

Title: 14/15 PSI - Quantum Field Theory I (A) - Lecture 4

Date: Oct 09, 2014 03:30 PM

URL: <http://pirsa.org/14100030>

Abstract:

$$\langle \Omega | T \varphi_1 \varphi_2 \dots \varphi_n | \Omega \rangle = G(x_1, \dots, x_n)$$

Interaction Picture

$$H = H_0 + H_{\text{int}}$$

$\uparrow$
 H_{KG}

$\uparrow$ e.g. $\int d^3x \frac{\lambda}{4!} \varphi^4(\vec{x})$

... x_n)

Schrödinger Picture:

$$i \frac{d}{dt} |\Psi\rangle_S = H |\Psi\rangle_S$$

Heisenberg Picture:

$$\begin{aligned} \mathcal{O}_H(t) &= e^{iHt} \mathcal{O}_S e^{-iHt} \\ &= U^\dagger(t, 0) \mathcal{O}_S U(t, 0) \end{aligned}$$

time evolution operator

$$|\Psi\rangle_H = e^{iHt} |\Psi\rangle_S = U^\dagger(t, 0) |\Psi\rangle_S$$

Interaction Picture

operators evolve with H_0

$$\mathcal{O}_I(t) =$$

$$| \Psi \rangle_S$$

$$\Theta_I(t) = e^{iH_0 t} \Theta_S e^{-iH_0 t}$$

$$| \Psi(t) \rangle_I = e^{iH_0 t} | \Psi(t) \rangle_S$$

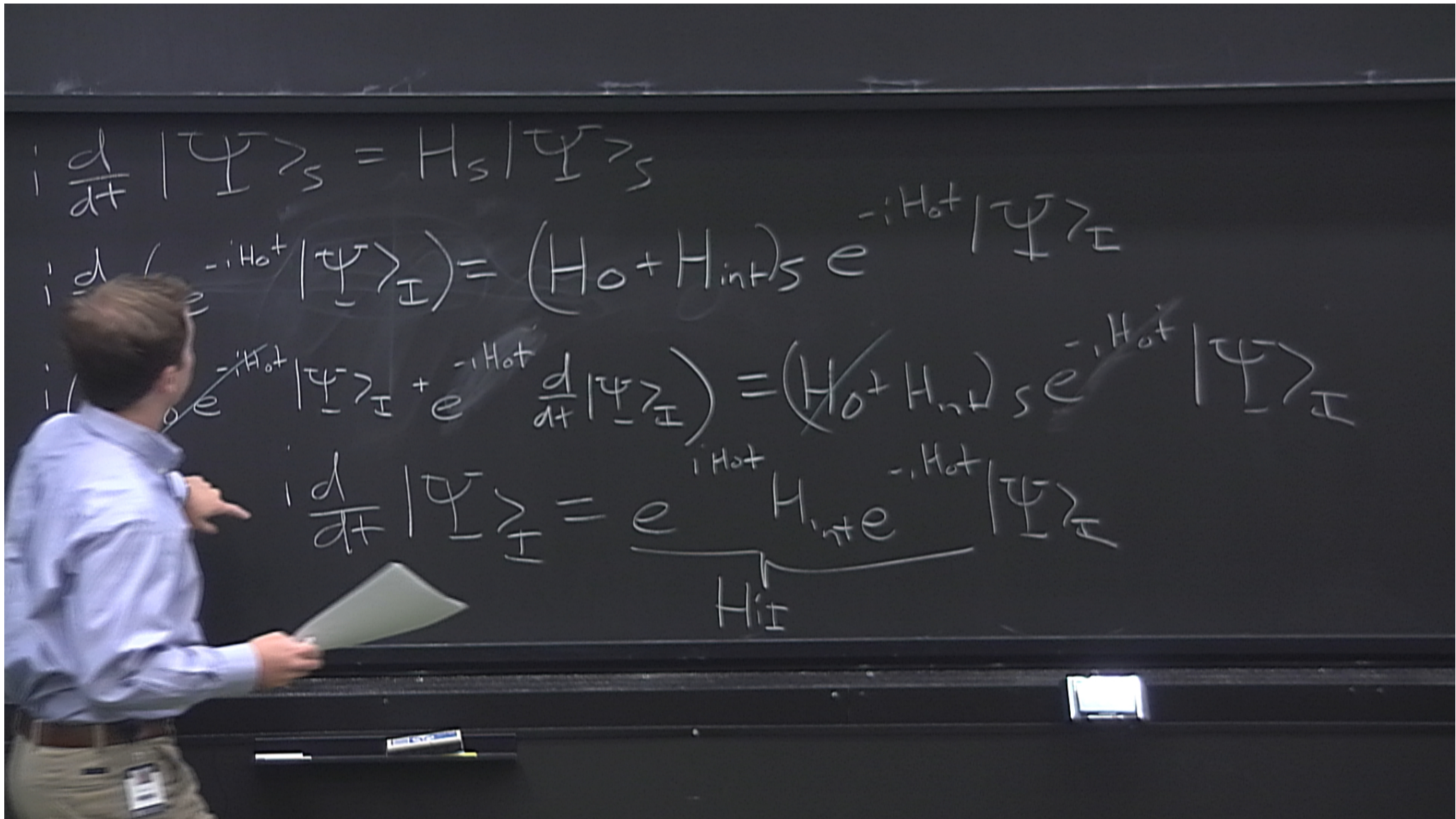
$$t, 0) |\Psi\rangle_S$$

H_0

H_{int}

$$O_I(t) = e^{iH_0 t} O_S e^{-iH_0 t}$$

$$|\Psi(t)\rangle_I = e^{iH_0 t} |\Psi(t)\rangle_S$$



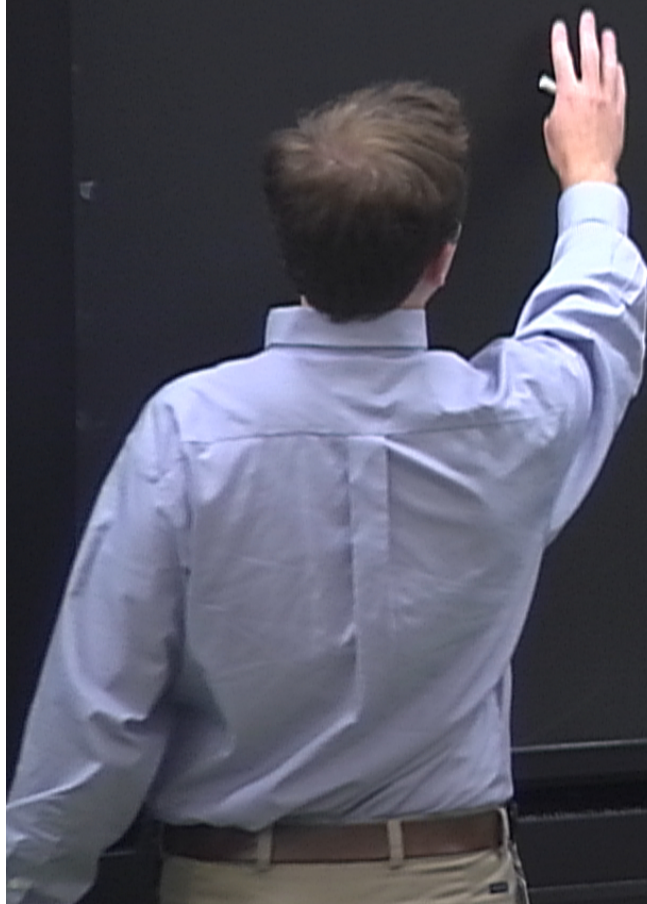
$$i \frac{d}{dt} |\Psi\rangle_S = H_S |\Psi\rangle_S$$

$$i \frac{d}{dt} (e^{-iH_0 t} |\Psi\rangle_I) = (H_0 + H_{int})_S e^{-iH_0 t} |\Psi\rangle_I$$

$$i \left(-iH_0 e^{-iH_0 t} |\Psi\rangle_I + e^{-iH_0 t} \frac{d}{dt} |\Psi\rangle_I \right) = (H_0 + H_{int})_S e^{-iH_0 t} |\Psi\rangle_I$$

$$i \frac{d}{dt} |\Psi\rangle_I = \underbrace{e^{iH_0 t} H_{int} e^{-iH_0 t}}_{H_I} |\Psi\rangle_I$$

$$i \frac{d}{dt} U_{\pm}(t, t_0) = H_{\pm}(t) U_{\pm}(t, t_0)$$



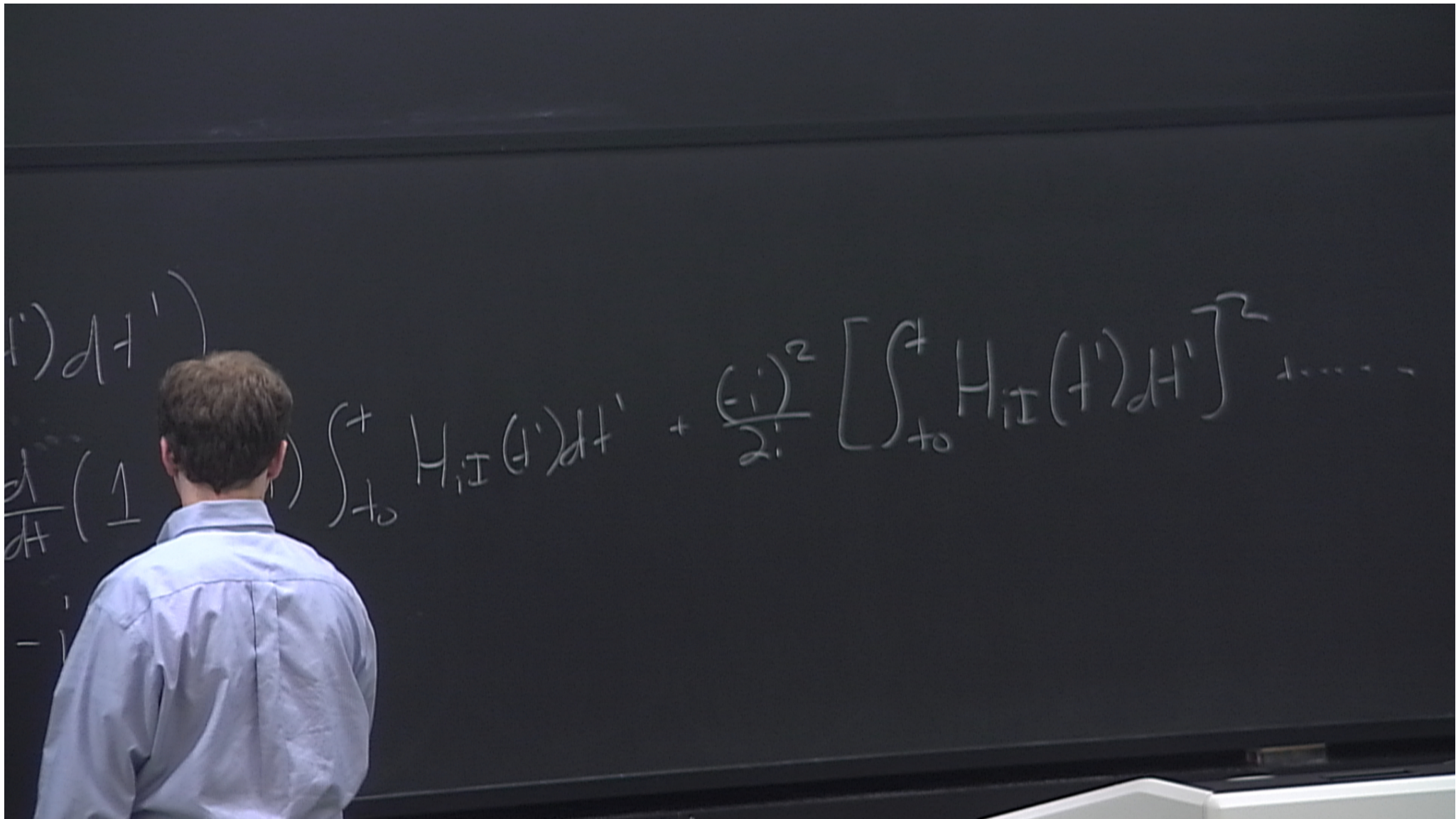
$$i \frac{d}{dt} U_{\pm}(t, t_0) = H_{\pm}(t) U_{\pm}(t, t_0)$$

$$U(t, t_0) \stackrel{?}{=} \exp\left(-i \int_{t_0}^t H_{\pm}(t') dt'\right)$$

$$i \frac{d}{dt} U_{\pm}(t, t_0) = H_{i\pm}(t) U_{\pm}(t, t_0)$$

$$U(t, t_0) \stackrel{?}{=} \exp\left(-i \int_{t_0}^t H_{i\pm}(t') dt'\right)$$

$$\frac{d}{dt} \left(\exp\left(-i \int_{t_0}^t H_{i\pm}(t') dt'\right) \right) = \frac{d}{dt} \left(1 + (-i) \int_{t_0}^t H_{i\pm}(t') dt' \right)$$



$1, t_0)$

$$H_{iI}(t') dt')$$

$$)) = \frac{d}{dt} \left(1 + (-i) \int_{t_0}^t H_{iI}(t') dt' + \frac{(-i)^2}{2!} \left[\int_{t_0}^t H_{iI}(t') dt' \right]^2 + \dots \right)$$

$$= -i H_{iI}(t) + \frac{(-i)^2}{2} \left(H_{iI}(t) \int_{t_0}^t H_{iI}(t') dt' + \int_{t_0}^t H_{iI}(t') H_{iI}(t) dt' \right) + \dots$$

Directly integrate:

$$U_I(t, t_0) = \underset{\uparrow}{\text{const}} - i \int_{t_0}^t H_I(t_1) U_I(t_1, t_0) dt_1$$

$$\int_{t_0}^t H_I(t_1) U_I(t_1, t_0) dt_1$$

because time evolving for 0 time does not change anything

Directly integrate:

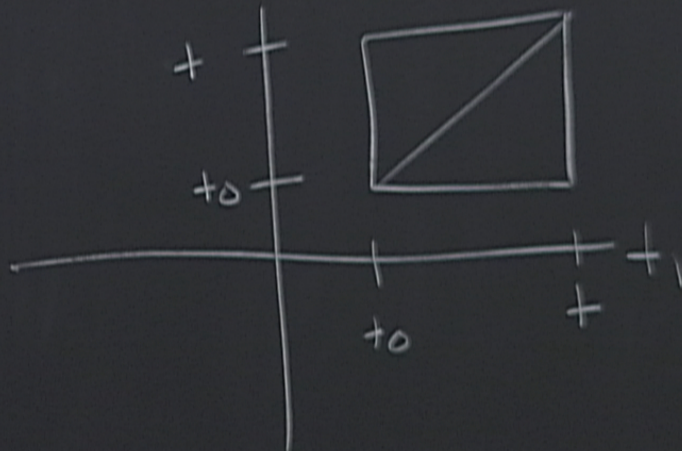
$$U_I(t, t_0) = \overset{1}{\text{const}} - i \int_{t_0}^t H_I(t_1) U_I(t_1, t_0) dt_1$$

$U(t, t) = 1$ because time evolving for 0

$$U_I(t, t_0) = \sum_{n=0}^{\infty} (-i)^n \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n$$

$$\int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_1} dt_2 T H_I(t_1) H_I(t_2) = \int$$

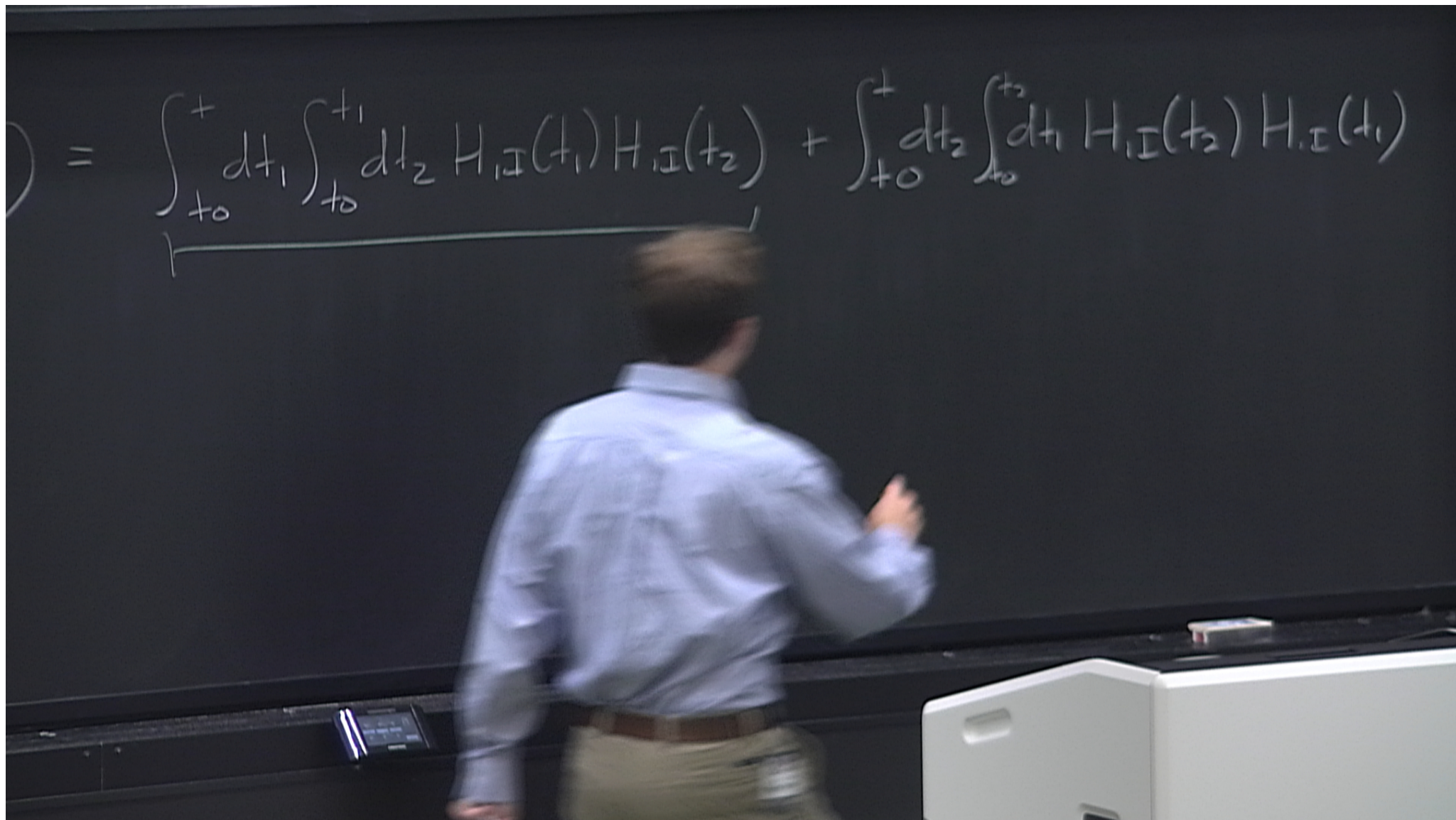
$$\int_{t_0}^{t_1} dt_1 \int_{t_0}^{t_2} dt_2 T H_I(t_1) H_I(t_2) = \int$$



$$I(t_1) H_{II}(t_2) = \int_{t_0}^{+} dt_1 \int_{t_0}^{t_1} dt_2 H_{II}(t_1) H_{II}(t_2)$$

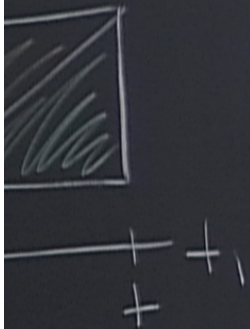
1
+ +
+ +

$$(t_2) = \underbrace{\int_{t_0}^{+} dt_1 \int_{t_0}^{t_1} dt_2 H_{I,I}(t_1) H_{I,I}(t_2)} + \int_{t_0}^{+} dt_2 \int_{t_0}^{t_2} dt_1 H_{I,I}(t_2) H_{I,I}(t_1)$$



$$H_I(t_1) H_I(t_2) = \underbrace{\int_{t_0}^{+} dt_1 \int_{t_0}^{t_1} dt_2 [H_I(t_1) H_I(t_2)]}_{\text{}} + \underbrace{\int_{t_0}^{+} dt_2 \int_{t_0}^{t_2} dt_1}_{\text{}}$$

$$U_I(t, t_0) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{t_0}^{+} dt_1 \int_{t_0}^{t_1} dt_2 \dots \int_{t_0}^{t_{n-1}} dt_n H_I(t_1) H_I(t_2) \dots H_I(t_n)$$



$$U_I(t_2) = \underbrace{\int_{t_0}^{+} dt_1 \int_{t_0}^{t_1} dt_2 \bar{H}_{I,I}(t_1) H_{I,I}(t_2)} + \underbrace{\int_{t_0}^{+} dt_2 \int_{t_0}^{t_2} dt_1 \bar{H}_{I,I}(t_2) H_{I,I}(t_1)}$$

$$U_I(t, t_0) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{t_0}^{+} dt_1 \cdots \int_{t_0}^{+} dt_n \bar{H}_{I,I}(t_1) \cdots H_{I,I}(t_n)$$

$$H_I(t_2) = \underbrace{\int_{t_0}^+ dt_1 \int_{t_0}^{t_1} dt_2 T H_I(t_1) H_I(t_2)} + \underbrace{\int_{t_0}^+ dt_2 \int_{t_0}^{t_2} dt_1 T H_I(t_2) H_I(t_1)}$$

$$U_I(t, t_0) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{t_0}^+ dt_1 \dots \int_{t_0}^+ dt_n T H_I(t_1) \dots H_I(t_n)$$

$$\boxed{U_I(t, t_0) = T \exp \left[-i \int_{t_0}^+ dt' H_I(t') \right]} \quad \text{Dyson's Formula}$$

Fields in interaction picture

$$\varphi(t_0, \vec{x}) = \int \frac{d^3p}{(2\pi)^2 E_p} \left(e^{i\vec{p}\cdot\vec{x}} + a_{\vec{p}} e^{-i\vec{p}\cdot\vec{x}} \right)$$

Fields in interaction picture

$$\varphi(t_0, \vec{x}) = \int \frac{d^3p}{(2\pi)^2 E_{\vec{p}}} \left(a_{\vec{p}} e^{i\vec{p} \cdot \vec{x}} + a_{\vec{p}}^\dagger e^{-i\vec{p} \cdot \vec{x}} \right)$$

$$\varphi_H(t, \vec{x}) =$$

Fields in interaction picture

$$\varphi(t_0, \vec{x}) = \int \frac{d^3p}{(2\pi)^2 E_p} (a_{\vec{p}} e^{i\vec{p} \cdot \vec{x}} + a_{\vec{p}}^\dagger e^{-i\vec{p} \cdot \vec{x}})$$
$$\varphi_H(t, \vec{x}) = e^{iH(t-t_0)} \varphi(t_0, \vec{x}) e^{-iH(t-t_0)}$$

Interaction picture

$$\int \frac{d^3p}{(2\pi)^3} \left(a_{\vec{p}} e^{i\vec{p}\cdot\vec{x}} + a_{\vec{p}}^\dagger e^{-i\vec{p}\cdot\vec{x}} \right)$$

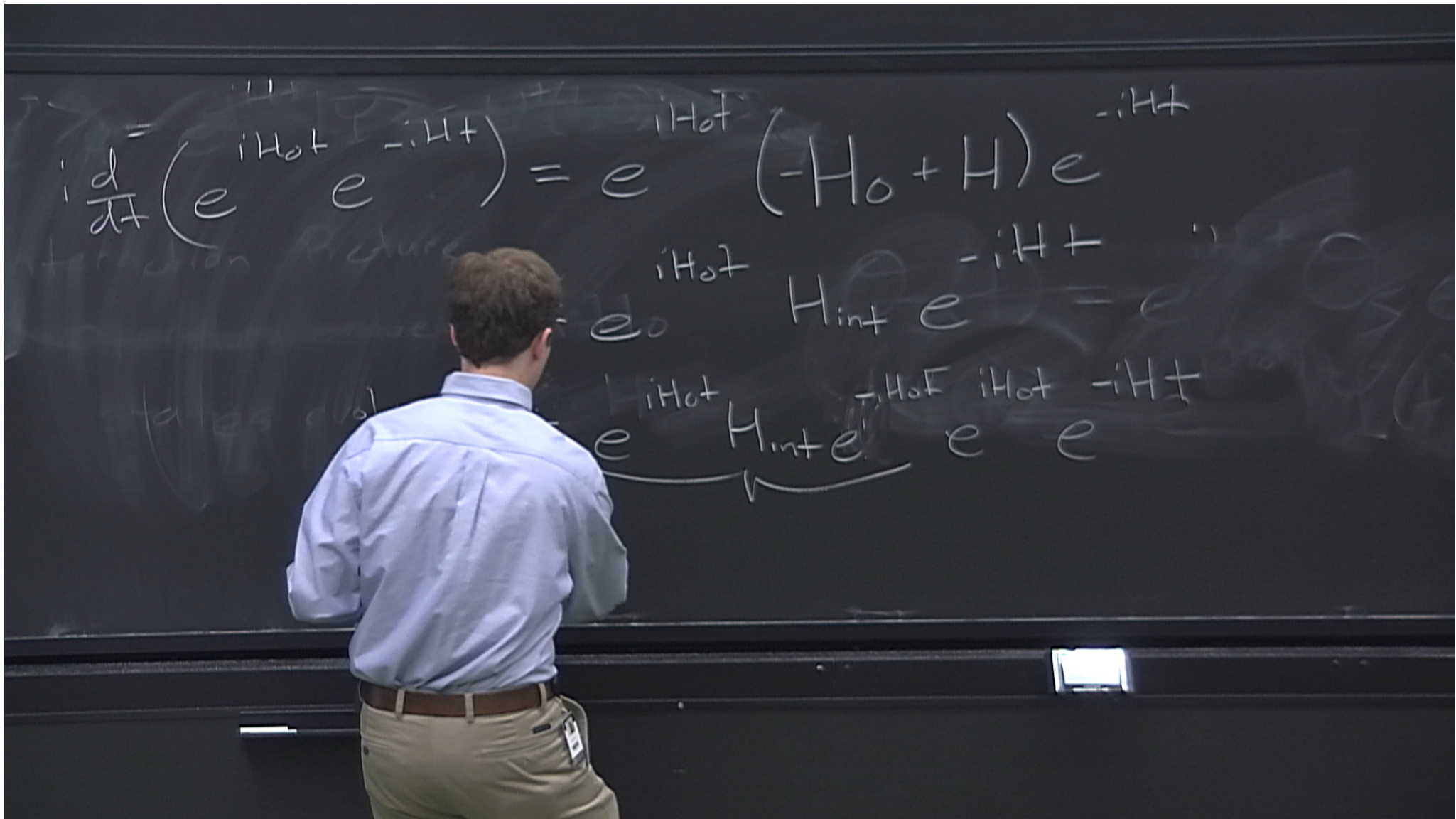
$$= e^{iH_0(t-t_0)} \varphi(t_0, \vec{x}) e^{-iH(t-t_0)}$$

$$= e^{iH_0\Delta t} \varphi(t_0, \vec{x}) e^{-iH\Delta t}$$

Schrödinger picture

$$\frac{d}{dt} \psi = -iH\psi$$

$$\psi(t) = e^{-iH(t-t_0)} \psi(t_0)$$



$$\begin{aligned}
 i \frac{d}{dt} \left(e^{iH_0 t} e^{-iH t} \right) &= e^{iH_0 t} (-H_0 + H) e^{-iH t} \\
 &= e^{iH_0 t} H_{int} e^{-iH t} \\
 &= e^{iH_0 t} H_{int} e^{-iH_0 t} e^{iH_0 t} e^{-iH t} \\
 &= H_{I,I} \cup I
 \end{aligned}$$

$$i \frac{d}{dt} |\Psi\rangle_S = H_S |\Psi\rangle_S$$

$$i \frac{d}{dt} (e^{-iH_0 t} |\Psi\rangle_I) = (H_0 + H_{int})_S e^{-iH_0 t} |\Psi\rangle_I$$

$$i \left(\cancel{-iH_0} e^{-iH_0 t} |\Psi\rangle_I + e^{-iH_0 t} \frac{d}{dt} |\Psi\rangle_I \right) = (\cancel{H_0} + H_{int})_S e^{-iH_0 t} |\Psi\rangle_I$$

$$i \frac{d}{dt} |\Psi\rangle_I = \underbrace{e^{iH_0 t} H_{int} e^{-iH_0 t}}_{H_I} |\Psi\rangle_I$$

$$i \frac{d}{dt} |\Psi\rangle_S = H_S |\Psi\rangle_S$$

$$i \frac{d}{dt} (e^{-iH_0 t} |\Psi\rangle_I) = (H_0 + H_{int})_S e^{-iH_0 t} |\Psi\rangle_I$$

$$i \left(-iH_0 e^{-iH_0 t} |\Psi\rangle_I + e^{-iH_0 t} \frac{d}{dt} |\Psi\rangle_I \right) = (H_0 + H_{int})_S e^{-iH_0 t} |\Psi\rangle_I$$

$$i \frac{d}{dt} |\Psi\rangle_I = \underbrace{e^{iH_0 t} H_{int} e^{-iH_0 t}}_{H_I} |\Psi\rangle_I$$

$$i \frac{d}{dt} |\Psi\rangle_S = H_S |\Psi\rangle_S$$

$$i \frac{d}{dt} (e^{-iH_0 t} |\Psi\rangle_I) = (H_0 + H_{int})_S e^{-iH_0 t} |\Psi\rangle_I$$

$$i \left(-iH_0 e^{-iH_0 t} |\Psi\rangle_I + e^{-iH_0 t} \frac{d}{dt} |\Psi\rangle_I \right) = (H_0 + H_{int})_S e^{-iH_0 t} |\Psi\rangle_I$$

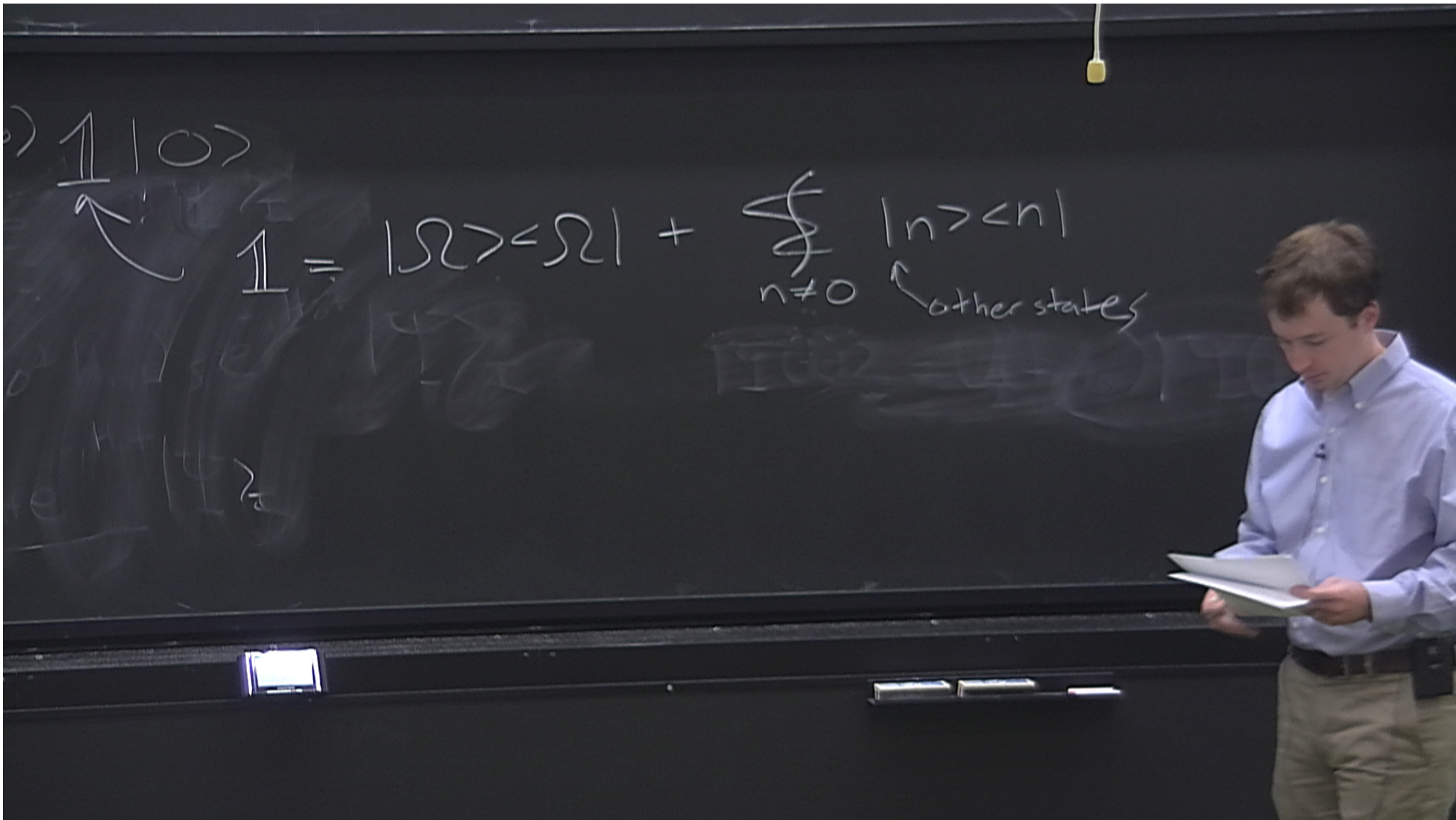
$$i \frac{d}{dt} |\Psi\rangle_I = \underbrace{e^{iH_0 t} H_{int} e^{-iH_0 t}}_{H_I} |\Psi\rangle_I$$

Interacting vacuum:

$$e^{iH(T+t_0)}|0\rangle = e^{iH(T+t_0)}\mathbb{1}|0\rangle$$

$$\mathbb{1} = 15$$

$\underline{1} | 0 \rangle$
 $\underline{1} = |\Omega\rangle\langle\Omega| + \sum_{n \neq 0} |n\rangle\langle n|$
 other states



vacuum:

$$|0\rangle = e^{-iH(T+t_0)} \mathbb{1} |0\rangle$$

$$\mathbb{1} = |\Omega\rangle\langle\Omega| + \sum_{n \neq 0} |n\rangle\langle n|$$

$$= e^{-iE_0(T+t_0)} |\Omega\rangle\langle\Omega| + \sum_{n \neq 0} e^{-iE_n(T+t_0)} |n\rangle\langle n|$$

$H|1\rangle$

vacuum:

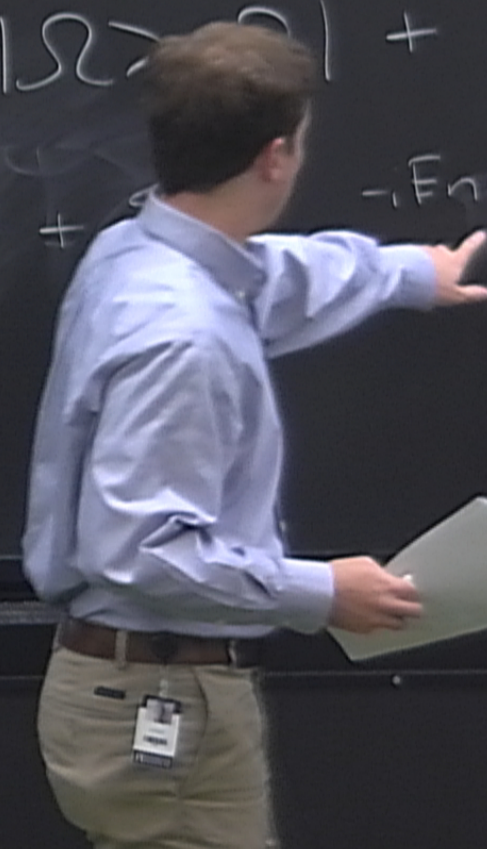
$$|0\rangle = e^{-iH(T+t_0)} \mathbb{1} |0\rangle$$

$$\mathbb{1} = |0\rangle\langle 0| + \sum_{n \neq 0} |n\rangle\langle n|$$

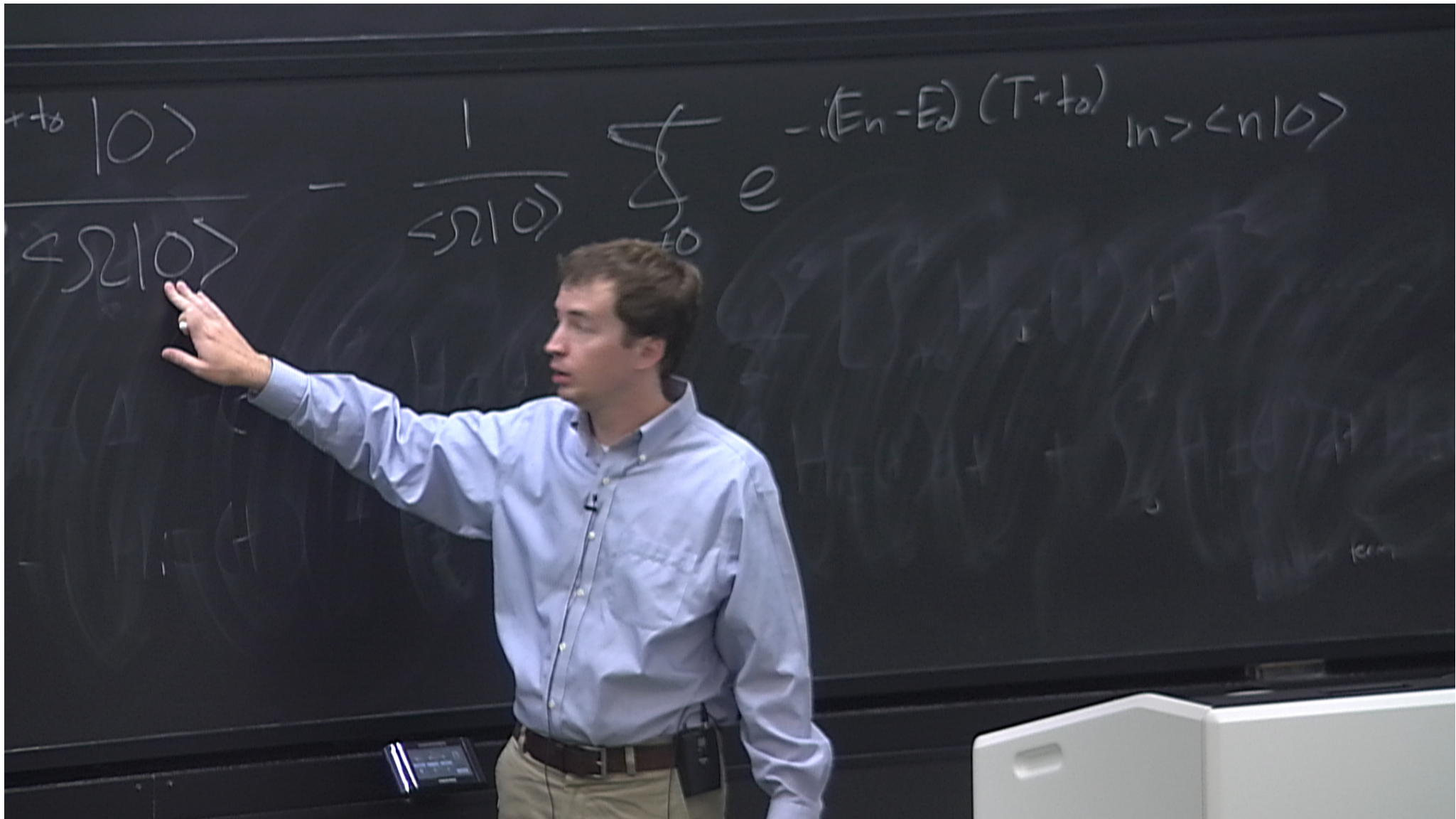
$$= e^{-iE_0(T+t_0)} |0\rangle\langle 0| + \sum_{n \neq 0} e^{-iE_n(T+t_0)} |n\rangle\langle n|$$

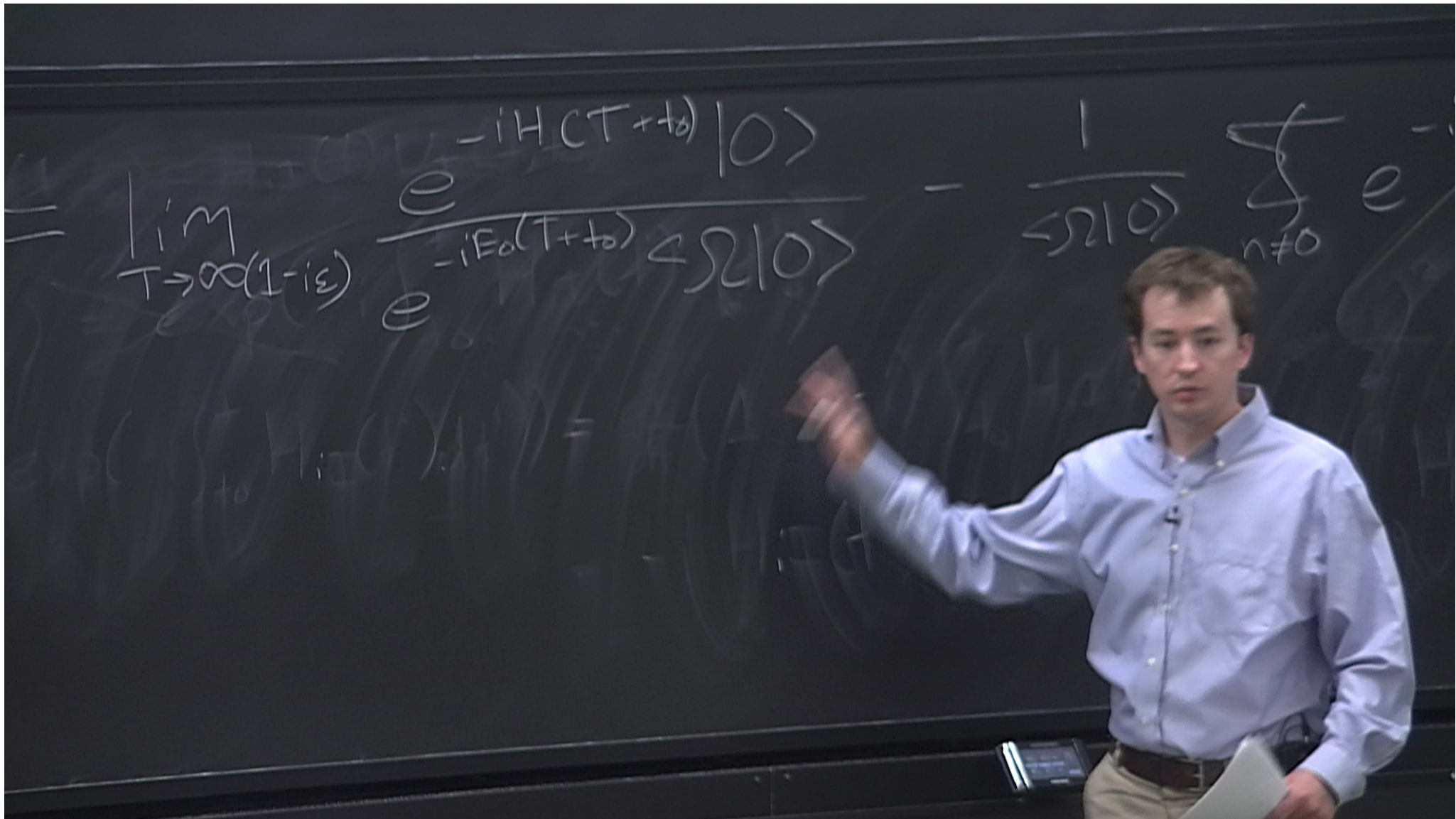
$$E_0 = \langle 0 | H | 0 \rangle$$

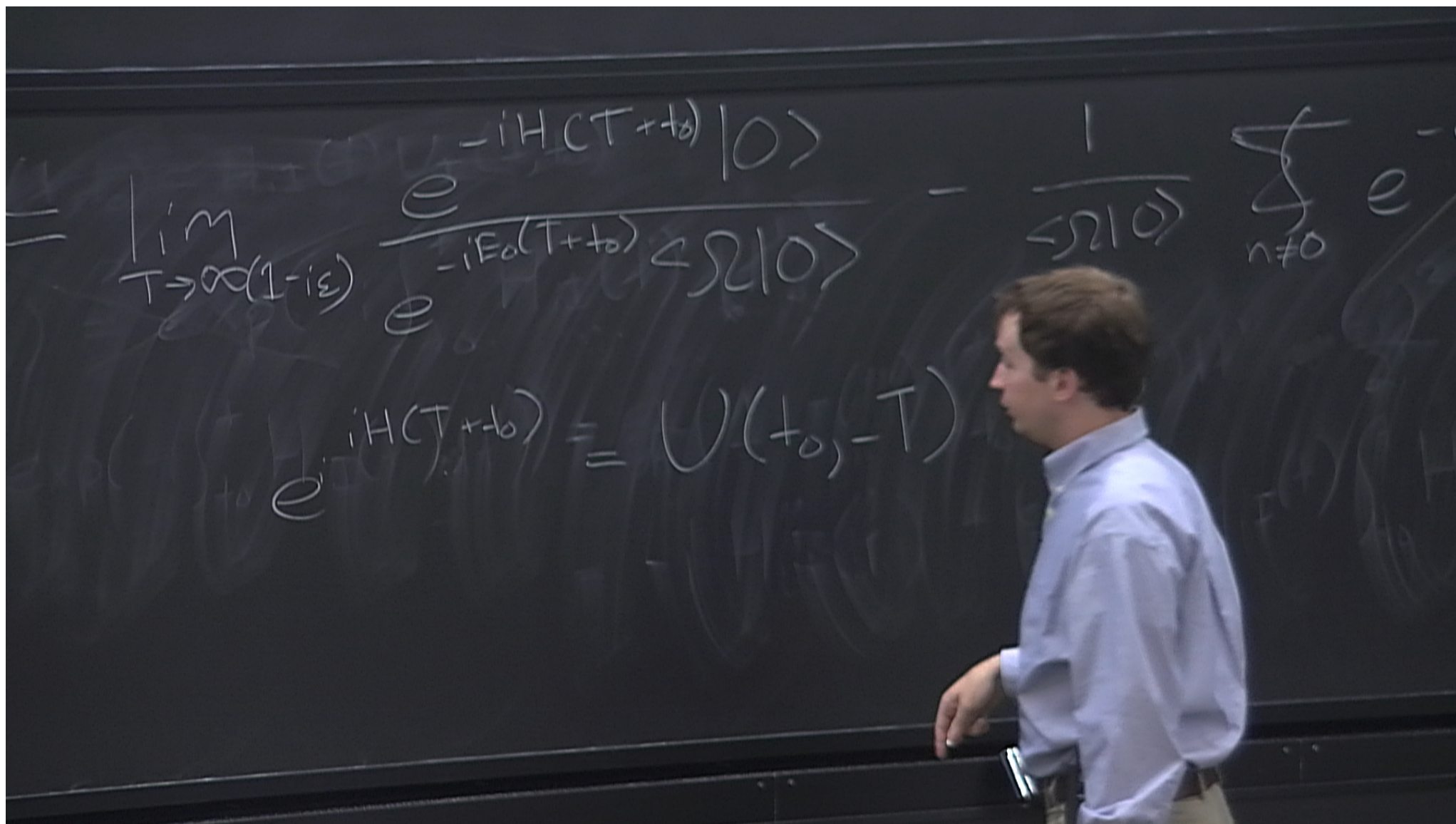
$$H_0 |0\rangle = 0$$



$$|\Omega\rangle = \frac{e^{-iH(T+t_0)}|0\rangle}{e^{-iE_0(T+t_0)}\langle\Omega|0\rangle}$$







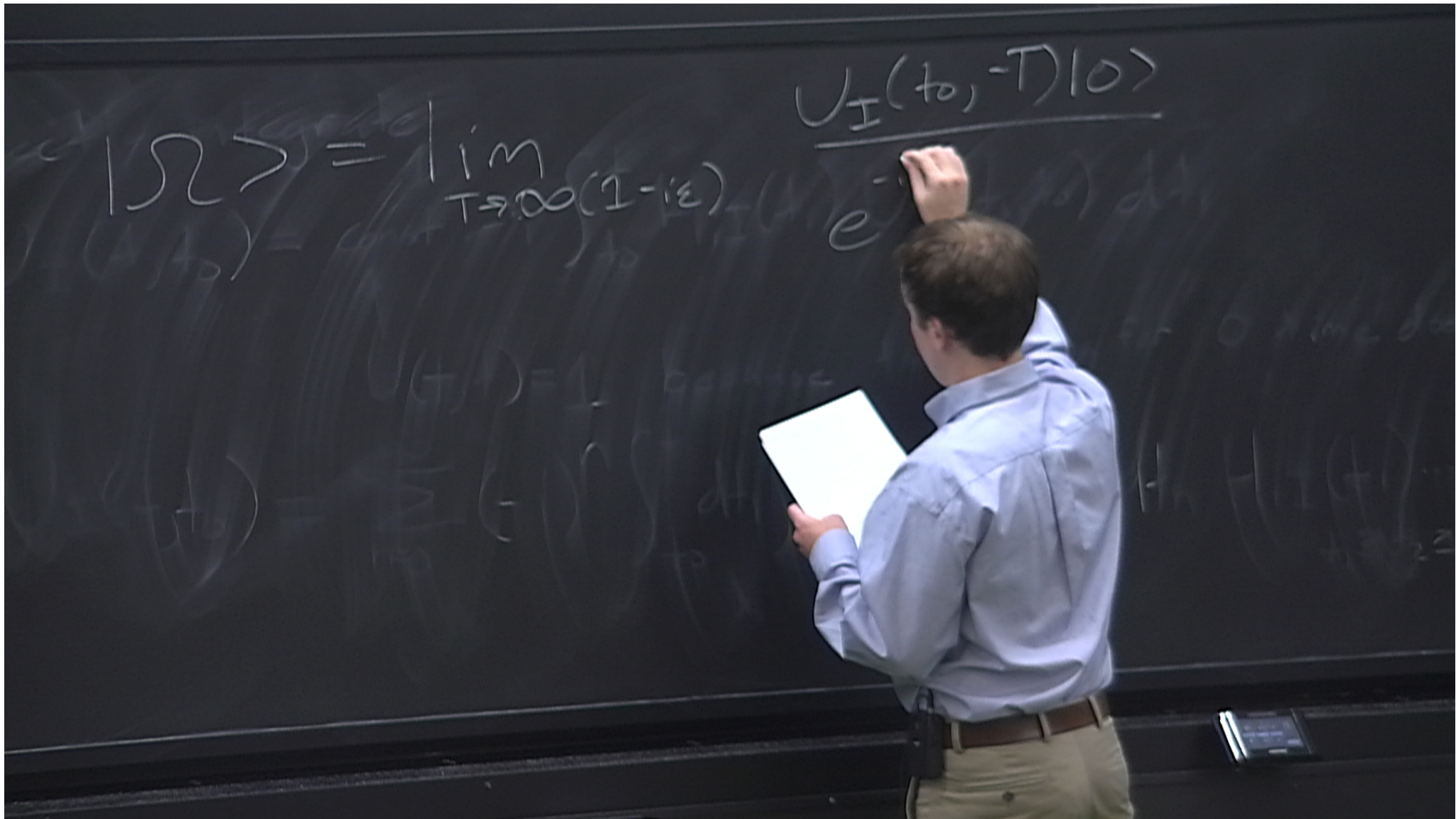
$$= \lim_{T \rightarrow \infty (1-i\epsilon)} \frac{e^{-iH(T+t_0)} |0\rangle}{e^{-iE_0(T+t_0)} \langle 0|0\rangle} - \frac{1}{\langle 0|0\rangle} \sum_{n \neq 0} e^{-iE_n(T+t_0)} \langle 0|n\rangle \langle n|0\rangle$$

$$e^{-iH(T+t_0)} = U(t_0, -T)$$

$$H_0 |0\rangle = 0$$

$$e^{-iH_0 T} |0\rangle = |0\rangle$$

$$U(t_0, T) |0\rangle =$$



$$|\Omega\rangle = \lim_{T \rightarrow \infty} (1 - i\varepsilon)$$

$$\frac{U_I(t_0, -T)|0\rangle}{e^{-iH_I(t_0, -T)}}$$

$$|\Omega\rangle = \lim_{T \rightarrow \infty} (1 - i\varepsilon)$$

$$\langle \Omega | = \lim_{T \rightarrow \infty} (1 - i\varepsilon)$$

$$\frac{U_I(t_0, -T)|0\rangle}{e^{-iE_0(T+t_0)}\langle \Omega|0\rangle}$$

$$\frac{\langle 0|U_I(T, t_0)}{e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle}$$

Example $G(x,y) = \langle \Sigma \mid T, \varphi(x) \varphi(y) \mid \Sigma \rangle$

$$G(x,y) =$$

$$x^0 > y^0 > t_0$$

Example $G(x,y) = \langle \Omega | T, \phi(x) \phi(y) | \Omega \rangle$

$$G(x,y) = \lim_{T \rightarrow \infty (1-i\epsilon)} \frac{e^{-iE_0(T+t_0)} \langle \Omega | 0 \rangle}{e}$$

$$G(x,y) = \langle \Omega | T, \phi(x) \phi(y) | \Omega \rangle \quad x^0 > y^0 > t_0$$

$$= \lim_{T \rightarrow \infty (1-i\epsilon)} \frac{e^{-iE_0(T+t_0)} \langle \Omega | 0 \rangle}{e^{-iE_0(1-t_0)} \langle 0 | \Omega \rangle}$$

$$x^0 > y^0 > t_0$$

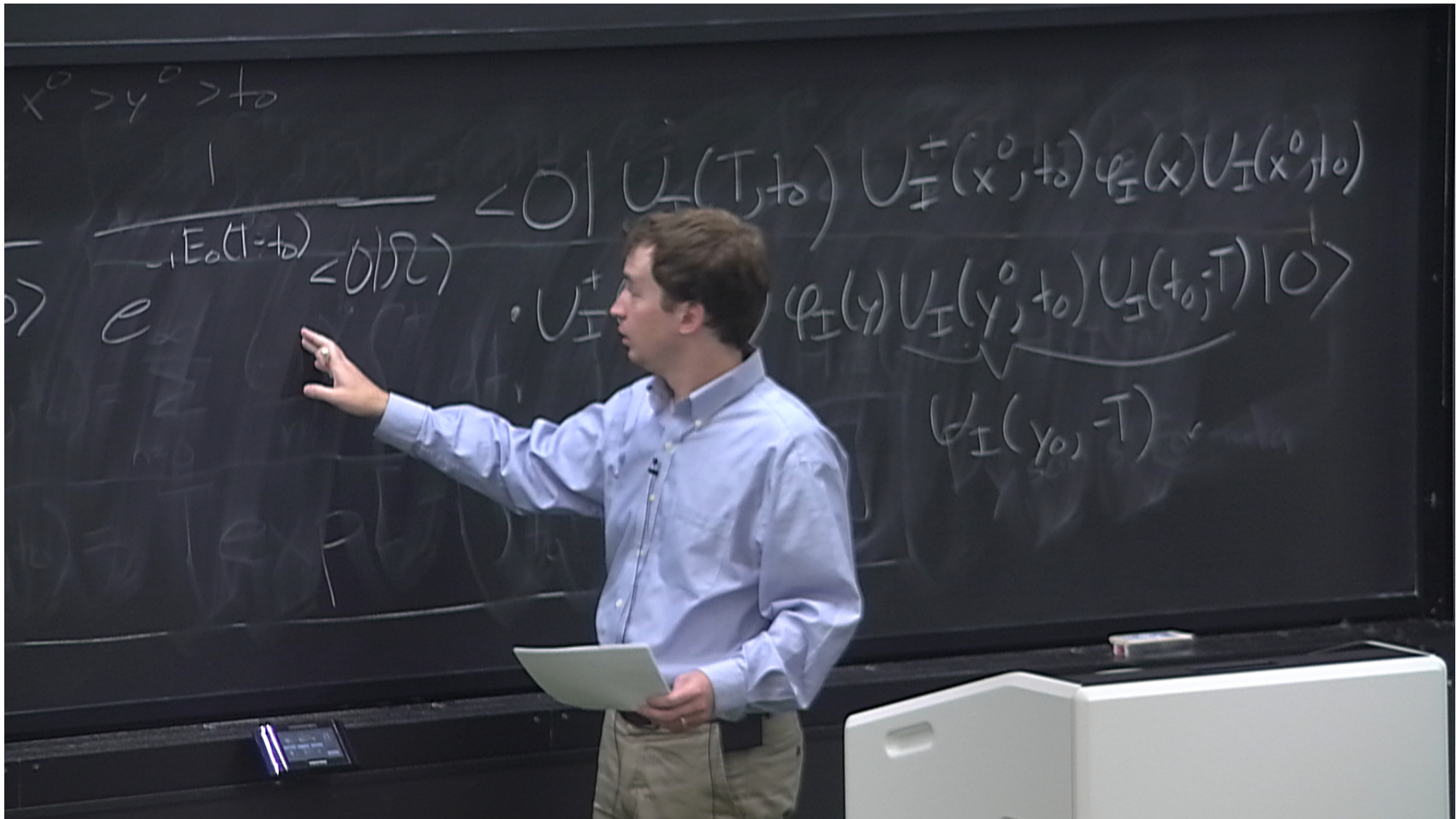
$$\langle \Omega | 0 \rangle = \frac{1}{e^{-iE_0(t-t_0)} \langle 0 | \Omega \rangle} \langle 0 | U_I(T, t_0)$$

$$U_I(t, t_0) = T \exp \left(-i \int_{t_0}^t H_I(t') dt' \right)$$

Example $G(x,y) = \langle \Omega | T \phi(x) \phi(y) | \Omega \rangle$

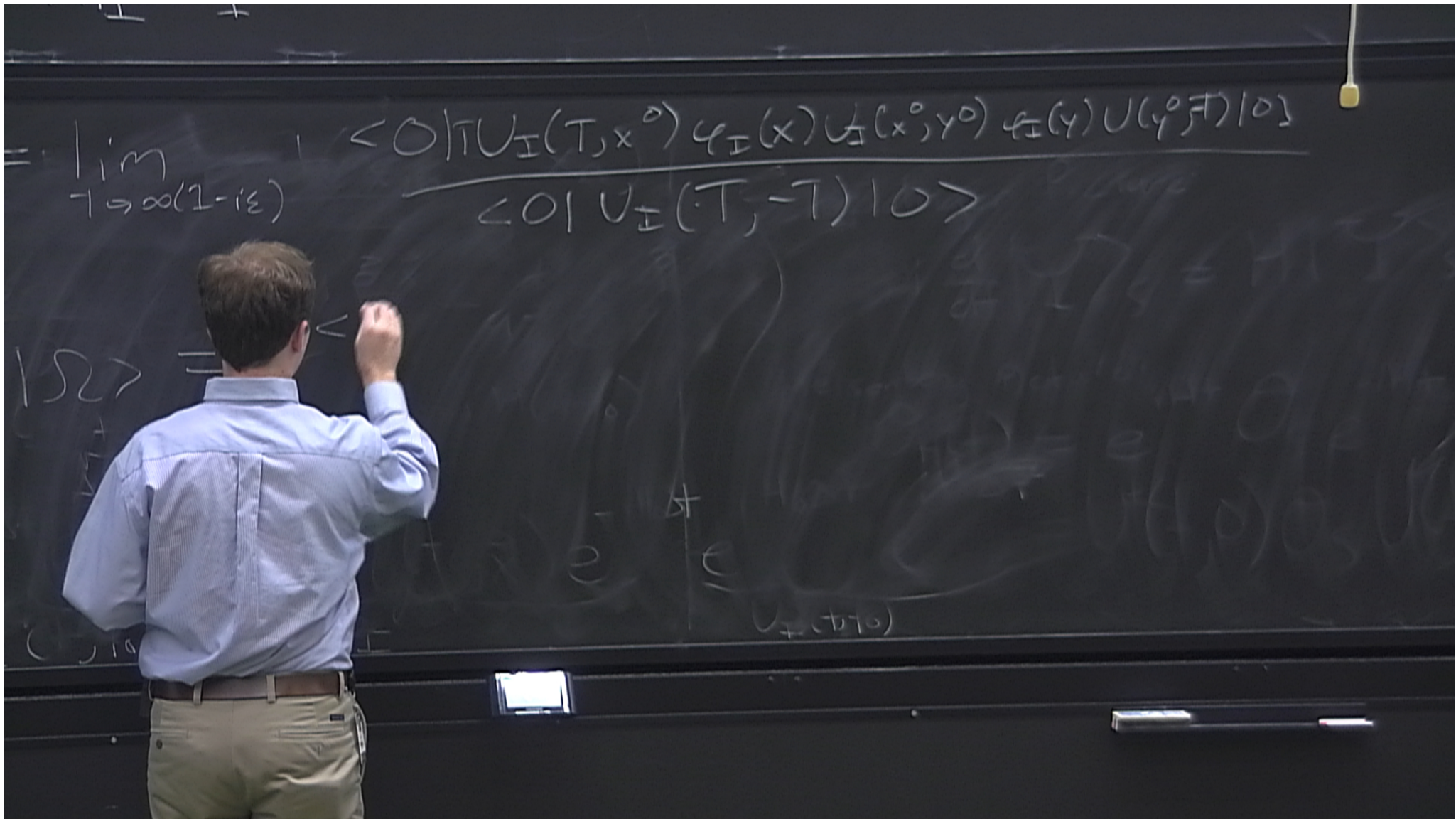
$$G(x,y) = \lim_{T \rightarrow \infty (1-i\epsilon)} \frac{e^{-iE_0(T+t_0)} \langle \Omega | 0 \rangle}{e^{-iE_0(1-t_0)}}$$

$$\phi_H = U_I^\dagger \phi_\pm U_I$$



$$\begin{aligned}
 & x^0 > y^0 > t_0 \\
 & \frac{1}{e^{-iE_0(t-t_0)} \langle 0 | \Omega \rangle} \langle 0 | U_I(T, t_0) U_I^\dagger(x^0, t_0) \varphi_I(x) U_I(x^0, t_0) \\
 & \cdot U_I^\dagger(y^0, t_0) \varphi_I(y) U_I(y^0, t_0) U_I(t_0, -T) | 0 \rangle \\
 & \quad \quad \quad \underbrace{U_I(y^0, -T)}_{\text{or } U_I(y_0, -T)}
 \end{aligned}$$

$$\langle \Omega | T \varphi(x) \varphi(y) | \Omega \rangle = \lim_{T \rightarrow \infty (1-i\epsilon)} \langle 0 | U_I(T, x)$$



$$= \lim_{T \rightarrow \infty (1-i\epsilon)} \frac{\langle 0 | T U_I(T, x^0) \psi_I(x) U_I(x^0, y^0) \psi_I(y) U_I(y^0, T) | 0 \rangle}{\langle 0 | U_I(-T, -T) | 0 \rangle}$$

$$|\Omega\rangle =$$

$$\begin{aligned}
 & \frac{\langle 0 | T U_I(T, x^0) \varphi_I(x) U_I(x^0, y^0) \varphi_I(y) U_I(y^0, T) | 0 \rangle}{\langle 0 | U_I(T, -T) | 0 \rangle} \\
 &= \frac{\langle 0 | T \varphi_I(x) \varphi_I(y) \exp \left[-i \int_{-\infty}^{\infty} dt H_I(t) \right] | 0 \rangle}{\langle 0 | T \exp \left[-i \int_{-\infty}^{\infty} dt H_I(t) \right] | 0 \rangle}
 \end{aligned}$$

$$\langle \Omega | T \varphi(x) \varphi(y) | \Omega \rangle = \lim_{T \rightarrow \infty (1-i\epsilon)} \frac{\langle 0 | T U_I(T, x^0) \varphi_I(x) U_I^\dagger(T, y^0) \varphi_I(y) | 0 \rangle}{\langle 0 | U_I(T, x^0) U_I^\dagger(T, y^0) | 0 \rangle}$$

$$\langle \Omega | T \varphi(x) \varphi(y) | \Omega \rangle = \frac{\langle 0 | T \varphi_I(x) \varphi_I(y) | 0 \rangle}{\langle 0 | T 1 | 0 \rangle}$$

$$\text{Hint} = \int d^3x \frac{1}{4} F_{\mu\nu}^2(x) \Rightarrow$$



$$\langle 0 | T U_I(T, x^0) \varphi_I(x) U_I(x^0, y^0) \varphi_I(y) U_I(y^0, T) | 0 \rangle$$

$$\langle 0 | U_I(T, -T) | 0 \rangle$$

$$\langle 0 | T \varphi_I(x) \varphi_I(y) \exp \left[-i \int_{-\infty}^{\infty} dt H_{I,I}(t) \right] | 0 \rangle$$

$$\langle 0 | T \exp \left[-i \int_{-\infty}^{\infty} dt H_{I,I}(t) \right] | 0 \rangle$$

$$\Rightarrow H_{I,I} = \int d^3x \frac{\lambda}{4!} \varphi_I^4$$

$$\varphi_I = e^{iH_0 t} \varphi_S e^{-iH_0 t}$$

$$\frac{\langle 0 | T U_I(T, x^0) \varphi_I(x) U_I(x^0, y^0) \varphi_I(y) U_I(y^0, T) | 0 \rangle}{\langle 0 | U_I(T, -T) | 0 \rangle}$$

$$\langle 0 | T \varphi_I(x) \varphi_I(y) \exp \left[-i \int_{-\infty}^{\infty} dt H_{I,I}(t) \right] | 0 \rangle$$

$$\langle 0 | T \exp \left[-i \int_{-\infty}^{\infty} dt H_{I,I}(t) \right] | 0 \rangle$$

$$\Rightarrow H_{I,I} = \int d^3x \frac{\lambda}{4!} \varphi_I^4$$

$$\varphi_I = e^{i H_0 t} \varphi_S e^{-i H_0 t}$$

$$\frac{\langle 0 | T U_I(T, x^0) \varphi_I(x) U_I(x^0, y^0) \varphi_I(y) U_I(y^0, T) | 0 \rangle}{\langle 0 | U_I(T, -T) | 0 \rangle}$$

$$\langle 0 | T \varphi_I(x) \varphi_I(y) \exp \left[-i \int_{-\infty}^{\infty} dt H_{I,I}(t) \right] | 0 \rangle$$

$$\langle 0 | T \exp \left[-i \int_{-\infty}^{\infty} dt H_{I,I}(t) \right] | 0 \rangle$$

$$H_{I,I} = \int d^3x \frac{\lambda}{4!} \varphi_I^4$$

$$\varphi_I = e^{iH_0 t} \varphi_S e^{-iH_0 t}$$

Interacting vacuum

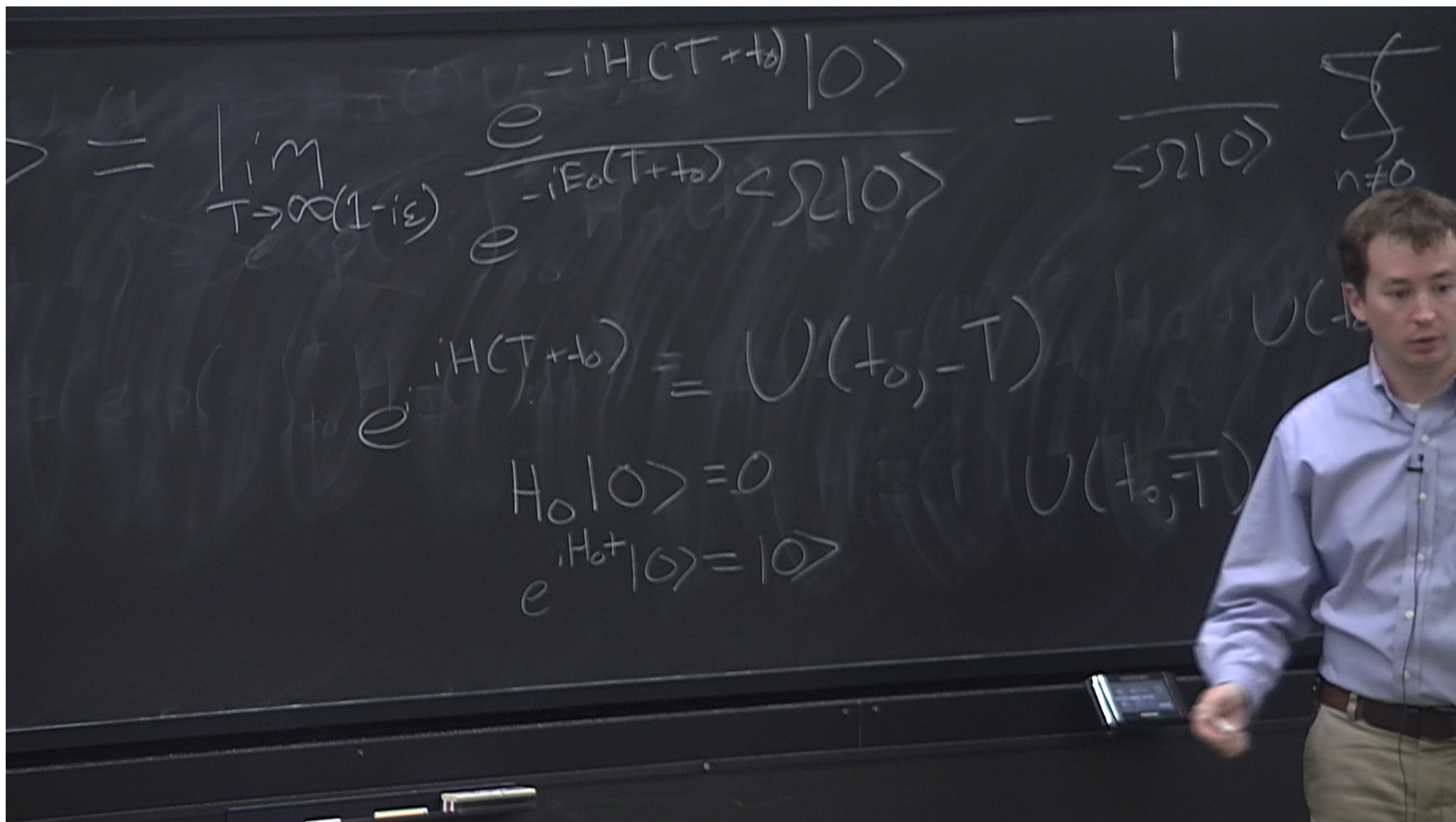
$$e^{-iH(T+t_0)}|0\rangle = e^{-iH(T+t_0)} \mathbb{1}|0\rangle$$

$\mathbb{1} =$

$$= e^{-iE_0(T+t_0)}|\Omega\rangle\langle\Omega|0\rangle$$

$$E_0 = \langle\Omega|H|\Omega\rangle$$

$$H_0|0\rangle = 0$$



Wick's Theorem

$$\langle 0 | T \varphi_1 \dots \varphi_n | 0 \rangle$$

Wick's Theorem

$$\langle 0 | T \varphi_1 \dots \varphi_n | 0 \rangle$$

$$\varphi(x) = \varphi^+(x) + \varphi^-(x)$$

$$\varphi^+(x) = \int \frac{d^3 p}{(2\pi)^3 2E_p} a_{\vec{p}} e^{-i p \cdot x}$$

$$\varphi^-(x) = \int \frac{d^3 p}{(2\pi)^3 2E_p} a_{\vec{p}}^\dagger e^{+i p \cdot x}$$

$$x^0 > y^0$$

$$T \varphi$$

$$x^0 > y^0$$

$$T \varphi(x) \varphi(y) = \varphi(x) \varphi(y)$$

$$= (\varphi^+(x) + \varphi^-(x)) (\varphi^+(y) + \varphi^-(y))$$

$$= \varphi^+(x) \varphi^+(y) + \varphi^-(y) \varphi^+(x) + [\varphi^+(x), \varphi^-(y)]$$

$$+ \varphi^-(x) \varphi^-(y) + \varphi^-(x) \varphi^-(y)$$

$$1) \int \varphi(x) \varphi(y) = \oplus (x^0 - y^0) [\varphi^+(x), \varphi^-(y)] = \oplus (y^0 - x^0) [\varphi^+(y), \varphi^-(x)]^{(T)}$$