

Title: 14/15 PSI - Quantum Field Theory I (A) - Lecture 1

Date: Oct 06, 2014 03:30 PM

URL: <http://pirsa.org/14100027>

Abstract:

Quantum Field Theory I

Why QFT?

- QM + special relativity imply that par

Field Theory I

?
special relativity imply that particle # is not conserved

Quantum Field Theory I

Why QFT?

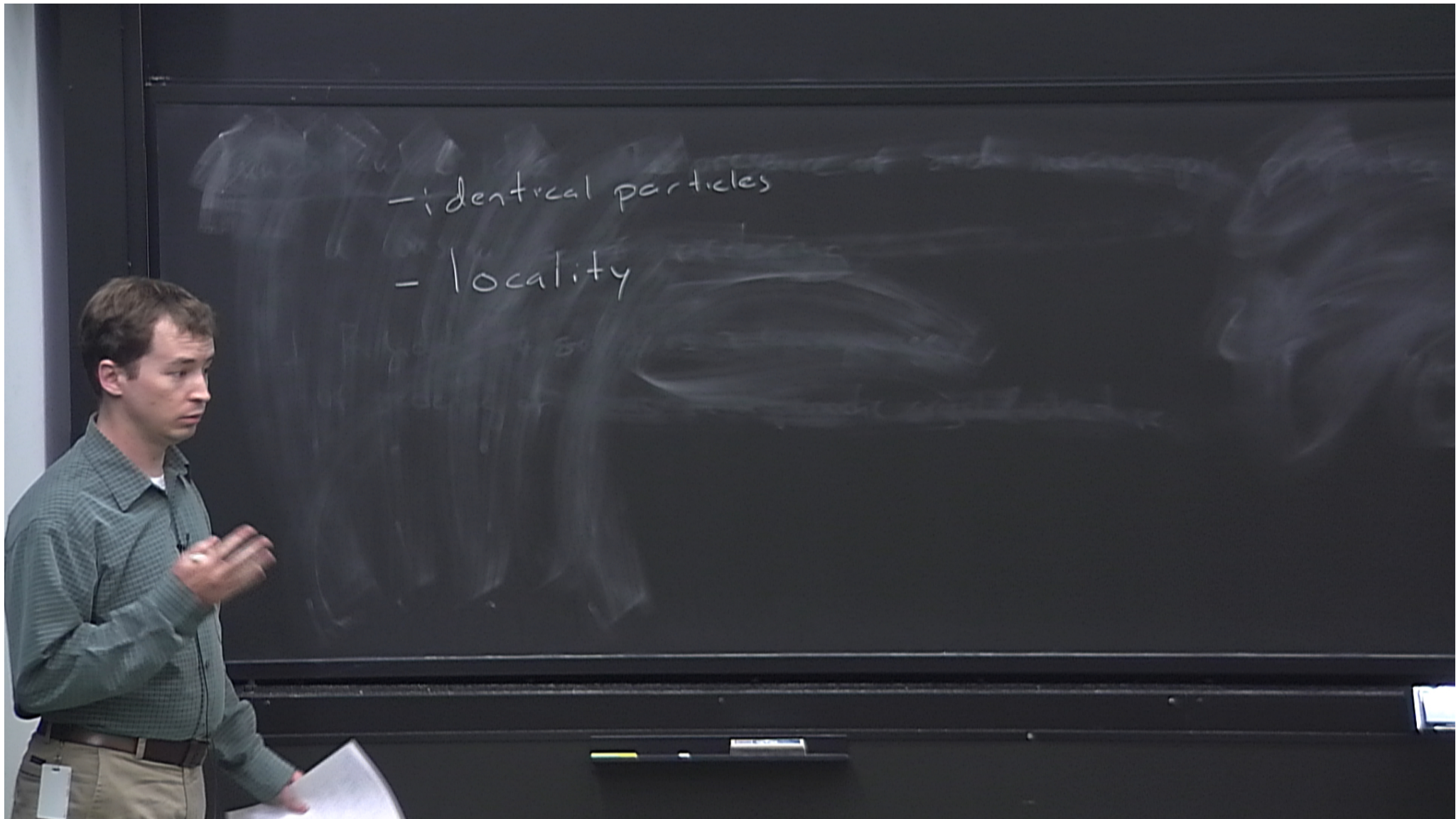
- QM + special relativity imply the

box of size L

$$\Delta p \geq \frac{\hbar}{L}$$

$$\Delta E \geq \frac{\hbar c}{L}$$

$$\Delta E \geq 2$$



- identical particles

- locality

Outline:

Spin-0

Spin- $\frac{1}{2}$

Spin-1

Spin-0

icles

Spin-0

Spin- $\frac{1}{2}$ } Tibra
Spin-1 }

Spin-0

Classical Field Theory

Canonical Quantization

LSZ Reduction Formula

Interaction Picture + Wick's Theorem

Feynman Diagrams + Scattering Amplitudes

Cross Sections + Decay Rates

Introduction to Loops

icles

Spin-0

Spin- $\frac{1}{2}$ } Tibra
Spin-1 }

Spin-0

Classical Field Theory

Canonical Quantization

LSZ Reduction Formula

Interaction Picture + Wick's Theorem

Feynman Diagrams + Scattering Amplitudes

Cross Sections + Decay Rates

Introduction to Loops

Classical Field Theory

$$\varphi_a(t, \vec{x}) = \varphi_a(x)$$

generalizes

$$q_a(t)$$

Introduction to Loops

theory
generalizes

$q_a(t)$

finite # of generalized
coordinates

Introduction to Loops

generalizes

$$q_a(t)$$

$$q_{a,\vec{x}}(t)$$

finite # of generalized
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Introduction to Loops

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Classical Field Theory

$$\varphi_a(t, \vec{x}) = \varphi_a(x)$$

generalizes

$$q_a(t)$$

finite #
coord

$$q_{a,\vec{x}}(t)$$

$$S = \int dt L(q_a, \dot{q}_a) \rightarrow S = \int dt \int d^3x \mathcal{L}(\varphi_a, \partial_\mu \varphi_a)$$

Introduction to Loops

Field Theory

$\varphi_a(x)$

generalizes

$q_a(t)$

finite # of generalized coordinates

$q_{a,\vec{x}}(t)$

$$L(q_a, \dot{q}_a) \rightarrow S = \int dt + \underbrace{\int d^3x}_{\text{locality}} \mathcal{L}(\varphi_a(x), \partial_\mu \varphi_a(x))$$

Introduction to Loops

d Theory

$\varphi_a(x)$

generalizes

$q_a(t)$

finite # of generalized coordinates

$q_{a,\vec{x}}(t)$

$(q_a, \dot{q}_a)$

$\rightarrow$

$$S = \int dt \underbrace{\int d^3x}_{\text{locality}} \mathcal{L}(\varphi_a(x), \partial_\mu \varphi_a(x))$$

Lorentz invariance

$$\mathcal{L} \supset \partial_\mu \varphi_a \partial^\mu \varphi_a$$

Introduction to Loops

Field Theory

$\phi_a(x)$

generalizes

$q_a(t)$

finite # of generalized coordinates

$q_{a,\vec{x}}(t)$

$L(q_a, \dot{q}_a)$

$\rightarrow$

$$S = \int dt \underbrace{\int d^3x}_{\text{locality}} \mathcal{L}(\phi_a(x), \partial_\mu \phi_a(x))$$

Lorentz invariance

$$\mathcal{L} = \partial_\mu \phi_a \partial^\mu \phi_a$$

on to Loops

finite # of generalized
coordinates

$\mathcal{L}(\varphi_a(x), \partial_M \varphi_a(x))$

↖ Lorentz
invariance

$\partial_M \varphi_a \partial^M \varphi_a$

$$S = \int dt \int_{\text{non-local}} d^3x \int d^3y \mathcal{L}(\varphi_a(x), \varphi_a(y))$$

$$\varphi \rightarrow \varphi'(x) = \varphi(\Lambda^{-1}x) \quad \text{active transformations}$$

Λ Lorentz transformation

$$x^\mu \rightarrow (x')^\mu = \Lambda^\mu{}_\nu x^\nu$$

where $\Lambda^\mu{}_\sigma \eta^{\sigma\rho} \Lambda^\nu{}_\rho = \eta^{\mu\nu}$

$$\eta^{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} +1 & & \\ & -1 & \\ & & -1 \end{pmatrix}$$

$$\varphi \rightarrow \varphi'(x) = \varphi(\Lambda^{-1}x)$$

active transformations

Λ Lorentz transformation

$$x^\mu \rightarrow (x')^\mu = \Lambda^\mu{}_\nu x^\nu$$

$$\varphi'(x') = \varphi(x)$$

$$\varphi'(\Lambda x) = \varphi(x)$$

relabel x

where $\Lambda^\mu{}_\sigma \eta^{\sigma\rho} \Lambda^\nu{}_\rho = \eta^{\mu\nu}$

$$\eta^{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

Klein-Gordon

$$\mathcal{L}_{KG} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} m^2 \varphi^2$$

$$\hbar = c = 1$$

Gordon

$$= \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} m^2 \varphi^2$$

$$\hbar = c = 1$$

Gordon

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$$\hbar = c = 1$$

Klein-Gordon

$$\mathcal{L}_{KG} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} m^2 \varphi^2$$

$$\hbar = c =$$

EOM: Classical particle mechanics

$$\frac{\delta S}{\delta q_a(t)} = 0$$

finite # of generalized
coordinates

↙ Lorentz
invariance

$$(\varphi_a(x), \partial_M \varphi_a(x))$$

$$\mathcal{L} \supset \partial_M \varphi_a \partial^M \varphi_a$$

$$S = \int dt \int d^3x \int d^3y \underbrace{\mathcal{L}(\varphi_a(x), \varphi_a(y))}_{\text{non-local}} \varphi(x) \varphi(y)$$

$$\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 \quad \hbar = c = 1$$

ssical mechanics

$$\frac{\partial L}{\partial q_a} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_a} = 0 \quad \text{E-L eqn}$$

Klein-Gordon

$$\mathcal{L}_{KG} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2$$

$$\hbar = c$$

EOM: Classical particle mechanics

$$\frac{\delta S}{\delta q_a(t)} = 0$$

Field Theory

$$\frac{\delta S}{\delta \phi_a(x)} = 0$$

$$\frac{\partial \mathcal{L}}{\partial q_a} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}_a}$$

$$\frac{\partial \mathcal{L}}{\partial \phi_a} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_a}$$

$$\phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2$$

$$\hbar = c = 1$$

particle mechanics

$$\frac{\partial L}{\partial q_a} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_a} = 0 \quad \text{E-L eqn}$$

$$\left| \frac{\partial \mathcal{L}}{\partial \phi_a} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_a)} \right) = 0 \right|$$

$\delta\varphi(x)$

KG: $\boxed{\partial_\mu \partial^\mu \varphi + m^2 \varphi = 0}$

Noether's Theorem: Every continuous symmetry c

$$Q = \int_{\mathbb{R}^3} d^3x j^0$$

$$\frac{dQ}{dt} = \int_{\mathbb{R}^3} d^3x \frac{dj^0}{dt} = - \int_{\mathbb{R}^3} d^3x \vec{\nabla} \cdot \vec{j} =$$

Introduction to Loops

$\times j^0$ Theorem I

$$\times \frac{dj^0}{dt} = - \int_{\mathbb{R}^3} d^3x \vec{\nabla} \cdot \vec{j} = - \int \vec{j} \cdot d\vec{S} \stackrel{!}{=} 0$$

assuming $\vec{j} \rightarrow 0$ su

Proof: If we have a continuous symmetry,
we can consider an infinitesimal transformation

$$\delta\varphi(x) = Y_a(\varphi)$$

Proof: If we have a continuous symmetry,
we can consider an infinitesimal transformation

$$\delta\varphi_a(x) = Y_a(\varphi)$$

This is a symmetry transformation if

$$\delta\mathcal{L} = \partial_\mu F^\mu(\varphi)$$

metry,
+ transformation

Under an arbitrary transformation, $\delta\varphi_a$

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\varphi_a} \delta\varphi_a + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_a)} \partial_\mu(\delta\varphi_a)$$

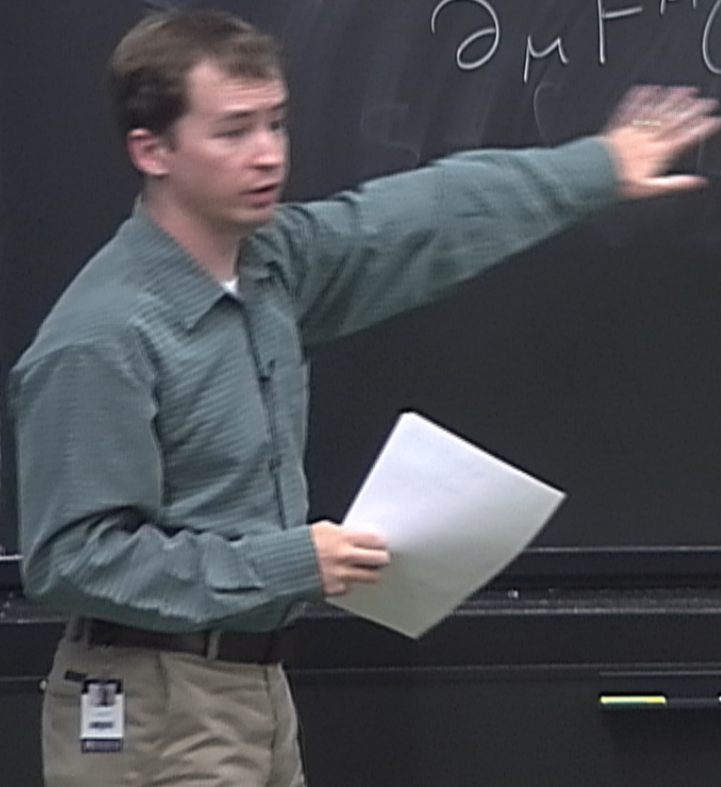
if

$$= \left(\frac{\partial\mathcal{L}}{\partial\varphi_a} - \partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_a)} \right) \delta\varphi_a$$

$$+ \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_a)} \delta\varphi_a \right)$$

If $\delta\varphi_a$ is a symmetry

$$\partial_\mu F^M(\varphi) = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi_a)} Y_a(\varphi) \right)$$



If $\delta\varphi_a$ is a symmetry

$$\partial_\mu F^\mu(\varphi) = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_a)} Y_a(\varphi) \right)$$

$$\partial_\mu j^\mu = 0$$

$$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_a)} Y_a - F^\mu(\varphi)$$

Example: Translations

$$x^{\nu} \rightarrow x^{\nu} - \epsilon^{\nu}$$

ϵ infinitesimal

$$\varphi_a(x) \rightarrow \varphi_a(x) + \epsilon^{\nu} \partial_{\nu} \varphi_a(x)$$

active transformations

Example: Translations

$$x^v \rightarrow x^v - \epsilon^v$$

ϵ infinitesimal

$$\varphi_a(x) \rightarrow \varphi_a(x) + \epsilon^v \partial_v \varphi_a(x)$$

active transformations

$$\mathcal{L}(x) \rightarrow \mathcal{L}(x) + \epsilon^v \partial_v \mathcal{L}(x)$$

Translations

$$x^{\nu} \rightarrow x^{\nu} + \epsilon^{\nu} \quad \epsilon^{\nu} \text{ infinitesimal}$$

$$\rightarrow \varphi_a(x) + \epsilon^{\nu} \partial_{\nu} \varphi_a(x)$$

active transformations

$$\rightarrow \mathcal{L}(x) + \epsilon^{\nu} \partial_{\nu} \mathcal{L}(x)$$

$$H = \frac{\partial \mathcal{L}}{\partial(\partial_{\nu} \varphi_a)} \partial_{\nu} \varphi_a - \int d^3x \mathcal{L} \equiv T^M_2$$

energy + momentum

$$Y_a = \epsilon^{\nu} \partial_{\nu} \varphi_a$$

$$F^M = \epsilon^{\nu} \partial_{\nu} \mathcal{L}(x)$$

4-momentum

$$P^\mu = \int d^3x T^{0\mu}$$

c/a

4-momentan

$$P^\mu = \int d^3x T^{0\mu}$$

c.a.

4-momentum

$$P^\mu = \int d^3x T^{0\mu}$$

Example: $U(1)$ Internal Symmetry

Complex Field

$$\Phi(x) = \frac{1}{\sqrt{2}} (\phi_1(x) + i\phi_2(x))$$

4-momentum

$$P^\mu = \int d^3x T^{0\mu}$$

Example: $U(1)$ Internal Symmetry global

Complex Field

$$\Phi(x) = \frac{1}{\sqrt{2}} (\phi_1(x) + i\phi_2(x))$$

$$\mathcal{L} = \partial_\mu \Phi^\dagger \partial^\mu \Phi - V(|\Phi|^2)$$

$$\Phi \rightarrow e^{i\theta} \Phi \quad \Phi^\dagger \rightarrow e^{-i\theta} \Phi^\dagger$$

$$\delta \Phi = i \Theta \Phi$$

$$\delta \Phi^\dagger = -i \Theta \Phi^\dagger$$

$$\delta \Phi = i \Theta \Phi$$

$$\delta \Phi^* = -i \Theta \Phi^*$$

$$\textcircled{2} j^M = \frac{\partial \mathcal{L}}{\partial (\partial_n \Phi)} \delta \Phi + \frac{\partial \mathcal{L}}{\partial (\partial_n \Phi^*)} \delta \Phi^*$$

$$j^M = i (\partial^M \Phi^*) \Phi - i (\partial^M \Phi) \Phi^*$$

$$\delta \Phi = i \Theta \Phi$$

$$\delta \Phi^* = -i \Theta \Phi^*$$

$$\textcircled{2} j^M = \frac{\partial \mathcal{L}}{\partial (\partial_\mu \Phi)} \delta \Phi + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \Phi^*)} \delta \Phi^*$$

$$j^M = i (\partial^\mu \Phi^*) \Phi - i (\partial^\mu \Phi) \Phi^*$$

Hamiltonian Formalism

$$\pi^a(x) = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}^a}$$

momentum conjugate

$$\mathcal{L}_{KG} = \frac{1}{2} \dot{\varphi}^2 - \frac{1}{2} (\nabla \varphi)^2 - \frac{1}{2} m^2 \varphi^2$$

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = \dot{\varphi}$$

$$H = \int d^3x ($$

$$\mathcal{L}_{KG} = \frac{1}{2} \dot{\varphi}^2 - \frac{1}{2} (\nabla \varphi)^2 - \frac{1}{2} m^2 \varphi^2$$

$$\pi_1 = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = \dot{\varphi}$$

$$H = \int d^3x \left(\frac{1}{2} \pi_1^2 + \frac{1}{2} (\nabla \varphi)^2 + \frac{1}{2} m^2 \varphi^2 \right)$$

Solution to KG equation (classical)

$$\partial^2 \varphi + m^2 \varphi = 0$$

$$\varphi(t, \vec{x}) = \int \frac{d^3 k}{(2\pi)^3} e^{i \vec{k} \cdot \vec{x} - i \omega(t, \vec{k}) t}$$

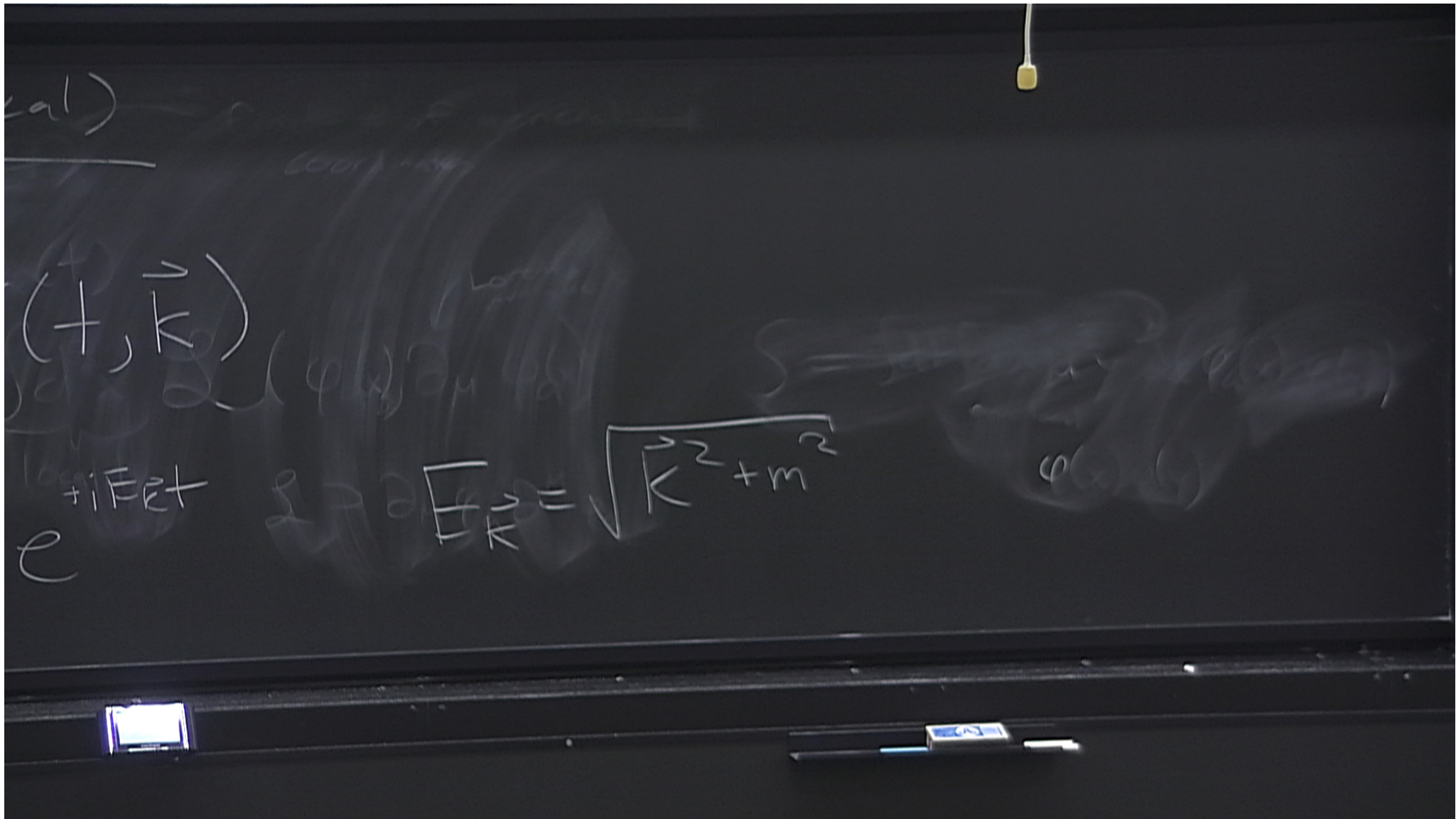
Solution to KG equation (classical)

$$\partial^2 \varphi + m^2 \varphi = 0$$

$$\varphi(t, \vec{x}) = \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \tilde{\varphi}(t, \vec{k})$$

$$\ddot{\tilde{\varphi}} + \vec{k}^2 \tilde{\varphi} + m^2 \tilde{\varphi} = 0$$

$$\tilde{\varphi}(t, \vec{k}) = A(\vec{k}) e^{-iE_{\vec{k}} t} + B(\vec{k}) e^{+iE_{\vec{k}} t}$$



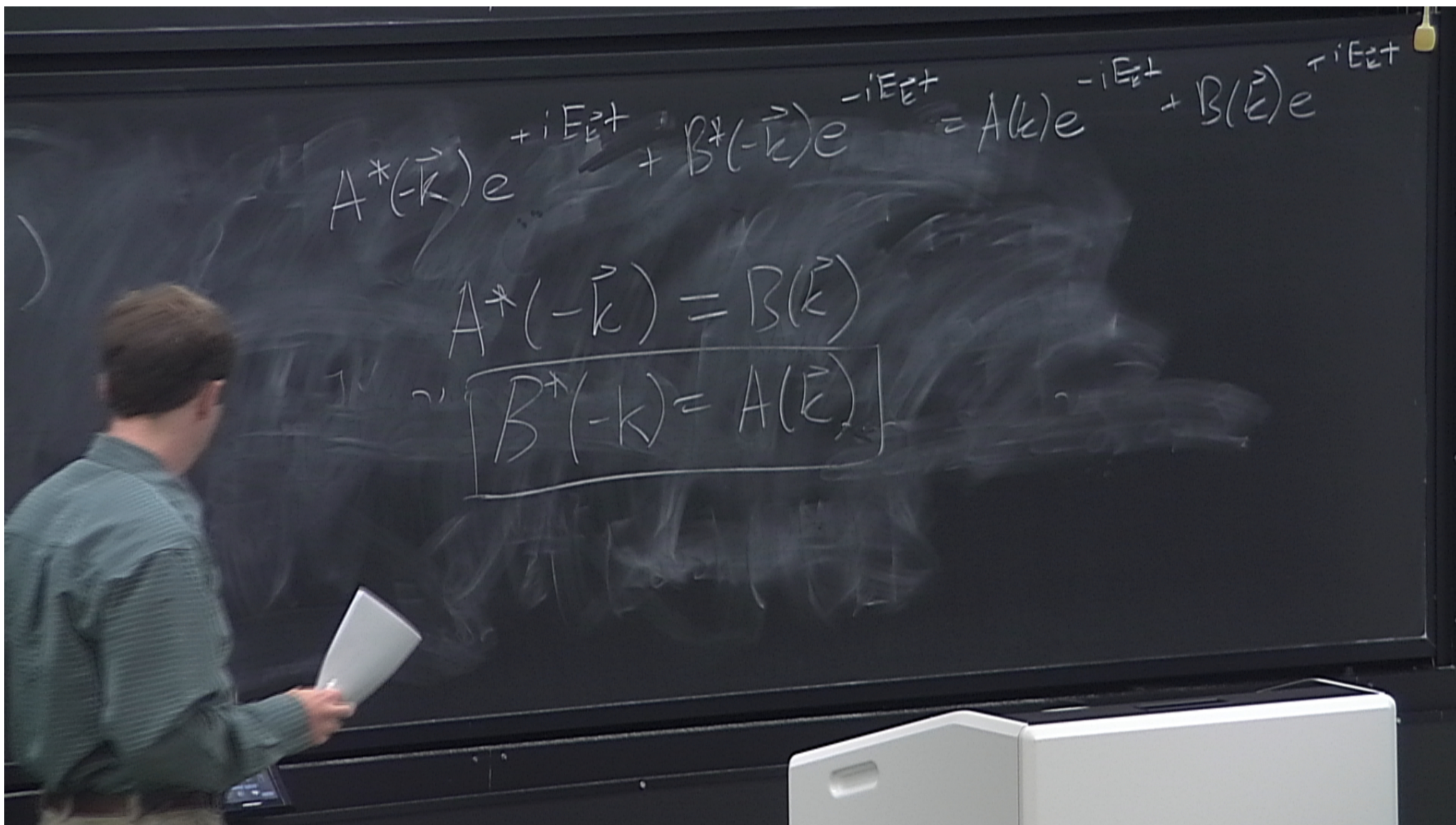
$$\varphi = \varphi^*$$

$$\varphi(t, \vec{x}) = \int \frac{d^3 k}{(2\pi)^3} e^{-i\vec{k} \cdot \vec{x}} \tilde{\varphi}^*(t, \vec{k})$$

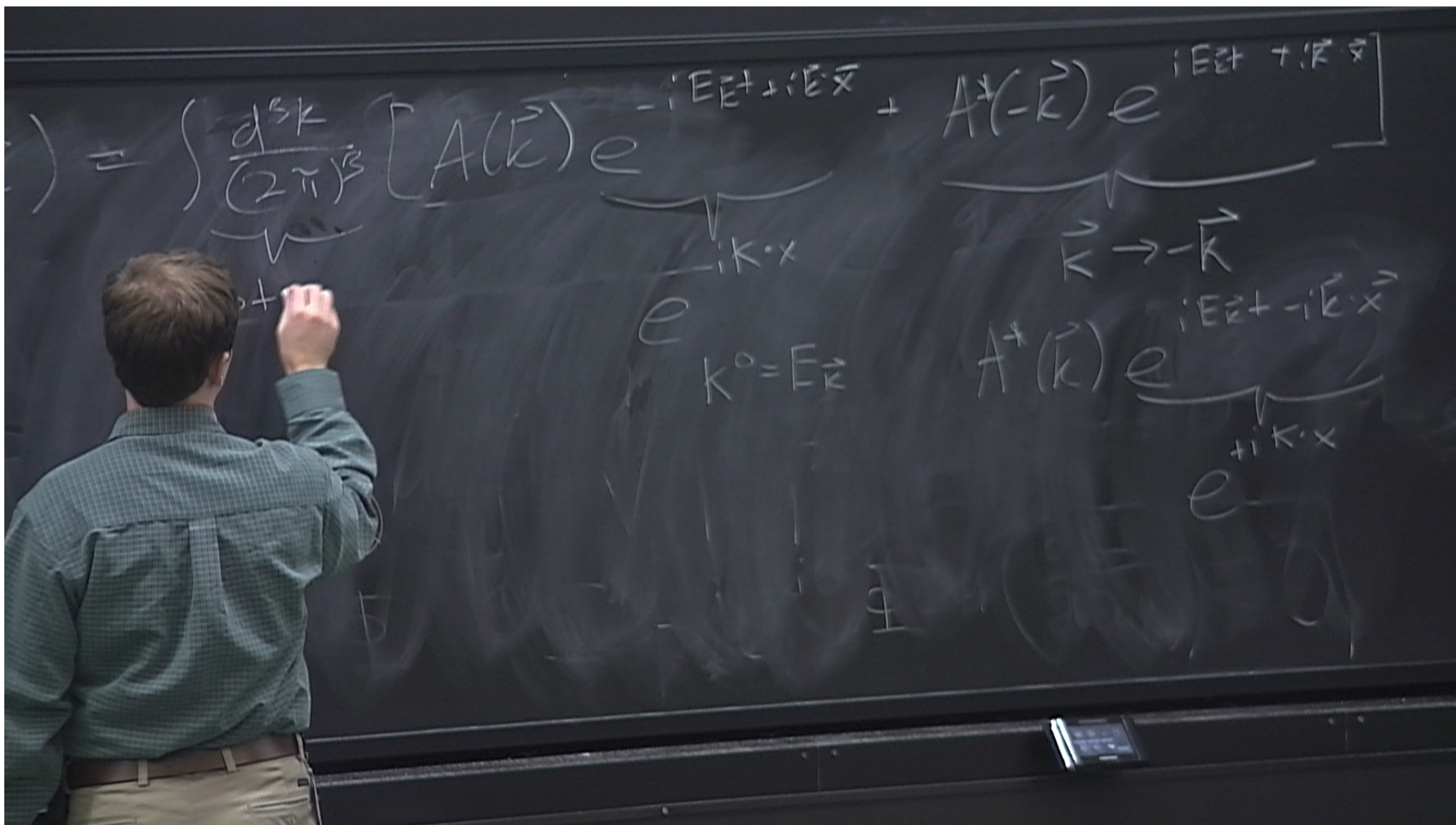
$$\vec{k} \rightarrow -\vec{k}$$

$$= \int \frac{d^3 k}{(2\pi)^3} e^{+i\vec{k} \cdot \vec{x}} \tilde{\varphi}^*(t, -\vec{k})$$

$$\tilde{\varphi}^*(t, -\vec{k}) = \tilde{\varphi}(t, \vec{k})$$



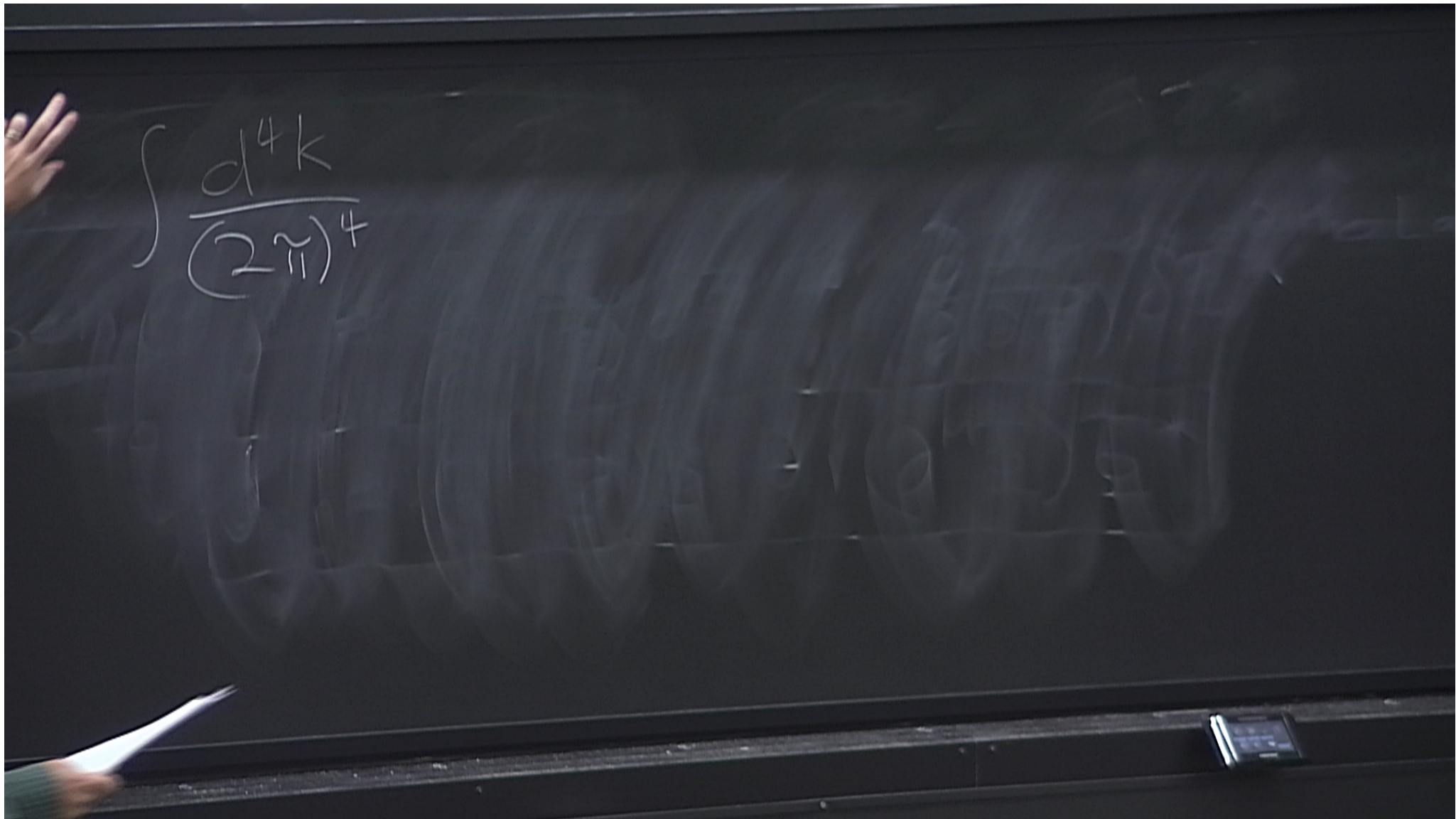
$$\varphi(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \left[A(\vec{k}) e^{-iE_{\vec{k}}t + i\vec{k} \cdot \vec{x}} + A^\dagger(-\vec{k}) \right]$$



$$) = \int \frac{d^3k}{(2\pi)^3} \left[A(\vec{k}) e^{-iE_k t + i\vec{k} \cdot \vec{x}} + A^\dagger(-\vec{k}) e^{iE_k t + i\vec{k} \cdot \vec{x}} \right]$$

$\underbrace{\frac{d^3k}{(2\pi)^3}}_{\text{not Lorentz invariant}}$

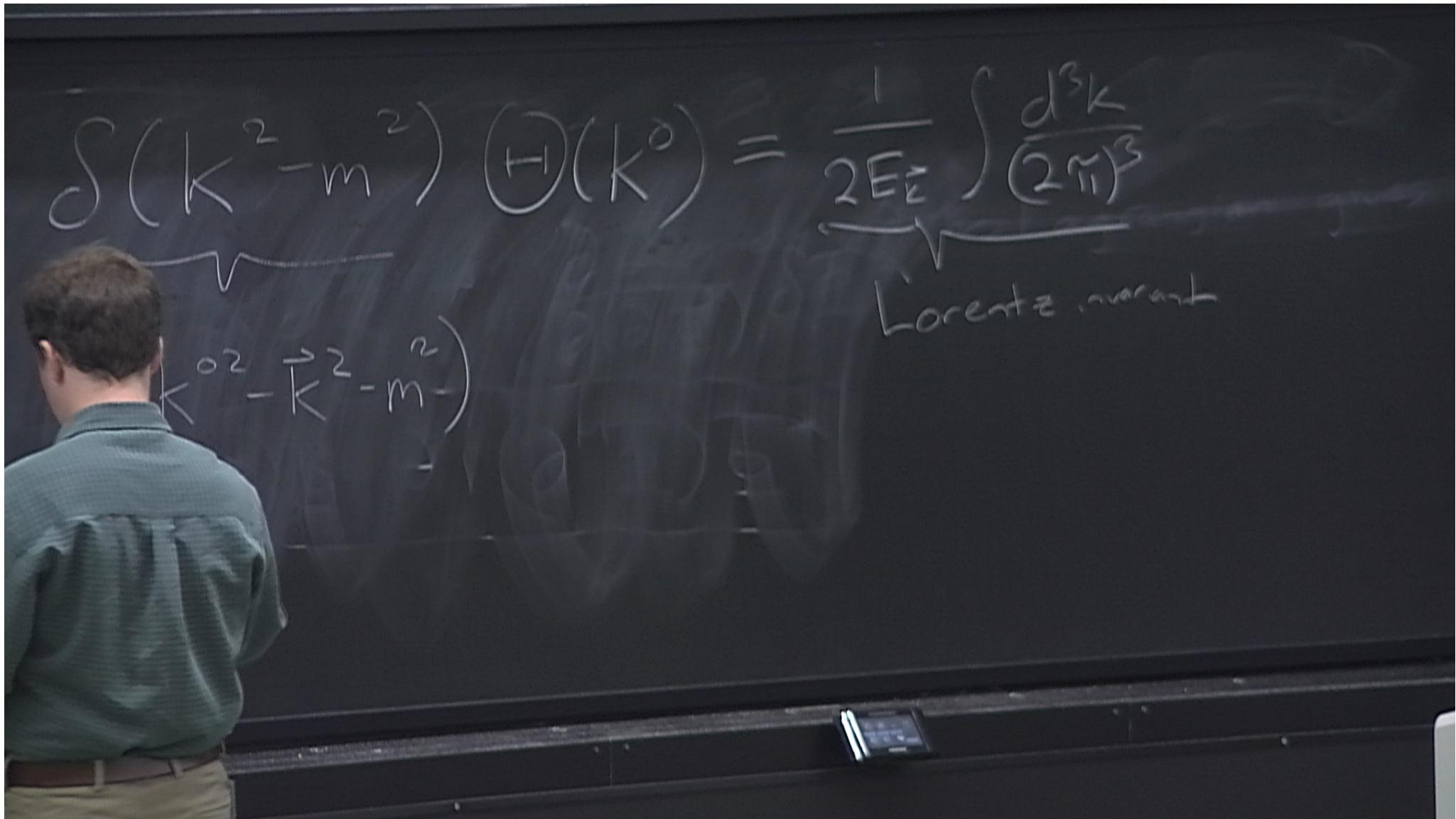
$\underbrace{e^{-iE_k t + i\vec{k} \cdot \vec{x}}}_{\substack{\vec{k} \cdot \vec{x} \\ k^0 = E_{\vec{k}}}} \quad \underbrace{A^\dagger(-\vec{k}) e^{iE_k t + i\vec{k} \cdot \vec{x}}}_{\substack{\vec{k} \rightarrow -\vec{k} \\ A^\dagger(\vec{k}) e^{iE_k t - i\vec{k} \cdot \vec{x}}}}$



$$\int \frac{d^4 k}{(2\pi)^4} \underbrace{\delta(k^2 - m^2)}_{\delta(k^0{}^2 - \vec{k}^2 - m^2)} \Theta(k^0)$$

$$\delta(k^0{}^2 - \vec{k}^2 - m^2)$$

$$\int \frac{d^4 k}{(2\pi)^4} \underbrace{\delta(k^2 - m^2)}_{\delta(k^0{}^2 - \vec{k}^2 - m^2)} \Theta(k^0) = \frac{1}{2E_k} \int \frac{d^3 k}{2}$$



$$\frac{d^4 k}{(2\pi)^4} \underbrace{\delta(k^2 - m^2)}_{\text{Lorentz invariant}} \Theta(k^0) = \frac{1}{2E_{\vec{k}}} \int \frac{d^3 k}{(2\pi)^3}$$

$$\delta(k^0{}^2 - \vec{k}^2 - m^2)$$

$$A(\vec{k}) = \frac{a(\vec{k})}{2E_{\vec{k}}}$$

$$\vec{x}) = \int \frac{d^3 k}{(2\pi)^3 2E_{\vec{k}}} \left[a(\vec{k}) e^{-ik \cdot x} + a^*(\vec{k}) e^{ik \cdot x} \right]$$

$$\frac{d^4 k}{(2\pi)^4} \underbrace{\delta(k^2 - m^2)}_{\text{Lorentz invariant}} \Theta(k^0) = \frac{1}{2E_{\vec{k}}} \int \frac{d^3 k}{(2\pi)^3}$$

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