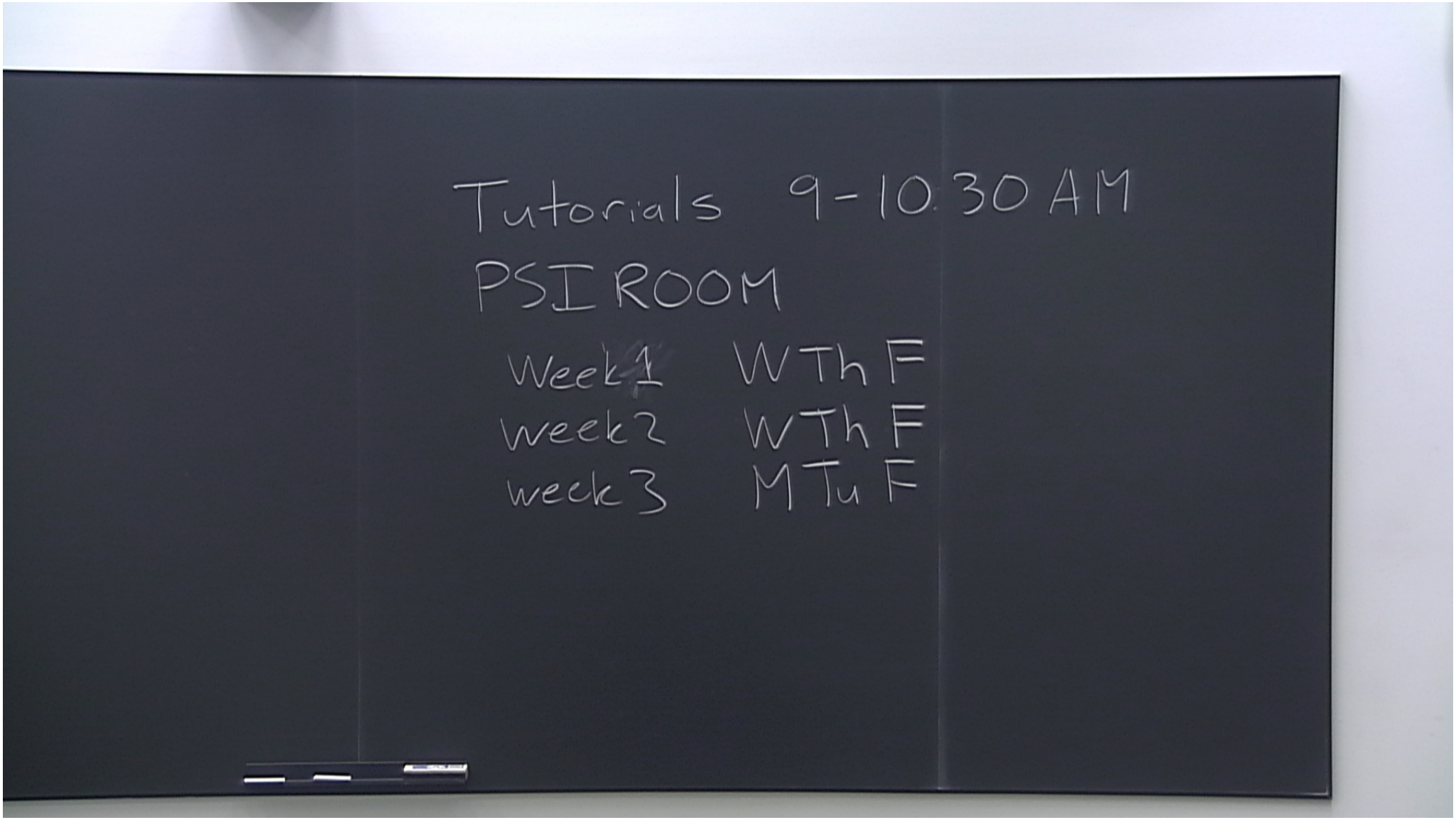
Title: 14/15 PSI - Quantum Field Theory I (A) - Lecture 2

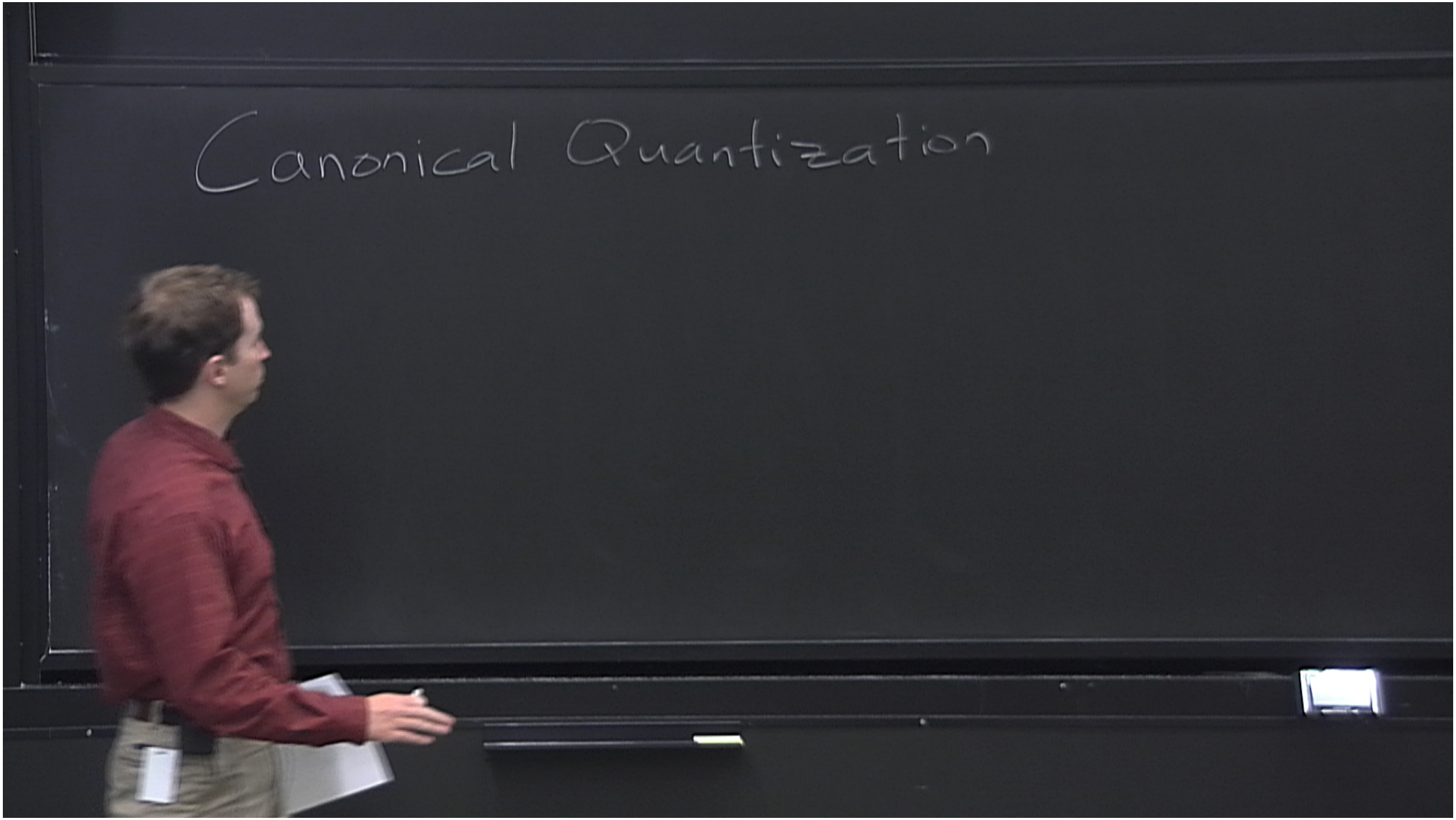
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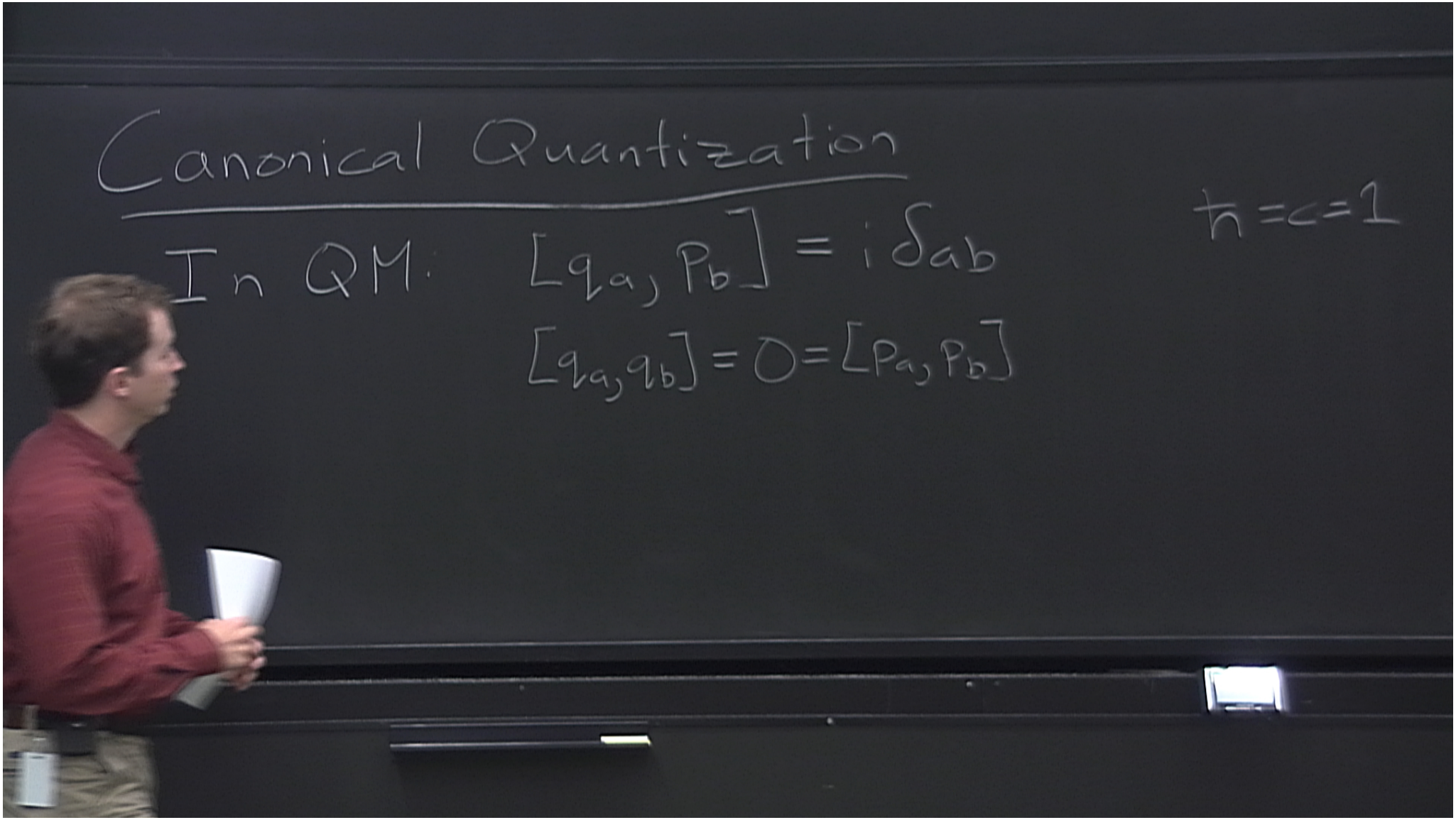
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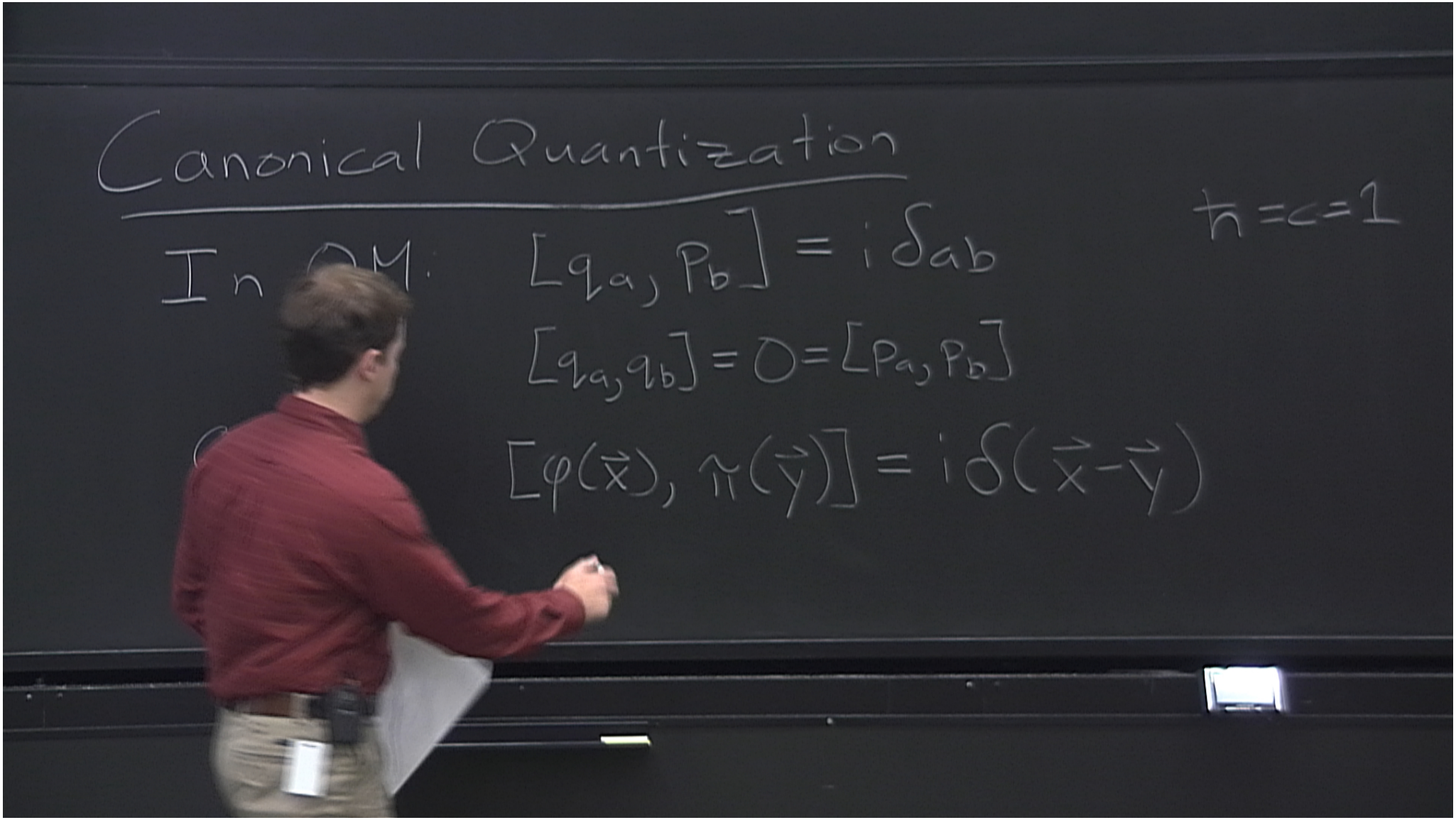
Abstract:

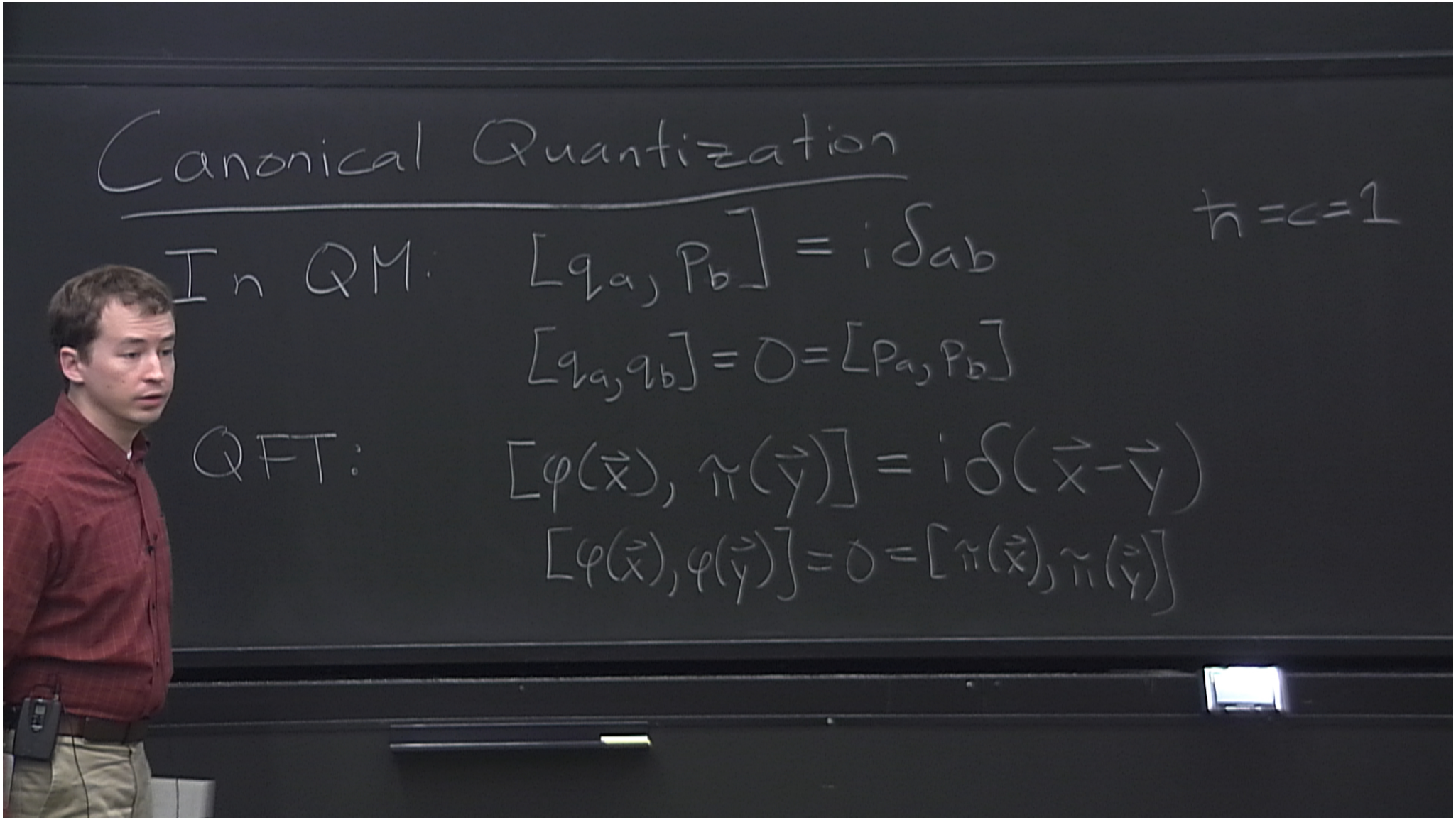


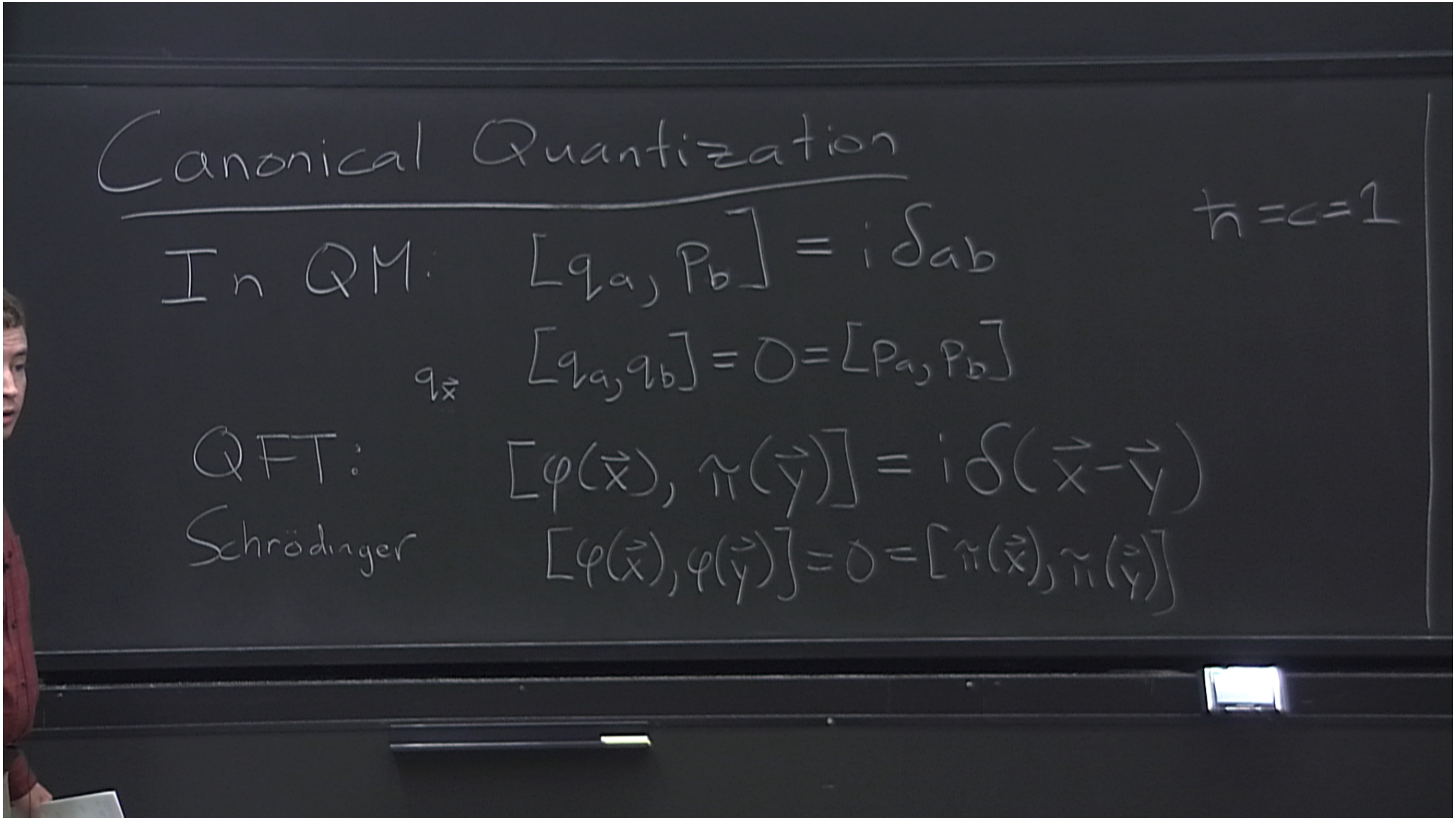

Tutorials 9-10:30 AM
PSI ROOM
Week 1 W Th F
week 2 W Th F
week 3 M Tu F

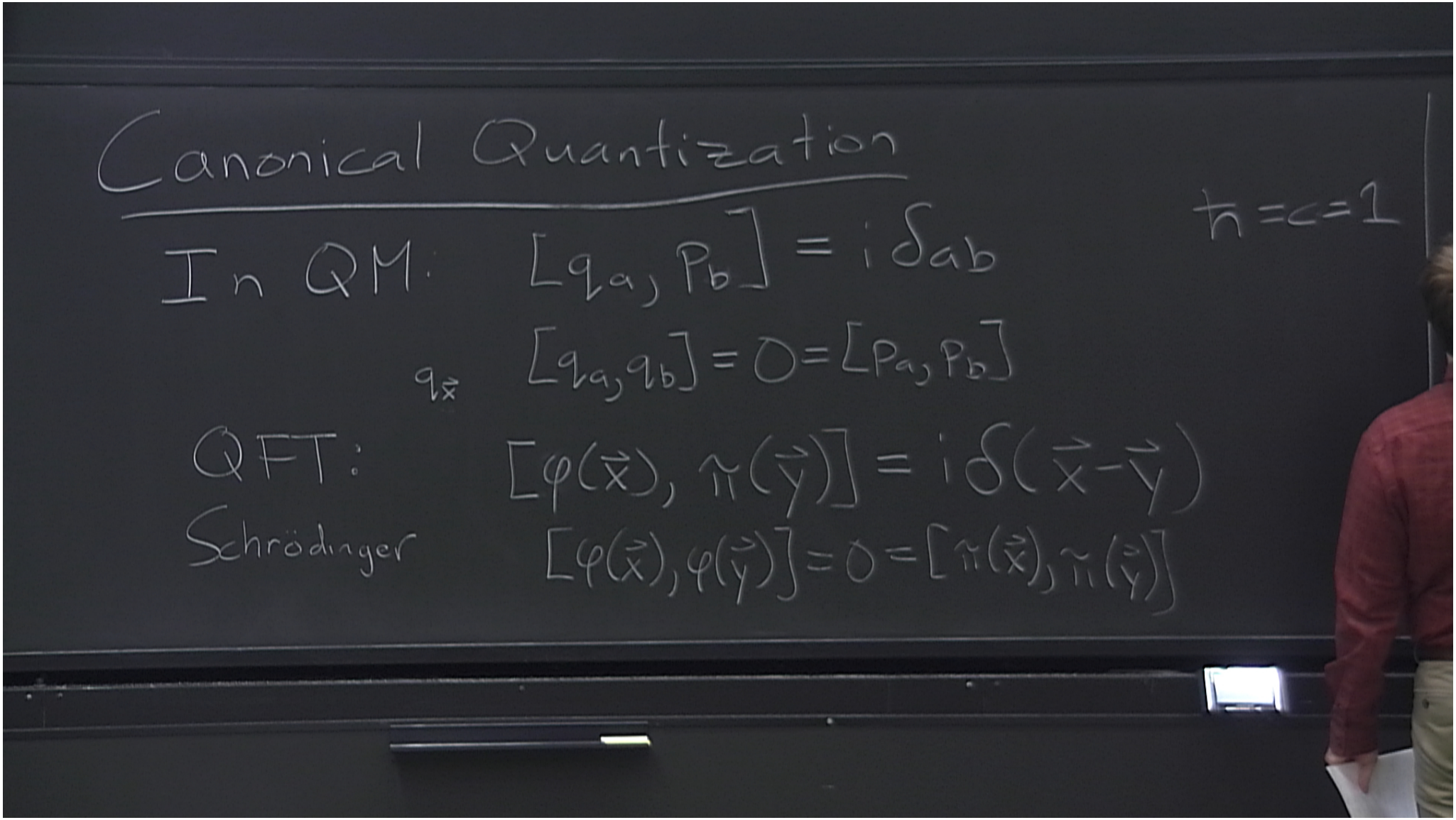

Canonical Quantization

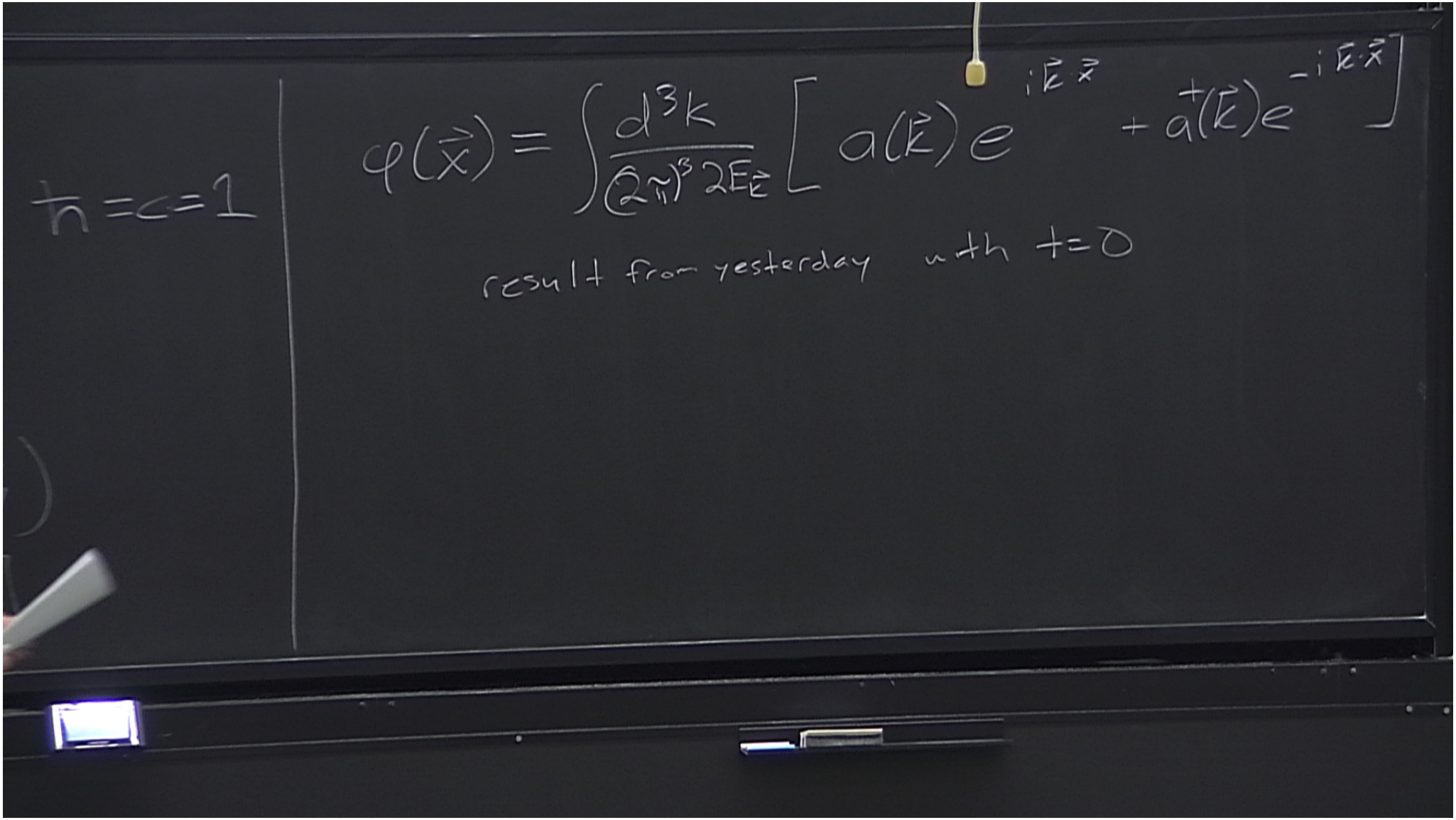

Canonical Quantization
In QM: $[q_a, p_b] = i\delta_{ab}$
$[q_a, q_b] = 0 = [p_a, p_b]$
$\hbar = c = 1$

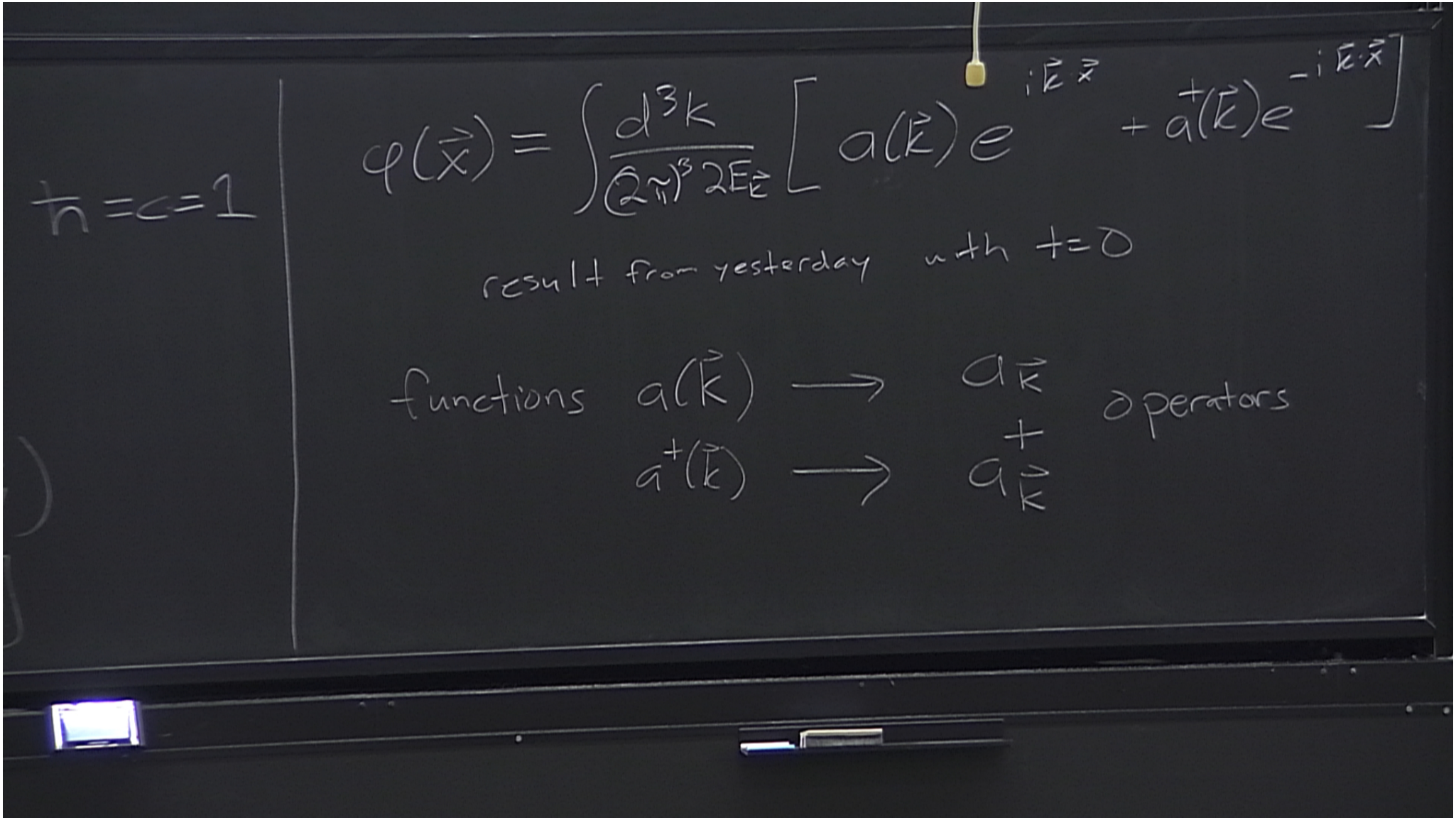

Canonical Quantization
In QM:
$[q_a, p_b] = i\delta_{ab}$
$[q_a, q_b] = 0 = [p_a, p_b]$
$[\varphi(\vec{x}), \pi(\vec{y})] = i\delta(\vec{x}-\vec{y})$
$\hbar = c = 1$

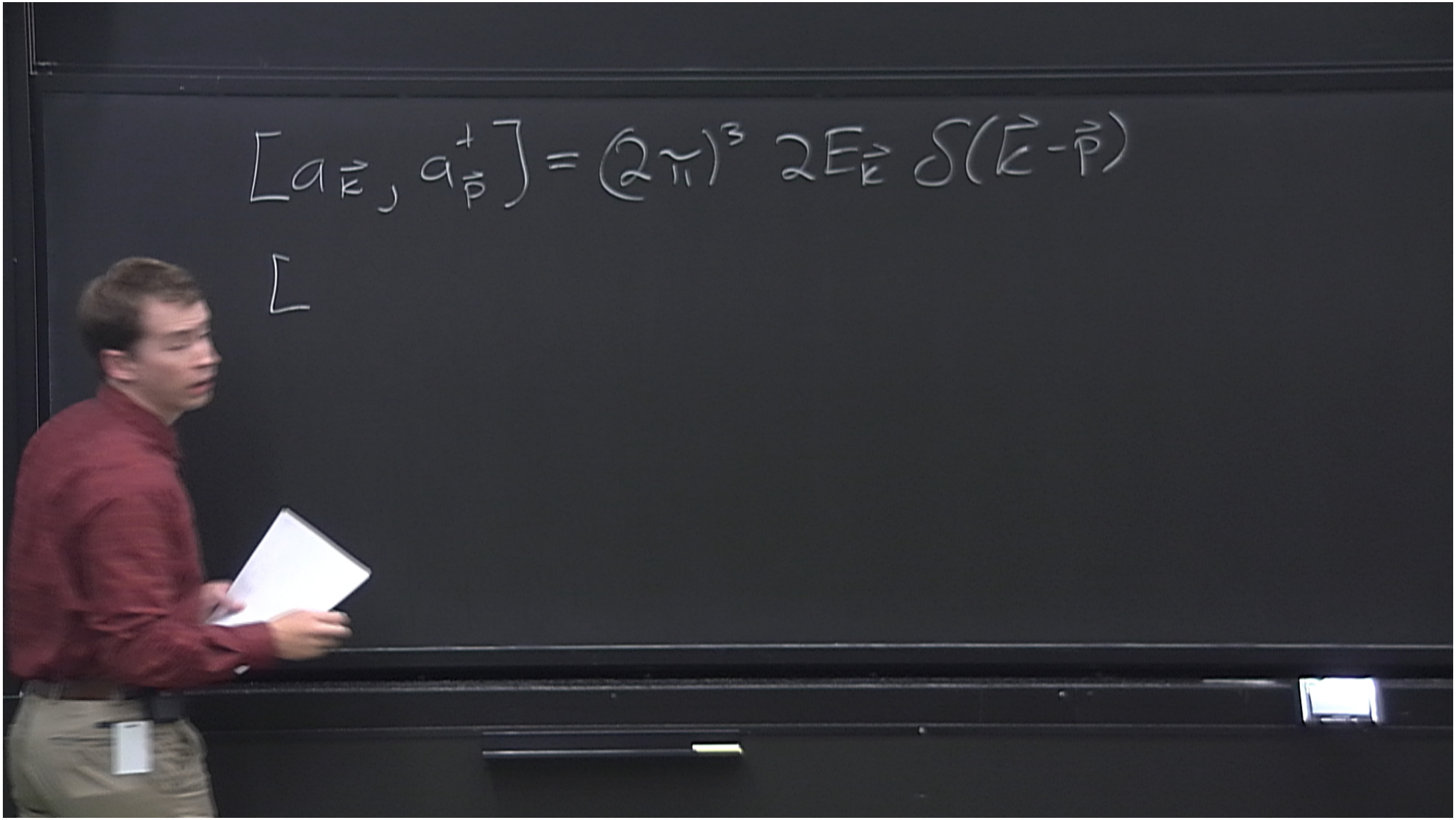

Canonical Quantization
$\hbar = c = 1$
In QM: $[q_a, p_b] = i\delta_{ab}$
$[q_a, q_b] = 0 = [p_a, p_b]$
QFT: $[\varphi(\vec{x}), \pi(\vec{y})] = i\delta(\vec{x}-\vec{y})$
$[\varphi(\vec{x}), \varphi(\vec{y})] = 0 = [\pi(\vec{x}), \pi(\vec{y})]$

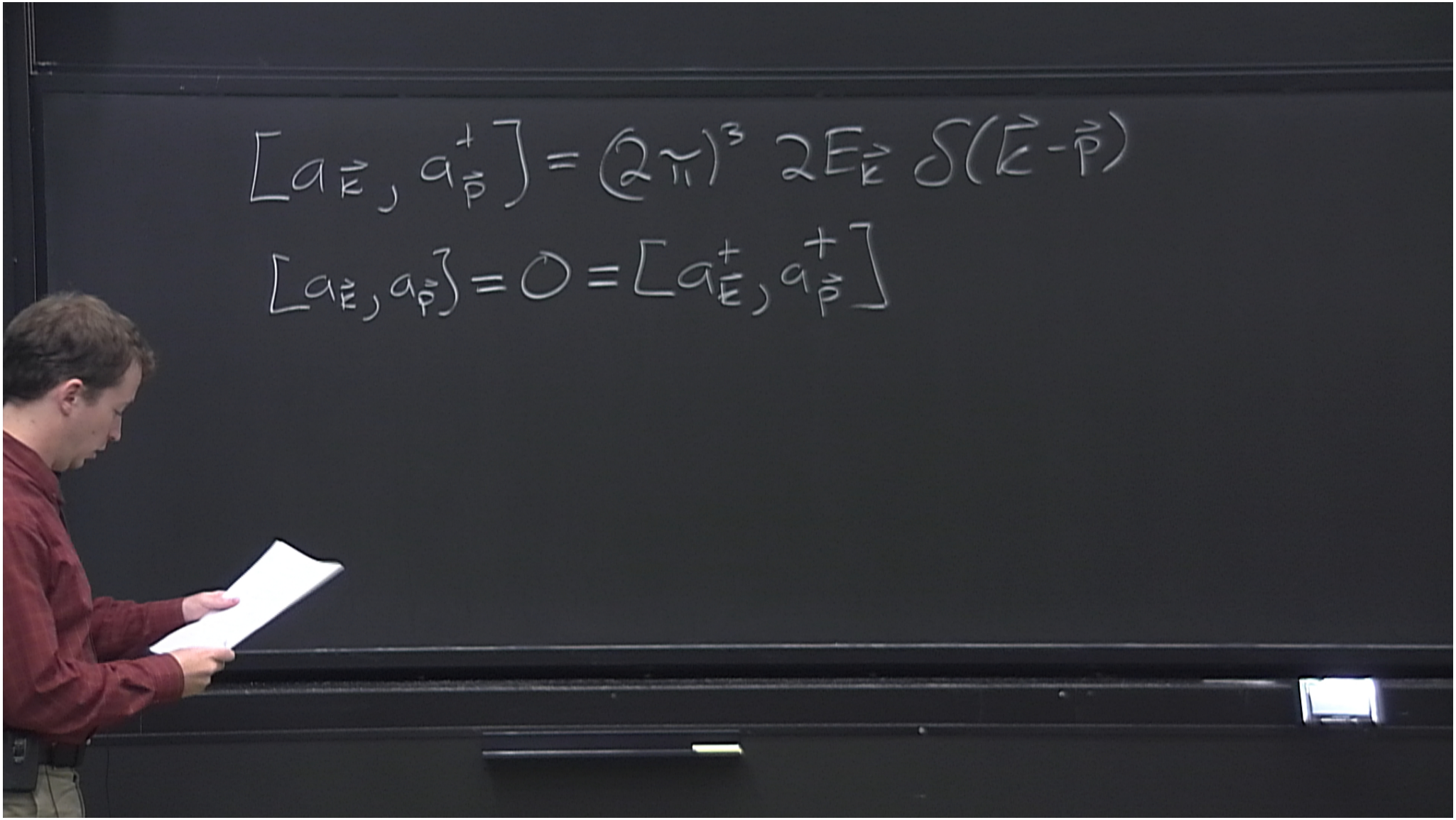

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Schrödinger $[\varphi(\vec{x}), \varphi(\vec{y})] = 0 = [\pi(\vec{x}), \pi(\vec{y})]$


$\hbar = c = 1$
$\varphi(\vec{x}) = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \left[a(\vec{k}) e^{i\vec{k}\cdot\vec{x}} + a^\dagger(\vec{k}) e^{-i\vec{k}\cdot\vec{x}} \right]$
result from yesterday with $t=0$


$\hbar = c = 1$
$\varphi(\vec{x}) = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \left[a(\vec{k}) e^{i\vec{k}\cdot\vec{x}} + a^\dagger(\vec{k}) e^{-i\vec{k}\cdot\vec{x}} \right]$
result from yesterday with $t=0$
functions $a(\vec{k}) \longrightarrow a_{\vec{k}}$
$a^\dagger(\vec{k}) \longrightarrow a^\dagger_{\vec{k}}$
operators


$[a_{\vec{k}}, a^{\dagger}_{\vec{p}}] = (2\pi)^3 \, 2E_{\vec{k}} \, \delta(\vec{k} - \vec{p})$
[


$[a_{\vec{k}}, a^{\dagger}_{\vec{p}}] = (2\pi)^3 \, 2E_{\vec{k}} \, \delta(\vec{k} - \vec{p})$
$[a_{\vec{k}}, a_{\vec{p}}] = 0 = [a^{\dagger}_{\vec{k}}, a^{\dagger}_{\vec{p}}]$

$$[a_{\vec{k}}, a^{\dagger}_{\vec{p}}] = (2\pi)^3 \, 2E_{\vec{k}} \, \delta(\vec{k}-\vec{p})$$

$$[a_{\vec{k}}, a_{\vec{p}}] = 0 = [a^{\dagger}_{\vec{k}}, a^{\dagger}_{\vec{p}}]$$

} tutorial tomorrow

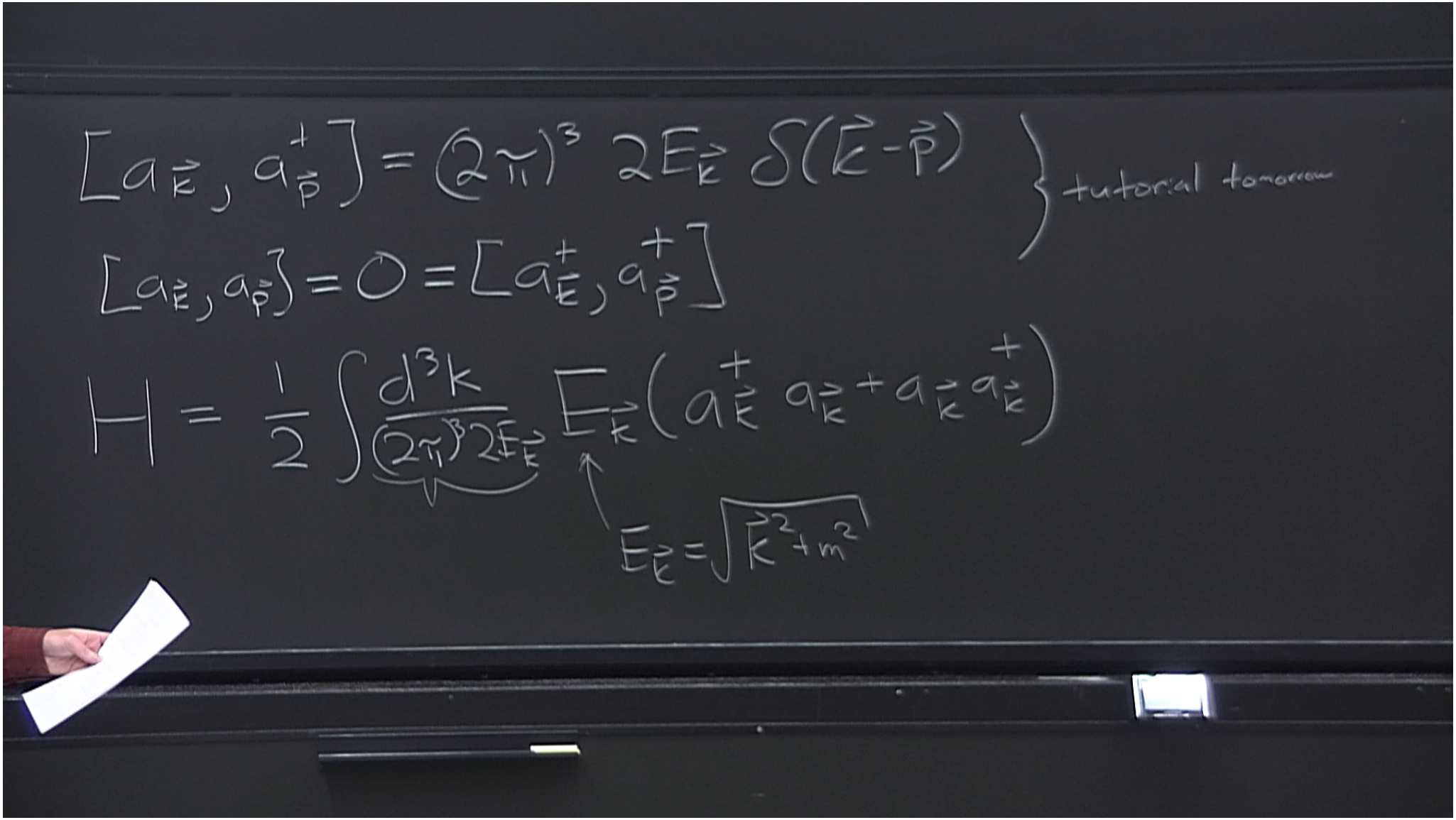
$$H = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} E_{\vec{k}} \left(a^{\dagger}_{\vec{k}} a_{\vec{k}} + a_{\vec{k}} a^{\dagger}_{\vec{k}} \right)$$

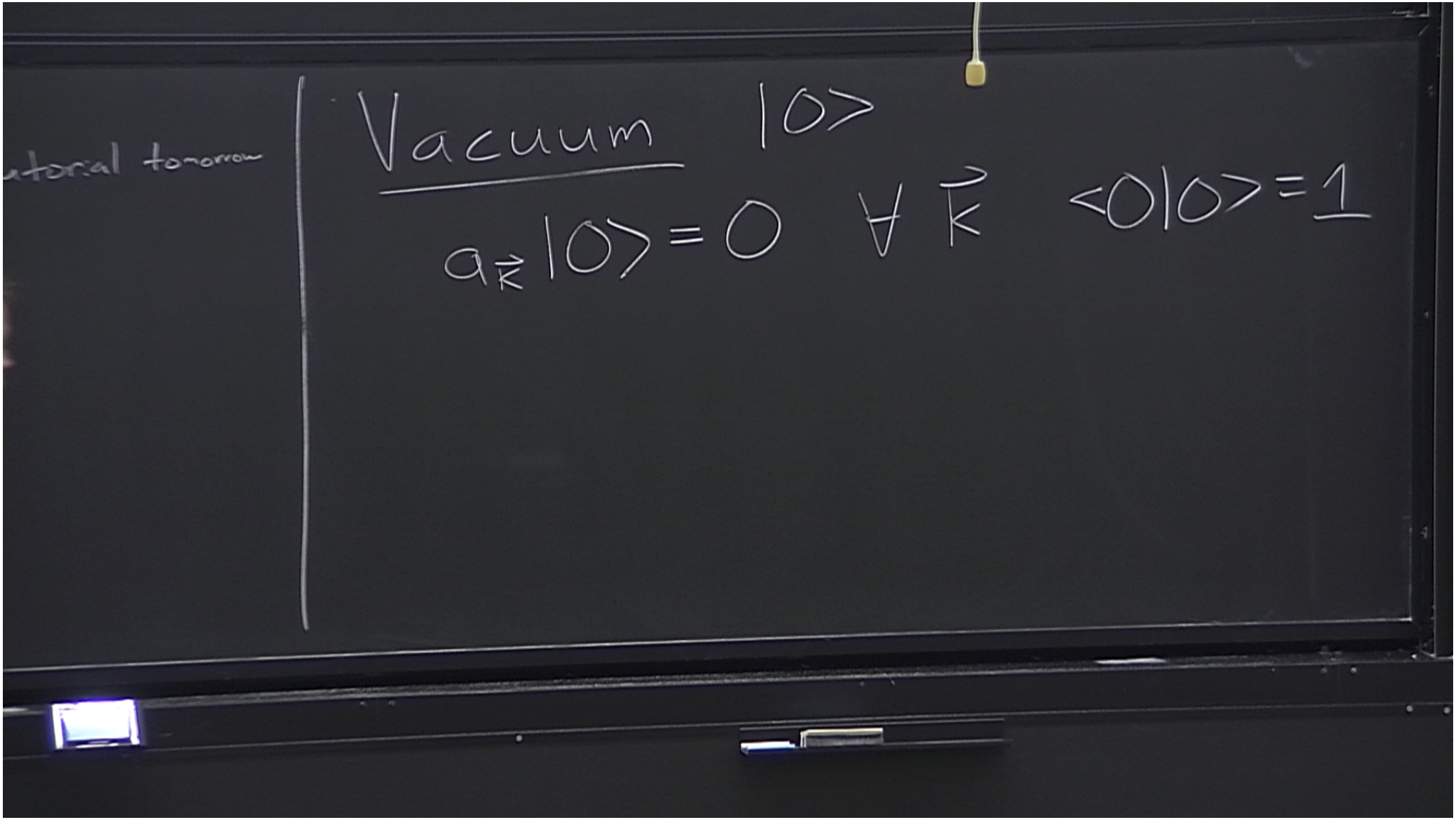
$$[a_{\vec{k}}, a^{\dagger}_{\vec{p}}] = (2\pi)^3 \, 2E_{\vec{k}} \, \delta(\vec{k}-\vec{p})$$

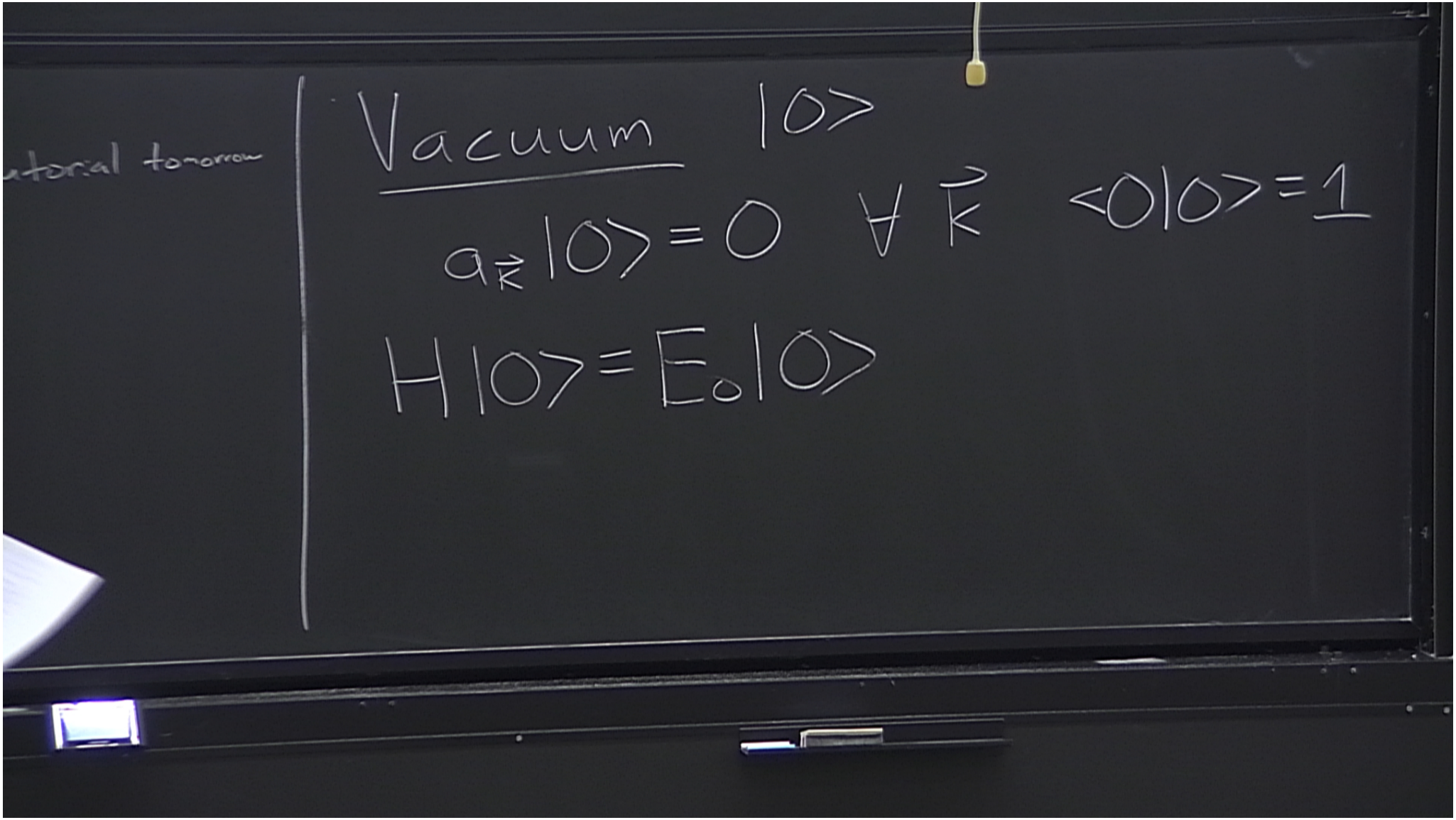
$$[a_{\vec{k}}, a_{\vec{p}}] = 0 = [a^{\dagger}_{\vec{k}}, a^{\dagger}_{\vec{p}}]$$

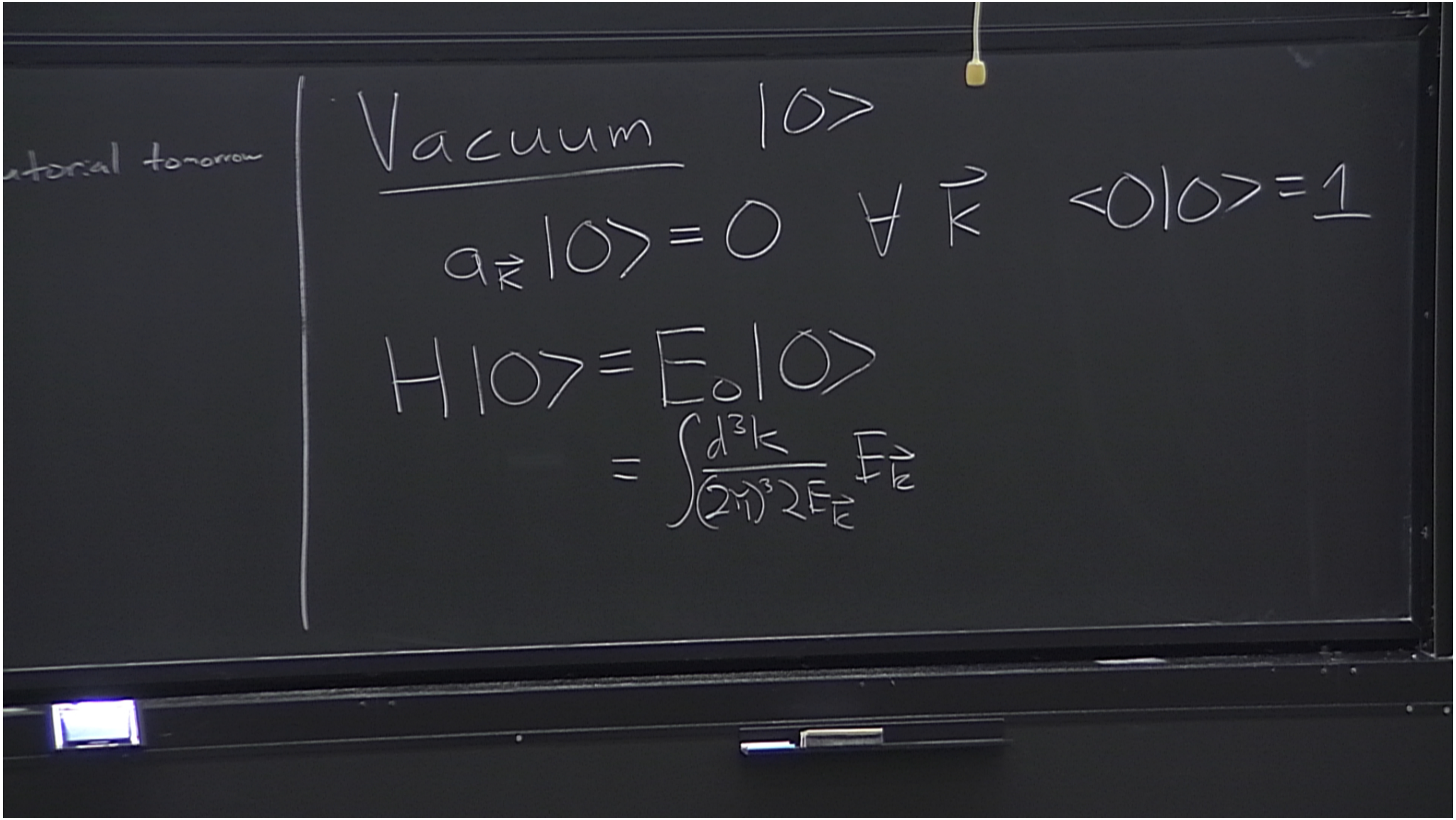
} tutorial tomorrow

$$H = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} E_{\vec{k}} \left(a^{\dagger}_{\vec{k}} a_{\vec{k}} + a_{\vec{k}} a^{\dagger}_{\vec{k}} \right)$$


$[a_{\vec{k}}, a^{\dagger}_{\vec{p}}] = (2\pi)^3 \, 2E_{\vec{k}} \, \delta(\vec{k}-\vec{p})$
tutorial tomorrow
$[a_{\vec{k}}, a_{\vec{p}}] = 0 = [a^{\dagger}_{\vec{k}}, a^{\dagger}_{\vec{p}}]$
$H = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} E_{\vec{k}} \left(a^{\dagger}_{\vec{k}} a_{\vec{k}} + a_{\vec{k}} a^{\dagger}_{\vec{k}} \right)$
$E_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$


utorial tomorrow
Vacuum |0>
$a_{\vec{k}}|0\rangle = 0 \quad \forall \vec{k} \quad \langle 0|0\rangle = 1$


utorial tomorrow
Vacuum |0⟩
$a_{\vec{k}}|0\rangle = 0 \quad \forall \vec{k} \quad \langle 0|0\rangle = 1$
$H|0\rangle = E_0|0\rangle$


utorial tomorrow
Vacuum $|0\rangle$
$a_{\vec{k}}|0\rangle = 0 \quad \forall \vec{k} \quad \langle 0|0\rangle = 1$
$H|0\rangle = E_0|0\rangle$
$= \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} F_{\vec{k}}$

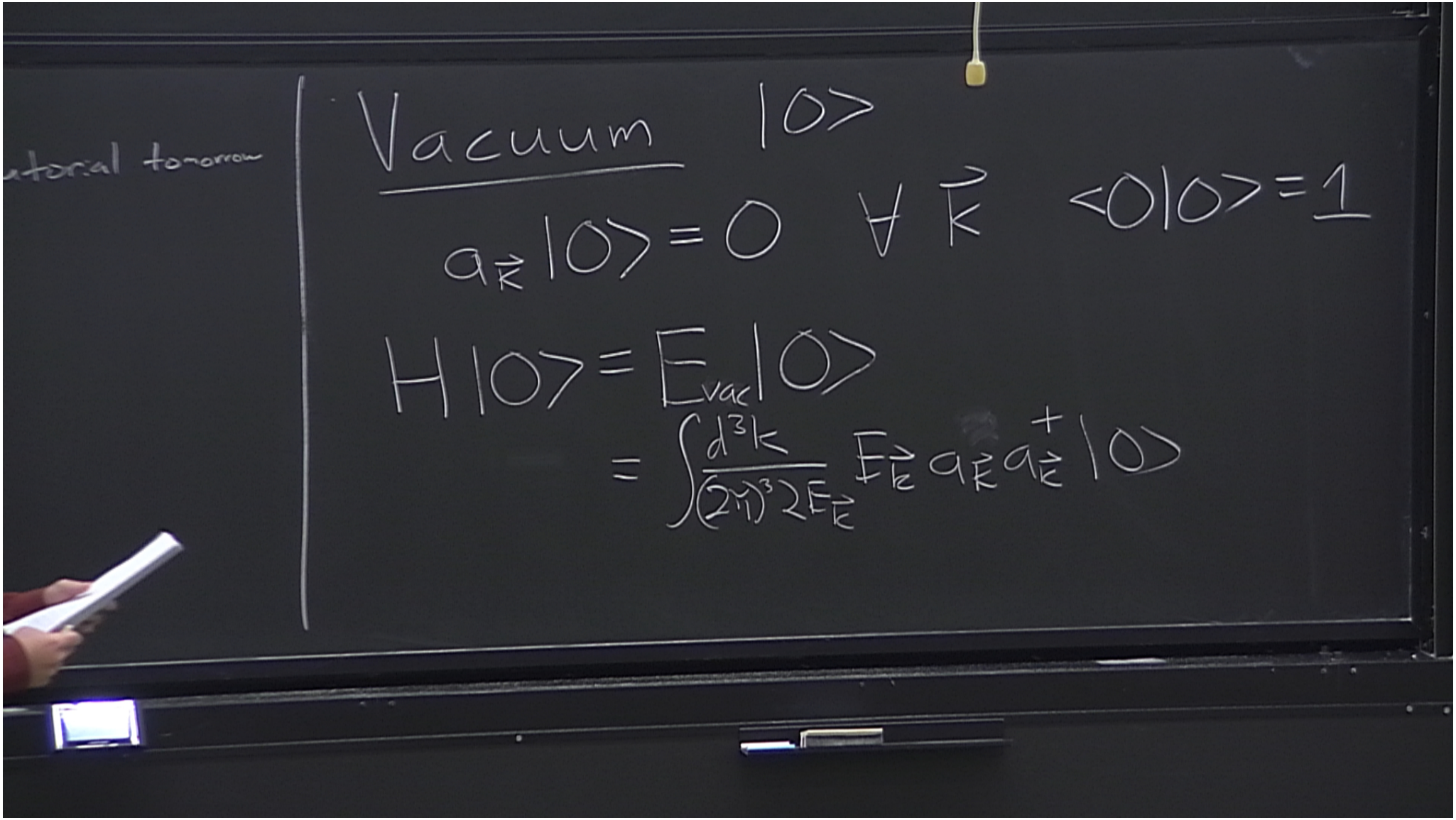
utorial tomorrow

Vacuum $|0\rangle$

$$a_{\vec{k}}|0\rangle = 0 \quad \forall \vec{k} \qquad \langle 0|0\rangle = 1$$

$$H|0\rangle = E_0|0\rangle$$

$$= \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} F_{\vec{k}}\, a^{+}_{\vec{k}} a_{\vec{k}} |0\rangle$$


utorial tomorrow
Vacuum $|0\rangle$
$a_{\vec{k}}|0\rangle = 0 \quad \forall \vec{k} \qquad \langle 0|0\rangle = 1$
$H|0\rangle = E_{vac}|0\rangle$
$= \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} F_{\vec{k}}\, a_{\vec{k}} a^{+}_{\vec{k}} |0\rangle$

torial tomorrow

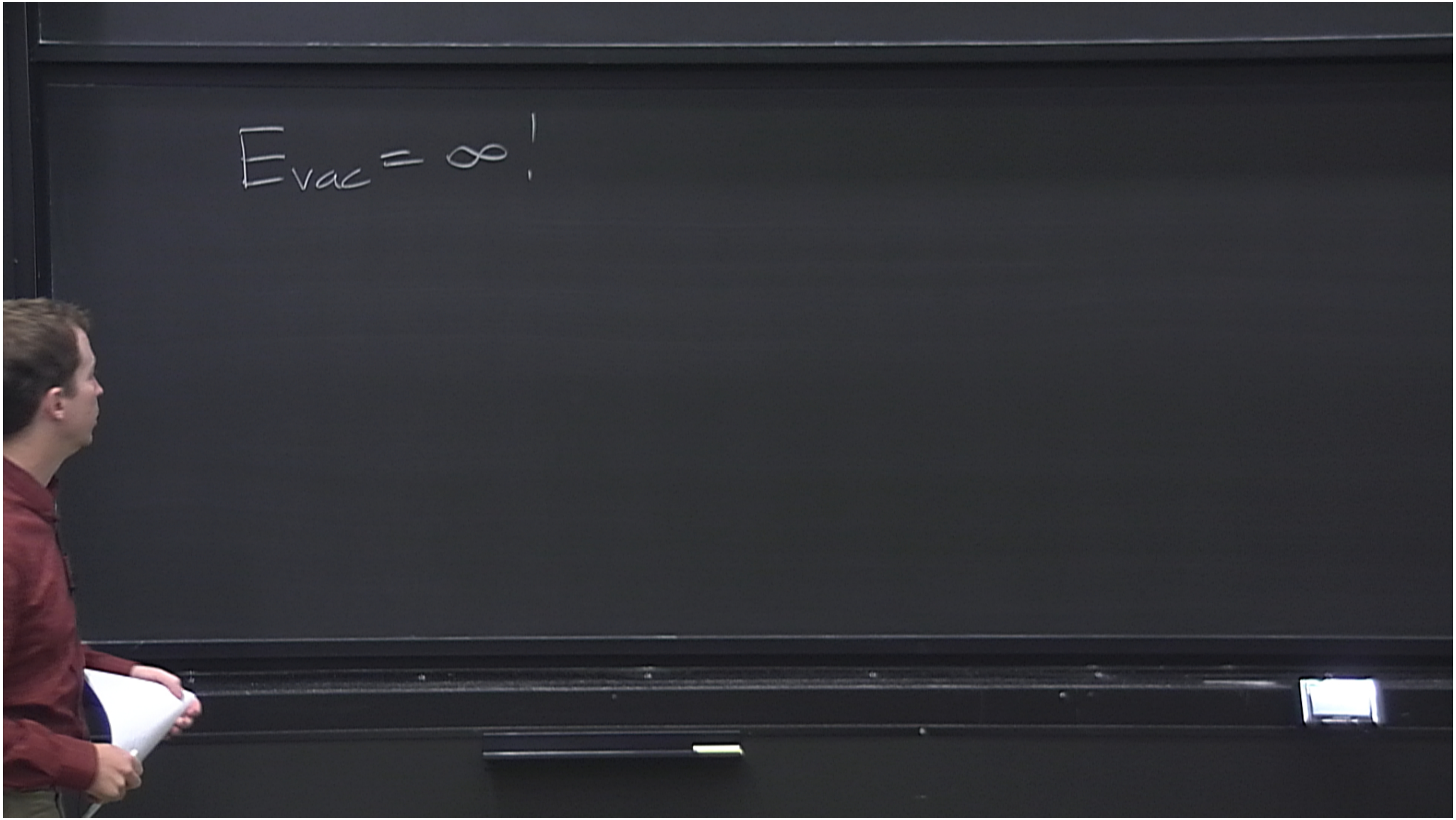
Vacuum $|0\rangle$

$a_{\vec{k}}|0\rangle = 0 \quad \forall \vec{k} \qquad \langle 0|0\rangle = 1$

$$H|0\rangle = E_{vac}|0\rangle$$

$$= \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} E_{\vec{k}}\, a_{\vec{k}} a^{+}_{\vec{k}}|0\rangle$$

$$= \int d^3k\, E_{\vec{k}}\, \delta(\vec{k}-\vec{k})|0\rangle$$


$E_{vac} = \infty$!

$$H = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} E_{\vec{k}} \left(a^\dagger_{\vec{k}} a_{\vec{k}} + a_{\vec{k}} a^\dagger_{\vec{k}}\right)$$

$$E_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$$

$$H|0\rangle = E_{vac}|0\rangle$$

$$= \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} E_{\vec{k}}$$

$$= \int d^3k \, E_{\vec{k}}$$

$E_{vac} = \infty$!

IR Divergence

Put theory in a box of size L w/ periodic bd conditions

$(2\pi)^3 \delta^{(3)}(\vec{0})$

$$H = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} E_{\vec{k}} \left(a^\dagger_{\vec{k}} a_{\vec{k}} + a_{\vec{k}} a^\dagger_{\vec{k}}\right)$$

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$$H|0\rangle = E_{vac}|0\rangle$$

$$= \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} E_{\vec{k}}$$

$$= \int d^3k \, E_{\vec{k}}$$

$E_{vac} = \infty$!

IR Divergence

Put theory in a box of size L w/ periodic bd [illegible]tions

$$(2\pi)^3 \delta^{(3)}(\vec{0}) = \lim_{L\to\infty} \int_{-\frac{L}{2}}^{\frac{L}{2}} d^3x \, e^{i\vec{x}\cdot\vec{p}}\Big|_{\vec{p}=0} = [illegible]$$

$$H = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} E_{\vec{k}} \left(a^\dagger_{\vec{k}} a_{\vec{k}} + a_{\vec{k}} a^\dagger_{\vec{k}}\right)$$

$$E_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$$

$$H|0\rangle = E_{vac}|0\rangle$$

$$= \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} E_{\vec{k}}$$

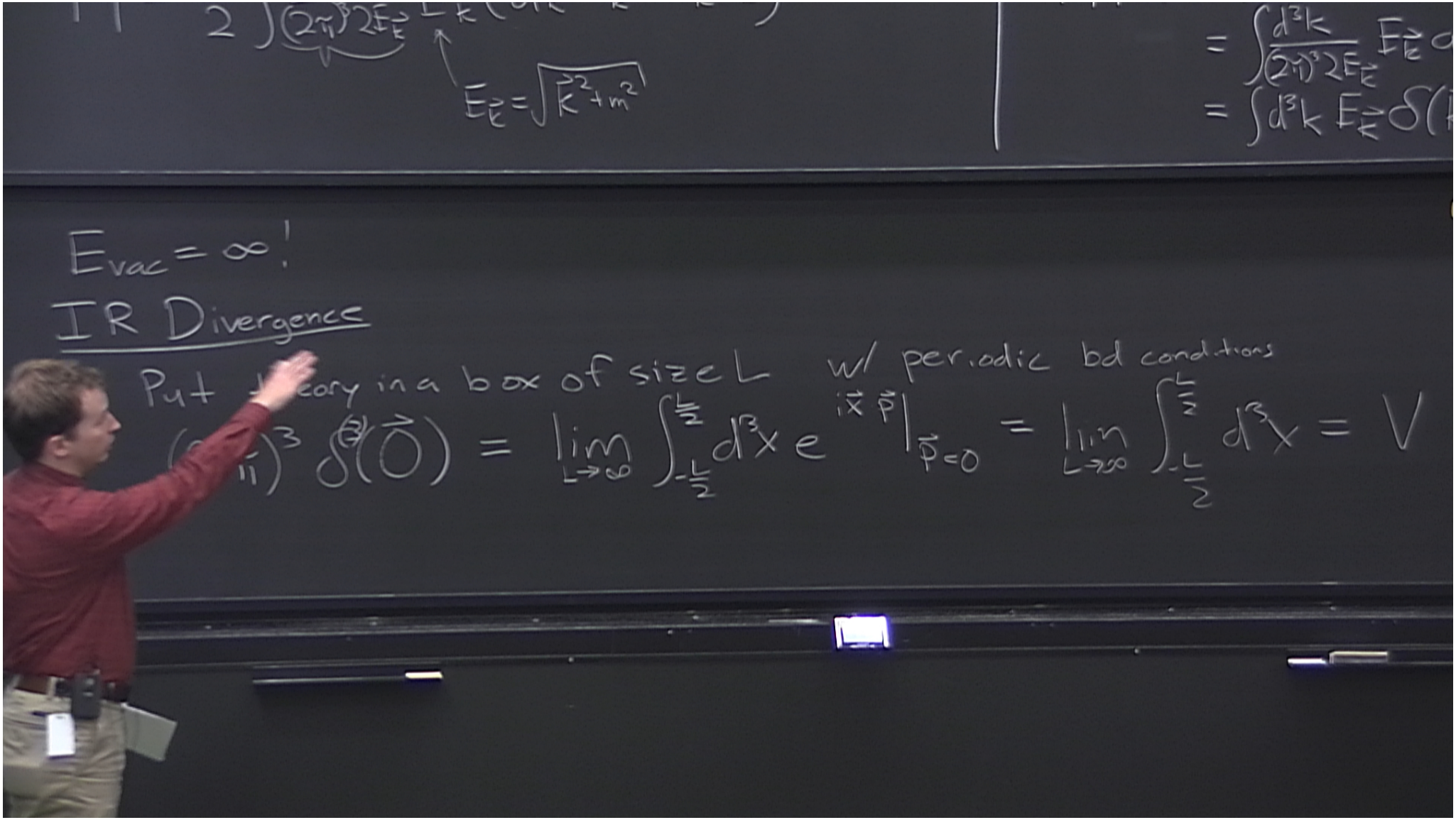
$$= \int d^3k \, E_{\vec{k}}$$

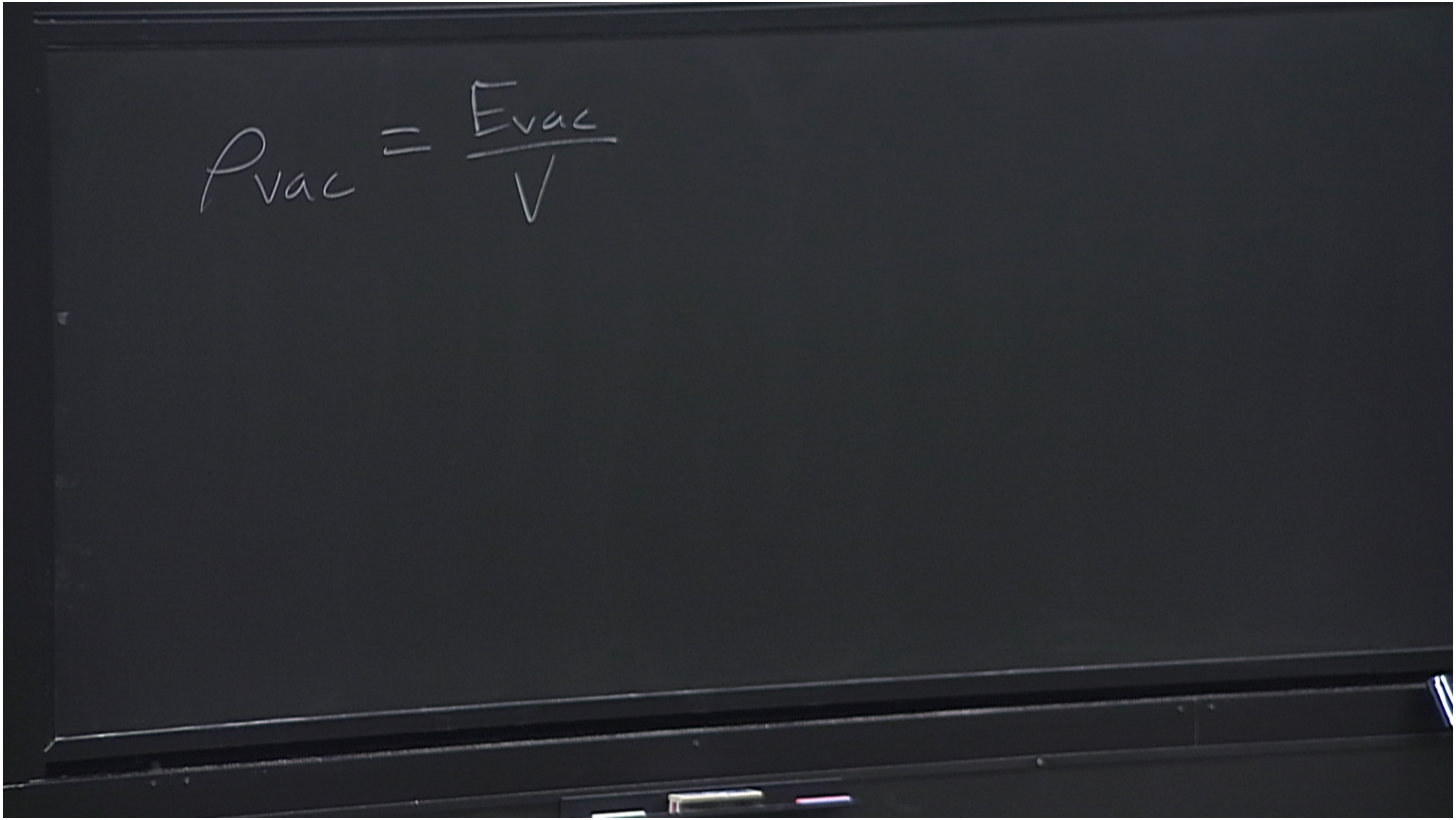
$E_{vac} = \infty$!

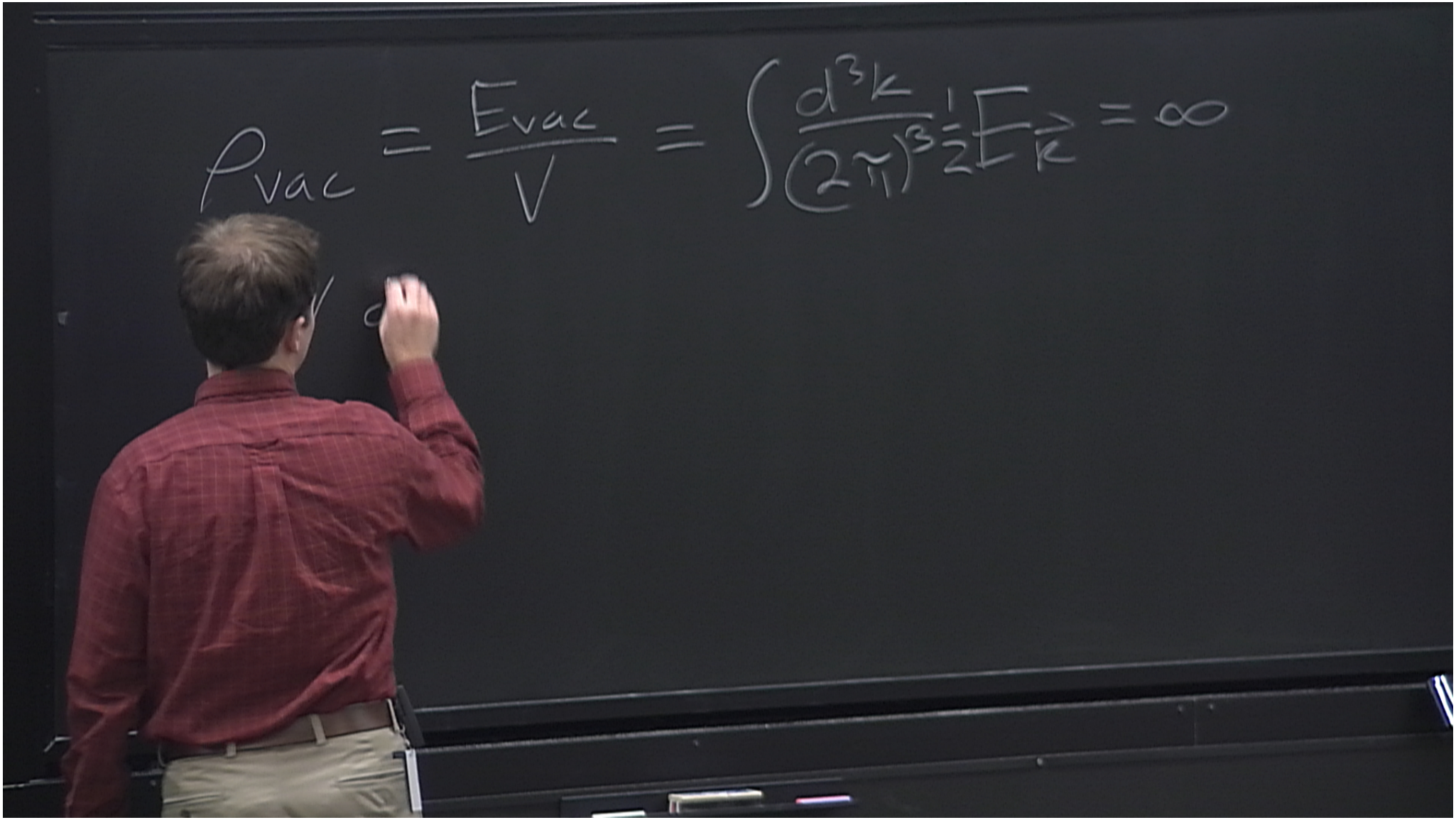
IR Divergence

Put theory in a box of size L periodic bd conditions

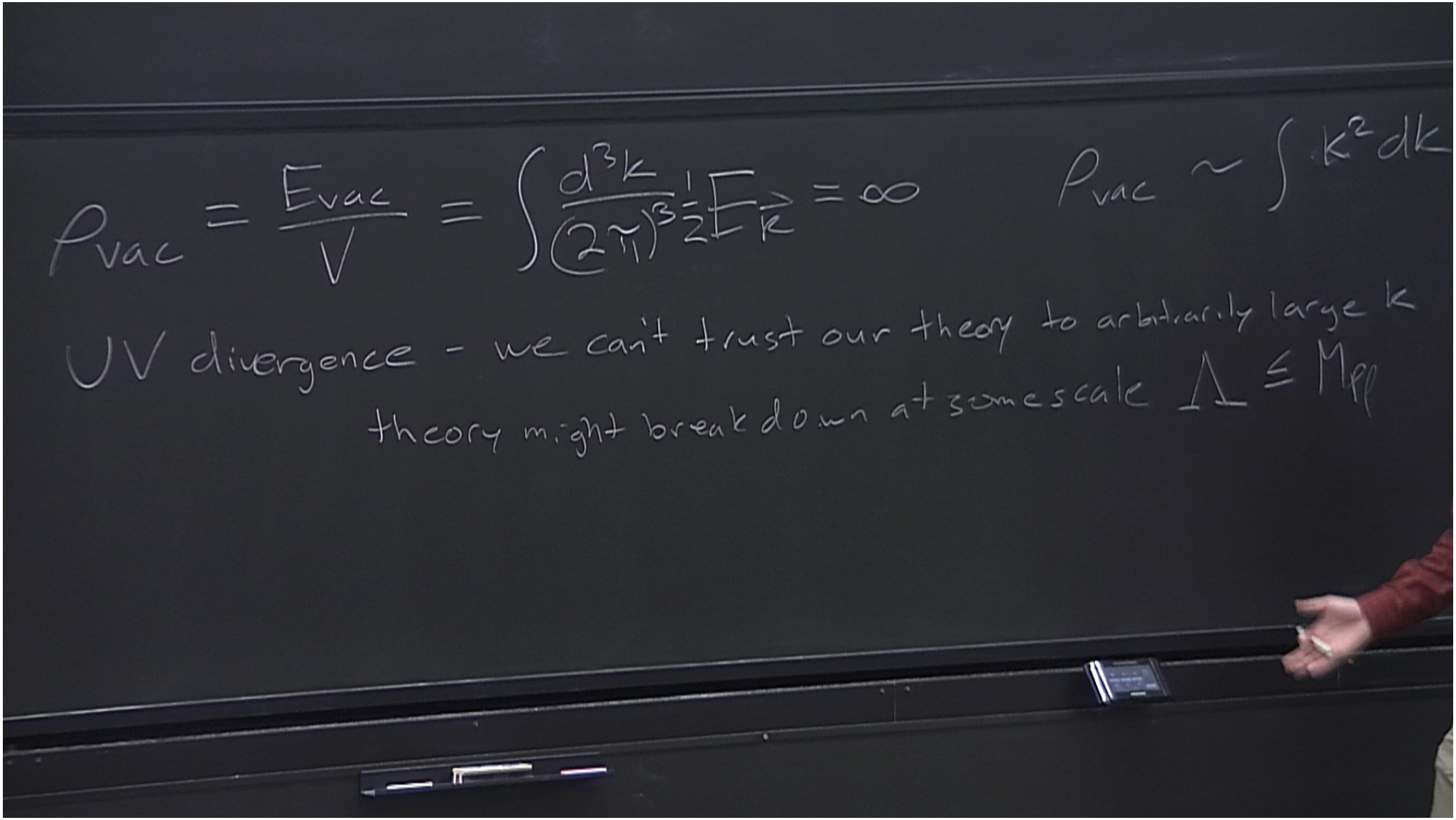
$$(2\pi)^3 \delta^{(3)}(\vec{0}) = \lim_{L\to\infty} \int_{-\frac{L}{2}}^{\frac{L}{2}} d^3x \Big|_{\vec{p}=0} = \lim_{L\to\infty} \int_{-\frac{L}{2}}^{\frac{L}{2}} d^3x$$


$E_{vac} = \infty$!
IR Divergence
periodic bd conditions
$\delta^{(3)}(\vec{0}) = \lim_{L\to\infty} \int_{-\frac{L}{2}}^{\frac{L}{2}} d^3x\, e^{i\vec{x}\cdot\vec{p}}\Big|_{\vec{p}=0} = \lim_{L\to\infty} \int_{-\frac{L}{2}}^{\frac{L}{2}} d^3x = V$


$\rho_{vac} = \frac{E_{vac}}{V}$


$\rho_{vac} = \frac{E_{vac}}{V} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} E_{\vec{k}} = \infty$


$\rho_{vac} \sim \int k^2 dk \cdot k$

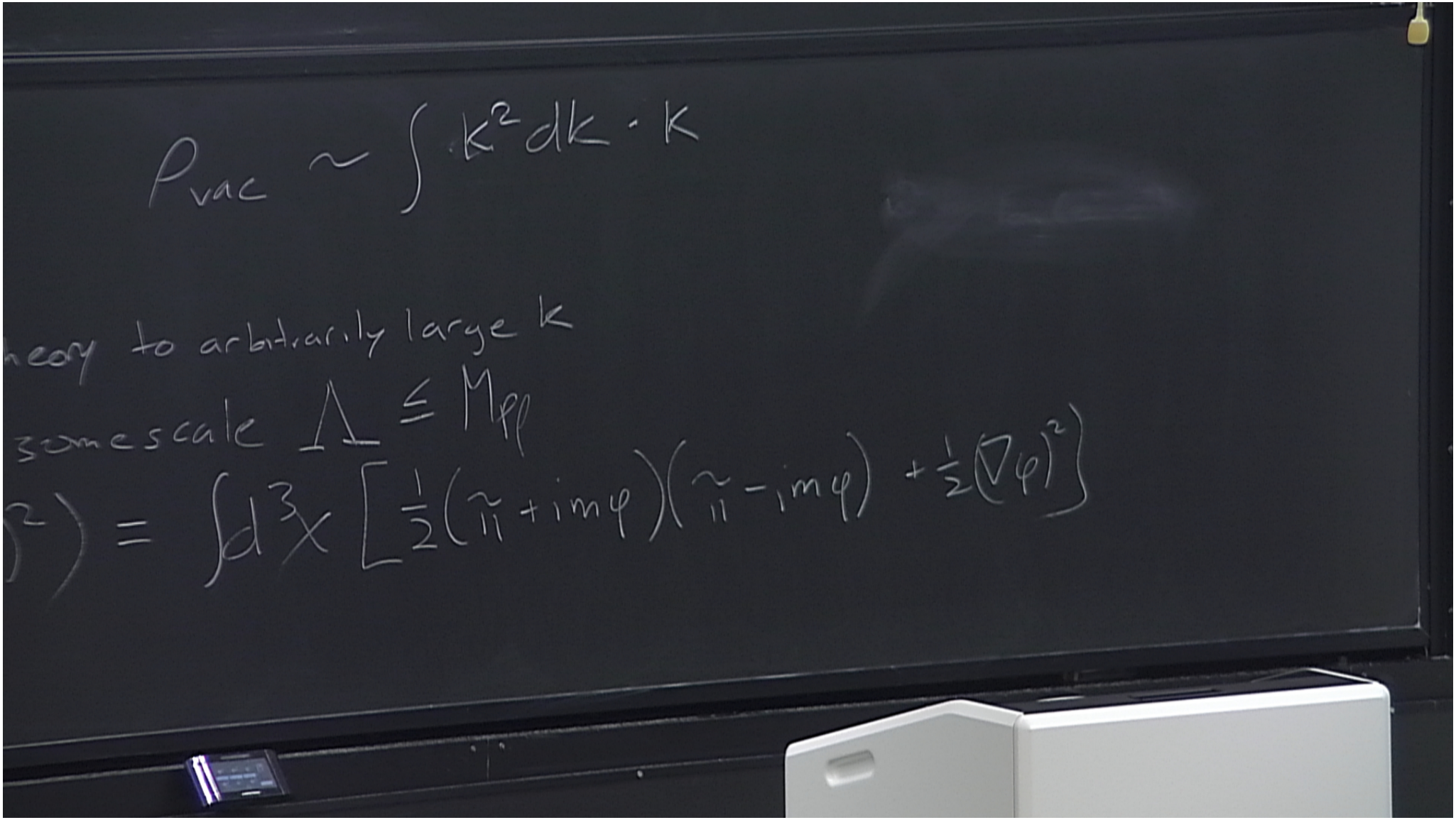

$\rho_{vac} = \frac{E_{vac}}{V} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} E_{\vec{k}} = \infty$
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UV divergence - we can't trust our theory to arbitrarily large k
theory might break down at some scale $\Lambda \leq M_{pl}$

$$\rho_{vac} = \frac{E_{vac}}{V} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} E_{\vec{k}} = \infty \qquad \rho_{vac} \sim \int k^2 dk$$

UV divergence - we can't trust our theory to arbitrarily large k

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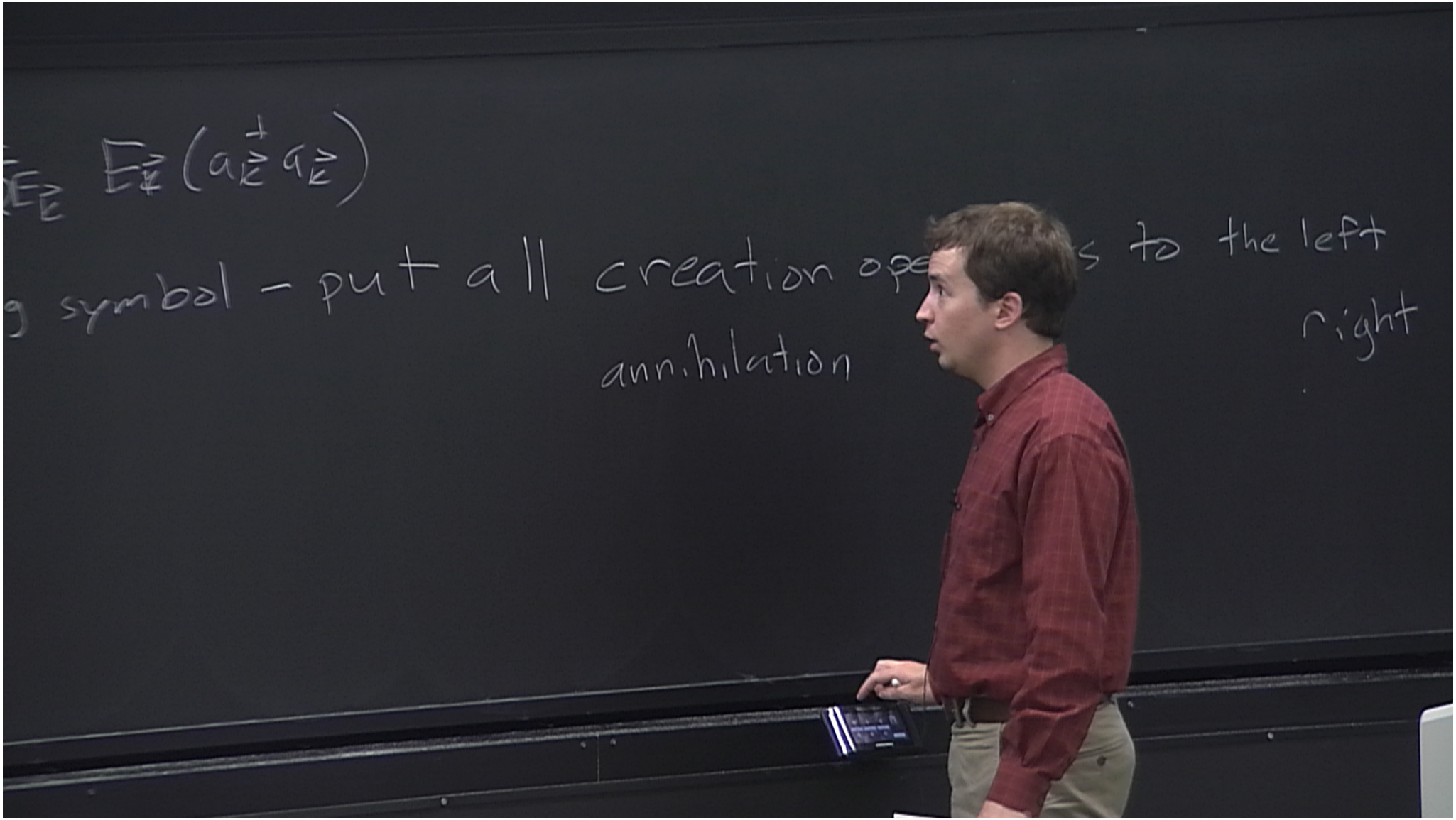
$$H = \int d^3x \left(\frac{1}{2}\pi^2 + \frac{1}{2}m^2\varphi^2 + \frac{1}{2}(\nabla\varphi)^2 \right) = \int d^3x \left[\frac{1}{2}(\pi + im\varphi) \right.$$


$\rho_{vac} \sim \int k^2 dk \cdot k$
heory to arbitrarily large k
somescale $\Lambda \leq M_{pl}$
$= \int d^3x \left[\frac{1}{2}(\tilde{\pi} + im\varphi)(\tilde{\pi} - im\varphi) + \frac{1}{2}(\nabla\varphi)^2 \right]$

$$:H: = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} E_{\vec{k}} \left(a^\dagger_{\vec{k}} a_{\vec{k}}\right)$$

normal ordering symbol – put all creation op

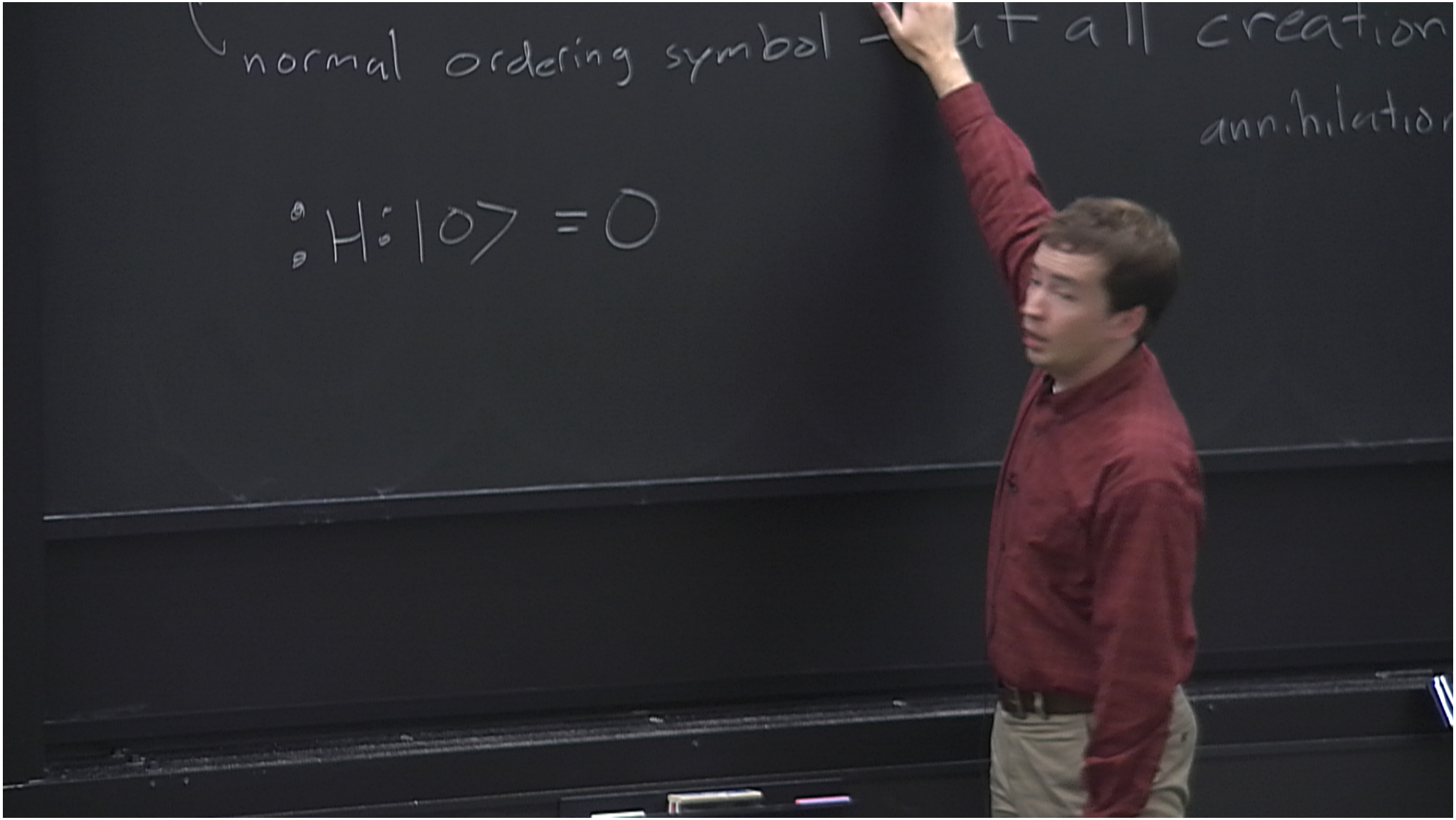
annihilation


$E_{\vec{k}}\left(a^{\dagger}_{\vec{k}} a_{\vec{k}}\right)$
g symbol - put all creation
annihilation
to the left
right

$E_{\vec{k}}$ $E_{\vec{k}}(a^{\dagger}_{\vec{k}} a_{\vec{k}})$

symbol - put all creation operators to the left

annihilation right


normal ordering symbol
creation
annihilation
:H:|0> = 0

$$\frac{E_{vac}}{V} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} E_{\vec{k}} = \infty \qquad \rho_{vac} \sim \int k^2 dk \cdot k$$

ence - we can't trust our theory to arbitrarily large k

theory might break down at some scale $\Lambda \leq M_{pl}$

$$\int d^3x \left(\frac{1}{2}\tilde{\pi}^2 + \frac{1}{2}m^2\varphi^2 + \frac{1}{2}(\nabla\varphi)^2\right) = \int d^3x \left[\frac{1}{2}(\tilde{\pi} + im\varphi)(\tilde{\pi} - im\varphi) + \frac{1}{2}(\nabla\varphi)^2\right]$$

$$= \int d^3x \left\{\frac{1}{2}\tilde{\pi}^2 + \frac{m}{2}\varphi^2 + \frac{im}{2}\underbrace{(\varphi\tilde{\pi} - \tilde{\pi}\varphi)}_{[\varphi,\tilde{\pi}]} + \frac{1}{2}(\nabla\varphi)^2\right\}$$

$$\int d^3k \quad \Gamma_{\vec{k}}(a^\dagger_{\vec{k}}(t))$$

$$\rho_{vac} = \frac{E_{vac}}{V} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} E_{\vec{k}} = \infty \qquad \rho_{vac} \sim \int k^2 dk \cdot k$$

UV divergence - we can't trust our theory to arbitrarily large k

theory might break down at some scale $\Lambda \leq M_{pl}$

$$H = \int d^3x \left(\frac{1}{2}\tilde{\pi}^2 + \frac{1}{2}m^2\varphi^2 + \frac{1}{2}(\nabla\varphi)^2 \right) = \int d^3x \left[\frac{1}{2}(\tilde{\pi} + im\varphi)(\tilde{\pi} - im\varphi) + \frac{1}{2}(\nabla\varphi)^2 \right]$$

$$= \int d^3x \left[\frac{1}{2}\tilde{\pi}^2 + \frac{m}{2}\varphi^2 + \frac{im}{2}(\varphi\tilde{\pi} - \tilde{\pi}\varphi) + \frac{1}{2}(\nabla\varphi)^2 \right]$$

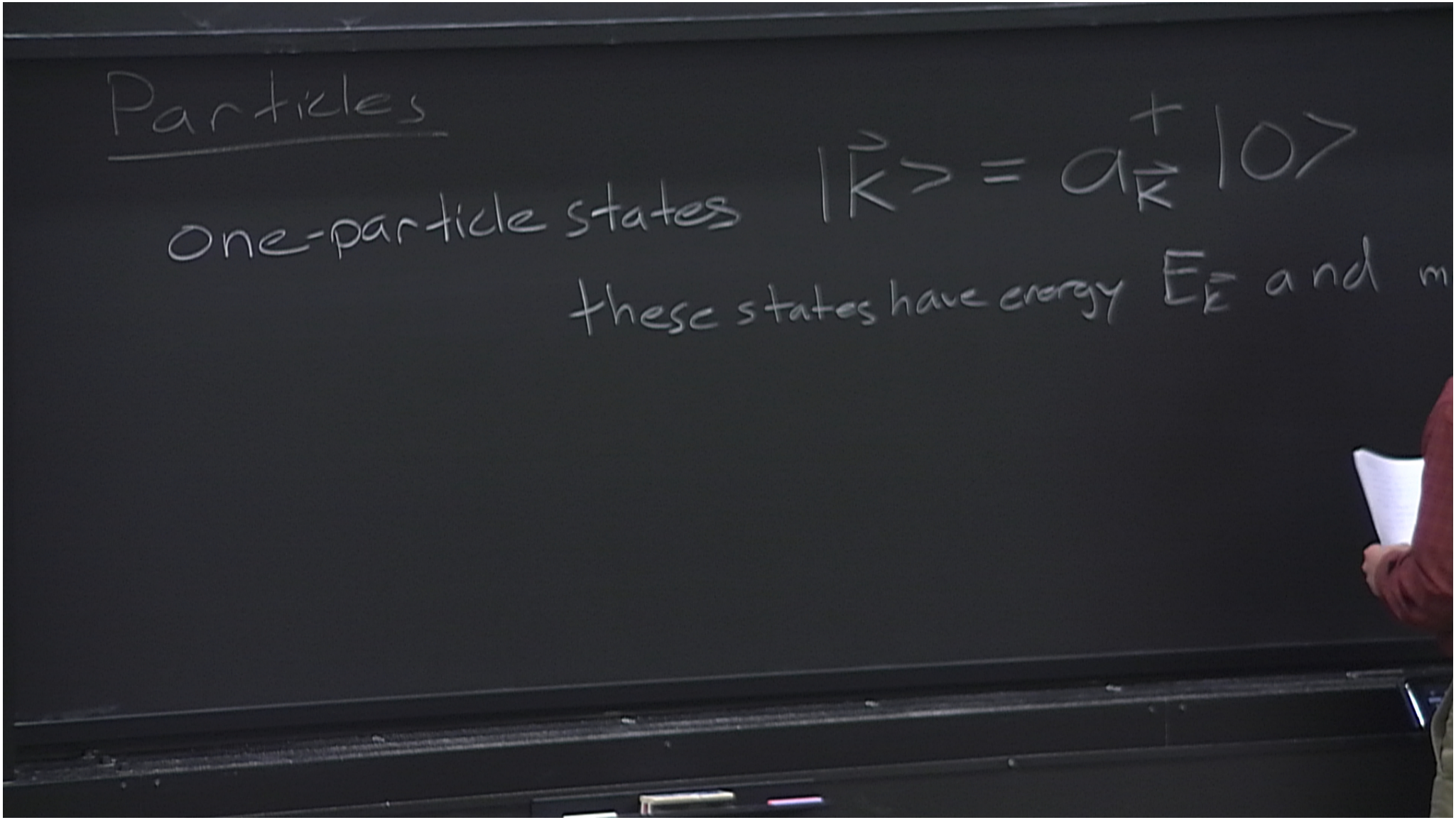
$[\varphi, \tilde{\pi}]$

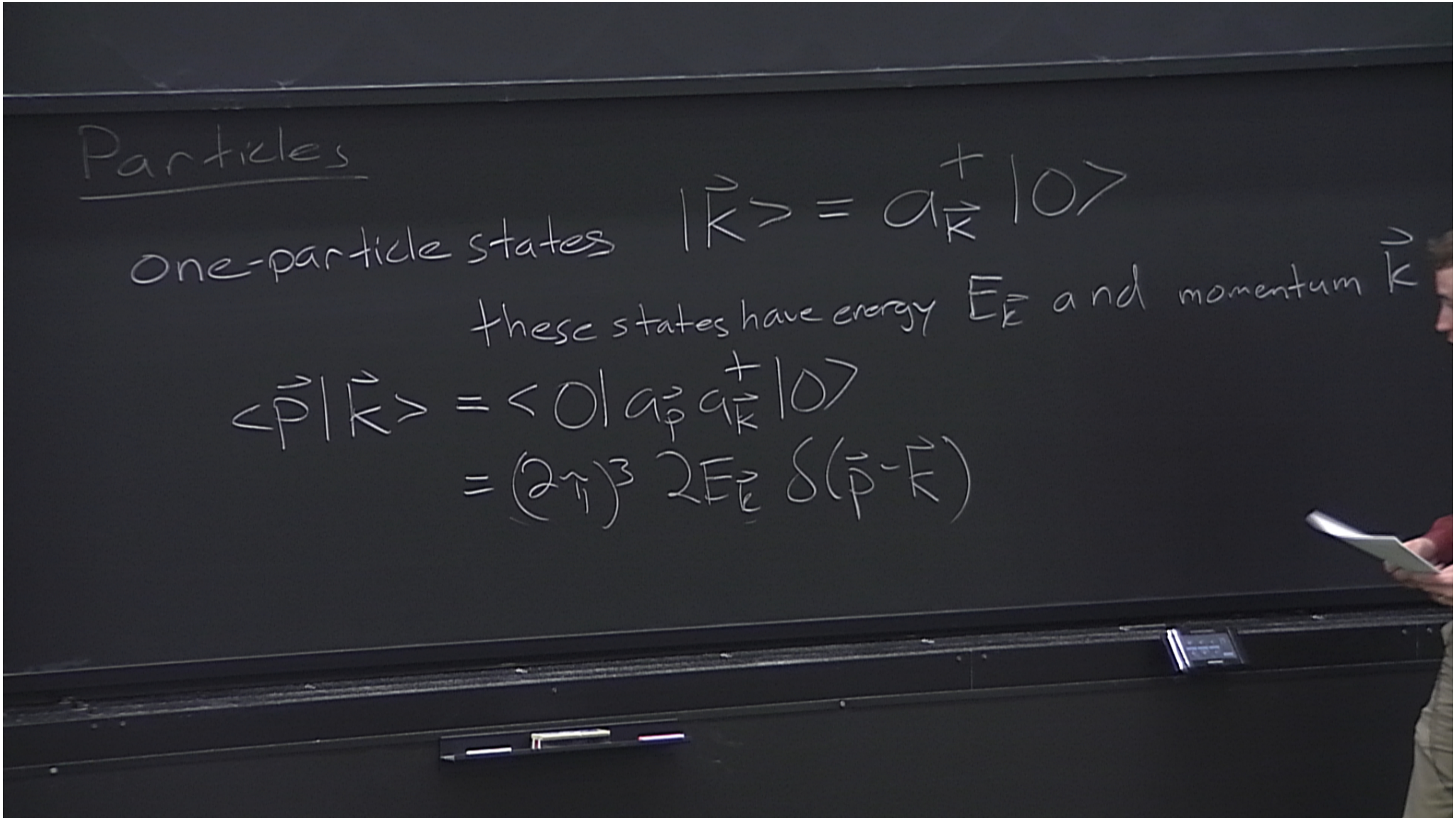
$$:H: = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} E_{\vec{k}} \left(a^\dagger_{\vec{k}} a_{\vec{k}} \right)$$

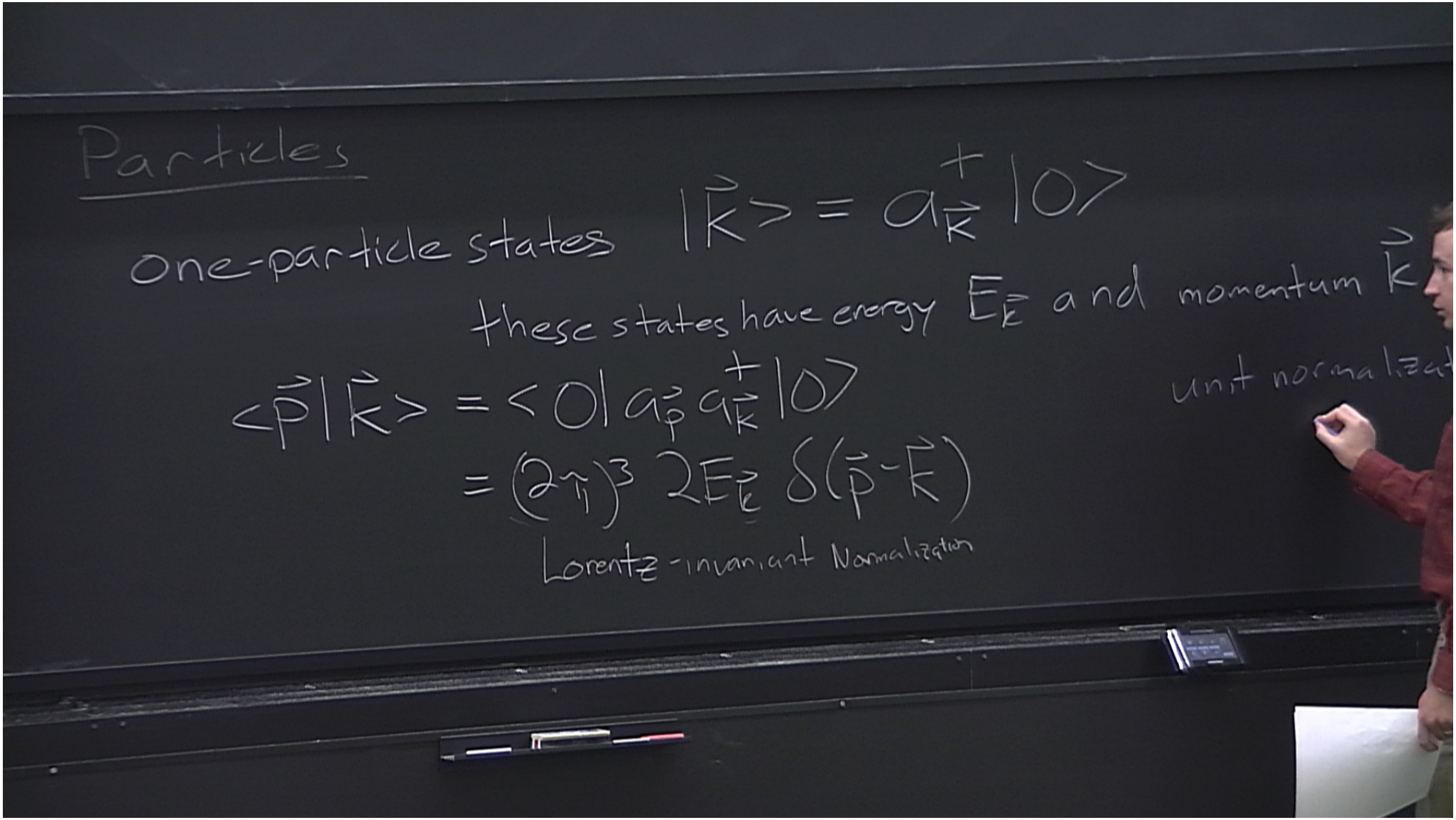
normal ordering symbol - put all creation operators to the left

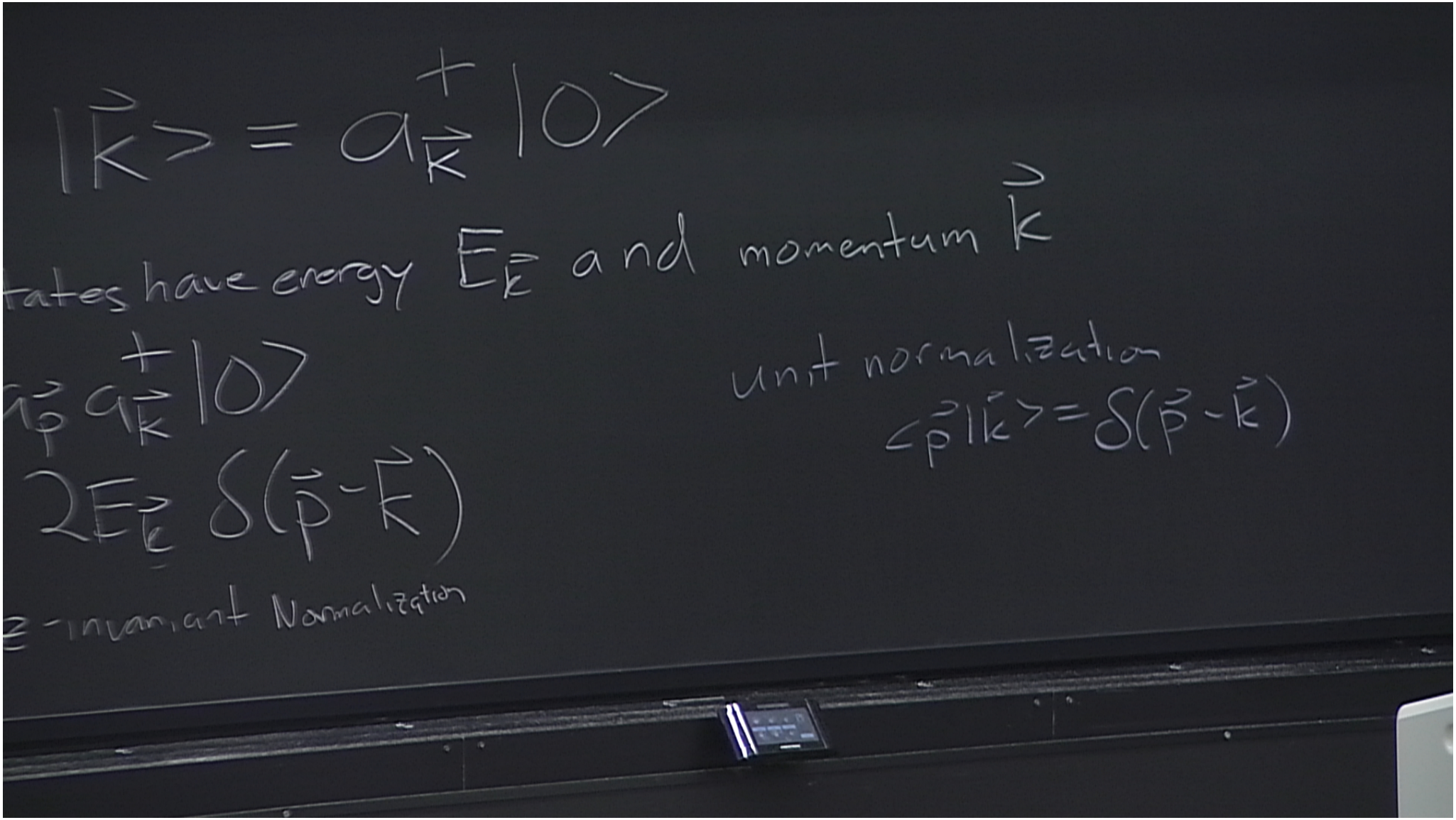
annihilat... right

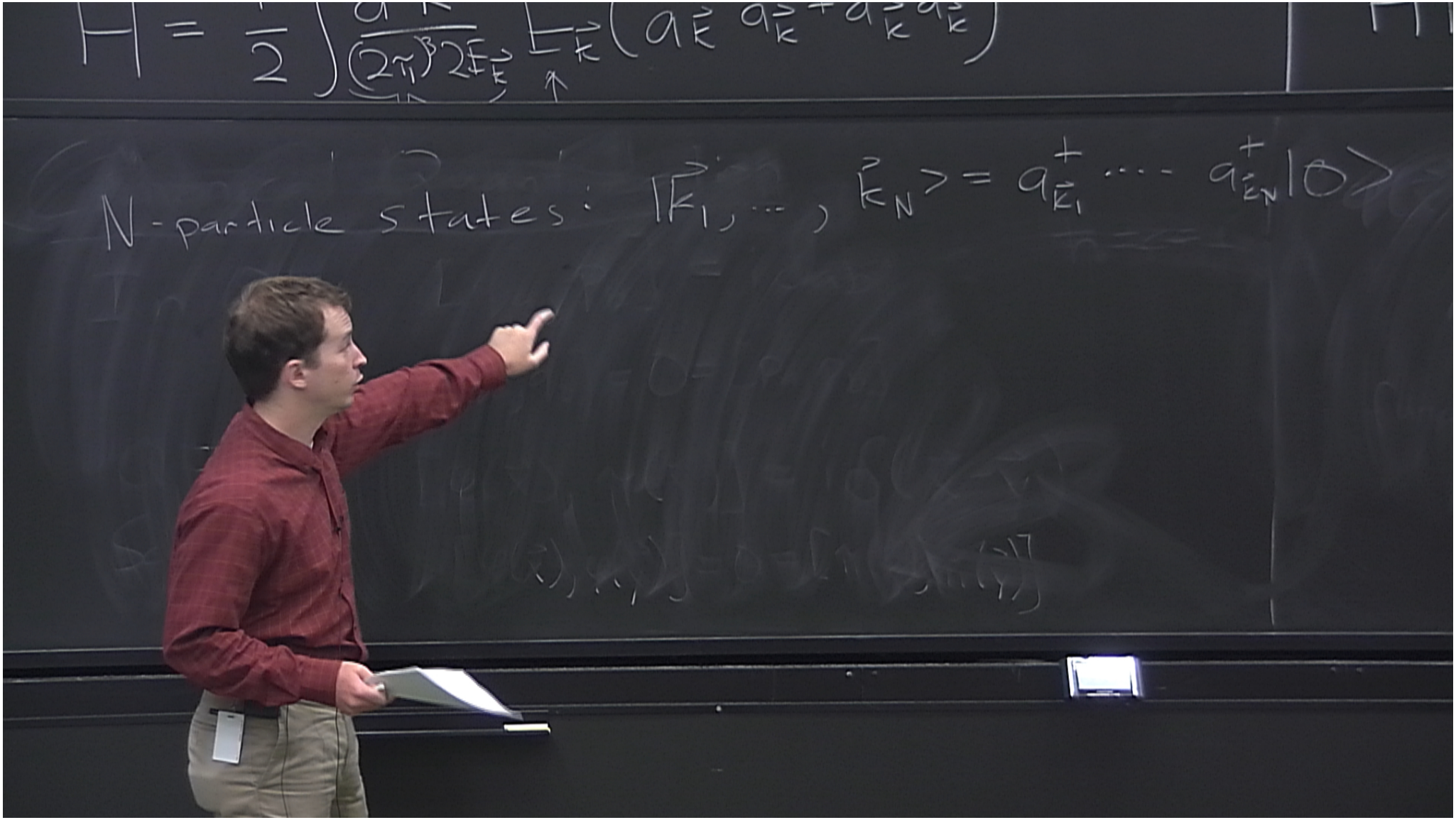
$$:H:|0\rangle = 0$$

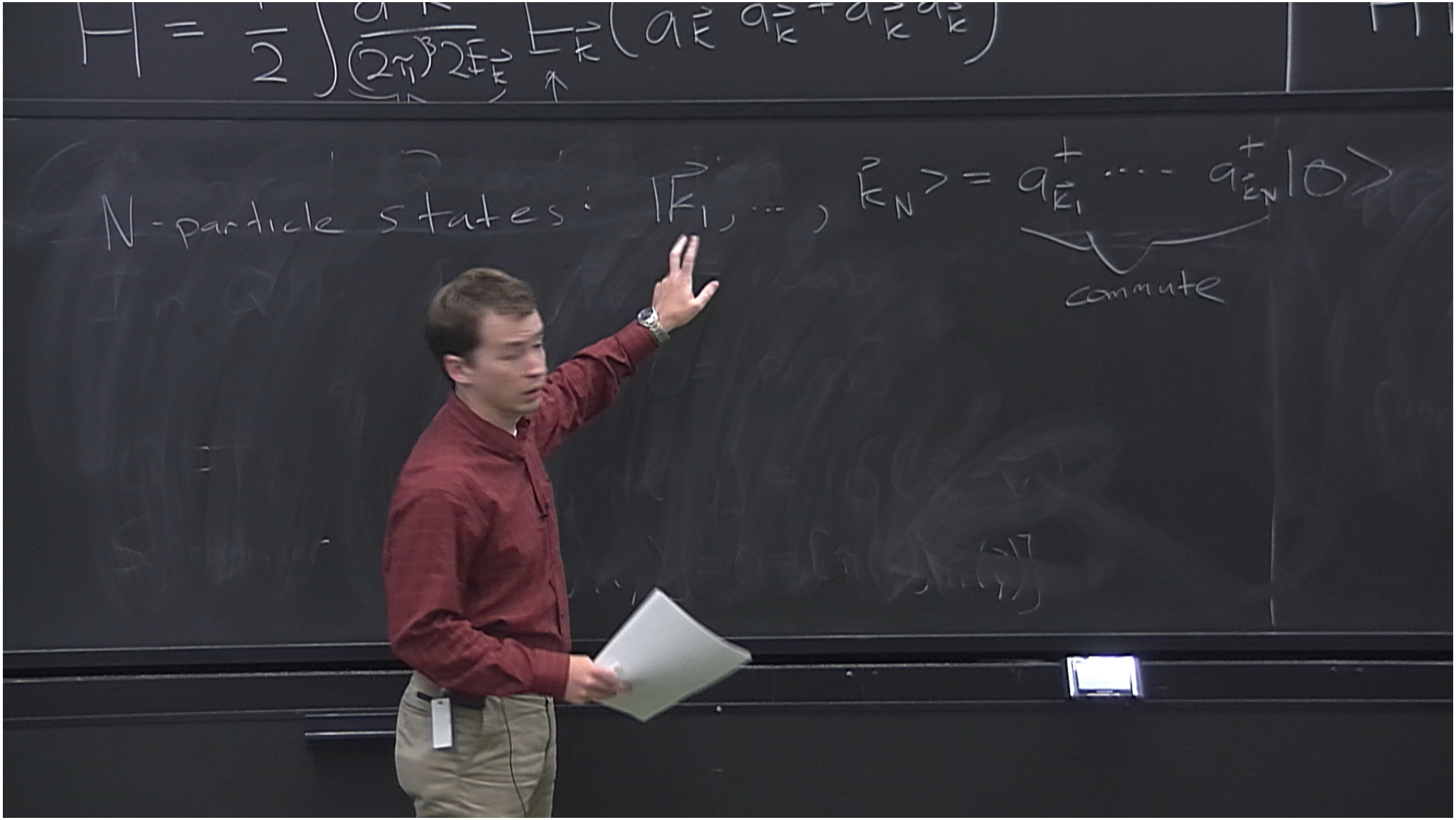

Particles
one-particle states $|\vec{k}\rangle = a^{\dagger}_{\vec{k}}|0\rangle$
these states have energy $E_{\vec{k}}$ and m

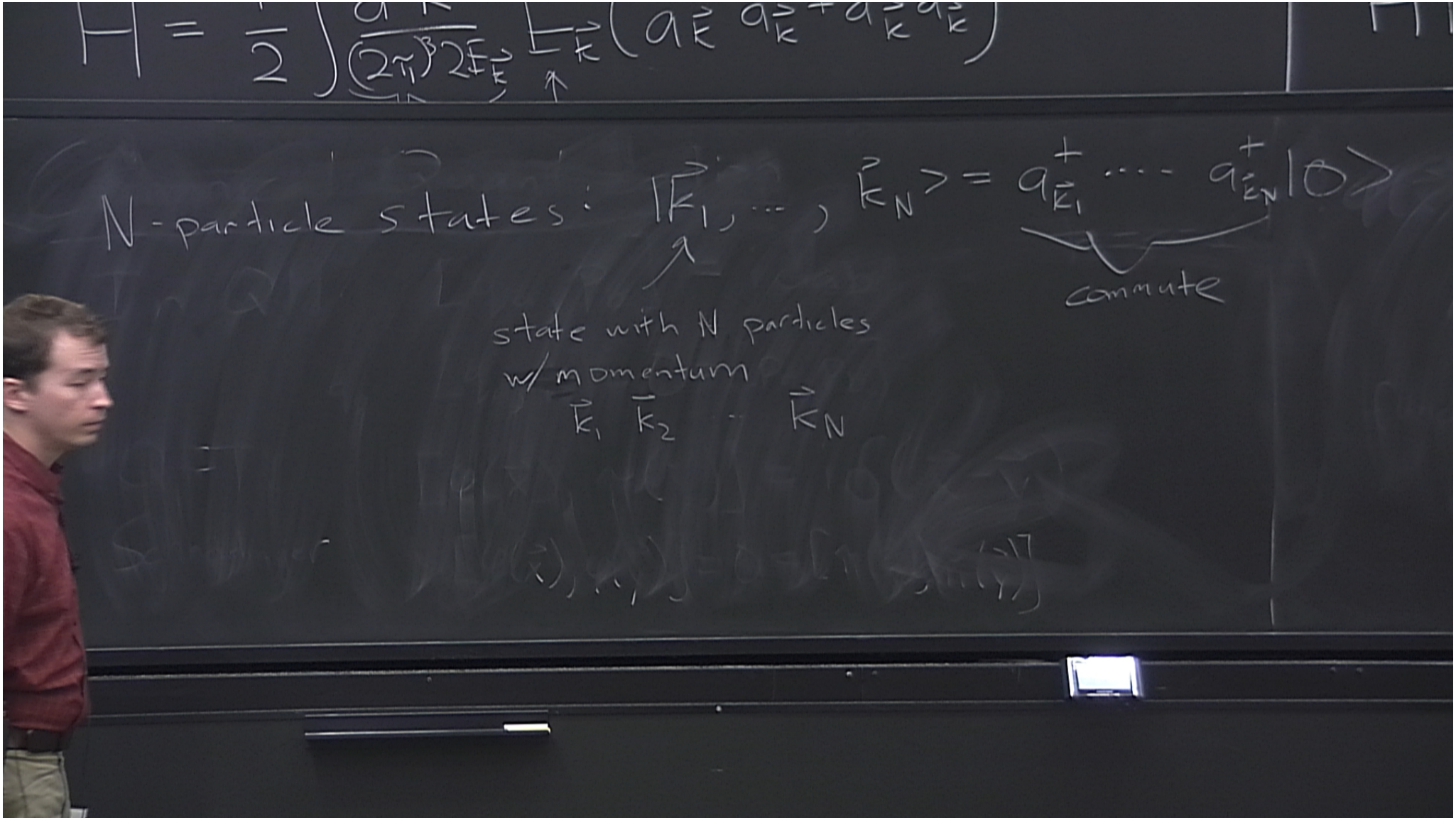

Particles
one-particle states $|\vec{k}\rangle = a^\dagger_{\vec{k}}|0\rangle$
these states have energy $E_{\vec{k}}$ and momentum $\vec{k}$
$\langle\vec{p}|\vec{k}\rangle = \langle 0|a_{\vec{p}}\, a^\dagger_{\vec{k}}|0\rangle$
$= (2\pi)^3 2E_{\vec{k}}\,\delta(\vec{p}-\vec{k})$

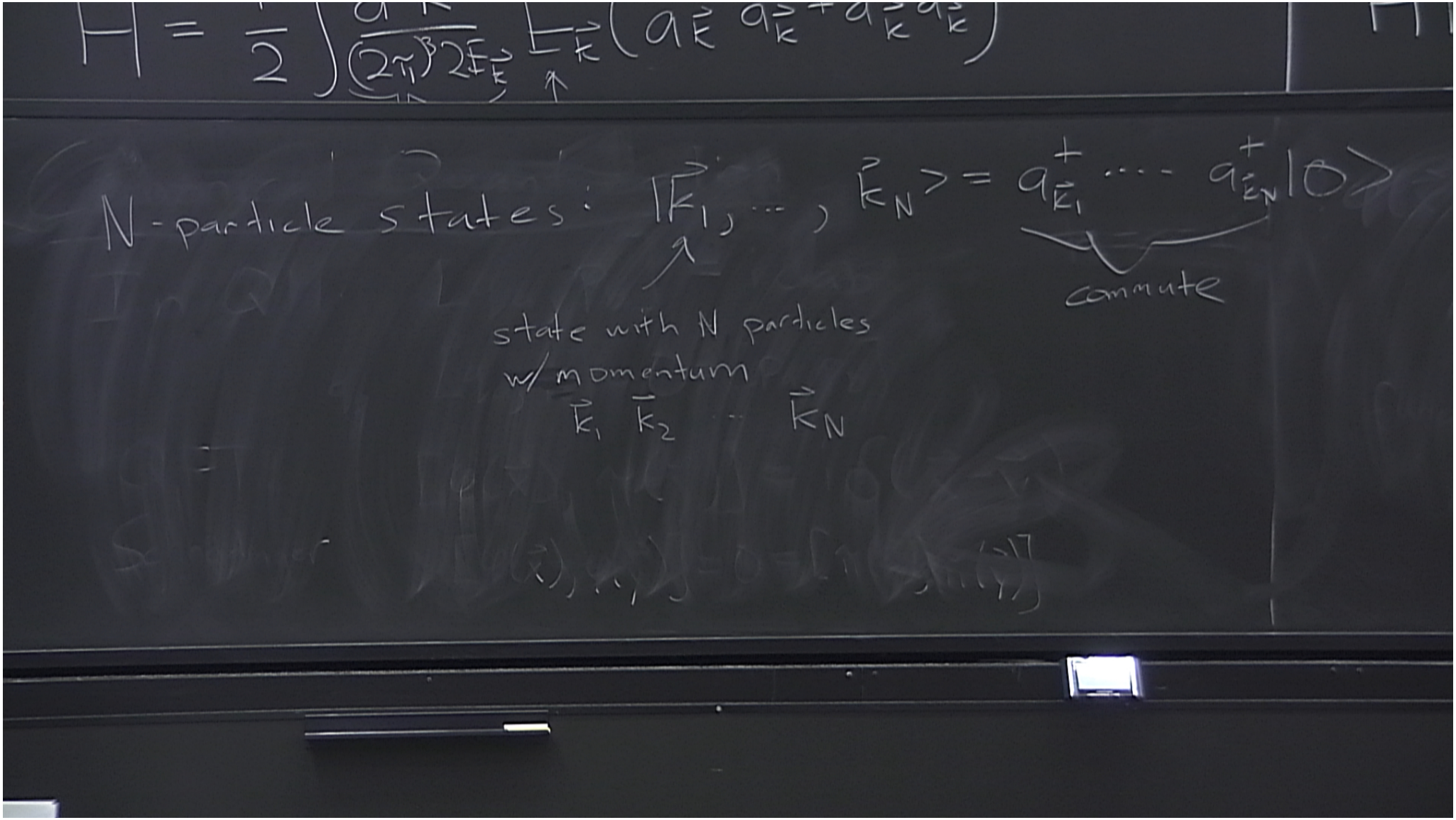

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Lorentz-invariant Normalization
unit normaliza

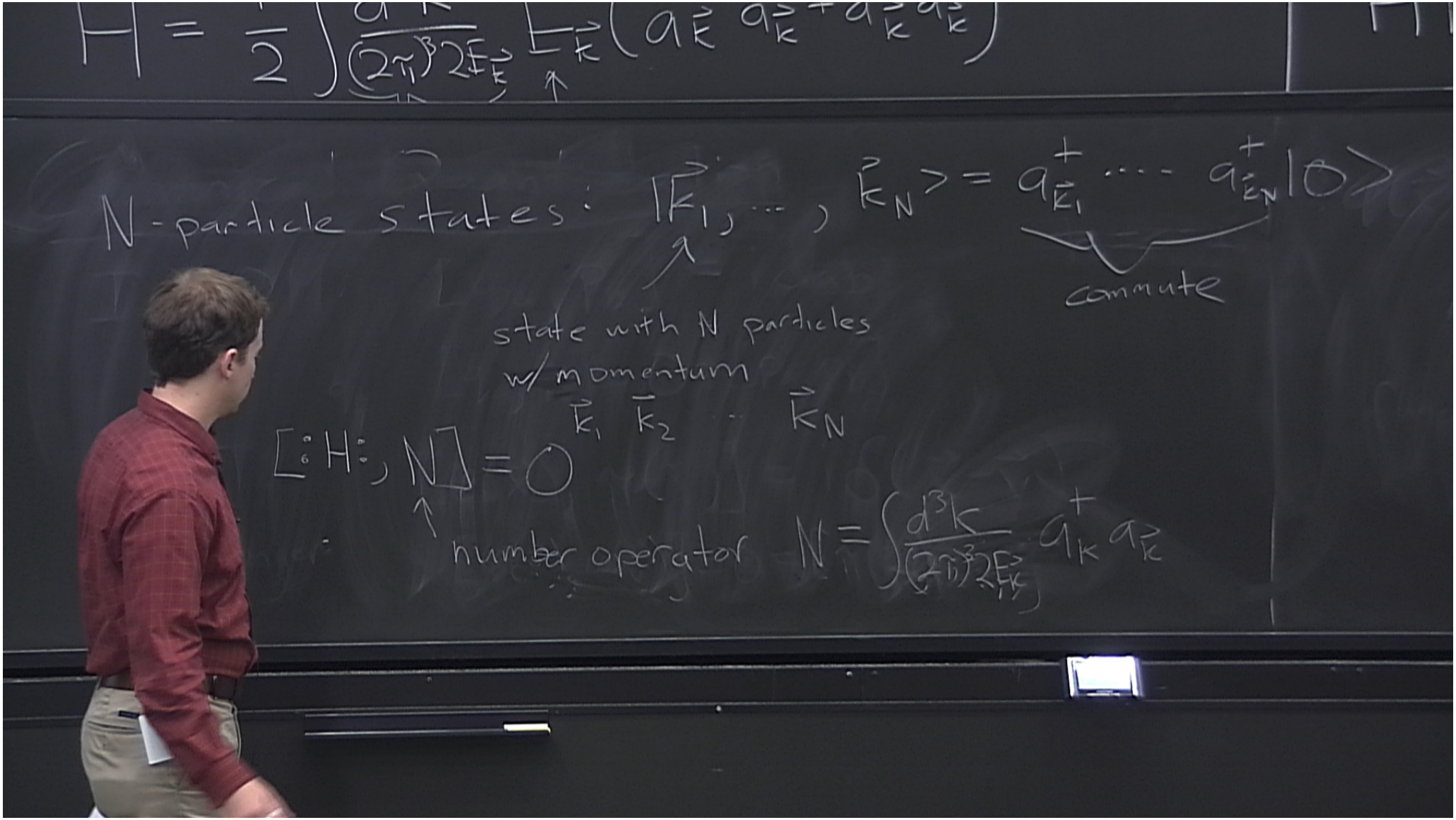

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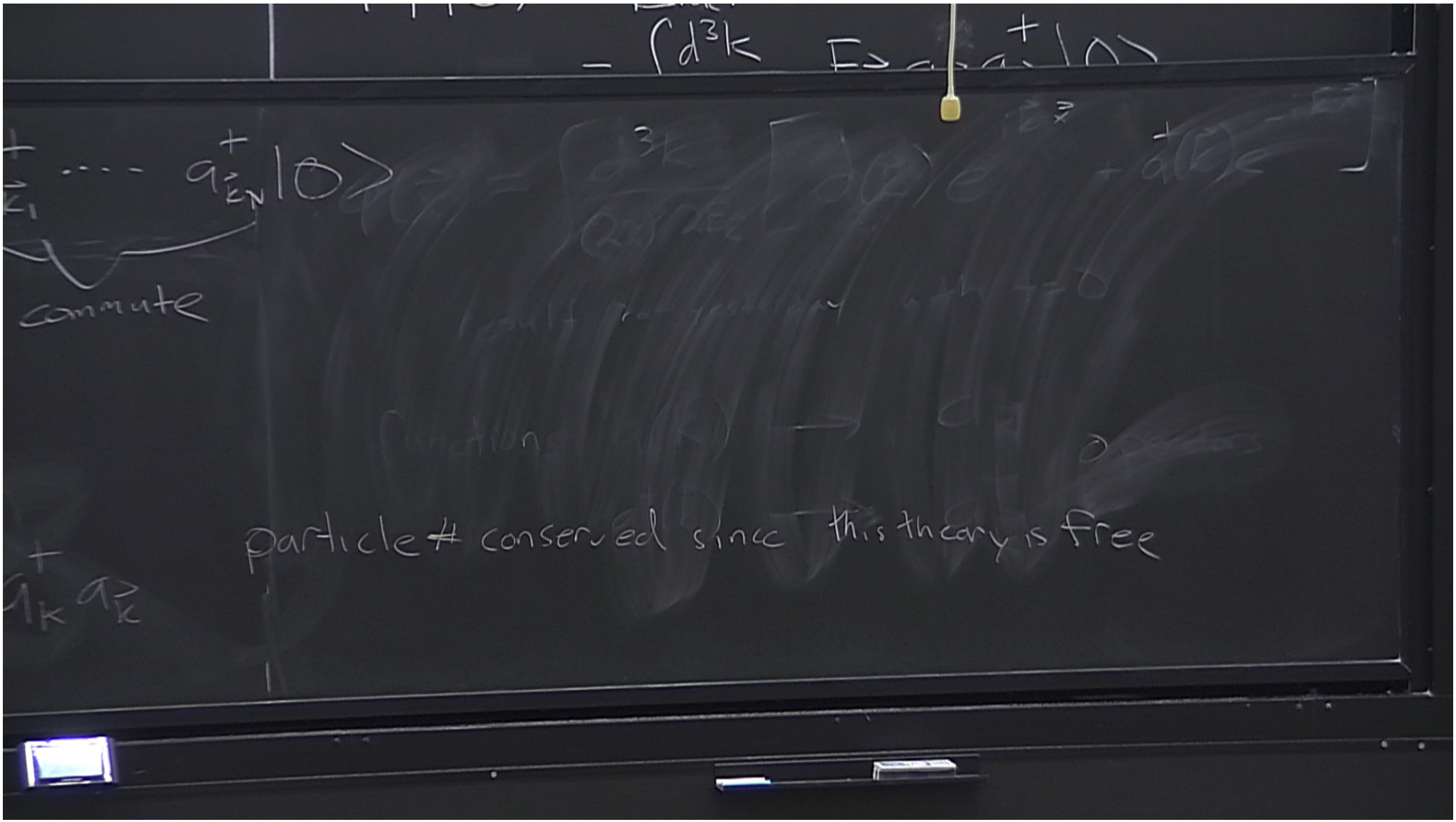

N-particle states: $|\vec{k}_1, \cdots, \vec{k}_N\rangle = a^\dagger_{\vec{k}_1} \cdots a^\dagger_{\vec{k}_N} |0\rangle$

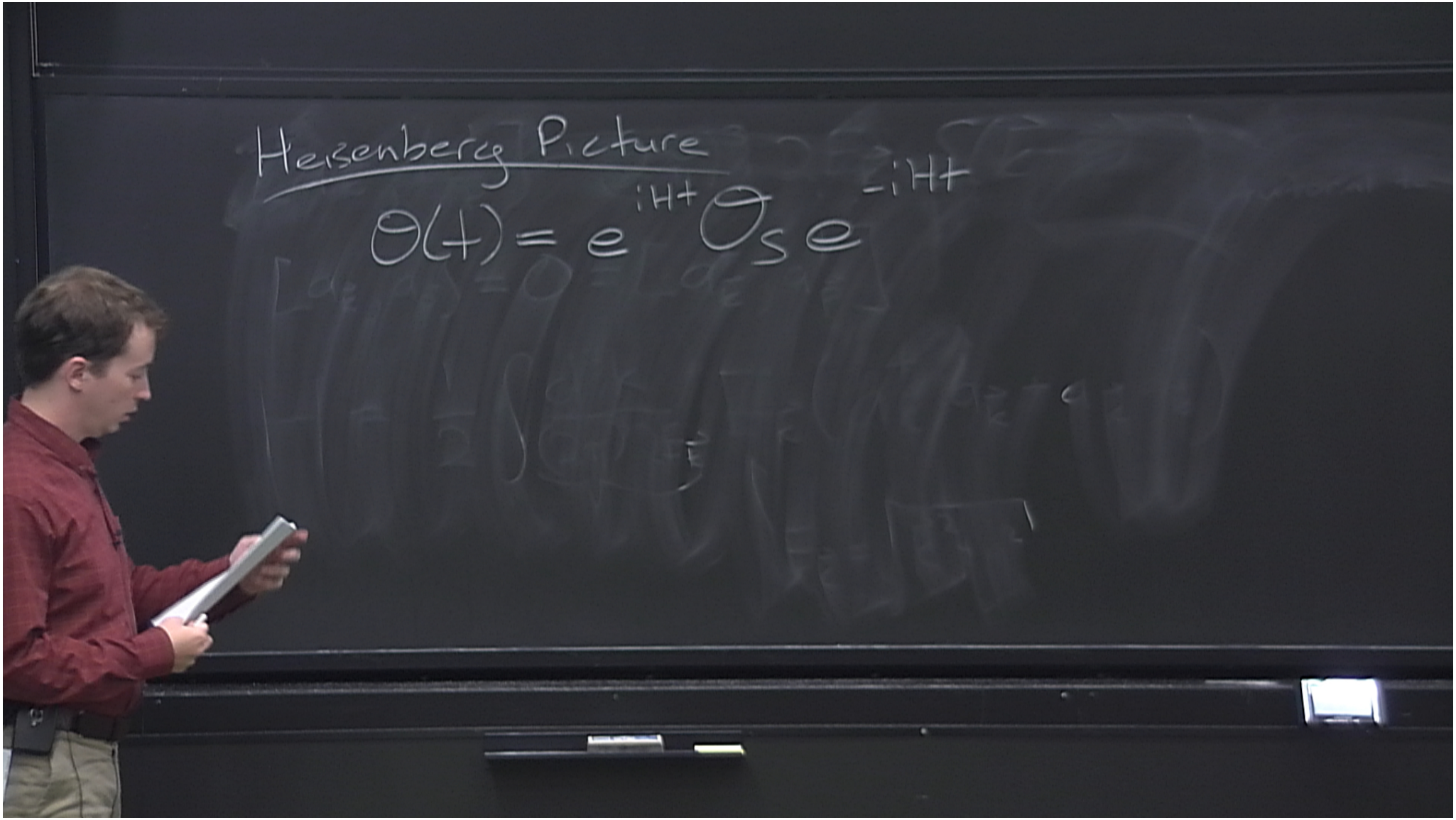

N-particle states: $|\vec{k}_1, \cdots, \vec{k}_N\rangle = a^\dagger_{\vec{k}_1} \cdots a^\dagger_{\vec{k}_N}|0\rangle$
commute

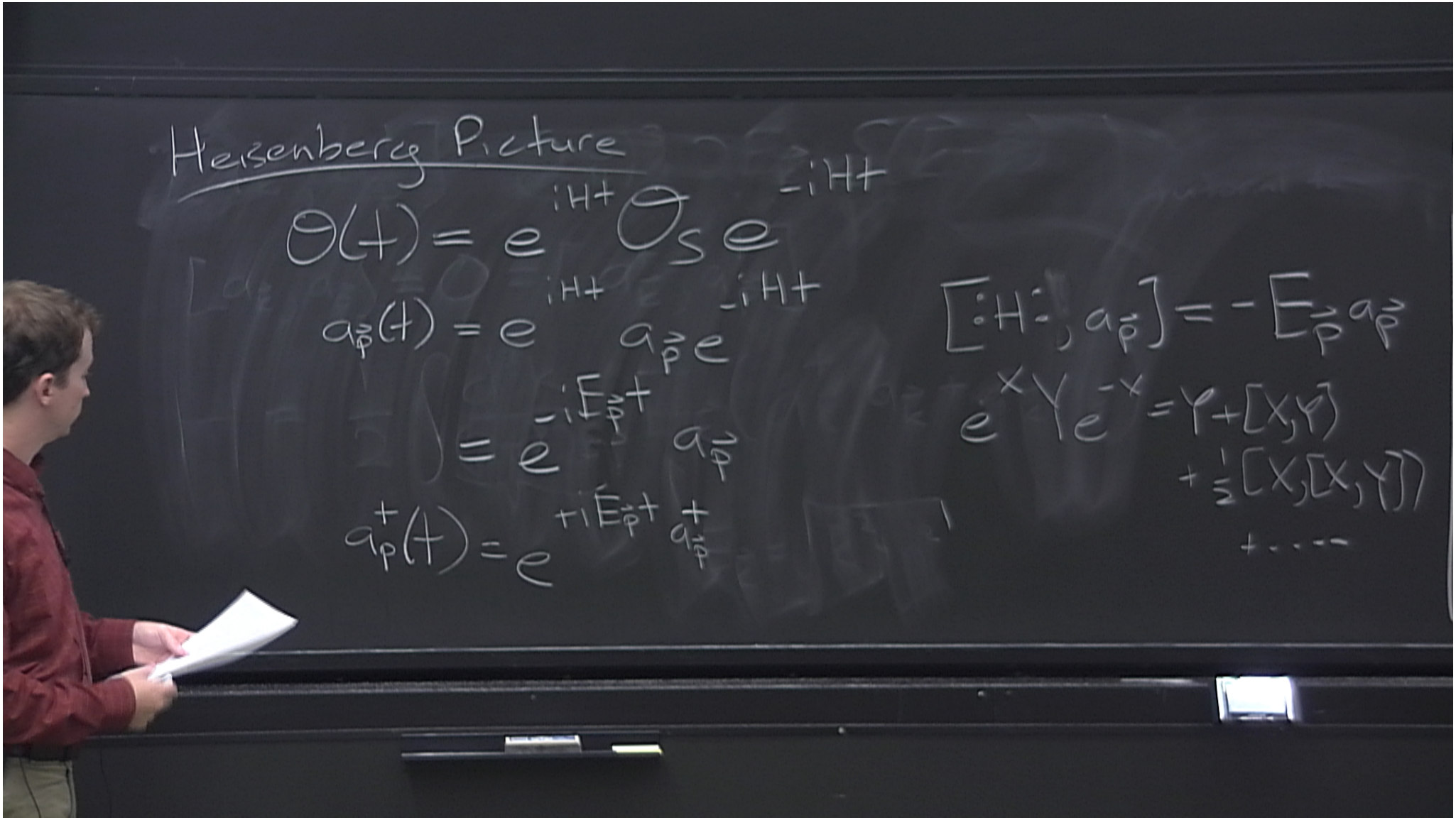

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commute
state with N particles
w/ momentum
$\vec{k}_1$ $\vec{k}_2$ $\vec{k}_N$

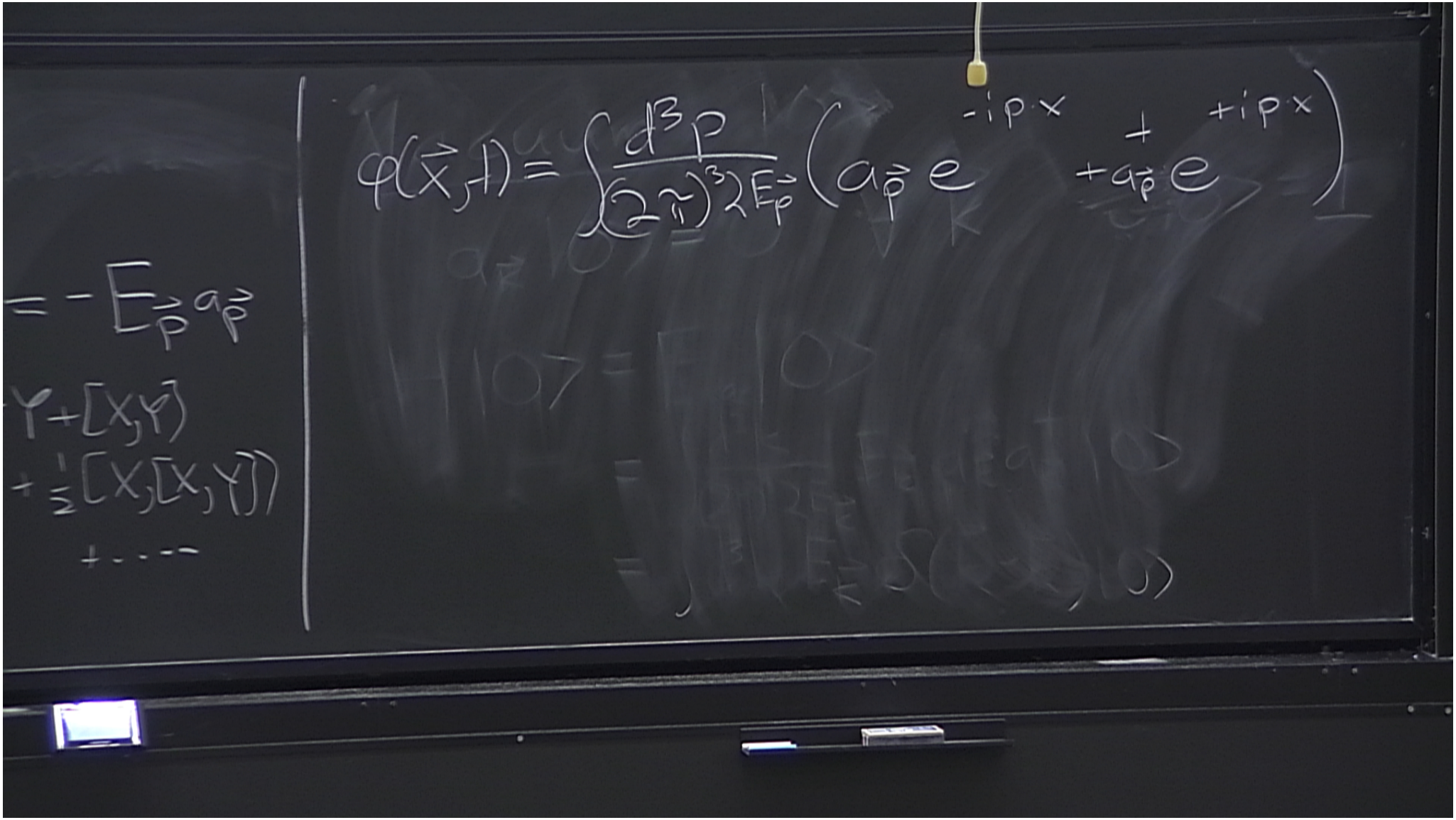

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$\vec{k}_1$ $\vec{k}_2$ $\cdots$ $\vec{k}_N$

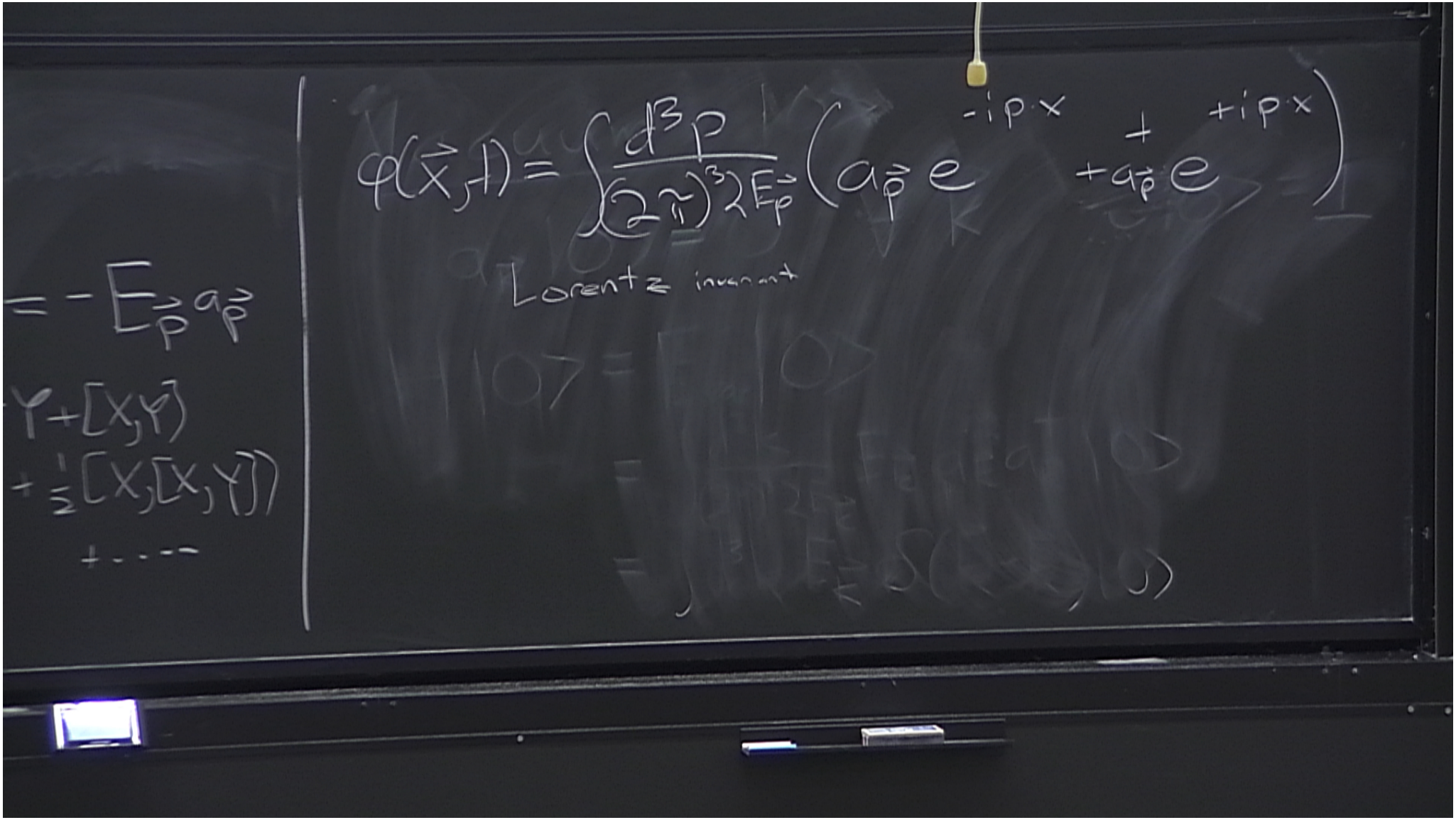

N-particle states: $|\vec{k}_1, \cdots, \vec{k}_N\rangle = a^\dagger_{\vec{k}_1} \cdots a^\dagger_{\vec{k}_N} |0\rangle$
commute
state with N particles w/ momentum $\vec{k}_1$ $\vec{k}_2$. $\vec{k}_N$
$[:H:, N] = 0$
number operator $N = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} a^\dagger_{\vec{k}} a_{\vec{k}}$

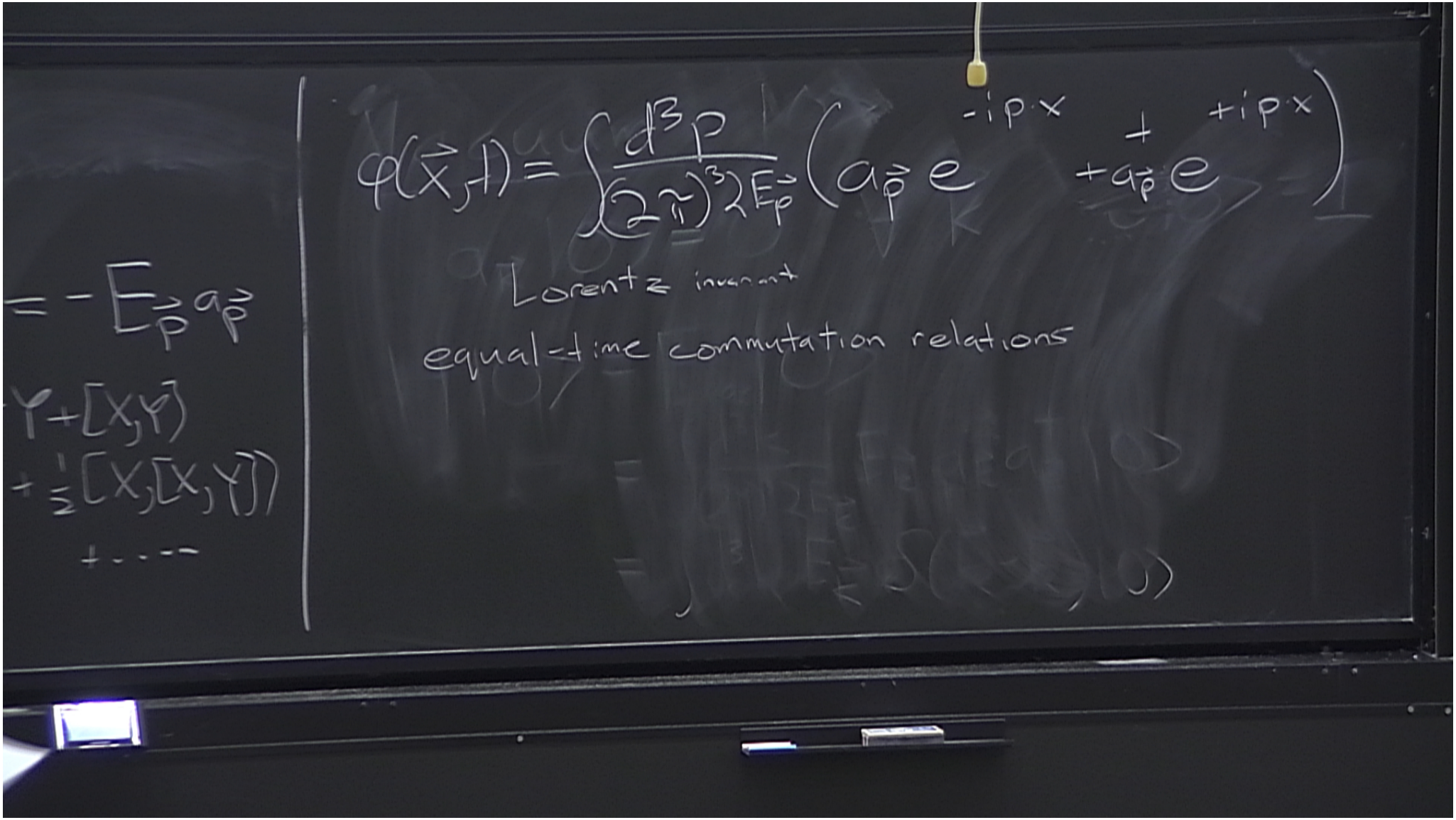

commute
particle # conserved since this theory is free

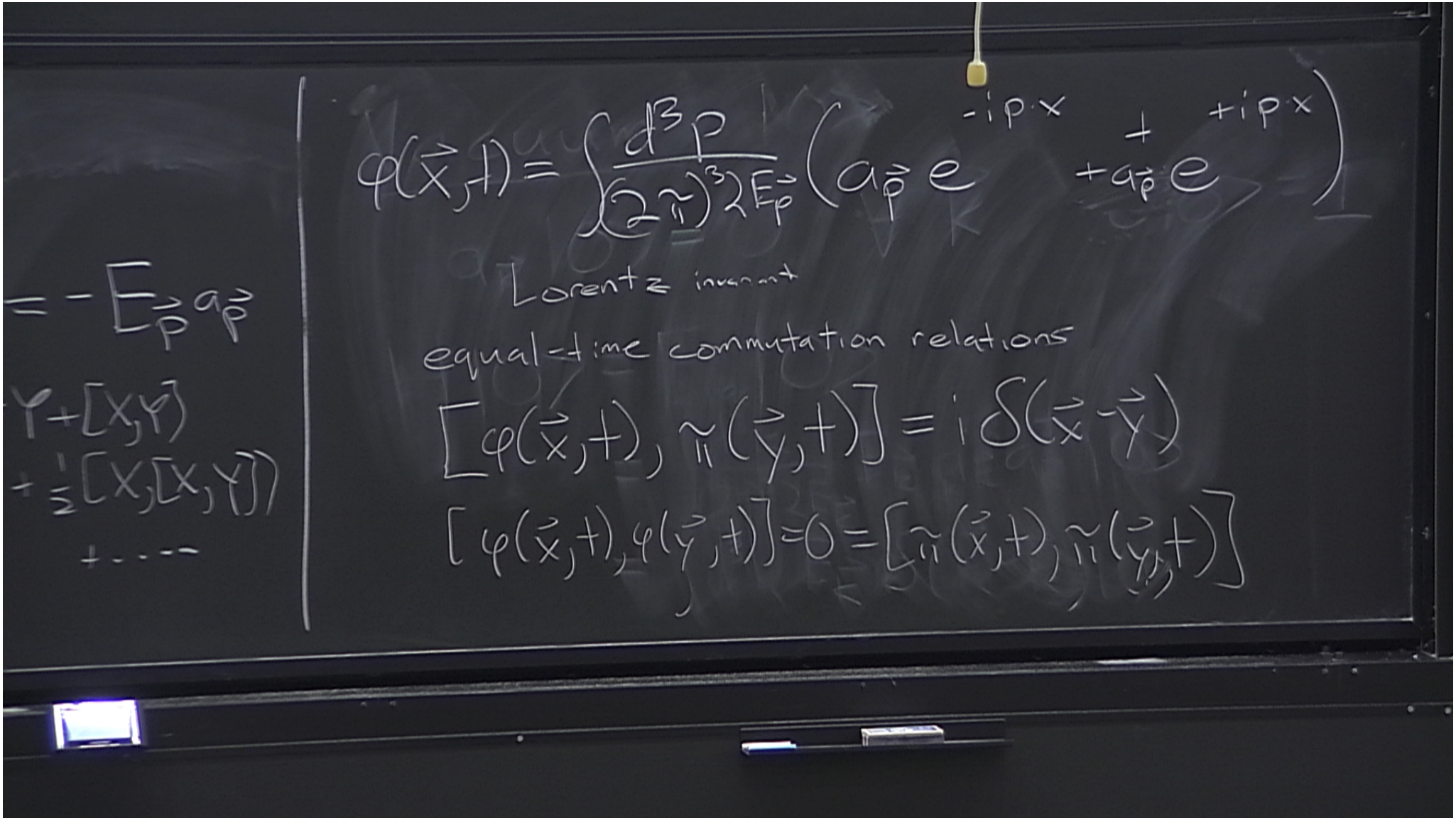

Heisenberg Picture
$O(t) = e^{iHt} O_S e^{-iHt}$

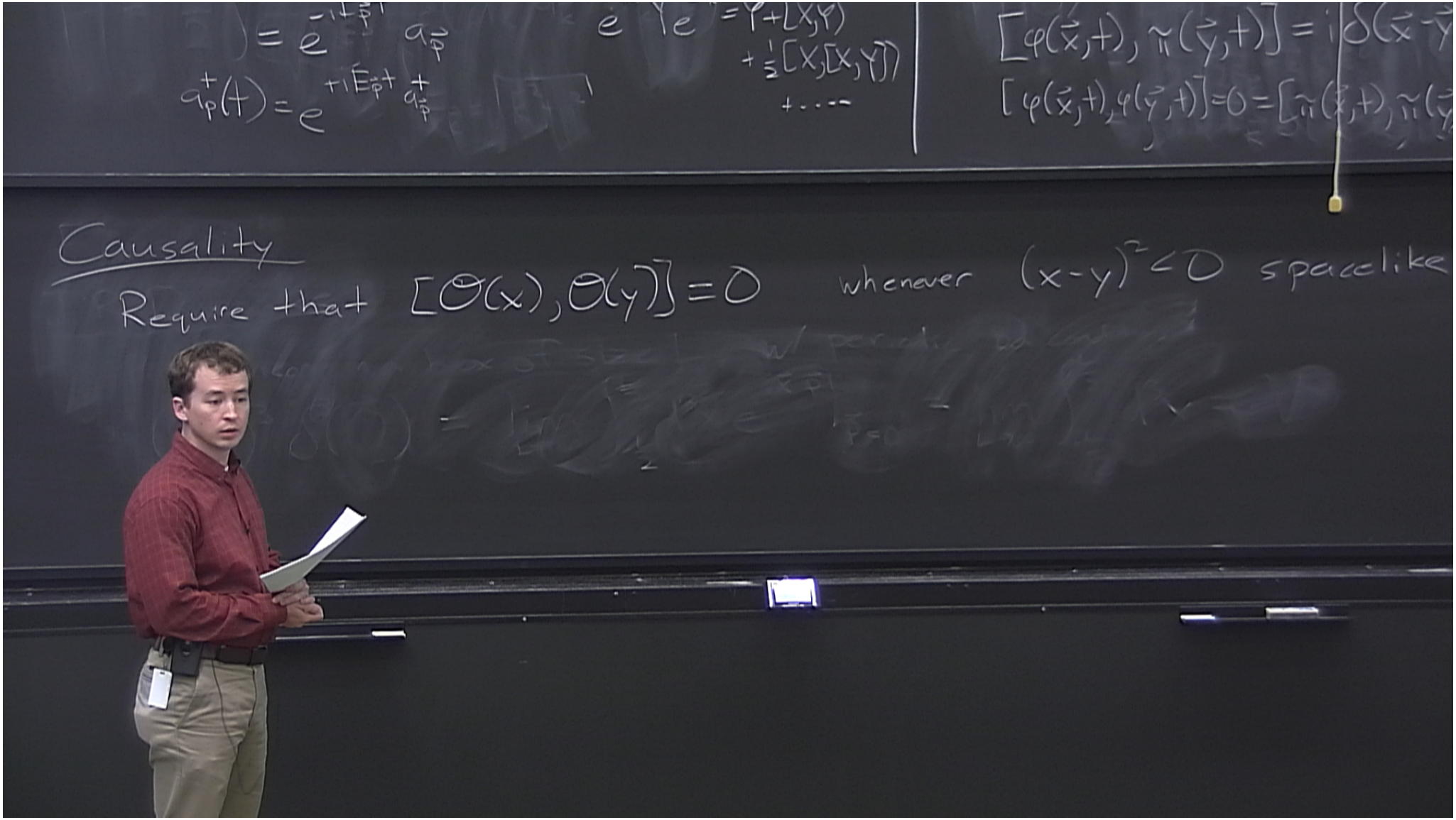

Heisenberg Picture
$\mathcal{O}(t) = e^{iHt}\mathcal{O}_S e^{-iHt}$
$a_{\vec{p}}(t) = e^{iHt} a_{\vec{p}} e^{-iHt}$
$= e^{-iE_{\vec{p}}t} a_{\vec{p}}$
$a^\dagger_{\vec{p}}(t) = e^{+iE_{\vec{p}}t} a^\dagger_{\vec{p}}$
$[:H:, a_{\vec{p}}] = -E_{\vec{p}} a_{\vec{p}}$
$e^X Y e^{-X} = Y + [X,Y] + \frac{1}{2}[X,[X,Y]] + \cdots$


$\varphi(\vec{x},t) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} \left(a_{\vec{p}}\, e^{-ip\cdot x} + a^{\dagger}_{\vec{p}}\, e^{+ip\cdot x} \right)$
$= -E_{\vec{p}}\, a_{\vec{p}}$
$Y + [X,Y]$
$+ \frac{1}{2}[X,[X,Y]]$
$+ \ldots$


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Lorentz invariant
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Lorentz invariant
equal-time commutation relations


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$+ \ldots$
$\varphi(\vec{x},t) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} \left(a_{\vec{p}}\, e^{-ip\cdot x} + a^{\dagger}_{\vec{p}}\, e^{+ip\cdot x} \right)$
Lorentz invariant
equal-time commutation relations
$[\varphi(\vec{x},t), \tilde{\pi}(\vec{y},t)] = i\delta(\vec{x}-\vec{y})$
$[\varphi(\vec{x},t), \varphi(\vec{y},t)] = 0 = [\tilde{\pi}(\vec{x},t), \tilde{\pi}(\vec{y},t)]$


Causality
Require that $[\Theta(x), \Theta(y)] = 0$ whenever $(x-y)^2 < 0$ spacelike

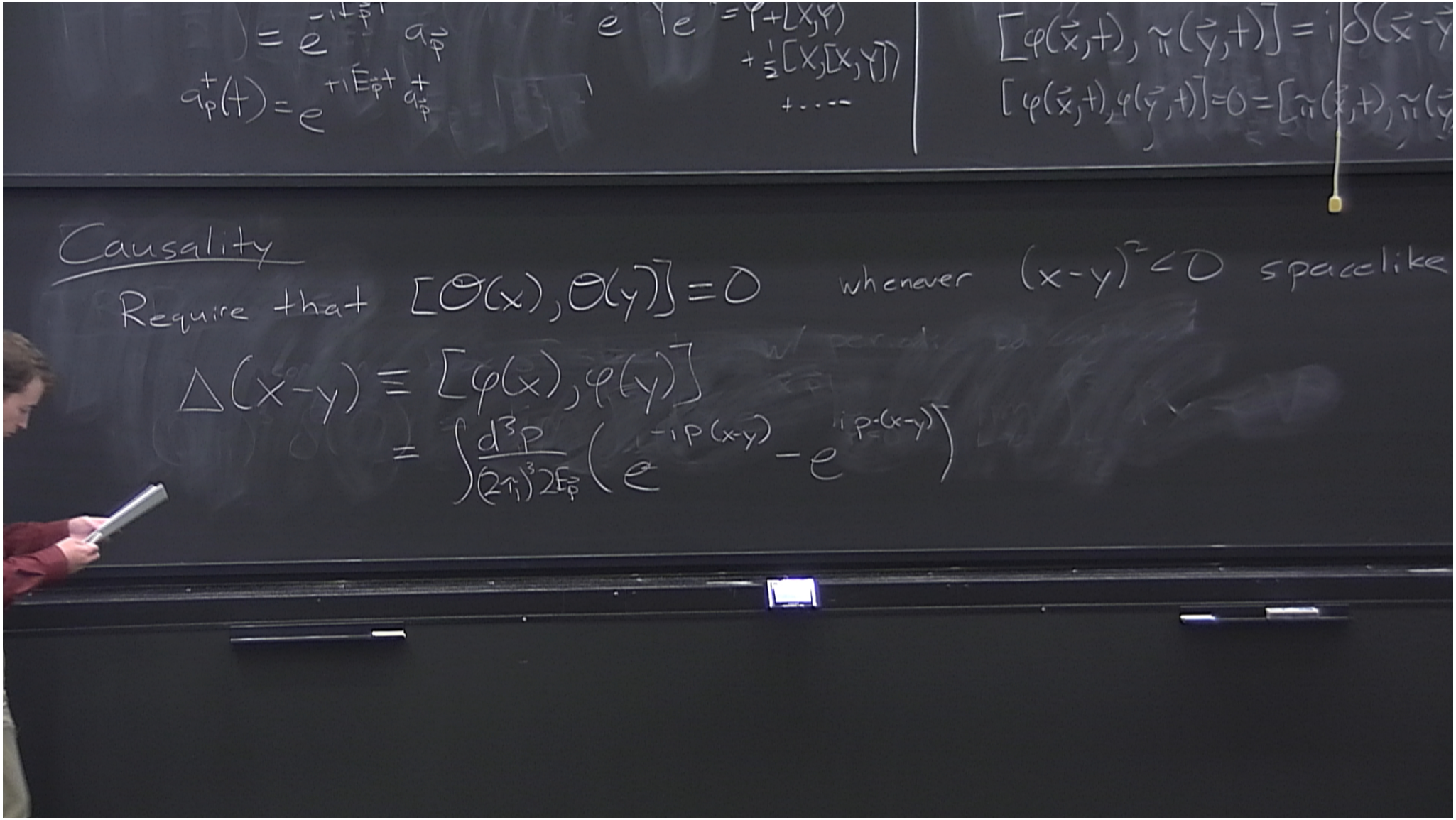
$$\varphi(\vec{x},t) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} \left(a_{\vec{p}}\, e^{-ip\cdot x} + a^{\dagger}_{\vec{p}}\, e^{+ip\cdot x} \right)$$

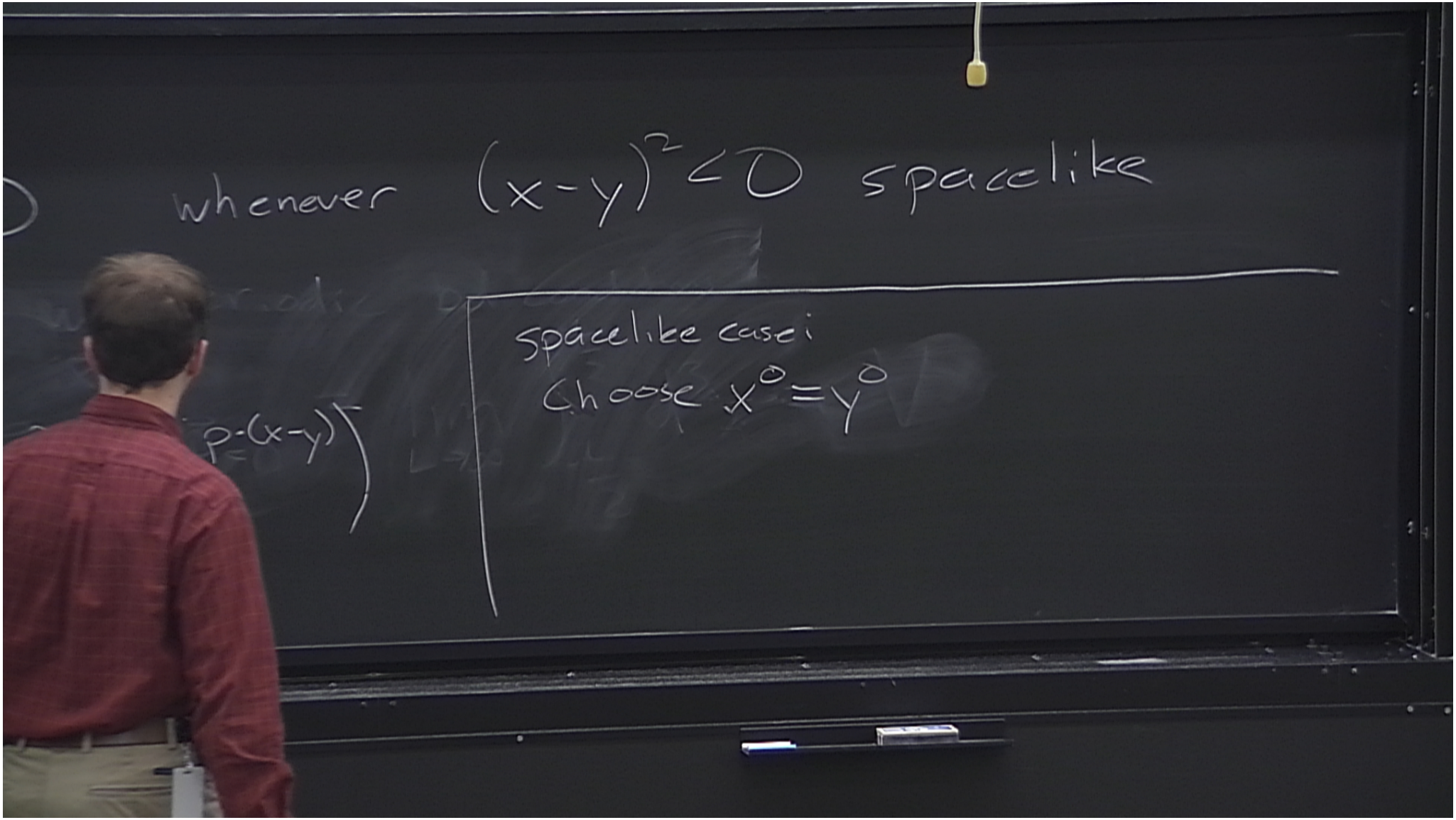
Lorentz invariant

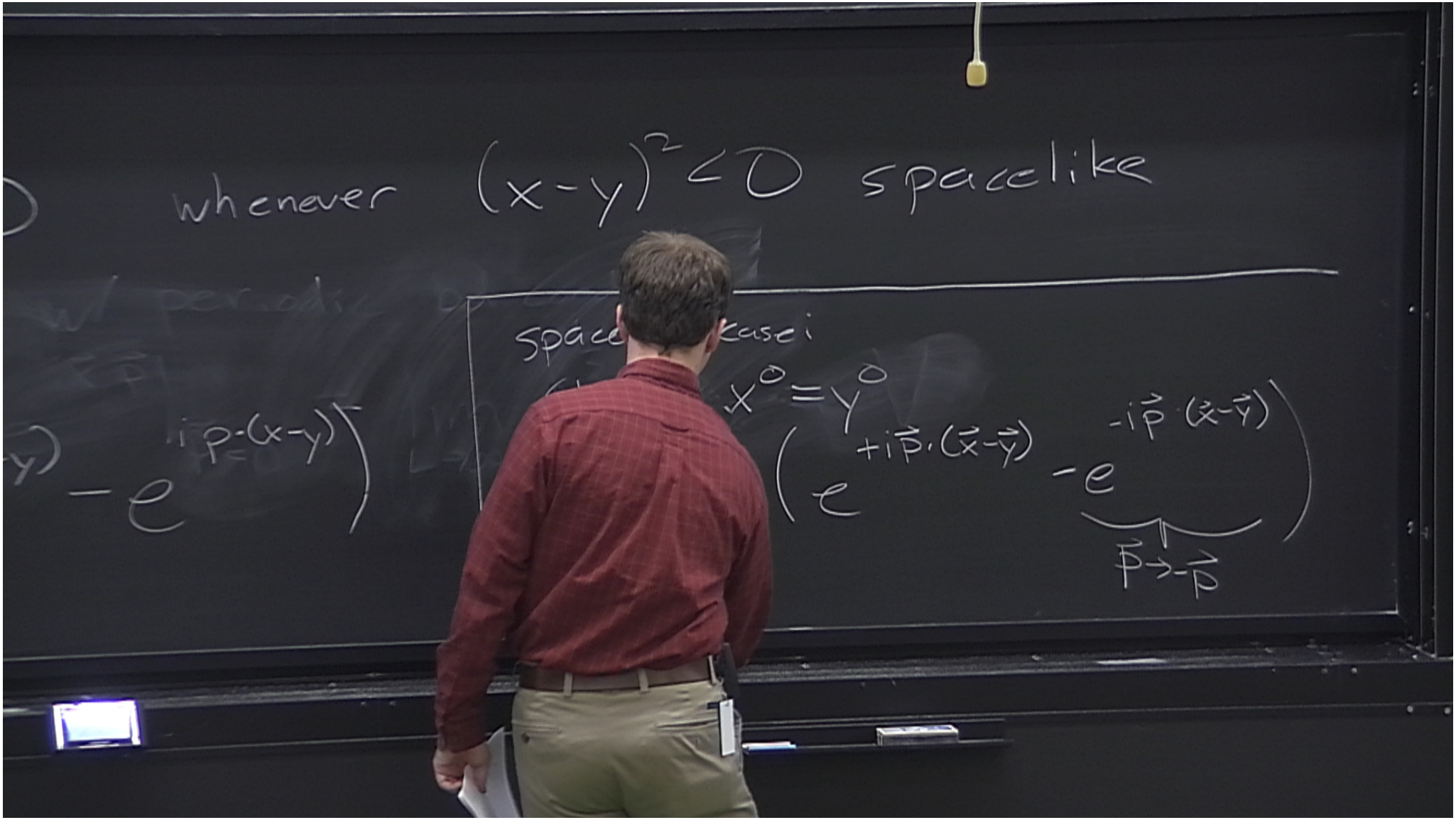
equal-time commutation relations

$$[\varphi(\vec{x},t), \tilde{\pi}(\vec{y},t)] = i\delta(\vec{x}-\vec{y})$$

$$[\varphi(\vec{x},t), \varphi(\vec{y},t)] = 0 = [\tilde{\pi}(\vec{x},t), \tilde{\pi}(\vec{y},t)]$$


Causality
Require that $[\Theta(x), \Theta(y)] = 0$ whenever $(x-y)^2 < 0$ spacelike
$\Delta(x-y) \equiv [\varphi(x), \varphi(y)]$
$= \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} \left(e^{-ip(x-y)} - e^{ip(x-y)} \right)$


whenever $(x-y)^2 < 0$ spacelike
spacelike case i
Choose $x^0 = y^0$


whenever $(x-y)^2 < 0$ spacelike
$x^0 = y^0$
$+i\vec{p}\cdot(\vec{x}-\vec{y})$
$-i\vec{p}\cdot(\vec{x}-\vec{y})$
$\vec{p} \to -\vec{p}$

whenever $(x-y)^2 < 0$ spacelike

$$-e^{ip\cdot(x-y)}\Big)$$

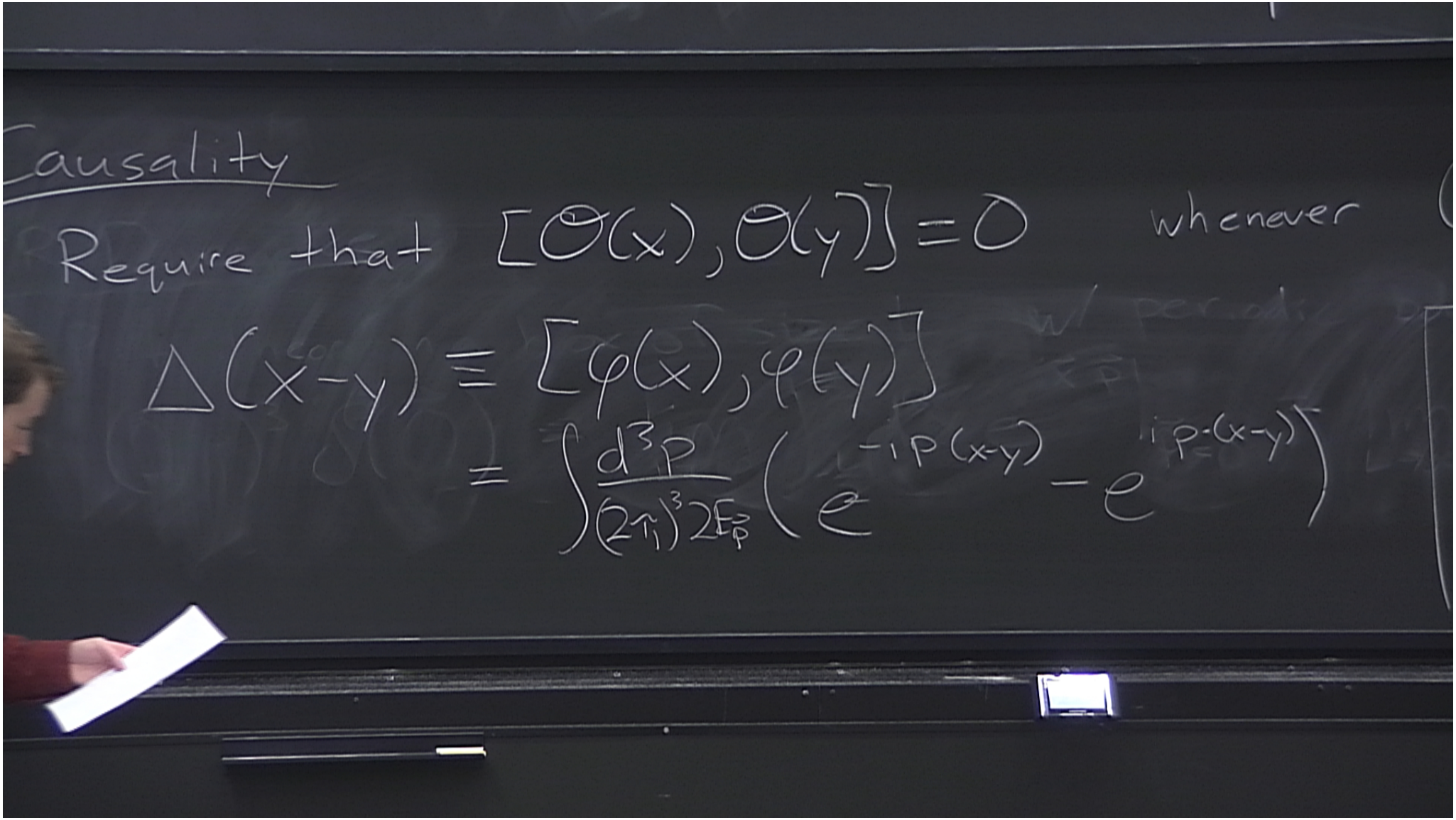
spacelike case:

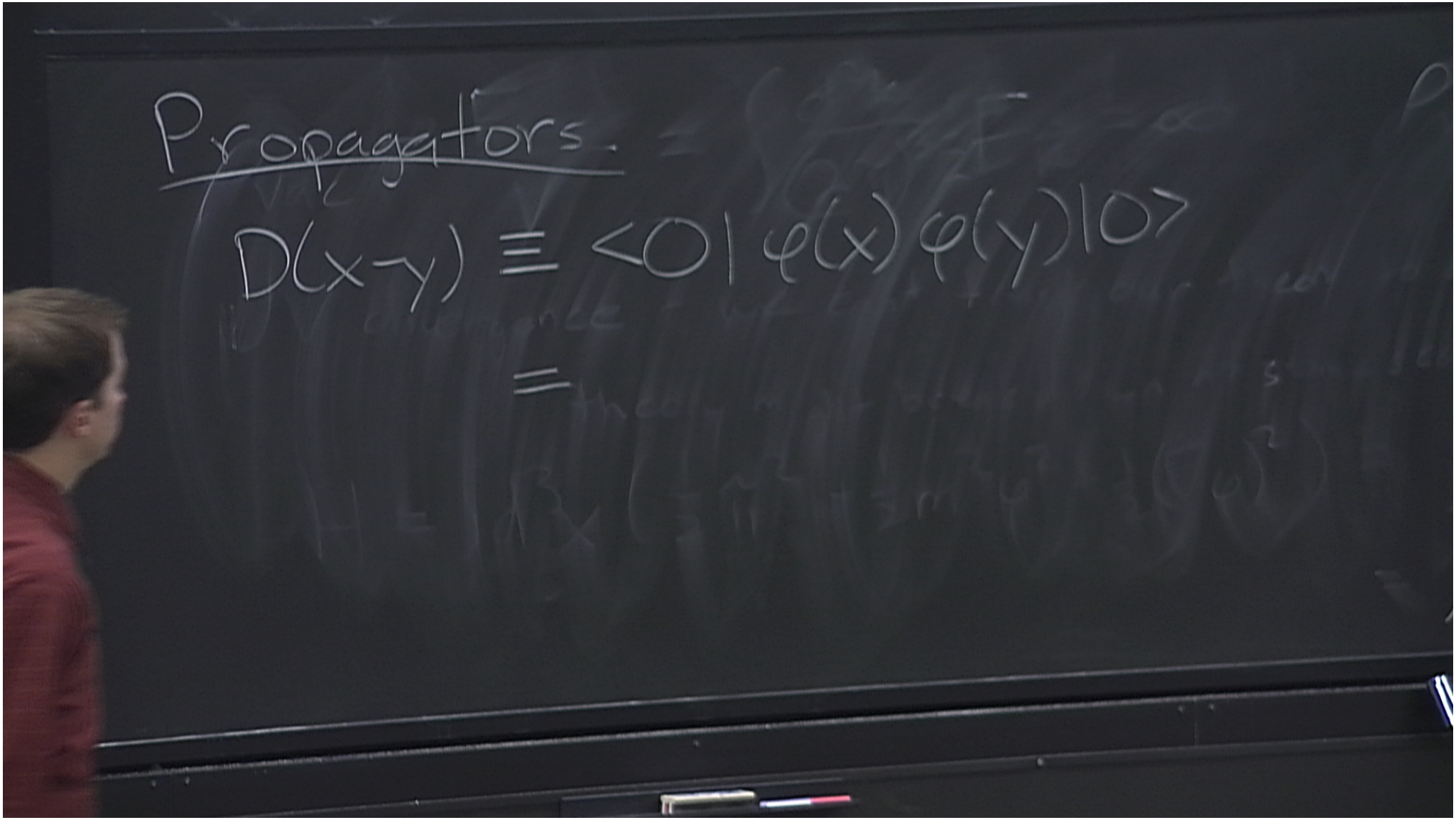
Choose $x^0 = y^0$

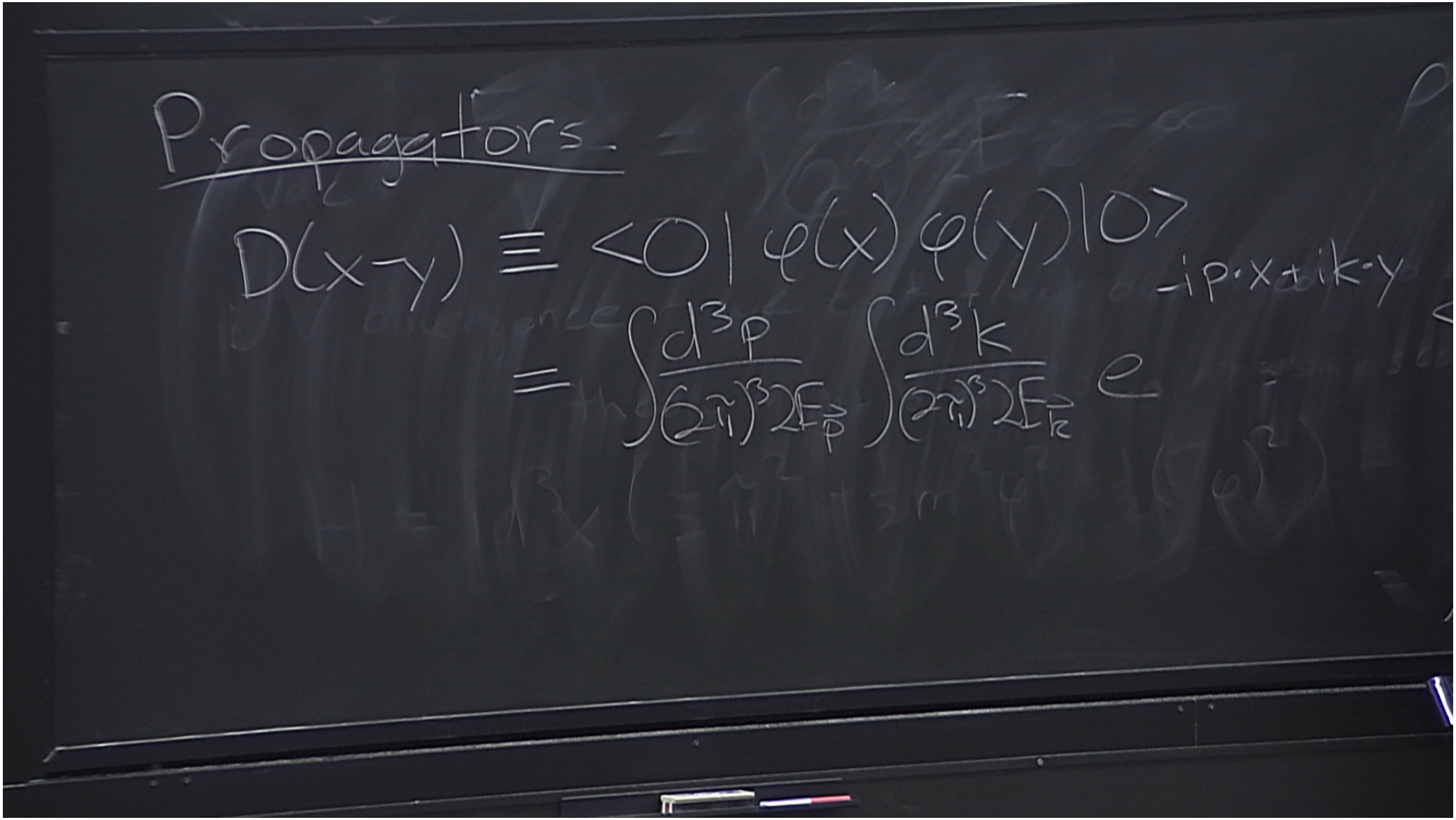
$$\Delta = \int \frac{d^3p}{(2\pi)^3 2E_p}\left(e^{+i\vec{p}\cdot(\vec{x}-\vec{y})} - e^{-i\vec{p}\cdot(\vec{x}-\vec{y})}\right)$$

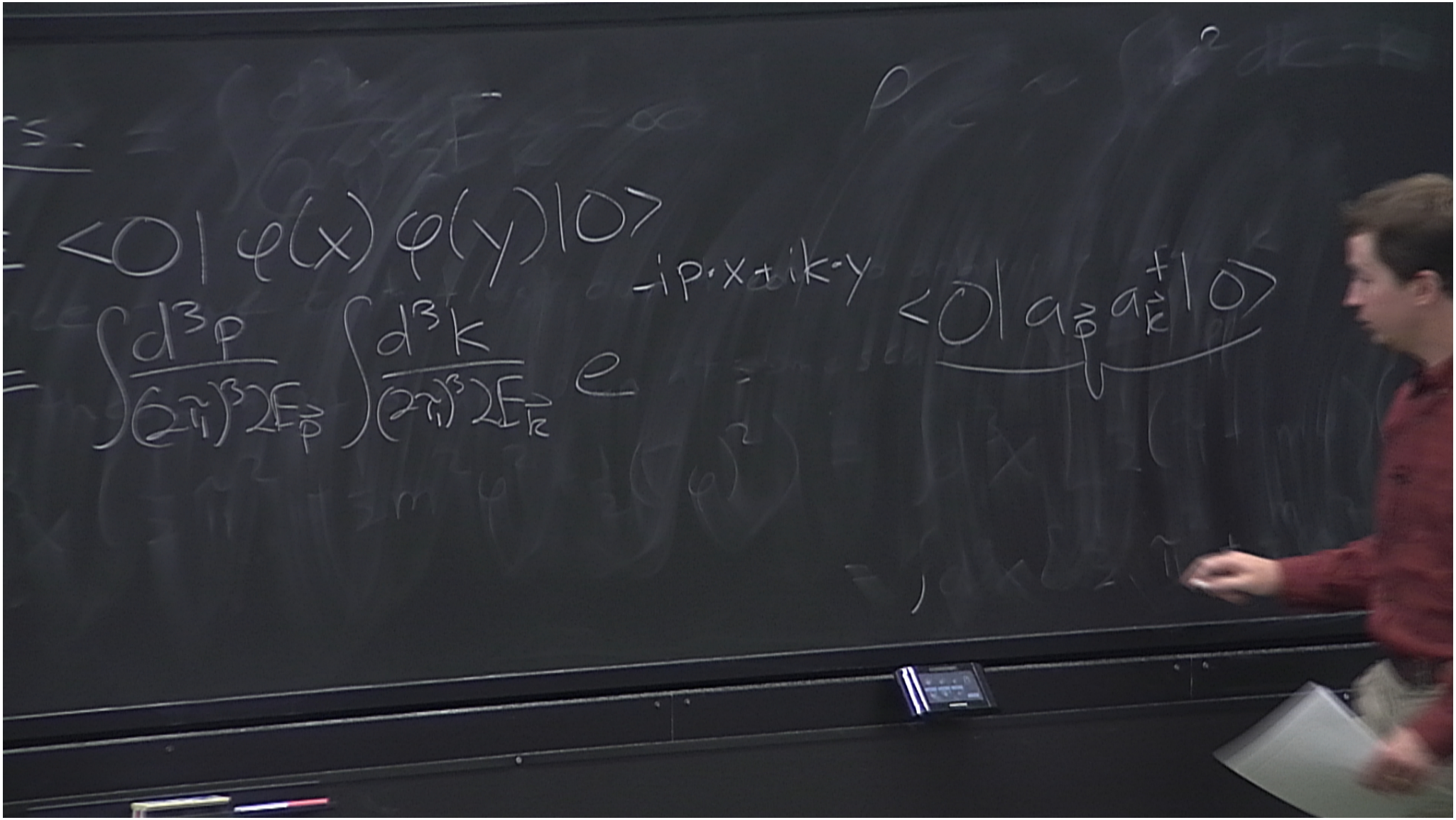
$\vec{p} \to -\vec{p}$

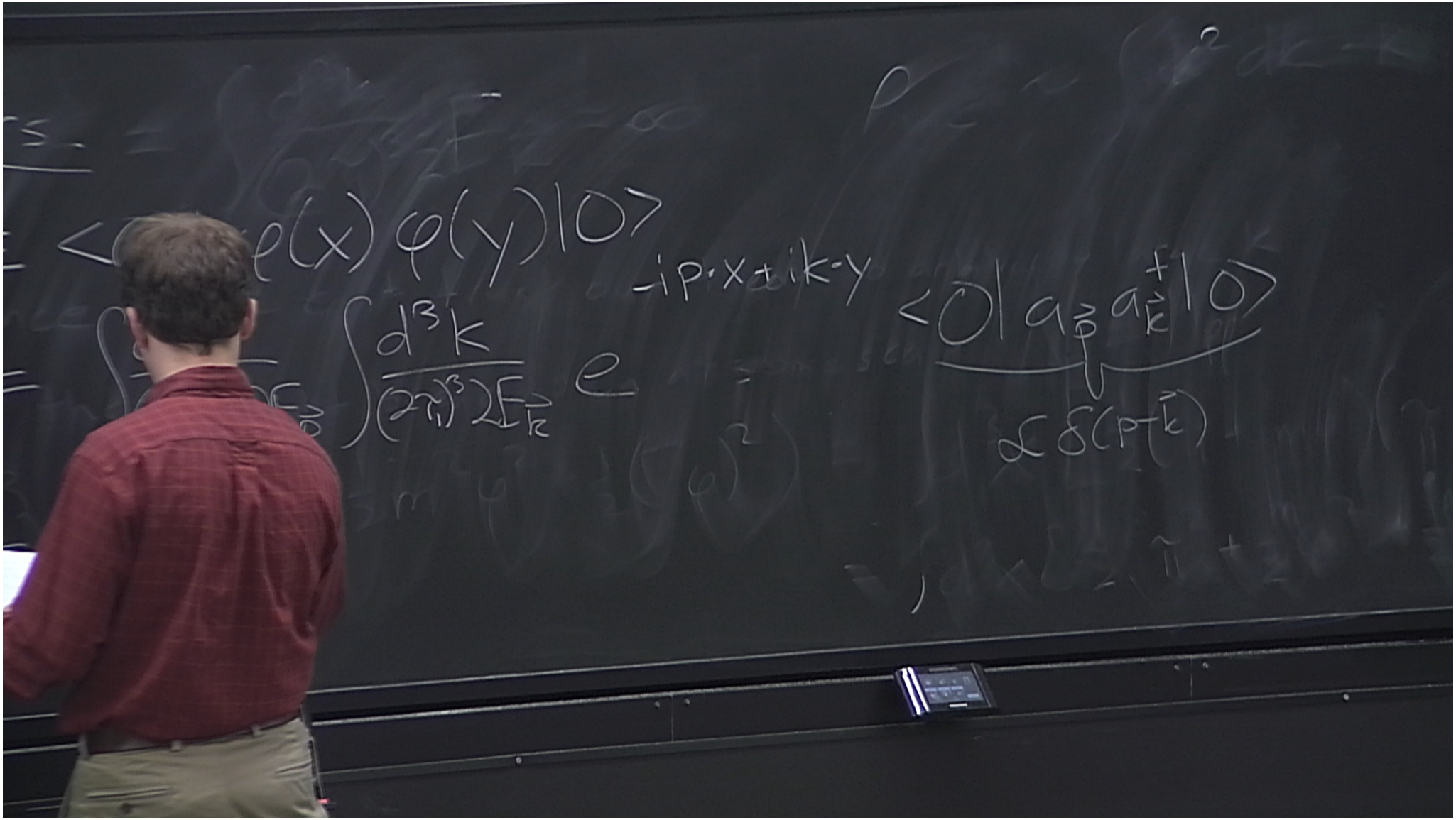
$= 0$

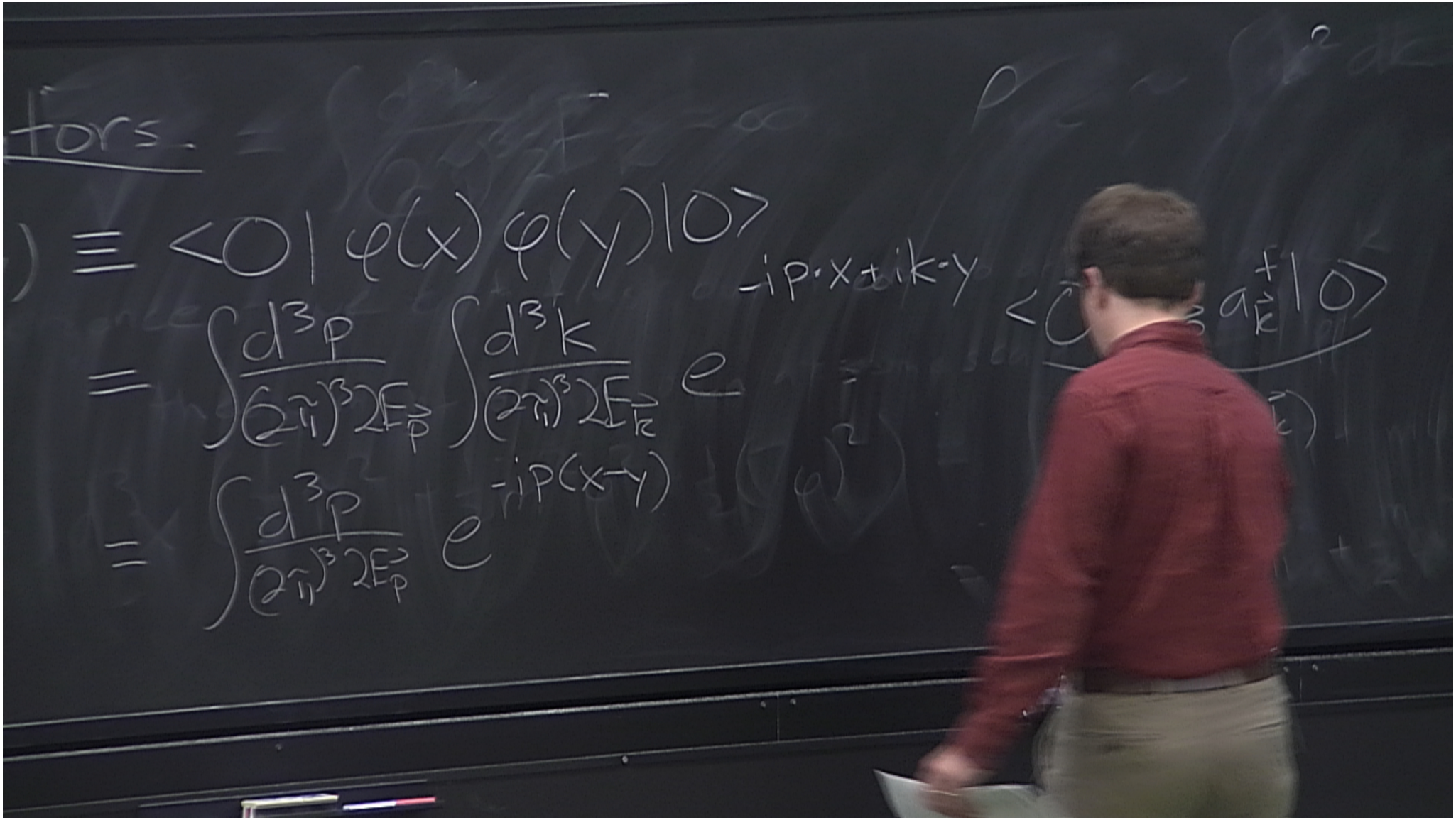

Causality
Require that $[\phi(x), \phi(y)] = 0$ whenever
$\Delta(x-y) \equiv [\varphi(x), \varphi(y)]$
$= \int \frac{d^3p}{(2\pi)^3 2E_p} \left(e^{-ip(x-y)} - e^{ip\cdot(x-y)} \right)$

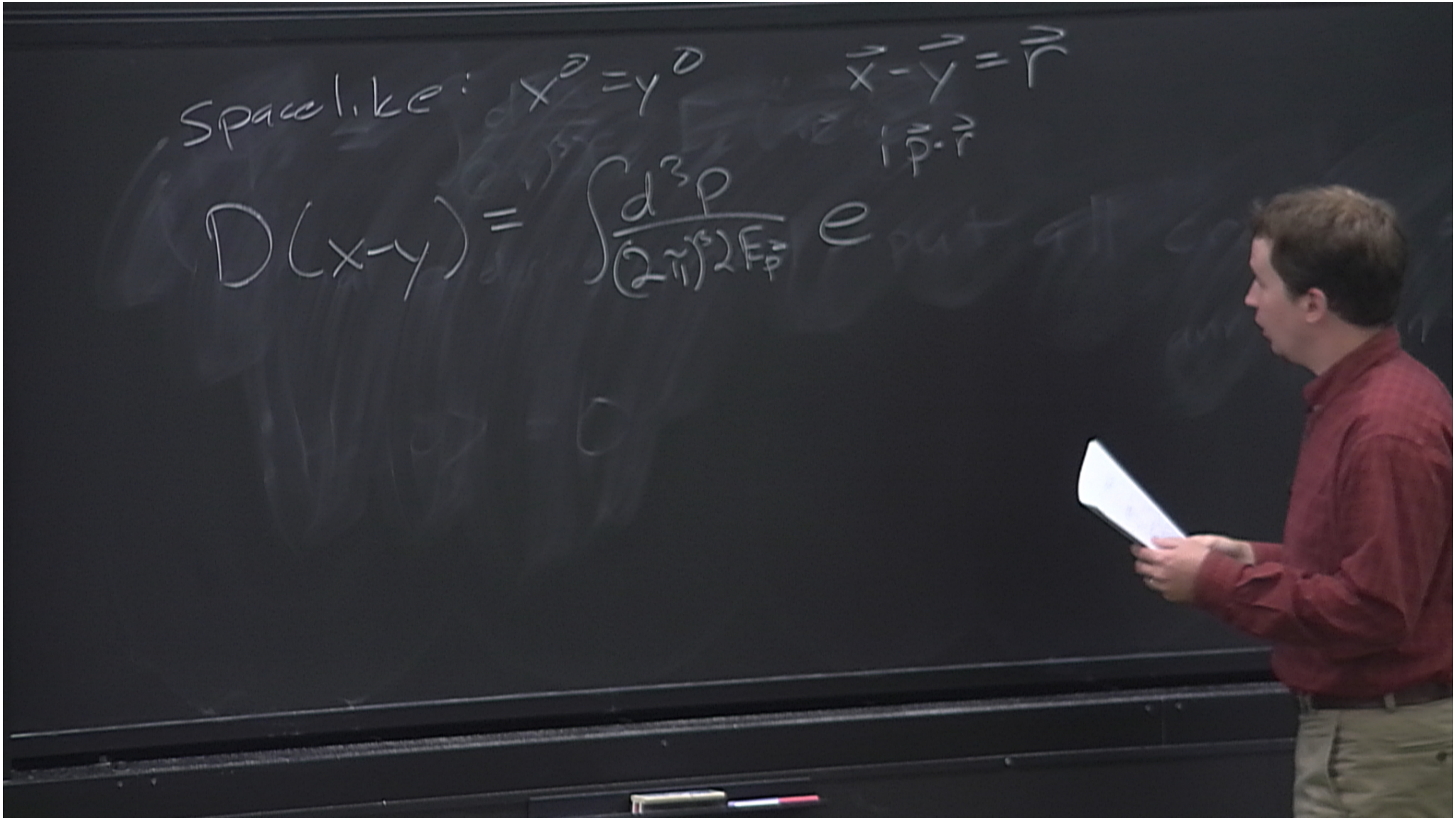

Propagators.
D(x-y) ≡ <0| φ(x) φ(y) |0>

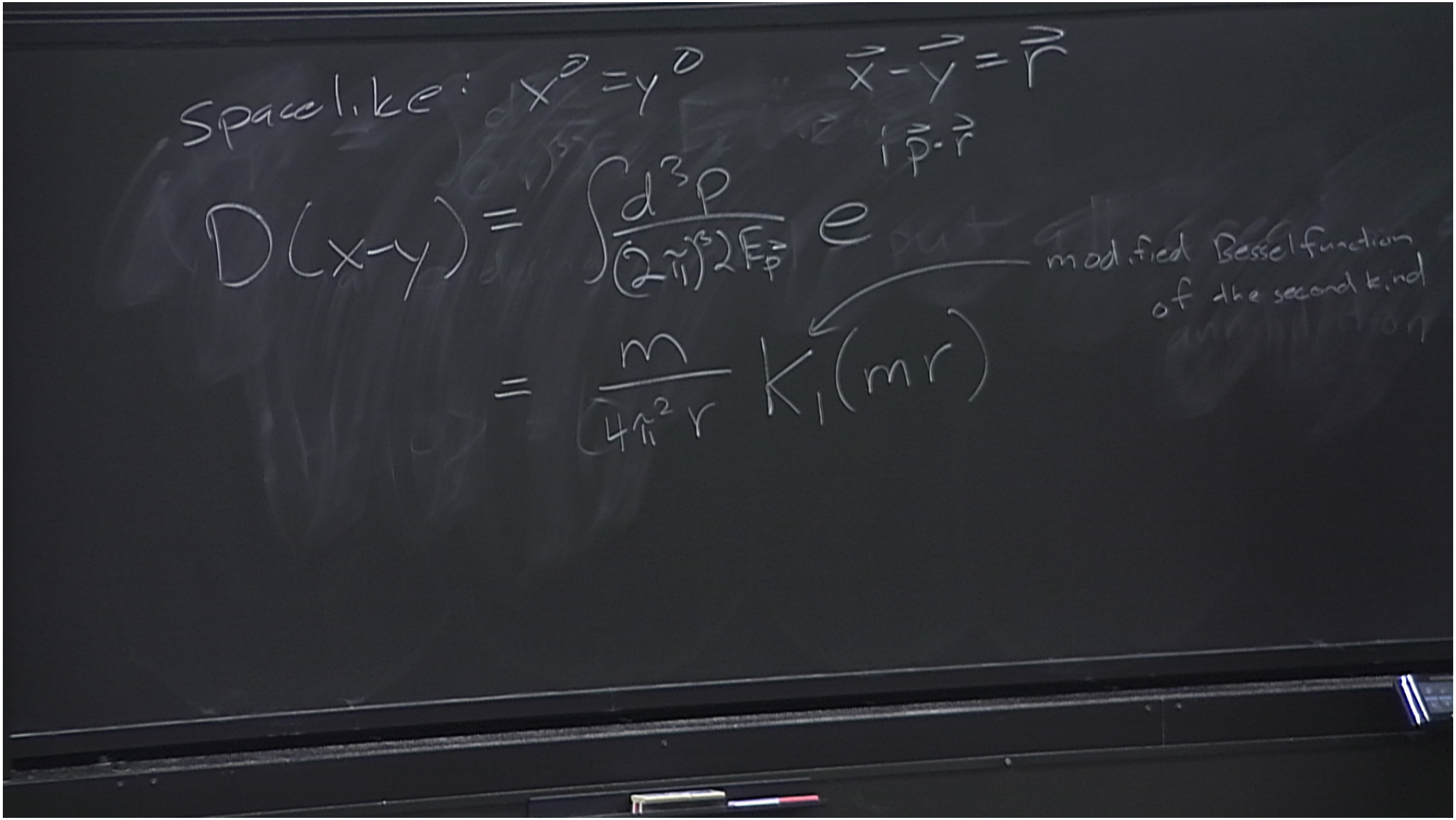

Propagators.
$D(x-y) \equiv \langle 0|\varphi(x)\varphi(y)|0\rangle$
$= \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} e$
$-ip\cdot x + ik\cdot y$

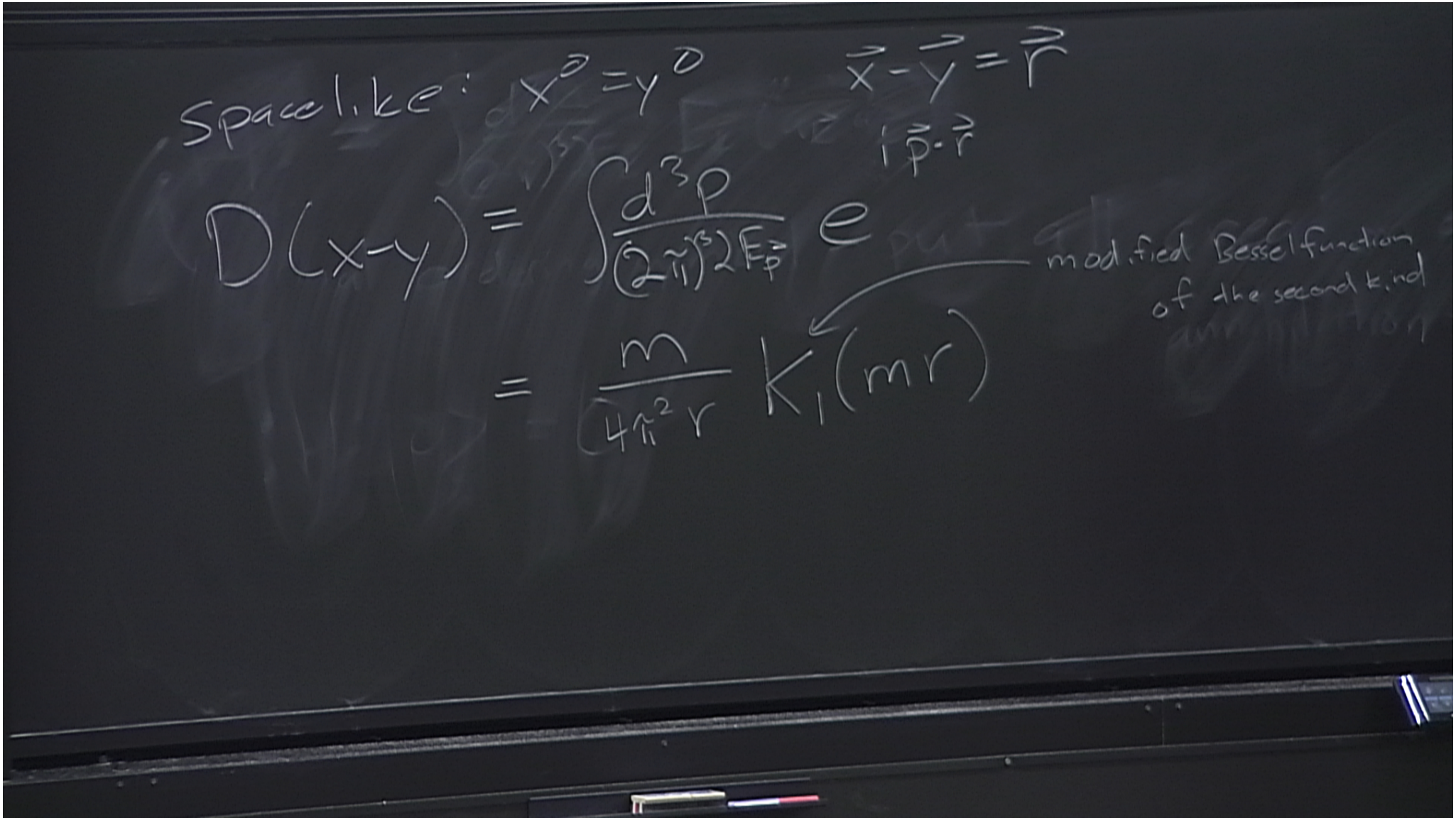

$\langle 0|\varphi(x)\varphi(y)|0\rangle$
$\int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} e^{-ip\cdot x + ik\cdot y} \langle 0|a_{\vec{p}}\, a^{\dagger}_{\vec{k}}|0\rangle$

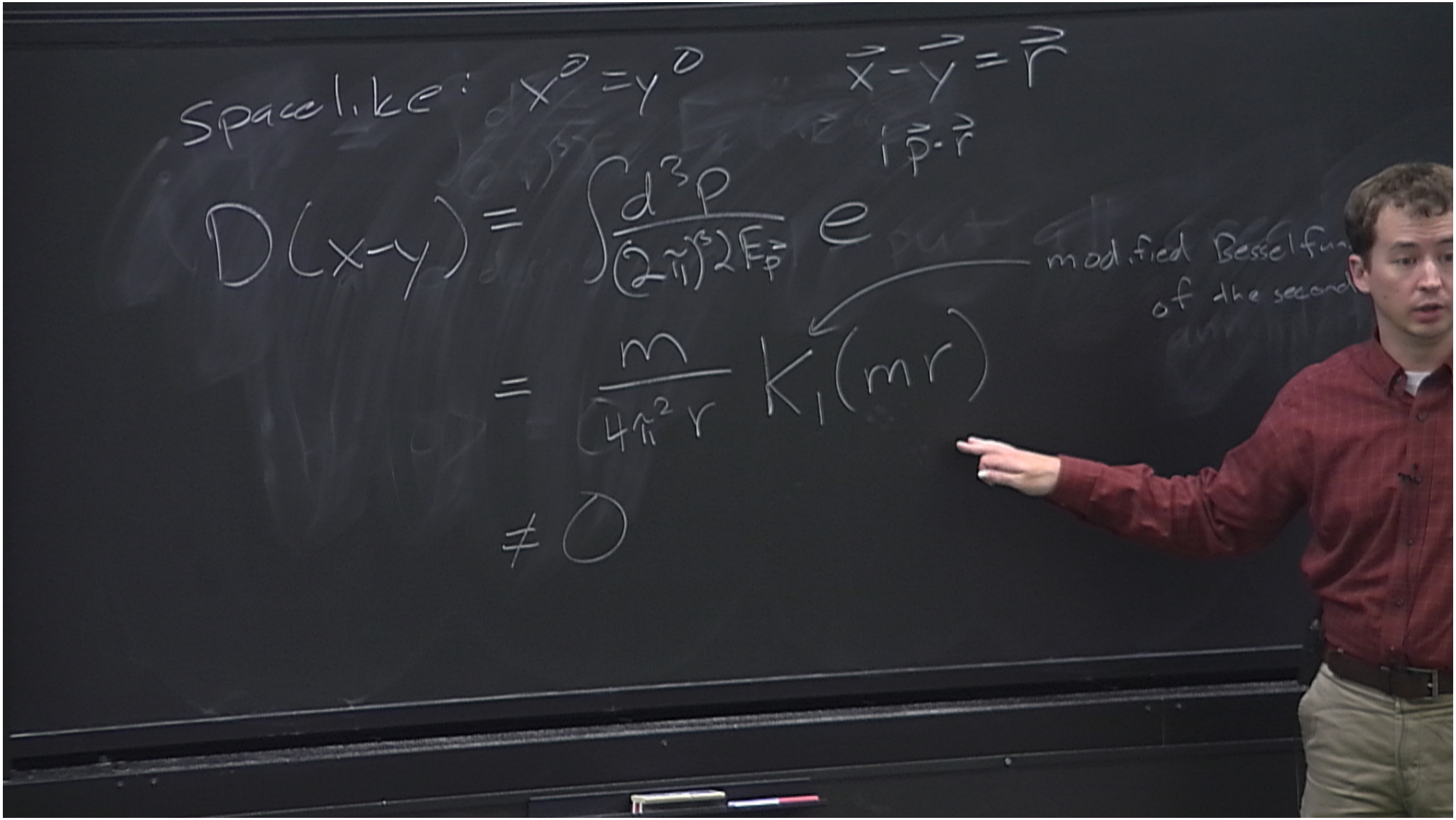

$\varphi(x)\,\varphi(y)|0\rangle$
$\int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}}\, e^{-ip\cdot x + ik\cdot y}$
$\langle 0|\, a_{\vec{p}}\, a^{\dagger}_{\vec{k}}\,|0\rangle$
$\propto \delta(p-k)$

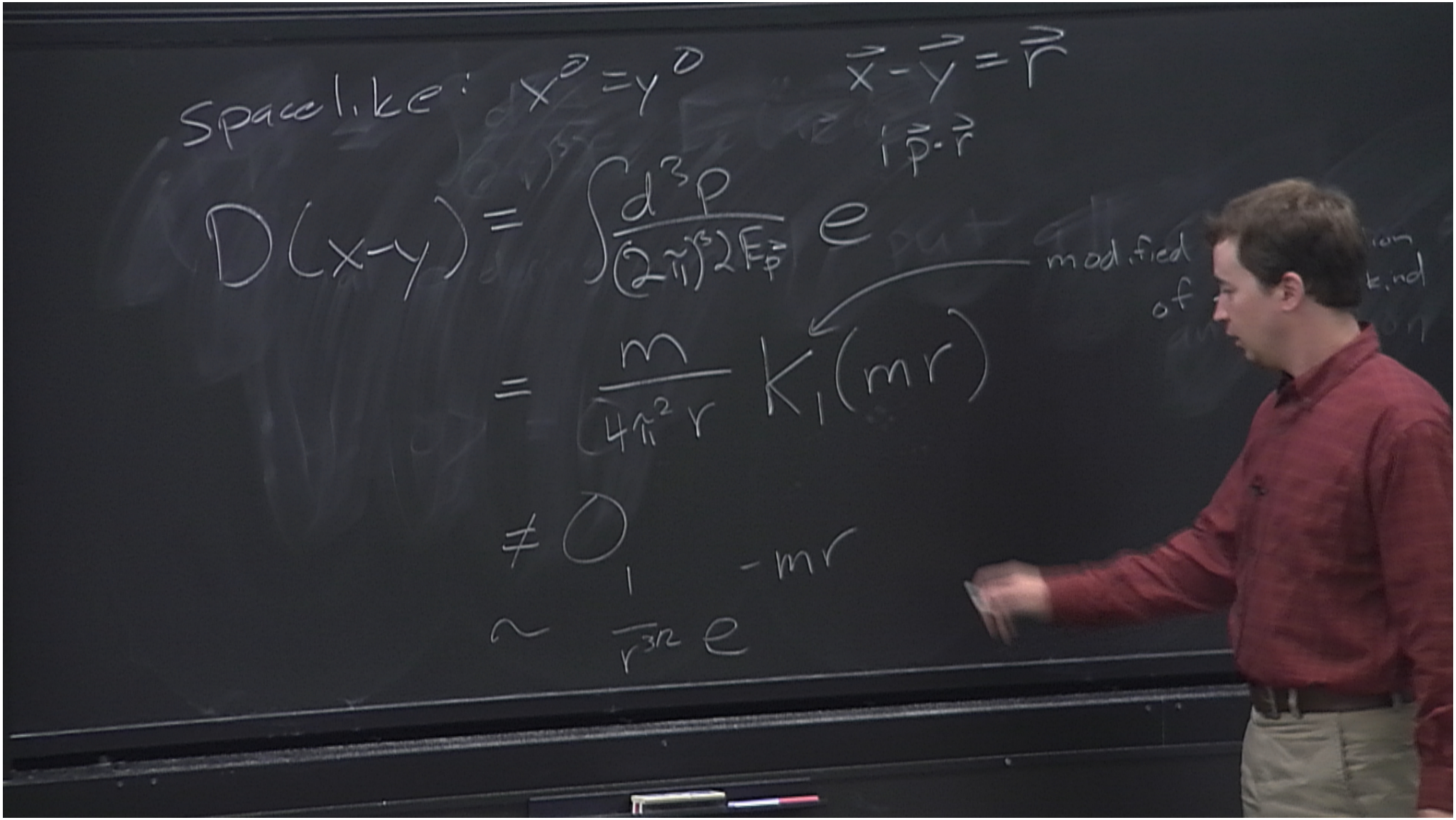

tors.
$\equiv \langle 0|\varphi(x)\varphi(y)|0\rangle$
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$= \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} e^{-ip(x-y)}$

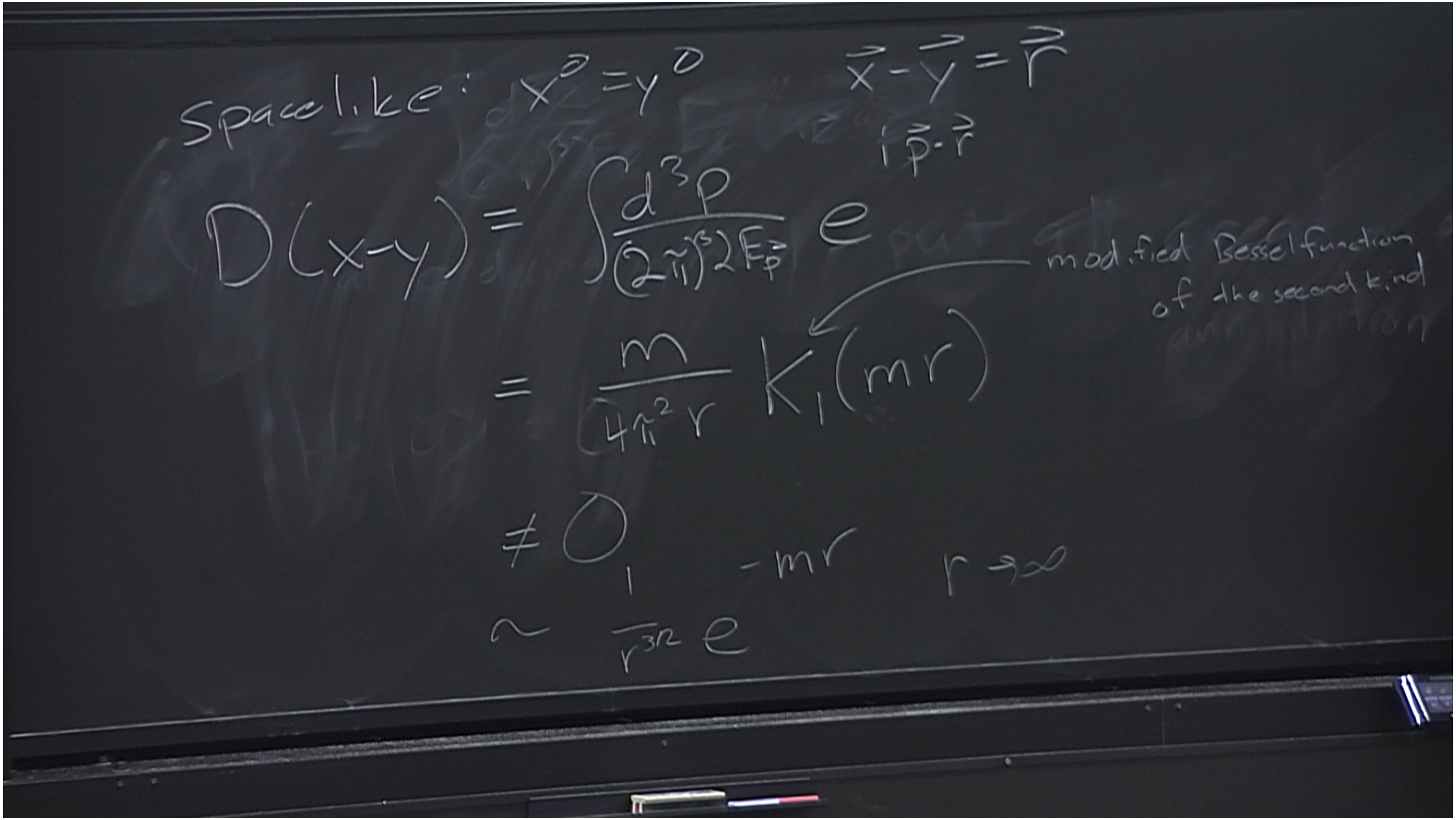

spacelike: $x^0 = y^0$
$\vec{x} - \vec{y} = \vec{r}$
$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} e^{i\vec{p}\cdot\vec{r}}$

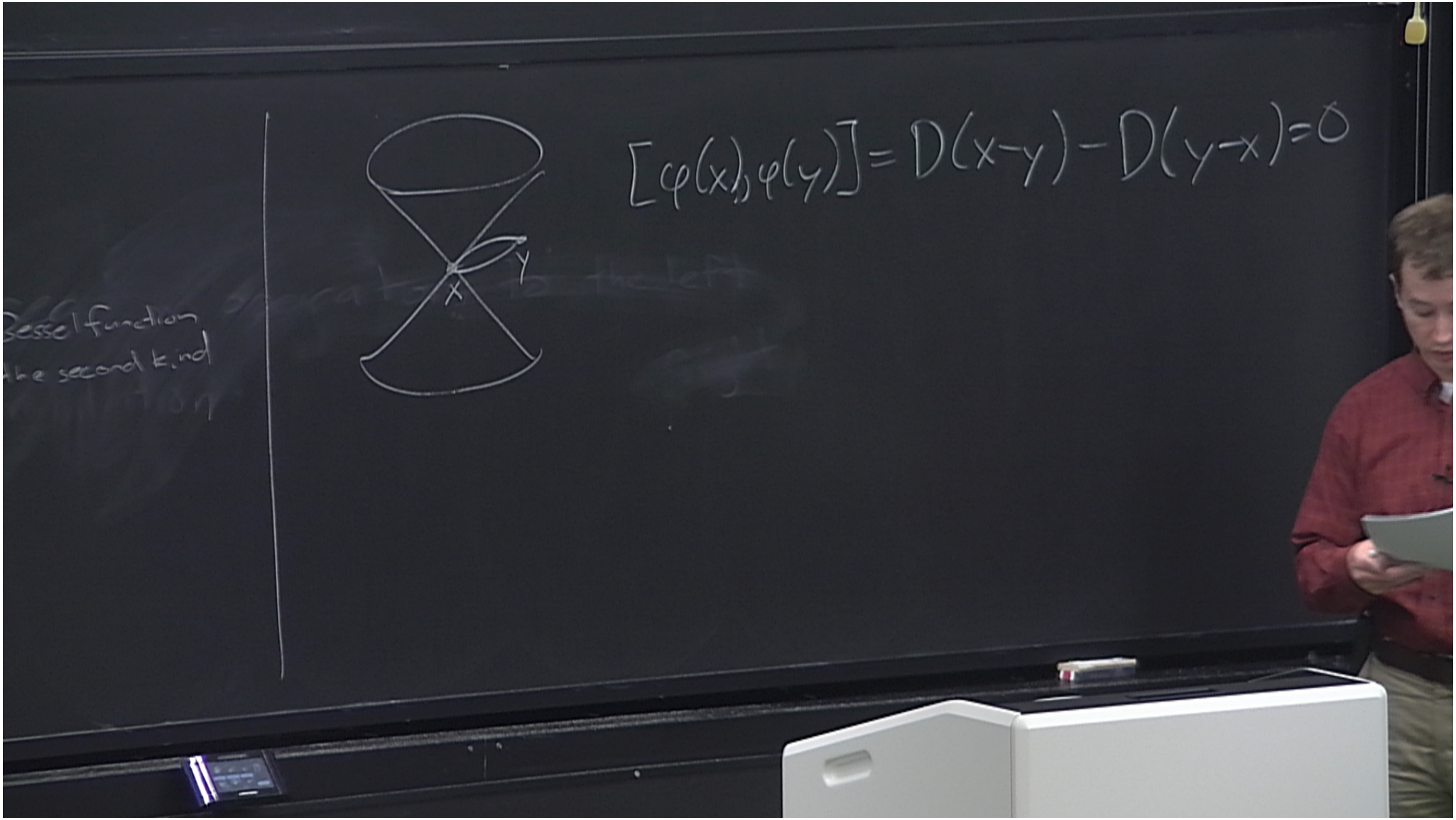

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$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} e^{i\vec{p}\cdot\vec{r}}$
$= \frac{m}{4\pi^2 r} K_1(mr)$
modified Bessel function of the second kind

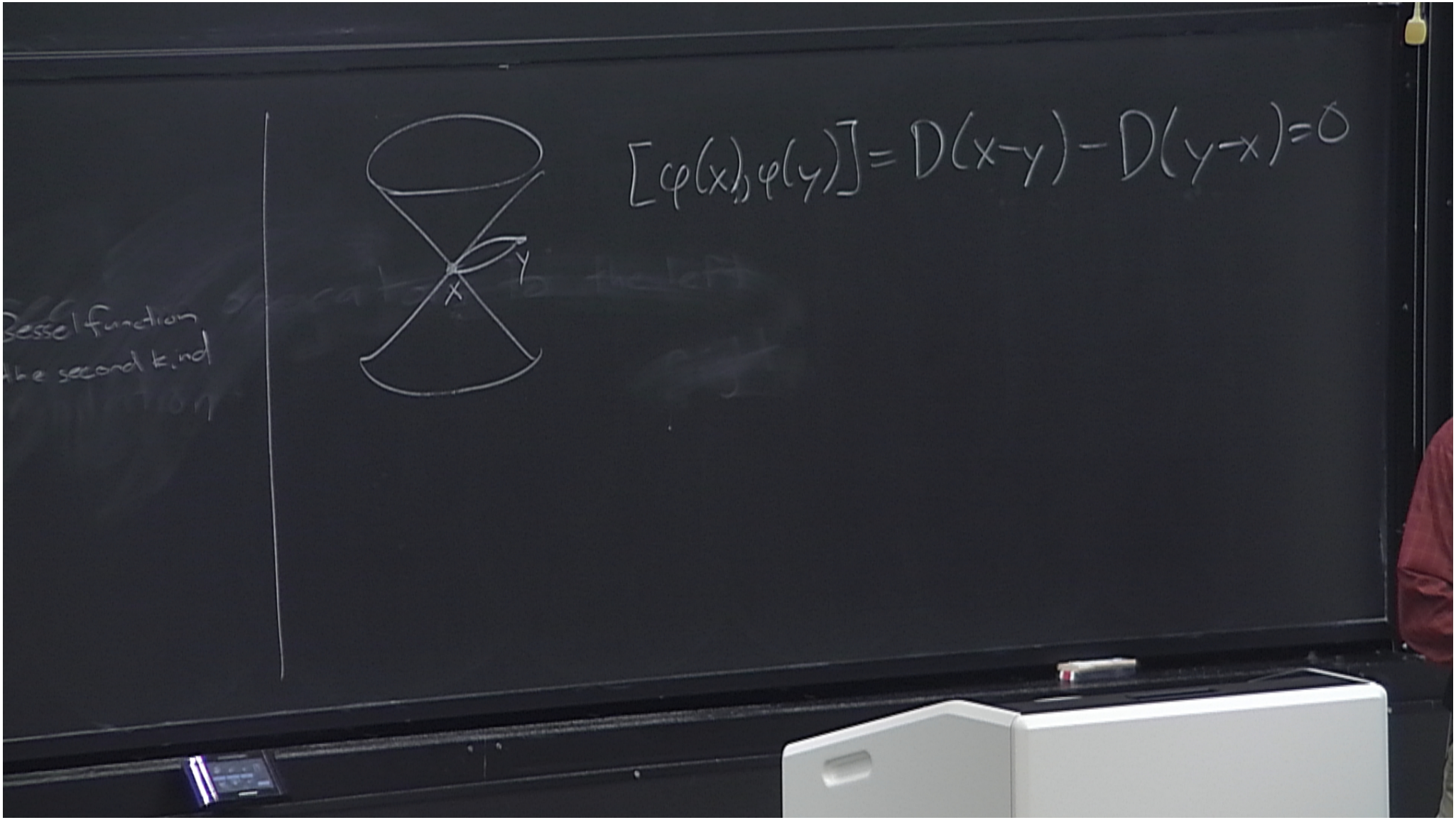

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modified Bessel function of the second kind

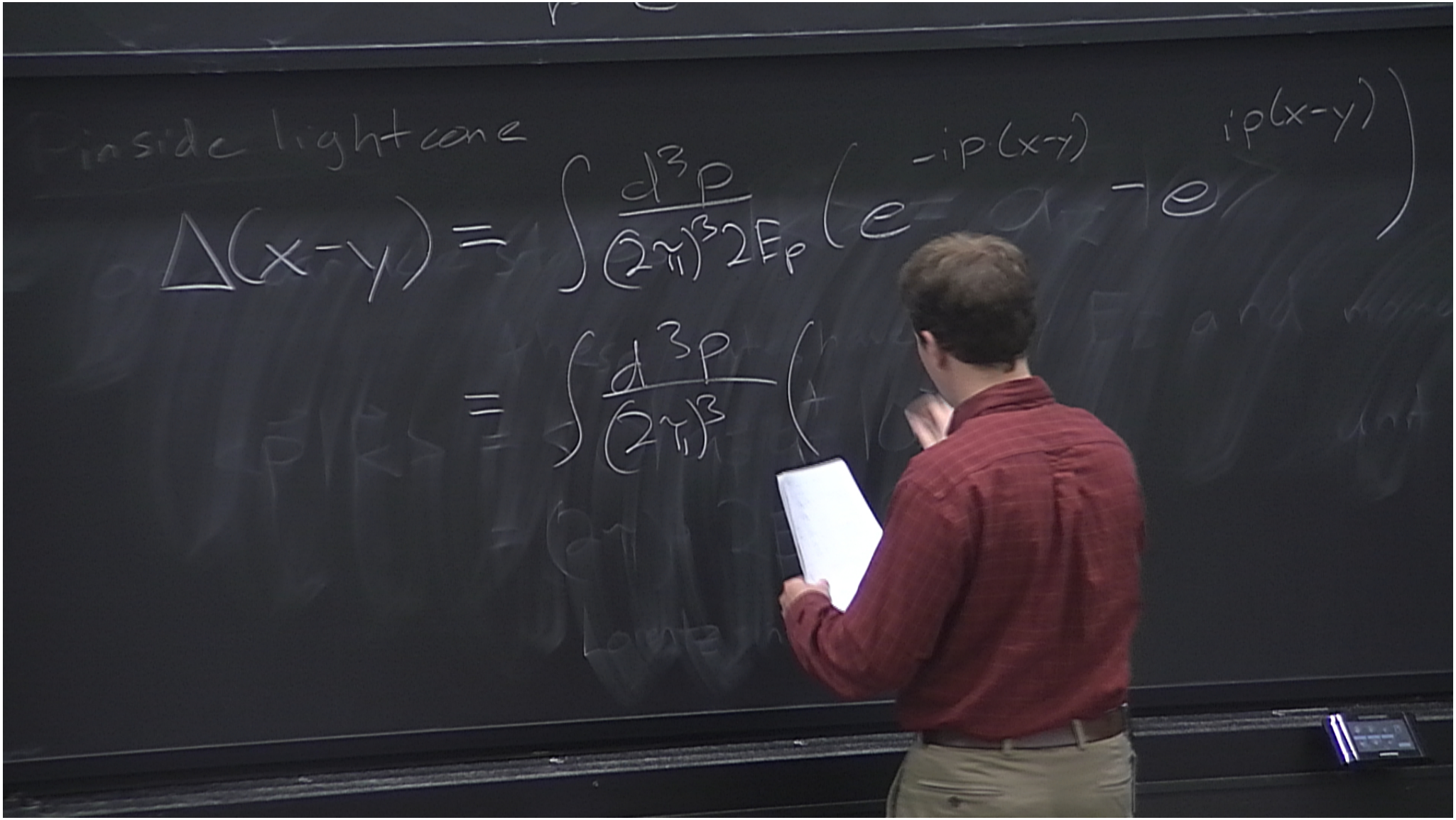

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$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_p} e^{i\vec{p}\cdot\vec{r}}$
$= \frac{m}{4\pi^2 r} K_1(mr)$
$\neq 0$
modified Bessel fu
of the second

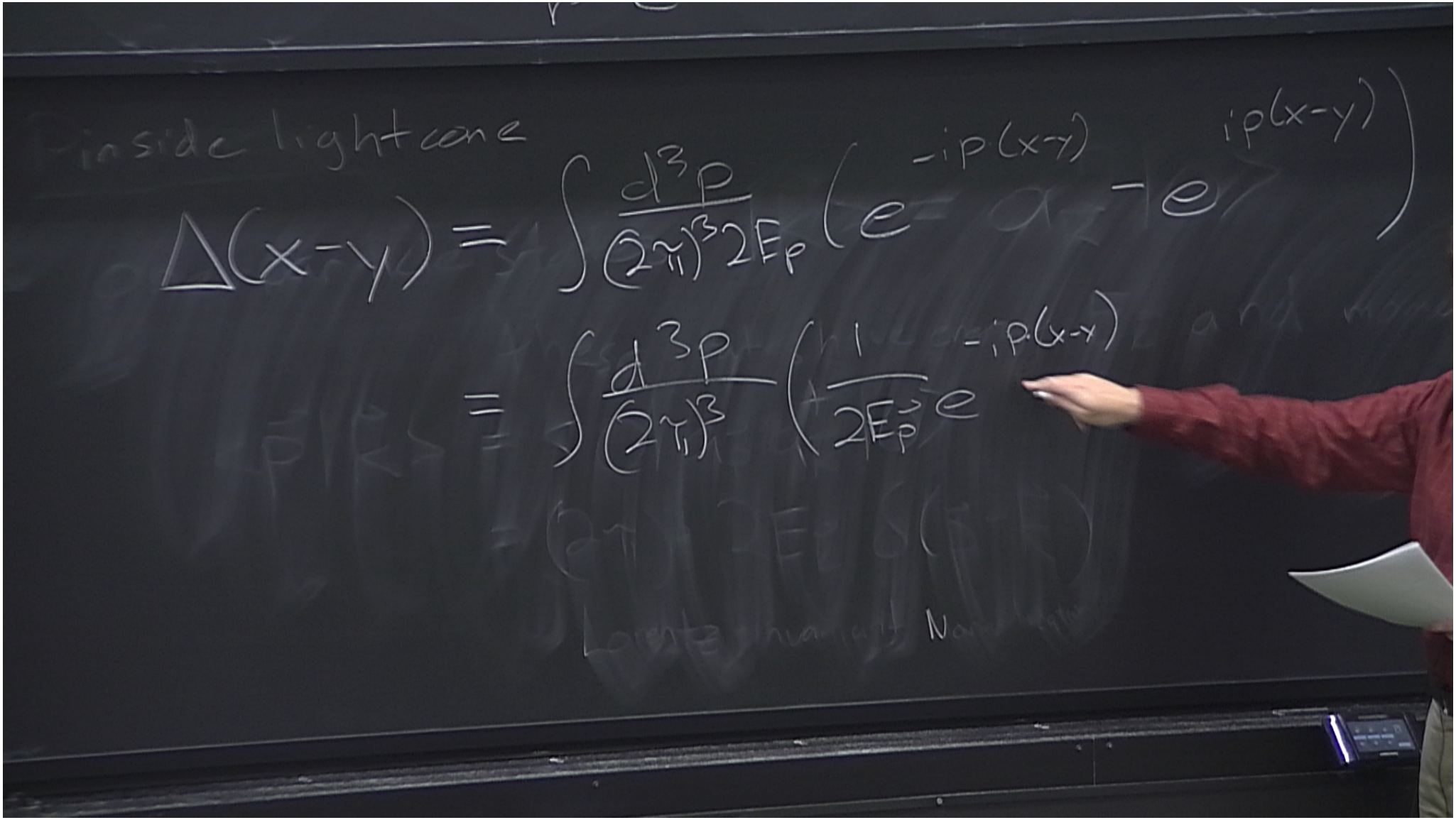

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$\vec{x} - \vec{y} = \vec{r}$
$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} e^{i\vec{p}\cdot\vec{r}}$
modified
of
$= \frac{m}{4\pi^2 r} K_1(mr)$
$\neq 0$
$\sim \frac{1}{r^{3/2}} e^{-mr}$

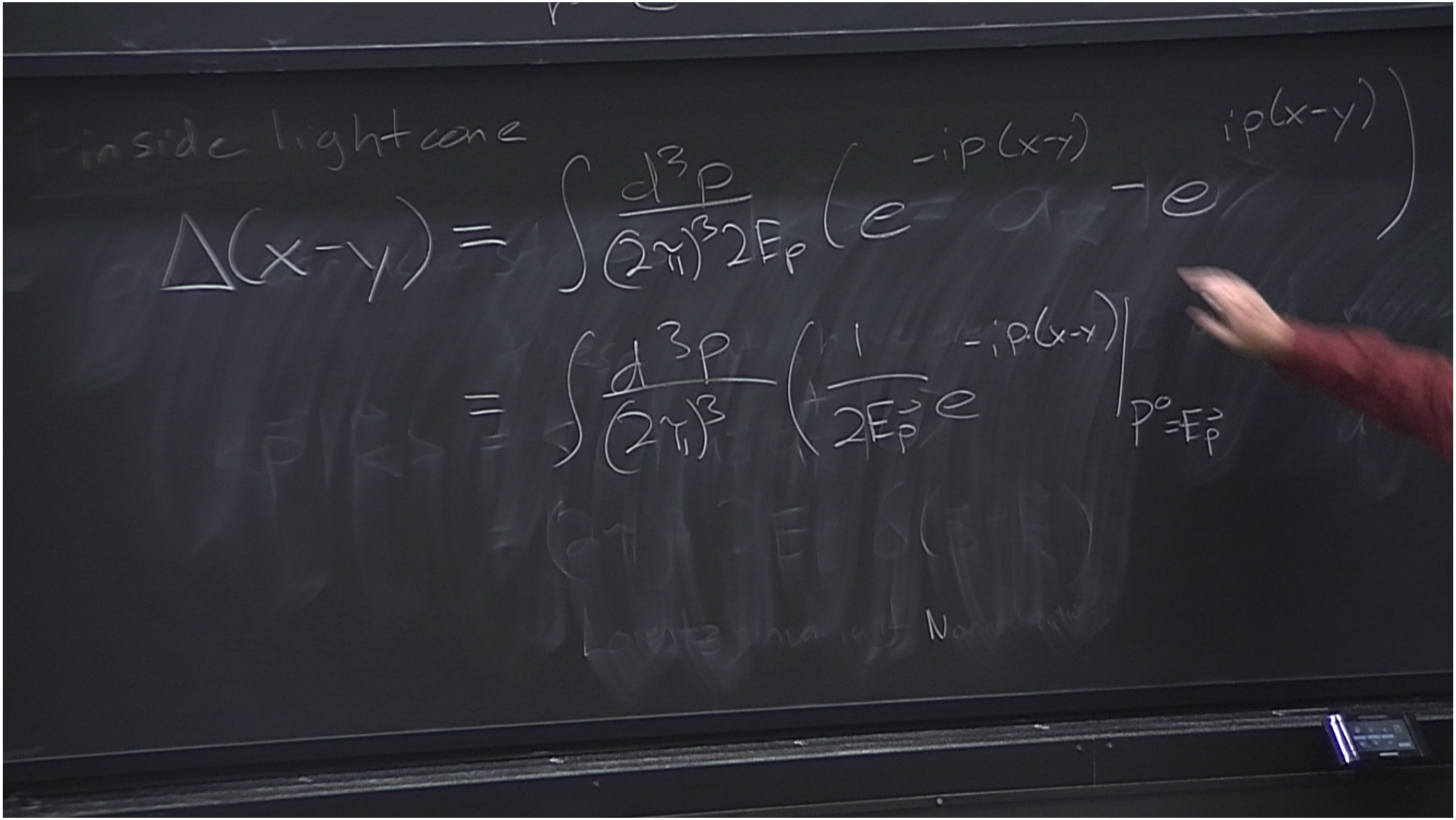

spacelike: $x^0 = y^0$ $\vec{x} - \vec{y} = \vec{r}$
$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} e^{i\vec{p}\cdot\vec{r}}$
modified Bessel function of the second kind
$= \frac{m}{4\pi^2 r} K_1(mr)$
$\neq 0$
$\sim \frac{1}{r^{3/2}} e^{-mr}$ $r \to \infty$

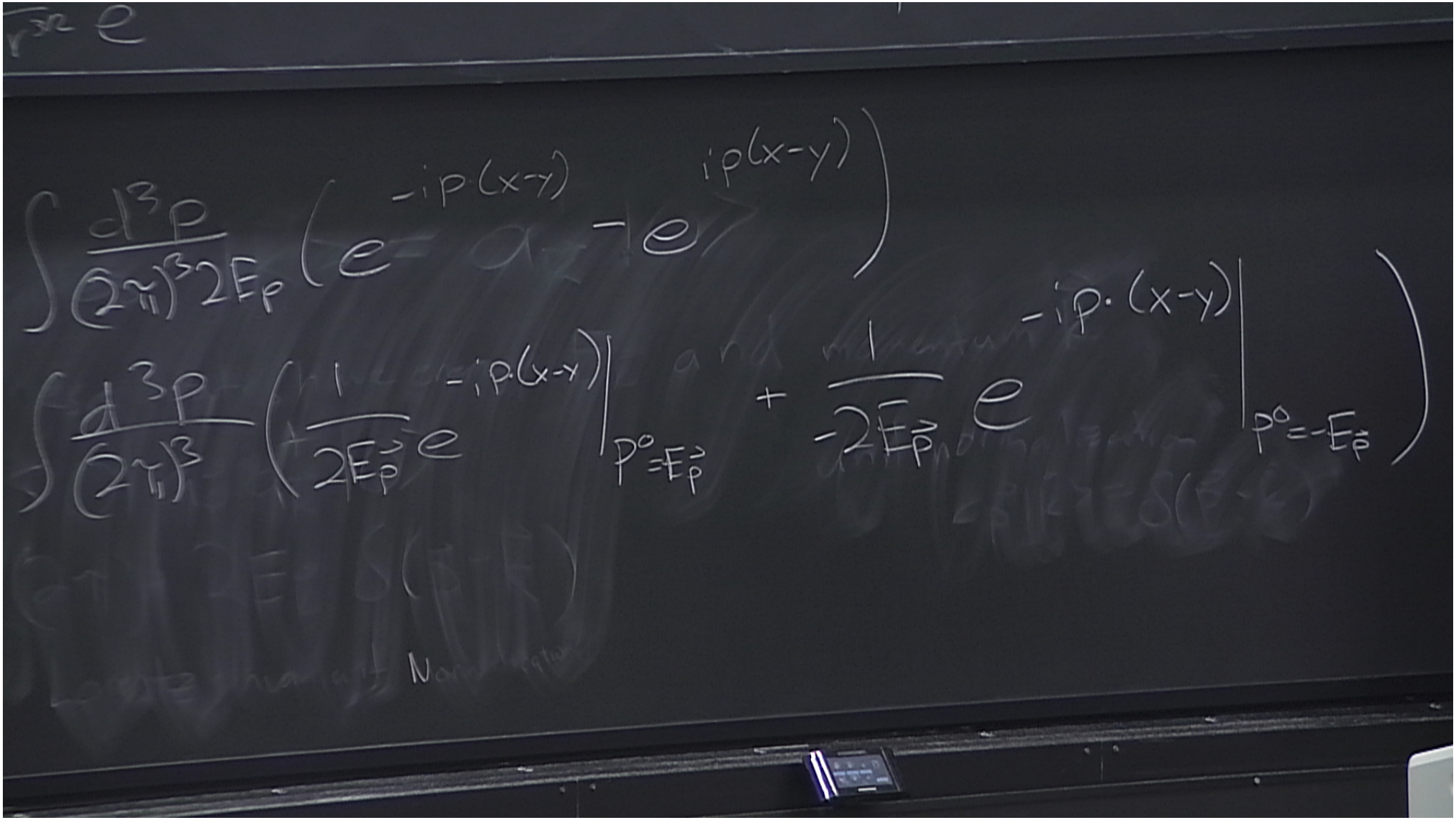

$[\varphi(x), \varphi(y)] = D(x-y) - D(y-x) = 0$
Y
X
Bessel function
the second kind

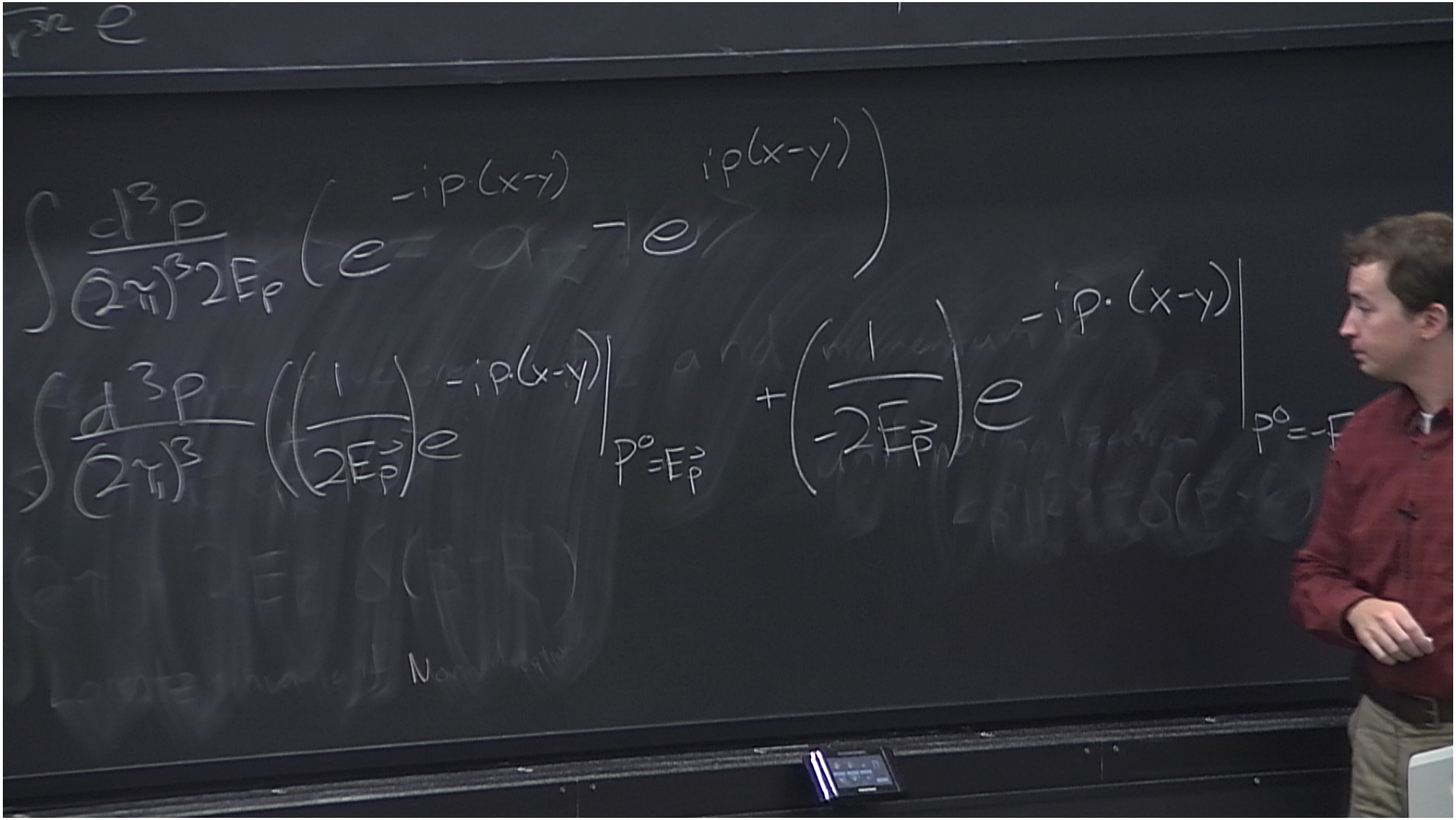

$[\varphi(x), \varphi(y)] = D(x-y) - D(y-x) = 0$
Bessel function
the second kind
x
y

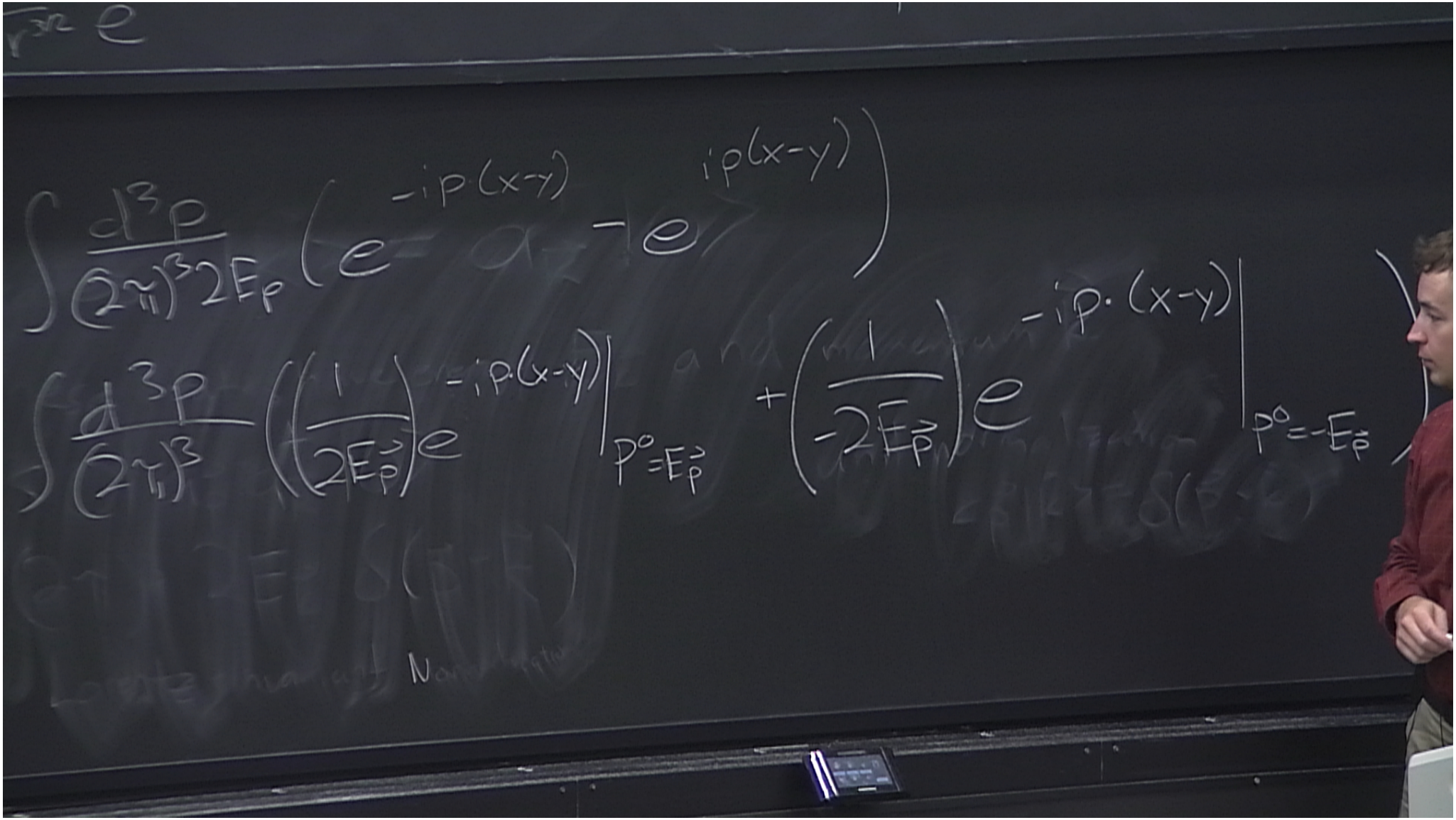

inside light cone
$\Delta(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_p} \left(e^{-ip(x-y)} - e^{ip(x-y)} \right)$
$= \int \frac{d^3p}{(2\pi)^3}$

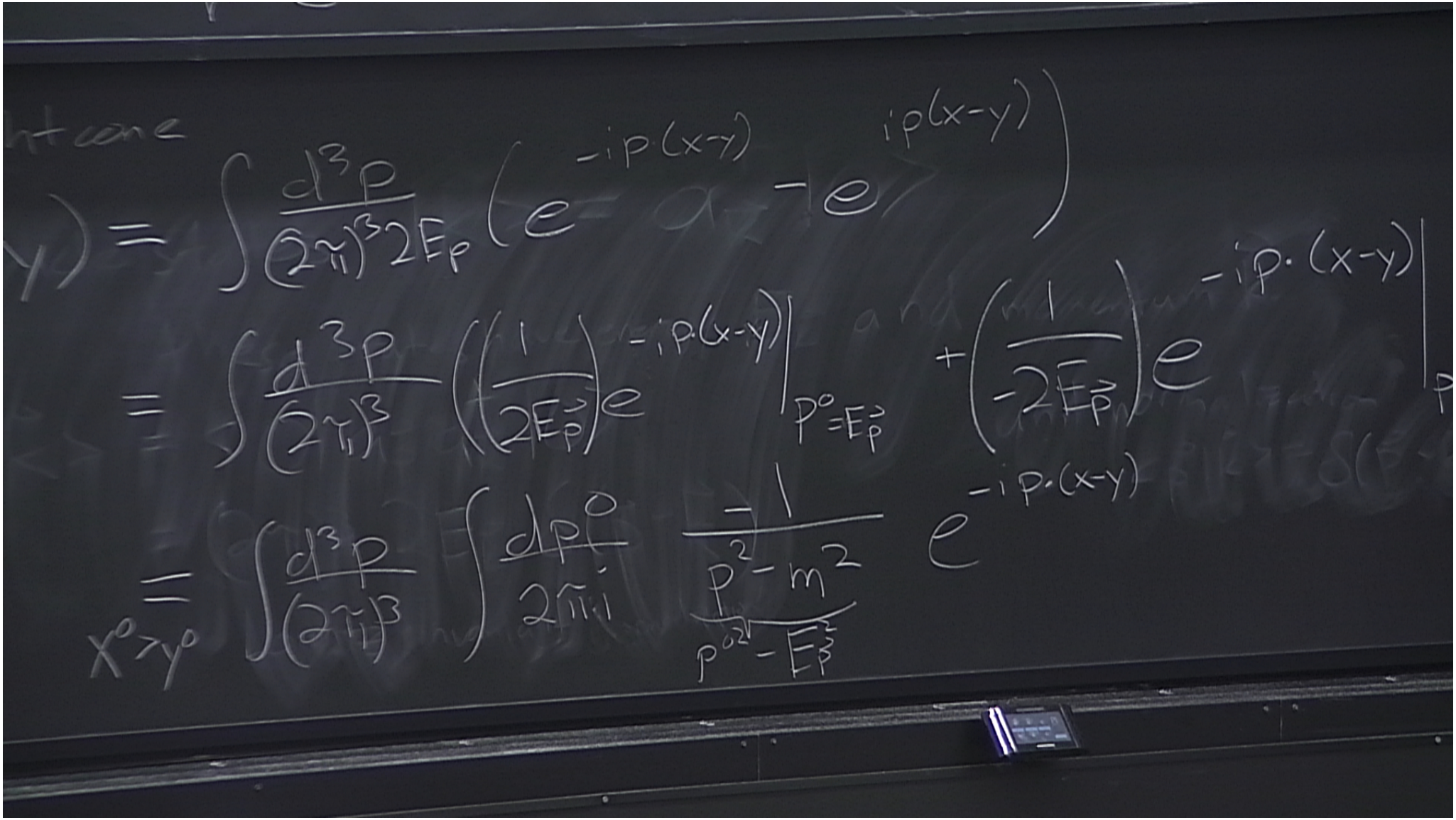

inside light cone
$\Delta(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_p} \left(e^{-ip(x-y)} - e^{ip(x-y)} \right)$
$= \int \frac{d^3p}{(2\pi)^3} \left(\frac{1}{2E_p} e^{-ip\cdot(x-y)} \right)$

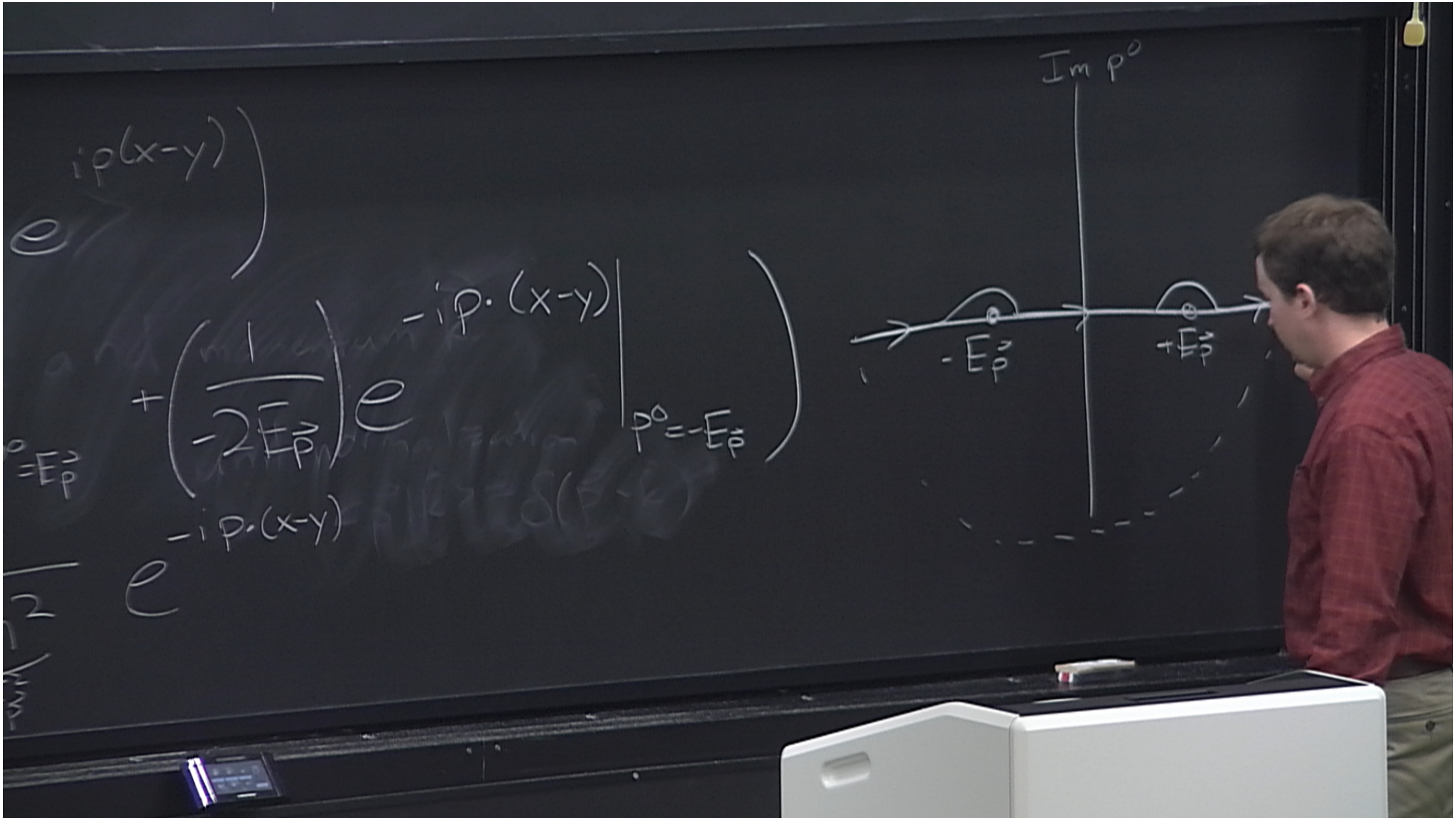

inside light cone
$\Delta(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_p} \left(e^{-ip(x-y)} - e^{ip(x-y)} \right)$
$= \int \frac{d^3p}{(2\pi)^3} \left(\frac{1}{2E_{\vec{p}}} e^{-ip\cdot(x-y)} \right)\Big|_{p^0=E_{\vec{p}}}$


$\int \frac{d^3p}{(2\pi)^3 2E_p} \left(e^{-ip\cdot(x-y)} - e^{ip(x-y)} \right)$
$\int \frac{d^3p}{(2\pi)^3} \left(\frac{1}{2E_{\vec{p}}} e^{-ip\cdot(x-y)} \Big|_{p^0=E_{\vec{p}}} + \frac{1}{-2E_{\vec{p}}} e^{-ip\cdot(x-y)} \Big|_{p^0=-E_{\vec{p}}} \right)$

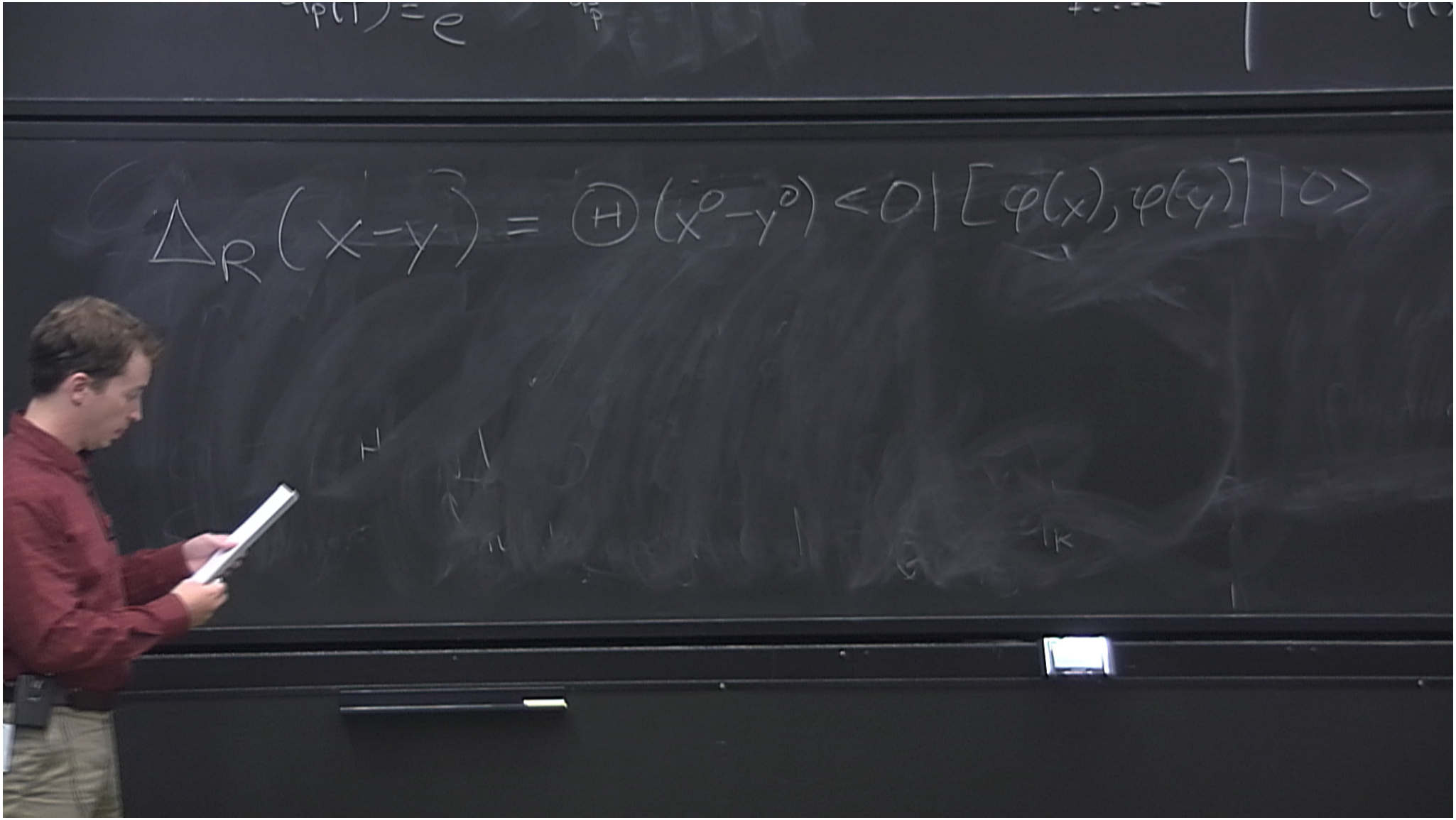

$\int \frac{d^3p}{(2\pi)^3 2E_p} \left(e^{-ip\cdot(x-y)} - e^{ip\cdot(x-y)} \right)$
$\int \frac{d^3p}{(2\pi)^3} \left(\left(\frac{1}{2E_{\vec{p}}} \right) e^{-ip\cdot(x-y)} \Big|_{p^0=E_{\vec{p}}} + \left(\frac{1}{-2E_{\vec{p}}} \right) e^{-ip\cdot(x-y)} \Big|_{p^0=-E}$

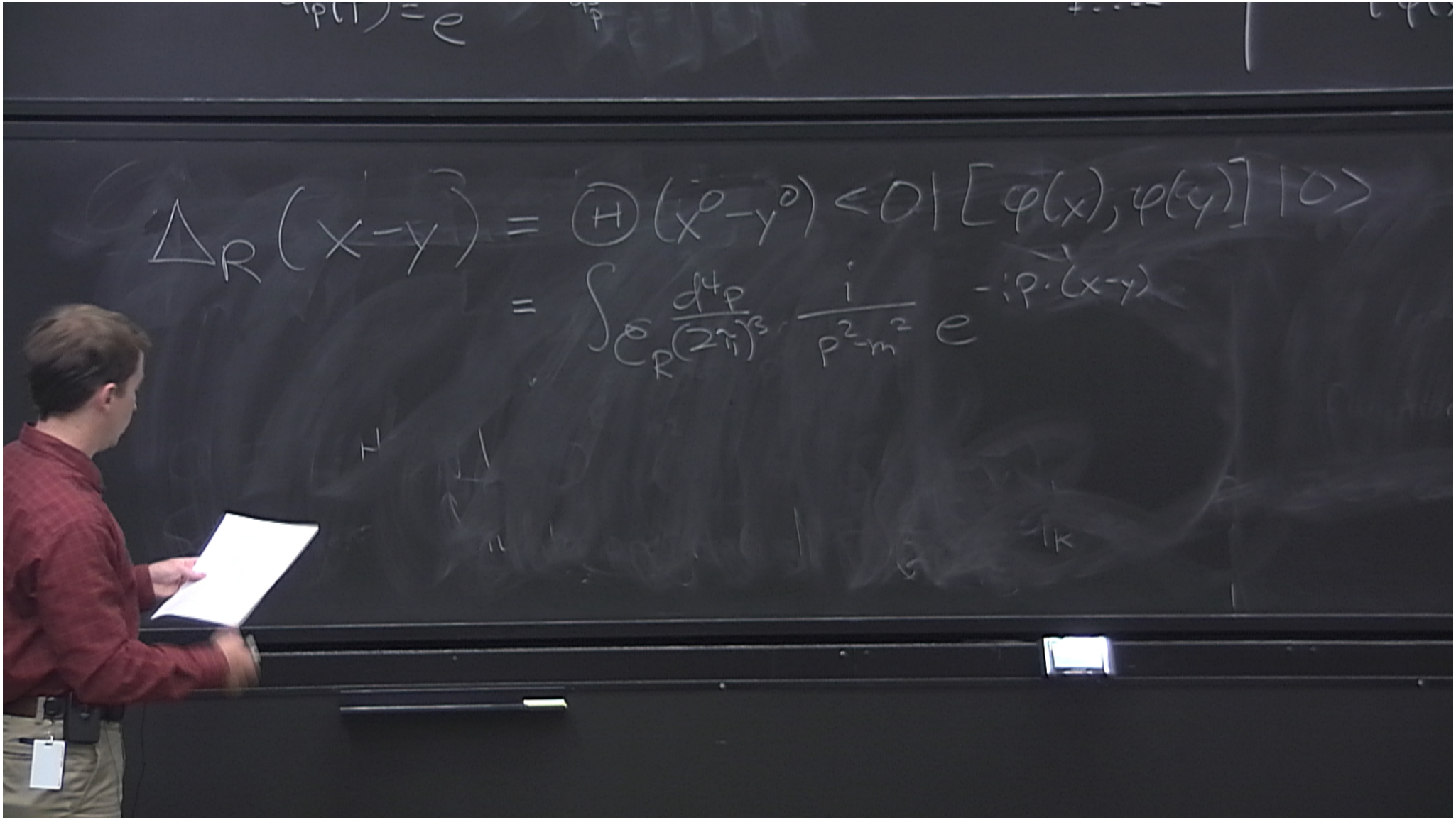

$\int \frac{d^3p}{(2\pi)^3 2E_p} \left(e^{-ip\cdot(x-y)} - e^{ip\cdot(x-y)} \right)$
$\int \frac{d^3p}{(2\pi)^3} \left(\left(\frac{1}{2E_{\vec{p}}} \right) e^{-ip\cdot(x-y)} \Big|_{p^0 = E_{\vec{p}}} + \left(\frac{1}{-2E_{\vec{p}}} \right) e^{-ip\cdot(x-y)} \Big|_{p^0 = -E_{\vec{p}}} \right)$

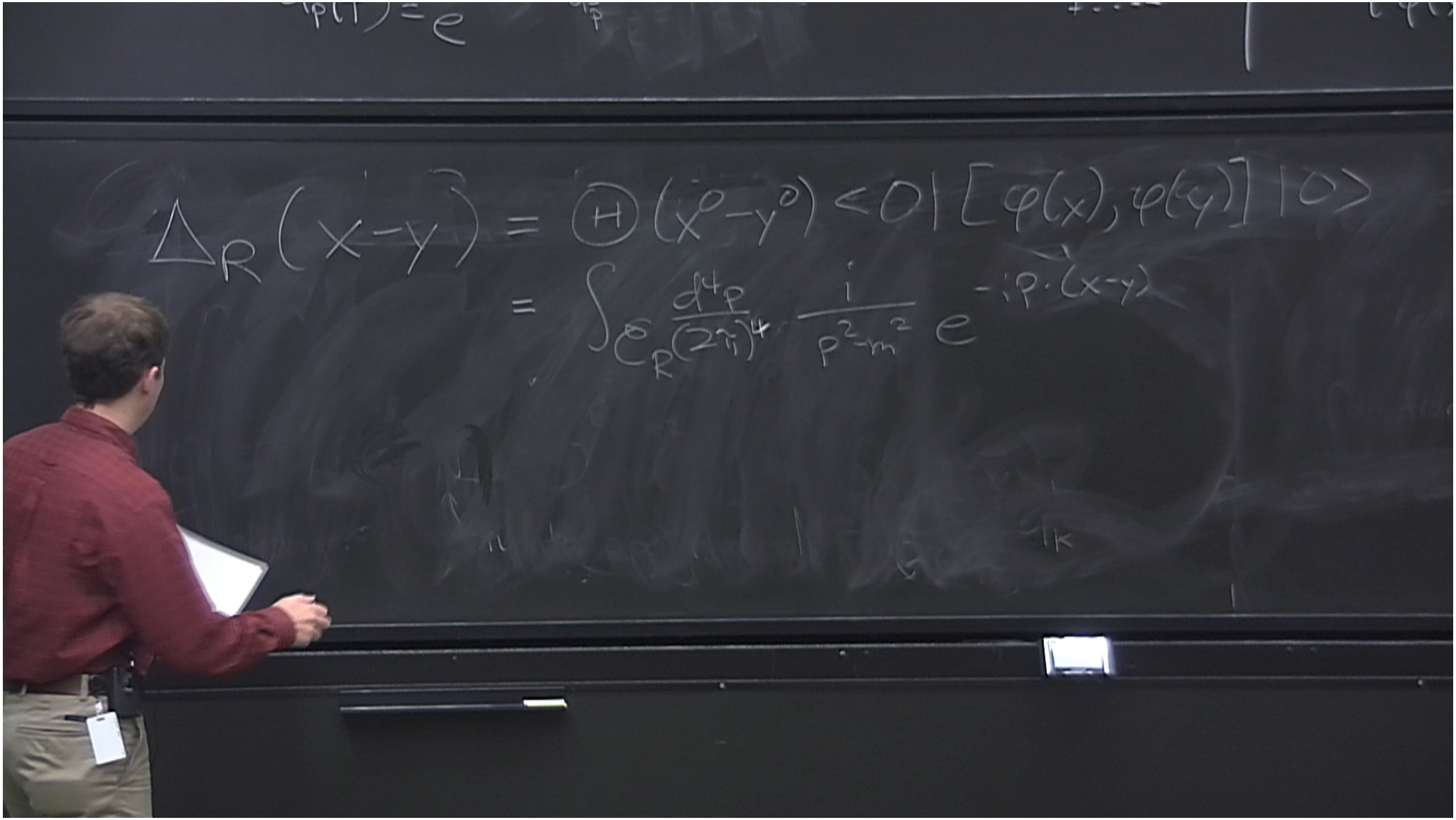

$= \int \frac{d^3p}{(2\pi)^3 2E_p} \left(e^{-ip(x-y)} - e^{ip(x-y)} \right)$
$= \int \frac{d^3p}{(2\pi)^3} \left(\left(\frac{1}{2E_p} \right) e^{-ip(x-y)} \Big|_{p^0 = E_p} + \left(\frac{1}{-2E_p} \right) e^{-ip\cdot(x-y)} \Big|_{p^0 = -E_p} \right)$
$= \int \frac{d^3p}{(2\pi)^3} \int \frac{dp^0}{2\pi i} \frac{-1}{p^2 - m^2} e^{-ip\cdot(x-y)}$
$x^0 > y^0$
$p^{0^2} - E_p^2$

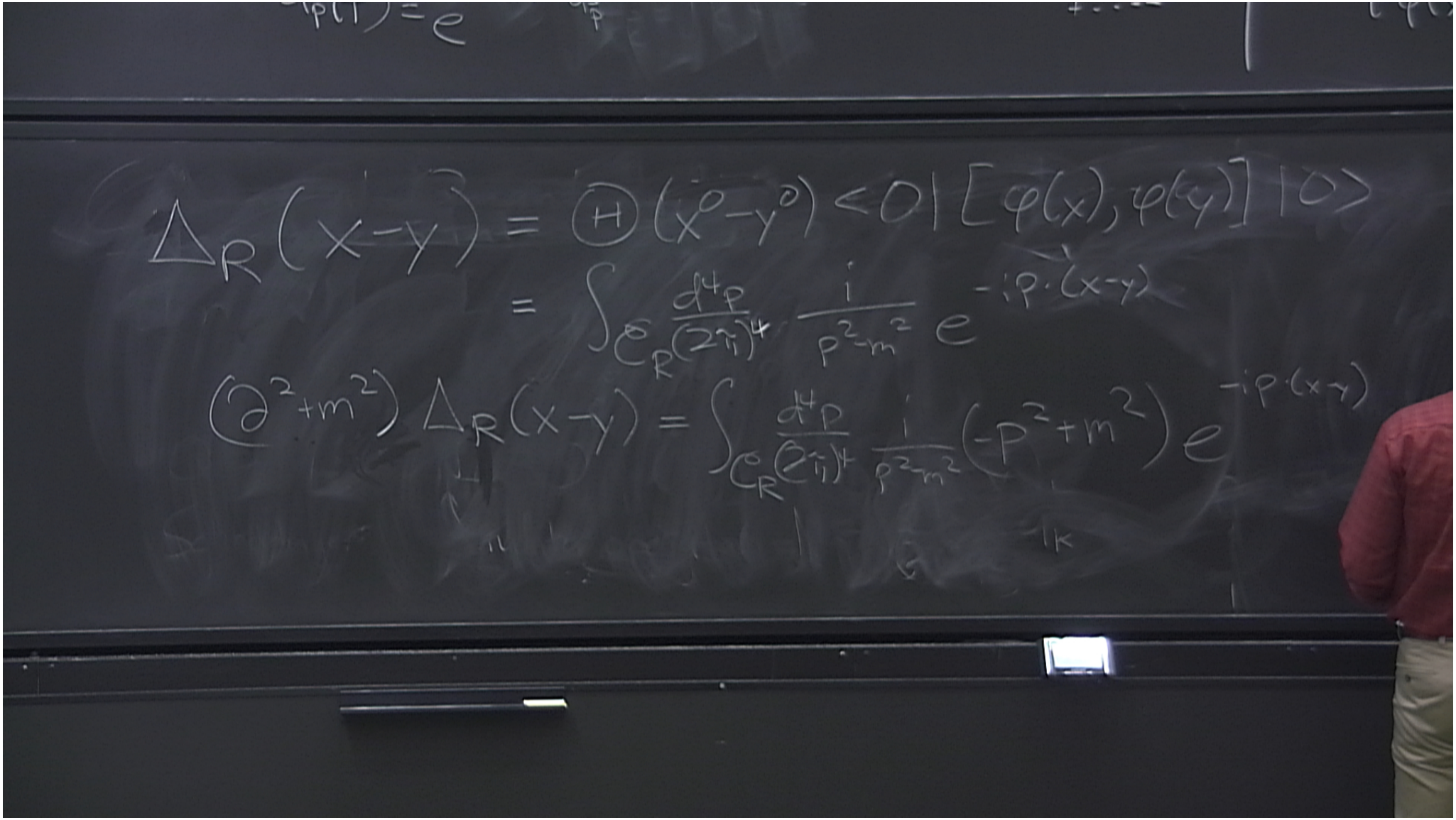

Im p⁰
-E⃗p
+E⃗p
p⁰=-E⃗p

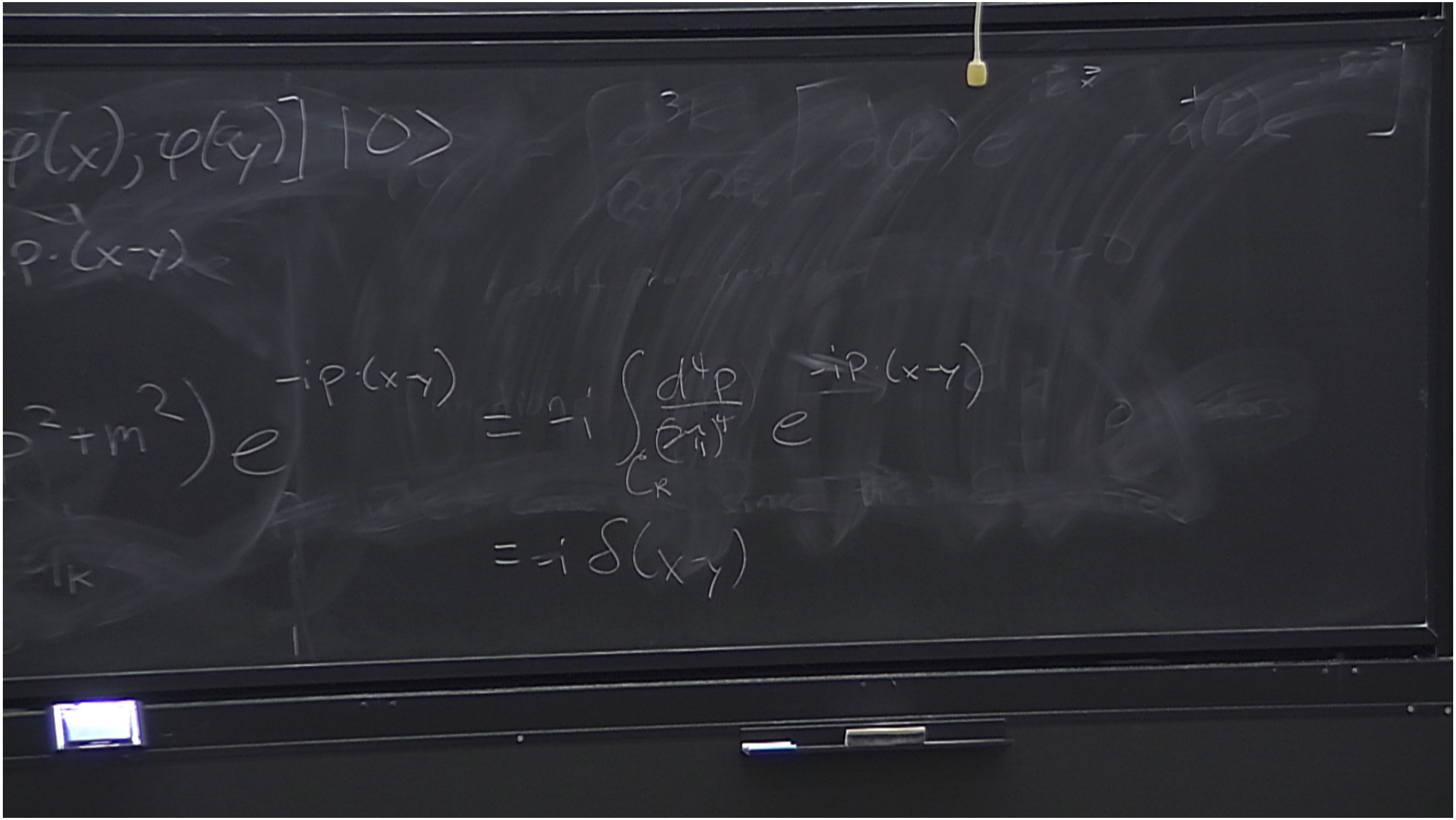

Im p°
Re p°
$-E_{\vec{p}}$
$+E_{\vec{p}}$
$e^{ip(x-y)}$
$+\left(\frac{1}{-2E_{\vec{p}}}\right)e^{-ip\cdot(x-y)}$
$e^{-ip\cdot(x-y)}$


$\Delta_R(x-y) = \theta(x^0-y^0)\langle 0|[\varphi(x),\varphi(y)]|0\rangle$


$\Delta_R(x-y) = \theta(x^0-y^0)\langle 0|[\varphi(x),\varphi(y)]|0\rangle$
$= \int_{C_R} \frac{d^4p}{(2\pi)^3} \frac{i}{p^2-m^2} e^{-ip\cdot(x-y)}$


$\Delta_R(x-y) = \theta(x^0-y^0)\langle 0|[\varphi(x),\varphi(y)]|0\rangle$
$= \int_{C_R} \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2} e^{-ip\cdot(x-y)}$


$\Delta_R(x-y) = \theta(x^0-y^0)\langle 0|[\varphi(x),\varphi(y)]|0\rangle$
$= \int_{C_R} \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2} e^{-ip\cdot(x-y)}$
$(\partial^2+m^2)\Delta_R(x-y) = \int_{C_R} \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2}(-p^2+m^2) e^{-ip\cdot(x-y)}$


$\varphi(x), \varphi(y)] |0\rangle$
$p \cdot (x-y)$
$(\partial^2 + m^2) e^{-ip\cdot(x-y)} = -i \int_{C_R} \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)}$
$= -i\,\delta(x-y)$