Title: 14/15 PSI - Quantum Field Theory I (A) - Lecture 3

Date: Oct 08, 2014 03:30 PM

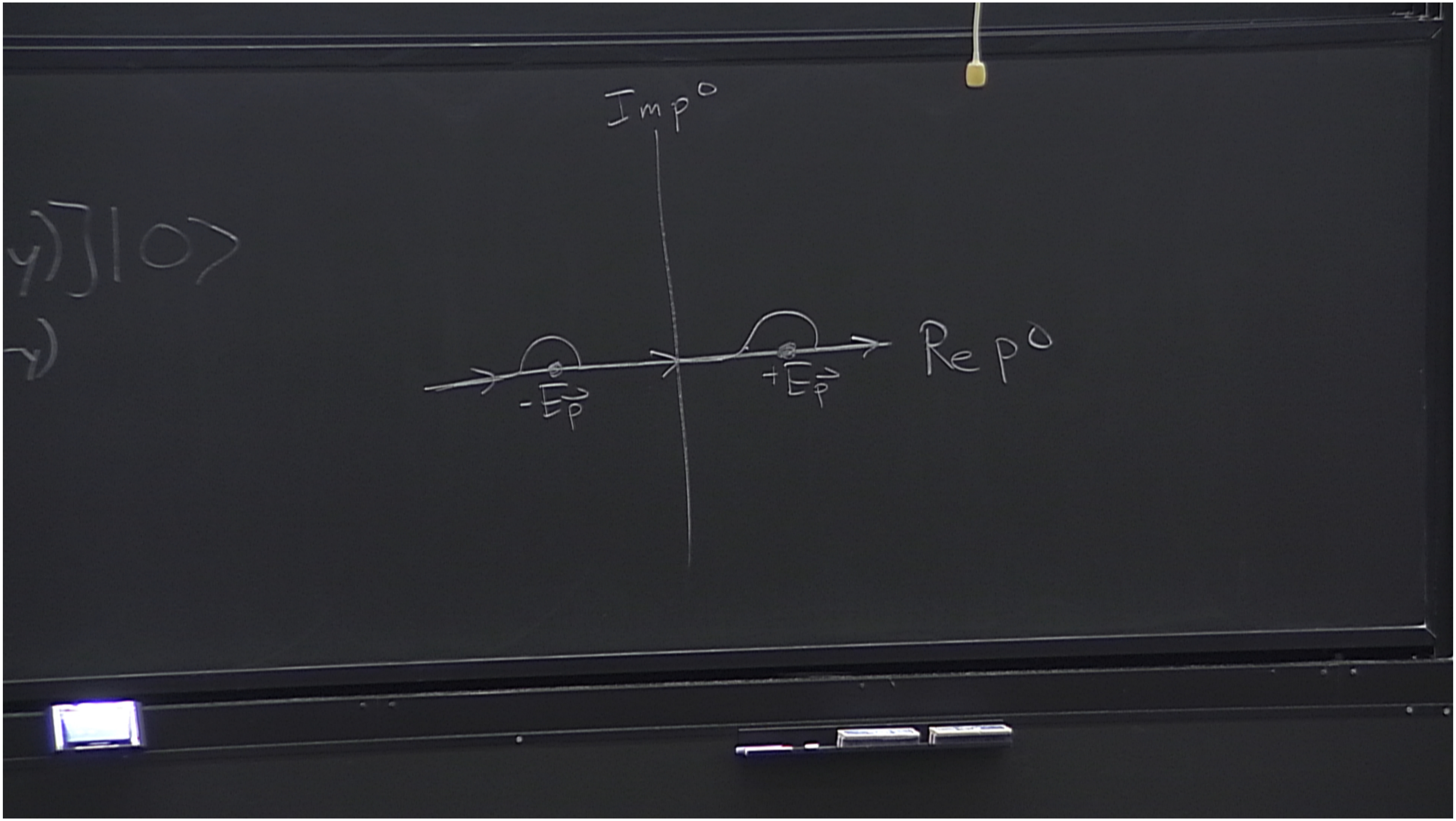
URL: http://pirsa.org/14100029

Abstract:

Propagators

$$\Delta_R(x-y) = \Theta(x-y)\,\langle 0 | [\varphi(x), \varphi(y)] | 0 \rangle$$

$$= \int_{\mathcal{C}_R} \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2} e^{-ip\cdot(x-y)}$$


$y)]|0\rangle$
$y)$
$\mathrm{Im}\, p^0$
$\mathrm{Re}\, p^0$
$-E_{\vec{p}}$
$+E_{\vec{p}}$

Propagators

$$\Delta_R(x-y) = \Theta(x^0-y^0)\langle 0|[\varphi(x),\varphi(y)]|0\rangle$$

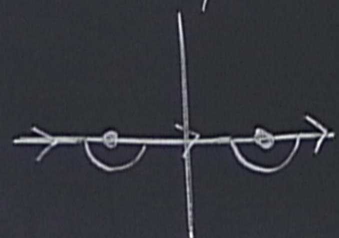
$$= \int_{C_R} \frac{d^4p}{(2\pi)^4}\,\frac{i}{p^2-m^2}\,e^{-ip\cdot(x-y)}$$

$$(\partial^2+m^2)\Delta_R = -i\,\delta(x-y)$$

$$\partial^2 \varphi + m^2 \varphi = J(x)$$

use Δ_R to solve inhomogeneous KG when we impose k

$$\Delta_A(x-y) = \Theta(y^0 - x^0) \langle 0| \varphi(x), \varphi(y)] |0\rangle$$

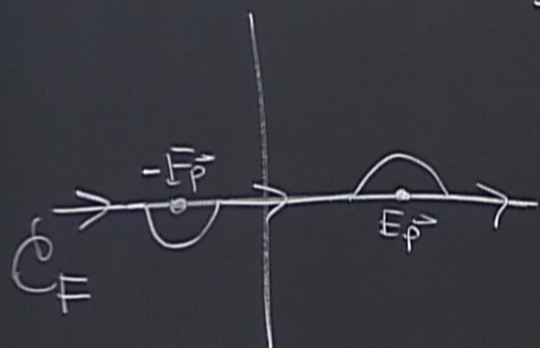


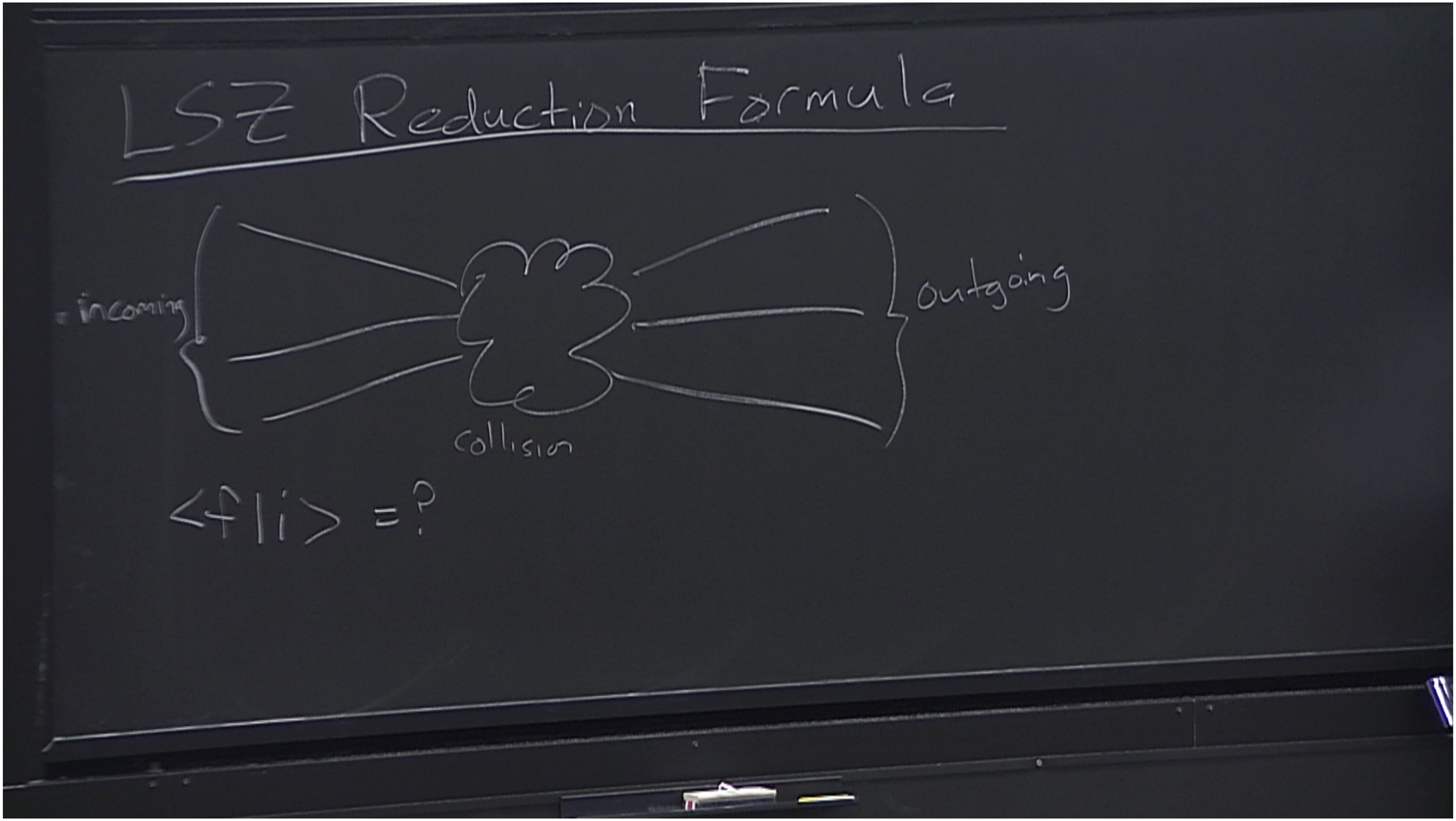
$$\Delta_F(x-y) = \Theta(x^0-y^0)\langle 0|\varphi(x)\varphi(y)|0\rangle + \Theta(y^0-x$$

$$= \langle 0|T\,\varphi(x)\,\varphi(y)|0\rangle$$

time-ordering

put latest to the left




LSZ Reduction Formula
incoming
outgoing
collision
<f|i> = ?

Free theory: (KG theory)

one particle states $|k\rangle = a^{\dagger}_{\vec{k}} |0\rangle$

where $a_{\vec{k}} |0\rangle = 0$

$\langle 0|0\rangle = 1$

$\langle \vec{k}|\vec{k}'\rangle = (2\pi)^3 2E_{\vec{k}} \delta(\vec{k} - \vec{k}')$

Define: $a_1^\dagger = \int d^3k\, f_1(\vec{k})\, a_k^\dagger$

$\uparrow$ want this to create a 1-particle state

that is

- localized in momentum space near $\vec{k}_1$
- localized in position space near origin

$$f_1 \propto \exp\left[-(\vec{k}-\vec{k}_1)^2/4\sigma^2\right]$$

e state

e near $\vec{k}_1$

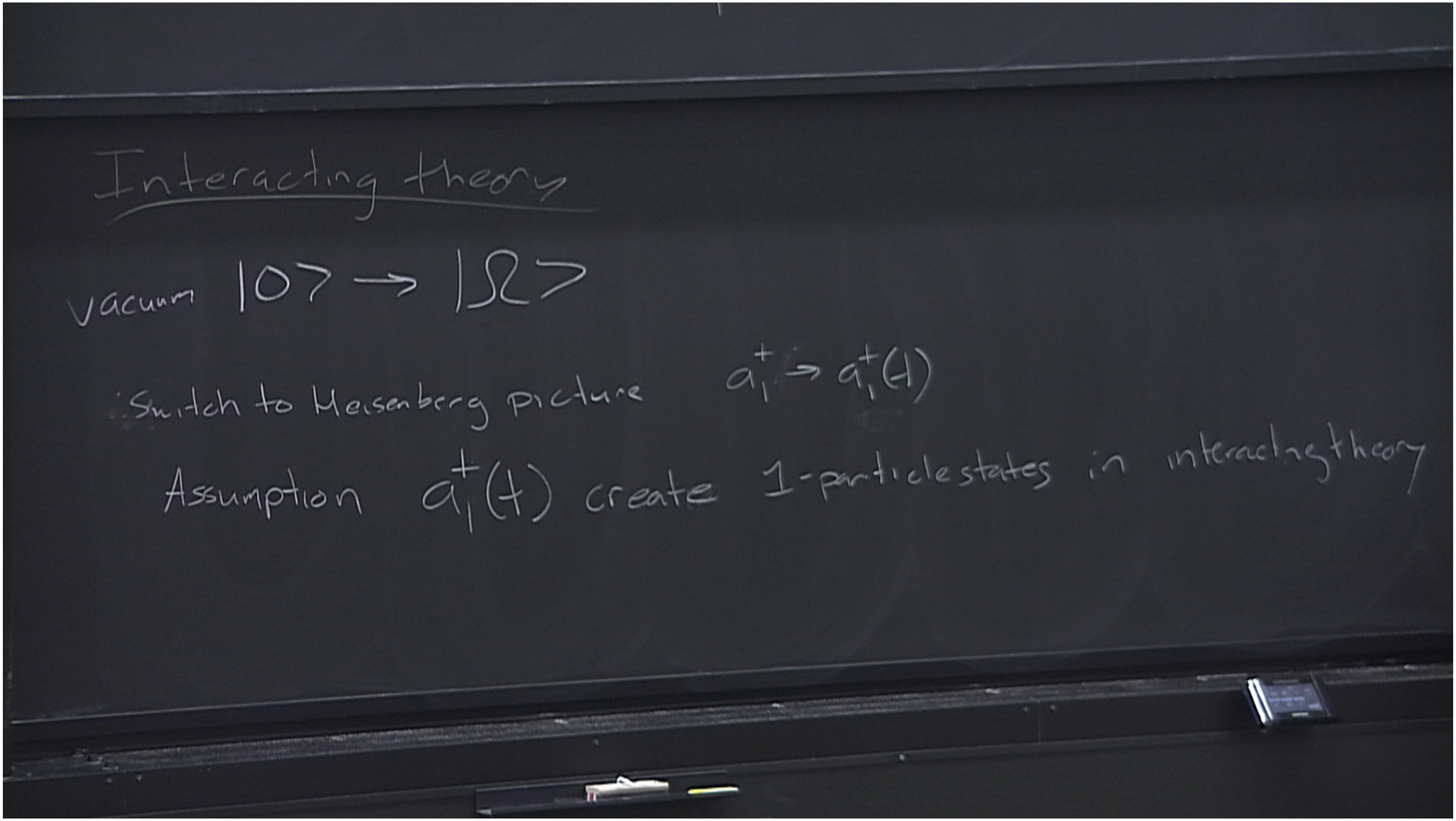
ar origin at $t=0$

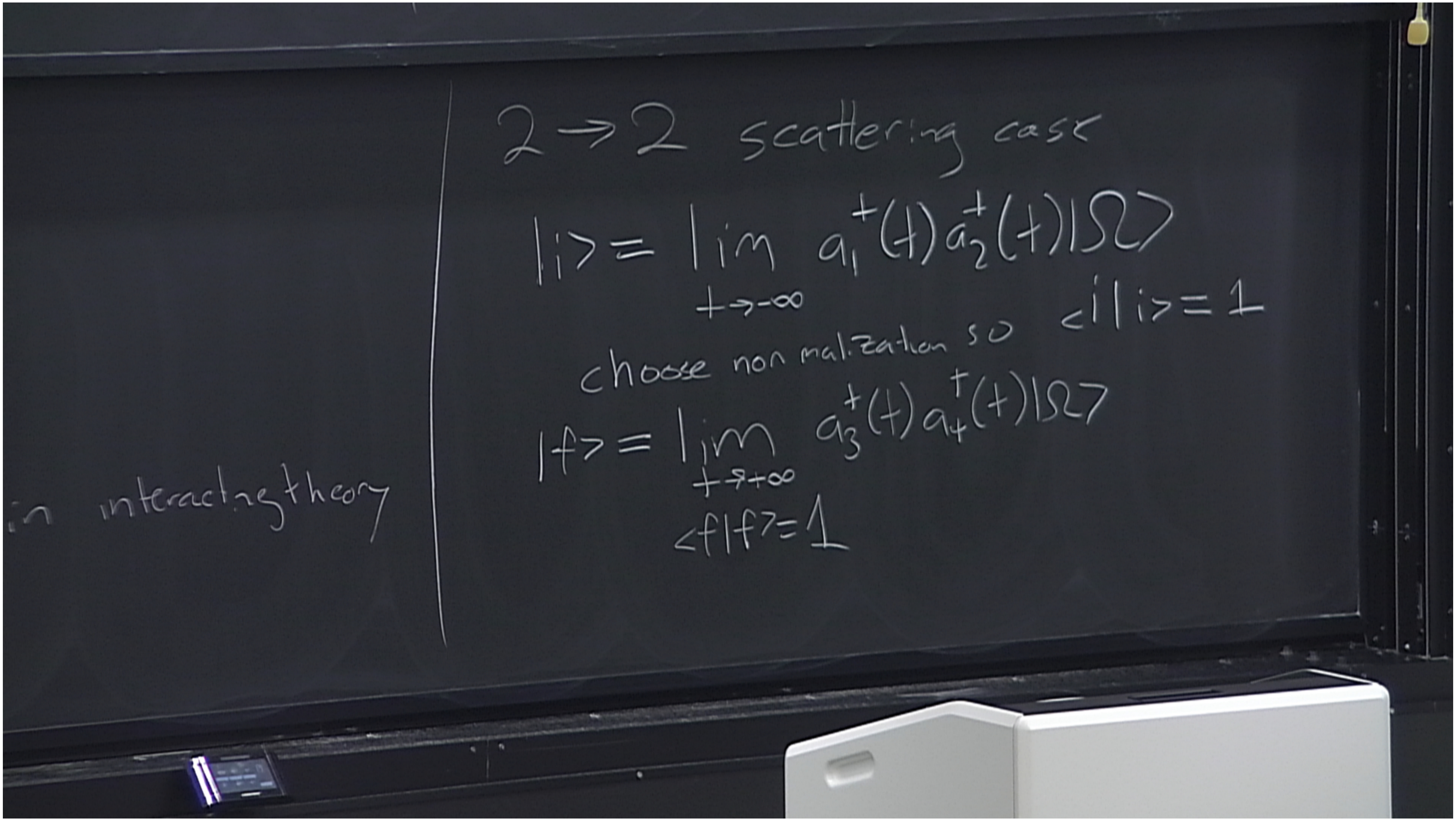
Evolve $a_1^\dagger|0\rangle \to$ spread out

$a_1^\dagger a_2^\dagger|0\rangle$ in far past

$\vec{k}_1 \neq \vec{k}_2$

particles become widely separated


Interacting theory
vacuum $|0\rangle \to |\Omega\rangle$
Switch to Heisenberg picture $a_1^\dagger \to a_1^\dagger(t)$
Assumption $a_1^\dagger(t)$ create 1-particle states in interacting theory



in interacting theory
2→2 scattering case
$|i\rangle = \lim_{t\to-\infty} a_1^\dagger(t)\, a_2^\dagger(t)|\Omega\rangle$
choose normalization so $\langle i|i\rangle = 1$
$|f\rangle = \lim_{t\to+\infty} a_3^\dagger(t)\, a_4^\dagger(t)|\Omega\rangle$
$\langle f|f\rangle = 1$

Goal: $\langle f | i \rangle$

Useful identity: $a_1^\dagger(+\infty) - a_1^\dagger(-\infty) = \int_{-\infty}^{\infty} dt\, \partial_0 a_1^\dagger(t)$

$$= -i \int d^3k\, f_1(\vec{k}) \int d^4$$

tutorial

Assumption: $a_k^\dagger = -i \int d^3x\, e^{-ikx} \left(\partial_0 \varphi(x) + i E_{\vec{k}} \varphi(x) \right)$

$$= \int_{-\infty}^{\infty} dt\, \partial_0 a_1^{\dagger}(t)$$

$$= -i \int d^3k\, f_1(\vec{k}) \int d^4x\, e^{-ik\cdot x} (\partial^2 + m^2)\varphi(x) = I_1^{\dagger}$$

$$i E_{\vec{k}}\, \varphi(x)))$$

ntity: $a_1^\dagger(+\infty) - a_1^\dagger(-\infty) = \int_{-\infty}^{\infty} dt\, \partial_0 a_1^\dagger(t)$

$$= -i \int d^3k\, f_1(\vec{k}) \int d^4x\, e^{-ik\cdot x}$$

tutorial

$$a_k^\dagger = -i \int d^3x\, e^{-ikx} \left(\partial_0 \varphi(x) + i E_{\vec{k}} \varphi(x) \right)$$

$$a_1(+\infty) = a_1(-\infty) + I_1$$

$$(-\infty)|\Omega\rangle$$

$$(-\infty)|\Omega\rangle$$

$$a_2^\dagger(+\infty)|\Omega\rangle + \cdots + \langle\Omega|T\, I_4\, I_3\, (-I_1^\dagger)(-I_2^\dagger)|\Omega\rangle$$

$$\langle\Omega|T\, a_4(-\infty)\, I_3\, a_1^\dagger(+\infty)\, a_2^\dagger(+\infty)|\Omega\rangle$$

$$a_1(+\infty) = a_1(-\infty) + I_1$$

$$\langle f|i\rangle = \langle\Omega|\, a_4(+\infty)\, a_3(+\infty)\, a_1^\dagger(-\infty)\, a_2^\dagger(-\infty)\,|\Omega\rangle$$

$$= \langle\Omega|\, T\, a_4(+\infty)\, a_3(+\infty)\, a_1^\dagger(-\infty)\, a_2^\dagger(-\infty)\,|\Omega\rangle$$

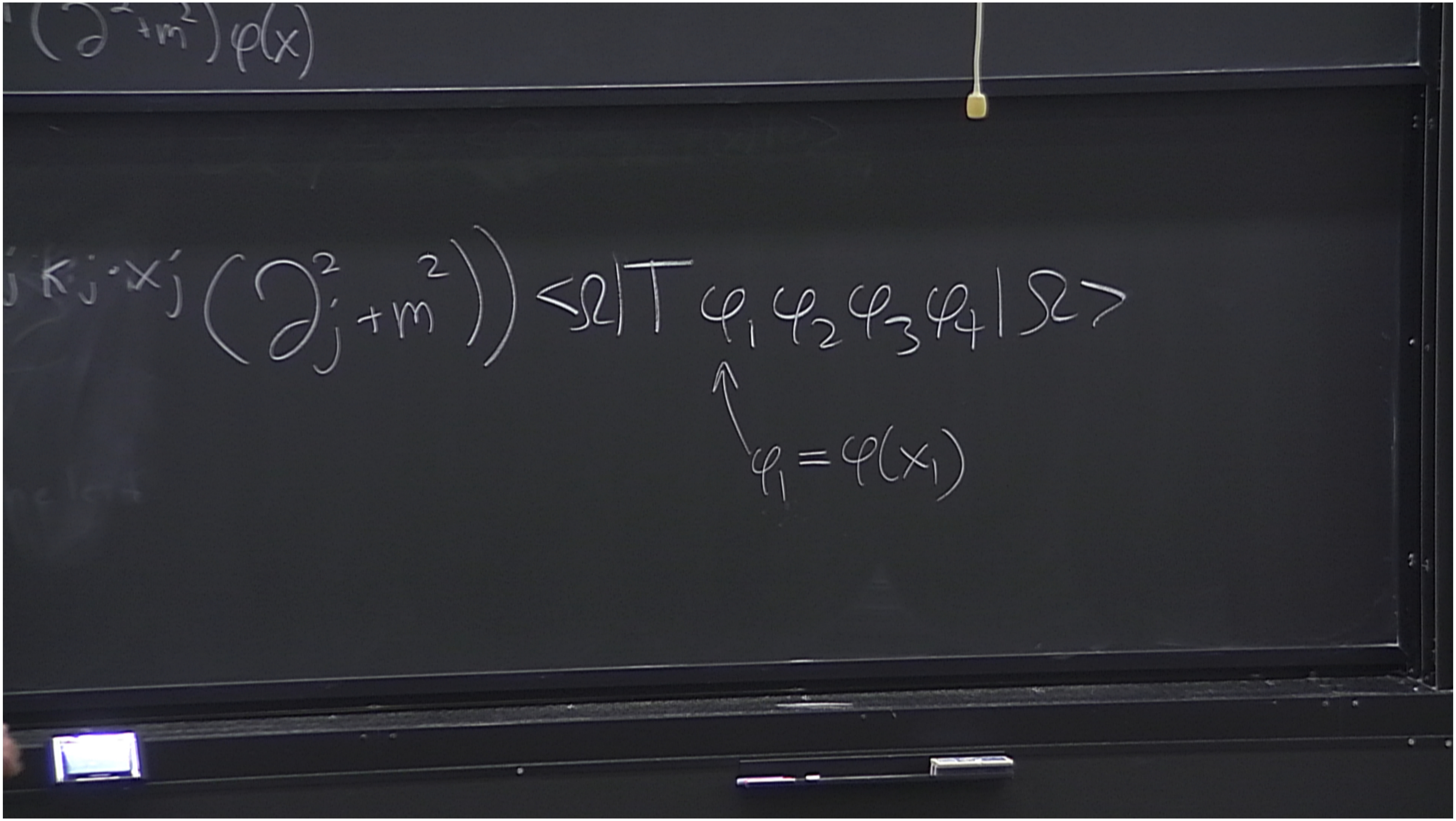
$$= \langle\Omega|\, T\, a_4(-\infty)\, a_3(-\infty)\, a_1^\dagger(+\infty)\, a_2^\dagger(+\infty)\,|\Omega\rangle \to 0$$

take limit: $f_1(\vec{k}) \to \delta(\vec{k} - \vec{k}_1)$

$$-I_1^\dagger \to +i\int d^4x_1\, e^{-ik_1\cdot x_1}\,(\partial^2 + m^2)\,\varphi(x)$$

$\langle\Omega$

$$\Delta_F(x-y) = \Theta(x^0 - y^0)\,\langle 0|\varphi(x)\varphi(y)|0\rangle + \Theta(y^0 - y$$



$(\partial^2 + m^2)\varphi(x)$
$k_j \cdot x_j (\partial_j^2 + m^2)) \langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle$
$\varphi_1 = \varphi(x_1)$

$-f_1^{*} \to +i\int d^4x_1\, e^{-ik_1\cdot x_1}(\partial^2+m^2$

LSZ Reduction Formula

$$\boxed{\langle f|i\rangle = i^{2+2}\left(\prod_{j=1}^{4}\int d^4x_j\, e^{-i\lambda_j k_j\cdot x_j}\right.}$$

$\lambda_j = -1$ final

$+1$ initial

$(\partial^2 + m^2)\varphi(x)$

$$\ldots k_j \cdot x_j \left(\partial_j^2 + m^2\right)\Big) \langle \Omega | T\, \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle$$

$$\partial_j^2 = \eta^{\mu\nu} \frac{\partial}{\partial x_j^\mu} \frac{\partial}{\partial x_j^\nu} \qquad \varphi_1 = \varphi(x_1)$$

-1 final state

$+1$ initial state

$$-f_1^\dagger \to +i\int d^4x_1\, e^{-ik_1\cdot x_1}(\partial^2+m^2)\varphi(x)$$

LSZ Reduction Formula

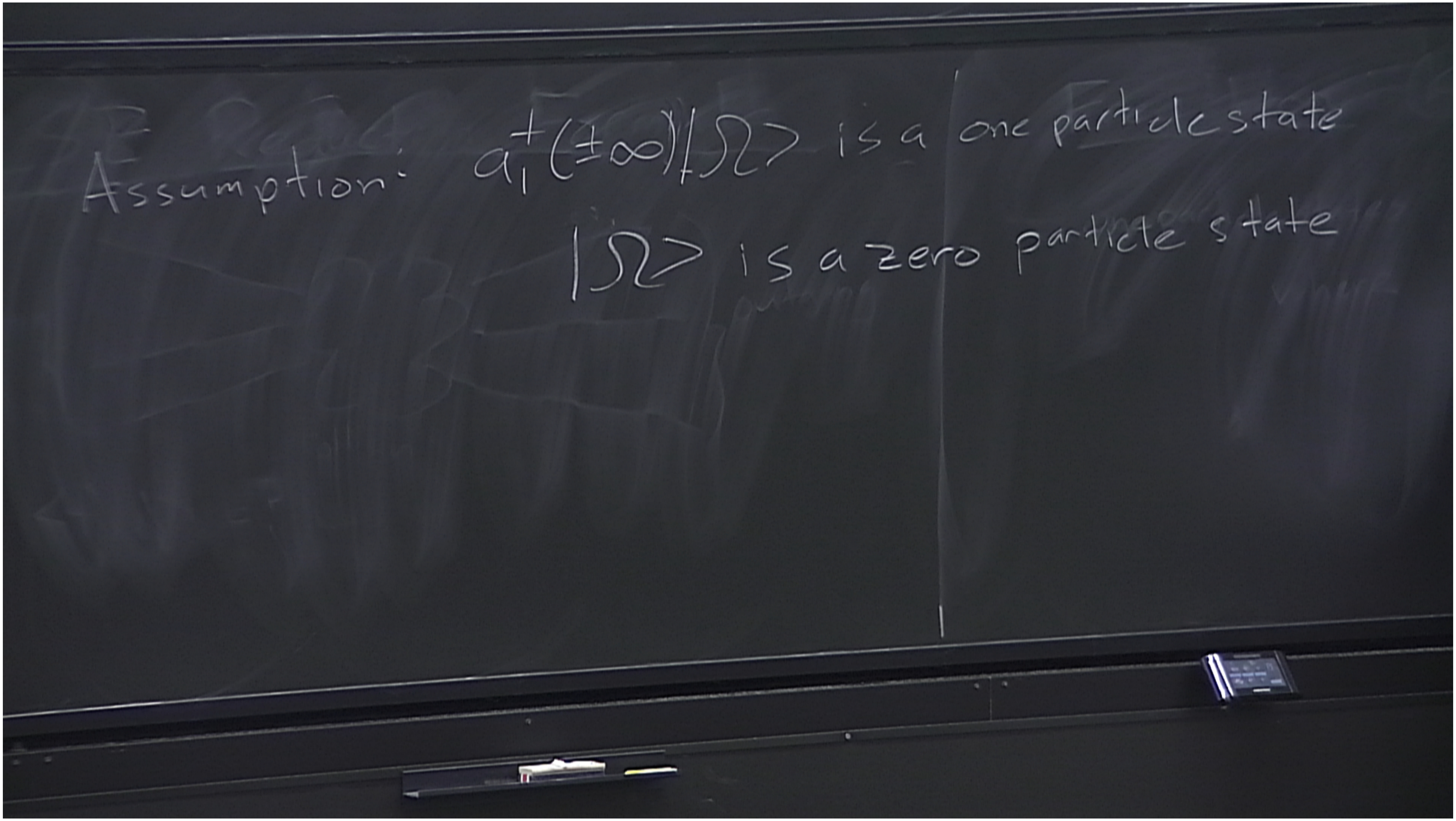
$$\boxed{\langle f|i\rangle = i^{2+2}\left(\prod_{j=1}^{4}\int d^4x_j\, e^{-i\lambda_j k_j\cdot x_j}(\partial_j^2 \ldots\right.}$$

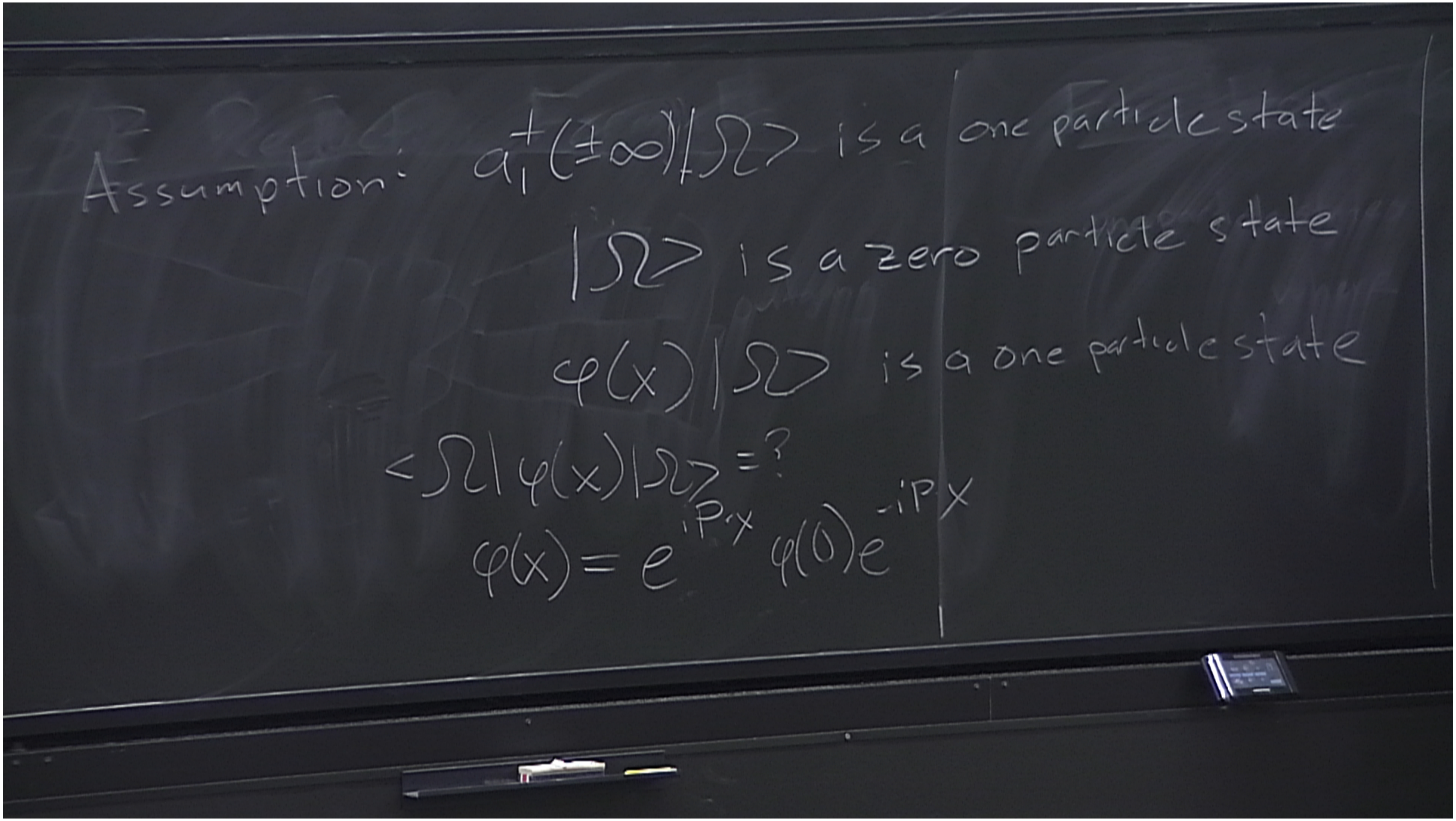
$\lambda_j = -1$ final state

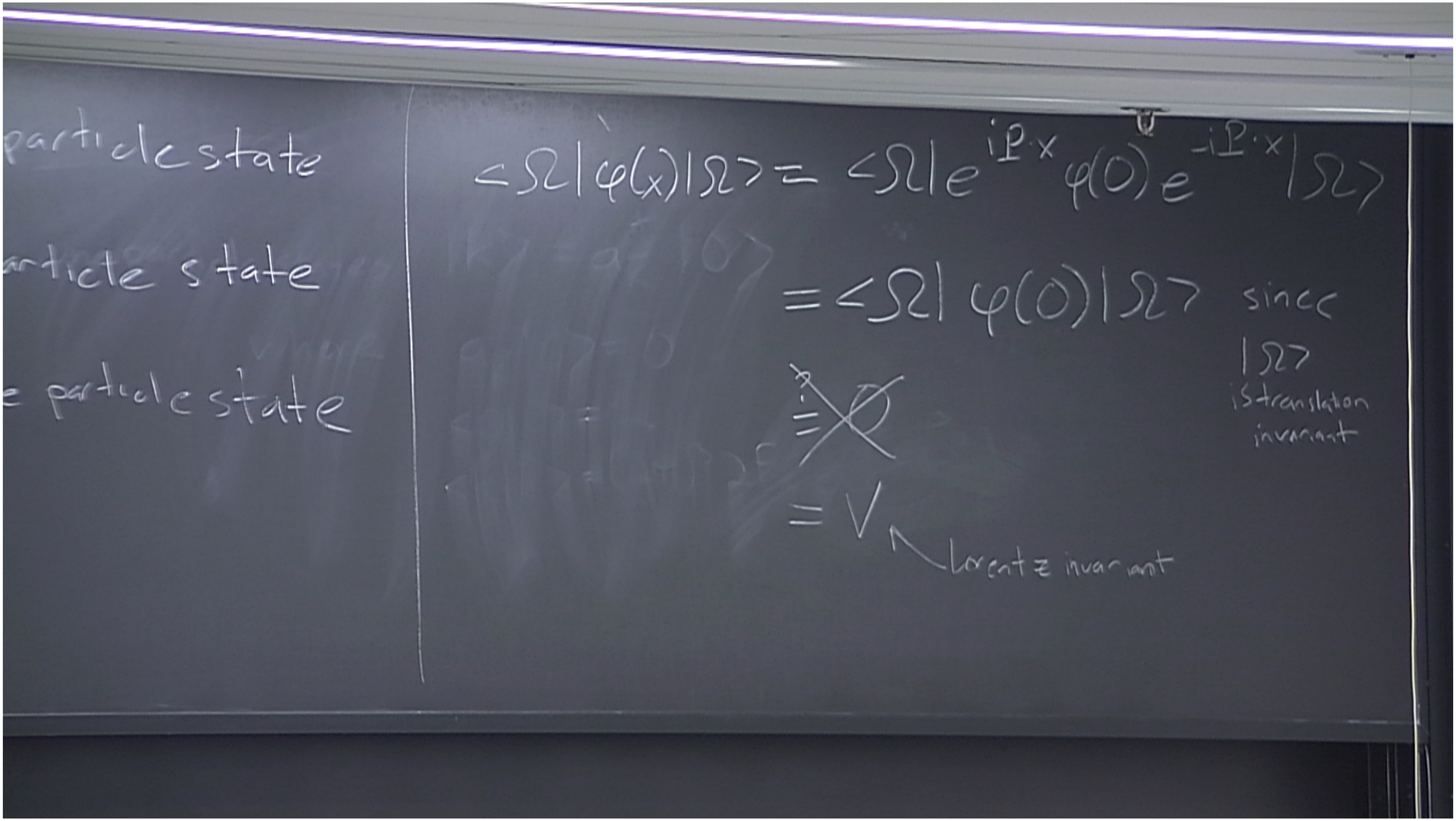
$+1$ initial state

works for $n \to m$ scattering

as well w/ obvious modifications


Assumption: $a_1^{\dagger}(\pm\infty)|\Omega\rangle$ is a one particle state
$|\Omega\rangle$ is a zero particle state


Assumption: $a_1^\dagger(\pm\infty)|\Omega\rangle$ is a one particle state
$|\Omega\rangle$ is a zero particle state
$\varphi(x)|\Omega\rangle$ is a one particle state
$\langle\Omega|\varphi(x)|\Omega\rangle \overset{?}{=}$
$\varphi(x) = e^{iP\cdot x}\varphi(0)e^{-iP\cdot x}$


particle state
particle state
particle state
$\langle\Omega|\varphi(x)|\Omega\rangle = \langle\Omega|e^{iP\cdot x}\varphi(0)e^{-iP\cdot x}|\Omega\rangle$
$= \langle\Omega|\varphi(0)|\Omega\rangle$
since $|\Omega\rangle$ is translation invariant
$\overset{?}{=} \cancel{0}$
$= V$
Lorentz invariant

$$\langle \Omega | \varphi(x) | \Omega \rangle = v$$

Redefine $\tilde{\varphi}(x) = \varphi(x) - v$

$$\langle \Omega | \tilde{\varphi}(x) | \Omega \rangle = 0$$

Drop $\sim$ in what follows

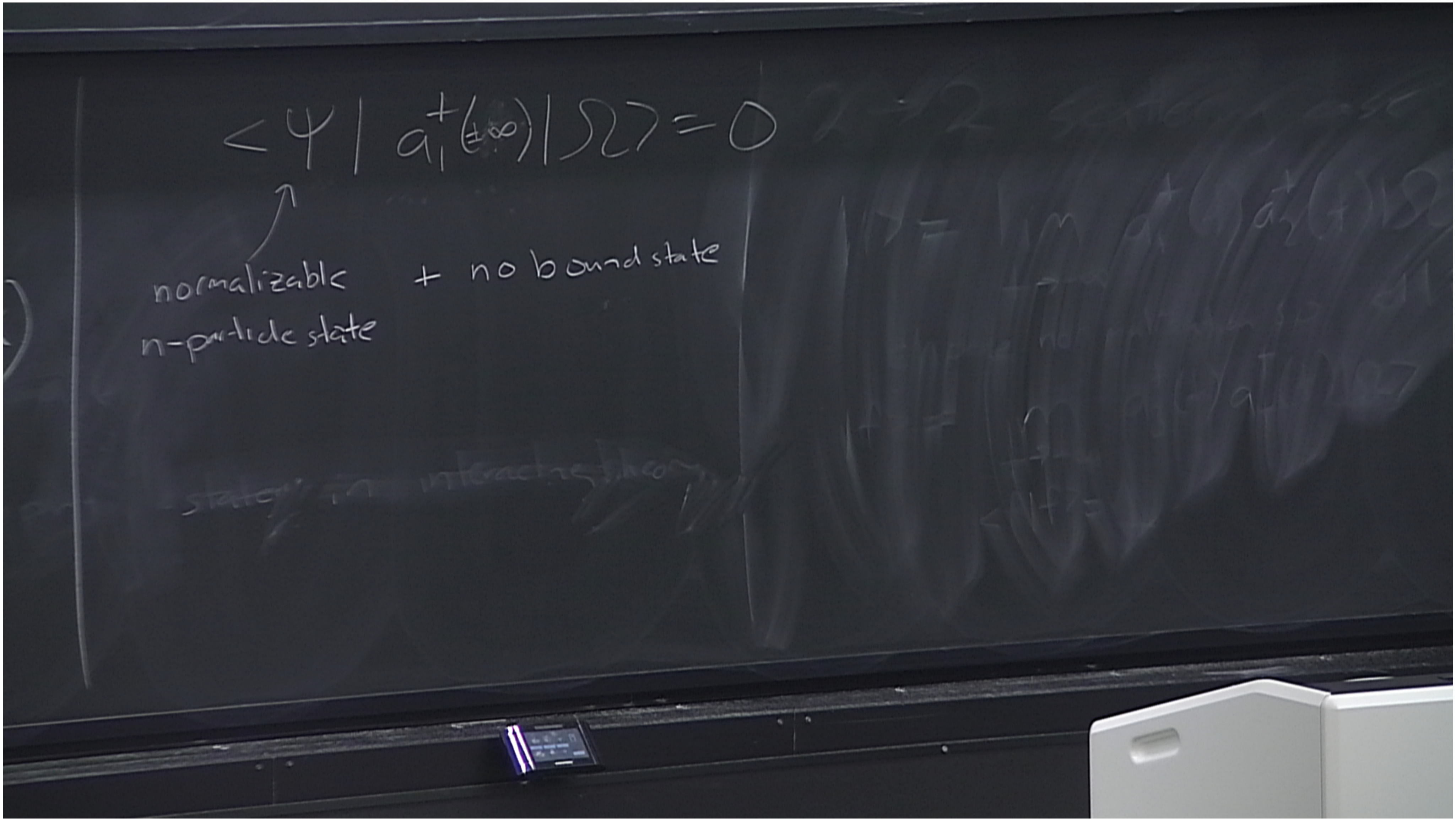
Free theory form factor

$$\langle \vec{k} \,|\, \varphi(x) \,|\, \Omega \rangle = e^{ik\cdot x}$$

Interacting theory:

$$\langle \vec{k} | \varphi(x) | \Omega \rangle = \langle \vec{k} | \, e^{iP\cdot x} \varphi(0) e^{-iP\cdot x} | \Omega \rangle$$

$$= e^{ik\cdot x} \underbrace{\langle \vec{k} | \varphi(0) | \Omega \rangle}_{\text{want this to be 1}}$$


$\langle \Psi | a_1^{+}(\pm\infty) | \Omega \rangle = 0$
normalizable
n-particle state
+ no bound state

$$\langle \vec{k} | \varphi(0) | \Omega \rangle = \frac{1}{N}$$

$$\tilde{\varphi}(x) = N \varphi(x)$$

$$\langle \vec{k} | \tilde{\varphi}(0) | \Omega \rangle = 1$$

$$\langle \Psi | \, a_1^{\dagger}(t\to\infty) \, |$$

normalizable n-particle state

\+ no b

take limit $f_1(k) \to \delta(k-k_1)$

$$-I_1^+ \to +i\int d^4x_1\, e^{-ik_1\cdot x_1}(\partial^2+m^2)\varphi(x)$$

Suppose we start with

$$\mathcal{L}_{\text{naive}} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 + \frac{1}{3!}g\varphi^3$$

Shift + rescale + rename

new factors

new term

$$\mathcal{L} = \frac{1}{2}Z_\varphi(\partial\varphi)^2 - \frac{1}{2}Z_m m^2\varphi^2 + \frac{1}{3!}Z_g g\varphi^3 + Y\varphi$$

LSZ valid

$(\partial^2 + m^2)\varphi(x)$

$\langle \Omega | \varphi(x_1) | \Omega \rangle \neq 0$

φ^3

new factors

new term

$m^2 \varphi^2 + \frac{1}{3!} Z_g g \varphi^3 + Y \varphi$

We will set

$Z = 1 + O(g^2)$

$Y = 0 + O(g)$

not generally valid