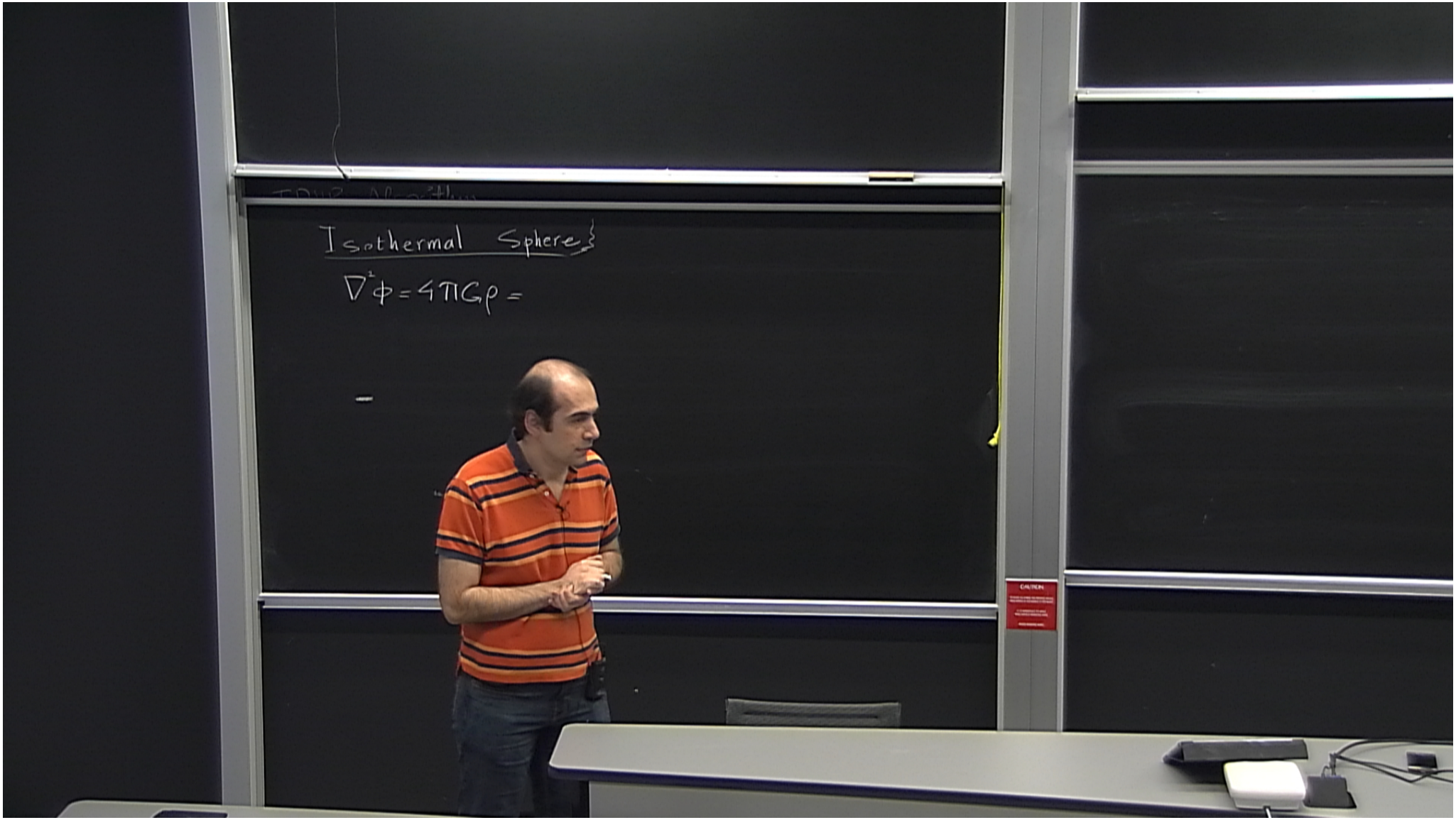
Title: PHYS 781 - Lecture 8

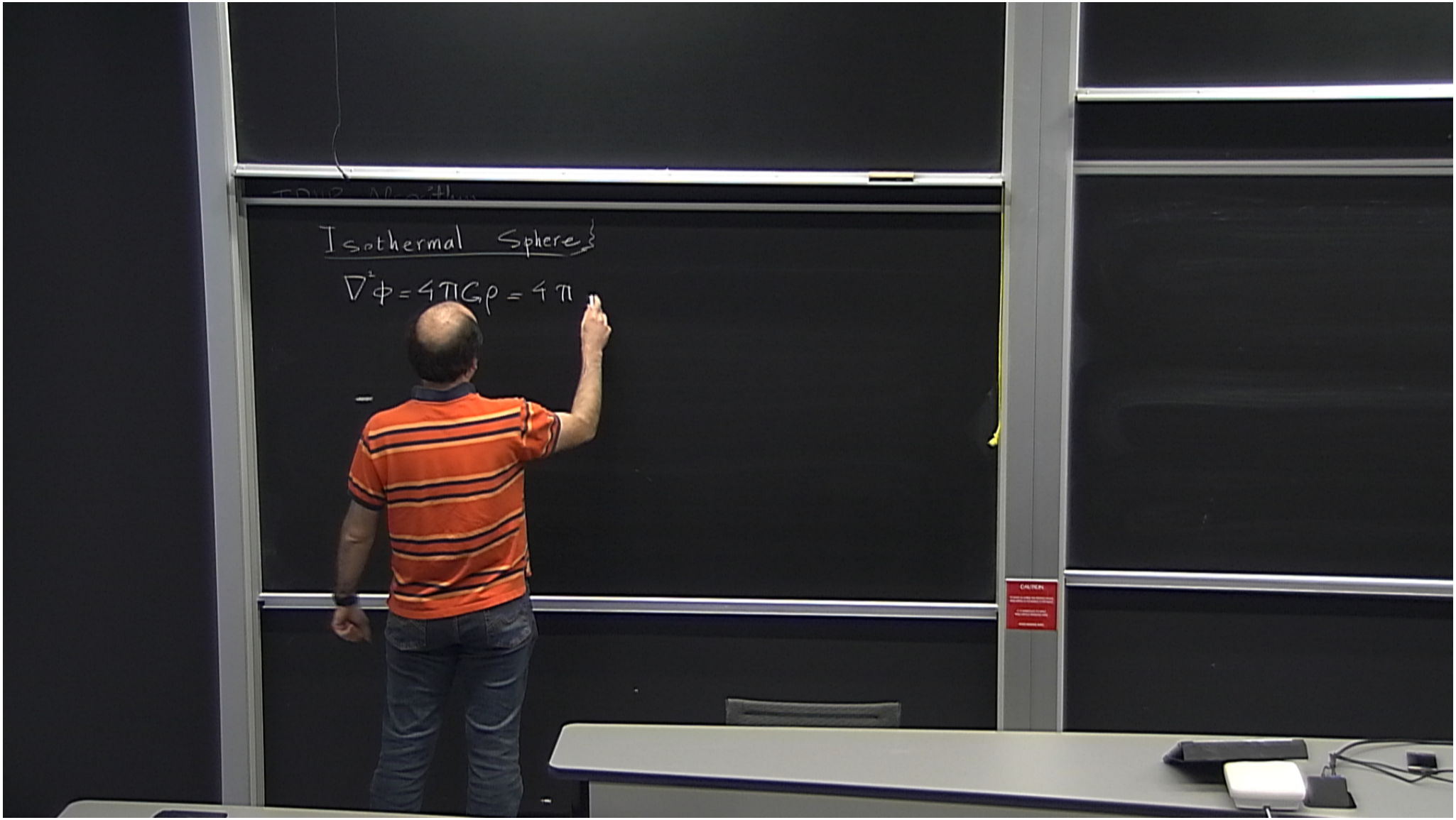
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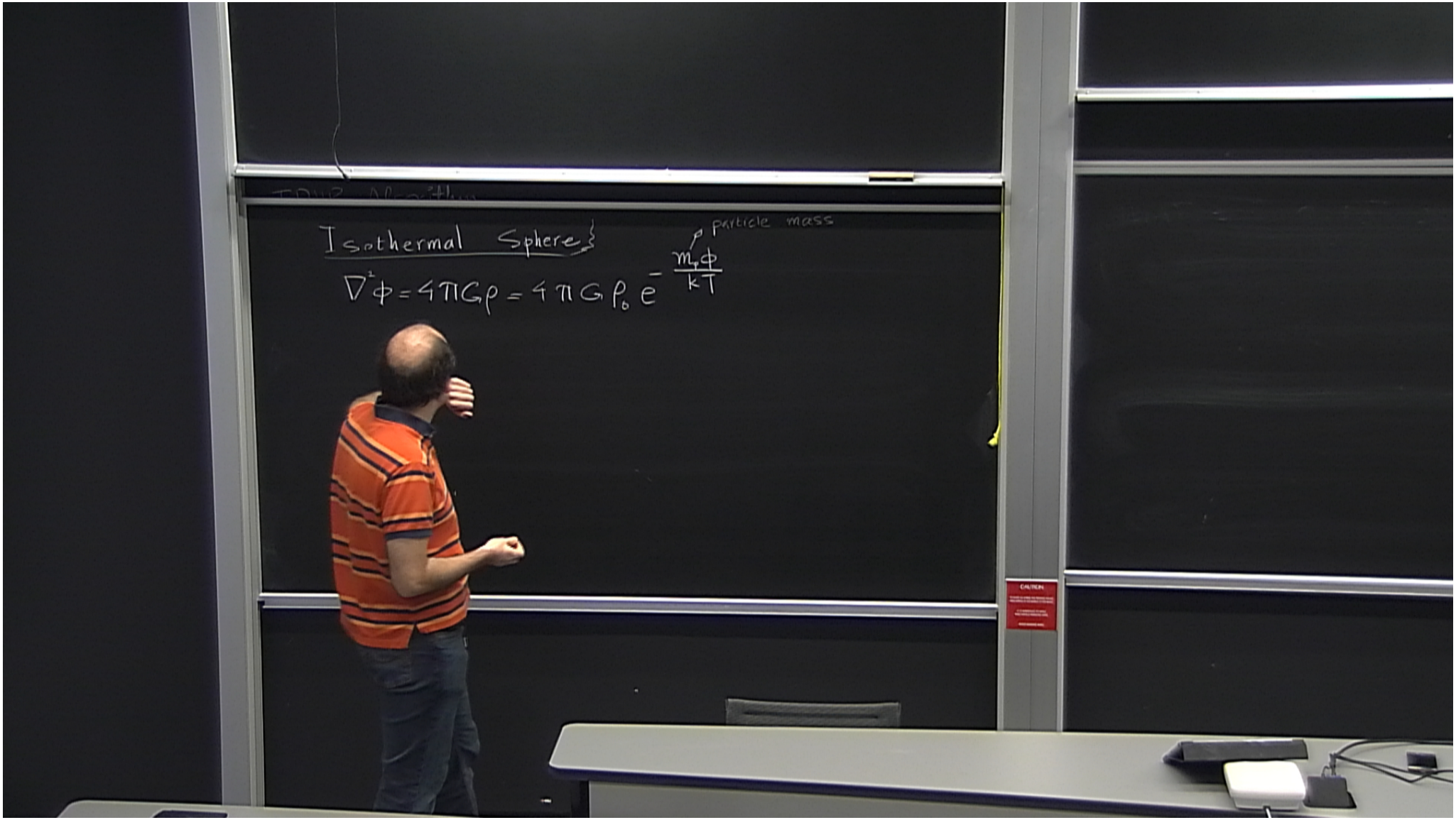
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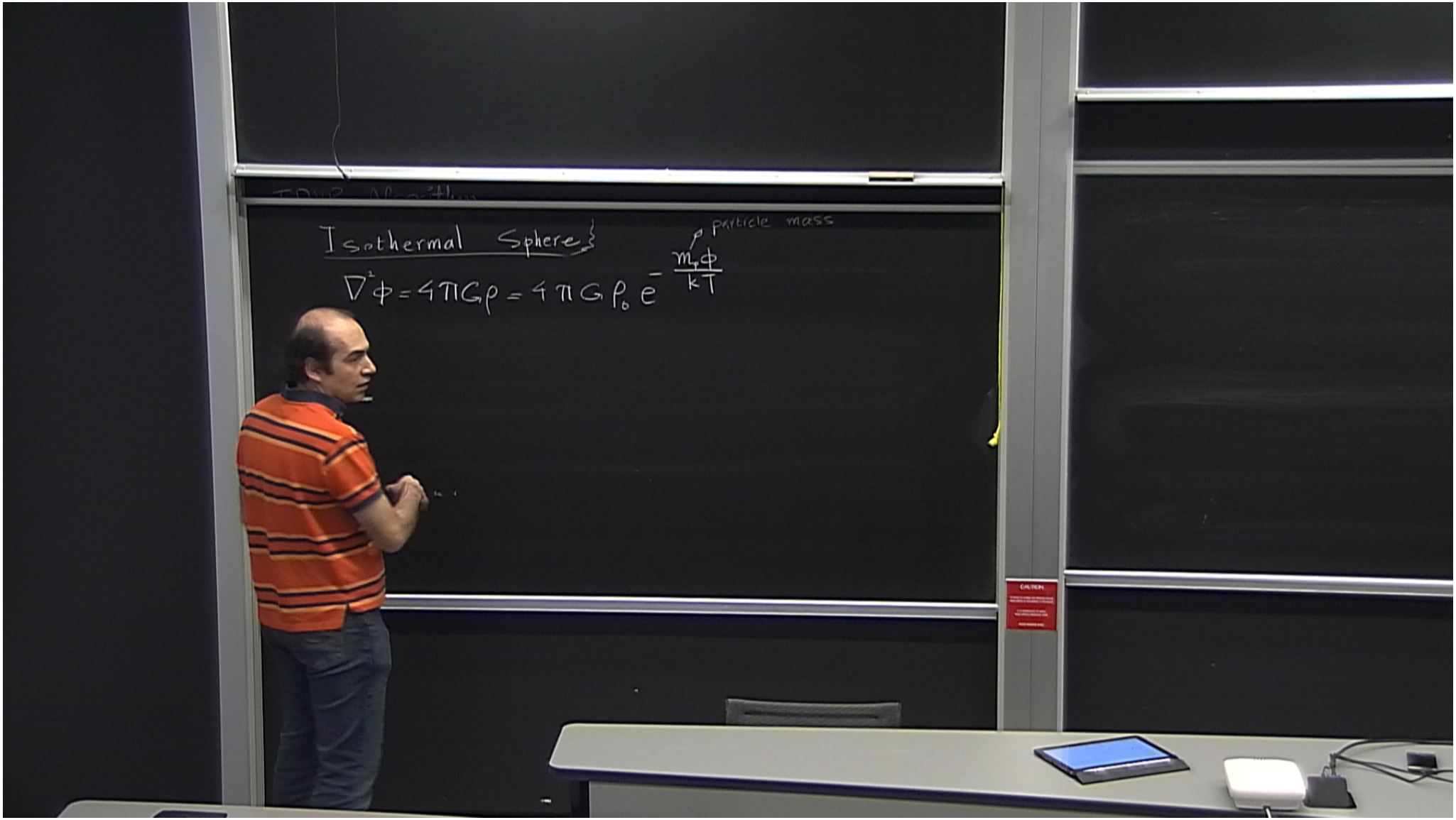
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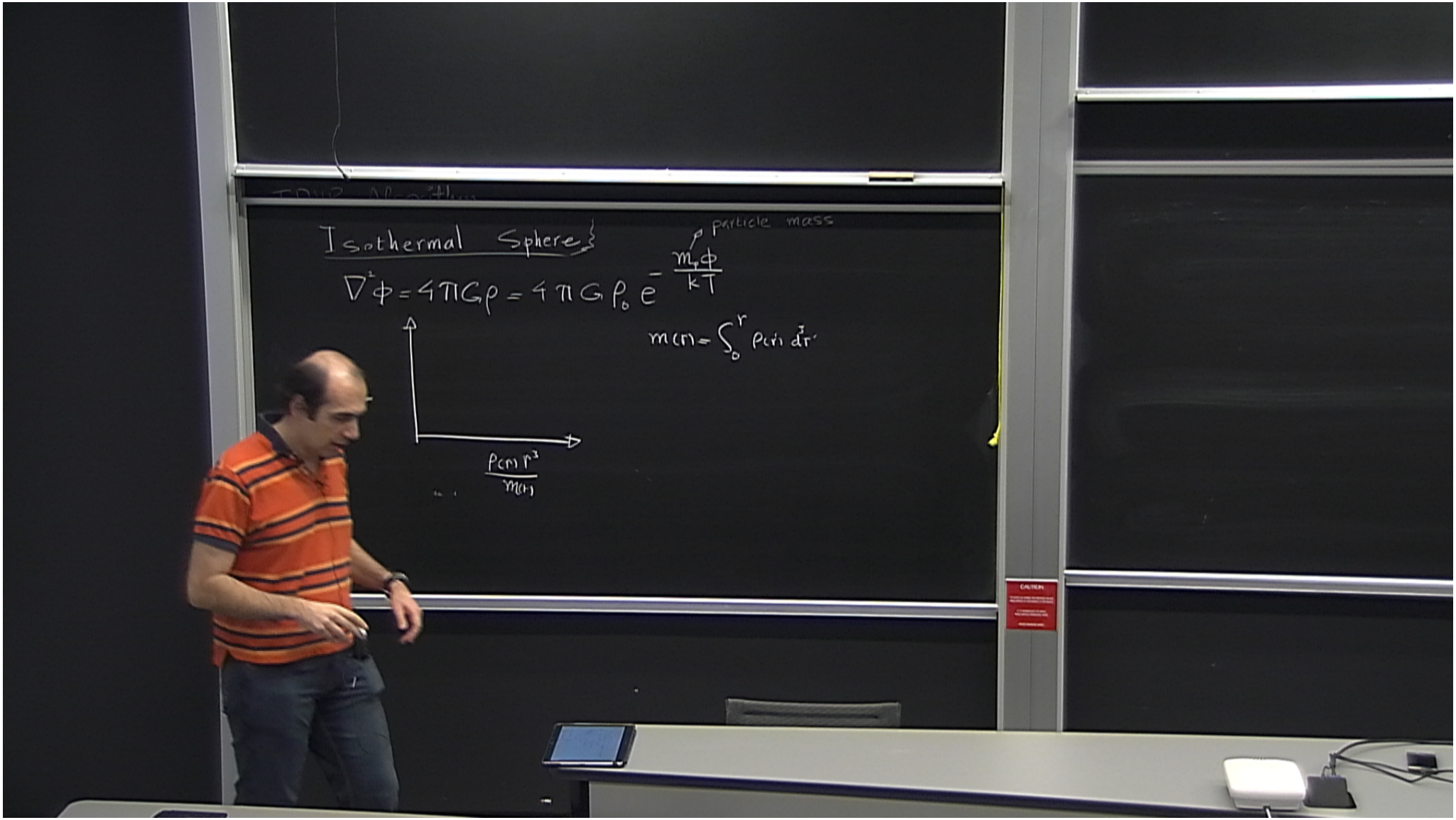

Isothermal Sphere

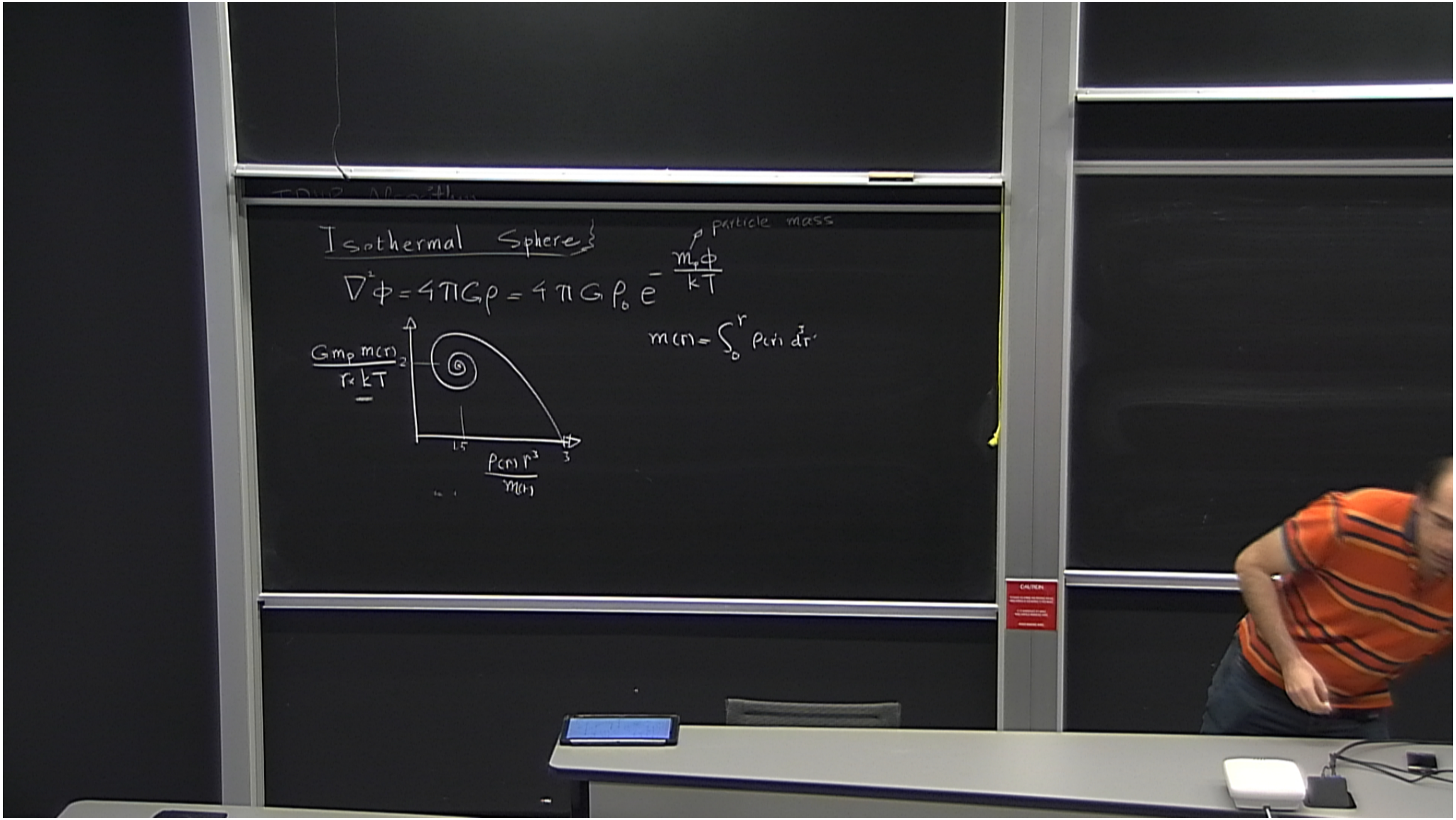

Isothermal Sphere
$\nabla^2\phi = 4\pi G\rho =$

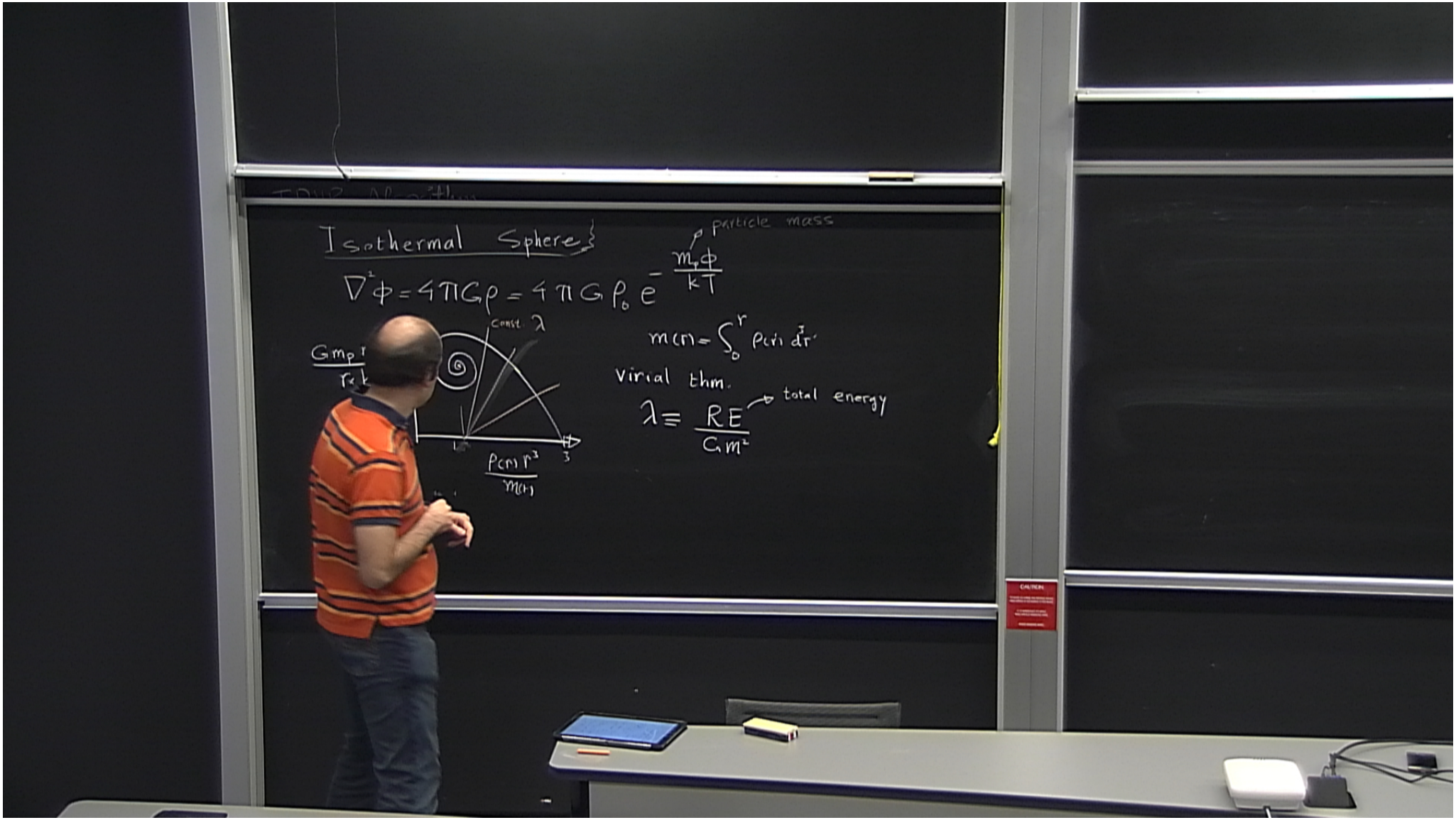

Isothermal Sphere
$\nabla^2\phi = 4\pi G\rho = 4\pi$

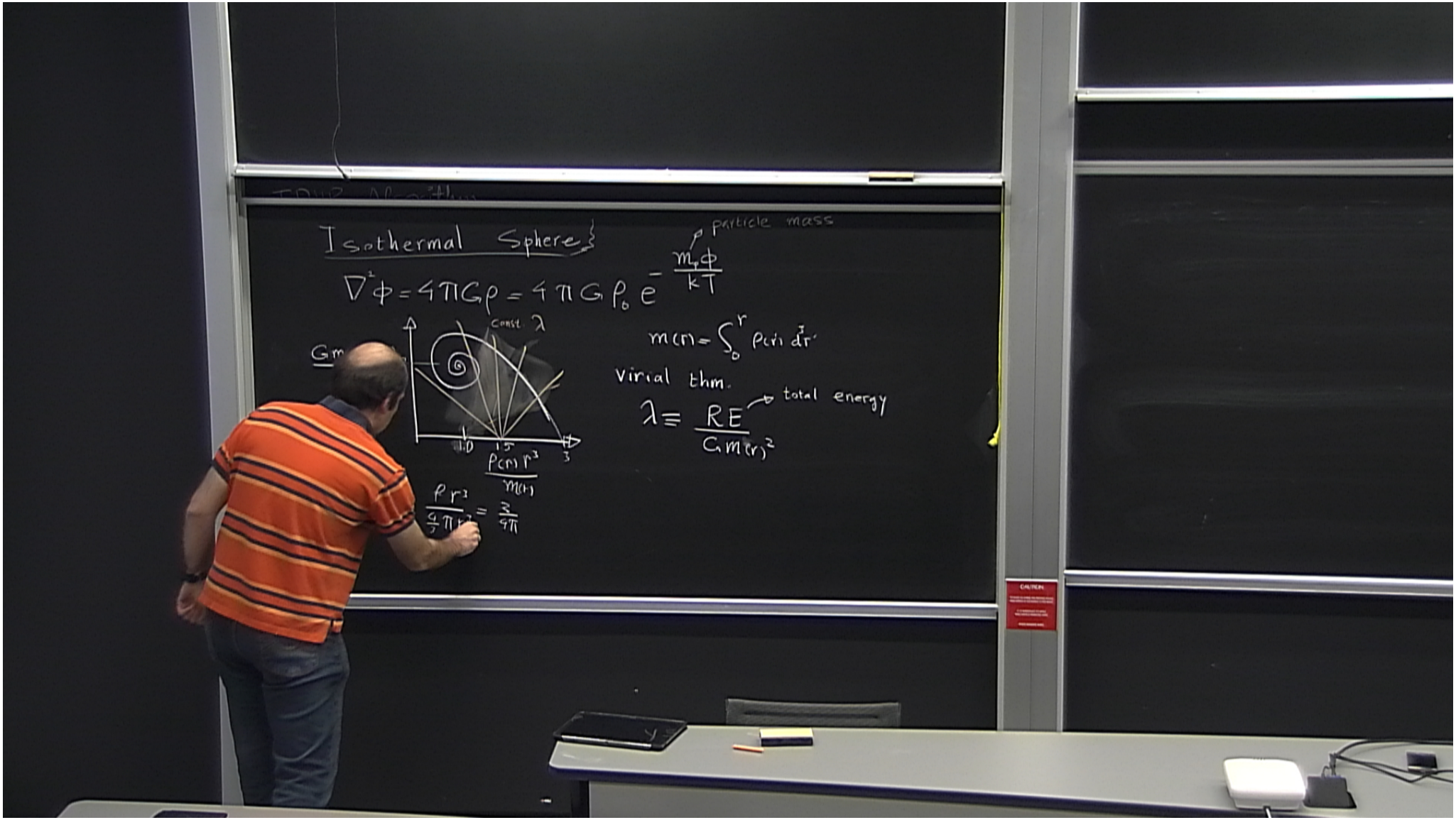

Isothermal Sphere
particle mass
$\nabla^2\phi = 4\pi G\rho = 4\pi G\rho_0 e^{-\frac{m_p\phi}{kT}}$

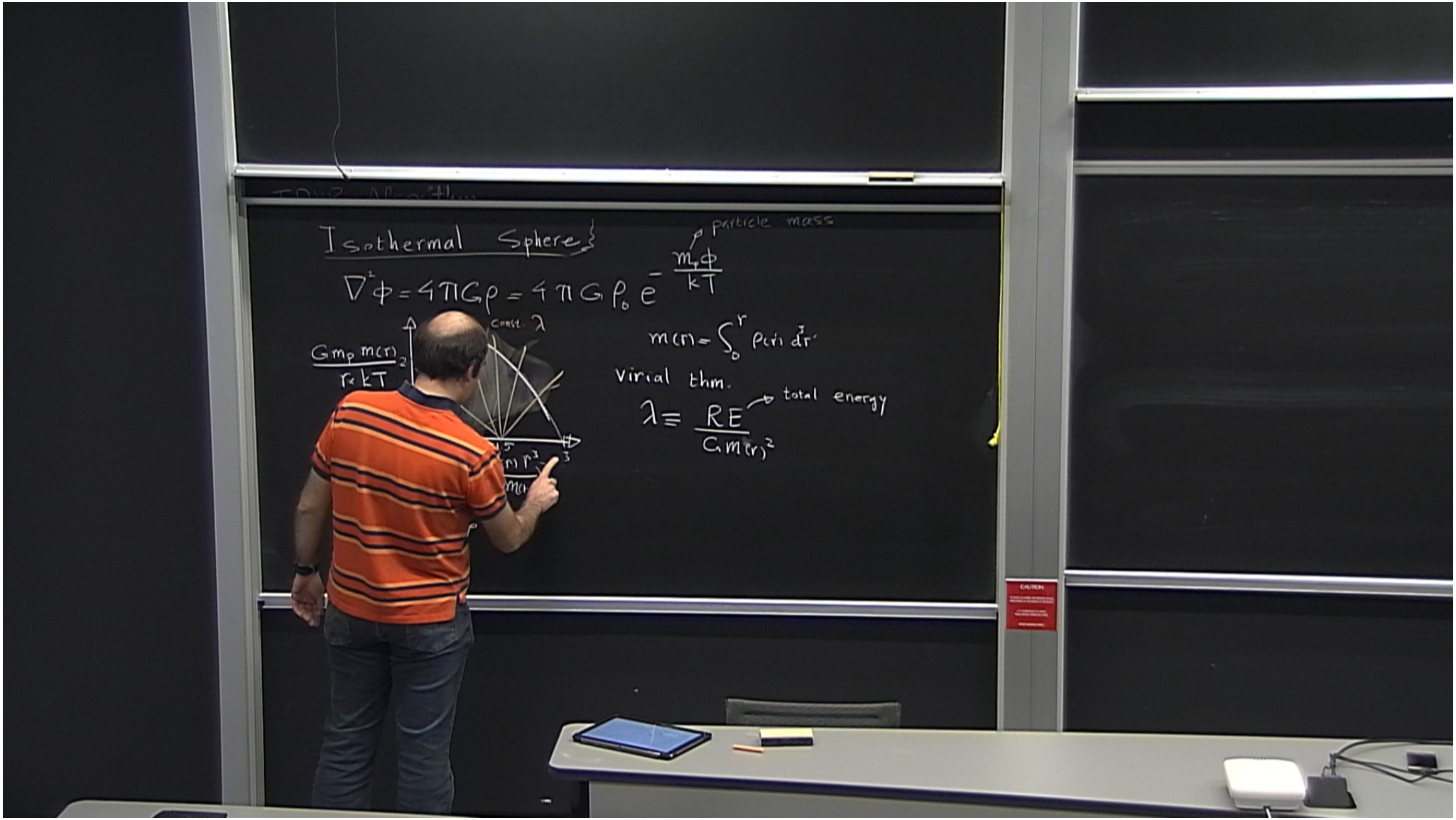

Isothermal Sphere
$\nabla^2\phi = 4\pi G\rho = 4\pi G\rho_0 e^{-\frac{m_p\phi}{kT}}$
particle mass

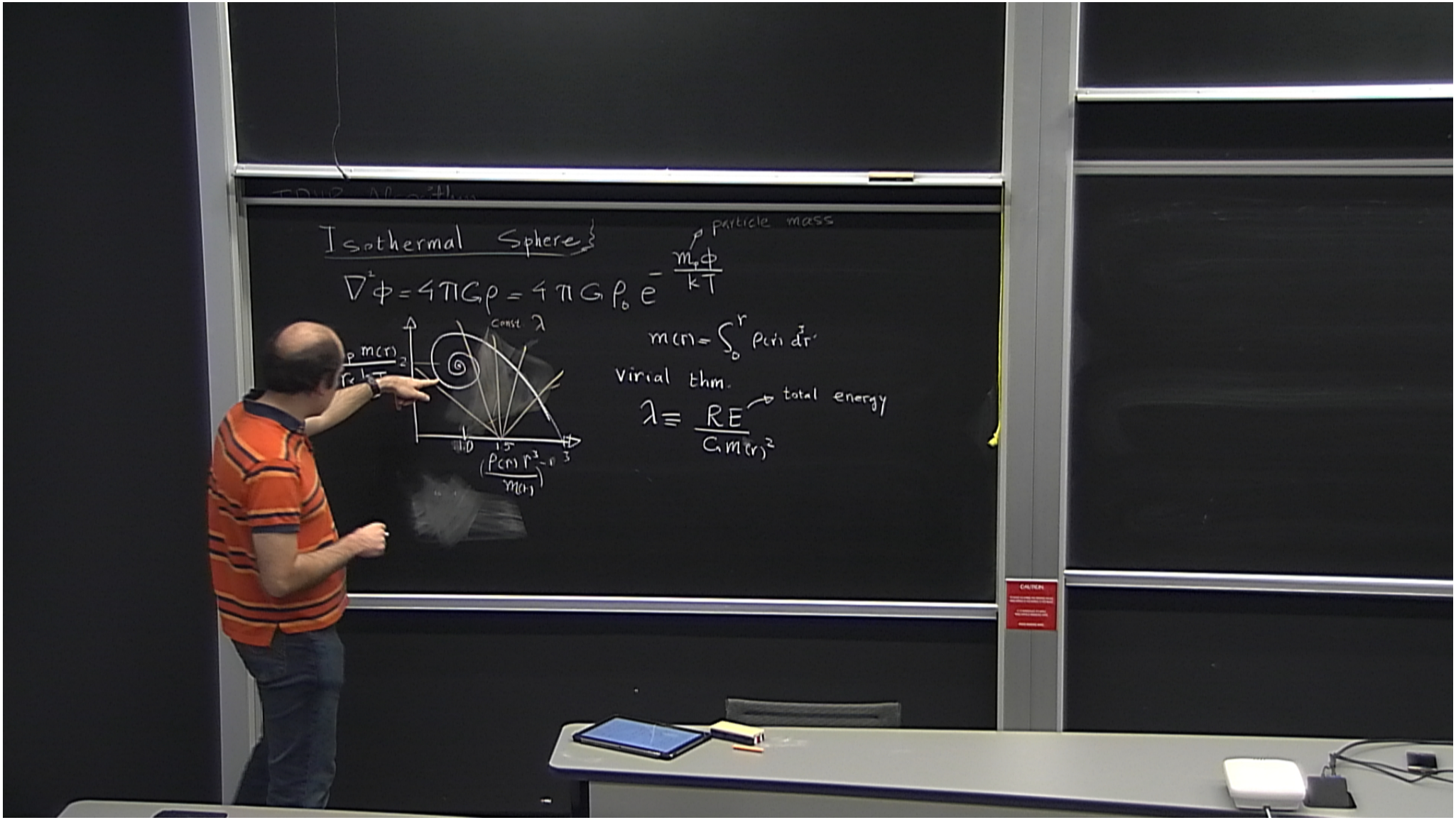

Isothermal Sphere
particle mass
$\nabla^2\phi = 4\pi G\rho = 4\pi G\rho_0 e^{-\frac{m_p\phi}{kT}}$
$m(r) = \int_0^r \rho(r)\, d\vec{r}'$
$\frac{\rho(r)r^2}{m(r)}$

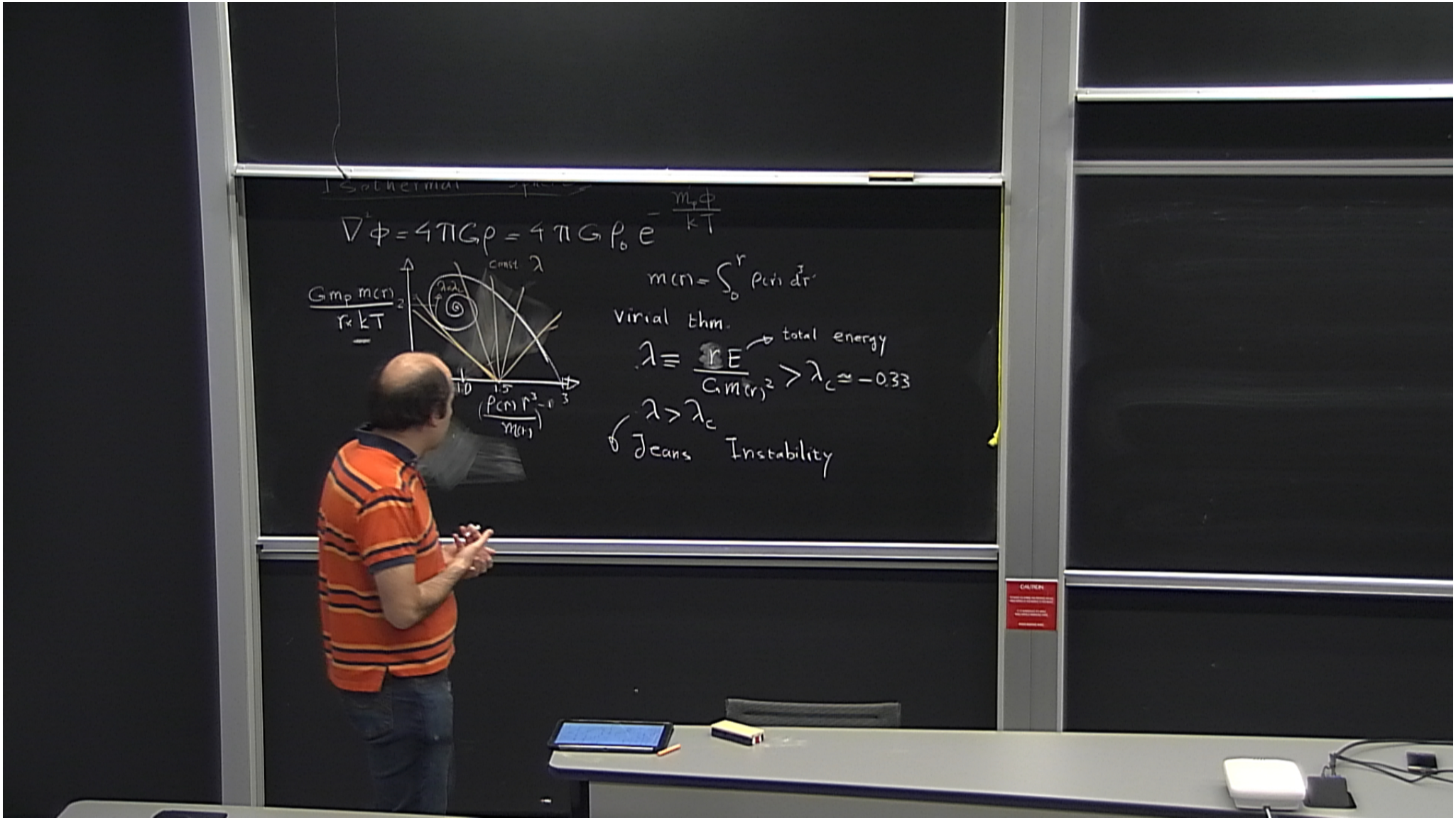

Isothermal Sphere
particle mass
$\nabla^2\phi = 4\pi G\rho = 4\pi G\rho_0 e^{-\frac{m_p\phi}{kT}}$
$\frac{G m_p m(r)}{r k T}$
2
1.5
3
$\frac{\rho(r) r^3}{m(r)}$
$m(r) = \int_0^r \rho(r)\, d\vec{r}'$

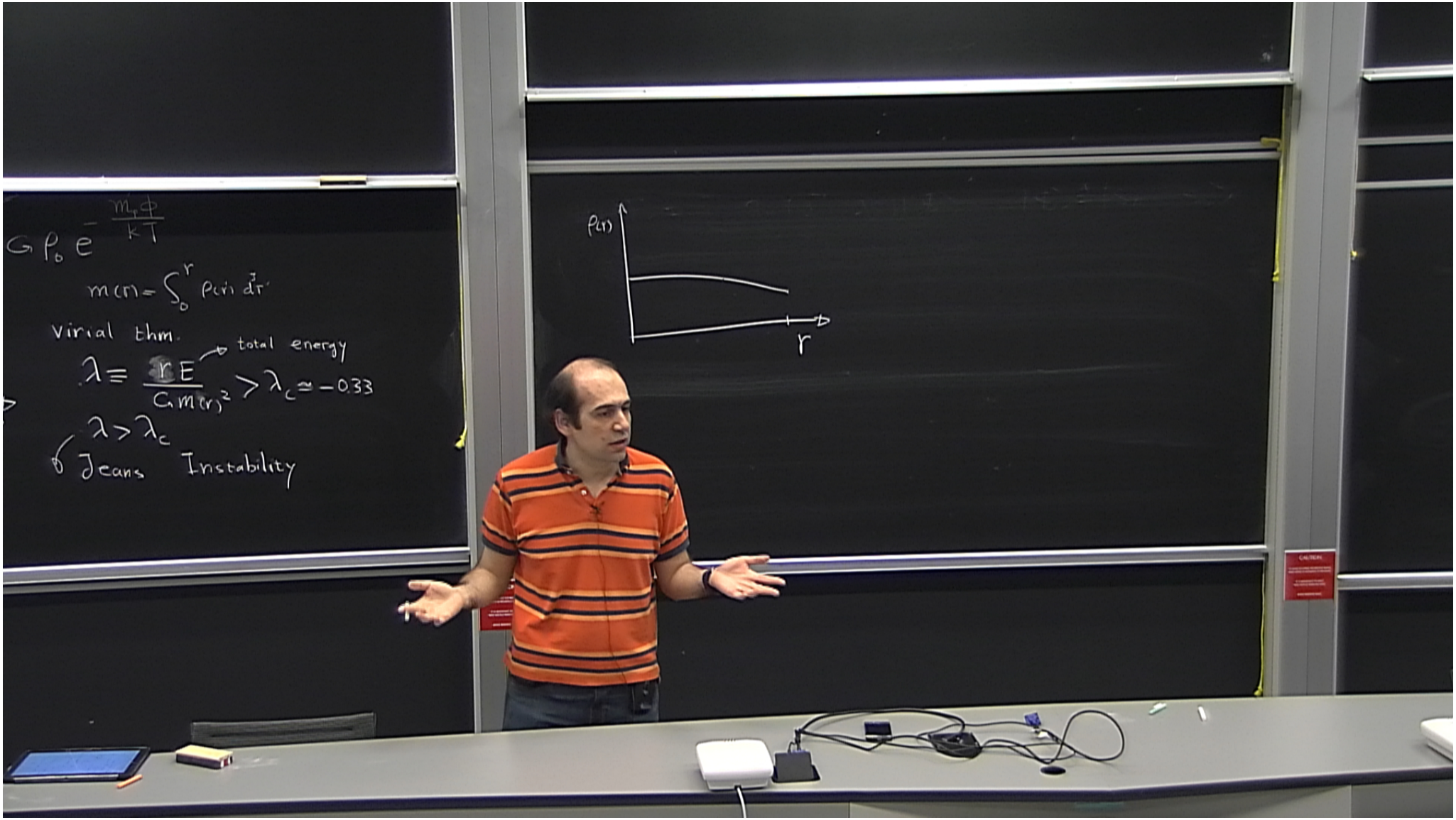

Isothermal Sphere
$\nabla^2\phi = 4\pi G\rho = 4\pi G\rho_0 e^{-\frac{m_p\phi}{kT}}$
particle mass
$\frac{Gmp^2}{r^2}$
const. λ
$\frac{\rho(r)r^3}{m(r)}$
$m(r) = \int_0^r \rho(r)\, d\tau$
Virial thm.
$\lambda \equiv \frac{RE}{Gm^2}$
total energy

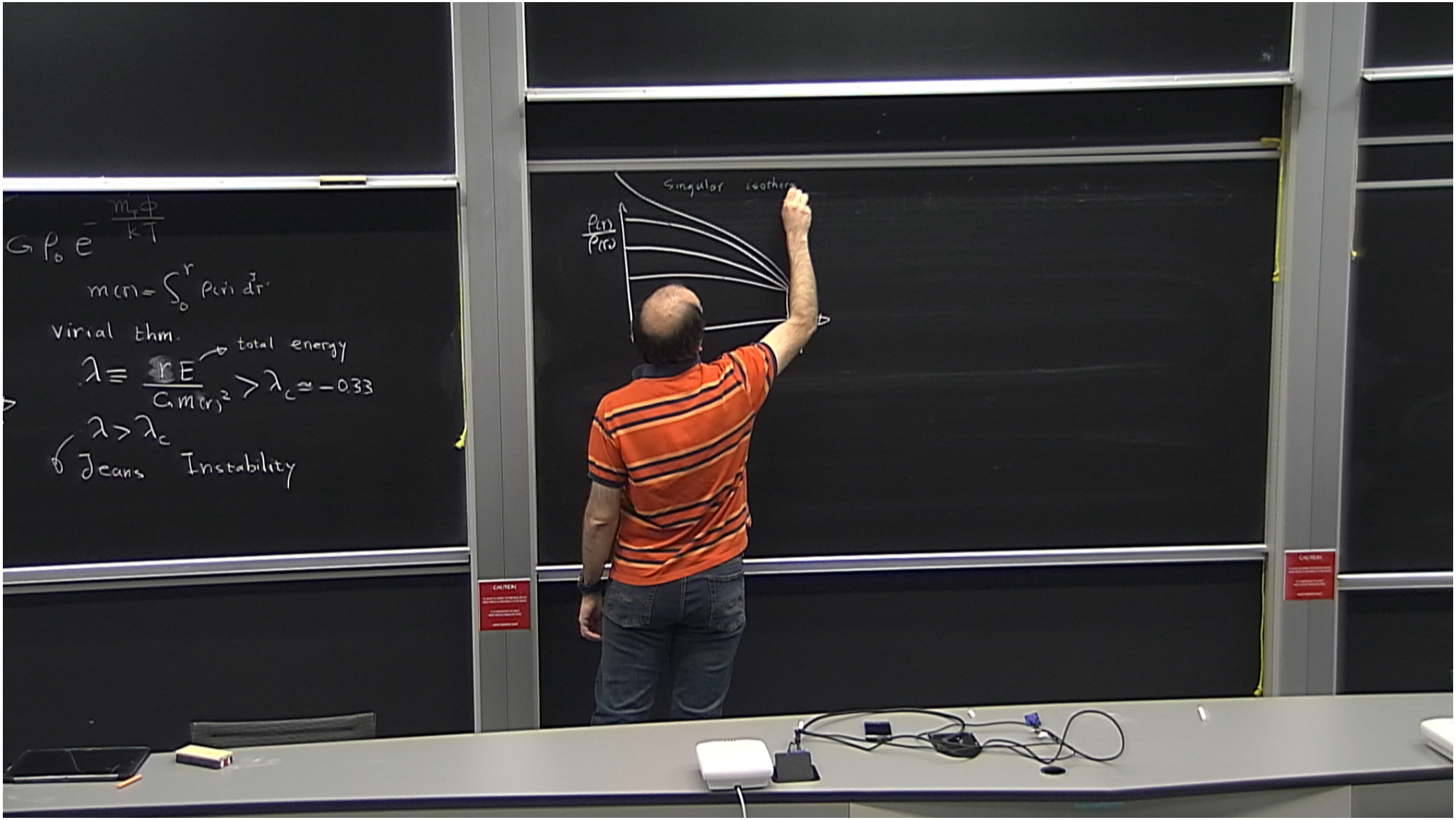

Isothermal Sphere
particle mass
$\nabla^2\phi = 4\pi G\rho = 4\pi G\rho_0 e^{-\frac{m_p\phi}{kT}}$
const. λ
$m(r) = \int_0^r \rho(r)\, dr^3$
Virial thm.
$\lambda \equiv \frac{RE}{GM(r)^2}$
total energy
1.0
1.5
3
$\frac{\rho(r)\, r^2}{m(r)}$
$\frac{\rho r^2}{\frac{G}{2}\pi} = \frac{3}{4\pi}$

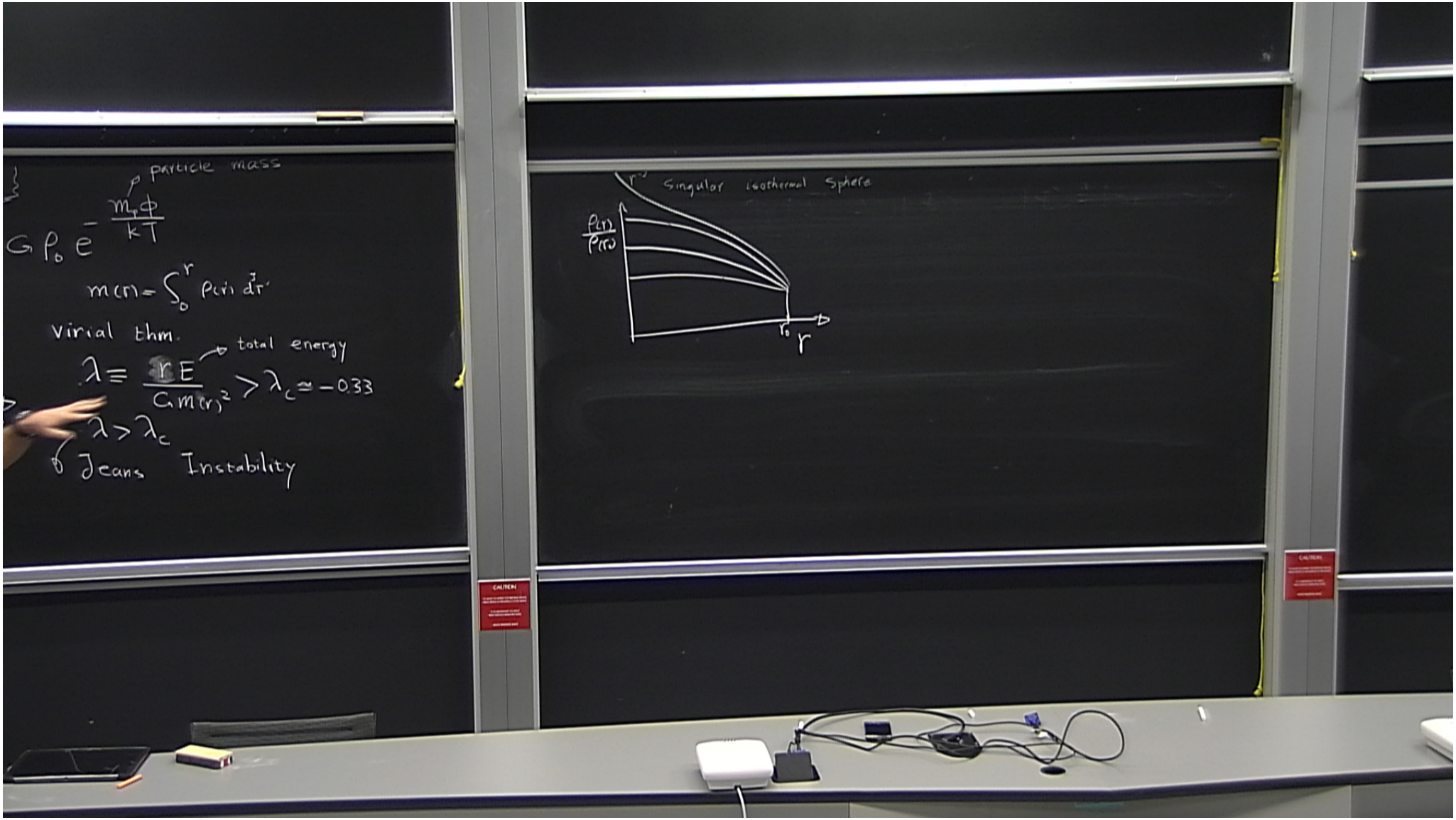

Isothermal Sphere
particle mass
$\nabla^2\Phi = 4\pi G\rho = 4\pi G\rho_0 e^{-\frac{m_p\Phi}{kT}}$
$\frac{G m_p m(r)}{r^2 kT}$
const. λ
$m(r) = \int_0^r \rho(r')\, d\tau'$
Virial thm.
$\lambda \equiv \frac{RE}{G m(r)^2}$
total energy

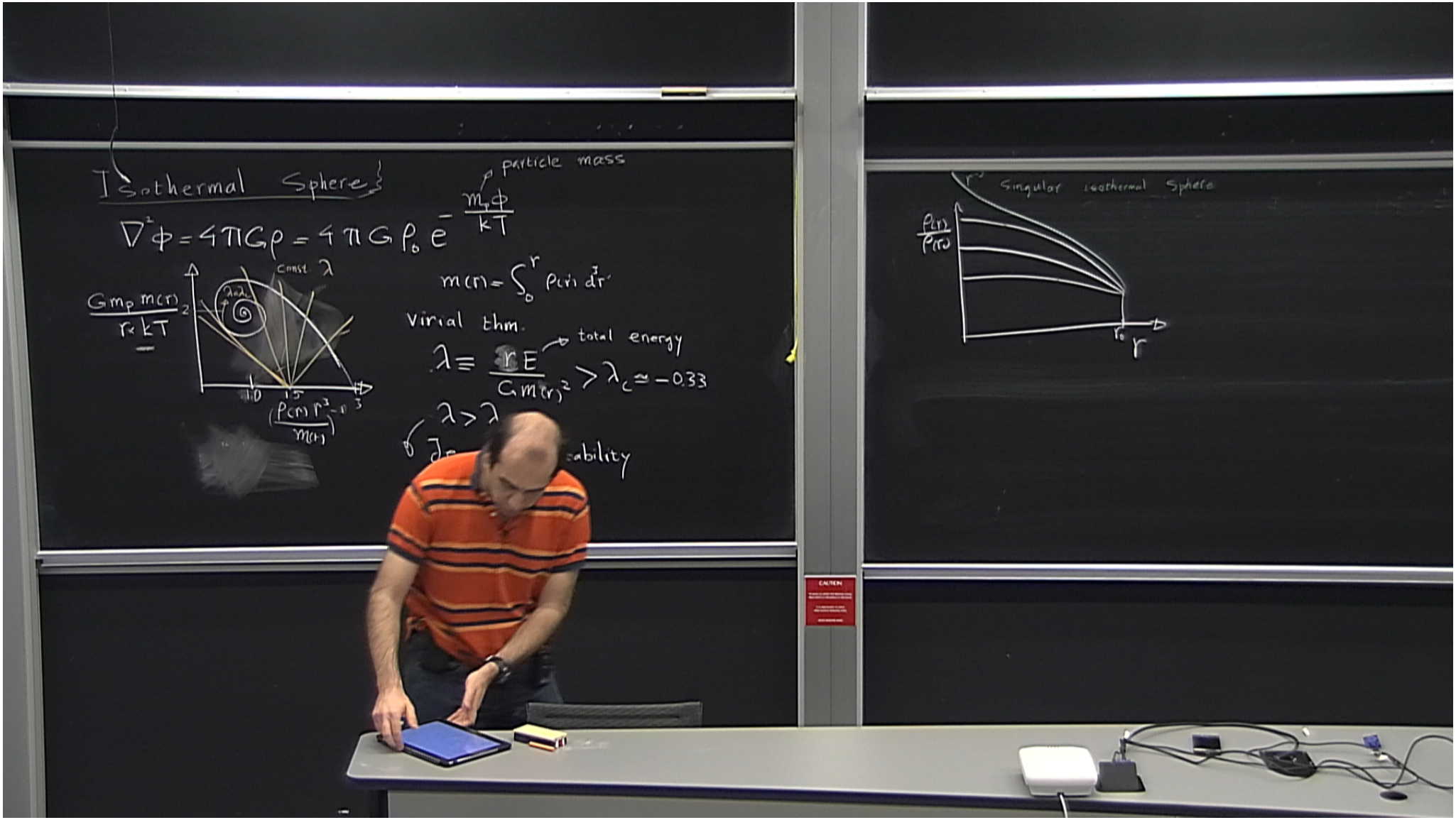

Isothermal Sphere
particle mass
$\nabla^2\phi = 4\pi G\rho = 4\pi G\rho_0 e^{-\frac{m_p\phi}{kT}}$
const λ
$m(r) = \int_0^r \rho(r)\, dr^3$
Virial thm.
$\lambda \equiv \frac{RE}{GM(r)^2}$
total energy
1.0
1.5
3

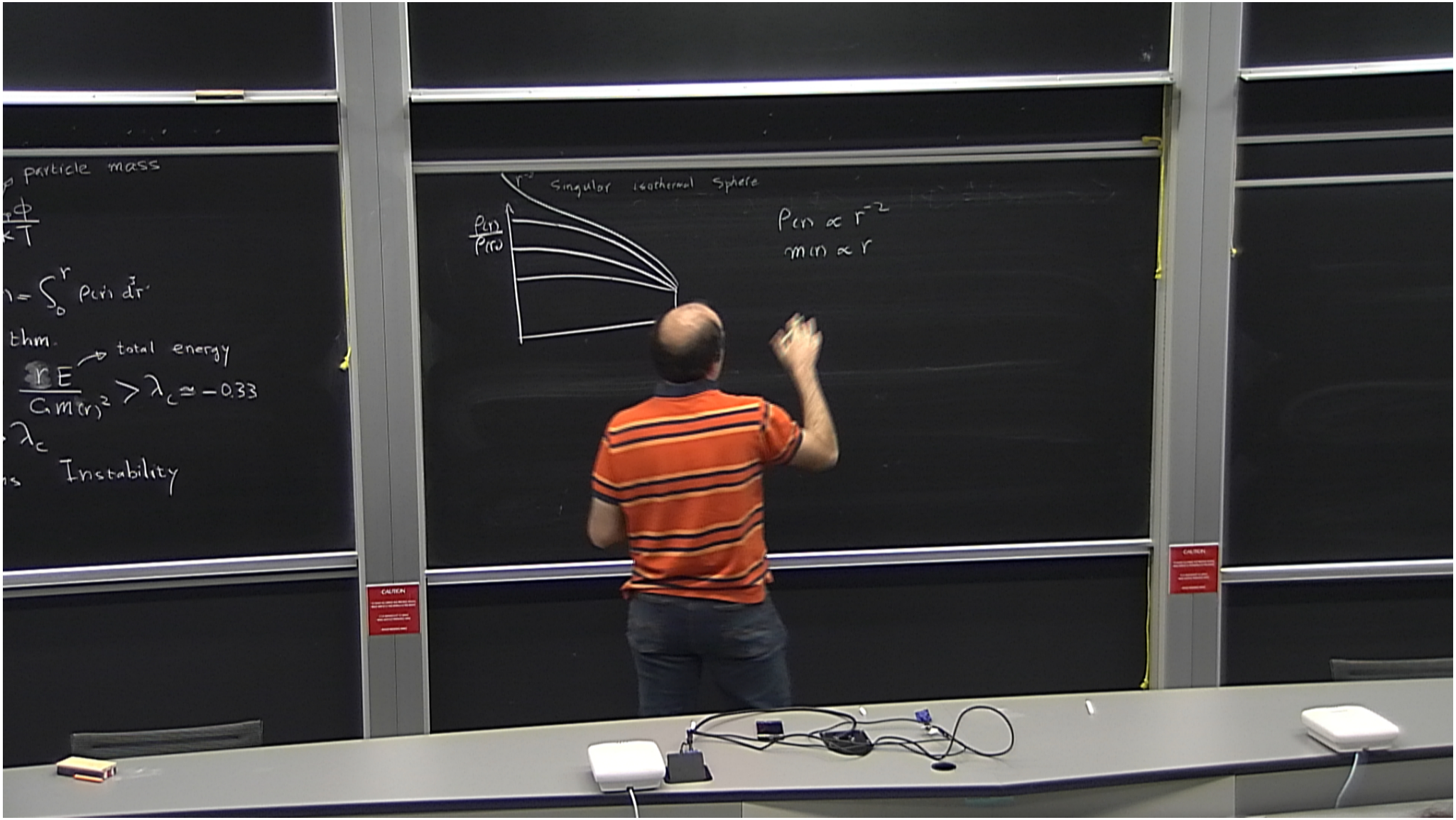

Isothermal
$\nabla^2\phi = 4\pi G\rho = 4\pi G\rho_0 e^{-\frac{m_p\phi}{kT}}$
$\frac{G m_p m(r)}{r k T}$
const λ
$m(r) = \int_0^r \rho(r)\, dr$
Virial thm.
$\lambda \equiv \frac{rE}{G m(r)^2} > \lambda_c \simeq -0.33$
total energy
$\lambda > \lambda_c$
Jeans Instability

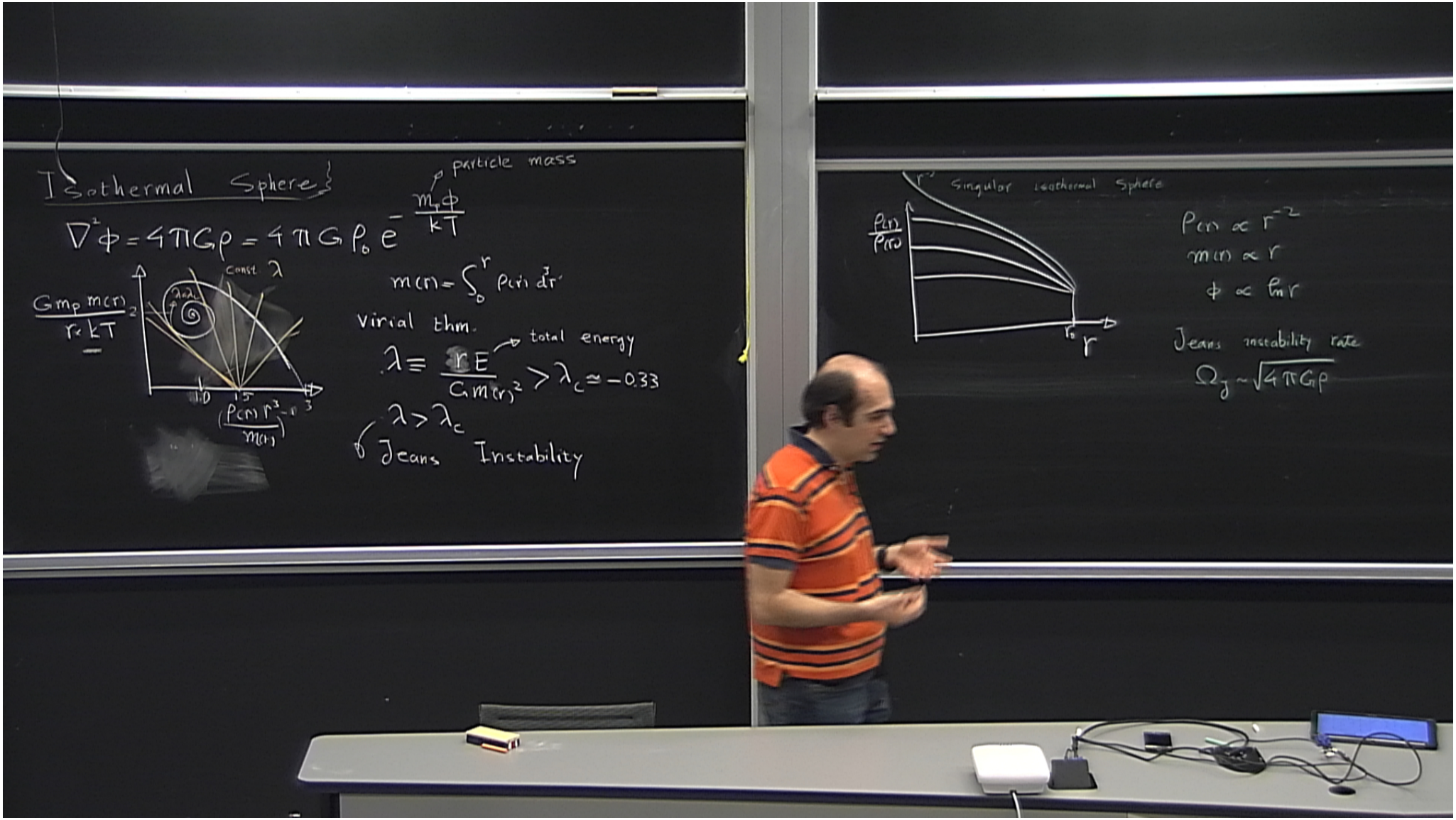

$G\rho_0 e^{-\frac{m_1\phi}{kT}}$
$m(r) = \int_0^r \rho(r)\, dV$
Virial thm.
total energy
$\lambda \equiv \frac{rE}{G m(r)^2} > \lambda_c \cong -0.33$
$\lambda > \lambda_c$
Jeans Instability
$\rho(r)$
r

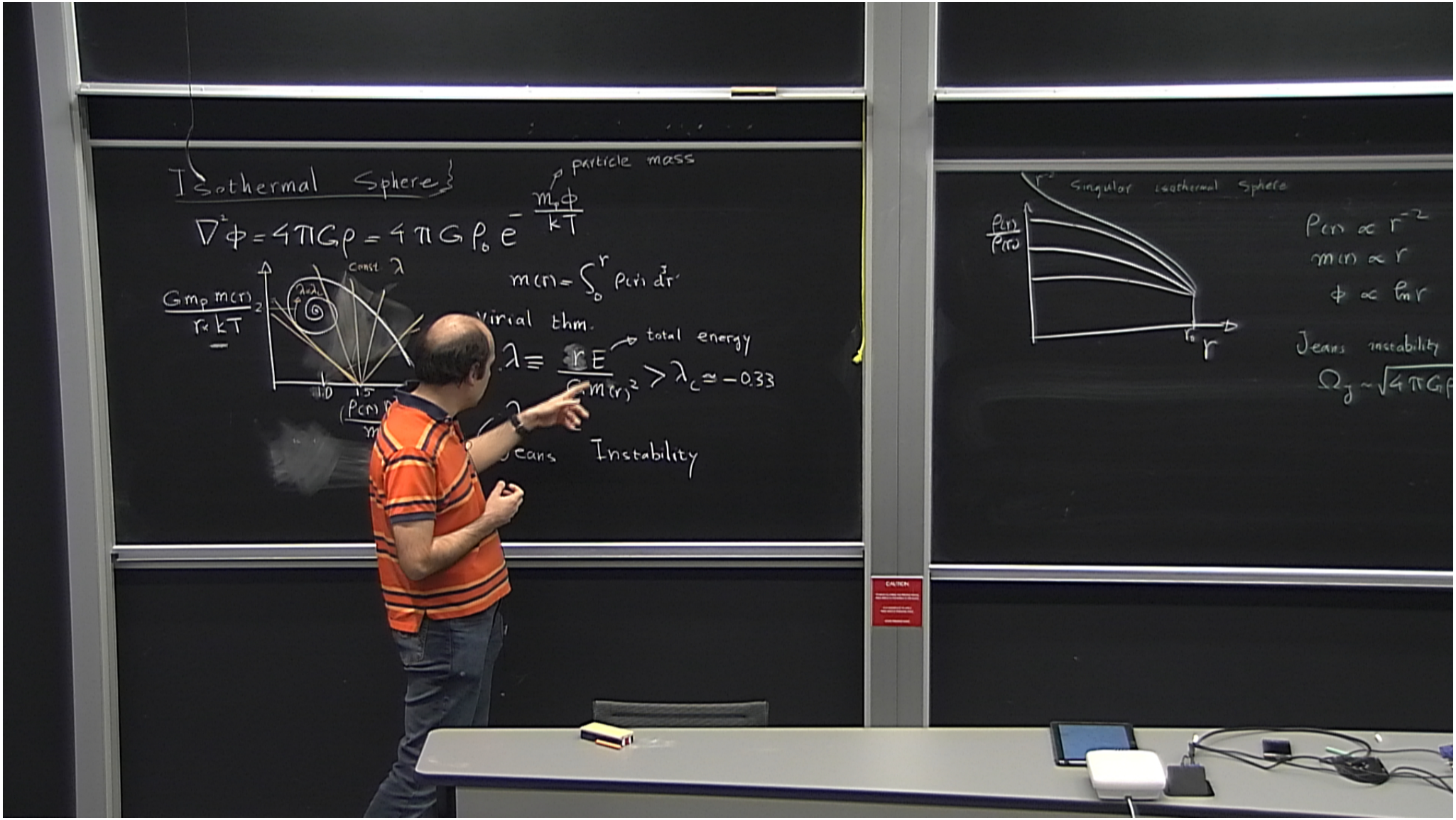

$G\rho_0 e^{-\frac{m_r\Phi}{kT}}$
$m(r) = \int_0^r \rho(r)\, dr$
Virial thm.
total energy
$\lambda \equiv \frac{rE}{G m(r)^2} > \lambda_c \cong -0.33$
$\lambda > \lambda_c$
Jeans Instability
Singular isother
$\frac{\rho(r)}{\rho(r)}$

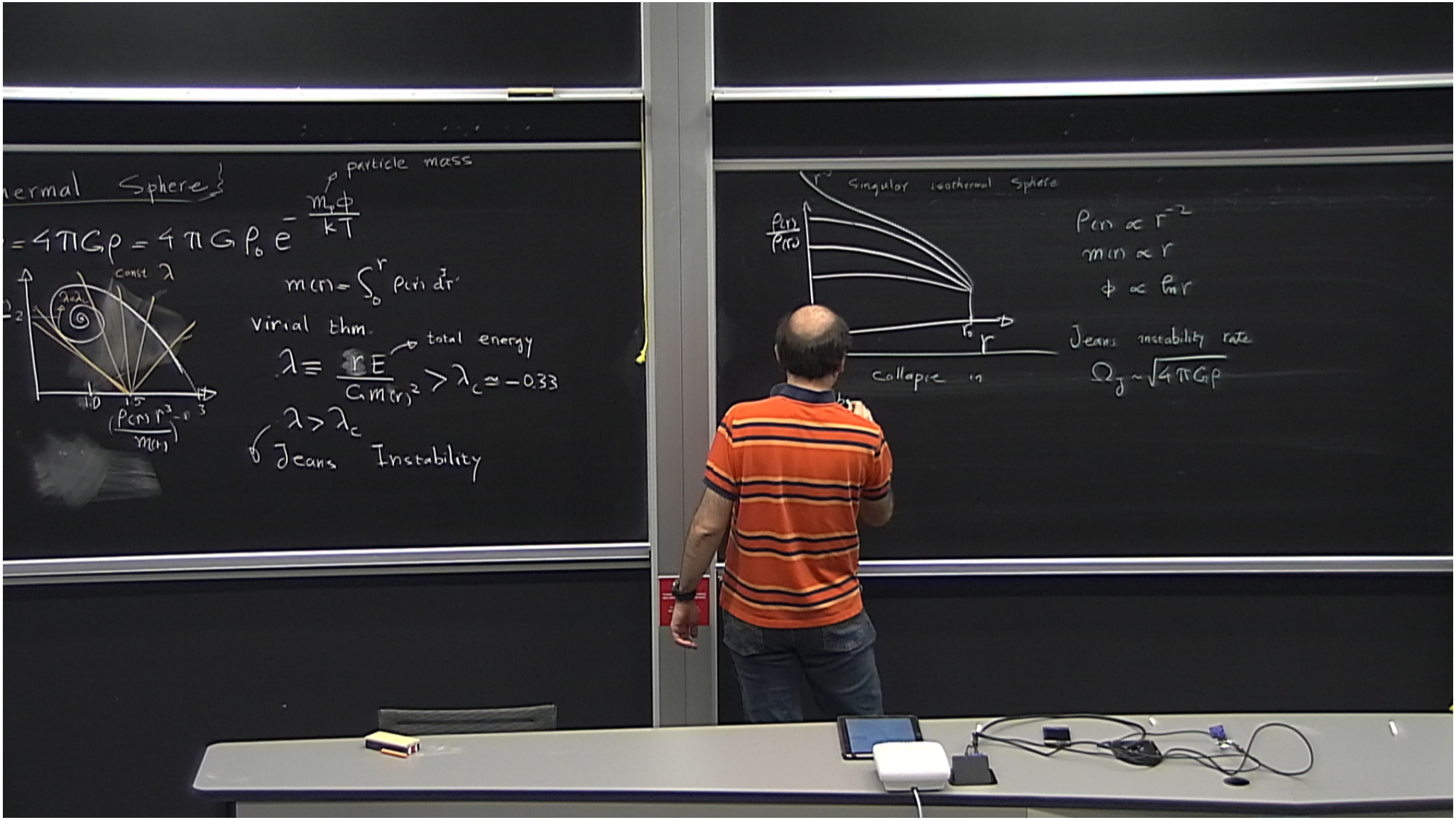

particle mass
$G\rho_0 e^{-\frac{m_p\phi}{kT}}$
$m(r) = \int_0^r \rho(r)\, d\vec{r}$
Virial thm.
total energy
$\lambda \equiv \frac{rE}{G m(r)^2} > \lambda_c \cong -0.33$
$\lambda > \lambda_c$
Jeans Instability
Singular isothermal Sphere
$\frac{\rho(r)}{\rho(r_0)}$
r_0
r

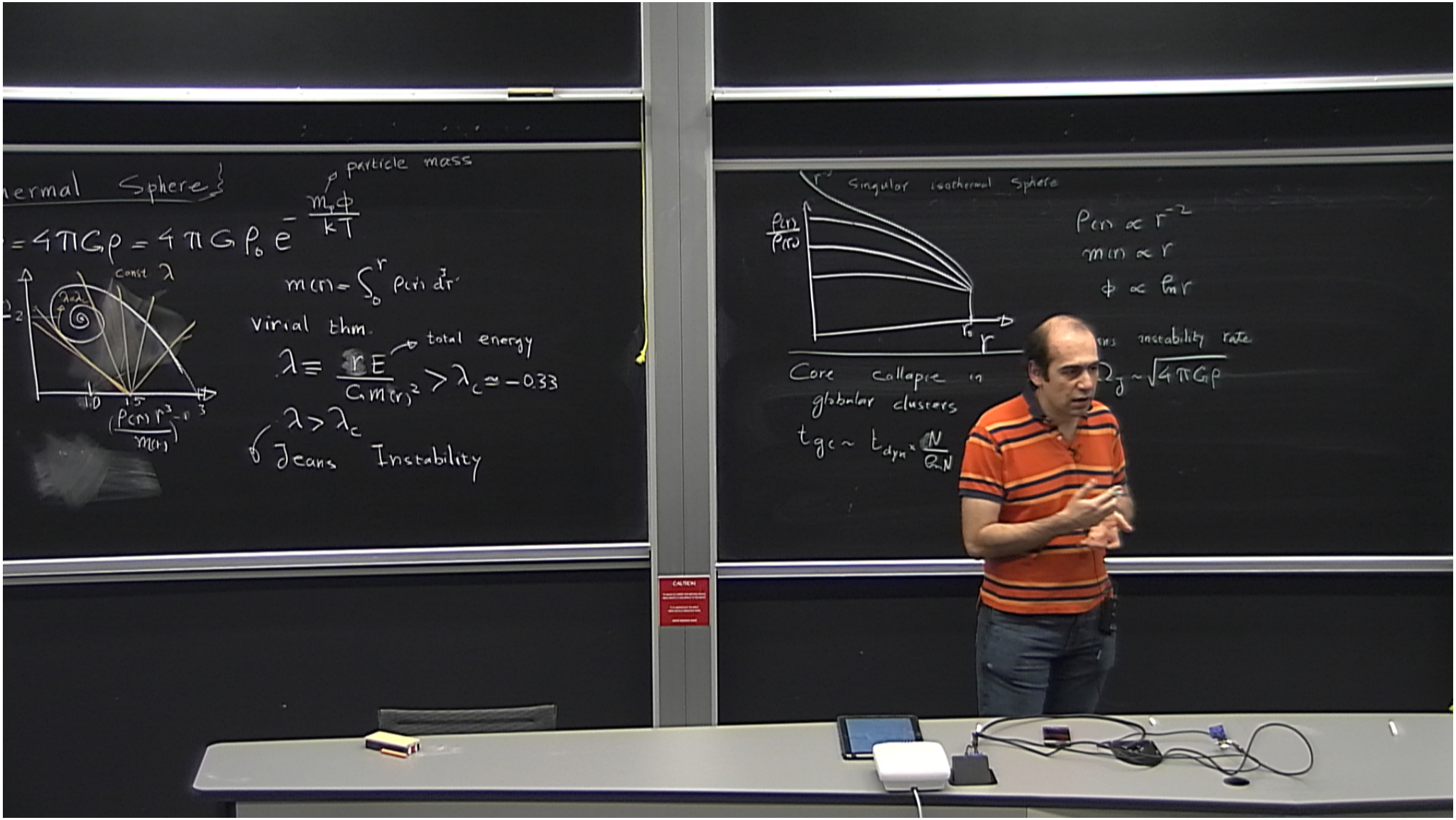

Isothermal Sphere
particle mass
$\nabla^2\phi = 4\pi G\rho = 4\pi G\rho_0 e^{-\frac{m_p\phi}{kT}}$
$\frac{G m_p m(r)}{r^2 kT}$
const λ
$m(r) = \int_0^r \rho(r')\, dr'$
Virial thm.
$\lambda \equiv \frac{rE}{G m(r)^2} > \lambda_c \simeq -0.33$
total energy
$\lambda > \lambda$
$\frac{\rho(r) r^2}{m(r)}$
Singular isothermal Sphere
$\frac{\rho(r)}{\rho(r_0)}$
r_0
r

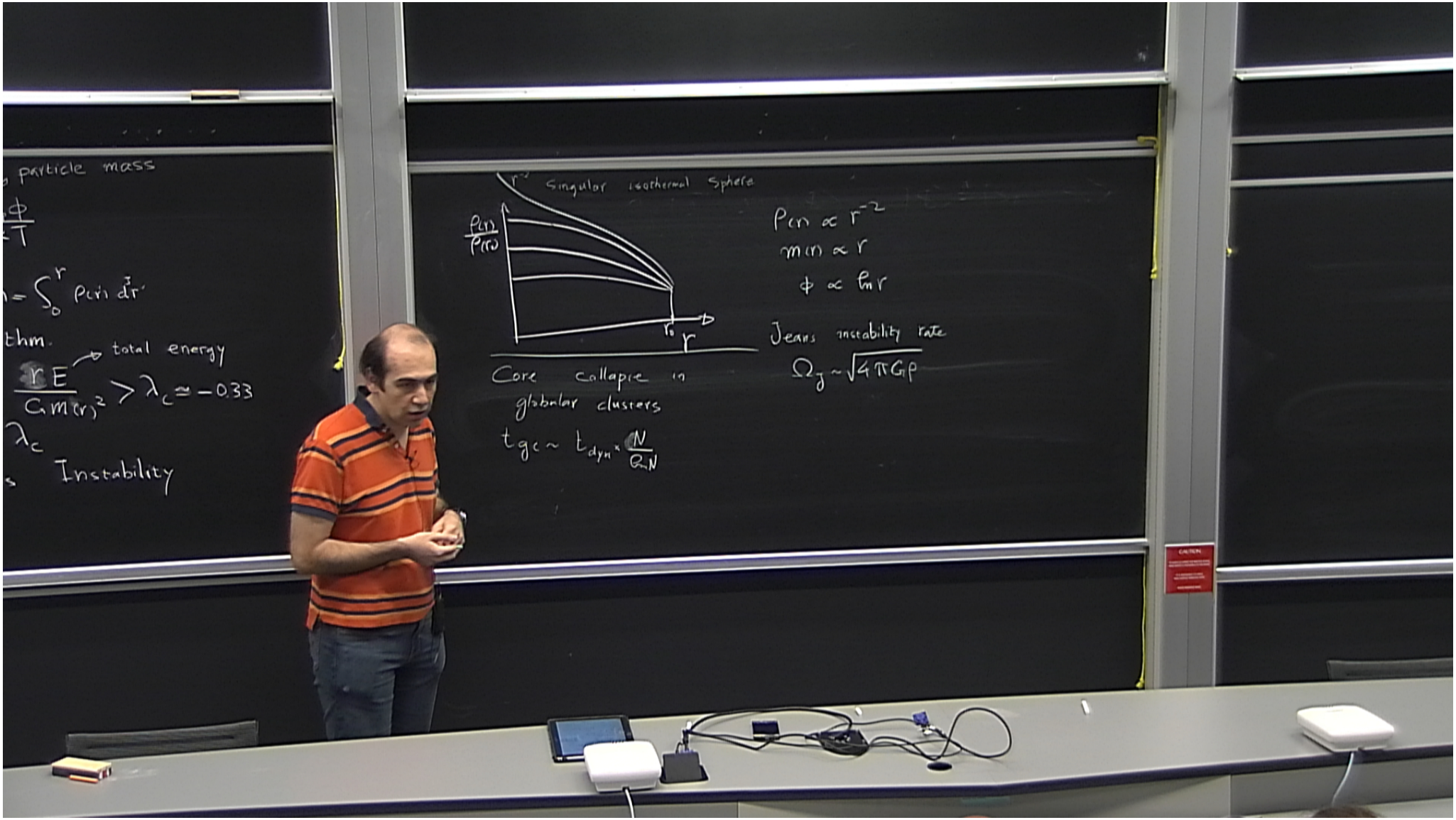

particle mass
thm.
total energy
Instability
Singular isothermal Sphere
$\rho(r) \propto r^{-2}$
$m(r) \propto r$

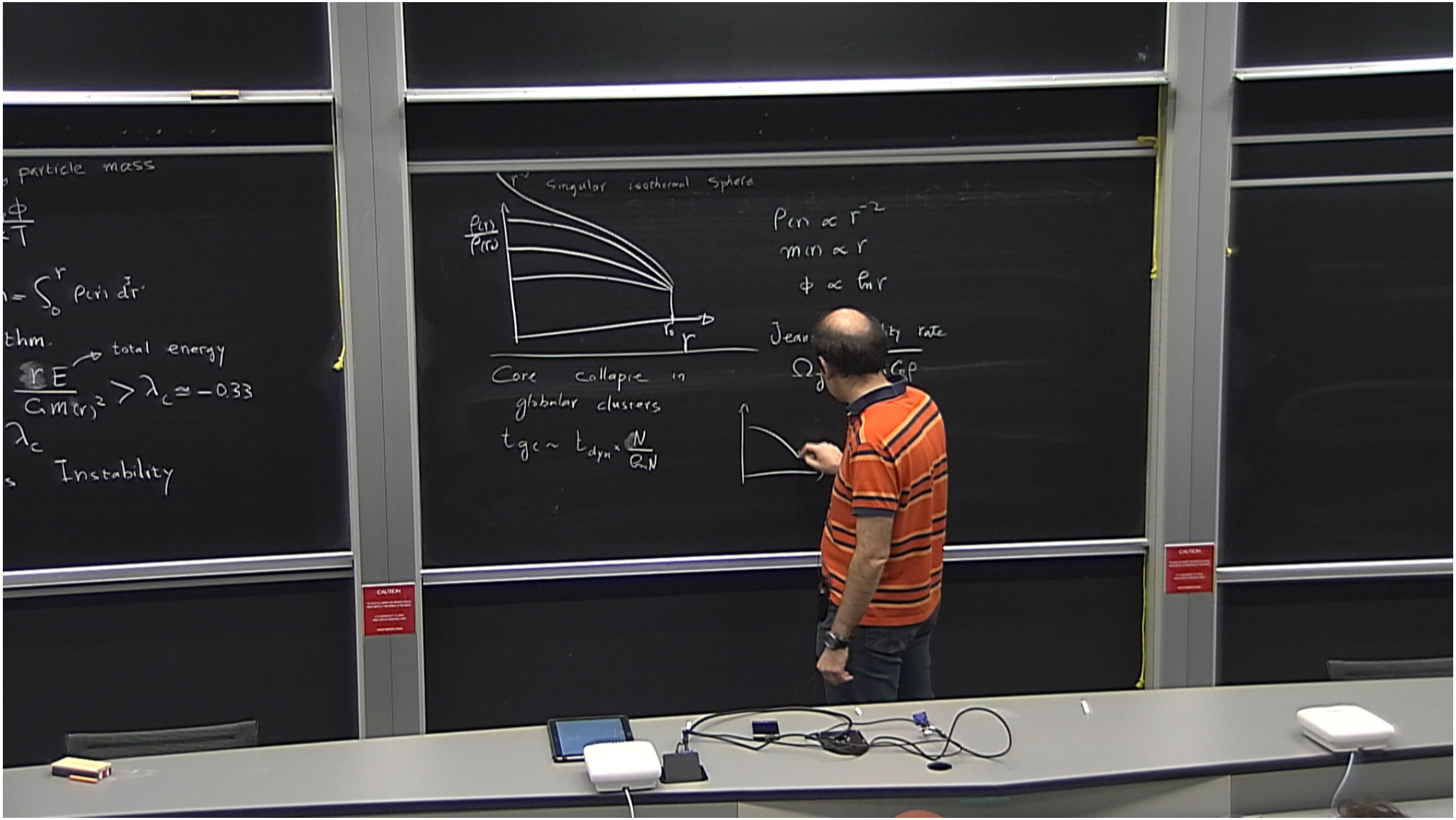

Isothermal Sphere
particle mass
$\nabla^2\Phi = 4\pi G\rho = 4\pi G\rho_0 e^{-\frac{m_p\Phi}{kT}}$
$\frac{G m_p m(r)}{r k T} = 2$
const λ
1.0
1.5
3
$\frac{\rho(r) r^3}{m(r)} = 0$
$m(r) = \int_0^r \rho(r) dr$
Virial thm.
$\lambda \equiv \frac{rE}{Gm(r)^2} > \lambda_c \simeq -0.33$
total energy
$\lambda > \lambda_c$
Jeans Instability
Singular isothermal Sphere
$\frac{\rho(r)}{\rho(r_0)}$
r_0
r
$\rho(r) \propto r^{-2}$
$m(r) \propto r$
$\Phi \propto \ln r$
Jeans instability rate
$\Omega_J \sim \sqrt{4\pi G\rho}$

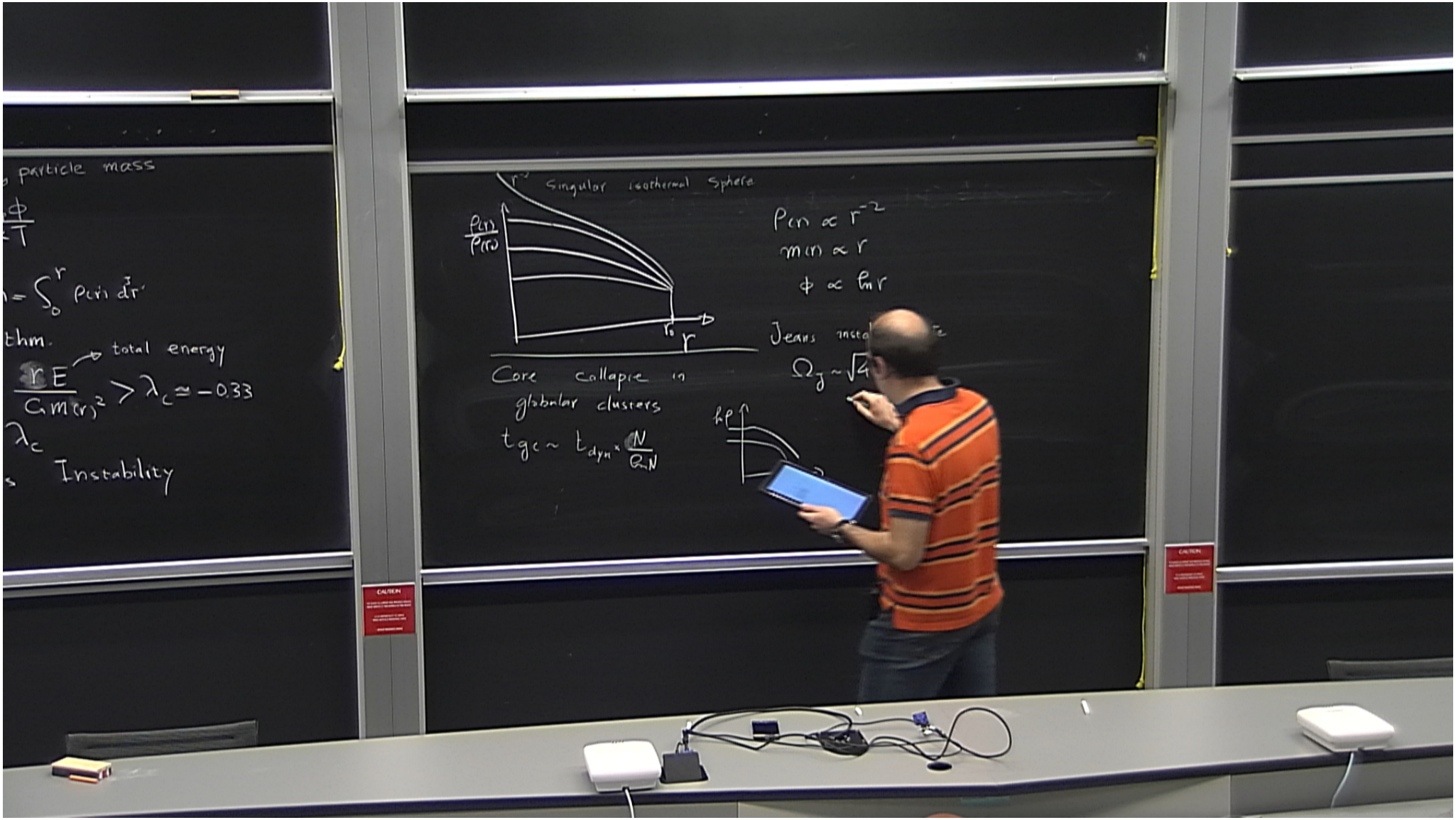

Isothermal Sphere
particle mass
$\nabla^2\phi = 4\pi G\rho = 4\pi G\rho_0 e^{-\frac{m_p\phi}{kT}}$
$\frac{G m_p m(r)}{r^2 kT}$
const λ
$m(r) = \int_0^r \rho(r)\, dr$
virial thm.
$\lambda \equiv \frac{rE}{Gm(r)^2} > \lambda_c \simeq -0.33$
total energy
Jeans Instability
Singular Isothermal Sphere
$\rho(r) \propto r^{-2}$
$m(r) \propto r$
$\phi \propto \ln r$
Jeans instability
$\Omega_J \sim \sqrt{4\pi G\rho}$

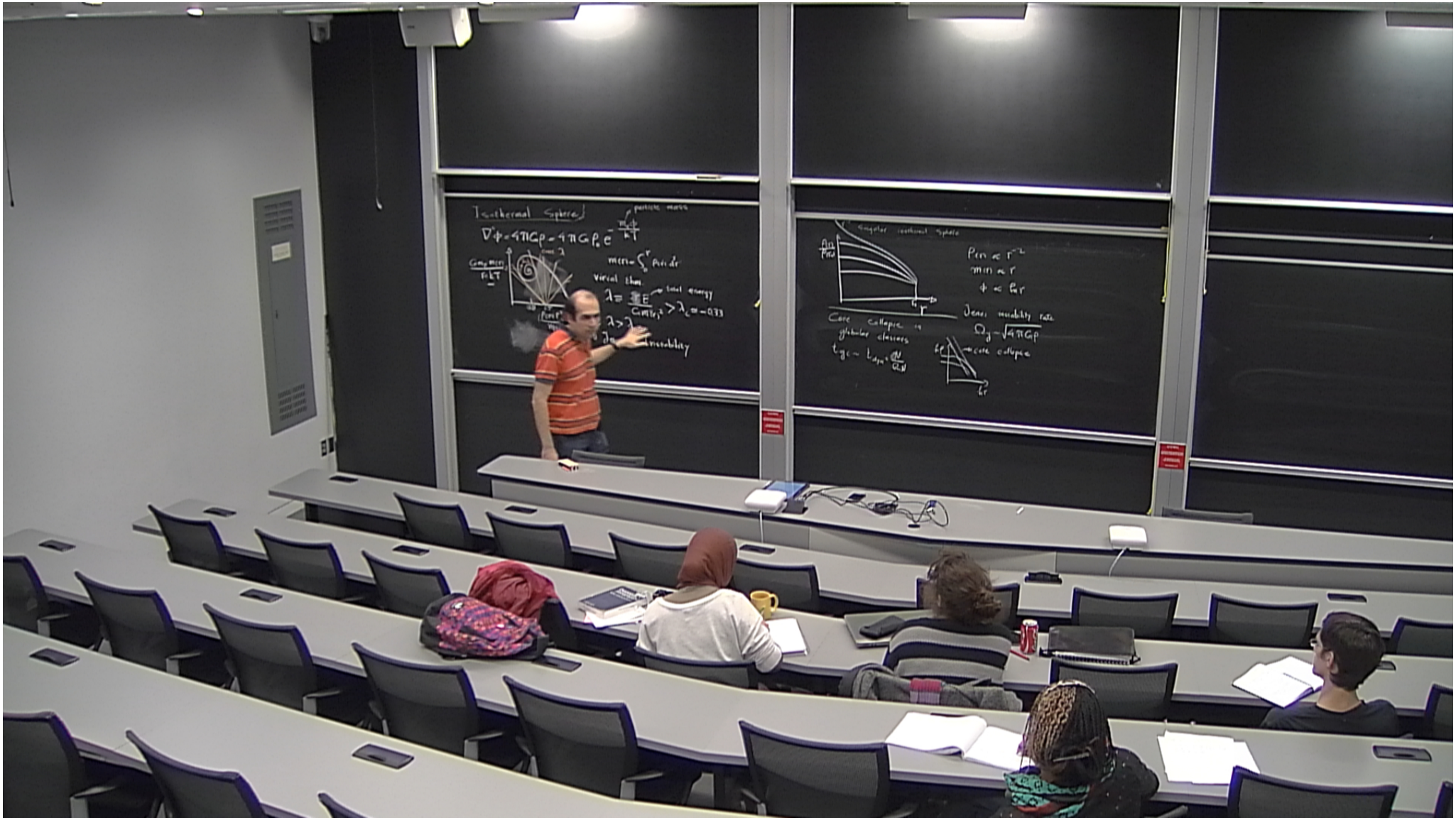

particle mass
Const λ
total energy
Virial thm.
Jeans Instability
Singular isothermal Sphere
Jeans instability rate
collapse in

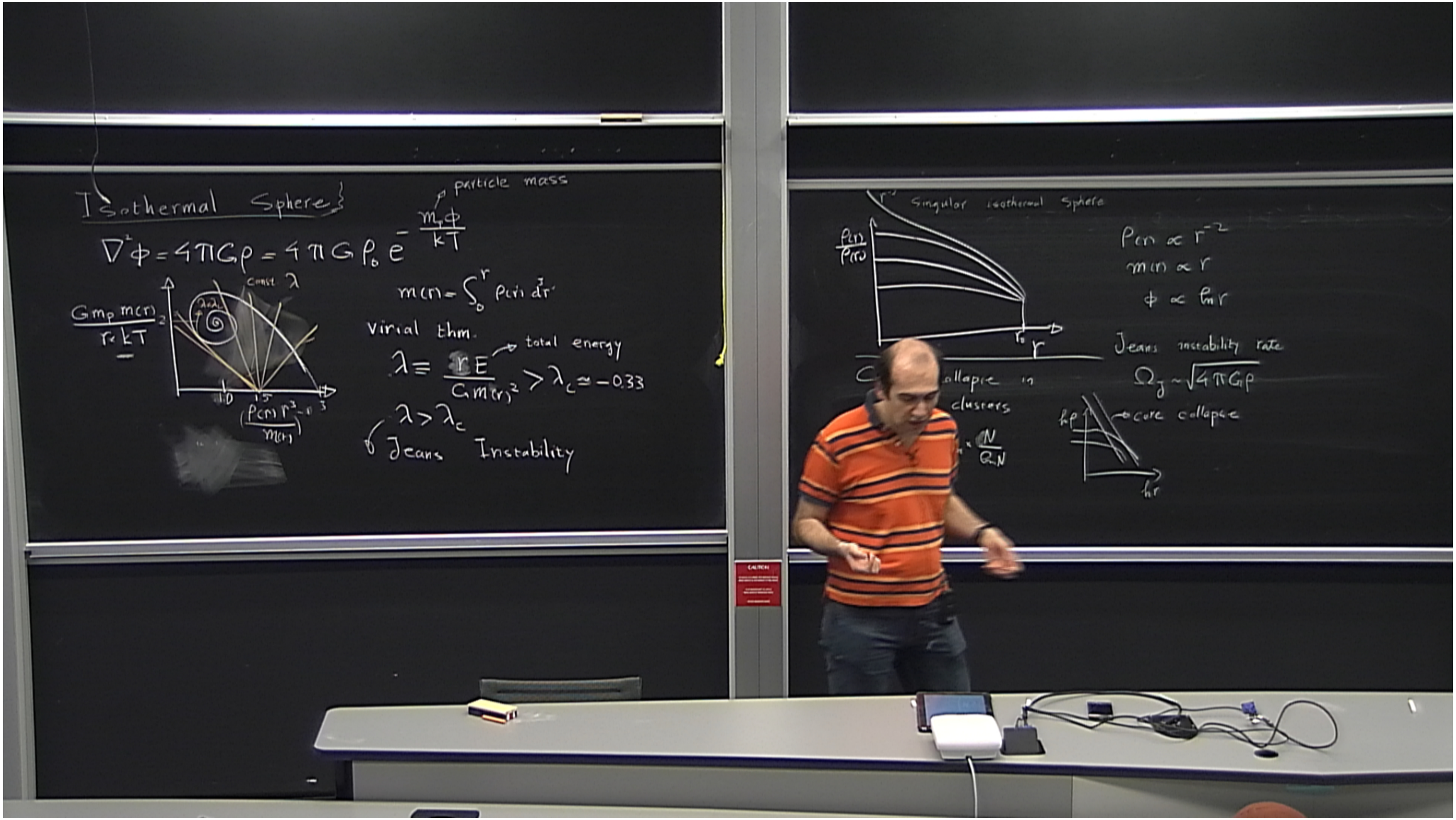

particle mass
Const λ
total energy
Virial thm.
Jeans Instability
Singular isothermal Sphere
Core collapse in globular clusters
CALTECH

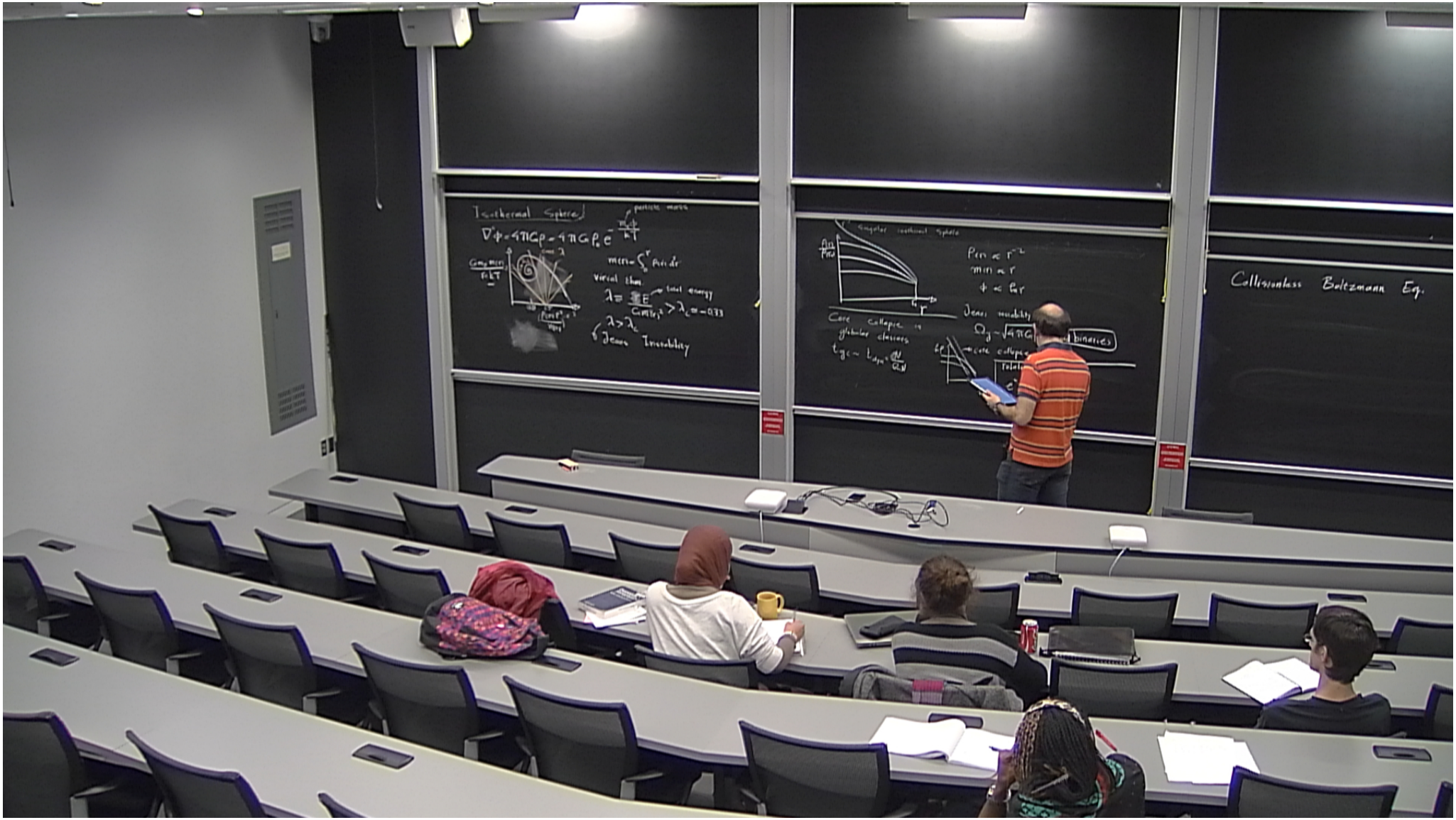

particle mass
thm.
total energy
Instability
r^-2 Singular isothermal Sphere
ρ(r)/ρ(r0)
r0
r
ρ(r) ∝ r^-2
m(r) ∝ r
Φ ∝ ln r
Jeans instability rate
Ω_J ~ √(4πGρ)
Core collapse in globular clusters
t_gc ~ t_dyn × N/Q_N

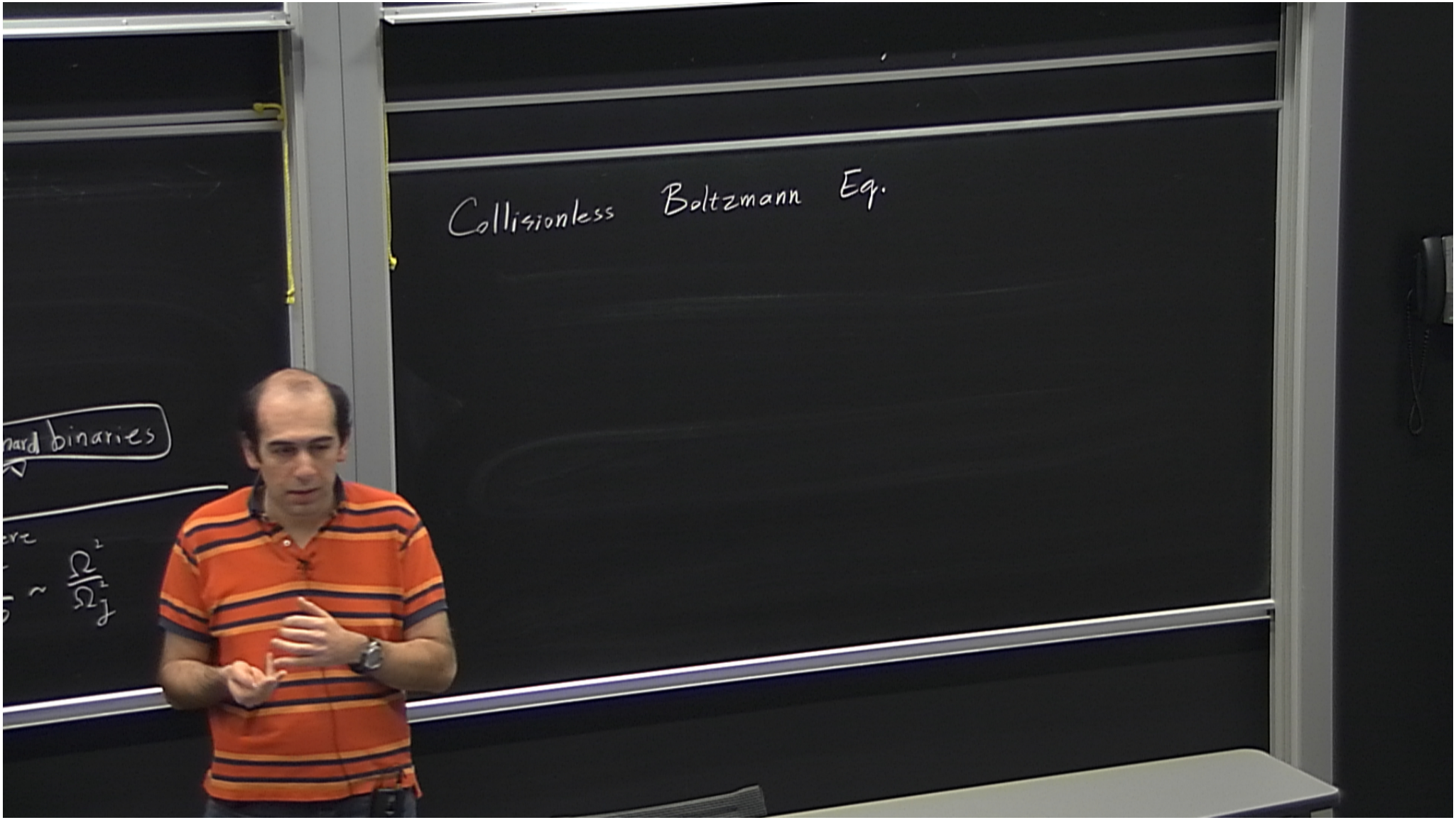

particle mass
thm.
total energy
Instability
Singular isothermal Sphere
Core collapse in globular clusters

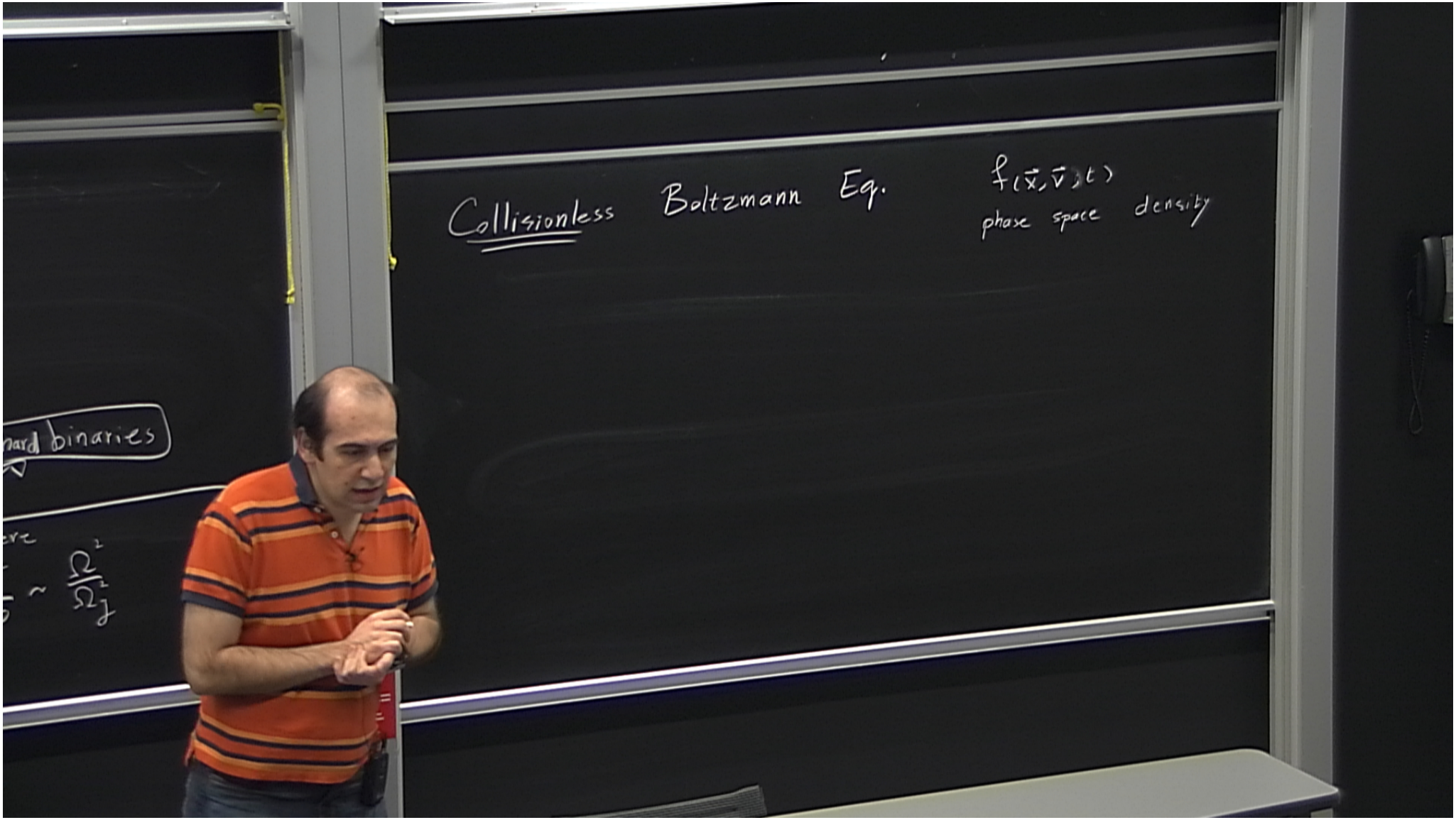

particle mass
thm.
total energy
Instability
Singular isothermal Sphere
Core collapse in globular clusters

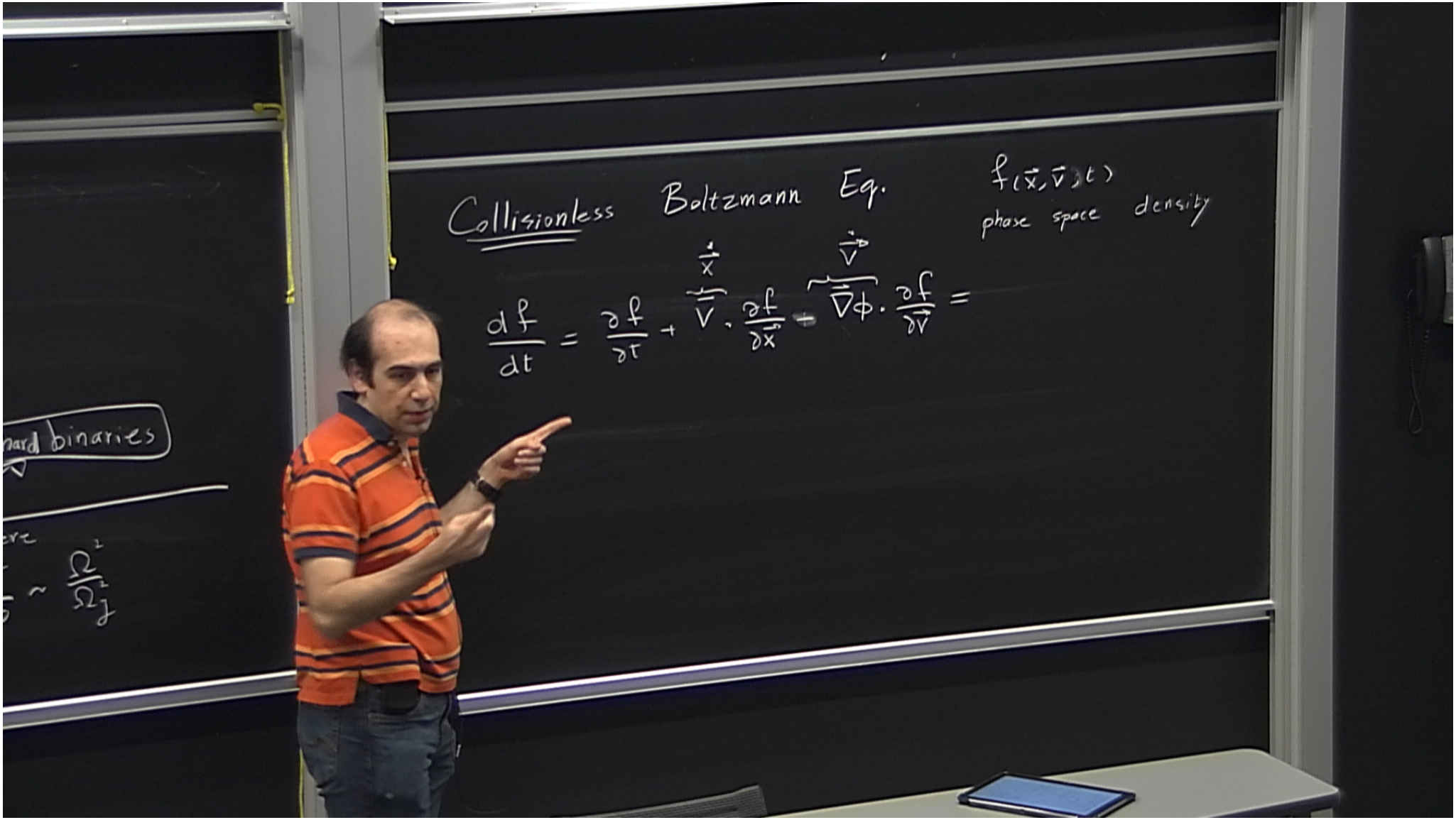

Isothermal Spheres
particle mass
virial theorem
total energy
Jeans instability
Singular isothermal Sphere
Core collapse
globular clusters
core collapse

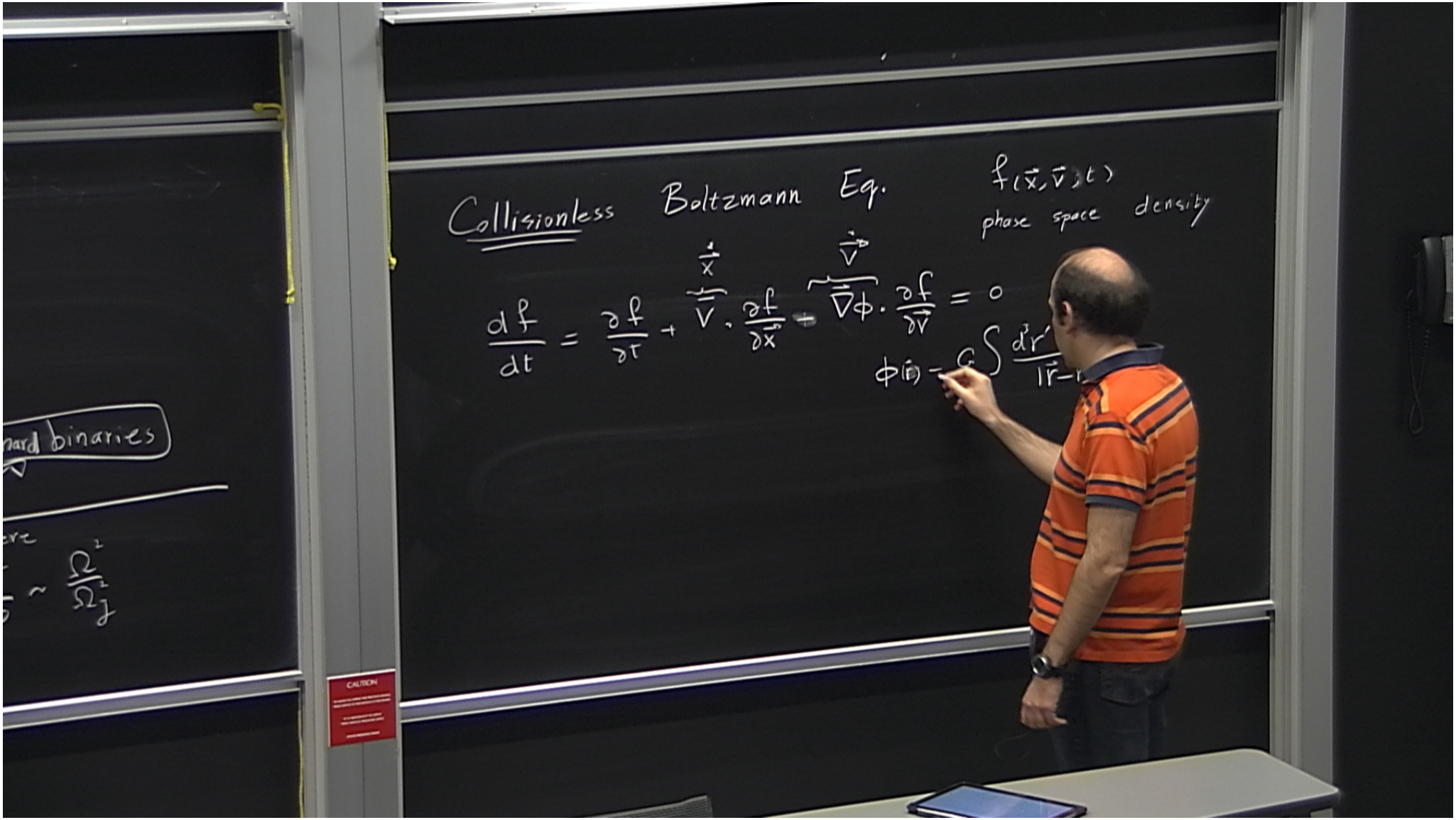

Isothermal Sphere
particle mass
$\nabla^2\phi = 4\pi G\rho = 4\pi G\rho_0 e^{-\frac{m_p\phi}{kT}}$
const. λ
$\frac{G m_p m(r)}{r k T}$
$\frac{\rho(r) r^3}{m(r)}$
$m(r) = \int_0^r \rho(r) dr'$
Virial thm.
$\lambda \equiv \frac{rE}{GM(r)^2} > \lambda_c \simeq -0.33$
total energy
$\lambda > \lambda_c$
Jeans Instability
Singular isothermal Sphere
$\rho(r) \propto r^{-2}$
$m(r) \propto r$
$\phi \propto \ln r$
Jeans instability rate
$\Omega_J \sim \sqrt{4\pi G\rho}$
core collapse
collapse in
clusters
$\frac{N}{Q_c N}$
hr

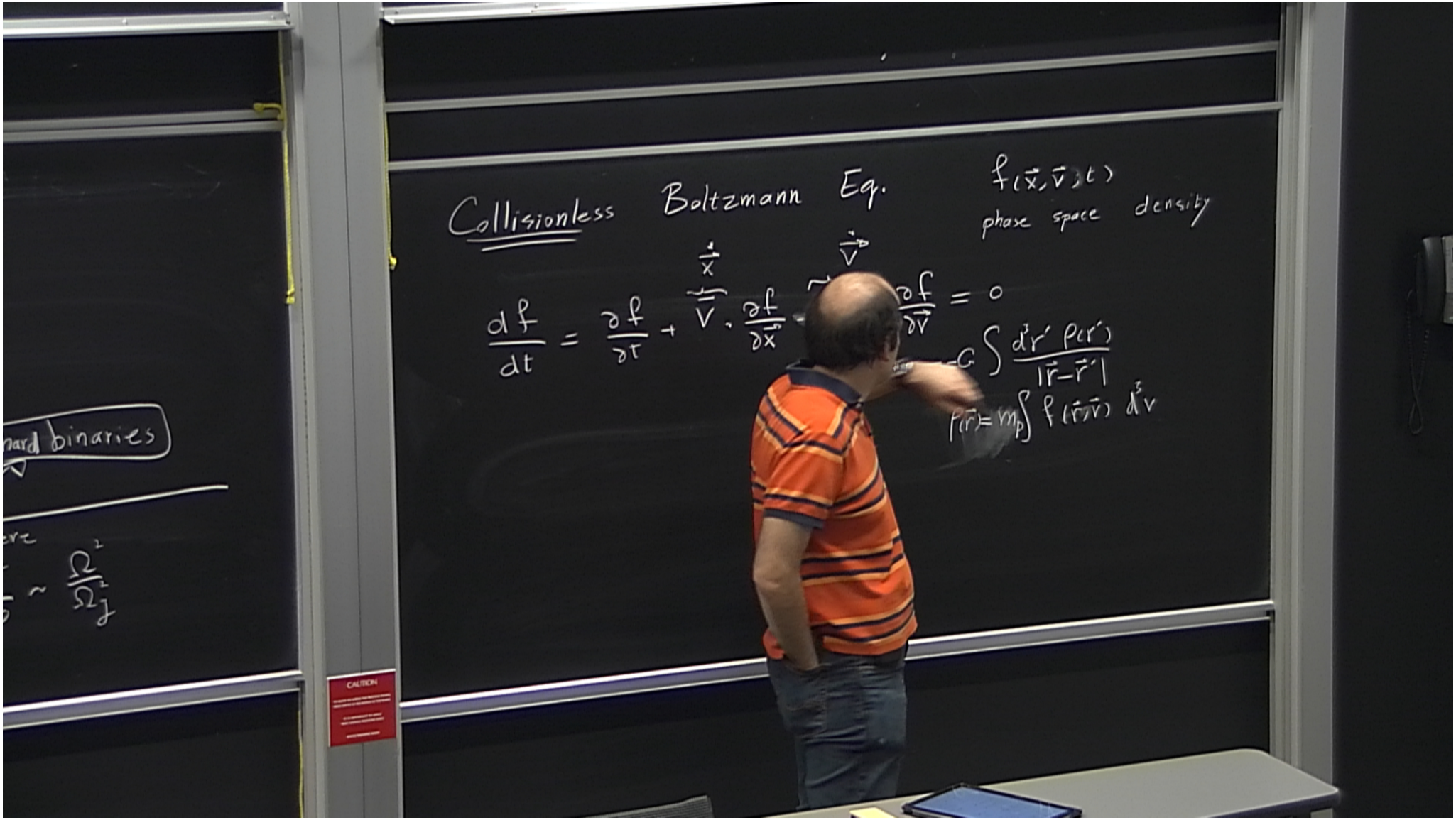

Isothermal Spheres
Collisionless Boltzmann Eq.

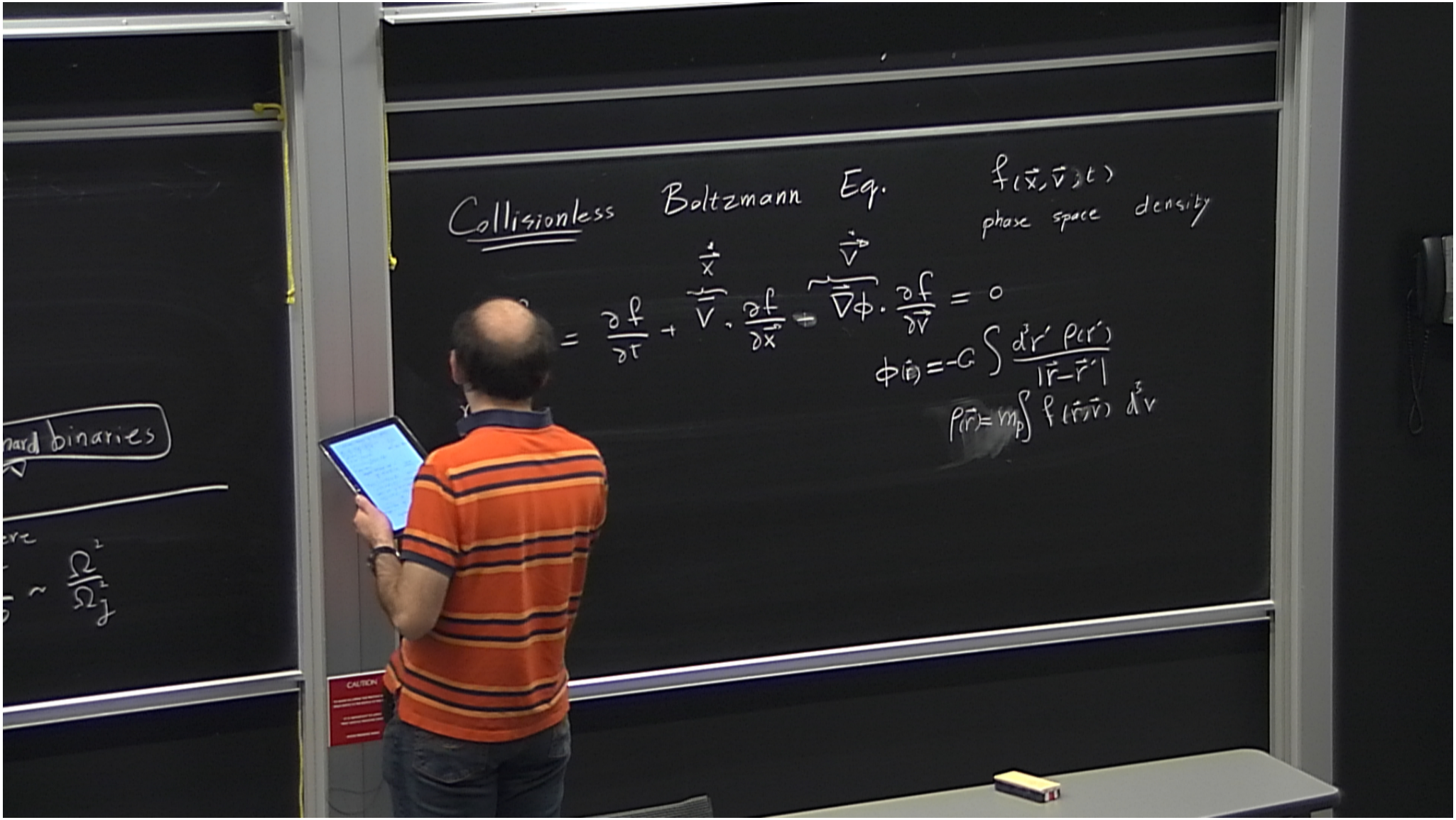

Collisionless Boltzmann Eq.
hard binaries

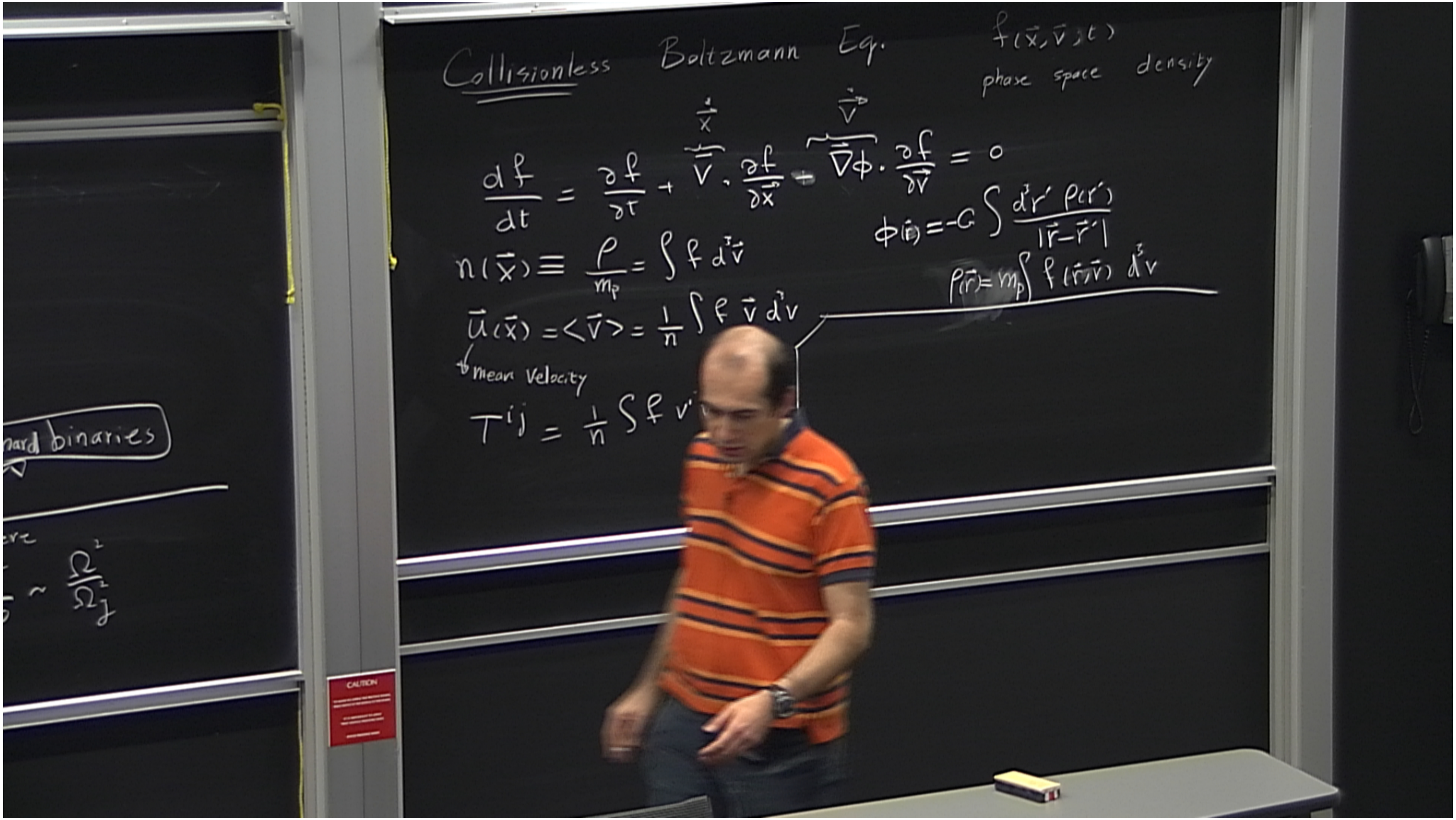

Collisionless Boltzmann Eq.
$f(\vec{x},\vec{v},t)$
phase space density
hard binaries

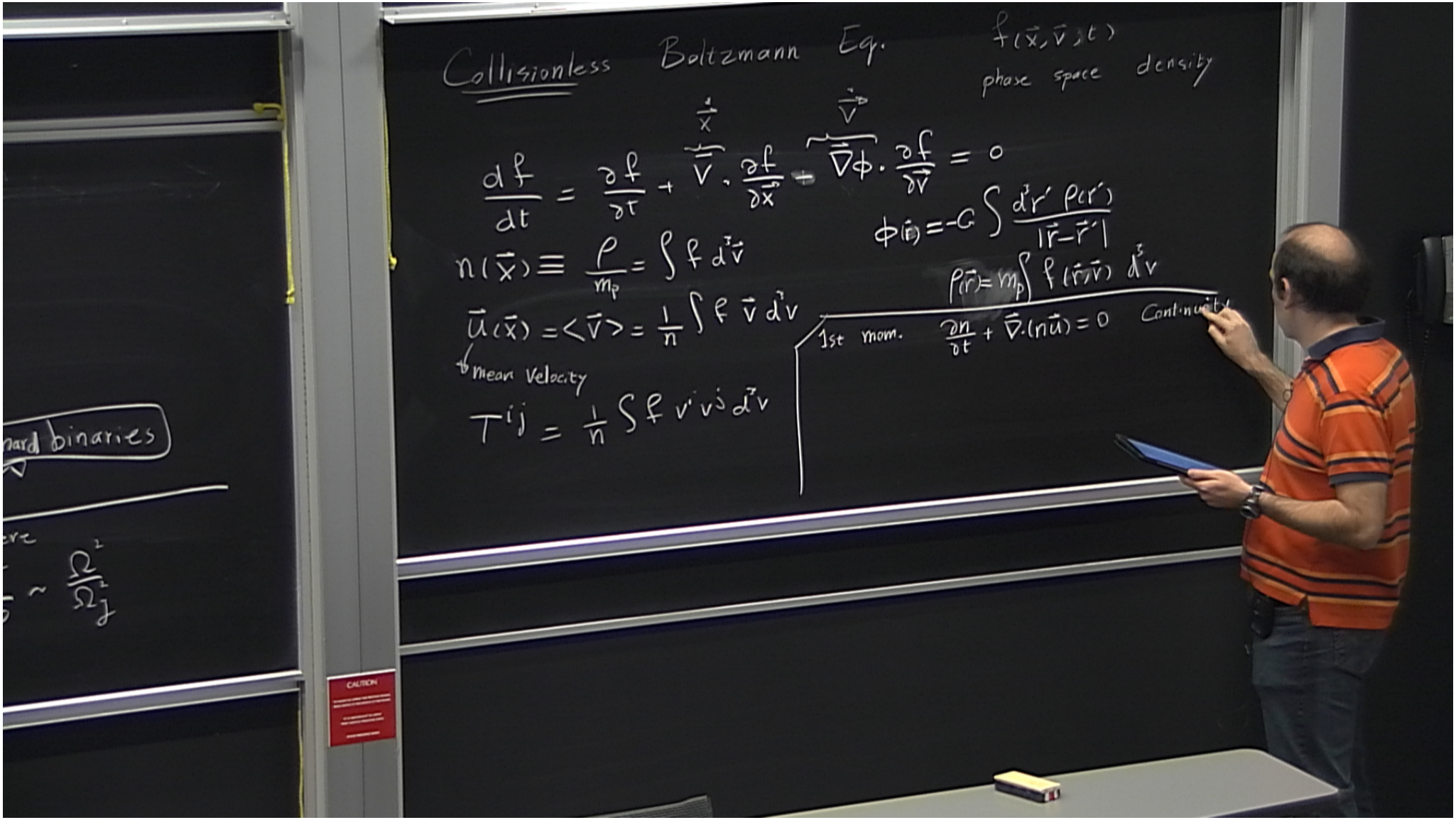

Collisionless Boltzmann Eq.
$f(\vec{x}, \vec{v}, t)$
phase space density
$\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} - \nabla\phi \cdot \frac{\partial f}{\partial \vec{v}} =$
hard binaries

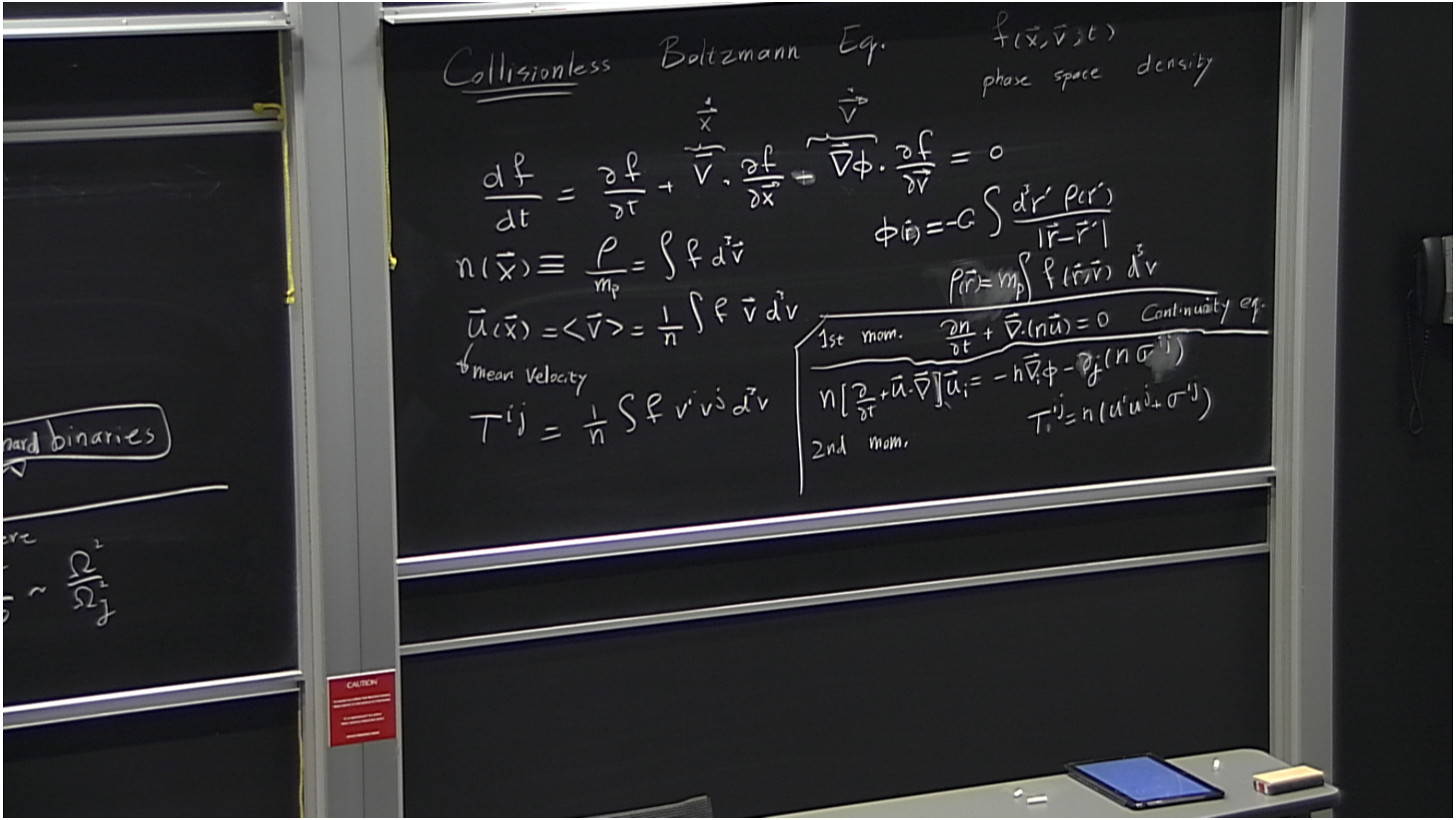

Collisionless Boltzmann Eq.
$f(\vec{x},\vec{v},t)$
phase space density
$\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{v}\cdot\frac{\partial f}{\partial \vec{x}} - \nabla\phi\cdot\frac{\partial f}{\partial \vec{v}} = 0$
$\phi(\vec{r}) = -G\int \frac{d^3r'}{|\vec{r}-\vec{r}'|}$
hard binaries
$\sim \frac{\Omega^2}{\Omega_J^2}$
CAUTION

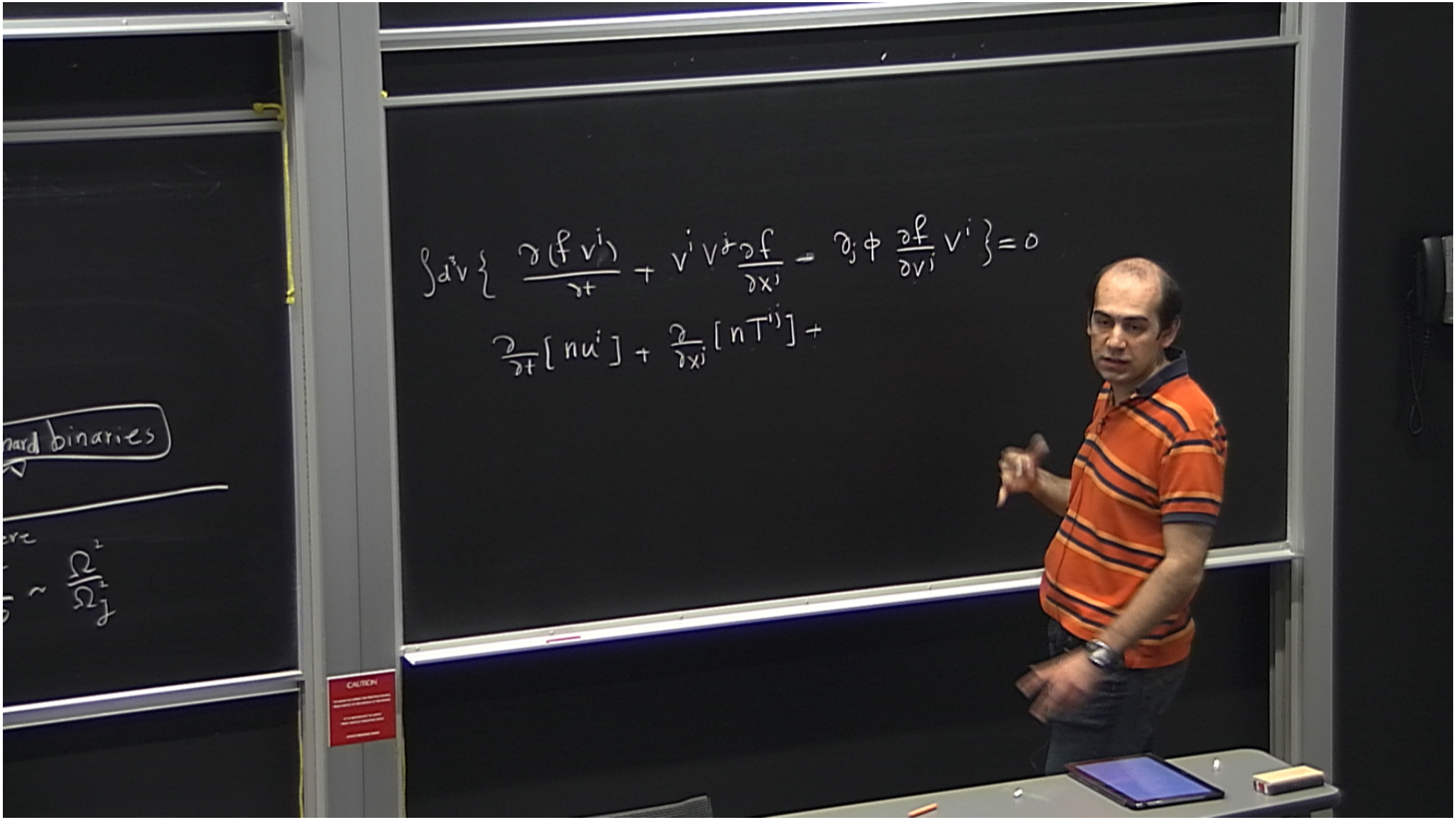

Collisionless Boltzmann Eq.
f(x, v, t)
phase space density
df/dt = ∂f/∂t + v · ∂f/∂x
∂f/∂v = 0
G ∫ d³r' ρ(r') / |r − r'|
ρ(r) = m_p ∫ f(r, v) d³v
hard binaries
CAUTION

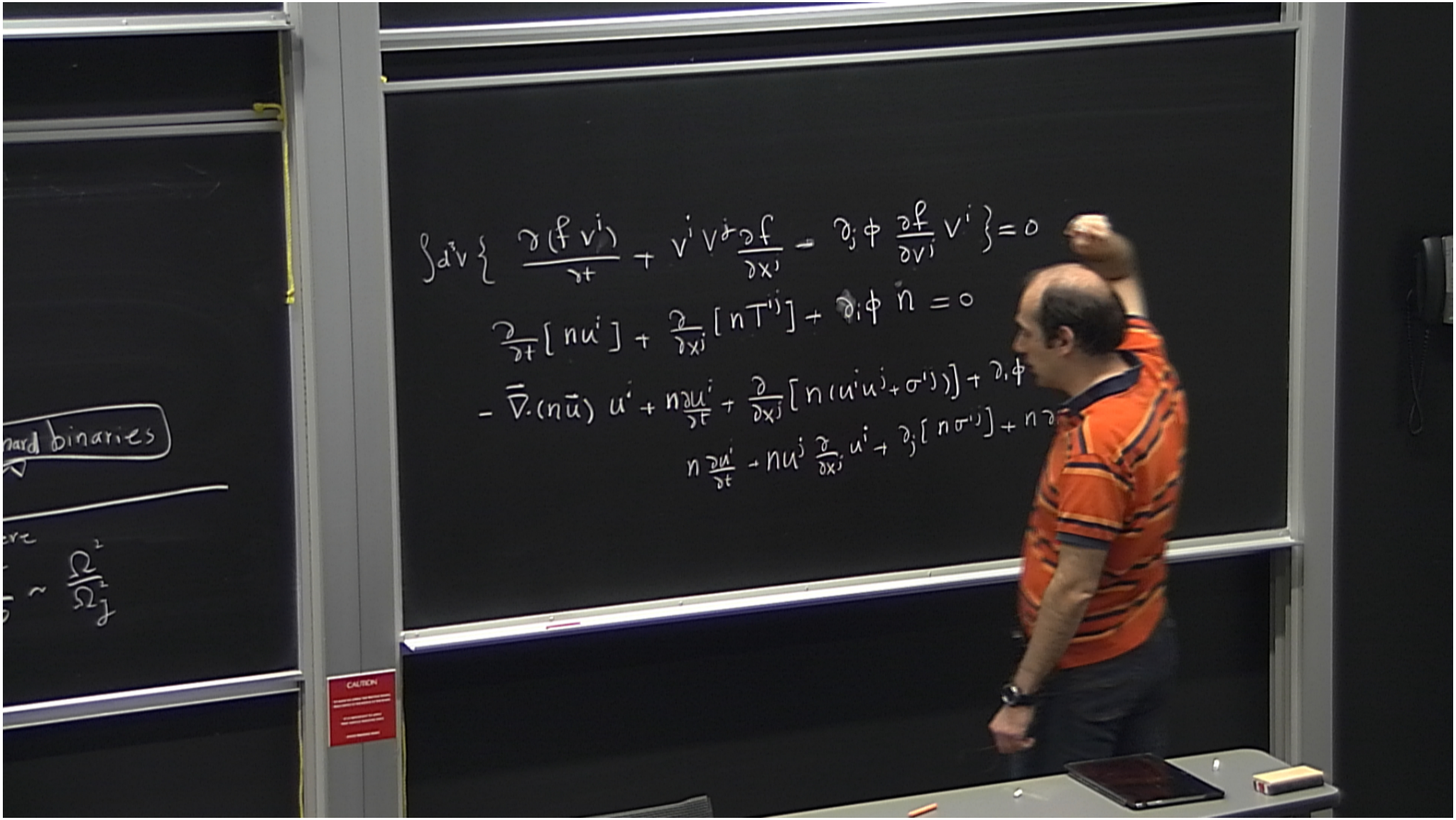

Collisionless Boltzmann Eq.
$f(\vec{x},\vec{v},t)$
phase space density
$\frac{\partial f}{\partial t} + \vec{v}\cdot\frac{\partial f}{\partial \vec{x}} - \nabla\phi\cdot\frac{\partial f}{\partial \vec{v}} = 0$
$\phi(\vec{r}) = -G\int \frac{d^3r'\,\rho(r')}{|\vec{r}-\vec{r}'|}$
$\rho(\vec{r}) = m_p\int f(\vec{r},\vec{v})\,d^3v$
hard binaries
$\sim \frac{\Omega^2}{\Omega_J^2}$
CAUTION

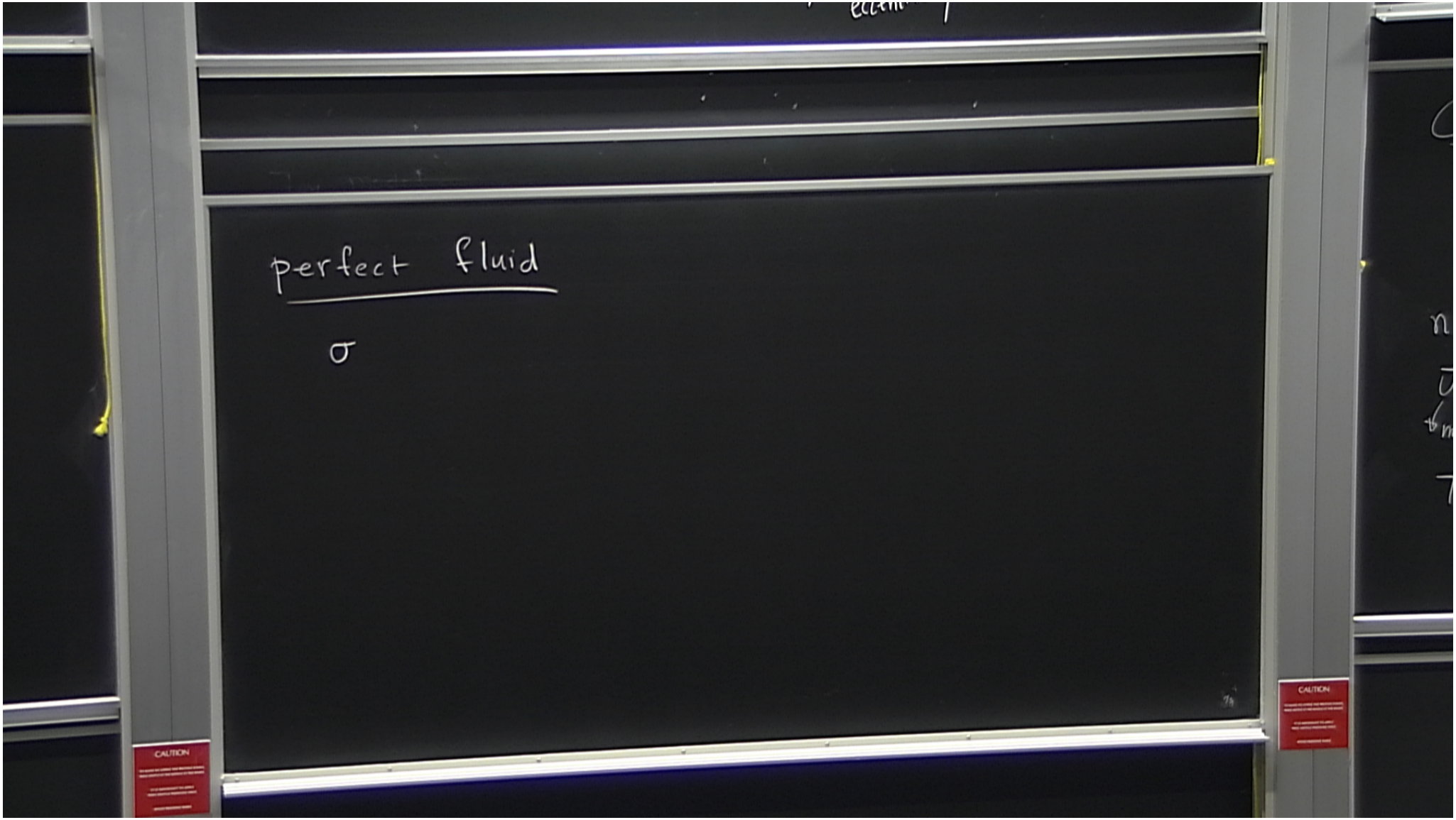

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mean Velocity
$T^{ij} = \frac{1}{n}\int f\, v^i$
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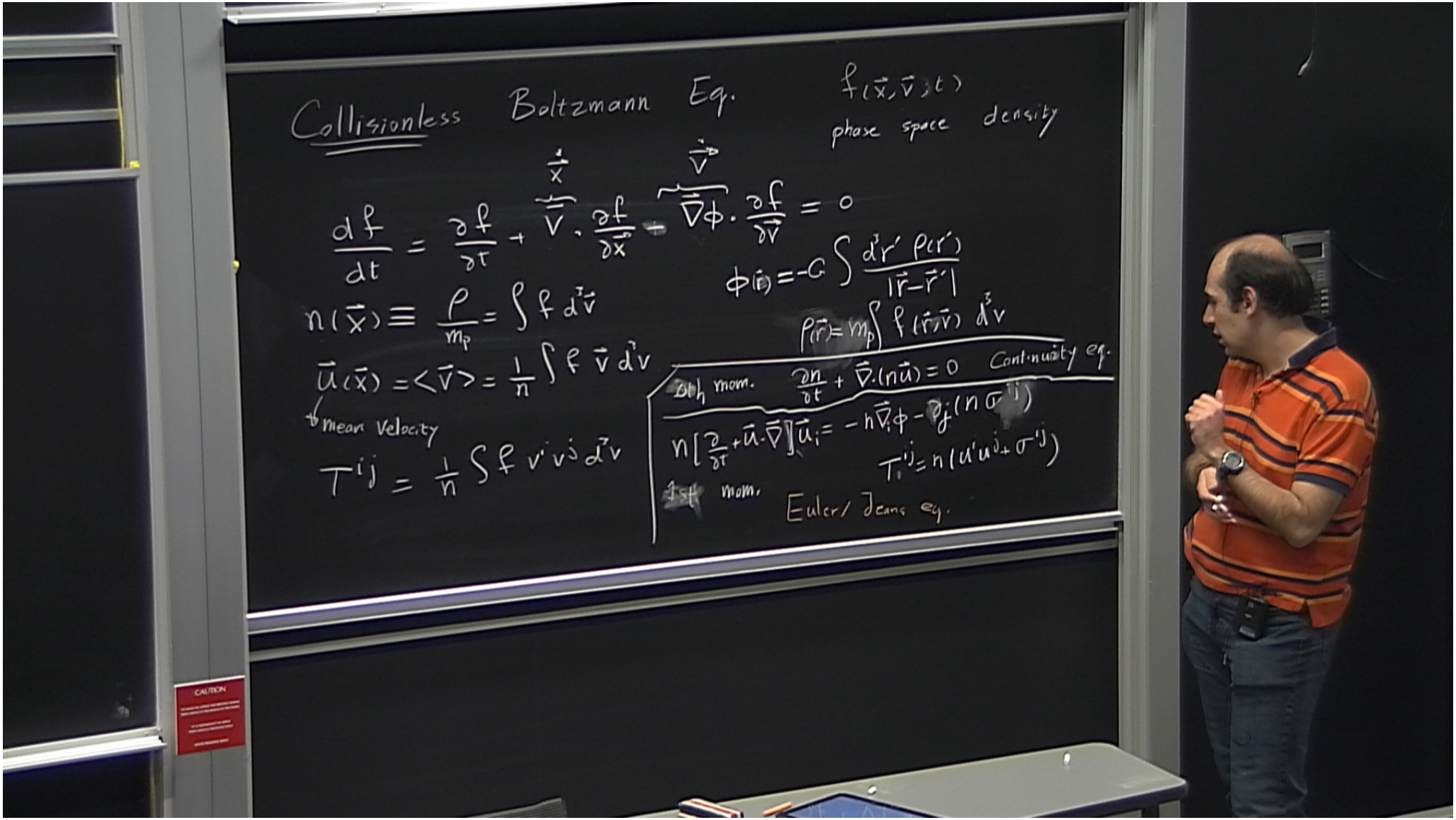

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1st mom. $\frac{\partial n}{\partial t} + \vec{\nabla}\cdot(n\vec{u}) = 0$ Continuity
hard binaries
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CAUTION

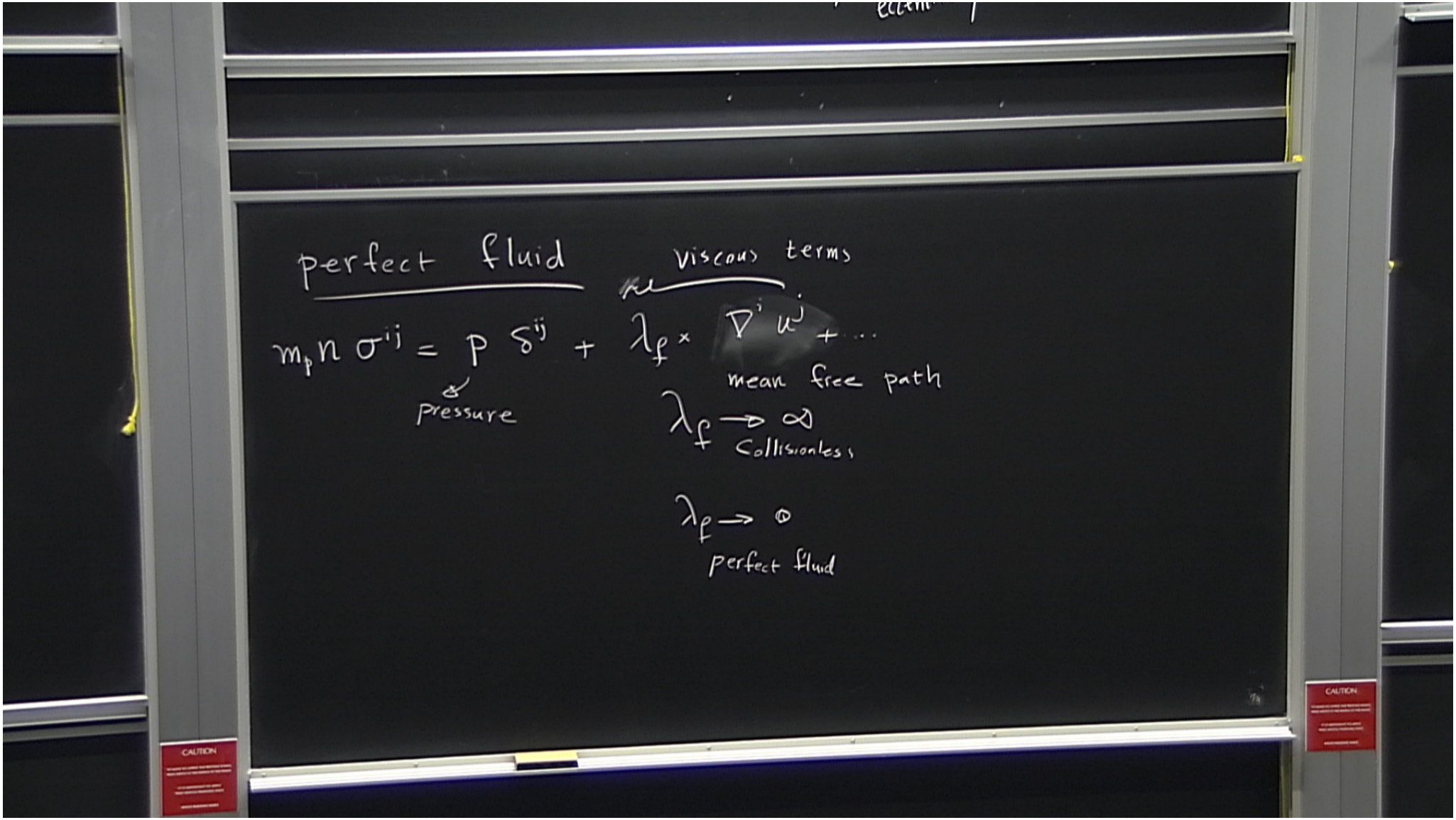

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$n\left[\frac{\partial}{\partial t} + \vec{u}\cdot\vec{\nabla}\right]\vec{u}_i = -n\vec{\nabla}_i\phi - \partial_j(n\sigma^{ij})$
2nd mom.
$T^{ij} = n(u^iu^j + \sigma^{ij})$
hard binaries
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CAUTION

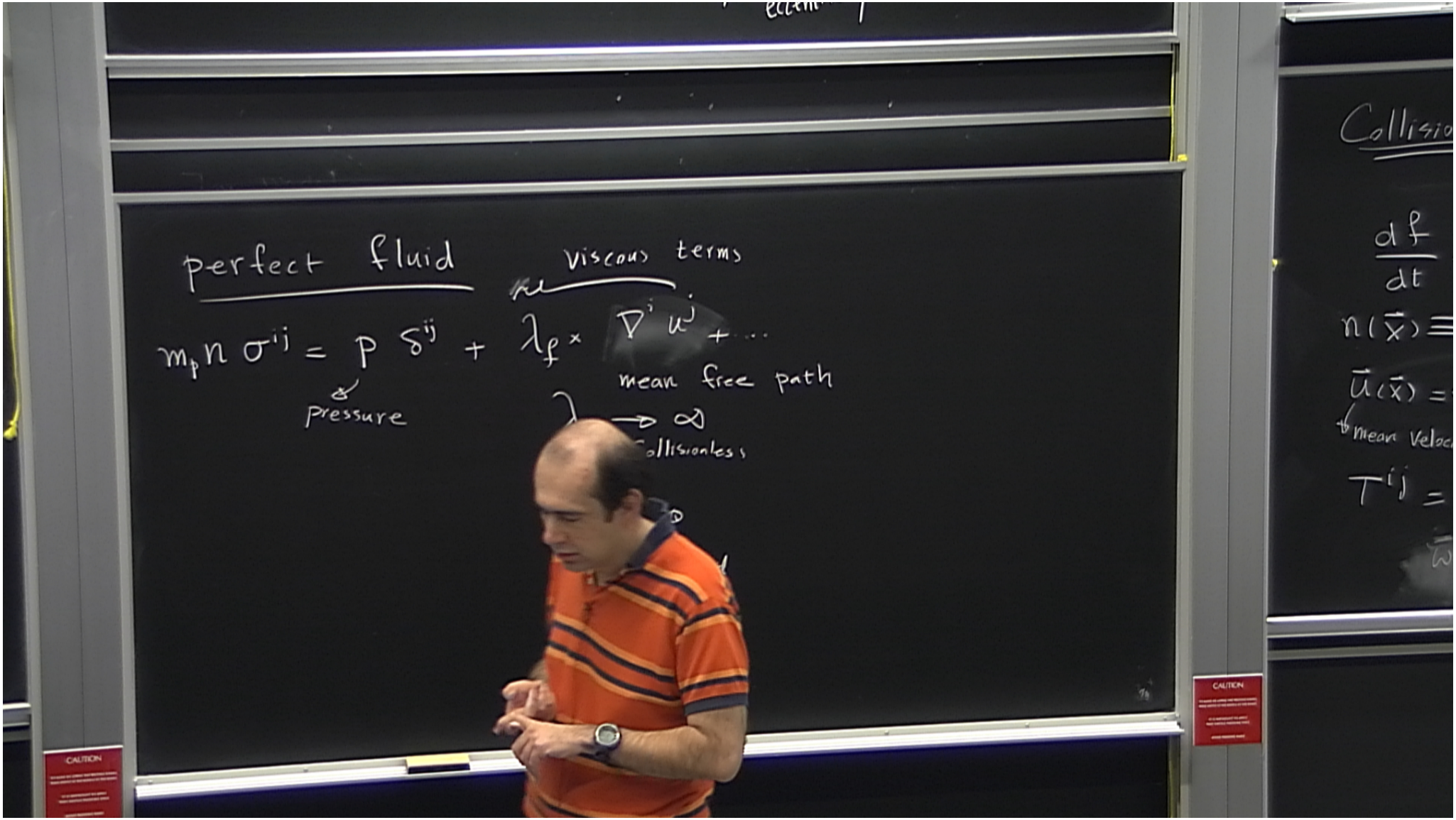

$\int d^3v \left\{ \frac{\partial (f v^i)}{\partial t} + v^i v^j \frac{\partial f}{\partial x^j} - \partial_j \phi \frac{\partial f}{\partial v_i} v^i \right\} = 0$
$\frac{\partial}{\partial t}[n u^i] + \frac{\partial}{\partial x^j}[n T^{ij}] +$
hard binaries
$\sim \frac{\Omega^2}{\Omega_J^2}$
CAUTION

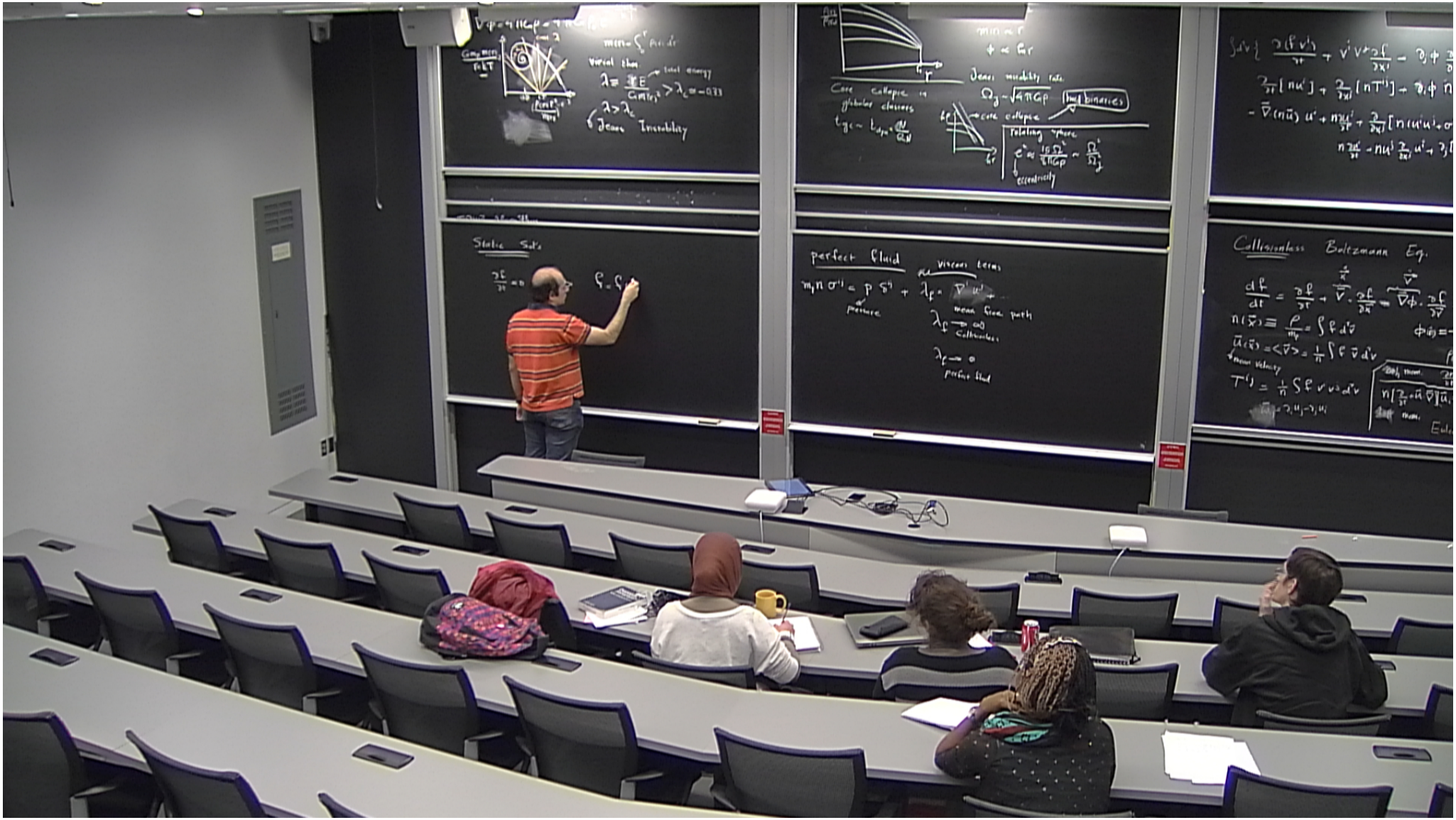

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$- \vec{\nabla}\cdot(n\vec{u})\, u^i + n \frac{\partial u^i}{\partial t} + \frac{\partial}{\partial x^j}[n(u^i u^j + \sigma^{ij})] + \partial_i \phi$
$n \frac{\partial u^i}{\partial t} + n u^j \frac{\partial}{\partial x^j} u^i + \partial_j [n \sigma^{ij}] + n \partial$
hard binaries
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CAUTION

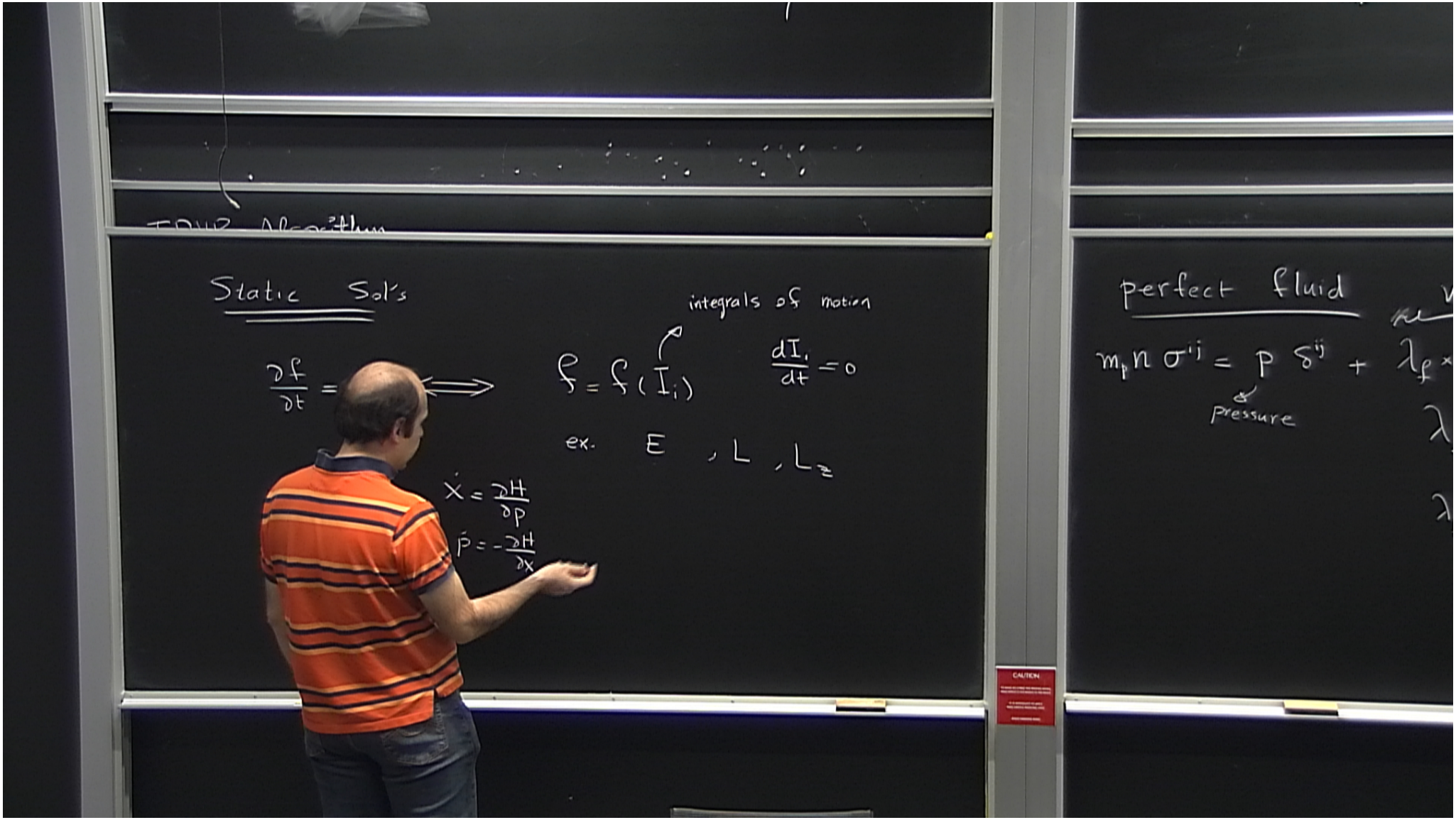

perfect fluid
σ

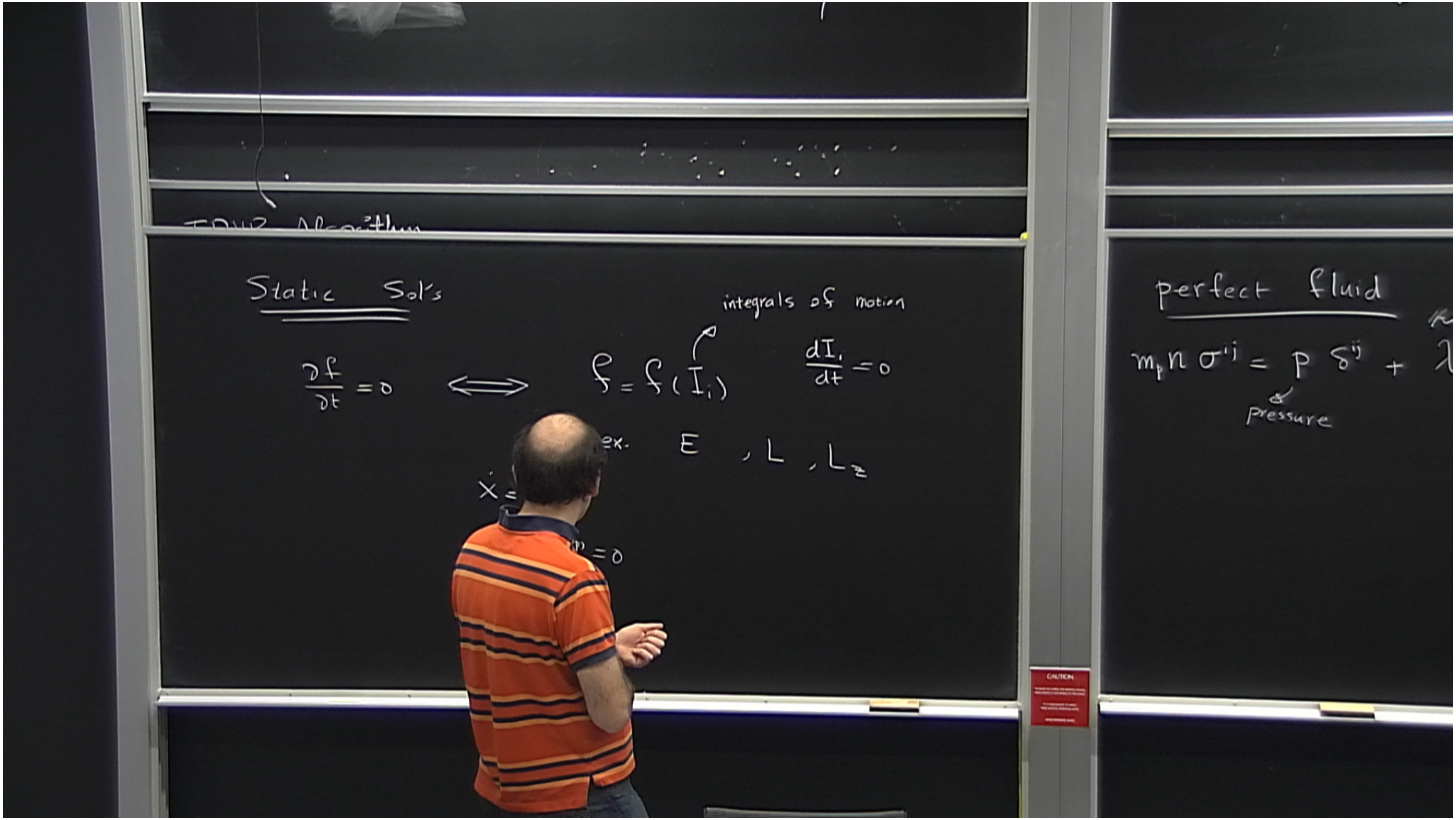

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$T^{ij} = n(u^iu^j + \sigma^{ij})$
1st mom.
Euler/Jeans eq.

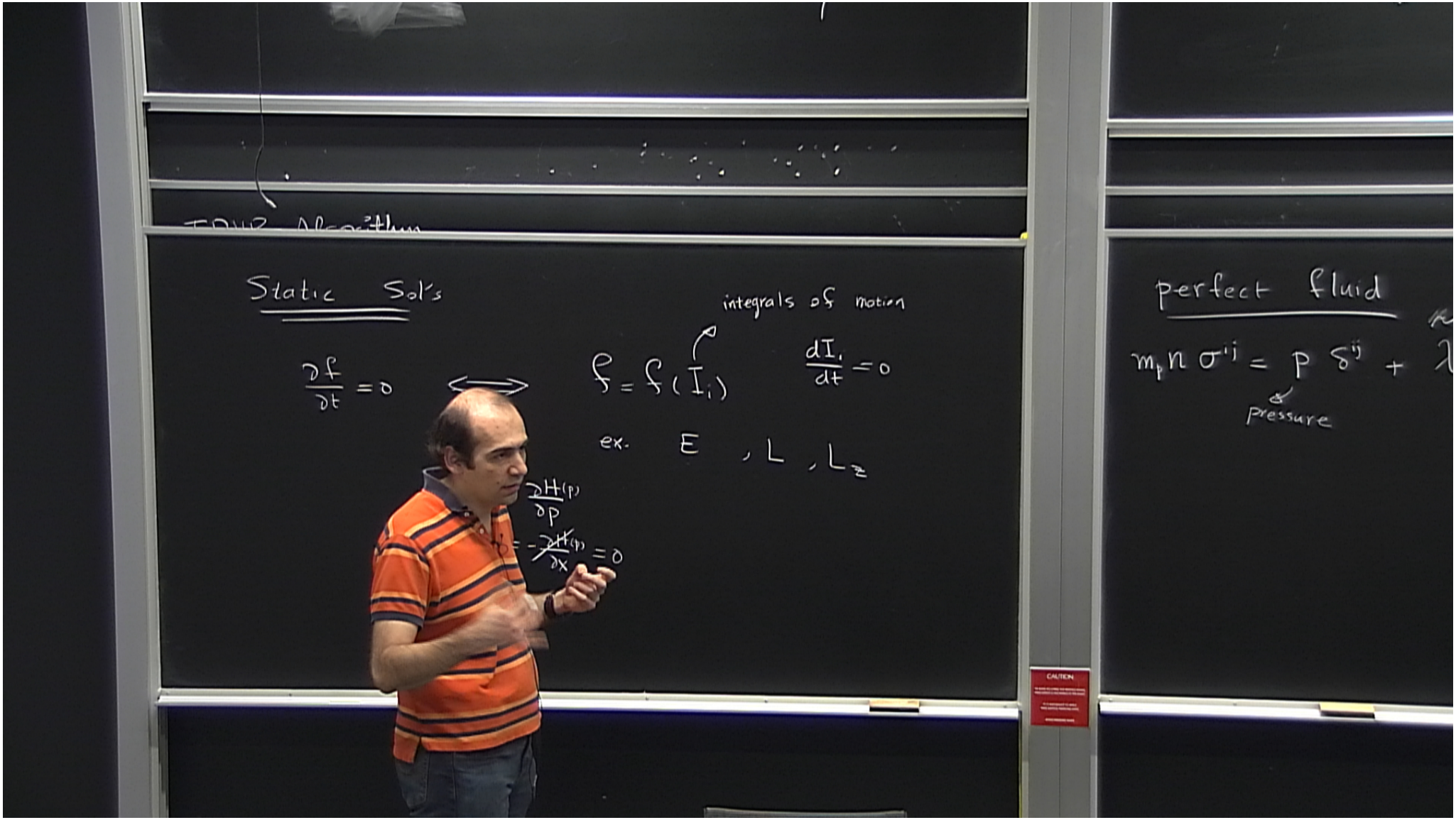

perfect fluid
viscous terms
$m_p n \sigma^{ij} = p \delta^{ij} + \lambda_f \times \nabla^i u^j + \ldots$
pressure
mean free path
$\lambda_f \to \infty$
Collisionless
$\lambda_f \to 0$
perfect fluid

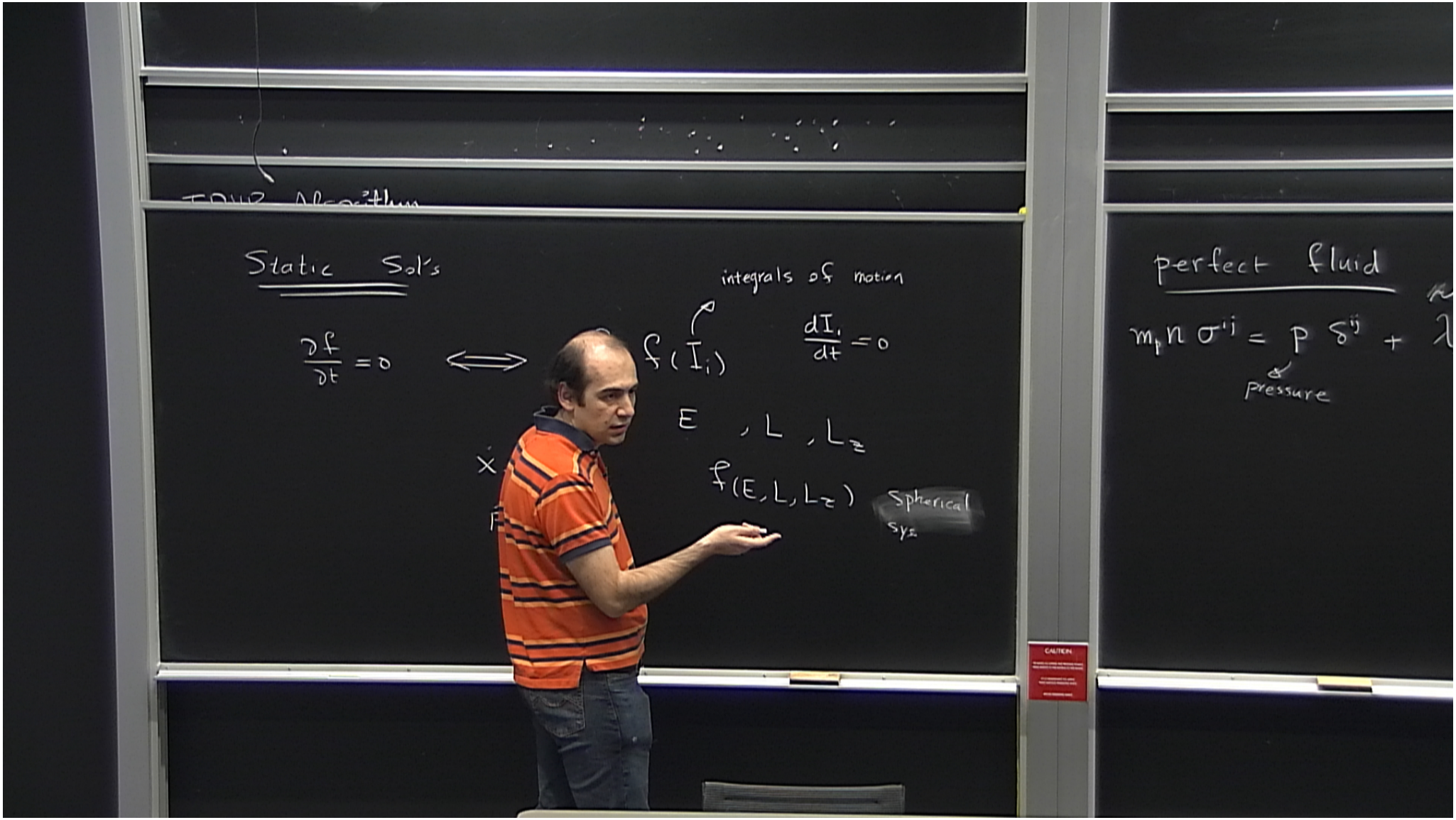

perfect fluid
viscous terms
$m_p n \sigma^{ij} = p \delta^{ij} + \lambda_f \times \nabla^i u^j + \dots$
pressure
mean free path
$\lambda \to \infty$
collisionless
Collisio
$\frac{df}{dt}$
$n(\vec{x}) \equiv$
$\vec{u}(\vec{x}) =$
mean Veloc
$T^{ij} =$
CAUTION
CAUTION

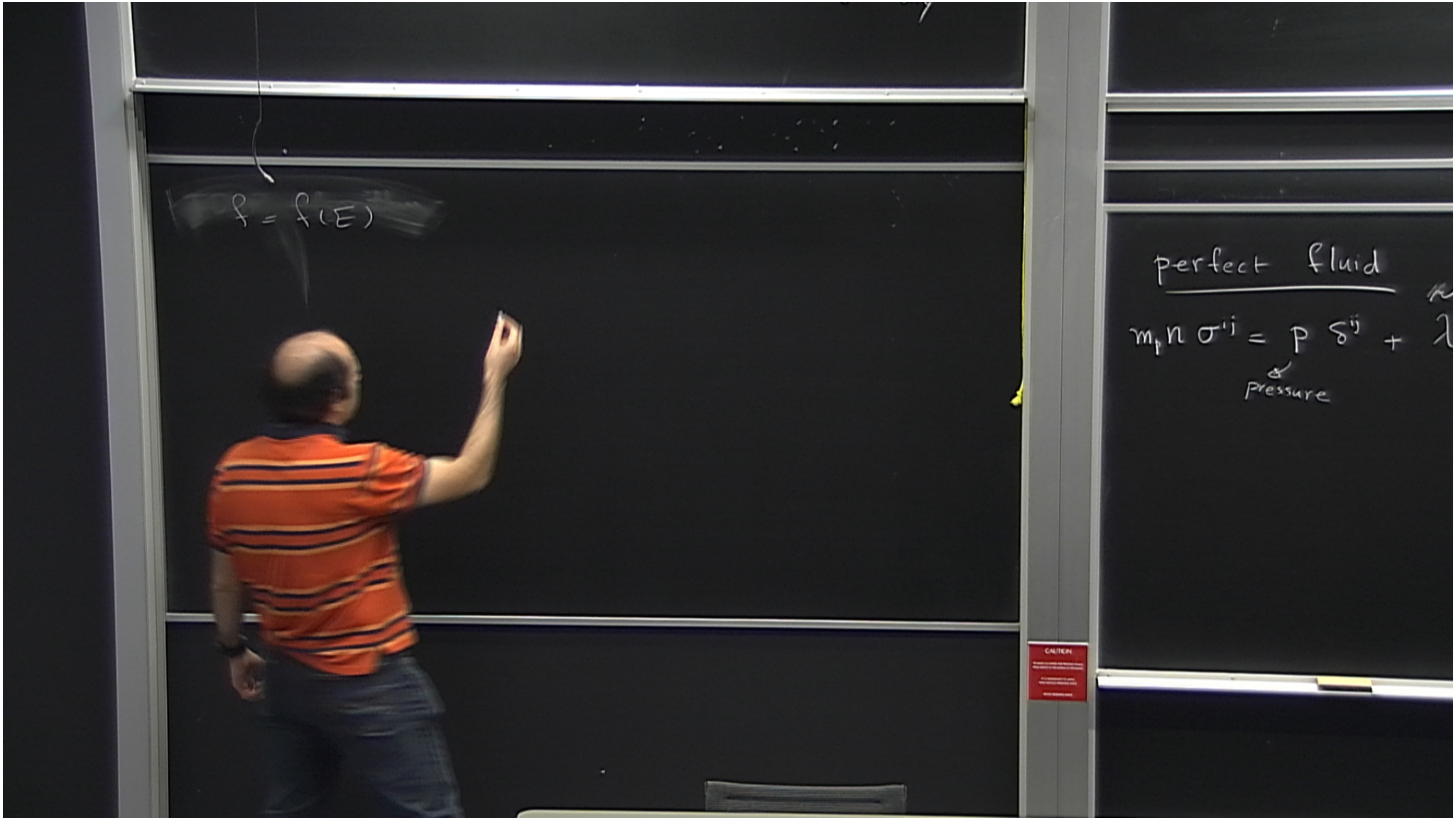

Static Sol's
perfect fluid
viscous terms
Collisionless Boltzmann Eq.
Jeans instability rate
Core collapse in globular clusters
core collapse
Rotating sphere
eccentricity
mean velocity

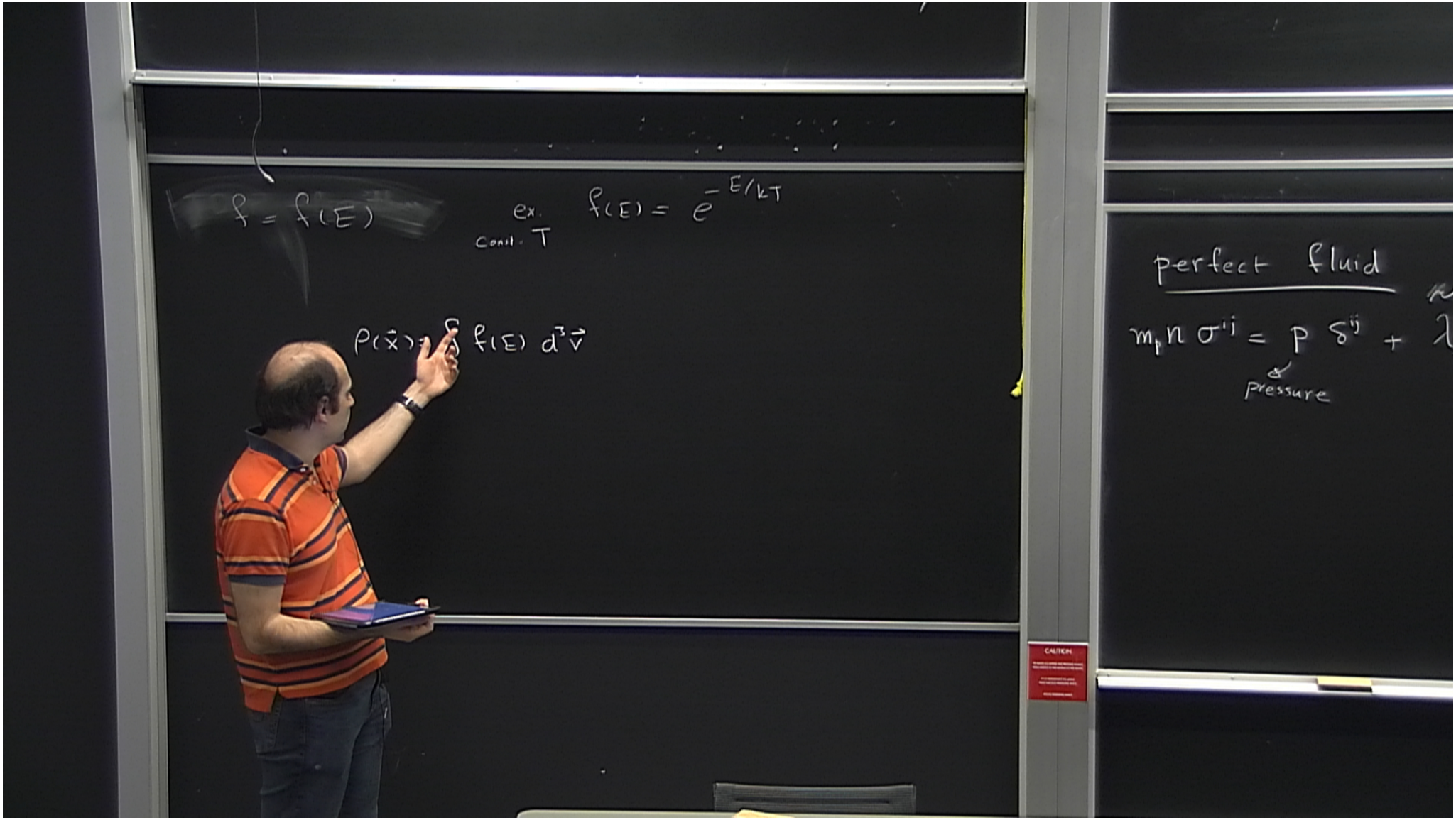

Static Sol's
integrals of motion
$\frac{\partial f}{\partial t} = 0 \Longrightarrow f = f(I_i)$
$\frac{dI_i}{dt} = 0$
ex. E , L , L_z
$\dot{X} = \frac{\partial H}{\partial p}$
$\dot{p} = -\frac{\partial H}{\partial x}$
perfect fluid
$m_p n \sigma^{ij} = p\,\delta^{ij} + \lambda_f$
pressure
CAUTION

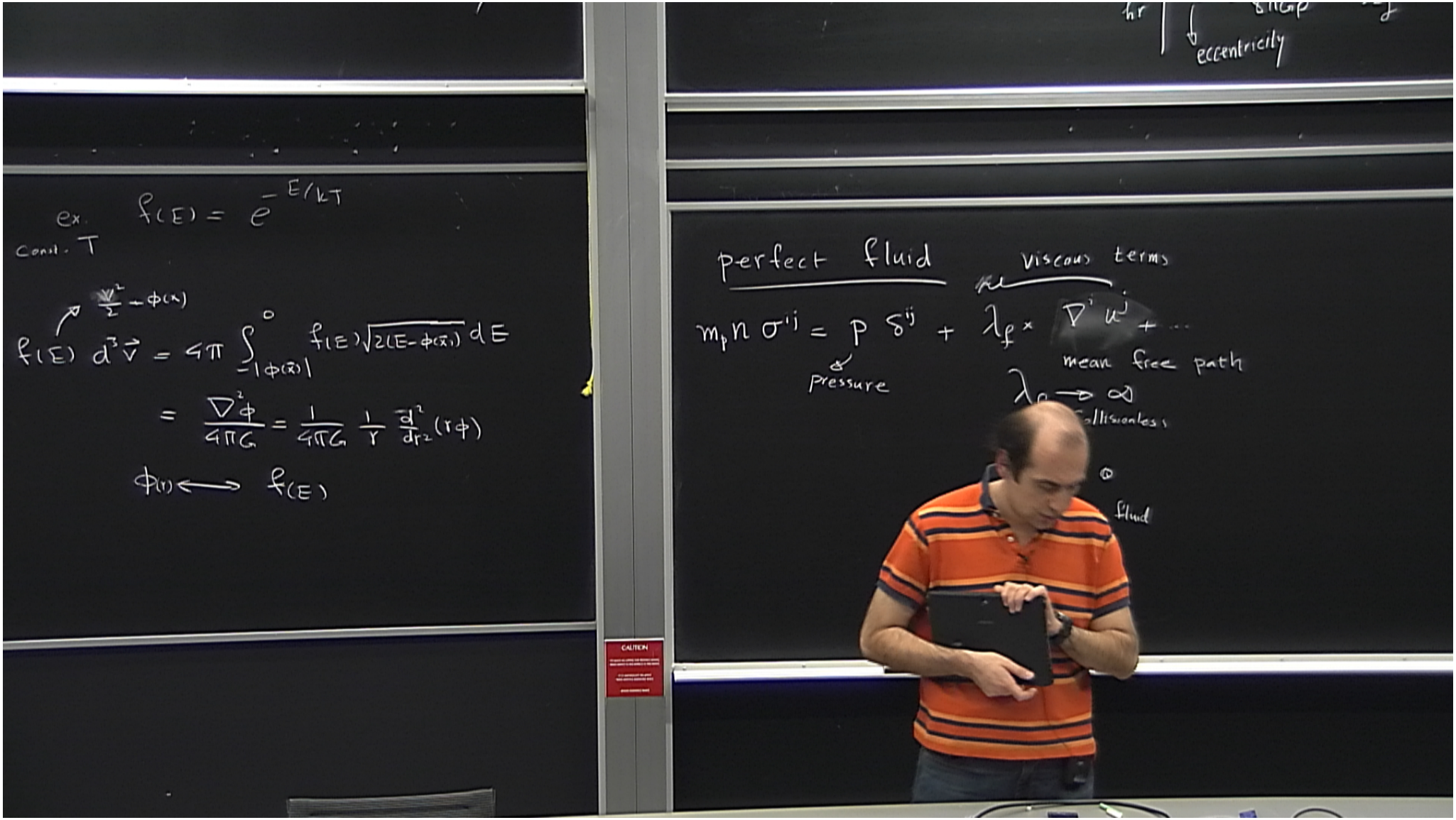

Static Sol's
integrals of motion
$\frac{\partial f}{\partial t} = 0 \iff f = f(I_i)$
$\frac{dI_i}{dt} = 0$
ex. E , L , L_z
perfect fluid
$m_p n \sigma^{ij} = p \delta^{ij} +$
pressure
CAUTION


Static Sol's
integrals of motion
$\frac{\partial f}{\partial t} = 0$
$f = f(I_i)$
$\frac{dI_i}{dt} = 0$
ex. E , L , L_z
$\frac{\partial H(p)}{\partial p}$
$\frac{\partial H(p)}{\partial x} = 0$
perfect fluid
$m_p n \sigma^{ij} = p \delta^{ij} +$
pressure
CAUTION


Static Sol's
integrals of motion
$\frac{\partial f}{\partial t} = 0 \iff f(I_i)$
$\frac{dI_i}{dt} = 0$
E , L , L_z
$f(E, L, L_z)$
Spherical
perfect fluid
$m_p n \sigma^{ij} = p \delta^{ij} +$
Pressure


f = f(E)
perfect fluid
$m_i n \sigma'^{ij} = p \delta^{ij} +$
Pressure


$f = f(E)$
ex. const. T
$f(E) = e^{-E/kT}$
$\rho(\vec{x}) = \int f(E)\, d^3\vec{v}$
perfect fluid
$m_p n \sigma^{ij} = p\, \delta^{ij} +$
pressure


ex. $f(E) = e^{-E/kT}$
const. T
$f(E)\, d^3\vec{v} = 4\pi \int_{-|\phi(\vec{x})|}^{0} f(E)\sqrt{2(E-\phi(\vec{x}))}\, dE$
$= \frac{\nabla^2\phi}{4\pi G} = \frac{1}{4\pi G}\frac{1}{r}\frac{d^2}{dr^2}(r\phi)$
$\phi(r) \longleftrightarrow f(E)$
eccentricity
perfect fluid
viscous terms
$m_p n \sigma^{ij} = p\,\delta^{ij} + \lambda_f \times \nabla^i u^j + \ldots$
pressure
mean free path
$\lambda_f \to \infty$
collisionless
fluid
CAUTION