Title: De Sitter Wavefunctionals and the Resummation of Time

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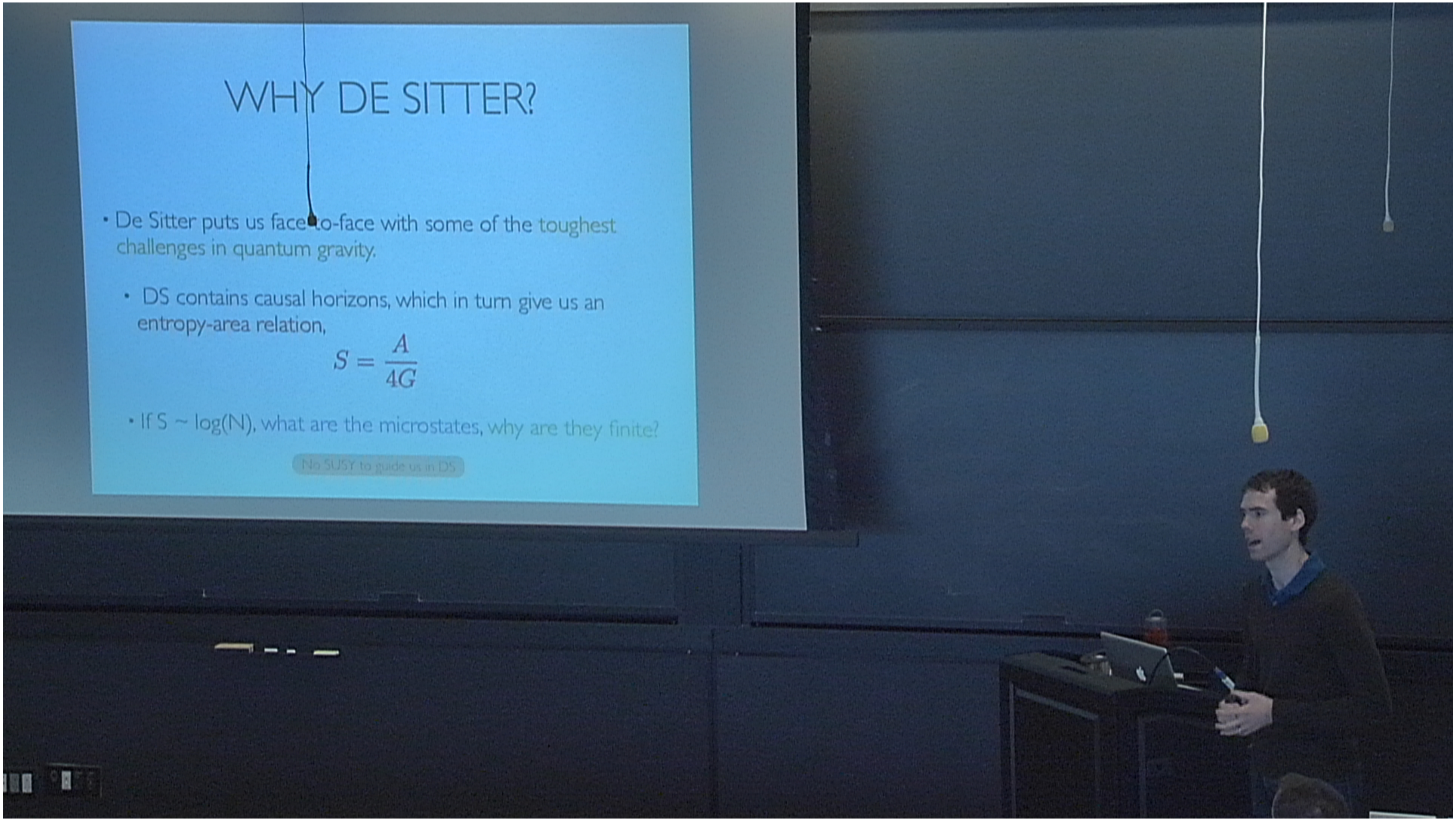
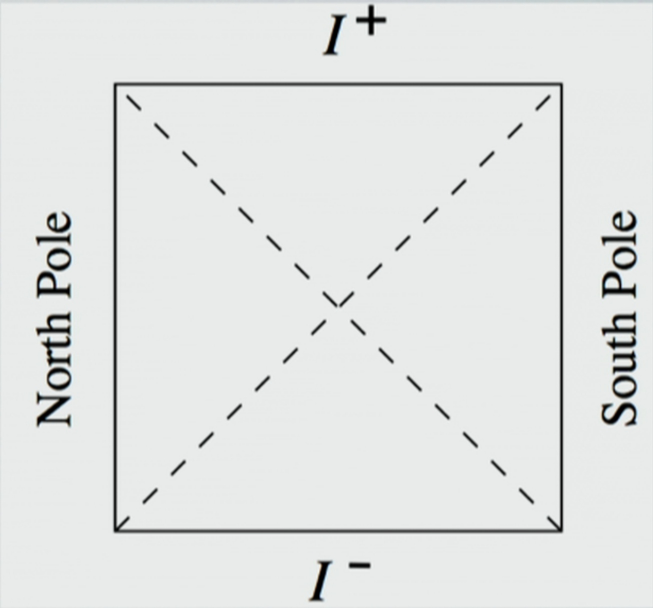
Abstract: <span>The holographic RG of Anti-De Sitter gives a powerful clue about the underlying AdS/CFT correspondence. The question is whether similar hints can be found for the heretofore elusive holographic dual of De Sitter. The framework of stochastic inflation uses nonperturbative insight to tame bad behavior in the perturbation series of a massless scalar in DS at late times. Remarkably, this fully quantum system loses phase information in the leading approximation, but retains a probabilistic character and allows for a controlled prediction of late time Green's functions. Recasting this as a "resummation of time", we wish understand whether the distributions that result can be thought of as an attractive UV fixed point of a theory<br> living on a spacelike slice of DS. We derive stochastic inflation via the wavefunctional approach to Quantum Field Theory. This allows for the straightforward implementation of corrections to the original framework.</span>


WHY DE SITTER?
• De Sitter puts us face-to-face with some of the toughest challenges in quantum gravity.
• DS contains causal horizons, which in turn give us an entropy-area relation,
$S = \frac{A}{4G}$
• If S ~ log(N), what are the microstates, why are they finite?
No SUSY to guide us in DS

# THE BOUNDARIES OF DE SITTER

- The horizon that gives us entropy also confuses the boundary formulation.
- Should we formulate the theory on the global boundary? horizon? complimentarity?
- Do we need to understand the emergence of a timelike dimension?




Penrose diagram of DS
Past and future infinities are spacelike
Observer only has causal contact in triangle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I+
North Pole
South Pole
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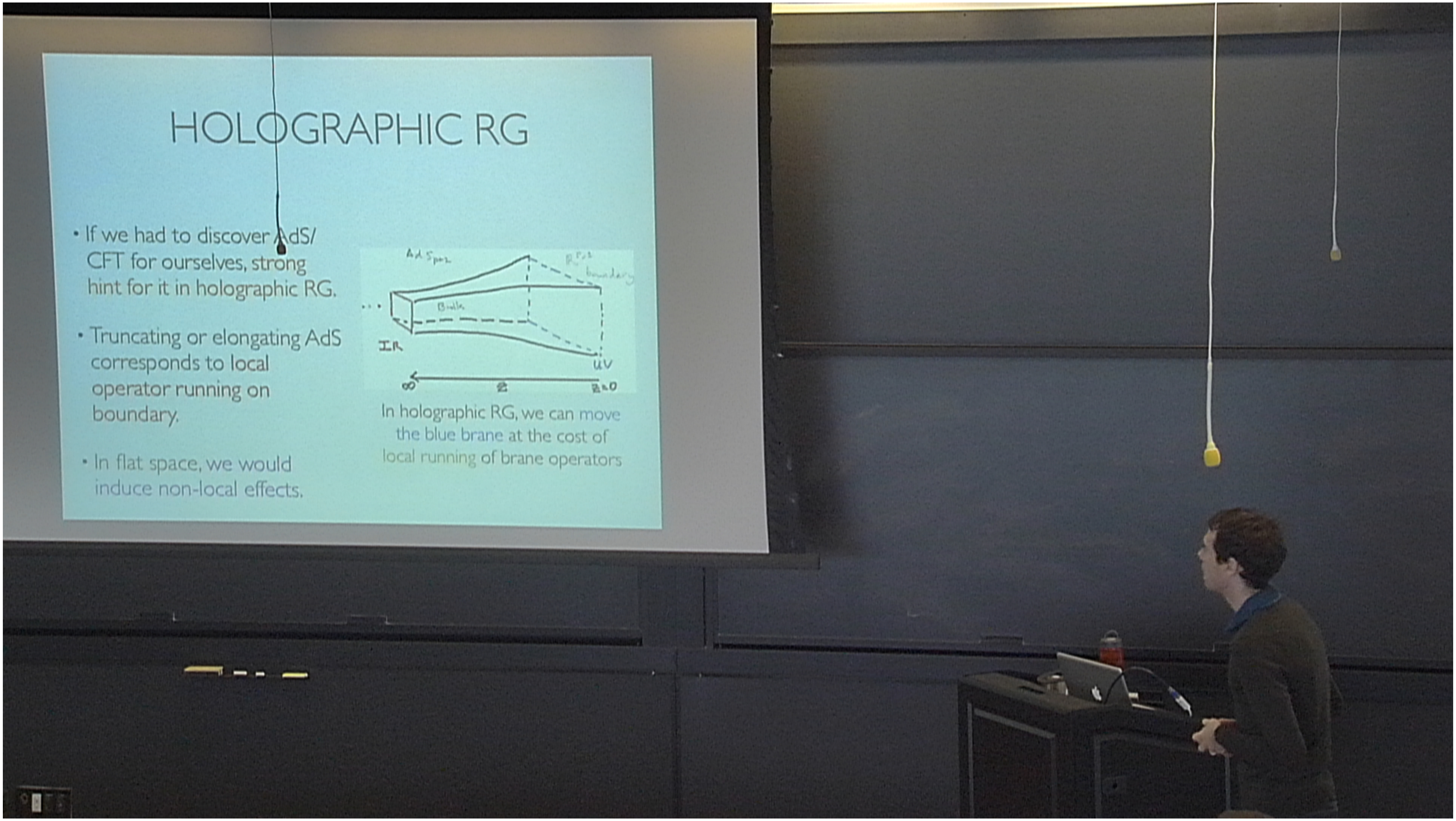
# DS/CFT CORRESPONDENCE?

- Perhaps DS has a CFT dual in analogy with its cousin, AdS.

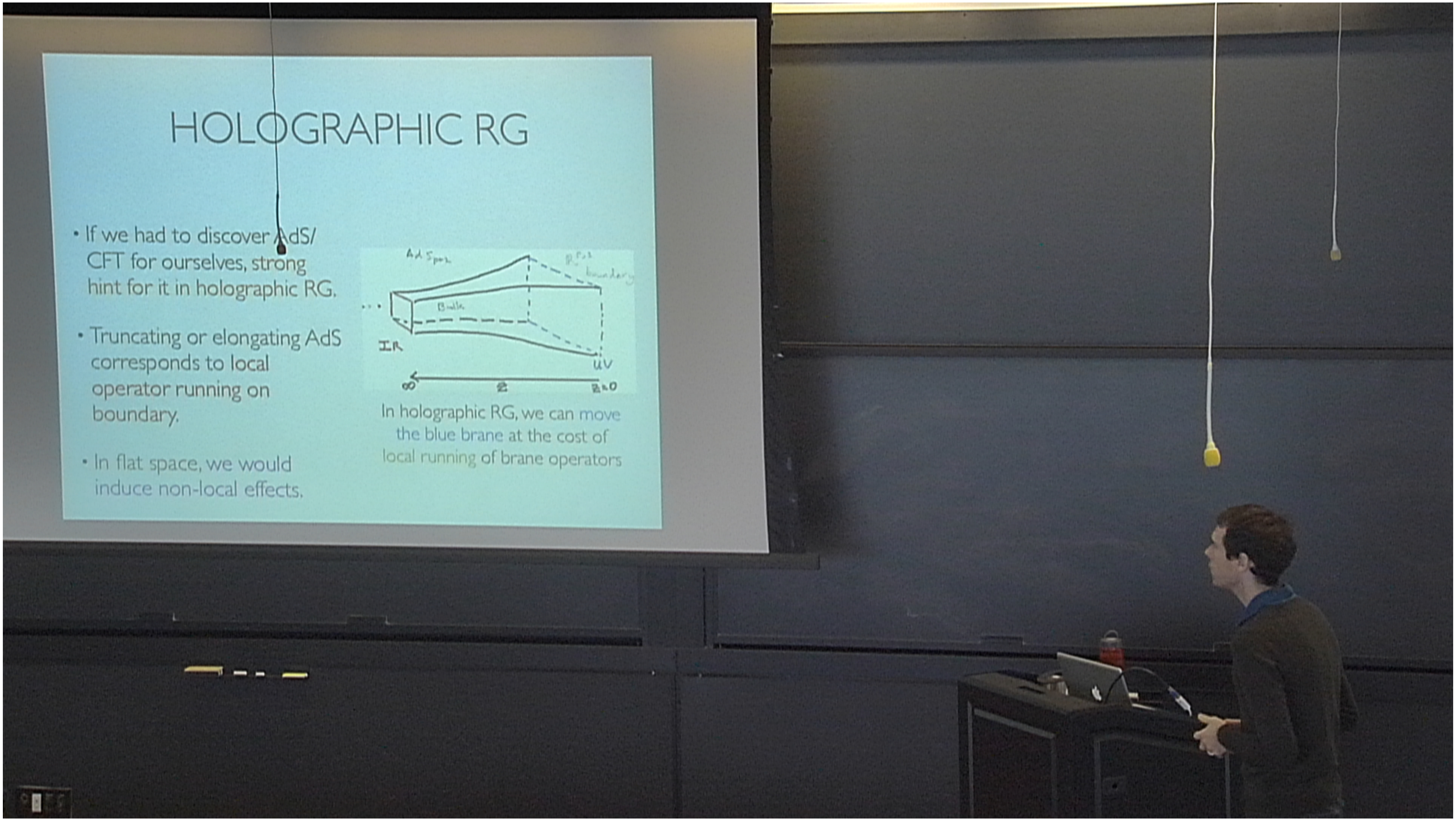
$$ds^2_{DS} = \frac{1}{(H\eta)^2}(d\eta^2 - d\vec{x}_i^2)$$

$$ds^2_{AdS} = \frac{1}{(kz)^2}(dt^2 - d\vec{x}_{i-1}^2 - dz^2)$$

- Boundary possesses SO(D,1) symmetry, the conformal group for $\boldsymbol{R}^{d-1}$.
- There are complications:
  - Where to define?
  - Naive analytic continuation gives wrong DS ground state, complex anomalous dimensions
  - CFT is non-unitary, best formulation?



HOLOGRAPHIC RG
• If we had to discover AdS/CFT for ourselves, strong hint for it in holographic RG.
• Truncating or elongating AdS corresponds to local operator running on boundary.
• In flat space, we would induce non-local effects.
AdS p+2
boundary
Bulk
IR
UV
∞
z
z=0
In holographic RG, we can move the blue brane at the cost of local running of brane operators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UV
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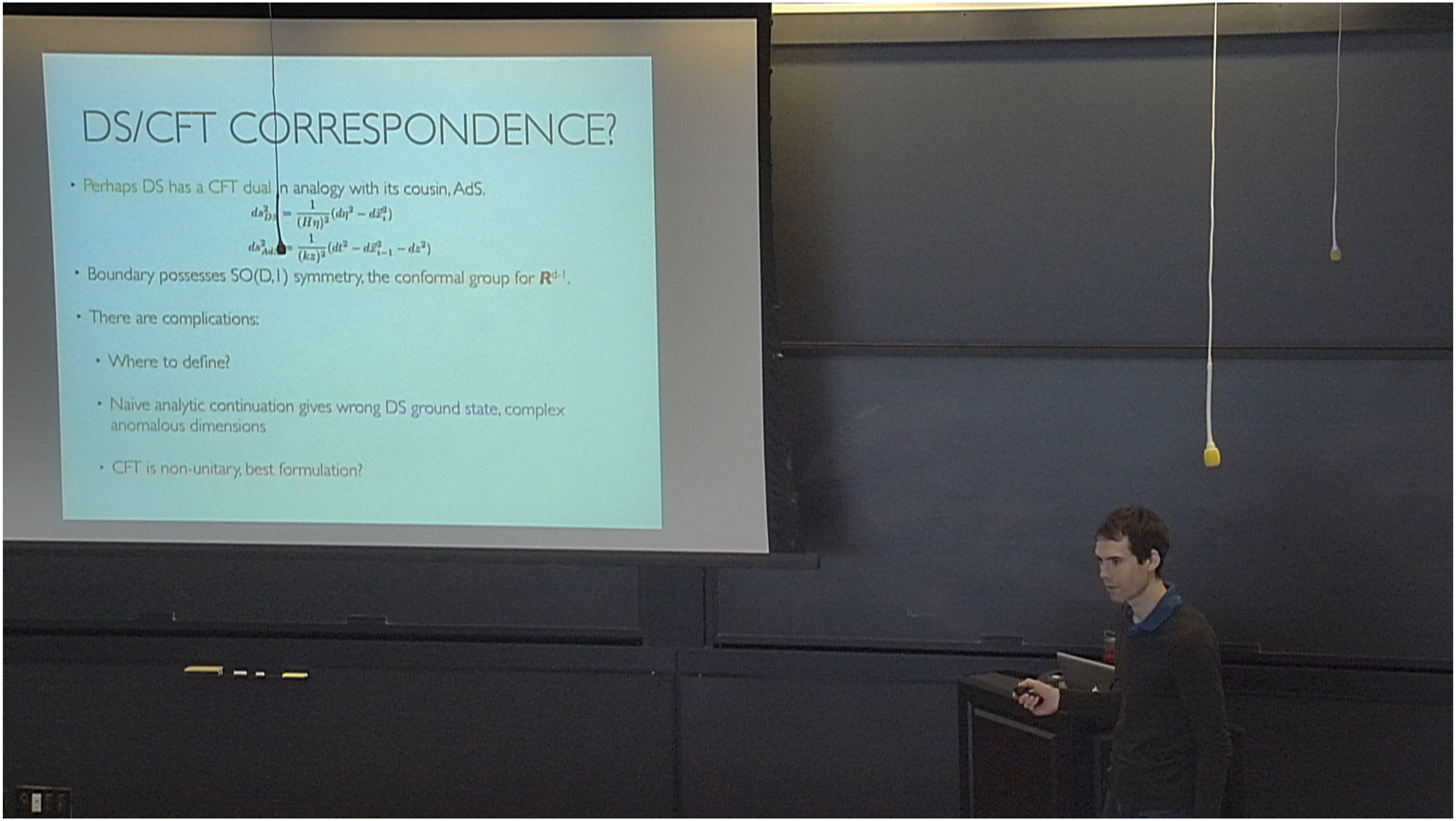
HOLOGRAPHIC RG IN DE SITTER?
• The analog of the radial direction in AdS is the timelike direction in DS.
• We are thus looking for a theory that exhibits the "resummation of time."
• Such an example has long been known, Stochastic Inflation (Starobinsky, 1986).



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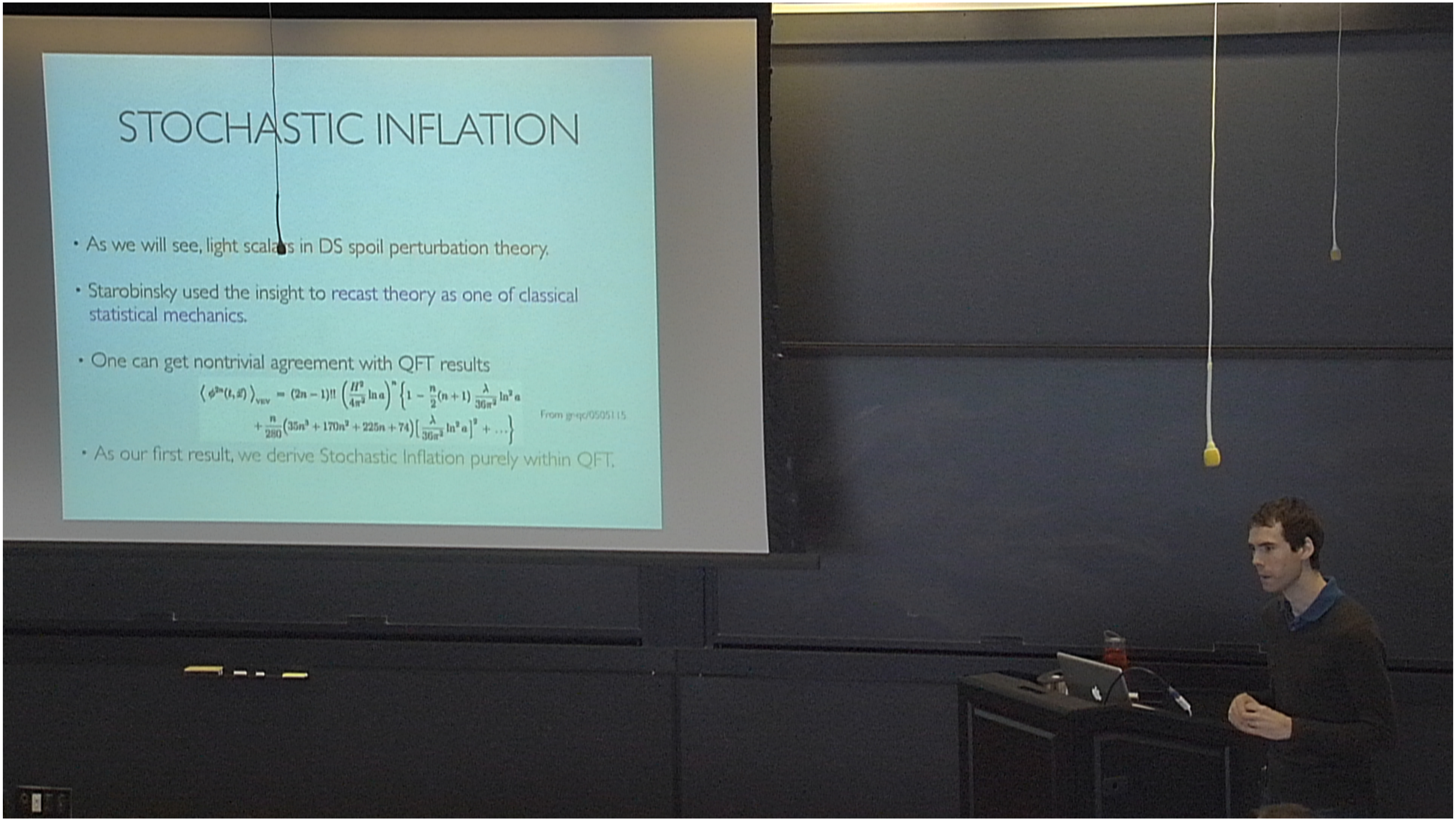
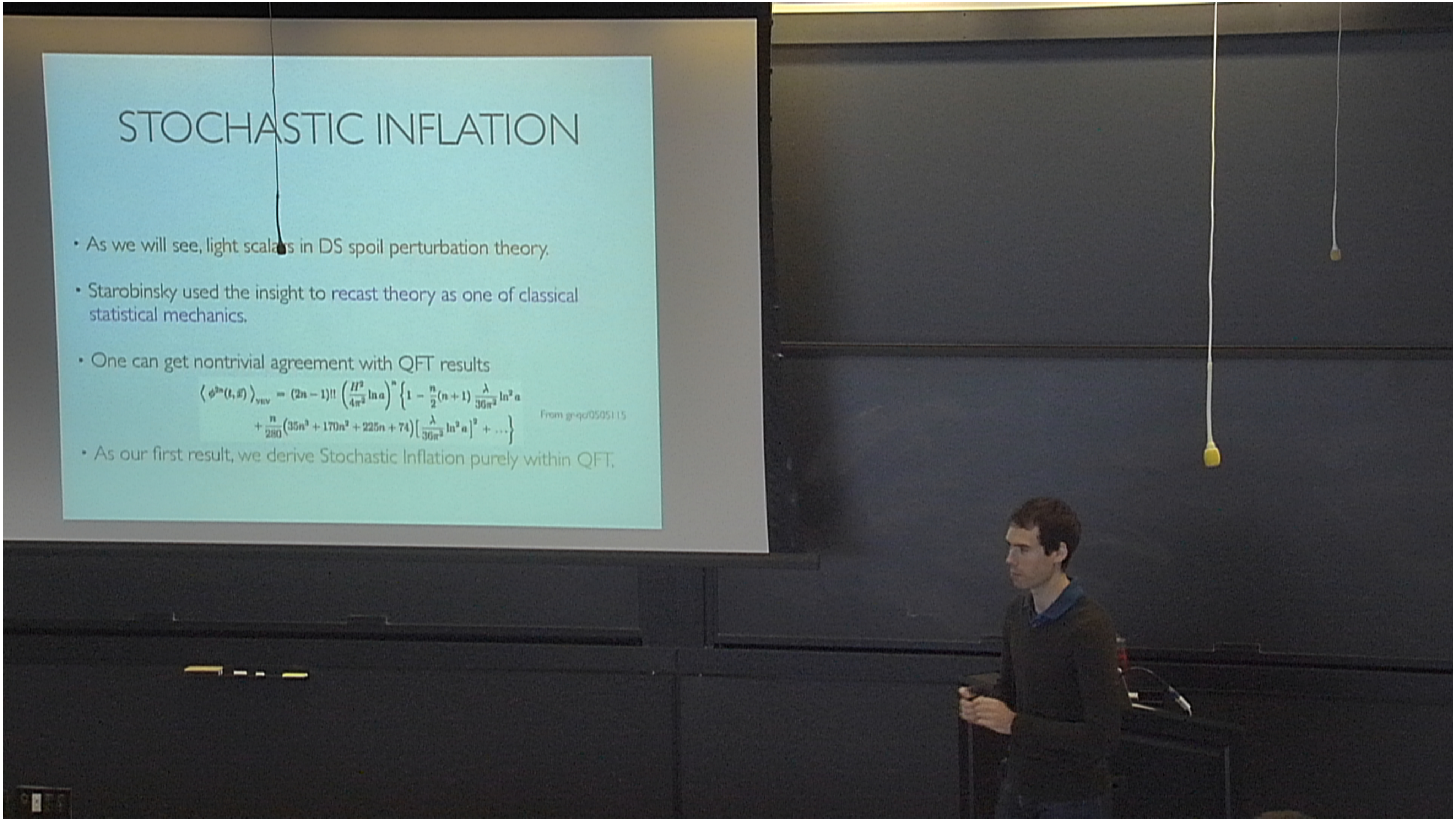

STOCHASTIC INFLATION
• As we will see, light scalars in DS spoil perturbation theory.
• Starobinsky used the insight to recast theory as one of classical statistical mechanics.
• One can get nontrivial agreement with QFT results
$\langle \phi^{2n}(t,\vec{x}) \rangle_{\text{VEV}} = (2n-1)!! \left(\frac{H^2}{4\pi^2}\ln a\right)^n \Big\{1 - \frac{n}{2}(n+1)\frac{\lambda}{36\pi^2}\ln^2 a + \frac{n}{280}(35n^3+170n^2+225n+74)\Big[\frac{\lambda}{36\pi^2}\ln^2 a\Big]^2 + \ldots\Big\}$
From gr-qc/0505115
• As our first result, we derive Stochastic Inflation purely within QFT.



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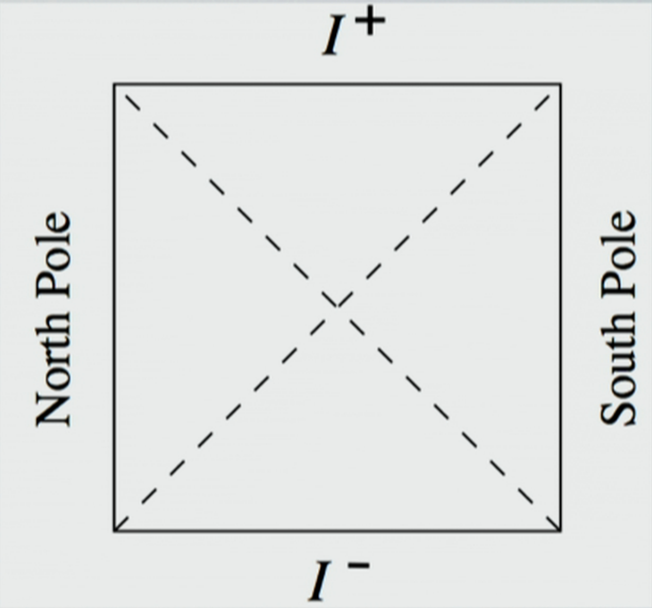


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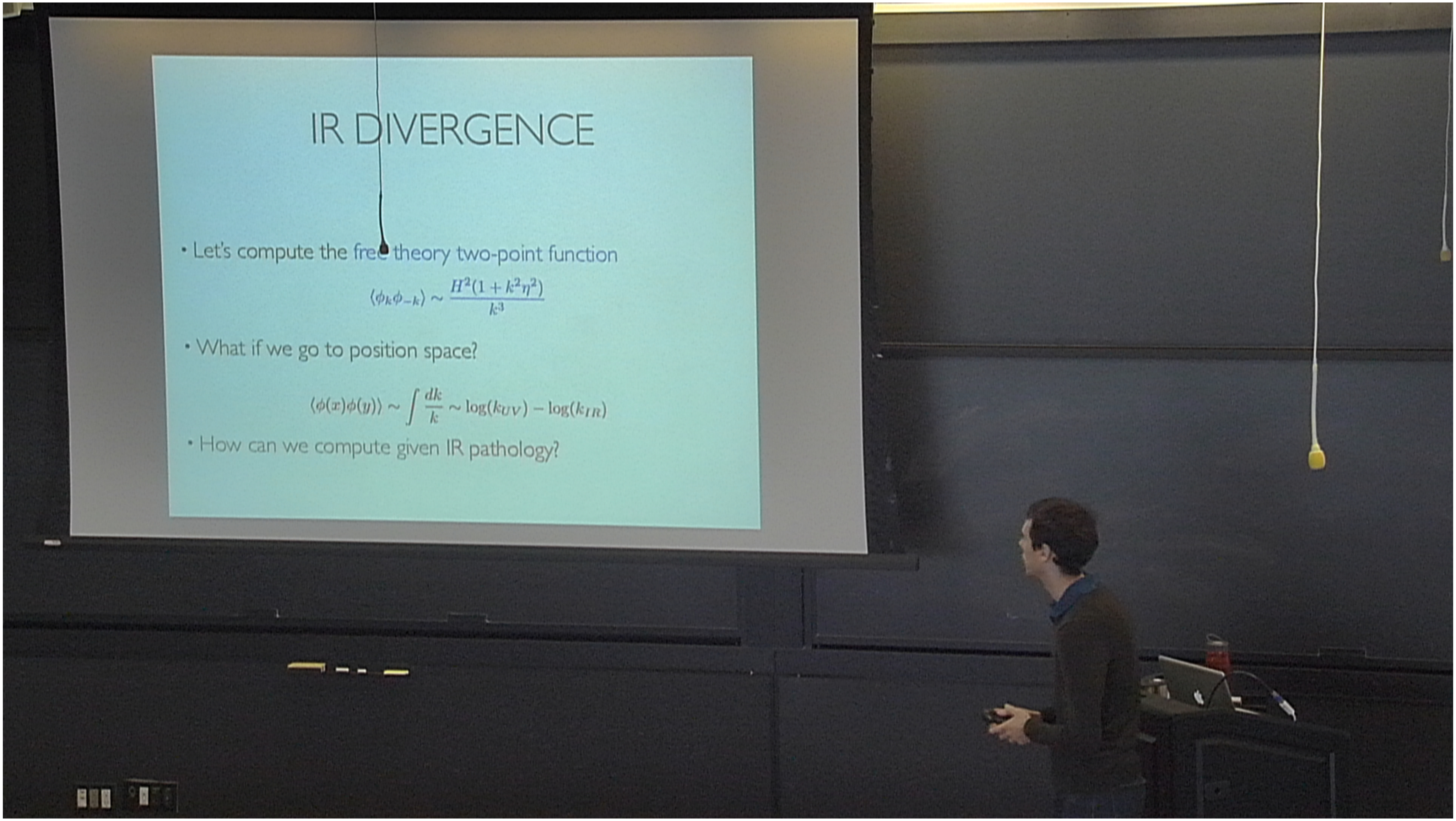
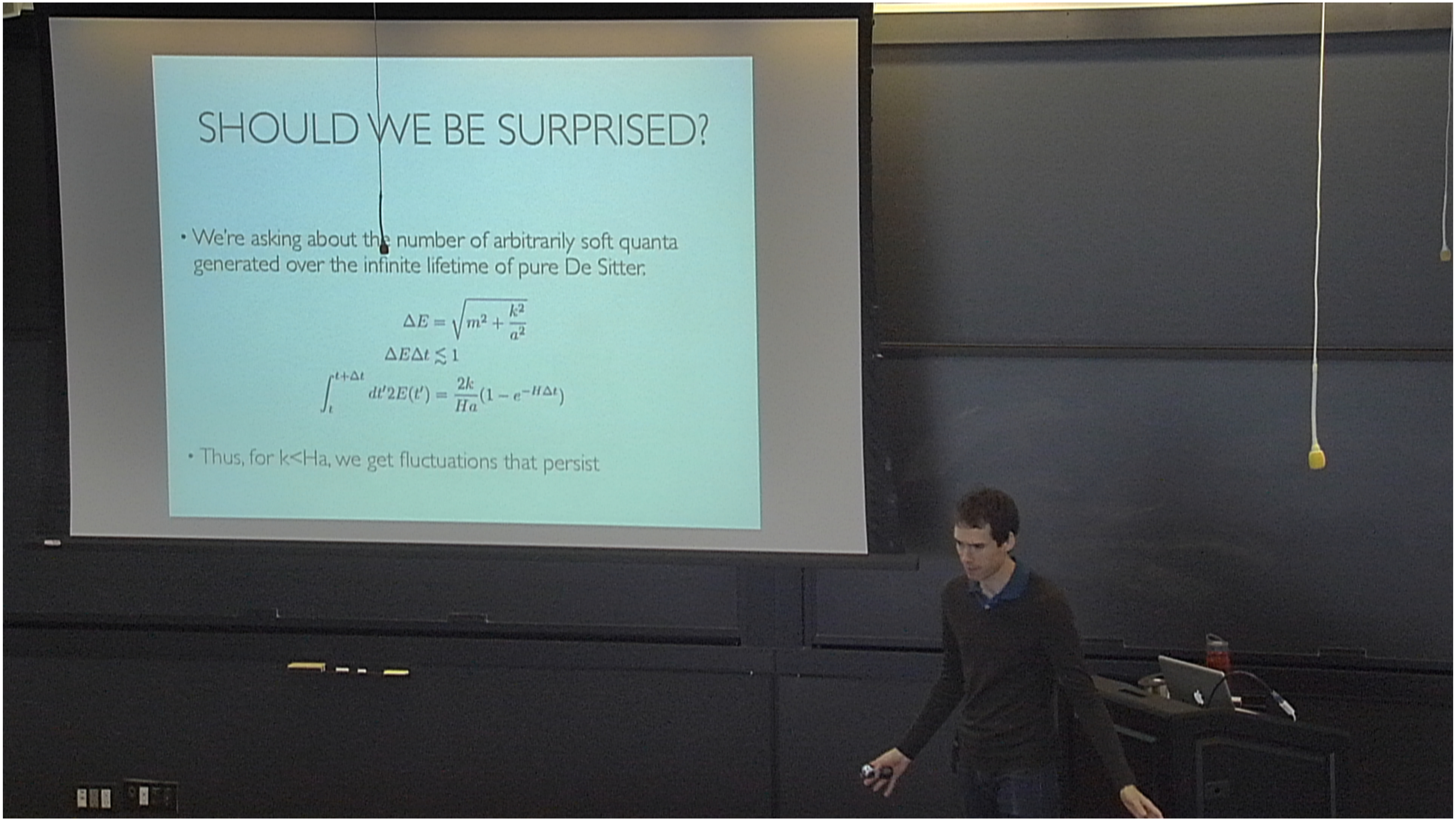

IR DIVERGENCE
• Let's compute the free theory two-point function
$\langle \phi_k \phi_{-k} \rangle \sim \frac{H^2(1+k^2\eta^2)}{k^3}$
• What if we go to position space?
$\langle \phi(x)\phi(y) \rangle \sim \int \frac{dk}{k} \sim \log(k_{UV}) - \log(k_{IR})$
• How can we compute given IR pathology?


SHOULD WE BE SURPRISED?
• We're asking about the number of arbitrarily soft quanta generated over the infinite lifetime of pure De Sitter.
$\Delta E = \sqrt{m^2 + \frac{k^2}{a^2}}$
$\Delta E \Delta t \lesssim 1$
$\int_t^{t+\Delta t} dt' 2E(t') = \frac{2k}{Ha}(1 - e^{-H\Delta t})$
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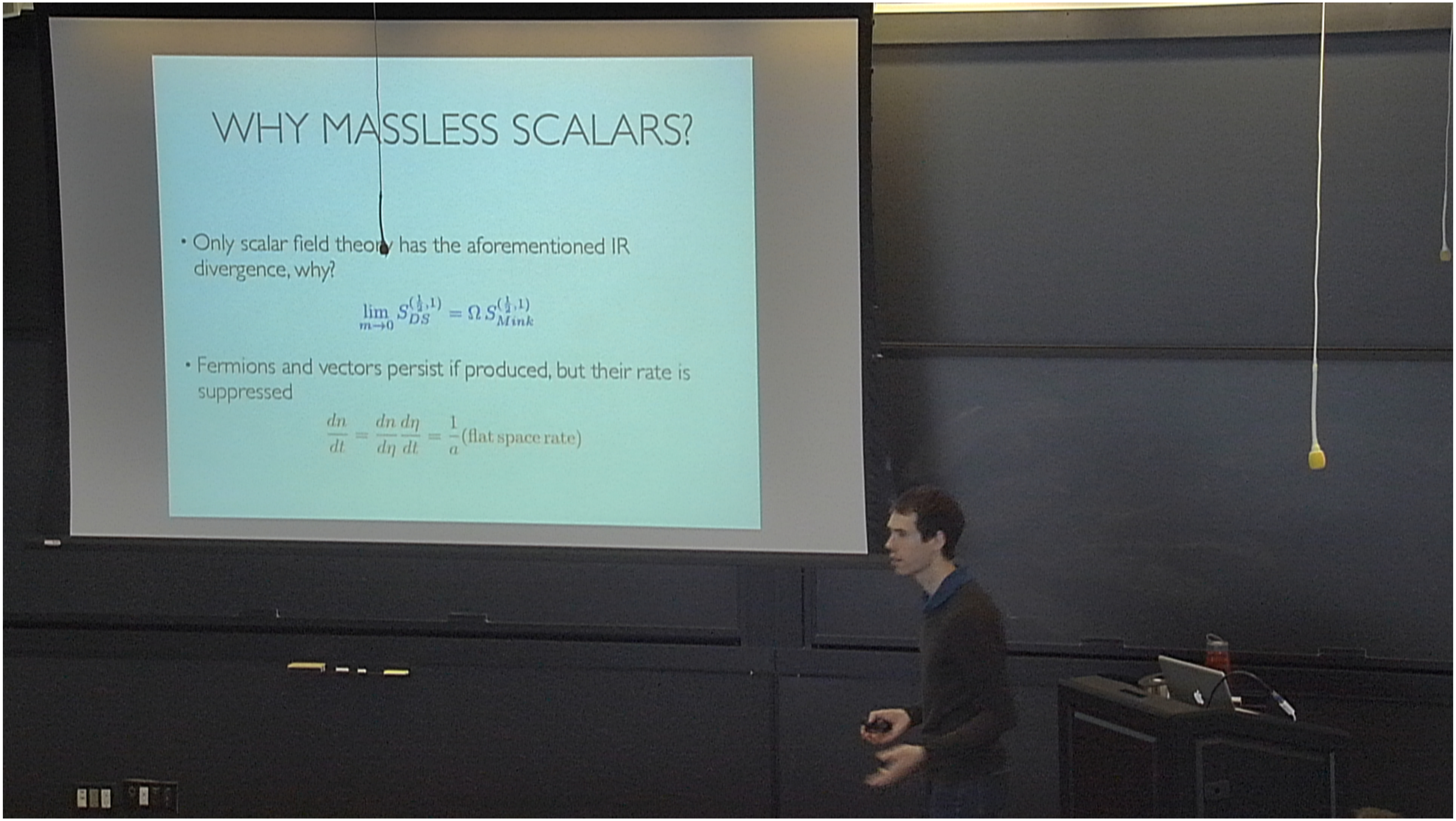
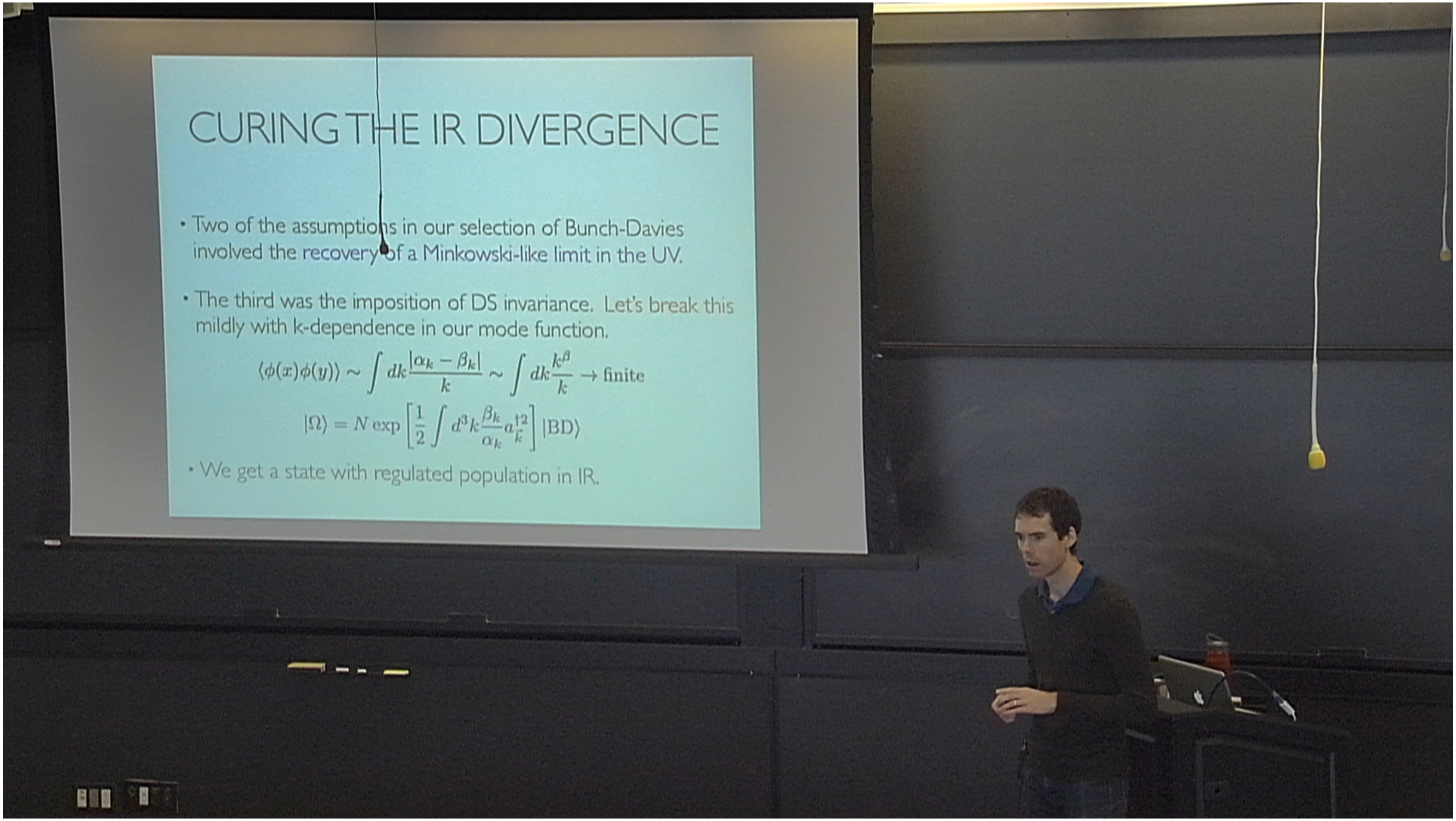
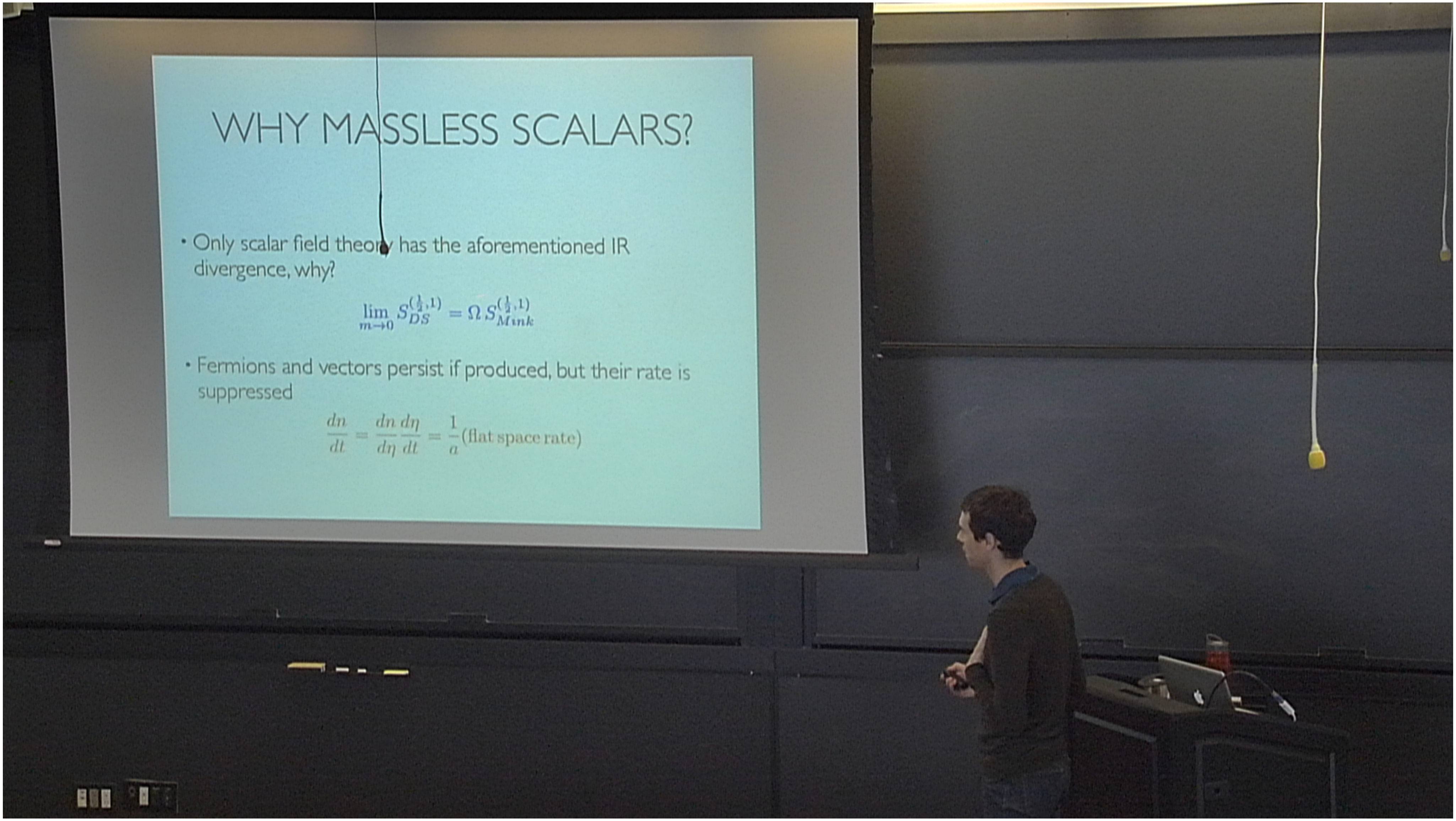

WHY MASSLESS SCALARS?
• Only scalar field theory has the aforementioned IR divergence, why?
$\lim_{m\to 0} S_{DS}^{(\frac{1}{2},1)} = \Omega\, S_{Mink}^{(\frac{1}{2},1)}$
• Fermions and vectors persist if produced, but their rate is suppressed
$\frac{dn}{dt} = \frac{dn}{d\eta}\frac{d\eta}{dt} = \frac{1}{a}(\text{flat space rate})$


CURING THE IR DIVERGENCE
• Two of the assumptions in our selection of Bunch-Davies involved the recovery of a Minkowski-like limit in the UV.
• The third was the imposition of DS invariance. Let's break this mildly with k-dependence in our mode function.
$\langle\phi(x)\phi(y)\rangle \sim \int dk \frac{|\alpha_k - \beta_k|}{k} \sim \int dk \frac{k^\beta}{k} \to \text{finite}$
$|\Omega\rangle = N \exp\left[\frac{1}{2}\int d^3k \frac{\beta_k}{\alpha_k} a_k^{\dagger 2}\right] |\text{BD}\rangle$
• We get a state with regulated population in IR.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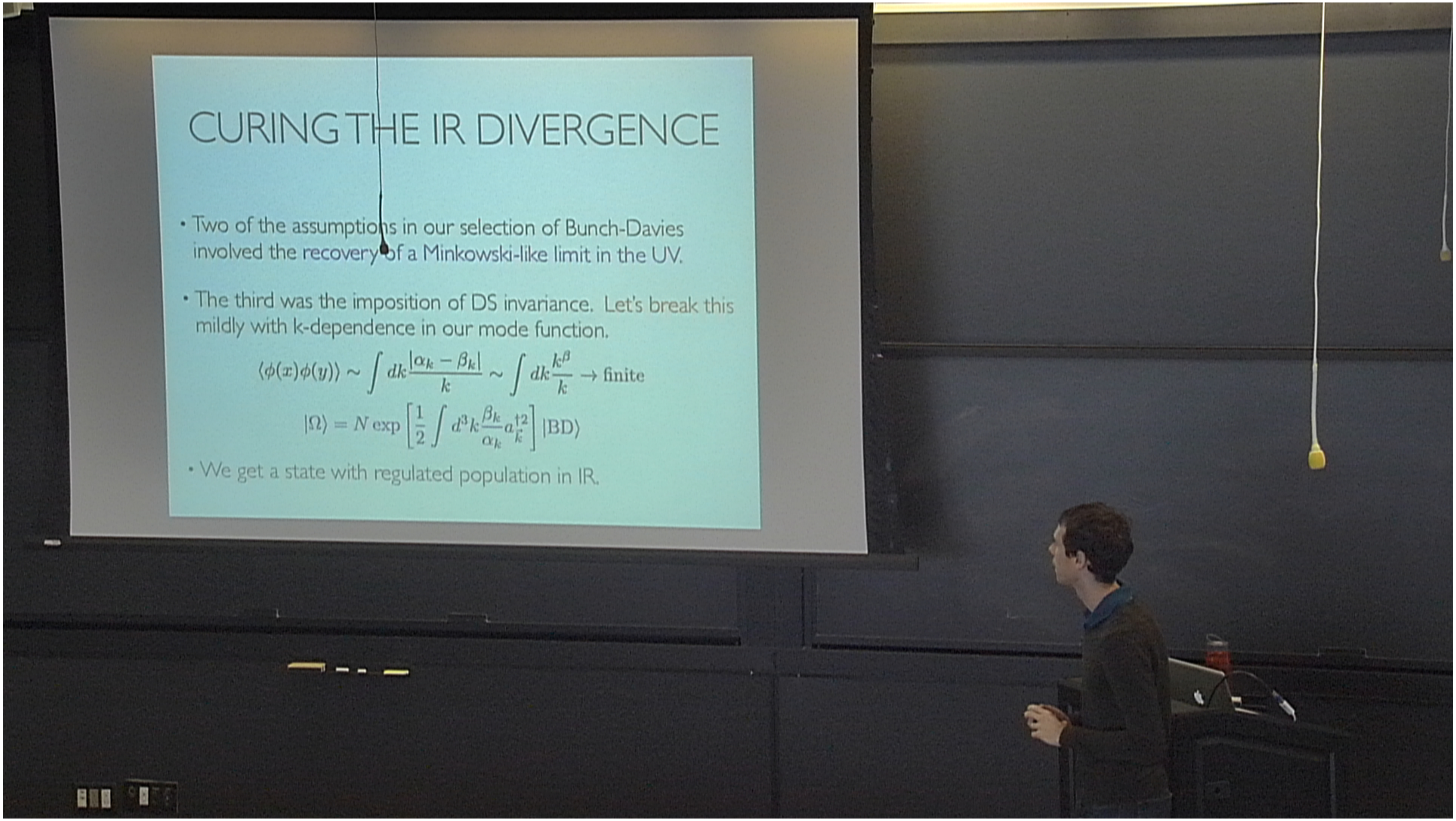
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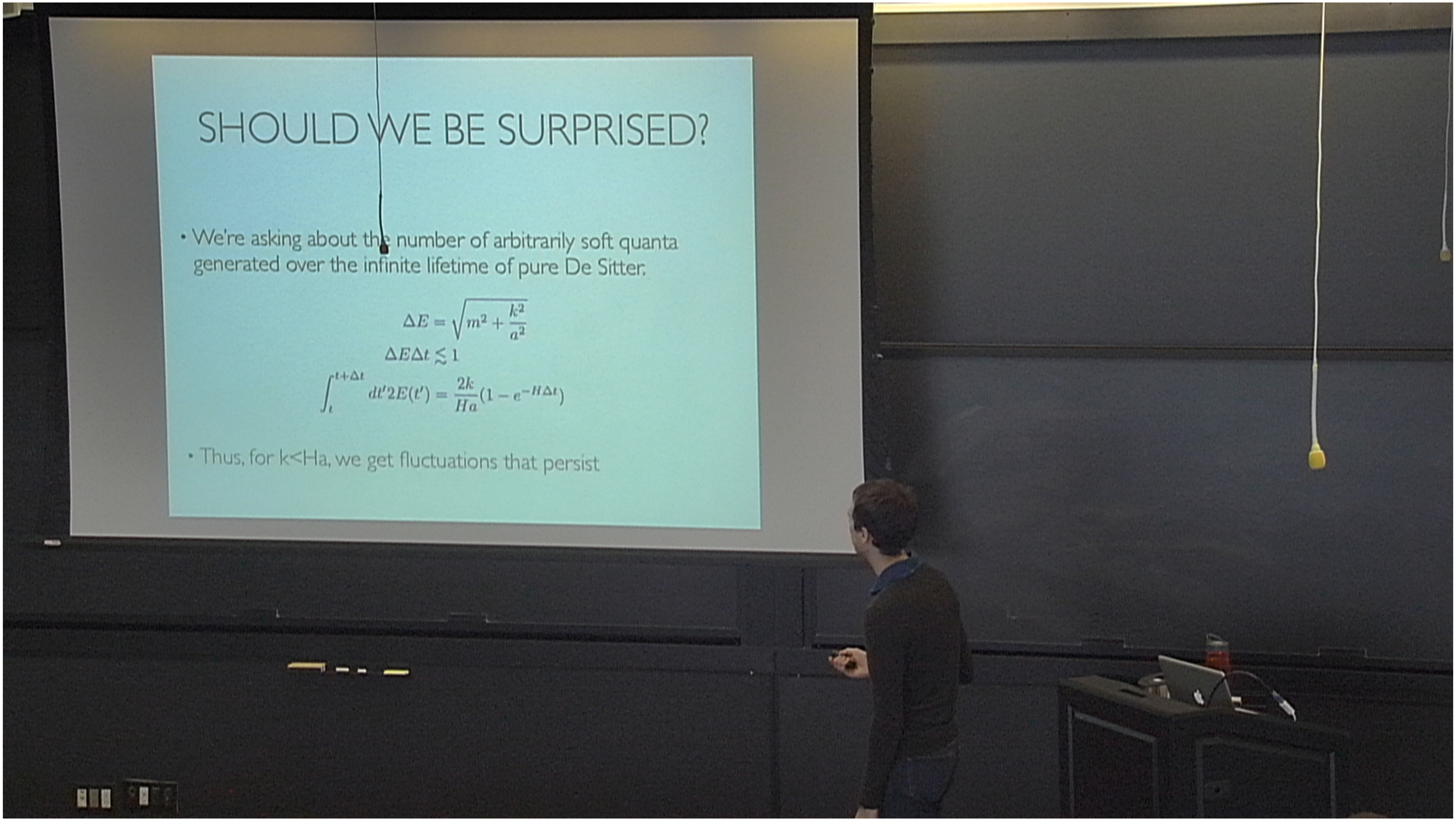
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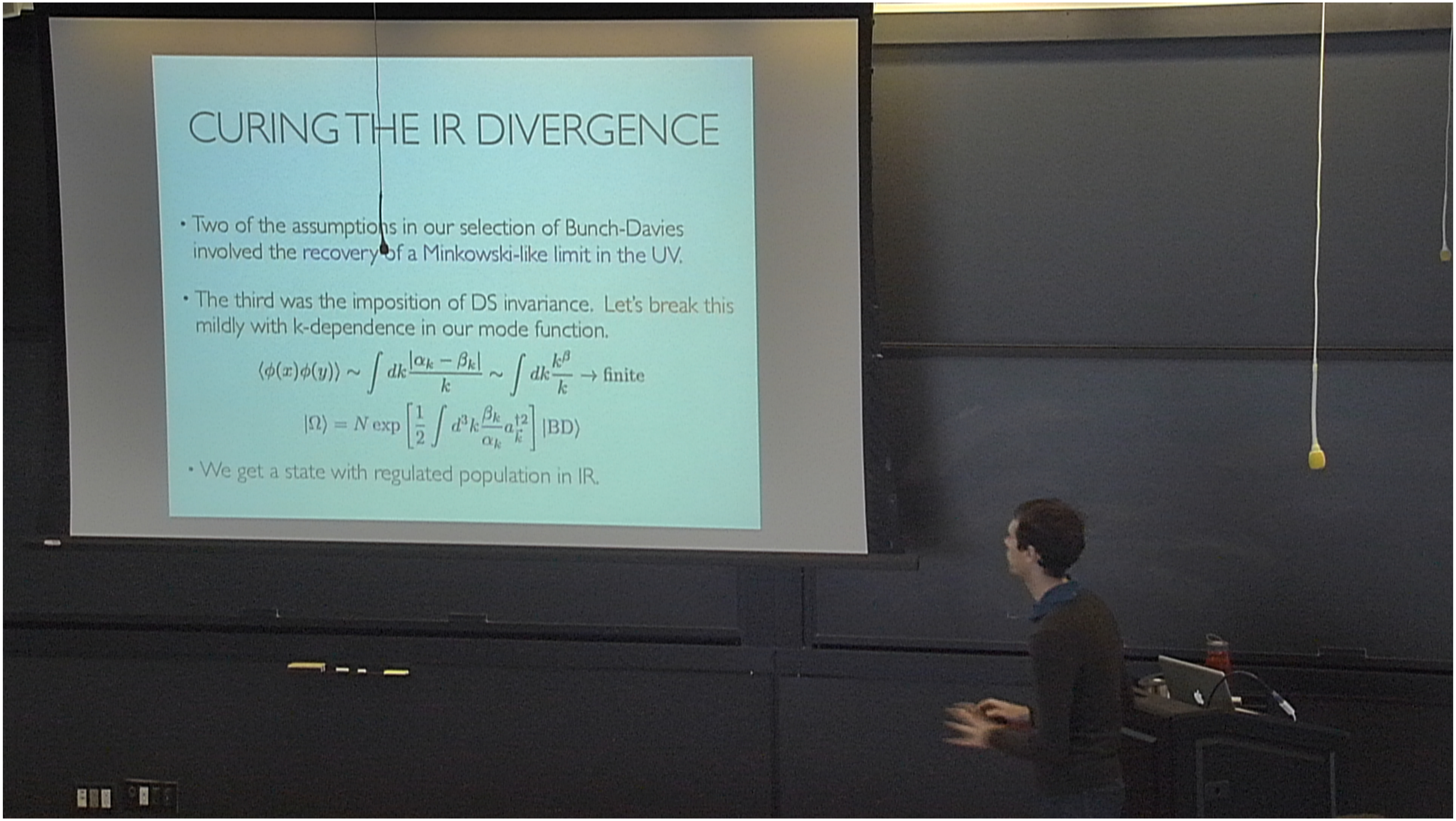
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CARTOON HISTORY OF AN IR-SAFE UNIVERSE
• In a more physical picture, the regulation arises from starting De Sitter at a finite time.
FRW
DS
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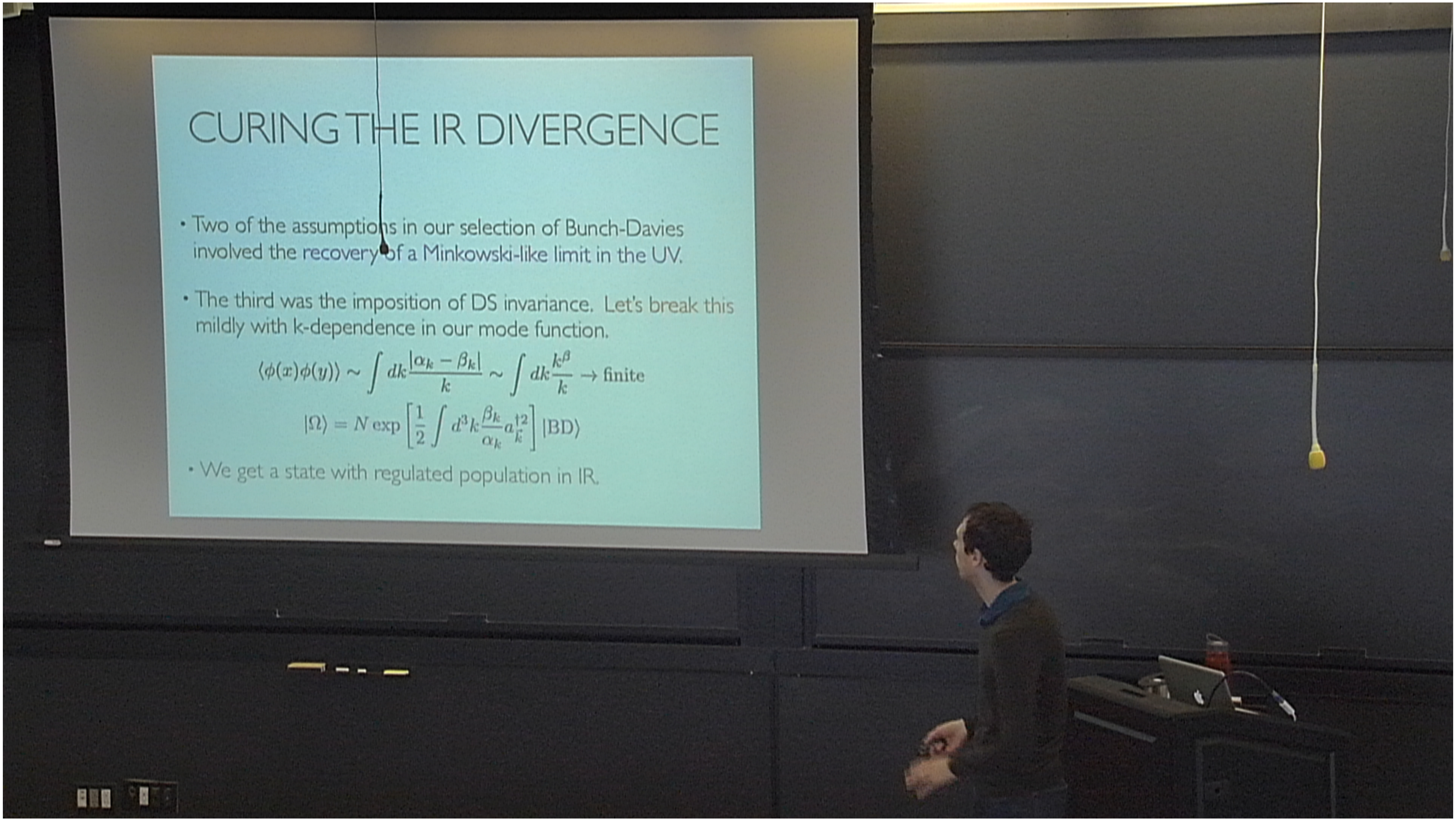
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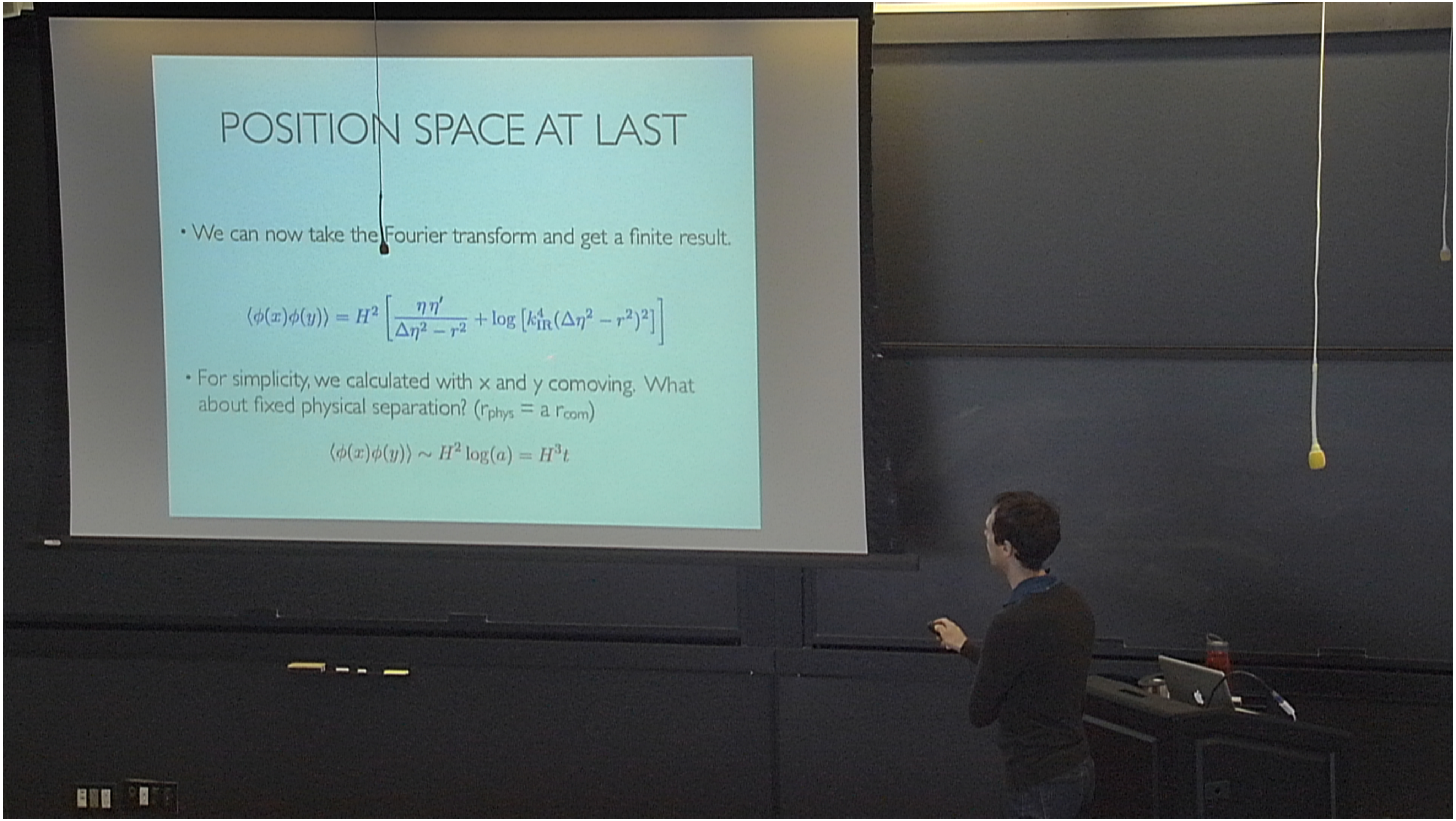
POSITION SPACE AT LAST
• We can now take the Fourier transform and get a finite result.
$\langle\phi(x)\phi(y)\rangle = H^2\left[\frac{\eta\eta'}{\Delta\eta^2 - r^2} + \log\left[k_{\rm IR}^4(\Delta\eta^2 - r^2)^2\right]\right]$
• For simplicity, we calculated with x and y comoving. What about fixed physical separation? ($r_{\rm phys}$ = a $r_{\rm com}$)
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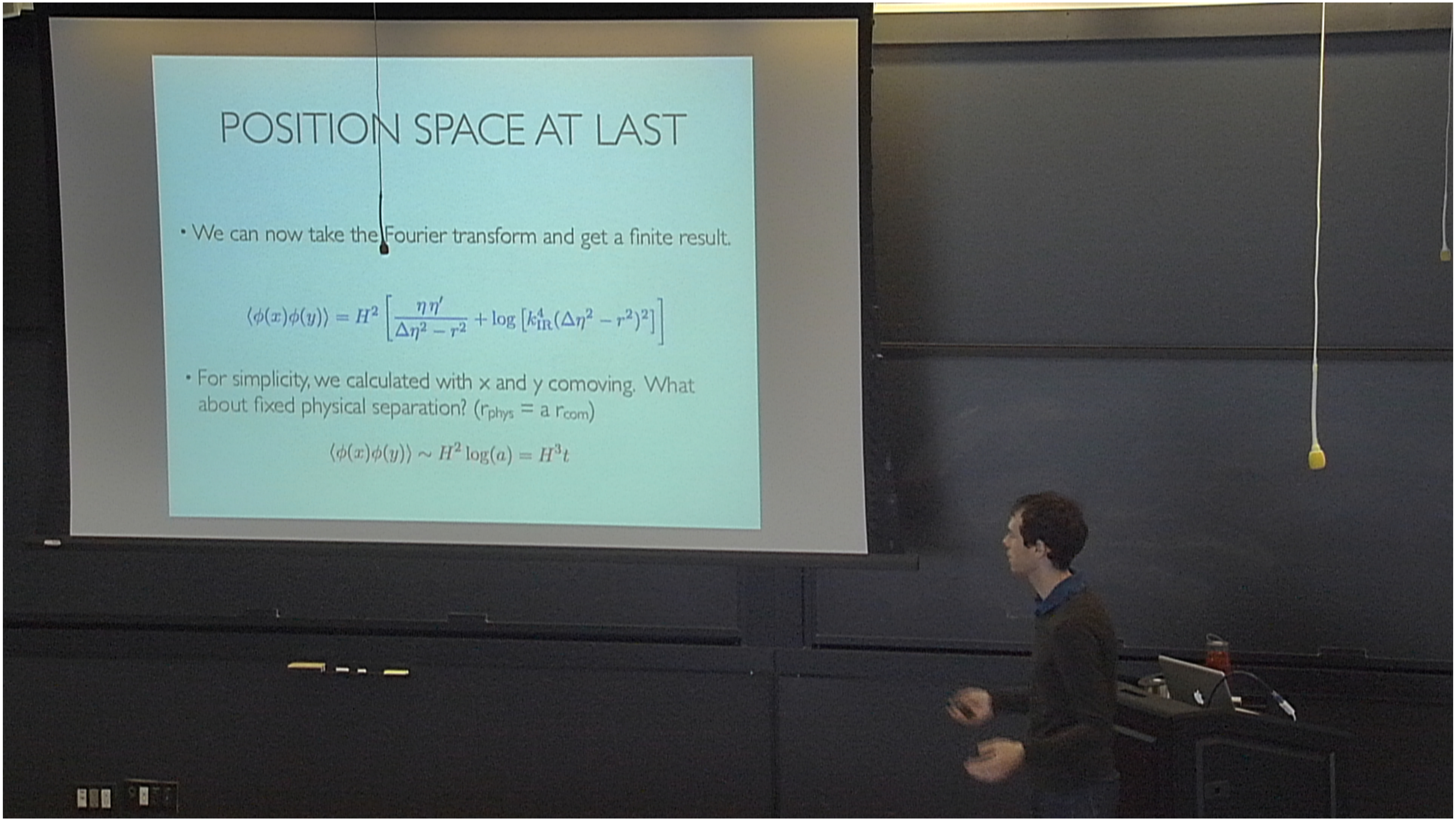
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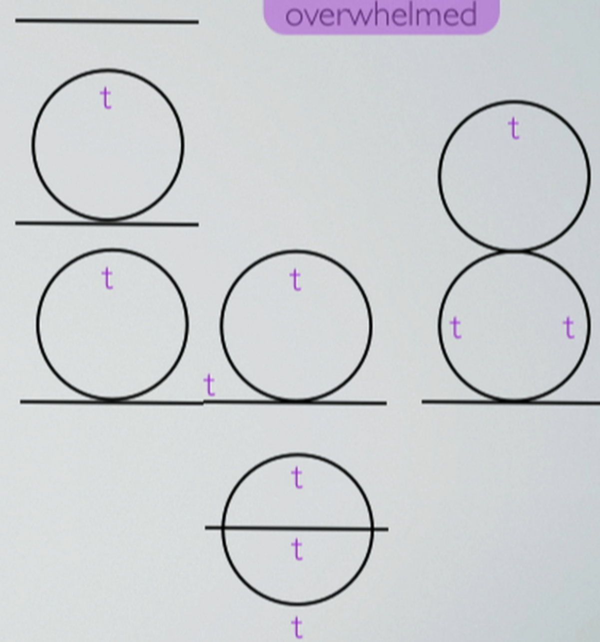
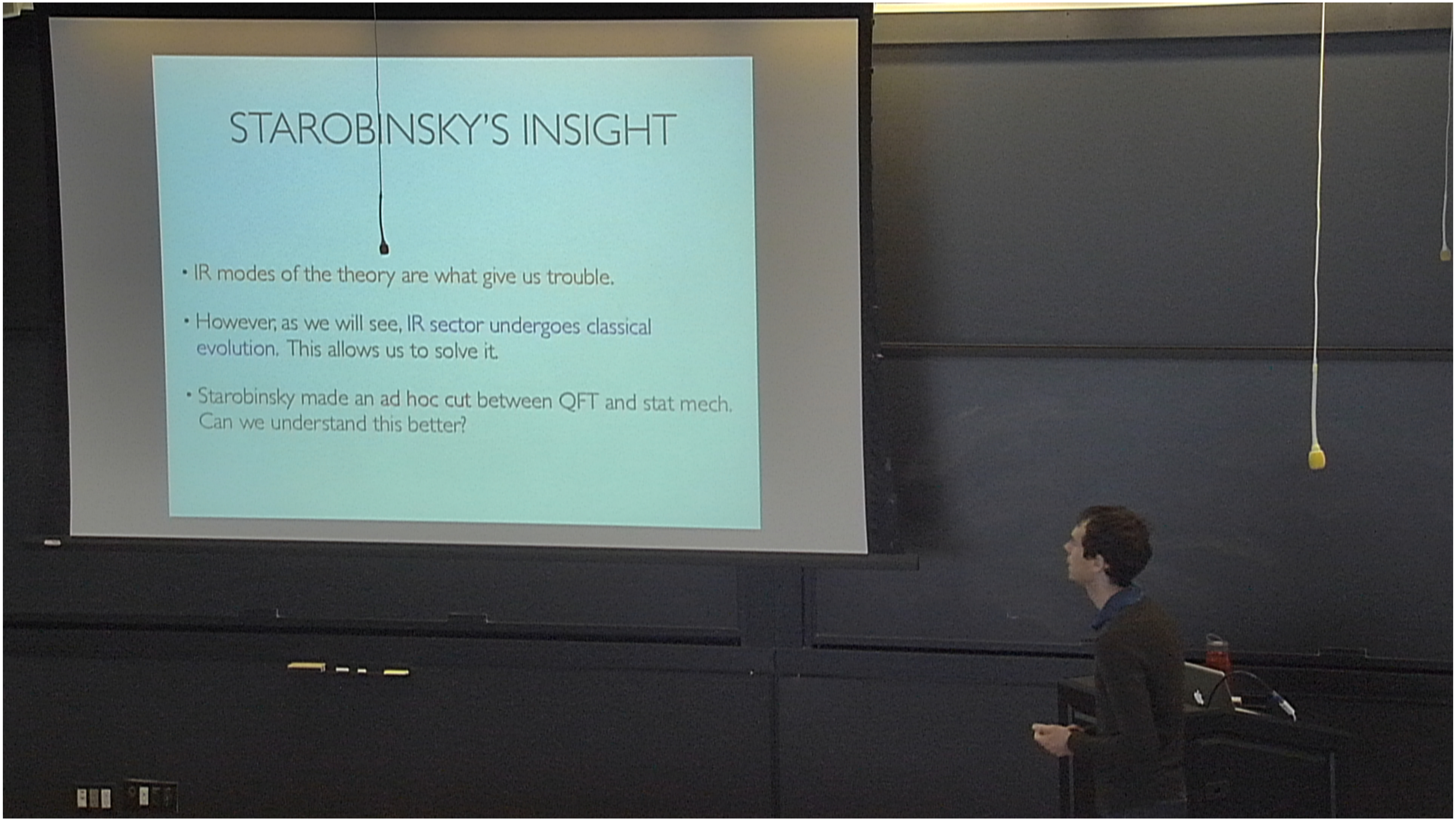
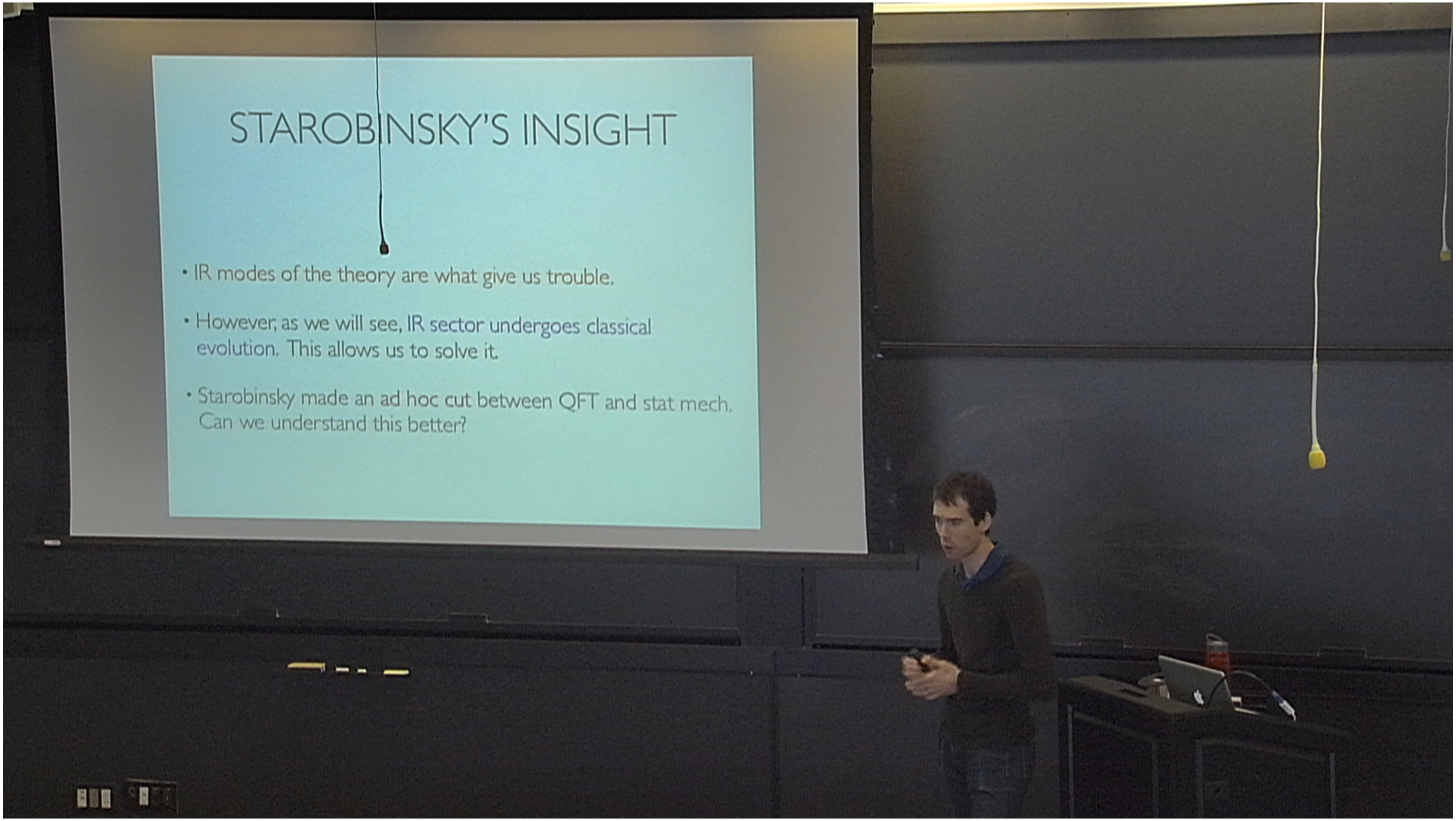
# PERTURBATION THEORY BREAKS DOWN

No matter how small $\lambda$, eventually overwhelmed



- We add interactions (for concreteness $\lambda\varphi^4$),
- Eventually theory becomes nonperturbative for arbitrarily small coupling.
- How to regain control?


STAROBINSKY'S INSIGHT
• IR modes of the theory are what give us trouble.
• However, as we will see, IR sector undergoes classical evolution. This allows us to solve it.
• Starobinsky made an ad hoc cut between QFT and stat mech. Can we understand this better?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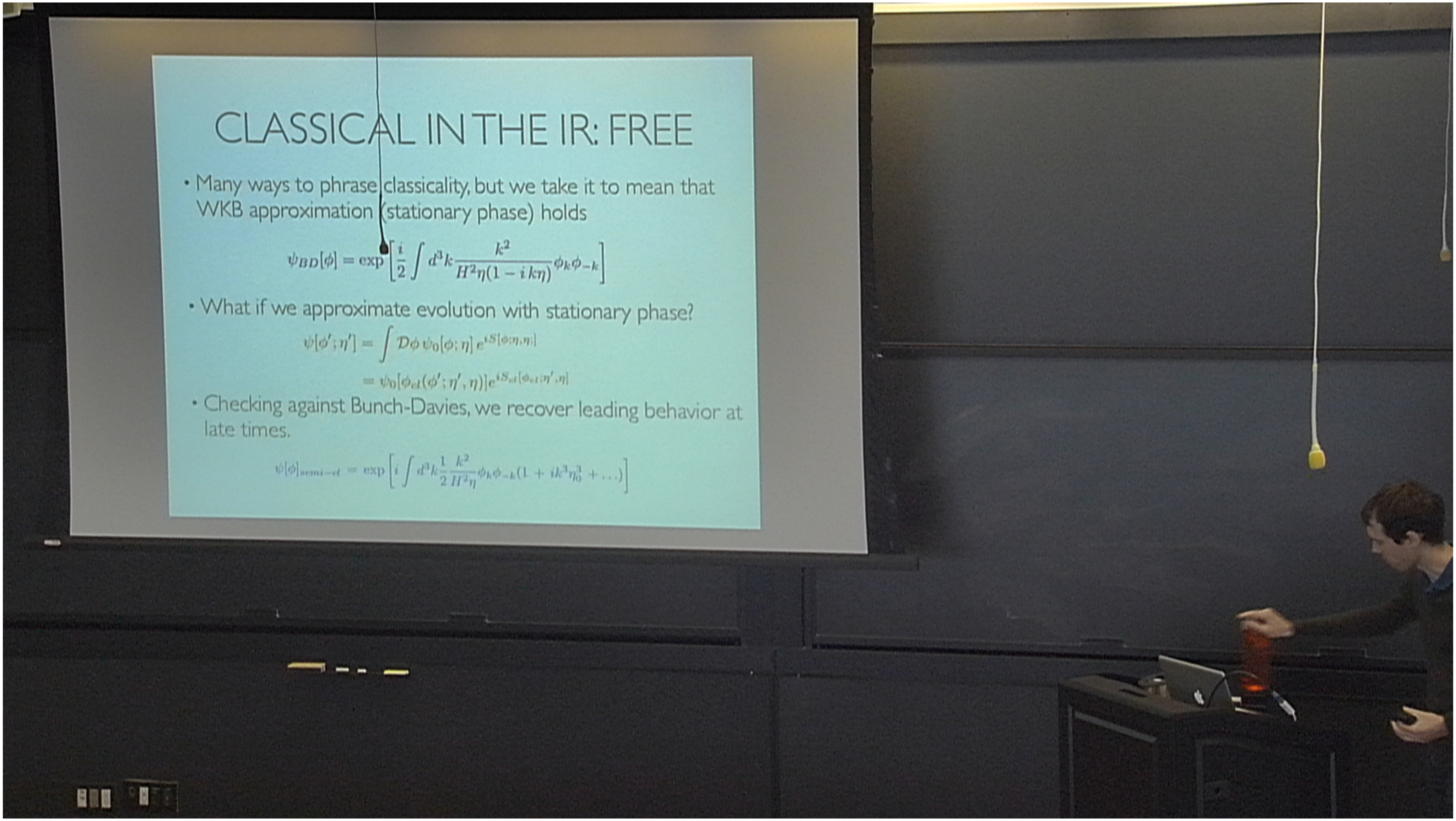
CLASSICAL IN THE IR: FREE
• Many ways to phrase classicality, but we take it to mean that WKB approximation (stationary phase) holds
$\psi_{BD}[\phi] = \exp\left[\frac{i}{2}\int d^3k \frac{k^2}{H^2\eta(1-ik\eta)}\phi_k\phi_{-k}\right]$
• What if we approximate evolution with stationary phase?
$\psi[\phi';\eta'] = \int \mathcal{D}\phi\, \psi_0[\phi;\eta]\, e^{iS[\phi;\eta,\eta']}$
$= \psi_0[\phi_{cl}(\phi';\eta',\eta)]e^{iS_{cl}[\phi_{cl};\eta',\eta]}$
• Checking against Bunch-Davies, we recover leading behavior at late times.
$\psi[\phi]_{semi-cl} = \exp\left[i\int d^3k \frac{1}{2}\frac{k^2}{H^2\eta}\phi_k\phi_{-k}(1 + ik^3\eta_0^3 + \ldots)\right]$

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# CLASSICAL IN THE IR: INTERACTING

- We want to make statements about a theory that *isn't just interacting but nonperturbative*.
- We map our field theory into a 1-D anharmonic oscillator with time-varying mass.

$$H = \frac{p^2}{2m} + \tilde{\lambda}\, m\, x^4$$

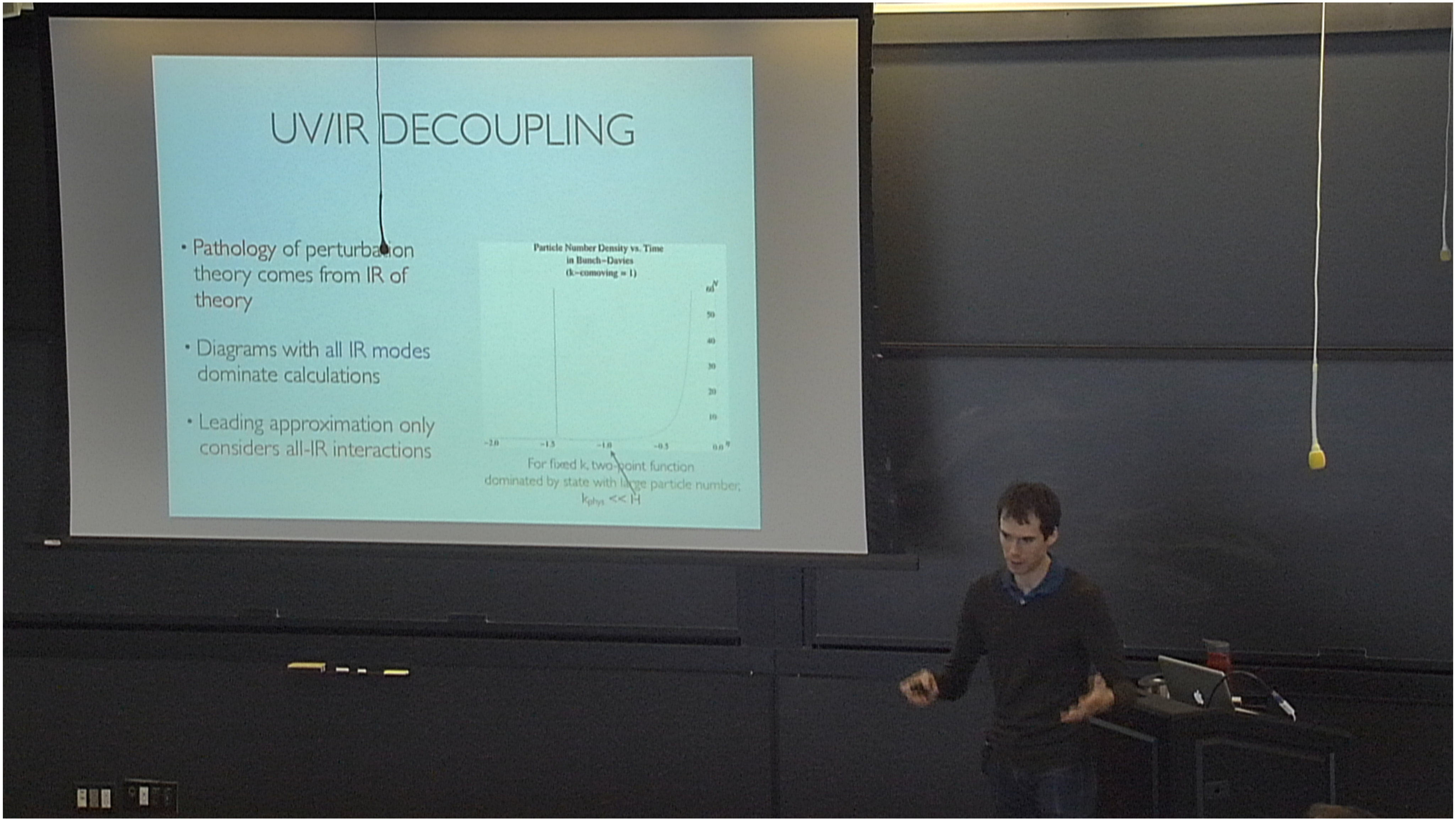
- When does WKB approximation hold?

$$p_{\rm TP}\, x_{\rm TP} \gg 1 \Rightarrow x_{\rm TP} \gg \left(\frac{1}{\sqrt{\tilde{\lambda}} m}\right)^{1/3} \Rightarrow$$

$$x_{\rm TP} \gg \eta/\lambda^{1/6}$$

- Under conservative assumptions, how much of state fails this condition?

$$\frac{\int_{-\eta/\lambda^{1/6}}^{\eta/\lambda^{1/6}} dx|\psi|^2}{\int dx|\psi|^2} \sim H\eta$$


UV/IR DECOUPLING
• Pathology of perturbation theory comes from IR of theory
• Diagrams with all IR modes dominate calculations
• Leading approximation only considers all-IR interactions
Particle Number Density vs. Time
in Bunch–Davies
(k–comoving = 1)
60
50
40
30
20
10
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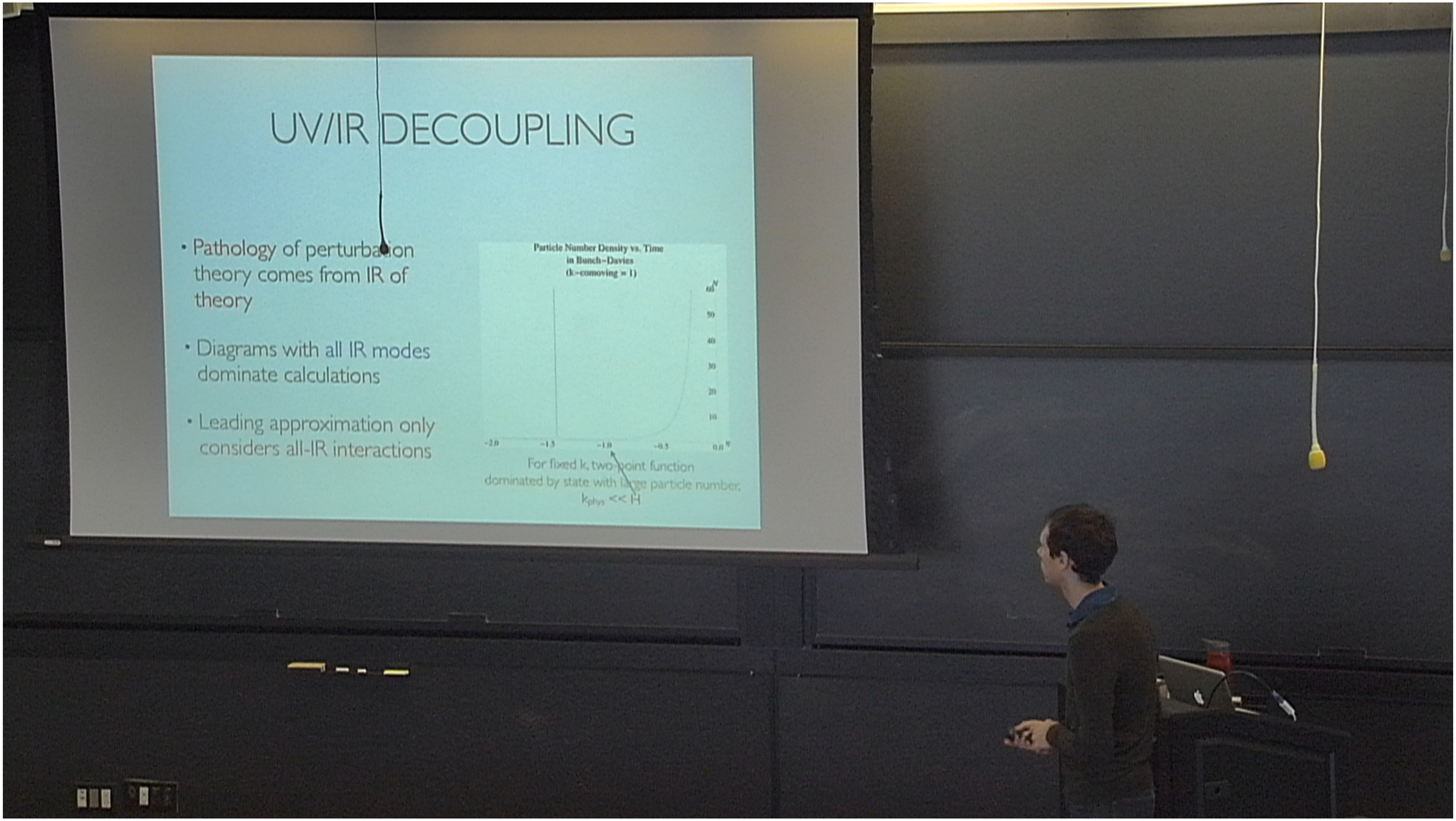
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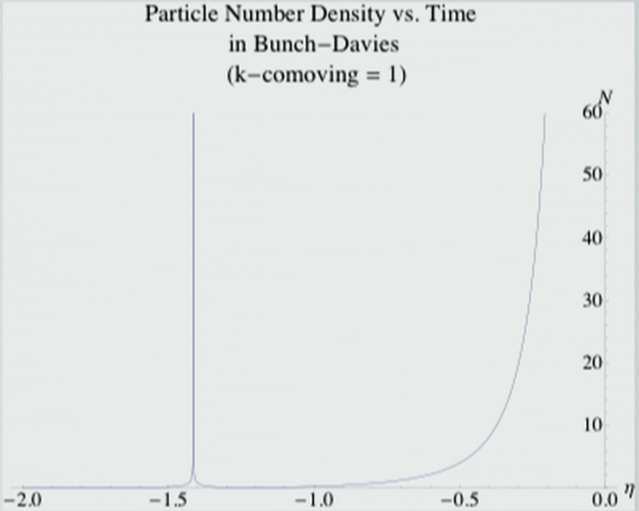


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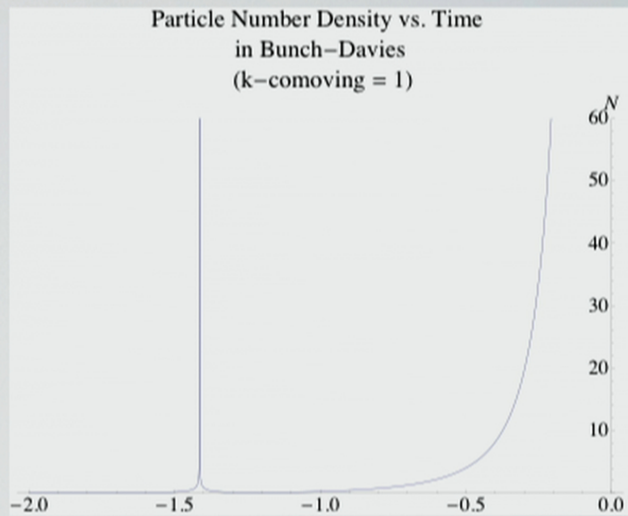
# UV/IR DECOUPLING

- Pathology of perturbation theory comes from IR of theory
- Diagrams with all IR modes dominate calculations
- Leading approximation only considers all-IR interactions

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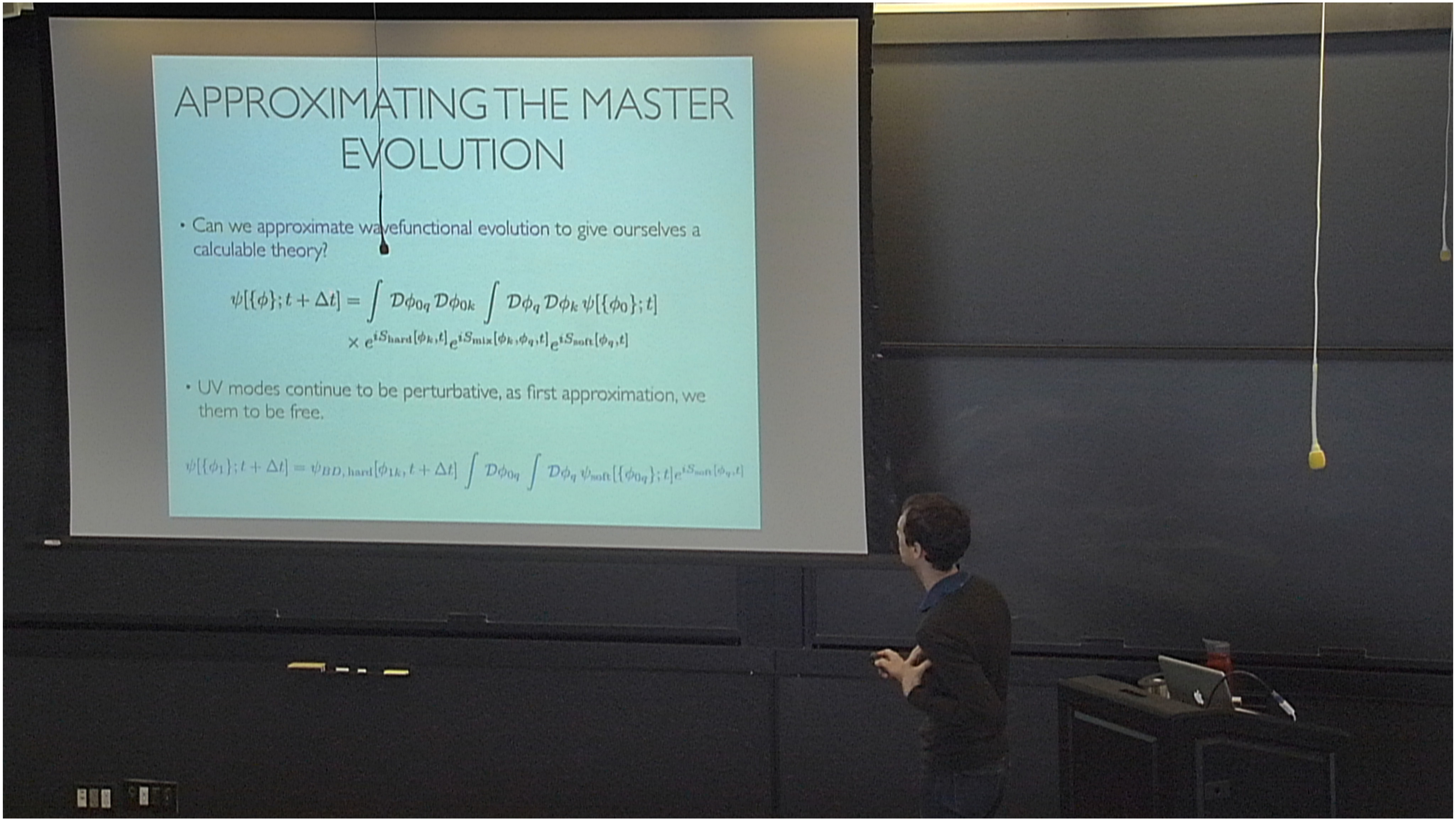
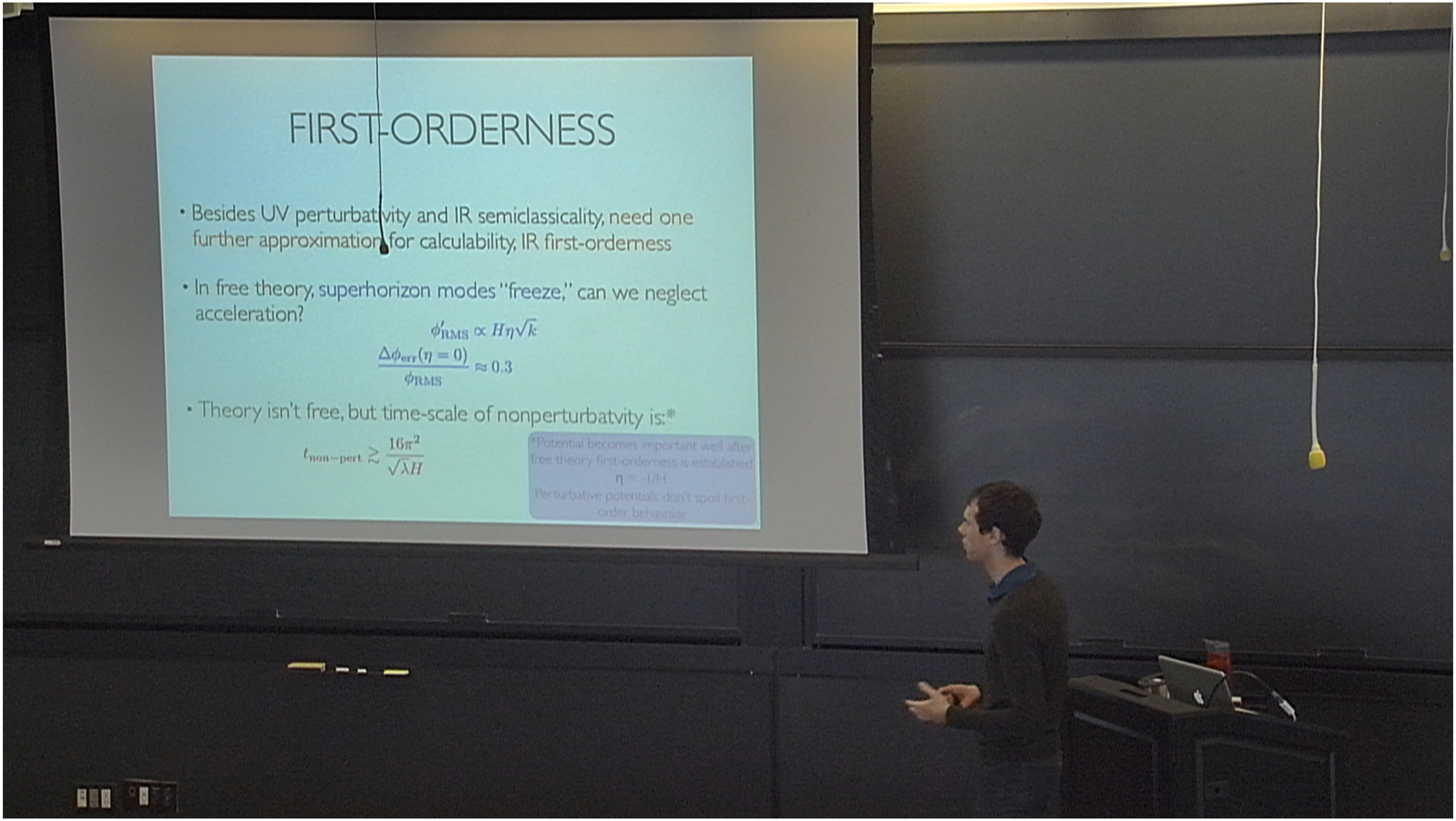

APPROXIMATING THE MASTER EVOLUTION
• Can we approximate wavefunctional evolution to give ourselves a calculable theory?
$\psi[\{\phi\};t+\Delta t] = \int \mathcal{D}\phi_{0q}\,\mathcal{D}\phi_{0k} \int \mathcal{D}\phi_q\,\mathcal{D}\phi_k\,\psi[\{\phi_0\};t] \times e^{iS_{\rm hard}[\phi_k,t]}e^{iS_{\rm mix}[\phi_k,\phi_q,t]}e^{iS_{\rm soft}[\phi_q,t]}$
• UV modes continue to be perturbative, as first approximation, we them to be free.
$\psi[\{\phi_1\};t+\Delta t] = \psi_{BD,\,\rm hard}[\phi_{1k},t+\Delta t] \int \mathcal{D}\phi_{0q} \int \mathcal{D}\phi_q\,\psi_{\rm soft}[\{\phi_{0q}\};t]e^{iS_{\rm soft}[\phi_q,t]}$


FIRST-ORDERNESS
• Besides UV perturbativity and IR semiclassicality, need one further approximation for calculability, IR first-orderness
• In free theory, superhorizon modes "freeze," can we neglect acceleration?
$\phi'_{\rm RMS} \propto H\eta\sqrt{k}$
$\frac{\Delta\phi_{\rm err}(\eta=0)}{\phi_{\rm RMS}} \approx 0.3$
• Theory isn't free, but time-scale of nonperturbatvity is:*
$t_{\rm non-pert} \gtrsim \frac{16\pi^2}{\sqrt{\lambda}H}$
*Potential becomes important well after free theory first-orderness is established
η = -1/H
Perturbative potentials don't spoil first-order behavior

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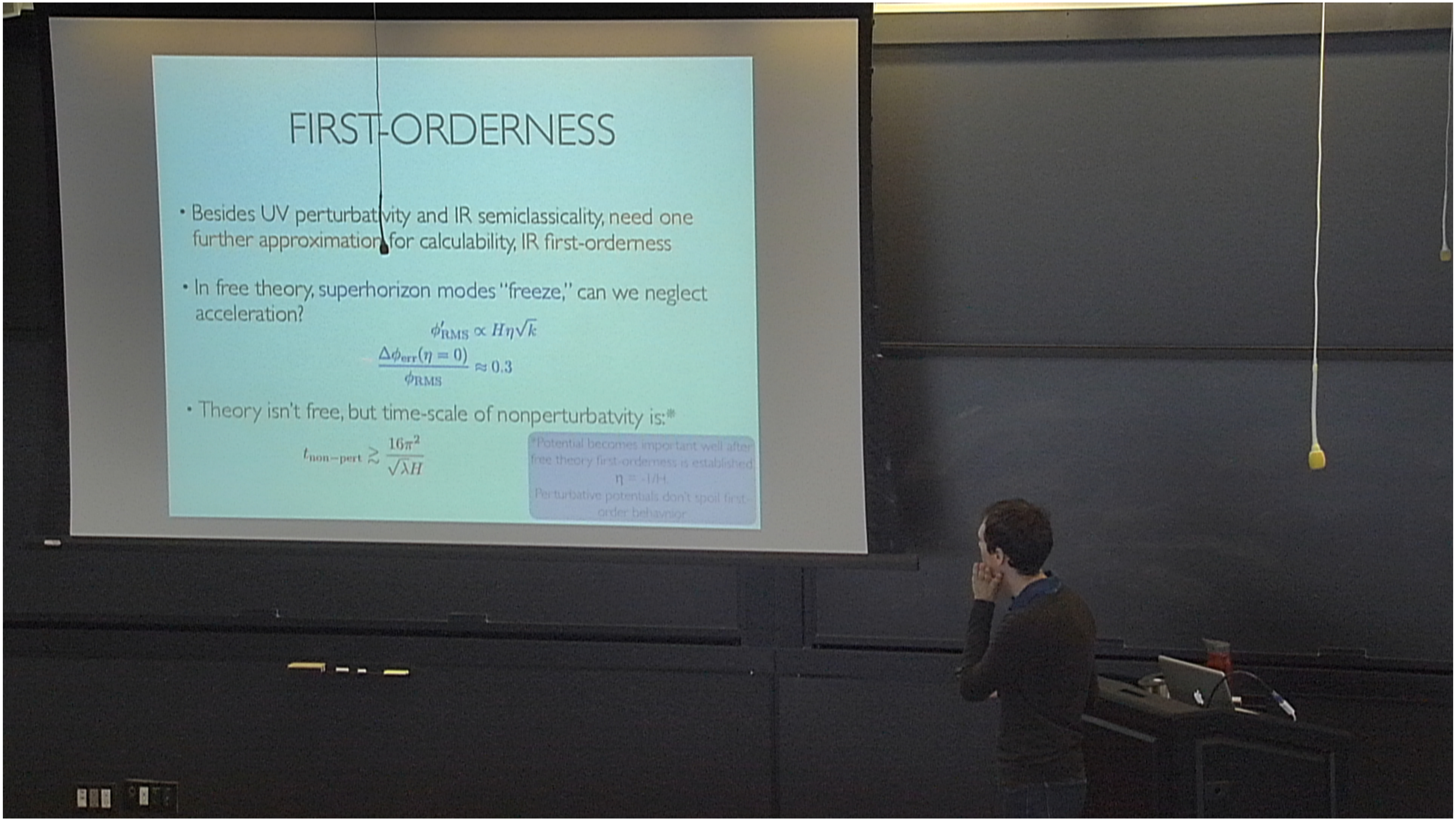
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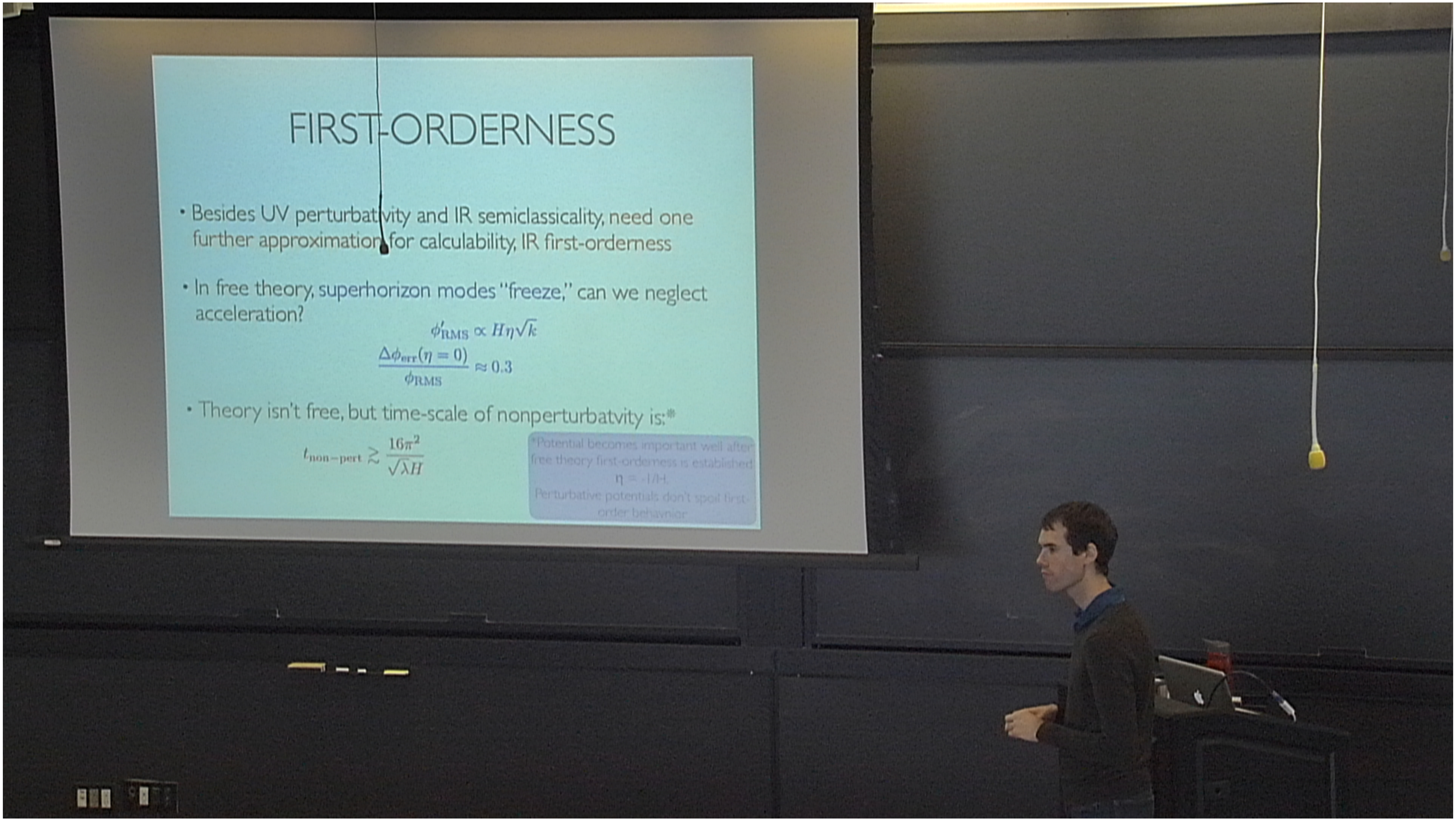
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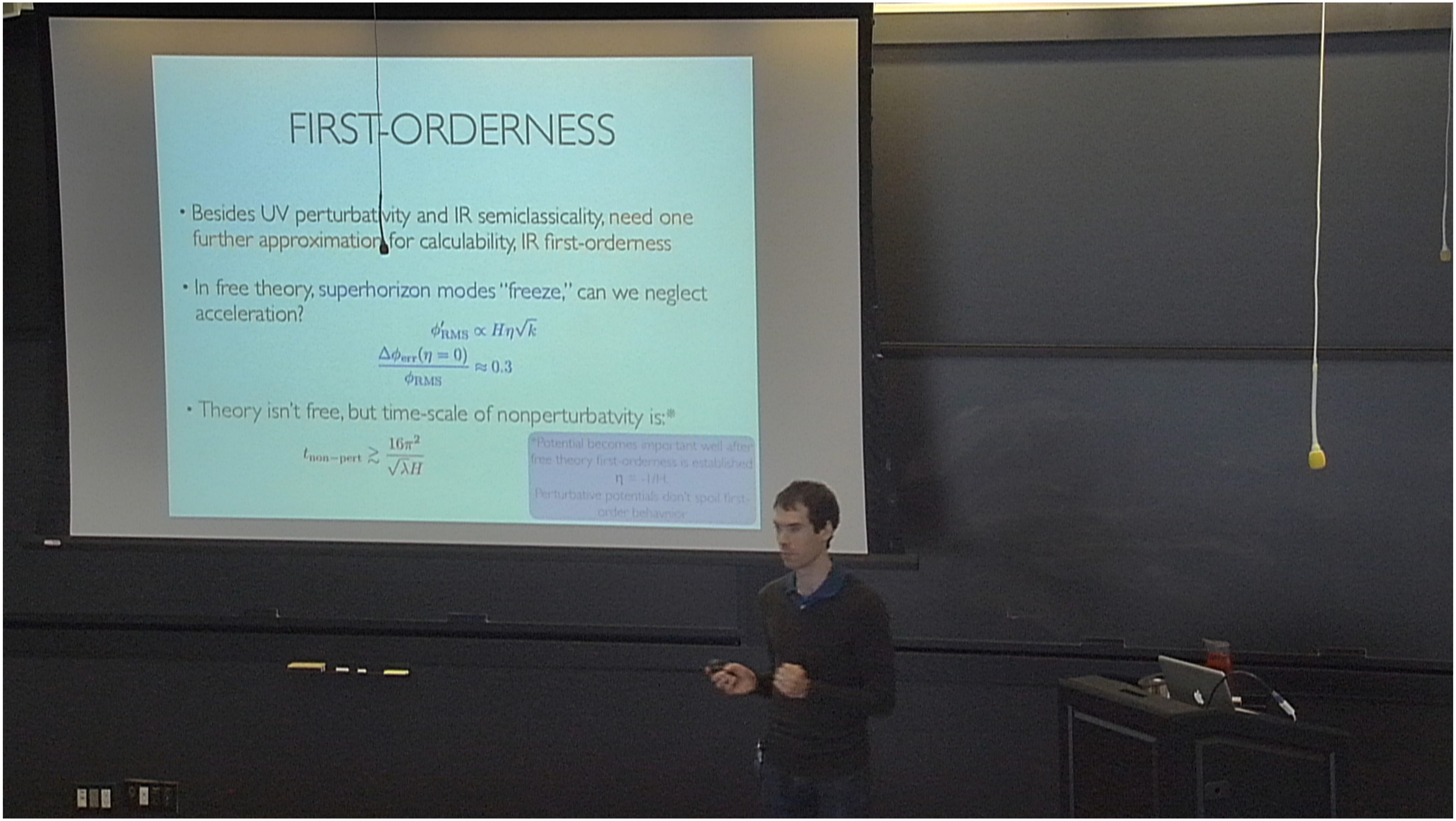
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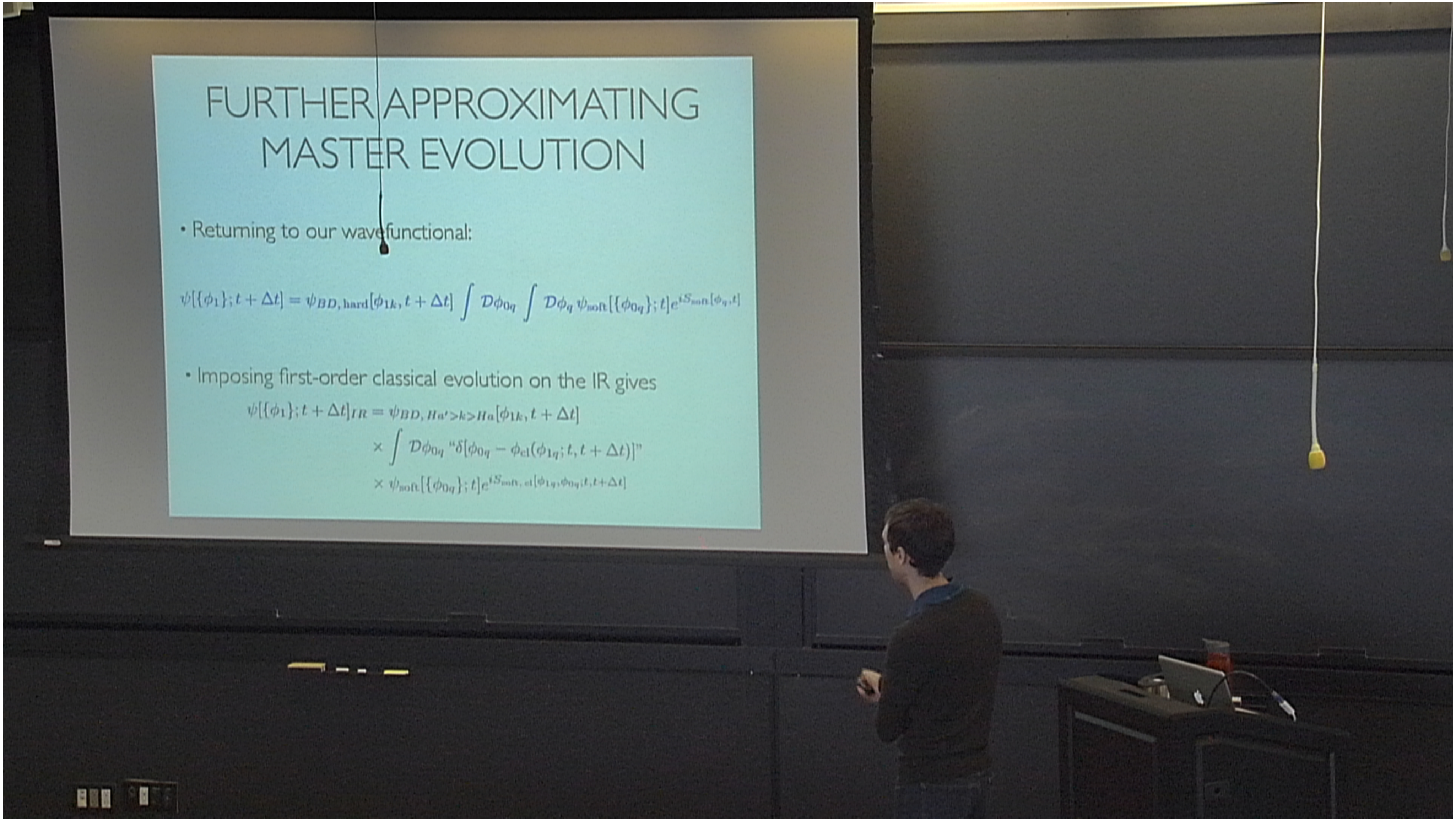
FURTHER APPROXIMATING MASTER EVOLUTION
• Returning to our wavefunctional:
$\psi[\{\phi_1\};t+\Delta t] = \psi_{BD,\,\mathrm{hard}}[\phi_{1k}, t+\Delta t] \int \mathcal{D}\phi_{0q} \int \mathcal{D}\phi_q \, \psi_{\mathrm{soft}}[\{\phi_{0q}\};t] e^{iS_{\mathrm{soft}}[\phi_q,t]}$
• Imposing first-order classical evolution on the IR gives
$\psi[\{\phi_1\};t+\Delta t]_{IR} = \psi_{BD,\,Ha'>k>Ha}[\phi_{1k}, t+\Delta t]$
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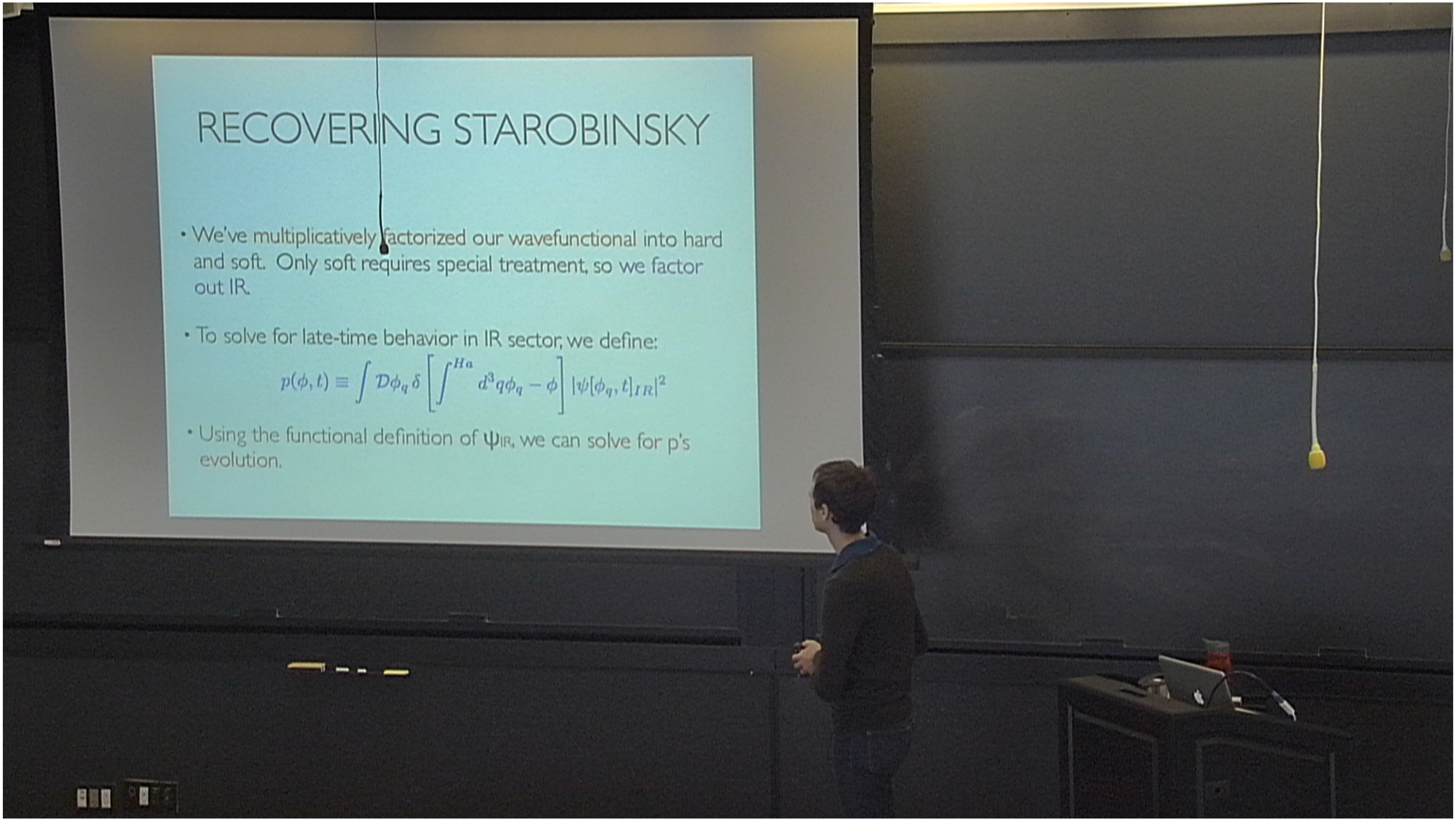
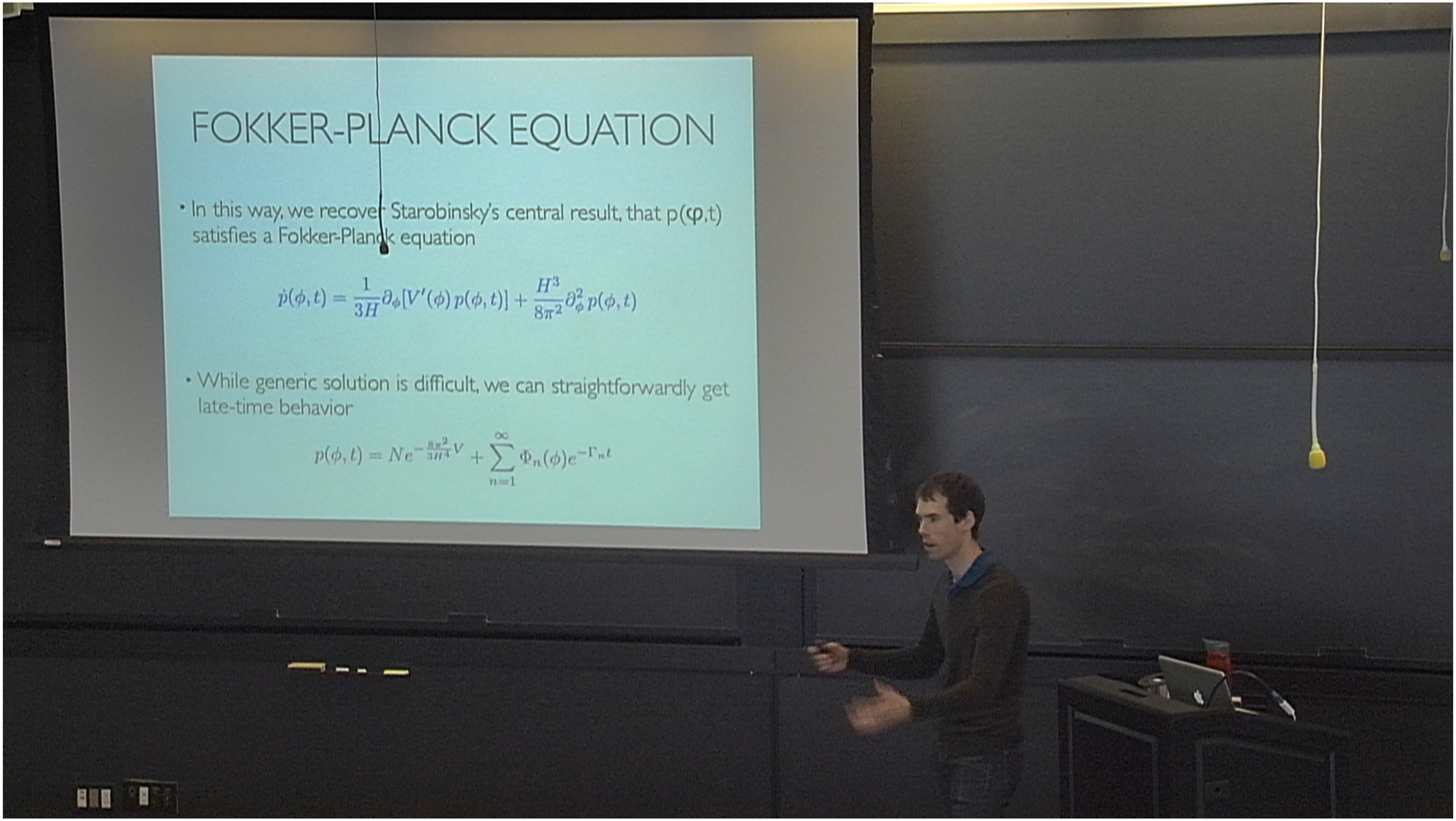
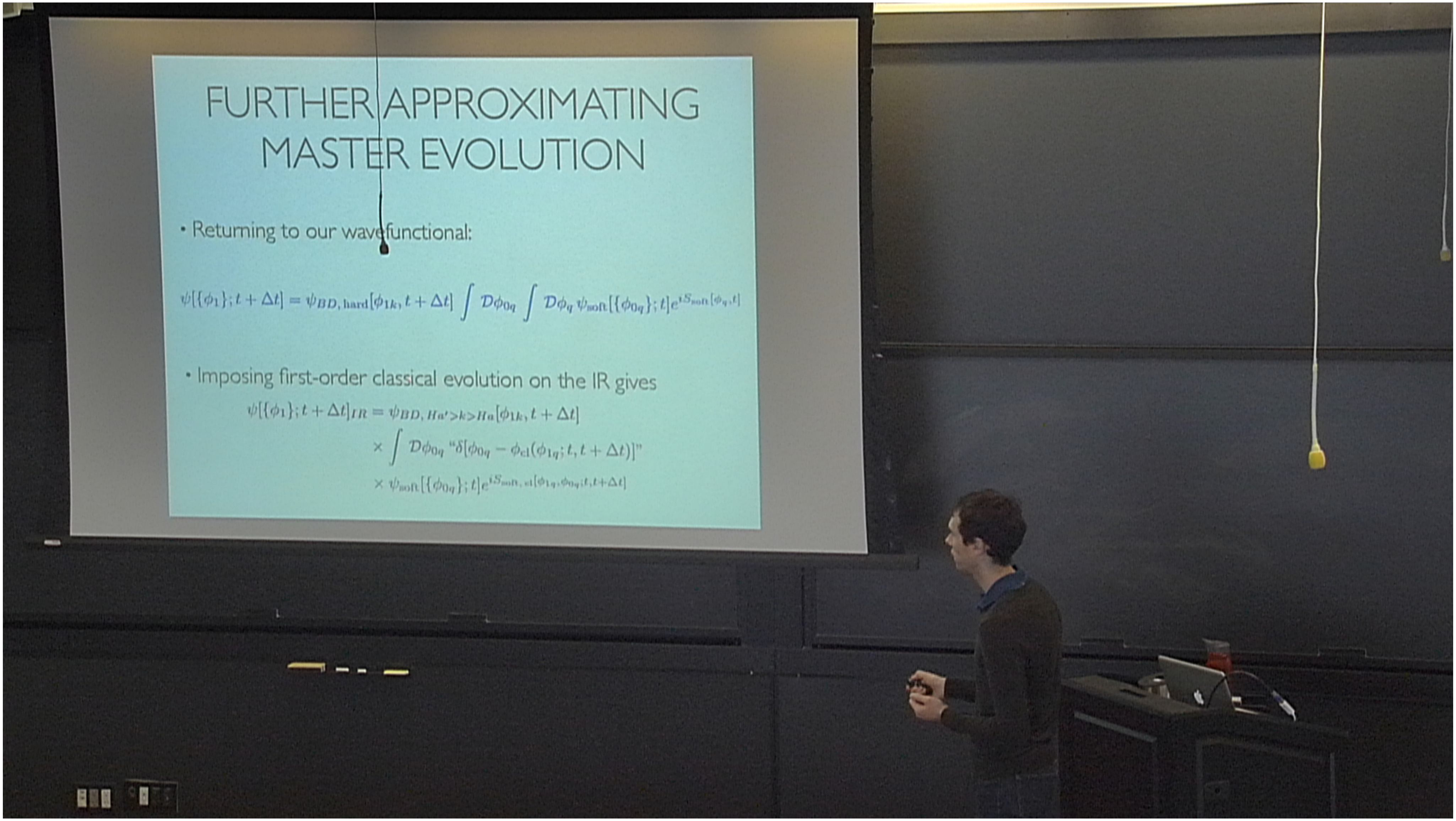

RECOVERING STAROBINSKY
• We've multiplicatively factorized our wavefunctional into hard and soft. Only soft requires special treatment, so we factor out IR.
• To solve for late-time behavior in IR sector, we define:
$p(\phi, t) \equiv \int \mathcal{D}\phi_q \, \delta\left[\int^{Ha} d^3q\phi_q - \phi\right] |\psi[\phi_q, t]_{IR}|^2$
• Using the functional definition of $\psi_{IR}$, we can solve for p's evolution.


FOKKER-PLANCK EQUATION
• In this way, we recover Starobinsky's central result, that p(φ,t) satisfies a Fokker-Planck equation
$\dot{p}(\phi,t) = \frac{1}{3H}\partial_\phi[V'(\phi)\,p(\phi,t)] + \frac{H^3}{8\pi^2}\partial_\phi^2\,p(\phi,t)$
• While generic solution is difficult, we can straightforwardly get late-time behavior
$p(\phi,t) = Ne^{-\frac{8\pi^2}{3H^4}V} + \sum_{n=1}^{\infty}\Phi_n(\phi)e^{-\Gamma_n t}$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# LATE TIME LIMIT

- Fokker-Planck solution has zero eigenvalue with all others positive.
- At very late times, any dependence on initial conditions is washed out. We flow to distribution dictated by interaction alone.
- Correlators stay finite $\langle \phi^2 \rangle_{t \to \infty} \sim \frac{H^2}{\sqrt{\lambda}}$

# THE DS/PS CORRESPONDENCE?

- At present, we still lack a firmly holographic understanding of stochastic inflation.
- However, we have recovered behavior reminiscent of the parton shower,* is this a hint of strong dynamics?
  - Both DS and PS have leading Markovian description
  - Probabilities flow in both (fixed point in DS) (Fokker-Planck vs. DGLAP)
  - Factorization in both (jets in QCD vs. Hubble patches)
- Proceeding, we hope to tighten the connection

*Similar perspective in Seery, 0903.2788

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