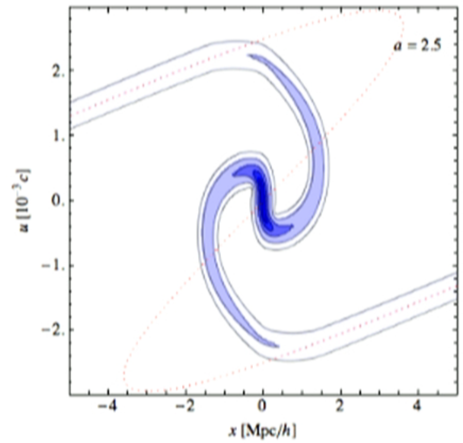

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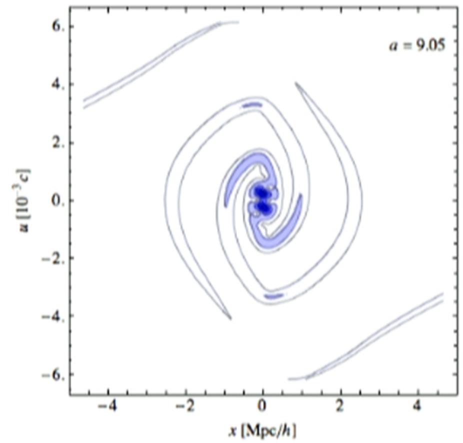

Coarse-grained dust model

- mass-weighted velocity
  - vorticity compatible with N-body

CU & Kopp (arXiv: 1407.4810)

Halo correlation in redshift space

- generalization of Gaussian streaming model
- truncated Post-Zel'dovich approximation

Kopp, CU, Haugg & Achitouv (in preparation)

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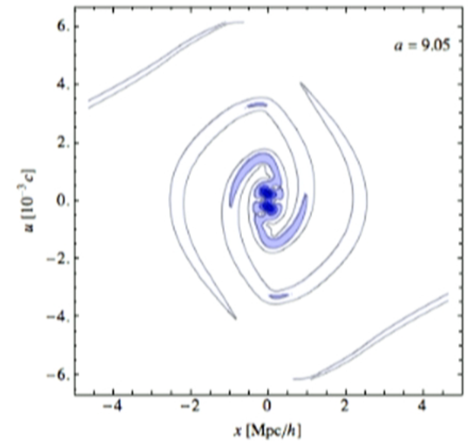

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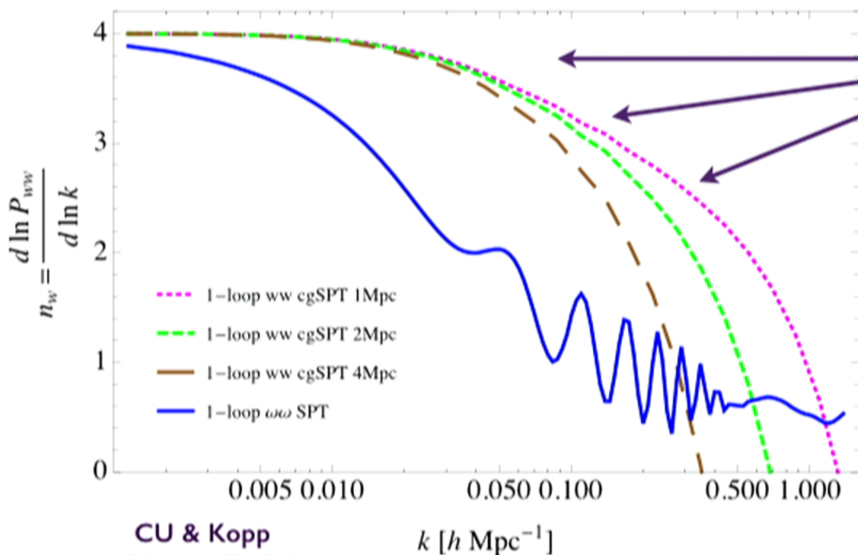
Spectral index of vorticity power spectrum



CU & Kopp
arXiv: 1407.4810

corresponding results EFT of LSS

Carrasco, Foreman, Green, Senatore
(2013, arXiv: 1310.0464)

$$n_{\boldsymbol{w}} = \begin{cases} 4 & \text{for} \quad k \lesssim 0.1 \\ 3.6 & \text{for} \quad 0.1 \lesssim k \lesssim 0.3 \\ 2.8 & \text{for} \quad 0.3 \lesssim k \lesssim 0.6 \end{cases}$$

usual estimate for vorticity
arising from mass-weighted velocity

$$\boldsymbol{\omega} \sim \frac{\boldsymbol{\nabla} \times [(1+\delta)\boldsymbol{v}]}{1+\delta}$$