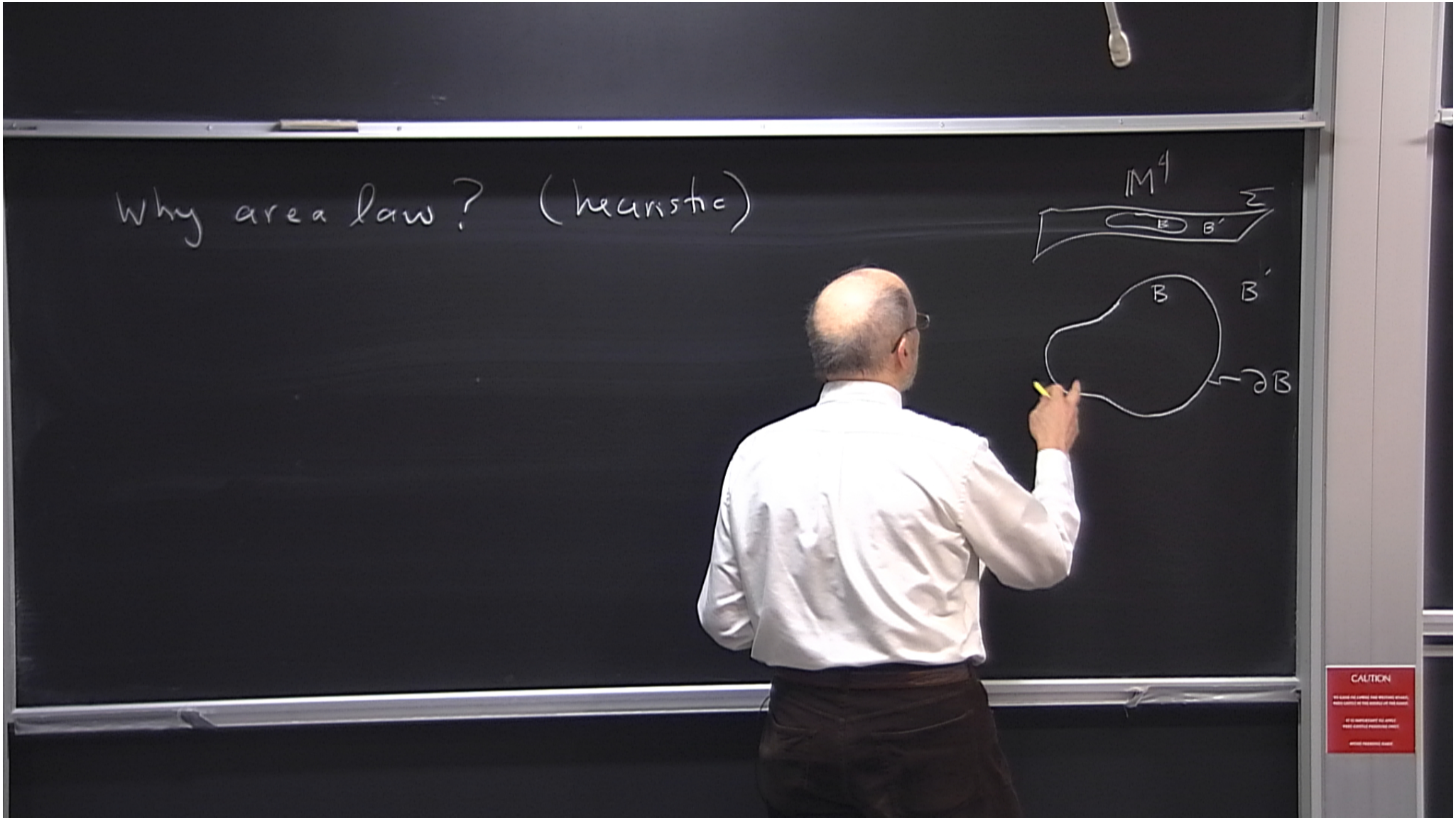
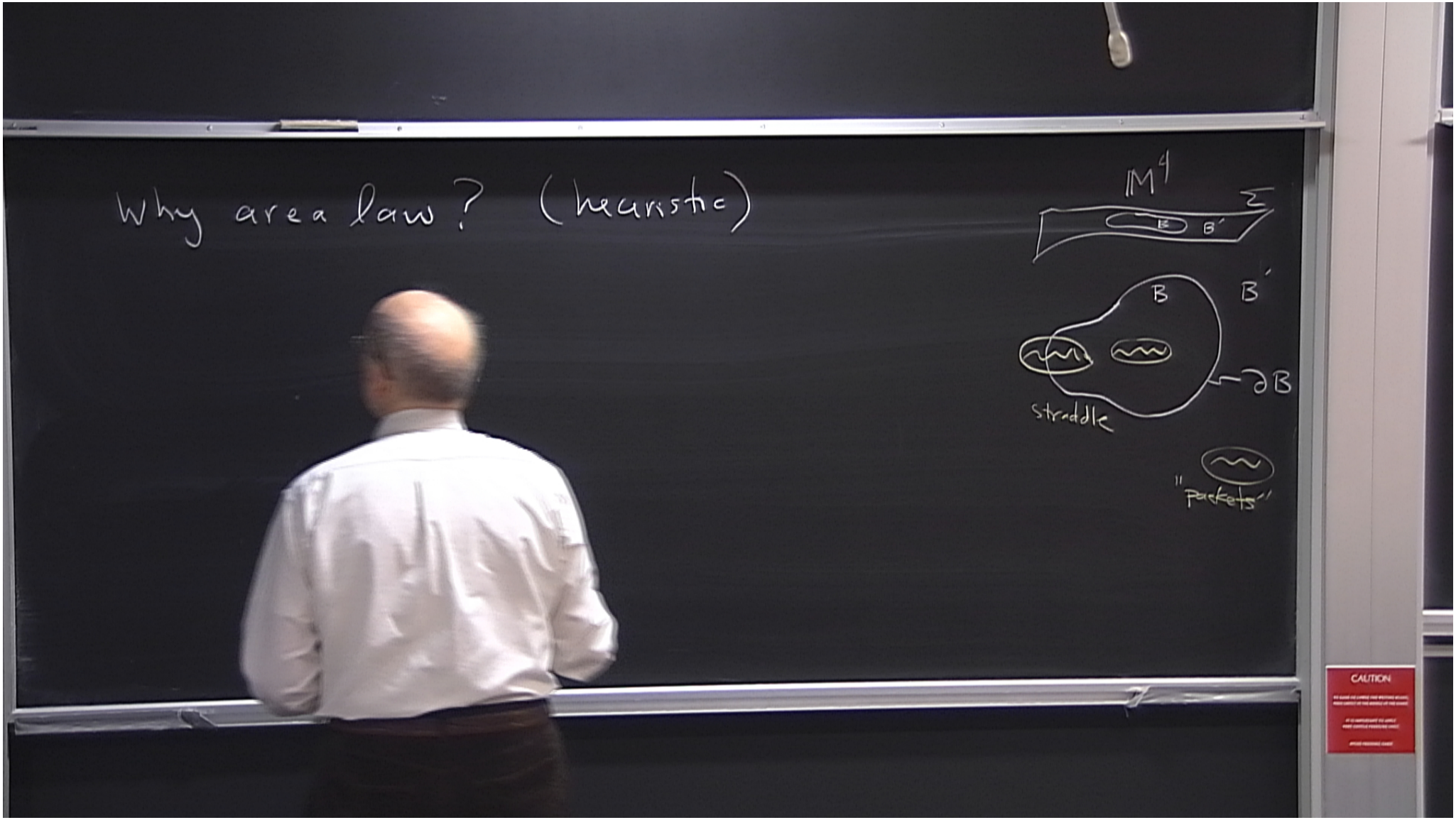
Title: Entanglement Entropy and the Area Law - Lecture 4

Date: Oct 28, 2014 05:00 PM

URL: http://pirsa.org/14100072

Abstract:

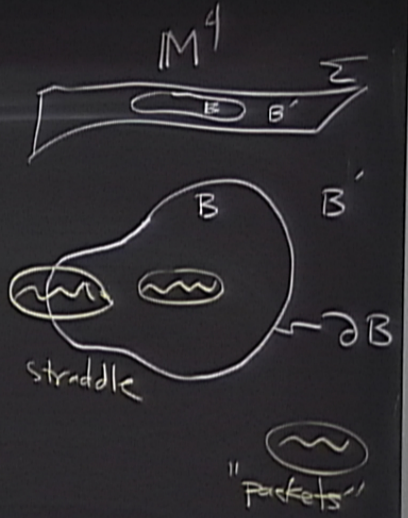

Why area law? (heuristic)
M^4
Σ
B
B'
B
B'
∂B
CAUTION


Why area law? (heuristic)
M^4
Σ
B
B'
B
B'
∂B
straddle
"Packets"
CAUTION

Why area law? (heuristic)

Assume $m^2 = 0$, each straddling mode $\longrightarrow \sim 1$ bit

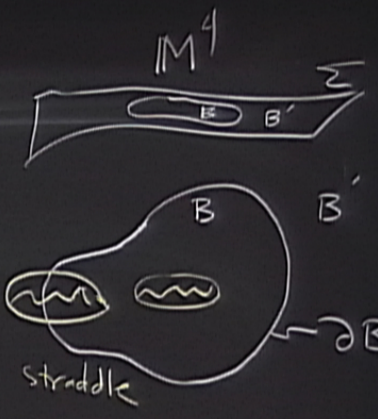
$\|\vec{k}\| = k = 2\pi/\lambda$

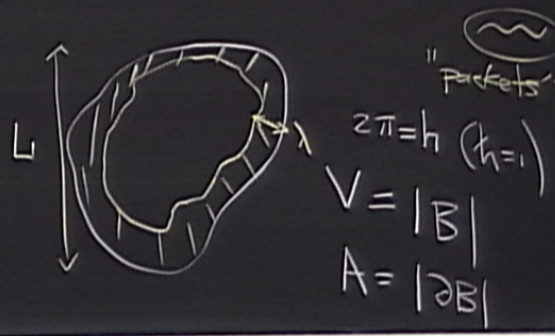


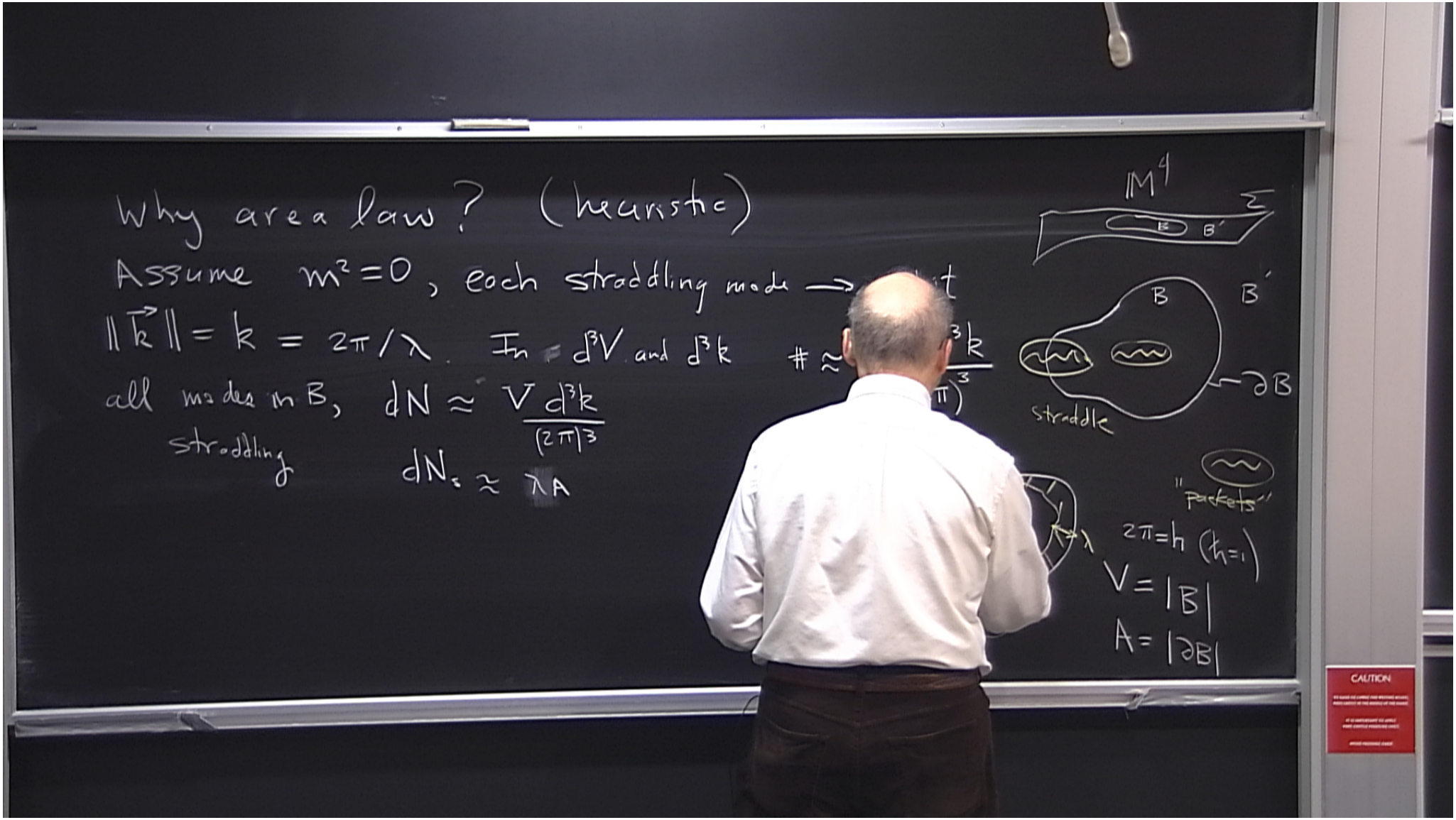
Why area law? (heuristic)

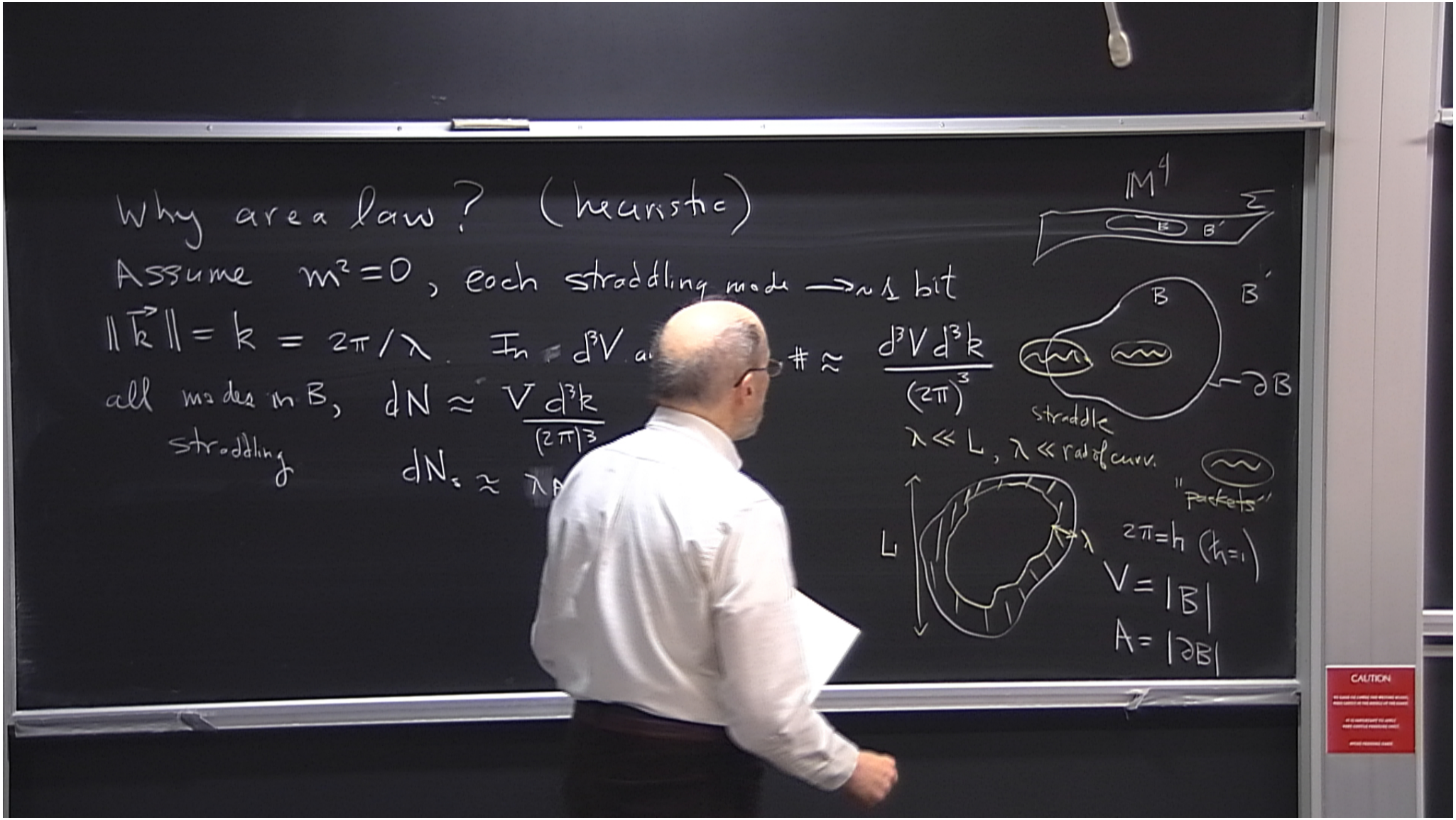
Assume $m^2 = 0$, each straddling mode $\longrightarrow \sim 1$ bit

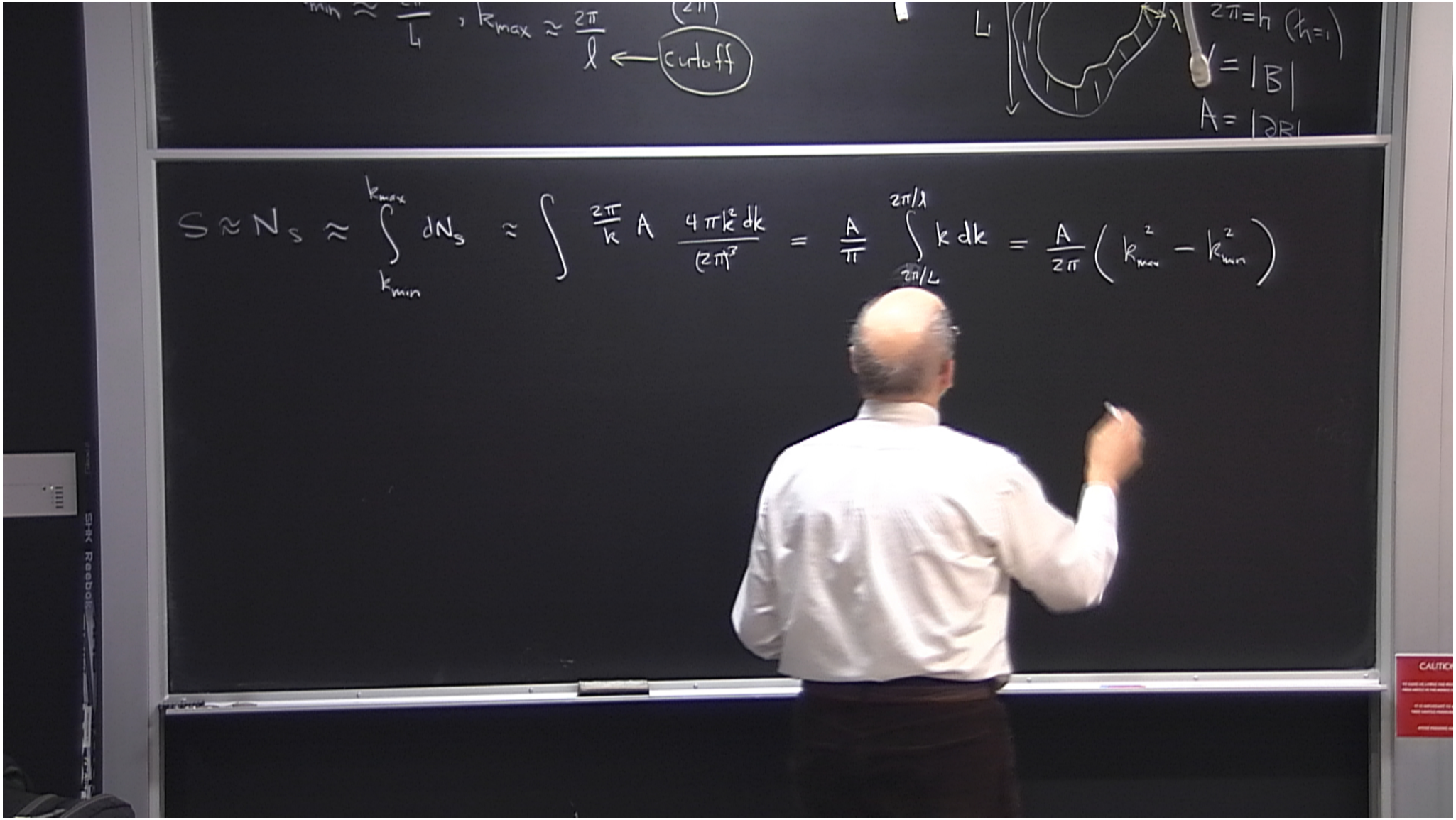
$\|\vec{k}\| = k = 2\pi/\lambda$ In d^3V and d^3k $\# \approx \frac{d^3V\, d^3k}{(2\pi)^3}$

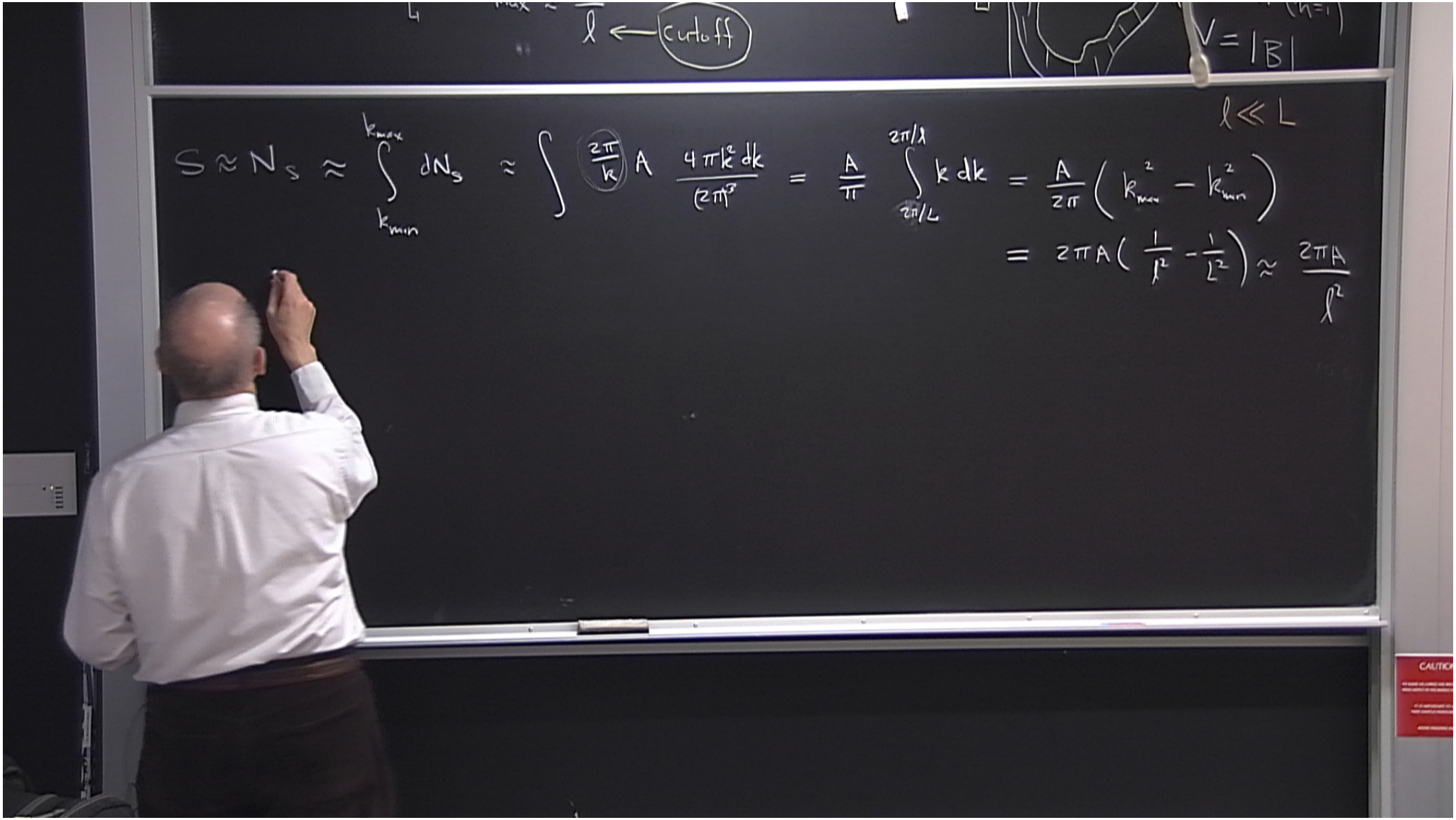


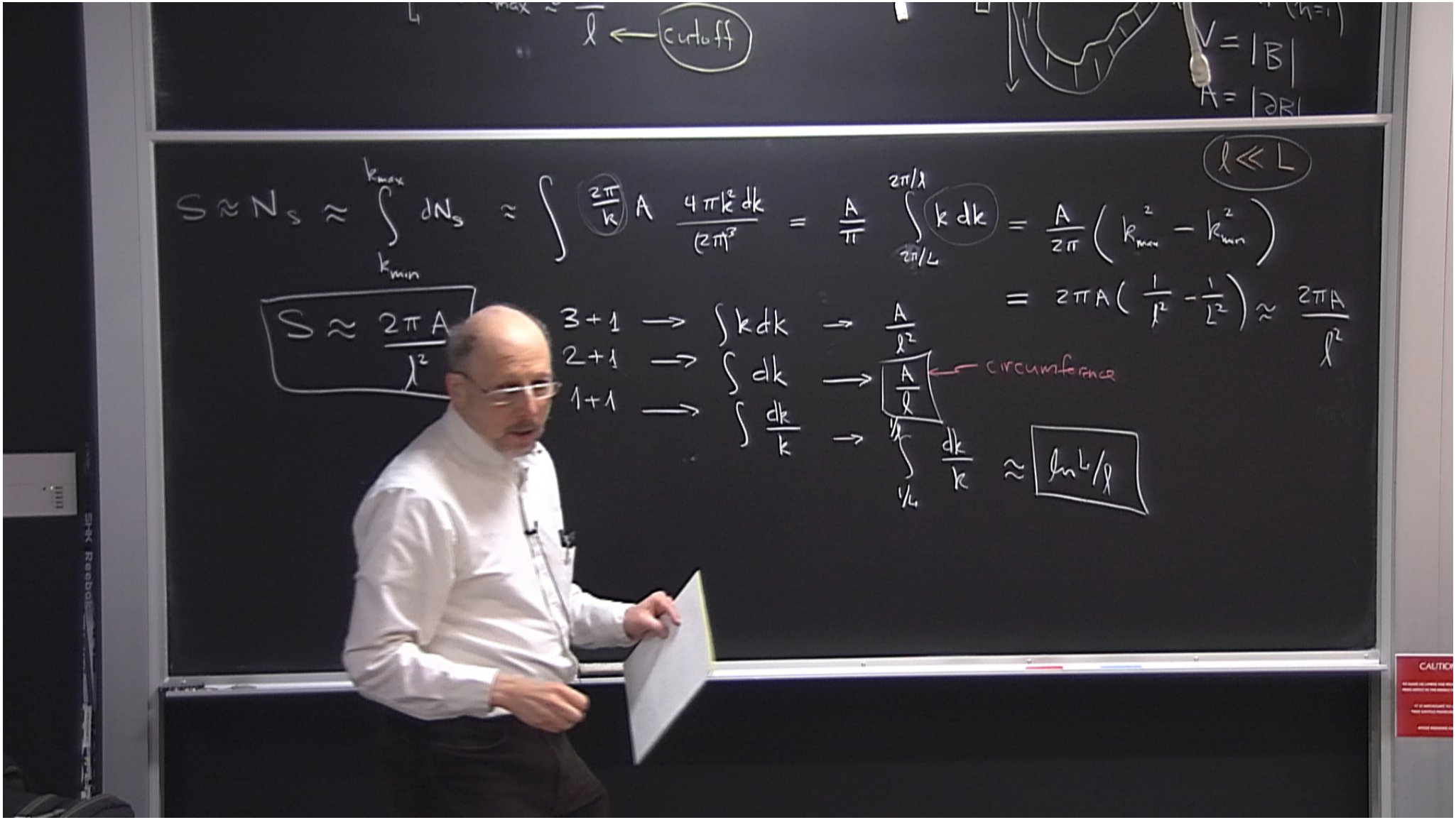


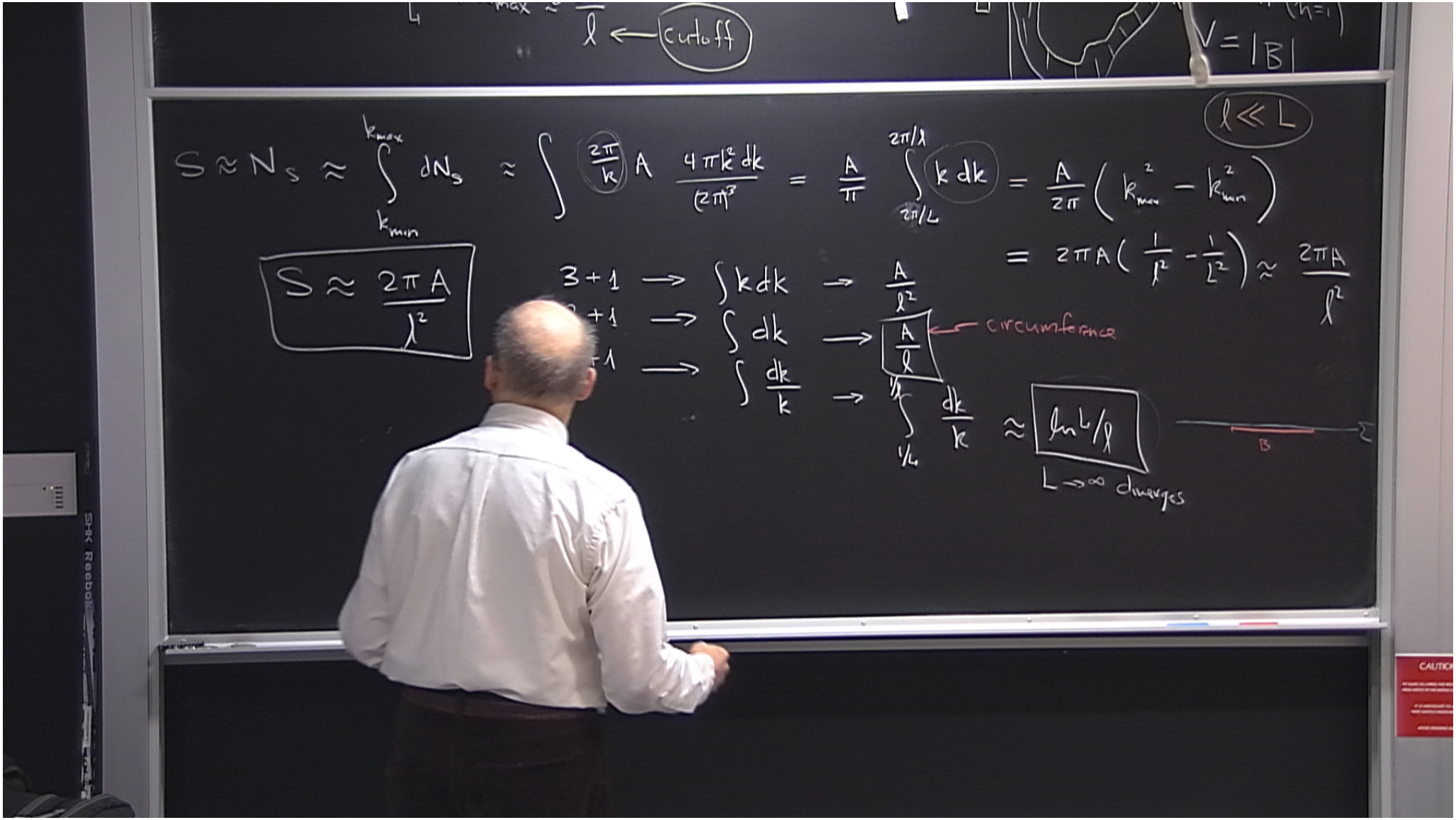

Why area law? (heuristic)
Assume $m^2=0$, each straddling mode →
$\|\vec{k}\| = k = 2\pi/\lambda$. In d^3V and d^3k # ≈
all modes in B, $dN \approx \frac{V\,d^3k}{(2\pi)^3}$
straddling $dN_s \approx \lambda A$
M^4
Σ
B
B'
∂B
straddle
"packets"
$2\pi=h$ ($\hbar=1$)
$V=|B|$
$A=|\partial B|$
CAUTION

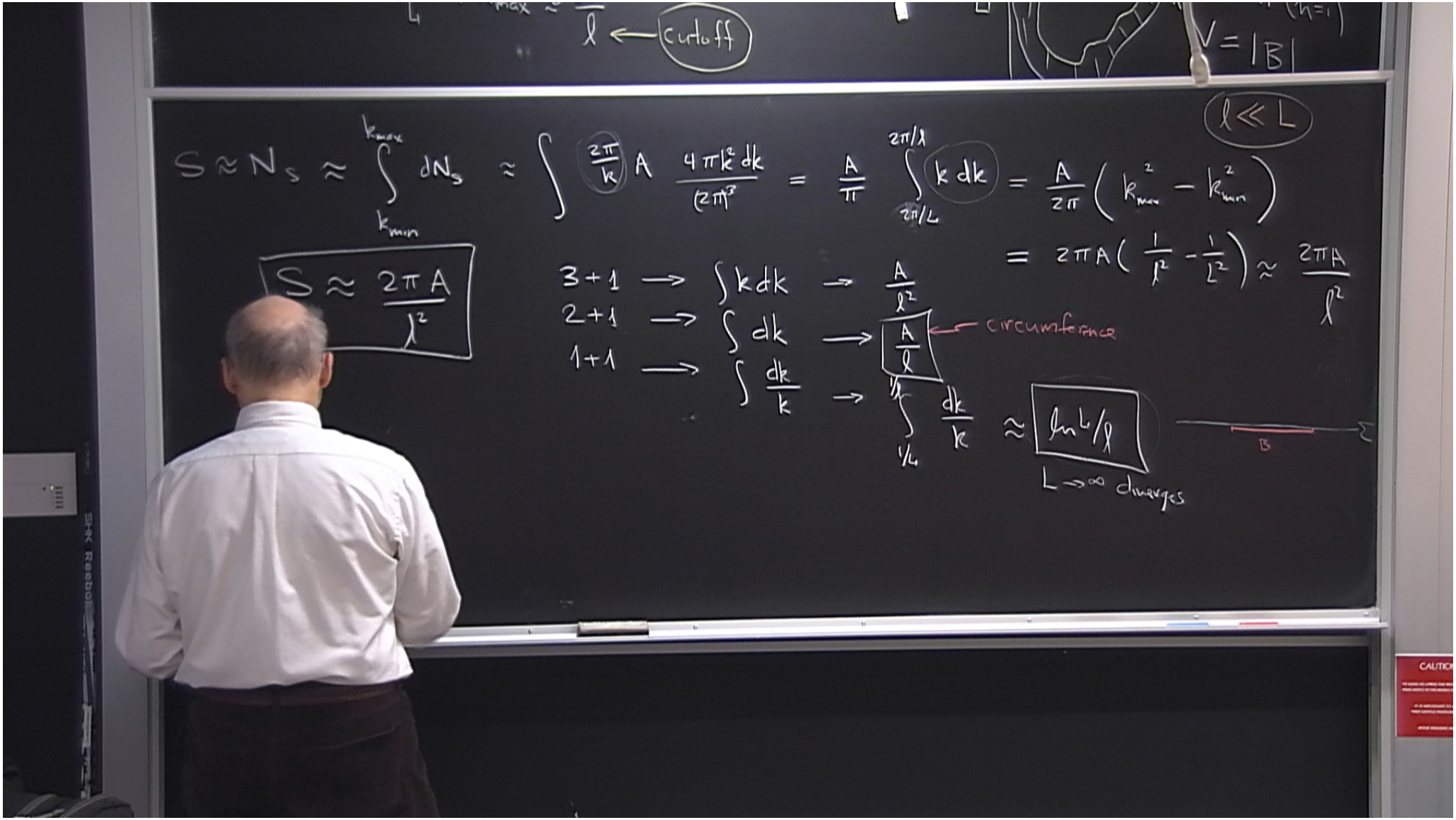

Why area law? (heuristic)
Assume $m^2=0$, each straddling mode $\longrightarrow \sim 1$ bit
$\|\vec{k}\| = k = 2\pi/\lambda$
all modes in B, $dN \approx \frac{V\,d^3k}{(2\pi)^3}$
straddling $dN_s \approx$
$\#\approx \frac{d^3V\,d^3k}{(2\pi)^3}$
M^4
Σ
B
B'
∂B
straddle
$\lambda \ll L$, $\lambda \ll$ rad of curv.
"packets"
L
λ
$2\pi = h$ $(\hbar = 1)$
$V = |B|$
$A = |\partial B|$
CAUTION

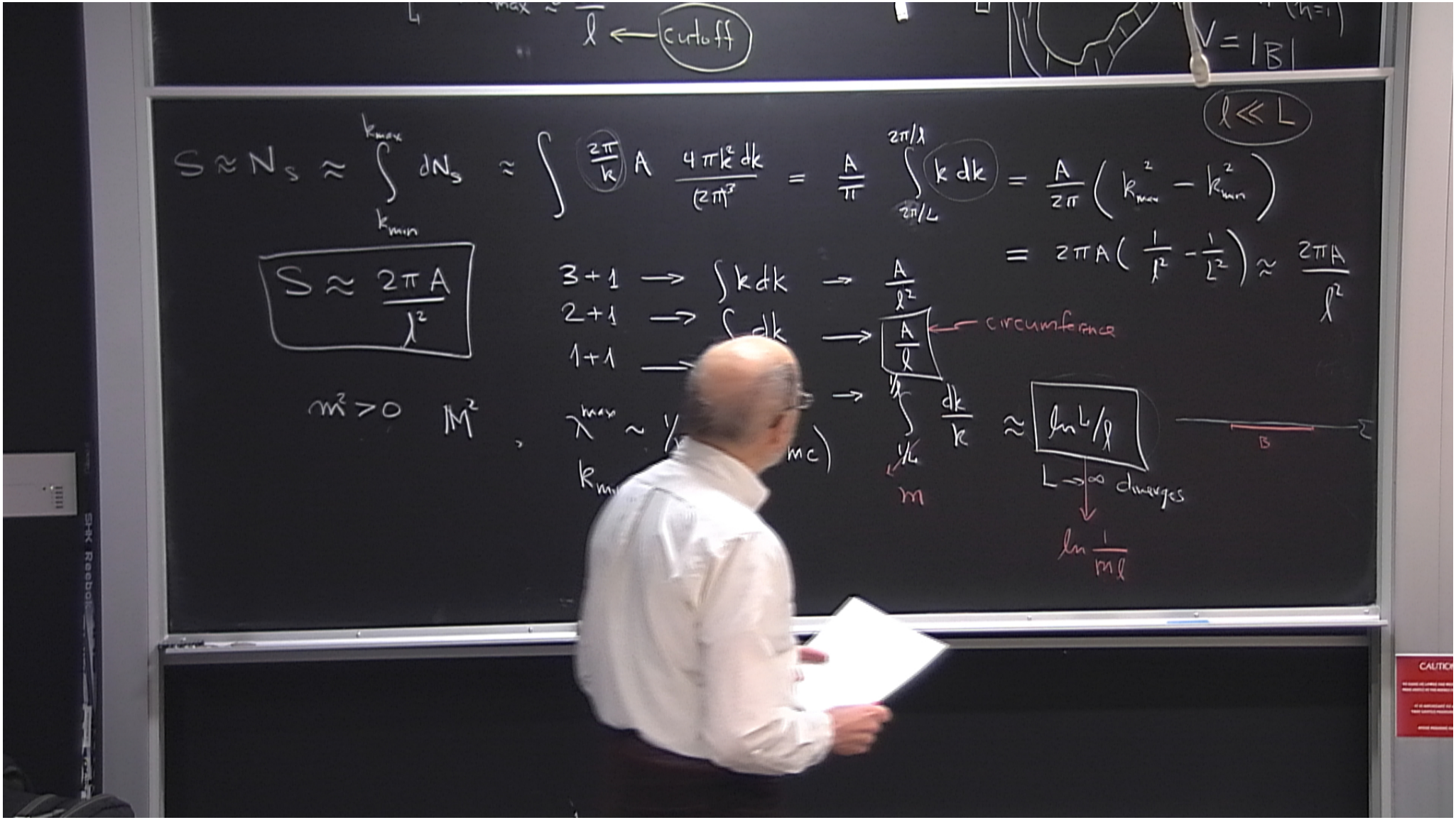

$k_{max} \approx \frac{2\pi}{\ell}$
cutoff
$2\pi = \hbar$ ($\hbar = 1$)
$V = |B|$
$A = |\partial B|$
$S \approx N_s \approx \int_{k_{min}}^{k_{max}} dN_s \approx \int \frac{2\pi}{k} A \frac{4\pi k^2 dk}{(2\pi)^2} = \frac{A}{\pi} \int_{2\pi/L}^{2\pi/\lambda} k\,dk = \frac{A}{2\pi}\left(k_{max}^2 - k_{min}^2\right)$

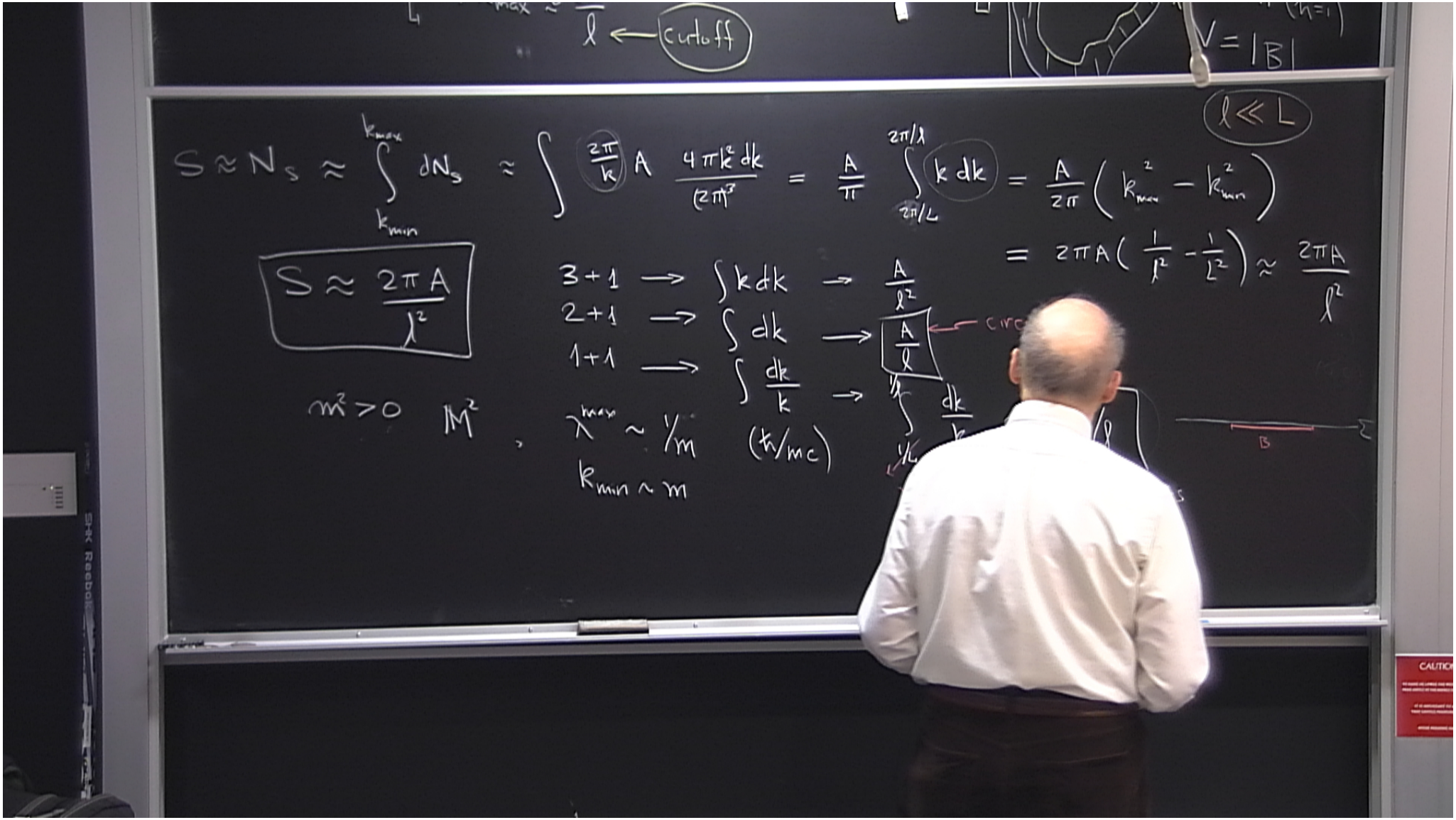

ℓ ← cutoff
V = |B|
ℓ ≪ L
$S \approx N_s \approx \int_{k_{min}}^{k_{max}} dN_s \approx \int \frac{2\pi}{k} A \frac{4\pi k^2 dk}{(2\pi)^2} = \frac{A}{\pi} \int_{2\pi/L}^{2\pi/\lambda} k\,dk = \frac{A}{2\pi}\left(k_{max}^2 - k_{min}^2\right)$
$= 2\pi A\left(\frac{1}{\ell^2} - \frac{1}{L^2}\right) \approx \frac{2\pi A}{\ell^2}$

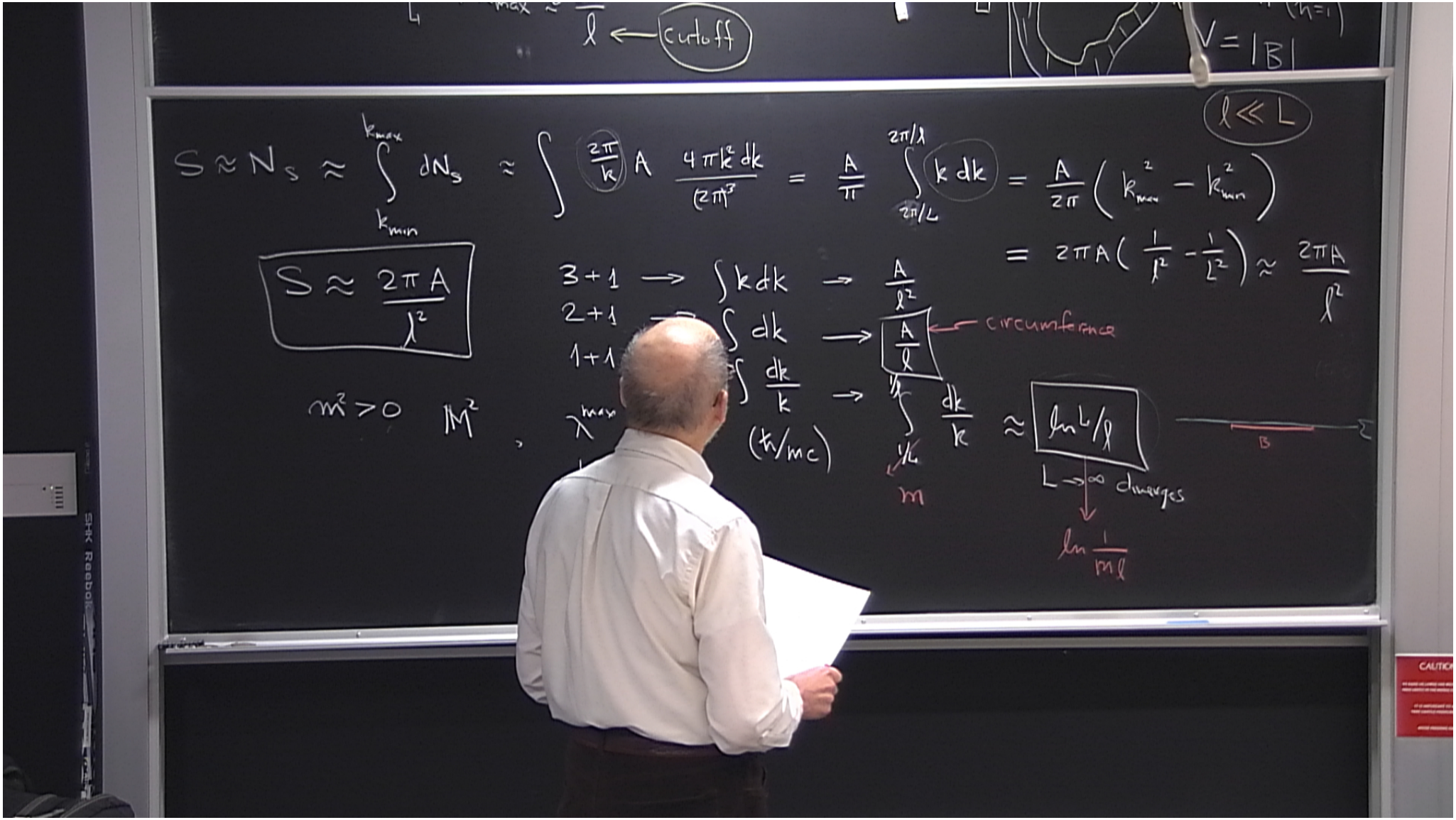

l ← cutoff
V = |B|
A = |∂R|
l ≪ L
$S \approx N_s \approx \int_{k_{min}}^{k_{max}} dN_s \approx \int \frac{2\pi}{k} A \frac{4\pi k^2 dk}{(2\pi)^3} = \frac{A}{\pi} \int_{2\pi/L}^{2\pi/l} k\,dk = \frac{A}{2\pi}\left(k_{max}^2 - k_{min}^2\right)$
$= 2\pi A\left(\frac{1}{l^2} - \frac{1}{L^2}\right) \approx \frac{2\pi A}{l^2}$
$S \approx \frac{2\pi A}{l^2}$
3+1 → ∫ k dk → $\frac{A}{l^2}$
2+1 → ∫ dk → $\frac{A}{l}$ ← circumference
1+1 → $\int \frac{dk}{k}$ → $\int_{1/L}^{1/l} \frac{dk}{k} \approx \ln L/l$

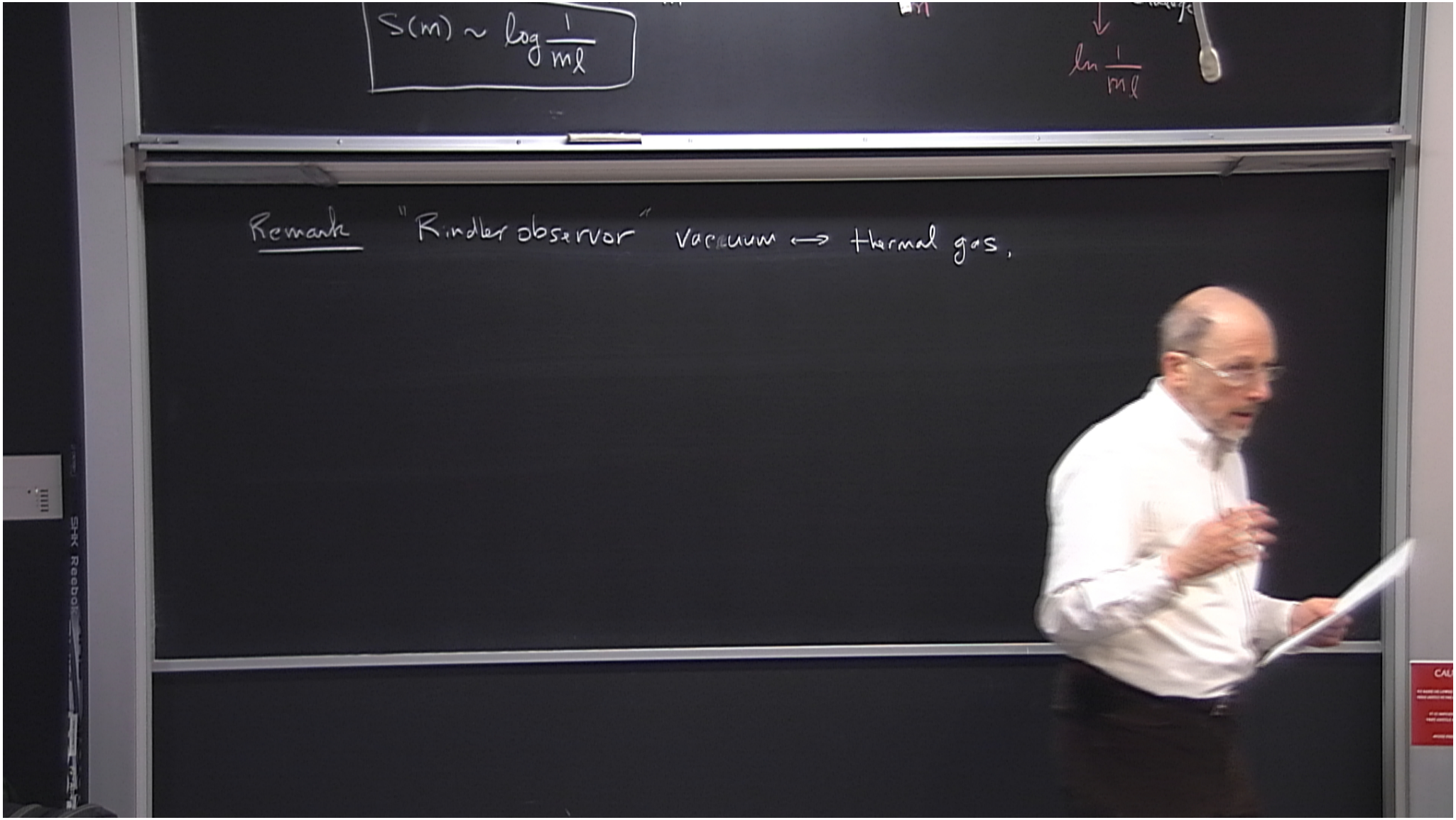

ℓ ← cutoff
V = |B|
ℓ ≪ L
$S \approx N_s \approx \int_{k_{min}}^{k_{max}} dN_s \approx \int \frac{2\pi}{k} A \frac{4\pi k^2 dk}{(2\pi)^3} = \frac{A}{\pi} \int_{2\pi/L}^{2\pi/\lambda} k\,dk = \frac{A}{2\pi}\left(k_{max}^2 - k_{min}^2\right)$
$= 2\pi A\left(\frac{1}{\lambda^2} - \frac{1}{L^2}\right) \approx \frac{2\pi A}{\lambda^2}$
$S \approx \frac{2\pi A}{\lambda^2}$
3+1 → ∫ k dk → $\frac{A}{\ell^2}$
2+1 → ∫ dk → $\frac{A}{\ell}$
circumference
1 → ∫ $\frac{dk}{k}$ → $\int_{1/L}^{1/\ell} \frac{dk}{k} \approx \ln L/\ell$
L→∞ diverges
B

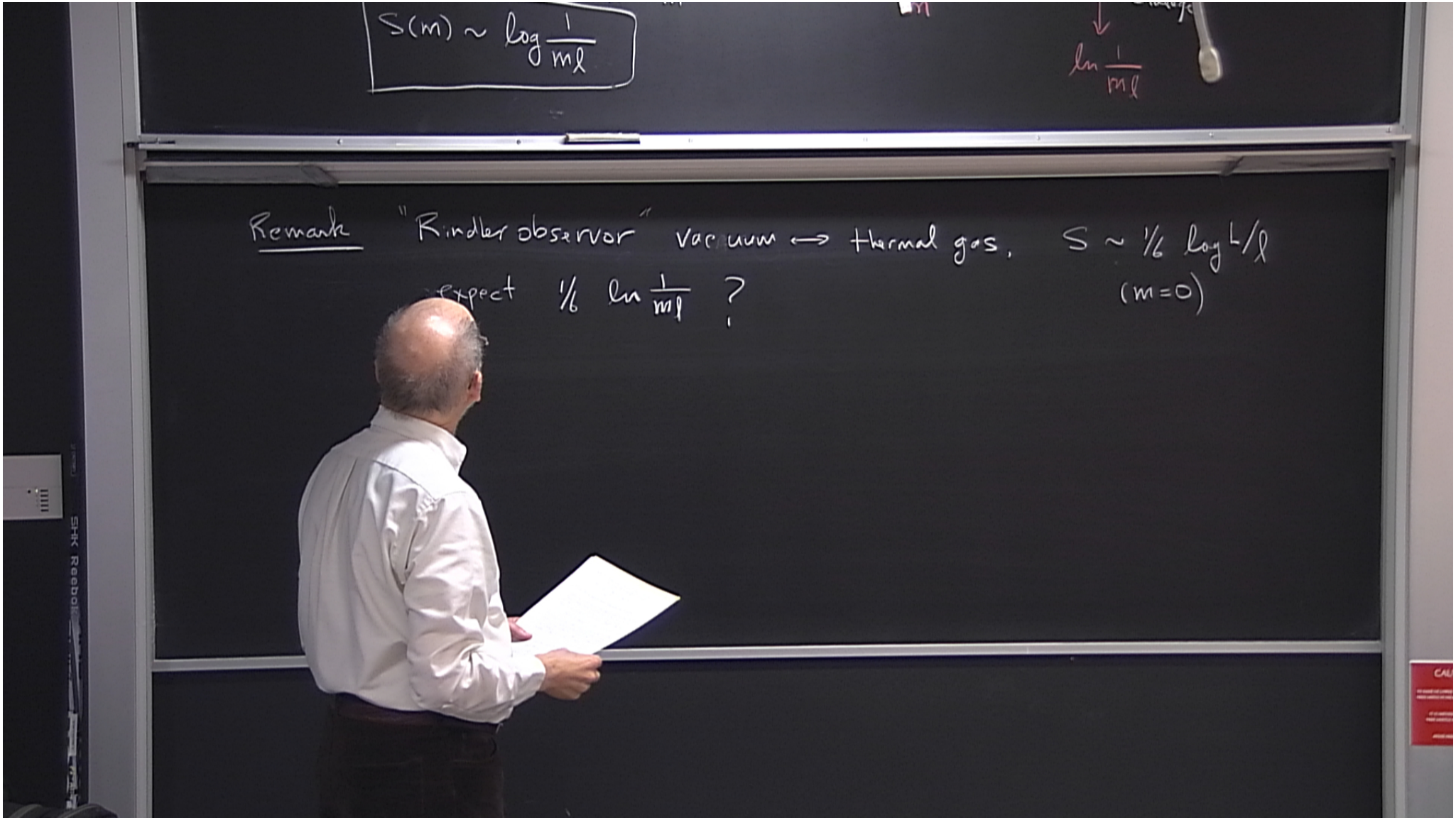

$\ell \leftarrow$ cutoff
$V = |B|$
$\ell \ll L$
$S \approx N_s \approx \int_{k_{min}}^{k_{max}} dN_s \approx \int \frac{2\pi}{k} A \frac{4\pi k^2 dk}{(2\pi)^3} = \frac{A}{\pi} \int_{2\pi/L}^{2\pi/\ell} k\,dk = \frac{A}{2\pi}\left(k_{max}^2 - k_{min}^2\right)$
$= 2\pi A\left(\frac{1}{\ell^2} - \frac{1}{L^2}\right) \approx \frac{2\pi A}{\ell^2}$
$S \approx \frac{2\pi A}{\lambda^2}$
3+1 → $\int k\,dk$ → $\frac{A}{\ell^2}$
2+1 → $\int dk$ → $\frac{A}{\ell}$ ← circumference
1+1 → $\int \frac{dk}{k}$ → $\int_{1/L}^{1/\ell} \frac{dk}{k} \approx \ln L/\ell$
$L \to \infty$ diverges
B

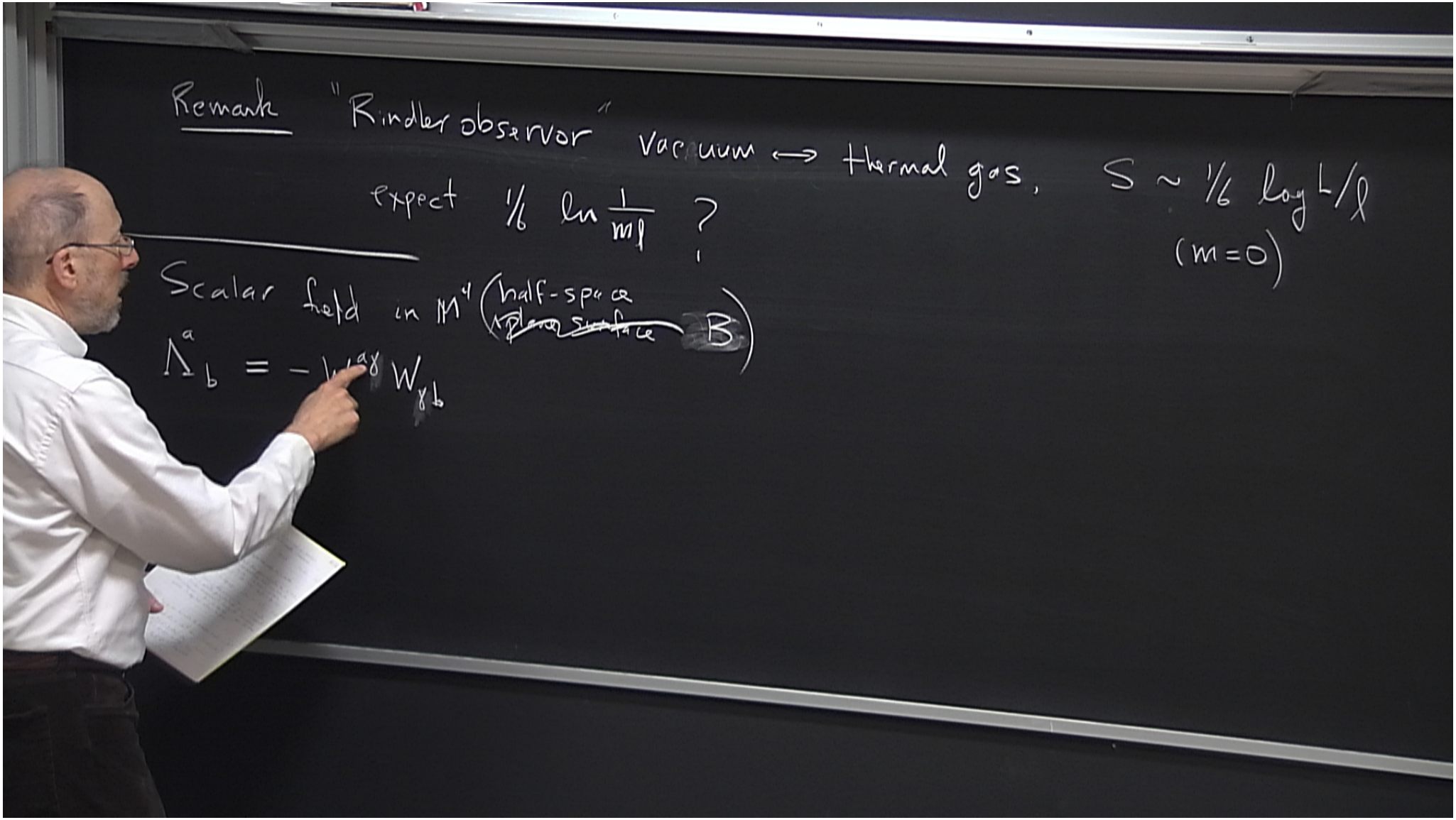

cutoff
$V = |B|$
$\ell \ll L$
$S \approx N_s \approx \int_{k_{min}}^{k_{max}} dN_s \approx \int \frac{2\pi}{k} A \frac{4\pi k^2 dk}{(2\pi)^3} = \frac{A}{\pi} \int_{2\pi/L}^{2\pi/\lambda} k\,dk = \frac{A}{2\pi}\left(k_{max}^2 - k_{min}^2\right)$
$= 2\pi A\left(\frac{1}{\lambda^2} - \frac{1}{L^2}\right) \approx \frac{2\pi A}{\lambda^2}$
$S \approx \frac{2\pi A}{\lambda^2}$
$3+1 \rightarrow \int k\,dk \rightarrow \frac{A}{\lambda^2}$
$2+1 \rightarrow \int dk \rightarrow \frac{A}{\ell}$
circumference
$1+1 \rightarrow$
$\int_{1/L}^{1/\ell} \frac{dk}{k} \approx \ln L/\ell$
$m^2 > 0$ M^2
$\lambda^{max} \sim 1/$
$mc)$
k_{min}
m
$L \to \infty$ diverges
$\ln \frac{1}{m\ell}$
B


cutoff
$V = |B|$
$\ell \ll L$
$S \approx N_s \approx \int_{k_{min}}^{k_{max}} dN_s \approx \int \frac{2\pi}{k} A \frac{4\pi k^2 dk}{(2\pi)^3} = \frac{A}{\pi} \int_{2\pi/L}^{2\pi/\lambda} k\,dk = \frac{A}{2\pi}\left(k_{max}^2 - k_{min}^2\right)$
$= 2\pi A\left(\frac{1}{\lambda^2} - \frac{1}{L^2}\right) \approx \frac{2\pi A}{\lambda^2}$
$S \approx \frac{2\pi A}{\lambda^2}$
$3+1 \rightarrow \int k\,dk \rightarrow \frac{A}{\lambda^2}$
$2+1 \rightarrow \int dk \rightarrow \frac{A}{\ell}$
$1+1 \rightarrow \int \frac{dk}{k} \rightarrow$
$m^2 > 0$ M^2
$\lambda^{max} \sim 1/m$ $(\hbar/mc)$
$k_{min} \sim m$
B


cutoff
$\ell \ll L$
$S \approx N_s \approx \int_{k_{min}}^{k_{max}} dN_s \approx \int \frac{2\pi}{k} A \frac{4\pi k^2 dk}{(2\pi)^3} = \frac{A}{\pi} \int_{2\pi/L}^{2\pi/\lambda} k\,dk = \frac{A}{2\pi}\left(k_{max}^2 - k_{min}^2\right)$
$= 2\pi A\left(\frac{1}{\ell^2} - \frac{1}{L^2}\right) \approx \frac{2\pi A}{\ell^2}$
$S \approx \frac{2\pi A}{\lambda^2}$
$3+1 \rightarrow \int k\,dk \rightarrow \frac{A}{\ell^2}$
$2+1 \rightarrow \int dk \rightarrow \frac{A}{\ell}$
$1+1 \rightarrow \int \frac{dk}{k} \rightarrow \int_{1/L}^{1/\ell} \frac{dk}{k} \approx \ln L/\ell$
circumference
$m^2 > 0$
M^2
λ^{max}
$(\hbar/mc)$
m
$L \to \infty$ diverges
$\ln \frac{1}{m\ell}$
B
$V = |B|$


$S(m) \sim \log \frac{1}{m\ell}$
$\ln \frac{1}{m\ell}$
Remark "Rindler observer" vacuum ⟷ thermal gas.


$S(m) \sim \log \frac{1}{ml}$
$\ln \frac{1}{ml}$
Remark "Rindler observer" vacuum ⟷ thermal gas. $S \sim \frac{1}{6} \log L/\ell$
expect $\frac{1}{6} \ln \frac{1}{ml}$?
$(m=0)$


Remark "Rindler observer" vacuum ⟷ thermal gas. $S \sim \frac{1}{6} \log L/\ell$ $(m=0)$
expect $\frac{1}{6} \ln \frac{1}{m\ell}$?
Scalar field in M^4 (half-space ~~plane surface~~ B)
$\Lambda^a{}_b = -\lambda^{ax} W_{xb}$

$S(m) \sim \log \frac{1}{m\ell}$

$\ln \frac{1}{m\ell}$

Remark "Rindler observer" vacuum $\longleftrightarrow$ thermal gas. $S \sim \frac{1}{6} \log L/\ell$ $(m=0)$

expect $\frac{1}{6} \ln \frac{1}{m\ell}$?

Scalar field in M^4 (half-space ~~Rindler Surface~~ B)

$\Lambda^a{}_b = -W^{ax} W_{xb}$ $\quad L = \int \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}|\vec{\nabla}\phi|^2 \, d^3x$ $(m=0)$

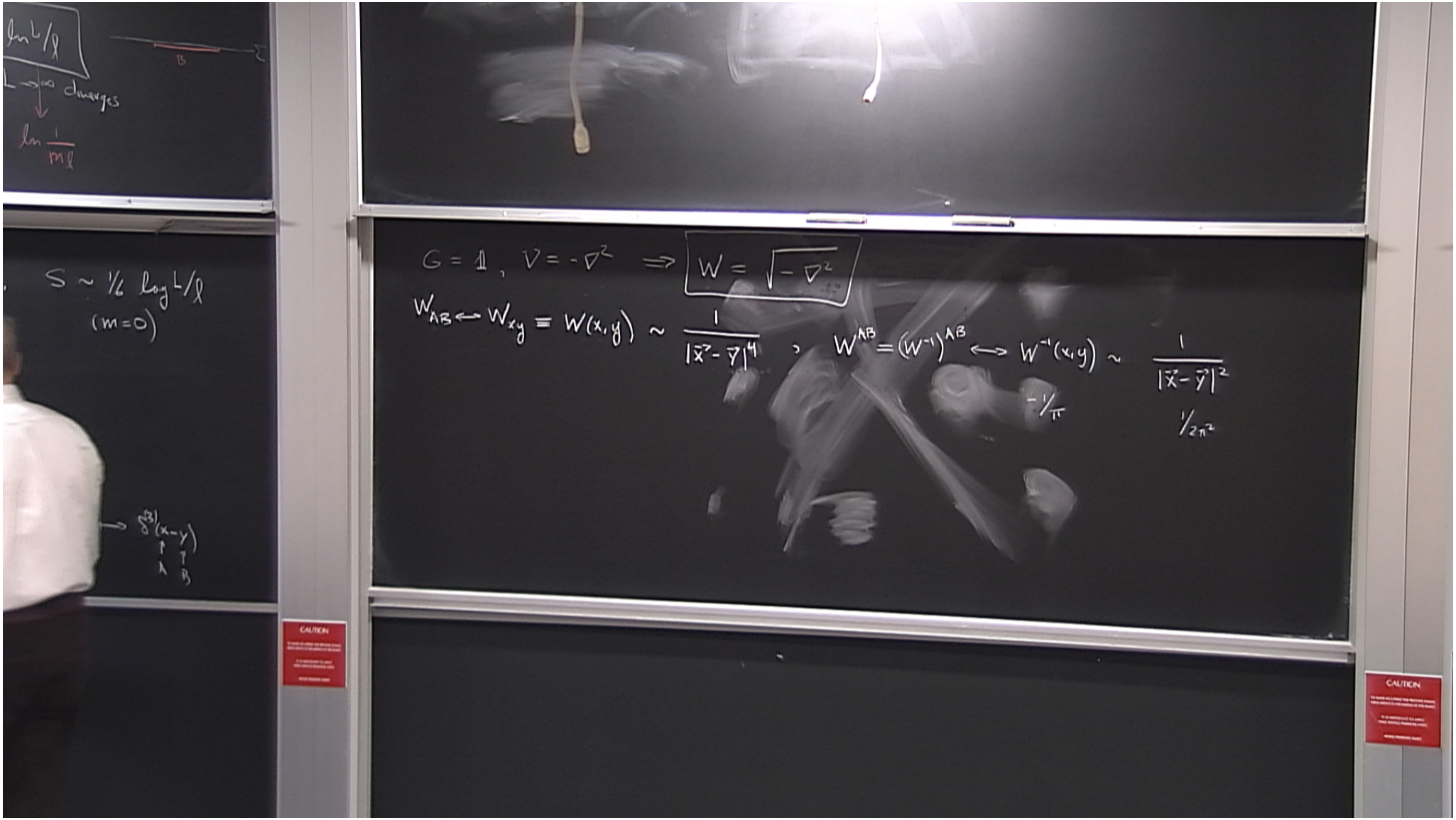
$a,b, \sim x$

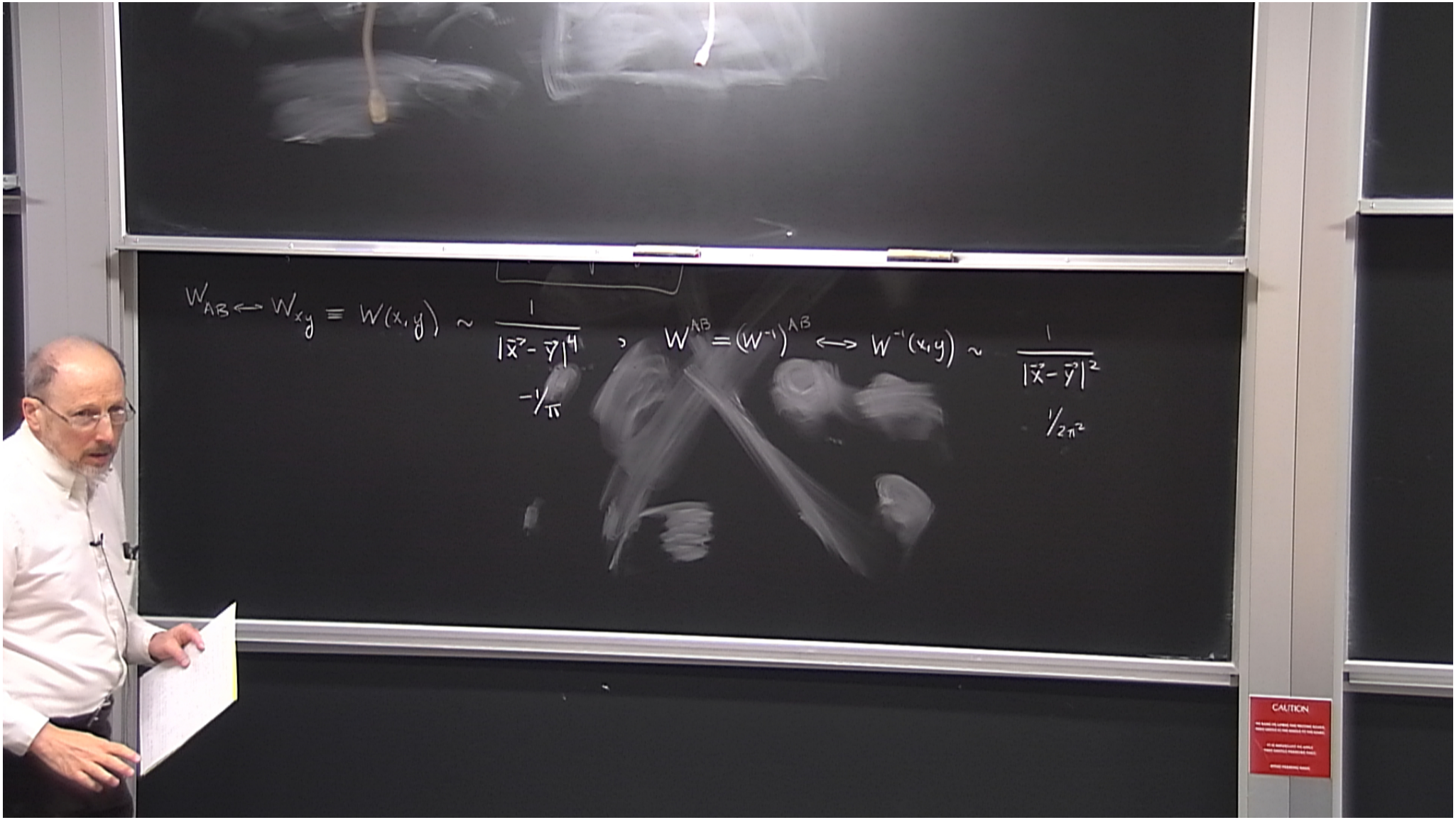
A,B

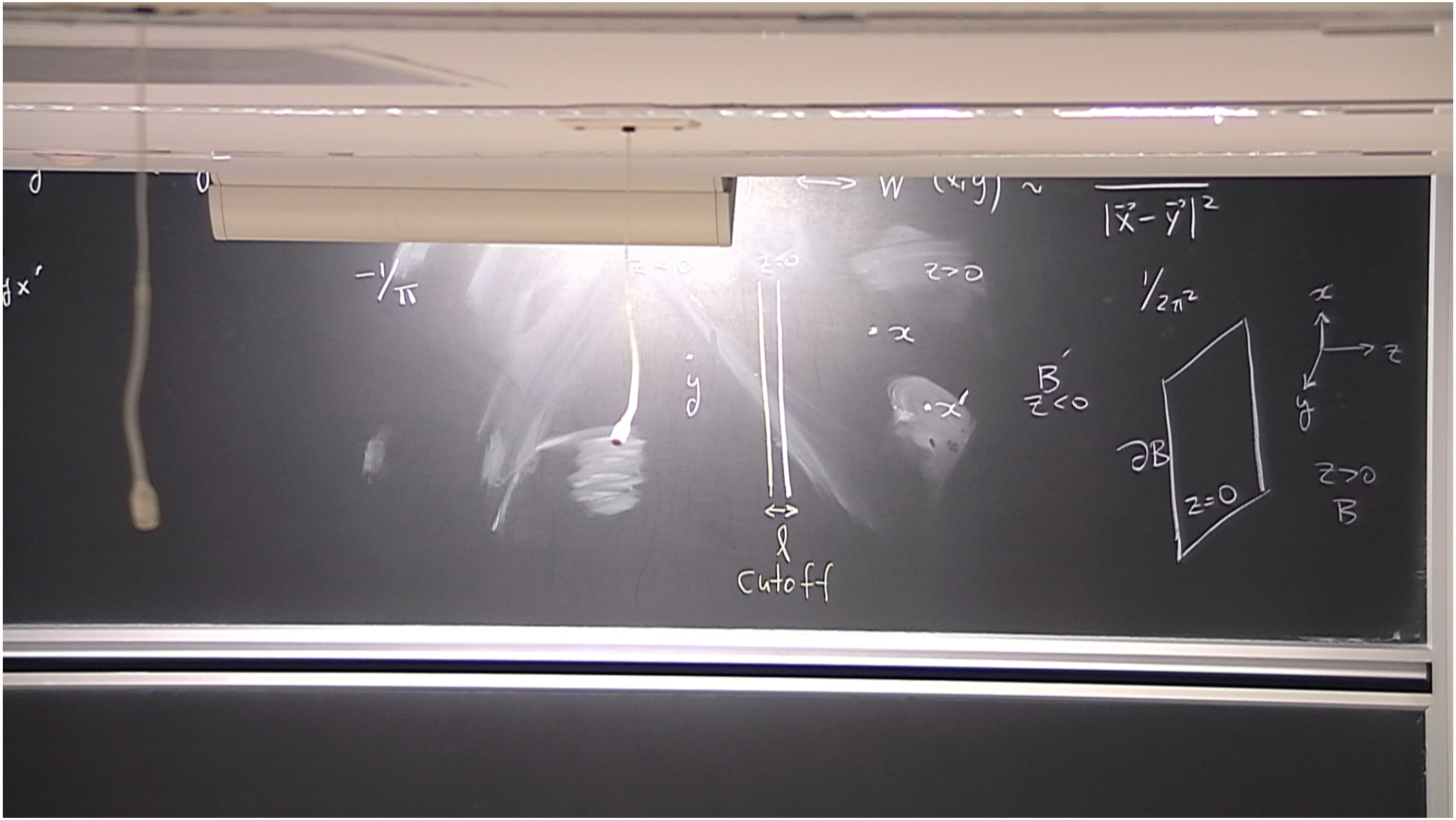
$H = \frac{1}{2} \int \dot{\phi}(x)\dot{\phi}(x) \, |\vec{\nabla}\phi|^2 \, d^3x$

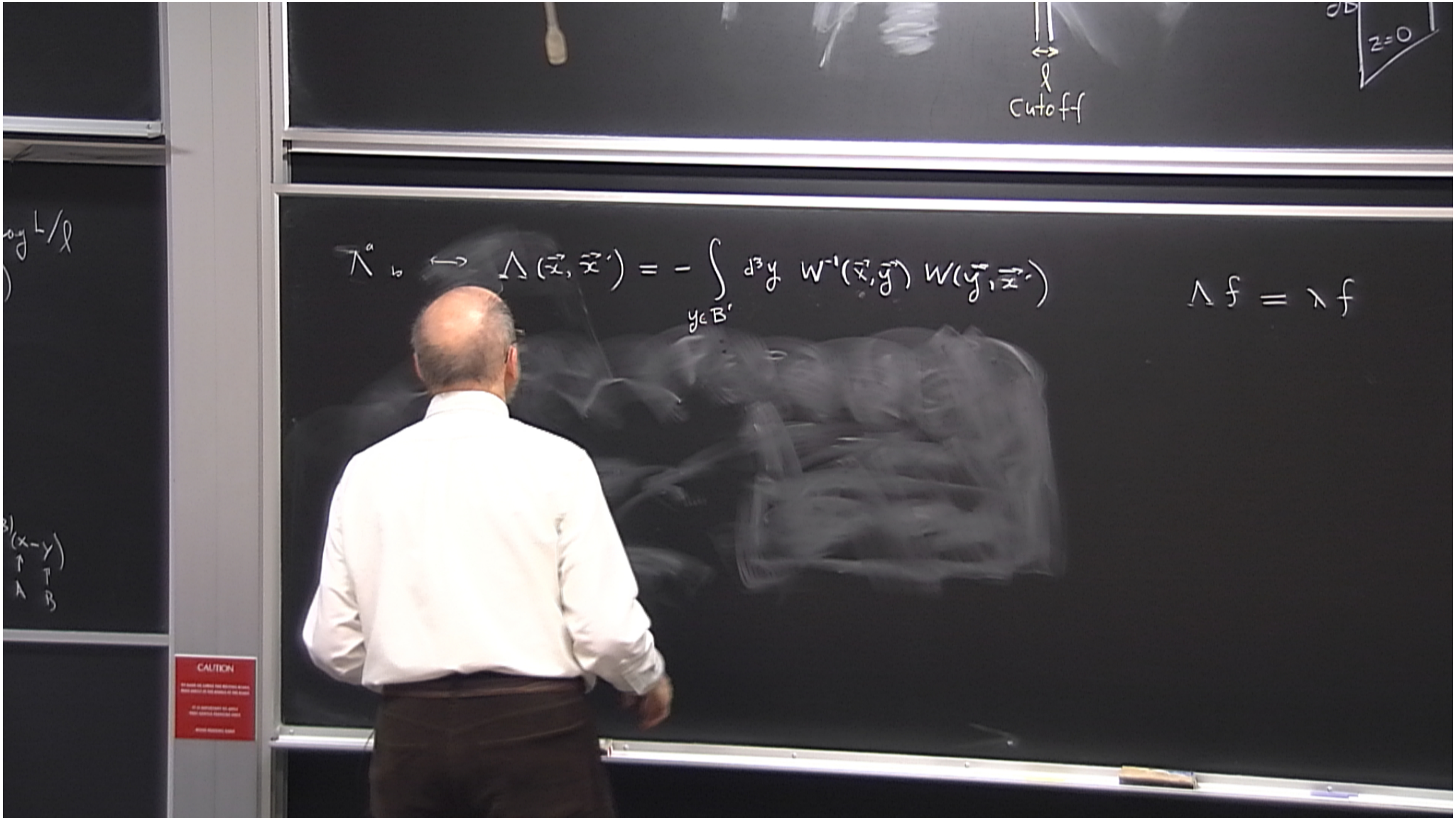
$G_{AB}\, \dot{\phi}^A \dot{\phi}^B$ $\qquad V_{AB}\, \phi^A \phi^B$

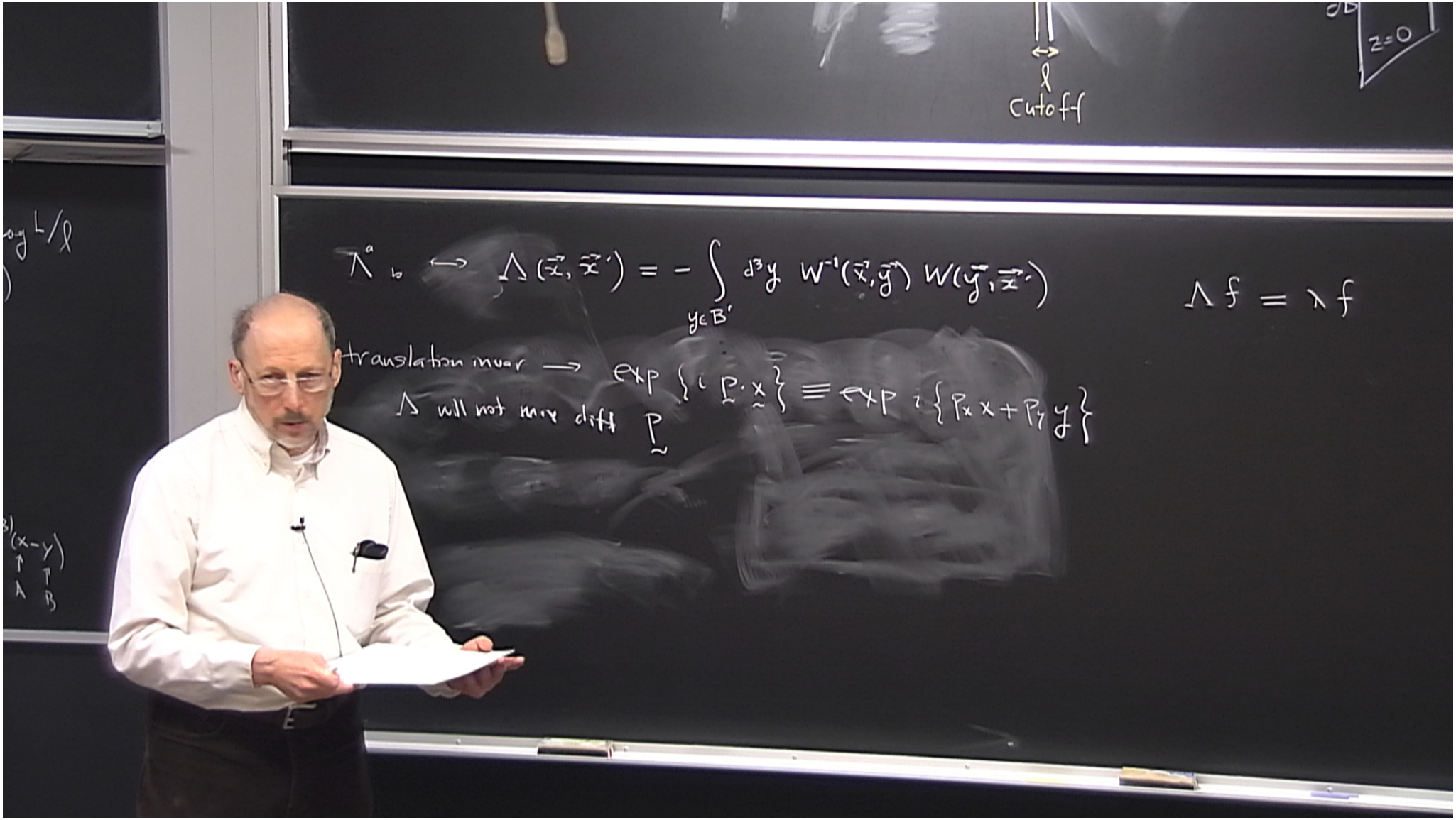
$G_{AB} \longleftrightarrow \delta^3(x-y)$ (A, B)

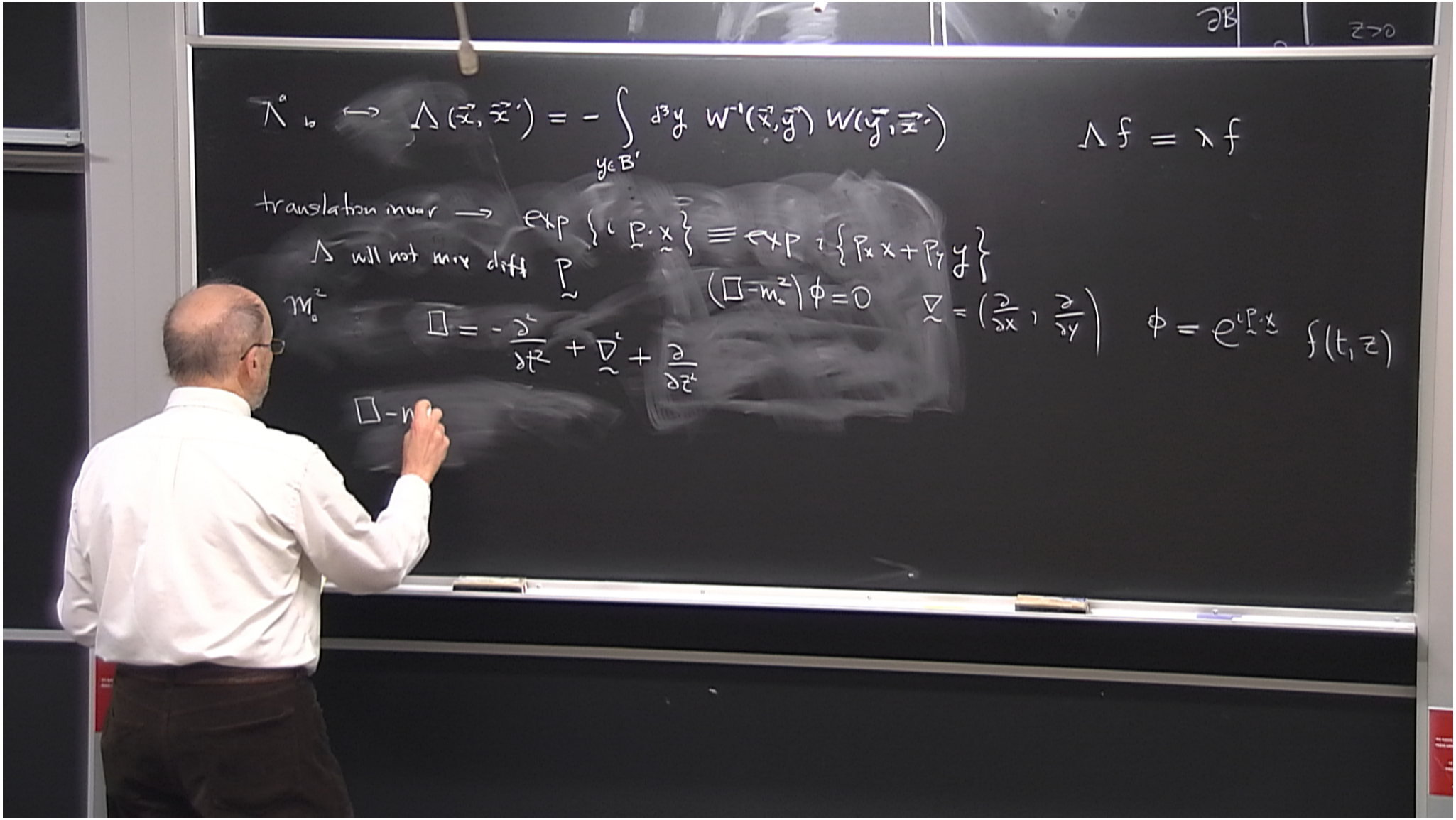

$\ln L/\ell$
$L \to \infty$ diverges
$\ln \frac{1}{m\ell}$
$S \sim \frac{1}{6} \log L/\ell$
$(m=0)$
$G = \mathbb{1}$, $V = -\nabla^2$ $\Rightarrow$ $W = \sqrt{-\nabla^2}$
$W_{AB} \leftrightarrow W_{xy} \equiv W(x,y) \sim \frac{1}{|\vec{x}-\vec{y}|^4}$; $W^{AB} = (W^{-1})^{AB} \leftrightarrow W^{-1}(x,y) \sim \frac{1}{|\vec{x}-\vec{y}|^2}$
$-1/\pi$
$1/2\pi^2$
$\delta^{(3)}(x-y)$
A B
CAUTION
CAUTION

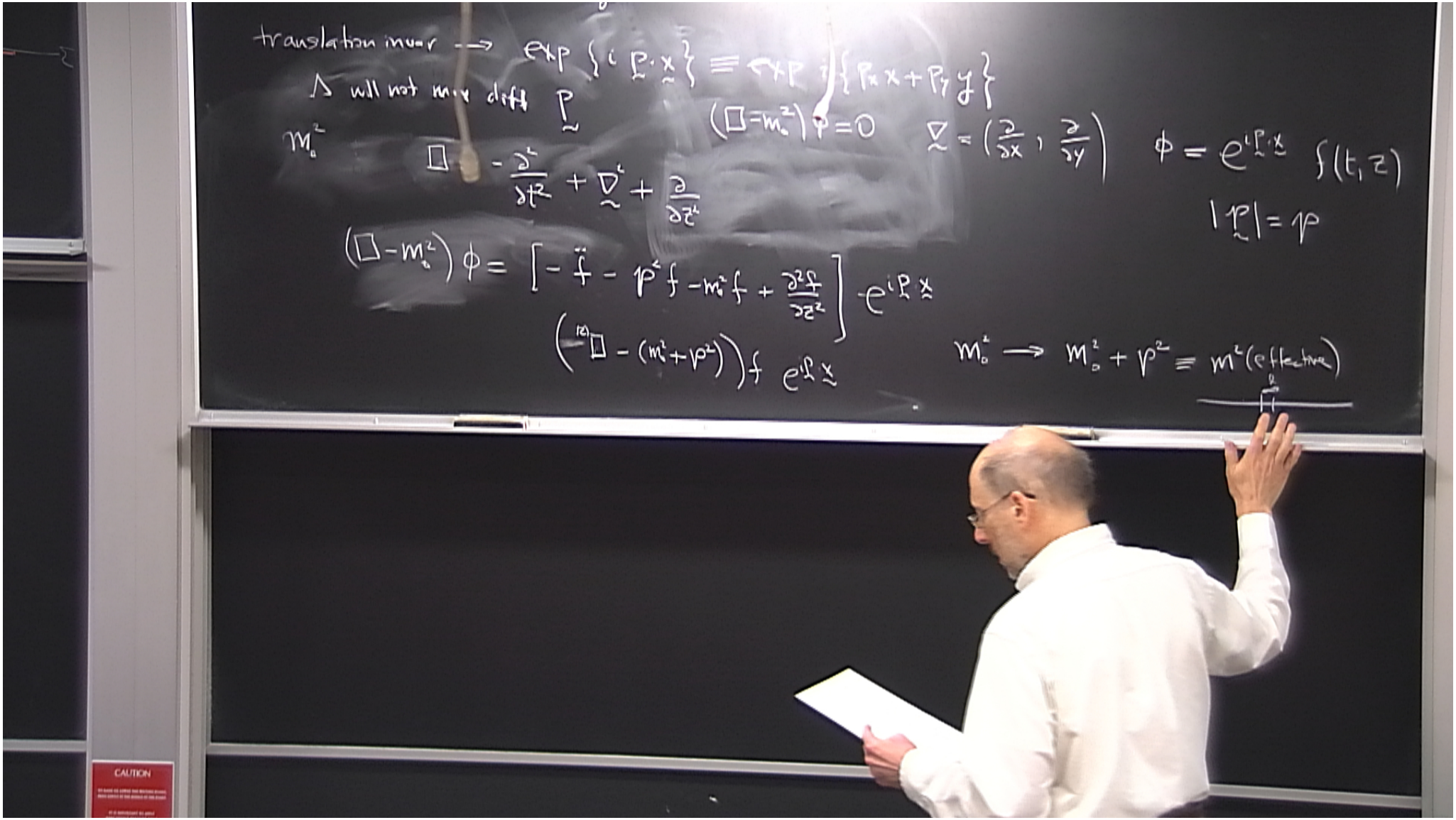

$W_{AB} \leftrightarrow W_{xy} \equiv W(x,y) \sim \frac{1}{|\vec{x}-\vec{y}|^4}$
$-1/\pi$
$W^{AB} = (W^{-1})^{AB} \leftrightarrow W^{-1}(x,y) \sim \frac{1}{|\vec{x}-\vec{y}|^2}$
$1/2\pi^2$
CAUTION

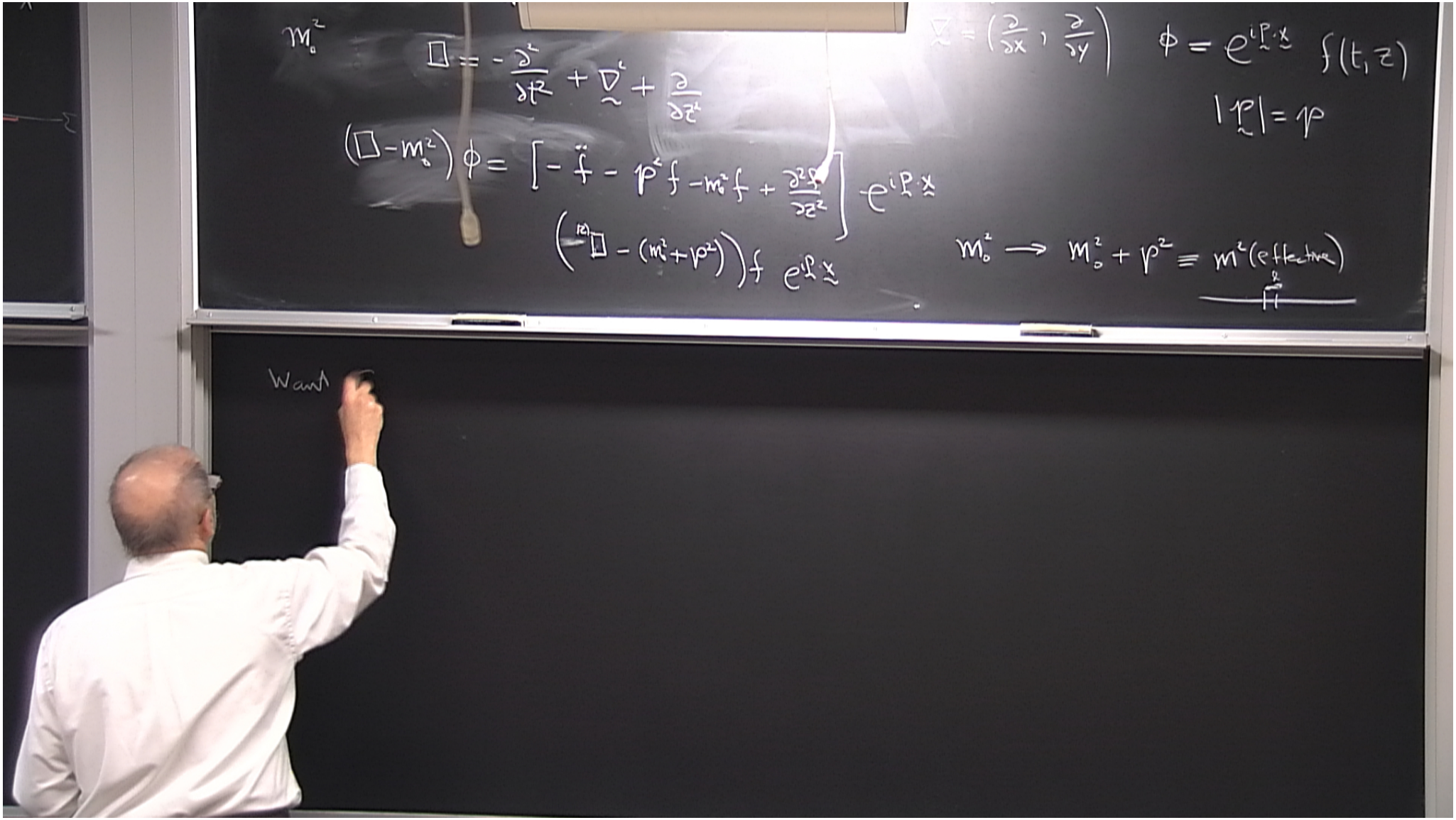

$\frac{1}{|\vec{x}-\vec{y}|^2}$
$-1/\pi$
$z<0$
$z>0$
$z=0$
$\frac{1}{2\pi^2}$
x
y
x'
B
$z<0$
∂B
$z=0$
$z>0$
B
ℓ
Cutoff

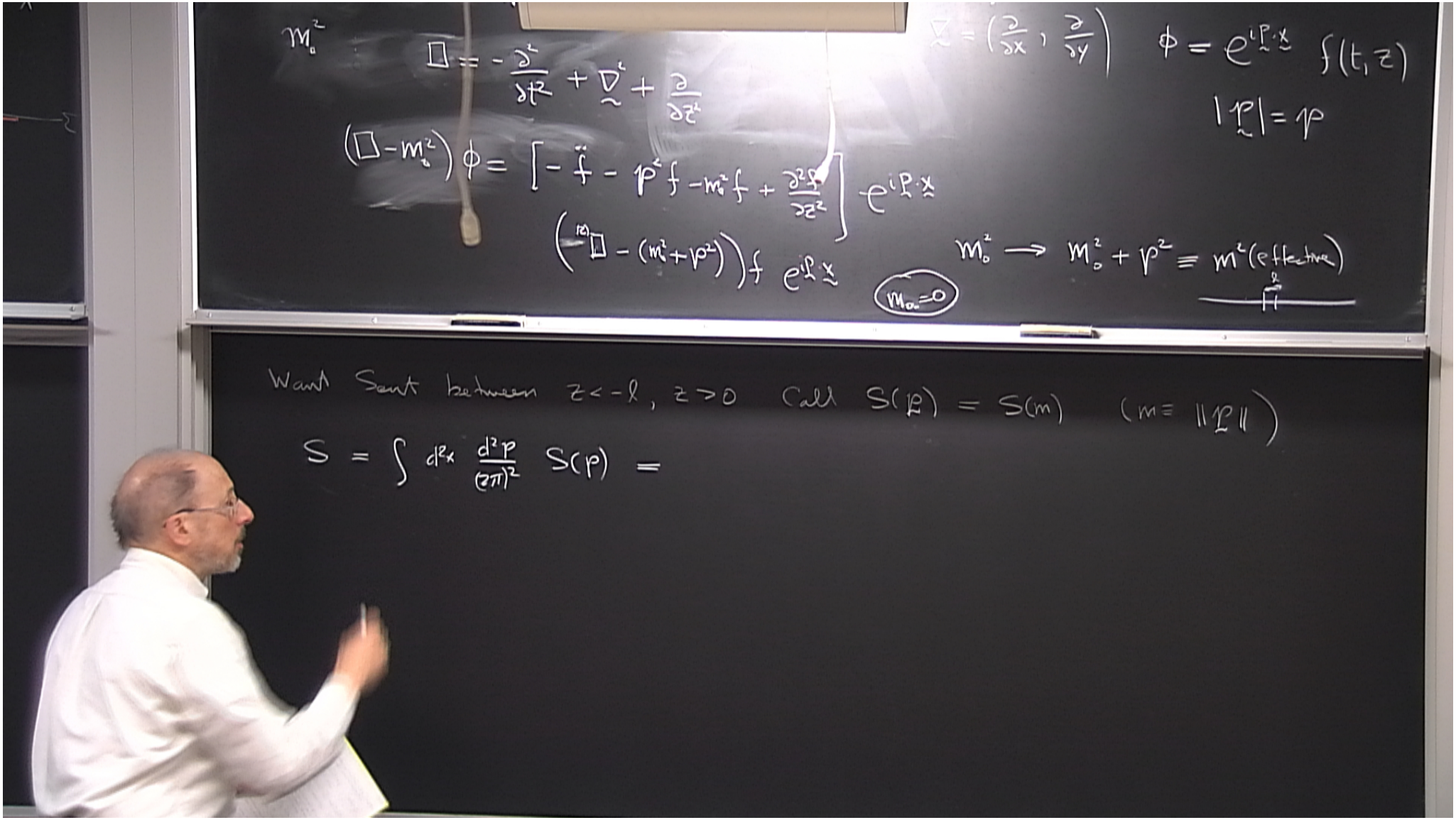

Cutoff
z=0
$\Lambda^a{}_b \longleftrightarrow \Lambda(\vec{x},\vec{x}') = -\int_{y\in B'} d^3y\, W^{-1}(\vec{x},\vec{y})\, W(\vec{y},\vec{x}')$
$\Lambda f = \lambda f$
CAUTION

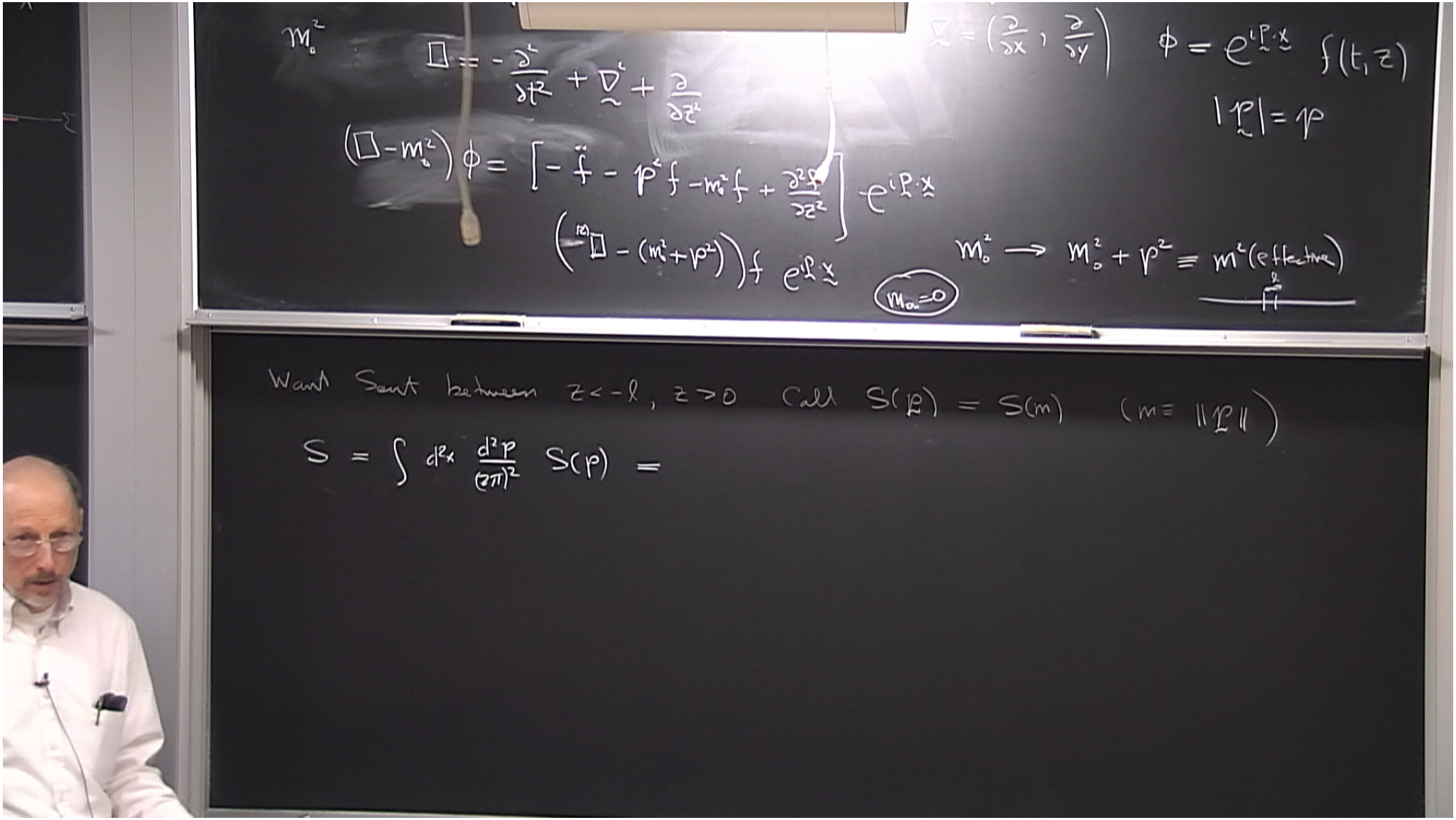

Cutoff
z=0
$\Lambda^a{}_b \longleftrightarrow \Lambda(\vec{x},\vec{x}') = -\int_{y\in B'} d^3y\, W^{-1}(\vec{x},\vec{y})\, W(\vec{y},\vec{x}')$
$\Lambda f = \lambda f$
translation invar $\longrightarrow$ $\exp\{i\,\underset{\sim}{P}\cdot\underset{\sim}{x}\} \equiv \exp i\{P_x x + P_y y\}$
Λ will not mix diff $\underset{\sim}{P}$

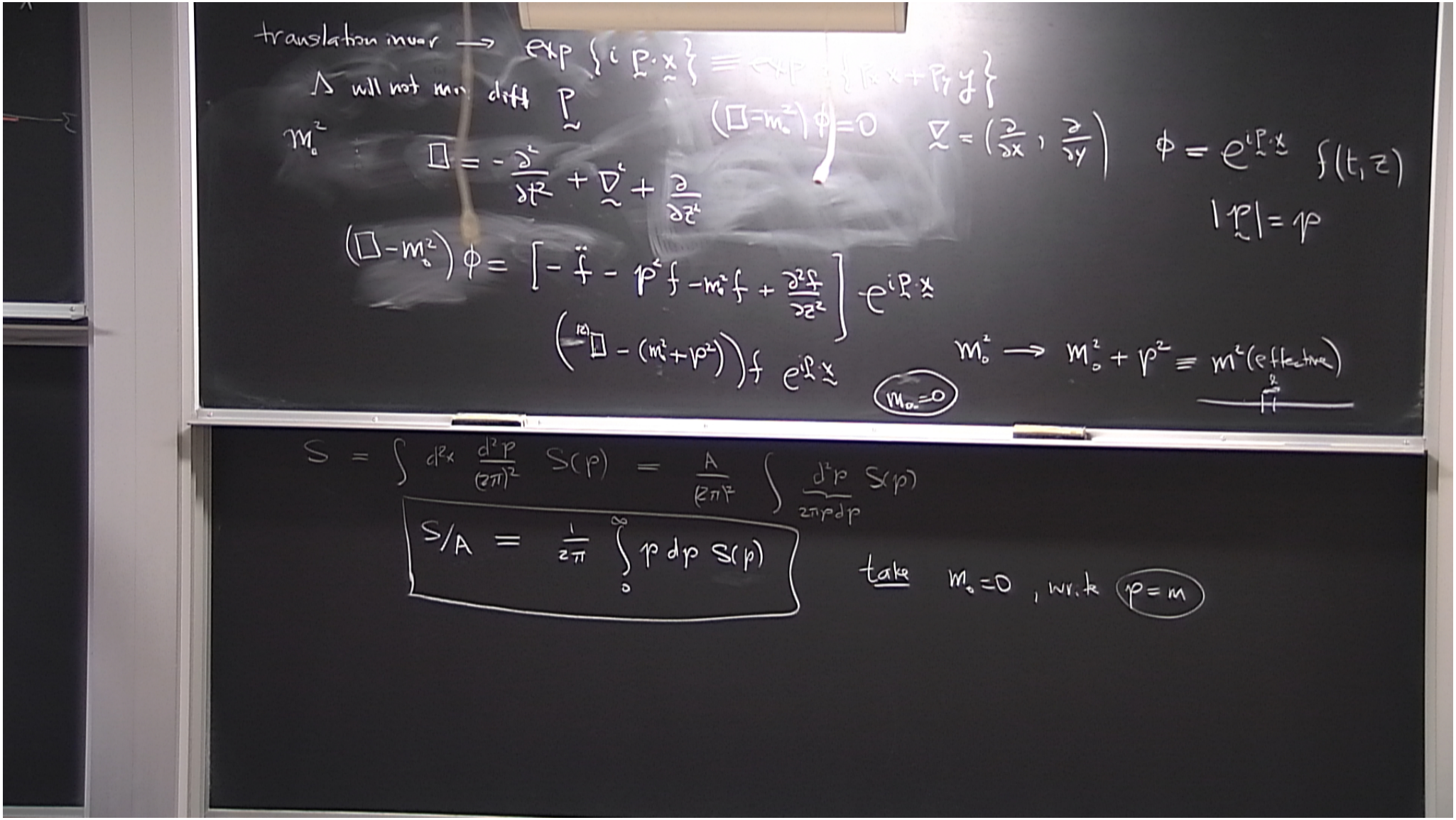

$\Lambda^a{}_b \longleftrightarrow \Lambda(\vec{x},\vec{x}') = -\int_{y\in B'} d^3y\; W^{-1}(\vec{x},\vec{y})\, W(\vec{y},\vec{x}')$
$\Lambda f = \nu f$
translation invar $\longrightarrow$ $\exp\{i\,\underset{\sim}{P}\cdot\underset{\sim}{x}\} \equiv \exp i\{p_x x + p_y y\}$
Λ will not mix diff $\underset{\sim}{P}$
m_0^2
$(\Box - m_0^2)\phi = 0$
$\underset{\sim}{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$
$\phi = e^{i\underset{\sim}{P}\cdot\underset{\sim}{x}} f(t,z)$
$\Box = -\frac{\partial^2}{\partial t^2} + \underset{\sim}{\nabla}^2 + \frac{\partial}{\partial z^2}$
$\Box - m$

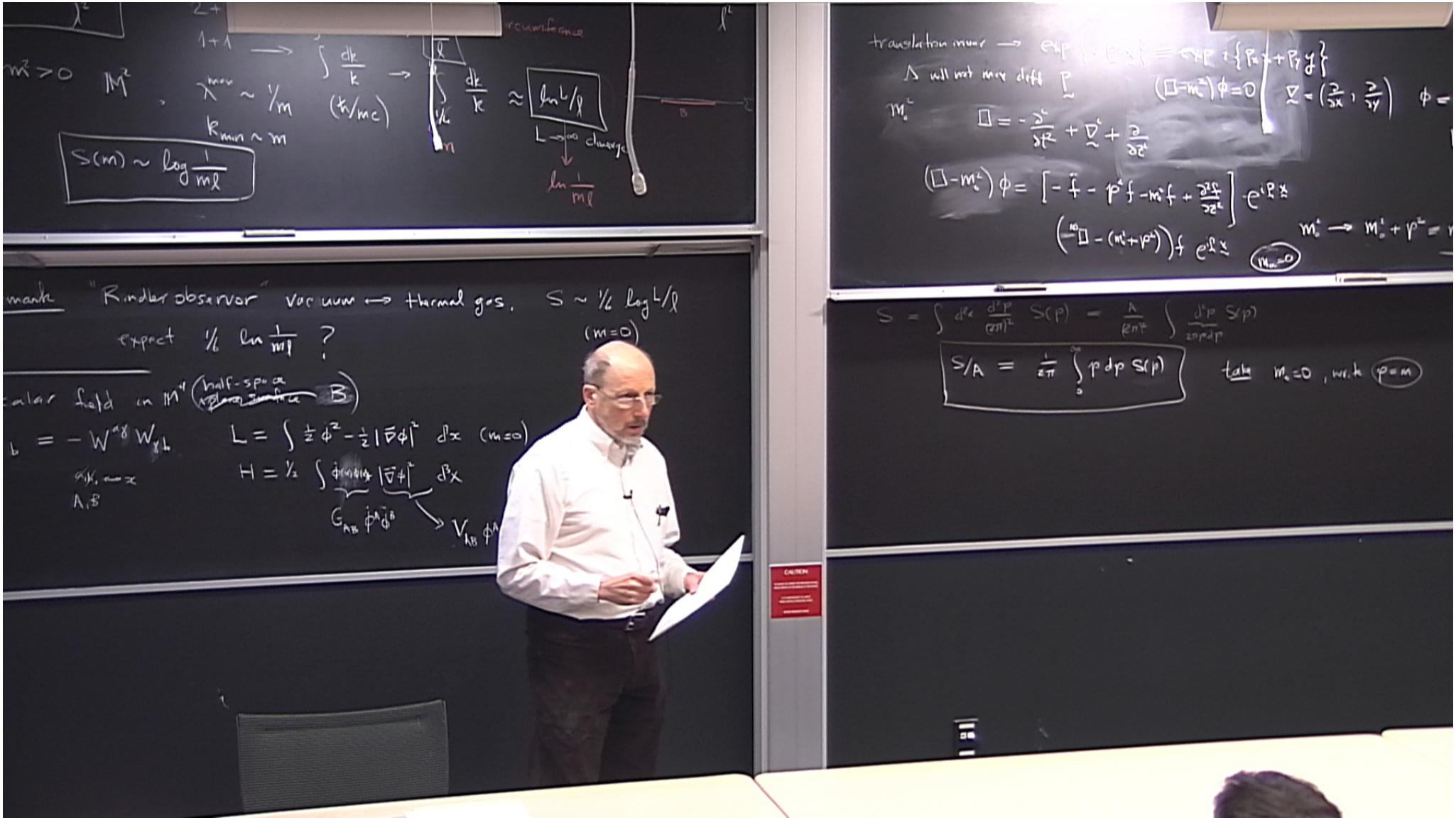

translation invar → exp{i p·x} ≡ exp{i(p_x x + p_y y)}
Λ will not mix diff p
m₀²
□ = −∂²/∂t² + ∇² + ∂²/∂z²
(□ − m₀²)φ = 0
∇ = (∂/∂x, ∂/∂y)
φ = e^{i p·x} f(t, z)
|p| = p
(□ − m₀²)φ = [−f̈ − p²f − m₀²f + ∂²f/∂z²] e^{i p·x}
(□ − (m₀² + p²)) f e^{i p·x}
m₀² → m₀² + p² ≡ m²(effective)
CAUTION

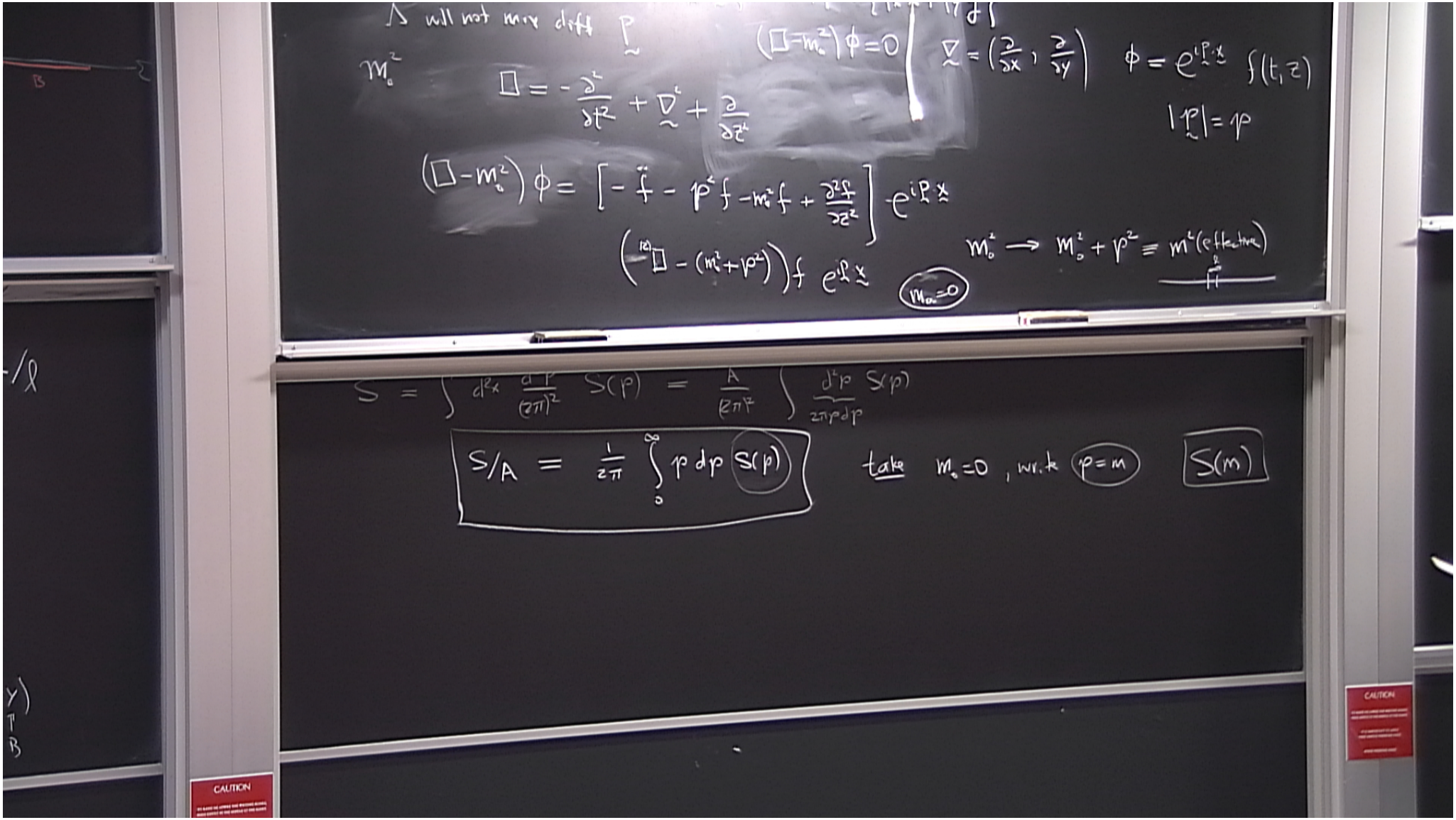

m_0^2
$\Box = -\frac{\partial^2}{\partial t^2} + \nabla^2 + \frac{\partial}{\partial z^2}$
$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$
$\phi = e^{i\vec{p}\cdot\vec{x}} f(t,z)$
$|\vec{p}| = p$
$(\Box - m_0^2)\phi = \left[-\ddot{f} - p^2 f - m^2 f + \frac{\partial^2 f}{\partial z^2}\right] e^{i\vec{p}\cdot\vec{x}}$
$\left(-\Box - (m^2 + p^2)\right) f\, e^{i\vec{p}\cdot\vec{x}}$
$m_0^2 \longrightarrow m_0^2 + p^2 \equiv m^2(\text{effective})$
Want

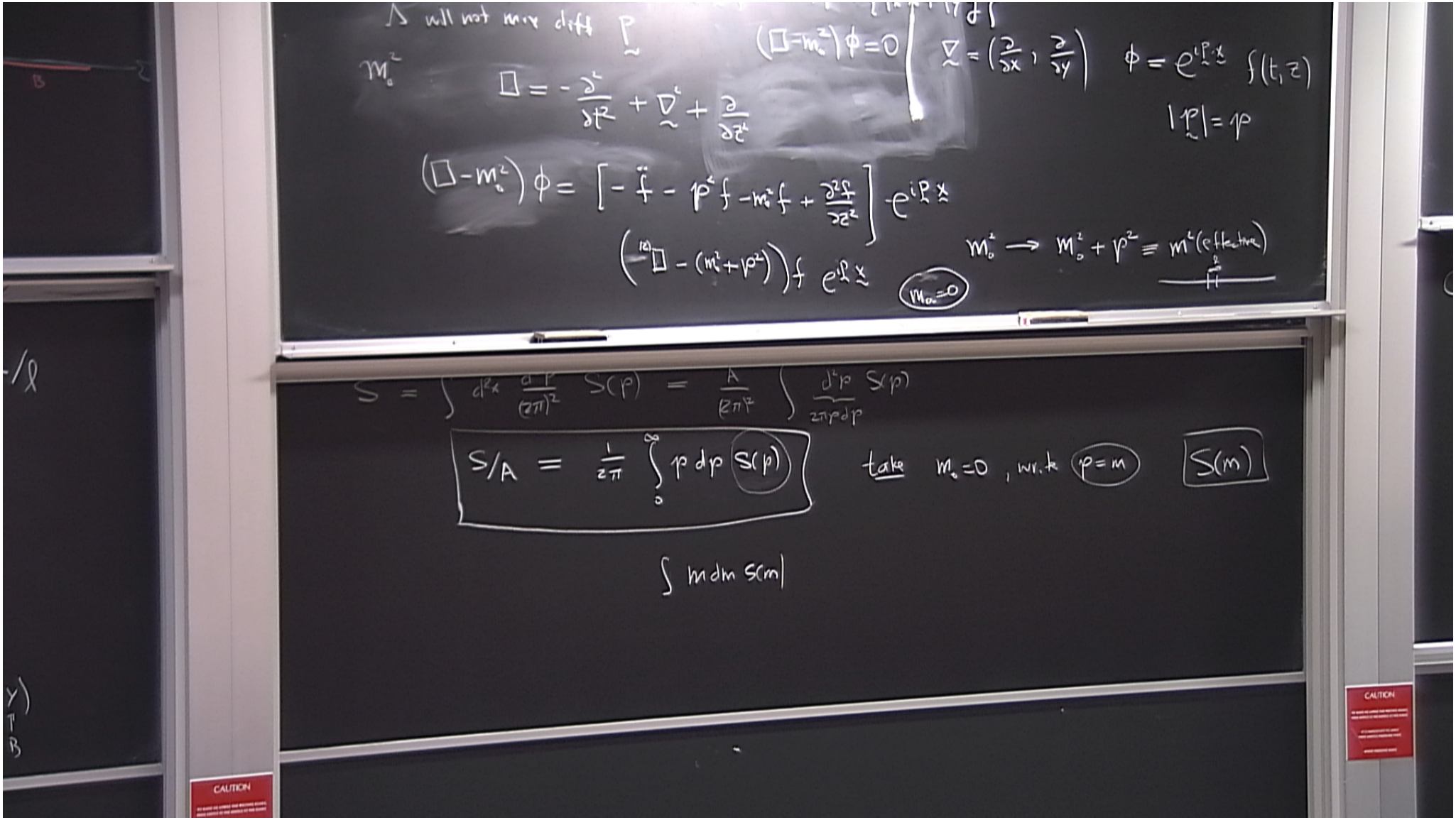

m_0^2
$\Box = -\frac{\partial^2}{\partial t^2} + \nabla^2 + \frac{\partial}{\partial z^2}$
$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$
$\phi = e^{i\vec{p}\cdot\vec{x}} f(t,z)$
$|\vec{p}| = p$
$(\Box - m_0^2)\phi = \left[-\ddot{f} - p^2 f - m_0^2 f + \frac{\partial^2 f}{\partial z^2}\right] e^{i\vec{p}\cdot\vec{x}}$
$\left(-\frac{\partial^2}{\partial t^2} - (m_0^2 + p^2)\right) f\, e^{i\vec{p}\cdot\vec{x}}$
$m_0 = 0$
$m_0^2 \longrightarrow m_0^2 + p^2 \equiv m^2$ (effective)
Want S out between $z < -\ell$, $z > 0$ Call $S(\vec{p}) = S(m)$ $(m \equiv \|\vec{p}\|)$
$S = \int d^2x \frac{d^2p}{(2\pi)^2} S(p) =$

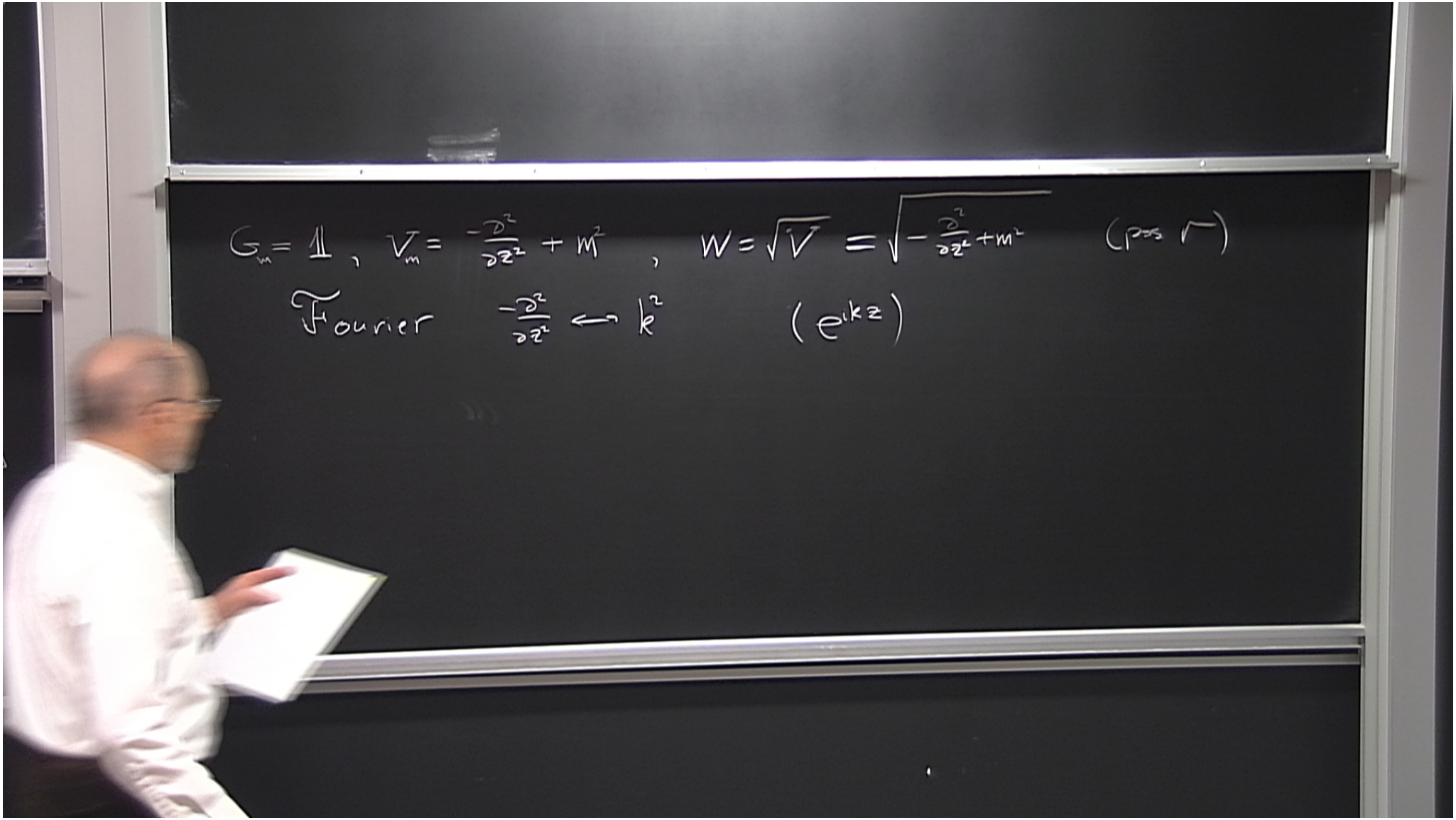

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$\Box = -\frac{\partial^2}{\partial t^2} + \nabla^2 + \frac{\partial}{\partial z^2}$
$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$
$\phi = e^{i\vec{p}\cdot\vec{x}} f(t,z)$
$|\vec{p}| = p$
$(\Box - m_0^2)\phi = \left[-\ddot{f} - p^2 f - m_0^2 f + \frac{\partial^2 f}{\partial z^2}\right] e^{i\vec{p}\cdot\vec{x}}$
$\left(-\Box - (m_0^2 + p^2)\right) f\, e^{i\vec{p}\cdot\vec{x}}$
$m_0 = 0$
$m_0^2 \longrightarrow m_0^2 + p^2 \equiv m^2(\text{effective})$
FT
Want S_{ant} between $z < -\ell$, $z > 0$ Call $S(\vec{p}) = S(m)$ $(m \equiv \|\vec{p}\|)$
$S = \int d^2x \frac{d^2p}{(2\pi)^2} S(p) =$

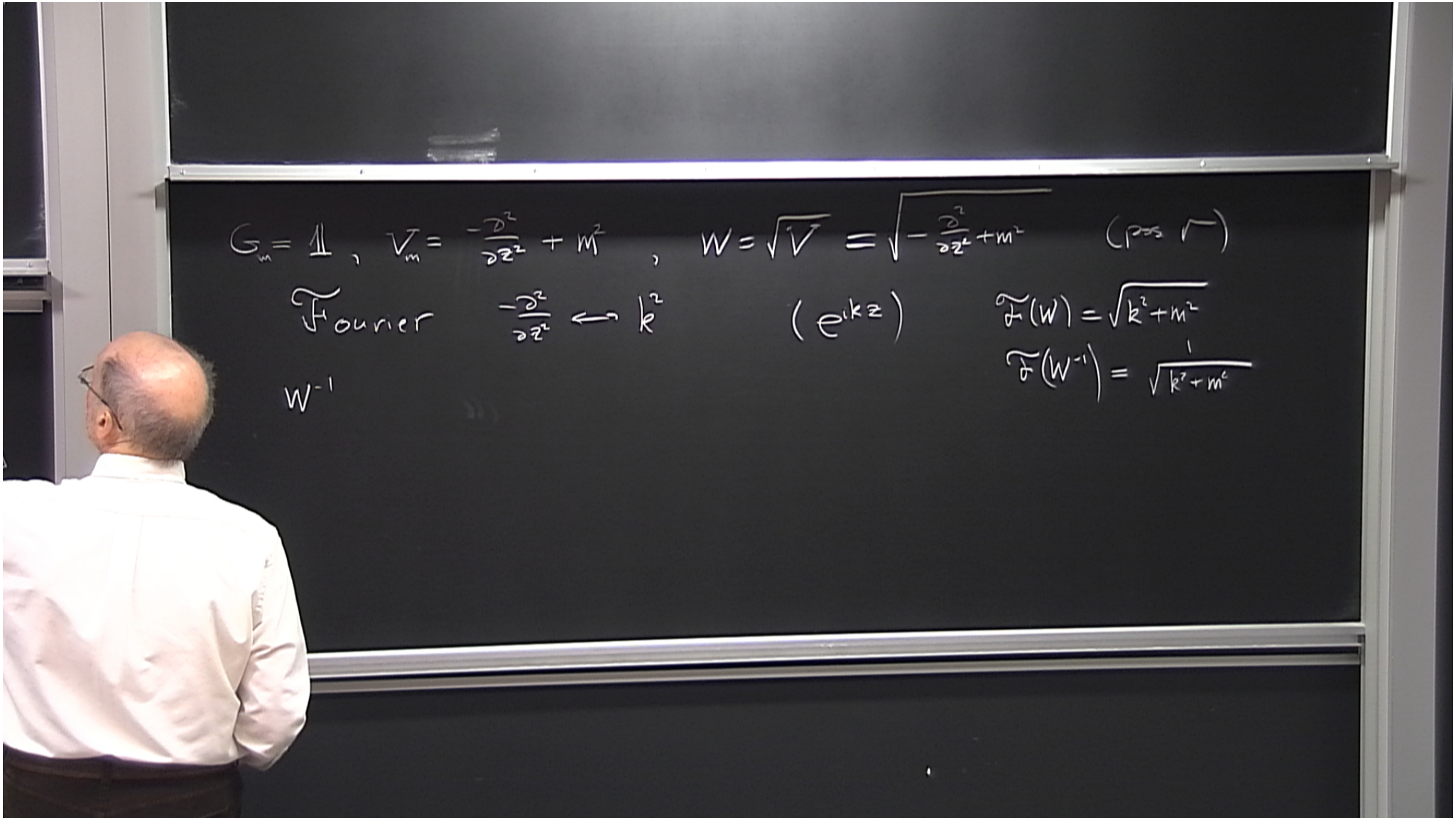

translation invar → exp{i p·x} = exp{p_x x + p_y y}
Λ will not mix diff p
m₀²
□ = −∂²/∂t² + ∇² + ∂²/∂z²
(□ − m₀²)φ = 0
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(□ − m₀²)φ = [−f̈ − p²f − m₀²f + ∂²f/∂z²] e^{i p·x}
(□ − (m₀² + p²)) f e^{i p·x}
m₀ = 0
m₀² → m₀² + p² ≡ m²(effective)
S = ∫ d²x d²p/(2π)² S(p) = A/(2π)² ∫ d²p/(2π p dp) S(p)
S/A = 1/(2π) ∫₀^∞ p dp S(p)
take m₀ = 0, write p = m

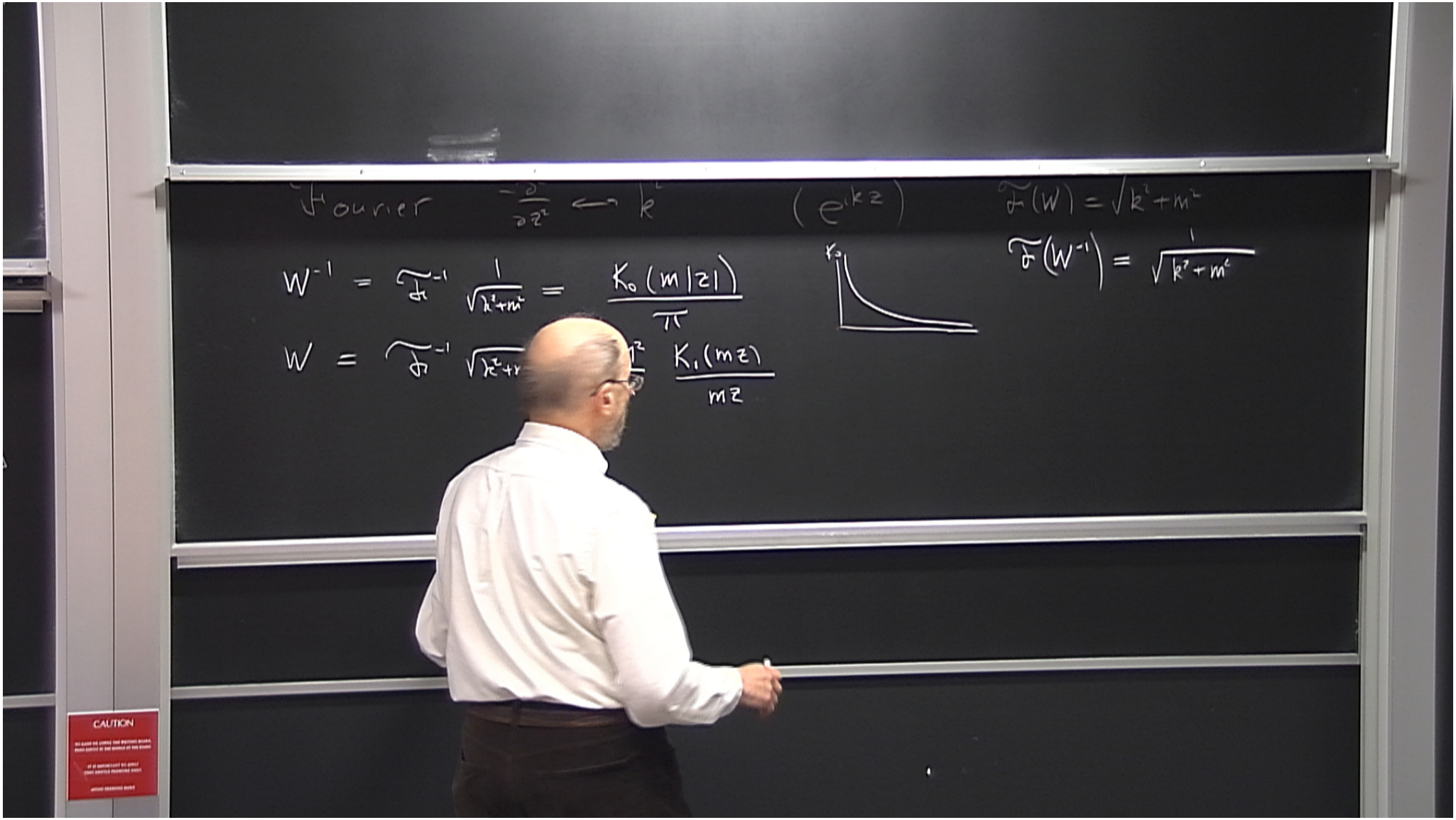

$S(m) \sim \log \frac{1}{m\ell}$
"Rindler observer" vacuum ⟷ thermal gas. $S \sim \frac{1}{6} \log L/\ell$
expect $\frac{1}{6} \ln \frac{1}{m\ell}$?
$(m=0)$
$L = \int \frac{1}{2}\phi^2 - \frac{1}{2}|\vec{\nabla}\phi|^2 \, d^2x$ $(m=0)$
$H = \frac{1}{2} \int \dot{\phi}\dot{\phi} \; |\vec{\nabla}\phi|^2 \, d^2x$
$S/A = \frac{1}{2\pi} \int_0^\infty p \, dp \, S(p)$
take $m_0 = 0$, with $p = m$
CAUTION

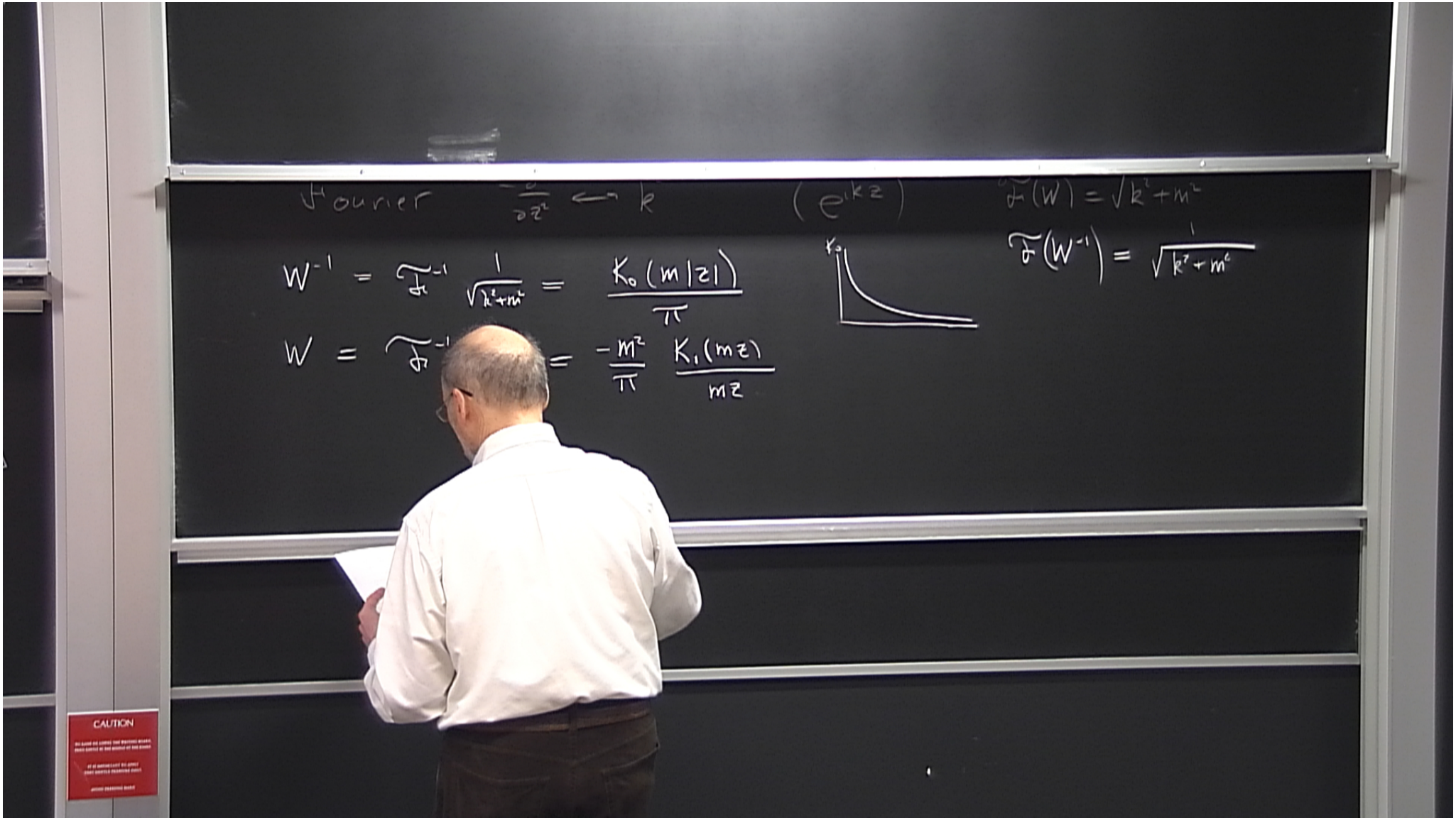

Λ will not mix diff $\underset{\sim}{p}$
m_0^2
$\Box = -\frac{\partial^2}{\partial t^2} + \underset{\sim}{\nabla}^2 + \frac{\partial}{\partial z^2}$
$(\Box - m_0^2)\phi = 0$
$\underset{\sim}{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$
$\phi = e^{i\underset{\sim}{p}\cdot\underset{\sim}{x}} f(t,z)$
$|\underset{\sim}{p}| = p$
$(\Box - m_0^2)\phi = \left[-\ddot{f} - p^2 f - m_0^2 f + \frac{\partial^2 f}{\partial z^2}\right] e^{i\underset{\sim}{p}\cdot\underset{\sim}{x}}$
$\left(\frac{\partial^2}{\partial t^2}\Box - (m_0^2 + p^2)\right) f\, e^{i\underset{\sim}{p}\cdot\underset{\sim}{x}}$
$m_{0_R} = 0$
$m_0^2 \longrightarrow m_0^2 + p^2 \equiv m^2$ (effective)
$S = \int d^2x \frac{d^2p}{(2\pi)^2} S(p) = \frac{A}{(2\pi)^2} \int \underbrace{d^2p}_{2\pi p\, dp} S(p)$
$S/A = \frac{1}{2\pi} \int_0^\infty p\, dp\, S(p)$
take $m_0 = 0$, write $p = m$
$S(m)$

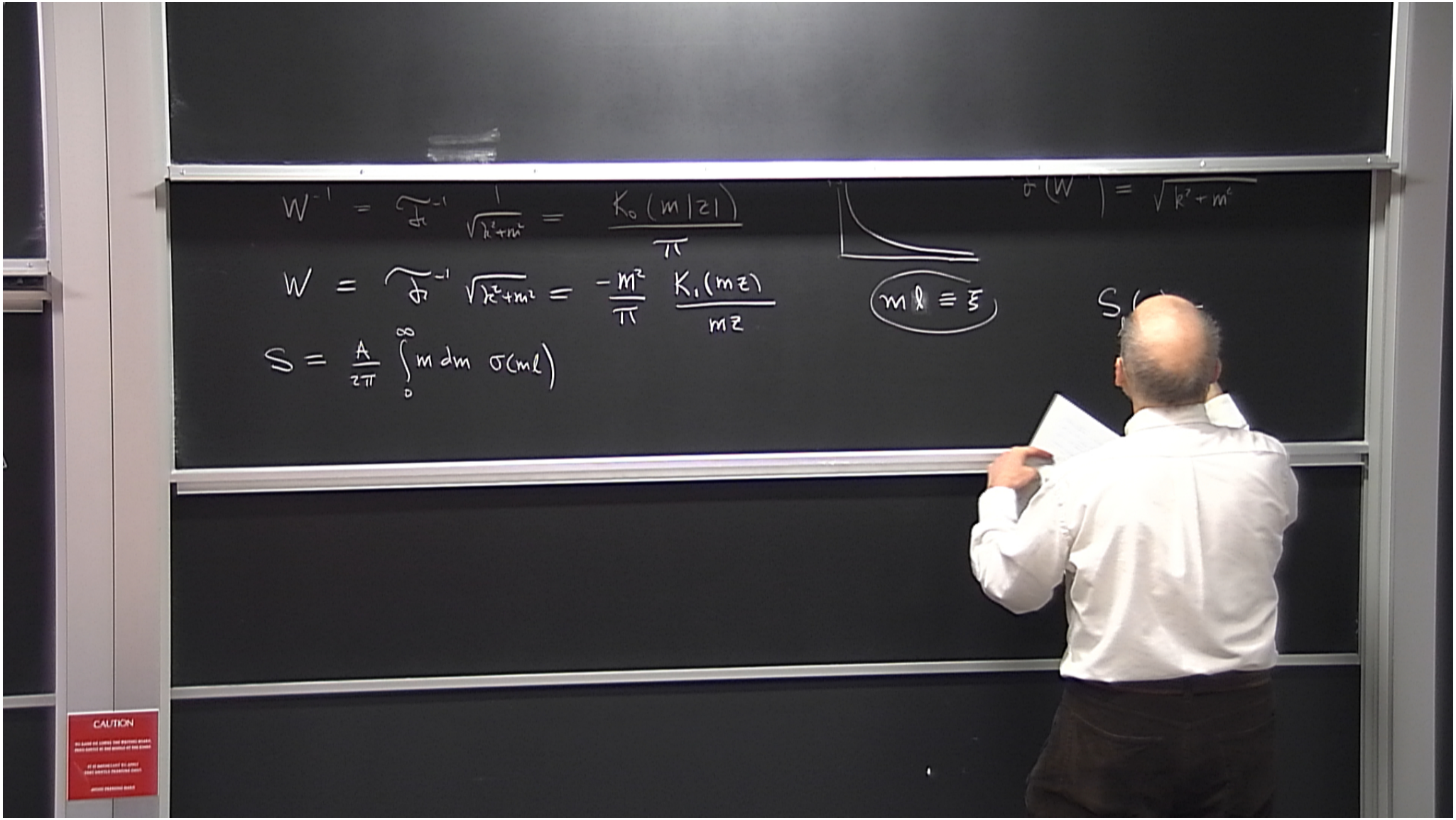

Λ will not mix diff $\vec{p}$
m_0^2
$\Box = -\frac{\partial^2}{\partial t^2} + \vec{\nabla}^2 + \frac{\partial}{\partial z^2}$
$(\Box - m_0^2)\phi = 0$
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$(\Box - m_0^2)\phi = \left[-\ddot{f} - p^2 f - m_0^2 f + \frac{\partial^2 f}{\partial z^2}\right] e^{i\vec{p}\cdot\vec{x}}$
$\left(\frac{1}{c^2}\Box - (m^2 + p^2)\right) f\, e^{i\vec{p}\cdot\vec{x}}$
$m_0 = 0$
$m_0^2 \longrightarrow m_0^2 + p^2 \equiv m^2(\text{effective})$
$S = \int d^2x \frac{d^2p}{(2\pi)^2} S(p) = \frac{A}{(2\pi)^2} \int \frac{d^2p}{2\pi p dp} S(p)$
$S/A = \frac{1}{2\pi} \int_0^\infty p\, dp\, S(p)$
take $m_0 = 0$, write $p = m$
$S(m)$
$\int m\, dm\, S(m)$
CAUTION
CAUTION

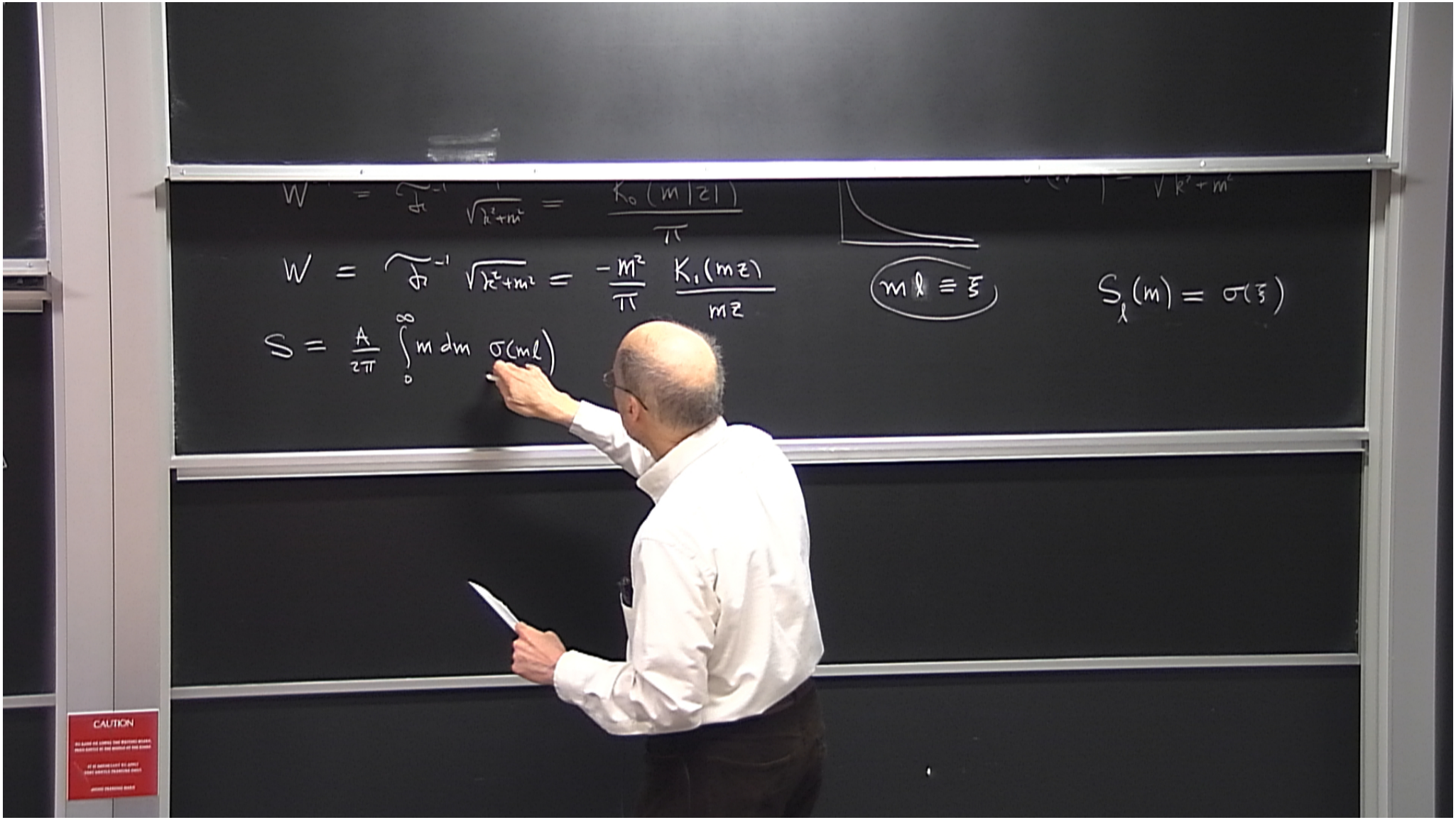

$G_m = \mathbb{1}$, $V_m = \frac{-\partial^2}{\partial z^2} + m^2$, $W = \sqrt{V} = \sqrt{-\frac{\partial^2}{\partial z^2} + m^2}$ (pos ↑)
Fourier $\frac{-\partial^2}{\partial z^2} \longleftrightarrow k^2$ (e^{ikz})

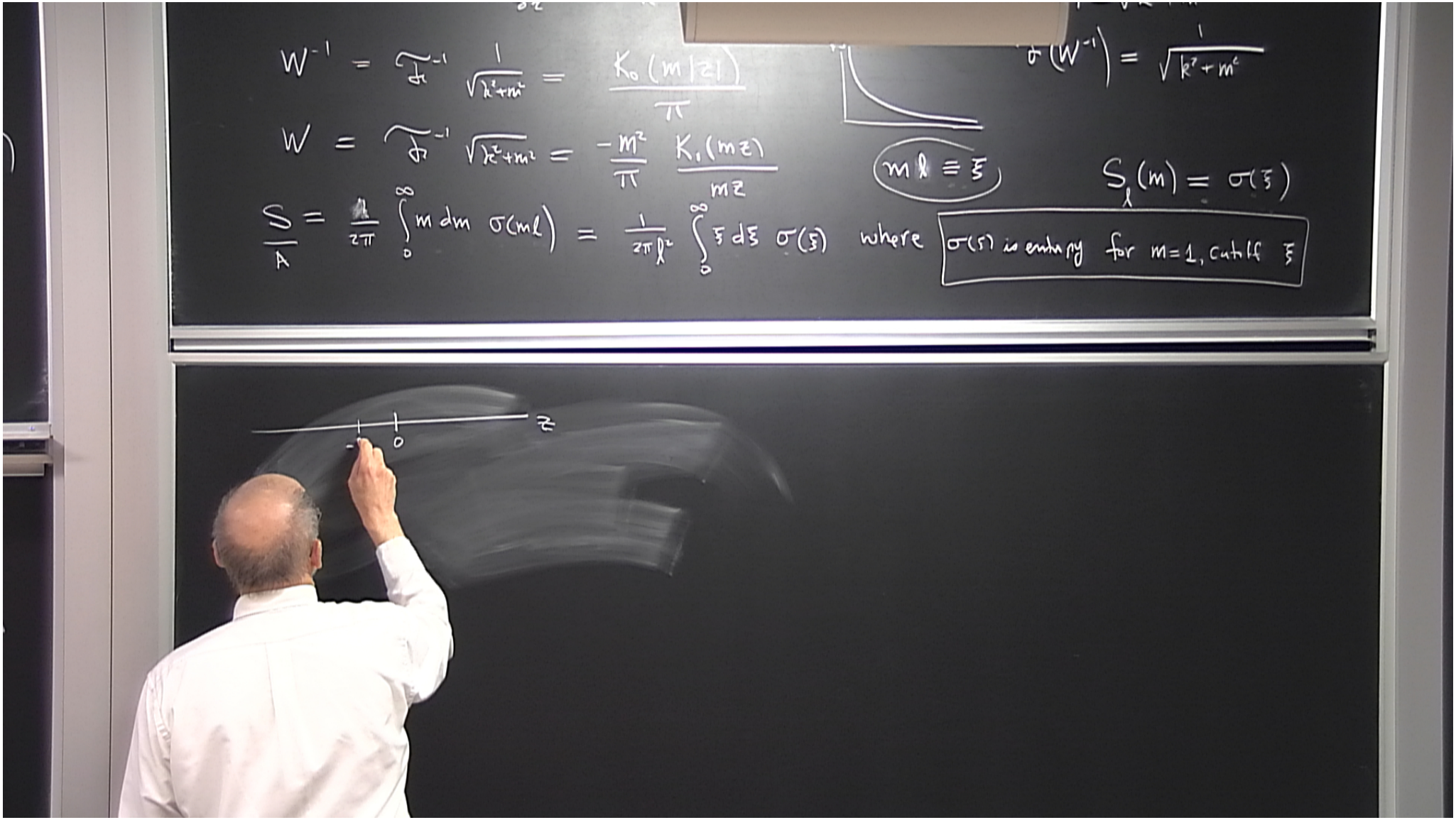

$G_m = \mathbb{1}$, $V_m = \frac{-\partial^2}{\partial z^2} + m^2$, $W = \sqrt{V} = \sqrt{-\frac{\partial^2}{\partial z^2} + m^2}$ (pos Γ)
Fourier $\frac{-\partial^2}{\partial z^2} \leftrightarrow k^2$ (e^{ikz})
$\tilde{f}(W) = \sqrt{k^2 + m^2}$
$\tilde{f}(W^{-1}) = \frac{1}{\sqrt{k^2 + m^2}}$
W^{-1}

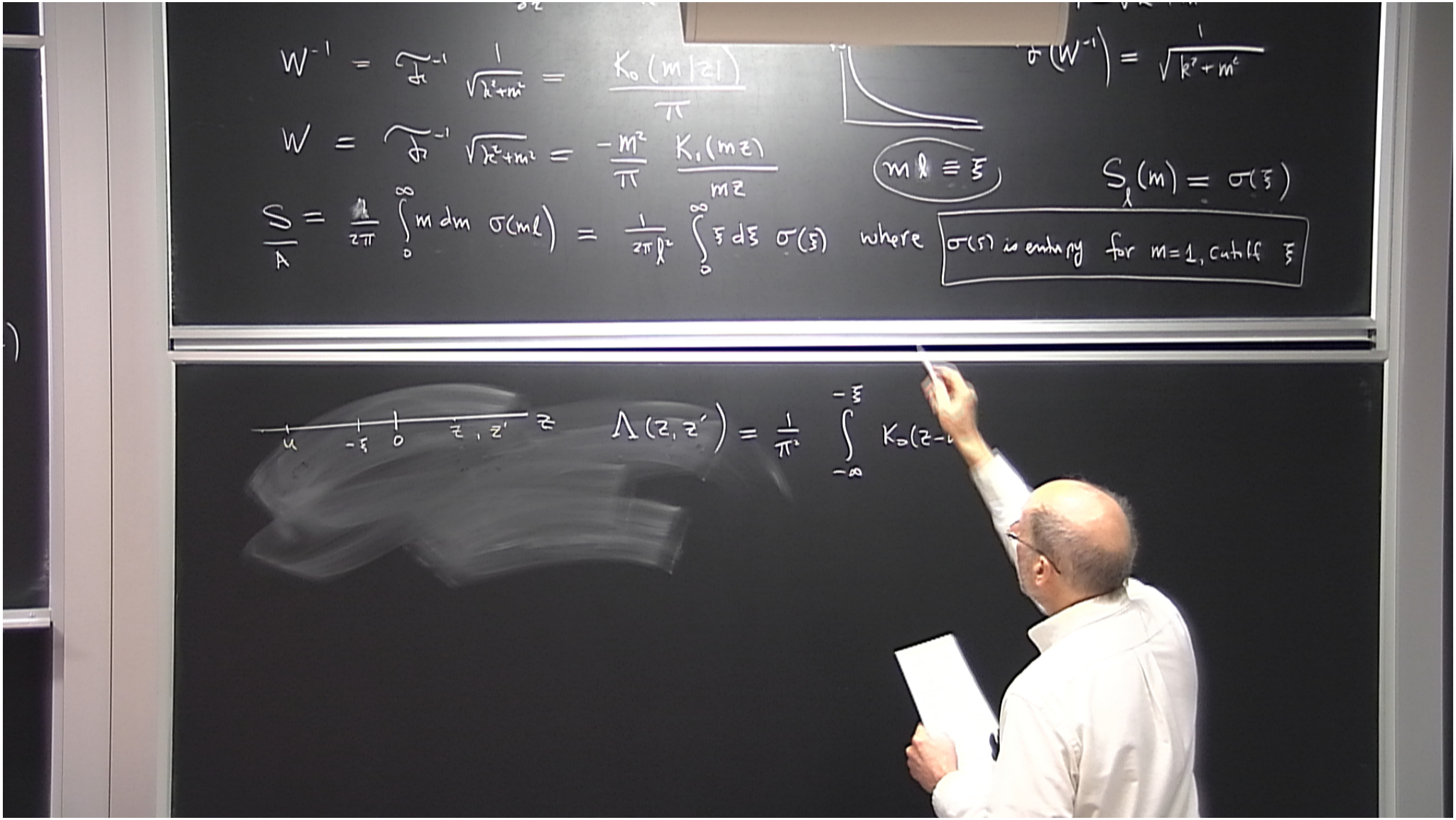

Fourier
$-\frac{\partial^2}{\partial z^2} \leftrightarrow k^2$
(e^{ikz})
$\tilde{f}(W) = \sqrt{k^2+m^2}$
$\tilde{f}(W^{-1}) = \frac{1}{\sqrt{k^2+m^2}}$
$W^{-1} = \tilde{f}^{-1} \frac{1}{\sqrt{k^2+m^2}} = \frac{K_0(m|z|)}{\pi}$
$W = \tilde{f}^{-1} \sqrt{k^2+m^2} = \frac{K_1(mz)}{mz}$
K_0
CAUTION


Fourier
$\frac{\partial}{\partial z^2} \leftrightarrow k^2$
(e^{ikz})
$\tilde{f}(W) = \sqrt{k^2+m^2}$
$W^{-1} = \mathcal{F}^{-1} \frac{1}{\sqrt{k^2+m^2}} = \frac{K_0(m|z|)}{\pi}$
$\tilde{f}(W^{-1}) = \frac{1}{\sqrt{k^2+m^2}}$
$W = \mathcal{F}^{-1}$ $= \frac{-m^2}{\pi} \frac{K_1(mz)}{mz}$
K_0
CAUTION


$W^{-1} = \widetilde{\Delta}^{-1} \frac{1}{\sqrt{k^2+m^2}} = \frac{K_0(m|z|)}{\pi}$
$W = \widetilde{\Delta}^{-1} \sqrt{k^2+m^2} = -\frac{m^2}{\pi} \frac{K_1(mz)}{mz}$
$S = \frac{A}{2\pi} \int_0^\infty m\,dm\ \sigma(ml)$
$\sigma(W^{-1}) = \sqrt{k^2+m^2}$
$ml \equiv \xi$
CAUTION


$W = \frac{K_0(m|z|)}{\pi}$
$W = \widetilde{\Delta}^{-1}\sqrt{k^2+m^2} = \frac{-m^2}{\pi}\frac{K_1(mz)}{mz}$
$ml \equiv \xi$
$S_l(m) = \sigma(\xi)$
$S = \frac{A}{2\pi}\int_0^\infty m\,dm\;\sigma(ml)$
CAUTION


$W^{-1} = \widetilde{\Delta}^{-1} \frac{1}{\sqrt{k^2+m^2}} = \frac{K_0(m|z|)}{\pi}$
$W = \widetilde{\Delta}^{-1} \sqrt{k^2+m^2} = -\frac{m^2}{\pi} \frac{K_1(mz)}{mz}$
$\sigma(W^{-1}) = \frac{1}{\sqrt{k^2+m^2}}$
$m\ell \equiv \xi$
$S_\ell(m) = \sigma(\xi)$
$\frac{S}{A} = \frac{1}{2\pi} \int_0^\infty m\,dm\ \sigma(m\ell) = \frac{1}{2\pi\ell^2} \int_0^\infty \xi\,d\xi\ \sigma(\xi)$ where $\sigma(\xi)$ is entropy for $m=1$, cutoff ξ
-1
0
z


$W^{-1} = \widetilde{\frac{1}{k^2}}^{-1} \frac{1}{\sqrt{k^2+m^2}} = \frac{K_0(m|z|)}{\pi}$
$\sigma(W^{-1}) = \frac{1}{\sqrt{k^2+m^2}}$
$W = \widetilde{\frac{1}{k^2}}^{-1} \sqrt{k^2+m^2} = -\frac{m^2}{\pi} \frac{K_1(mz)}{mz}$
$m\ell \equiv \xi$
$S_\ell(m) = \sigma(\xi)$
$\frac{S}{A} = \frac{1}{2\pi} \int_0^\infty m\,dm\ \sigma(m\ell) = \frac{1}{2\pi\ell^2} \int_0^\infty \xi\,d\xi\ \sigma(\xi)$ where
$\sigma(\xi)$ is entropy for $m=1$, cutoff ξ
$\Lambda(z,z') = \frac{1}{\pi^2} \int_{-\infty}^{-\xi} K_0(z-$
∞ $-\xi$ 0 z,z' z

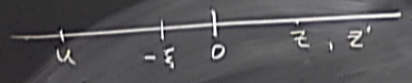
$W^{-1} = \widetilde{\frac{1}{k^2}}^{-1} \frac{1}{\sqrt{k^2+m^2}} = \frac{K_0(m|z|)}{\pi}$

$\sigma(W^{-1}) = \frac{1}{\sqrt{k^2+m^2}}$

$W = \widetilde{\frac{1}{k^2}}^{-1} \sqrt{k^2+m^2} = -\frac{m^2}{\pi} \frac{K_1(mz)}{mz}$

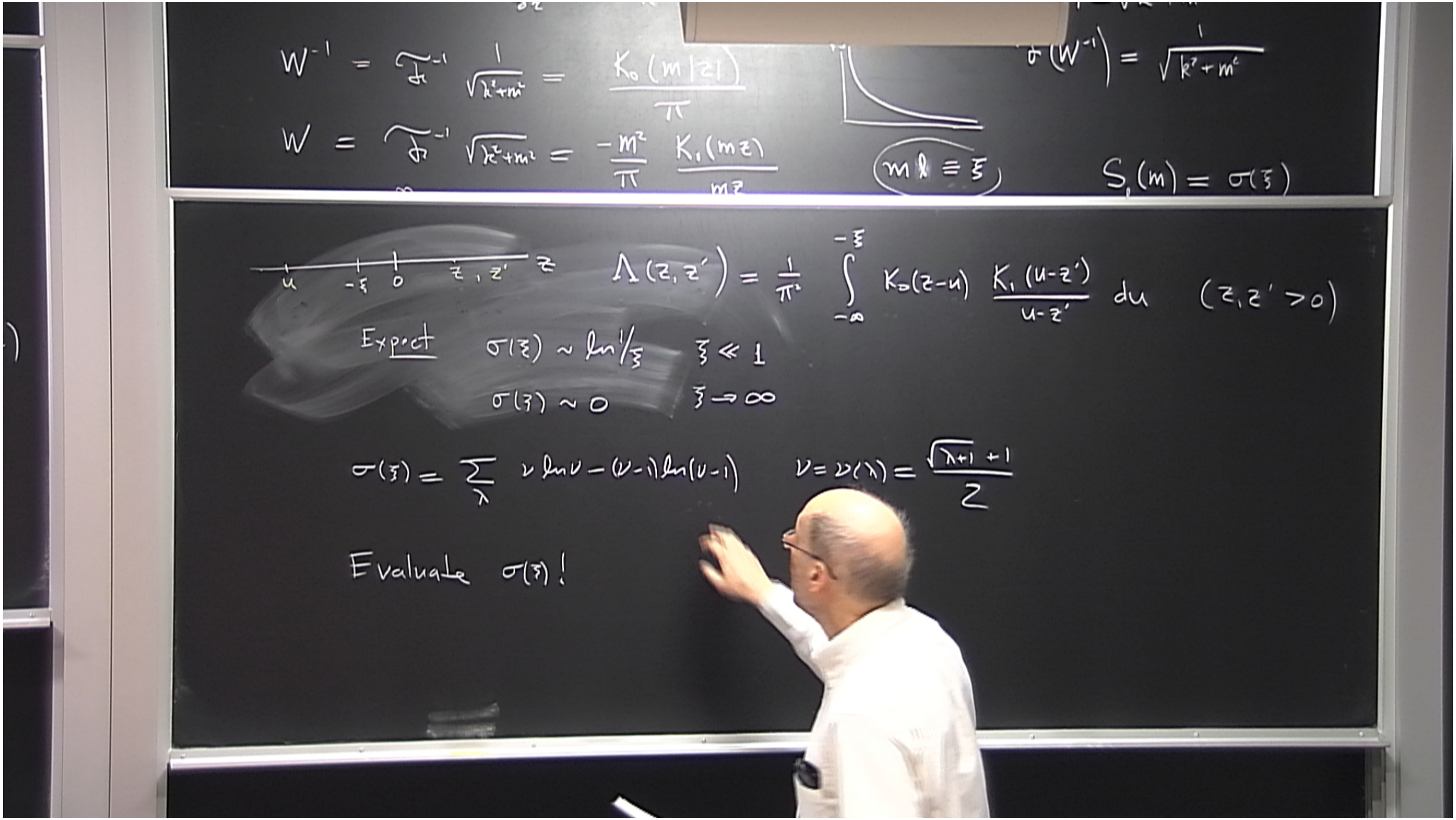
$m\ell \equiv \xi$ $\qquad S_1(m) = \sigma(\xi)$

$\frac{S}{A} = \frac{1}{2\pi} \int_0^\infty m\,dm\ \sigma(m\ell) = \frac{1}{2\pi\ell^2} \int_0^\infty \xi\,d\xi\ \sigma(\xi)$ where $\sigma(\xi)$ is entropy for $m=1$, cutoff ξ



$\Lambda(z,z') = \frac{1}{\pi^2} \int_{-\infty}^{-\xi} K_0(z-u) \frac{K_1(u-z')}{u-z'}\,du \quad (z,z'>0)$

Expect $\sigma(\xi) \sim \ln 1/\xi$


$W^{-1} = \widetilde{\Delta}^{-1} \frac{1}{\sqrt{k^2+m^2}} = \frac{K_0(m|z|)}{\pi}$
$W = \widetilde{\Delta}^{-1} \sqrt{k^2+m^2} = -\frac{m^2}{\pi} \frac{K_1(mz)}{mz}$
$\sigma(W^{-1}) = \frac{1}{\sqrt{k^2+m^2}}$
$mk \equiv \xi$
$S_1(m) = \sigma(\xi)$
$u \quad -\xi \quad 0 \quad z, z' \quad z$
$\Lambda(z,z') = \frac{1}{\pi^2} \int_{-\infty}^{-\xi} K_0(z-u) \frac{K_1(u-z')}{u-z'} du \quad (z,z'>0)$
Expect $\sigma(\xi) \sim \ln 1/\xi \quad \xi \ll 1$
$\sigma(\xi) \sim 0 \quad \xi \to \infty$
$\sigma(\xi) = \sum_\lambda \nu \ln \nu - (\nu-1)\ln(\nu-1) \quad \nu = \nu(\lambda) = \frac{\sqrt{\lambda+1}+1}{2}$
Evaluate $\sigma(\xi)$!

R.P.

① Prove $\lambda_n \to 0$ fast enough s.t. $\sigma(\xi) < \infty$ $(\forall \xi > 0)$

② " $\sigma(\xi) \to 0$ exponentially at $\xi \to \infty$

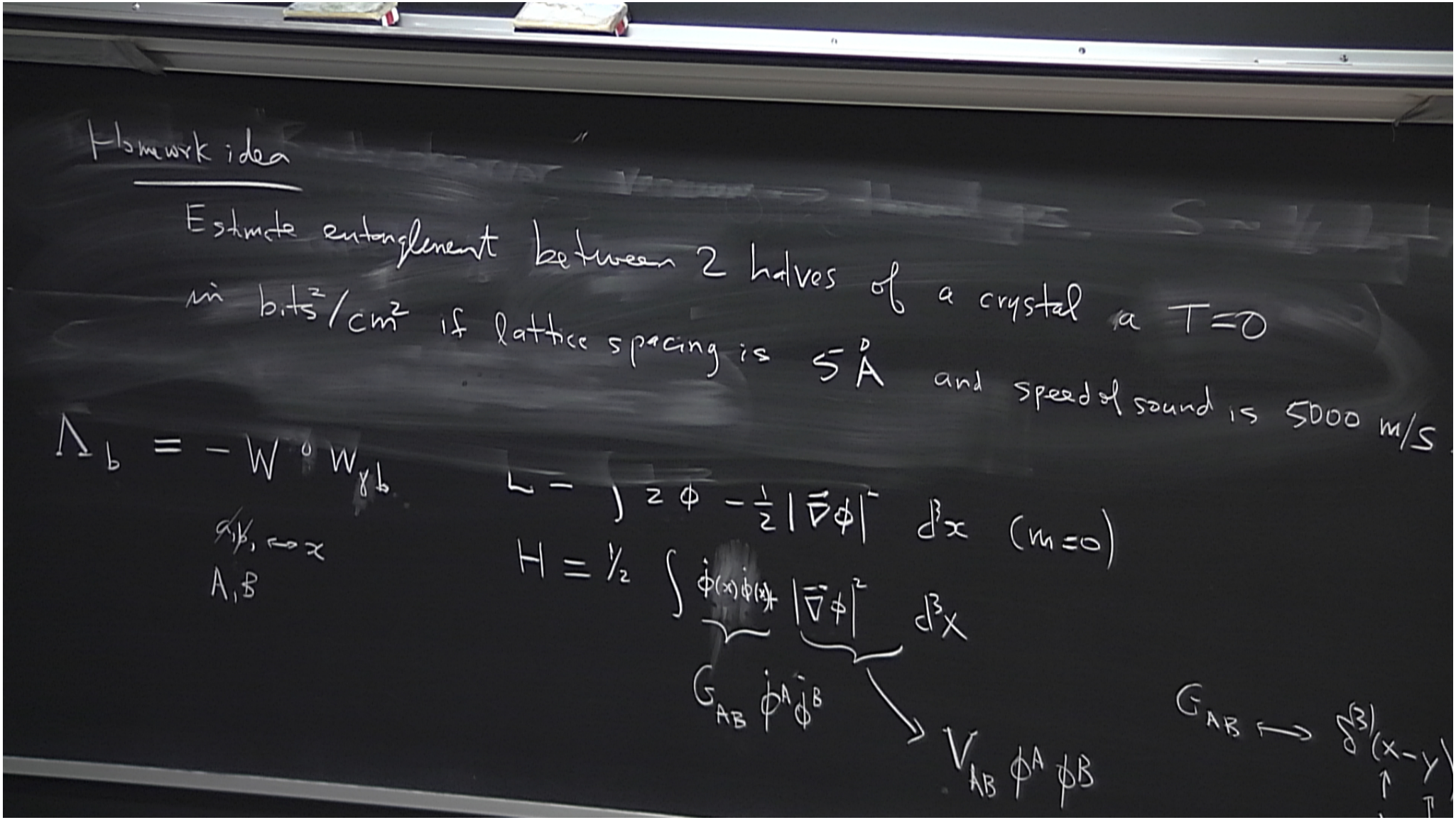
③ " $\sigma(\xi) \sim c\, k_n l / \xi$ near $\xi = 0$ find c

} → Area law

④ Eval $\int_0^\infty \xi\, d\xi\ \sigma(\xi)$ to get coeff in area law (for this cutoff)

⑤ Generalize Rindler-gas estimate to $m > 0$

⑥ " $\Lambda_-(x,y)$ etc to $m > 0$


Homework idea
Estimate entanglement between 2 halves of a crystal at T=0
in bits²/cm² if lattice spacing is 5 Å and speed of sound is 5000 m/s
$\Lambda_b = -W \circ W_{\gamma b}$
$L = \int \frac{1}{2}\dot{\phi} - \frac{1}{2}|\vec{\nabla}\phi|^2 \, d^3x \quad (m=0)$
$H = \frac{1}{2} \int \dot{\phi}(x)\dot{\phi}(x) + |\vec{\nabla}\phi|^2 \, d^3x$
$G_{AB}\,\dot{\phi}^A\dot{\phi}^B$
$V_{AB}\,\phi^A\phi^B$
$G_{AB} \longleftrightarrow \delta^3(x-y)$