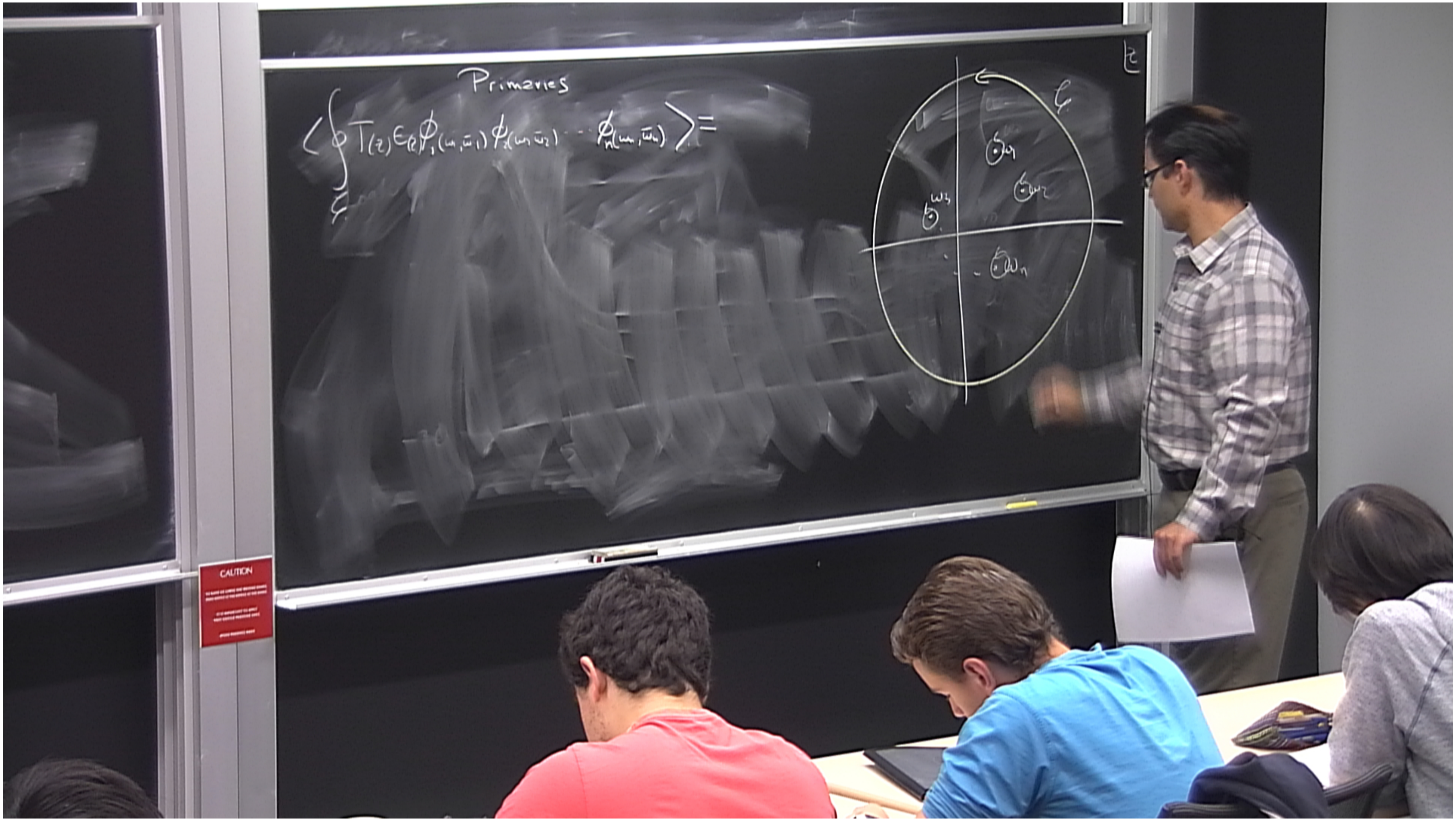
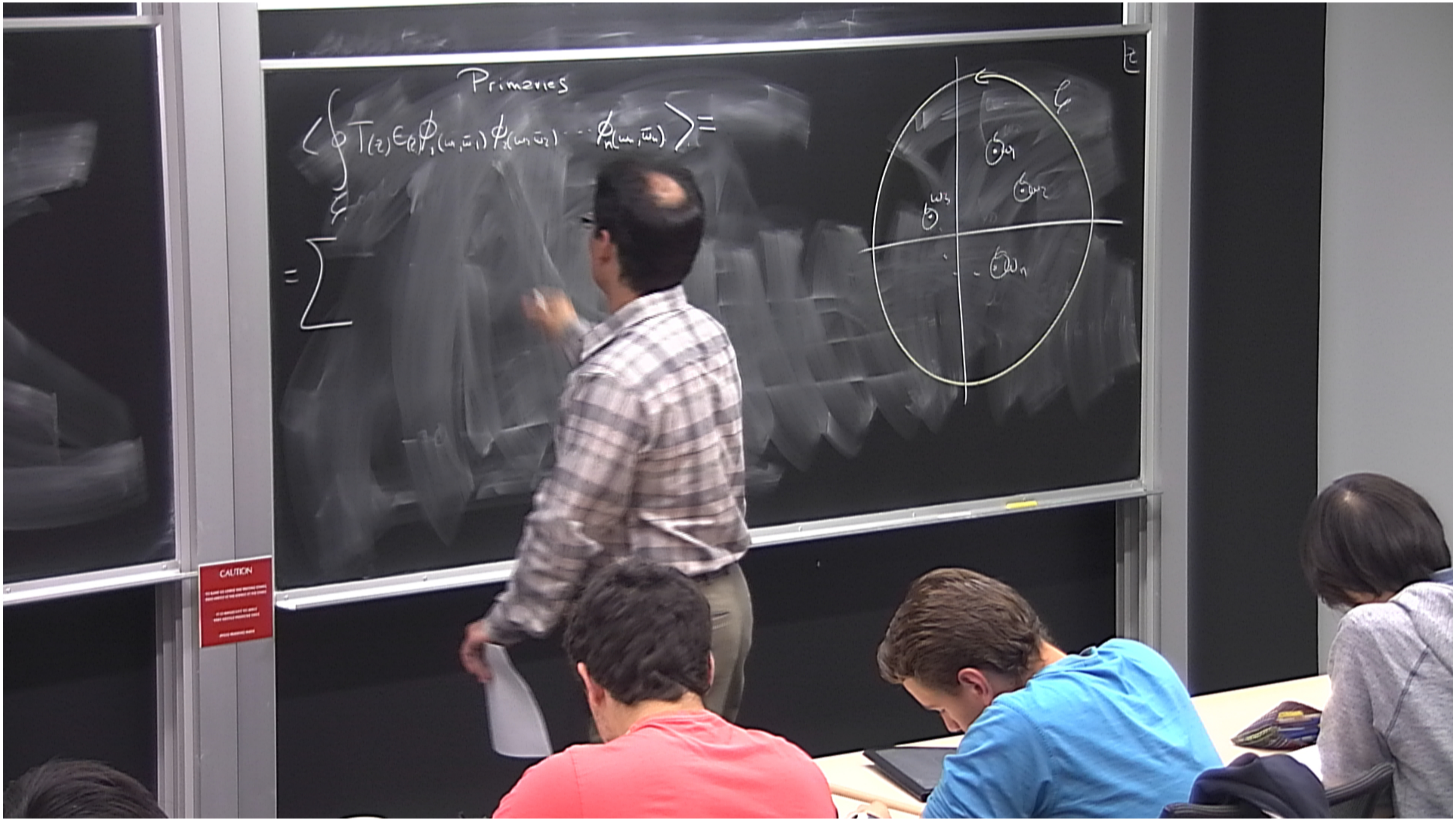
Title: 14/15 PSI - Conformal Field Theory - Lecture 14

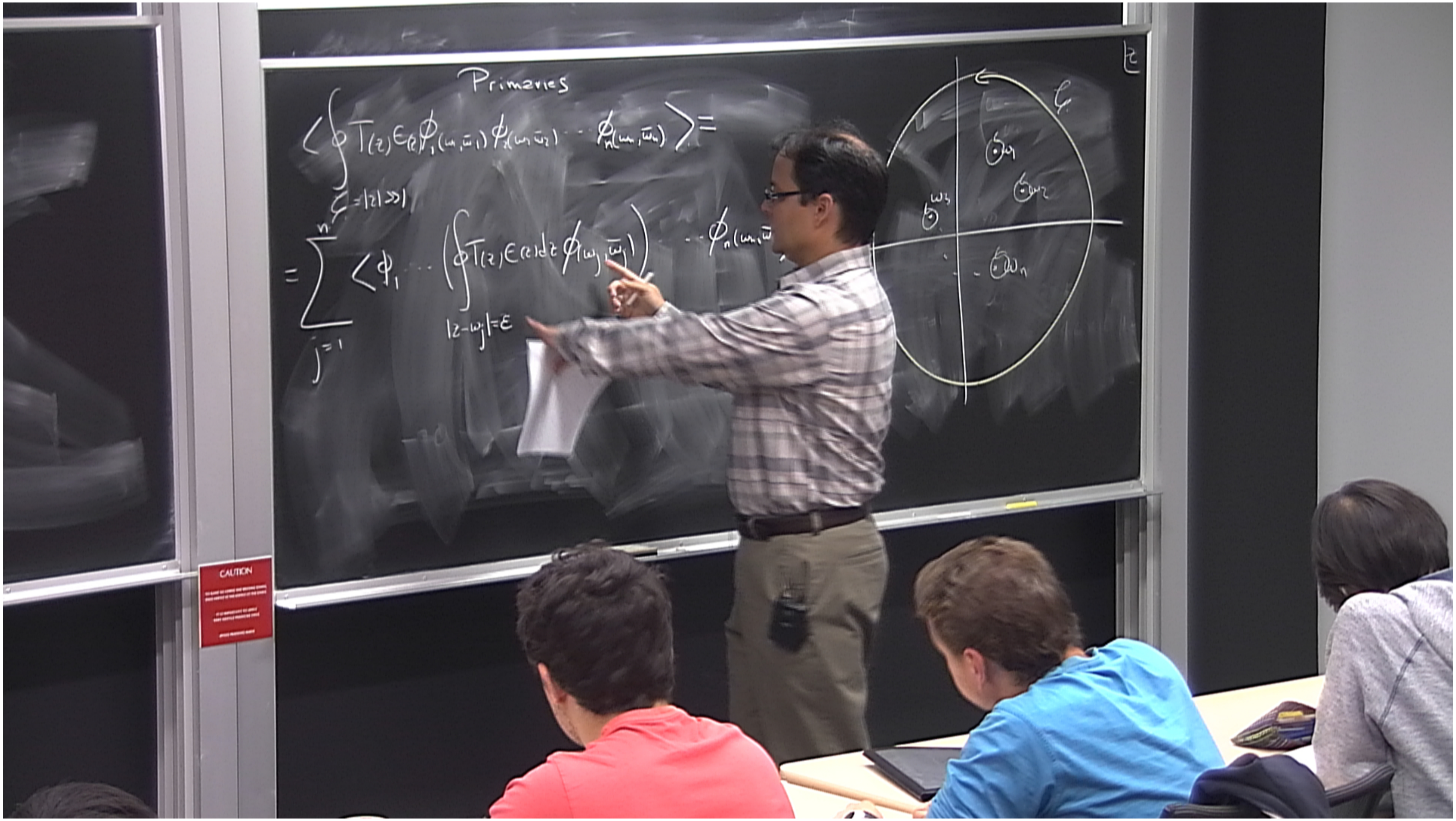
Date: Oct 24, 2014 03:30 PM

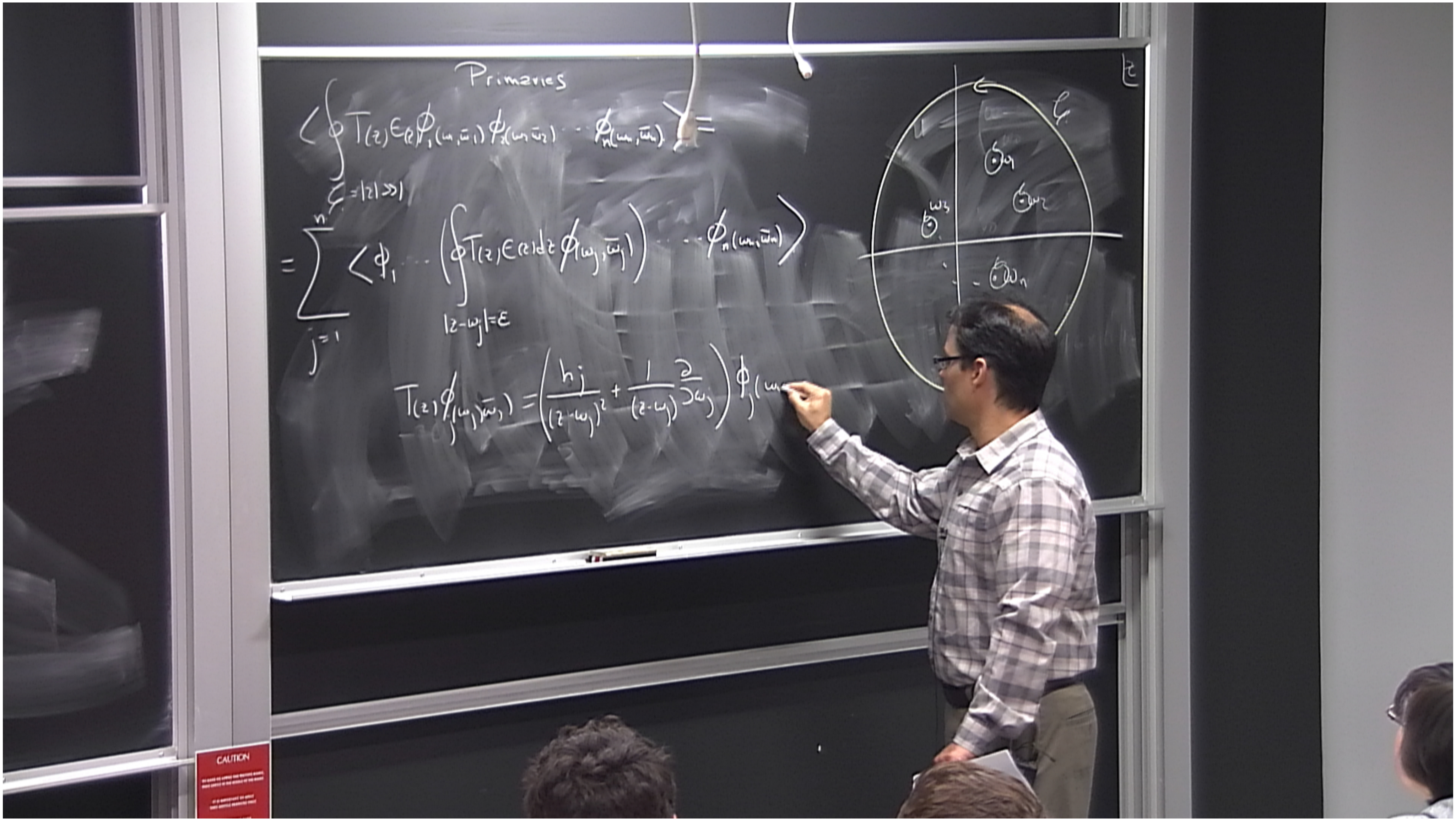
URL: http://pirsa.org/14100026

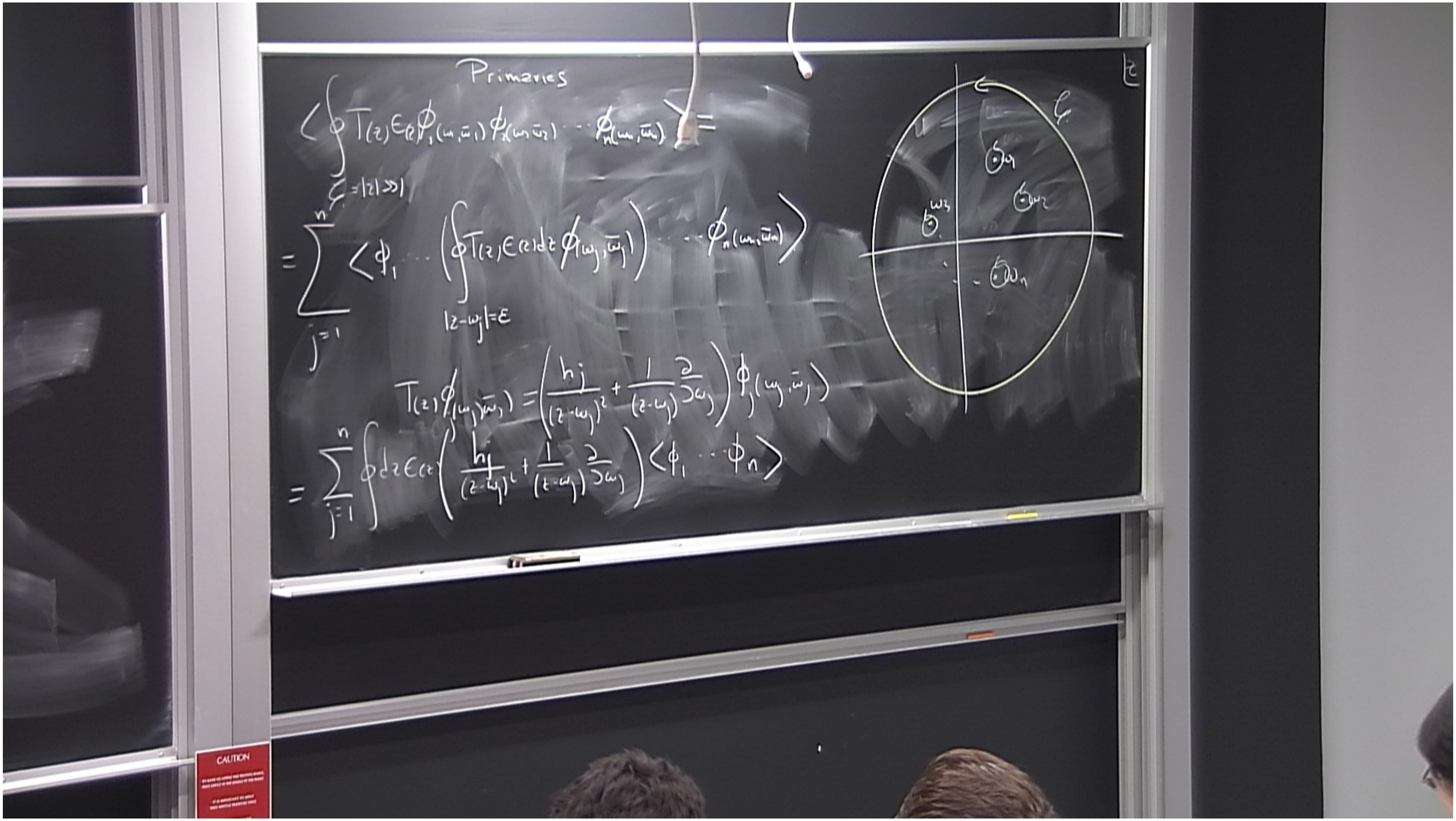
Abstract:

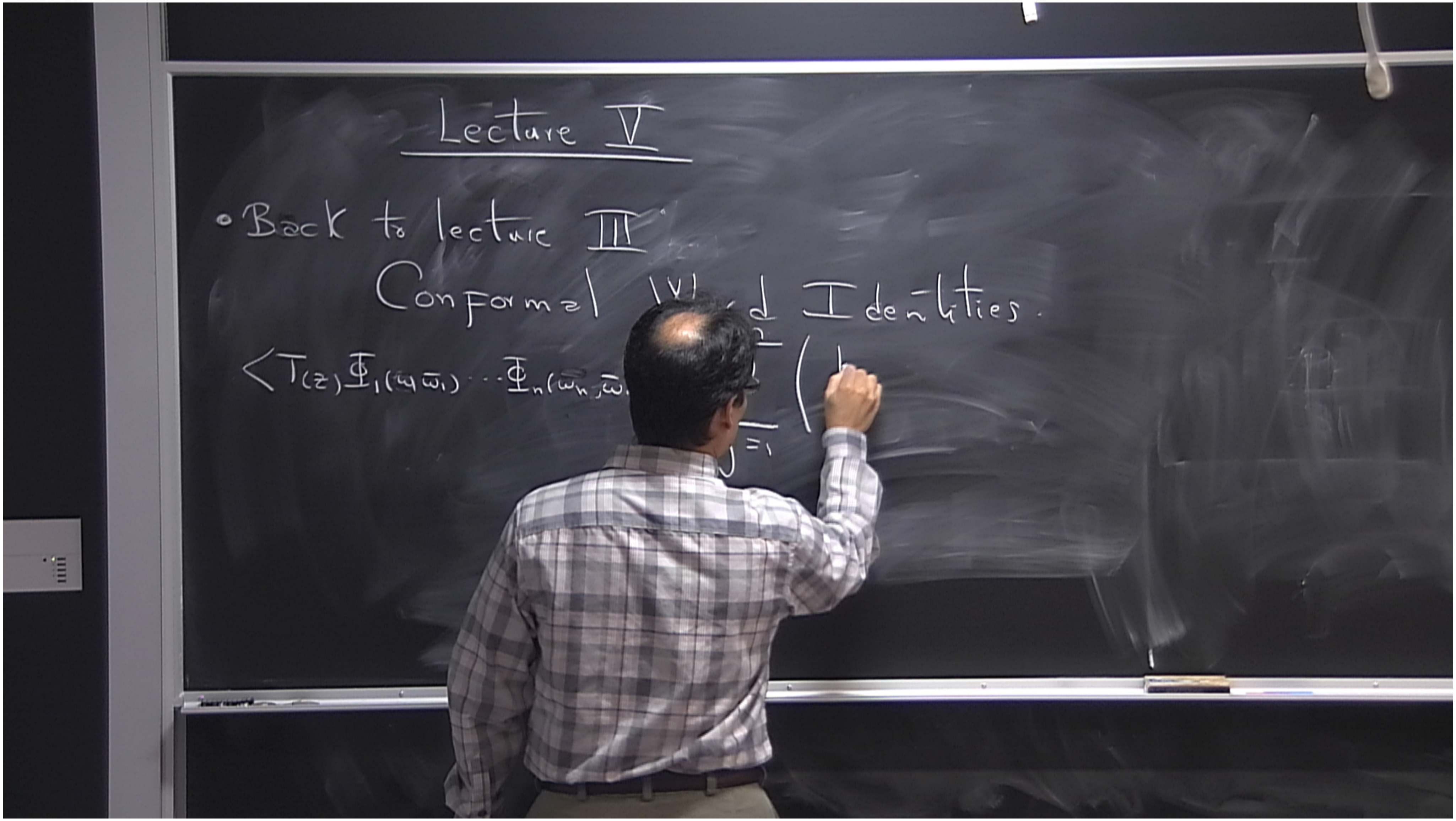

Primaries
$\langle \oint_C T(z)\,\epsilon(z)\,\phi_1(w_1,\bar{w}_1)\,\phi_2(w_2,\bar{w}_2)\cdots\phi_n(w_n,\bar{w}_n)\rangle =$
z
C
w_1
w_2
w_3
w_n
CAUTION

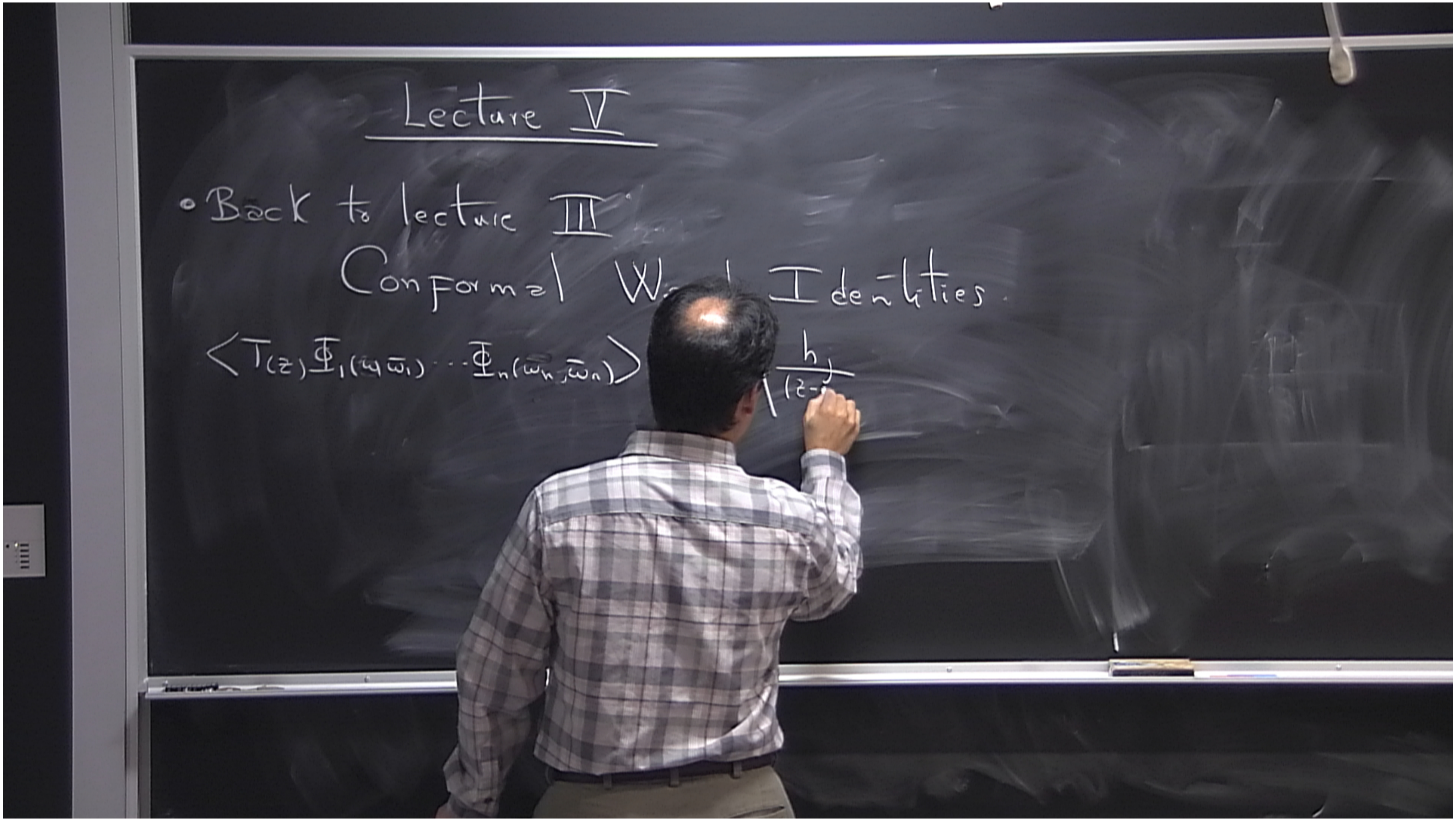

Primaries
CAUTION

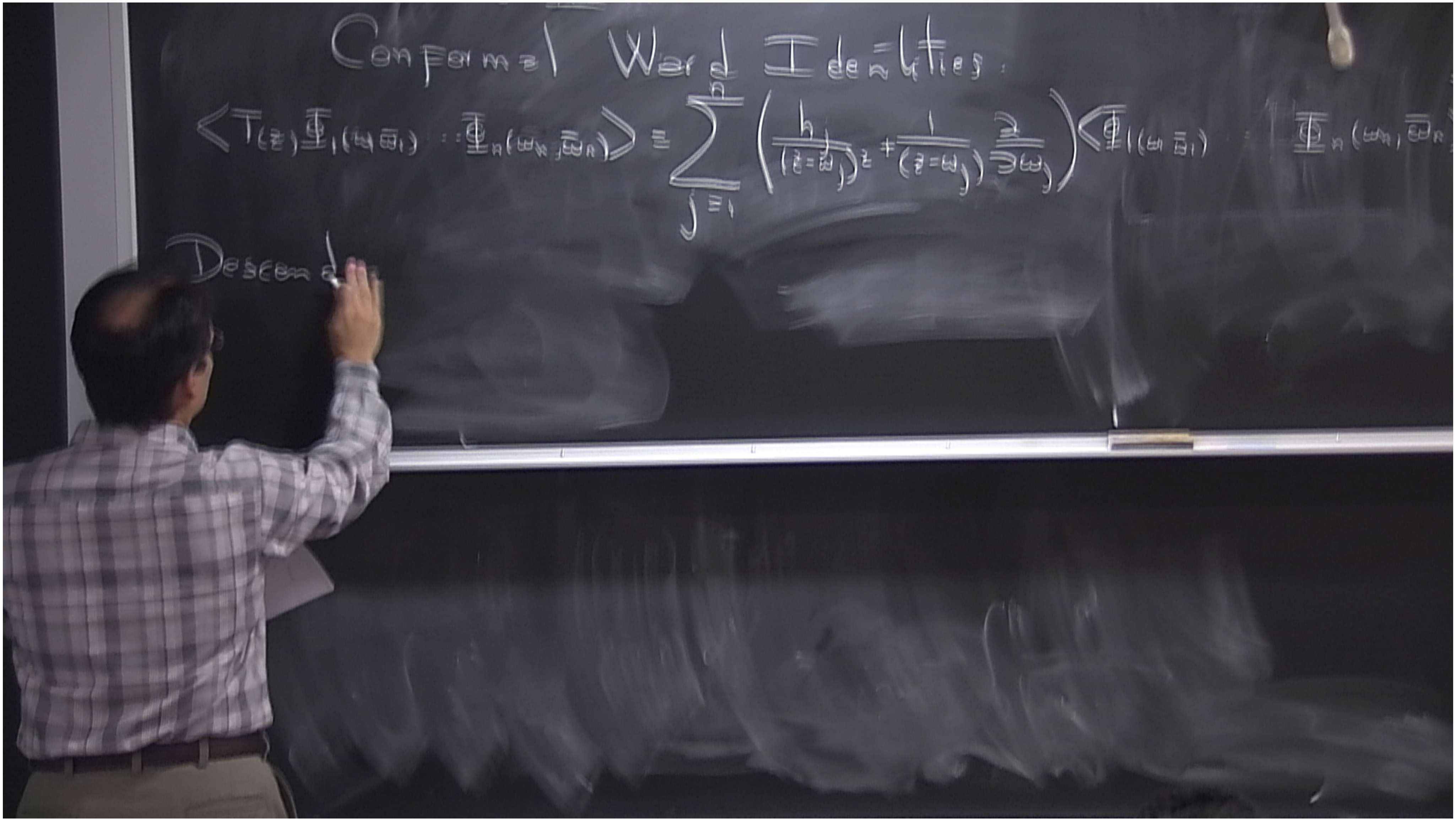

Primaries
CAUTION

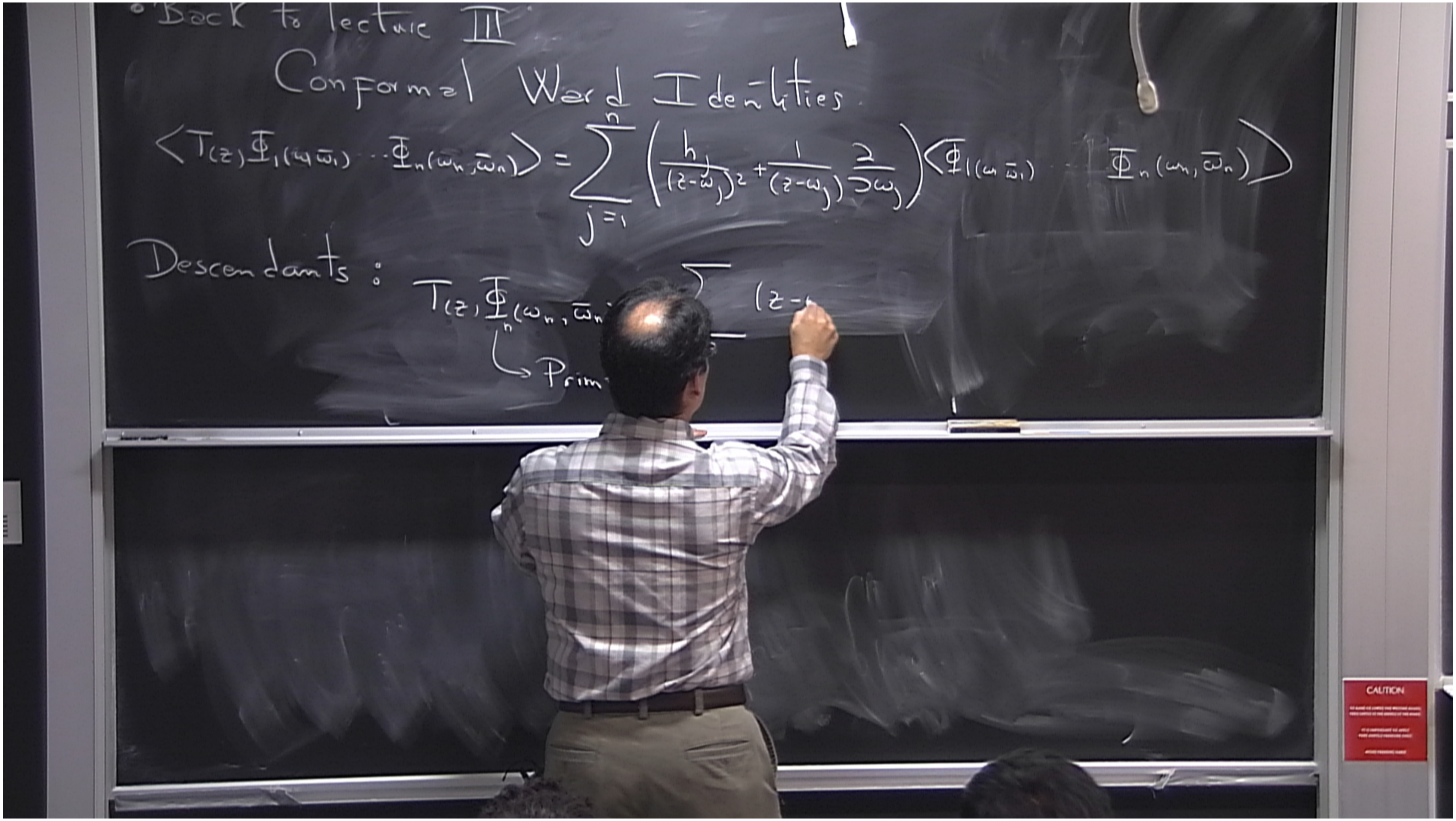

Primaries
$\langle \oint_{\zeta = |z| \gg 1} T(z)\epsilon(z)\phi_1(w_1,\bar{w}_1)\phi_2(w_2,\bar{w}_2)\cdots\phi_n(w_n,\bar{w}_n)\rangle =$
$= \sum_{j=1}^{n} \langle \phi_1 \cdots \left(\oint_{|z-w_j|=\epsilon} T(z)\epsilon(z)dz\,\phi(w_j,\bar{w}_j)\right)\cdots\phi_n(w_n,\bar{w}_n)\rangle$
$T(z)\phi_j(w_j,\bar{w}_j) = \left(\frac{h_j}{(z-w_j)^2} + \frac{1}{(z-w_j)}\frac{\partial}{\partial w_j}\right)\phi_j(w$
ζ
w_1
w_2
w_3
w_n
CAUTION

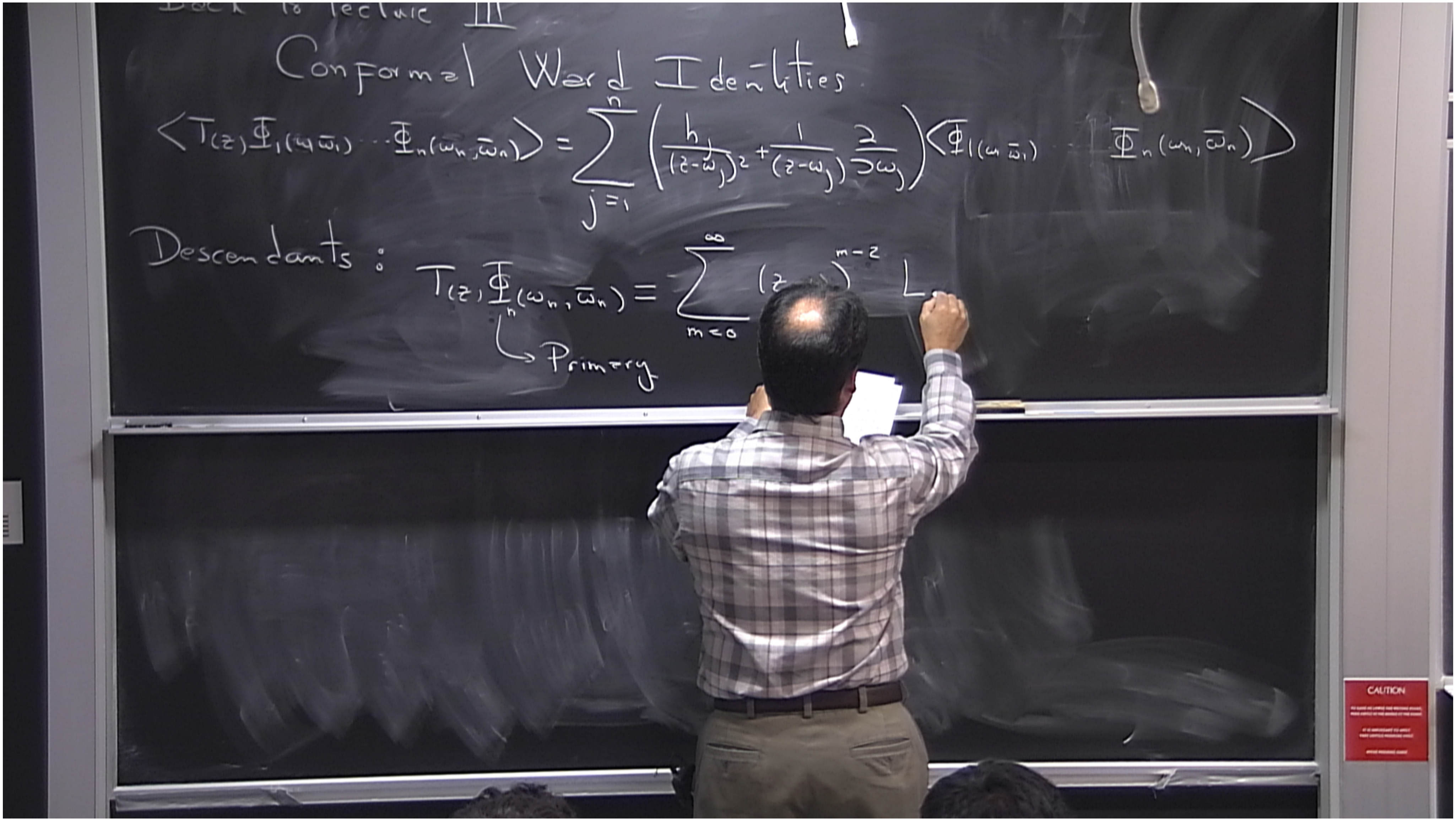

Primaries
$\left\langle \oint_{C = |z| \gg 1} T(z)\epsilon(z)\phi_1(w_1,\bar{w}_1)\phi_2(w_2,\bar{w}_2)\cdots\phi_n(w_n,\bar{w}_n) \right\rangle =$
$= \sum_{j=1}^{n} \left\langle \phi_1 \cdots \left(\oint_{|z-w_j|=\epsilon} T(z)\epsilon(z)dz\, \phi(w_j,\bar{w}_j) \right) \cdots \phi_n(w_n,\bar{w}_n) \right\rangle$
$T(z)\phi_j(w_j,\bar{w}_j) = \left(\frac{h_j}{(z-w_j)^2} + \frac{1}{(z-w_j)}\frac{\partial}{\partial w_j} \right) \phi_j(w_j,\bar{w}_j)$
$= \sum_{j=1}^{n} \oint dz\, \epsilon(z) \left(\frac{h_j}{(z-w_j)^2} + \frac{1}{(z-w_j)}\frac{\partial}{\partial w_j} \right) \langle \phi_1 \cdots \phi_n \rangle$
z
C
w_1
w_2
w_3
w_n

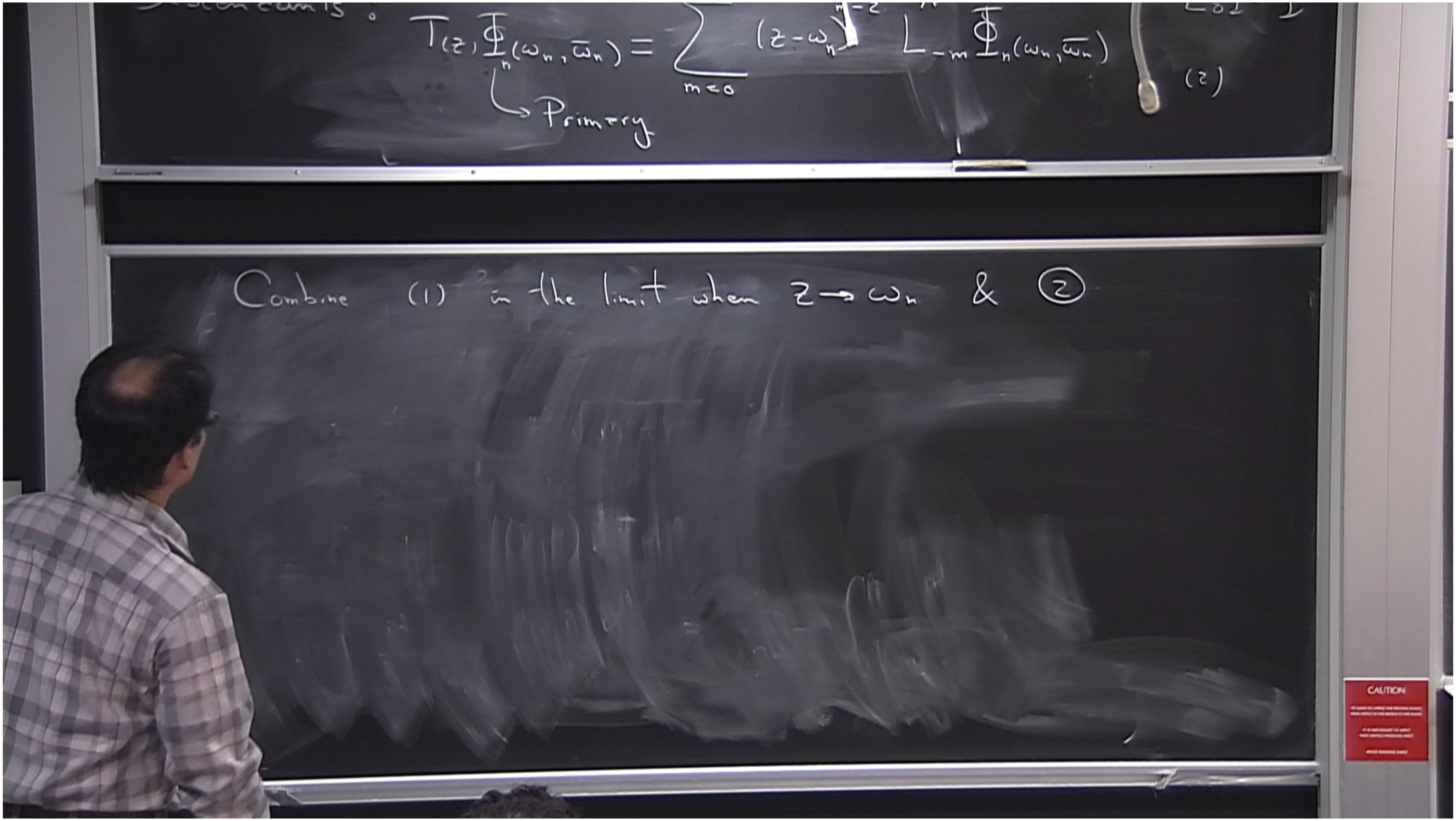

Lecture V
• Back to lecture III
Conformal
Identities.


Lecture V
• Back to lecture III
Conformal W
Identities
$\langle T(z)\,\Phi_1(w_1,\bar{w}_1)\cdots\Phi_n(w_n,\bar{w}_n)\rangle$

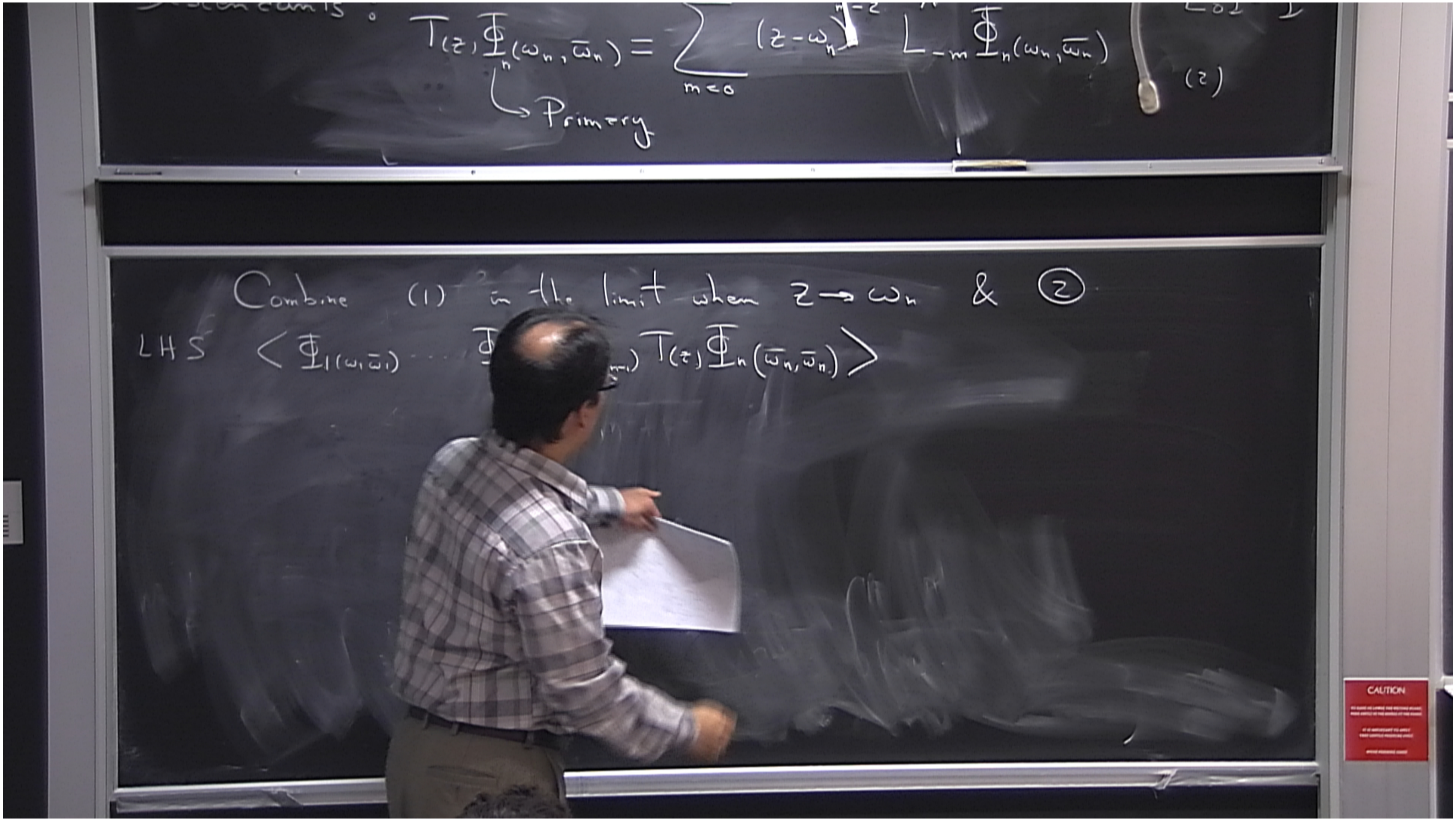

Conformal Ward Identities:
$\langle T(z)\Phi_1(w_1,\bar{w}_1)\cdots\Phi_n(w_n,\bar{w}_n)\rangle = \sum_{j=1}^{n}\left(\frac{h_j}{(z-w_j)^2}+\frac{1}{(z-w_j)}\frac{\partial}{\partial w_j}\right)\langle\Phi_1(w_1,\bar{w}_1)\cdots\Phi_n(w_n,\bar{w}_n)\rangle$
Descen


• Back to lecture III
Conformal Ward Identities
$\langle T(z)\Phi_1(w_1,\bar{w}_1)\cdots\Phi_n(w_n,\bar{w}_n)\rangle = \sum_{j=1}^{n}\left(\frac{h_j}{(z-w_j)^2}+\frac{1}{(z-w_j)}\frac{\partial}{\partial w_j}\right)\langle\Phi_1(w_1,\bar{w}_1)\cdots\Phi_n(w_n,\bar{w}_n)\rangle$
Descendants :

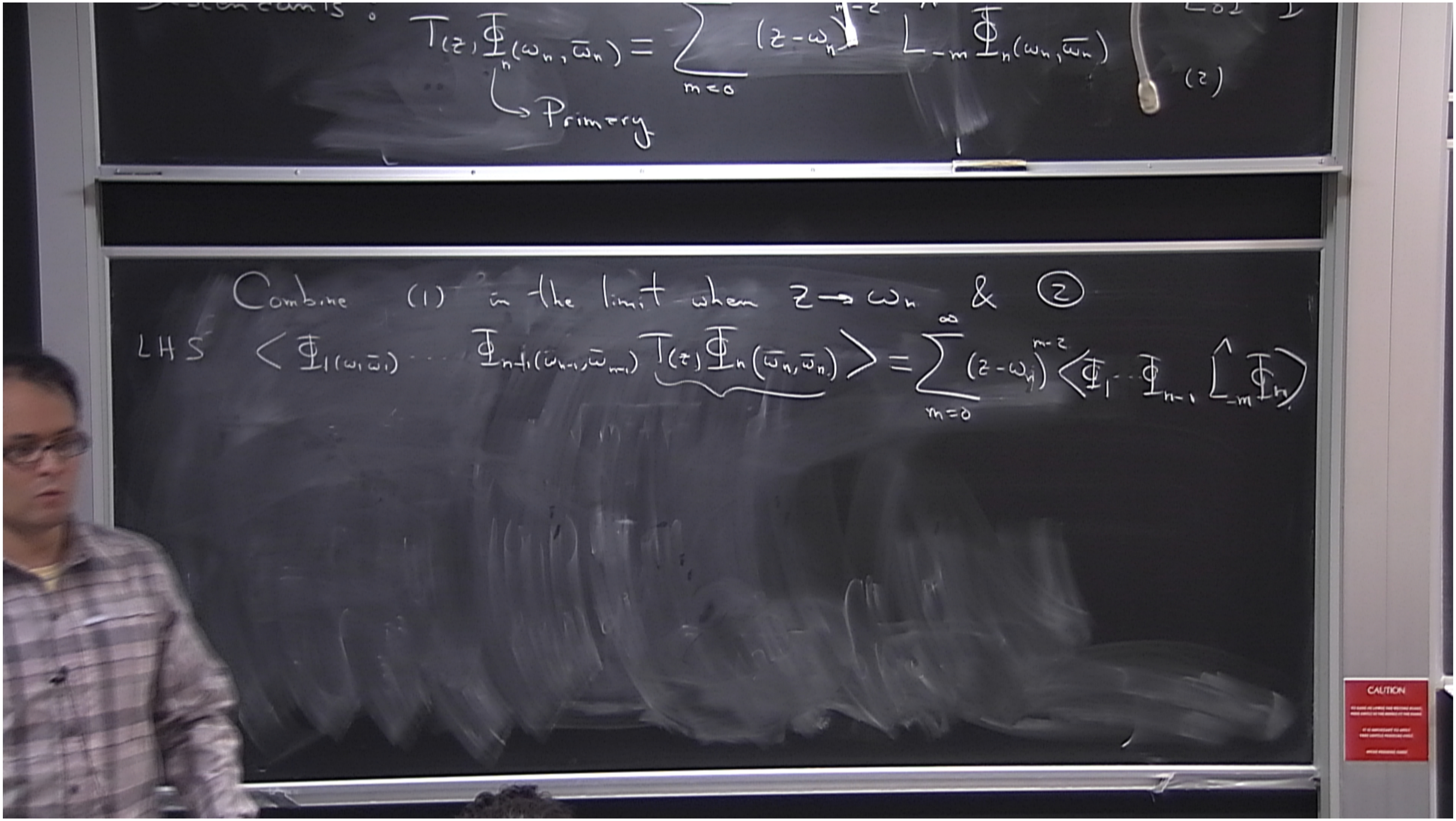

Conformal Ward Identities
$\langle T(z)\Phi_1(w_1\bar{w}_1)\cdots\Phi_n(w_n,\bar{w}_n)\rangle = \sum_{j=1}^{n}\left(\frac{h_j}{(z-w_j)^2}+\frac{1}{(z-w_j)}\frac{\partial}{\partial w_j}\right)\langle\Phi_1(w_1\bar{w}_1)\cdots\Phi_n(w_n,\bar{w}_n)\rangle$
Descendants :
Primary

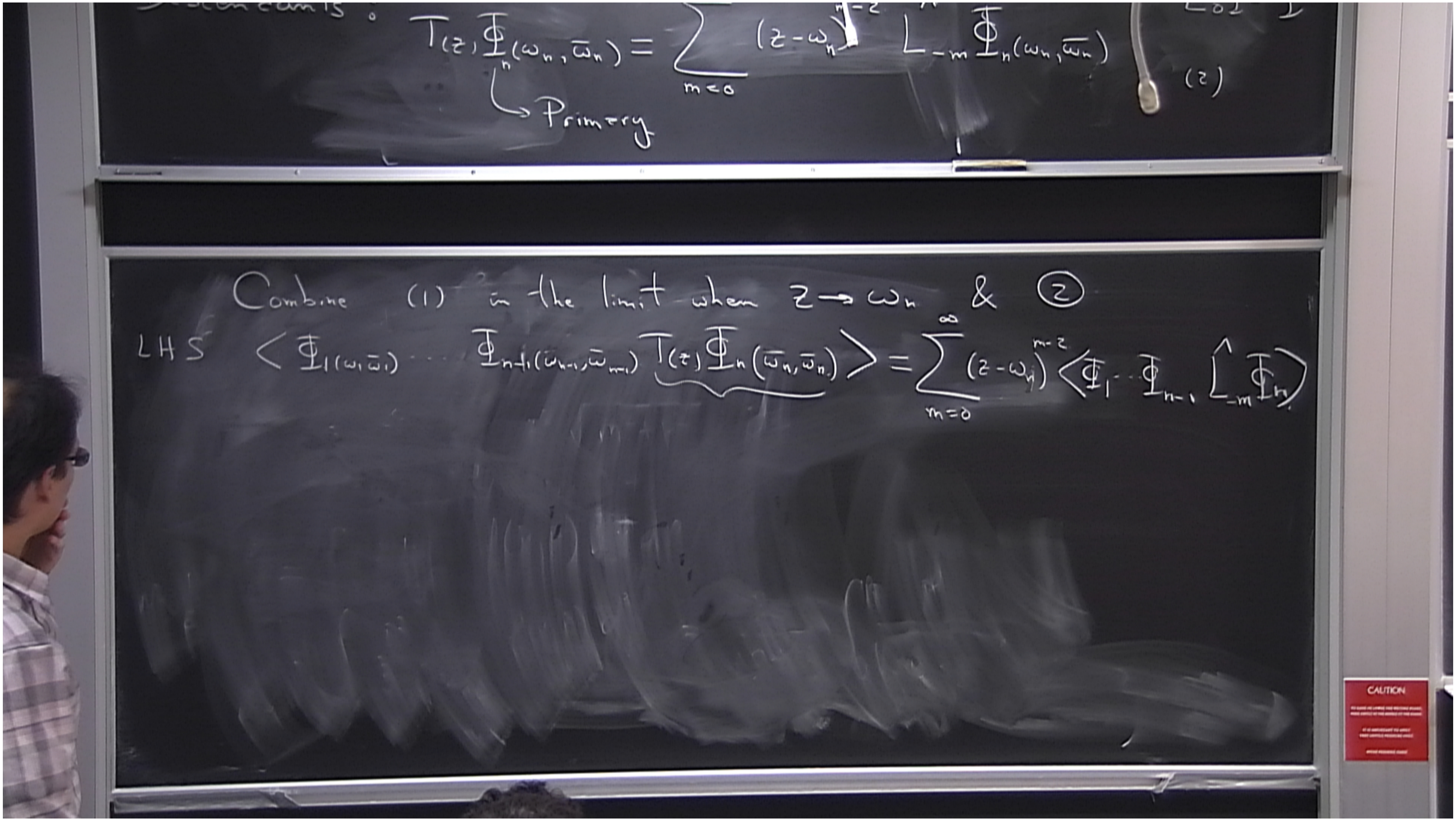


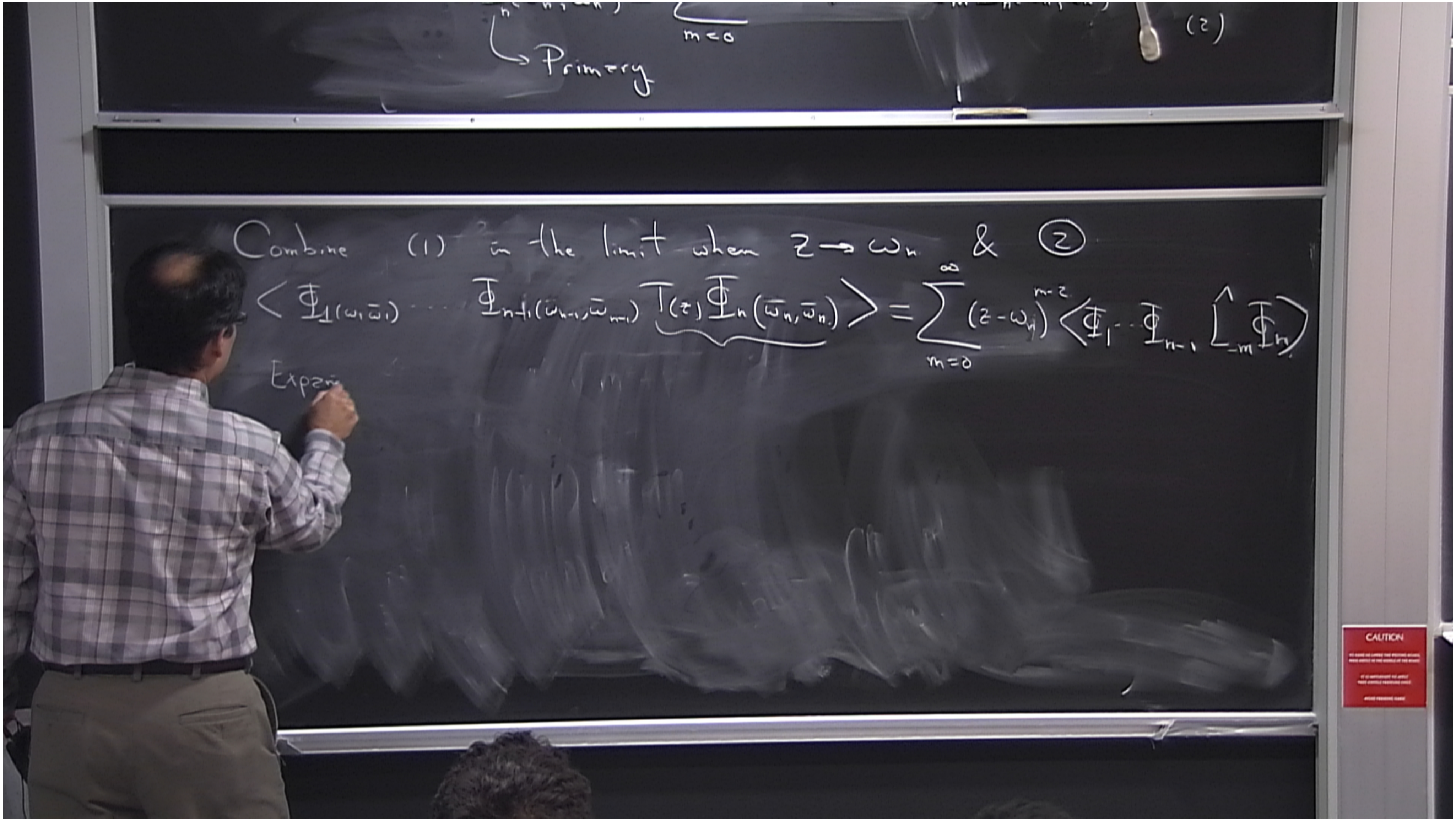
$T(z)\Phi_n(\omega_n,\bar{\omega}_n) = \sum_{m<0} (z-\omega_n) \; L_{-m}\Phi_n(\omega_n,\bar{\omega}_n)$
Primary
(2)
Combine (1) in the limit when $z \to \omega_n$ & 2

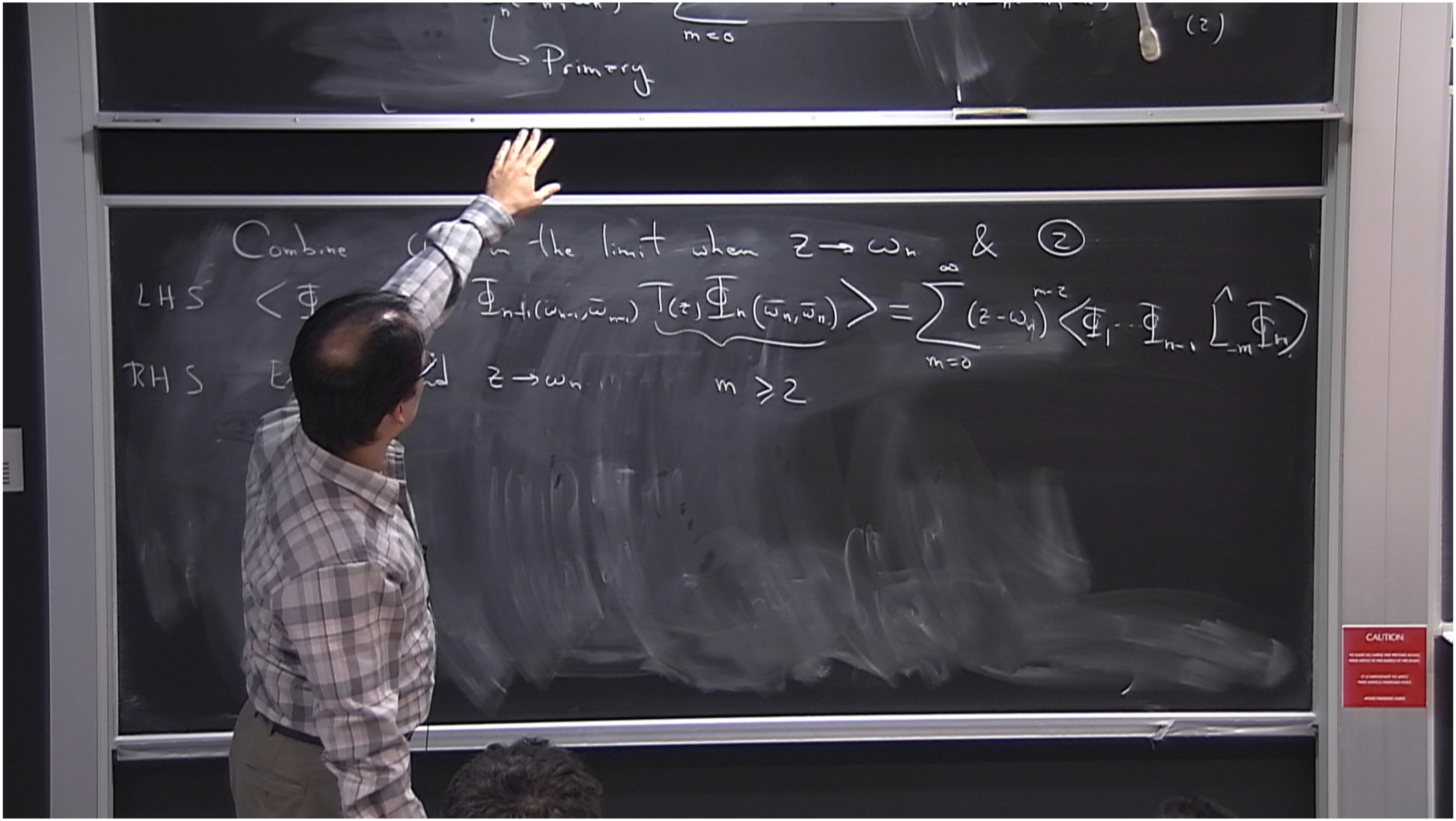


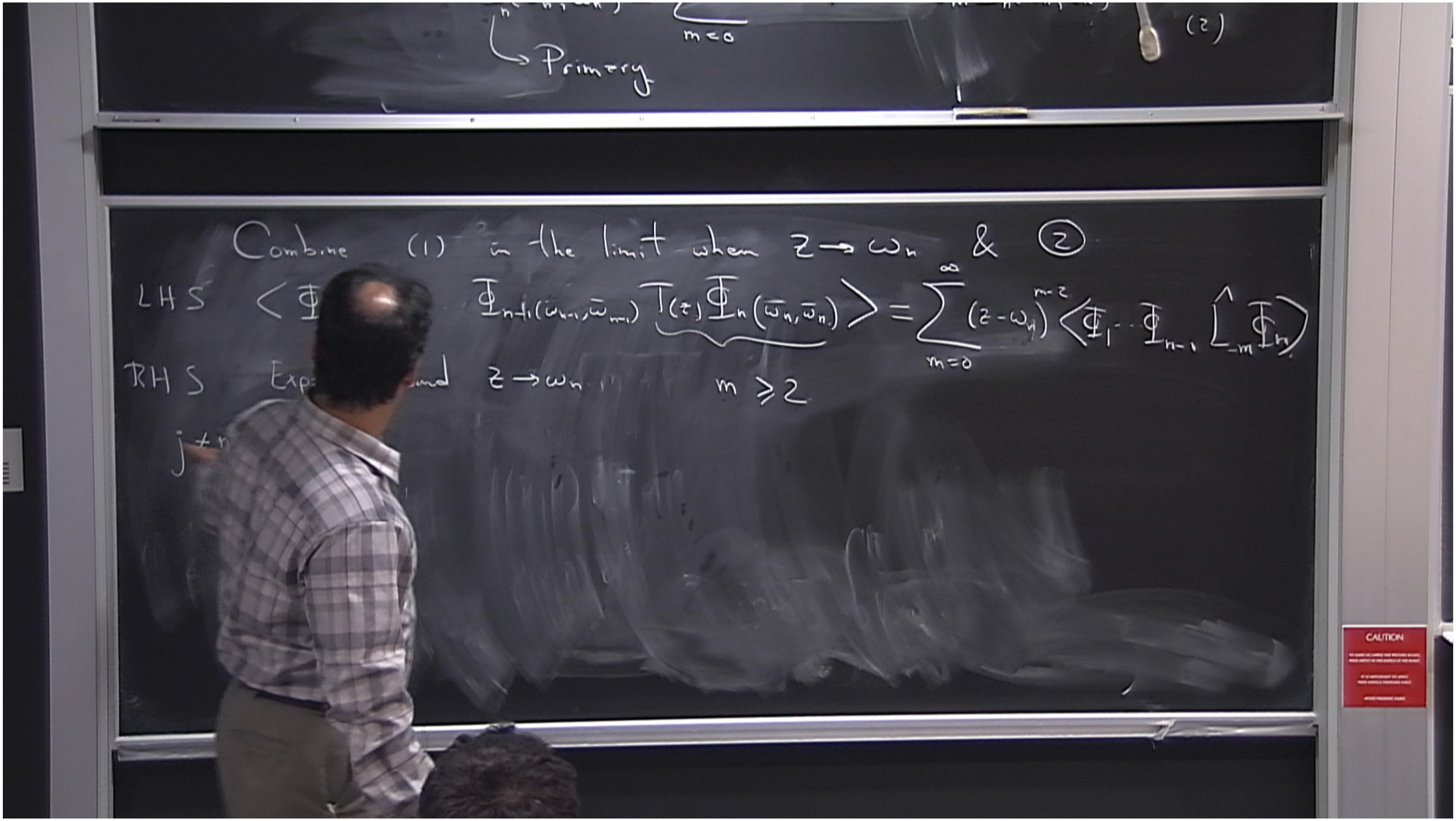
$T(z)\Phi_n(\omega_n,\bar{\omega}_n) = \sum_{m\leq 0} (z-\omega_n) \quad L_{-m}\Phi_n(\omega_n,\bar{\omega}_n)$
(2)
Primary
Combine (1) in the limit when $z \to \omega_n$ & 2
LHS $\langle \Phi_1(\omega_1,\bar{\omega}_1) \cdots T(z)\Phi_n(\bar{\omega}_n,\bar{\omega}_n) \rangle$

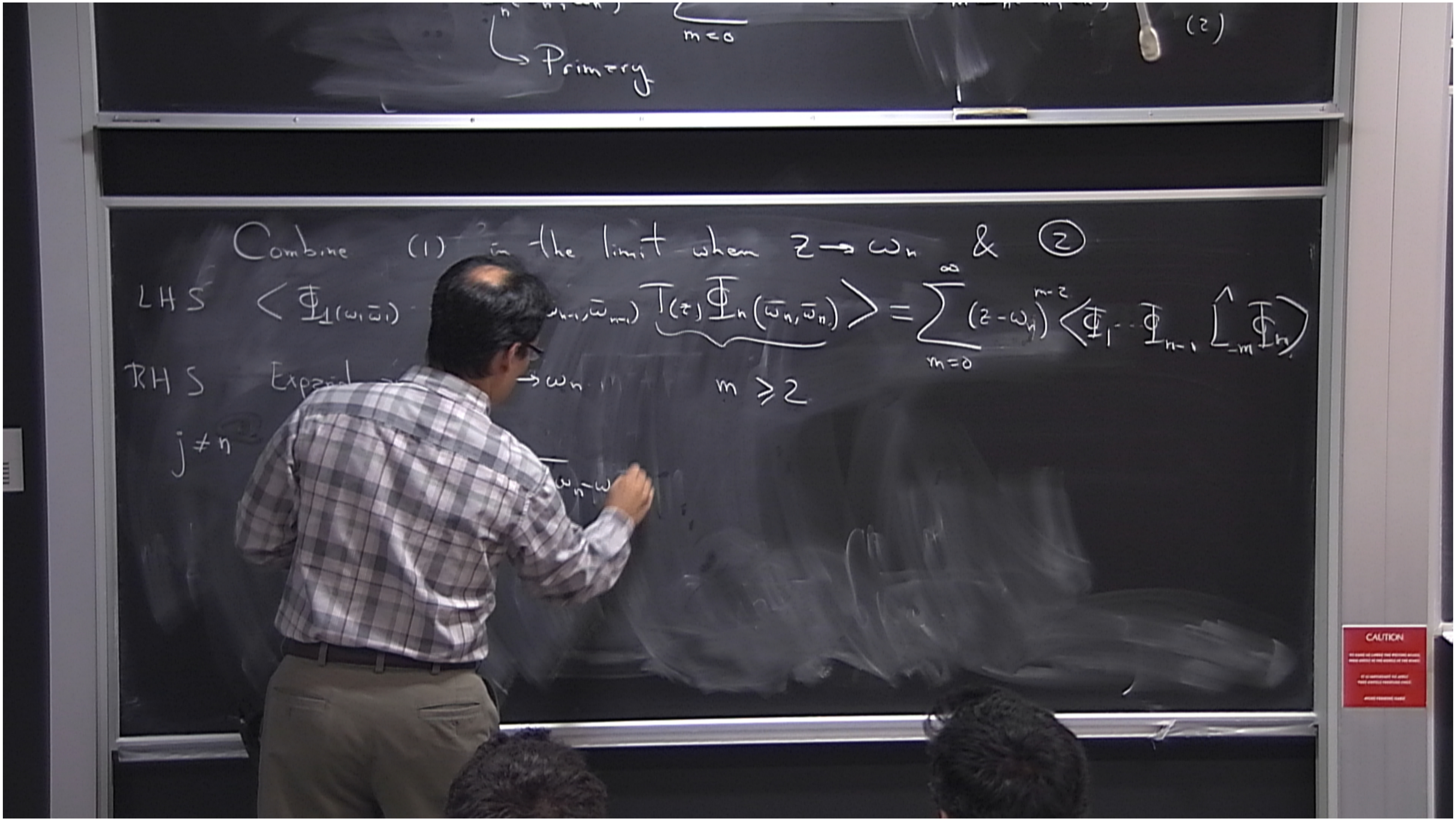

T(z) Φn(ωn, ω̄n) = Σ m≤0 (z - ωn)^(m-2) L-m Φn(ωn, ω̄n)
(2)
Primary
Combine (1) in the limit when z → ωn & (2)
LHS < Φ1(ω1, ω̄1) ... Φn-1(ωn-1, ω̄n-1) T(z) Φn(ωn, ω̄n) > = Σ m=0 ∞ (z - ωn)^(m-2) < Φ1 ... Φn-1 L-m Φn >
CAUTION

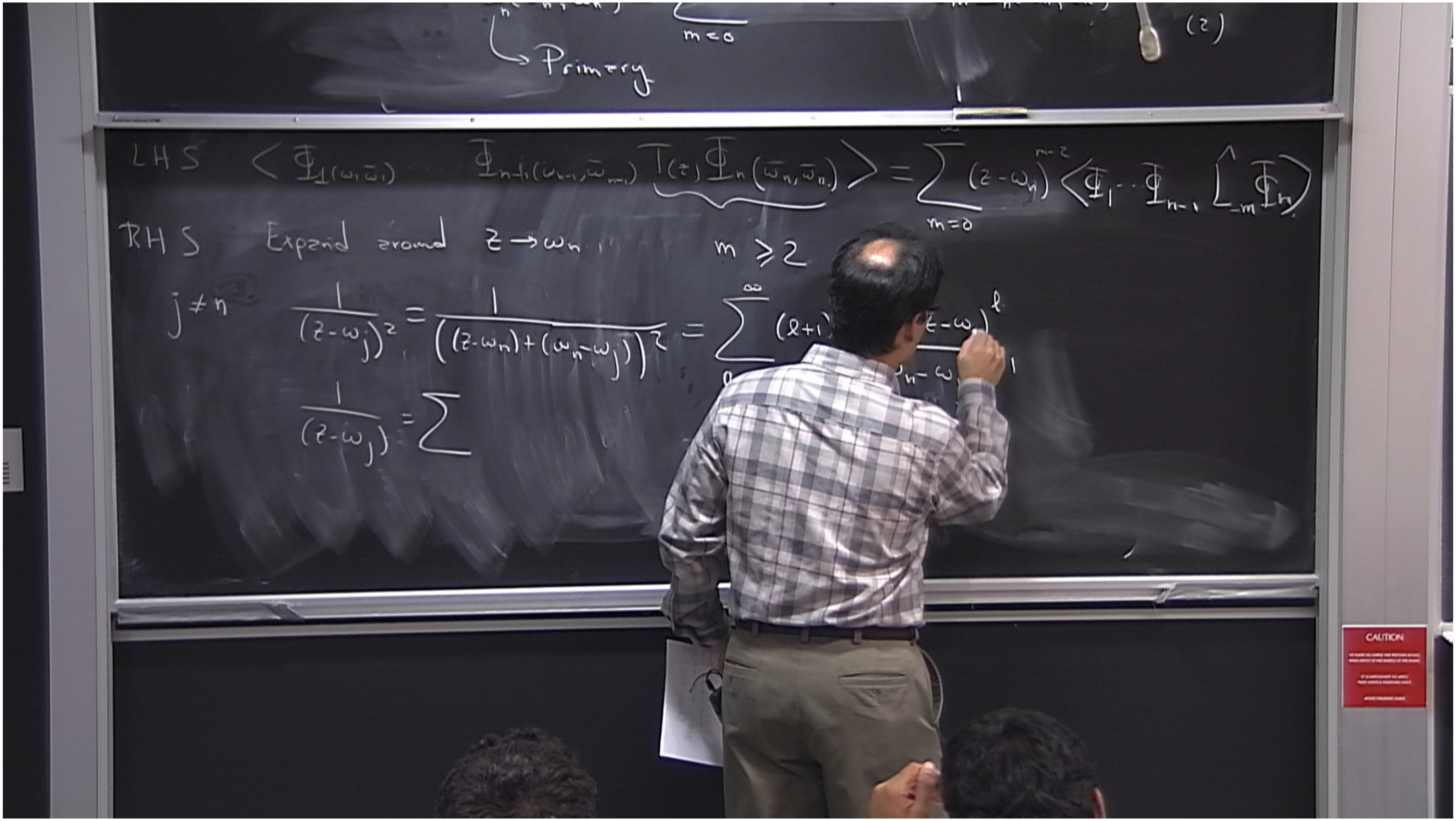

$T(z)\Phi_n(w_n,\bar{w}_n) = \sum_{m\geq 0} (z-w_n)^{m-2} L_{-m}\Phi_n(w_n,\bar{w}_n)$
Primary
(2)
Combine (1) in the limit when $z \to w_n$ & (2)
LHS $\langle \Phi_1(w_1,\bar{w}_1) \cdots \Phi_{n-1}(w_{n-1},\bar{w}_{n-1})\, T(z)\Phi_n(w_n,\bar{w}_n)\rangle = \sum_{m=0}^{\infty} (z-w_n)^{m-2} \langle \Phi_1 \cdots \Phi_{n-1} L_{-m}\Phi_n\rangle$

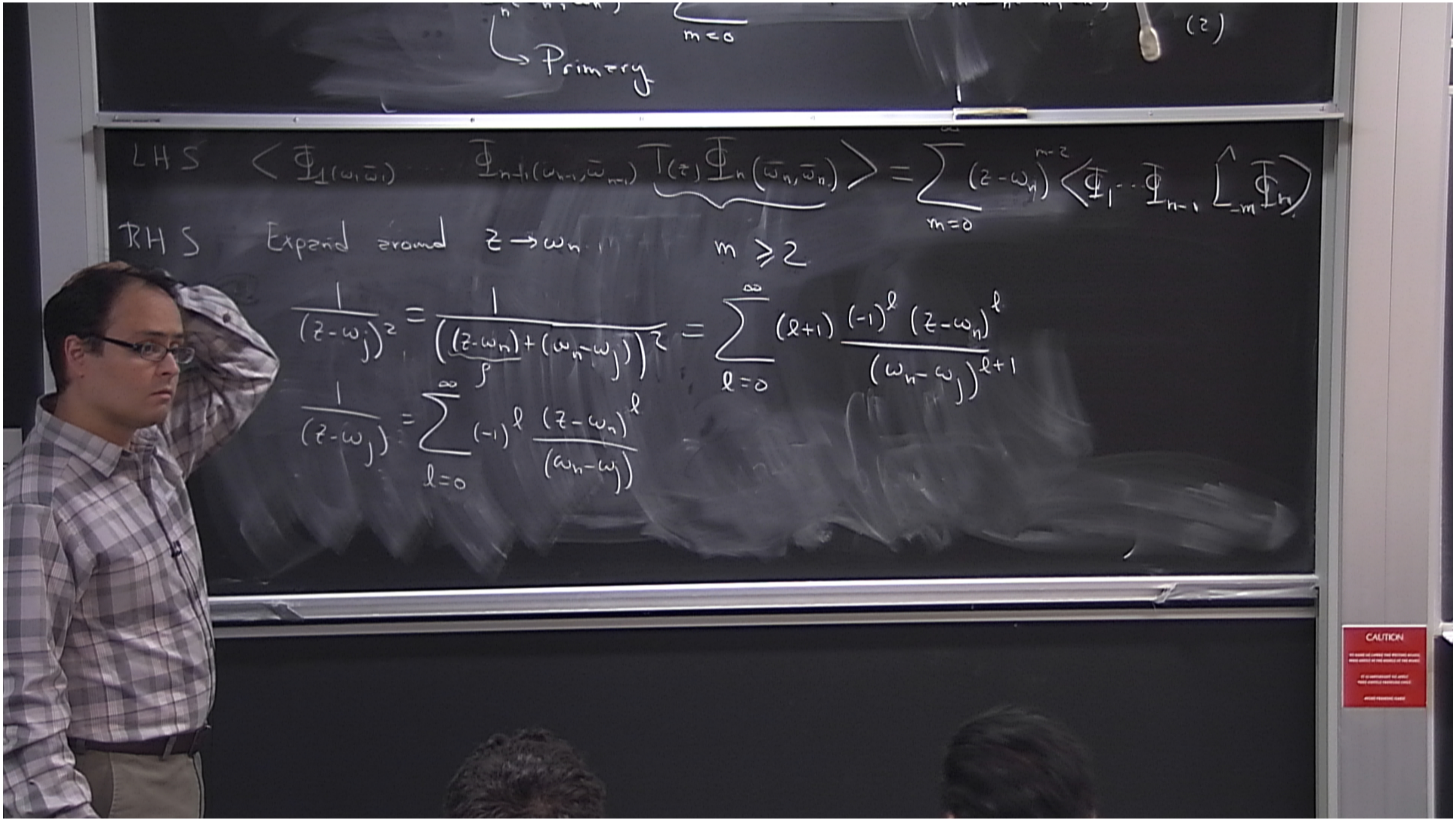

Primary
(2)
Combine (1) in the limit when $z \to \omega_n$ & (2)
$\langle \Phi_1(\omega_1,\bar\omega_1) \cdots \Phi_{n-1}(\omega_{n-1},\bar\omega_{n-1})\, T(z)\, \Phi_n(\omega_n,\bar\omega_n) \rangle = \sum_{m=0}^{\infty} (z-\omega_n)^{m-2} \langle \Phi_1 \cdots \Phi_{n-1}\, \hat{L}_{-m} \Phi_n \rangle$
Expan
CAUTION


Primary
(2)
Combine
the limit when z → ωn & 2
LHS
RHS
z → ωn
m ≥ 2

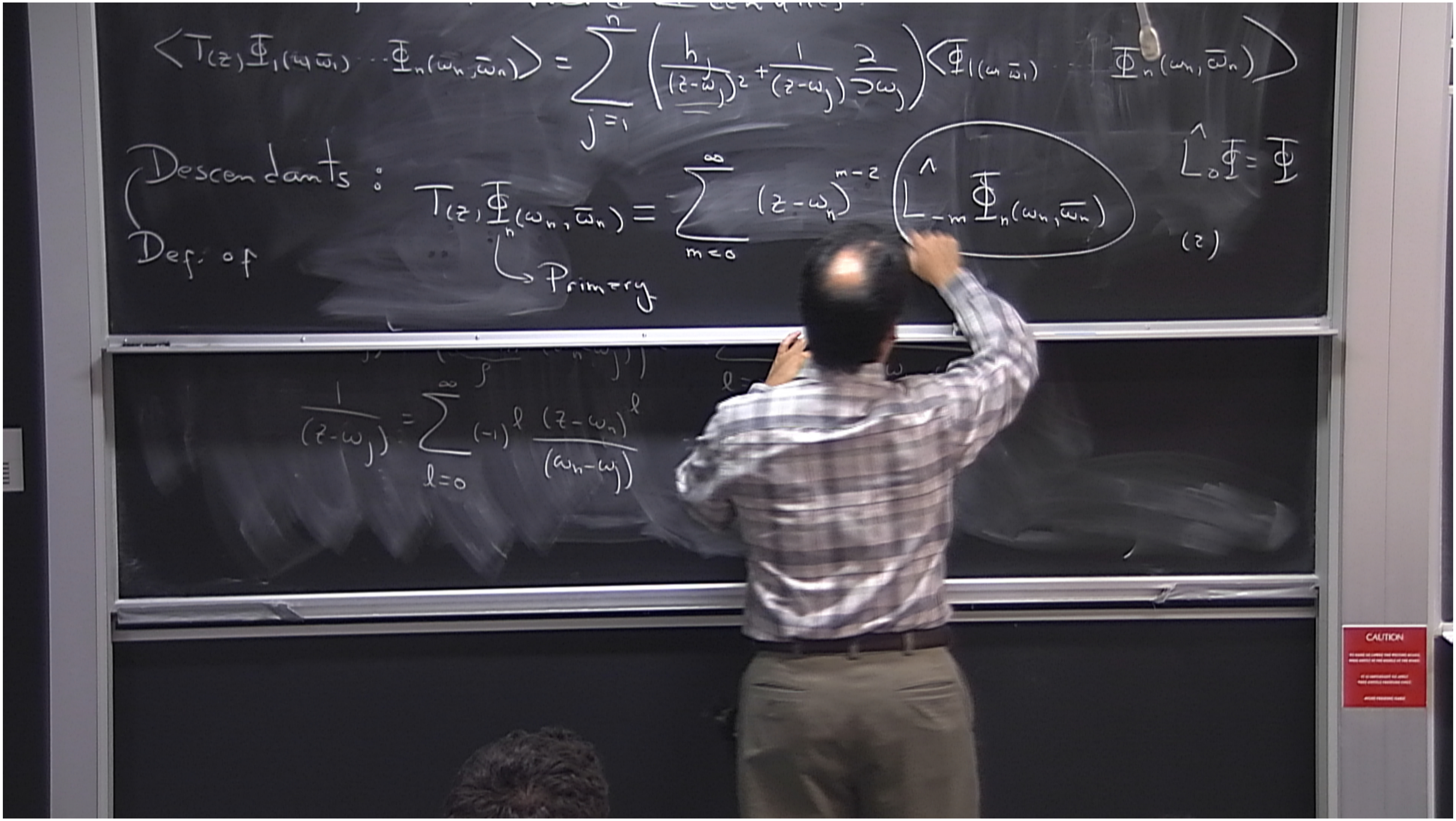

Primary
m<0
(2)
Combine (1) in the limit when z → ωn & 2
LHS
= Σ (z-ωn)^(m-2)
m=0
RHS
z → ωn
m ≥ 2

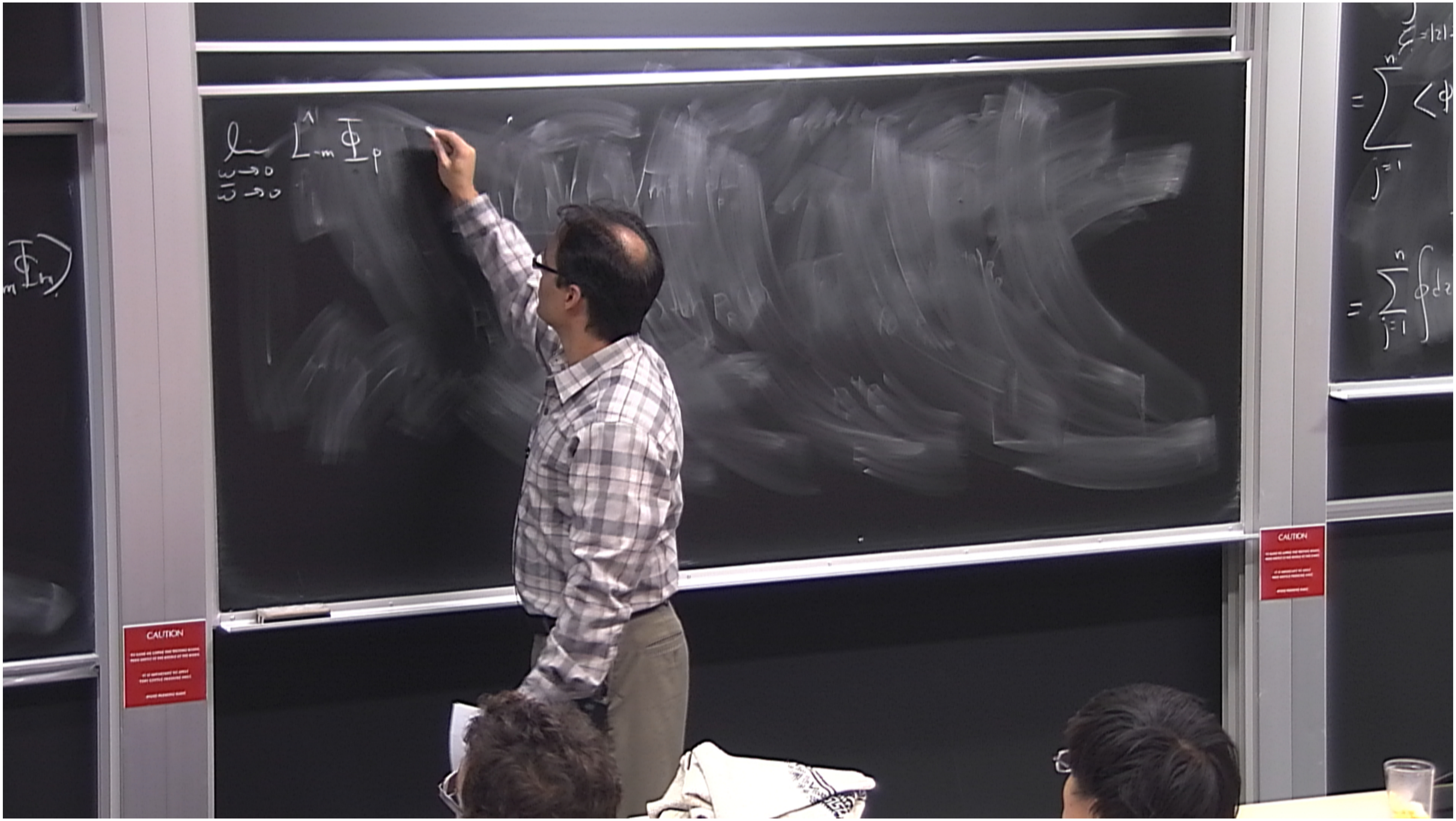

Primary
m<0
(2)
Combine (1) in the limit when z → ωn & (2)
LHS
RHS Expand
m ≥ 2
j ≠ n


Primary
LHS
RHS
Expand around $z \to w_n$
$m \geq 2$
$j \neq n$

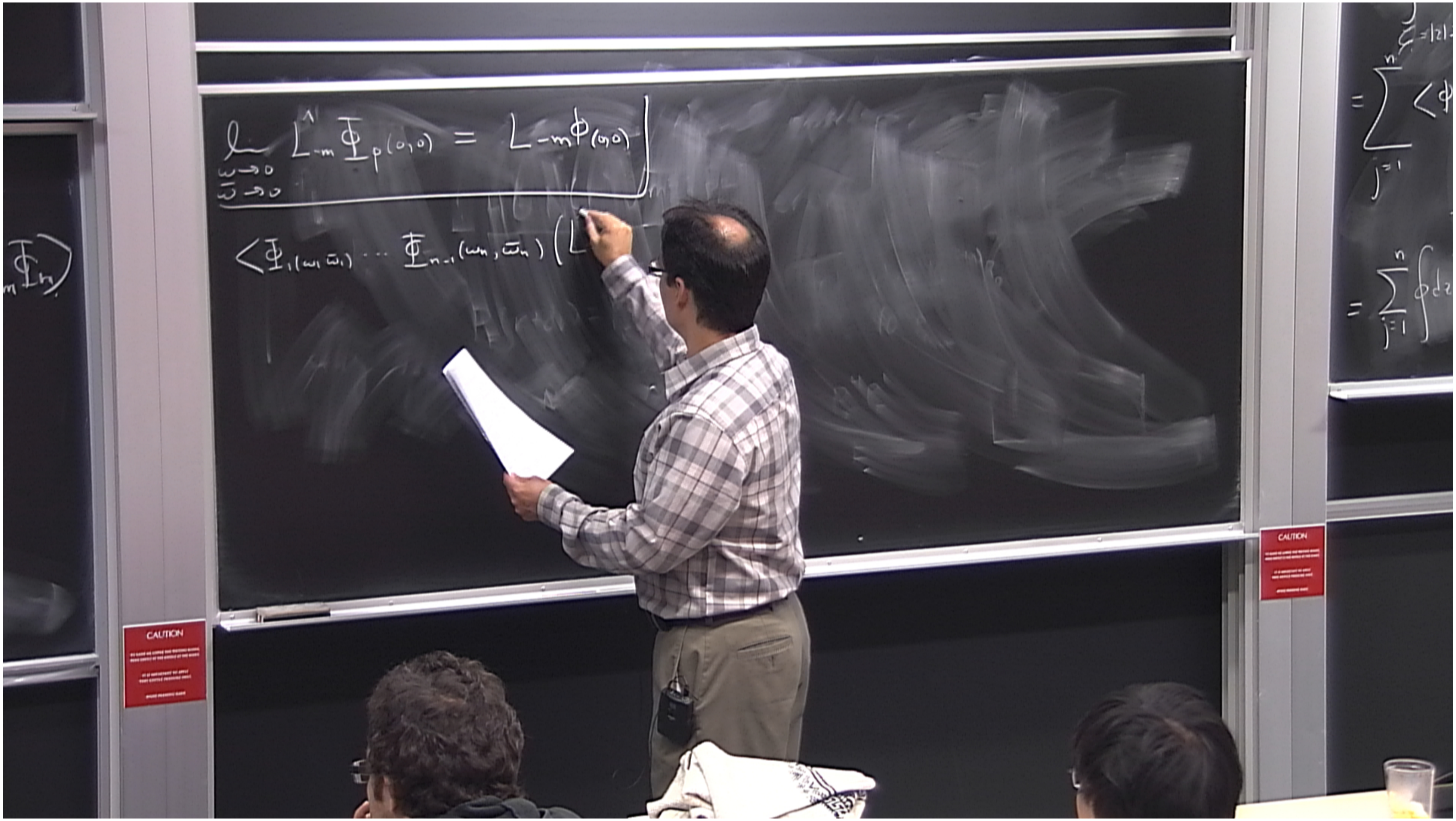


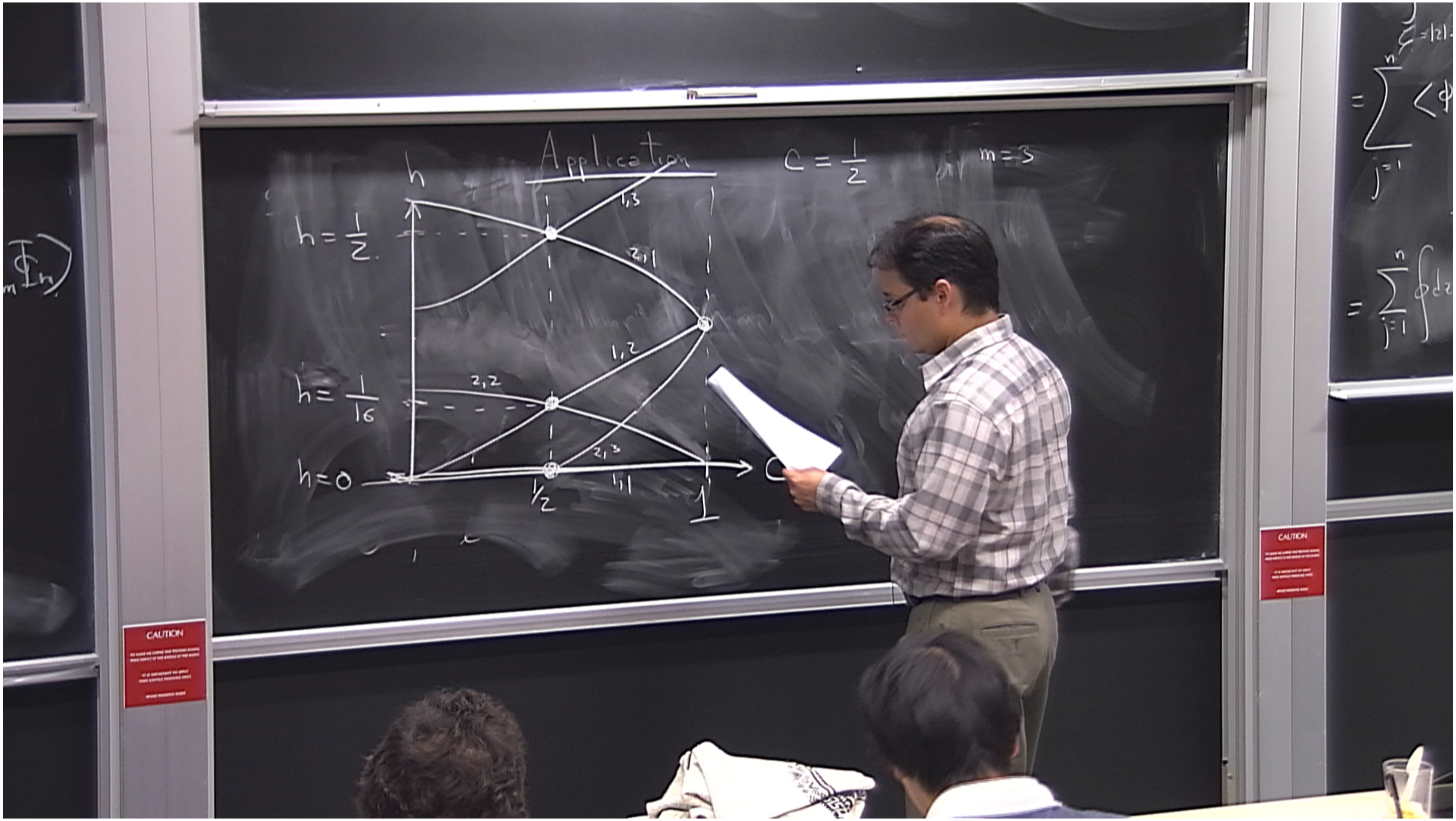
Primary
LHS $\langle \Phi_1(w_1,\bar w_1) \cdots \Phi_{n-1}(w_{n-1},\bar w_{n-1}) T(z)\Phi_n(w_n,\bar w_n) \rangle = \sum_{m=0}^{\infty} (z-w_n)^{m-2} \langle \Phi_1 \cdots \Phi_{n-1} L_{-m}\Phi_n \rangle$
RHS Expand around $z \to w_n$ $m \geq 2$
$\frac{1}{(z-w_j)^2} = \frac{1}{((z-w_n)+(w_n-w_j))^2} = \sum_{\ell=0}^{\infty} (\ell+1) \frac{(-1)^\ell (z-w_n)^\ell}{(w_n-w_j)^{\ell+1}}$
$\frac{1}{(z-w_j)} = \sum_{\ell=0}^{\infty} (-1)^\ell \frac{(z-w_n)^\ell}{(w_n-w_j)}$
CAUTION

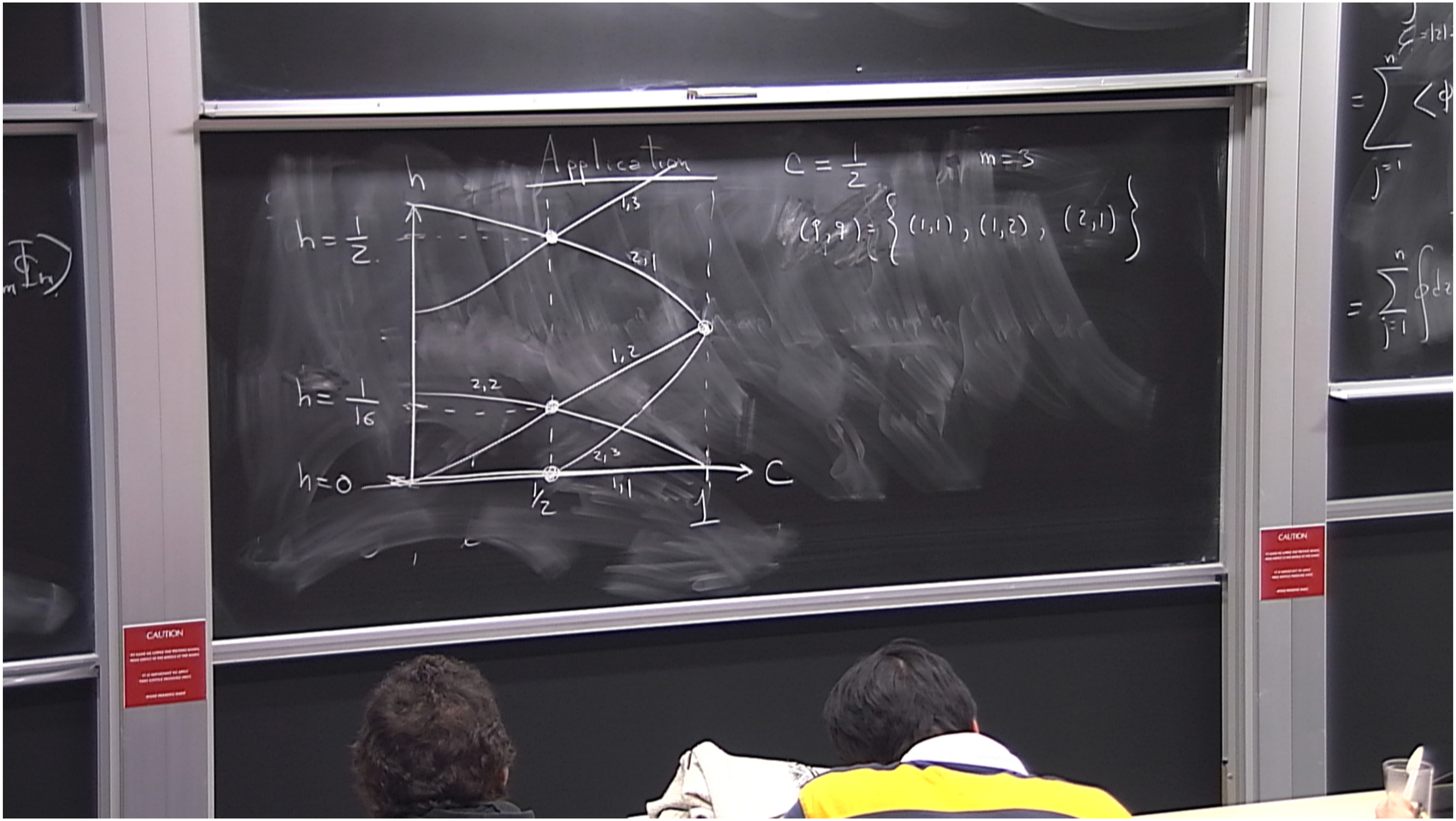

$\langle T(z)\Phi_1(w_1\bar{w}_1)\cdots\Phi_n(w_n,\bar{w}_n)\rangle = \sum_{j=1}^{n}\left(\frac{h_j}{(z-w_j)^2}+\frac{1}{(z-w_j)}\frac{\partial}{\partial w_j}\right)\langle\Phi_1(w_1\bar{w}_1)\cdots\Phi_n(w_n,\bar{w}_n)\rangle$
Descendants :
Def. of
$T(z)\Phi_n(w_n,\bar{w}_n) = \sum_{m<0}^{\infty}(z-w_n)^{m-2}\,\hat{L}_{-m}\Phi_n(w_n,\bar{w}_n)$
Primary
$\hat{L}_0\Phi = \Phi$
(z)
$\frac{1}{(z-w_j)} = \sum_{l=0}^{\infty}(-1)^l\frac{(z-w_n)^l}{(w_n-w_j)}$

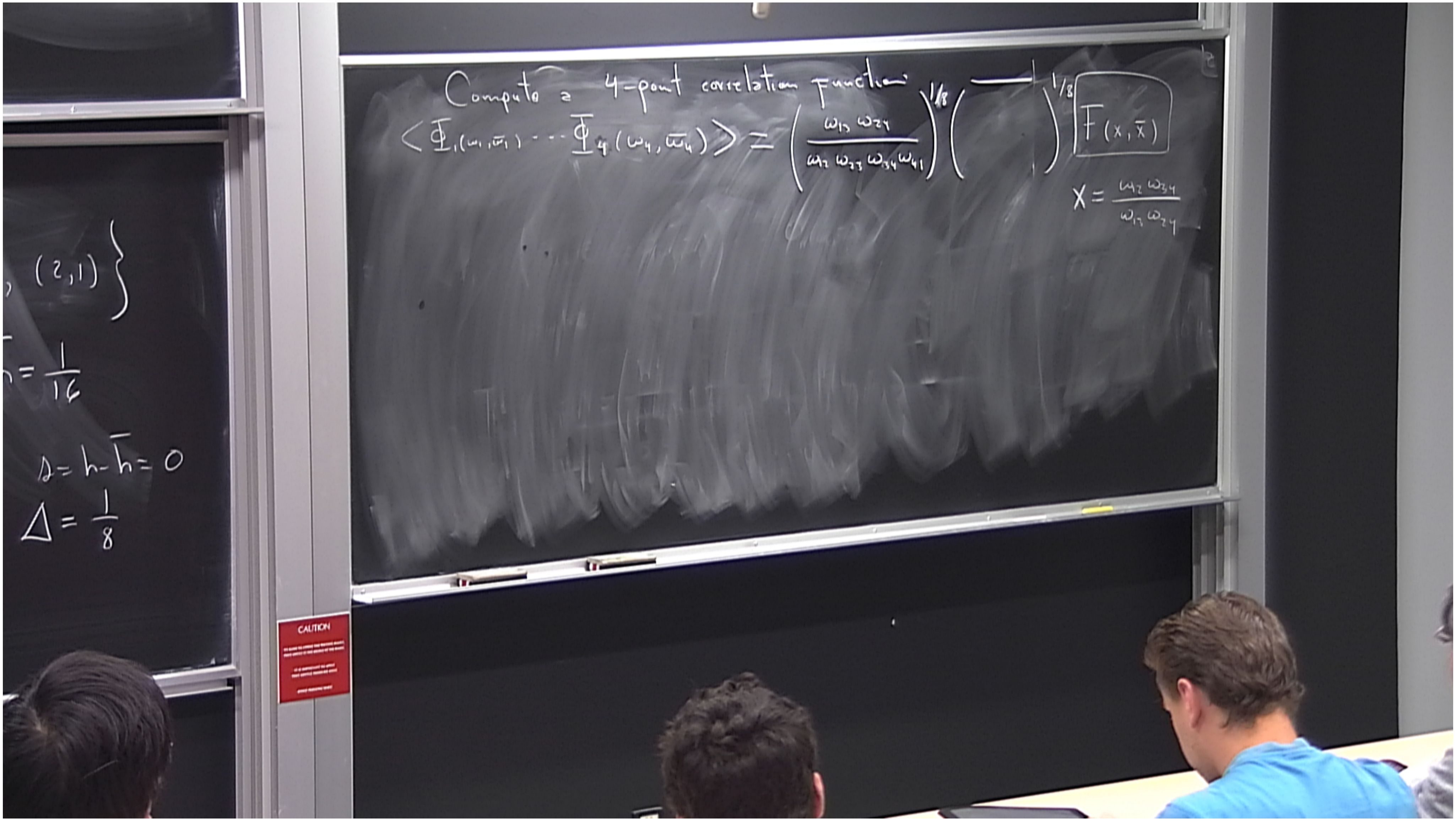


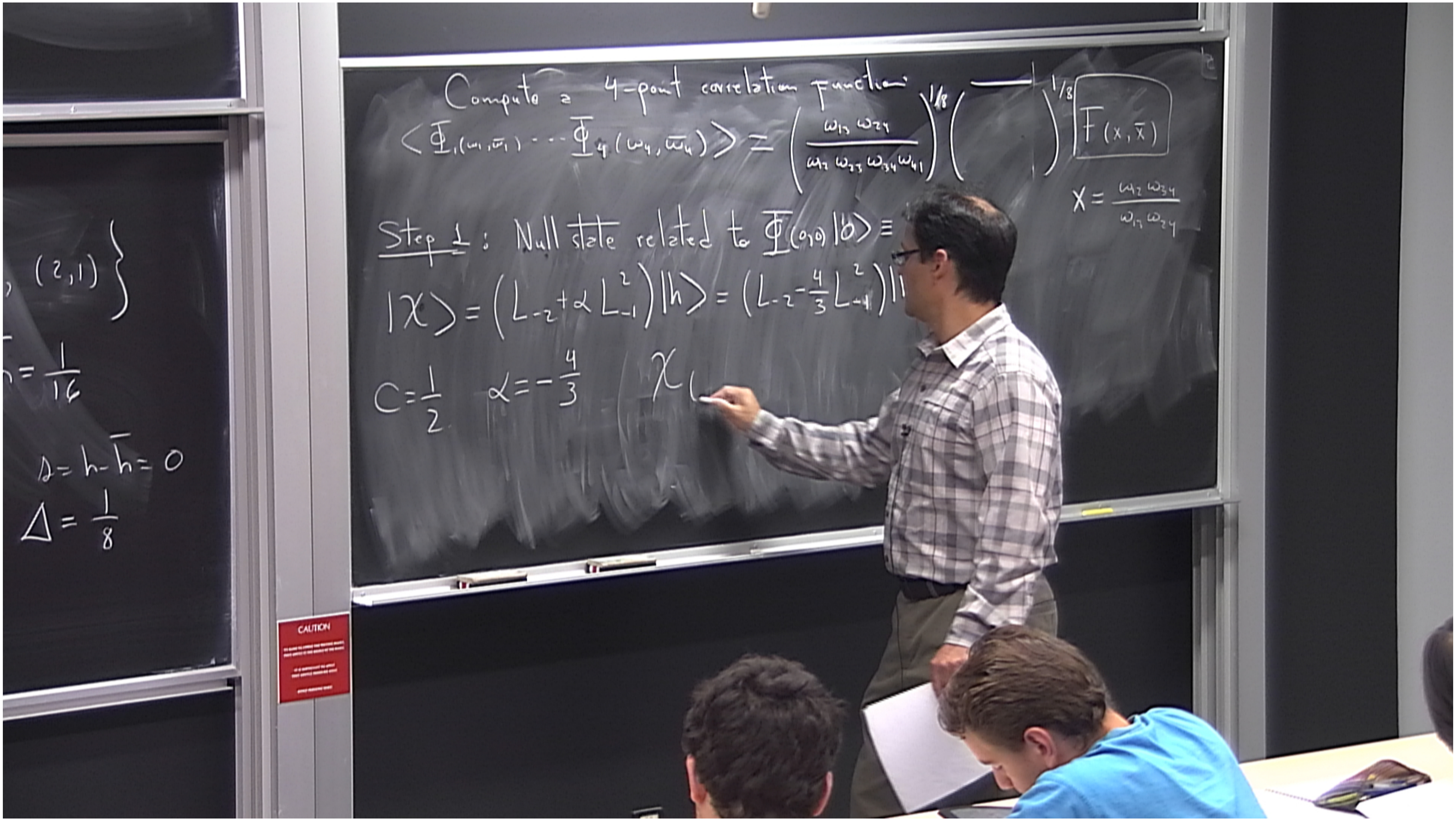
$\lim_{\substack{\omega \to 0 \\ \bar{\omega} \to 0}} \hat{L}_{-m} \Phi_p$

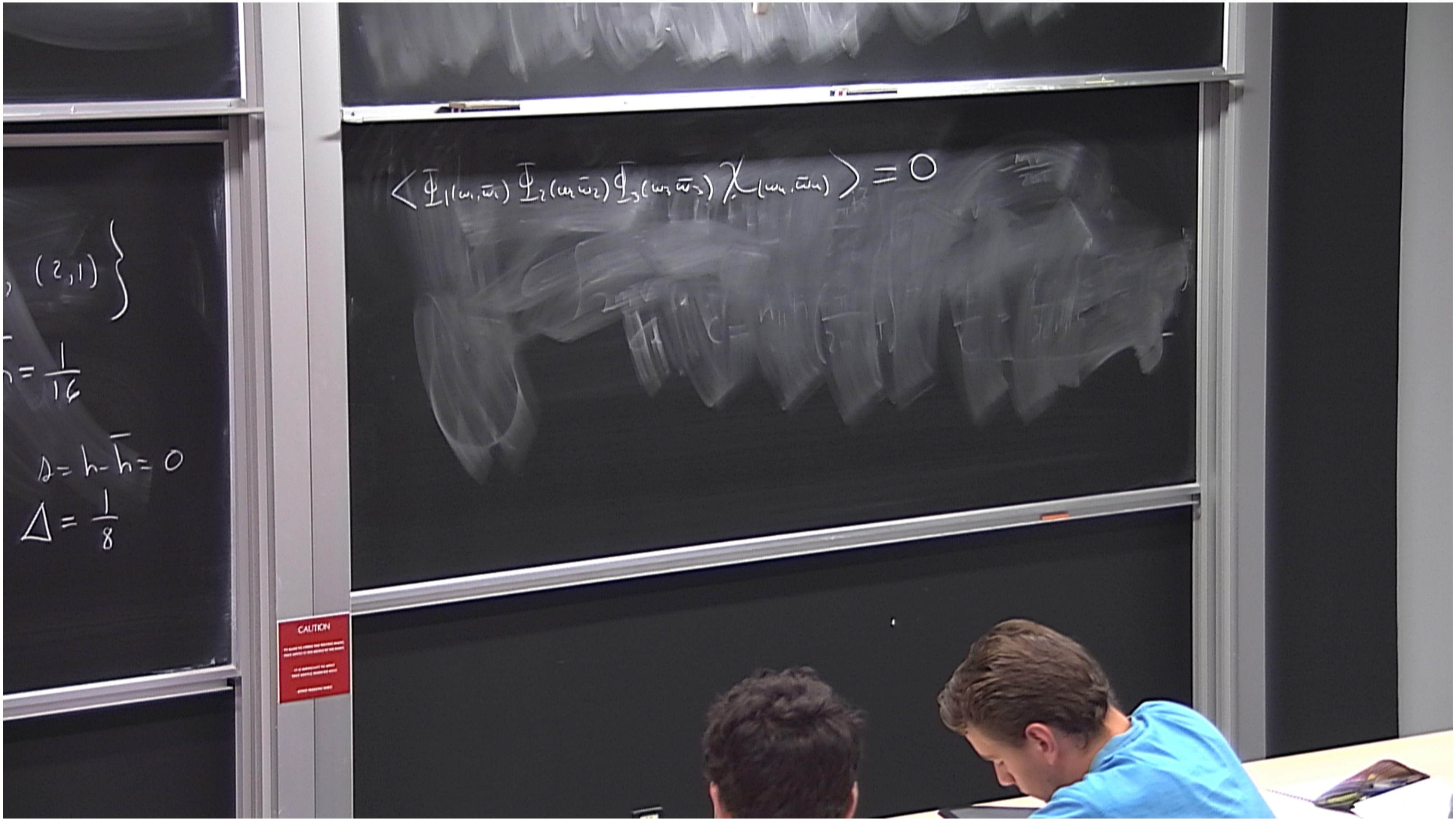

$\lim_{\substack{w \to 0 \\ \bar{w} \to 0}} \hat{L}_{-m} \Phi_p(0,0) = L_{-m} \phi(0,0)$
$\langle \Phi_1(w_1 \bar{w}_1) \cdots \Phi_{n-1}(w_n, \bar{w}_n) \big(L$
CAUTION
CAUTION

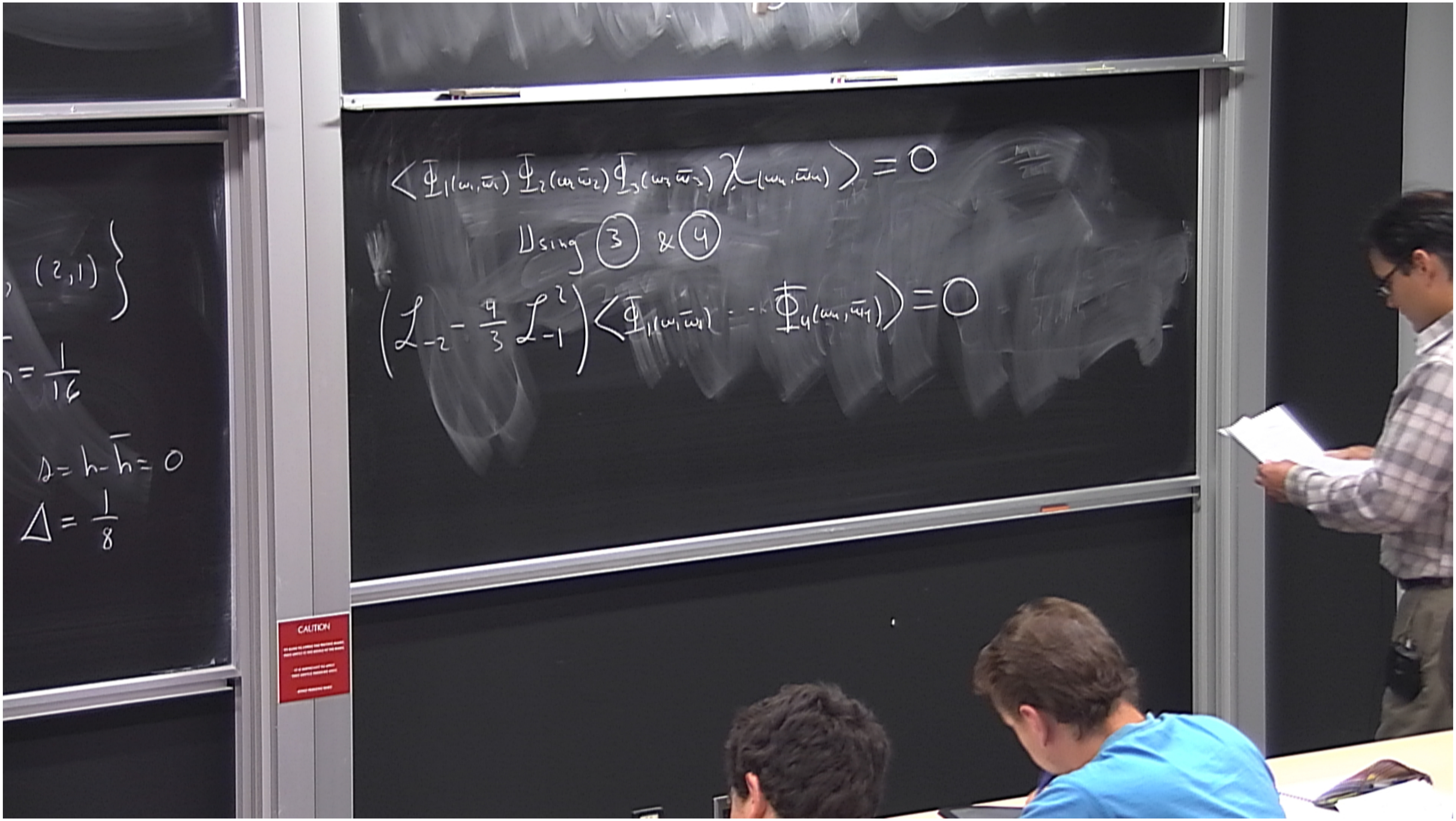

Application
c = 1/2
m = 3
h = 1/2
h = 1/16
h = 0
1,3
2,1
1,2
2,2
2,3
1,1
1/2
1
c

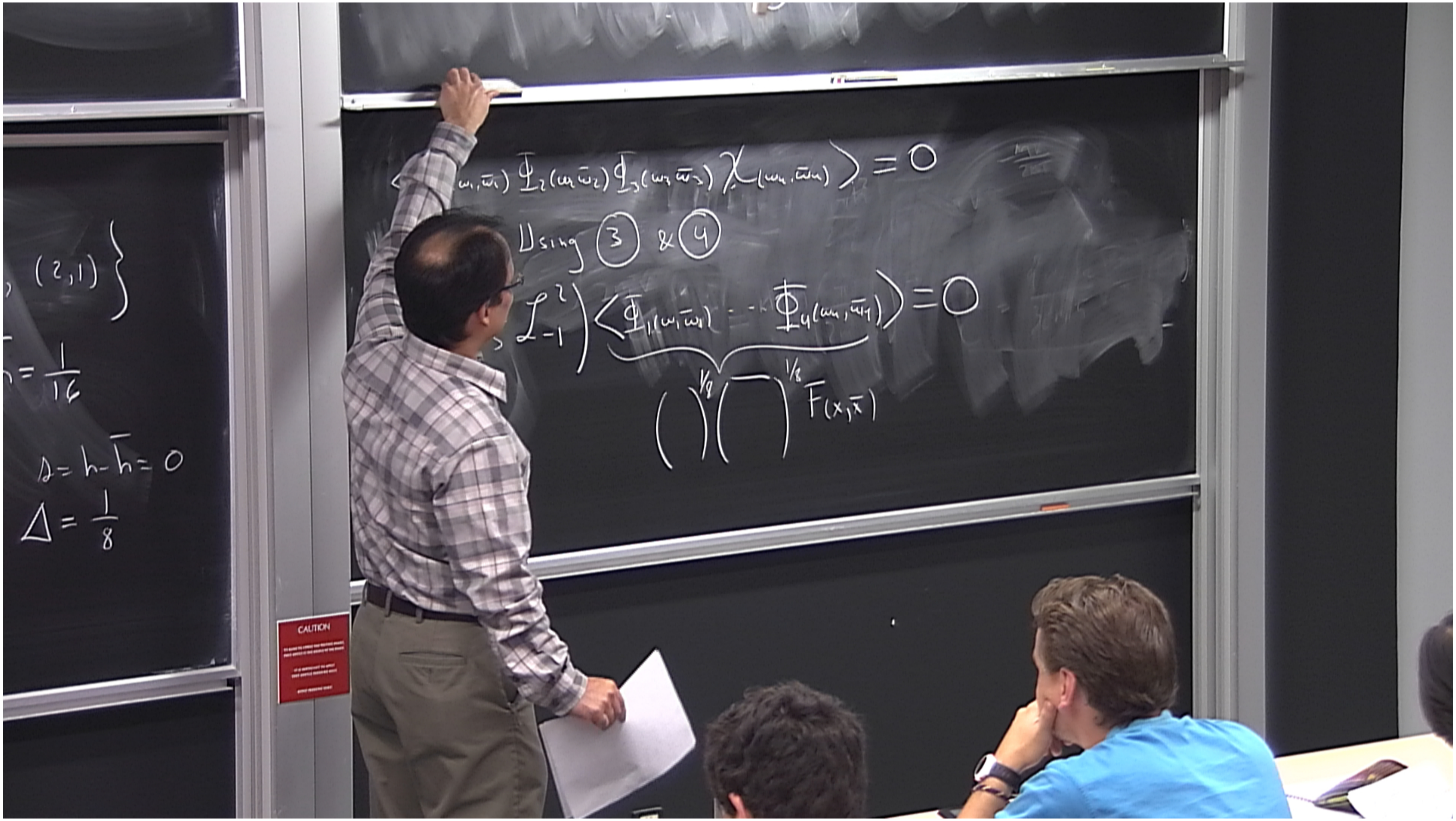

Application
c = 1/2
m = 3
(p, q) = {(1,1), (1,2), (2,1)}
h
h = 1/2
h = 1/16
h = 0
1,3
2,1
1,2
2,2
2,3
1,1
1/2
1
c

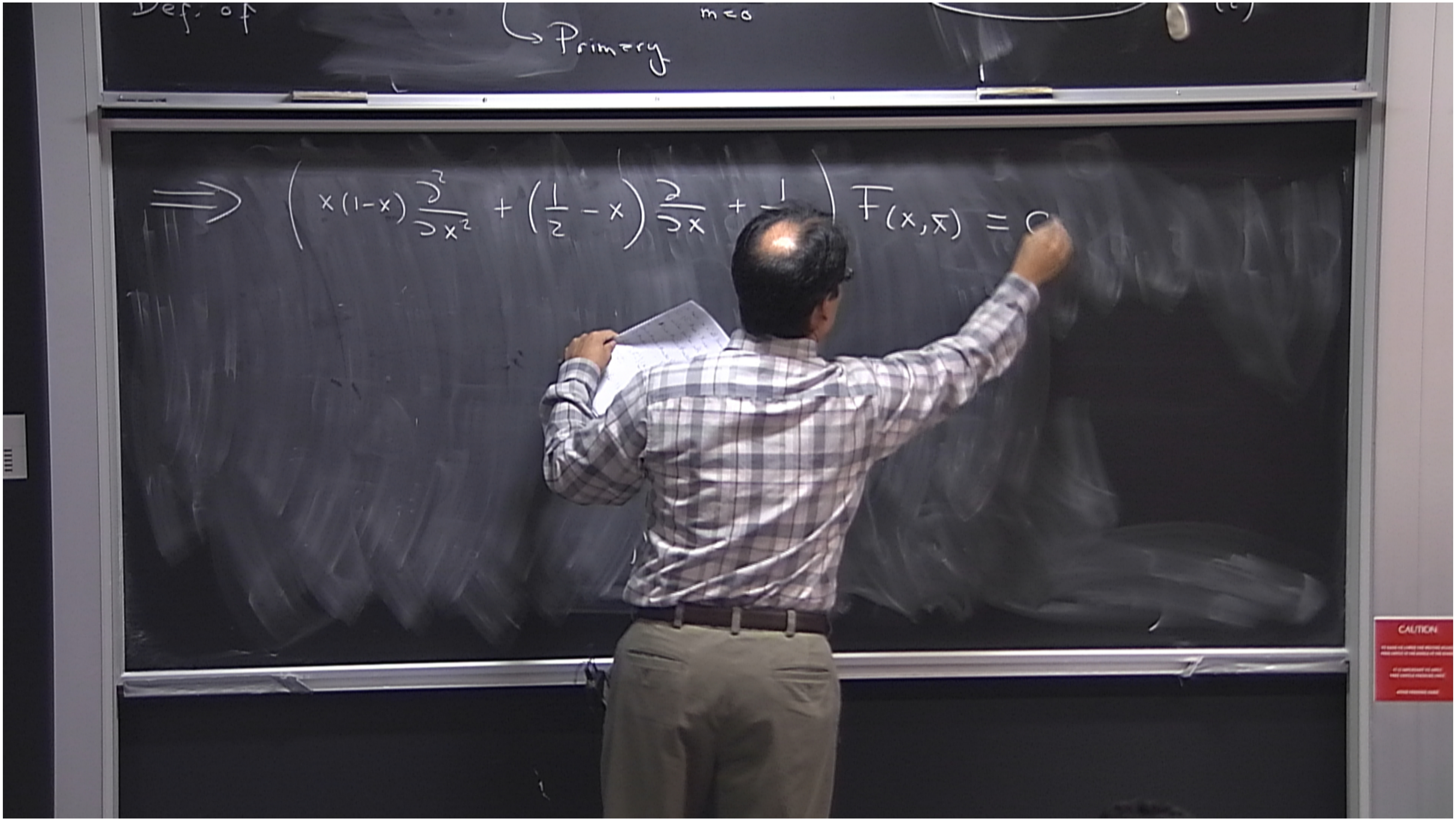

Compute a 4-point correlation function
$\langle \Phi_1(w_1,\bar{w}_1) \cdots \Phi_4(w_4,\bar{w}_4) \rangle = \left(\frac{w_{13} w_{24}}{w_{12} w_{23} w_{34} w_{41}} \right)^{1/8} \left(\right)^{1/8} \boxed{F(x,\bar{x})}$
$x = \frac{w_{12} w_{34}}{w_{13} w_{24}}$
$(2,1)$
$= \frac{1}{16}$
$s = h - \bar{h} = 0$
$\Delta = \frac{1}{8}$
CAUTION

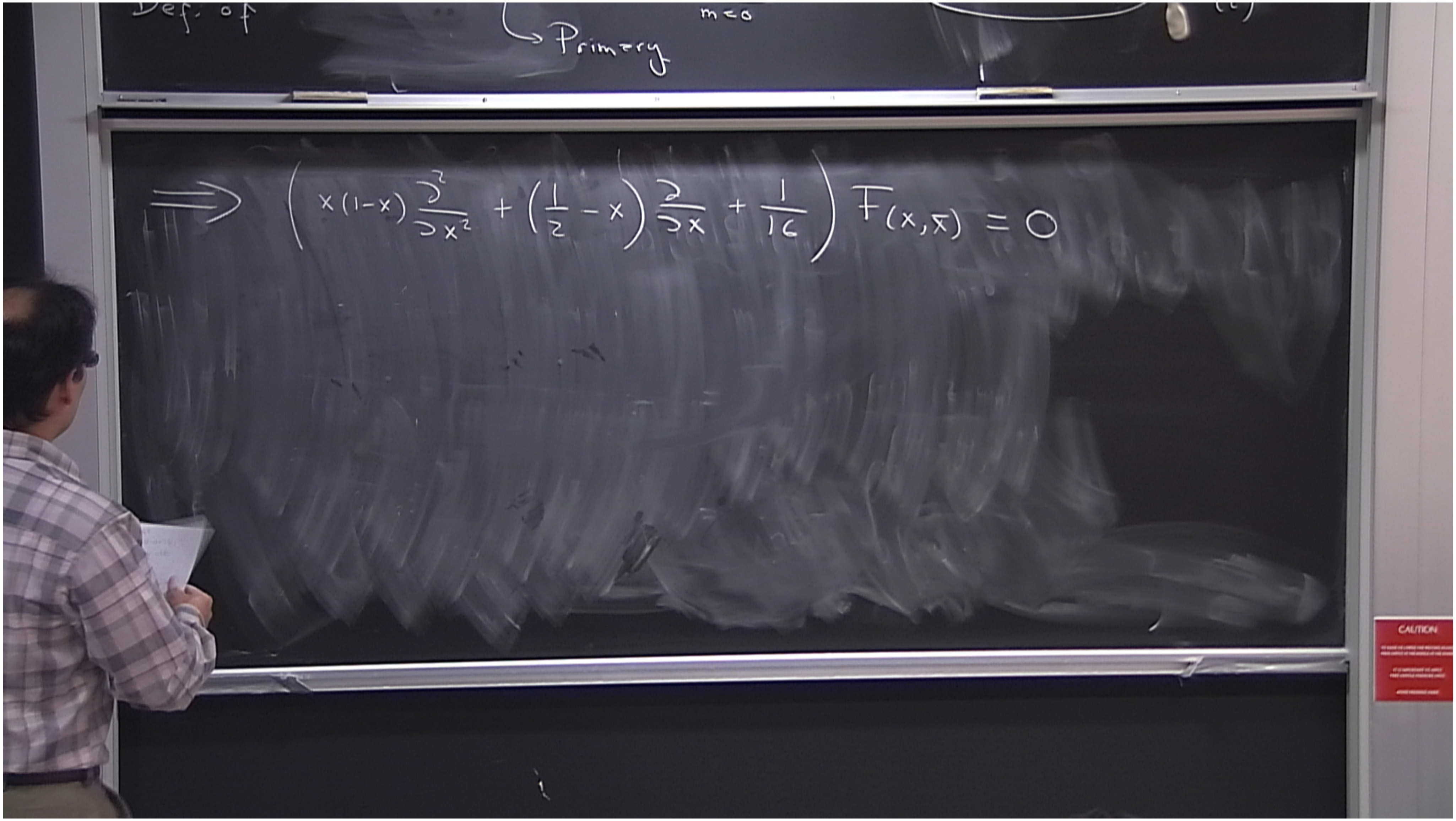

Compute a 4-point correlation function
$\langle \Phi_1(w_1,\bar{w}_1)\cdots\Phi_4(w_4,\bar{w}_4)\rangle = \left(\frac{w_{13}w_{24}}{w_{12}w_{23}w_{34}w_{41}}\right)^{1/8}\left(\quad\right)^{1/8} \boxed{F(x,\bar{x})}$
$x = \frac{w_{12}w_{34}}{w_{13}w_{24}}$
Step 1: Null state related to $\Phi(0,0)|0\rangle \equiv$
$|\chi\rangle = (L_{-2}+\alpha L_{-1}^2)|h\rangle = (L_{-2}-\frac{4}{3}L_{-1}^2)|h\rangle$
$c=\frac{1}{2}$ $\alpha=-\frac{4}{3}$ $\chi($
$(2,1)$
$h=\frac{1}{16}$
$s=h-\bar{h}=0$
$\Delta=\frac{1}{8}$

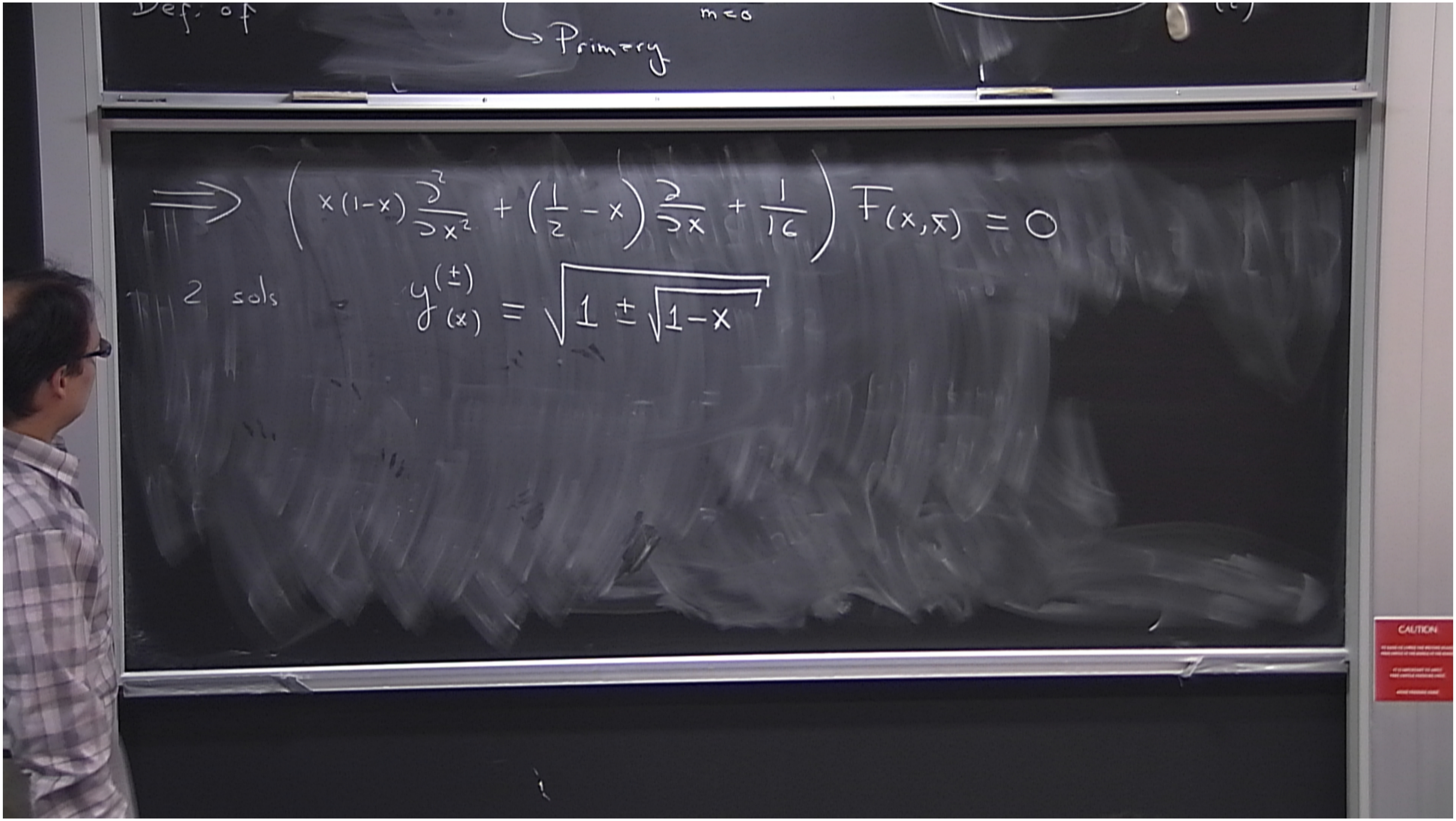

⟨Φ1(w1,w̄1) Φ2(w2,w̄2) Φ3(w3,w̄3) χ(w4,w̄4)⟩ = 0
(2,1)
= 1/16
s = h - h̄ = 0
Δ = 1/8
CAUTION

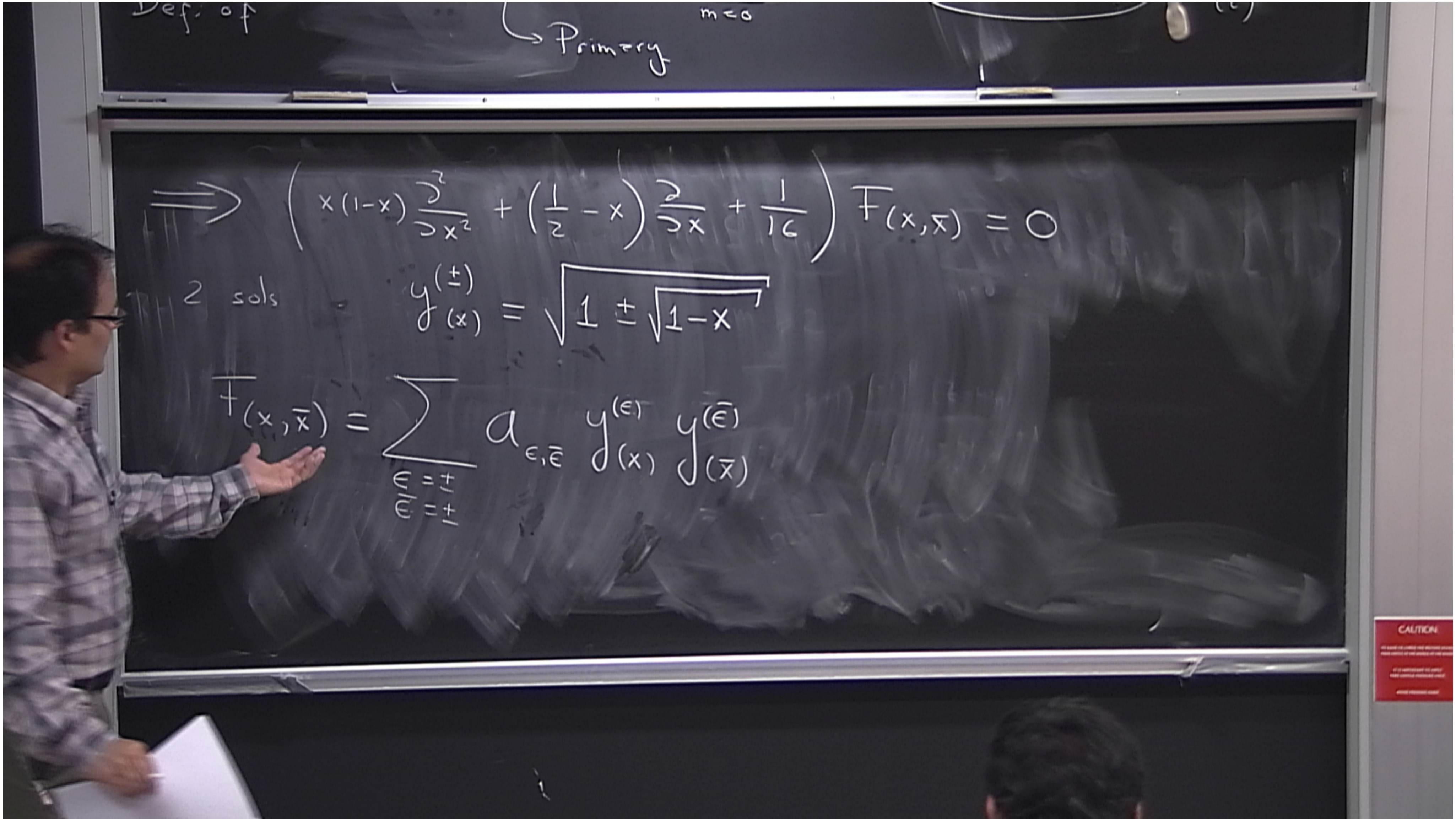

$\langle \Phi_1(w_1,\bar{w}_1)\,\Phi_2(w_2,\bar{w}_2)\,\Phi_3(w_3,\bar{w}_3)\,\chi(w_4,\bar{w}_4)\rangle = 0$
Using 3 & 4
$\left(\mathcal{L}_{-2} - \frac{4}{3}\mathcal{L}_{-1}^2\right)\langle \Phi_1(w_1,\bar{w}_1)\cdots\Phi_4(w_4,\bar{w}_4)\rangle = 0$
(2,1)
$= \frac{1}{16}$
$s = h - \bar{h} = 0$
$\Delta = \frac{1}{8}$
CAUTION

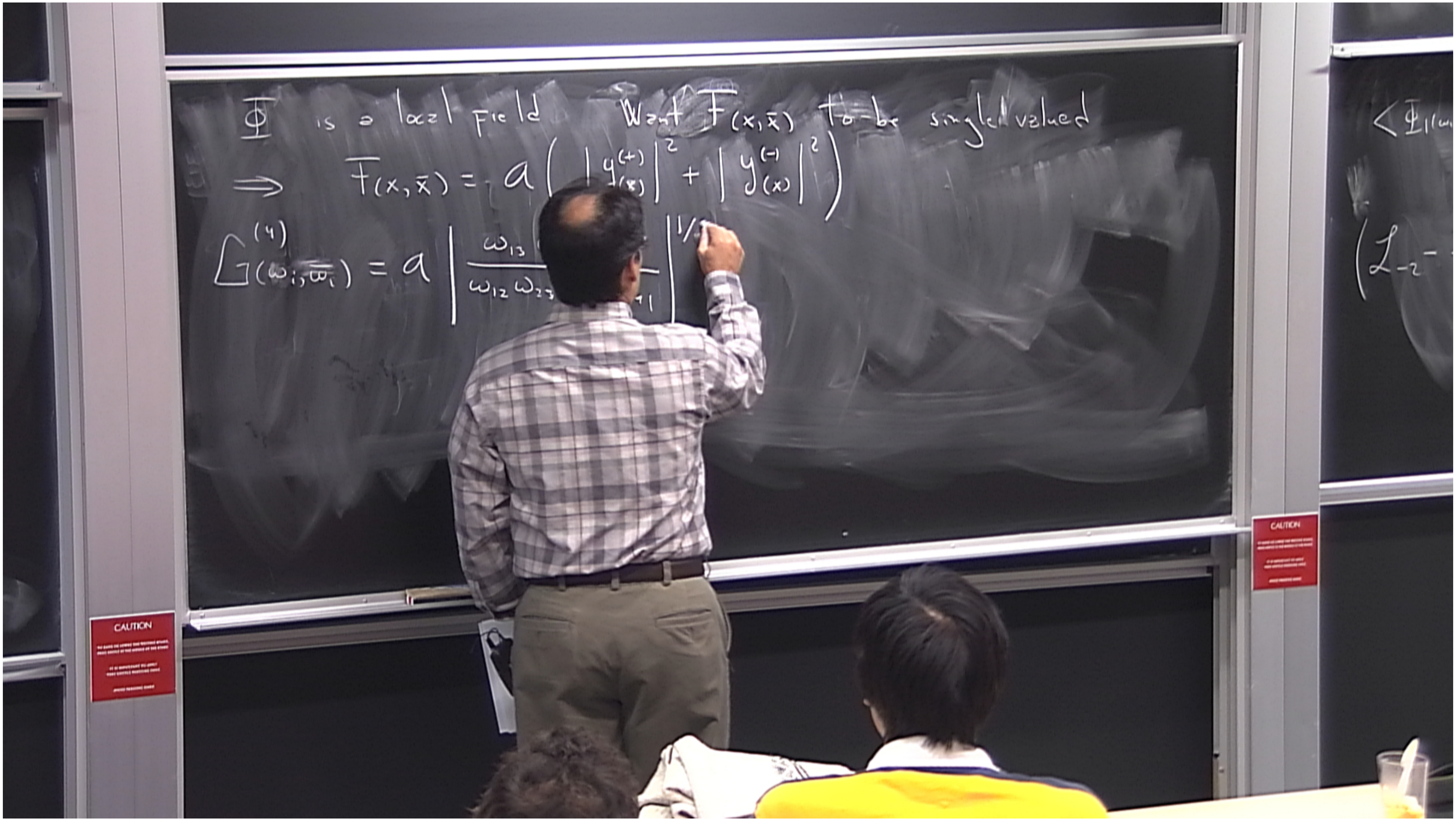

Using 3 & 4
(2,1)
F(x, x̄)
Δ = 1/8
CAUTION

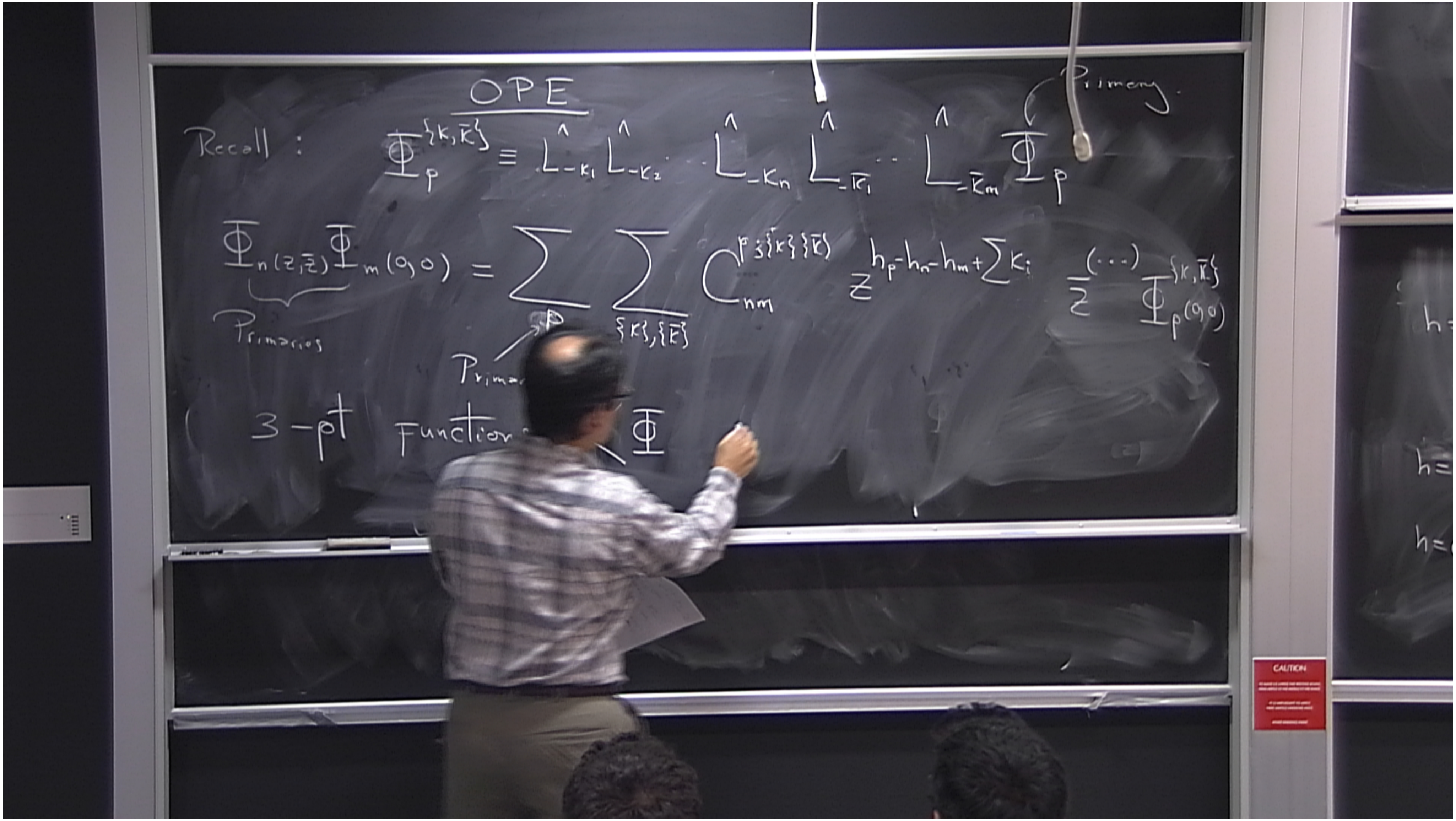

Def. of
Primary
$m=0$
$\Rightarrow \left(x(1-x)\frac{\partial^2}{\partial x^2} + \left(\frac{1}{2}-x\right)\frac{\partial}{\partial x} + \right) F(x,\bar{x}) =$
CAUTION

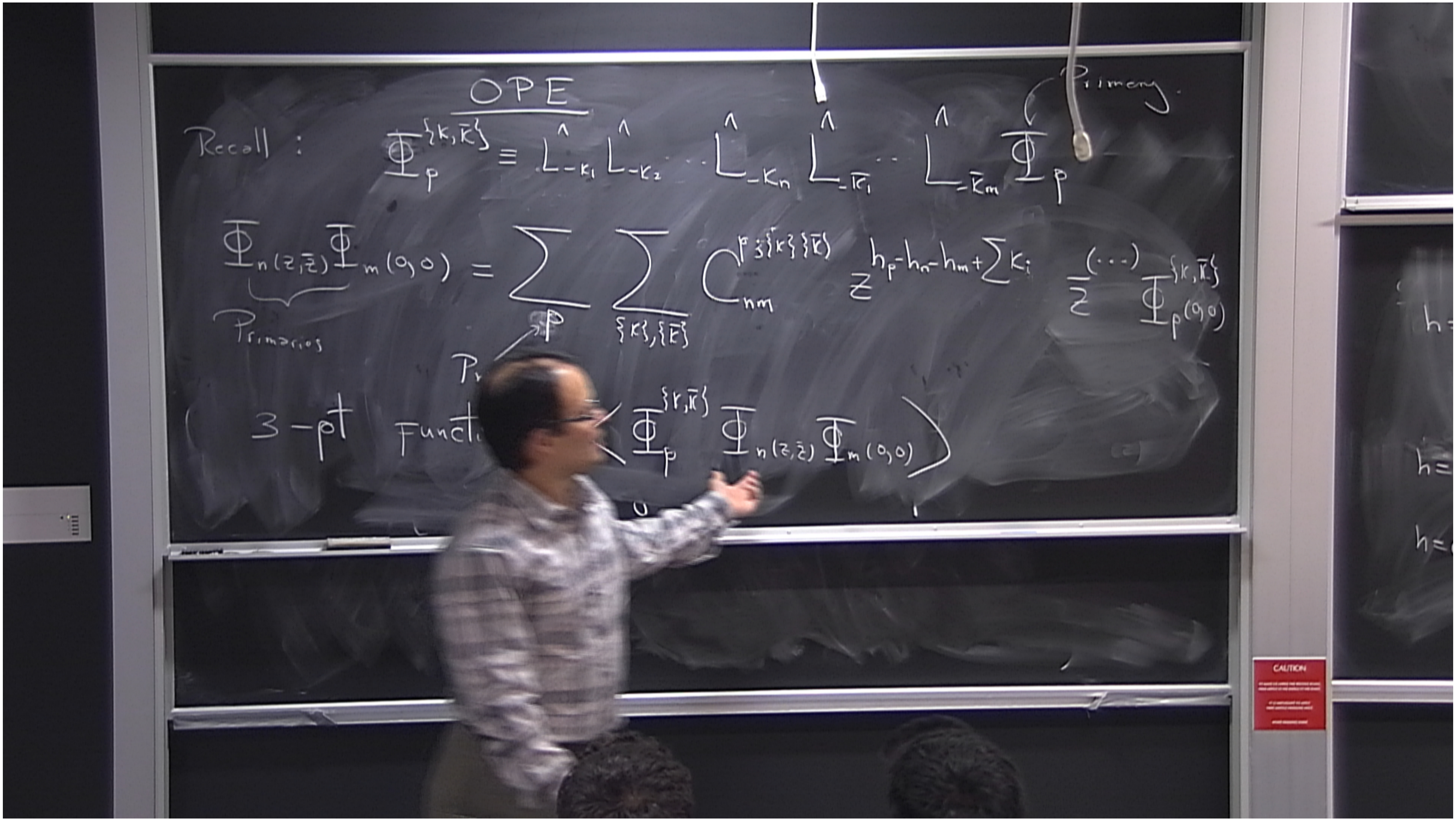

Def. of
$m<0$
Primary
$$\Longrightarrow \left(x(1-x)\frac{\partial^2}{\partial x^2} + \left(\frac{1}{2} - x\right)\frac{\partial}{\partial x} + \frac{1}{16} \right) F(x,\bar{x}) = 0$$

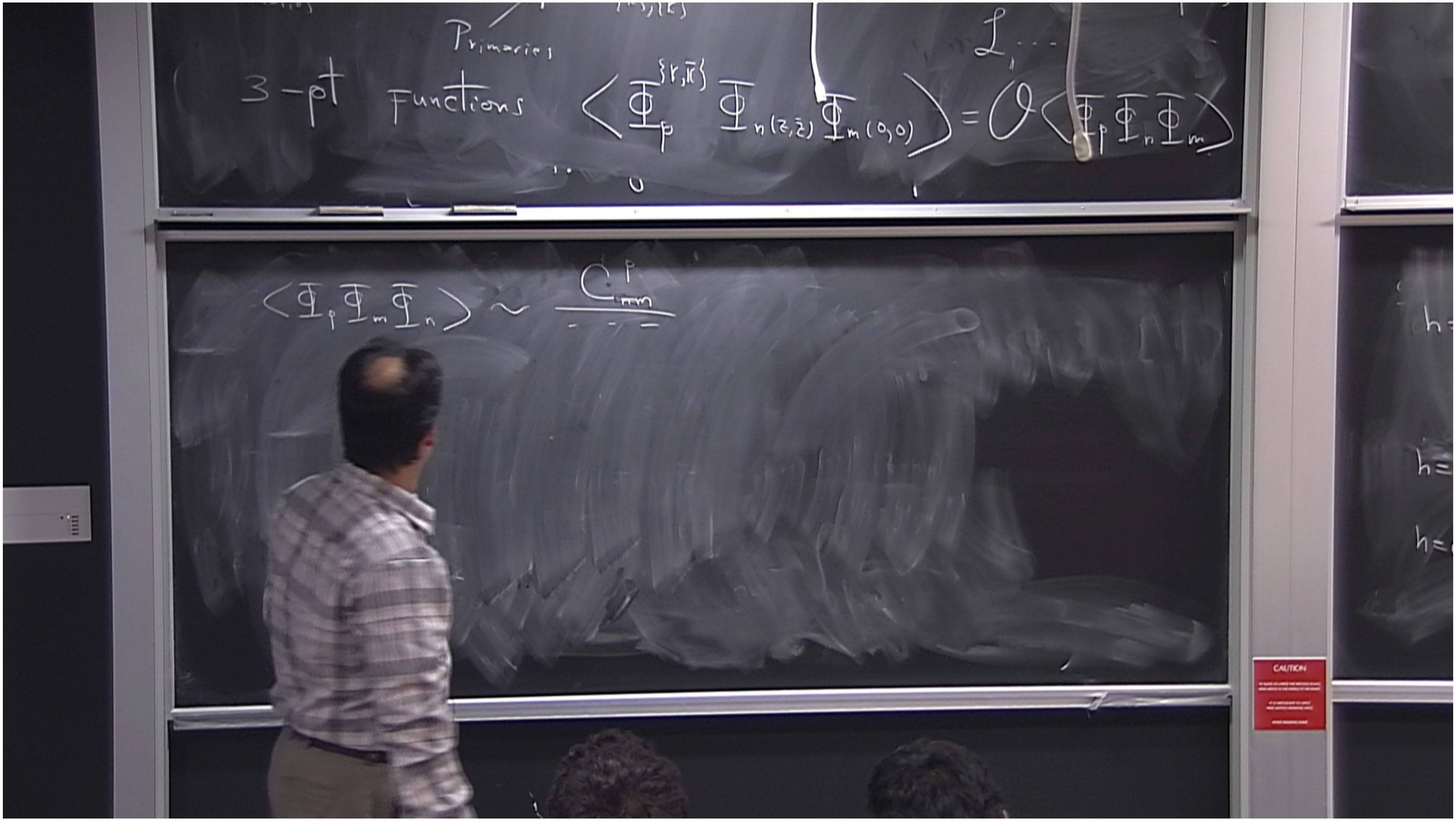

Def: of
Primary
$m=0$
$\Longrightarrow \left(x(1-x)\frac{\partial^2}{\partial x^2} + \left(\frac{1}{2} - x\right)\frac{\partial}{\partial x} + \frac{1}{16} \right) F(x,\bar{x}) = 0$
2 sols
$y^{(\pm)}(x) = \sqrt{1 \pm \sqrt{1-x}}$

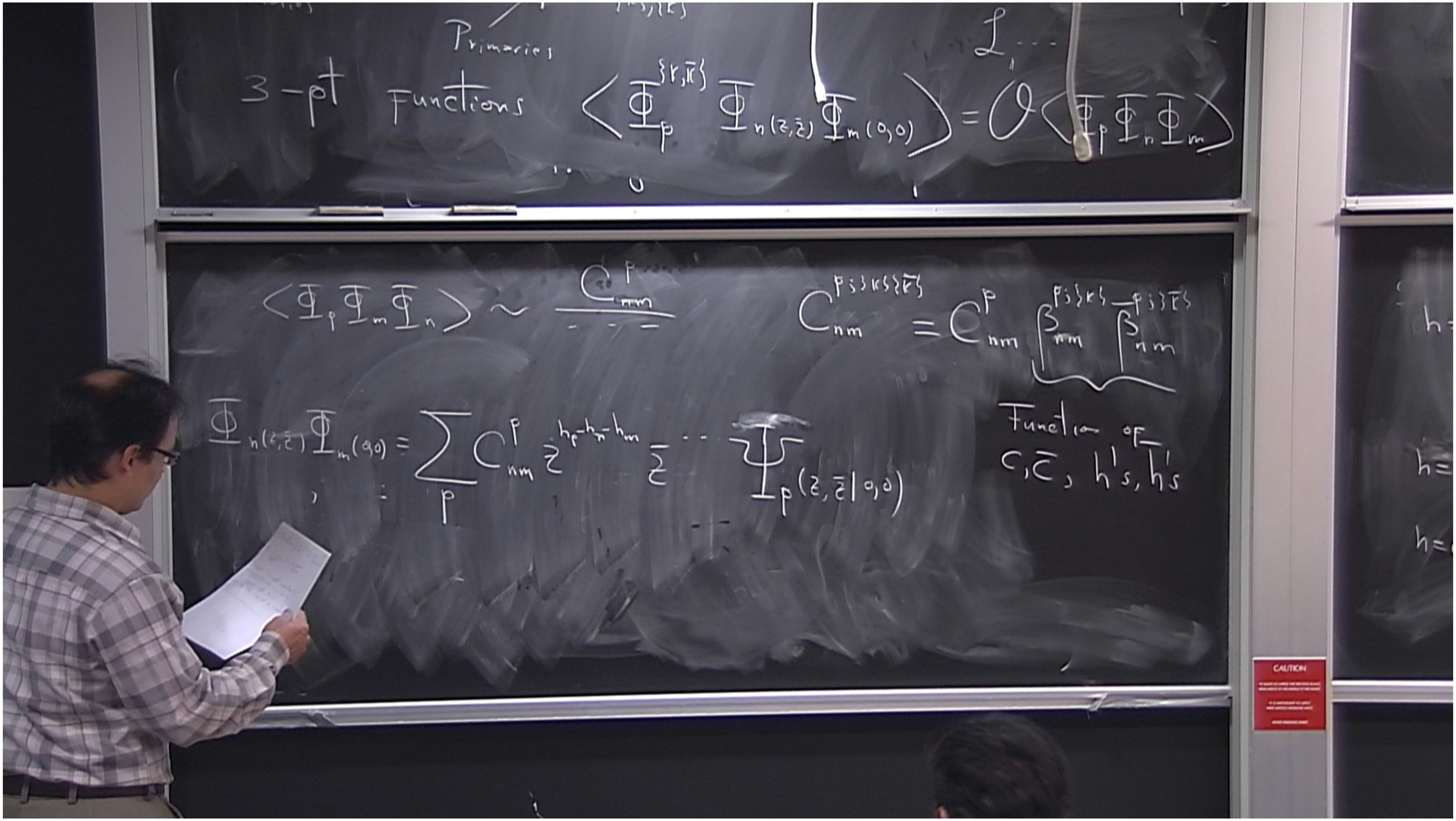

Def. of
Primary
$m=0$
$$\Longrightarrow \left(x(1-x)\frac{\partial^2}{\partial x^2} + \left(\frac{1}{2} - x\right)\frac{\partial}{\partial x} + \frac{1}{16} \right) F(x,\bar{x}) = 0$$
2 sols $\quad y^{(\pm)}(x) = \sqrt{1 \pm \sqrt{1-x}}$
$$F(x,\bar{x}) = \sum_{\substack{\epsilon=\pm \\ \bar{\epsilon}=\pm}} a_{\epsilon,\bar{\epsilon}}\, y^{(\epsilon)}(x)\, y^{(\bar{\epsilon})}(\bar{x})$$
CAUTION

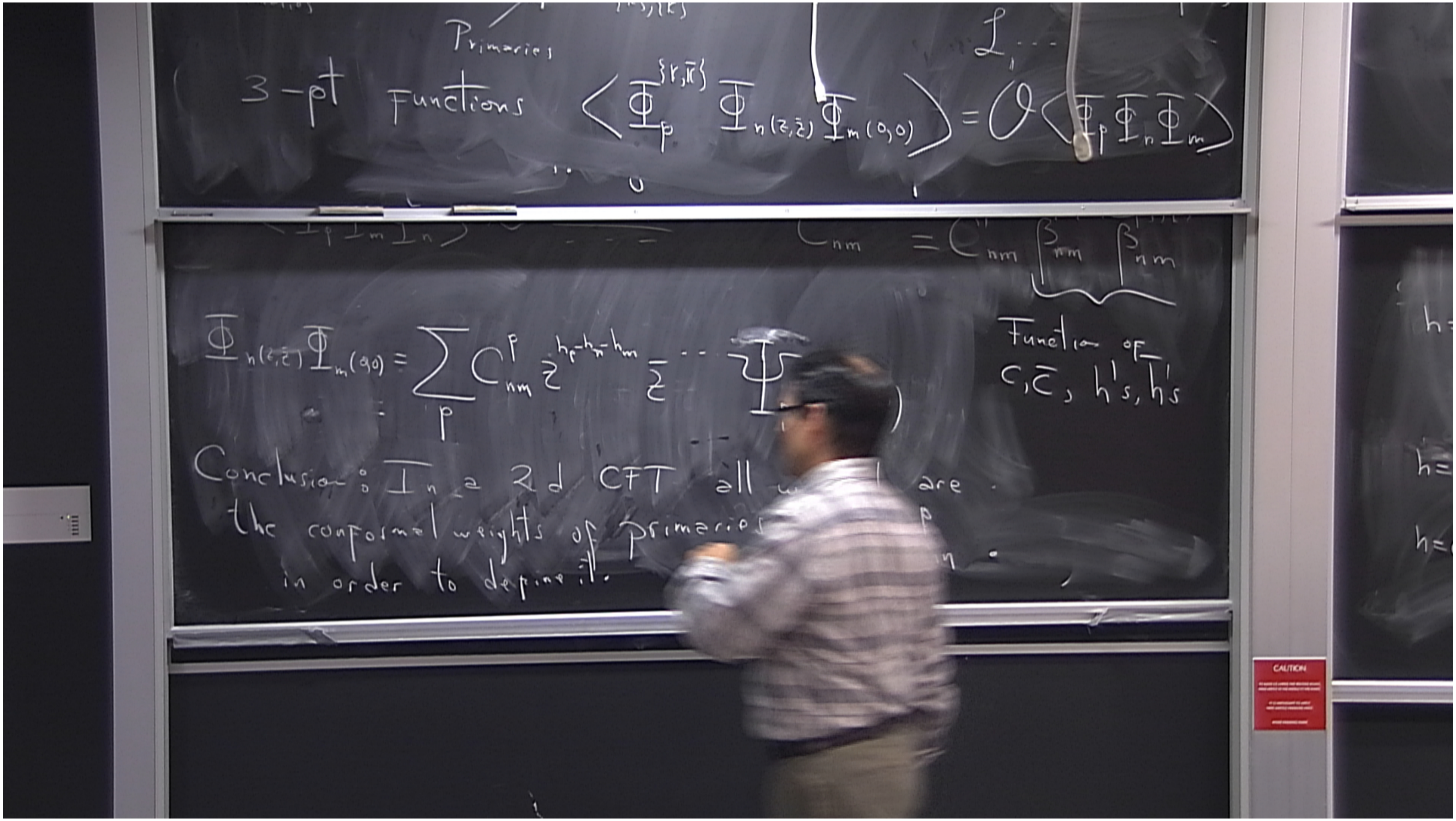


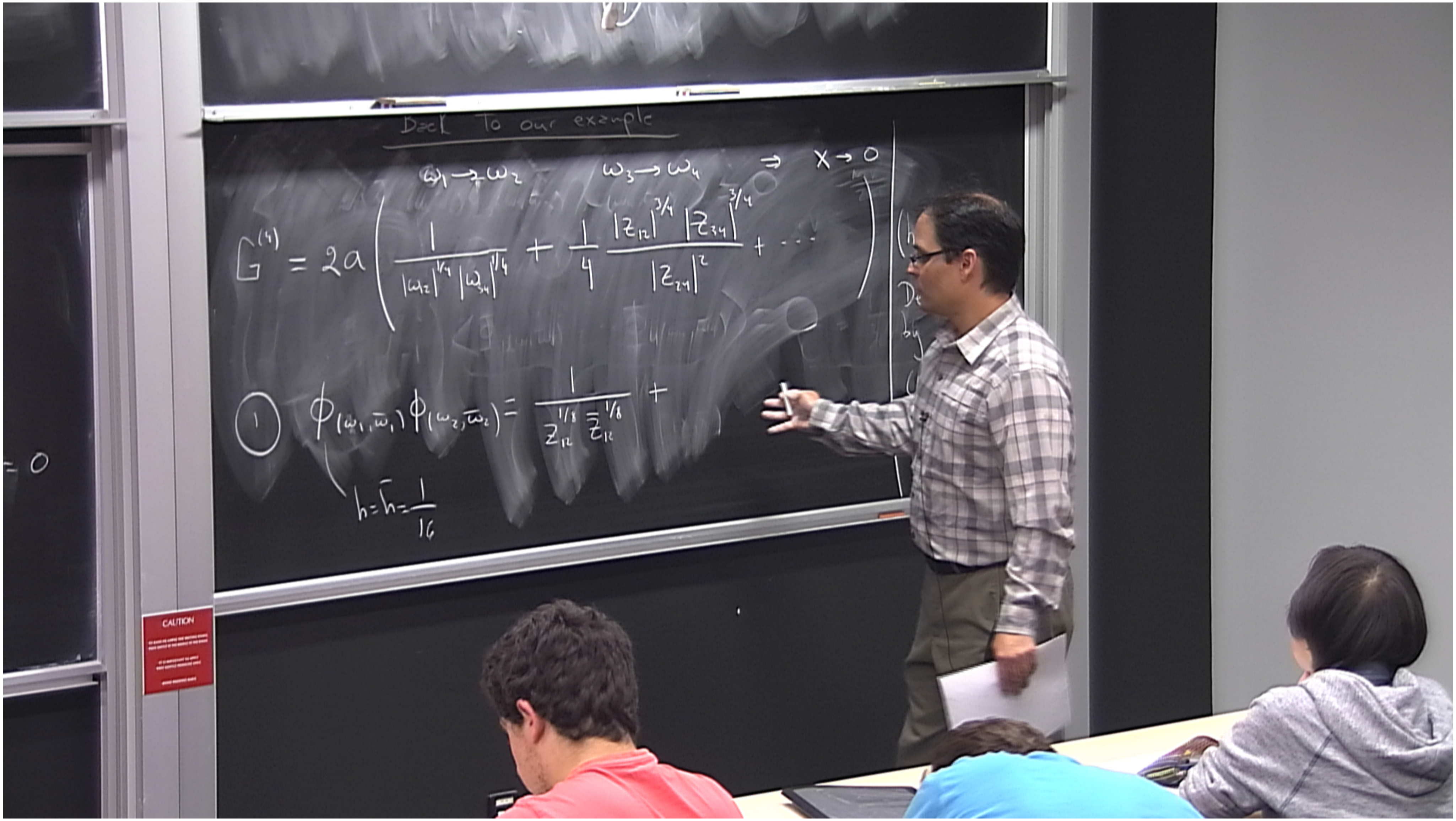

OPE
Primary
Recall : $\Phi_p^{\{k,\bar{k}\}} \equiv \hat{L}_{-k_1}\hat{L}_{-k_2}\cdots\hat{L}_{-k_n}\hat{L}_{-\bar{k}_1}\cdots\hat{L}_{-\bar{k}_m}\Phi_p$
$\underbrace{\Phi_n(z,\bar{z})\Phi_m(0,0)}_{\text{Primaries}} = \sum_p \sum_{\{k\},\{\bar{k}\}} C_{nm}^{p;\{k\}\{\bar{k}\}} z^{h_p-h_n-h_m+\sum k_i} \bar{z}^{(\cdots)} \Phi_p^{\{k,\bar{k}\}}(0,0)$
Primar
3-pt Function

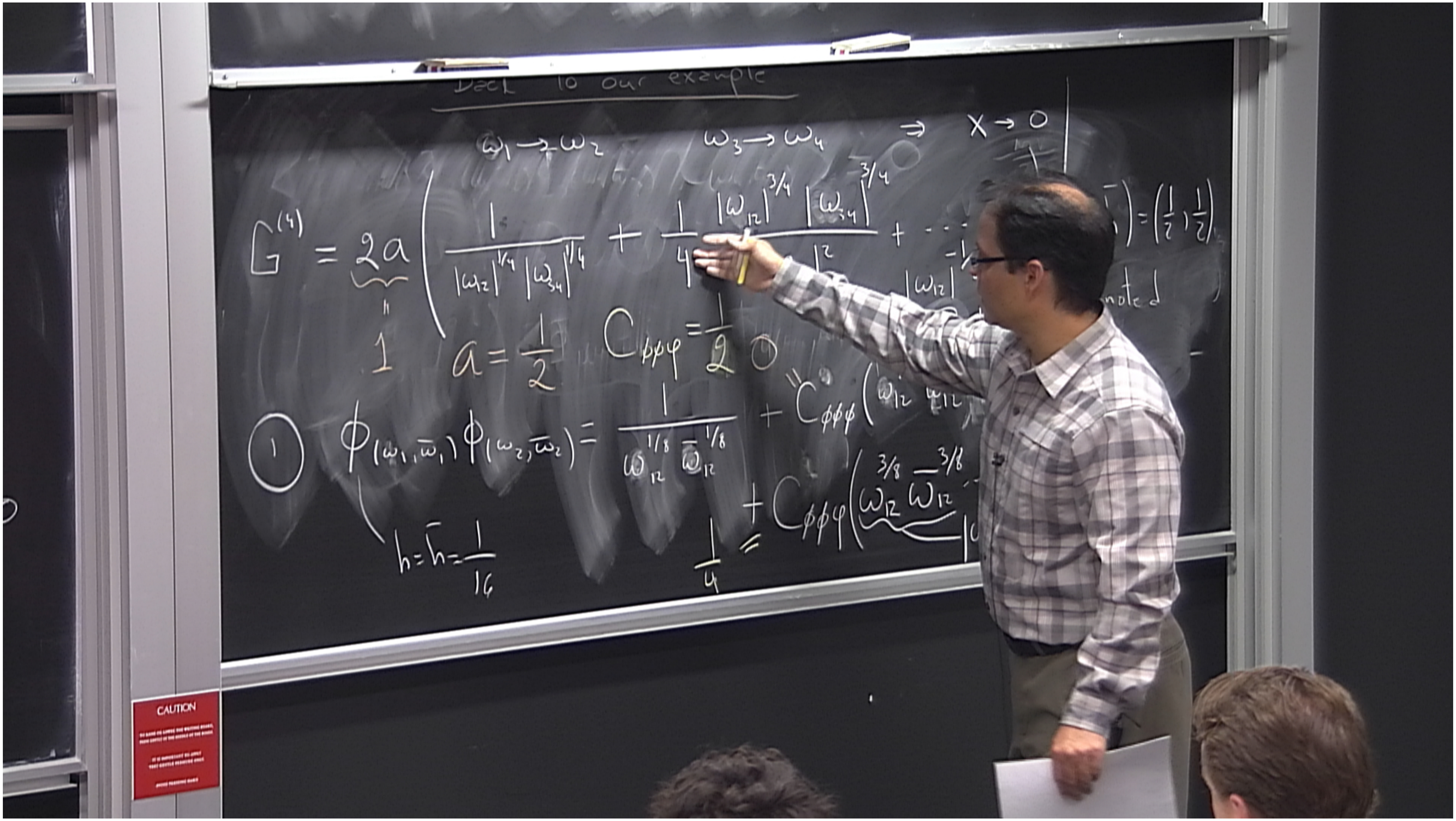

OPE
Primary
Recall : $\Phi_p^{\{k,\bar{k}\}} \equiv \hat{L}_{-k_1} \hat{L}_{-k_2} \cdots \hat{L}_{-k_n} \hat{L}_{-\bar{k}_1} \cdots \hat{L}_{-\bar{k}_m} \Phi_p$
$\Phi_{n}(z,\bar{z}) \Phi_{m}(0,0) = \sum_{p} \sum_{\{k\},\{\bar{k}\}} C_{nm}^{p;\{k\}\{\bar{k}\}} z^{h_p-h_n-h_m+\sum k_i} \bar{z}^{(\cdots)} \Phi_p^{\{k,\bar{k}\}}(0,0)$
Primaries
3-pt Funct
$\langle \Phi_p^{\{k,\bar{k}\}} \Phi_{n}(z,\bar{z}) \Phi_{m}(0,0) \rangle$
CAUTION

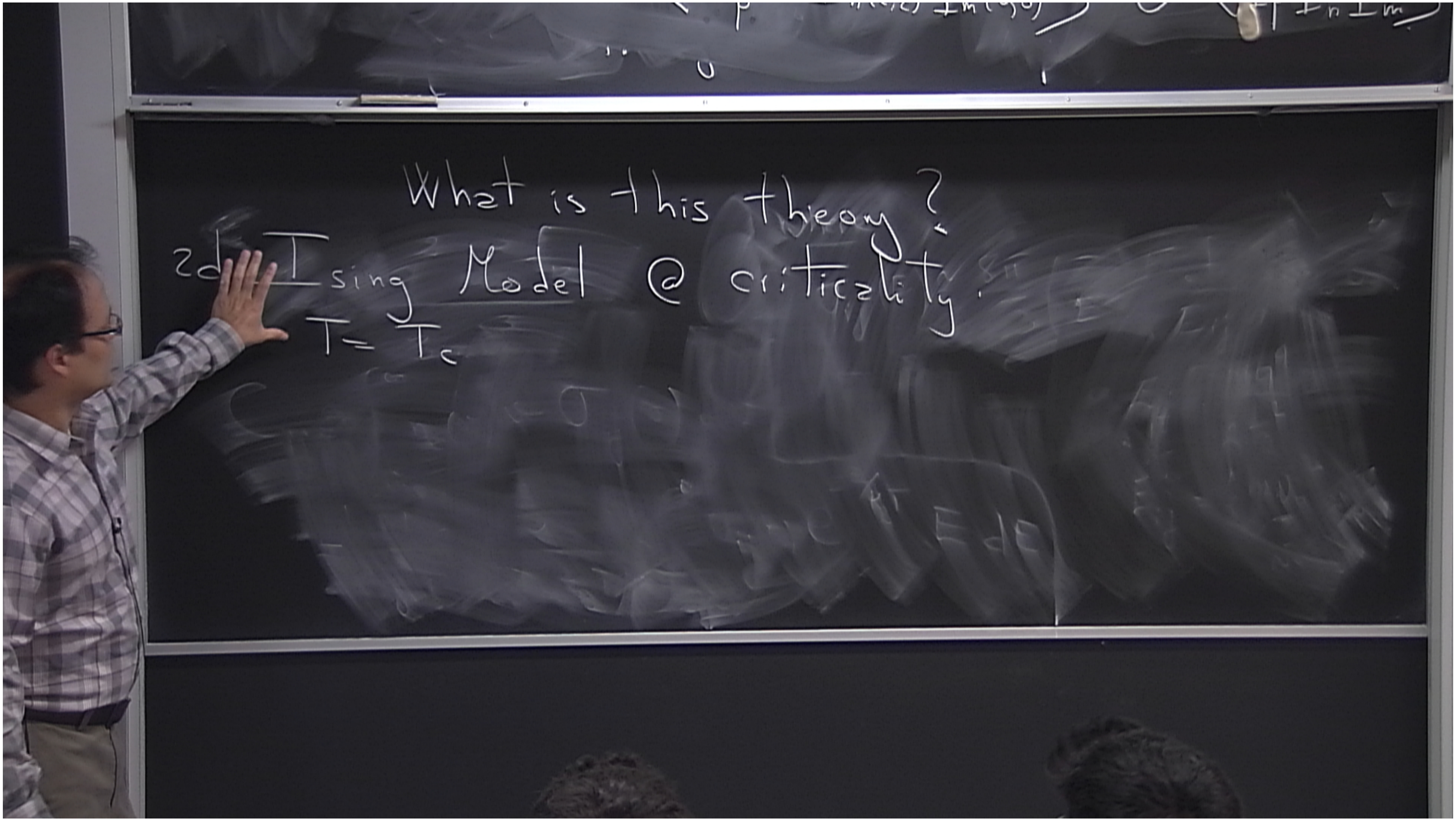

Primaries
3-pt Functions $\langle \Phi_p^{\{r,\bar{r}\}} \Phi_{n(z,\bar{z})} \Phi_{m(0,0)} \rangle = \mathcal{O} \langle \Phi_p \Phi_n \Phi_m \rangle$
$\langle \Phi_p \Phi_m \Phi_n \rangle \sim \frac{C^p_{mn}}{\quad}$
CAUTION

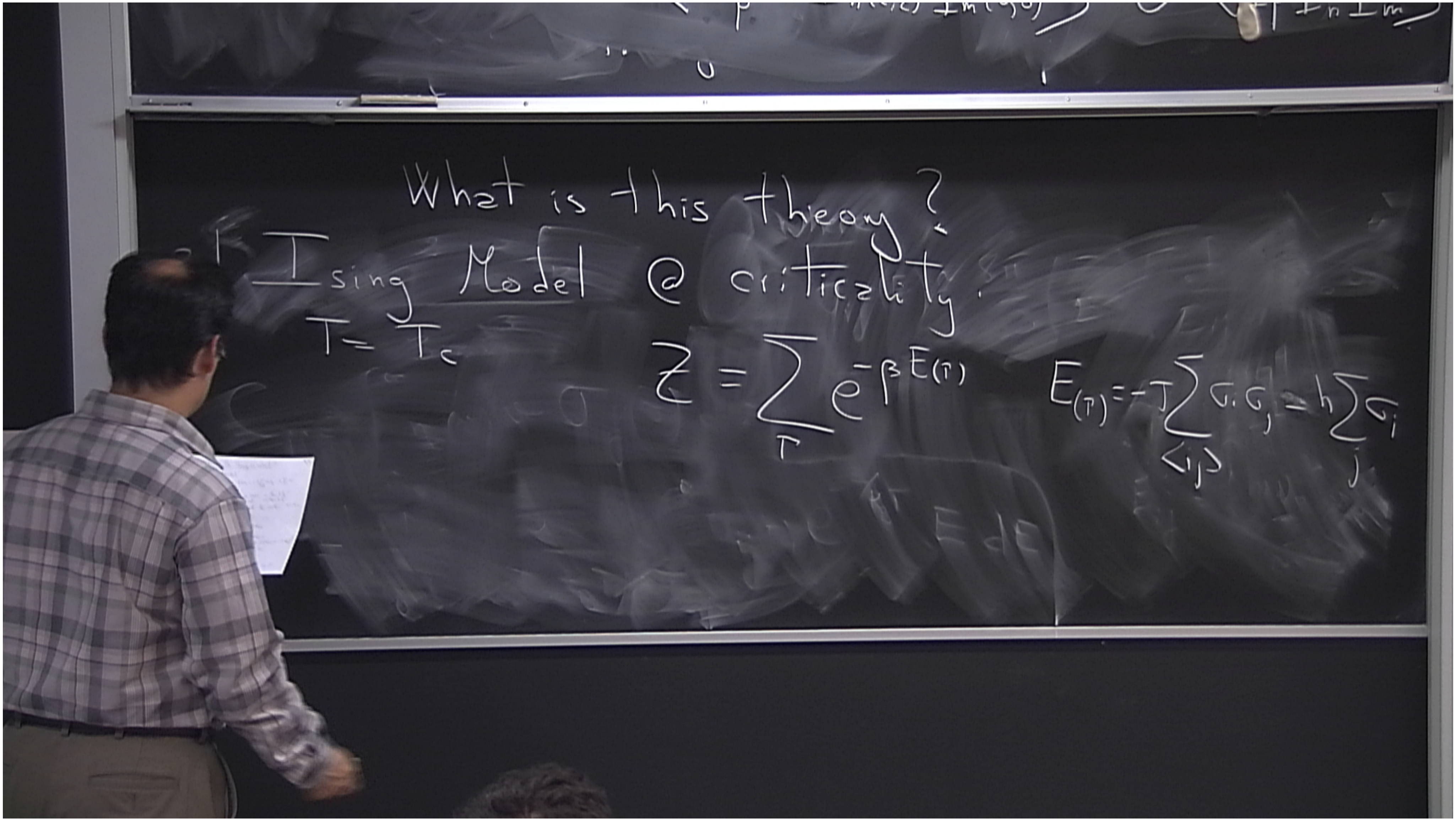

Primaries
3-pt Functions $\langle \Phi_p^{\{r,\bar{r}\}} \Phi_{n(z,\bar{z})} \Phi_{m(0,0)} \rangle = \mathcal{O}\langle \Phi_p \Phi_n \Phi_m \rangle$
$\langle \Phi_p \Phi_m \Phi_n \rangle \sim C^p_{nm}$
$C^{p;\{r\};\{\bar{r}\}}_{nm} = C^p_{nm} \beta^{p;\{r\}}_{nm} \bar{\beta}^{p;\{\bar{r}\}}_{nm}$
Function of $c, \bar{c}$, h's, $\bar{h}$'s
$\Phi_{n(z,\bar{z})} \Phi_{m(0,0)} = \sum_p C^p_{nm} z^{h_p-h_n-h_m} \bar{z}^{\cdots} \Psi_p(z,\bar{z}|0,0)$

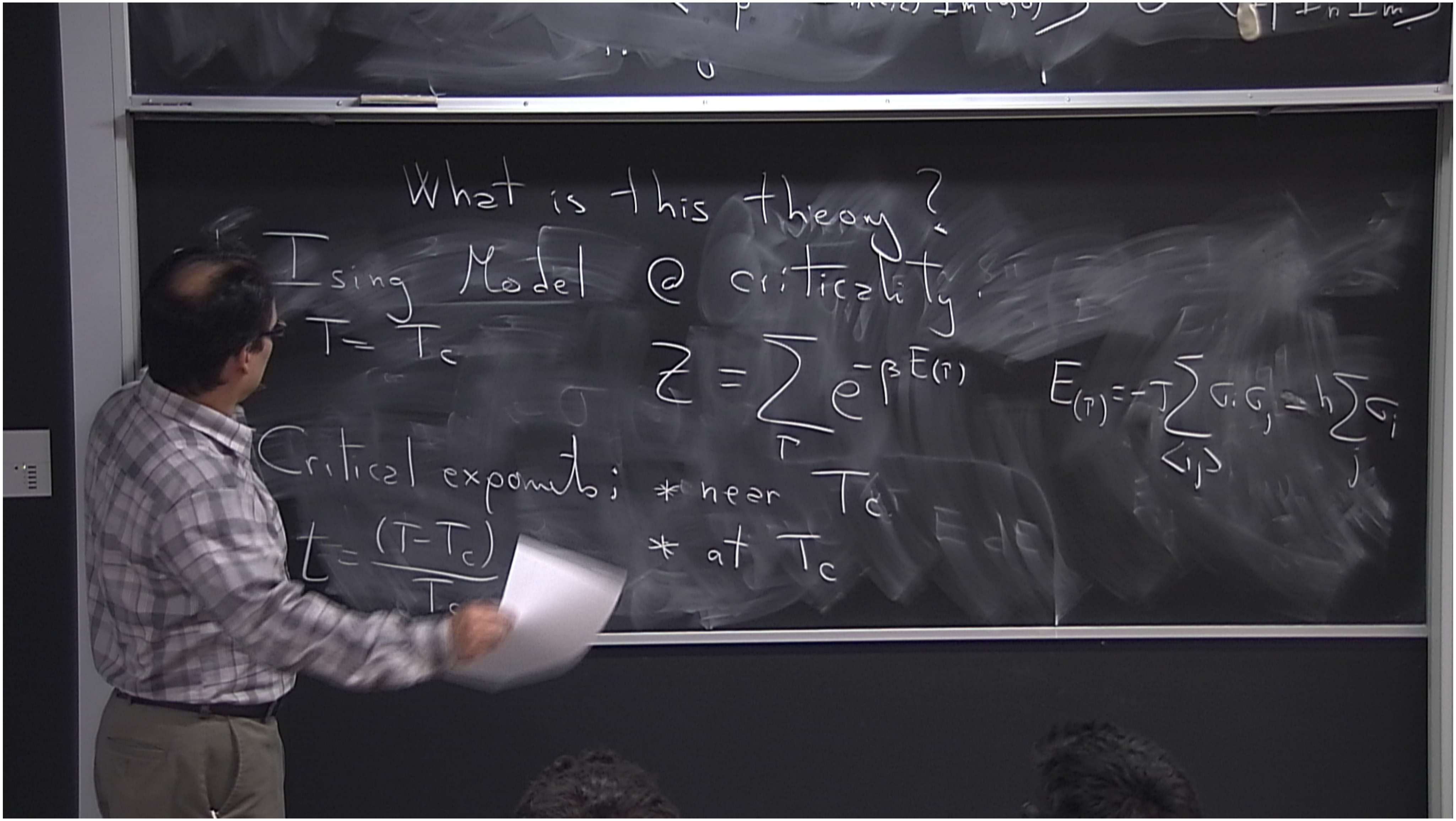

Primaries
3-pt Functions
Function of
c, c̄, h's, h̄'s
Conclusion: In a 2d CFT all ... are
the conformal weights of primaries
in order to define it.


Back to our example
ω1 → ω2   ω3 → ω4   ⇒   x → 0
G^(4) = 2a (1 / (|ω12|^1/4 |ω34|^1/4) + 1/4 (|z12|^3/4 |z34|^3/4) / |z24|^2 + ...)
1   φ(ω1, ω̄1) φ(ω2, ω̄2) = 1 / (z12^1/8 z̄12^1/8) +
h = h̄ = 1/16


to our example
ω1 → ω2
ω3 → ω4
⇒ x → 0
C φφφ = 1/2
a = 1/2
h = h̄ = 1/16
noted
CAUTION


What is this theory?
2d Ising Model @ criticality.
T = T_c


What is this theory?
Ising Model @ criticality.
$T = T_c$
$Z = \sum_{\tau} e^{-\beta E(\tau)}$
$E(\tau) = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j - h \sum_j \sigma_j$


What is this theory?
Ising Model @ criticality.
T = Tc
Z = Σσ e^{-βE(σ)}
E(σ) = -J Σ<ij> σi σj - h Σi σi
Critical exponents; * near Tc
* at Tc
t = (T-Tc)/Tc

What is this theory?

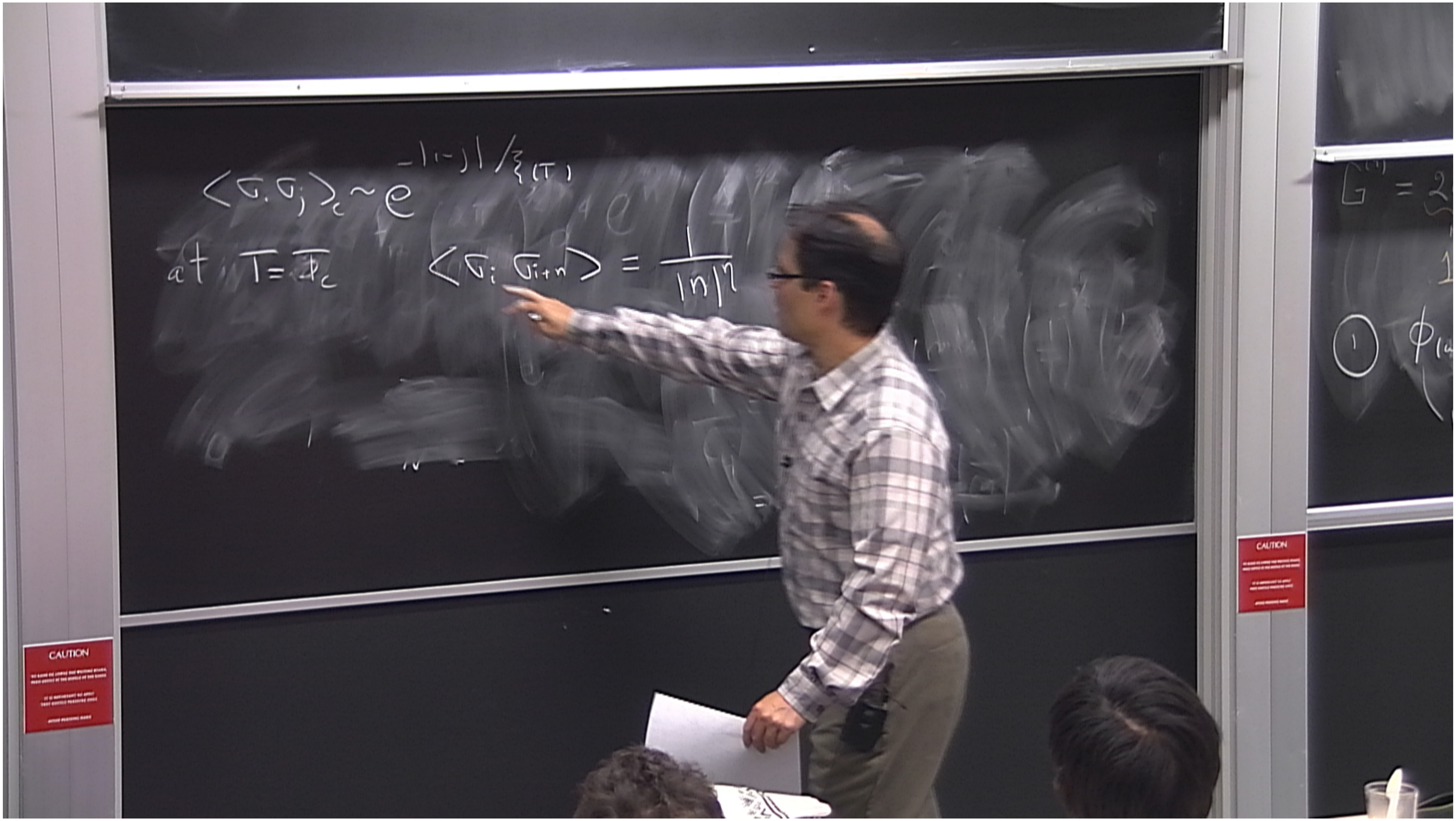
2d Ising Model @ criticality.

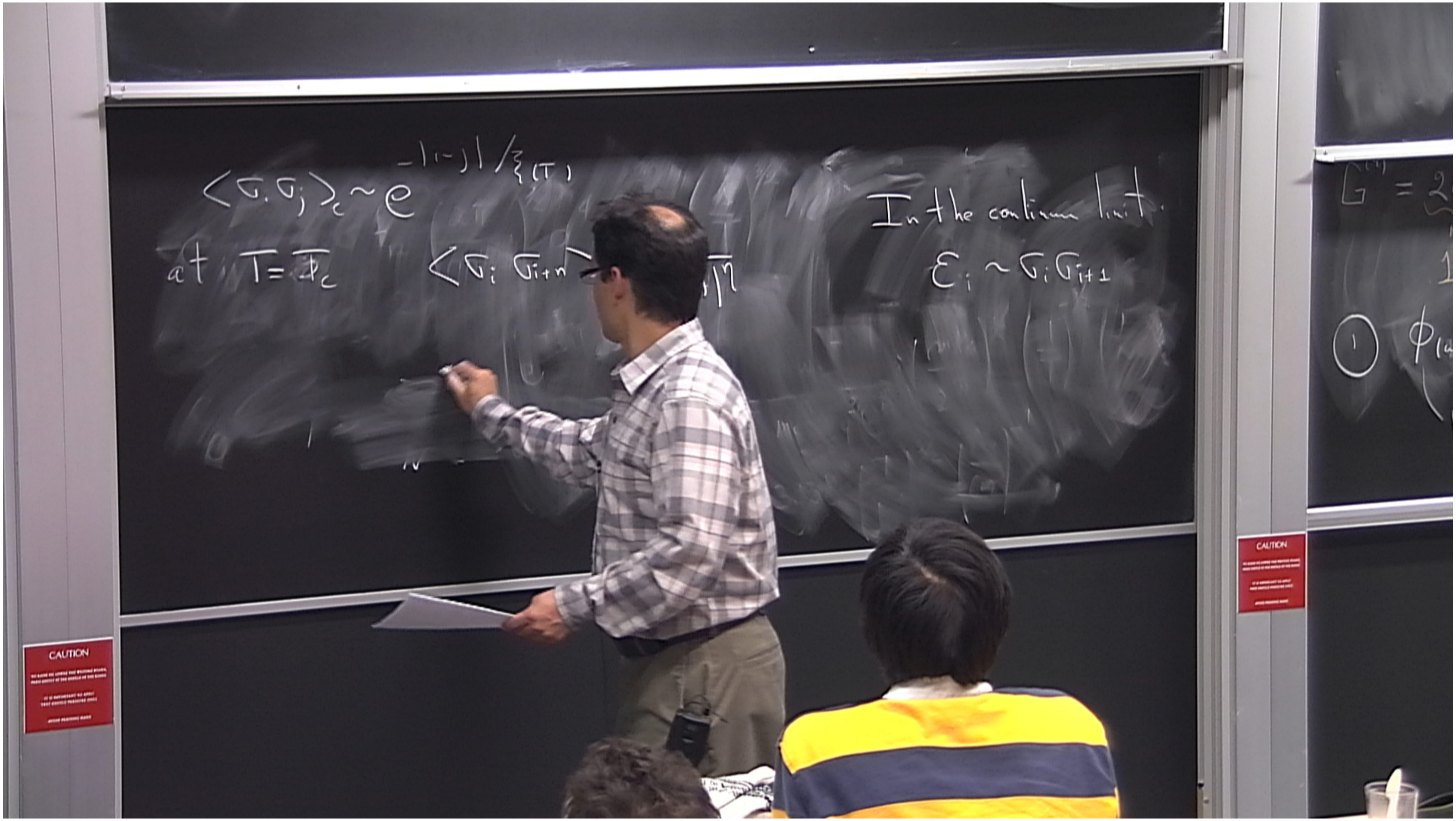
$T = T_c$

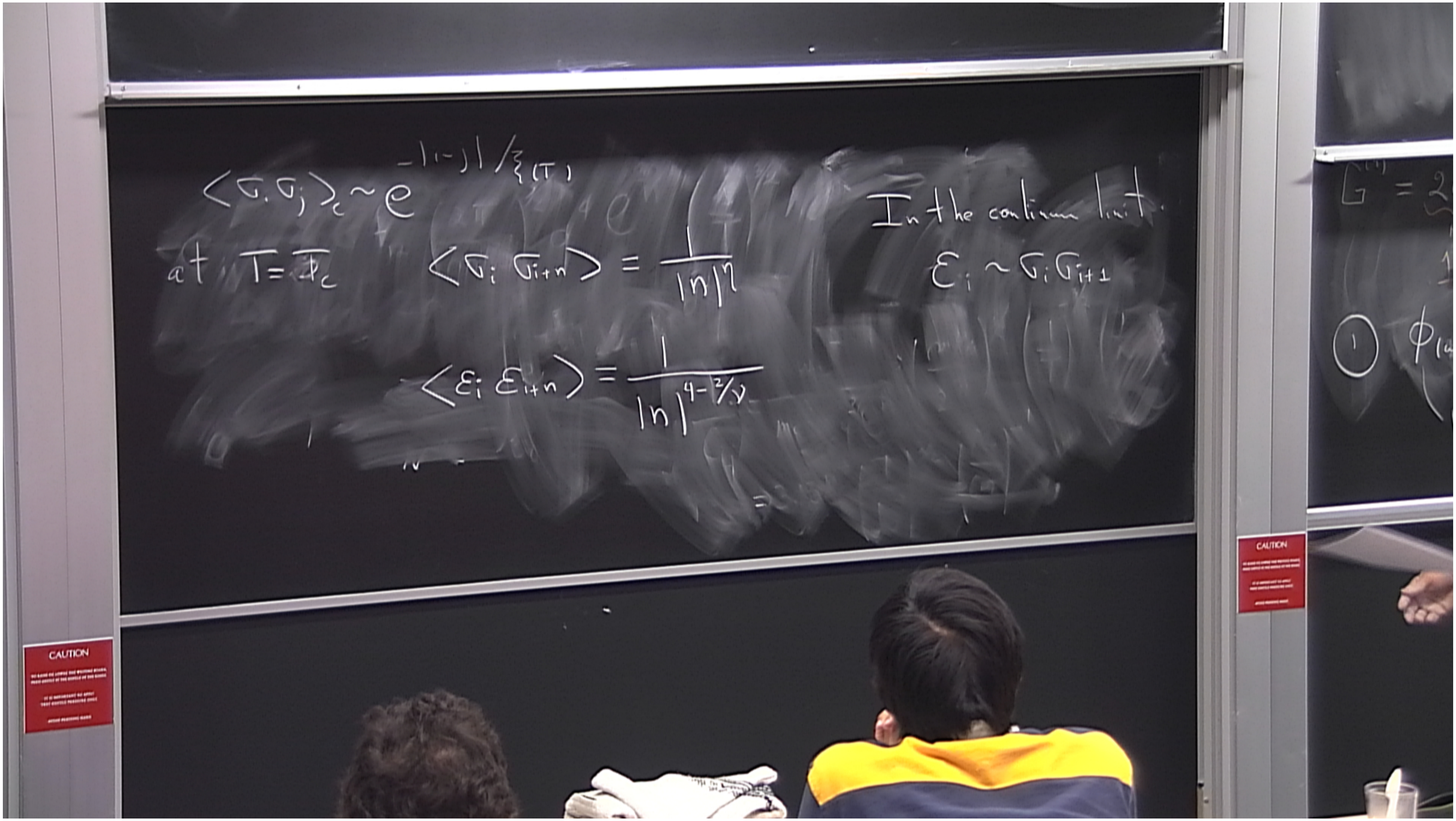
$$Z = \sum_{\sigma} e^{-\beta E(\sigma)} \qquad E(\sigma) = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j - h \sum_i \sigma_i$$

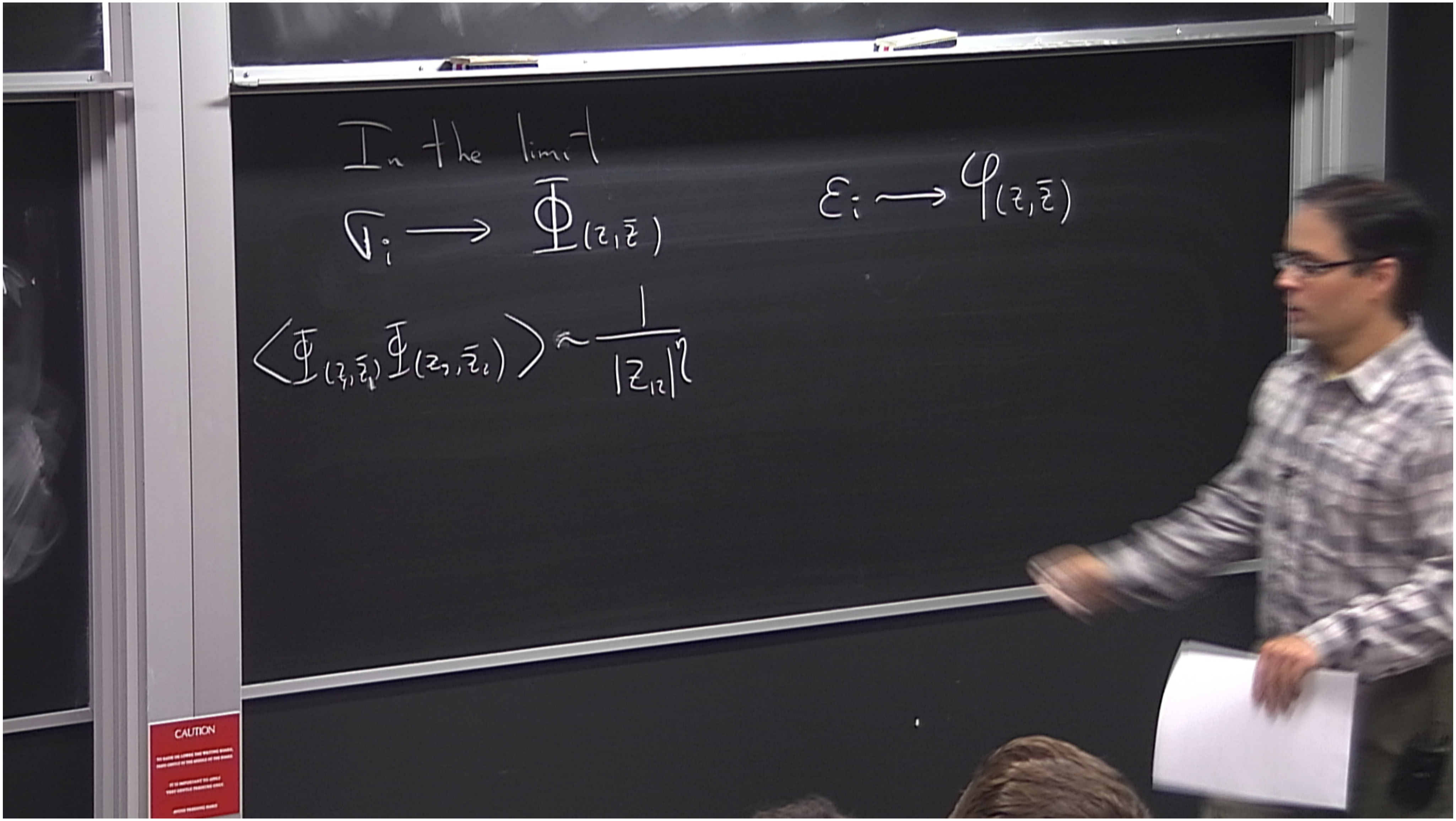
Critical exponents; * near T_c

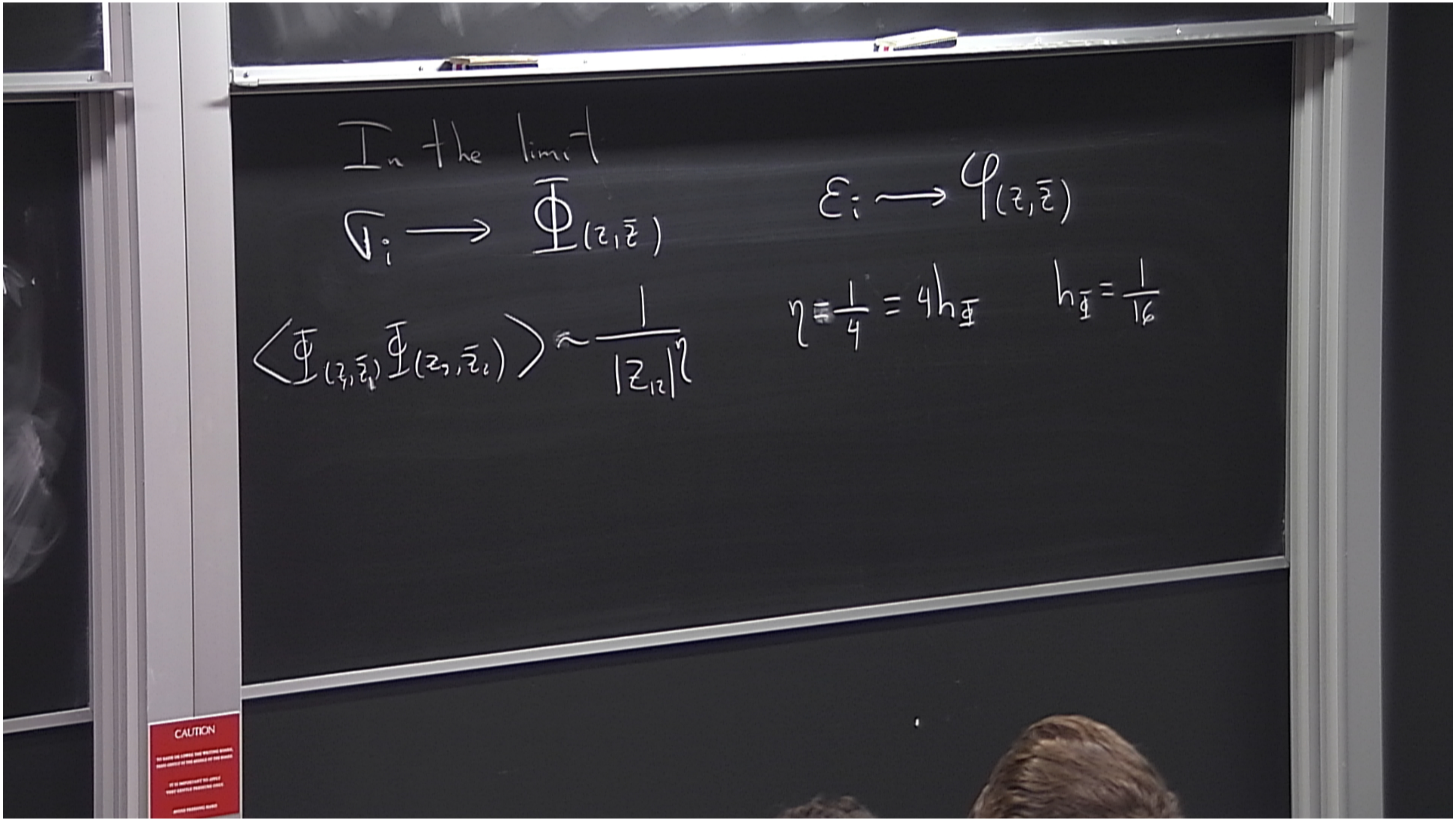
$t = \frac{(T-T_c)}{T_c}$ * at T_c


$\langle \sigma_i \sigma_j \rangle_c \sim e^{-|i-j|/\xi(T)}$
at $T = T_c$ $\langle \sigma_i \sigma_{i+n} \rangle = \frac{1}{|n|^\eta}$
CAUTION
CAUTION


⟨σ_i σ_j⟩_c ~ e^{-|i-j|/ξ(T)}
at T = T_c
In the continuum limit.
ε_i ~ σ_i σ_{i+1}
CAUTION
CAUTION


$\langle \sigma_i \sigma_j \rangle_c \sim e^{-|i-j|/\xi(T)}$
at $T = T_c$
$\langle \sigma_i \sigma_{i+n} \rangle = \frac{1}{|n|^\eta}$
$\langle \varepsilon_i \varepsilon_{i+n} \rangle = \frac{1}{|n|^{4-2/\nu}}$
In the continuum limit:
$\varepsilon_i \sim \sigma_i \sigma_{i+1}$


In the limit
$\sigma_i \longrightarrow \Phi(z,\bar{z})$
$\varepsilon_i \longrightarrow \varphi(z,\bar{z})$
$\langle \Phi(z_1,\bar{z}_1)\Phi(z_2,\bar{z}_2)\rangle \approx \frac{1}{|z_{12}|^{\eta}}$
CAUTION

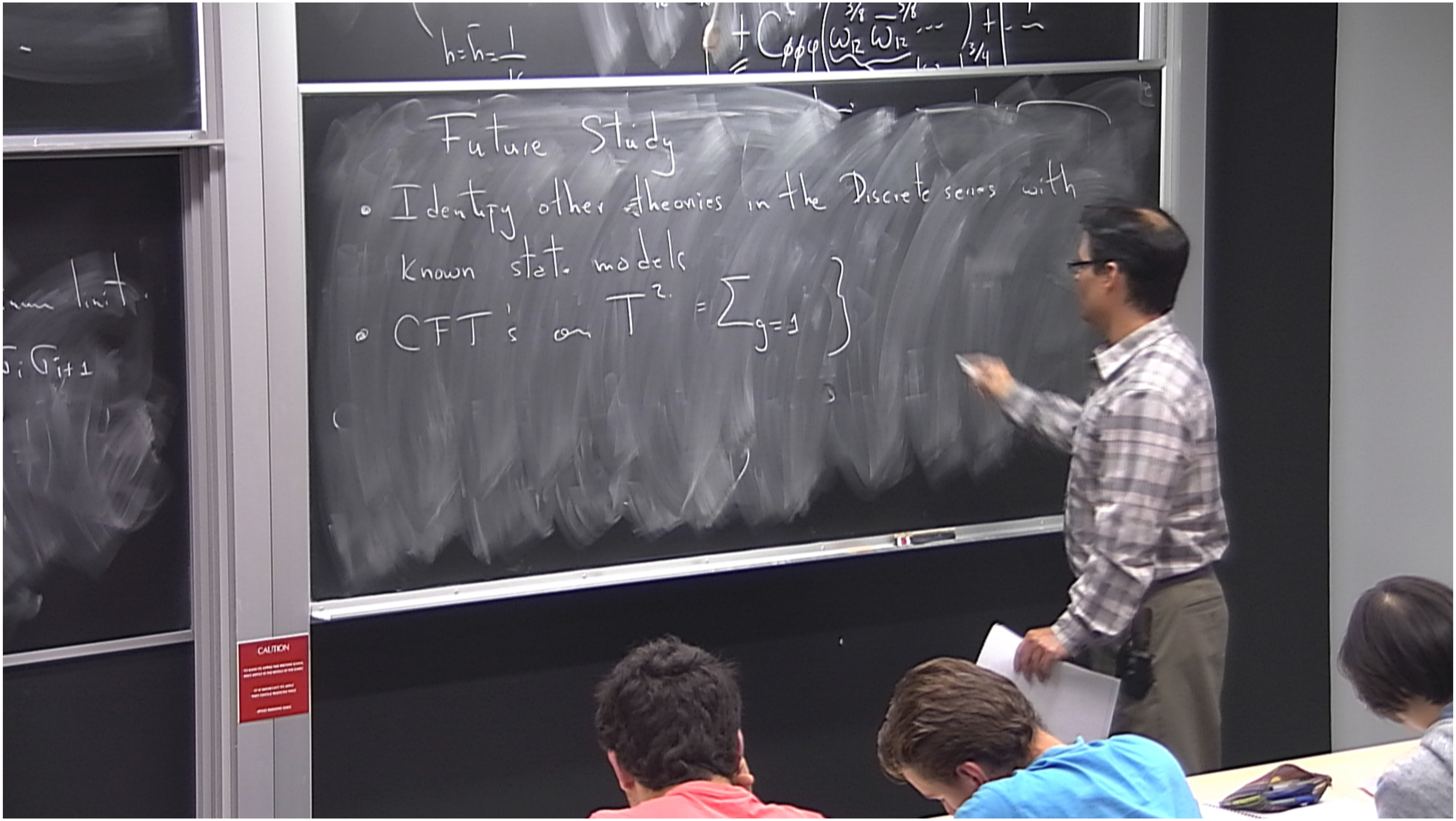

In the limit
$\sigma_i \longrightarrow \Phi(z,\bar{z})$
$\varepsilon_i \longrightarrow \varphi(z,\bar{z})$
$\langle \Phi(z_1,\bar{z}_1)\Phi(z_2,\bar{z}_2)\rangle \sim \frac{1}{|z_{12}|^{\eta}}$
$\eta = \frac{1}{4} = 4h_{\Phi}$
$h_{\Phi} = \frac{1}{16}$
CAUTION

Fusion Rules OPE

$$[\Phi][\Phi] = \mathbb{1} + [\varphi]$$

$$[\Phi][\varphi] = [\Phi]$$

$$[\varphi][\varphi] = \mathbb{1}$$


Future Study
• Identify other theories in the Discrete series with
known stat. models
• CFT's on T2
CAUTION