

Title: 14/15 PSI - Quantum Field Theory I (A) - Lecture 6

Date: Oct 14, 2014 03:30 PM

URL: <http://pirsa.org/14100035>

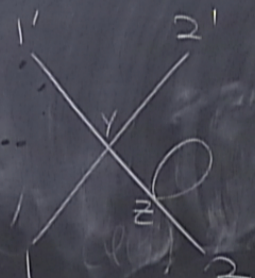
Abstract:

Last time:

$$\begin{aligned}
 \text{LSZ: } \langle f | i \rangle &= i^4 \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 e^{i(p_1 x_1 + p_2 x_2 - p_1' x_3 - p_2' x_4)} \\
 &\quad \cdot (\partial_1^2 + m_1^2)(\partial_2^2 + m_2^2)(\partial_3^2 + m_1'^2)(\partial_4^2 + m_2'^2) \langle S | T \varphi_1 \varphi_2 \varphi_1' \varphi_2' | S \rangle
 \end{aligned}$$

Last time:

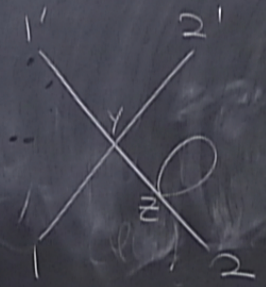
$$\begin{aligned}
 \text{LSZ: } \langle f|i \rangle &= i^4 \int d^4x_1 d^4x_2 d^4x_1' d^4x_2' e^{i(p_1 \cdot x_1 + p_2 \cdot x_2 - p_1' \cdot x_1' - p_2' \cdot x_2')} \\
 &\quad \cdot (\partial_1^2 + m_1^2)(\partial_2^2 + m_2^2)(\partial_{1'}^2 + m_1'^2)(\partial_{2'}^2 + m_2'^2) \langle S|T\phi_1\phi_2\phi_1'\phi_2' \rangle \\
 &= (-i\lambda)^2 \int d^4y d^4z \Delta_{1y} \Delta_{1y} \Delta_{2y} \Delta_{2y}
 \end{aligned}$$

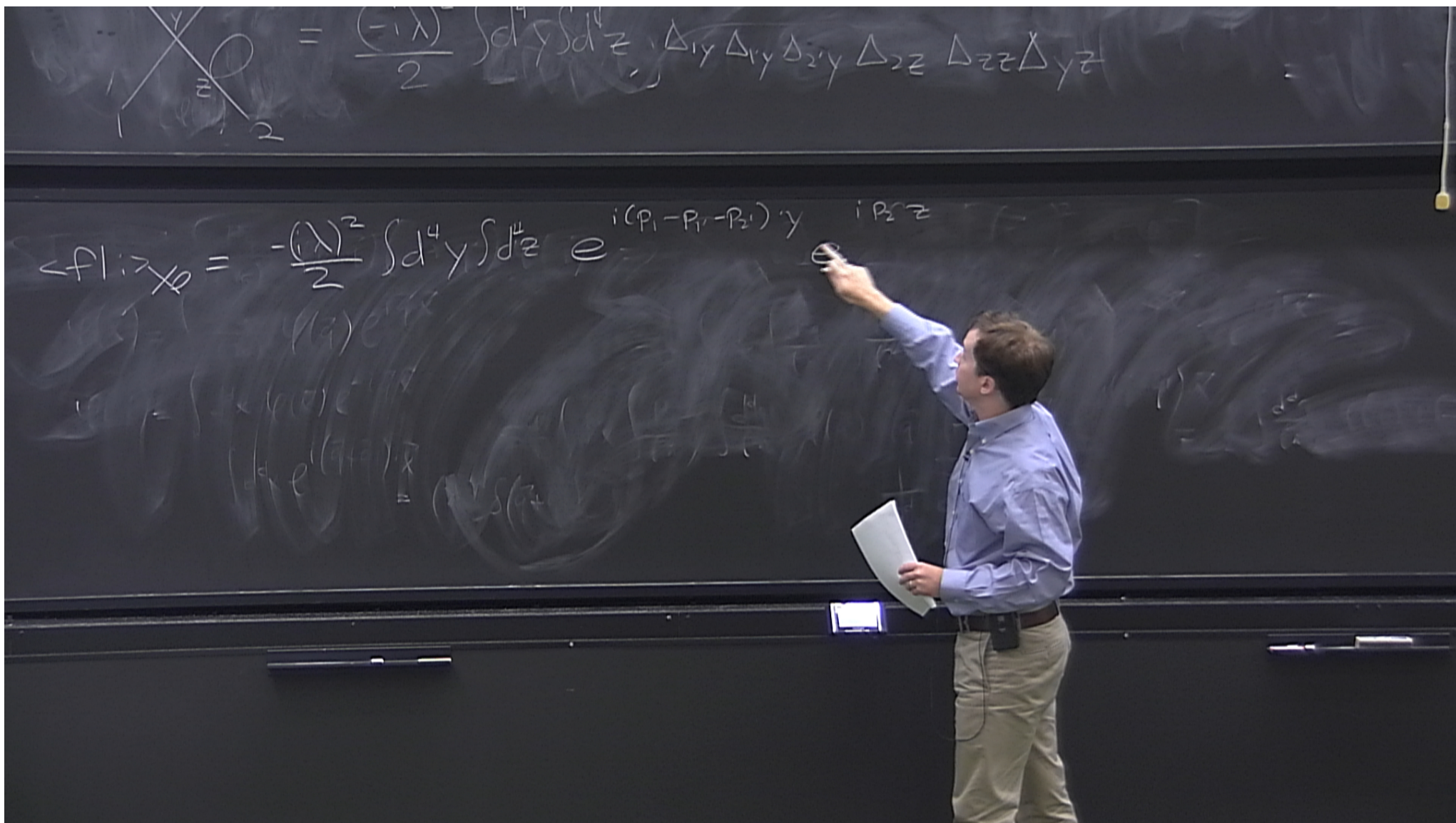


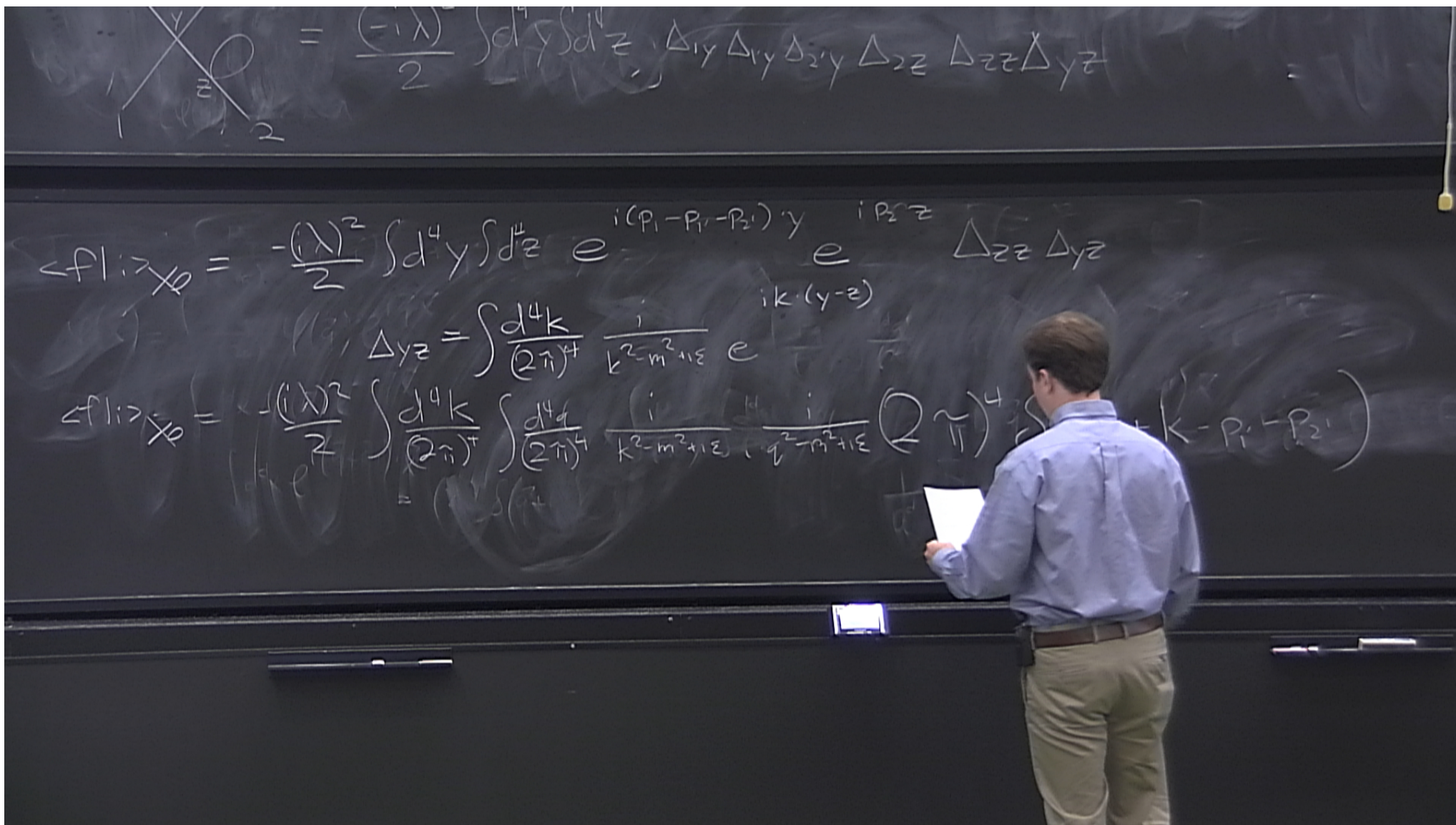
Last time:

$$i\mathcal{L}Z: \langle f|_1 \rangle = i^4 \int d^4x_1 d^4x_2 d^4x_1' d^4x_2' e^{i(p_1 \cdot x_1 + p_2 \cdot x_2 - p_1' \cdot x_1' - p_2' \cdot x_2')} \cdot (\partial_1^2 + m_1^2)(\partial_2^2 + m_2^2)(\partial_{1'}^2 + m_{1'}^2)(\partial_{2'}^2 + m_{2'}^2) \langle S \rangle$$

$$= (-i\lambda)^2 \int d^4y d^4z \Delta_{1y} \Delta_{1y} \Delta_{2y} \Delta_{2z} \Delta_{zz} \Delta_{yz}$$





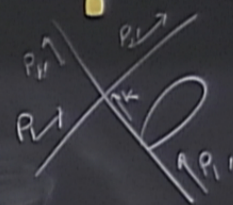


$$\frac{-i\lambda}{2} \int d^4y \int d^4z \Delta_{1y} \Delta_{1y} \Delta_{2y} \Delta_{2z} \Delta_{zz} \Delta_{yz}$$

$$\int d^4y \int d^4z e^{i(p_1 - p_1' - p_2') \cdot y} e^{i p_2 \cdot z} \Delta_{zz} \Delta_{yz}$$

$$\Delta_{yz} = \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} e^{i k \cdot (y - z)}$$

$$\int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{q^2 - m^2 + i\epsilon} \underbrace{(2\pi)^4 \delta(p_1 + k - p_1' - p_2')}_{y \text{ integral}} \underbrace{(2\pi)^4 \delta(p_2 - k)}_{z \text{ integral}}$$



$$\langle T \rangle = \frac{(i\lambda)^2}{2} \int \frac{d^4 k}{(2\pi)^4} \int \frac{d^4 q}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{q^2 - m^2 + i\epsilon} (2\pi)^4 \delta(p_1 + k - p_1' - p_2') (2\pi)^4 \delta(p_2 - k)$$

$$\int d^4 k \delta^{(4)}(p_2 - k) \frac{1}{k^2 - m^2 + i\epsilon} = \frac{1}{p_2^2 - m^2} = \frac{1}{0} \quad \text{! integral}$$

$$\psi_k(\vec{x}) = \int_0^{N/b} \frac{d^d k}{(2\pi)^d} \psi(\vec{k}) e^{i\vec{k}\vec{x}}$$

$$\psi_s(\vec{x}) = \int_{N/b}^{\Lambda} \frac{d^d k}{(2\pi)^d} \psi(\vec{k}) e^{i\vec{k}\vec{x}}$$

$\psi(\vec{x}) = \psi_s(\vec{x}) + \psi_k(\vec{x})$
 $\uparrow$ $\uparrow$
 slow modes fast modes

$b > 1$; $\frac{\Lambda}{b} < \Lambda$

Momentum Space Feynman Rules (Scattering Amplitudes)

$i\mathcal{M}$ = sum of all completely connected, amputated diagrams

1. internal propagator

$$\frac{1}{\vec{p}} = \frac{1}{p^2 - m^2 + i\epsilon}$$

2. vertex

$$X = -i\lambda$$

3. external line

$$= 1$$

4. momentum conservation at each vertex

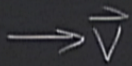
5. Divide by symmetry factor

6. Integrate over undetermined momenta

$$\int \frac{d^4 p}{(2\pi)^4}$$

Cross Sections + Decay Rates

point
masses



$$l_B = vT$$

spheres
of radius r

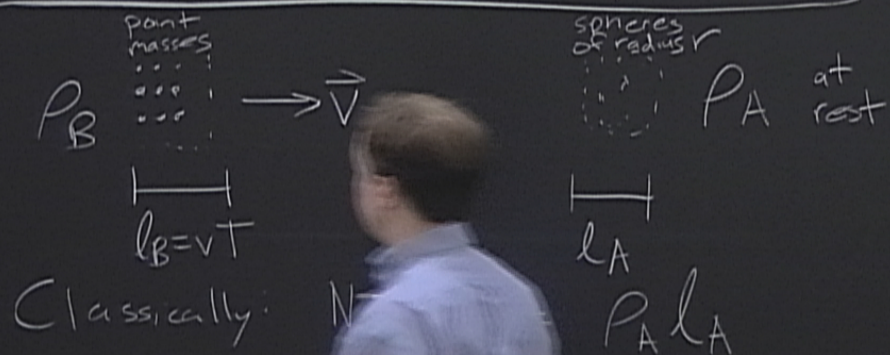


ρ_A at rest

$$l_A$$

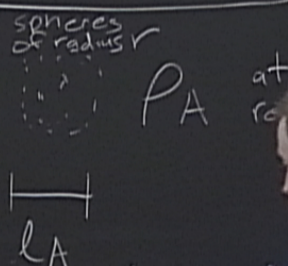
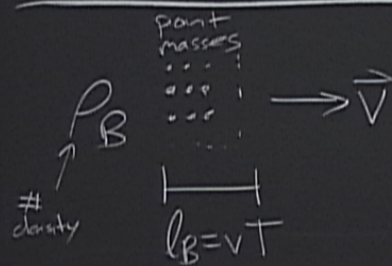
A = cross sectional area common

Cross Sections + Decay Rates



$A =$ cross sectional area common to both beams

Cross Sections + Decay Rates



A = cross sectional area common to both

Classically: $N_{\text{collisions}} = \sigma \rho_B l_A$

cross-section σ

+ Decay Rates

spheres
of radius r

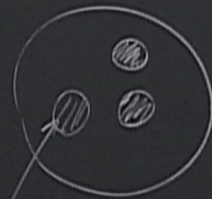
P_A at rest

l_A

$$\text{ions} = \sigma P_A l_A P_B l_B A$$

cross-section $\sigma = \pi r^2$

A = cross sectional area common to both beams

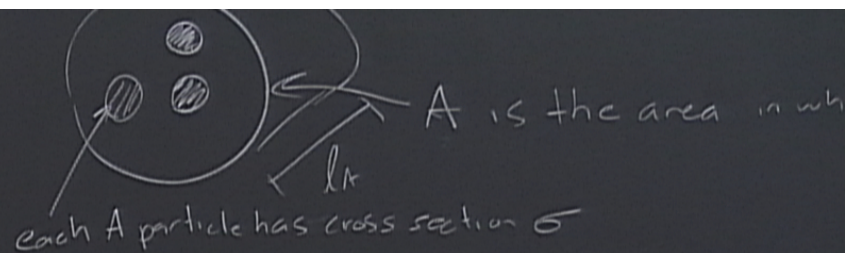


A is the area in which there are A particles

each A particle has cross section σ

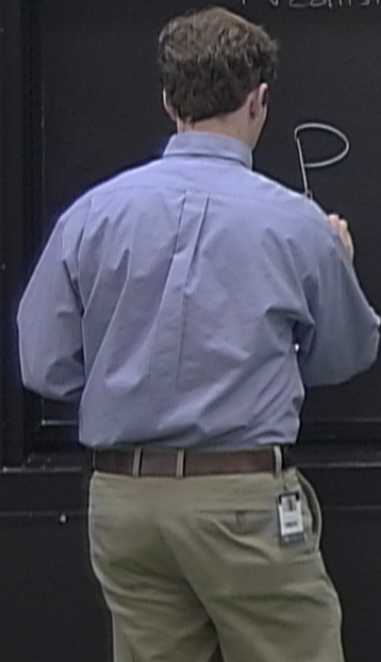
density
 $l_B = vT$

Classically: $N_{\text{collisions}} = \sigma P_A l_A P_B l_B A$
 cross-section $\sigma = \pi r^2$



$$N_{\text{collisions}} = \frac{\sigma N_A N_B}{A} = \frac{\sigma N_A N_B \cdot v T}{\text{vol}}$$

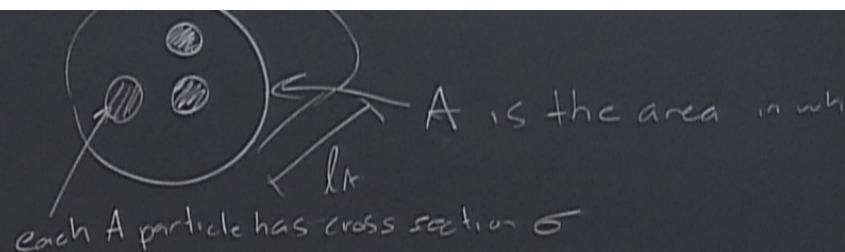
$N_B = P_B l_B A$ $l_B A$



density
 $l_B = vT$

Classically: $N_{\text{collisions}} = \sigma P_A l_A P_B l_B A$

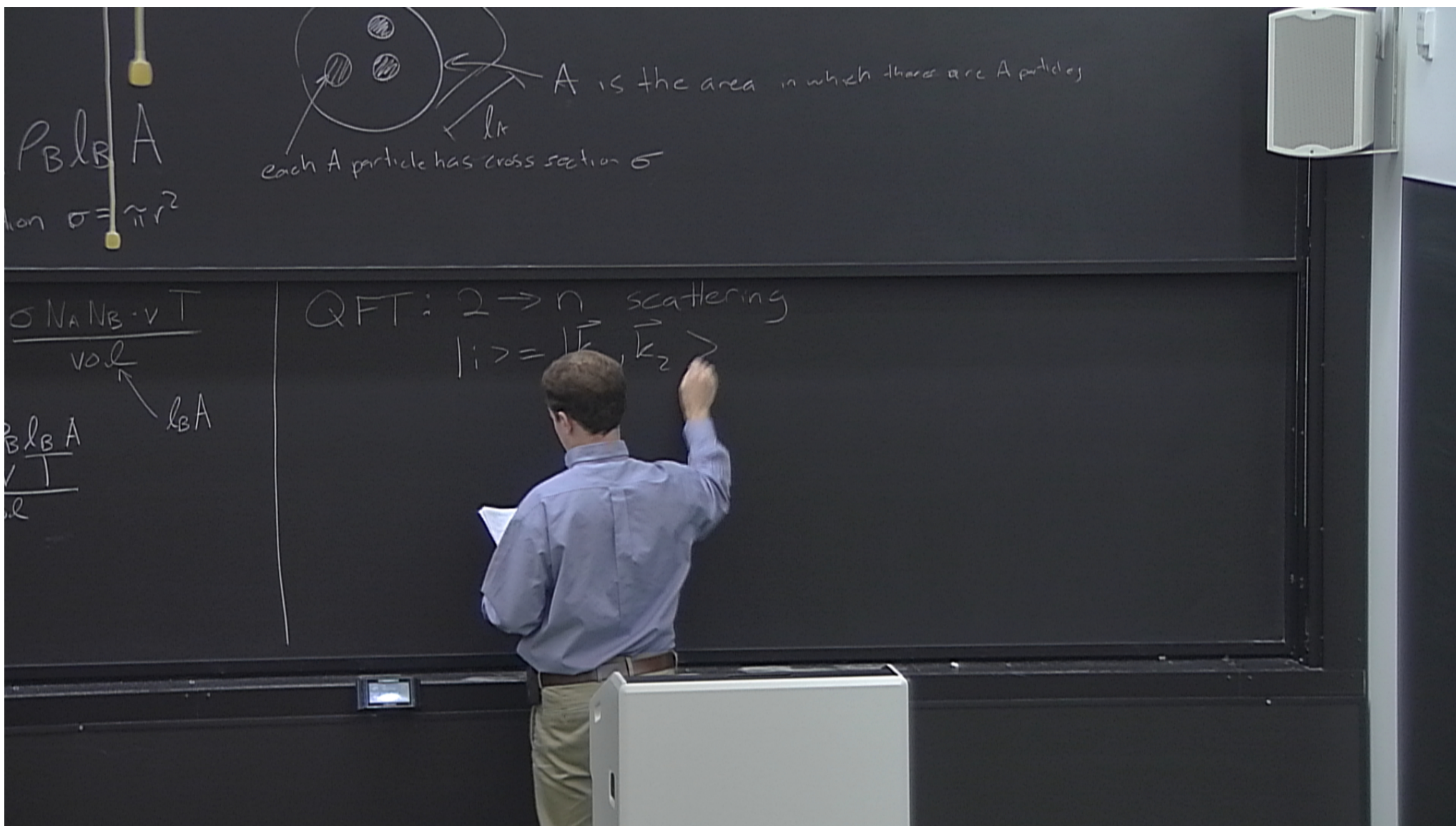
cross-section $\sigma = \pi r^2$

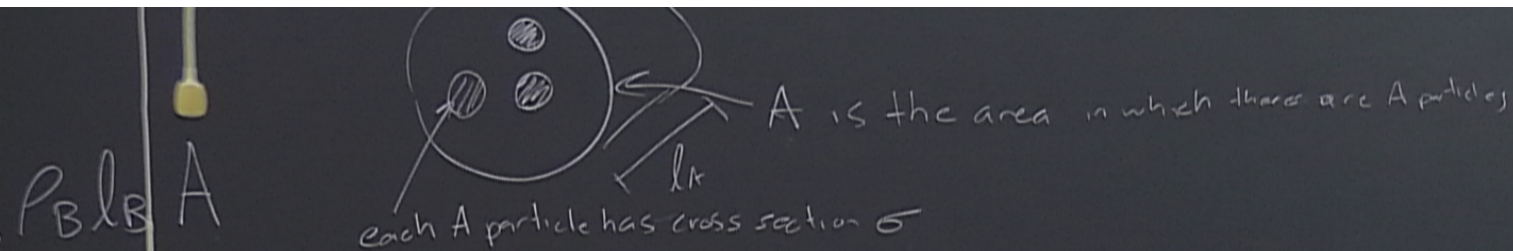


$$N_{\text{collisions}} = \frac{\sigma N_A N_B}{A} = \frac{\sigma N_A N_B \cdot vT}{\text{vol}}$$

$$N_B = P_B l_B A$$

probability $P = \frac{N_{\text{collisions}}}{N_A N_B}$





$$P_B l_B A$$

$$\sigma = \pi r^2$$

$$\frac{\sigma N_A N_B \cdot V T}{\text{vol}}$$

$\swarrow$ $l_B A$

$$\frac{P_B l_B A}{V T}$$

QFT: $2 \rightarrow n$ scattering

$$|i\rangle = |\vec{k}_1, \vec{k}_2\rangle$$

$$|f\rangle = |\vec{p}_1, \vec{p}_n\rangle$$

$$dP = \frac{|<f|i>|^2}{T}$$

$$\langle k|k \rangle = (2\pi)^3 2E_k \delta^3(0) = 2E_k \cdot \text{vol}$$

$$\langle 1|1 \rangle = 2E_1 2E_2 \text{vol}^2$$

$$\langle f|1 \rangle = \prod_{j=1}^n (2E_{p_j} \text{vol})$$

$$dP = \underbrace{d\Phi}_{\text{phasespace}} \frac{|M|^2}{2E_1 2E_2} \frac{T}{\text{vol}}$$

$$d\Phi = (2\pi)^4 \delta^4\left(\sum_{j=1}^n p_j - k_1 - k_2\right) \prod_{j=1}^n \frac{d^3p_j}{(2\pi)^3 2E_{p_j}}$$

$$\langle k|k \rangle = (2\pi)^3 2E_k \delta^3(0) = 2E_k \cdot \text{vol}$$

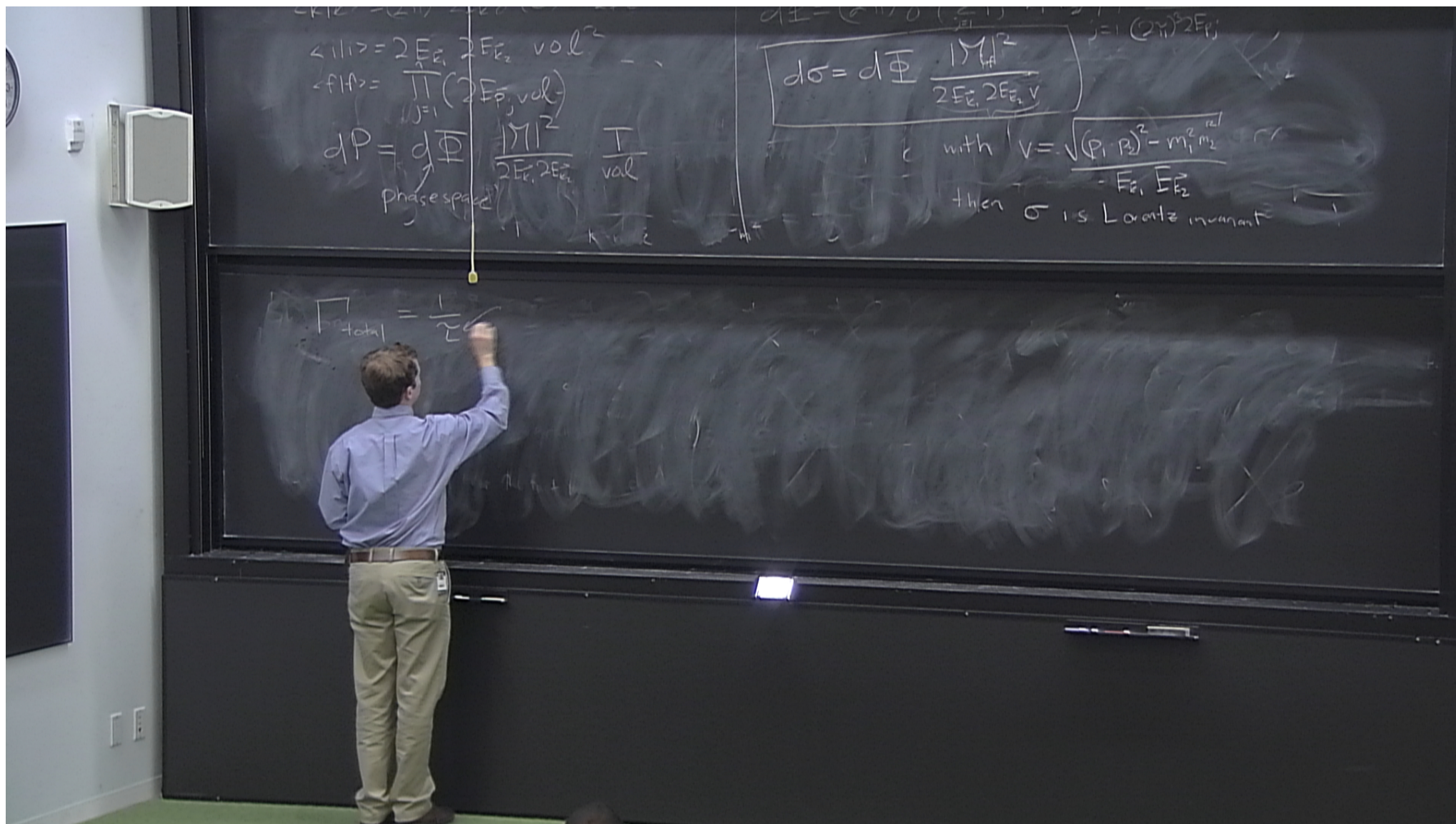
$$\langle ii \rangle = 2E_i 2E_i \text{ vol}^2$$

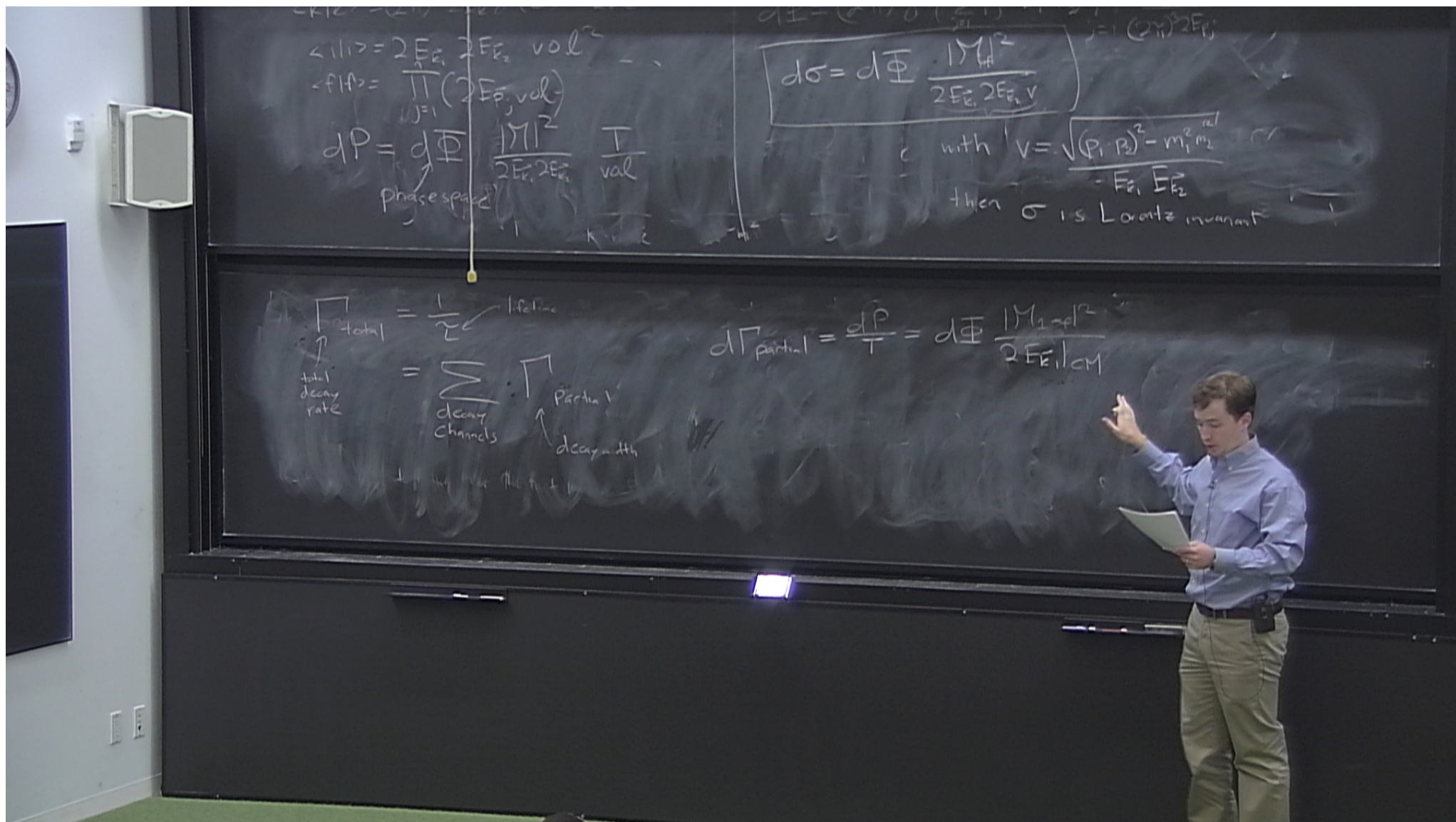
$$\langle f|f \rangle = \prod_{j=1}^n (2E_{p_j} \text{ vol})$$

$$dP = d\Phi \frac{|M|^2}{2E_1 2E_2} \frac{T}{\text{vol}}$$

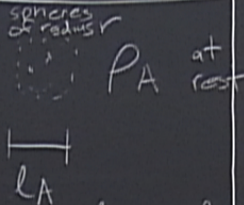
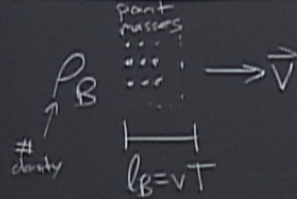
phase space

$$d\Phi = (2\pi)^4 \delta^4\left(\sum_{j=1}^n p_j - k_1 - k_2\right) \prod_{j=1}^n \frac{d^3 p_j}{(2\pi)^3 2E_{p_j}}$$





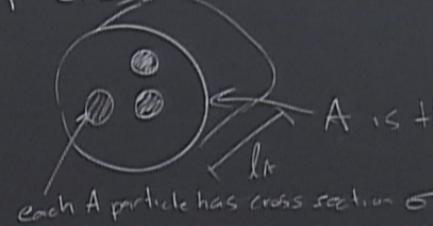
Cross Sections + Decay Rates



Classically: $N_{\text{collisions}} = \sigma P_A l_A P_B l_B A$

cross-section $\sigma = \pi r^2$

A = cross sectional area common to both beams



If we want T_{partial} or σ we should integrate over inequivalent final states

either:

- restrict integration
-

Scalar Nucleon-Meson Theory

$$\mathcal{L} = -$$

Scalar Nucleon-Meson Theory

$$\mathcal{L} = \partial_\mu \rho^\dagger \partial^\mu \rho + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - M^2 \rho^\dagger \rho - \frac{1}{2} m^2 \varphi^2 - g \rho^\dagger \rho \varphi$$

$\rho \rightarrow e^- \rho$
 $\varphi \rightarrow \gamma$

meson

$g \propto M, m$

Scalar Nucleon-Meson Theory

$$\mathcal{L} = \partial_\mu \rho^\dagger \partial^\mu \rho + \frac{1}{2} \partial_\mu \psi \partial^\mu \psi - M^2 \rho^\dagger \rho - \frac{1}{2} m^2 \psi^2 - g \rho^\dagger \rho \psi$$

$\rho \rightarrow e$
 $\psi \rightarrow \gamma$

$g \propto M, m$

Scalar Nucleon-Meson Theory

$$\mathcal{L} = \partial_\mu \rho^\dagger \partial^\mu \rho + \frac{1}{2} \partial_\mu \psi \partial^\mu \psi - M^2 \rho^\dagger \rho - \frac{1}{2} m^2 \psi^2 - g \rho^\dagger \rho \psi$$

$\rho \rightarrow e$
 $\psi \rightarrow \gamma$

nucleon
i0

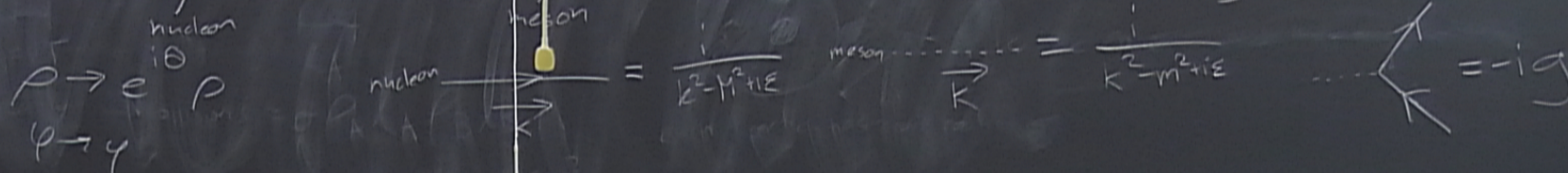
me

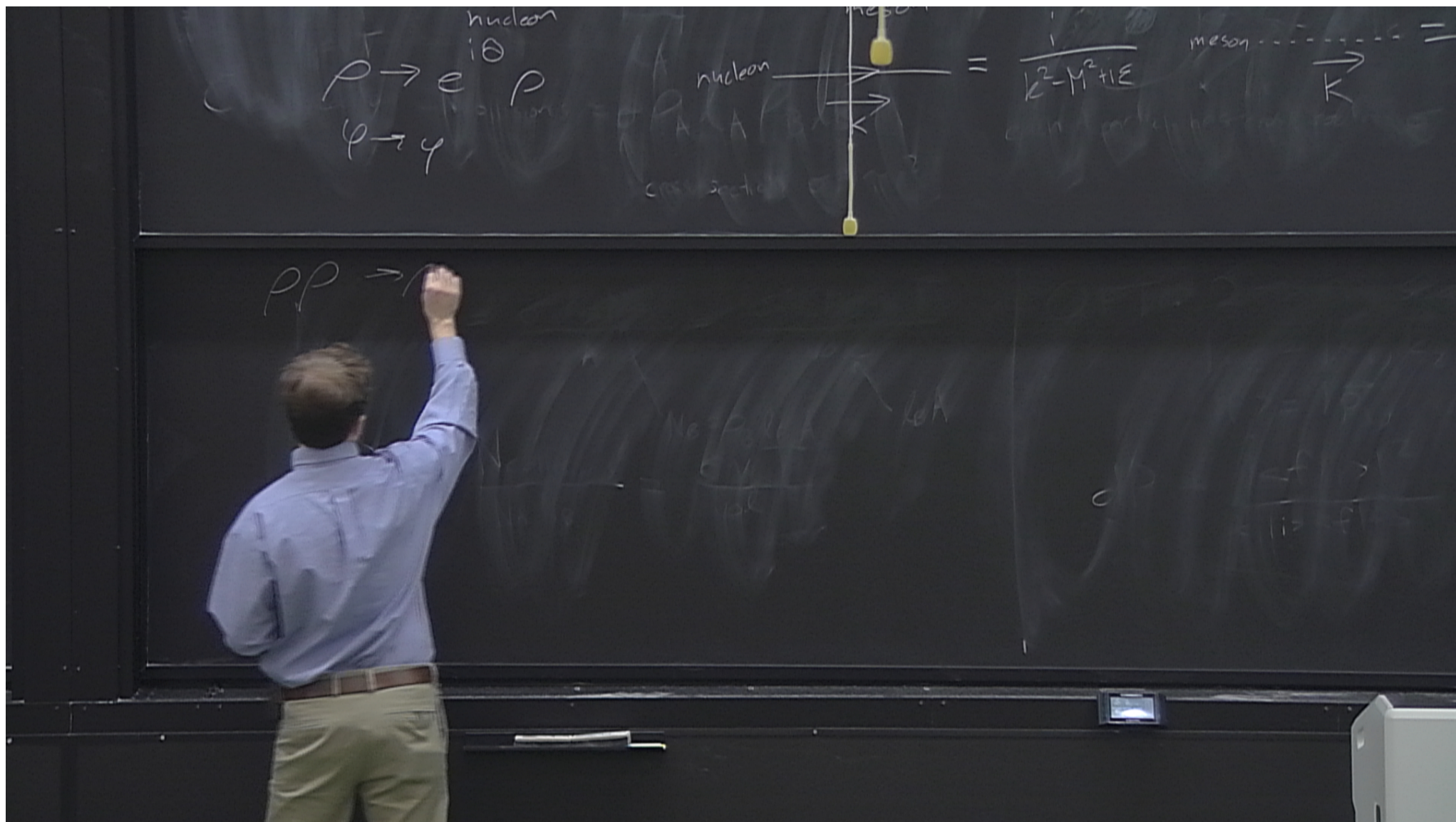
$g \propto M, m$

Scalar Nucleon Meson

$$\mathcal{L} = \partial_\mu \rho^\dagger \partial^\mu \rho + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - M^2 \rho^\dagger \rho - \frac{1}{2} m^2 \varphi^2 - g \rho^\dagger \rho \varphi$$

$g \ll M, m$





$\rho \rightarrow e$ ρ $\rho \rightarrow \gamma$

nucleon $\rightarrow$ nucleon $= \frac{1}{k^2 - M^2 + i\epsilon}$ meson $\rightarrow$ meson $=$

$\rho \rightarrow \gamma$

cross section

$pp \rightarrow pp$ Cross Section

$+ \uparrow$

$M = (-ig)^2 \left[\frac{i}{(k_1 - p_1)^2 - m^2 + i\epsilon} + \frac{i}{(k_1 - p_2)^2 - m^2 + i\epsilon} \right] + \mathcal{O}(g^4)$

$\rho \rightarrow e$ nucleon
 $\psi \rightarrow \psi$ 10
 ρ

Diagram illustrating the scattering process:

$$\sigma = \frac{1}{K^2 - M^2 + iE}$$

Labels in the diagram:

- nucleon
- meson
- K

$$dP = d\Phi \frac{|M|^2}{2E_1 2E_2} \frac{T}{\text{vol}}$$

phase space

with $v = \frac{\sqrt{(\vec{p}_1 \cdot \vec{p}_2)^2 - m_1^2 m_2^2}}{E_1 E_2}$
 then σ is Lorentz invariant

$$d\Phi_2 = (2\pi)^4 \delta^4(p_1 + p_2 - k_1 - k_2) \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2}$$

Work in the Center of mass frame $\vec{k}_1 = -\vec{k}_2$ $k_1^0 = k_2^0 = E$

Take relativistic limit $E \gg M, m$

$$\frac{1}{(2\pi)^2 2E_1 2E_2} \delta(E_{\vec{p}_1} + E_{\vec{p}_2} - 2E) d^3 p_1 \Big|_{\vec{p}_1 = -\vec{p}_2}$$

relativistic limit $E_{\vec{p}_1} = |\vec{p}_1|$

Take relativistic limit $E \gg m$

$$d\Phi_2 = \frac{1}{(2\pi)^2 2E_1 2E_2} \delta(E_{\vec{p}_1} + E_{\vec{p}_2} - 2E) d^3p_1 \bigg|_{\vec{p}_1 = -\vec{p}_2} \quad \text{relativistic limit } E_{\vec{p}_1} = |\vec{p}_1|$$

$$= \frac{1}{16\pi^2 |\vec{p}_1|^2} d\Omega |\vec{p}_1|^2 d|\vec{p}_1| \delta(2|\vec{p}_1| - 2E)$$

$$d\Phi_2 = \frac{d\Omega}{32\pi^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{g^4}{256\pi^2 E^2} \left(\frac{1}{(k_1 - p_1)^2 - m^2} + \frac{1}{(k_1 - p_2)^2 - m^2} \right)^2$$

Take relativistic limit $E \gg M, m$

$$d\Phi_2 = \frac{1}{(2\pi)^2 2E_{\vec{p}_1} 2E_{\vec{p}_2}} \delta(E_{\vec{p}_1} + E_{\vec{p}_2} - 2E) d^3p_1 \Big|_{\vec{p}_1 = -\vec{p}_2} \quad \text{relativistic limit } E_{\vec{p}_1} = |\vec{p}_1|$$

$$= \frac{1}{16\pi^2 |\vec{p}_1|^2} d\Omega |\vec{p}_1|^2 d|\vec{p}_1| \delta(2|\vec{p}_1| - 2E)$$

$$d\Phi_2 = \frac{d\Omega}{32\pi^2}$$

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$$d\sigma = d\Phi \frac{|M|^2}{4(E_1 p_2)^2 - m_1^2 m_2^2}$$

Take relativistic limit $E \gg M, m$

$$d\Phi_2 = \frac{1}{(2\pi)^2 2E_{\vec{p}_1} 2E_{\vec{p}_2}} \delta(E_{\vec{p}_1} + E_{\vec{p}_2} - 2E) d^3p_1 \bigg|_{\vec{p}_1 = -\vec{p}_2} \quad \text{relativistic limit } |E_{\vec{p}_1}| = |\vec{p}_1|$$

$$= \frac{1}{16\pi^2 |\vec{p}_1|^2} d\Omega |\vec{p}_1|^2 d|\vec{p}_1| \delta(2|\vec{p}_1| - 2E)$$

$$d\Phi_2 = \frac{d\Omega}{32\pi^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{g^4}{256\pi^2 E^2} \left(\frac{1}{(k_1 - p_1)^2 - m^2} + \frac{1}{(k_1 - p_2)^2 - m^2} \right)^2$$

$$d\sigma = d\Phi \frac{|M|^2}{4(E_1 p_2)^2 - m_1^2 m_2^2}$$

$$d\Phi_2 = \frac{d\Omega}{32\pi^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{g^4}{256\pi^2 E^2} \left(\frac{1}{(k_1 - p_1)^2 - m^2} + \frac{1}{(k_1 - p_2)^2 - m^2} \right)^2$$

$$d\sigma = d\Phi \frac{|M|^2}{4(E_1 p_1)^2 - m_1^2 m_2^2}$$

constraints
from delta
functions
in $d\Phi$

$$\frac{d\Omega}{d\Omega} = \frac{1}{256\pi^2 E^2} ((k_1 - p_1)^2 - m^2) ((k_1 - p_2)^2 - m^2)$$

constraints
from delta
functions
in dE

$$\Gamma_{\text{total}} = \frac{1}{\tau} \quad \text{lifetime}$$

total decay rate

$$= \sum_{\text{decay channels}} \Gamma_{\text{partial}}$$

decay width

$$d\Gamma_{\text{partial}} = \frac{dP}{T}$$

$$d\Gamma_{\text{partial}} = \frac{|M_{1 \rightarrow f}|^2}{2m_1}$$

$$2E_{\text{CM}}$$

$$\frac{d\Omega}{d\Omega} = \frac{1}{256\pi^2 E^2} ((k_1 - p_1)^2 - m^2) ((k_1 - p_2)^2 - m^2)$$

constraints
from delta
functions
in $d\Phi$

$$d\Gamma_{\text{partial}} = d\Phi \frac{|M_{1 \rightarrow f}|^2}{2m_1}$$

$2E_{\text{CM}}$

$$\frac{d\sigma}{d\Omega} = \frac{g^4}{256\pi^2 E^2} \left(\frac{1}{(k_1 - p_1)^2 - m^2} + \frac{1}{(k_1 - p_2)^2 - m^2} \right)^2$$

constraints
from delta
functions
in dE

$$k_1 = (E, \vec{k})$$

$$k_2 = (E, -\vec{k})$$

$$p_1 = (E, \vec{p})$$

$$p_2 = (E, -\vec{p})$$

$$k_1^2 = k_2^2 = p_1^2 = p_2^2 = 0$$

$$k_1 \cdot p_1 = E^2 - \vec{p} \cdot \vec{k} = E^2(1 - \cos\theta)$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{g^4}{256\pi^2 E^2} \left(\frac{\cos^2\theta}{1 - \cos^2\theta} \right)^2}$$

