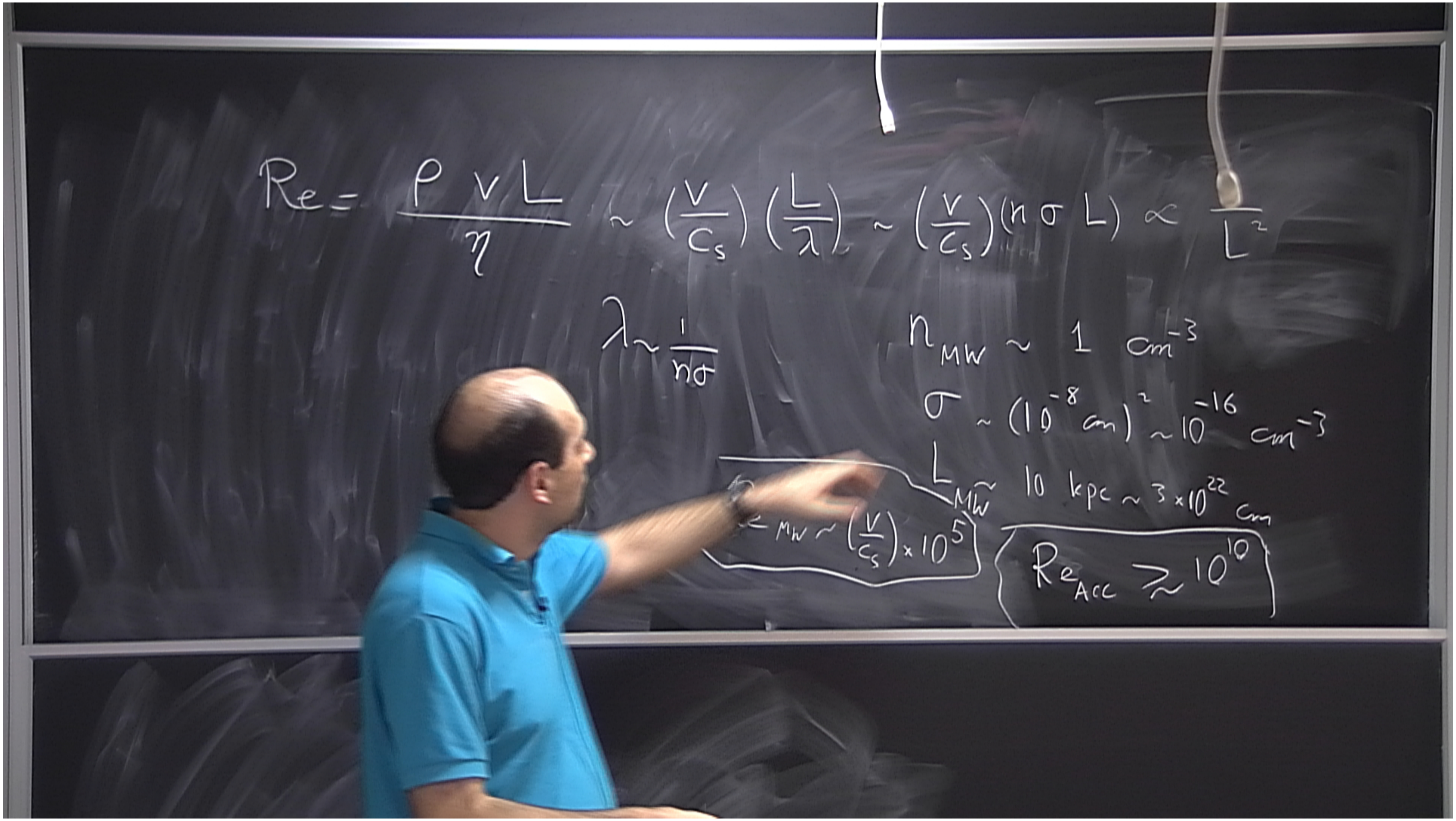
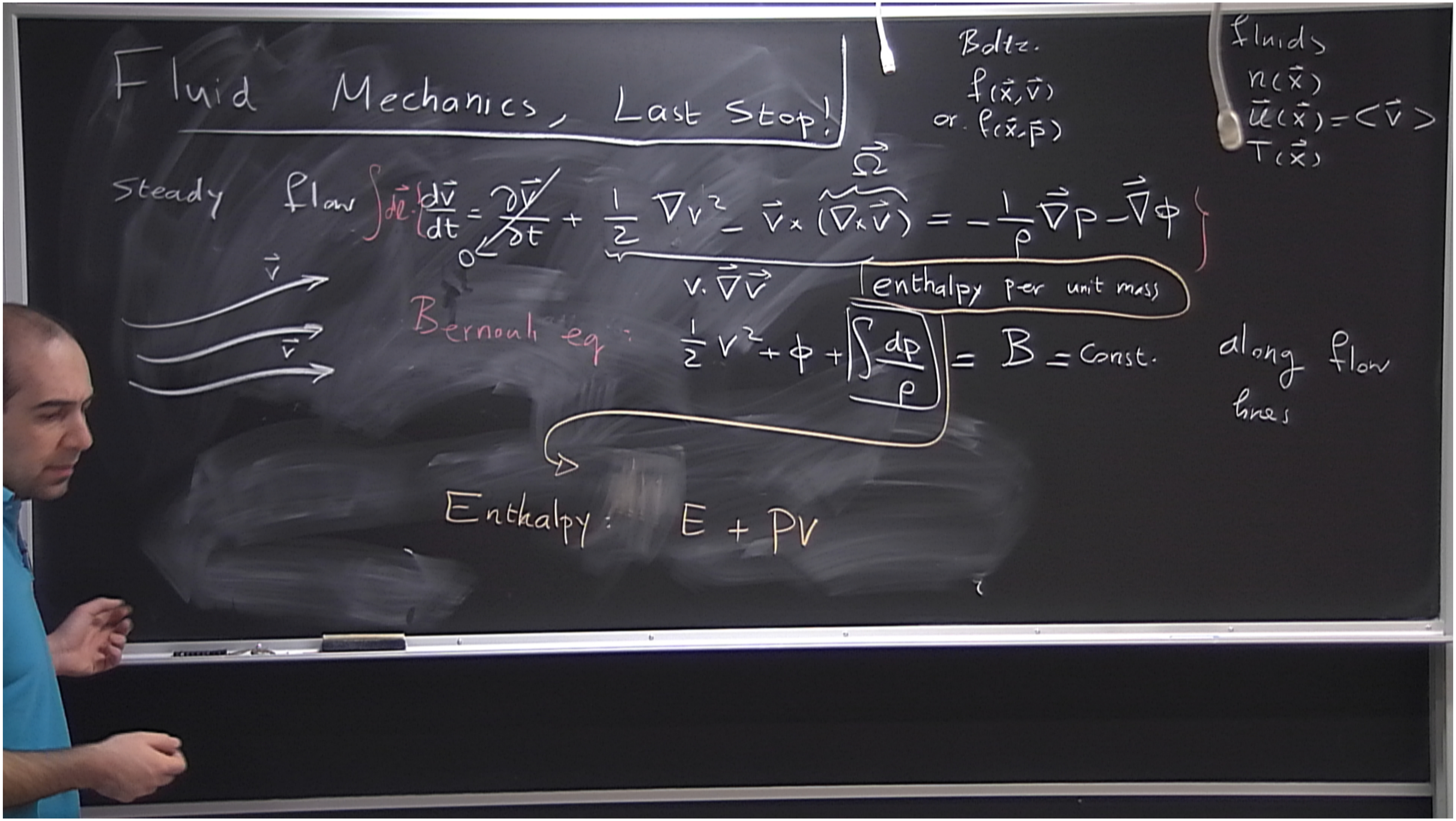
Title: PHYS 781 - Lecture 13

Date: Oct 31, 2014 12:30 PM

URL: http://pirsa.org/14100065

Abstract:


$Re = \frac{\rho v L}{\eta} \sim \left(\frac{v}{c_s}\right)\left(\frac{L}{\lambda}\right) \sim \left(\frac{v}{c_s}\right)(n \sigma L) \propto L^2$
$\lambda \sim \frac{1}{n\sigma}$
$n_{MW} \sim 1 \ cm^{-3}$
$\sigma \sim (10^{-8} cm)^2 \sim 10^{-16} \ cm^{-3}$
$L_{MW} \sim 10 \ kpc \sim 3 \times 10^{22} \ cm$
$\left(\frac{v}{c_s}\right) \times 10^5$
$Re_{Acc} \gtrsim 10^{10}$


Fluid Mechanics, Last Stop!
Boltz.
$f(\vec{x},\vec{v})$
or. $f(\vec{x},\vec{p})$
fluids
$n(\vec{x})$
$\vec{u}(\vec{x}) = \langle \vec{v} \rangle$
$T(\vec{x})$
Steady flow
$\int d\vec{l} \cdot \left\{ \frac{d\vec{v}}{dt} = \cancel{\frac{\partial \vec{v}}{\partial t}} + \frac{1}{2}\nabla v^2 - \vec{v} \times \overbrace{(\nabla \times \vec{v})}^{\vec{\Omega}} = -\frac{1}{\rho}\vec{\nabla} p - \vec{\nabla}\phi \right\}$
$v \cdot \vec{\nabla}\vec{v}$
enthalpy per unit mass
Bernoulli eq:
$\frac{1}{2}v^2 + \phi + \boxed{\int \frac{dp}{\rho}} = B = \text{Const.}$
along flow lines
Enthalpy: $E + PV$

enthalpy / unit mass

$$W = \frac{E + PV}{\text{Mass}} = \epsilon + \frac{P}{\rho}$$

ϵ: energy / unit mass

$$dW = d\epsilon + d\left(\frac{P}{\rho}\right)$$

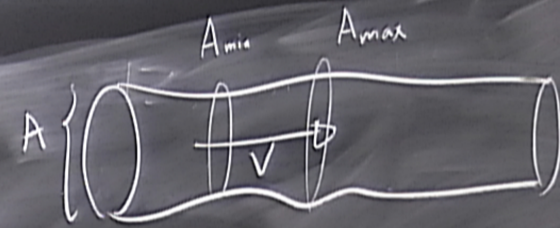
$$= d\epsilon + P\,d\left(\frac{1}{\rho}\right) + \frac{dP}{\rho} = \frac{dP}{\rho}$$

$Tds = 0$ for adiabtic $\Longleftrightarrow$ perfect fluid

adiabtic flows:

$$\frac{1}{2}v^2 + \phi + \epsilon + \frac{P}{\rho} = B$$

flow in a pipe



$$c_s^2 \equiv \frac{dP}{d\rho}$$

$$\rho v A = \dot{M} = \text{const.}$$

$$\frac{v^2}{2} + \int \frac{dP}{\rho} = B$$

$$v\,dv = -\frac{dP}{\rho} = -c_s^2(\rho)\frac{d\rho}{\rho}$$

$$\frac{d\rho}{\rho} = -\frac{dv}{v} - \frac{dA}{A}$$

$$\frac{v^2}{c_s^2}\frac{dv}{v} = \cancel{c_s^2}\left(\frac{dv}{v} + \frac{dA}{A}\right)$$

$$\left(1 - \frac{v^2}{c_s^2}\right)\frac{dv/dx}{v} = -\frac{dA/dx}{A} \Rightarrow \Rightarrow A \text{ is min. or max}$$

where $v = c_s$

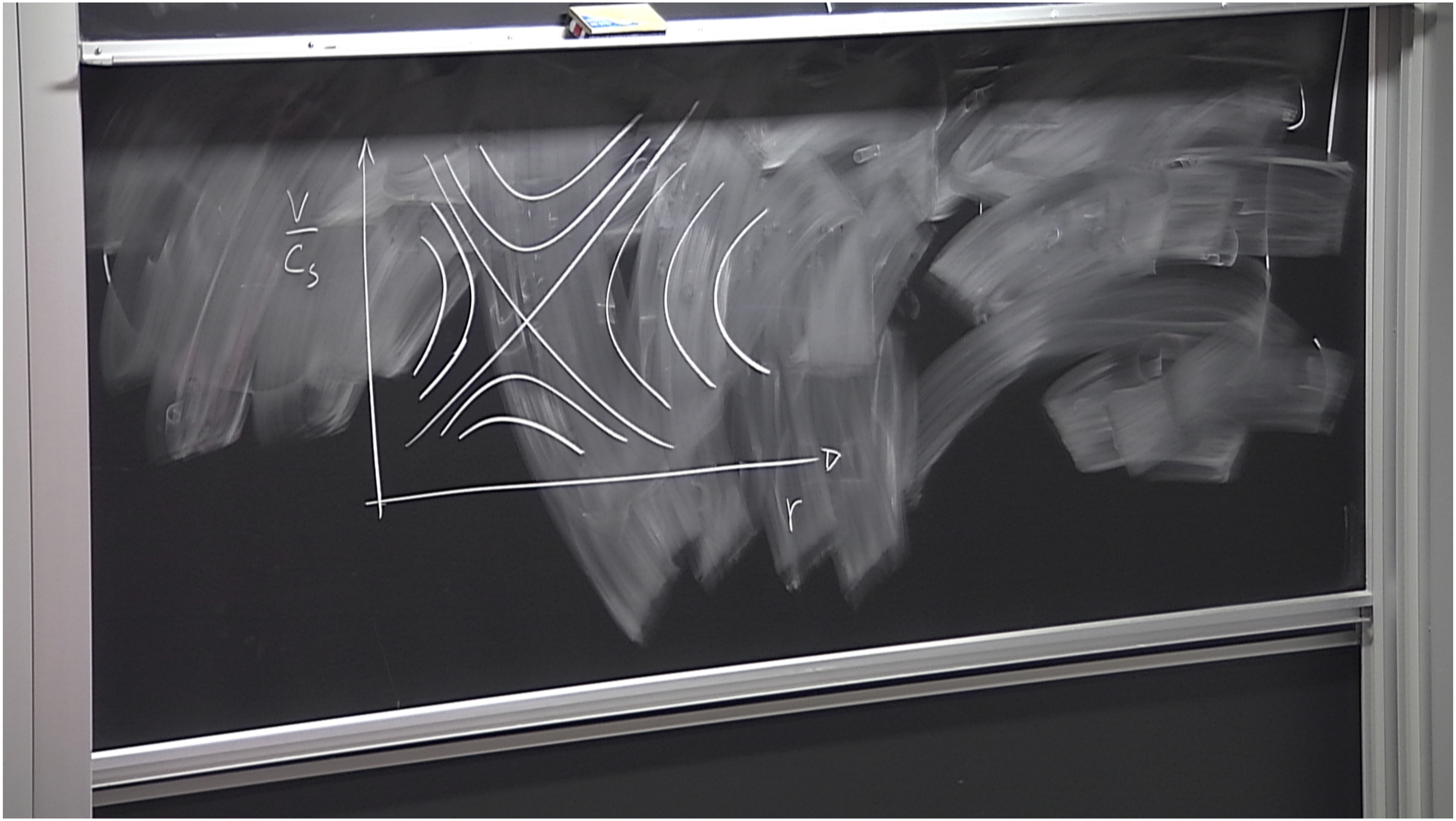
$$\dot{M} = \rho(v=c_s)\, c_s\, A_{min/max}$$

Bondi Accretion/outflow (spherical sym.)

$$A = 4\pi r^2 \qquad P \propto \rho^\gamma \qquad \int \frac{dP}{\rho} = \frac{C_s^2}{\gamma - 1} \qquad C_s^2 = \frac{dP}{d\rho} = \gamma \frac{P}{\rho}$$

$$4\pi \rho v r^2 = \dot{M}$$

$$\frac{v^2}{2} + \frac{C_s^2}{\gamma - 1} - \frac{GM}{r} = \text{Const.}$$


$\frac{v}{c_s}$
r

$$\frac{1}{2}\left(1-\frac{c_s^2}{v^2}\right)\frac{dv^2}{dr} = -\frac{GM}{r^2}\left(1-\frac{2c_s^2 r}{GM}\right)$$

When $v = c_s \iff r_s = \frac{GM}{2c_s^2}$

$$\frac{1}{2}\frac{dv^2}{dr} = -c_s^2\frac{d\rho/dr}{\rho} - \frac{GM}{r^2}$$

$$= c_s^2\left(\frac{v\,dv/dr}{v^2} + \frac{2}{r}\right) - \frac{GM}{r^2}$$

$$\frac{1}{2}\left(1-\frac{c_s^2}{v^2}\right)\frac{dv^2}{dr} = -\frac{GM}{r^2}\left[1-\frac{2c_s^2 r}{GM}\right]$$

Bondi acc. rate

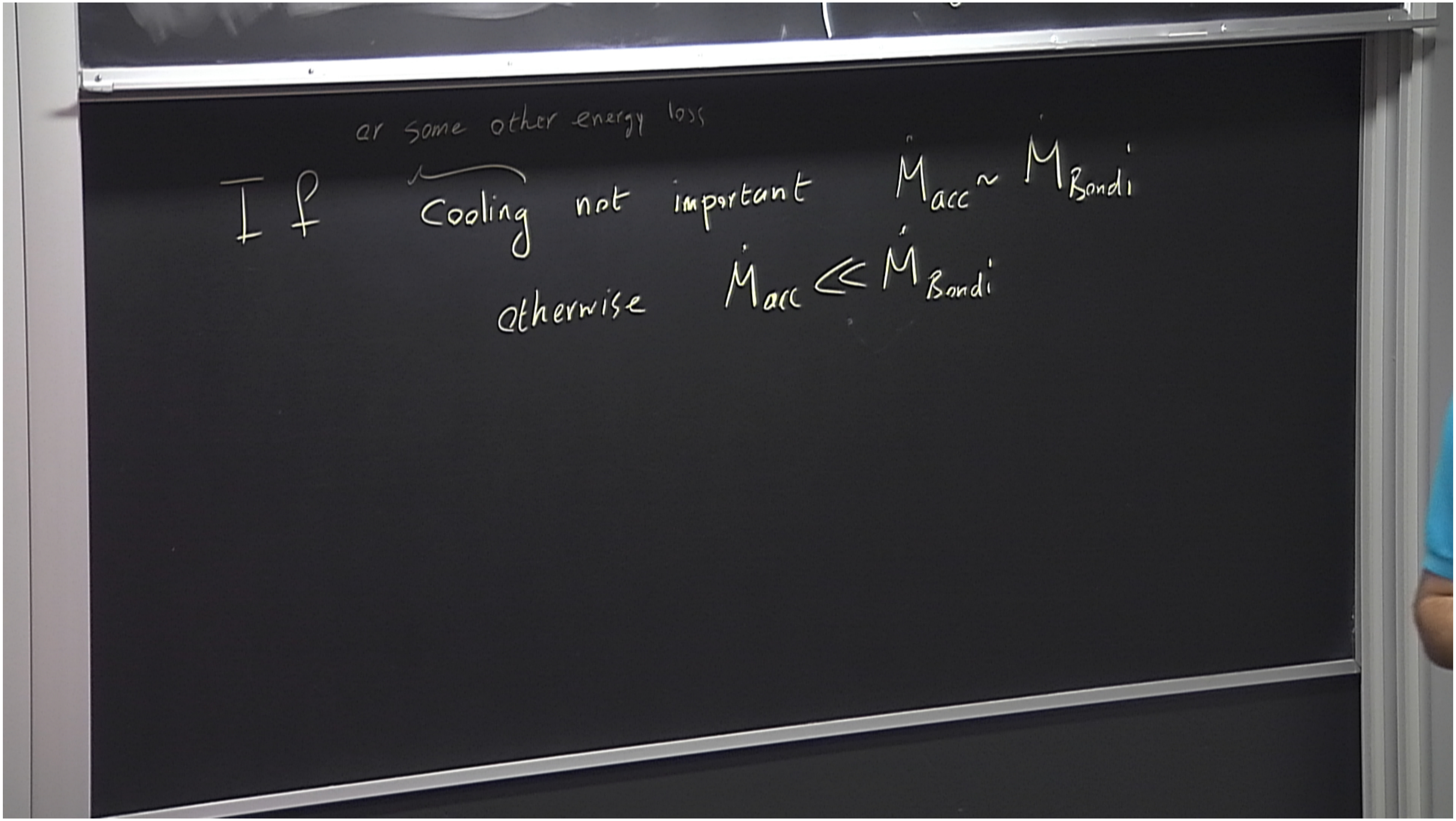
$$\dot{M} = \pi G^2 M^2 \frac{\rho_\infty}{c_s^3(\infty)} f(\gamma)$$

$$f(\gamma) = \left(\frac{2}{5-3\gamma}\right)^{(5-3\gamma)/2(\gamma-1)}$$

influence radius

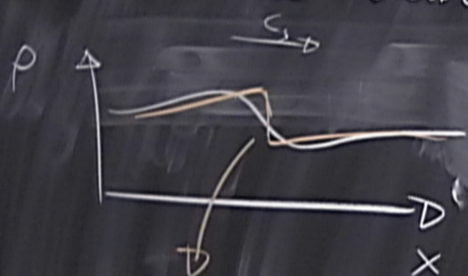
$$\dot{M} \sim 4\pi r_s^2 \times c_s \times \rho_\infty$$

Bondi radius $\sim \frac{GM}{c_s^2(\infty)}$


or some other energy loss
If cooling not important $\dot{M}_{acc} \sim \dot{M}_{Bondi}$
otherwise $\dot{M}_{acc} \ll \dot{M}_{Bondi}$

enthalpy / unit mass | adiabtic flows:

Shock waves: $c_s^2 = \frac{dp}{d\rho} \propto \gamma \rho^{\gamma-1}$



$v_s = \sqrt{g h}$ shallow

$\sqrt{g \lambda}$ deep

shock front forms within $\Delta t \sim \frac{\text{Period}}{\delta\rho/\rho}$

Shock jump conditions

$\rho_1 \; V_1 \; P_1 \longrightarrow \quad \Big| \quad \rho_2 \; V_2 \; P_2 \longrightarrow$

mom. cons.

$\rho V^2 + P = \text{Const.}$

$\rho V^2 \left[\frac{1}{2} V^2 + \epsilon + \frac{P}{\rho} \right] = \text{Const.}$

$T^{\mu j}{}_{,j} = 0$

$j^{i}{}_{,i} = 0 \longrightarrow \rho V = \text{Const}$

$\frac{P_2}{P_1} = \frac{\gamma(\gamma+1)}{2} \left(\frac{U}{c_1} \right)^2$

$U = V_1$

$v_s \overset{?}{=} V_1 - V_2$

$v_s = \frac{\gamma+1}{2} U$

$U \gg c_1$