Title: Complex Loop Quantum Cosmology

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Abstract: We study in the context of loop quantum cosmology the effect of the analytic continuation that sends the Barbero-Immirzi parameter to a purely imaginary value. We show that this construction leads once again to a bouncing scenario, in which however the contracting and expanding phases on each side of the bounce are not symmetrical. Moreover, the minimal volume reached by the universe and the critical matter density become naturally independent of the Barbero-Immirzi parameter. This analytic continuation was first proposed in the context of black hole entropy calculation, and constitutes a proposal for defining a theory of self-dual quantum gravity in terms of the complex Ashtekar connection and for solving the so-called reality conditions. We expect that the systematic investigation of this analytic continuation in various setups will eventually lead to new insights on the status of the quantum states of complex Ashtekar gravity.

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Motivation for working with $\gamma = i$

- γ doesn't play any role at the classical level
- The real Ashtekar Barbero connection doesn't respect the time diffeomorphism symmetry / the complex connection does
- Polynomial hamiltonian constraint
- The use of the complex variables lead to the right semi classical limit in black hole physics without any fine tuning
- Previous work on 2+1 gravity [BA, Geiller, Noui, Yu (2013)] show the irrelevance of γ at the quantum level

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General strategy

- Develop an analytic continuation from the real kinematical Hilbert space to the complex one
- Such procedure should solve the reality condition: real and positive area spectrum
- Systematically study this procedure and how it affects the results of real LQG and real LQC

Application of the analytic continuation to the real area spectrum

- The modification on the real area spectrum reads:

$$A(j) = 8\pi G\hbar\gamma\sqrt{j(j+1)} \rightarrow A(s) = 4\pi G\hbar\sqrt{s^2+1}$$

- The area spectrum remains positive and real even if $\gamma = \pm i$
- γ-independent and continuous spectrum but still an area gap
- The area spectrum is given by the Casimir of the $SU(1,1)$ group in the continuous representation
- Obvious difficulty to work with a non compact group: scalar product , non compact spin network [Freidel, Livine (2002)]
- Hence the interest to study complex LQC: quantum states, scalar product ...

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Result in black hole physics

- The transition from quantum to semi classical black hole offers an interesting arena to study the semi classical limit of LQG
- In real LQG, recovering the right Bekenstein Hawking area law requires an unnatural fine tuning on the Barbero Immirzi parameter:

$$\gamma \approx 0.2375...$$

- Surprisingly, working with the complex Ashtekar variables $\gamma = \pm i$ leads to the right semi classical result without any fine tuning [Frodden, Geiller, Noui, Perez (2012)] and the right logarithmic corrections [BA, Noui, Mouchet (2014)]
- A similar analytic continuation (from $\Lambda > 0$ to $\Lambda < 0$) leads to the BTZ black hole entropy from the Turaev-Viro model [Geiller, Noui (2013)]

Building the analytic continuation in black hole physics

- BHs in LQG are described by isolated horizons on which a Chern-Simons theory lives
- Constraint for the area of the BH:

$$A = \frac{2\pi\gamma\, k}{(1-\gamma^2)}$$

- Quantizing this Chern-Simons theory on the horizon leads to the quantum black hole
- Can we build a quantum back hole with $\gamma = \pm i$? [BA, Noui, Mouchet (2014)]
- First observation: if $\gamma = \pm i$, the Chern-Simons level has to be purely imaginary $k = i\,\lambda$ to keep A positive and real:

$$A = \frac{2\pi\gamma\, k}{(1-\gamma^2)} \quad \gamma \in \mathbb{R} \qquad A = \pi\,\lambda \quad \gamma = \pm i$$

Building the analytic continuation in black hole physics

- The number of states for the quantum BH is given by the Verlinde formula:

$$N_k(d_l) = \frac{2}{k+2}\sum_{d=1}^{k+1} sin^2\left(\frac{\pi d}{k+2}\right)\prod_{l=1}^{n}\frac{sin\left(\frac{\pi d d_l}{k+2}\right)}{sin\left(\frac{\pi d}{k+2}\right)} \quad d_l = 2j_l + 1$$

- First step: reinterprets such sum as a sum of residues of an holomorphic function on the complex plane:

$$I_k(d_l) = \frac{i}{\pi}\oint_{\mathcal{C}} sinh^2(z)\prod_{l=1}^{n}\frac{sinh(d\ d_l z)}{sinh(z)} coth(k+2)z \quad (k, d_l) \in \mathbb{N}$$

- Poles are on the imaginary axis and the contour encompass the imaginary axis btw $[0, i\pi]$
- The problem is reduced to an integration on the complex plane: i.e. to a choice of the contour $\mathcal{C}$

Building the analytic continuation in black hole physics

Definition of the number of microstates

- $\gamma = i$, $k = i\,\lambda$, $d_l = i\,s_l$
  For large black hole: $\lambda \gg 1$

$$I_\infty(s_l, n) = \frac{i}{\pi} \oint_{\mathcal{C}} sinh^2(z) \prod_{l=1}^{n} \frac{sinh(\,s_l\,z)}{sinh(z)}$$

The one color model

- Assume that all the edges are colored by the same spin.
- The number of microstate is given by:

$$I_\infty(s, n) = \frac{i}{\pi} \oint_{\mathcal{C}} sinh^2(z) e^{nS(z)} \quad S(z) = log \frac{sinh(\,s\,z)}{sinh(z)}$$

- For n large, i.e. in the thermodynamical limit, apply the stationary phase method (critical point dominates)

Building the analytic continuation in black hole physics

Result for the complex BH

- Compute the partition function in the grand canonical ensemble using the technics developped by [Frodden; Gosh, Perez (2010)]
- Evalutation of the entropy with a chemical potential μ:

$$S = \frac{A}{4l_p^2} - \frac{3}{2}log(\frac{A}{l_p^2}) \qquad \mu = 2T_U$$

[BA, Noui, Mouchet (2014)]

- Recovering the right semi classical limit without any fine tuning !
- Sign of radiation [Geiller, Noui (2013)], study the role of the chemical potential in the semi-classical limit in LQG [BA, Geiller, Noui, Perez (2014)]

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Looking for a general spin-j dependent hamiltonian

- To apply our analytic continuation in real LQC, we need to define the hamiltonian for any spin-j representation
- First result: a new close formula for such an hamiltonian [BA, Grain, Noui (2014)]
- Lead to interesting results for real LQC and generalize precedent work on real LQC for $j = 1$ [Vandersloot (2005)].

The new spin-j dependent hamiltonian

- The spin-j dependent curvature:

$$F^{l}_{ab} = -\epsilon^{l}_{ab} \frac{3}{d(d^2-1)} |p| \frac{sin^2(\lambda b)}{\lambda^2} \frac{1}{sin\theta} \partial_\theta \frac{sin(d\theta)}{sin\theta}$$

$$sin^2\theta/2 = sin^4 b/2 \qquad d = 2j+1$$

- Asking for $j = 1/2$ and for $j = 1$, we recover the usual results
- New j- dependent Hamiltonian :

$$C_{tot} = \frac{\Pi^2}{2V} - \frac{9V}{8\pi G\gamma^2\lambda^2 d(d^2-1)} \frac{sin^2(\lambda b)}{sin\theta} \partial_\theta \frac{sin(d\theta)}{sin\theta}$$

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The energy density ρ

- The energy density of matter is given by:

$$\rho = \frac{9}{8\pi G\gamma^2 d(d^2-1)} |p| \frac{sin^2(\lambda b)}{\lambda^2} \frac{1}{sin\theta} \partial_\theta \frac{sin(d\theta)}{sin\theta}$$

- It remains bounded (with the curvature): so for any j-representation there is a bounce.
- However, for $j > 1/2$, the density can become negative valued.
- For $j = 1/2$, the density is symmetric with regard to the bounce
- Is it a kind of selection of the case $j = 1/2$?

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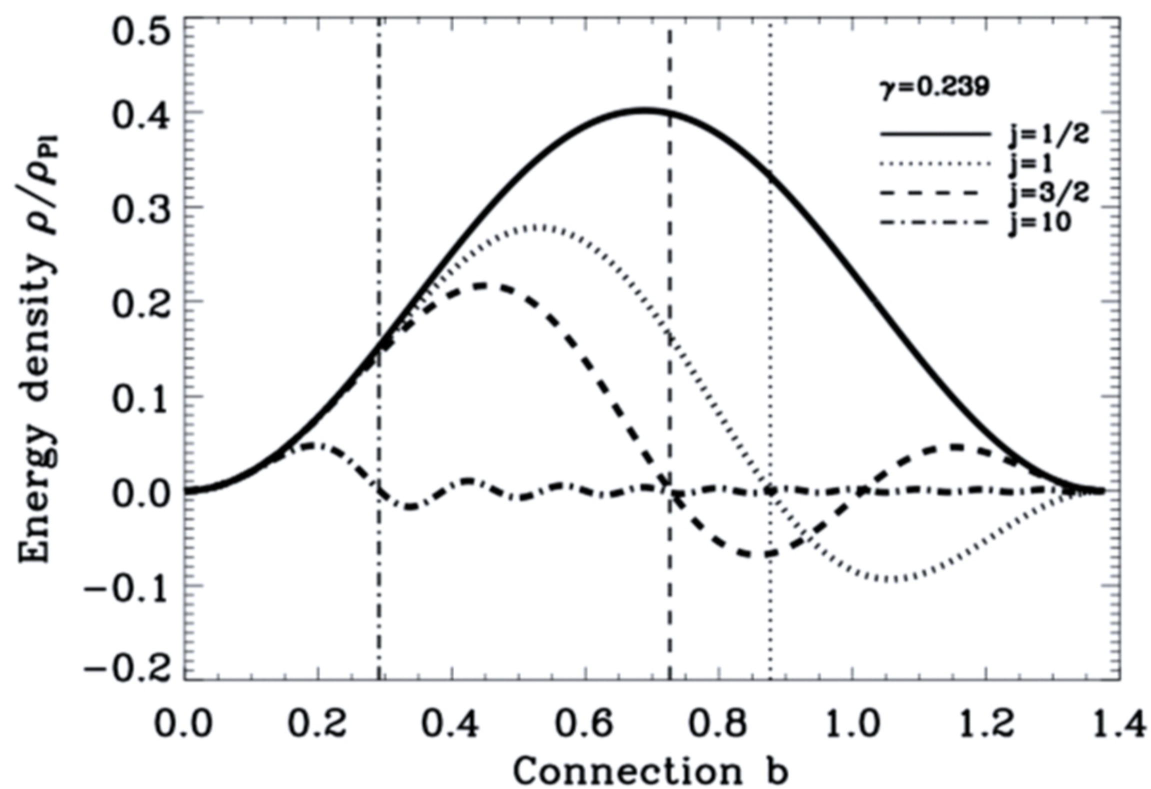
Plot of the density during the bouncing scenario



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- Defined the so called Self Dual Loop Quantum Cosmology
- Analytically continue the j-dependent hamiltonian
- Question: is this universe still a bouncing one ? What is its dynamics at large volume ?
- Work out the semi classical theory and the so called exactly solvable LQC

From real to complex variables

- The first step: make γ explicit in the different equations

$$b = \gamma \tilde{b} \to b \in i\,\mathbb{R}$$

- b is related to the variable θ by:

$$sin^4\theta/2 = sin^8 b/2 \to b \in i\,\mathbb{R} \quad sinh^4\theta/2 = sinh^8 b/2$$

  So θ become also purely imaginary for coherence.

- The spin j is traded for:

$$j \in \mathbb{N}/2 \to j = \frac{1}{2}(-1 + i\,s) \quad s \in \mathbb{R}^+ \qquad d = 2j + 1 = i\,s$$

The new complex spin-s dependent hamiltonian

- The spin-s dependent curvature:

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The asymmetric bouncing universe

Semi-classical limit

- For late time, after the bounce ($b \ll 1$), we recover the usual cosmology:

$$H^2 = \frac{8\pi G}{3}\rho$$

- For early time , before the bounce ($b \gg 1$), we find a new cosmology:

$$H \propto -\rho$$

- Contrary to real LQC, self dual LQC describes a non symmetric bouncing universe
- Such modified Friedman law has been also found in brane cosmology ...

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Phase space to quantize

- Canonical variables and the constraints

$$\{\phi, \Pi\} = 1 \qquad \{b, v\} = 1 \qquad H = \frac{1}{2}(\Pi^2 - (v\, X(b))^2) = 0$$

- Exactly solvable LQC : new polarization
- Quantum states : wave function $\Psi(b, \phi)$
- The quantum version reduces as usual to a Klein Gordon equation:

$$\frac{\partial^2}{\partial \phi^2}\Psi(b, \phi) = (X(b)\, \frac{\partial}{\partial b})^2 \Psi(b, \phi)$$

with a much more complicated function $X(b)$ than real LQC:

$$X(b) = 4\pi G \sqrt{2\rho}$$

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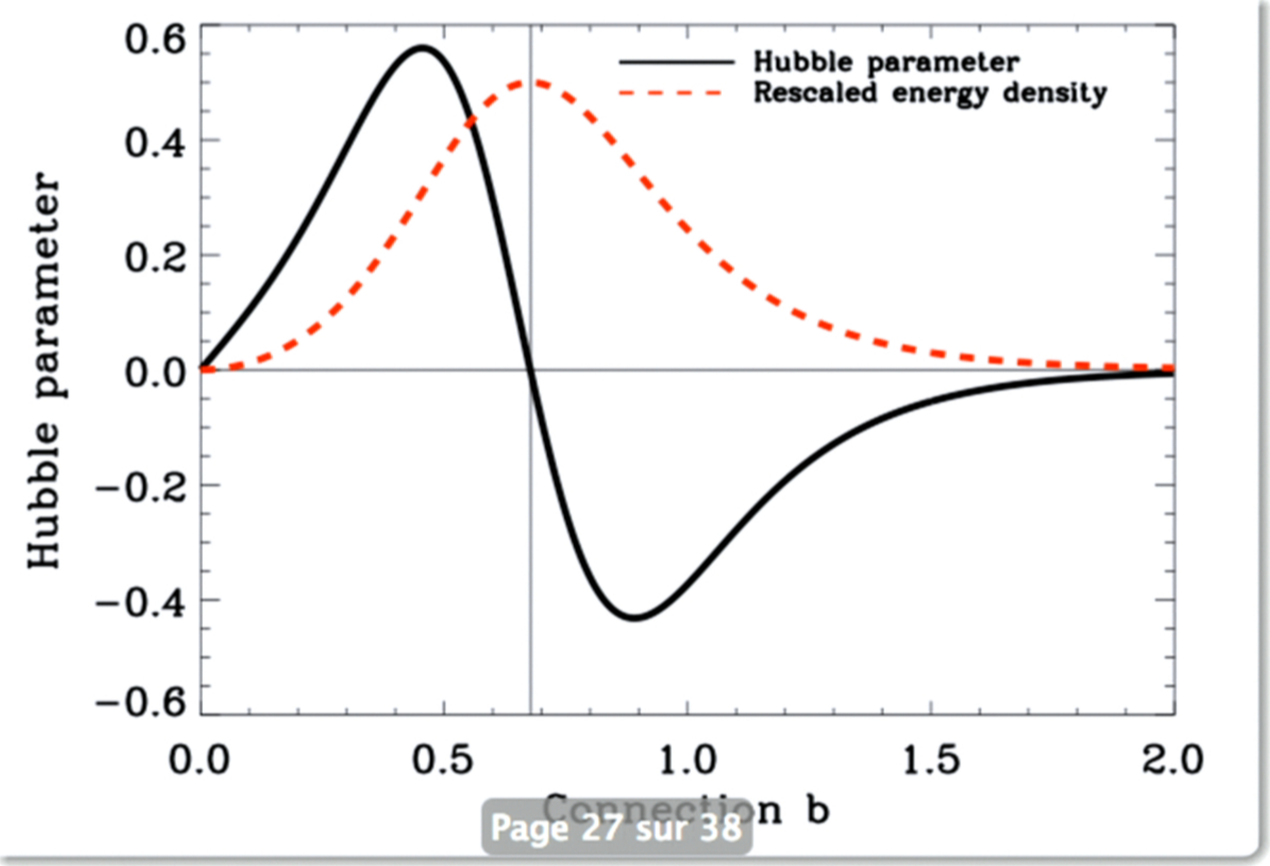

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- It remains bounded (with the curvature): so for any s-representation there is a bounce.
- However, for $s > 0$, the density can become negative valued.
- For any s, the density is asymmetric with regard to the bounce
- The case $s = 0$ seems to be the " dual" of the case $j = 1/2$

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Minimal value for the volume operator

- The volume operator is given by:

$$v = -i \frac{1}{X(x)} \partial_x$$

- The expectation value of the volume operator is given by :

$$< \xi, V \xi >= 4\pi G \int_{-\infty}^{+\infty} dx |\frac{\partial F}{\partial x}|^2 (\frac{1}{X(x-\phi)} - \frac{1}{X(\phi - x)})$$

Minimal value for the volume operator

- So X(x) admit a maximal value, falls of to zero when x goes to $\pm\infty$
- From this, the volume admit a minimal value at the quantum level:

$$< \xi, V\,\xi >\geqslant \frac{8\pi G}{X_{max}} \int_{-\infty}^{+\infty} dx \left|\frac{\partial F}{\partial x}\right|^2$$

Conclusion

- The initial singularity is also resolved in self dual LQC
- The analytic continuation successfully tested on black hole entropy preserves the bouncing scenario in LQC ! [BA, Grain, Noui (2014)]

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Perspectives

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- Go beyond the semi classical approach and define the full quantum theory of self dual LQC
- Study the quantum state of such a quantum cosmology
- Could lead to new insight about the quantum states of full self dual LQG

Conclusion

- To tackle the dynamic of LQG, it might be interesting to come back to the original complex Ashtekar variables
- To do so, we propose a consistent analytic continuation which could lead to the resolution of the reality condition
- We have shown that such procedure:
  - leads to the good semi-classcial limit without fine tuning in black hole physics
  - preserves the bounce scenario in quantum cosmology even if it describes an asymmetric bouncing universe
  - trades the discrete spectrum of geometric operator into continuous spectrum
  - seems to trade SU(2) spin network for non compact (SU(1,1)-continuous) spin network
- Such analytic continuation could lead to the definition of Quantum Self Dual A Page 37 sur 38 y

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  - seems to trade SU(2) spin network for non compact (SU(1,1)-continuous) spin network
- Such analytic continuation could lead to the definition of Quantum Self Dual Ashtekar Gravity

Thank you for your attention