

Title: 14/15 PSI - Quantum Field Theory I (A) - Lecture 9

Date: Oct 17, 2014 03:30 PM

URL: <http://pirsa.org/14100038>

Abstract:

$$[\Sigma^{\mu\nu}, \gamma^\rho] = \Lambda_\sigma{}^\rho \gamma^\sigma$$

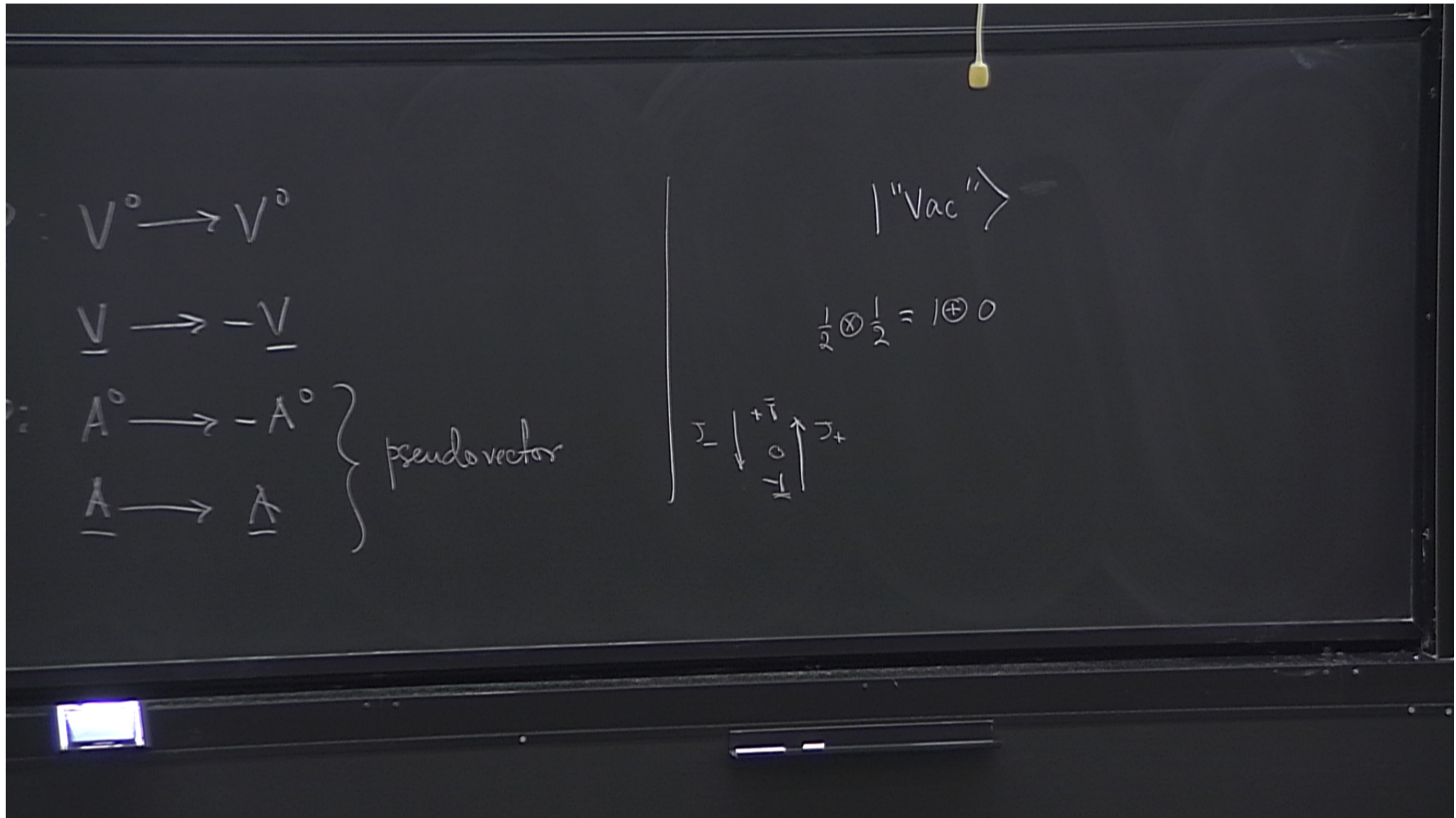
$$S(\Lambda)^{-1} \gamma^\rho S(\Lambda) = \Lambda_\sigma{}^\rho \gamma^\sigma$$

$$S(\Lambda) = \exp\left[-\frac{i}{2} \omega_{\mu\nu} \Sigma^{\mu\nu}\right], \quad \Lambda_\rho{}^\sigma = \exp\left[-\frac{i}{2} \omega_{\mu\nu} \mathcal{M}^{\mu\nu}\right]$$

$$P: \psi_{\vec{p}}(\vec{x}, t) \rightarrow \gamma^0 \psi_{\vec{p}}(-\vec{x}, t)$$

$$\begin{aligned} \bar{\psi} \psi & \quad V^\mu = \bar{\psi} \gamma^\mu \psi \rightarrow \bar{\psi} S^{-1} \gamma^\mu S \psi \\ \bar{\psi} \gamma_5 \psi & \quad = (\bar{\psi} \gamma^5 \psi) \Lambda^\mu{}_\sigma \\ & \quad A^\mu = \bar{\psi} \gamma^\mu \gamma_5 \psi \end{aligned}$$

$$\left. \begin{aligned} P: V^0 & \rightarrow V^0 \\ \underline{V} & \rightarrow -\underline{V} \\ P: A^0 & \rightarrow -A^0 \\ \underline{A} & \rightarrow \underline{A} \end{aligned} \right\} \text{pseudovector}$$



Lagrangians for spinors

$$\mathcal{L}_{\text{kin}} = i \bar{\psi}_D \gamma^m \partial_m \psi_D$$

$$\begin{array}{c} \uparrow \\ \cancel{\partial_n \bar{\psi} \partial^m \psi} \end{array}$$

$$S = \int d^4x \mathcal{L}_{\text{kin}}, \quad \hbar = c = 1$$

$$[d^4x] = -4$$

$$[\partial_m] = +1$$

$$[\psi] = [\bar{\psi}] = \frac{3}{2}$$

$$\mathcal{L}_{\text{mass}} = -m \bar{\psi} \psi$$

$$\mathcal{L}_{\text{DIRAC}} = i \bar{\psi} \not{\partial} \psi - m \bar{\psi} \psi$$

$$\not{\partial} = \gamma^\mu \partial_\mu$$

$$\bar{\Psi}_a: \frac{\partial \mathcal{L}}{\partial \bar{\Psi}_a} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \bar{\Psi}_a)} \right) = 0$$

$$(i \not{\partial} - m) \psi(x) = 0$$

$$(i \not{\partial} + m) (i \not{\partial} - m) \psi(x) = 0$$

$$(\gamma^\mu \gamma^\nu \partial_\mu \partial_\nu + m^2) \psi(x) = 0$$

$$(\partial^2 + m^2) \psi(x) = 0$$

$= 0$

$$\psi_D = \begin{pmatrix} \chi_L \\ \psi_R \end{pmatrix}$$

$$\bar{\psi}_D = \psi_D^\dagger \gamma^0 = (\psi_R^\dagger, \chi_L^\dagger)$$

$$S'(\lambda) = \begin{pmatrix} S'_L(\lambda) & 0 \\ 0 & S_R(\lambda) \end{pmatrix}$$

$$\psi_D \rightarrow S'(\lambda) \psi_D$$

$$\chi_L \rightarrow S'_L \chi_L$$

$$\chi_L^\dagger \rightarrow \chi_L^\dagger S_R$$

$$\psi_D = \begin{pmatrix} \chi_L \\ \varphi_R \end{pmatrix} \quad \bar{\psi}_D = \psi_D^\dagger \gamma^0 = (\varphi_R^\dagger, \chi_L^\dagger)$$

$$S(\Lambda) = \begin{pmatrix} S_L(\Lambda) & 0 \\ 0 & S_R(\Lambda) \end{pmatrix}$$

$$\psi_D \rightarrow S(\Lambda) \psi_D$$

$$\chi_L \rightarrow S_L^{-1} \chi_L$$

$$\chi_L^\dagger \rightarrow \chi_L^\dagger S_R$$

$$S_L^{-1} \sigma^\mu S_R = \Lambda^\mu_\nu \sigma^\nu$$

$$S_R^{-1} \bar{\sigma}^\mu S_L = \Lambda^\mu_\nu \bar{\sigma}^\nu$$

$$\begin{aligned}
\mathcal{L}_{\text{kin}} &= i \bar{\psi} \gamma^m \partial_m \psi \\
&= i (\varphi_R^\dagger, \chi_L^\dagger) \begin{pmatrix} 0 & \sigma^m \\ \bar{\sigma}^m & 0 \end{pmatrix} \partial_m \begin{pmatrix} \chi_L \\ \varphi_R \end{pmatrix} \\
&= \underbrace{i \chi_L^\dagger \bar{\sigma}^m \partial_m \chi_L} + \underbrace{i \varphi_R^\dagger \sigma^m \partial_m \varphi_R}
\end{aligned}$$

$$\bar{\psi}_D = \psi_D^\dagger \gamma^0 = (\psi_R^\dagger, \chi_L^\dagger)$$

$$\begin{pmatrix} 0 \\ S_R(\lambda) \end{pmatrix}$$

$$\psi_D \rightarrow S(\lambda) \psi_D$$

$$\chi_L \rightarrow S_L^\dagger \chi_L$$

$$\chi_L^\dagger \rightarrow \chi_L^\dagger S_R$$

$$\psi_L \rightarrow \left(1 + \frac{k \cdot \sigma}{2} - \frac{\theta \cdot \sigma}{2} \right) \psi_L$$

$$= \Lambda^\mu \cdot \sigma^\nu$$

$$= \Lambda^\mu \cdot \bar{\sigma}^\nu$$

$$\psi_D = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$\psi_L = \chi$$

$$\psi_R = -i\sigma^2 \chi^*$$

$$\psi_D^* = \underbrace{\begin{pmatrix} 0 & i\sigma^2 \\ -i\sigma^2 & 0 \end{pmatrix}}_e \psi_D$$

$$e^\dagger e = 1$$

Define, for any Dirac spinor ψ , a charge conjugate $\psi^{(c)}$ by

$$\psi^{(c)} \equiv C\psi, \quad C^T C = 1, \quad C^T \gamma^\mu C = -\gamma^{\mu*}$$

A Majorana spinor is one which satisfies

$$\psi^{(c)} = \psi$$

Classical solutions of the D.E.

$$\psi_p(x)_a = U(p)_a e^{-ip \cdot x}$$

$$i \frac{\partial}{\partial t}$$

$$U(p) =$$

$$\begin{pmatrix} U_1(p) \\ U_2(p) \end{pmatrix}$$

$$, p^0 > 0$$

$$\begin{pmatrix} \not{p} \cdot \sigma u_2(p) \\ \not{p} \cdot \bar{\sigma} u_1(p) \end{pmatrix} = m \begin{pmatrix} u_1(p) \\ u_2(p) \end{pmatrix}$$

In the limit $p^\mu = (m, 0, 0, 0) \equiv p_0^\mu$

$$u_1(p_0) = u_2(p_0) = \sqrt{m} \xi, \quad \xi^\dagger \xi = 1$$

$$\begin{pmatrix} u_2(p) \\ u_1(p) \end{pmatrix} = m \begin{pmatrix} u_1(p) \\ u_2(p) \end{pmatrix}$$

$$\text{unit } p^\mu = (m, 0, 0, 0) \equiv p_0^\mu$$

$$u_2(p_0) = \sqrt{m} \xi, \quad \xi^\dagger \xi = 1$$

$$p_b^\mu \rightarrow \Lambda^\mu_\nu p_0^\nu = p^\mu$$

$$u(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi \\ \sqrt{p \cdot \bar{\sigma}} \xi \end{pmatrix}$$

$$(p \cdot \sigma)(p \cdot \bar{\sigma}) = p^2 = m^2$$

$$\bar{u} = u^\dagger \gamma^0$$

$$\bar{u}(p) u(p) = 2m \xi^\dagger \xi = 2m$$

$$\xi^{r\dagger} \xi^s = \delta^{rs}$$

$$u^s(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^s \\ \sqrt{p \cdot \bar{\sigma}} \xi^s \end{pmatrix}$$

$$\bar{u}^s(p) u^r(p) = 2m \delta^{rs}$$

$$u^{s\dagger}(p) u^r(p) = 2E_p \delta^{rs}$$

$$\psi_p(x) = \mathcal{U}(p) e^{+ip \cdot x}$$

$$p \cdot x = p_\mu x^\mu$$

$$= p^0 x^0 - \underline{p} \cdot \underline{x}$$

$$\mathcal{U}^s(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \eta^s \\ -\sqrt{p \cdot \bar{\sigma}} \eta^s \end{pmatrix}$$

$$\bar{\mathcal{U}}^r(p) \mathcal{U}^s(p) = -2m \delta^{rs}$$

$$\mathcal{U}^{tr}(p) \mathcal{U}^s(p) = +2E_p \delta^{rs}$$

$$\bar{u}^s(p) \not{v}^r(p) = \bar{v}^s(p) \not{u}^r(p) = 0$$

$$u^{st}(p) \not{v}^r(p) \neq 0$$

$$v^{st}(p) \not{u}^r(p) \neq 0$$

$$u^{st}(-p) \not{v}^r(p) = 0$$

$$v^{st}(-p) \not{u}^r(p) = 0$$

$$\sum_{s=1}^2 u^s(p) \bar{u}^s(p) = \begin{pmatrix} m & \not{p} \cdot \sigma \\ \not{p} \cdot \bar{\sigma} & m \end{pmatrix} = \not{p} + m$$

$$\sum_{s=1}^2 v^s(p) \bar{v}^s(p) = \not{p} - m$$

$$\psi(x) = \int d^3k$$

$$\psi(x) = \int \frac{d^3k}{(2\pi)^3 2E_k} \sum_{s=1}^2 \left\{ u^s(p) b_s(p) e^{-ip \cdot x} + v^s(p) c_s^*(p) e^{ip \cdot x} \right\}$$

Noether's Theorem: $\delta\phi_a = X_a(\phi)$, $\delta\mathcal{L} = \partial_\mu F^\mu$

$$\partial_\mu J^\mu = 0, \quad J^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_a)} X_a(\phi) - F^\mu(\phi)$$

1. Global $U(1)$ symmetry $\left. \begin{array}{l} \psi(x) \rightarrow e^{i\alpha} \psi(x) \\ \bar{\psi}(x) \rightarrow e^{-i\alpha} \bar{\psi}(x) \end{array} \right\} \text{ for } \alpha \rightarrow \text{constant}$

$$\begin{aligned} \delta\psi &= i\alpha\psi \\ \delta\bar{\psi} &= -i\alpha\bar{\psi} \end{aligned}$$

$$\delta\mathcal{L} = 0$$

$$J^\mu = \bar{\psi} \gamma^\mu \psi \quad J^0 = \psi^\dagger \psi$$