#### Title: Cosmological Constraints on theories of Modified Gravity

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Abstract: <span>Recently, research in cosmology has seen a growing interest in theories of gravity beyond General Relativity (GR). From an observational point of view, there are two main reasons for this. Firstly, the law of gravity has never been directly tested on scales larger than the Solar System. Hence, by understanding better the various signatures that different gravity models can leave on cosmological observables, one can improve the chances of identifying any departures from GR, or alternatively, extend the model's observational success into a whole new regime. Secondly, theories of modified gravity can arise also as an alternative to the cosmological constant (or any other form of dark energy) to explain the current accelerated expansion of the Universe. Using my results from suitably modified Boltzmann, N-body codes and semi-analytical models of structure formation, I will describe the way modified gravity models typically impact a series of cosmological observables. I will use two popular models as examples, which are known as Galileon and Nonlocal Gravity. In the Galileon model, the modifications to gravity on large scales are driven by nonlinear derivative interactions of a scalar field, which can nevertheless be suppressed in the Solar System by means of a mechanism known as Vainshtein screening. This model can provide a good fit to the latest CMB, BAO and SNIa data, although with different cosmological parameters than the standard LCDM model. Specifically, unlike LCDM, the Galileon model predicts nonzero neutrino masses (over 5sigma) and the constraints on the Hubble rate are compatible with its local determinations. The results from my N-body simulations and Halo Occupation Distribution analysis also show that the model can describe the measured clustering amplitude of Luminous Red Galaxies and that the screening mechanism can be very efficient in "hiding" the modifications to gravity on small scales. However, these results also show that the observational viability of the model may be under pressure due to the combined constraints derived from the sign of the ISW effect and from Solar System tests. In the Nonlocal model, the acceleration of the universe is driven by terms that involve the inverse of a derivative operator acting on curvature tensors. This model is also likely to pass large-scale structure constraints with the same flying colors as Galileon gravity, but the lack of a screening mechanism in this model makes it unclear on whether or not it is able to satisfy Solar System constraints. These steps I will describe for the cases of the Galileon and Nonlocal models can be viewed as guidelines for one to place constraints on other (or not yet invented) models of modified gravity. References: The results I will describe are based on the following papers: arXiv:1208.0600, arXiv:1302.6241, arXiv:1306.3219, arXiv:1308.3699, arXiv:1401.1497, arXiv:1404.1365, arXiv:1406.0485, arXiv:1408.1084</span>

# Why modified gravity



- Preserve past inflationary, radiation and matter dominated eras.
- Modifications have to be small in the Solar System, where GR is very successful.

# Welcome to the Jungle

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + \mathcal{L}_m \right] \qquad \qquad \blacktriangleright \text{ No acceleration}$$

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + \mathcal{L}_m + \frac{\Lambda}{8\pi G} \right] \longrightarrow \text{Standard LCDM model}$$

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + \mathcal{L}_m + \frac{\nabla_\mu \varphi \nabla^\mu \varphi}{2} + V(\varphi) \right]$$

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + \mathcal{L}_m + f \left( \nabla_\mu \varphi \nabla^\mu \varphi \right) \right]$$

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + \mathcal{L}_m + f(R) \right]$$
$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + \mathcal{L}_m + f(\varphi)R + \frac{w(\varphi)\nabla_\mu \varphi \nabla^\mu \varphi}{2} + V(\varphi) \right]$$

)

# Welcome to the Jungle



# Welcome to the Jungle

## **Galileon origins**

Galilean shift transformation

$$\partial_\mu \varphi \longrightarrow \partial_\mu \varphi + b_\mu$$

In 4D Minkowski

Only 5 galilean-invariant Lagrangians that lead to EoM kept up to second order.

Nicolis et al. (2009)











Nicolis et al. (2009)

After covariantization Defayet et al. 2009

Acceleration of the Universe after radiation and matter domination.

Vainshtein (1972)

Vainshtein screening suppresses

modifications on small scales.

De Felice & Tsujikawa (2010)

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - \frac{1}{2} \sum_{i=1}^5 c_i \mathcal{L}_i - \mathcal{L}_m \right] \qquad M^3 \equiv M_{\rm Pl} H_0^2$$

$$\begin{aligned} \mathcal{L}_1 &= M^3 \varphi, \\ \mathcal{L}_2 &= \nabla_\mu \varphi \nabla^\mu \varphi, \\ \mathcal{L}_3 &= 2 \Box \varphi \nabla_\mu \varphi \nabla^\mu \varphi / M^3, \end{aligned}$$

$$\mathcal{L}_{4} = \nabla_{\mu}\varphi\nabla^{\mu}\varphi \left[2(\Box\varphi)^{2} - 2(\nabla_{\mu}\nabla_{\nu}\varphi)(\nabla^{\mu}\nabla^{\nu}\varphi) - R\nabla_{\mu}\varphi\nabla^{\mu}\varphi/2\right]/M^{6},$$

$$\mathcal{L}_{5} = \nabla_{\mu}\varphi\nabla^{\mu}\varphi \left[ (\Box\varphi)^{3} - 3(\Box\varphi)(\nabla_{\mu}\nabla_{\nu}\varphi)(\nabla^{\mu}\nabla^{\nu}\varphi) + 2(\nabla_{\mu}\nabla^{\nu}\varphi)(\nabla_{\nu}\nabla^{\rho}\varphi)(\nabla_{\rho}\nabla^{\mu}\varphi) - 6(\nabla_{\mu}\varphi)(\nabla^{\mu}\nabla^{\nu}\varphi)(\nabla^{\rho}\varphi)G_{\nu\rho} \right] / M^{9}.$$

$$\begin{aligned} & \mathcal{K}inetic Braiding \\ S &= \int d^{4}x \sqrt{-g} \left[ \frac{R}{16\pi G} - \frac{1}{2} \begin{array}{c} & \inf flat space, need explicit \\ & \operatorname{coupling to introduce fifth forces.} \end{array} \right] \\ & \mathcal{L} = -\frac{c_{2}}{2} \partial_{\mu} \varphi \partial^{\mu} \varphi - \frac{c_{3}}{\mathcal{M}^{3}} \partial^{2} \varphi \partial_{\mu} \varphi \partial^{\mu} \varphi - \frac{g}{M_{\mathrm{Pl}}} \varphi T_{\mu}^{\mu} \\ & \frac{c_{2}}{2} \partial^{2} \varphi + \frac{2c_{3}}{\mathcal{M}^{3}} \left[ (\partial^{2} \varphi)^{2} - \partial_{\mu} \partial_{\nu} \varphi \partial^{\mu} \partial^{\nu} \varphi \right] \left( \mathbf{x} - \frac{g}{M_{\mathrm{Pl}}} T_{\mu}^{\mu} \right) \\ & \mathcal{L}_{4} &= \nabla_{\mu} \varphi \nabla^{\mu} \varphi \left[ 2(\Box \varphi)^{2} - 2(\nabla_{\mu} \nabla_{\nu} \varphi)(\nabla^{\mu} \nabla^{\nu} \varphi) \\ & -R \nabla_{\mu} \varphi \nabla^{\mu} \varphi \right] / M^{6}, \\ & \mathcal{L}_{5} &= \nabla_{\mu} \varphi \nabla^{\mu} \varphi \left[ (\Box \varphi)^{3} - 3(\Box \varphi)(\nabla_{\mu} \nabla_{\nu} \varphi)(\nabla^{\mu} \nabla^{\nu} \varphi) \\ & + 2(\nabla_{\mu} \nabla^{\nu} \varphi)(\nabla_{\nu} \nabla^{\rho} \varphi)(\nabla_{\rho} \nabla^{\mu} \varphi) \\ & - 6(\nabla_{\mu} \varphi)(\nabla^{\mu} \nabla^{\nu} \varphi)(\nabla^{\rho} \varphi) G_{\nu\rho} \right] / M^{9}. \end{aligned}$$

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - \frac{1}{2} \sum_{i=1}^5 c_i \mathcal{L}_i - \mathcal{L}_m \right] \qquad M^3 \equiv M_{\rm Pl} H_0^2$$

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 $\mathcal{L}_1 = M^3 \varphi$ , Potential term; not interested.

$$\mathcal{L}_{2} = \nabla_{\mu} \varphi \nabla^{\mu} \varphi,$$

$$\mathcal{L}_{3} = 2 \Box \varphi \nabla_{\mu} \varphi \nabla^{\mu} \varphi / M^{3},$$
Cubic Galileon
(c4=c5=0)
Quartic Galileon
(c5=0)

$$\mathcal{L}_{4} = \nabla_{\mu}\varphi\nabla^{\mu}\varphi \left[2(\Box\varphi)^{2} - 2(\nabla_{\mu}\nabla_{\nu}\varphi)(\nabla^{\mu}\nabla^{\nu}\varphi) - R\nabla_{\mu}\varphi\nabla^{\mu}\varphi/2\right]/M^{6},$$

$$\mathcal{L}_{5} = \nabla_{\mu}\varphi\nabla^{\mu}\varphi \left[(\Box\varphi)^{3} - 3(\Box\varphi)(\nabla_{\mu}\nabla_{\nu}\varphi)(\nabla^{\mu}\nabla^{\nu}\varphi)\right]$$

$$+2(\nabla_{\mu}\nabla^{\nu}\varphi)(\nabla_{\nu}\nabla^{\rho}\varphi)(\nabla_{\rho}\nabla^{\mu}\varphi) +6(\nabla_{\mu}\varphi)(\nabla^{\mu}\nabla^{\nu}\varphi)(\nabla^{\rho}\varphi)G_{\nu\rho}]/M^{9}.$$

Quintic Galileon (Most general)



(Most general)

Parameter s	space
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 $\left\{\Omega_{b0}h^2, \Omega_{c0}h^2, h, n_s, A_s, \tau, \Sigma m_{\nu}\right\}$ 

 $\{c_2, c_3, c_4, c_5, \dot{\varphi}_i\}$ 

Parameter space of LCDM

Galileon parameters: can fix three.



















#### **Massive neutrinos**

- Over 6-sigma evidence;
- Driven by tension in H0 from CMB and BAO peaks;
- Earth-based neutrino experiments (KATRIN e.g.) can say more.

#### H0 compatibility

• Unlike LCDM, the constraints are compatible with local determinations (see vertical bars).

#### Low clustering amplitude

: Galileon

:  $\nu$ Galileon

1.4

1.6

- Despite enhanced gravity, clustering can be weaker than in LCDM:
- Mainly due to massive neutrino fraction.

### **Best-fitting cosmologies**



CMB TT spectrum

Possibility for lower ISW power on large scales.

Same goodness-of-fit as LCDM.

### **Best-fitting cosmologies**









### **Tension with ISW effect**



Lensing potential deepens at late times, contrary to LCDM.

The ISW effect is therefore negative in Galileon gravity.

### **Tension with ISW effect**



### Thus far ...

#### **CMB + BAO constraints**

Galileon model with massive neutrinos fits the data with the same flying colors as LCDM;



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#### **CMB + BAO constraints**

Galileon model with massive neutrinos fits the data with the same flying colors as LCDM;





#### Use of galaxy clustering data

Requires proper modeling of nonlinear structure formation;



### **Galileon gravity:**

# Targeted tests with spherical symmetry

Barreira et al. arXiv:1308.3699 (JCAP) Barreira et al. arXiv:1401.1497 (JCAP)

### Spherically Symmetric Eqs.

• Perturbed FRW – Newtonian Gauge

- Weak-field:  $\Phi, \partial \Phi \ll \partial \partial \Phi$
- $ds^{2} = [1 + 2\Psi] a(t)^{2} [1 2\Phi] \gamma_{ij} dx^{i} dx^{j}$
- Quasi-static:  $\dot{\Phi}, \dot{\delta \varphi} 
  ightarrow 0$
# Spherically Symmetric Eqs.

• Perturbed FRW – Newtonian Gauge

$$ds^{2} = [1 + 2\Psi] - a(t)^{2} [1 - 2\Phi] \gamma_{ij} dx^{i} dx^{j}$$

From (r,r) component:

$$\frac{\Psi_{,\chi}}{\chi} = \frac{B_0 \left(\Phi_{,\chi}/\chi\right) + B_1 \left(\varphi_{,\chi}/\chi\right) + B_2 \left(\varphi_{,\chi}/\chi\right)^2}{B_3 - B_4 \left(\varphi_{,\chi}/\chi\right)}$$

#### From (0,0) component:

$$\frac{\Phi_{,\chi}}{\chi} = \frac{\Omega_{m0}\delta a^{-3} + A_1\left(\varphi_{,\chi}/\chi\right) + A_2\left(\varphi_{\chi}/\chi\right)^2 + A_3\left(\varphi_{\chi}/\chi\right)^3}{A_4 + A_5\left(\varphi_{\chi}/\chi\right)}$$

From field EoM (six order algebraic equation):

$$0 = \eta_{02}\delta^{2} + \eta_{01}\delta + (\eta_{11}\delta + \eta_{10})\left[\frac{\varphi_{,\chi}}{\chi}\right] + (\eta_{21}\delta + \eta_{20})\left[\frac{\varphi_{,\chi}}{\chi}\right]^{2} + (\eta_{31}\delta + \eta_{30})\left[\frac{\varphi_{,\chi}}{\chi}\right]^{3} + \eta_{40}\left[\frac{\varphi_{,\chi}}{\chi}\right]^{4} + \eta_{50}\left[\frac{\varphi_{,\chi}}{\chi}\right]^{5} + \eta_{60}\left[\frac{\varphi_{,\chi}}{\chi}\right]^{6}.$$

• Weak-field: 
$$\Phi, \partial \Phi \ll \partial \partial \Phi$$

• Quasi-static: 
$$\dot{\Phi}, \dot{\delta arphi} 
ightarrow 0$$

$$\chi = aH_0r$$

$$\delta \to \text{Top hat}$$

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# **Spherically Symmetric Eqs.**

- $ds^{2} = [1 + 2\Psi] a(t)^{2} [1 2\Phi] \gamma_{ij} dx^{i} dx^{j}$ From (r,r) component:  $\frac{\Psi_{,\chi}}{\chi} = \frac{B_0 \left(\Phi_{,\chi}/\chi\right) + B_1 \left(\varphi_{,\chi}/\chi\right) + B_2 \left(\varphi_{,\chi}/\chi\right)^2}{B_3 - B_4 \left(\varphi_{,\chi}/\chi\right)}$ From (0,0) component:  $\frac{\Omega_{m0}\delta a^{-3}+A_1\left(\varphi_{,\chi}/\chi\right)+A_2\left(\varphi_{\chi}/\chi\right)^2+A_3\left(\varphi_{\chi}/\chi\right)^3}{A_4+A_5\left(\varphi_{\chi}/\chi\right)}$  $\Phi_{,\chi}$  $\chi$
- Weak-field:  $\Phi, \partial \Phi \ll \partial \partial \Phi$

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ightarrow 0$$

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#### From field EoM (six order algebraic equation):

Perturbed FRW – Newtonian Gauge

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• Vainshtein screening mechanism at work:

$$\frac{\Psi_{,\chi}}{\chi} = \frac{\Omega_{m0}\delta}{2}a^{-3} + \frac{A_1}{2}\left[\frac{\varphi_{,\chi}}{\chi}\right]$$



$$\frac{\Psi_{,\chi}}{\chi} = \frac{\Omega_{m0}\delta}{2}a^{-3} + \underbrace{\frac{A_1}{2} \left[\frac{\varphi_{,\chi}}{\chi}\right]} \left[\frac{\varphi_{,\chi}}{\chi}\right] \sim \sqrt{\delta}, \quad \delta \gg 1$$

Fifth force term gets suppressed since it grows slower with the density.















# **Quintic Galileon**

From field EoM (six order algebraic equation):

$$0 = \eta_{02}\delta^{2} + \eta_{01}\delta + (\eta_{11}\delta + \eta_{10})\left[\frac{\varphi_{,\chi}}{\chi}\right] + (\eta_{21}\delta + \eta_{20})\left[\frac{\varphi_{,\chi}}{\chi}\right]^{2} + (\eta_{31}\delta + \eta_{30})\left[\frac{\varphi_{,\chi}}{\chi}\right]^{3} + \eta_{40}\left[\frac{\varphi_{,\chi}}{\chi}\right]^{4} + \eta_{50}\left[\frac{\varphi_{,\chi}}{\chi}\right]^{5} + \eta_{60}\left[\frac{\varphi_{,\chi}}{\chi}\right]^{6}.$$

- $\begin{array}{c}
  0.2 \\
  a = 1 \\
  0.0 \\
  \hline \\
  0.0$
- Density-dependence of the Galileon gradient today

#### **Physical branch**

$$\frac{\varphi_{\chi}}{\chi} \left( \delta \to 0 \right) \to 0$$

# **Quintic Galileon**

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- $\begin{array}{c} 0.2 \\ 0.0 \\$
- Density-dependence of the Galileon gradient today

**Physical branch** 

 $\frac{\varphi_{,\chi}}{\chi} \left( \delta \to 0 \right) \to 0$ 

Physical solution becomes a complex number !!

1) Breakdown of the weakfield or quasi-static limits?

2) Breakdown of the model?

# **Galileon gravity:**

# **Targeted tests with N-body simulations**

Barreira et al.arXiv: 1306.3219(JCAP)Li & Barreira, et al.arxiv: 1308.3491 (JCAP)Barreira et al.arXiv: 1401.1497(JCAP)

## Simulations

- ECOSMOG N-body code (AMR); Li et al. (2011); arXiv:1110.1379
- Np = 512^3 particles; L = 200Mpc/h;
- Initial conditions determined with the linear matter power spectrum of the model at z = 49;

Model	Expansion history	Force law
Full model	Galileon	GR + Screened fifth force
Linear model	Galileon	GR + linear fifth force
$\operatorname{QCDM}$	Galileon	GR

# **Simulation results**



Halo mass function







## **Halo Occupation Distribution**





## **Halo Occupation Distribution**

• Postulate shape for the mean number of galaxies, N, in haloes of mass M.

$$\langle N|M\rangle = \frac{1}{2} \left[ 1 + \operatorname{erf}\left(\frac{\log_{10}\left(M/M_{\min}\right)}{\sigma_{\log_{10}M}}\right) \right]$$

## **Galileon Summary**



## The Maggiore&Mancarella model

Nonlocal action (Maggiore&Mancarella 2014)

$$A = \frac{1}{2\kappa} \int \mathrm{d}x^4 \sqrt{-g} \left[ R - \frac{m^2}{6} R \Box^{-2} R - \mathcal{L}_m \right]$$

Auxiliary scalar quantities

$$U = -\Box^{-1}R, \quad S = -\Box^{-1}U$$

Integral (nonlocal) solutions

$$U(x) = U_{\text{hom}}(x) - \int dy^4 \sqrt{-g(y)} G(x,y) R(y)$$

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Impose "by hand" that these are retarded operators, to

keep causality.

Model is specified by a choice of the homogeneous solution.

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Impose "by hand" that these are retarded operators, to

keep causality.

Model is specified by a choice of the homogeneous solution.

"Localized" version of the action: Scalar-Tensor theory, easier to work with.

$$A = \frac{1}{2\kappa} \int \mathrm{d}x^4 \sqrt{-g} \left[ \left( 1 - \frac{m^2}{6} S \right) R - \frac{\xi_1}{\xi_1} \left( \Box U + R \right) - \frac{\xi_2}{\xi_2} (\Box S + U) - \mathcal{L}_m \right]$$
 Lagrange multipliers

### **Structure formation equations**



### **Structure formation equations**



### **Nonlocal: Prospect constraints**



Barreira et al. arXiv: 1408.1084 (JCAP)












