Title: Quantum Fields and Strings at Finite Coupling

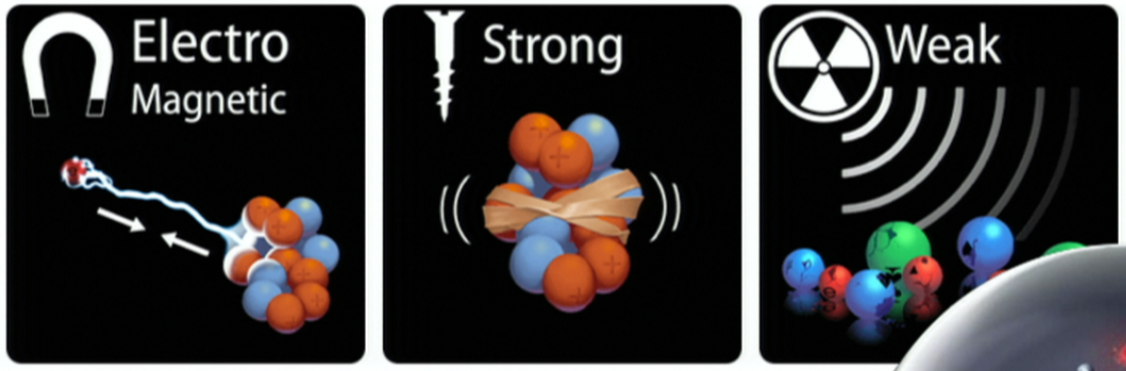
Date: Oct 15, 2014 02:00 PM

URL: http://pirsa.org/14100050

Abstract: Gauge theories lie at the heart of our understanding of three of the four known forces in nature: the electromagnetic, weak and strong forces. Moreover, our best understood non-perturbative definition of a theory of quantum gravity is also given by a gauge theory. Yet, despite their absolutely central role in physics, gauge theories are still far from being tamed with our current theoretical tools. In this colloquium we shall explore the realm of quantum fields and strings at finite coupling and survey some of the exciting recent developments which are improving this state of affairs. The main character in our incursions will be a most symmetric gauge theory known as N=4 super Yang-Mills theory, often referred to as the harmonic oscillator of the twenty first century or, as it was most recently coined, as the Darth Vader theory.

Gauge Theories and Nature

- Gauge Theories describe three of the four known forces in Nature

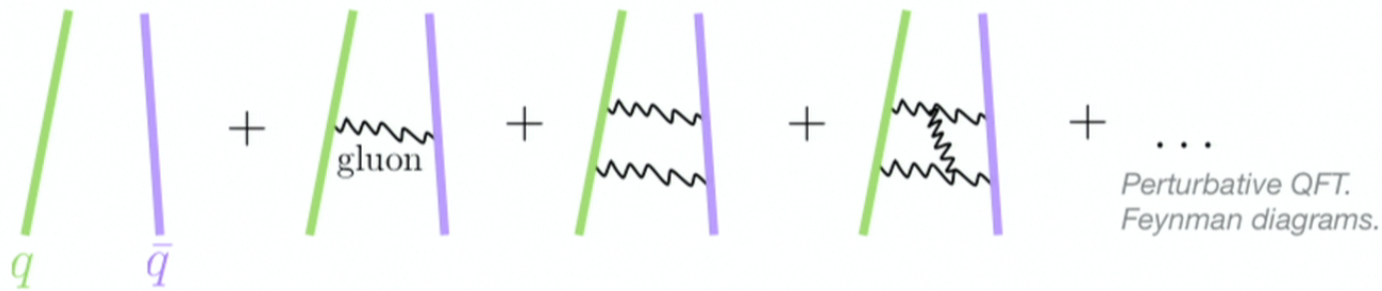


- Currently, they are our best non-perturbative definition of a theory of quantum gravity

... and yet we are not good enough for them.

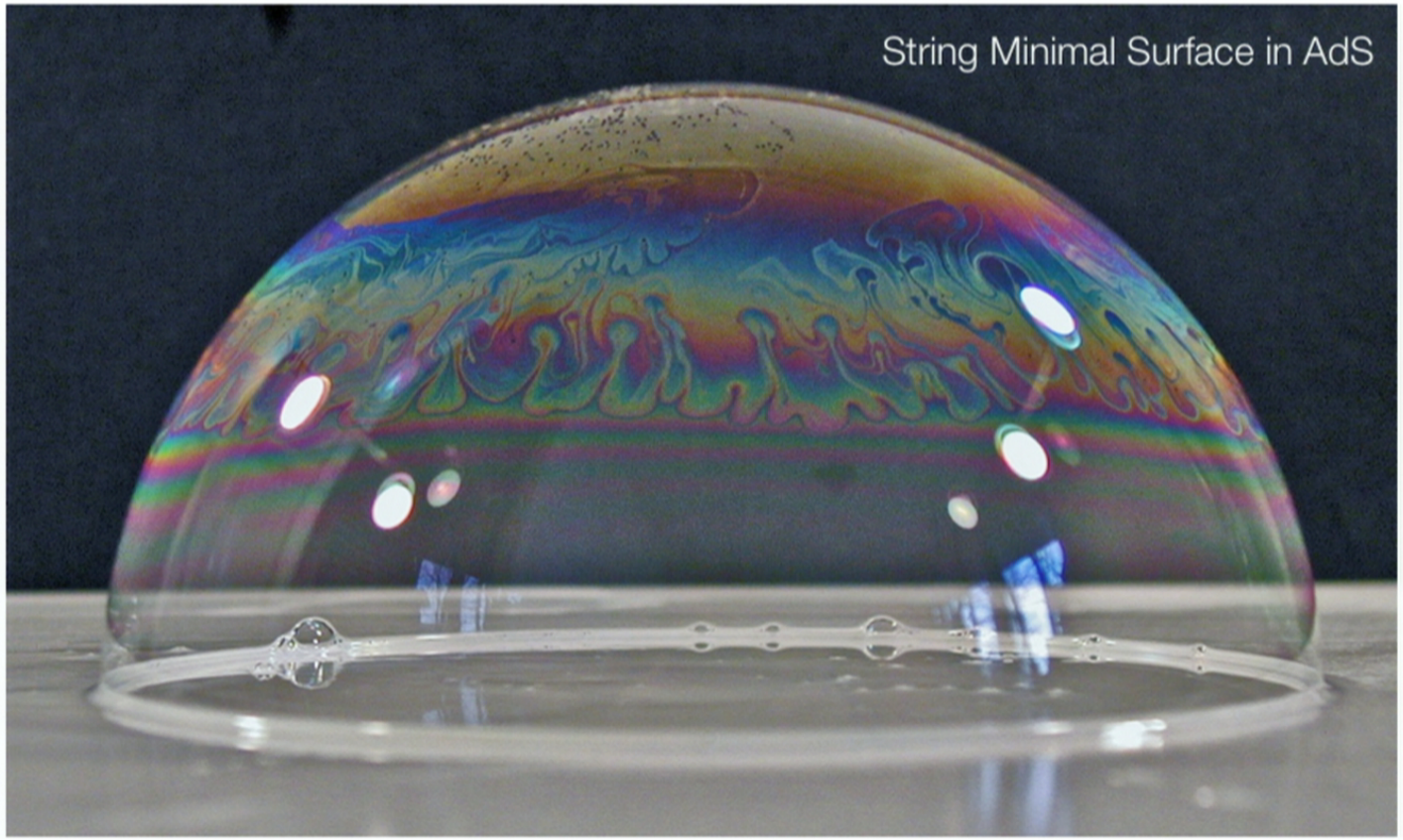
How to describe gauge theories at finite coupling?
How does a finite coupling QFT look like?
How do gauge/gravity dualities work/emerge?

Life is (somehow) simple at weak coupling



Perturbative QFT.
Feynman diagrams.

And sometimes also at strong coupling



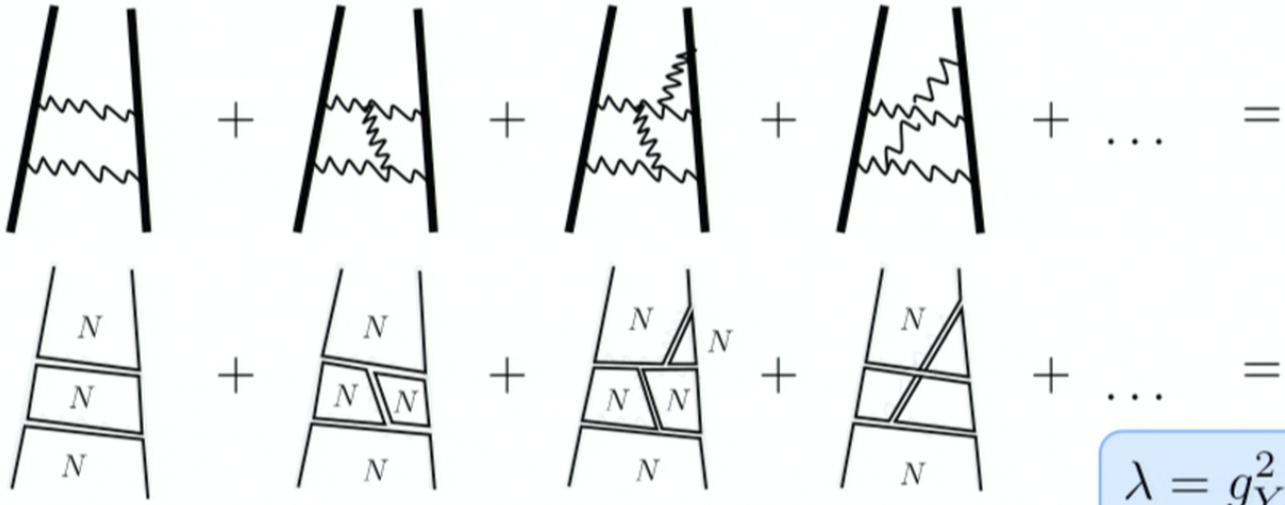

STRING WORLD
?
GLUON REALM

Outline

- Large N Gauge Theories and Holography.
- Integrable ‘t Hooft World-sheets and the Darth Vader theory.
- The Finite Coupling Spectrum. Gauge meets Strings.
- Scattering Amplitudes at Finite Coupling. Pentagons and the Flux-Tube gas.
- Outlook.

Large N Gauge Theories are String Theories

['t Hooft]

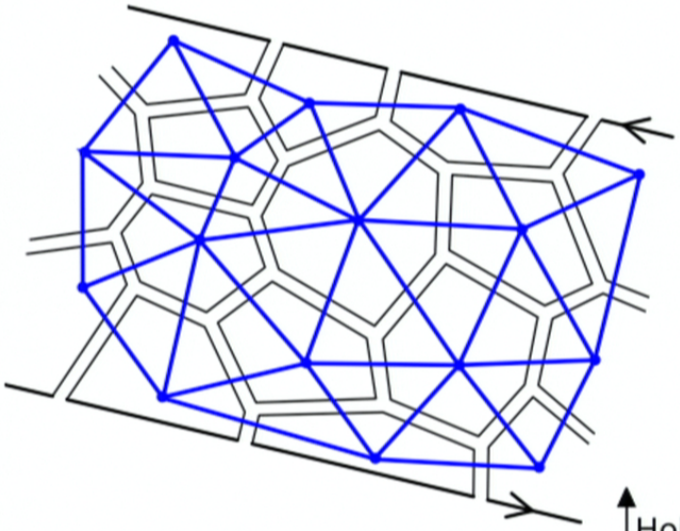


$$\lambda = g_{YM}^2 N$$

$$(\cdots + \lambda^2 + \lambda^3 + \lambda^4 + \ldots) + \frac{1}{N^2}(\cdots + \lambda^3 + \ldots) + \ldots \quad =$$

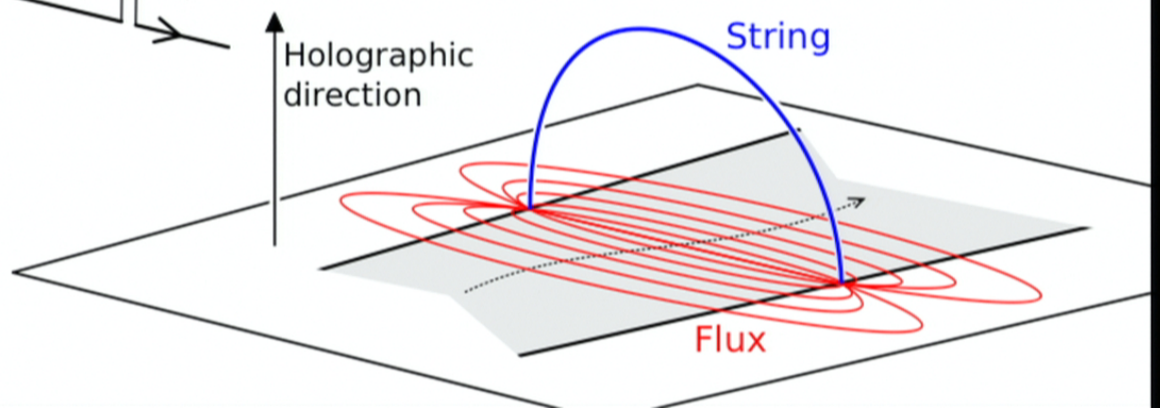
String Genus Expansion with tension related to λ and string coupled related to $1/N$

Holography (and the Holographic Flux Tube) ['t Hooft, Polyakov, Maldacena,...]



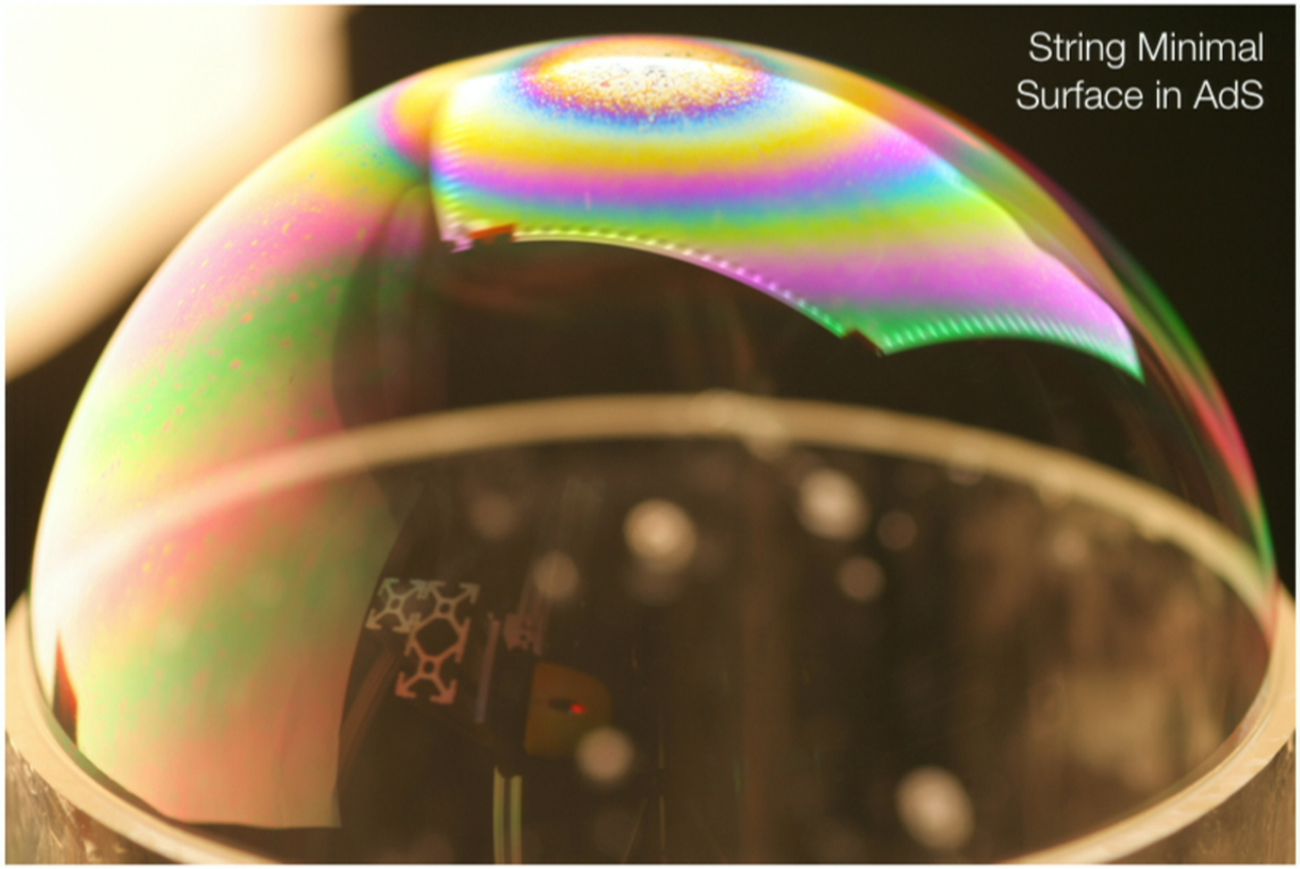
$$\text{string tension} = \sqrt{\lambda}$$
$$\text{string coupling} = 1/N$$

$=$



At large ‘t Hooft coupling the string tension is large and classical string surfaces dominate.

I.e. in such theories life is simple(r) *both* at weak and at strong coupling.



A Very Special World-Sheet

The basic message of the previous cartoons is that one should be able to trade the **4d** Feynman diagrammatic description by a **2d** description of the dual string, i.e. of the dual flux tube. This is of course a great tradeoff.

We could even be more greedy and ask ourselves whether a four dimensional gauge theory exists for which (a) we have absolute control over its associated chromodynamic flux tube and (b) the dual two dimensional string description is Integrable*.

Such gem exists. It is a gauge theory with gluons, scalars and fermions known as N=4 super Yang-Mills theory.

We believe we should be able to solve this theory. If so, this will be a most valuable first solution of a fully interactive quantum field theory.

* There are infinitely many definitions of quantum integrability. One definition is human-centric and simply stands for exact solvability. A second definition, a more useful one, has to do with factorization of the scattering of many particles which we shall now recall.

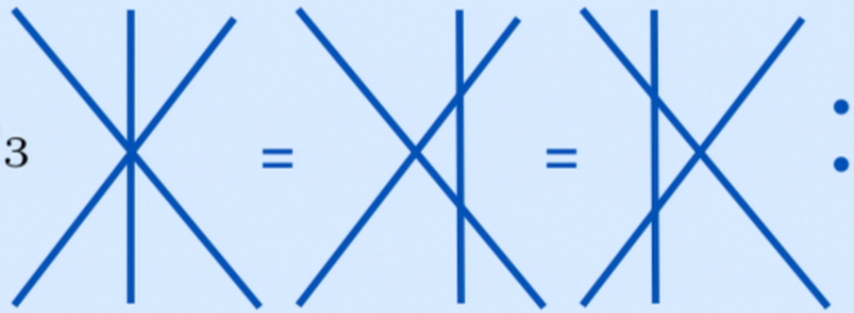
A Very Special World-Sheet

Integrability Interlude:

In 1+1D $\quad Q_1 = \sum p_j \ , \ Q_2 = \sum p_j^2 \ , \ \Rightarrow \ \{p_1, p_2\} = \{p_1', p_2'\}$

Integrability : If $\quad \exists Q_3 = \sum p_j^3 \ \Rightarrow \ \{p_1, p_2, p_3\} = \{p_1', p_2', p_3'\}$

$e^{i\alpha Q_3}$



- Factorized scattering.
- S-matrices obey YB.

Powerful. Often synonym to exact solvability.

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A Very Special World-Sheet

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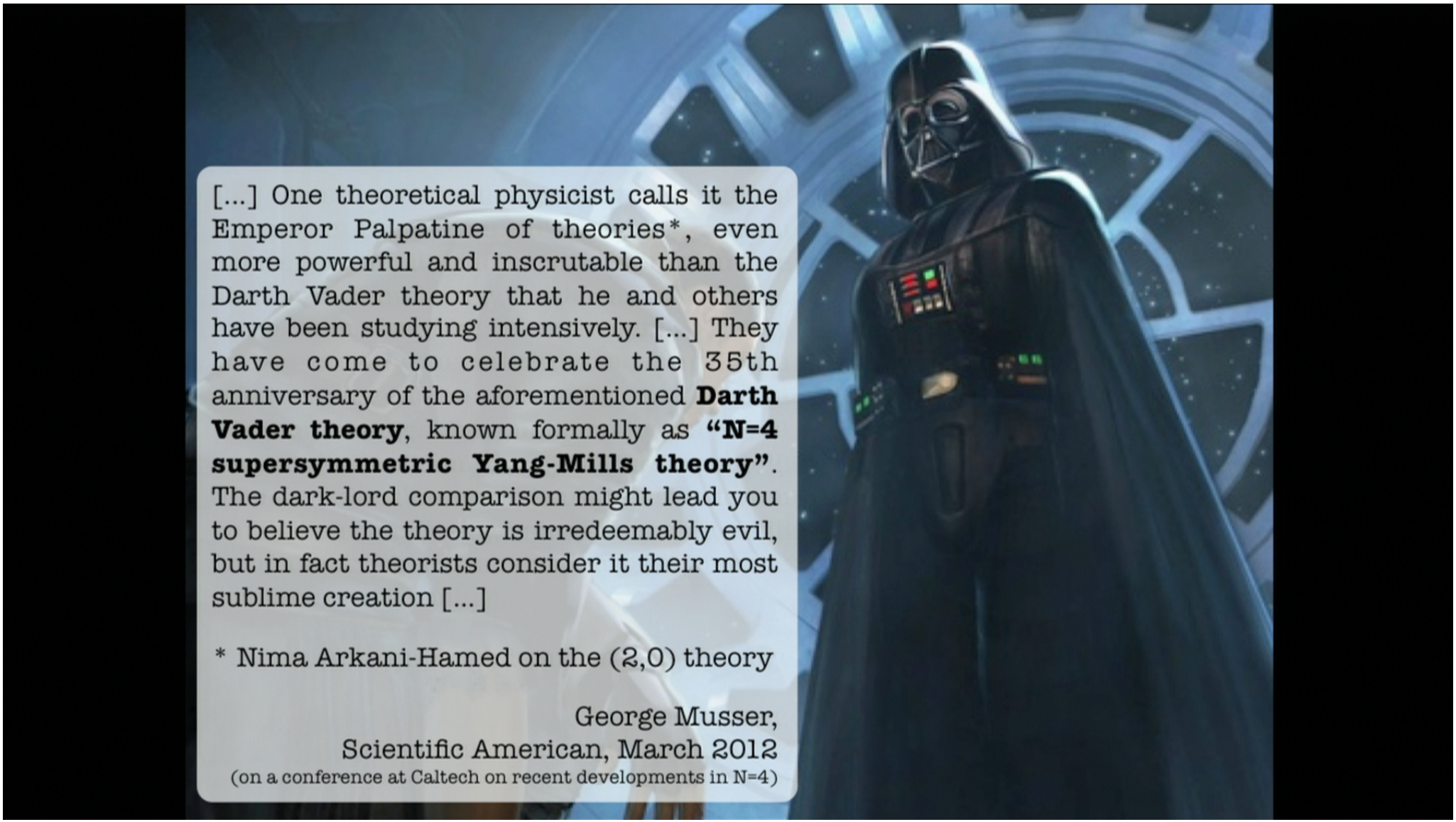
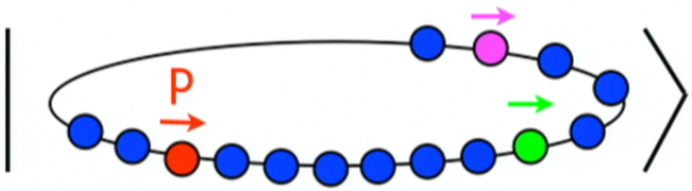

[...] One theoretical physicist calls it the Emperor Palpatine of theories*, even more powerful and inscrutable than the Darth Vader theory that he and others have been studying intensively. [...] They have come to celebrate the 35th anniversary of the aforementioned **Darth Vader theory**, known formally as **"N=4 supersymmetric Yang-Mills theory"**. The dark-lord comparison might lead you to believe the theory is irredeemably evil, but in fact theorists consider it their most sublime creation [...]
* Nima Arkani-Hamed on the (2,0) theory
George Musser,
Scientific American, March 2012
(on a conference at Caltech on recent developments in N=4)

I like to think of it as the Ising model of QFT's

(more at the end)

Integrable Spin Chains at Weak Coupling, **Integrable** Classical Ripples at Strong Coupling

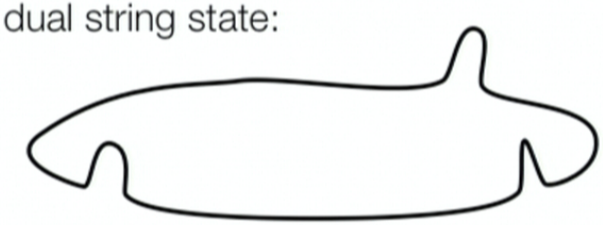
composite operator in the gauge theory:



Integrable Spin Chain

[Minahan, Zarembo; Beisert, Staudacher]

dual string state:

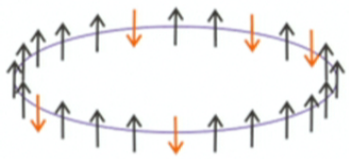


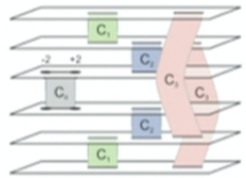
Integrable Classical String

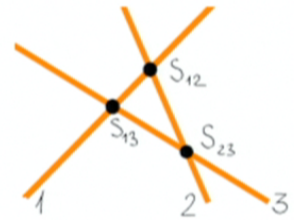
[Benna Polchinski Roiban]

A most natural conjecture (with amazing body of evidence) is then that at any coupling integrability persists:

The Planar Spectrum of a Gauge Theory



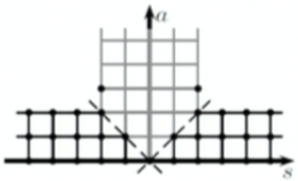


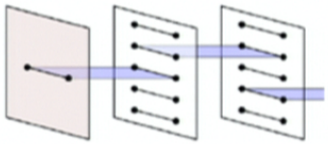


Integrable Spin Chains in SYM → String Classical Integrability → S-matrix, Asymptotic Bethe Ansatz. →

2002	2003-2008	2004-2006
Minahan,Zarembo, Beisert,Kristijanssen,Staudacher ...	Bena,Polchinski,Roiban Kazakov,Marshakov, Minahan, Zarembo, Frolov, Tseytlin Schafer-Nameki Beisert,Kazakov,Sakai,Zarembo Gromov,PV ...	Arutyunov,Frolov,Staudacher Staudacher, Beisert Janik Hernandez,Lopez Roiban, Tseytlin Beisert,Eden,Staudacher ...

The Planar Spectrum of a Gauge Theory





Y-system, TBA, Konishi Plot.

Quantum spectral curve

?

2009-2010

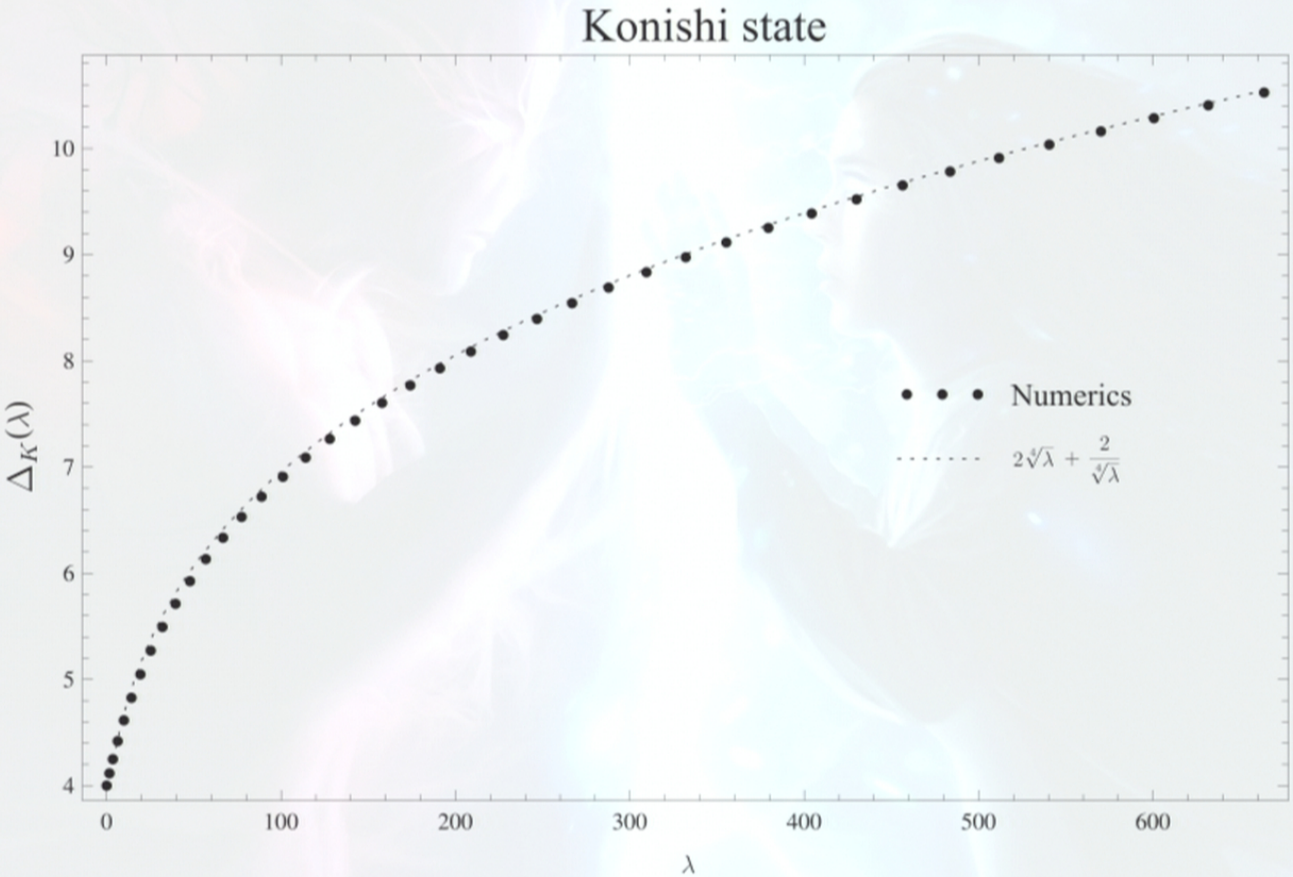
Gromov , Kazakov, PV
Bombardelli,Fioravanti, Tateo
Gromov , Kazakov,PV
Arutyunov, Frolov
...

2011-2014

Gromov, Kazakov, Leurent, Volin
...

Gauge Theory meets Strings Theory

[Gromov,Kazakov,PV]



So it can be done. Gauge theories can be tamed.

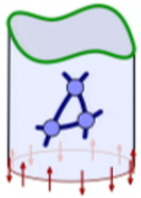
Not so long ago, this would probably be seen as naive wishful thinking

arXiv:1012.3982v5 [hep-th] 21 Feb 2012

Review of AdS/CFT Integrability: An Overview

Niklas Beisert[†,1], Changrim Ahn[2], Luis F. Alday[3,4], Zoltán Bajnok[5], James M. Drummond[6,7], Lisa Freyhult[8], Nikolay Gromov[9,10], Romuald A. Janik[11], Vladimir Kazakov[12,13], Thomas Klose[8,14], Gregory P. Korchemsky[15], Charlotte Kristjansen[16], Marc Magro[17,1], Tristan McLoughlin[1], Joseph A. Minahan[8], Rafael I. Nepomechie[18], Adam Rej[19], Radu Roiban[20], Sakura Schäfer-Nameki[9,21], Christoph Sieg[22,23], Matthias Staudacher[22,1], Alessandro Torrielli[24,25], Arkady A. Tseytlin[19], Pedro Vieira[26], Dmytro Volin[20] and Konstantinos Zoubos[16]

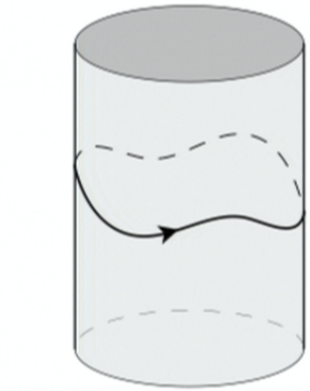
[†]corresponding author, e-mail address: nbeisert@aei.mpg.de



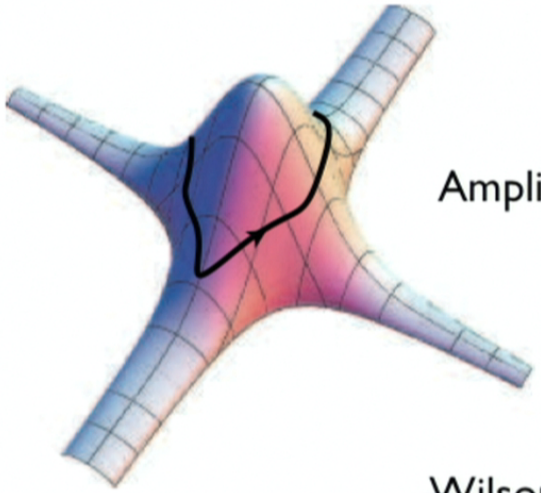
Abstract: This is the introductory chapter of a review collection on integrability in the context of the AdS/CFT correspondence. It provides a review of the achievements and the status of this subject as of the year 2010.

What comes next?

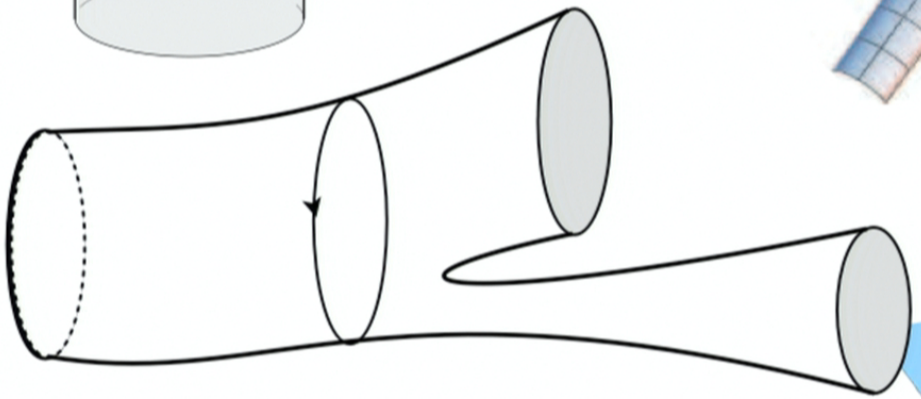
More Dynamical Observables



spectrum

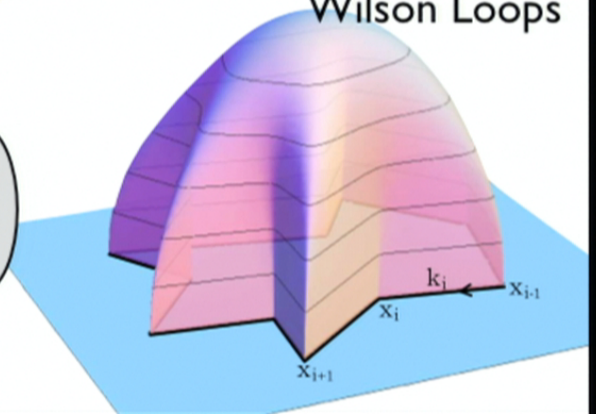


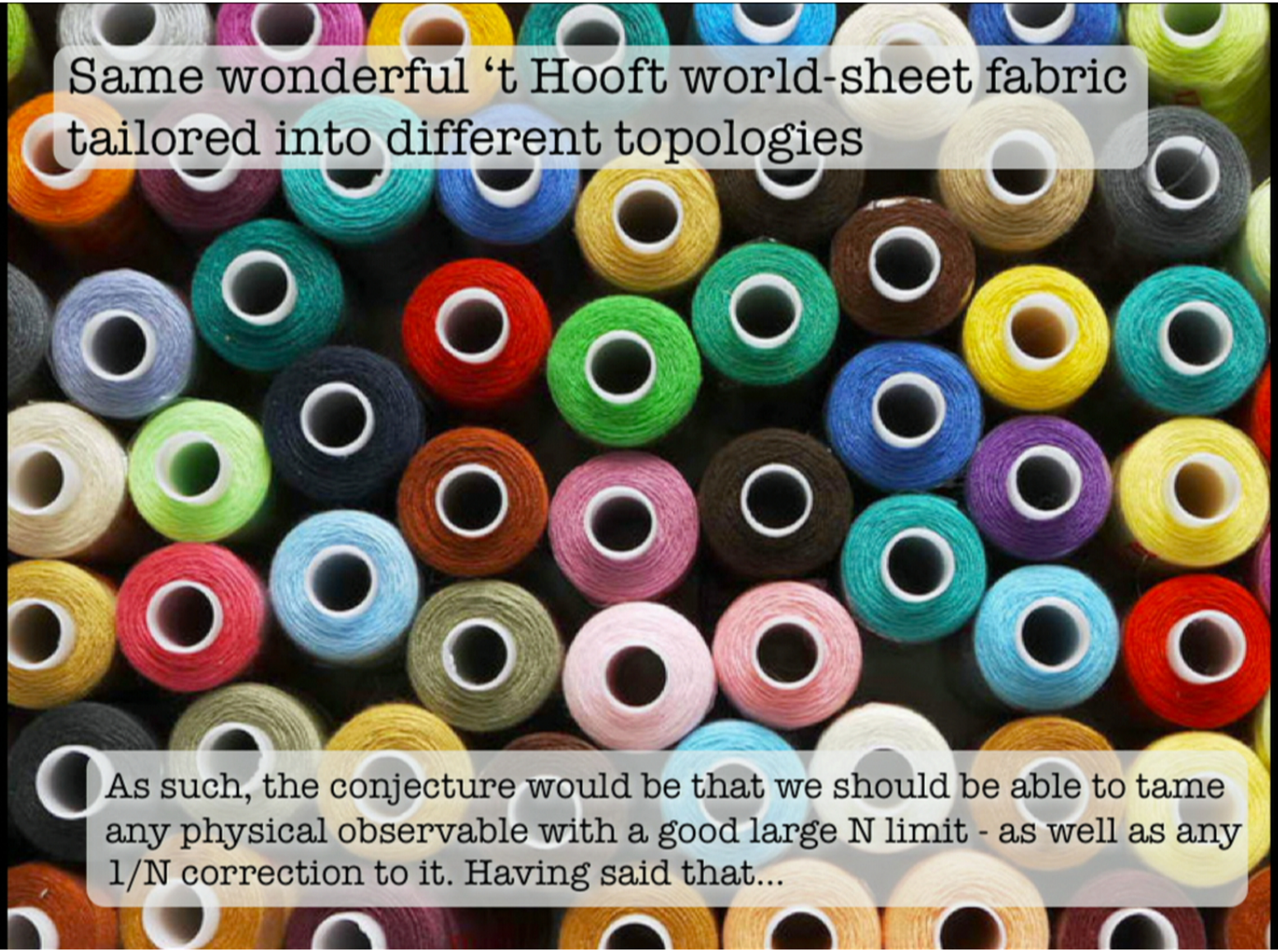
Amplitudes



(3 pt) correlation functions

Wilson Loops



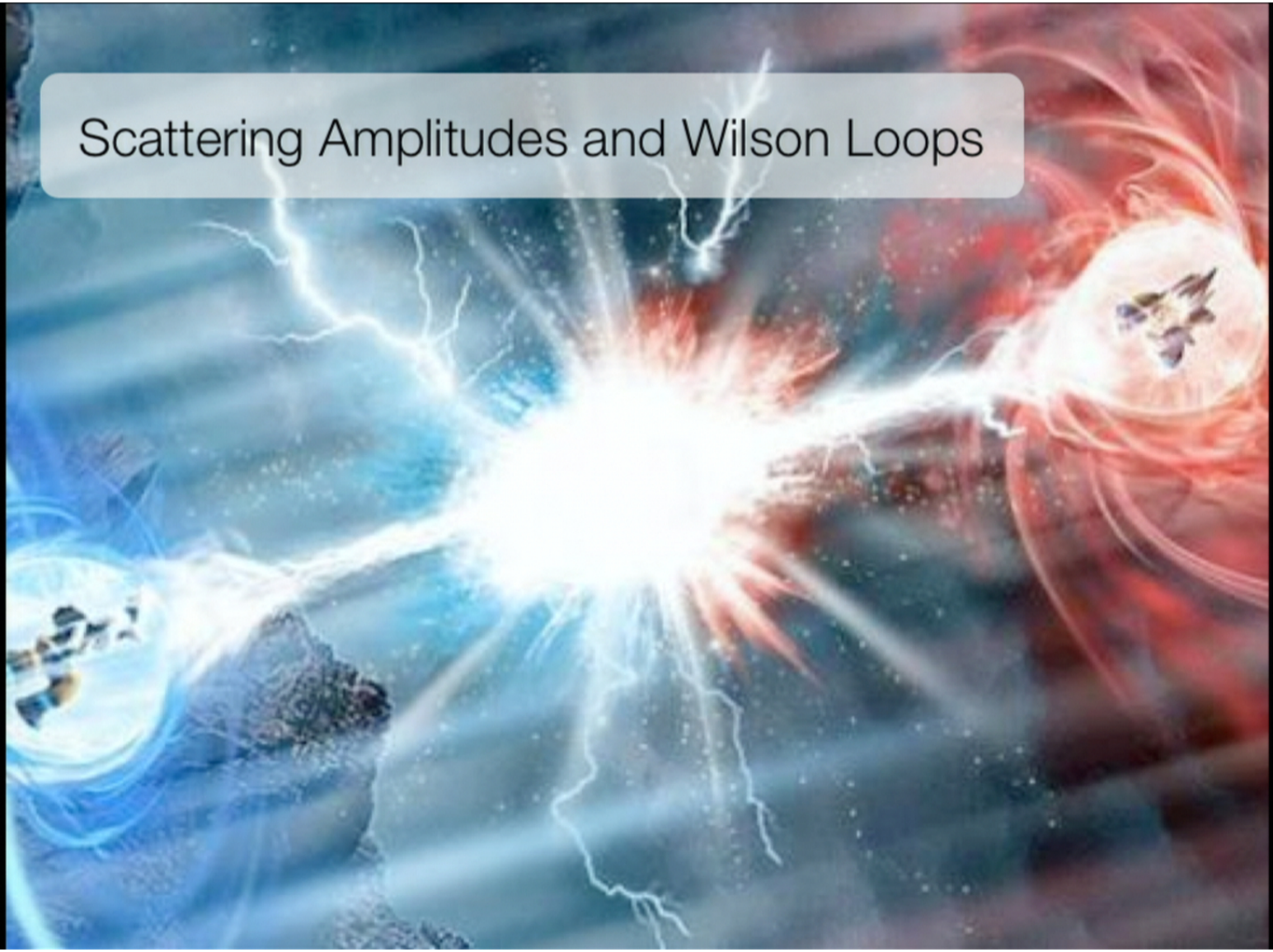

Same wonderful 't Hooft world-sheet fabric tailored into different topologies
As such, the conjecture would be that we should be able to tame any physical observable with a good large N limit - as well as any 1/N correction to it. Having said that...

... with hindsight, the spectrum was in The Book.

In The Book, one can find a the recipe for computing the spectrum of an Integrable theory at finite volume.

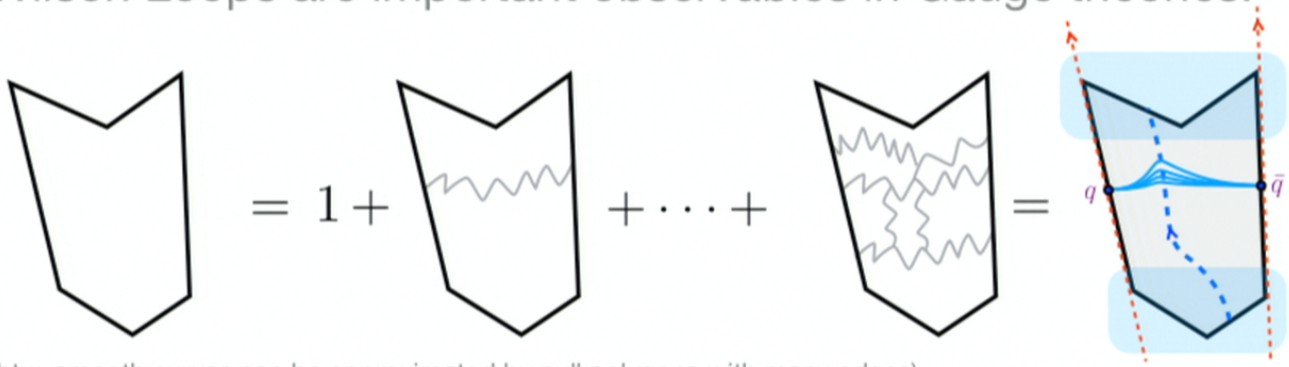
Of course, The AdS/CFT Integrable theory is not a random theory but probable The richest amongst all Integrable theories. Still.

On the other hand, what comes next for these more dynamical quantities is brand new and that is very exciting.


Scattering Amplitudes and Wilson Loops

(Another) Remarkable Duality

- Wilson Loops are important observables in Gauge theories.

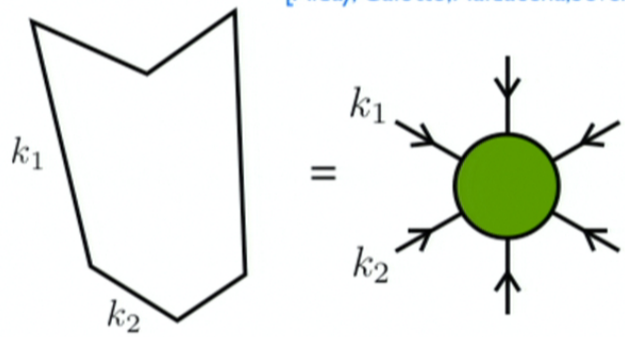


(btw, smooth curves can be approximated by null polygons with many edges)

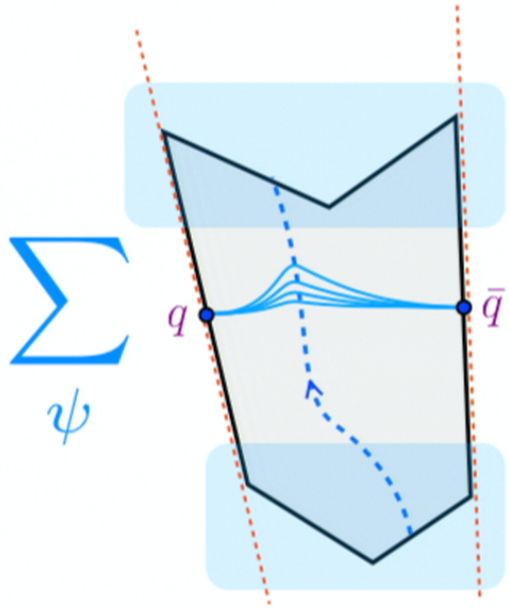
[Alday, Gaiotto,Maldacena,Sever,PV]

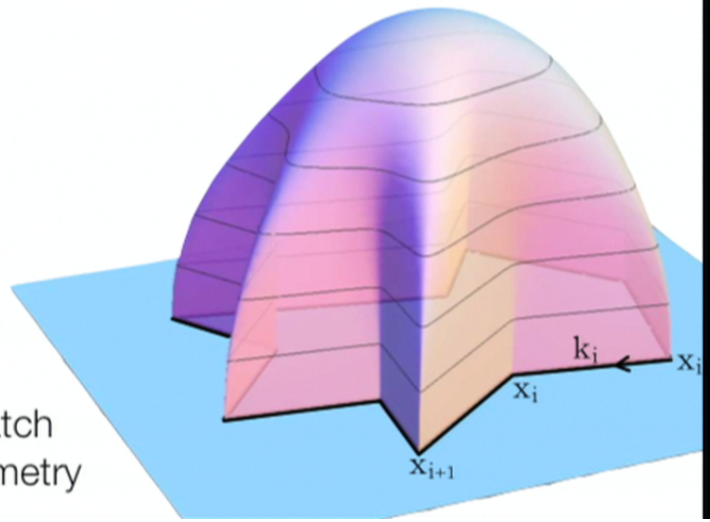
- In planar N=4 SYM they are *even* more so as
  WL = Scattering Amplitudes

  [Alday, Maldacena; Drummond, Korchemsky, Sokatchev; Brandhuber, Heslop, Travaglini; Drummond, Henn, Korchemsky, Sokatchev; Berkovits, Maldacena]



Amplitudes = Sum over Flux Tube states
= Open String Partition Function



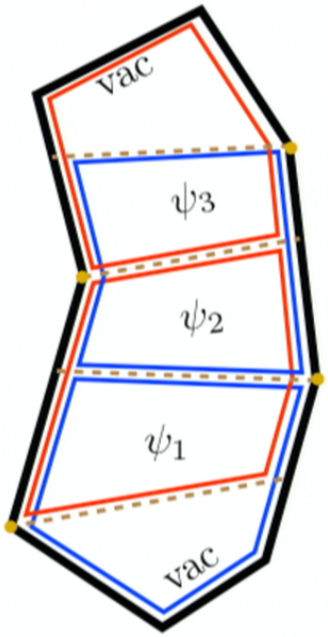


Basic ideas

1. Use the spectrum knowledge to describe the propagation
2. Tesselate the flux tube world-sheet like we patch a quilt as to tame the involved polygonal geometry

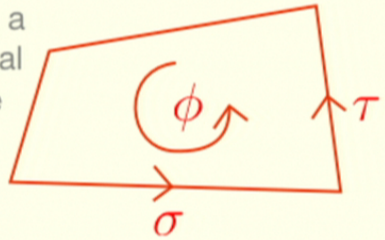
[Basso,Sever,PV]

The Quantum Quilt. Amplitudes as a Flux Tube Gas



To measure some charge of the states flowing from A to B we act with the symmetry generators (corresponding to that charge) on A (or on B). See e.g. the usual OPE for 4pt correlation functions where we act with dilatations on two points to measure what flows from two operators to the other two. (Of course acting on both A and B means doing nothing by definition of a symmetry.)

Similarly, each middle square in our tessellation has 3 symmetries corresponding to a time translation, a space translation and a rotation of the orthogonal directions. We act with those symmetries on the cusps below each that square. In this way we measure the energy, momentum and angular momentum flowing in each middle square.



There are n-5 middle squares so the 3(n-5) parameters which neatly parametrize all conformally inequivalent polygons.

$$\mathcal{W} = \sum_{\psi_i} \left[\prod_{j=1}^{n-5} e^{-E_j \tau_j + i p_j \sigma_j + i m_j \phi_j} \right] P(0|\psi_1) P(\psi_1|\psi_2) \ldots P(\psi_{n-6}|\psi_{n-5}) P(\psi_{n-5}|0)$$

[Basso,Sever,PV]

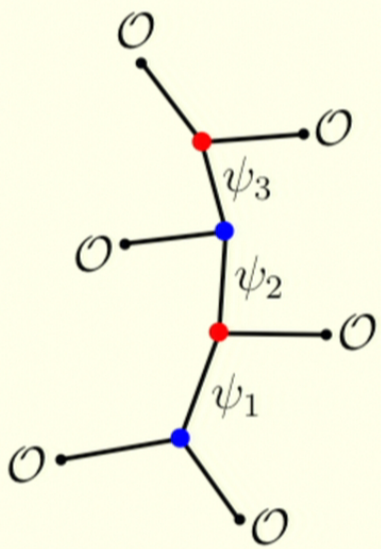
The Quantum Quilt. Amplitudes as a Flux Tube Gas

To measure some charge of the states flowing from A to B we act with

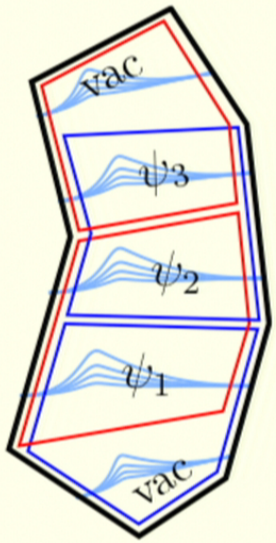
[Basso,Sever,PV]

The Quantum Quilt. Amplitudes as a Flux Tube Gas

The analogue of the usual OPE for correlation functions in a CFT:



$\longleftrightarrow$



OPE for Correlation Functions

OPE for Wilson Loops

$$\mathcal{W} = \sum_{\psi_i} \left[\prod_{j=1}^{n-5} e^{-E_j \tau_j + i p_j \sigma_j + i m_j \phi_j} \right] P(0|\psi_1) P(\psi_1|\psi_2) \ldots P(\psi_{n-6}|\psi_{n-5}) P(\psi_{n-5}|0)$$

So far only we used conformal symmetry. Integrability comes in now.

There are several *equivalent* descriptions of the excited flux tube. As null Wilson lines with adjoint insertions, as large spin operators,

$$\mathcal{O} = \mathrm{tr}\,(Z\,DDDD\ldots DDDD\,\vec{F}\,DDDD\ldots DDDD\,\vec{F}\,DDDD\ldots DDDD\,Z)$$

or an excited GKP [Gubser,Klebanov,Polyakov] folded string.

These states have a *fixed* number of excitations with given momenta and we know their spectrum exactly from Integrability [Basso 2010]

(we also know how these excitations scatter amongst themselves [Basso,Rej;Fioravanti,Piscaglia,Rossi;Basso,Sever,PV]).

Finding the Pentagons is the most interesting part.

$$\mathcal{W} = \sum_{\psi_i} \left[\prod_{j=1}^{n-5} e^{-E_j\tau_j + ip_j\sigma_j + im_j\phi_j} \right] P(0|\psi_1)P(\psi_1|\psi_2)\ldots P(\psi_{n-6}|\psi_{n-5})P(\psi_{n-5}|0)$$

So far only we used conformal symmetry. Integrability comes in now.

Example. Mass of a Gluon Excitation:

Inverse comes from going from the BMN to the GKP vacuum

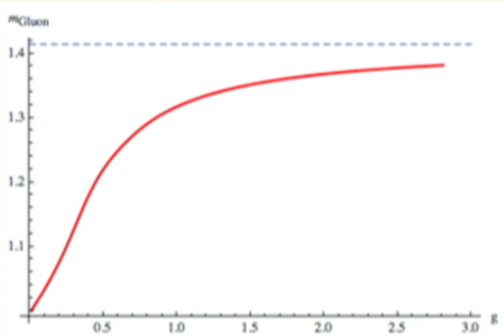
```
ω1 = 15; ω2 = 200;
```

```
K[g_] := K[g] = ParallelTable[2 j (-1)^((i+1) j) NIntegrate[BesselJ[i, 2 g t] BesselJ[j, 2 g t] / ((e^t - 1) t), {t, 0, ω2}], {i, ω1}, {j, ω1}]

χG[g_] := χG[g] = ParallelTable[NIntegrate[-1/t (BesselJ[i, 2 g t] (e^((-1)^i t/2) - BesselJ[0, 2 g t])) / (e^t - 1), {t, 0, ω2}, WorkingPrecision → 20], {i, ω1}]
```

```
mGauge[g_] := mGauge[g] = 1 + 4 g (Inverse[IdentityMatrix[ω1] + K[g]].χG[g])[[1]]
```

```
GaugePlot =
 Show[
  Monitor[Table[{g, mGauge[g]}, {g, 1/100, 3, 1/5}] //
    ListPlot[#, PlotStyle → {Thick, Red}, Joined → True, InterpolationOrder → 4, AxesLabel → {"g", "m_Gluon"}] &,
   ProgressIndicator[g, {1/100, 3}]], Plot[√2, {x, 0, 3}, PlotStyle → Dashed], PlotRange → All]
```



So far only we used conformal symmetry. Integrability comes in now.

There are several *equivalent* descriptions of the excited flux tube. As null Wilson lines with adjoint insertions, as large spin operators,

$$\mathcal{O} = \mathrm{tr}\,(Z\,DDDD\ldots DDDD\,\vec{F}\,DDDD\ldots DDDD\,\vec{F}\,DDDD\ldots DDDD\,Z)$$

or an excited GKP [Gubser,Klebanov,Polyakov] folded string.

These states have a *fixed* number of excitations with given momenta and we know their spectrum exactly from Integrability [Basso 2010]

(we also know how these excitations scatter amongst themselves [Basso,Rej;Fioravanti,Piscaglia,Rossi;Basso,Sever,PV]).

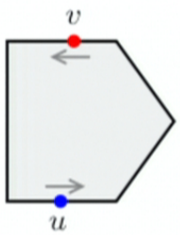
Finding the Pentagons is the most interesting part.

$$\mathcal{W} = \sum_{\psi_i} \left[\prod_{j=1}^{n-5} e^{-E_j\tau_j + ip_j\sigma_j + im_j\phi_j} \right] P(0|\psi_1)P(\psi_1|\psi_2)\ldots P(\psi_{n-6}|\psi_{n-5})P(\psi_{n-5}|0)$$

The Pentagon Bootstrap

[Basso,Sever,PV]

The main idea is to postulate a set of Bootstrap axioms that the pentagons ought to satisfy. They take the form of functional equations. For example, for the simplest possible transition



$P(u|v)$

we propose:

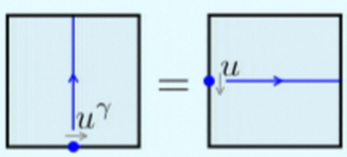
$$\text{I. } P(u|v) = P(-v|-u)$$

$$\text{II. } P(u|v) = S(u,v)P(v|u)$$

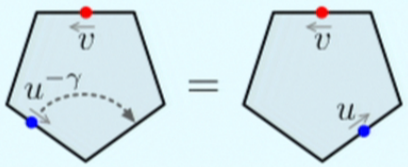
$$\text{III. } P(u^{-\gamma}|v) = P(v|u)$$

- Axiom I is the obvious reflection symmetry of the pentagon.
- Axiom III comes from the mirror symmetry of the flux tube. There exists a non-perturbative path in the rapidity u which implements the Wick rotation:

$$E(u^\gamma) = ip(u)$$
$$p(u^\gamma) = iE(u)$$

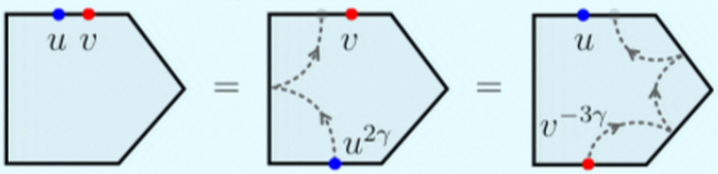


and thus



which is nothing but axiom III

- Axiom II, together with the other two, implies Watson's equation $P(0|u,v) = S(v,u)P(0|v,u)$ where P(0|u,v) is given by



This is a nice self-consistency check and is the main motivation for Axiom II.

The Solution

[Basso,Sever,PV]

- We can solve the bootstrap equations. E.g., a solution for the scalar excitations is

$$P(u|v)^2 = \frac{S(u,v)}{g^2(u-v)(u-v+i)S(u^\gamma,v)}$$

It provides a precise connection between the space-time and the flux tube S-matrices.

- Multi-particles are built in terms of the single particle transitions:

$$P(\{u_i\}|\{v_j\}) = \frac{\prod_{i,j} P(u_i|v_j)}{\prod_{i>j} P(u_i|u_j) \prod_{i<j} P(v_i|v_j)}$$ x (Group theory matrix part)

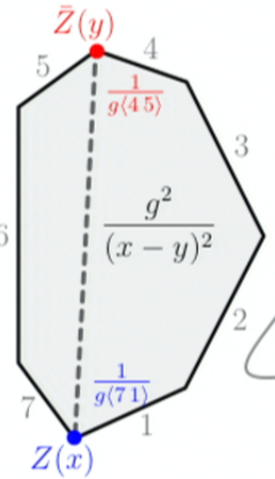
For example, for two scalars

$$P(\mathbf{u}|\mathbf{v})^{j_1 j_2}_{i_1 i_2} = P_{\rm dyn}(\mathbf{u}|\mathbf{v}) \left[\pi_1(\mathbf{u}|\mathbf{v})\delta^{j_1}_{i_1}\delta^{j_2}_{i_2} + \pi_2(\mathbf{u}|\mathbf{v})\delta^{j_2}_{i_1}\delta^{j_1}_{i_2} + \pi_3(\mathbf{u}|\mathbf{v})\delta_{i_1 i_2}\delta^{j_1 j_2}\right]$$

where the matrix functions π_j are simple, rational functions of the rapidities, independent of the coupling.

Stepping Back. Tree Level and Half Santa Claus

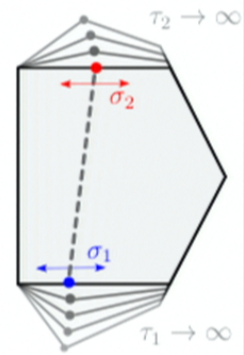
$$\mathcal{R}^{(7145)}_{\text{tree}} =$$



$$= \frac{1}{\langle 7\,1\rangle (x-y)^2 \langle 4\,5\rangle} = e^{-\tau_1-\tau_2} \times \frac{1}{e^{\sigma_1-\sigma_2}+e^{\sigma_2-\sigma_1}+e^{\sigma_1+\sigma_2}} + \cdots =$$

Express it using the OPE coordinates

From your favorite tree-level generator, BCFW for example.



$$\int \frac{dp_1\,dp_2}{16\pi^2} e^{-ip_1\sigma_1+ip_2\sigma_2} \Gamma\left(\frac{1}{2}-\frac{ip_1}{2}\right)\Gamma\left(\frac{1}{2}+\frac{ip_2}{2}\right)\Gamma\left(\frac{ip_1}{2}-\frac{ip_2}{2}\right)$$

and this is half Santa. Indeed...

From Integrability, a *totally* different computation yielded

$$S(p_1,p_2) = \frac{\Gamma\left(\frac{1}{2}-\frac{ip_1}{2}\right)\Gamma\left(\frac{1}{2}+\frac{ip_2}{2}\right)\Gamma\left(\frac{ip_1}{2}-\frac{ip_2}{2}\right)}{\Gamma\left(\frac{1}{2}-\frac{ip_2}{2}\right)\Gamma\left(\frac{1}{2}+\frac{ip_1}{2}\right)\Gamma\left(\frac{ip_2}{2}-\frac{ip_1}{2}\right)}$$

in perfect agreement with axiom II

Stepping Back. Tree Level and Half Santa Claus

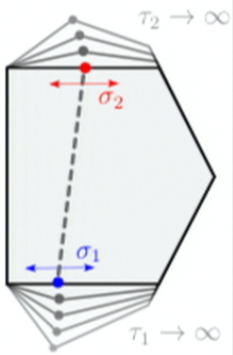
$\mathcal{R}^{(7145)}_{\text{tree}} =$

And indeed, we could do the Santa Claus test at 1 loop, 2 loops, ...

Express it using the OPE coordinates

$$= e^{-\tau_1-\tau_2} \times \frac{1}{e^{\sigma_1-\sigma_2} + e^{\sigma_2-\sigma_1} + e^{\sigma_1+\sigma_2}} + \cdots =$$

tree-FW



$$_2\sigma_2 \Gamma\left(\frac{1}{2} - \frac{ip_1}{2}\right)\Gamma\left(\frac{1}{2} + \frac{ip_2}{2}\right)\Gamma\left(\frac{ip_1}{2} - \frac{ip_2}{2}\right)$$

and this is half Santa. Indeed...

From Integrability, a *totally* different computation yielded

$$S(p_1, p_2) = \frac{\Gamma\left(\frac{1}{2} - \frac{ip_1}{2}\right)\Gamma\left(\frac{1}{2} + \frac{ip_2}{2}\right)\Gamma\left(\frac{ip_1}{2} - \frac{ip_2}{2}\right)}{\Gamma\left(\frac{1}{2} - \frac{ip_2}{2}\right)\Gamma\left(\frac{1}{2} + \frac{ip_1}{2}\right)\Gamma\left(\frac{ip_2}{2} - \frac{ip_1}{2}\right)}$$

in perfect agreement with axiom II

Stepping Forward.

*We must now put the pentagons to the test. At **weak**, **strong** and **finite** coupling.*

$$\mathcal{W} = \sum_{\psi_i} \left[\prod_{j=1}^{n-5} e^{-E_j \tau_j + i p_j \sigma_j + i m_j \phi_j} \right] P(0|\psi_1) P(\psi_1|\psi_2) \ldots P(\psi_{n-6}|\psi_{n-5}) P(\psi_{n-5}|0)$$

Weak Coupling. The Hexagon Function Program

$$\mathcal{W}_{\rm hex} = 1 + e^{-\tau}\left(e^{i\phi} + e^{-i\phi}\right)\mathcal{A} + e^{-2\tau}\left(e^{2i\phi} + e^{-2i\phi}\right)\mathcal{B} + e^{-2\tau}\mathcal{C} + \mathcal{O}(e^{-3\tau})$$

$$\mathcal{W} = a_1 f^{(1)}_{\rm hex}(\sigma,\tau,\phi) + a_2 f^{(2)}_{\rm hex}(\sigma,\tau,\phi) + a_3 f^{(3)}_{\rm hex}(\sigma,\tau,\phi) + \ldots$$

where the functions are a base of so-called iterated integrals of a certain degree (we can think of them as fancy generalizations of logarithms and polylogarithms). To fix the constants one can then "simply" expand the ansatz and compare with the OPE. Then it feeds back into the OPE as a powerful very powerful self-consistency check, *both* of the Hexagon ansatz and of the integrability based conjectures.

	3 loops (symbol) [7]	4 loops (symbol) [8]
# of constants before imposing (most of) OPE	2	80
# of constants after imposing $e^{-\tau\pm i\phi}$	0	4
# of constants after imposing $e^{-2\tau\pm 2i\phi}$	✓	0
# of constants after imposing $e^{-2\tau+0i\phi}$	✓	✓

This data is particularly efficient in conjunction with the hexagon function program by Dixon et al [Dixon,Drummond,Henn],[Dixon,Duhr,Pennington,Von Hippel], [Dixon, Drummond, Duhr,Pennington],[Dixon,Von Hippel,to appear]

Strong Coupling. The Emergence of Strings

$$\mathcal{W}^{\text{string}} \simeq \exp\left(-\frac{\sqrt{\lambda}}{2\pi} Y Y_c\right) = 1 - \frac{\sqrt{\lambda}}{2\pi}\left(e^{i\phi} + e^{-i\phi}\right) \int_{\mathbb{R}} \frac{d\theta}{\pi \cosh^2(2\theta)} e^{-\sqrt{2}\tau \cosh\theta + i\sqrt{2}\sigma \sinh\theta}$$

$$+ \frac{\sqrt{\lambda}}{2\pi} \int_{\mathbb{R}+i0} \frac{d\theta}{\pi \sinh^2(2\theta)} e^{-2\tau \cosh\theta + 2i\sigma \sinh\theta} + \dots .$$

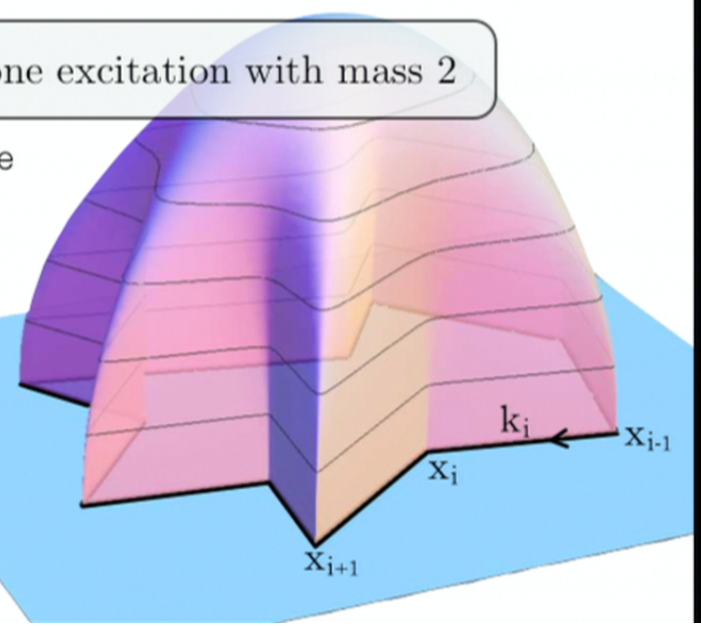
Direct computation of the Area.
(using classical Integrability of the string sigma model)
Purely Geometrical Problem.

We see two excitations with mass $\sqrt{2}$ and one excitation with mass 2

Looks good. A 2d worldsheet in (4+1) dimensional anti de Sitter has 3 transverse modes. They were first found by Frolov and Tseytlin when semi-classically quantizing the GKP folded string and perfectly match this spectrum.

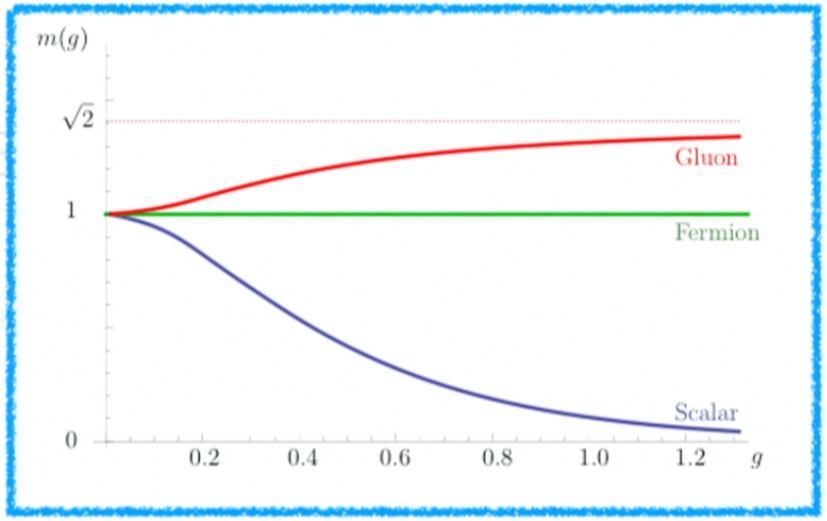
This result begs for a Flux Tube gas interpretation.
It *works* and we learn something very cute as a bonus...

[Alday,Maldacena] (4pt), **Area**
[Alday,Maldacena] (special case of 8pt),
[Alday,Gaiotto,Maldacena] (6pt)
[Alday,Maldacena,Sever,PV] (general configuration)



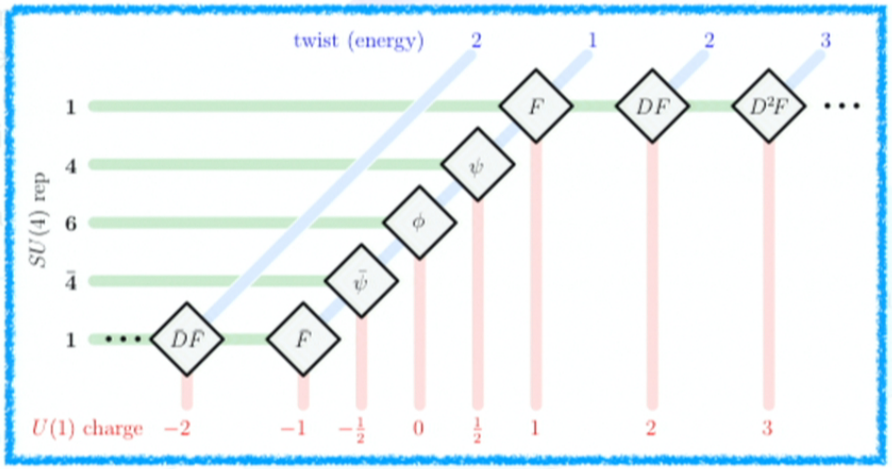
Strong Coupling. The E

The two lightest modes are the transverse gluonic excitations of the flux tube. The gluonic pentagons become 1 plus a small correction and the sum over multi-particle gluons exponentiates yielding precisely the corresponding terms in the Y-system.



We see two excitations with mass $\sqrt{2}$ and one excitation with mass 2

The mass 2 excitation is even more interesting. There is no fundamental excitation with that mass at strong coupling. Instead, this excitation, corresponding to the missing direction in AdS, is an *emergent* excitation which arises as a sort of bound-state made out of two *fermionic* excitations each of mass 1 which emerges at strong coupling. Taking them into account we get the remaining third of the Y-system minimal area result! [Basso,Sever,PV]



Just the beginning

As we see, we start to have full control over week and strong coupling and to understand what is the precise dynamical mechanism by which gluons become strings.

There is still *much* that we did not discuss and also *much* to be done.

What is the role of the sphere?, how do we describe super Wilson loops required to describe all possible scattering amplitudes in the theory?, What is the collinear behavior of scattering amplitudes at any coupling? What about their factorization behavior? What is the math behind the matrix part that we saw before for the scalars and fermions? What is the Regge behavior of scattering amplitudes at finite coupling? How does the flux tube description talk to other approaches to describing scattering amplitudes? Etcetera

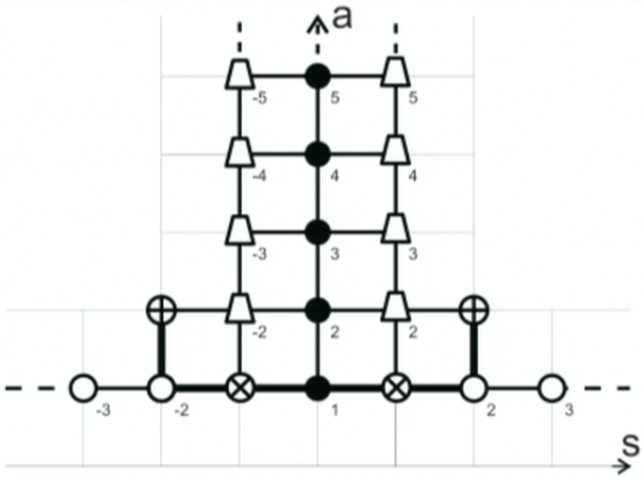
Equally important....

... Finite Coupling and the Flux Tube Gas

$$\mathcal{W} = \sum_{\psi_i} \left[\prod_{j=1}^{n-5} e^{-E_j \tau_j + i p_j \sigma_j + i m_j \phi_j} \right] P(0|\psi_1) P(\psi_1|\psi_2) \ldots P(\psi_{n-6}|\psi_{n-5}) P(\psi_{n-5}|0)$$

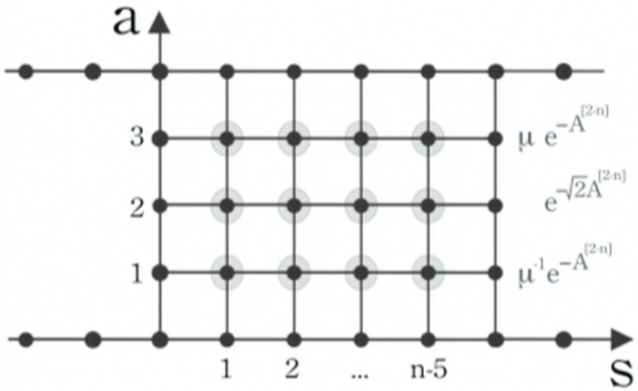
please let me in....

What lies beyond the gas? Any Master Object?



Y-system for the Spectrum at Finite Coupling

[Gromov,Kazakov,PV]]



Y-system for Scattering Amplitudes at Strong Coupling

[Alday,Maldacena,Sever,PV]

[Basso,Sever,PV]

$$\mathcal{W} = \sum_{\psi_i} \left[\prod_{j=1}^{n-5} e^{-E_j \tau_j + i p_j \sigma_j + i m_j \phi - j} \right] P(0|\psi_1) P(\psi_1|\psi_2) \ldots P(\psi_{n-6}|\psi_{n-5}) P(\psi_{n-5}|0)$$

Flux Tube Gas for Scattering Amplitudes at Finite Coupling. Can we do even better?

Zooming out

Spectrum
Scattering Amplitudes
Three Point Correlation Functions.
Four Point Correlation Functions.
Smooth Wilson Loops.
Form Factors.
Other Quantities
1/N Corrections.
Partial 1/N Re-summations.
Connections to Localization
Connections to the Conformal Bootstrap.
Connections to other theories. General lessons.
...

We will see what a QFT looks like. And then?

Then we don't know.

It is like predicting next month's weather. We believe it will probably be great.

Still, perhaps we can safely muse a little bit and propose an analogy with Onsager's 1944 solution of the 2D Ising model. What we know today is that whole fields came to existence as a direct consequence of this solution. The fields of critical phenomena and phase transitions, (2D) conformal field theory, transfer matrices and integrable models, dualities, the notion of finite size scaling are just a few examples of research topics that either appeared as a direct consequence of Onsager's breakthrough or where rapid advances came as a result of it. It was not even clear before this solutions whether phase transitions would be smoothened out in an exact solution of a statistical model! Probably the most conservative scenario is that we will experience at last a similar revolution in our understanding of Quantum Fields and String once we will have at our disposal the first clean solution of an interacting Gauge theory.

Thank you


ANSATZ
FINITE
Integrability
AMPLITUDES
String
BETHE
CHAIN
Spin
YANG
Conformal
CORRELATION
FACTORS
PLANAR
SCALAR
SPINONS
Functions
COUPLING
SPECTRUM
CLASSICS
FLUX
BAXTER
MAGNONS
quantum
LIMIT
Anti
SITTER
Loops
PERTURBATION
TUBE
SPECTRAL
SCATTERING
MODELS
ADS
Darth
Soap
Kohishi
GRAVITY
HOLOGRAPHY
INTEGRABILITY
BES