Title: 14/15 PSI - Quantum Field Theory I (A) - Lecture 5

Date: Oct 10, 2014 03:30 PM

URL: http://pirsa.org/14100031

Abstract:

Last time:

$$\varphi(x) = \varphi^{+}(x) + \varphi^{-}(x)$$

$$\varphi^{+}(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} \, a_{\vec{p}} \, e^{-ip\cdot x}$$

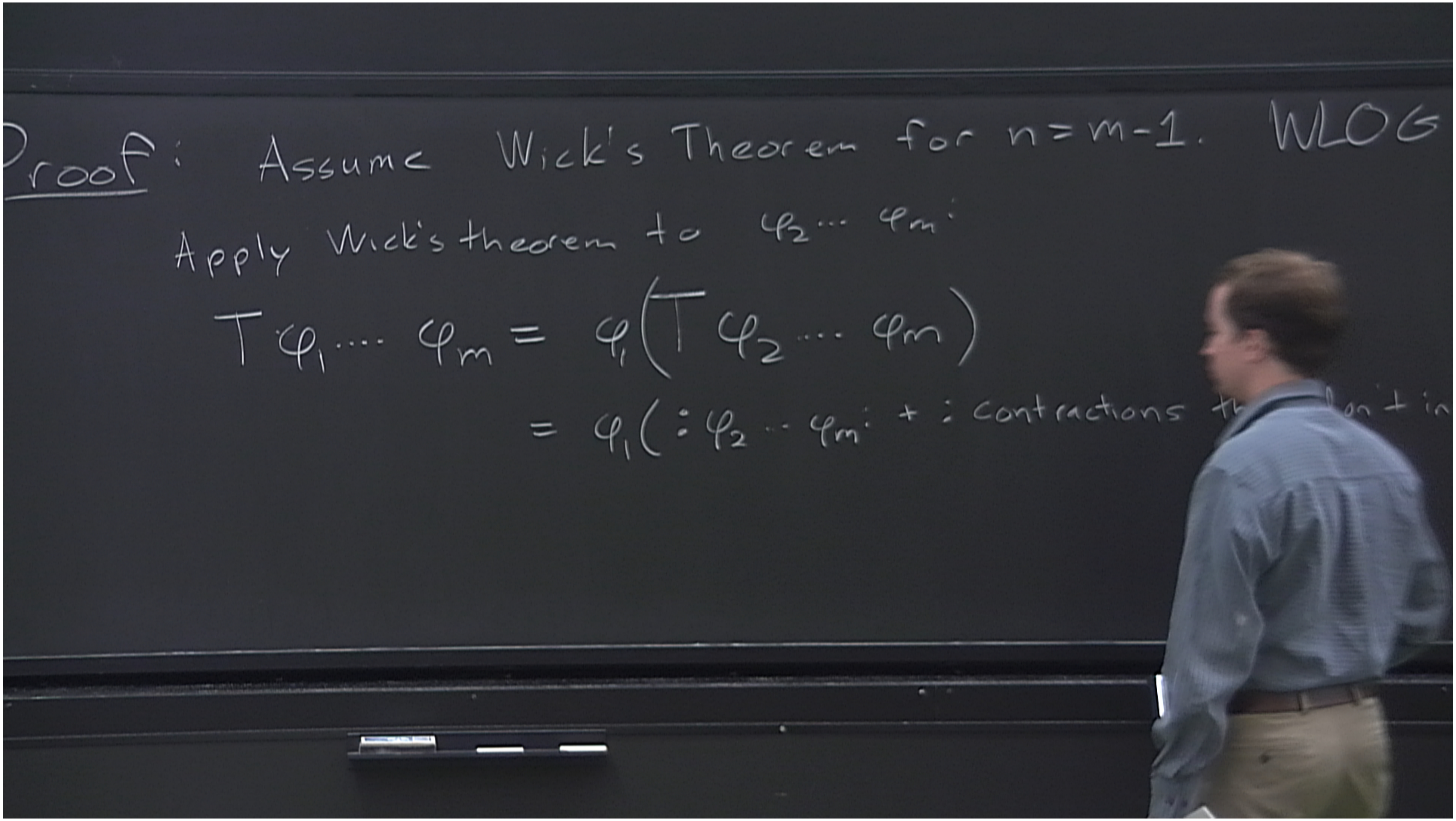
$$\varphi^{-}(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} \, a^{\dagger}_{\vec{p}} \, e^{+ip\cdot x}$$

$$\varphi(x)\varphi(y) = \varphi(x)\varphi(y) =$$

Wick's Theorem:

$$T\,\varphi_1 \cdots \varphi_n = :\varphi_1 \cdots$$

Proof: Assume Wick's Theorem for $n = m - 1$. W


Proof: Assume Wick's Theorem for $n=m-1$. WLOG
Apply Wick's theorem to $\varphi_2 \cdots \varphi_m$:
$T\varphi_1 \cdots \varphi_m = \varphi_1 \left(T \varphi_2 \cdots \varphi_m\right)$
$= \varphi_1 (:\varphi_2 \cdots \varphi_m: + :$ contractions

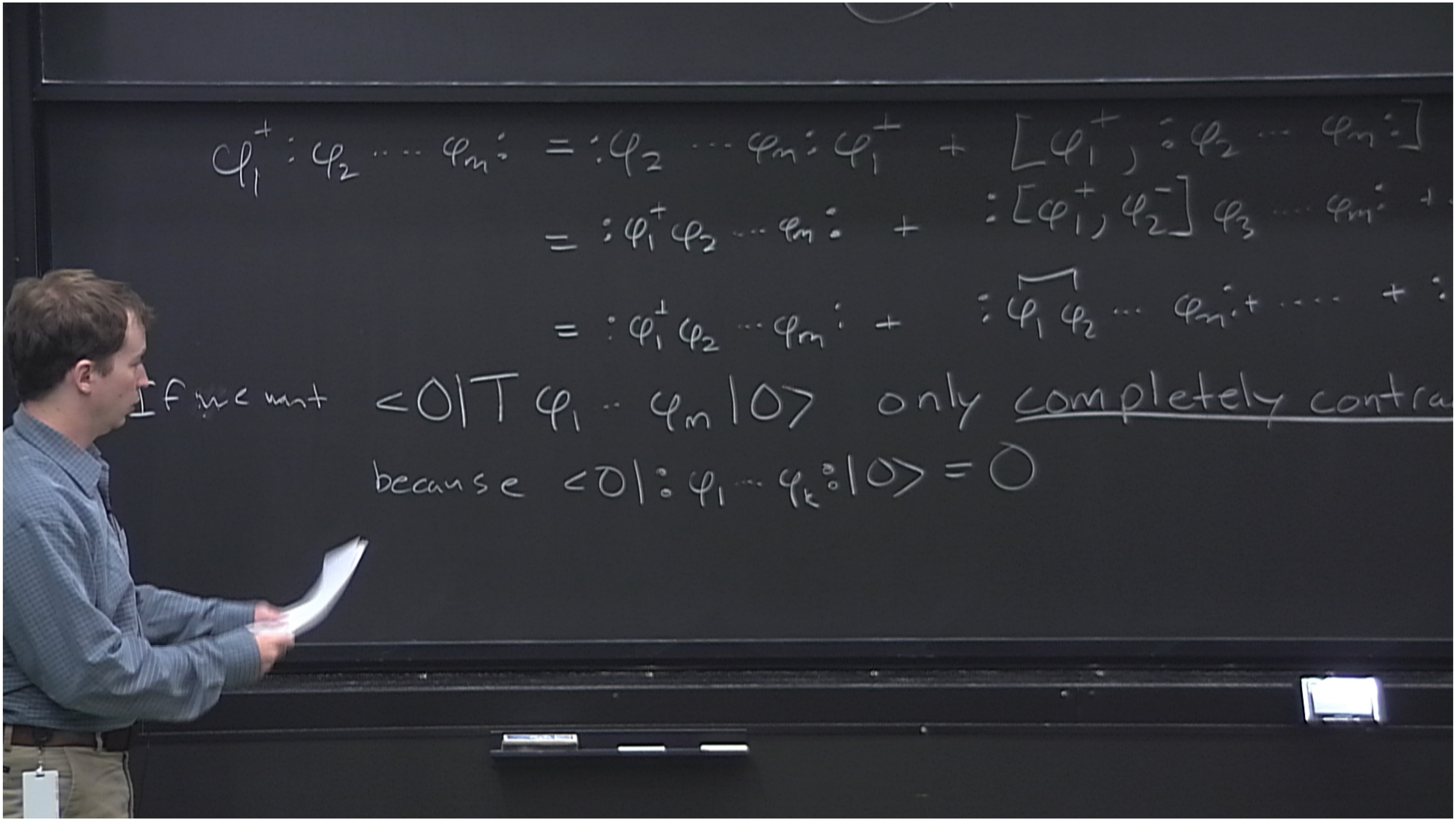
$$\varphi_1^+ :\varphi_2 \cdots \varphi_n: \;=\; :\varphi_2 \cdots \varphi_n:\varphi_1^+ \;+\; \left[\varphi_1^+ , :\varphi_2 \cdots \varphi_n:\right]$$

$$= :\varphi_1^+ \varphi_2 \cdots \varphi_n: \;+\; :\left[\varphi_1^+ , \varphi_2^-\right] \varphi_3 \cdots \varphi_n: \;+ \cdots +$$

$$\cdots \varphi_m : \varphi_1^+ + [\varphi_1^+, :\varphi_2 \cdots \varphi_m :]$$

$$\varphi_2 \cdots \varphi_m : + :[\varphi_1^+, \varphi_2^-]\varphi_3 \cdots \varphi_m : + \cdots + :\varphi_2 \cdots [\varphi_1^+, \varphi_m^-]:$$

$$\varphi_2 \cdots \varphi_m : + :\varphi_1 \varphi_2 \cdots \varphi_m : + \cdots + :\varphi_1 \varphi_2 \cdots \varphi_m :$$


$\varphi_1^+ :\varphi_2 \cdots \varphi_m: = :\varphi_2 \cdots \varphi_m: \varphi_1^+ + [\varphi_1^+, :\varphi_2 \cdots \varphi_m:]$
$= :\varphi_1^+ \varphi_2 \cdots \varphi_m: + :[\varphi_1^+, \varphi_2^-] \varphi_3 \cdots \varphi_m: + \cdots$
$= :\varphi_1^+ \varphi_2 \cdots \varphi_m: + :\overbrace{\varphi_1 \varphi_2} \cdots \varphi_m: + \cdots + :$
If we want $\langle 0| T \varphi_1 \cdots \varphi_m |0\rangle$ only completely contra
because $\langle 0| :\varphi_1 \cdots \varphi_k: |0\rangle = 0$

$$\varphi_1^+ :\varphi_2 \cdots \varphi_m: \; = \; :\varphi_2 \cdots \varphi_m: \varphi_1^+ + [\varphi_1^+, :\varphi_2 \cdots \varphi_m:]$$

$$= \; :\varphi_1^+ \varphi_2 \cdots \varphi_m: + :[\varphi_1^+, \varphi_2^-]\, \varphi_3 \cdots \varphi_m: + \cdots +$$

$$= \; :\varphi_1^+ \varphi_2 \cdots \varphi_m: + :\overbrace{\varphi_1 \varphi_2} \cdots \varphi_m: + \cdots + :\varphi_1 \varphi_2 \cdots$$

we want $\langle 0|T\varphi_1 \cdots \varphi_m|0\rangle$ only <u>completely contracted</u> ter

because $\langle 0| :\varphi_1 \cdots \varphi_k: |0\rangle = 0$

Example: $\langle 0|T\varphi_1\varphi_2\varphi_3\varphi_4|0\rangle = \Delta_{12}\Delta_{34}$

$\varphi_2 \cdots \varphi_m : \varphi_1^+ + [\varphi_1^+, :\varphi_2 \cdots \varphi_m:]$

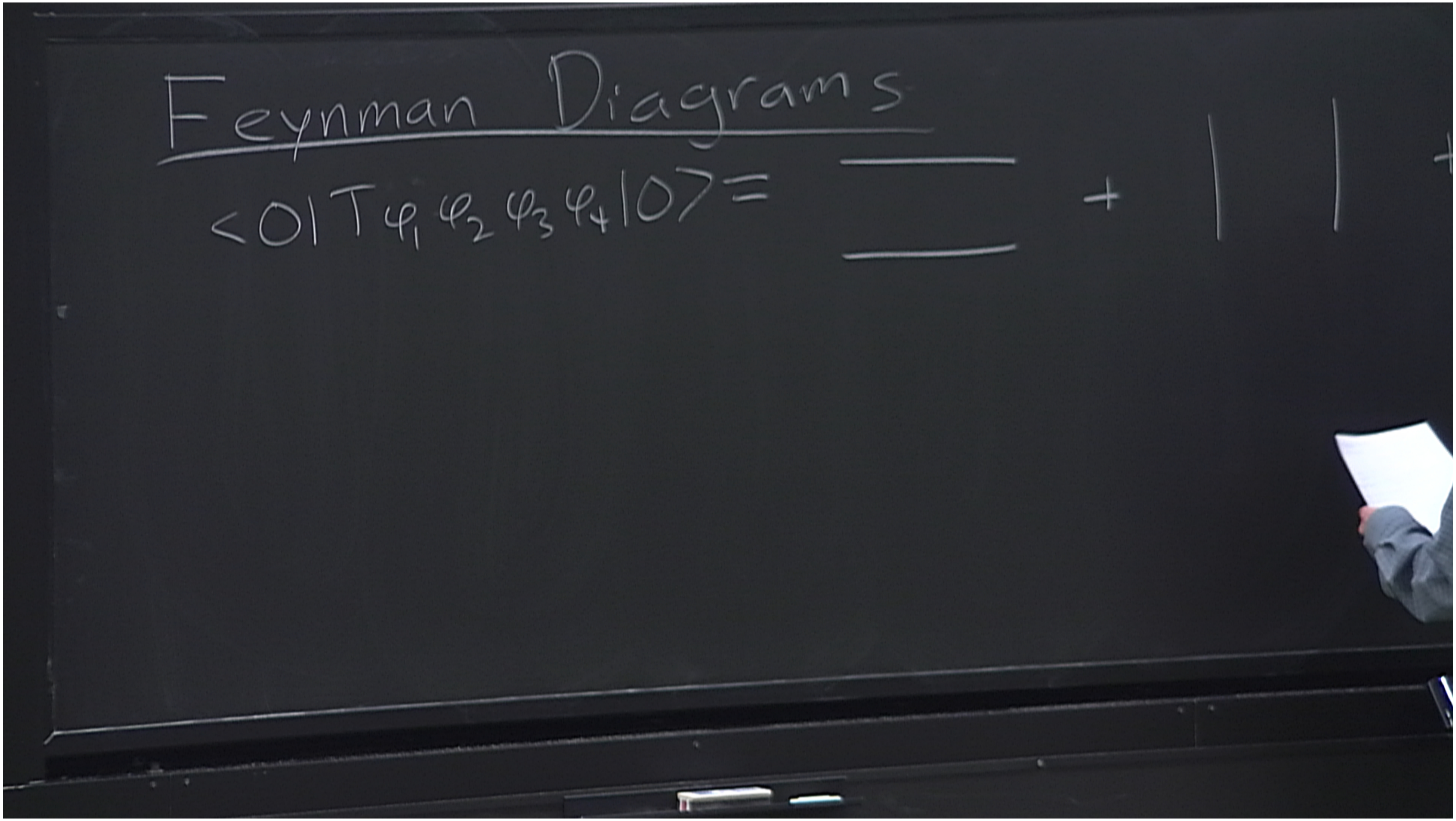
$:\varphi_1^+ \varphi_2 \cdots \varphi_m: + :[\varphi_1^+, \varphi_2^-]\varphi_3 \cdots \varphi_m: + \cdots + :\varphi_2 \cdots [\varphi_1^+, \varphi_m^-]:$

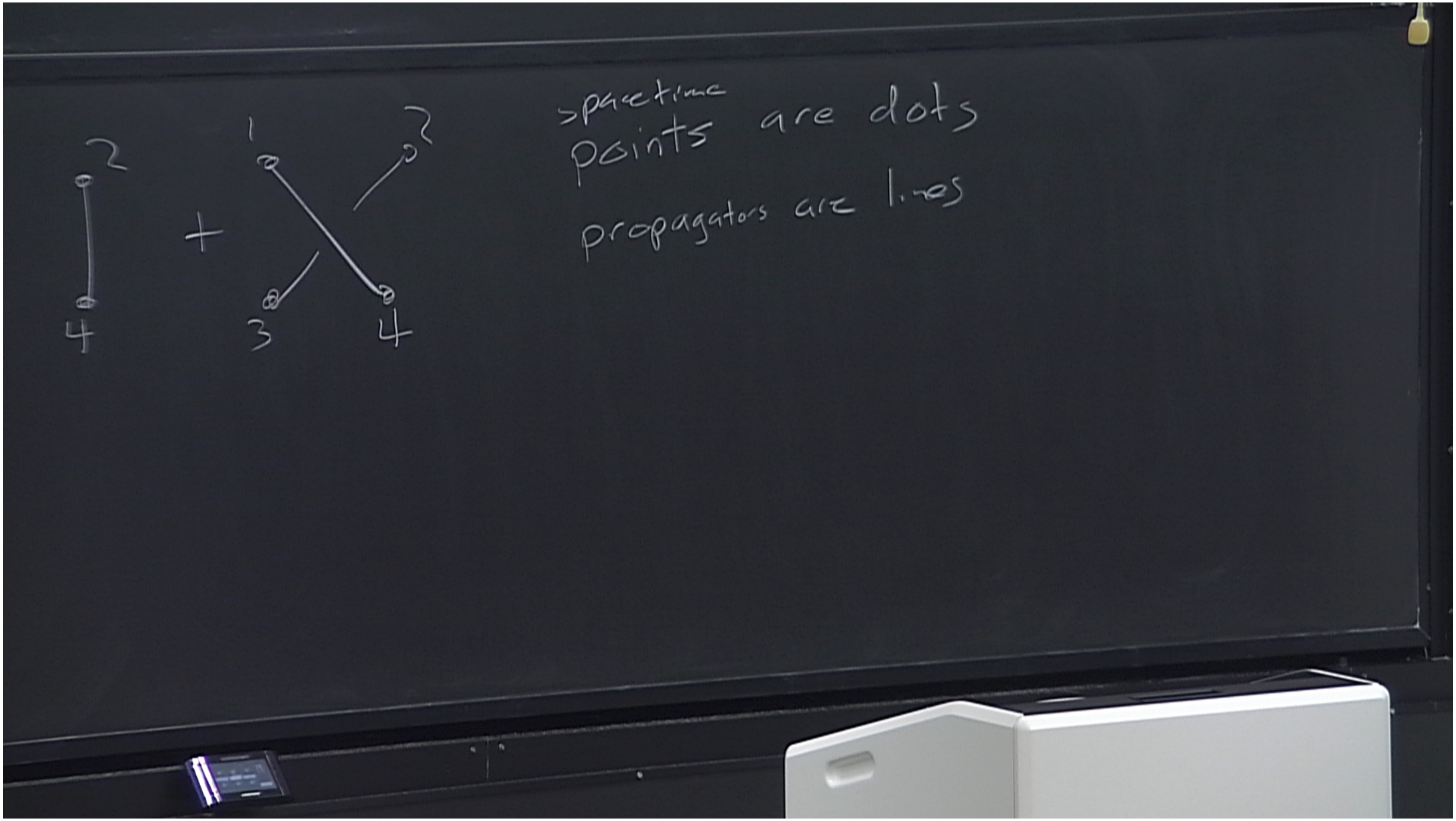
$:\varphi_1^+ \varphi_2 \cdots \varphi_m: + :\overbrace{\varphi_1 \varphi_2} \cdots \varphi_m: + \cdots + :\varphi_1 \varphi_2 \cdots \varphi_m:$

$\varphi_m |0\rangle$ only completely contracted terms surive!

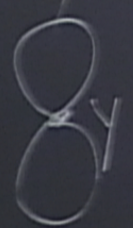
$\langle 0| :\varphi_1 \cdots \varphi_k: |0\rangle = 0$

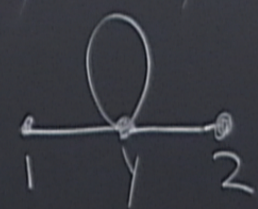
$\langle 0| \varphi_1 \varphi_2 \varphi_3 \varphi_4 |0\rangle = \Delta_{12}\Delta_{34} + \Delta_{13}\Delta_{24} + \Delta_{14}\Delta_{23}$

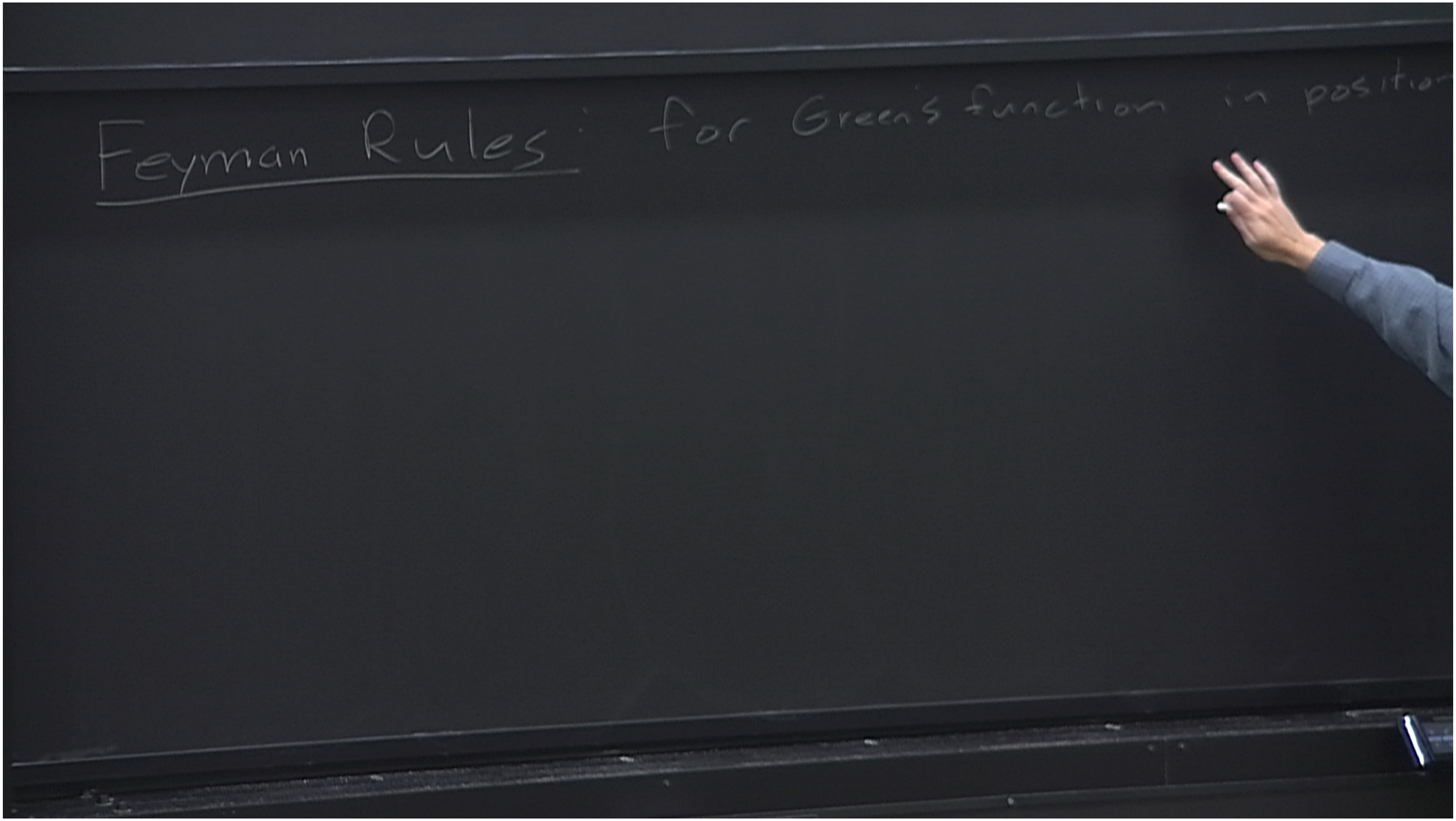

Feynman Diagrams
$\langle 0|T\varphi_1\varphi_2\varphi_3\varphi_4|0\rangle =$
+

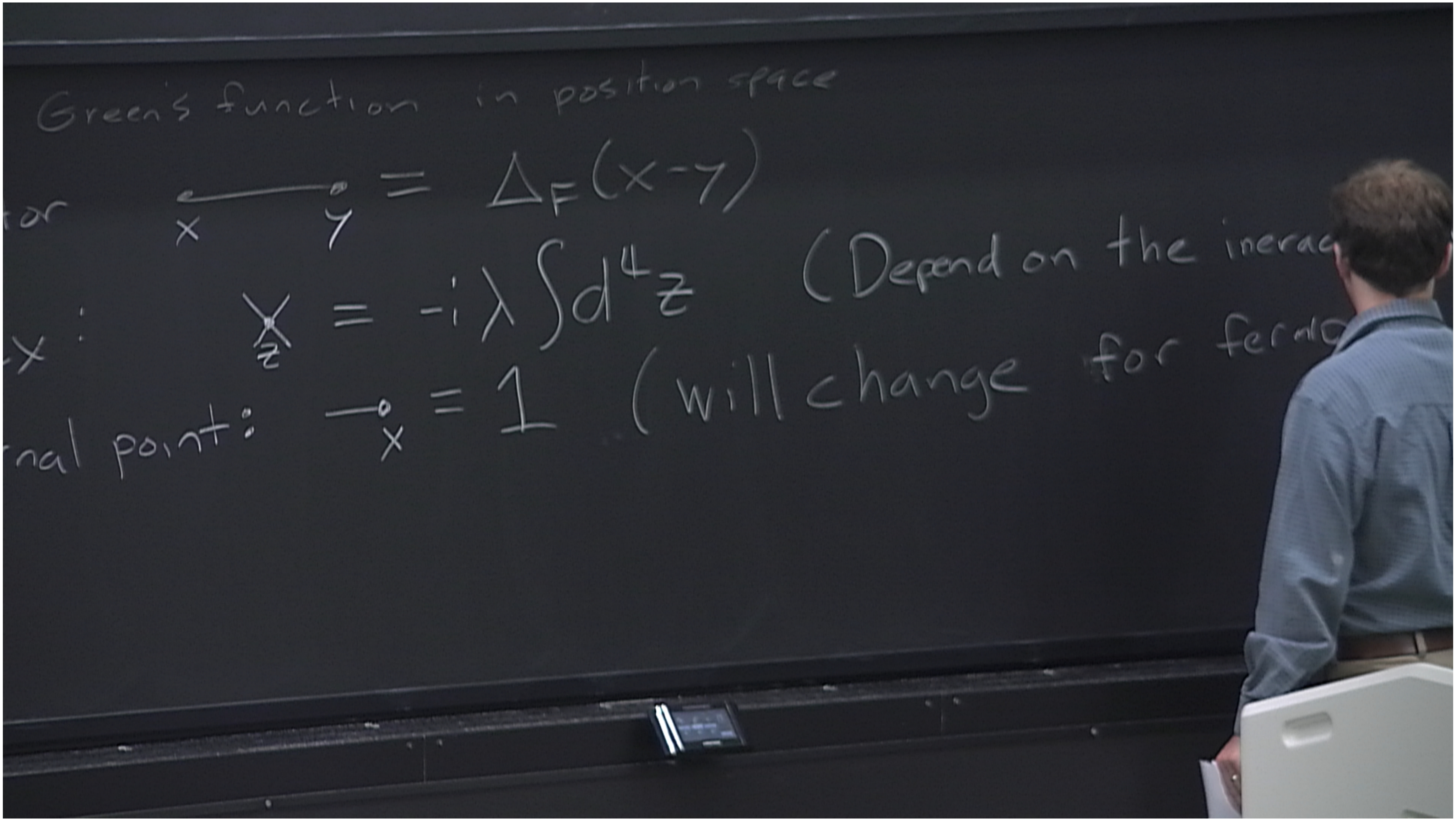

spacetime
points are dots
propagators are lines

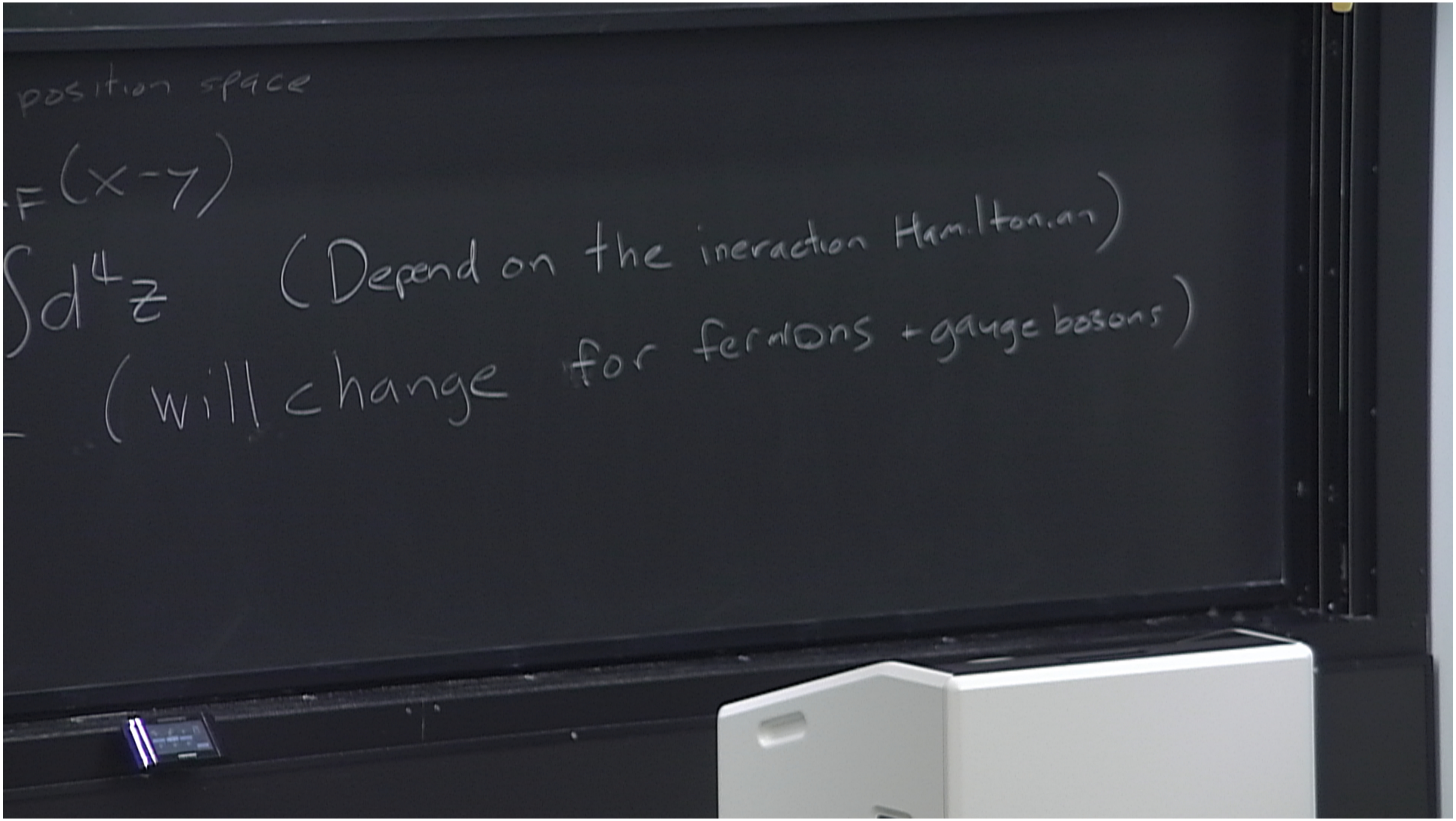
$$\int d^4y\, \Delta_{yy}\,\Delta_{yy} + \underbrace{4\cdot 3}_{\text{12 possible Wick contractions}} \frac{-i\lambda}{4!}\int d^4y\, \Delta_{1y}\,\Delta_{2y}\,\Delta_{yy}$$

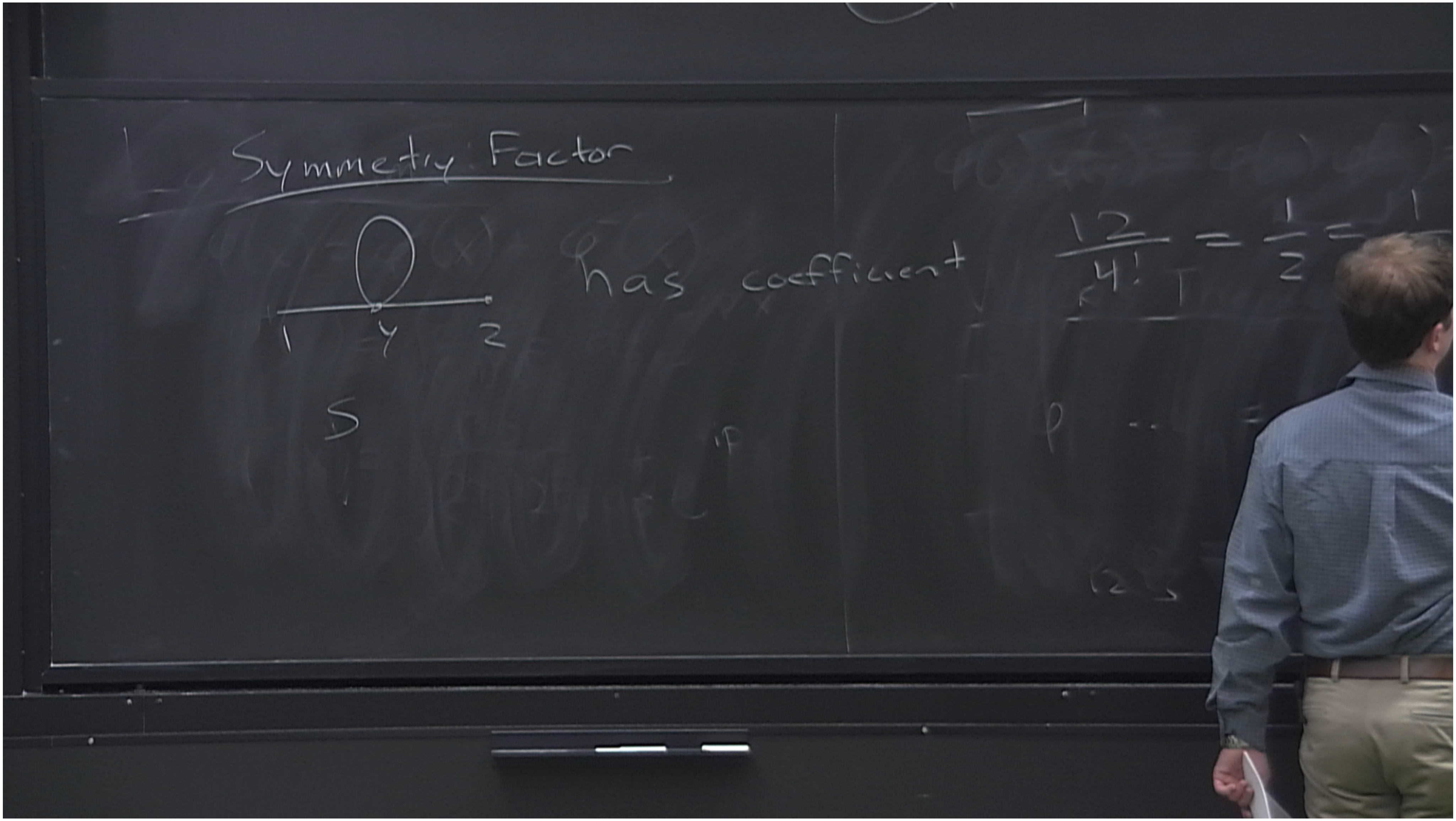





Feyman Rules: for Green's function in position

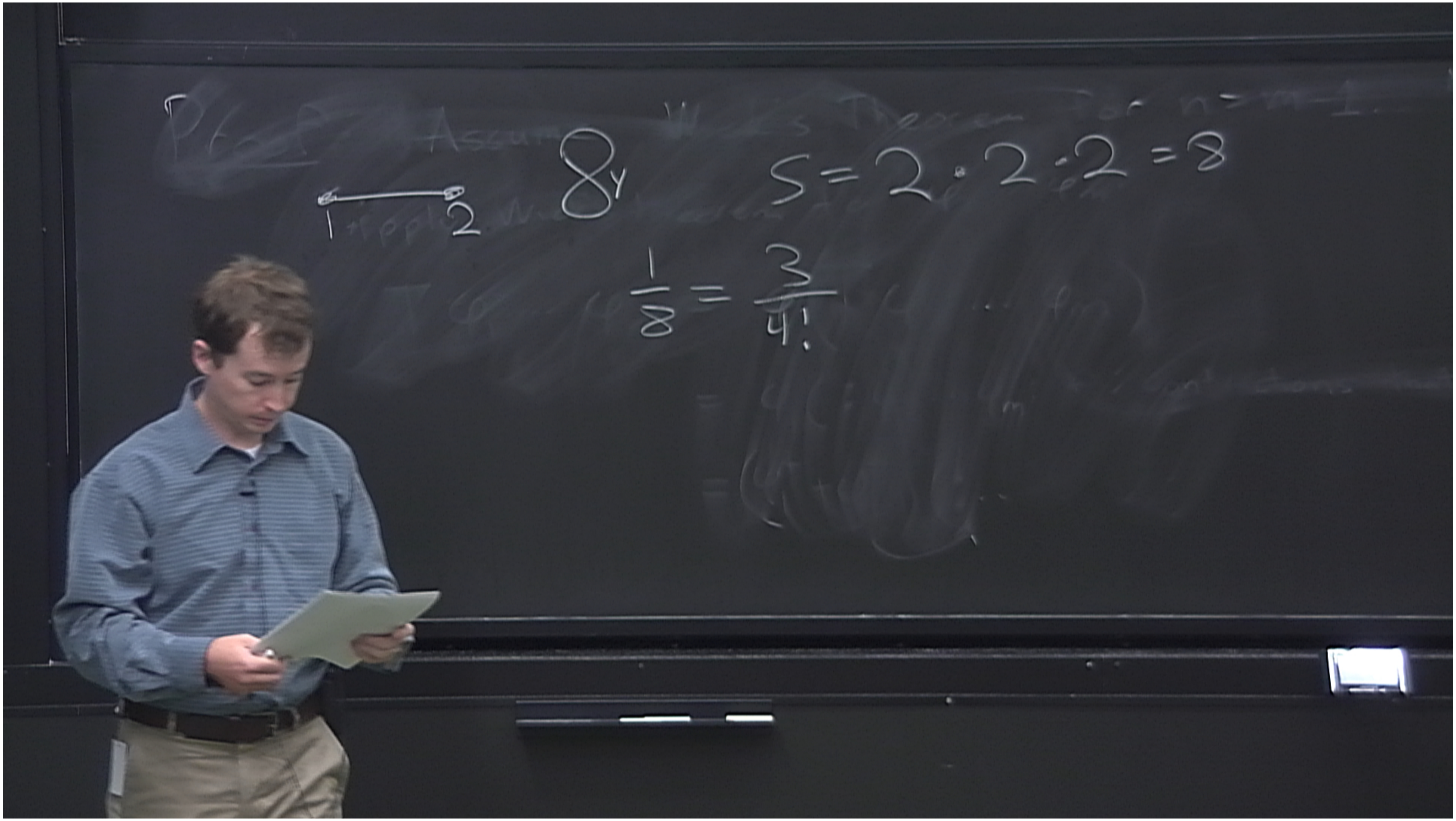

Green's function in position space
$= \Delta_F(x-y)$
$= -i\lambda\int d^4z$
(Depend on the inerac
(will change for fermio
nal point:
$= 1$

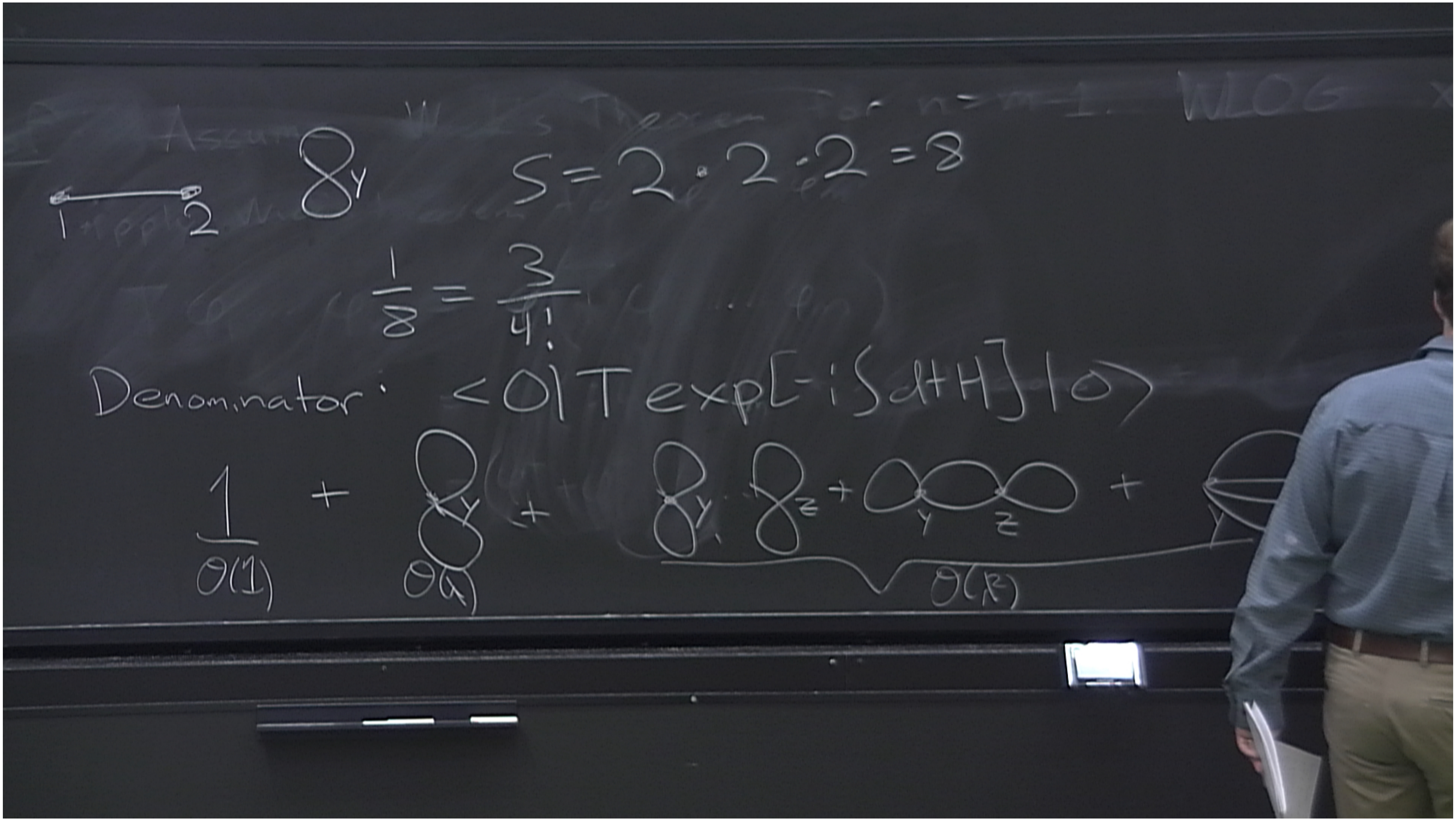

position space
$F(x-y)$
$\int d^4z$ (Depend on the ineraction Hamiltonian)
(will change for fermions + gauge bosons)


Symmetry Factor
has coefficient
$\frac{12}{4!} = \frac{1}{2} =$
1 y 2
S

$$\frac{12}{4!} = \frac{1}{2} = \frac{1}{S_k}$$ symmetry factor

components of diagram w/o changing the diagram


S = 2 · 2 · 2 = 8
1/8 = 3/4!


S = 2 · 2 · 2 = 8
1/8 = 3/4!
Denominator: <0|T exp[-i∫dt H]|0>
1 + … + … + … + …
O(1)
O(λ)
O(λ²)

$$\varphi_1^+ :\varphi_2 \cdots \varphi_m: \;=\; :\varphi_2 \cdots \varphi_m:\varphi_1^+ + [\varphi_1^+, :\varphi_2 \cdots \varphi_m:]$$

$$= :\varphi_1^+\varphi_2 \cdots \varphi_m: + :[\varphi_1^+, \varphi_2^-]\varphi_3 \cdots \varphi_m: + \cdots + :\varphi_2$$

$$= :\varphi_1^+\varphi_2 \cdots \varphi_m: + :\overbrace{\varphi_1\varphi_2}\cdots\varphi_m: + \cdots + :\varphi_1\varphi_2\cdots\varphi$$

want $\langle 0|T\varphi_1 \cdots \varphi_m|0\rangle$ only <u>completely contracted</u> terms

because $\langle 0|:\varphi_1 \cdots \varphi_k:|0\rangle = 0$

Example: $\langle 0|T\varphi_1\varphi_2\varphi_3\varphi_4|0\rangle = \Delta_{12}\Delta_{34} + \Delta_{13}\Delta_{24} + \Delta_{14}\Delta_{2}$

$$\varphi_1^+ :\varphi_2 \cdots \varphi_m: \;=\; :\varphi_2 \cdots \varphi_m: \varphi_1^+ + [\varphi_1^+, :\varphi_2 \cdots \varphi_m:]$$

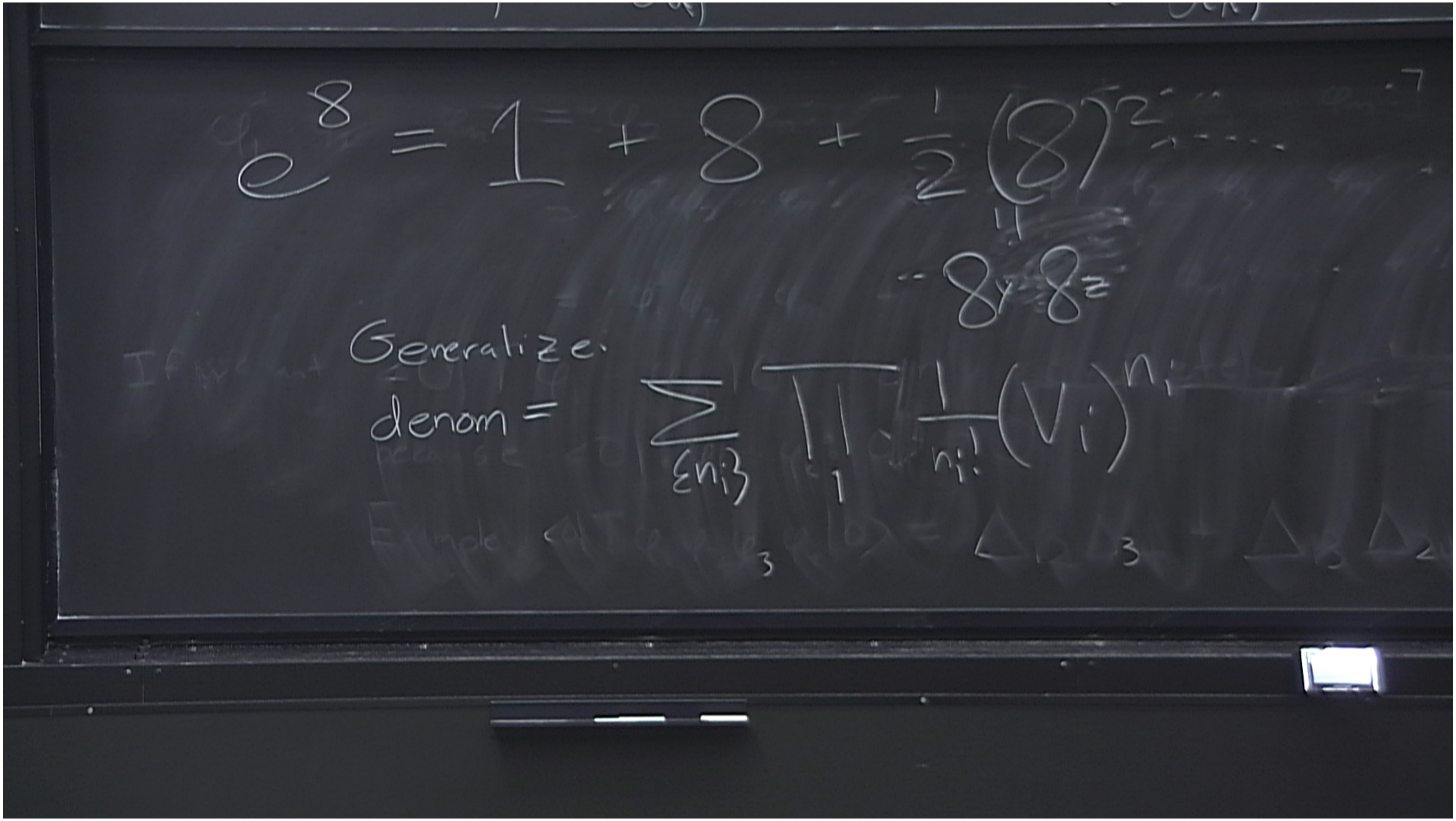
$$= :\varphi_1^+ \varphi_2 \cdots \varphi_m: + :[\varphi_1^+, \varphi_2^-]\varphi_3 \cdots \varphi_m: + \cdots + :\varphi_2 \cdots$$

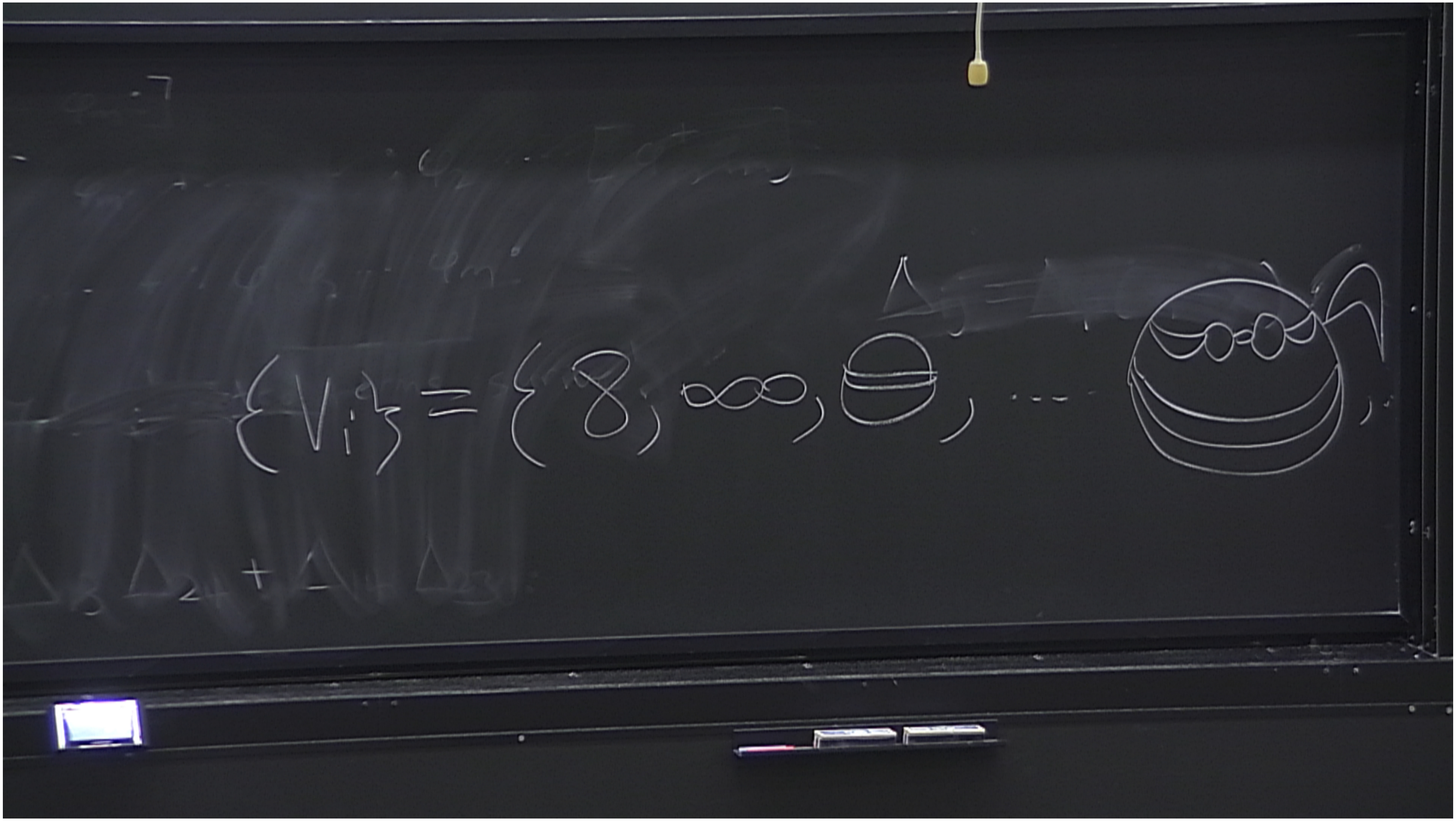
$$= :\varphi_1^+ \varphi_2 \cdots \varphi_m: + :\overbrace{\varphi_1 \varphi_2}\cdots \varphi_m: + \cdots + :\varphi_1 \varphi_2 \cdots \varphi \cdots$$

want $\langle 0 | T \varphi_1 \cdots \varphi_m | 0 \rangle$ only completely contracted terms

because $\langle 0 | :\varphi_1 \cdots \varphi_k: | 0 \rangle = 0$

Example: $\langle 0 | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | 0 \rangle = \Delta_{12}\Delta_{34} + \Delta_{13}\Delta_{24} + \Delta_{14}\Delta_{2 \cdots}$


$e^{8} = 1 + 8 + \frac{1}{2}(8)^2 + \cdots$
$8_y \, 8_z$
Generalize.
denom = $\sum_{\{n_i\}} \prod_i \frac{1}{n_i!}(V_i)^{n_i}$


{V_i} = {

$\mathcal{O}(1) \qquad \mathcal{O}(\lambda) \qquad \mathcal{O}(\lambda^2)$

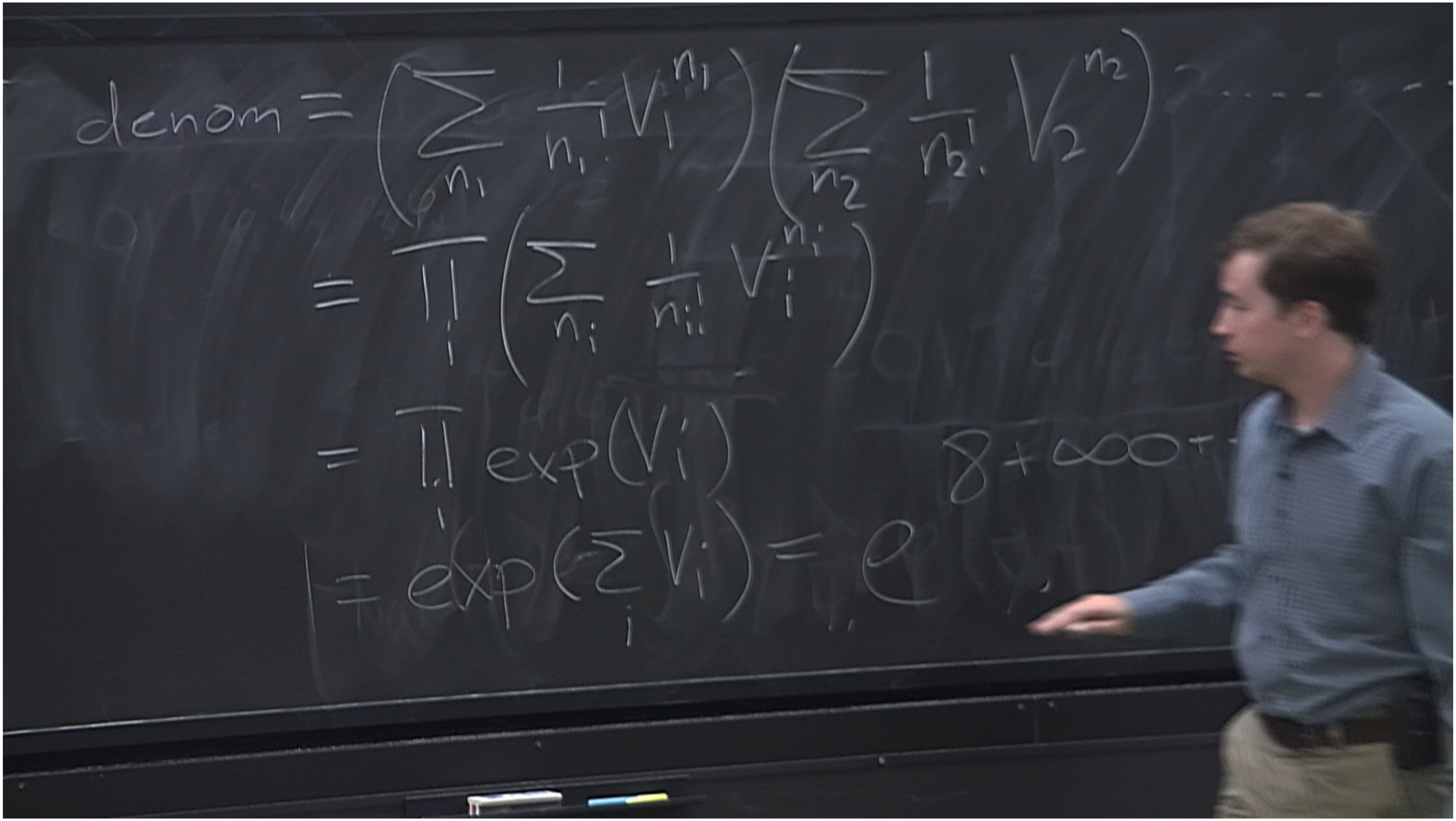
$$e^{\delta} = 1 + \delta + \frac{1}{2}(\delta)^2 + \cdots$$

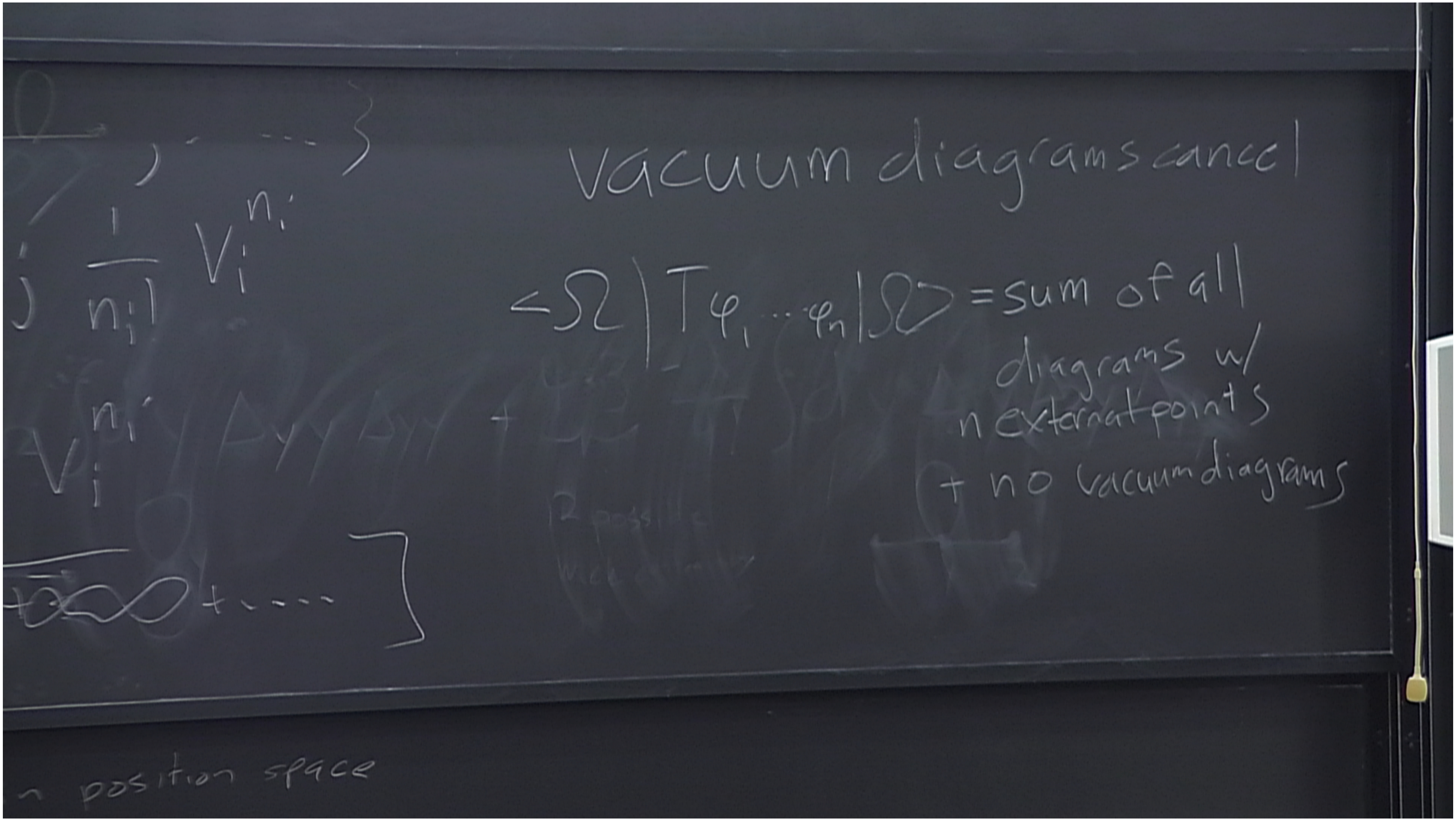
$$\cdots \delta_y \, \delta_z$$

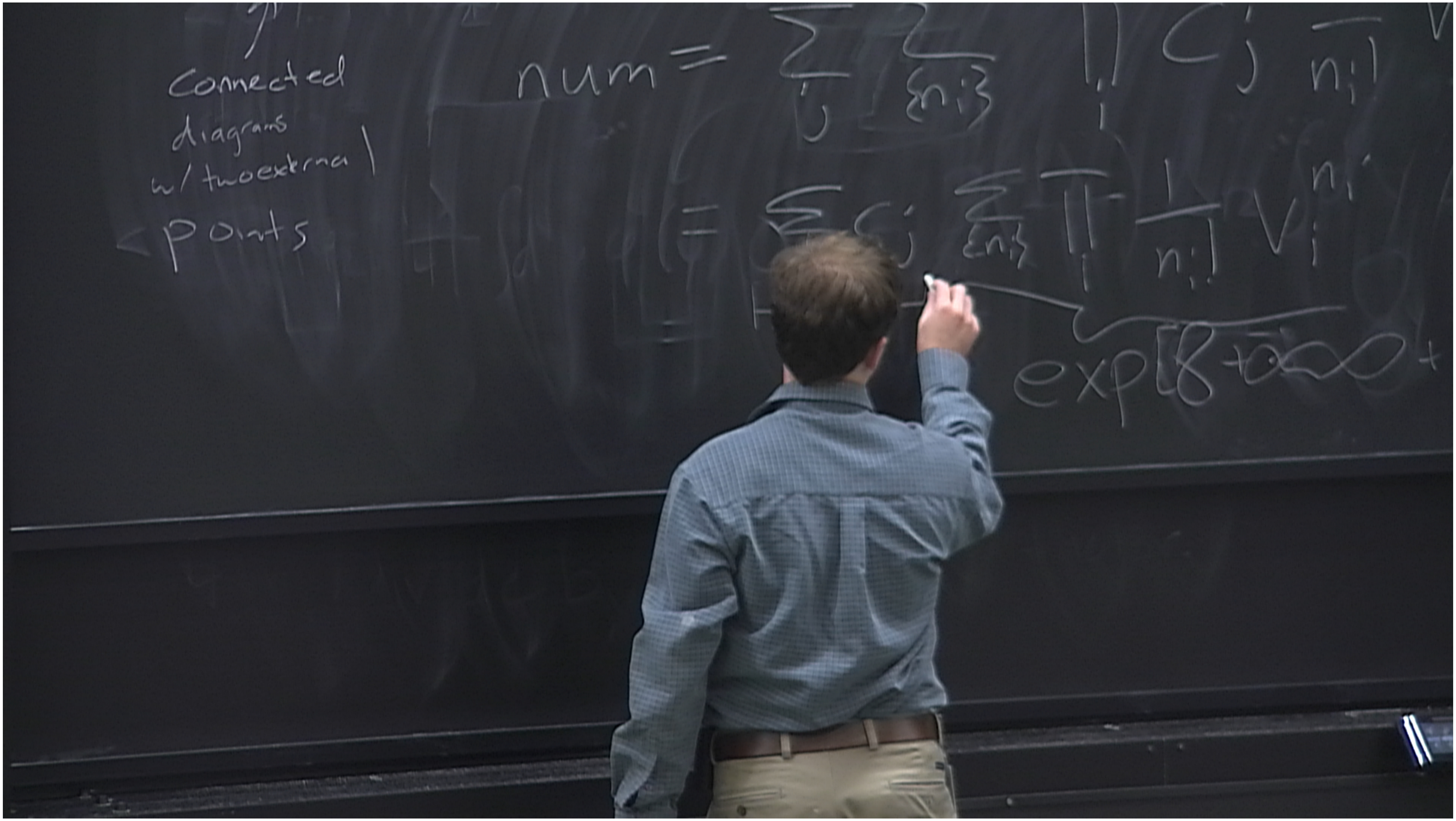
Generalize:

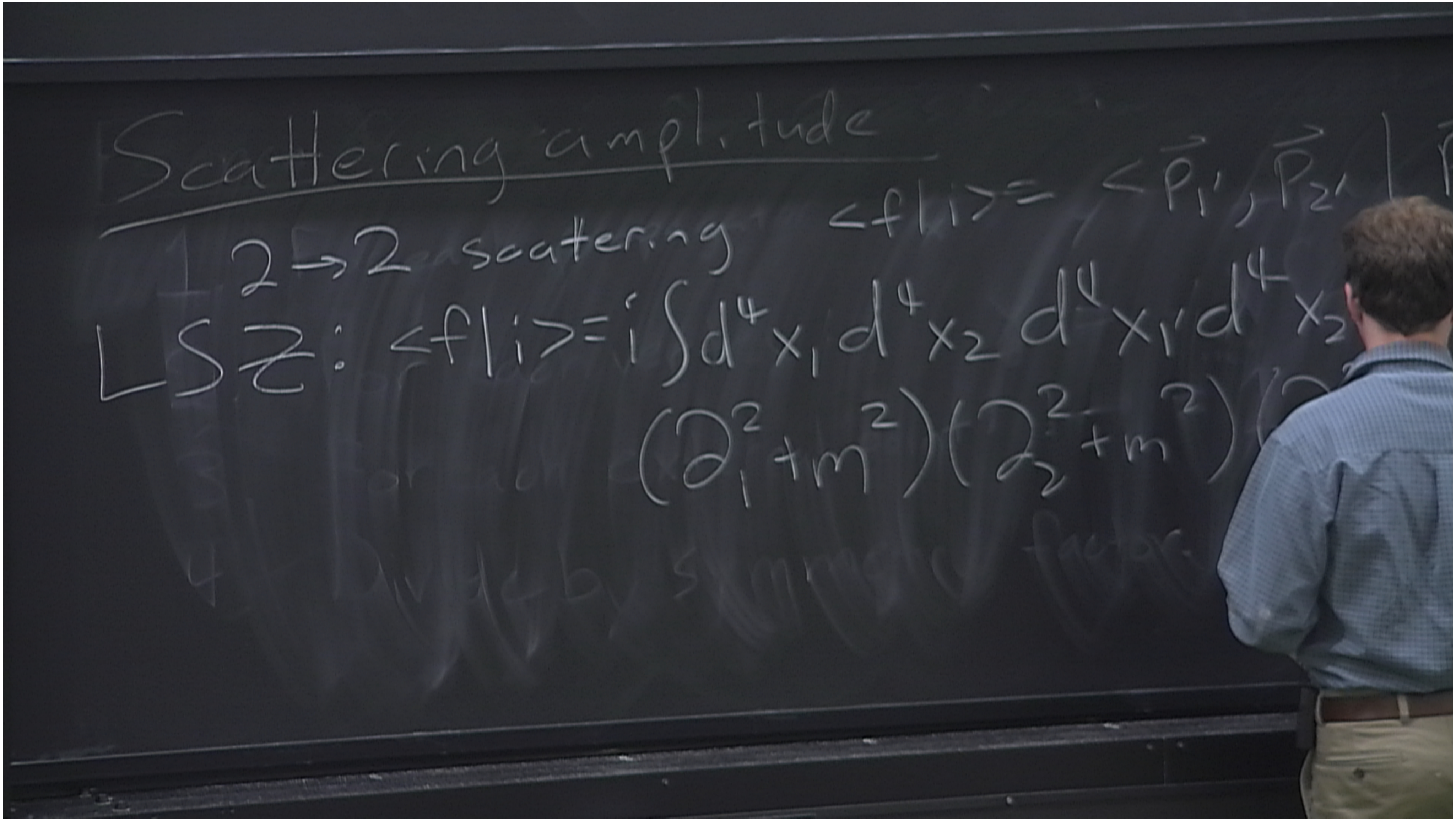
$$\text{denom} = \sum_{\{n_i\}} \prod_i \frac{1}{n_i!} (V_i)^{n_i}$$

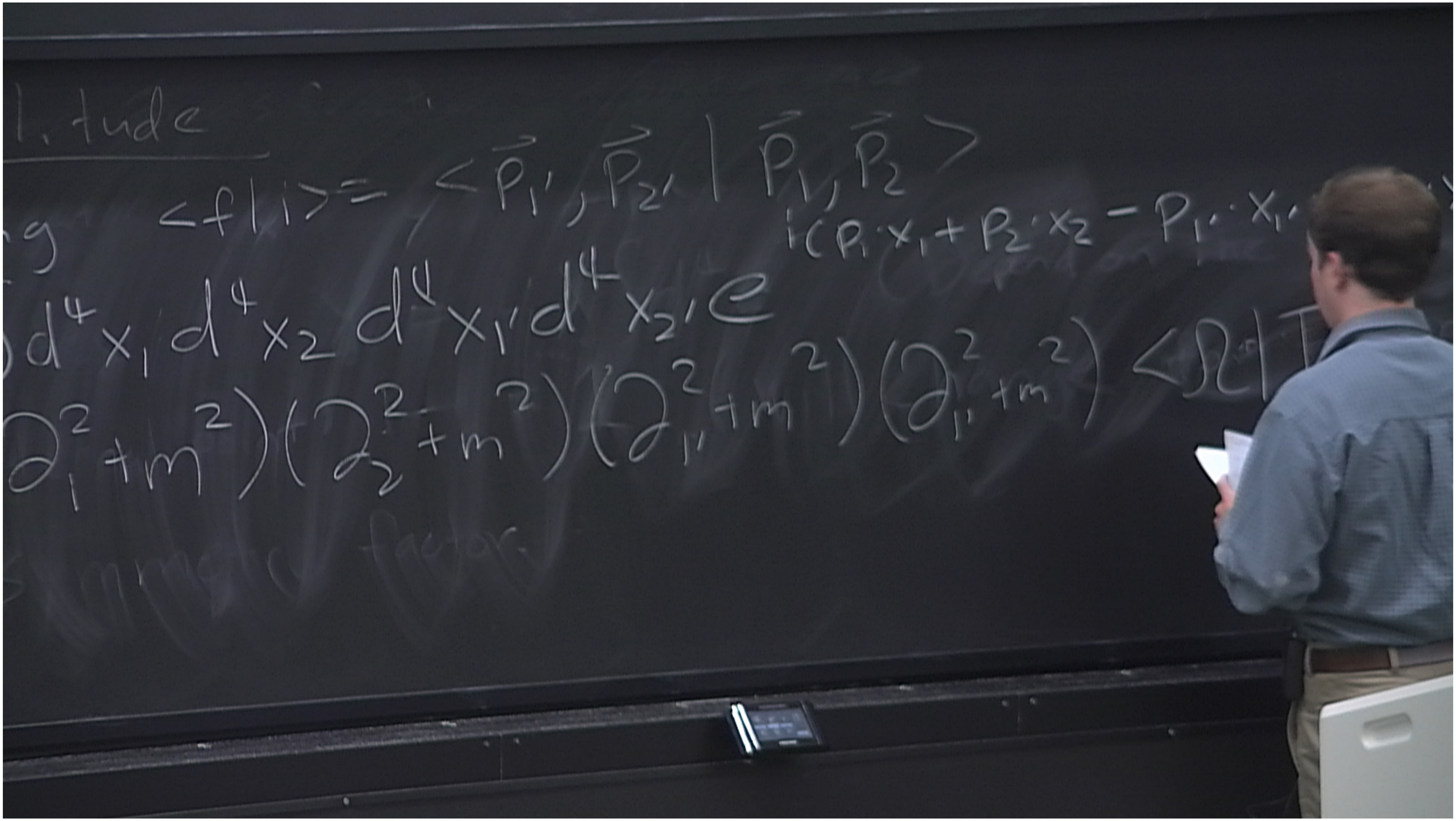
$$\text{denom} = \left(\sum_{n_1} \frac{1}{n_1!} V_1^{n_1}\right)\left(\sum_{n_2} \frac{1}{n_2!} V_2^{n_2}\right) \cdots$$

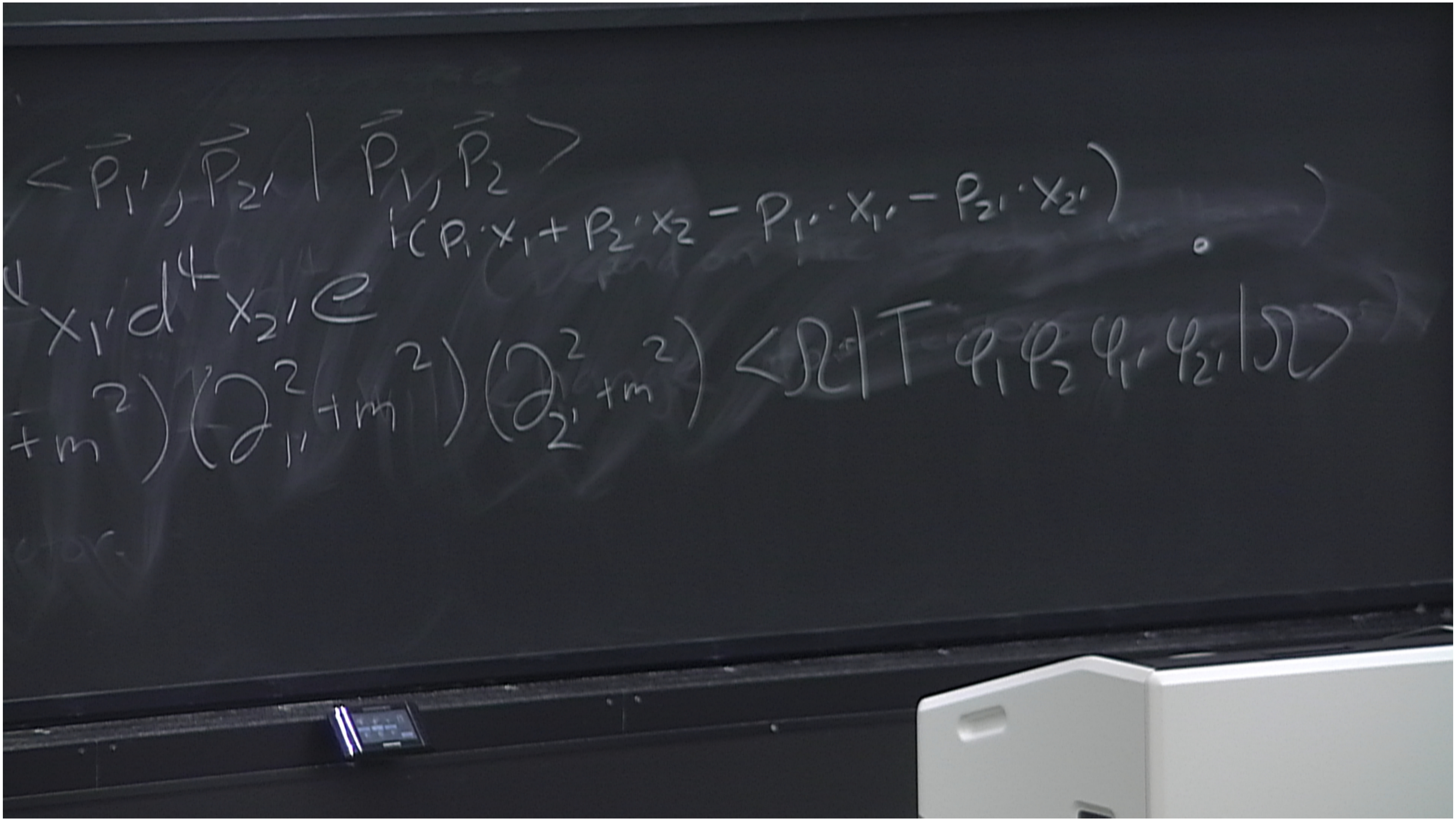

$\text{denom} = \left(\sum_{n_1} \frac{1}{n_1!} V_1^{n_1}\right)\left(\sum_{n_2} \frac{1}{n_2!} V_2^{n_2}\right) \cdots$
$= \prod_i \left(\sum_{n_i} \frac{1}{n_i!} V_i^{n_i}\right)$
$= \prod_i \exp(V_i)$
$= \exp\left(\sum_i V_i\right) = e$

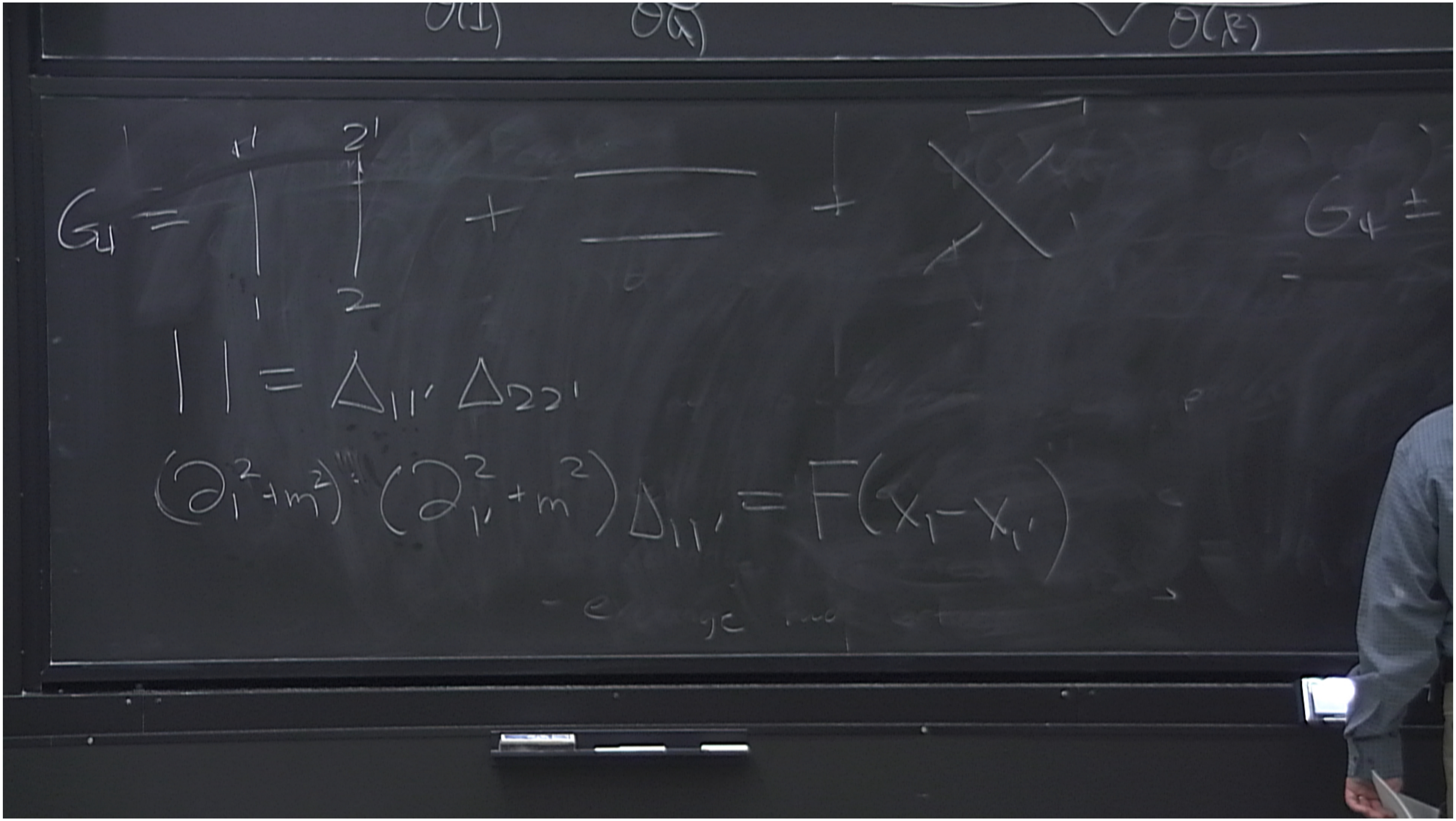

vacuum diagrams cancel
<Ω|Tφ1 ... φn|Ω> = sum of all
diagrams w/
n external points
+ no vacuum diagrams
position space

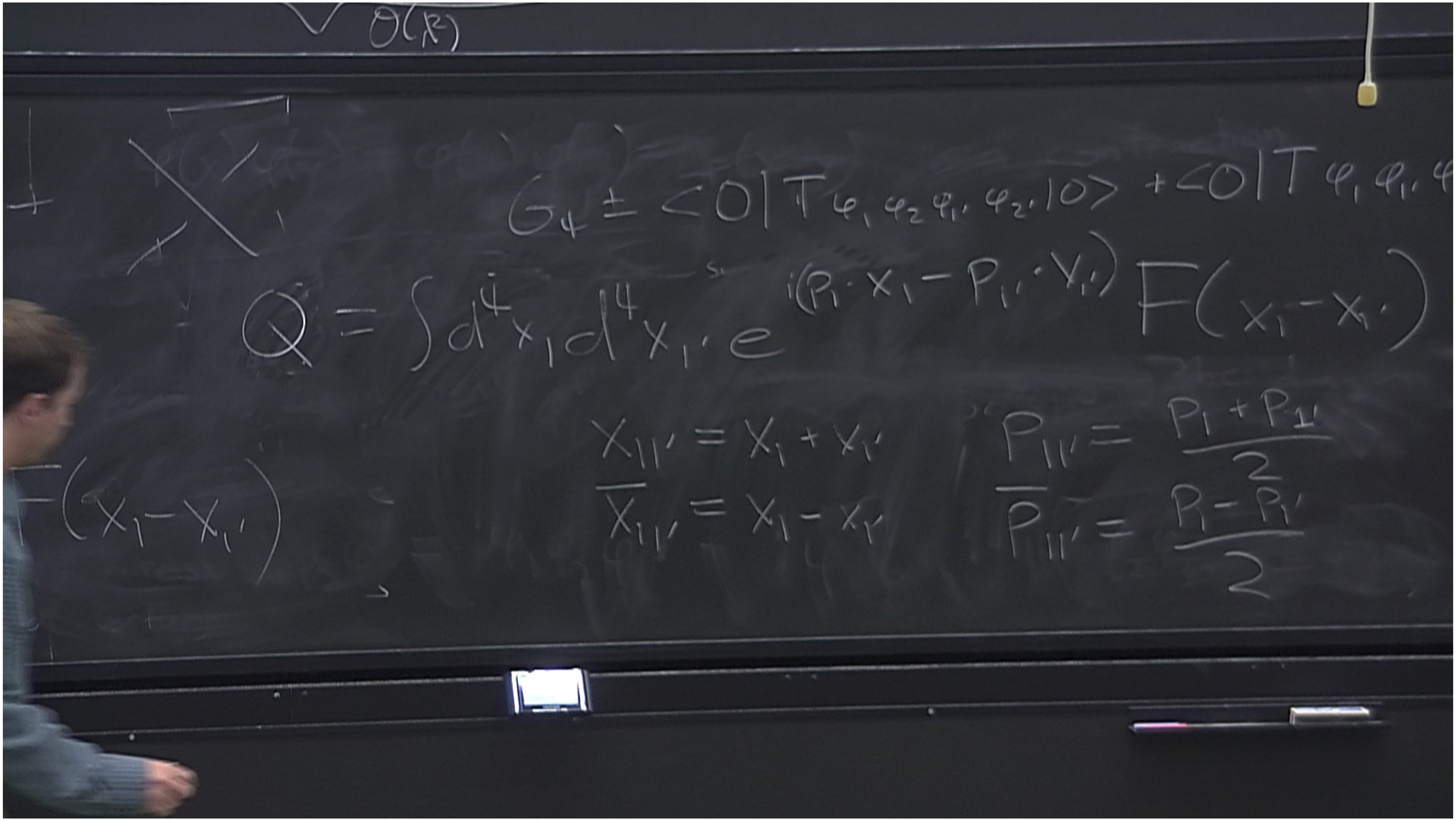

Connected
diagrams
w/ two external
points
num =
exp

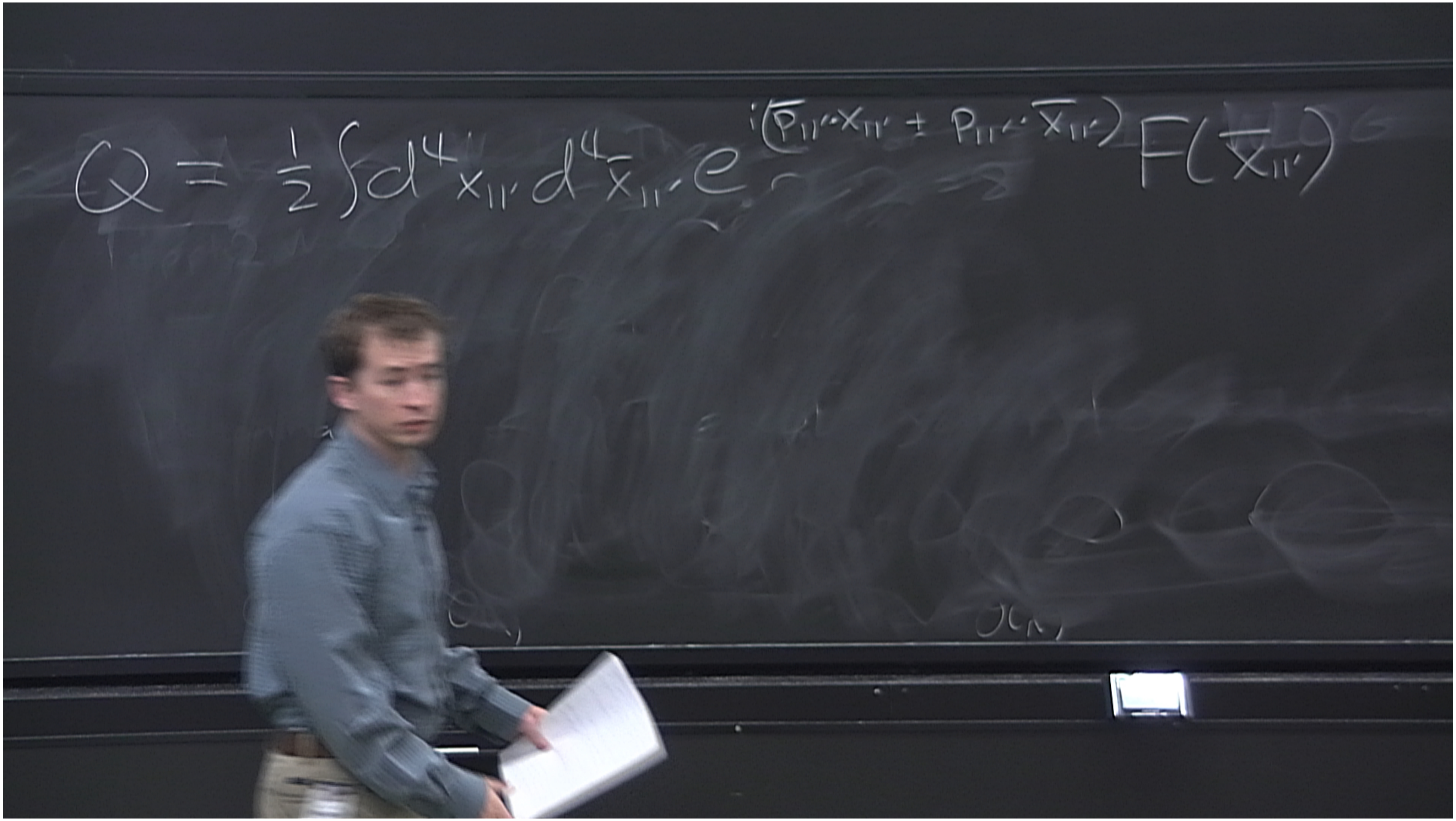

Scattering amplitude
2→2 scattering
<f|i>= <p1', p2' |
LSZ: <f|i>= i ∫ d4x1 d4x2 d4x1' d4x2
(∂1² + m²)(∂2² + m²)


l. tude
<f|i>= <p1', p2' | p1, p2>
i(p1·x1 + p2·x2 − p1'·x1'
d4x1 d4x2 d4x1' d4x2' e
(∂1² + m²)(∂2² + m²)(∂1'² + m²)(∂1'² + m²) <Ω|T
factor


$\langle \vec{p}_{1'}, \vec{p}_{2'} | \vec{p}_1, \vec{p}_2 \rangle$
$d^4 x_{1'} d^4 x_{2'} e^{i(p_1 \cdot x_1 + p_2 \cdot x_2 - p_{1'} \cdot x_{1'} - p_{2'} \cdot x_{2'})}$
$+m^2)(\partial_{1'}^2 + m^2)(\partial_{2'}^2 + m^2) \langle \Omega | T \varphi_1 \varphi_2 \varphi_{1'} \varphi_{2'} | \Omega \rangle$


G4 =
1'
2'
1
2
|| = Δ11' Δ22'
(∂1²+m²)(∂1'²+m²)Δ11' = F(x1−x1')
G4 ±


G4 ± <0|T φ1 φ2 φ1' φ2' |0> + <0|T φ1 φ1' φ
Q = ∫ d4x1 d4x1' e^(i(P1·x1 − P1'·x1')) F(x1 − x1')
X11' = X1 + X1'
P11' = (P1 + P1')/2
X̄11' = X1 − X1'
P̄11' = (P1 − P1')/2
F(X1 − X1')


$Q = \frac{1}{2}\int d^4x_{11'}\, d^4\bar{x}_{11'}\, e^{i(\bar{p}_{11'}\cdot x_{11'} \pm p_{11'}\cdot \bar{x}_{11'})} F(\bar{x}_{11'})$

$$Q = \frac{1}{2}\int d^4x_{11'}\, d^4\bar{x}_{11'}\, e^{i(\bar{p}_{11'}\cdot x_{11'} \pm p_{11'}\cdot \bar{x}_{11'})} F(\bar{x}_{11'})$$

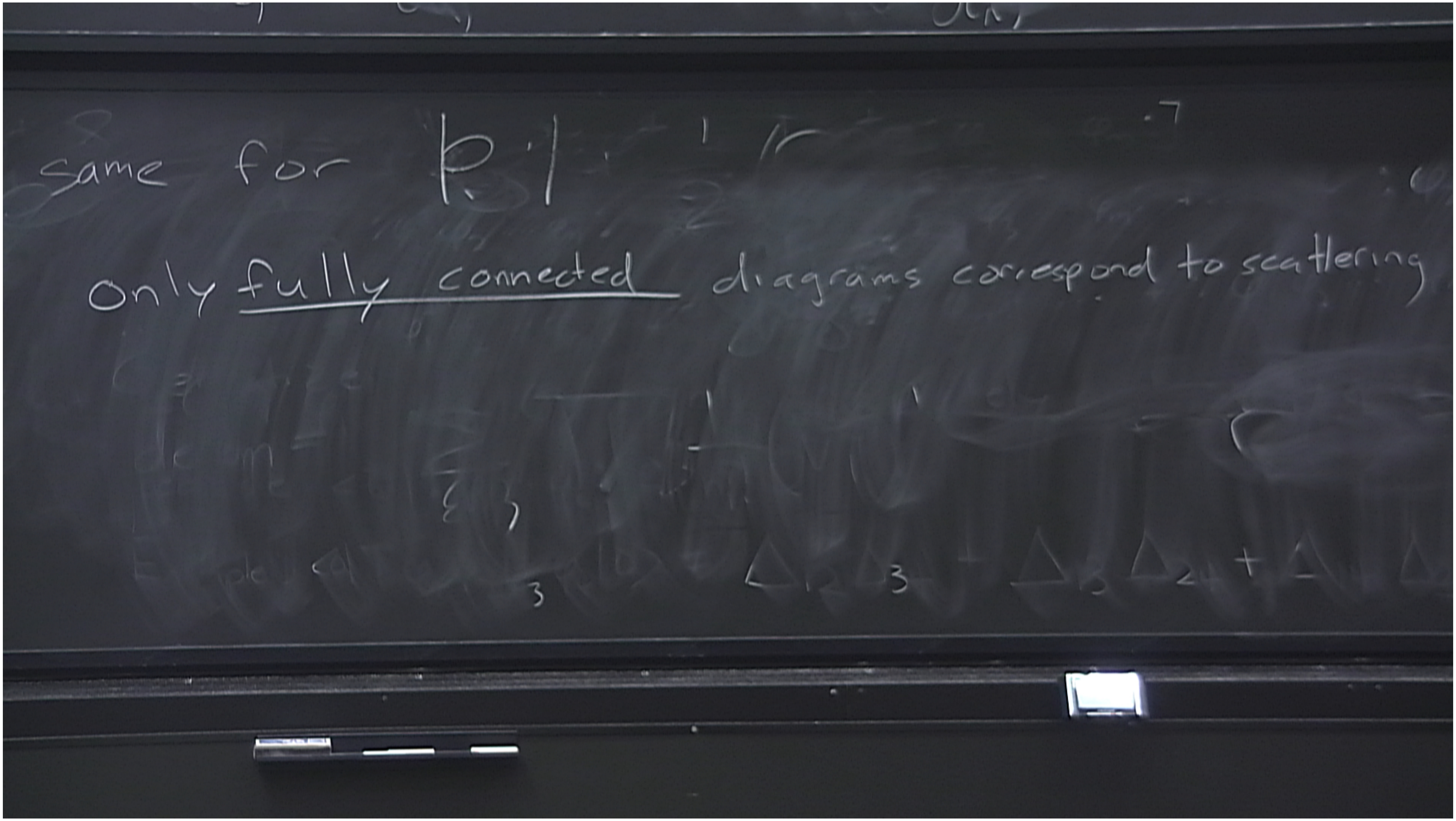
$$= \frac{1}{2}\int d^4x_{11'}\, e^{i p_{11'}\cdot x_{11'}}\, \tilde{F}(p_{11'})$$

$$= (2\pi)^4\, \delta^4(p_1 - p_{1'})\, \tilde{F}\left(\frac{p_1 + p_{1'}}{2}\right)$$

$$Q = \frac{1}{2}\int d^4x_{11'}\, d^4\bar{x}_{11'}\, e^{i(\bar{p}_{11'}\cdot x_{11'} + p_{11'}\cdot \bar{x}_{11'})} F(\bar{x}_{11'})$$

$$= \frac{1}{2}\int d^4 x_{11'}\, e^{i p_{11'}\cdot x_{11'}}\, \tilde{F}(p_{11'})$$

$$= (2\pi)^4\, \delta^4(p_1 - p_{1'})\, \tilde{F}\left(\frac{p_1 + p_{1'}}{2}\right)$$

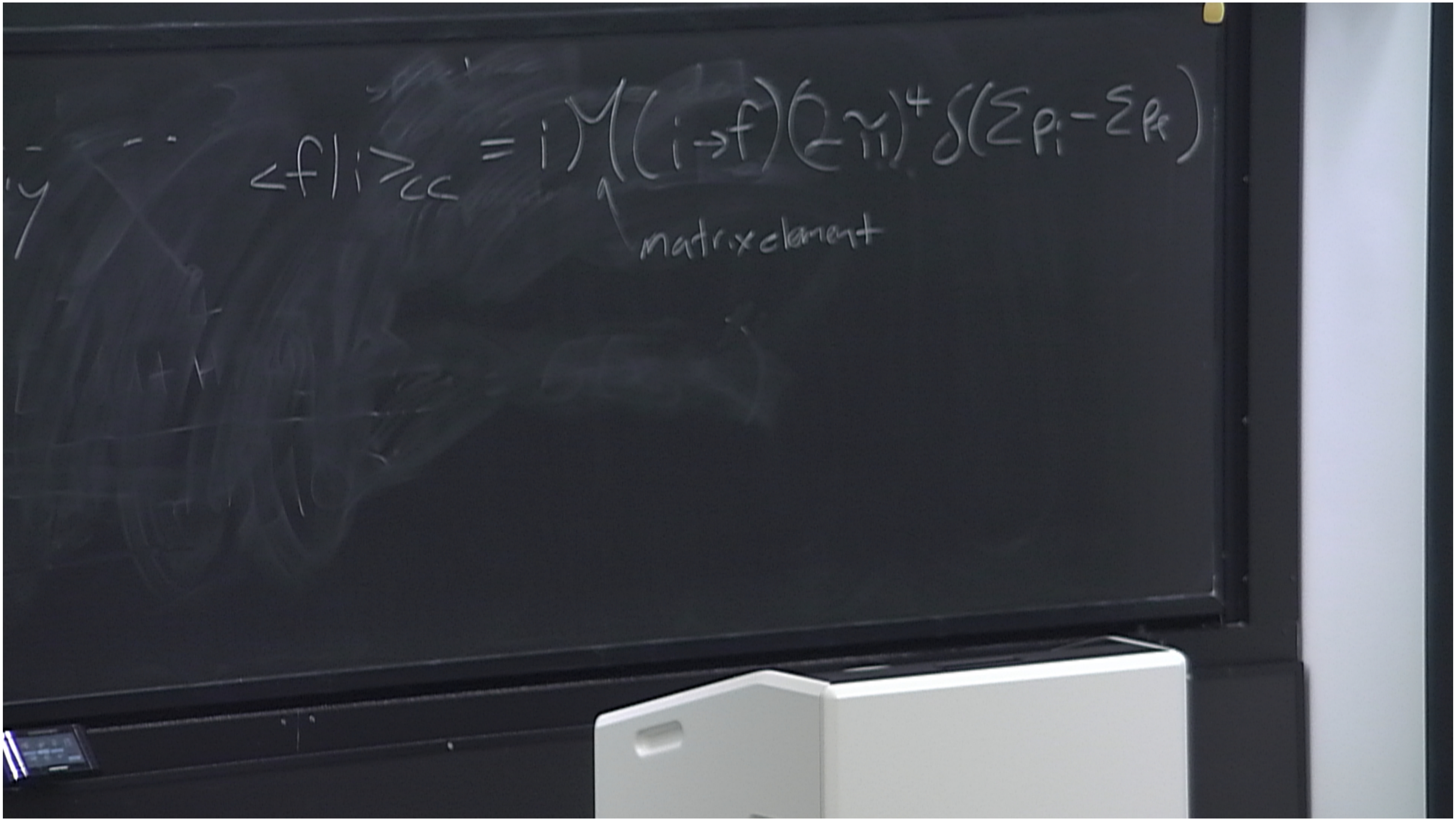

same for
only fully connected diagrams correspond to scattering

$$= -i\lambda \int d^4y \, \Delta_{1y} \Delta_{2y} \Delta_{1'y} \Delta_{2'y}$$

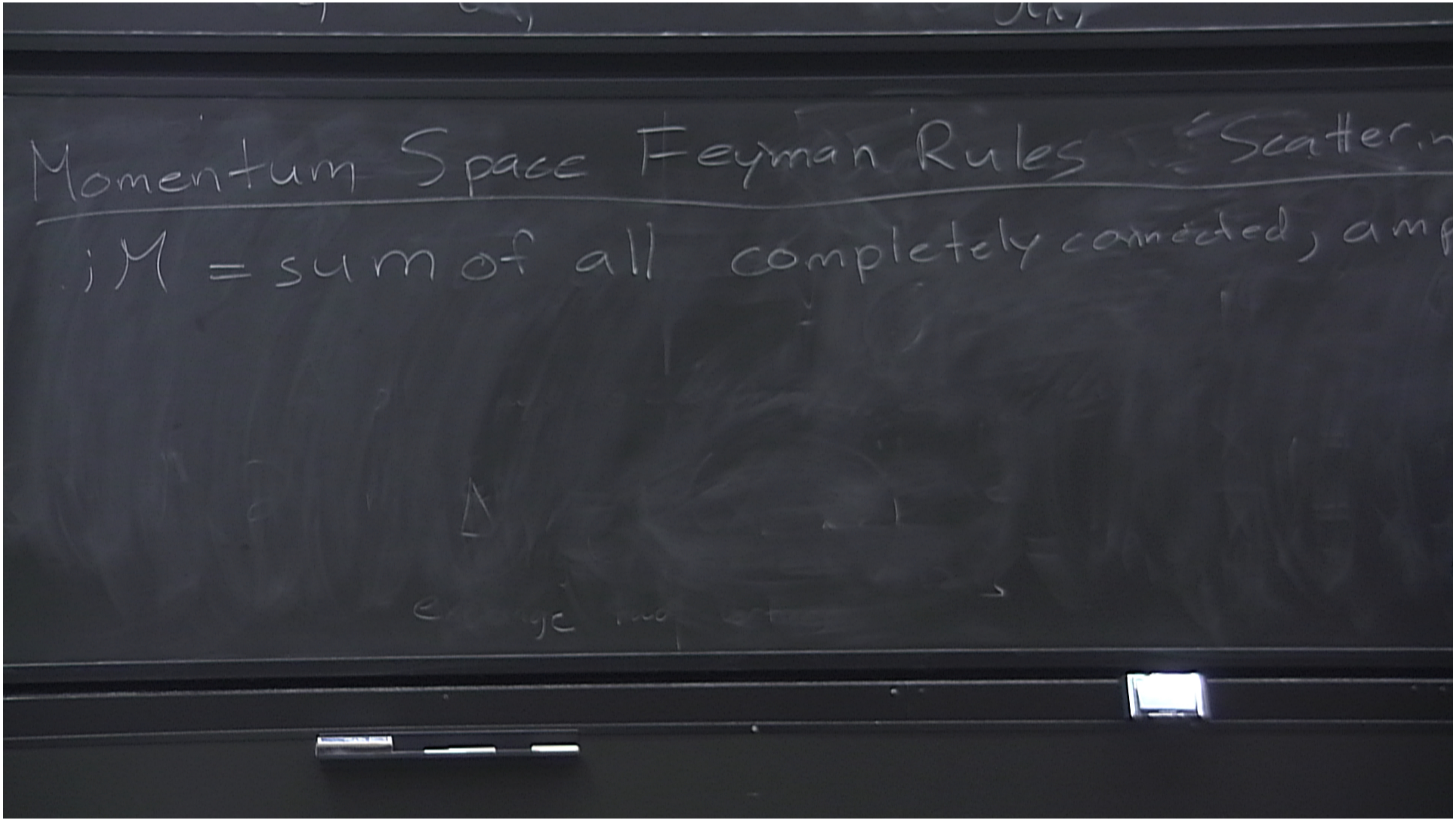
$$(\partial_1^2 + m^2)\Delta_{1y} = i\delta^4(x_1 - y)$$

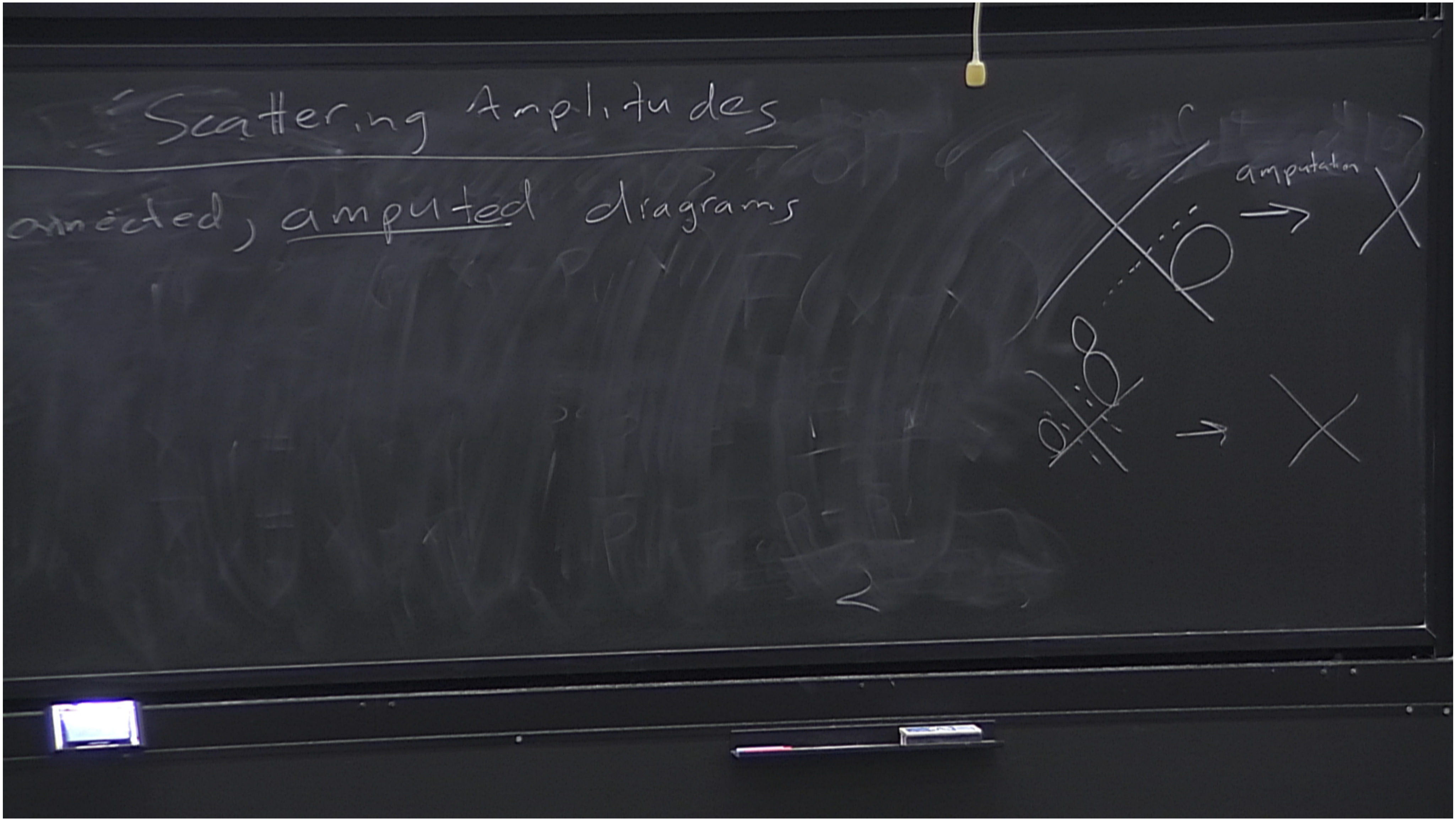
$$\langle f | i \rangle_x = -i\lambda \int d^4y \, e^{i(p_1 + p_2 - p_{1'} - p_{2'})\cdot y}$$

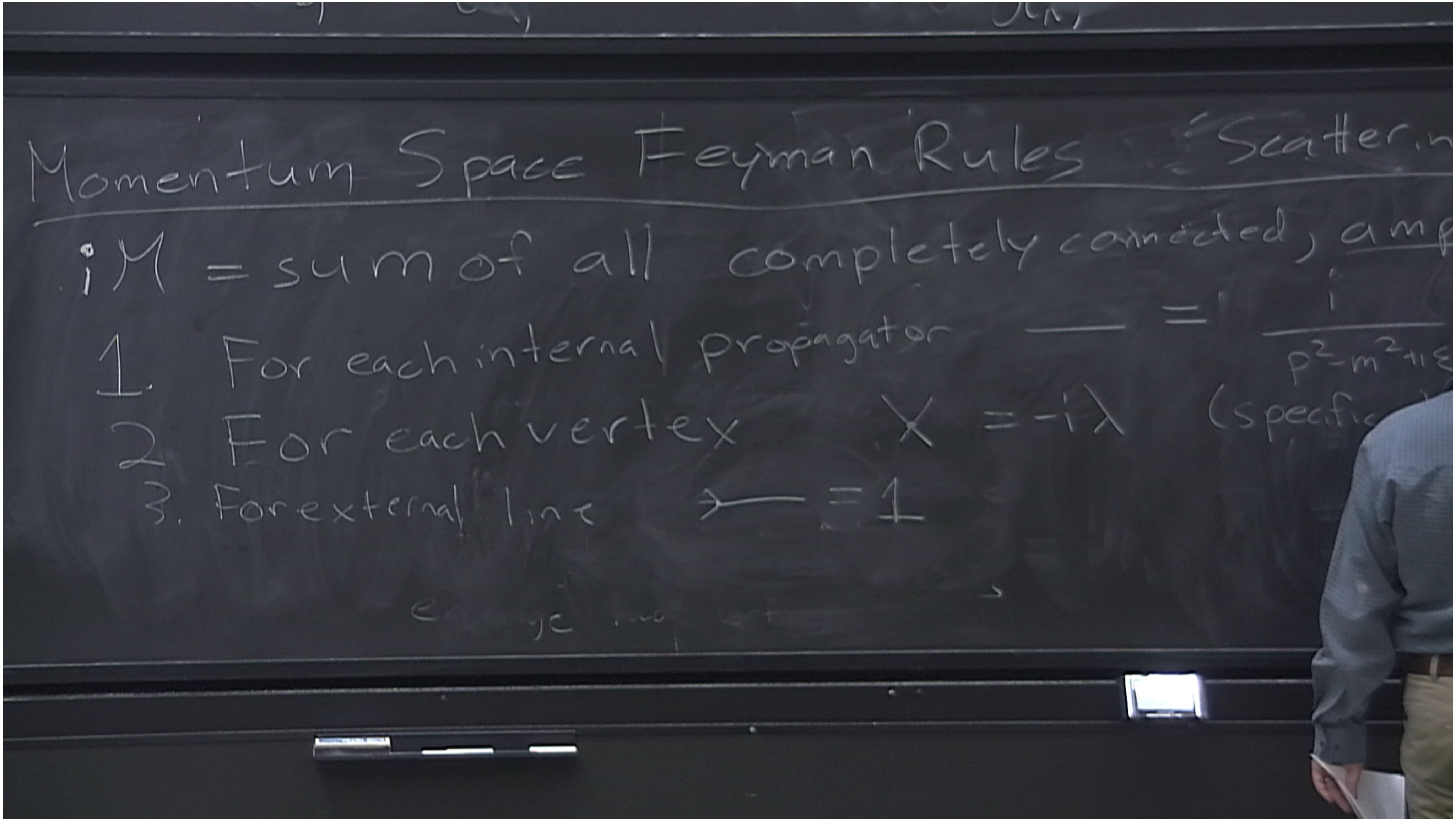
$$= -i\lambda (2\pi)^4 \delta^4(p_1 + p_2 - p_1' - p_2')$$


$\langle f | i \rangle_{cc} = i \mathcal{M}(i \to f)(2\pi)^4 \delta(\Sigma p_i - \Sigma p_f)$
matrix element


Momentum Space Feyman Rules


Momentum Space Feyman Rules
iM = sum of all completely connected,


Scattering Amplitudes
onnected, amputed diagrams
amputation



Momentum Space Feyman Rules
Scatter
iM = sum of all completely connected, amp
1. For each internal propagator = i / p²-m²+iε
2. For each vertex = -iλ (specific
3. For external line = 1

5. Divide by symmetry factor

6. Integrate over undermined momenta