Title: PHYS 781 - Lecture 9

Date: Oct 23, 2014 02:00 PM

URL: http://pirsa.org/14100059

Abstract:

$$\frac{\partial f}{\partial t} + \cdots$$

Cons.

$\int d^3v$ (Boltzmann eq.) $\longrightarrow$ Continuity eq

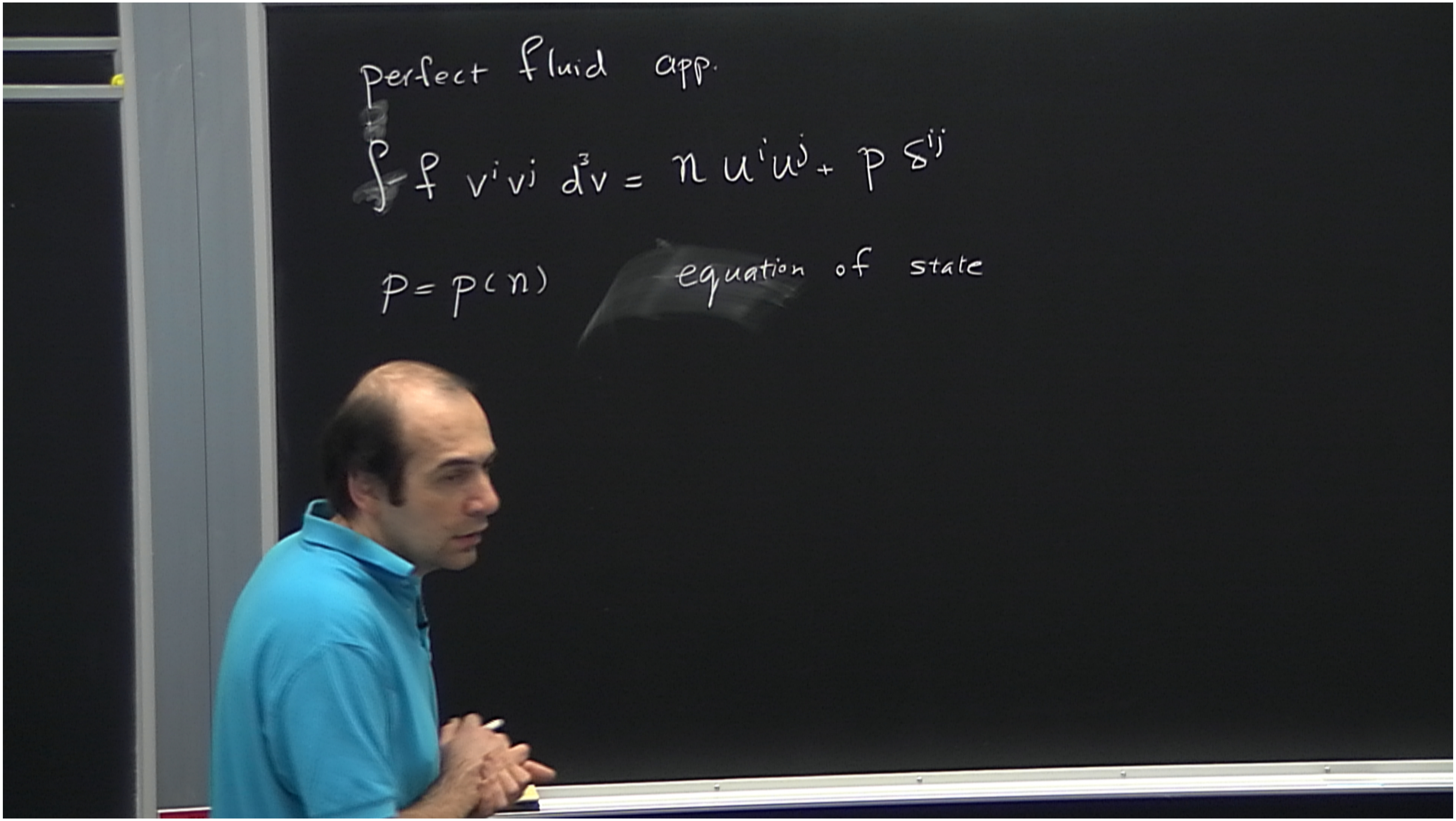
$\int d^3v \, v^i$ (" ") $\longrightarrow$ Euler/Jeans eq

Navier-Stokes

$\int d^3v \, v^i v^j$ (" ") Momentum cons.

$\longrightarrow$ Energy Cons.

$\vdots$


perfect fluid app.
$\int f\, v^i v^j\, d^3v = n\, u^i u^j + p\, \delta^{ij}$
$p = p(n)$
equation of state

perfect fluid app.

$$\int f\, v^i v^j\, d^3v = n\, u^i u^j + p\, \delta^{ij}$$

$p = p(n)$ equation of state

$p \propto n^{5/3}$ N.R. degenerate gas

$P = nkT$ ideal gas

perfect fluid app.

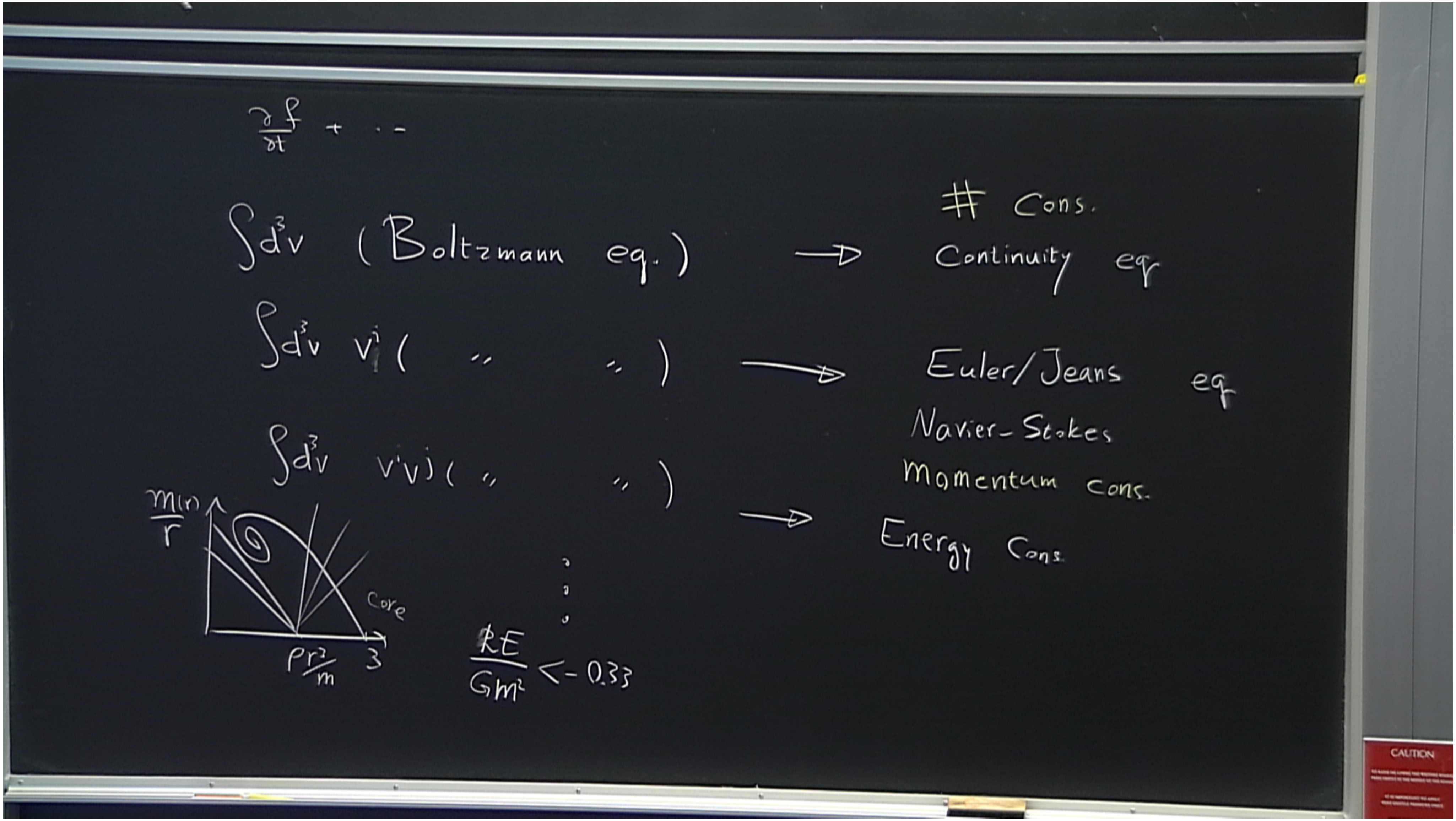
$$\int f \, v^i v^j \, d^3v = n u^i u^j + p \, \delta^{ij}$$

$p = p(n)$ equation of state

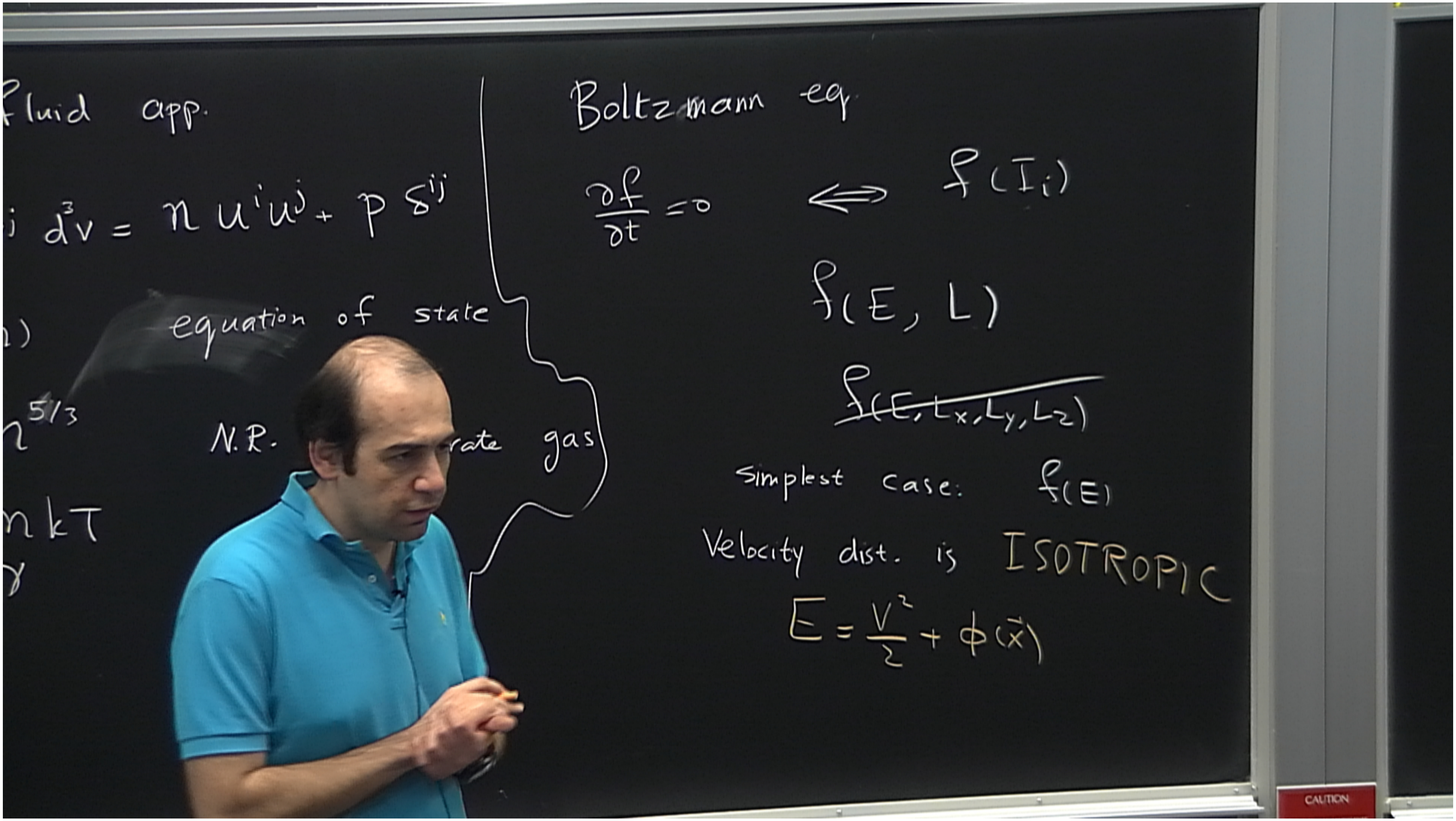
$P \propto n^{5/3}$ N.R. degenerate gas

$P = nkT$ ideal gas

$P \propto n^{\gamma}$



$\frac{\partial f}{\partial t} + \ldots$
Cons.
$\int d^3v$ (Boltzmann eq.) → Continuity eq
$\int d^3v\, v^i$ (" ") → Euler/Jeans eq
Navier-Stokes
$\int d^3v\, v^i v^j$ (" ")
Momentum cons.
→ Energy Cons
$\frac{m(r)}{r}$
Core
$\rho r^3/m$
3
$\frac{KE}{Gm^2} < -0.33$



fluid app.
$d^3v = n u^i u^j + p \delta^{ij}$
equation of state
$n^{5/3}$
N.R.
gas
nkT
Boltzmann eq.
$\frac{\partial f}{\partial t} = 0 \iff f(I_i)$
$f(E, L)$
$\cancel{f(E, L_x, L_y, L_z)}$
simplest case: $f(E)$
Velocity dist. is ISOTROPIC
$E = \frac{V^2}{2} + \phi(\vec{x})$
CAUTION

fluid app.

$\ldots\, d^3v = n u^i u^j + p\,\delta^{ij}$

$\ldots)$ equation of state

$n^{5/3}$ N.R. degenerate gas

$n k T$ ideal gas

γ

Boltzmann eq.

$$\frac{\partial f}{\partial t} = 0 \iff f(I_i)$$

$f(E, L)$ non-isotropic $L = r v_\perp$

$\cancel{f(E, L_x, L_y, L_z)}$

simplest case: $f(E)$

Velocity dist. is ISOTROPIC

$$E = \frac{v^2}{2} + \phi(\vec{x})$$

$$\rho = \int f(E)\,\frac{d^3v}{4\pi r^2 dv}$$

$$\nabla^2\phi = \frac{1}{r}\frac{d}{dr^2}(r\phi) = 4\pi G\rho = 16\pi^2 G\int_{-|\phi|}^{0} f(E)\sqrt{2(E-\phi)}\;dE$$

$$\phi(r) \longrightarrow \rho(r) \longrightarrow f(E)$$

Abel's integral: $$f(E) = \frac{1}{\sqrt{8}\,\pi^2}\frac{d}{dE}\int \frac{\left(\frac{d\rho}{dr}\right)dr}{\sqrt{E-\phi(r)}}$$

$$\rho = \int f(E)\, d^3v = \int f(E)\, 4\pi v^2 dv$$

$$\nabla^2 \phi = \frac{1}{r} \frac{d}{dr^2}(r\phi) = 4\pi G \rho = 16\pi^2 G \int_{-|\phi|}^{0} f(E) \sqrt{2(E-\phi)}\, dE$$

$$\phi(r) \longrightarrow \rho(r) \longrightarrow f(E)$$

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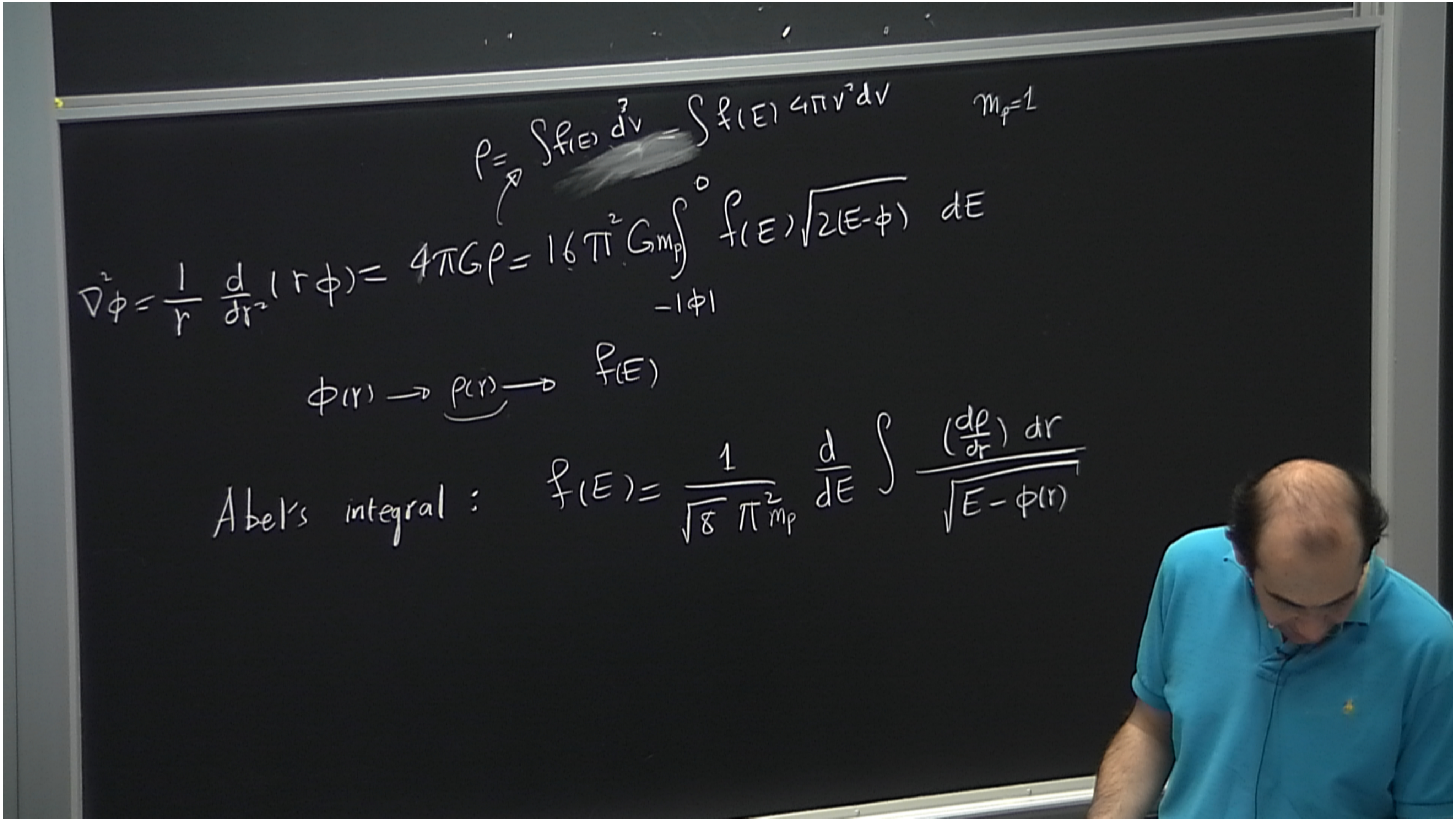
$\rho = \int f(E)\, d^3v = \int f(E)\, 4\pi v^2 dv$
$m_p = 1$
$\nabla^2\phi = \frac{1}{r}\frac{d}{dr^2}(r\phi) = 4\pi G\rho = 16\pi^2 G m_p \int_{-|\phi|}^{0} f(E)\sqrt{2(E-\phi)}\, dE$
$\phi(r) \longrightarrow \rho(r) \longrightarrow f(E)$
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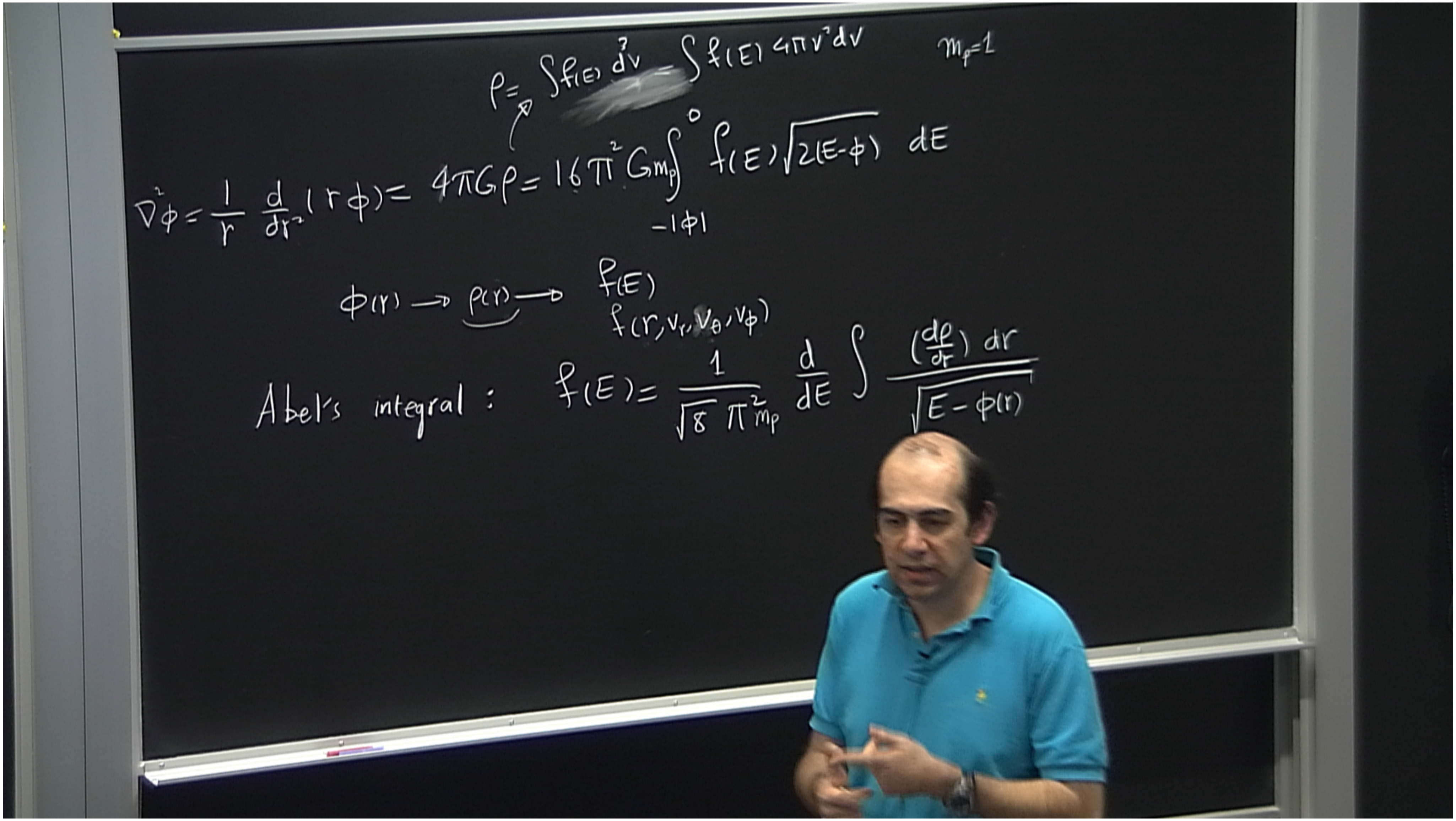
$\rho = \int f(E)\, d^3v = \int f(E)\, 4\pi v^2 dv$
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$\phi(r) \rightarrow \rho(r) \rightarrow f(E)$
$f(r, v_r, v_\theta, v_\phi)$
Abel's integral: $f(E) = \frac{1}{\sqrt{8}\,\pi^2 m_p}\frac{d}{dE}\int \frac{\left(\frac{d\rho}{dr}\right) dr}{\sqrt{E-\phi(r)}}$

$$\rho = \int f(E)\, d^3v = \int f(E)\, 4\pi v^2 dv \qquad m_p = 1$$

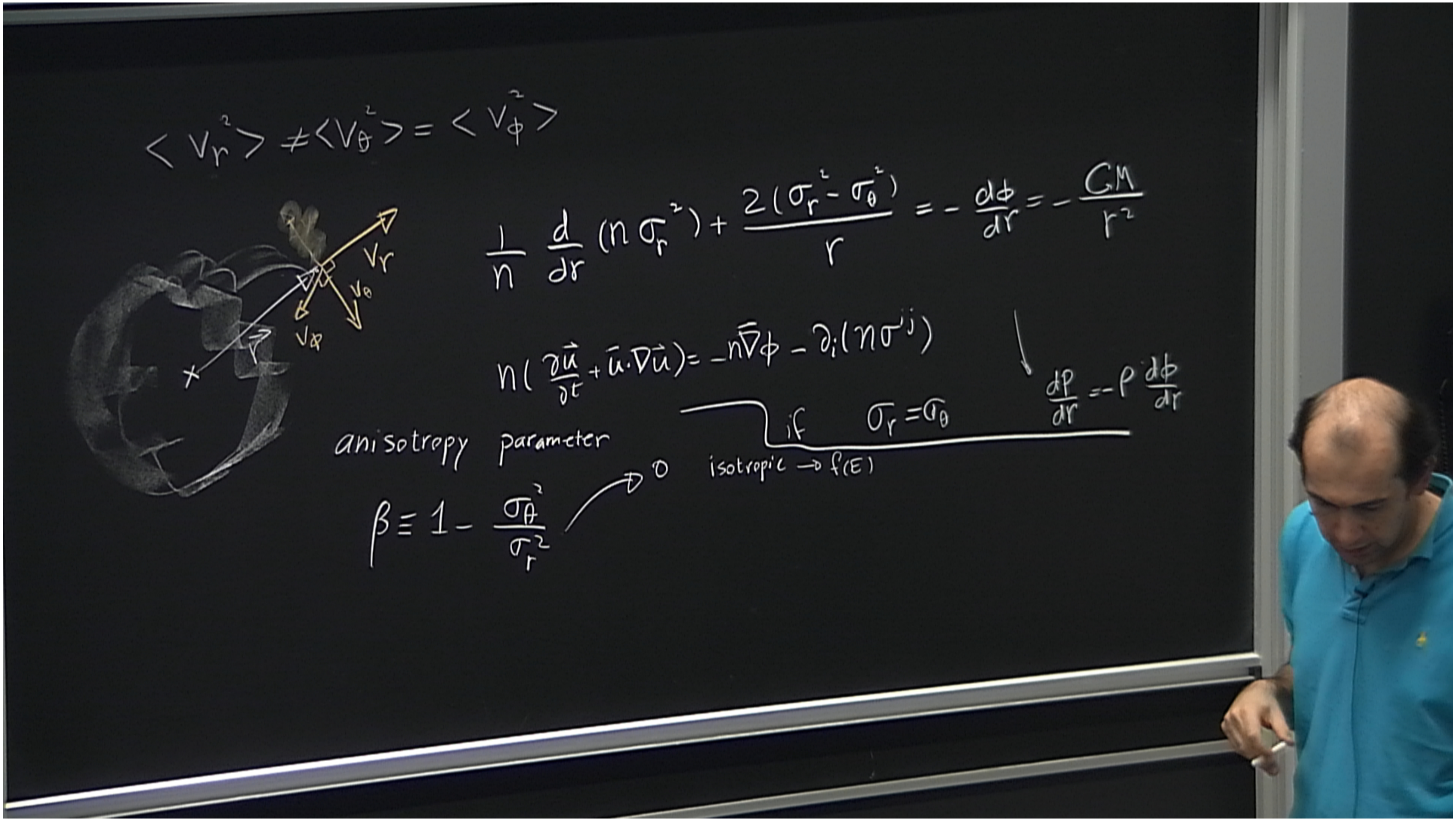
$$\nabla^2\phi = \frac{1}{r}\frac{d}{dr^2}(r\phi) = 4\pi G\rho = 16\pi^2 G m_p \int_{-|\phi|}^{0} f(E)\sqrt{2(E-\phi)}\, dE$$

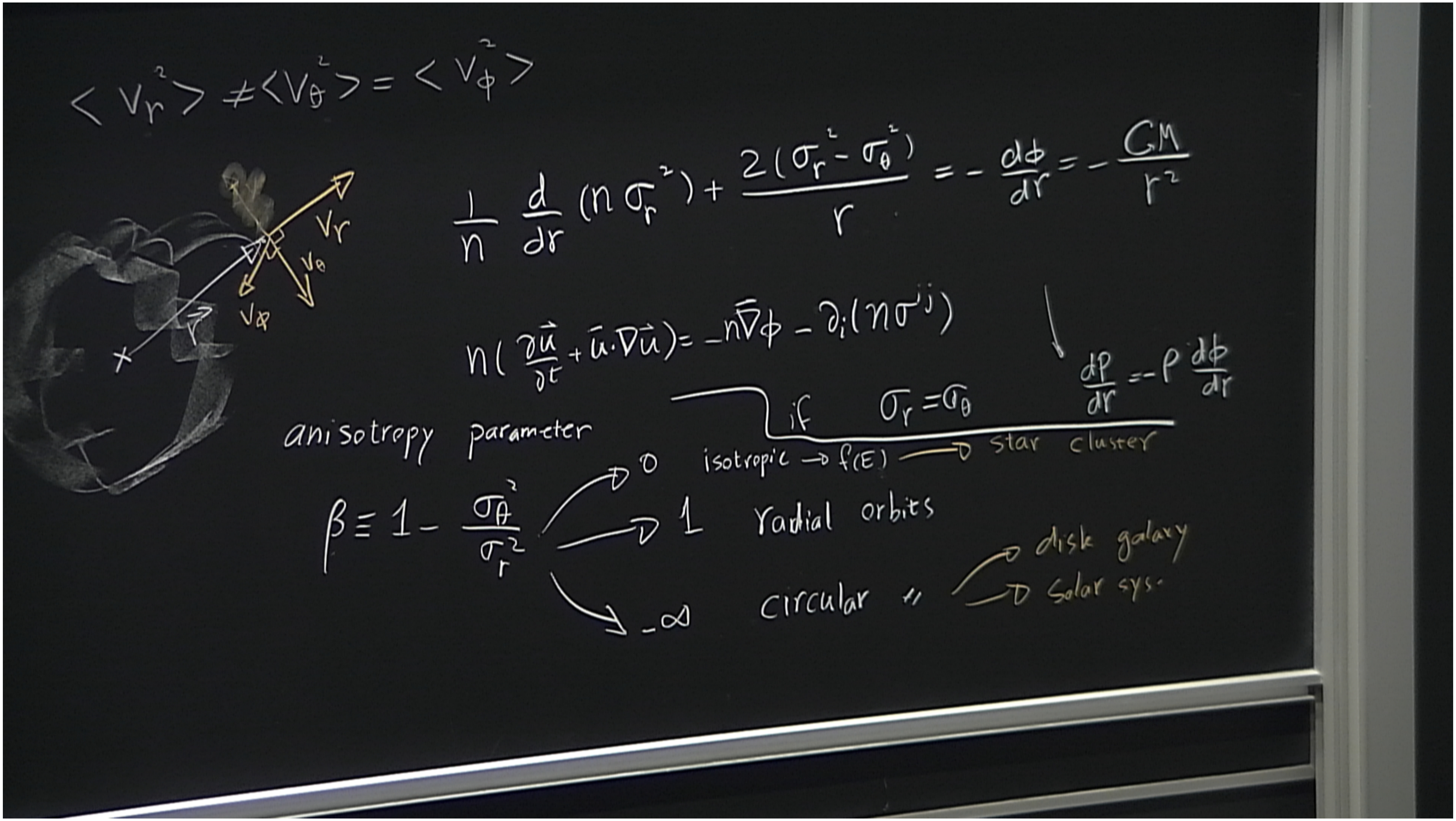
$$\phi(r) \longrightarrow \underbrace{\rho(r)}\longrightarrow \begin{matrix} f(E) \\ f(r, v_r, v_\theta, v_\phi) \end{matrix}$$

Abel's integral: $$f(E) = \frac{1}{\sqrt{8}\,\pi^2 m_p}\frac{d}{dE}\int \frac{\left(\frac{d\rho(r)}{dr}\right) dr}{\sqrt{E-\phi(r)}}$$

$$v^2 \sim \frac{GM}{R}$$

$$\langle K \rangle = -\frac{1}{2}\langle U \rangle$$


$\langle v_r^2 \rangle \neq \langle v_\theta^2 \rangle = \langle v_\phi^2 \rangle$
v_r
v_θ
v_ϕ
$\vec{r}$
$\frac{1}{n}\frac{d}{dr}(n\sigma_r^2) + \frac{2(\sigma_r^2 - \sigma_\theta^2)}{r} = -\frac{d\phi}{dr} = -\frac{GM}{r^2}$
$n\left(\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\nabla\vec{u}\right) = -n\vec{\nabla}\phi - \partial_i(n\sigma^{ij})$
$\frac{dP}{dr} = -\rho\frac{d\phi}{dr}$
if $\sigma_r = \sigma_\theta$
anisotropy parameter
$\beta \equiv 1 - \frac{\sigma_\theta^2}{\sigma_r^2} \to 0$
isotropic $\to f(E)$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v_r
v_θ
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$\vec{r}$
$\frac{1}{n}\frac{d}{dr}(n\sigma_r^2) + \frac{2(\sigma_r^2 - \sigma_\theta^2)}{r} = -\frac{d\Phi}{dr} = -\frac{GM}{r^2}$
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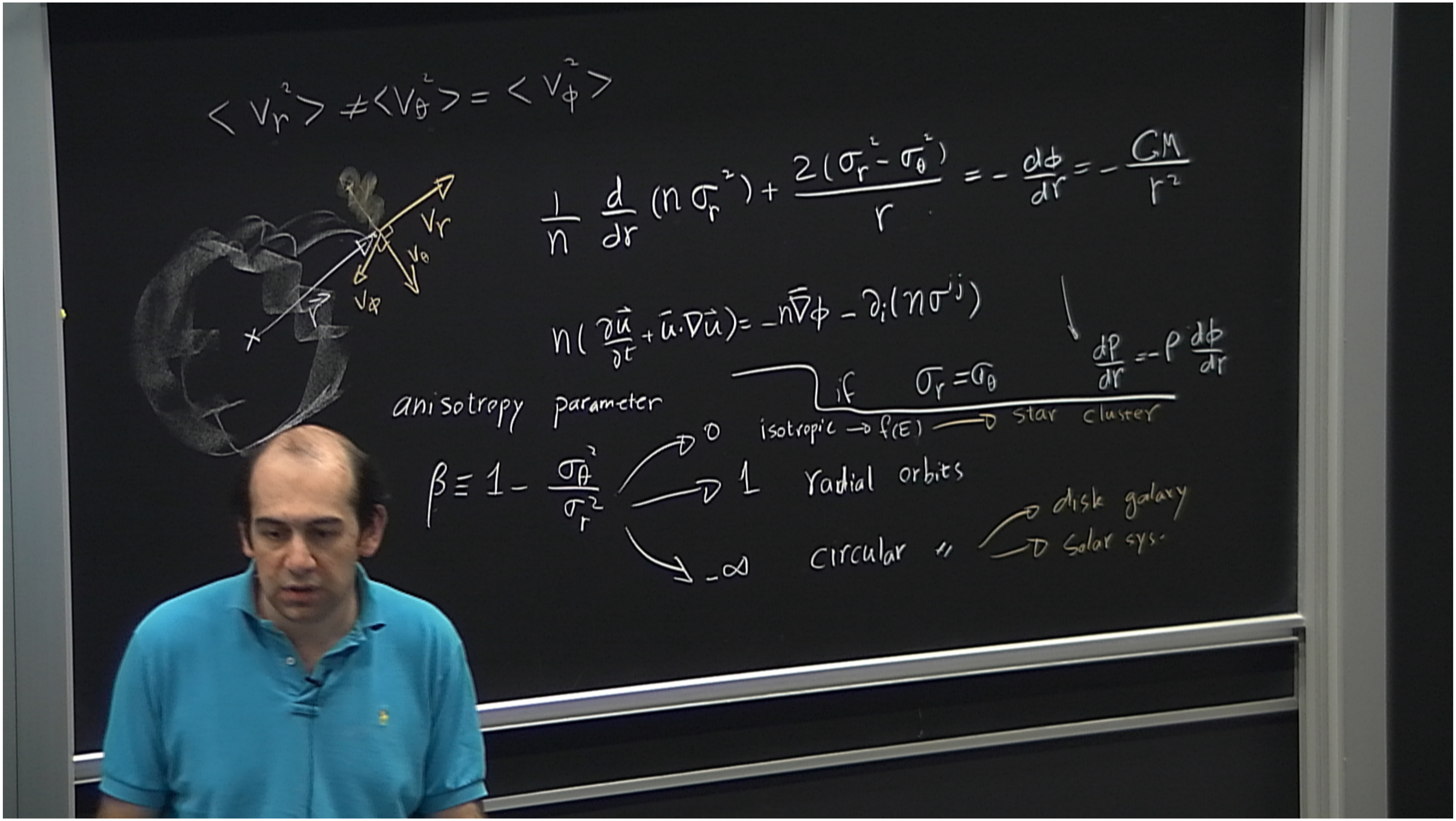
if $\sigma_r = \sigma_\theta$
anisotropy parameter
$\beta \equiv 1 - \frac{\sigma_\theta^2}{\sigma_r^2}$
0 isotropic → f(E) → star cluster
1 radial orbits
$-\infty$ circular " → disk galaxy → Solar sys.


$\langle v_r^2 \rangle \neq \langle v_\theta^2 \rangle = \langle v_\phi^2 \rangle$
v_r
v_θ
v_ϕ
r
$\frac{1}{n}\frac{d}{dr}(n\sigma_r^2) + \frac{2(\sigma_r^2 - \sigma_\theta^2)}{r} = -\frac{d\phi}{dr} = -\frac{GM}{r^2}$
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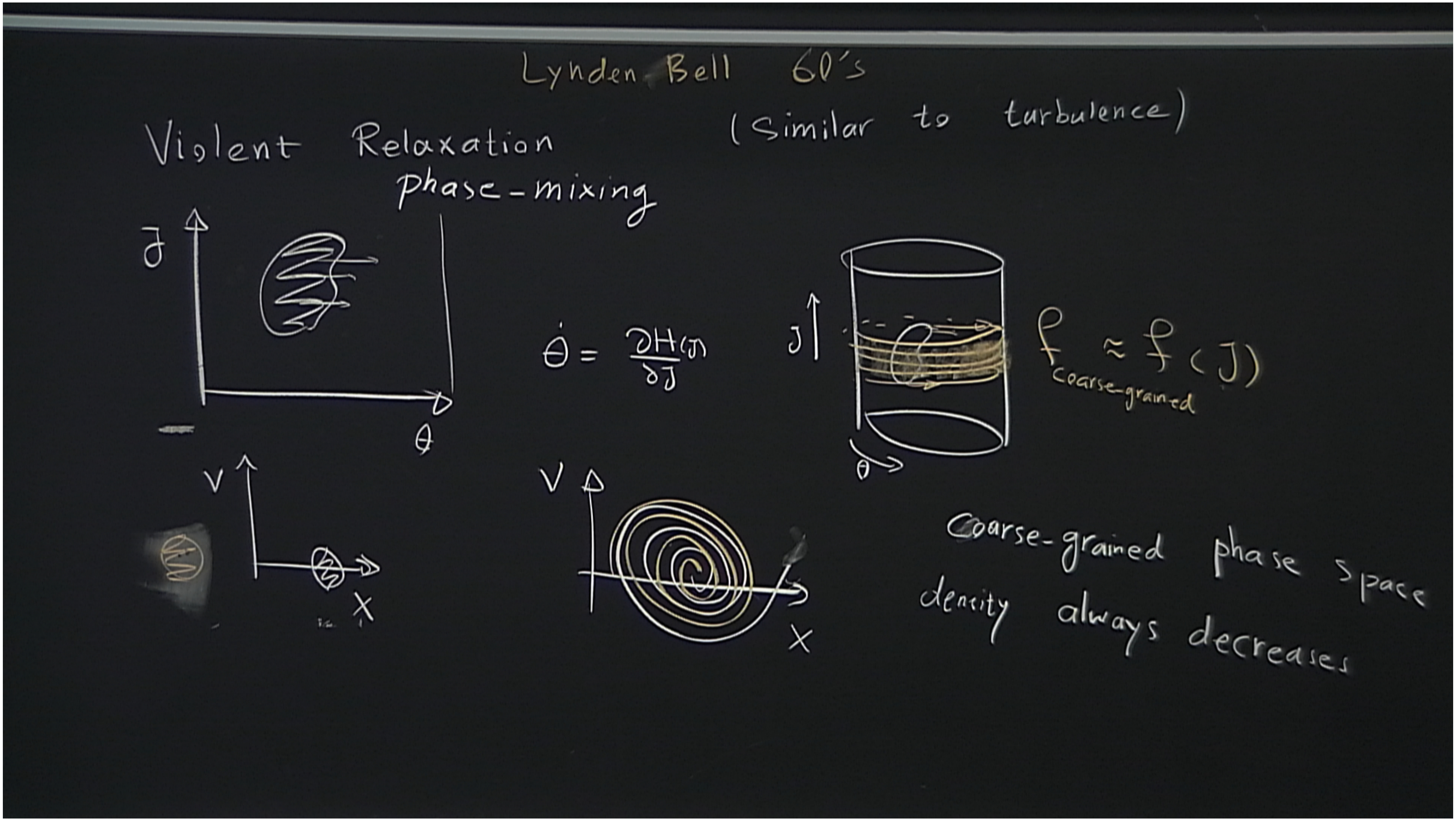


Lynden Bell 60's
Violent Relaxation
(Similar to turbulence)
phase-mixing
J
θ
$\dot{\theta} = \frac{\partial H(J)}{\partial J}$
J
θ
$f_{\text{coarse-grained}} \approx f(J)$
V
X
V
X
Coarse-grained phase space density always decreases

Collisional Relaxation

$$\Delta v_\perp^{kick} \approx \frac{G m_p}{b^2} \times \frac{2b}{v} \sim \frac{2 G m_p}{b v}$$

$$\Delta v_\perp^2 = \#\text{ kicks} \times \left(\frac{2 G m_p}{b v}\right)^2$$

$$= \text{time} \times \int n \times (2\pi b\, db)\, v \times \left(\frac{2 G m_p}{b v}\right)^2$$

$$= \text{time} \times \frac{8\pi^2 G^2 m_p^2}{v}\, n \ln\left(\frac{b_{max}}{b_{min}}\right)$$

Space

CAUTION

$$\text{time} = t_{gc} = \frac{V^3}{8\pi^2 G^2 m_p^2 \, n \ln(b_{max}/b_{min})}$$

$$\Delta V_\perp^2 \sim V^2$$

$$b_{max} \sim R$$

$$b_{min} \sim \frac{G m_p}{V^2}$$

$$\frac{b_{max}}{b_{min}} \sim \frac{R V^2}{G m_p} \sim \frac{M}{m_p} \overbrace{\left(\frac{R V^2}{GM}\right)}^{\sim 1} \sim N$$

$$t_{gc} \sim \underbrace{t_{dyn}}_{\left(\frac{GM}{R^3}\right)^{-1/2}} \times \frac{N}{\ln N}$$

$$\text{time} = t_{gc} = \frac{v^3}{8\pi^2 G^2 m_p^2 \, n \ln(b_{max}/b_{min})}$$

$$\Delta v_\perp^2 \sim v^2$$

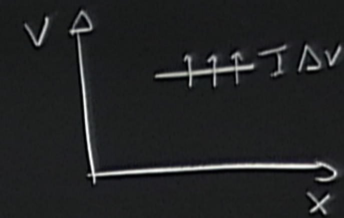
$$b_{max} \sim R$$

$$b_{min} \sim \frac{G m_p}{v^2}$$

$$\frac{b_{max}}{b_{min}} \sim \frac{R v^2}{G m_p} \sim \frac{M}{m_p} \underbrace{\left(\frac{R v^2}{GM}\right)}_{\sim 1} \sim N$$

$$t_{gc} \sim \underbrace{t_{dyn}}_{\left(\frac{GM}{R^3}\right)^{-1/2}} \times \frac{N}{\ln N}$$

$$\frac{\partial f}{\partial t} + v\cdot\frac{\partial f}{\partial x} - \nabla\phi\cdot\frac{\partial f(v)}{\partial \vec{v}} = -\frac{\partial J^i}{\partial v^i}$$



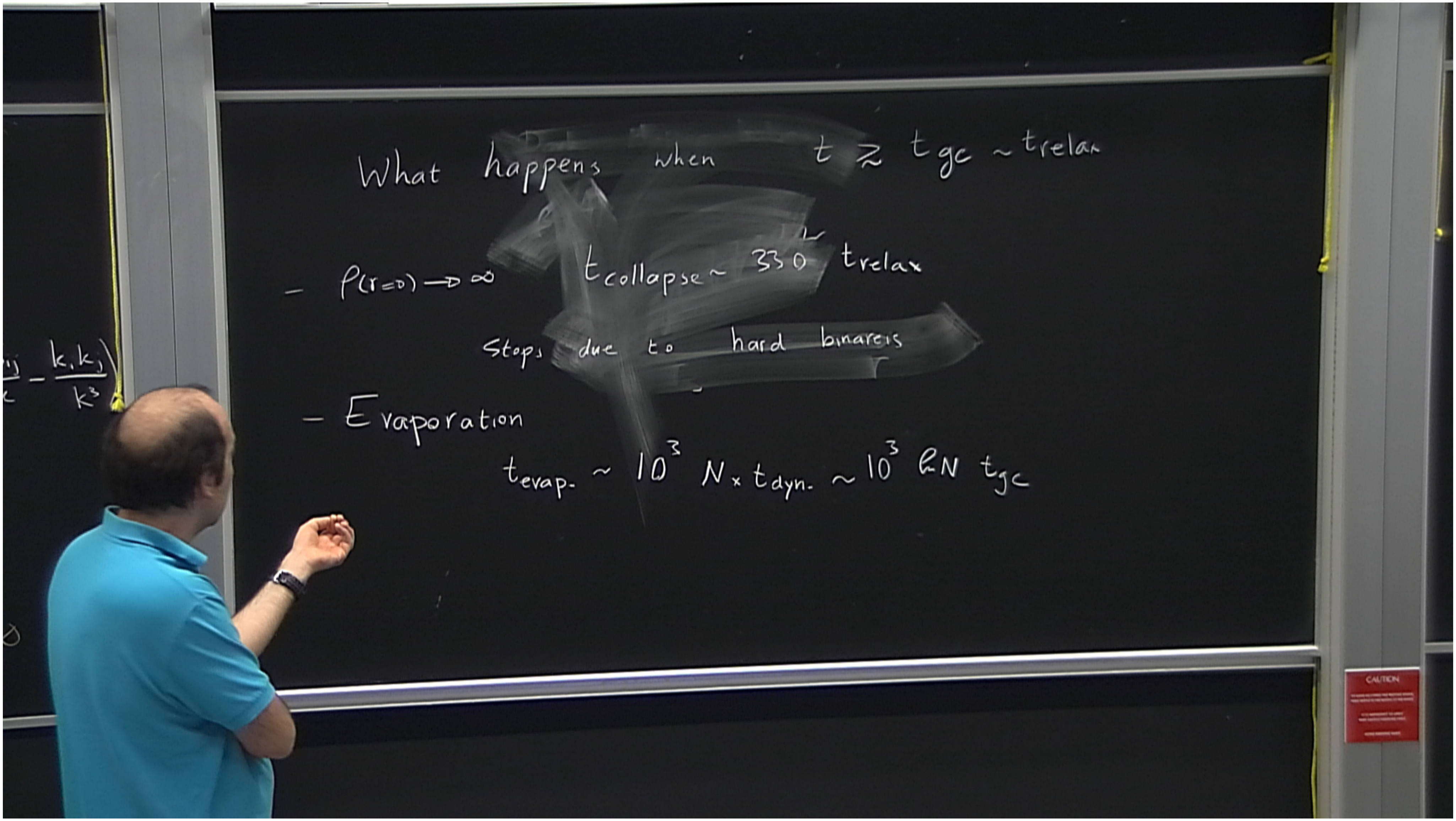
$$J_i = 2\pi G^2 m_p^5 \ln\left(\frac{b_{max}}{b_{min}}\right)\int d^3v'\left\{f(x,v)\frac{\partial f(x,v')}{\partial v'_j} - (\vec{v}\leftrightarrow\vec{v}')\right\}\left(\frac{\delta_{ij}}{k} - \frac{k_i k_j}{k^3}\right)$$

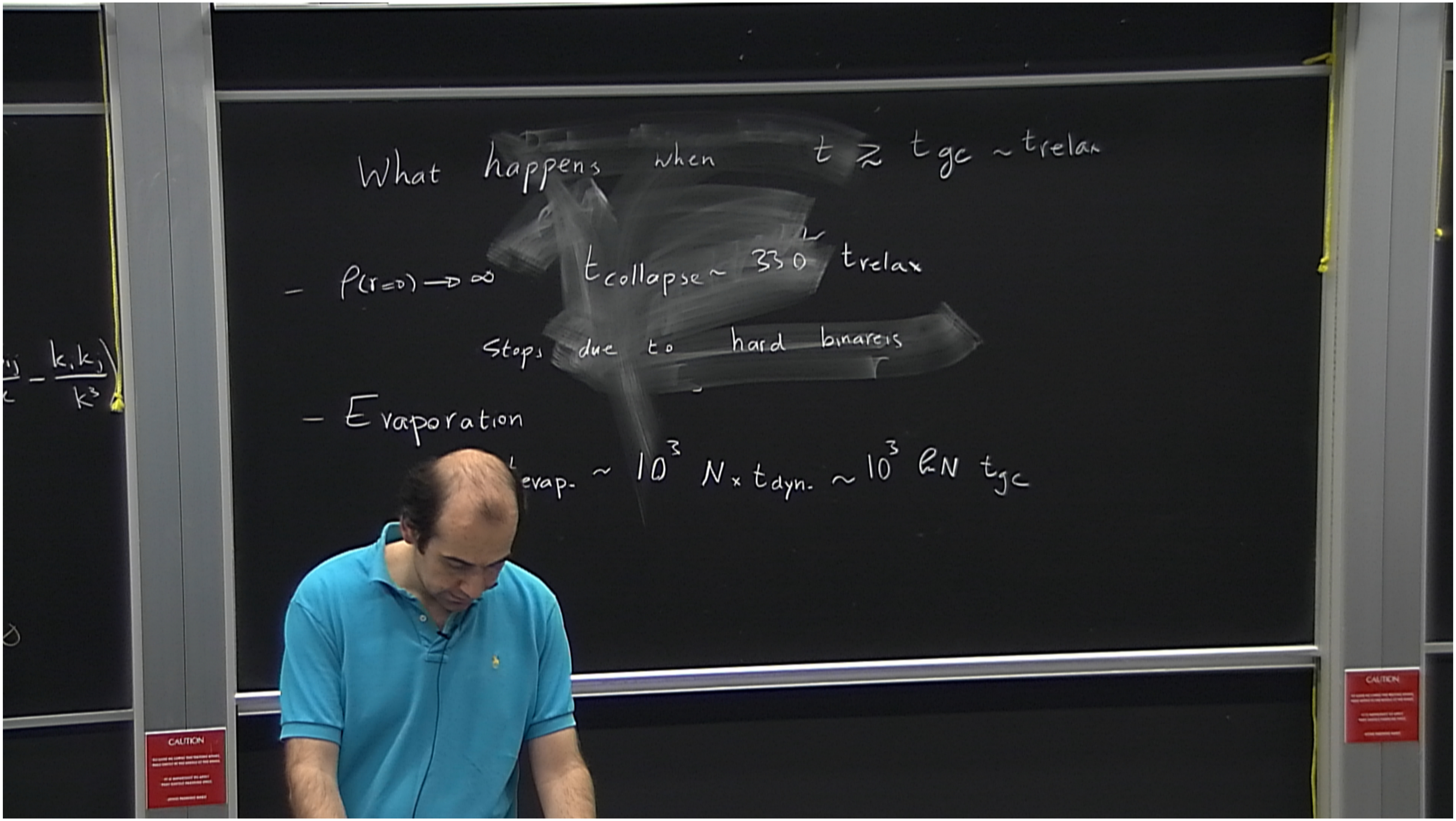
$$\vec{k} = \vec{v} - \vec{v}'$$

$$J_{dift} \propto \nabla n$$

$$f_{th}(v) \propto e^{-\frac{E}{\sigma^2}} = e^{-\frac{v^2}{2\sigma^2} - \frac{\phi(x)}{2\sigma^2}}$$

$$J \propto \int \cancel{d^3v'}\, d^3k\; f_{th}(v)\, f_{th}(v')\, \underbrace{(\vec{v} - \vec{v}')}_{\vec{k}} \left(\frac{\delta_{ij}}{k} - \frac{k_i k_j}{k^3}\right) = 0$$


What happens when t ≳ tgc ~ trelax
- ρ(r=0) → ∞ tcollapse ~ 330 trelax
Stops due to hard binaries
- Evaporation
tevap. ~ 10^3 N × tdyn. ~ 10^3 ln N tgc
CAUTION


What happens when t ≳ t_gc ~ t_relax
- ρ(r=0) → ∞   t_collapse ~ 330 t_relax
Stops due to hard binareis
- Evaporation
t_evap. ~ 10^3 N × t_dyn. ~ 10^3 ln N t_gc
CAUTION
CAUTION