

Title: Non local weak measurements

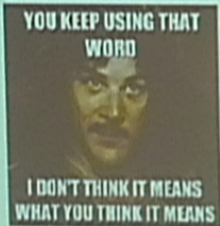
Date: Oct 07, 2014 03:30 PM

URL: <http://pirsa.org/14100073>

Abstract: Weak measurement is increasingly acknowledged as an important theoretical and experimental tool. Weak values- the results of weak measurements- are often used to understand seemingly paradoxical quantum behavior. Until now however, it was not known how to perform a weak non-local measurement of a general operator. Such a procedure is necessary if we are to take the associated `weak values' seriously as a physical quantity. We propose a novel scheme for performing non-local weak measurement which is based on the principle of quantum erasure. This method can be used for a large class of observables including those related to Hardy's paradox.

Terminology

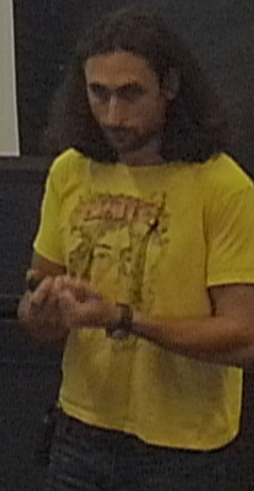
The term *measurement* means different things to different people



We will have three different terms used here

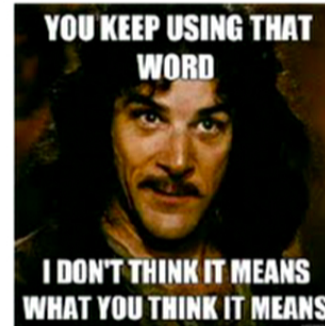
- Strong or von Neumann measurement - has a classical outcome (and a quantum outcome).
- Weak measurement - has no classical outcome.
- von Neumann scheme - can be used for both types of measurements.

I will sometimes say the word 'measurement' and mean von Neumann measurement.



Terminology

The term *measurement* means different things to different people



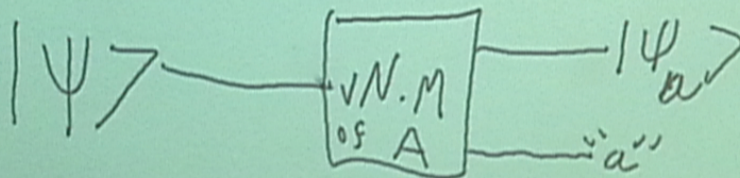
We will have three different terms used here

- Strong or von Neumann measurement - has a classical outcome (and a quantum outcome).
- Weak measurement - has no classical outcome.
- von Neumann scheme - can be used for both types of measurements.

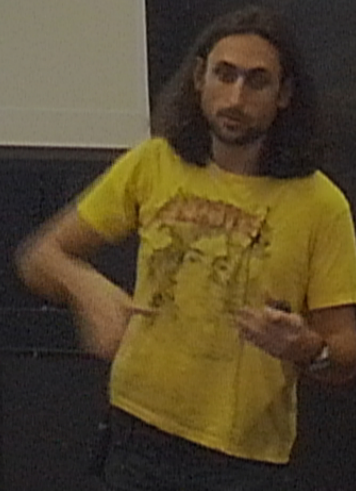
I will sometimes say the word 'measurement' and mean von Neumann measurement.

von Neumann measurements

The von Neumann measurement of an observable $A = \sum_a a \Pi_a$ where Π_a are orthogonal projections has a classical outcome a and an outgoing quantum state $|\psi_a\rangle$

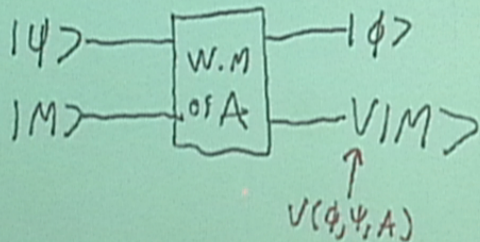


The probability for an outcome $a, |\psi_a\rangle$ is given by the Born rule $p(a) = \langle \psi | \Pi_a | \psi \rangle$ and the outgoing state is given by $|\psi\rangle_a = \frac{\Pi_a |\psi\rangle}{\sqrt{p(a)}}$



Post-selected weak measurements

A post selected weak measurement (details will come later) has as input: the state $|\psi\rangle$ and meter state $|M\rangle$. The output is a state $|\phi\rangle$ and an operator V acting on $|M\rangle$.

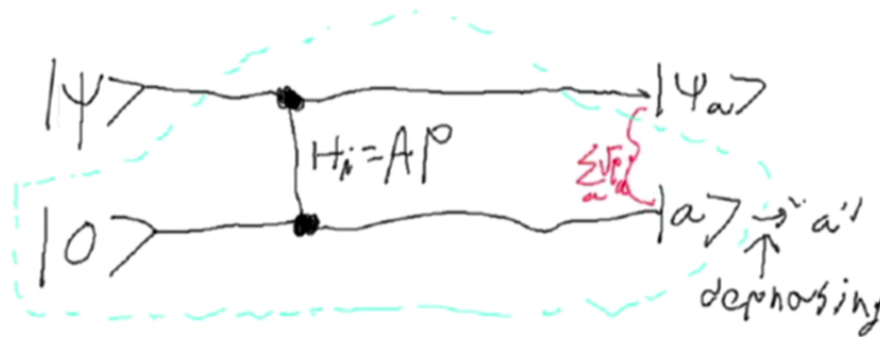


The procedure involves post-selection which happens with probability $\approx |\langle\phi|\psi\rangle|^2$.

In general V is a function of ϕ, ψ, A . To a good approximation it is a function of $\{A\}_w$.

The von Neumann scheme

It is possible to describe the von Neumann measurement in the following way.



where $e^{iaP} |0\rangle = |a\rangle$. So that $|\psi\rangle |0\rangle \rightarrow \sum_a \Pi_a |\psi\rangle |a\rangle$.

For a von Neumann measurement we need $|a\rangle$ to be an orthogonal set. After the interaction we dephase in this basis.

Non local measurements

- The von Neumann scheme does not always work.
- For example if we want to measure a non-local operator AB . The interaction AB is not physical so $H = ABP$ is not physical.
- Sometimes we can use a different measurement scheme.
- But sometimes a strong (von Neumann) measurement can be acausal.

Acausal measurements

- Take two qubits and the projectors A, B both onto the $|1\rangle$ state on their side.
- Imagine that a superior being can make a vN measurement of the degenerate operator AB .
- Note that $|1\rangle |1\rangle$ is an eigenvector with eigenvalue 1 and $|\psi\rangle |0\rangle$ are eigenvectors with degenerate eigenvalue 0.
- Alice prepares the state $|0 + 1\rangle$ and Bob can choose to prepare the state $|1\rangle$ or $|0\rangle$.
- $|0 + 1\rangle |0\rangle$ is an eigenstate but $|0 + 1\rangle |1\rangle = \frac{1}{\sqrt{2}}[|0\rangle |1\rangle + |1\rangle |1\rangle]$ is not.
- If the superior being makes a measurement, Alice's final state will depend on Bob's choice.

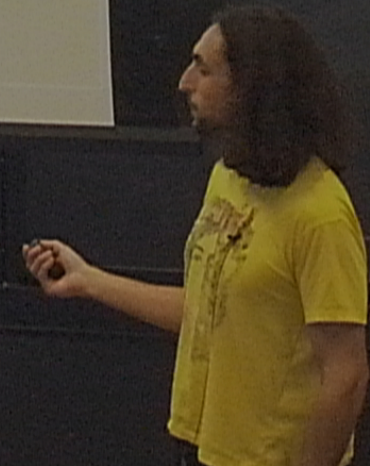
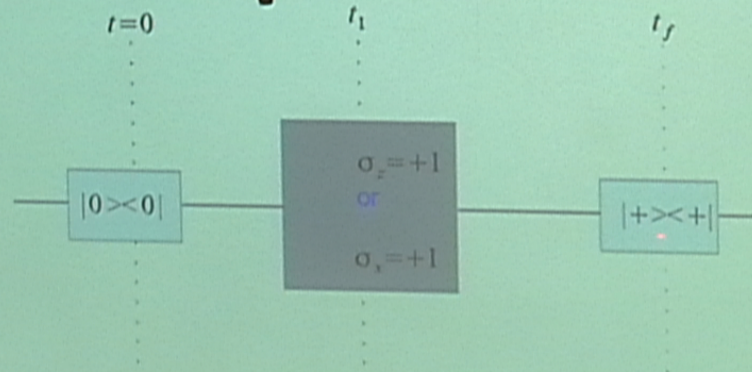
Acausal measurements

- Take two qubits and the projectors A, B both onto the $|1\rangle$ state on their side.
- Imagine that a superior being can make a vN measurement of the degenerate operator AB .
- Note that $|1\rangle |1\rangle$ is an eigenvector with eigenvalue 1 and $|\psi\rangle |0\rangle$ are eigenvectors with degenerate eigenvalue 0.
- Alice prepares the state $|0 + 1\rangle$ and Bob can choose to prepare the state $|1\rangle$ or $|0\rangle$.
- $|0 + 1\rangle |0\rangle$ is an eigenstate but $|0 + 1\rangle |1\rangle = \frac{1}{\sqrt{2}}[|0\rangle |1\rangle + |1\rangle |1\rangle]$ is not.
- If the superior being makes a measurement, Alice's final state will depend on Bob's choice.

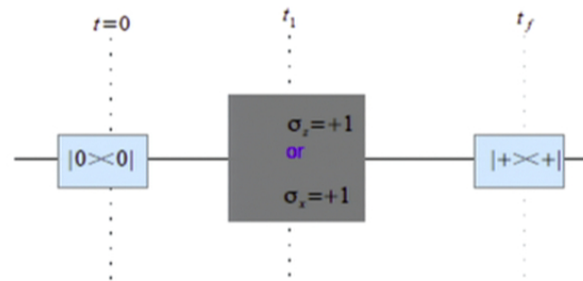
Non local weak measurements

- The (standard) weak measurement scheme is a weakened version of the νN scheme.
- So if we can't use the νN scheme we need to find some other way to make a weak measurement.
- But the usual schemes for νN measurements of non-local observables cannot be directly applied to weak measurements.
- To get around this we use the quantum eraser.
- **But let's first explain weak measurements**

What can we say about a quantum system at a time between two measurements?



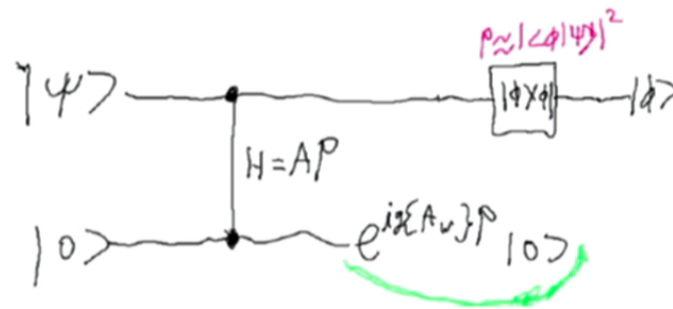
What can we say about a quantum system at a time between two measurements?



- The system seems to have a strange property: both X and Z measurements have deterministic outcomes.
- Hold on!... Didn't the measurement at t_1 change the probabilities for the desired outcome at t_f ?

A simple description for a measurement

A weak vN scheme with $g \ll 1$.



The weak measurement is a coherent channel for the meter. It involves both pre and post selection for the system.

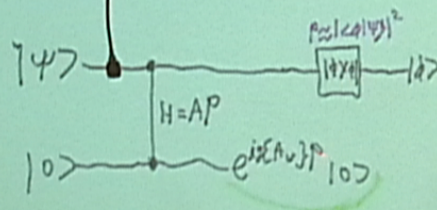
The evolution of the meter through the channel is given (up to second order in g) by the operator $e^{ig\{A\}_w}$

$$\{A\}_w = \frac{\langle\phi|A|\psi\rangle}{\langle\phi|\psi\rangle}$$



A simple description for a measurement

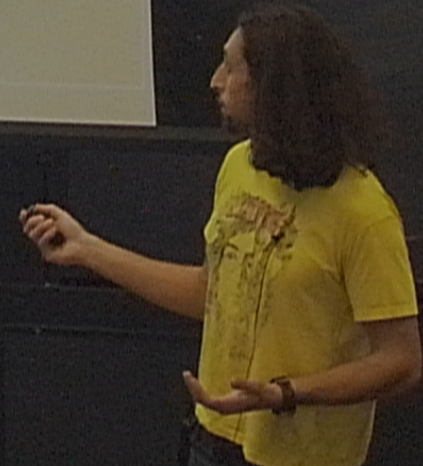
A weak vN scheme with $g \ll 1$.



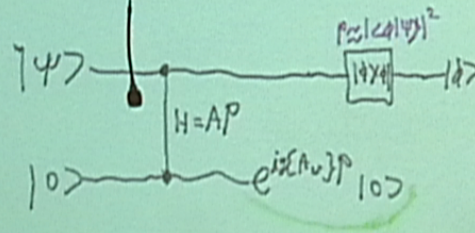
The weak measurement is a coherent channel for the meter. It involves both pre and post selection for the system.

The evolution of the meter through the channel is given (up to second order in g) by the operator $e^{ig(A)_w}$

$$\{A\}_w = \frac{\langle\phi|A|\psi\rangle}{\langle\phi|\psi\rangle}$$



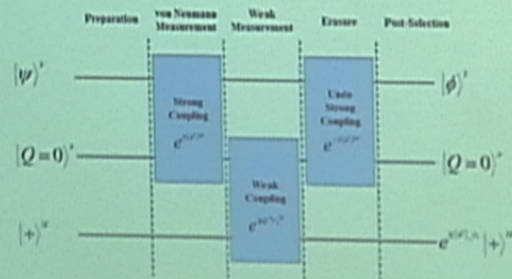
Some properties of weak measurements



- The correction to the probability for post selection due to the weak interaction is of order g^2 .
- The result is a (pseudo) potential - not a classical number.
- The evolution of the meter is determined by $\{A\}_w = \frac{\langle \phi | A | \psi \rangle}{\langle \phi | \psi \rangle}$.

Weak measurement of a strong measurement

It is possible to perform a weak measurement by making a strong measurement, measuring the measurement device and reversing the first measurement.



But reversing the measurement might not be possible due to operational constraints.

Measurement of $\sigma_z \sigma_z$

As an example of weak measurement via erasure we first look at a vN measurement of $\sigma_z \sigma_z$ that does not require communication.

- Alice and Bob want to measure $\sigma_z \sigma_z$ on a shared unknown state $|\psi\rangle$
- They share one entangled pair $|00 + 11\rangle$ - The meter
- Each couples to σ_z locally (CNOT)
- The joint state of the meter now holds the measurement result. Either $|00 + 11\rangle$ or $|01 + 10\rangle$
- They can now measure locally and later figure out what is the measurement result.

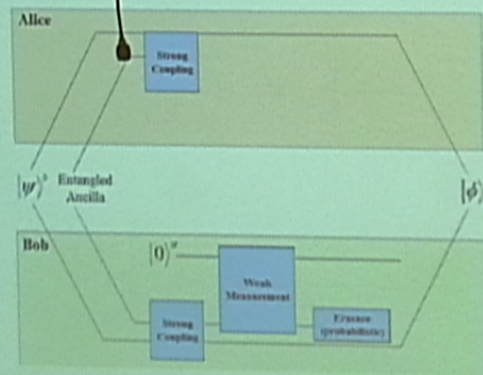
Measurement of $\sigma_z \sigma_z$

As an example of weak measurement via erasure we first look at a vN measurement of $\sigma_z \sigma_z$ that does not require communication.

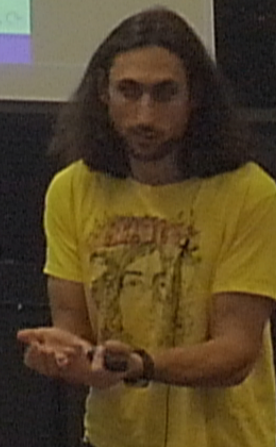
- Alice and Bob want to measure $\sigma_z \sigma_z$ on a shared unknown state $|\psi\rangle$
- They share one entangled pair $|00 + 11\rangle$ - The meter
- Each couples to σ_z locally (CNOT)
- The joint state of the meter now holds the measurement result. Either $|00 + 11\rangle$ or $|01 + 10\rangle$
- They can now measure locally and later figure out what is the measurement result.

Weak measurement of $\sigma_z \sigma_z$

Using this we can make a weak measurement of any product AB

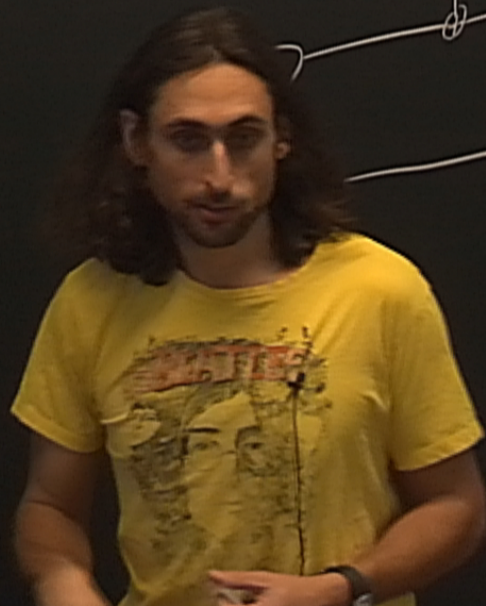
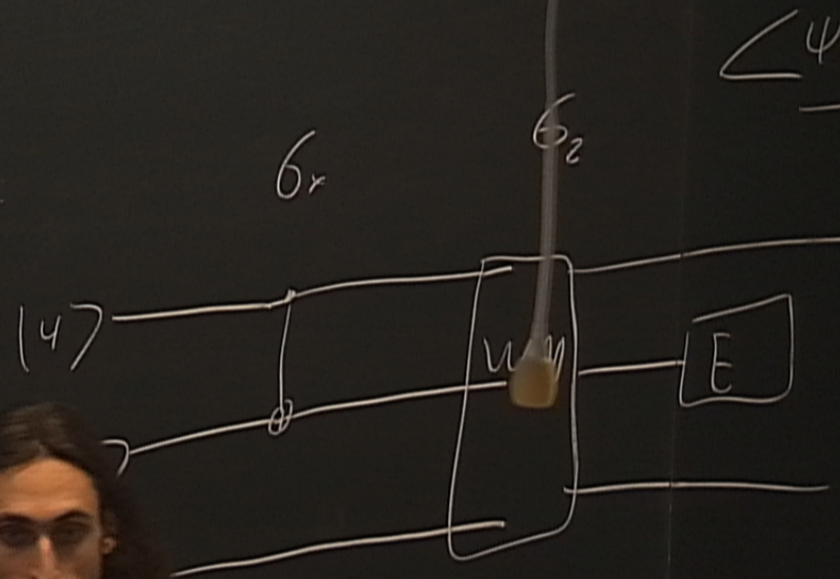
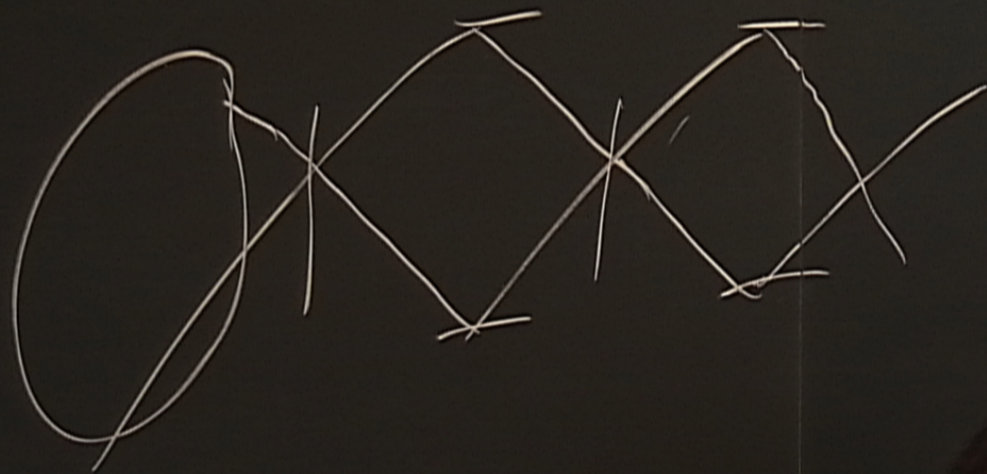


The scheme is probabilistic.



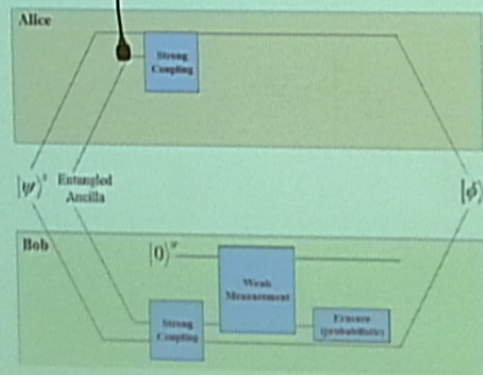
Final thoughts

- Using erasure it is possible to measure non-local weak values directly.
- This can also be applied for sequential weak measurements. (non Hermitian observables)
- It is still an open question if we can do this deterministically.
- Are there any other constraints?



Weak measurement of $\sigma_z \sigma_z$

Using this we can make a weak measurement of any product AB



The scheme is probabilistic.

