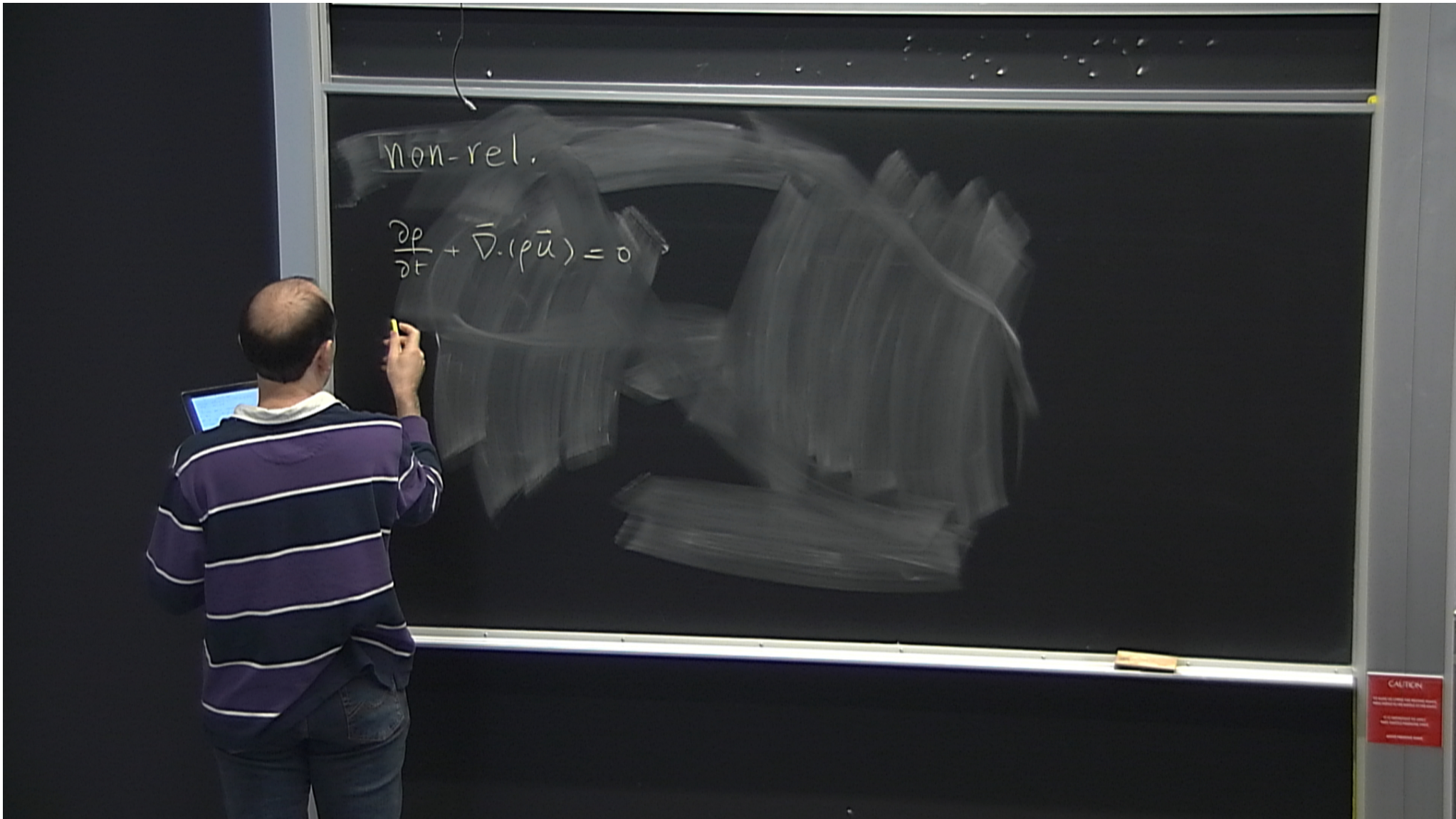


Title: PHYS 781 - Lecture 12

Date: Oct 30, 2014 02:00 PM

URL: <http://pirsa.org/14100060>

Abstract:



non-rel.

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

$$\frac{\partial}{\partial t} \left[\frac{1}{2} \rho u^2 + \epsilon \right] + \vec{\nabla} \cdot \vec{F} = 0$$

$$\frac{1}{2} \rho$$

non-rel.

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

$$\frac{\partial}{\partial t} \left[\frac{1}{2} \rho u^2 + \epsilon \right] + \vec{\nabla} \cdot \vec{F} = 0 \quad \vec{F} = \left[\frac{1}{2} \rho u^2 + \epsilon + p \right] \vec{u}$$

non-rel.

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

$$\frac{\partial}{\partial t} \left[\frac{1}{2} \rho u^2 + \epsilon \right] + \vec{\nabla} \cdot \vec{F} = 0$$
$$\vec{F} = \left(\frac{1}{2} \rho u^2 + \epsilon + p \right) \vec{u}$$

$$\frac{\partial}{\partial t} (\rho u_i) = - \frac{\partial T_{ij}}{\partial x_j}$$

$$T_{ij} = \rho u_i u_j + p \delta_{ij}$$

non-rel.

$$\# \left\{ \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0 \right.$$

$$E \left\{ \begin{aligned} \frac{\partial}{\partial t} \left[\frac{1}{2} \rho u^2 + \epsilon \right] + \vec{\nabla} \cdot \vec{F} &= 0 \\ \vec{F} &= \left(\frac{1}{2} \rho u^2 + \epsilon + p \right) \vec{u} \end{aligned} \right.$$

$$\vec{P} \left\{ \begin{aligned} \frac{\partial}{\partial t} (\rho u_i) &= - \frac{\partial T_{ij}}{\partial x_j} \\ T_{ij} &= \rho u_i u_j + p \delta_{ij} \end{aligned} \right.$$

relativistic

$$J^\mu = \int \frac{d^3 p}{E(p)} p^\mu f(x, p)$$

non-rel.

$$\# \left\{ \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0 \right.$$

$$\left. \frac{\partial}{\partial t} \left[\frac{1}{2} \rho u^2 + \epsilon \right] + \vec{\nabla} \cdot \vec{F} = 0 \right.$$

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relativistic

$$\vec{J}^\mu = \int \frac{d^3 p}{E(p)} p^\mu f(x, p)$$

$$T^{\mu\nu} = c^2 \int \frac{d^3 p}{E(p)} p^\mu p^\nu f(x, p)$$

non-rel.

$$\# \left\{ \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0 \right.$$

$$E \left\{ \begin{aligned} \frac{\partial}{\partial t} \left[\frac{1}{2} \rho u^2 + \epsilon \right] + \vec{\nabla} \cdot \vec{F} \\ \vec{F} = \left(\frac{1}{2} \rho u^2 + \epsilon + p \right) \vec{u} \end{aligned} \right.$$

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relativistic

$$\mathcal{J}^\mu = \int \frac{d^3 p}{E(p)} p^\mu f(x, p)$$

$$T^{\mu\nu} = c^2 \int \frac{d^3 p}{E(p)} p^\mu p^\nu f(x, p)$$

$$T^{\mu\nu}_{;\nu} = \frac{\partial T^{\mu\nu}}{\partial x^\nu} = 0$$

$$\mathcal{J}^\mu_{;\mu} = 0$$

non-rel.

$$\# \left\{ \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0 \right.$$

$$E \left\{ \begin{aligned} \frac{\partial}{\partial t} \left[\frac{1}{2} \rho u^2 + \epsilon \right] + \vec{\nabla} \cdot \vec{F} &= 0 \\ \vec{F} &= \left(\frac{1}{2} \rho u^2 + \epsilon + p \right) \vec{u} \end{aligned} \right.$$

$$\vec{P} \left\{ \begin{aligned} \frac{\partial}{\partial t} (\rho u_i) &= - \frac{\partial T_{ij}}{\partial x_j} \\ T_{ij} &= \rho u_i u_j + p \delta_{ij} \end{aligned} \right.$$

relativistic

$$\mathcal{J}^\mu = \int \frac{d^3 p}{E(p)} p^\mu f(x, p)$$

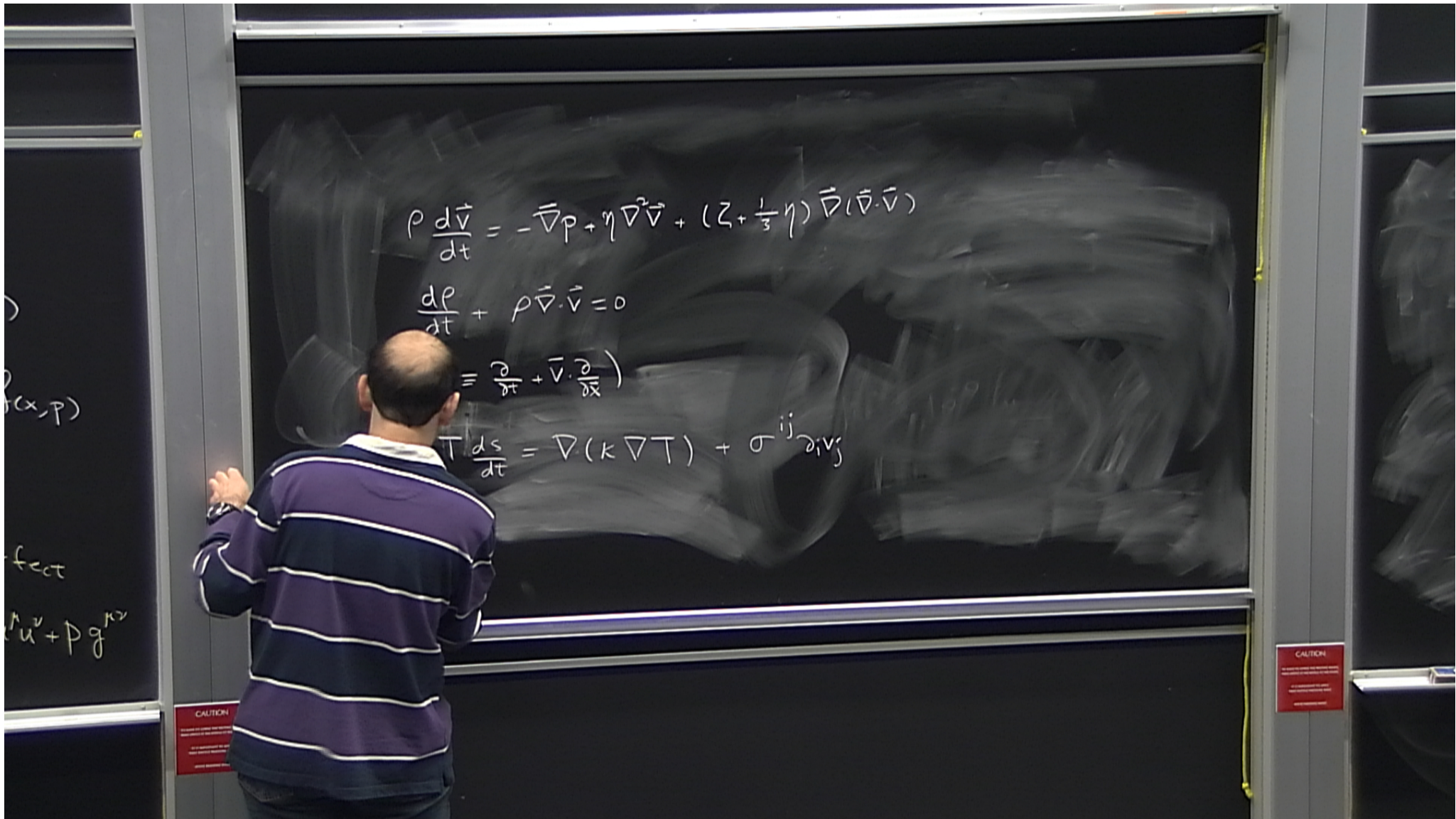
$$T^{\mu\nu} = c^2 \int \frac{d^3 p}{E(p)} p^\mu p^\nu f(x, p)$$

$$T^{\mu\nu}_{;\nu} = \frac{\partial T^{\mu\nu}}{\partial x^\nu} = 0$$

$$\mathcal{J}^\mu_{;\mu} = 0$$

perfect

$$\mathcal{J}^\mu = n u^\mu; T^{\mu\nu} = (P + \rho) u^\mu u^\nu + p g^{\mu\nu}$$



$$\rho \frac{d\vec{v}}{dt} = -\vec{\nabla} p + \eta \nabla^2 \vec{v} + \left(\zeta + \frac{1}{3}\eta\right) \vec{\nabla}(\vec{\nabla} \cdot \vec{v})$$

$$\frac{dp}{dt} + \rho \vec{\nabla} \cdot \vec{v} = 0$$

$$\equiv \frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{x}}$$

$$T \frac{ds}{dt} = \nabla \cdot (k \nabla T) + \sigma^{ij} \partial_i v_j$$

$$\rho \frac{d\vec{v}}{dt} = -\vec{\nabla} p + \eta \nabla^2 \vec{v} + \left(\zeta + \frac{1}{3}\eta\right) \vec{\nabla}(\vec{\nabla} \cdot \vec{v})$$

$$\frac{dp}{dt} + \rho \vec{\nabla} \cdot \vec{v} = 0$$

$$\left(\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{x}} \right)$$

$$\rho T \frac{ds}{dt} = \overbrace{\vec{\nabla} \cdot (k \vec{\nabla} T)}^{\text{heat flux}} + \overbrace{\sigma^{ij} \partial_i v_j}_{\text{viscous heating}}$$

s - entropy / unit mass

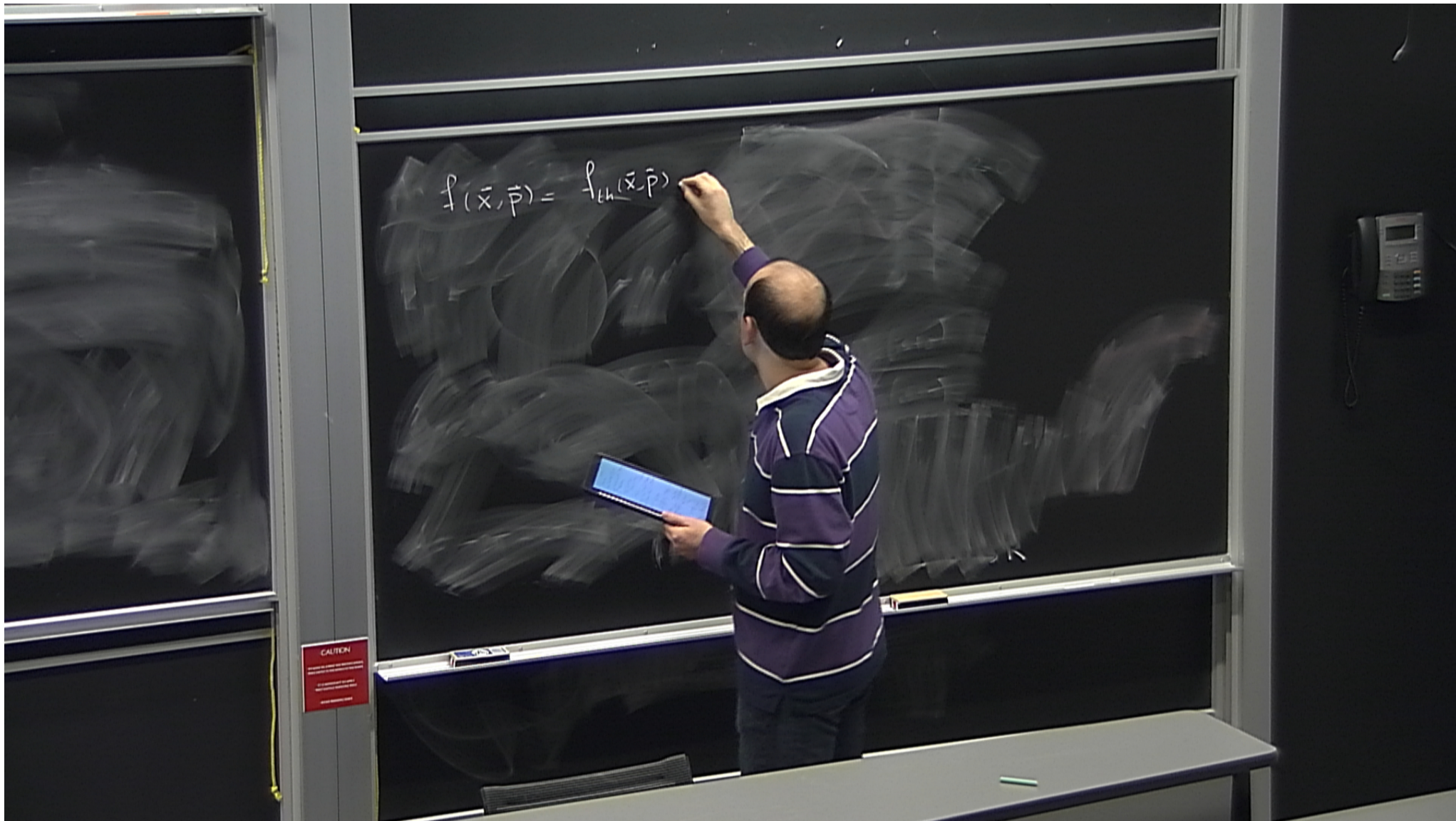
$$\rho \frac{d\vec{v}}{dt} = -\vec{\nabla} p + \eta \nabla^2 \vec{v} + \left(\zeta + \frac{1}{3}\eta\right) \vec{\nabla}(\vec{\nabla} \cdot \vec{v}) - \rho \vec{\nabla} \phi$$

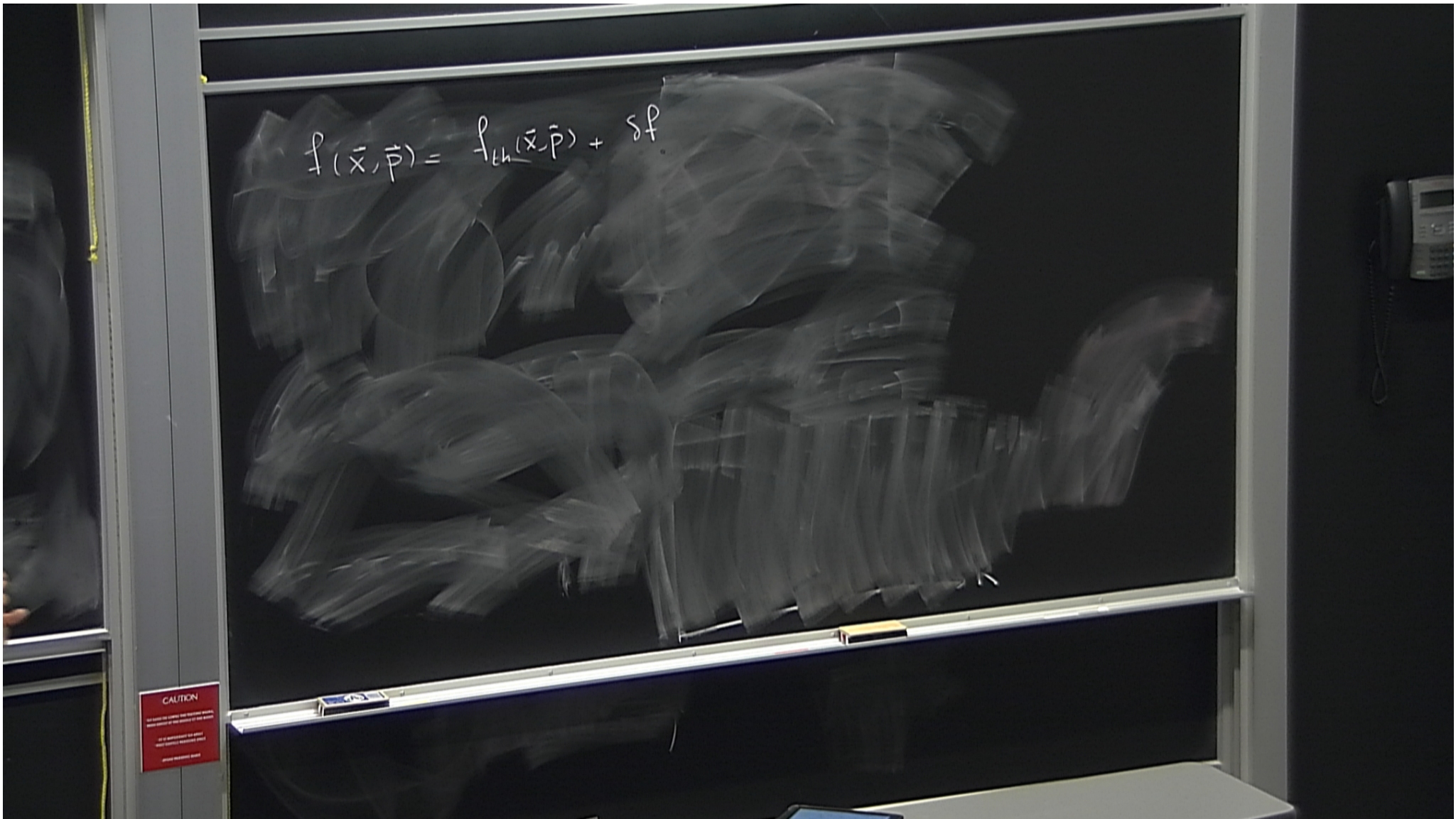
$$\frac{dp}{dt} + \rho \vec{\nabla} \cdot \vec{v} = 0$$

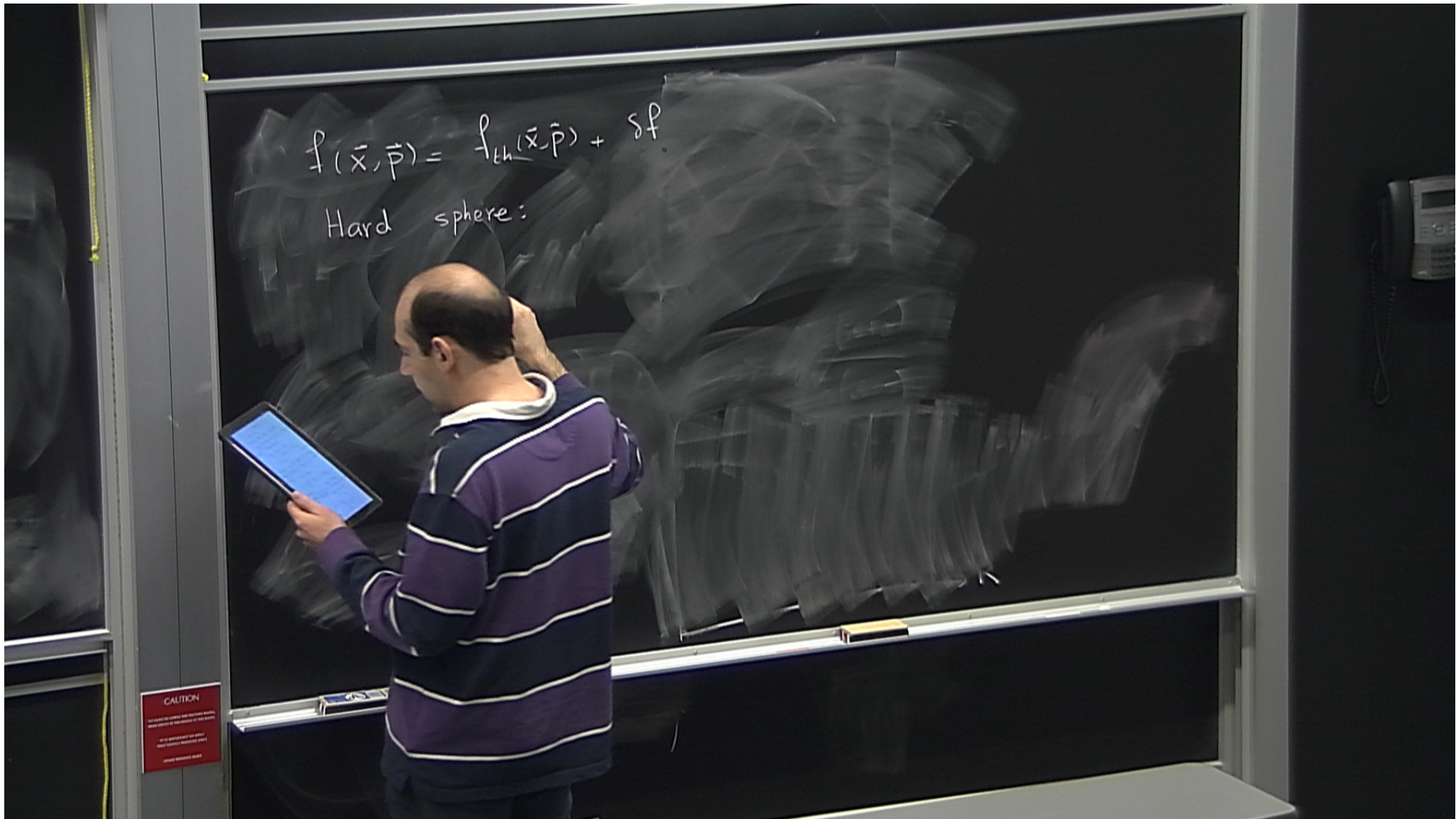
$$\left(\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{x}} \right)$$

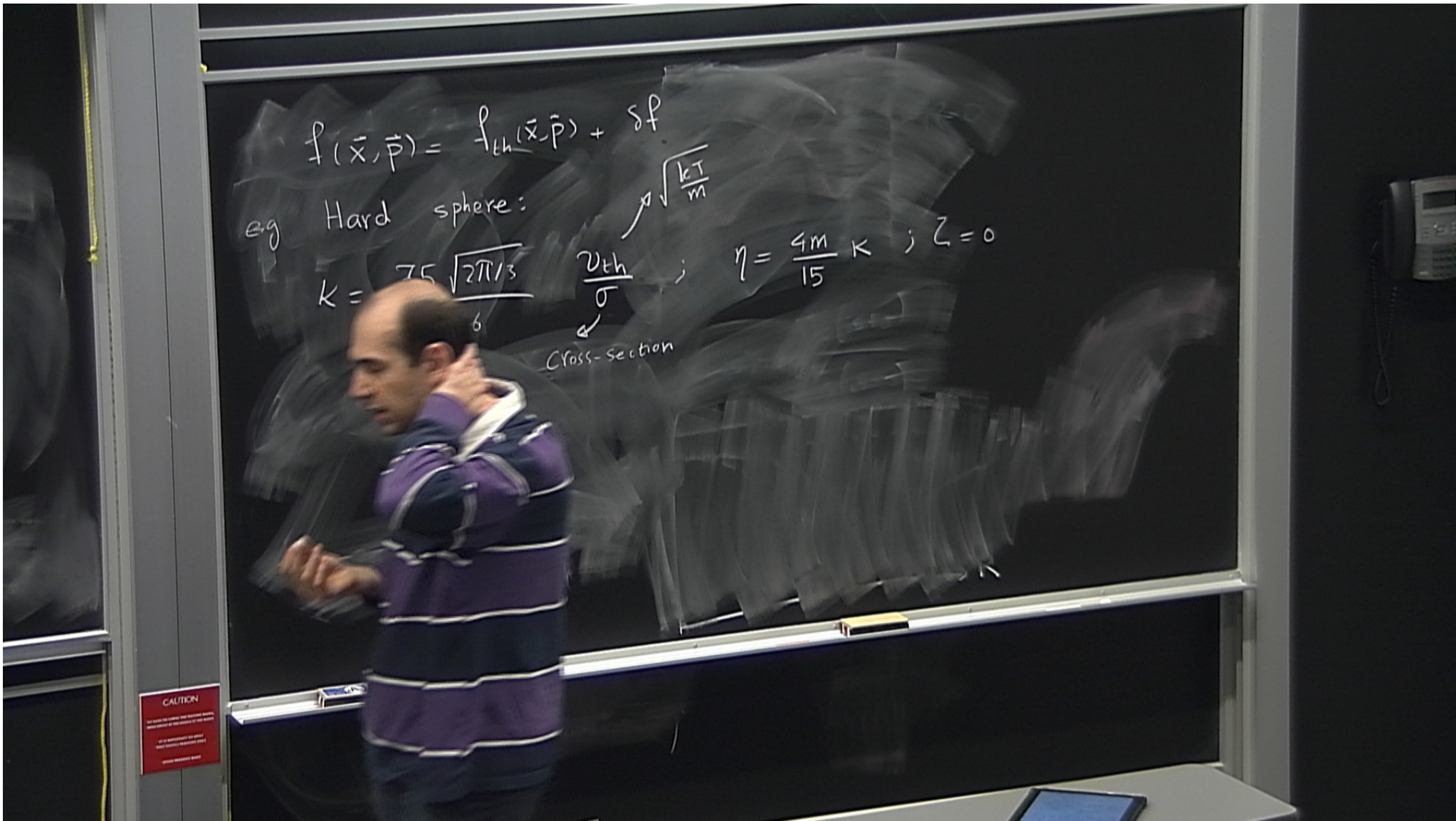
$$\rho T \frac{ds}{dt} = \overbrace{\vec{\nabla} \cdot (k \vec{\nabla} T)}^{\text{heat flux}} + \overbrace{\sigma^{ij} \partial_i v_j}_{\text{viscous heating}}$$

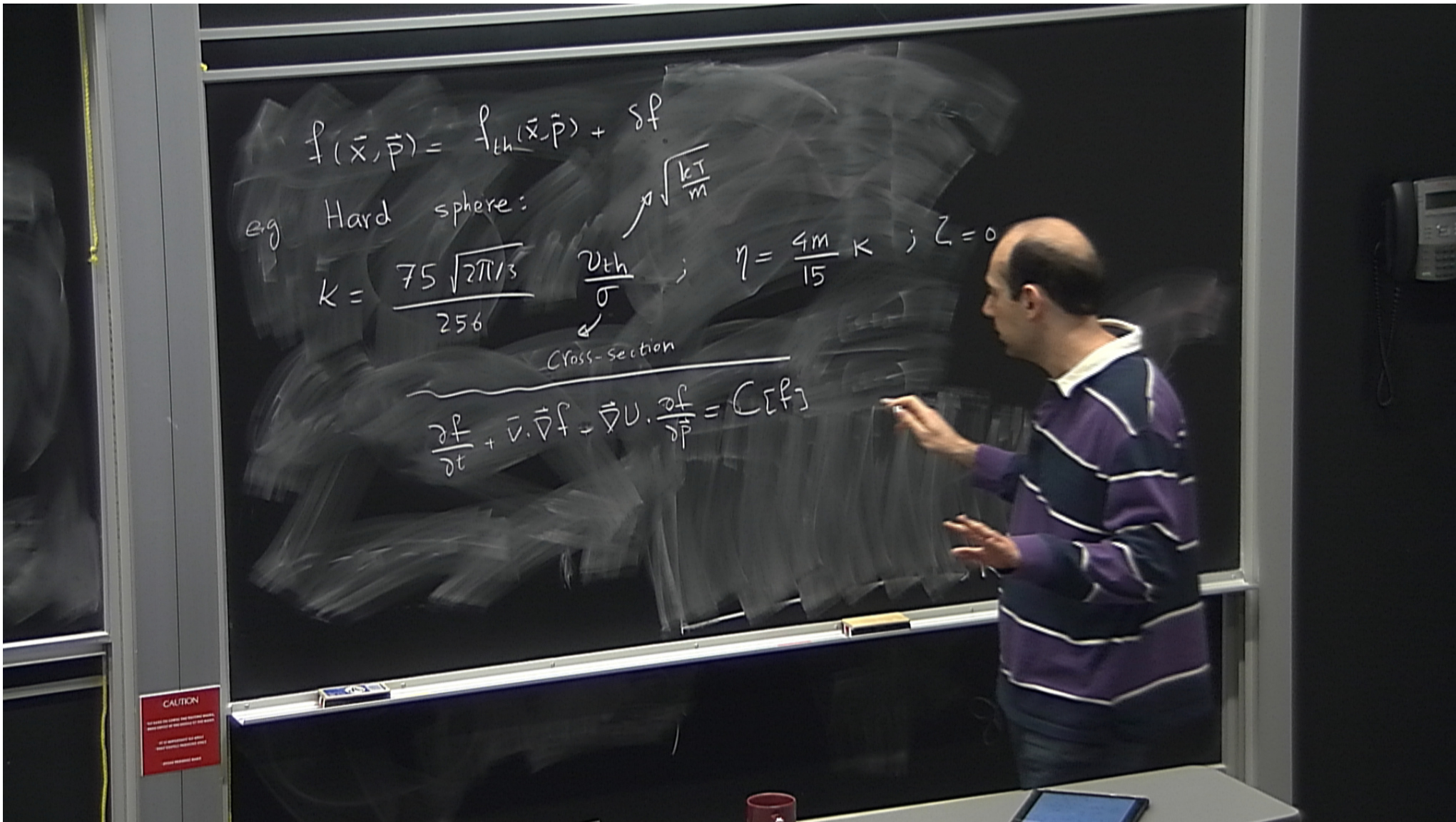
s : entropy / unit mass

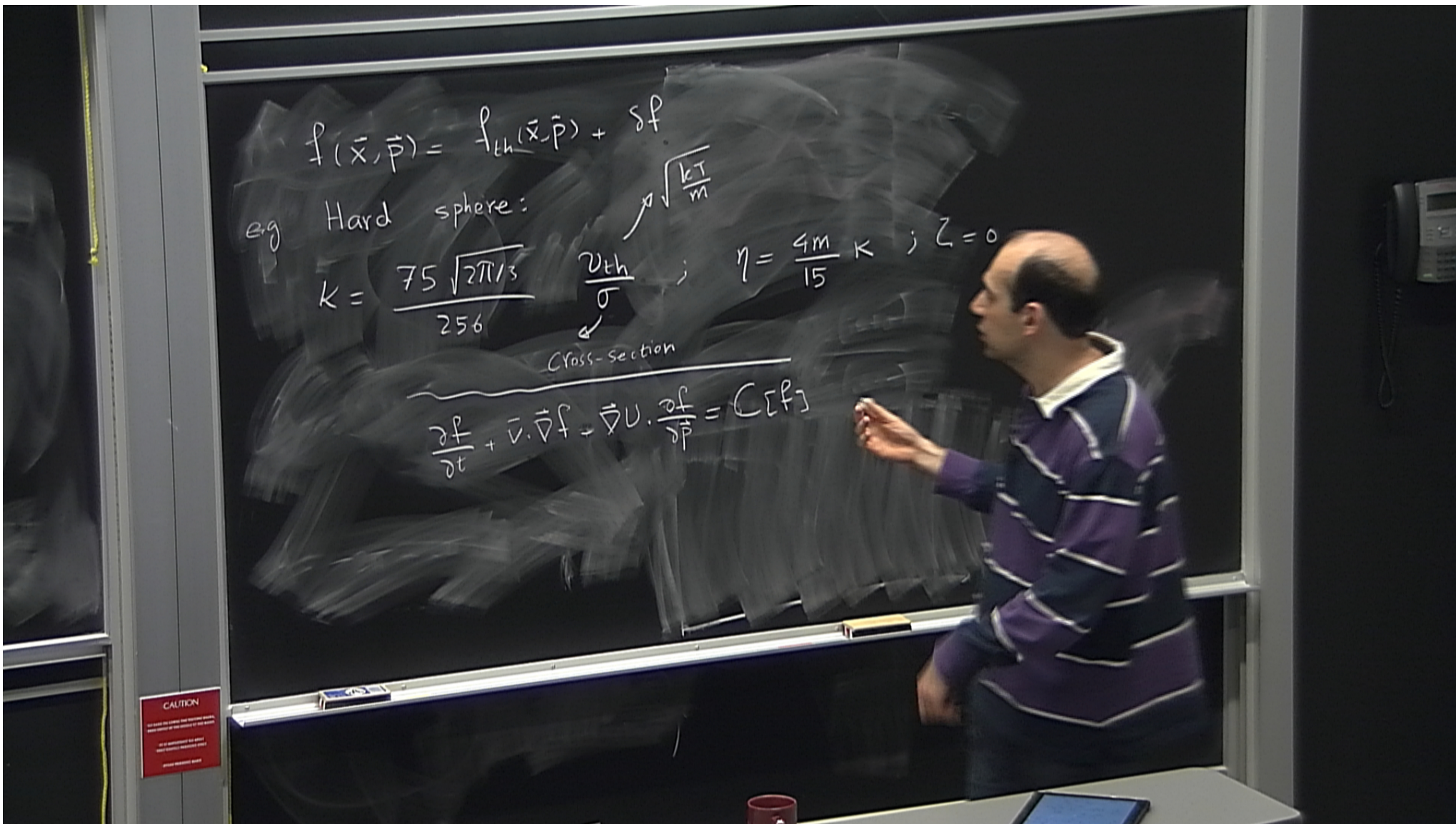


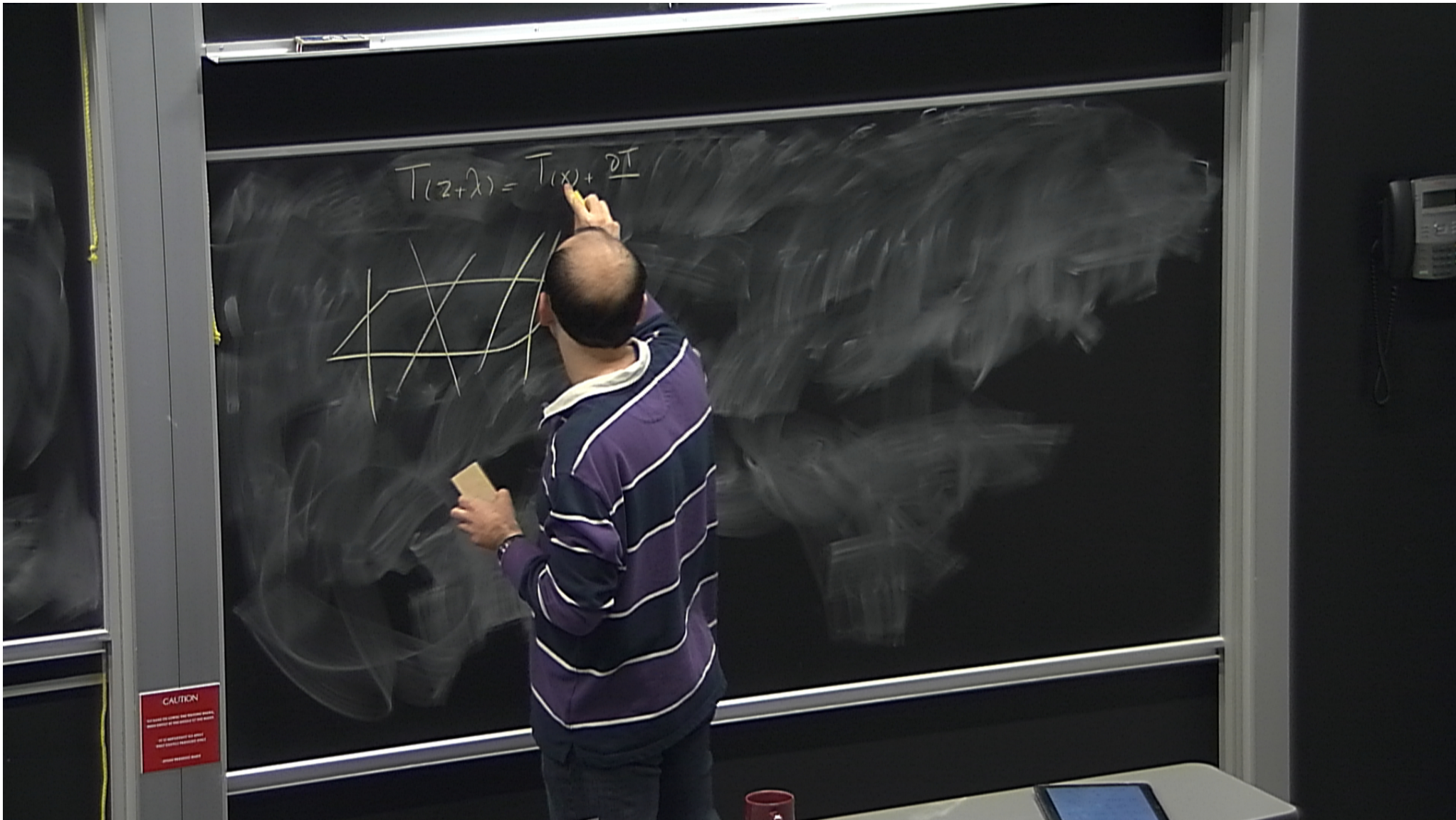


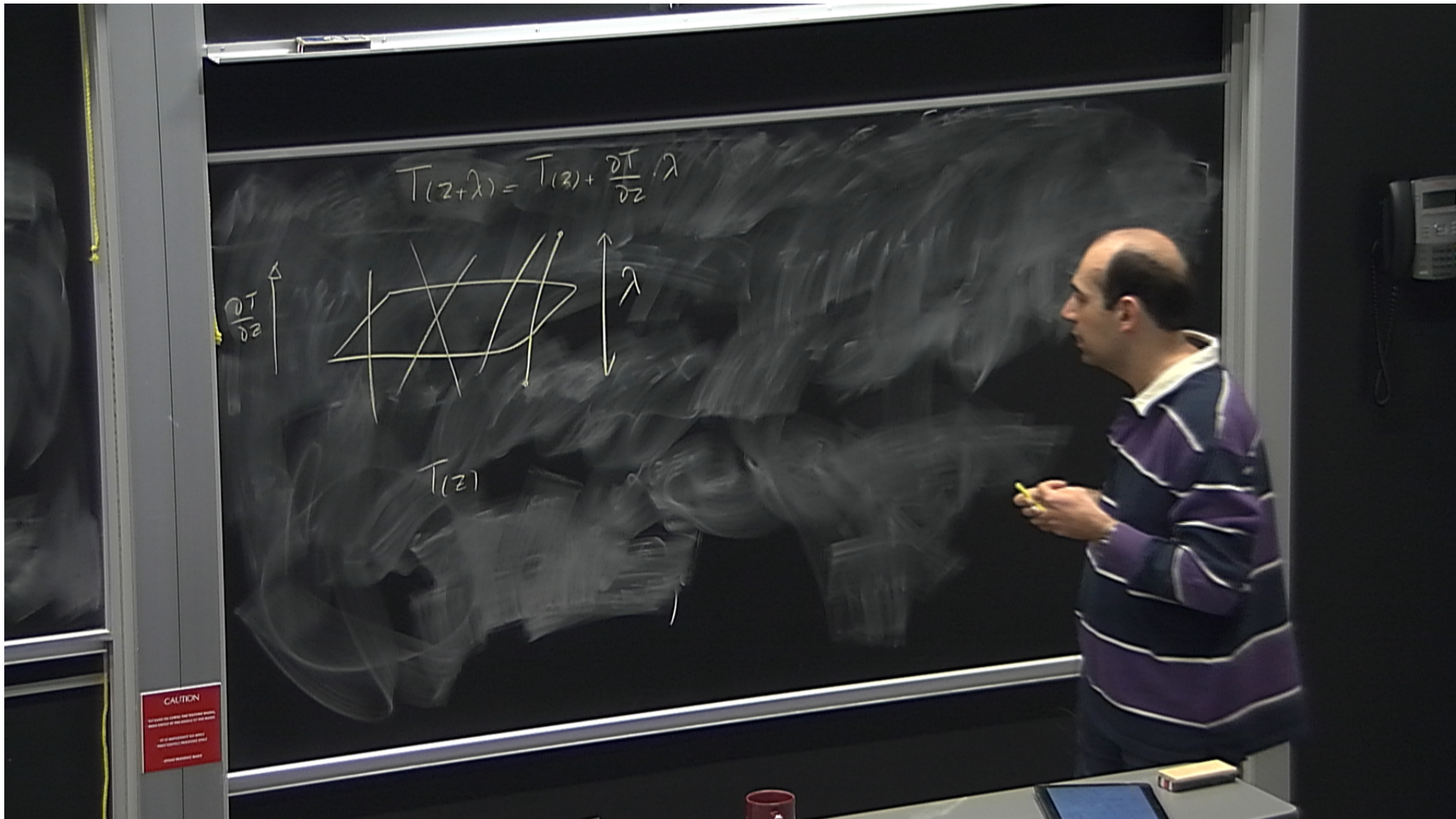


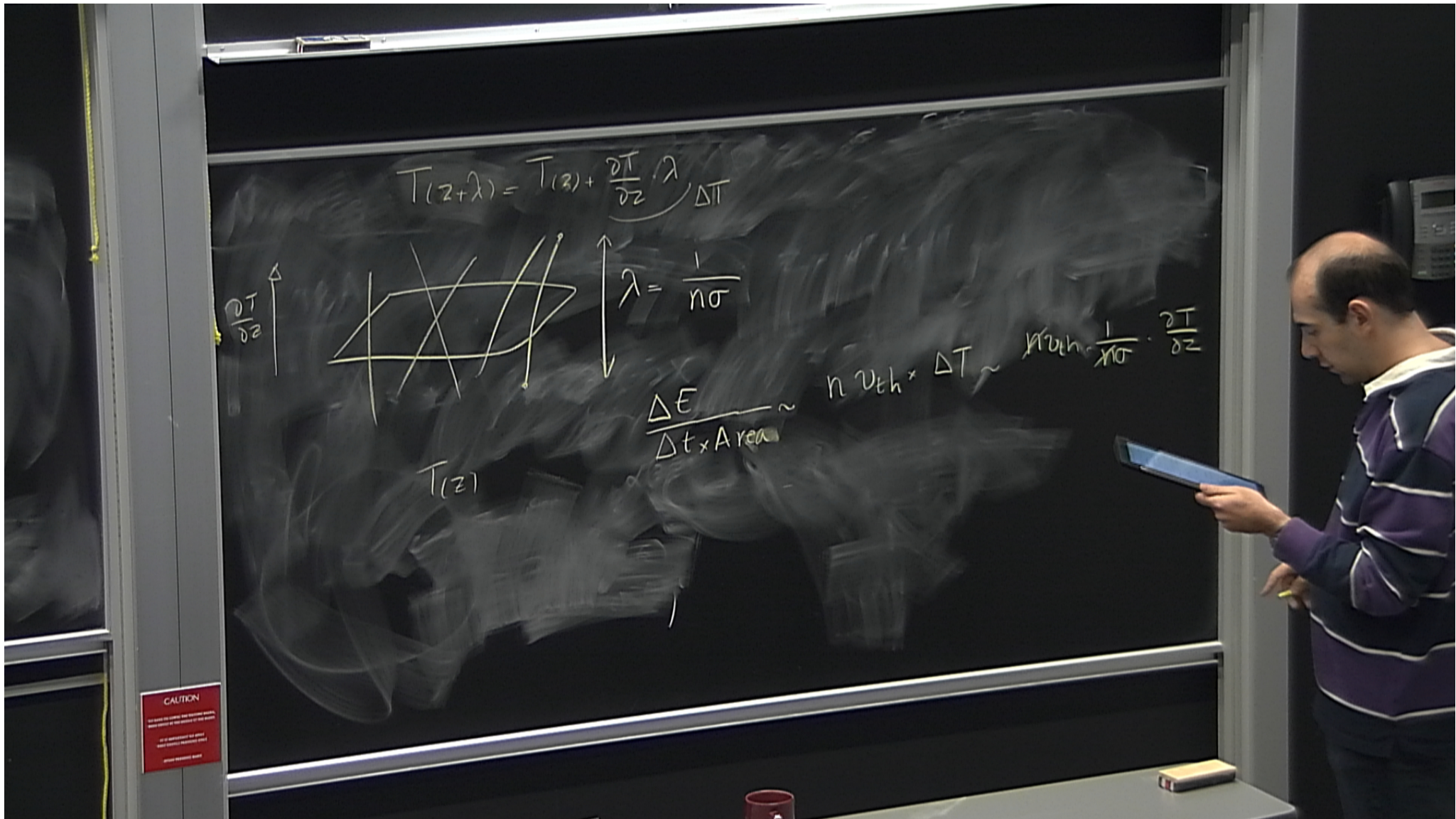


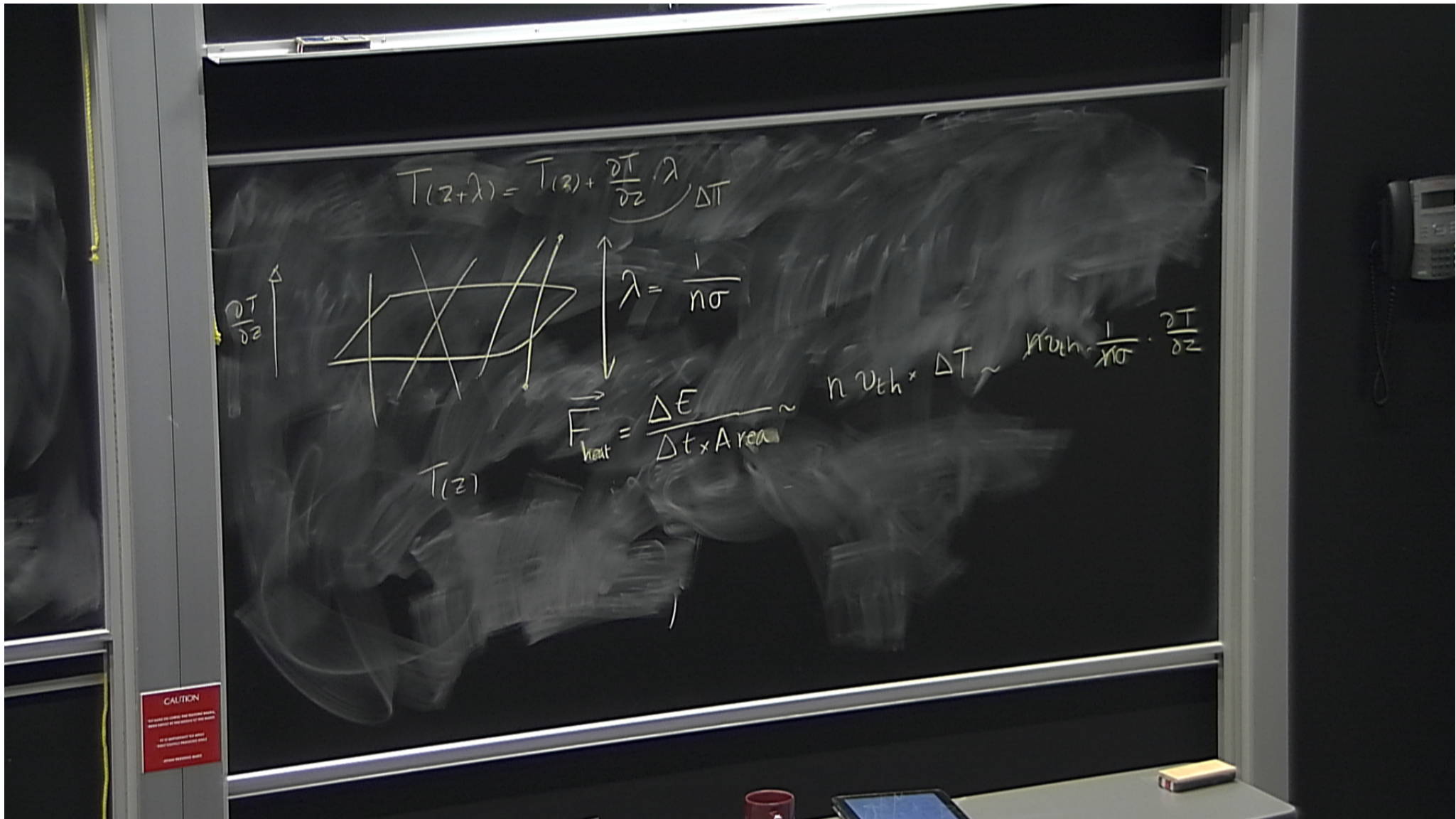


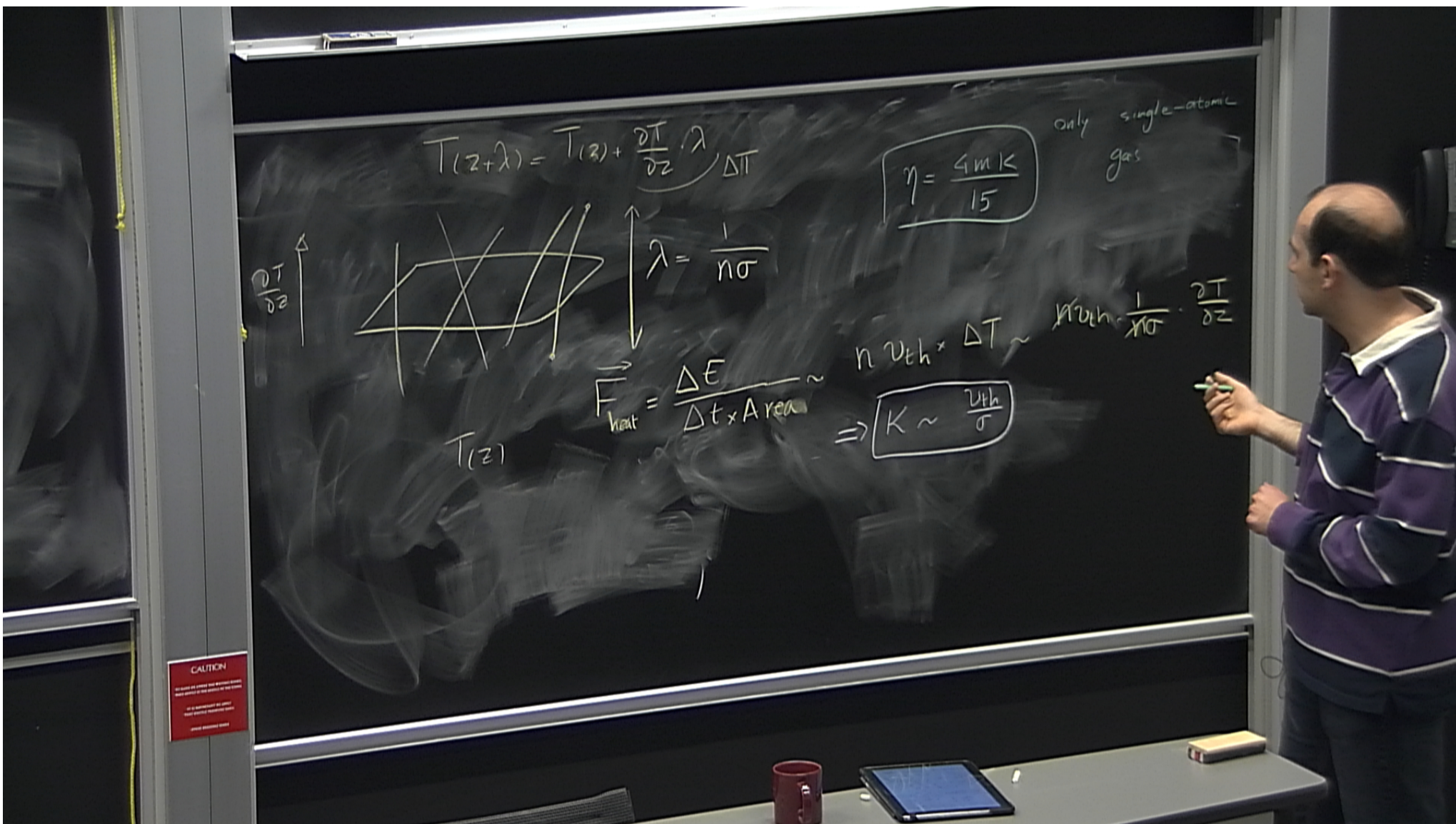


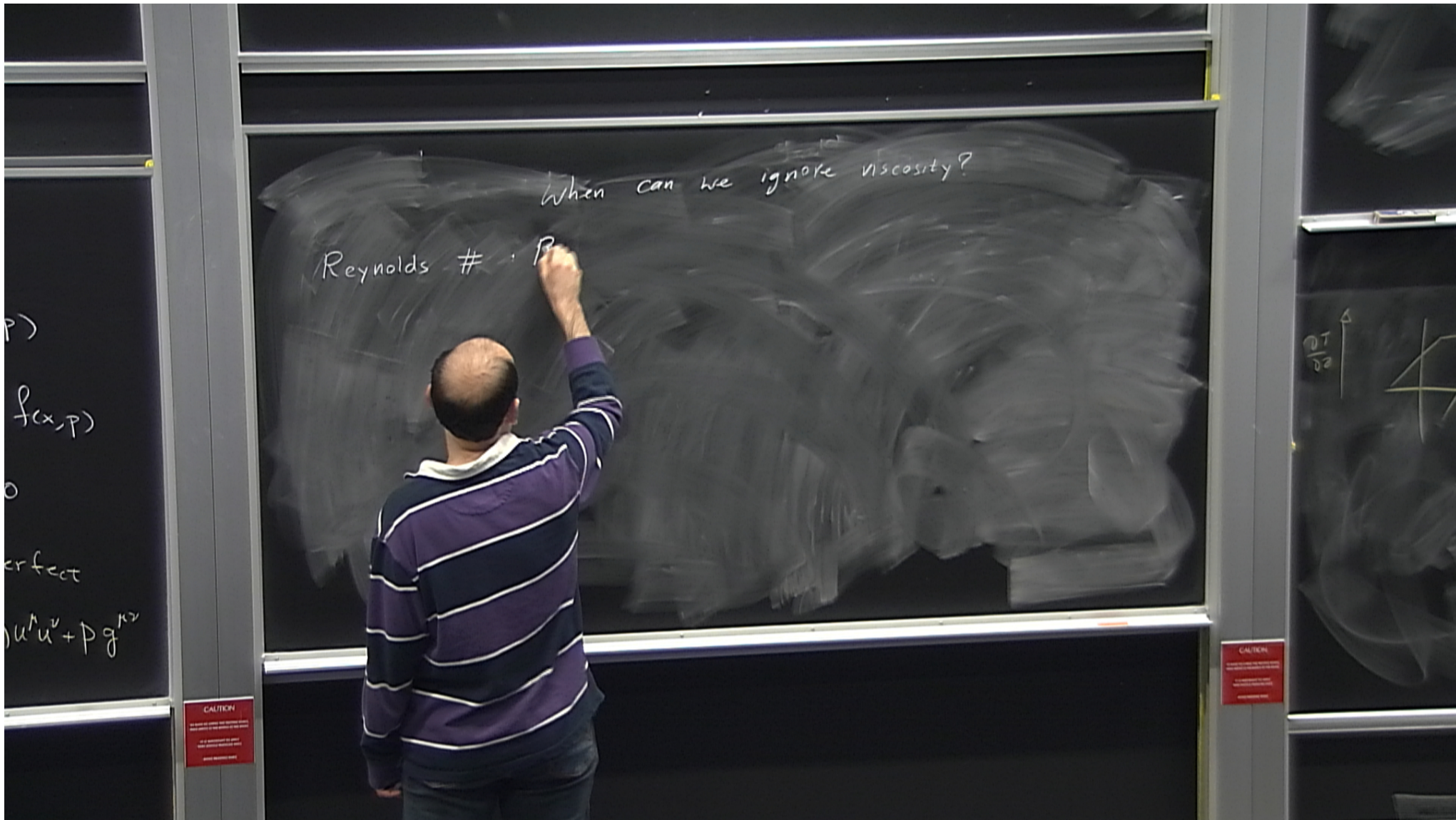


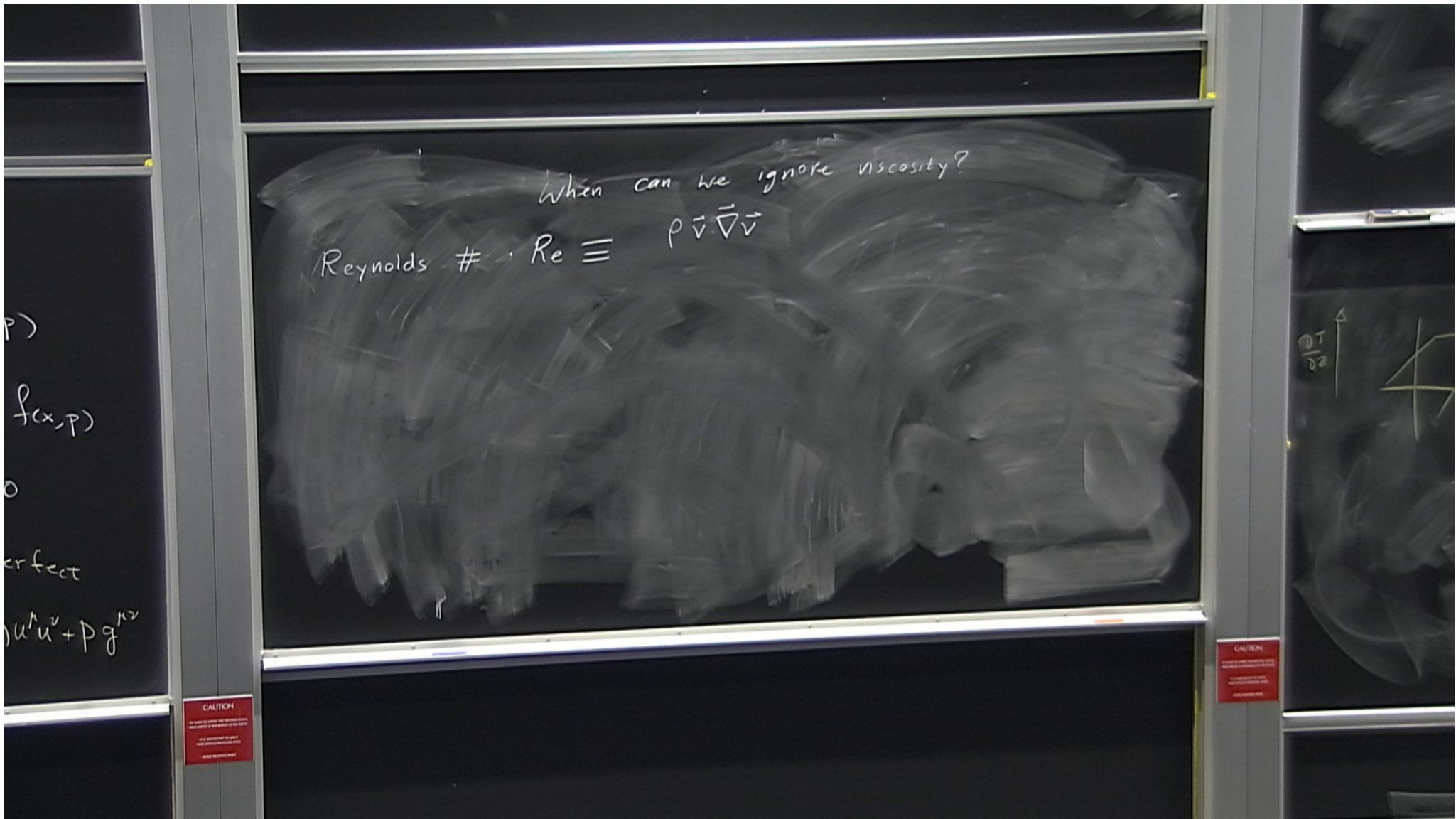












$$\rho \frac{d\vec{v}}{dt} = -\vec{\nabla} p + (\eta) \nabla^2 \vec{v} + \left(\frac{2}{3}\eta\right) \vec{\nabla}(\vec{\nabla} \cdot \vec{v}) - \rho \vec{\nabla} \phi$$

$$\frac{dp}{dt} + \rho \vec{\nabla} \cdot \vec{v} = 0$$

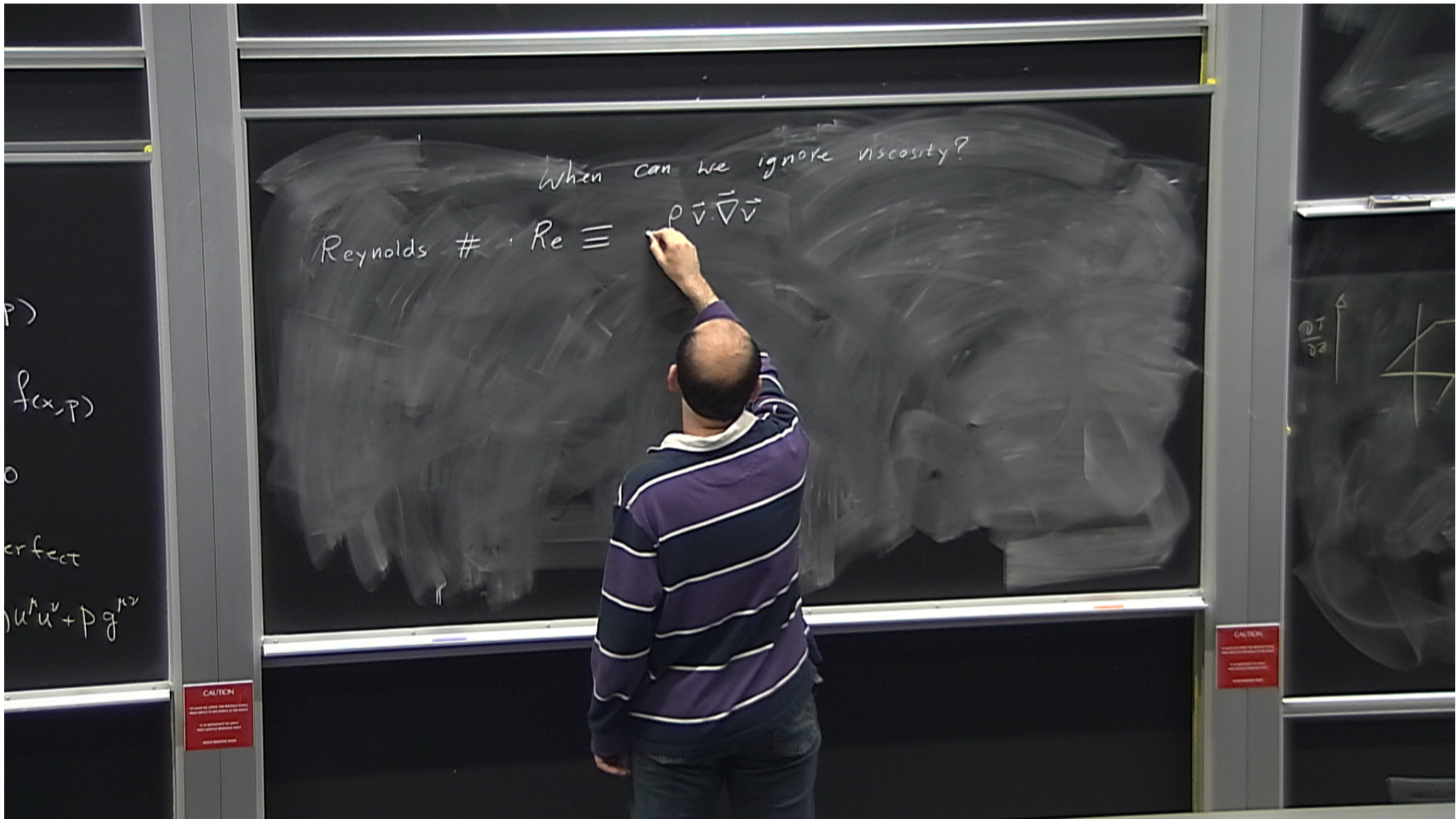
$$\left(\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{x}} \right)$$

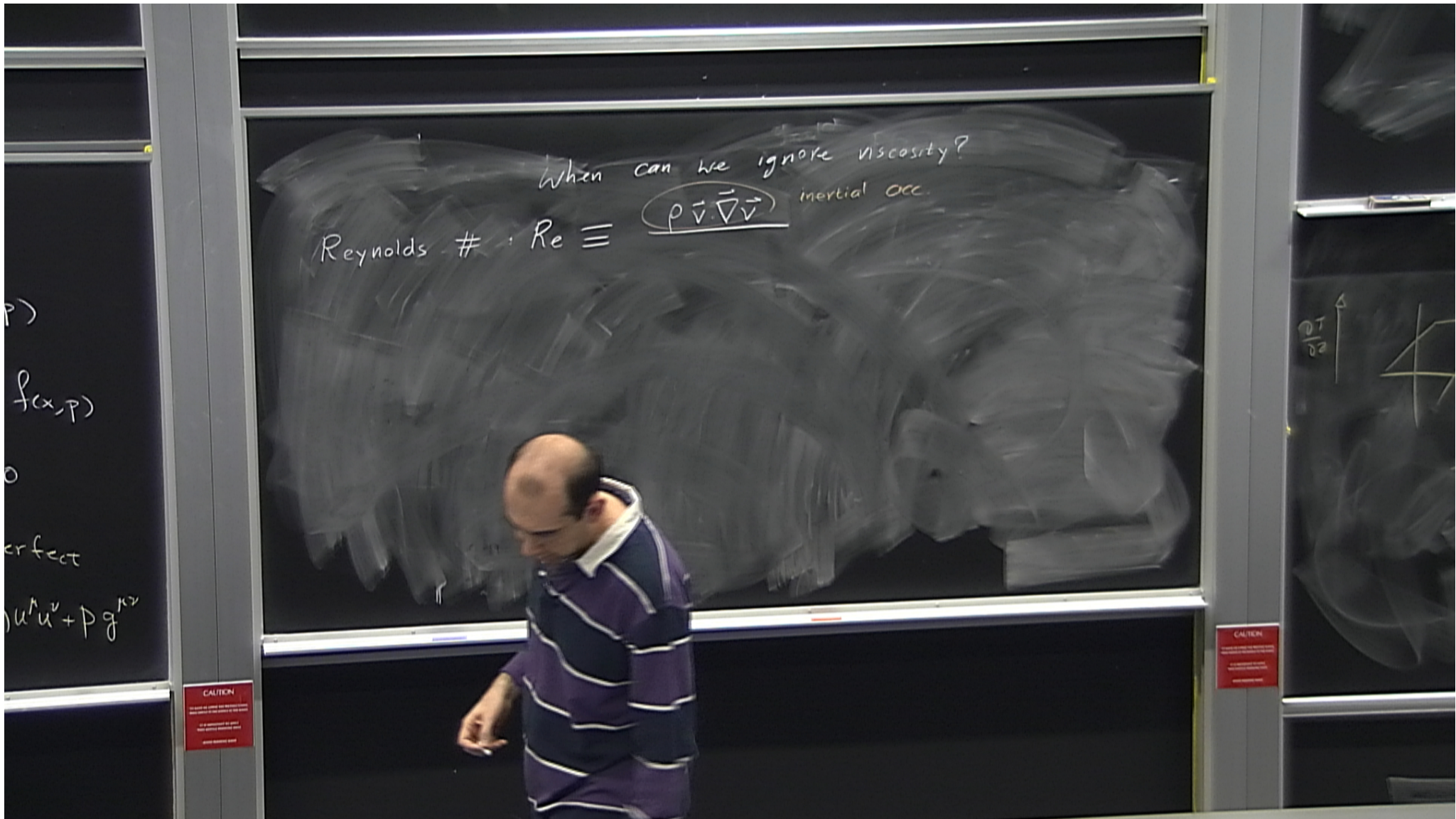
$$\rho T \frac{ds}{dt} = \underbrace{\vec{\nabla} \cdot (k \vec{\nabla} T)}_{\text{heat flux}} + \underbrace{\sigma^{ij} \partial_i v_j}_{\text{viscous heating}}$$

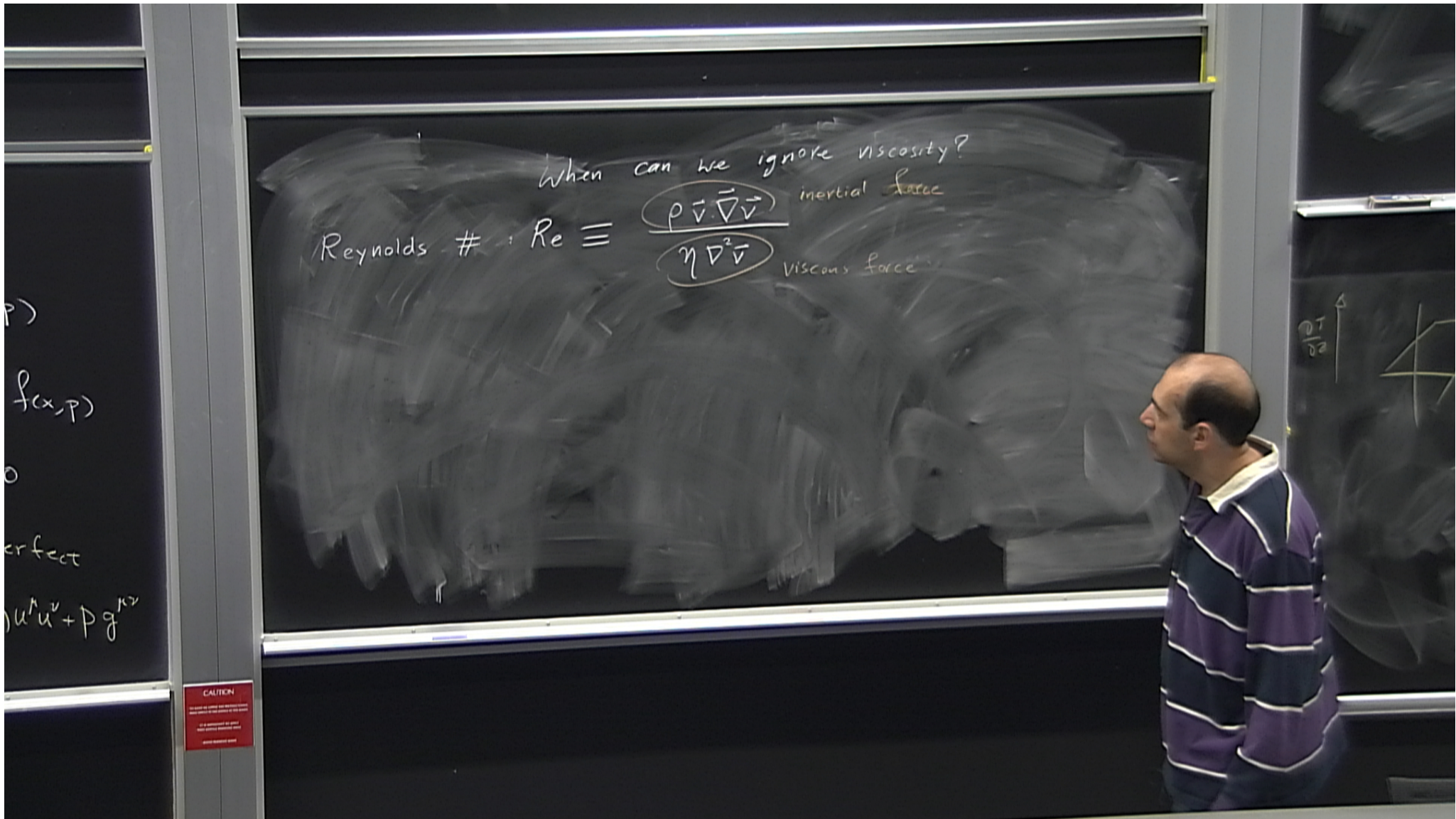
s: entropy unit mass

When can we ignore viscosity?

$$\rho - \rho \vec{v} \cdot \vec{\nabla} \vec{v}$$







When can we ignore viscosity?

$$\text{Reynolds \#} : Re \equiv \frac{\rho \vec{v} \cdot \vec{\nabla} \vec{v}}{\eta \nabla^2 \vec{v}}$$

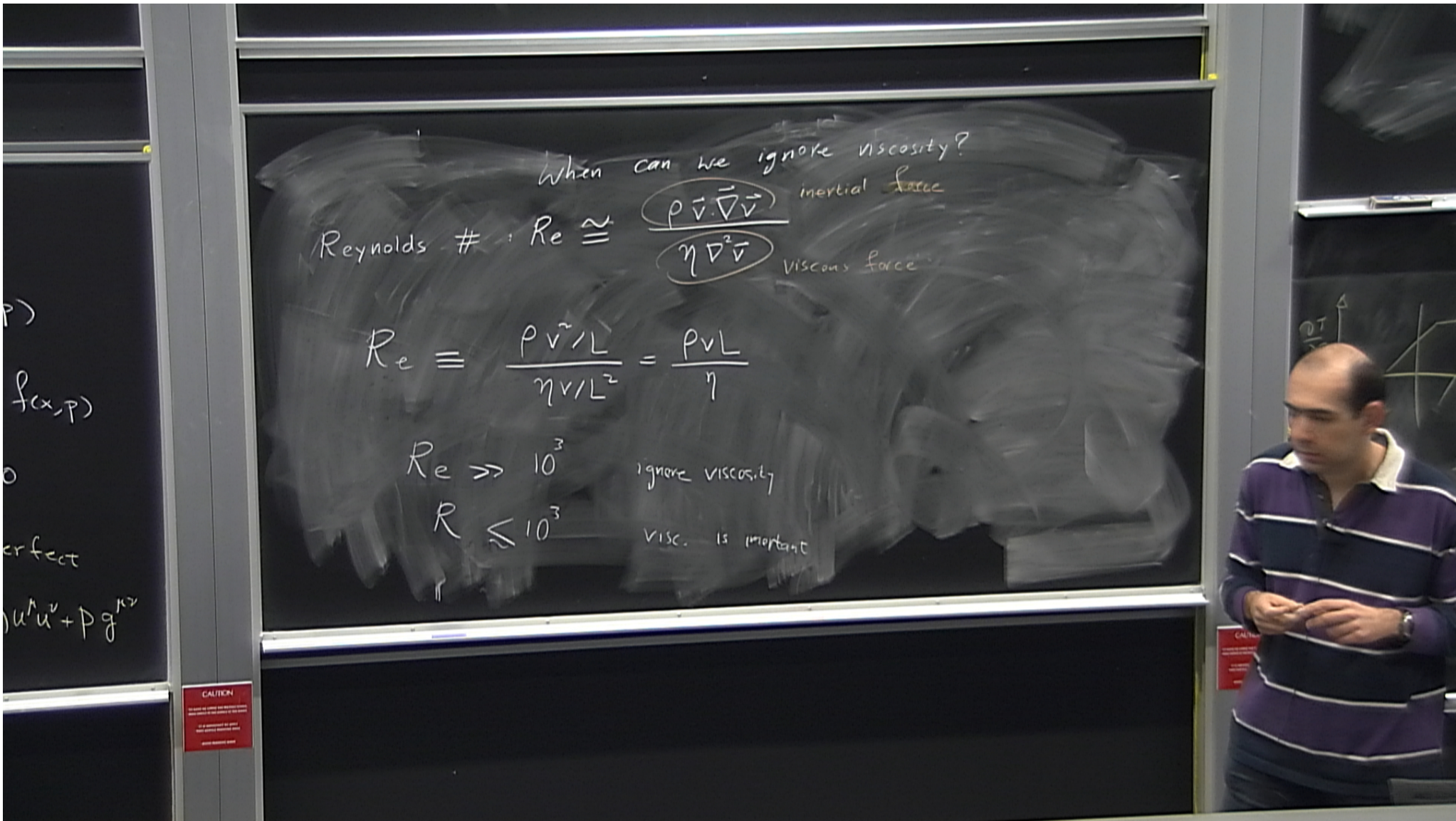
inertial force

viscous force

p)
f(x, p)
0
perfect
u^μu^ν + p g^{μν}

$\frac{dT}{dz}$
[Graph of T vs z]

CAUTION
Do not lean on the board
as this may damage the board
It is dangerous to play
with the board when it is
being used



$$S_{\text{tot}} = \int d^3x \, p \, s$$

$$\frac{dS_{\text{tot}}}{dt} = \int \left\{ k (\nabla h T)^2 + \frac{\eta}{2T} \sigma_{ij}^2 \right\} d^3x$$

Reynolds

$$Re \equiv$$

$$Re = \frac{R}{r}$$

$$S_{\text{tot}} = \int d^3x \rho s$$

$$\frac{d S_{\text{tot}}}{dt} = \int \left\{ k (\nabla^2 T)^2 + \frac{\eta}{2T} \sigma^{ij} \sigma^{ij} + \frac{\zeta}{T} (\nabla \cdot v)^2 \right\} d^3x + \text{surface terms}$$

$\sigma^{ij} = 2v^i v^j - \delta^{ij} v^k v^k - \frac{1}{3} (\nabla \cdot v) \delta^{ij}$

Reynolds

$$Re \equiv$$

$$Re =$$

$$R$$

$$S_{\text{tot}} = \int d^3x \rho s$$

$$\frac{dS_{\text{tot}}}{dt} = \int \left\{ k (\nabla^2 T)^2 + \frac{\eta}{2T} |\sigma^{ij}|^2 + \frac{\zeta}{T} (\nabla \cdot v)^2 \right\} d^3x + \text{surface terms}$$

turn $\checkmark$ bulk gradients $\rightarrow$ microscopic motion

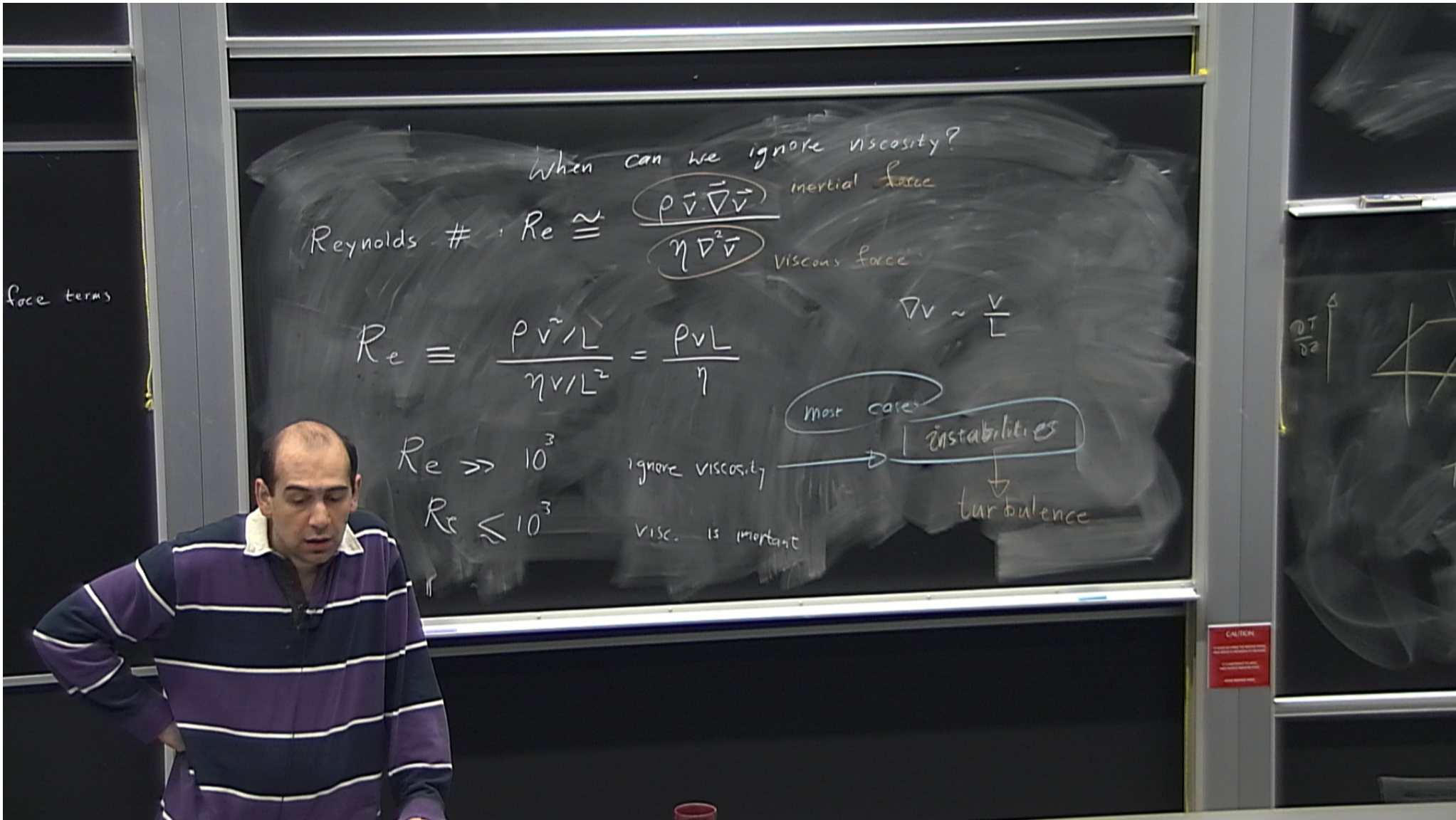
$$\sigma^{ij} = \partial_i v_j + \partial_j v_i - \frac{1}{3} (\nabla \cdot v) \delta_{ij}$$

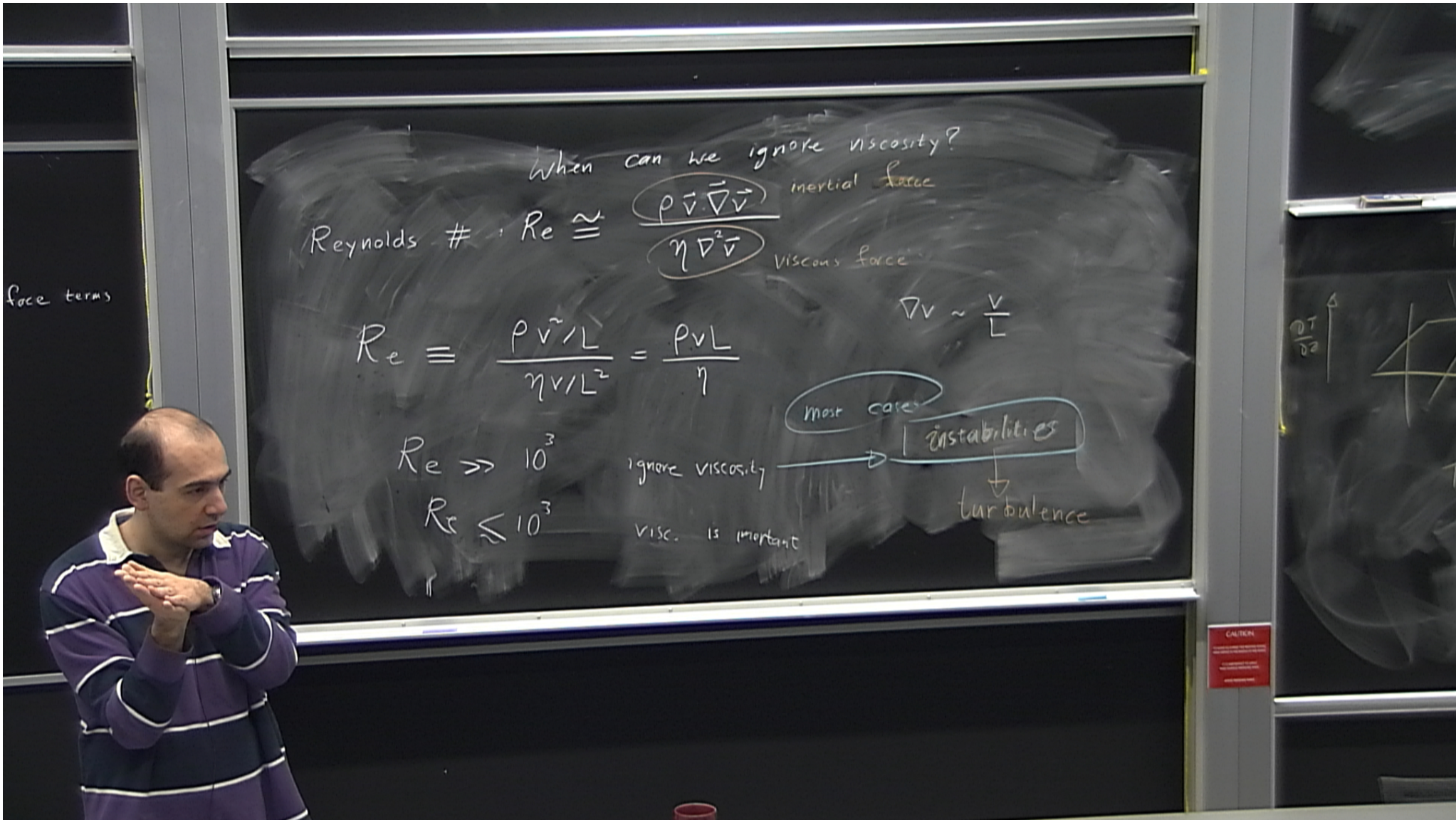
Reynolds #

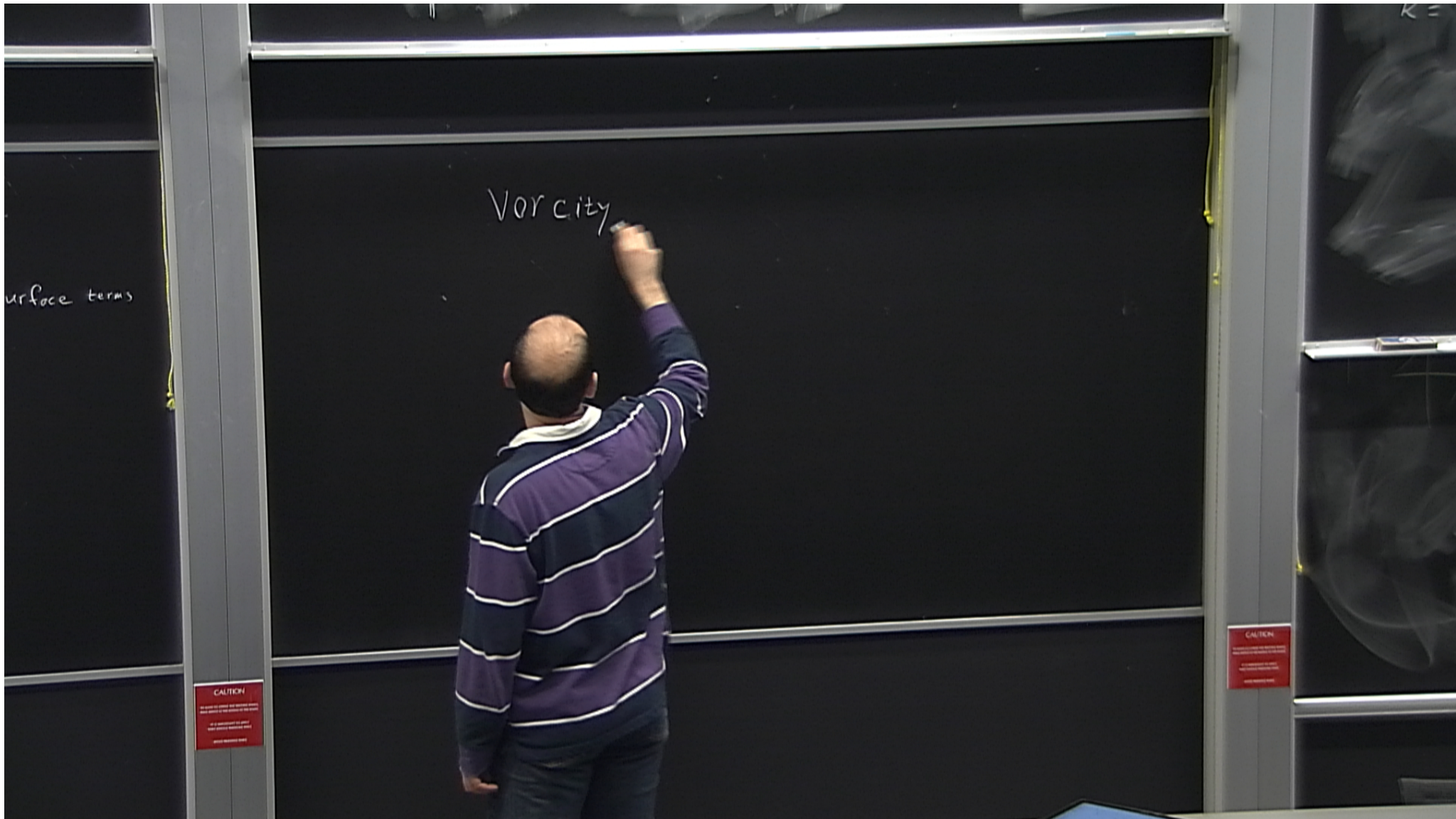
$$Re \equiv$$

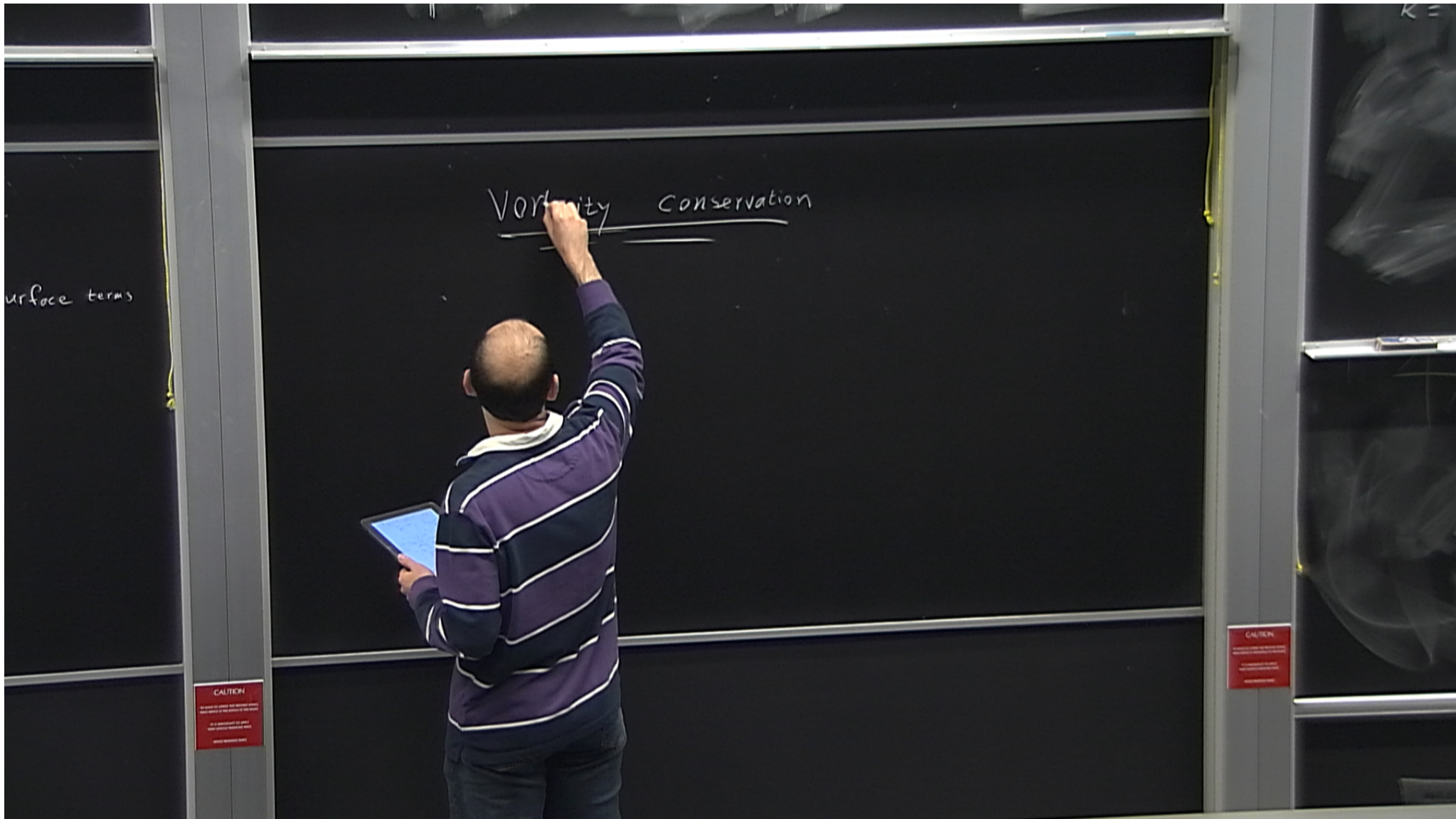
$$Re =$$

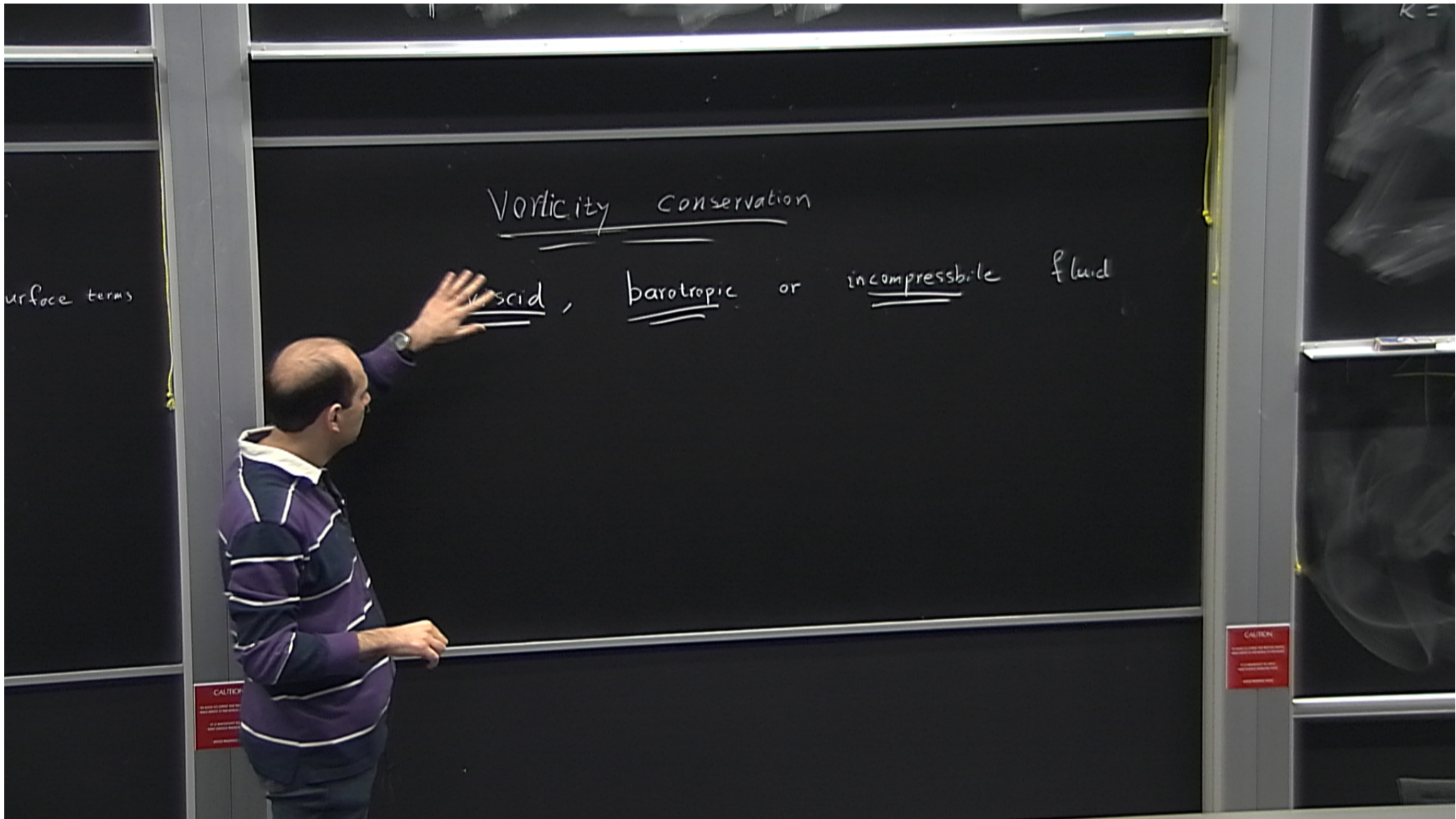
$$Re$$











Vorticity Conservation

inviscid, barotropic or incompressible fluid
 $\eta = 0$ $P = P(\rho)$ $\rho = \text{const.}$

Vorticity Conservation

$(\zeta = 0)$ inviscid, barotropic or incompressible fluid
 $P = P(\rho)$ $\rho = \text{const.}$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{\nabla P}{\rho} - \nabla \phi$$

$$\frac{\partial}{\partial t} (\nabla \times \vec{v}) = \frac{1}{\rho^2} (\nabla \rho \times \nabla P)$$

Vorticity Conservation

inviscid, barotropic or incompressible fluid
 $(\zeta \neq 0) \quad \eta = 0 \quad P = P(\rho) \quad \rho = \text{const}$

$$\frac{\partial \vec{v}}{\partial t} + \underbrace{\vec{v} \cdot \nabla \vec{v}} = -\frac{\nabla P}{\rho} - \nabla \phi$$

$$\frac{1}{2} \nabla v^2 = \vec{v} \times \nabla \times \vec{v}$$

$$\frac{\partial}{\partial t} \underbrace{(\nabla \times \vec{v})}_{\vec{\Omega}} - \nabla \times (\vec{v} \times \underbrace{(\nabla \times \vec{v})}_{\vec{\Omega}}) = \frac{1}{\rho^2} (\nabla \rho \times \nabla P)$$

↪ vorticity

Vorticity Conservation

$(\zeta \neq 0)$ inviscid, barotropic or incompressible fluid
 $\eta = 0$ $P = P(\rho)$ $\rho = \text{const.}$

$$\frac{\partial \vec{v}}{\partial t} + \underbrace{\vec{v} \cdot \nabla \vec{v}} = -\frac{\vec{\nabla} P}{\rho} - \vec{\nabla} \phi$$

$$\frac{1}{2} \vec{\nabla} v^2 = \vec{v} \times \vec{\nabla} \times \vec{v}$$

$$\frac{\partial}{\partial t} (\underbrace{\vec{\nabla} \times \vec{v}}_{\vec{\Omega}}) - \vec{\nabla} \times (\vec{v} \times (\underbrace{\vec{\nabla} \times \vec{v}}_{\vec{\Omega}})) = \frac{1}{\rho^2} (\cancel{\vec{\nabla} \rho \times \vec{\nabla} P}) \rightarrow \frac{\partial \vec{\Omega}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{\Omega})$$

↪ vorticity

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$(\zeta \neq 0)$ inviscid, barotropic or incompressible fluid
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↪ vorticity

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 $(\zeta \neq 0) \quad \eta = 0 \quad P = P(\rho) \quad \rho = \text{const.}$

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↪ vorticity

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 $(\zeta \neq 0) \quad \eta = 0 \quad P = P(\rho) \quad \rho = \text{const.}$

$$\frac{\partial \vec{v}}{\partial t} + \underbrace{\vec{v} \cdot \nabla \vec{v}} = -\frac{\nabla P}{\rho} - \nabla \phi$$

$$\frac{1}{2} \nabla v^2 = \vec{v} \times \vec{\Omega} \times \vec{v}$$

$$\frac{\partial}{\partial t} (\underbrace{\nabla \times \vec{v}}_{\vec{\Omega}}) - \nabla \times (\vec{v} \times (\underbrace{\nabla \times \vec{v}}_{\vec{\Omega}})) = \frac{1}{\rho^2} (\nabla \rho \times \nabla P) \rightarrow \frac{\partial \vec{\Omega}}{\partial t} = \nabla \times (\vec{v} \times \vec{\Omega})$$

$$\frac{d}{dt} \left(\oint_S \vec{\Omega} \cdot d\vec{A} \right) = \frac{d}{dt} \left(\oint \vec{v} \cdot d\vec{\ell} \right)$$

↪ vorticity



Vorticity Conservation

inviscid, barotropic or incompressible fluid
 $(\zeta \neq 0) \quad \eta = 0 \quad P = P(\rho) \quad \rho = \text{const.}$

$$\frac{\partial \vec{v}}{\partial t} + \underbrace{\vec{v} \cdot \nabla \vec{v}} = -\frac{\vec{\nabla} P}{\rho} - \vec{\nabla} \phi$$

$$\frac{1}{2} \vec{\nabla} v^2 = \vec{v} \times \vec{\nabla} \times \vec{v}$$

$$\frac{\partial}{\partial t} (\underbrace{\vec{\nabla} \times \vec{v}}_{\vec{\Omega}}) - \vec{\nabla} \times (\vec{v} \times (\underbrace{\vec{\nabla} \times \vec{v}}_{\vec{\Omega}})) = \frac{1}{\rho^2} (\vec{\nabla} \rho \times \vec{\nabla} P) \rightarrow \frac{\partial \vec{\Omega}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{\Omega})$$

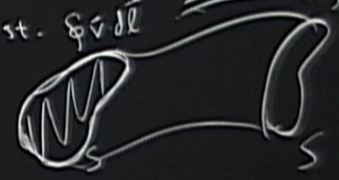
$$\frac{d}{dt} \left(\oint_S \vec{\Omega} \cdot d\vec{A} \right) = \frac{d}{dt} \left(\oint_S \vec{v} \cdot d\vec{\ell} \right) = 0$$

rotation



Vorticity Conservation

inviscid, barotropic or incompressible fluid
 $(\zeta \neq 0) \quad \eta = 0$ $P = P(\rho)$ $\rho = \text{const.} \quad \oint \vec{v} \cdot d\vec{\ell}$



$$\frac{\partial \vec{v}}{\partial t} + \underbrace{\vec{v} \cdot \nabla \vec{v}} = -\frac{\nabla P}{\rho} - \nabla \phi$$

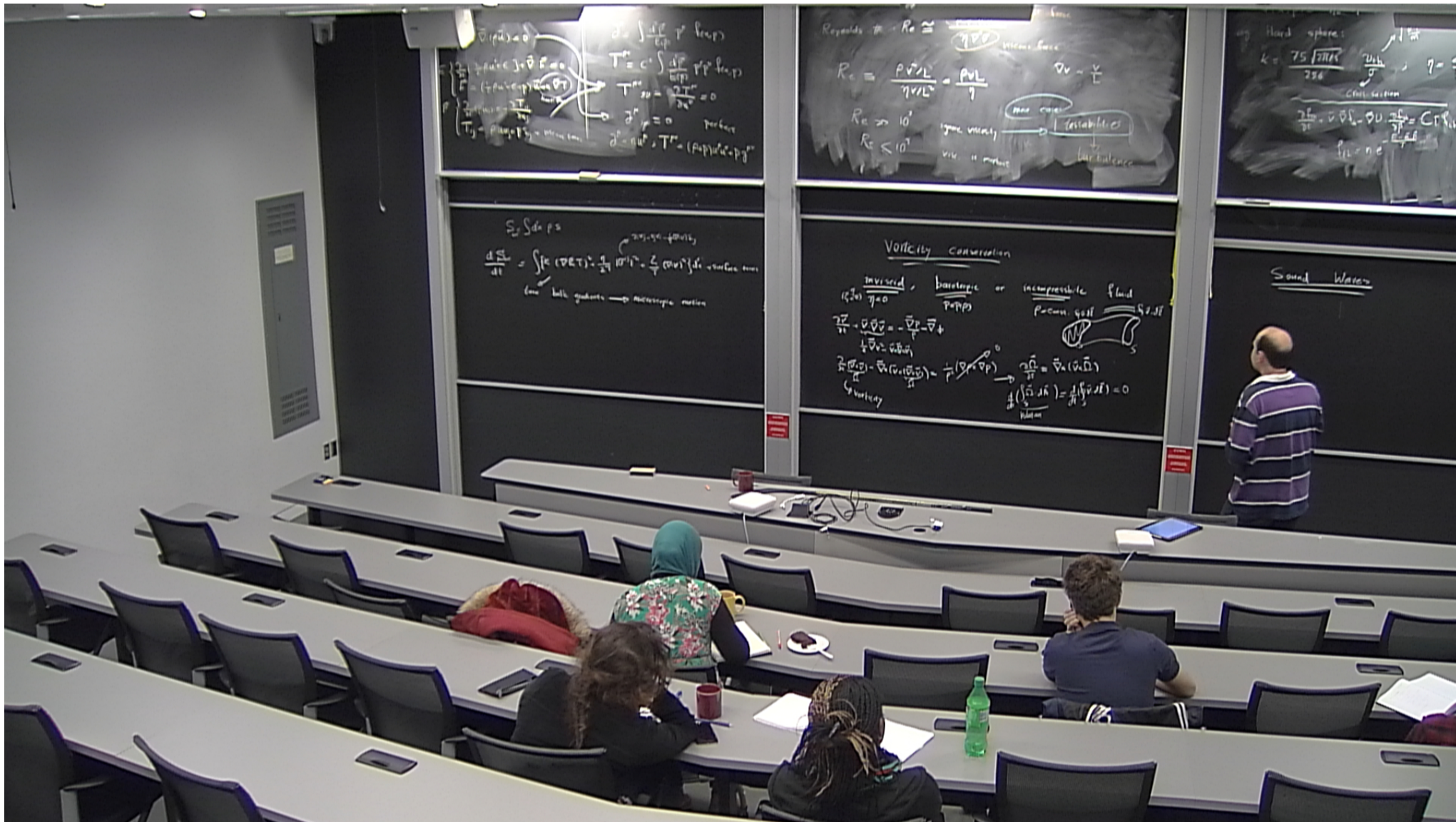
$$\frac{1}{2} \nabla v^2 = \vec{v} \times \nabla \times \vec{v}$$

$$\frac{\partial}{\partial t} (\underbrace{\nabla \times \vec{v}}_{\vec{\Omega}}) - \nabla \times (\vec{v} \times (\underbrace{\nabla \times \vec{v}}_{\vec{\Omega}})) = \frac{1}{\rho^2} (\nabla \rho \times \nabla P) \rightarrow \frac{\partial \vec{\Omega}}{\partial t} = \nabla \times (\vec{v} \times \vec{\Omega})$$

↙ vorticity

$$\frac{d}{dt} \left(\oint_S \vec{\Omega} \cdot d\vec{A} \right) = \frac{d}{dt} \left(\oint_S \vec{v} \cdot d\vec{\ell} \right) = 0$$

rotation



Sound Waves ; ignore visc.

$$\rho = \rho_0 + \delta \rho(x,t) \quad \cancel{\delta \vec{v} \cdot \vec{\nabla} \delta \rho} + \frac{\partial \delta \rho}{\partial t} + \rho_0 \nabla \cdot \delta \vec{v} = 0$$

$$P = P_0 + \delta P(x,t) \quad \cancel{\delta \vec{v} \cdot \vec{\nabla} \delta P} + \frac{\partial \delta P}{\partial t} + \frac{\vec{\nabla} \delta P \cdot \delta \vec{v}}{\rho_0} = 0$$

$$\vec{v} = \delta \vec{v}$$

$$\oint_S \vec{v} \cdot d\vec{\ell}$$

$$= 0$$

Sound Waves ; ignore visc.

$$\begin{aligned} \rho &= \rho_0 + \delta\rho(x,t) & \cancel{\delta\vec{v}} \cdot \vec{\nabla} \delta\rho + \frac{\partial \delta\rho}{\partial t} + \rho_0 \nabla \cdot \delta\vec{v} &= 0 \\ P &= P_0 + \delta P(x,t) & \cancel{\delta\vec{v}} \cdot \vec{\nabla} \delta P + \frac{\partial \delta P}{\partial t} + \frac{\vec{\nabla} \delta P}{\rho_0} &= 0 \\ \vec{v} &= \delta\vec{v} & \frac{\partial^2 \delta P}{\partial t^2} + \rho_0 & \end{aligned}$$

Sound Waves ; ignore visc.

assume

$$\rho = \rho_0 + \delta\rho(x,t) \quad \cancel{\delta\vec{v} \cdot \vec{\nabla} \delta\rho} + \frac{\partial \delta\rho}{\partial t} + \rho_0 \vec{\nabla} \cdot \delta\vec{v} = 0$$

$$P = P_0 + \delta P(x,t) \quad \cancel{\delta\vec{v} \cdot \vec{\nabla} \delta P} + \frac{\partial \delta P}{\partial t} + \frac{\vec{\nabla} \delta P \cdot \delta\vec{v}}{\rho_0}$$

$$\vec{v} = \delta\vec{v}$$

$$\frac{\partial^2 \delta P}{\partial t^2} + \rho_0 \vec{\nabla} \cdot \left[- \frac{\vec{\nabla} \delta P}{\rho_0} \right]$$

$$C_s^2 \equiv \frac{\delta P}{\delta \rho} = \text{const.}$$

Sound Waves ; ignore visc.

assume

$$\rho = \rho_0 + \delta\rho(x,t) \quad \cancel{\delta\vec{v} \cdot \vec{\nabla} \delta\rho} + \frac{\partial \delta\rho}{\partial t} + \rho_0 \vec{\nabla} \cdot \delta\vec{v} = 0$$

$$P = P_0 + \delta P(x,t) \quad \cancel{\delta\vec{v} \cdot \vec{\nabla} \delta P} + \frac{\partial \delta P}{\partial t} + \frac{\vec{\nabla} \delta P}{\rho_0} = 0$$

$$\vec{v} = \delta\vec{v}$$

$$\frac{\partial^2 \delta\rho}{\partial t^2} + \rho_0 \vec{\nabla} \cdot \left[-\frac{\vec{\nabla} \delta P}{\rho_0} \right] = 0$$

$$\Rightarrow \delta\rho'' = c_s^2 \nabla^2 \delta\rho$$

$$c_s^2 \equiv \frac{\delta P}{\delta \rho} = \text{const.}$$

e.g. $P \propto \rho^\Gamma$

Sound Waves ; ignore visc.

assume

$$\rho = \rho_0 + \delta\rho(x,t) \quad \cancel{\delta\vec{v} \cdot \vec{\nabla}} \delta\rho + \frac{\partial \delta\rho}{\partial t} + \rho_0 \vec{\nabla} \cdot \delta\vec{v} = 0$$

$$c_s^2 \equiv \frac{\delta P}{\delta \rho} = \text{const.}$$

$$P = P_0 + \delta P(x,t) \quad \cancel{\delta\vec{v} \cdot \vec{\nabla}} \delta\vec{v} + \frac{\partial \delta\vec{v}}{\partial t} + \frac{\vec{\nabla} \delta P}{\rho_0} = 0$$

$$\vec{v} = \delta\vec{v}$$

$$\frac{\partial^2 \delta\rho}{\partial t^2} + \rho_0 \vec{\nabla} \cdot \left[-\frac{\vec{\nabla} \delta P}{\rho_0} \right] = 0$$

$$\Rightarrow \delta\ddot{\rho} = c_s^2 \nabla^2 \delta\rho$$

e.g. $P \propto \rho^\Gamma$
 $c_s^2 = \Gamma \frac{P}{\rho} = \Gamma \frac{T}{m}$

ideal

CAUTION

Sound Waves ; ignore visc.

assume

$$\rho = \rho_0 + \delta \rho(x,t) \quad \cancel{\delta \vec{v} \cdot \vec{\nabla} \delta \rho} + \frac{\partial \delta \rho}{\partial t} + \rho_0 \vec{\nabla} \cdot \delta \vec{v} = 0$$

$$C_s^2 \equiv \frac{\delta P}{\delta \rho} = \text{const.}$$

$$P = P_0 + \delta P(x,t) \quad \cancel{\delta \vec{v} \cdot \vec{\nabla} \delta P} + \frac{\partial \delta P}{\partial t} + \frac{\vec{\nabla} \delta P \cdot \delta \vec{v}}{\rho_0} = 0$$

$$\vec{v} = \delta \vec{v}$$

$$\frac{\partial^2 \delta \rho}{\partial t^2} + \rho_0 \vec{\nabla} \cdot \left[- \frac{\vec{\nabla} \delta P}{\rho_0} \right] = 0$$

$$\Rightarrow \delta \ddot{\rho} = C_s^2 \nabla^2 \delta \rho$$

e.g. $P \propto \rho^\Gamma$
 $C_s^2 = \Gamma \frac{P}{\rho} = \Gamma \frac{T}{m}$

ideal gas
 single atom $C_s^2 = \frac{5}{3} \frac{kT}{m}$

