Title: 14/15 PSI - Quantum Field Theory I (A) - Lecture 10

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Abstract:

Symmetries of Dirac Theory

1. Global $U(1)$ symmetry

$$\psi(x) \longrightarrow e^{i\alpha}\psi(x)$$

$$\bar{\psi}(x) \longrightarrow e^{-i\alpha}\psi(x)$$

$\alpha \rightarrow$ Constant

$$J^\mu = -\bar{\psi}\gamma^\mu\psi(x)$$

2. In the limit $m = 0$ Chiral symmetry

$$\psi(x) \longrightarrow e^{i\alpha\gamma_5}\psi(x)$$

$$\bar{\psi}(x) \longrightarrow \bar{\psi}(x)\, e^{i\alpha\gamma_5}$$

$\alpha \rightarrow$ Constant

$$J^\mu = \bar{\psi}(x)\gamma^\mu\gamma_5\psi(x)$$

3.

...ral Symmetry

$\ldots i\alpha\gamma_5 \; \psi(x)$

$\bar\psi(x)\, e^{i\alpha\gamma_5}$

$\ldots\gamma^\mu\gamma_5\psi(x)$

3. Translational Symmetry

$$x^\mu \longrightarrow x^\mu - \varepsilon^\mu, \quad \varepsilon^\mu \longrightarrow \text{Constants}$$

$$T^{\mu\nu} = i\,\bar\psi\,\gamma^\mu\partial^\nu\psi$$

4. Lorentz transformation inv

$$x^\mu \longrightarrow \Lambda^\mu{}_\nu x^\nu$$

Conserved charges $J^{\mu\nu} = \int d^3x \; i\,\bar\psi\,\gamma^0\left[-\frac{i}{2}\Sigma^{\mu\nu} - \partial^\mu(x^\nu\mathcal{L})\right]\psi$

eory

try

$x)$

$\psi(x)$

$\psi(x)$

2. In the limit $m \simeq 0$ Chiral symmetry

$$\psi(x) \to e^{i\alpha\gamma_5}\psi(x)$$

$$\bar\psi(x) \to \bar\psi(x)\, e^{i\alpha\gamma_5}$$

$\alpha \to$ constant

$$J^\mu = \bar\psi(x)\gamma^\mu\gamma_5\psi(x)$$

$$\psi \to e^{i\gamma_5\alpha}\psi$$

$$\psi^\dagger \to \psi^\dagger e^{-i\gamma_5\alpha}$$

$$\bar\psi \to \psi^\dagger e^{-i\gamma_5\alpha}\gamma^0$$

$$= \bar\psi\, e^{i\gamma_5\alpha}$$

3. Translational Symmetry

$$x^\mu \to x^\mu - \varepsilon^\mu, \quad \varepsilon^\mu \to \text{co}$$

$$T^{\mu\nu} = i\bar\psi\gamma^\mu\partial^\nu\psi$$

4. Lorentz transformation inv

$$x^\mu \to \Lambda^\mu{}_\nu x$$

Conserved charges $J^{\mu\nu} = \int d^3x$

$$T^{\mu\nu} \sim \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}}$$

$j^\mu = -\bar{\psi}\gamma^\mu\psi(x)$ | $\psi^\dagger \to \psi^\dagger e^{-i\gamma_5\alpha}$ | $= \bar{\psi}\, e^{i\gamma_5\alpha}$

Quantization of Spinors

\# Matthew Schwartz — QFT & the SM.

Spin-Statistics Theorem: In (3+1)d integer spin fields satisfy canonical c…

half- " " " " " a…

Con…

FT & the SM.

& integer spin fields satisfy Canonical Commutation rel$\underline{\text{n}}$ (Bose statistics)

half- " " " " " anti- " " (Fermi Statistics)

Comments

#1. Lorentz invariance of S-matrix elements $\langle f | i \rangle$. Comput

#2 Causality: $[\mathcal{O}(x), \mathcal{O}(y)] = 0 \;, \quad (x-y)^2 < 0$

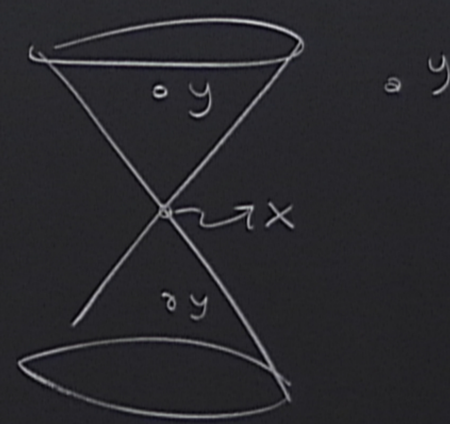
$$\mathcal{O}(x) = \bar{\psi}(x)\, \Gamma\, \psi(x)$$

Γ ↳ element of Clifford alg

#3. Stability: Impose $H \geq 0 \Rightarrow$ Spin-statistics. Sufficient.

e of S-matrix elements $\langle f | i \rangle$. Computed in terms $T(\hat{\varphi}_1 \cdots \hat{\varphi}_n)$.

$(x), \mathcal{O}(y)] = 0 \;, \quad (x-y)^2 < 0$



$= \overline{\psi}(x)\, \Gamma\, \psi(x)$

↳ element of Clifford alg

$\geq 0 \Rightarrow$ Spin-statistics Sufficient.

Dirac spinor

$$\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \psi_3(x) \\ \psi_4(x) \end{pmatrix}$$

$$\psi(x) = |\underbrace{s_0, s_1 \ldots}_{s_0 \quad s_k}\rangle$$

$$D = 2k+2$$

$$\shortparallel$$

$$4 = 2+2 \,, \; k=1$$

$$\psi_a(\underline{x}) = \int \frac{d^3p}{(2\pi)^3 2E_p} \sum_{s=1}^{2} \left\{ u_s(p)_a b_s(p) e^{ip\cdot \underline{x}} + v_s(p)_a c_s^*(p) e^{-ip\cdot \underline{x}} \right\}$$

$$\Pi_a(x) = \frac{\partial \mathcal{L}}{\partial \dot{\psi}_a} = i\psi_a^\dagger(x)$$

$$\{\psi_a(\underline{x}), \Pi_b(\underline{y})\} = i\delta_{ab}\,\delta^3(\underline{x}-\underline{y})\,\hbar$$

$$\{\psi_a(\underline{x}), \psi_b^\dagger(\underline{y})\} = \delta_{ab}\,\delta^3(\underline{x}-\underline{y})\,\hbar$$

Naive classical limit $\hbar \to 0$, $\{\psi_a(\underline{x}), \psi_b^\dagger(\underline{y})\} = 0$

$$\{\psi, \psi\} = \{\psi^\dagger, \psi^\dagger\} = 0$$

$$\{b_s(\underline{p}), b_r^\dagger(\underline{p}')\} = (2\pi)^3\, 2E_{\underline{p}}\, \delta_{rs}\, \delta^3(\underline{p} - \underline{p}')$$

$$\{c_s(\underline{p}), c_r^\dagger(\underline{p}')\} = (2\pi)^3\, 2E_{\underline{p}}\, \delta_{rs}\, \delta^3(\underline{p} - \underline{p}')$$

$\to$ Fermi-statistics

$\rightarrow \psi(x), \psi^\dagger(x) \rightsquigarrow$ Grassmann Variables

$$\psi(x) = \sum_n \alpha_n \phi_n(x) \quad , \alpha_n \rightarrow \text{Grassmann numbers.}$$

$\not{p} - \not{p}')$

$\rightarrow$ Fermi-Statistics

$(\not{p} - \not{p}')$

Hamiltonian

$$H = \int d^3x \left\{ \pi_a(x)\, \dot{\psi}_a(x) - \mathcal{L}(x) \right\}$$

$$= \int d^3x \left\{ -i\, \bar{\psi}\, \gamma^i \partial_i \psi + m \bar{\psi} \psi \right\}$$

$u, \dot{v},$

$$H = \int \frac{d^3p}{(2\pi)^3}\, \frac{1}{2} \sum_r \Big\{$$

$$H = \int \frac{d^3p}{(2\pi)^3}\,\frac{1}{2}\sum_r \left\{ b_r^\dagger(p)\, b_r(p) - c_r(p)\, c_r^\dagger(p) \right\}$$

$$\left\{ b_r^\dagger(p)\, b_r(p) + c_r^\dagger(p)\, c_r(p) - (2\pi)^3\, 2E_p\, \delta_{rr}\, \delta(0) \right\}$$

minus sign here

$$= \int \cdots\cdots \left\{ \underline{b_r^\dagger(p)\, b_r(p)} + \underline{c_r^\dagger(p)\, c_r(p)} \right.$$

$$:H: \longrightarrow \quad :a b^\dagger: = -:b^\dagger a: = -b^\dagger a$$

One particle state

$$b^\dagger_s(\underline{k})|0\rangle = |(\underline{k}, s)\rangle$$

$$:H:|(\underline{k}, s)\rangle = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} E_p \sum_r \left(b^\dagger_r(\underline{p}) b_r(\underline{p}) + c^\dagger_r(\underline{p}) c_r(\underline{p}) \right) b^\dagger_s(\underline{k})|0\rangle$$

$$= \int \qquad (2\pi)^3 \delta_{sr} \delta^{(3)}(\underline{p}-\underline{k}) 2E_p |0\rangle$$

$$= E_k |(\underline{k}, s)\rangle$$

$$:Q: = -e \int \frac{d^3p}{(2\pi)^3 2E_p} \sum_r \left(b_r^\dagger(\mathbf{p})\, b_r(\mathbf{p}) \underset{\text{minus}}{-} c_r^\dagger(\mathbf{p})\, c_r(\mathbf{p}) \right)$$

$$c^\dagger|0\rangle \quad , \quad b^\dagger|0\rangle$$

$$Q = -\int d^3x \; \psi^\dagger(x)\psi(x)$$

$$:Q:$$

$$\boxed{\partial_\mu J^\mu = 0}$$

$$\int_\Sigma d^3x \; J^0$$

$$:Q: = -\int \frac{d^3p}{(2\pi)^3 2E_p} \sum_r \left(\underbrace{b_r^\dagger(p)\, b_r(p)}\ \underset{\text{minus}}{-}\ \underbrace{c_r^\dagger(p)\, c_r(p)} \right)$$

$$c^\dagger|0\rangle \quad , \quad b^\dagger|0\rangle$$

$$\bar{\psi}\psi \qquad \bar{\psi}\gamma^\mu\psi$$

$b_r^\dagger(p) \longrightarrow$ creates a particle of spin r and momentum p

$c_r^\dagger(p) \longrightarrow$ " an anti- " " " r " " p

$b, c \longrightarrow$ destroy particles ntum p

p

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3 2E_p} \sum_r \left\{ b_r(p)\, u_r(p)\, e^{-ip\cdot x} + c_r^\dagger(p)\, v_r(p)\, e^{+ip\cdot x} \right\}$$

$$= \psi^{(+)}(x) + \psi^{(-)}(x)$$

$$\bar\psi(x) = \bar\psi^{(+)}(x) + \bar\psi^{(-)}(x)$$

tum $\underline{p}$

$\underline{p}$

$$\psi(x) = \int \frac{d^3\underline{p}}{(2\pi)^3 2E_{\underline{p}}} \sum_r \left\{ b_r(\underline{p})\, u_r(\underline{p})\, e^{-ip\cdot x} + c_r^\dagger(\underline{p})\, v_r(\underline{p})\, e^{+ip\cdot x} \right\}$$

$$= \psi^{(+)}(x) + \psi^{(-)}(x)$$

$$\bar{\psi}(x) = \bar{\psi}^{(+)}(x) + \bar{\psi}^{(-)}(x)$$

$$p^\mu = (E_{\underline{p}}, \underline{p})$$

$$p\cdot x = p_\mu x^\mu = \eta_{\mu\nu} p^\mu x^\nu = E_p t - \underline{p}\cdot\underline{x}$$

$$\{\psi_a(x), \bar{\psi}_b(y)\} = (i \partial\!\!\!/_x + m)_{ab} \left[D(x-y) - D(y-x) \right]$$

$$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_p} e^{-ip\cdot(x-y)} \quad , \; (x-y)^2 < 0$$

Propagators

$$[\phi(x), \phi(y)] = 0$$

$$\Rightarrow \{\psi_a(x), \bar{\psi}_b(y)\} = 0 \quad \text{for } (x-y)^2 < 0$$

$$\{\psi_a(x), \bar{\psi}_b(y)\} = ($$

$$D(x-y) = \int \frac{d^3 p}{(2\pi)^3 2E_p}$$

$$S_{F\,ab}(x-y) = \left[\langle 0|T\,\psi(x)\,\bar{\psi}(y)|0\rangle\right]_{ab}$$

$$= \begin{cases} \langle 0|\psi_a(x)\,\bar{\psi}_b(y)|0\rangle & \text{if } x^0 > y^0 \\ -\langle 0|\bar{\psi}_a(y)\,\psi_b(x)|0\rangle & \text{if } x^0 < y^0 \end{cases}$$

f spin r and momentum p

" " r " " p

$u_r(p)$ $\quad u_r(I) = \begin{pmatrix} \sqrt{\sigma\cdot p}\;\xi^r \\ \sqrt{\bar\sigma\cdot p}\;\xi^r \end{pmatrix}$

$\xi^{r\dagger}\xi^s = \delta^{rs}$

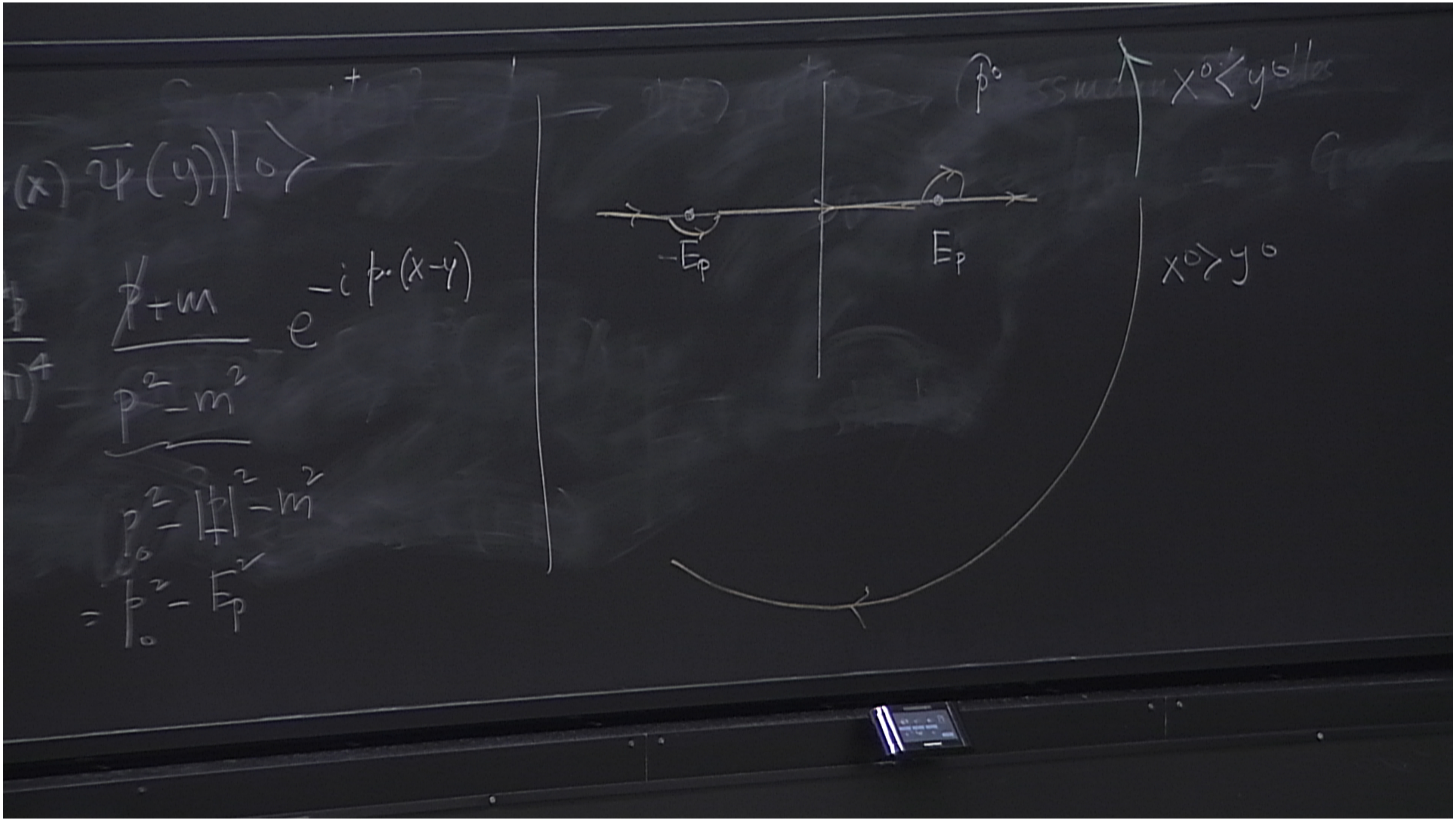
$$\psi(x) = \int \frac{d^3p}{(2\pi)^3 2E_p} \sum_r \Big\{ b_r(p)\, u_r(p)\, e^{-ip\cdot x} + c$$

$$v_r(p) = \begin{pmatrix} \sqrt{\sigma\cdot p}\;\xi^r \\ \sqrt{-\bar\sigma\cdot p}\;\xi^r \end{pmatrix}$$

$x^0 > y^0$

$$S_F(x-y) = \langle 0 | T(\psi(x)\,\bar{\psi}(y)) | 0 \rangle$$

$$= i \int \frac{d^4 p}{(2\pi)^4} \frac{\not{p} + m}{p^2 - m^2} e^{-i p \cdot (x-y)}$$



(x) ψ̄(y)|0⟩
p̸ + m
p² − m²
e^{−i p·(x−y)}
p₀² − |p|² − m²
= p₀² − E_p²
−E_p
E_p
p⁰
x⁰ < y⁰
x⁰ > y⁰