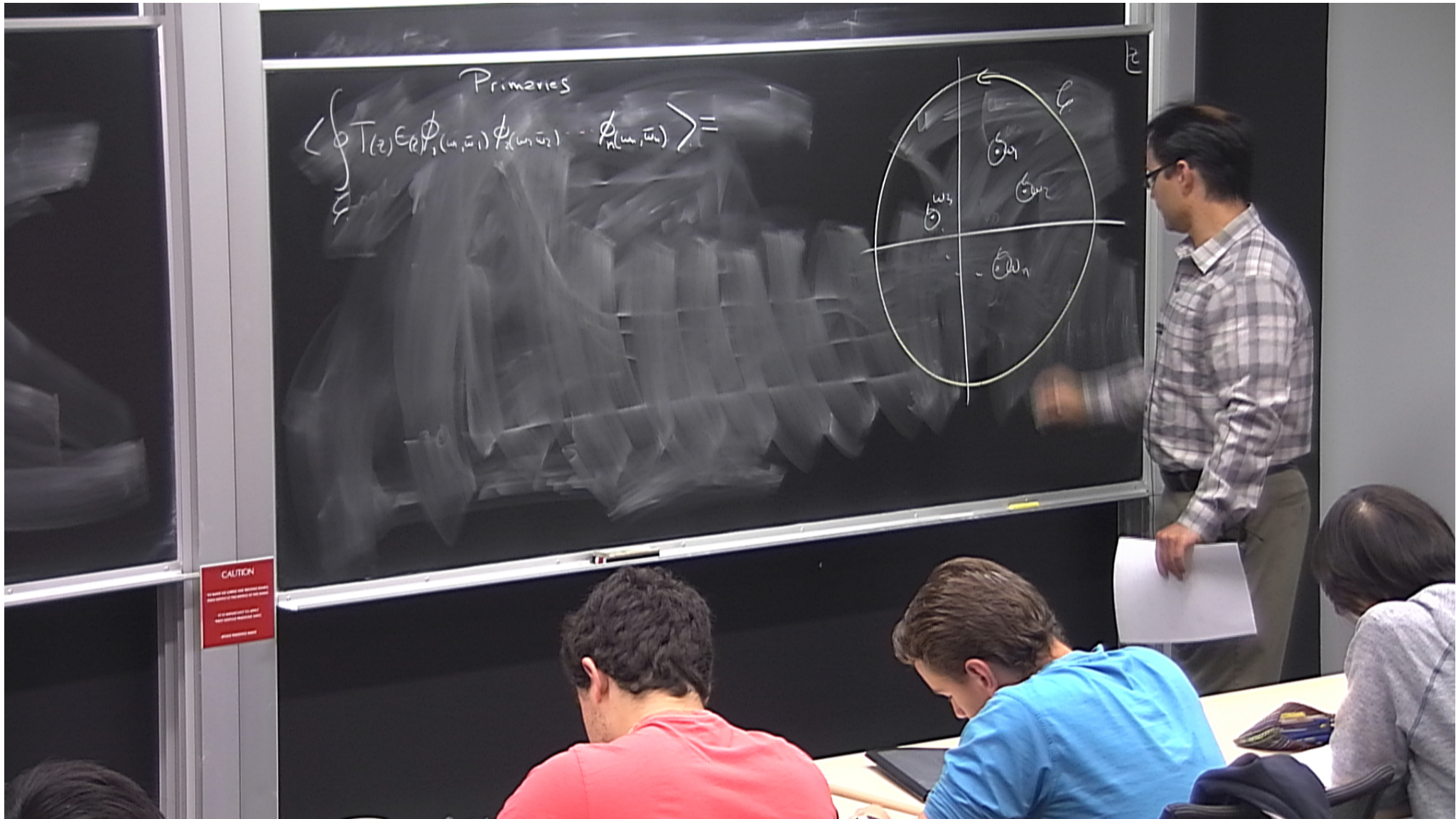


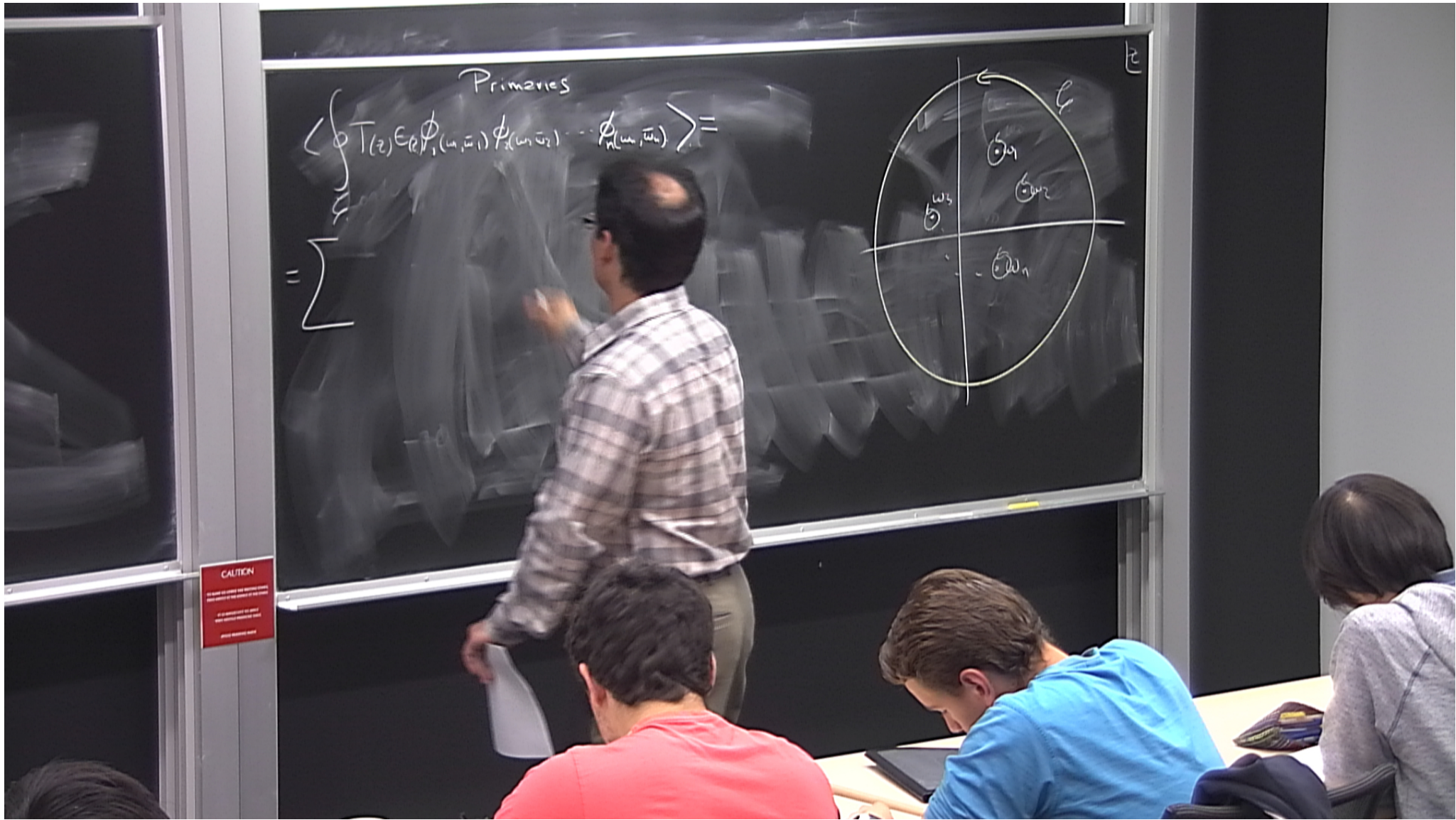
Title: 14/15 PSI - Conformal Field Theory - Lecture 14

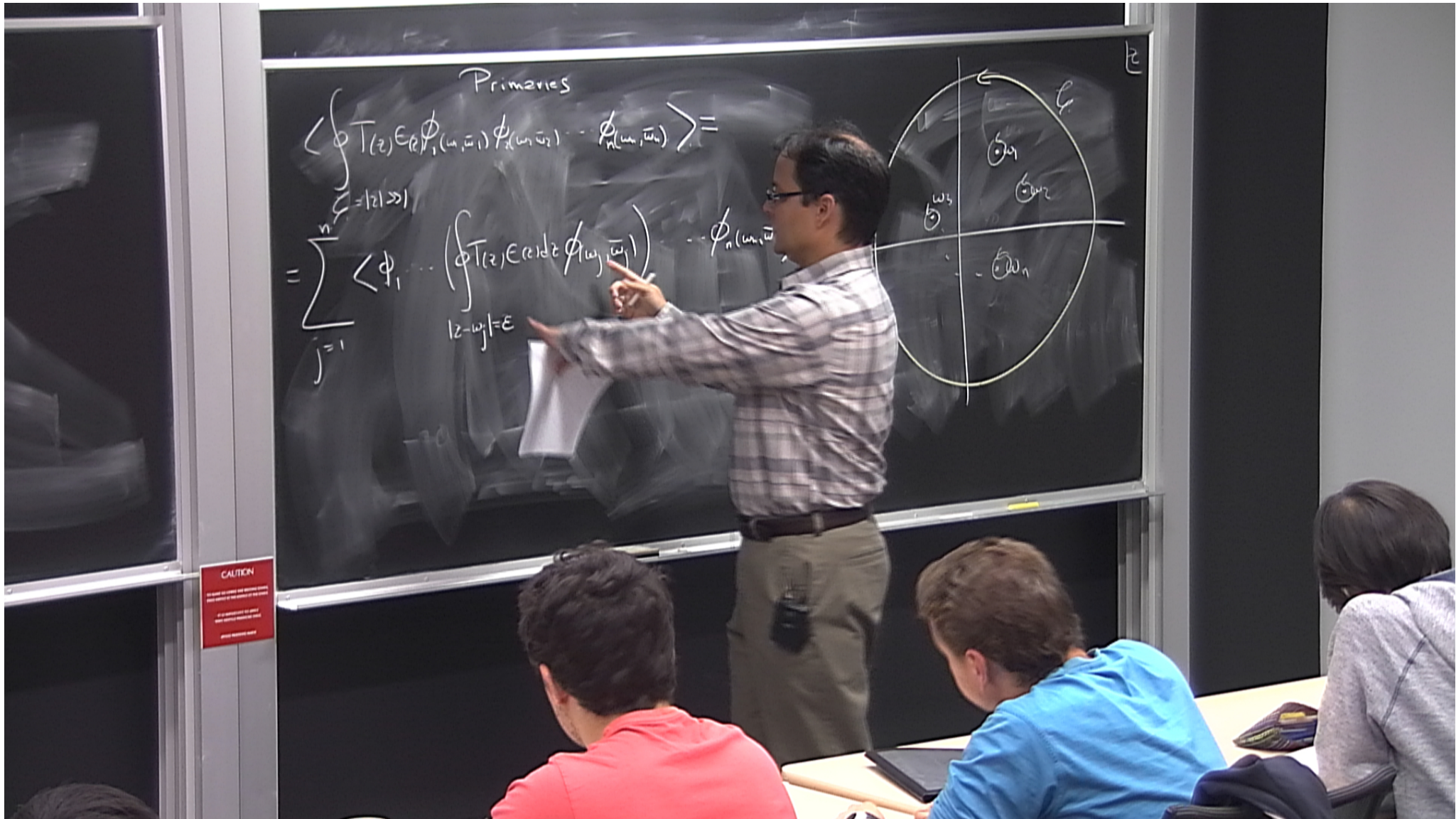
Date: Oct 24, 2014 03:30 PM

URL: <http://pirsa.org/14100026>

Abstract:





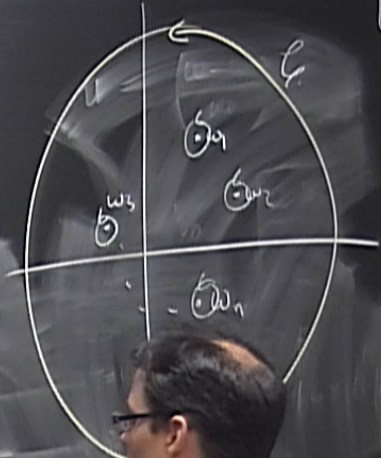


Primaries

$$\left\langle \oint_{\mathcal{C} = |z| \gg 1} T(z) \epsilon(z) \phi_1(\omega_1, \bar{\omega}_1) \phi_2(\omega_2, \bar{\omega}_2) \dots \phi_n(\omega_n, \bar{\omega}_n) \right\rangle =$$

$$= \sum_{j=1}^n \left\langle \phi_j \cdot \left(\oint_{|z-\omega_j|=\epsilon} T(z) \epsilon(z) dz \phi_1(\omega_1, \bar{\omega}_1) \dots \phi_n(\omega_n, \bar{\omega}_n) \right) \right\rangle$$

$$T(z) \phi_j(\omega_j, \bar{\omega}_j) = \left(\frac{h_j}{(z-\omega_j)^2} + \frac{1}{(z-\omega_j)} \frac{\partial}{\partial \omega_j} \right) \phi_j(\omega_j)$$



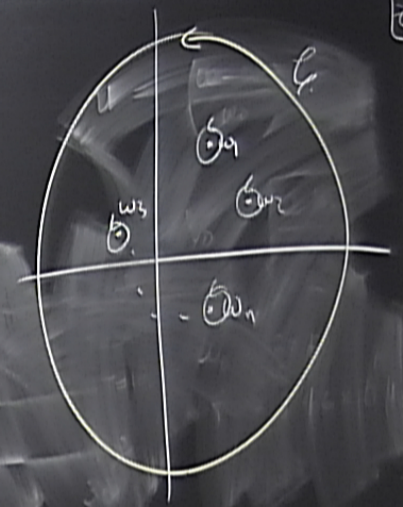
CAUTION
 ALL POINTS ON CORDON AND SERVICE POINTS
 MUST BE IN THE AREA OF THE BOARD

Primaries

$$\langle \phi | T(z) \epsilon(z) \phi_1(\omega_1, \bar{\omega}_1) \phi_2(\omega_2, \bar{\omega}_2) \dots \phi_n(\omega_n, \bar{\omega}_n) \rangle =$$

$$= \sum_{j=1}^n \langle \phi_1 \dots \left(\oint_{|z-\omega_j|=\epsilon} T(z) \epsilon(z) dz \phi_j(\omega_j, \bar{\omega}_j) \right) \dots \phi_n(\omega_n, \bar{\omega}_n) \rangle$$

$$T(z) \phi_j(\omega_j, \bar{\omega}_j) = \left(\frac{h_j}{(z-\omega_j)^2} + \frac{1}{(z-\omega_j)} \frac{\partial}{\partial \omega_j} \right) \phi_j(\omega_j, \bar{\omega}_j)$$

$$= \sum_{j=1}^n \oint dz \epsilon(z) \left(\frac{h_j}{(z-\omega_j)^2} + \frac{1}{(z-\omega_j)} \frac{\partial}{\partial \omega_j} \right) \langle \phi_1 \dots \phi_n \rangle$$


CAUTION
 Do not touch the board
 or the chalkboard eraser
 as they are very hot.

Lecture V

• Back to lecture III

Conformal \mathbb{R}^2 Identities.

$$\langle T(z), \Phi_1(\omega_1, \bar{\omega}_1) \cdots \Phi_n(\omega_n, \bar{\omega}_n) \rangle$$

$$\left(\prod_{j=1}^n \int_{\mathbb{R}^2} \right)$$

Lecture V

• Back to lecture III

Conformal World Identities

$$\langle T(z) \Phi_1(\omega_1, \bar{\omega}_1) \dots \Phi_n(\omega_n, \bar{\omega}_n) \rangle \frac{h}{(z - \omega_i)}$$

Conformal Ward Identities

$$\langle T_{\mu\nu}(x) \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle = \sum_{i=1}^n \left(\frac{h_{\mu\nu}(x_i)}{x_{i1}^2} + \dots \right) \langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle$$

Describe

• Back to lecture III

Conformal Ward Identities

$$\langle T(z) \Phi_1(\omega_1, \bar{\omega}_1) \cdots \Phi_n(\omega_n, \bar{\omega}_n) \rangle = \sum_{j=1}^n \left(\frac{h_j}{(z-\omega_j)^2} + \frac{1}{(z-\omega_j)} \frac{\partial}{\partial \omega_j} \right) \langle \Phi_1(\omega_1, \bar{\omega}_1) \cdots \Phi_n(\omega_n, \bar{\omega}_n) \rangle$$

Descendants :

$$T(z) \Phi_n(\omega_n, \bar{\omega}_n)$$

↙ Prim

↘ $(z-\omega_n)$

Back to lecture III

Conformal Ward Identities

$$\langle T(z) \Phi_1(\omega_1, \bar{\omega}_1) \dots \Phi_n(\omega_n, \bar{\omega}_n) \rangle = \sum_{j=1}^n \left(\frac{h_j}{(z-\omega_j)^2} + \frac{1}{(z-\omega_j)} \frac{\partial}{\partial \omega_j} \right) \langle \Phi_1(\omega_1, \bar{\omega}_1) \dots \Phi_n(\omega_n, \bar{\omega}_n) \rangle$$

Descendants :

$$T(z) \Phi_n(\omega_n, \bar{\omega}_n) = \sum_{m=0}^{\infty} (z-\omega_n)^{m-2} L_m \Phi_n(\omega_n, \bar{\omega}_n)$$

Primary

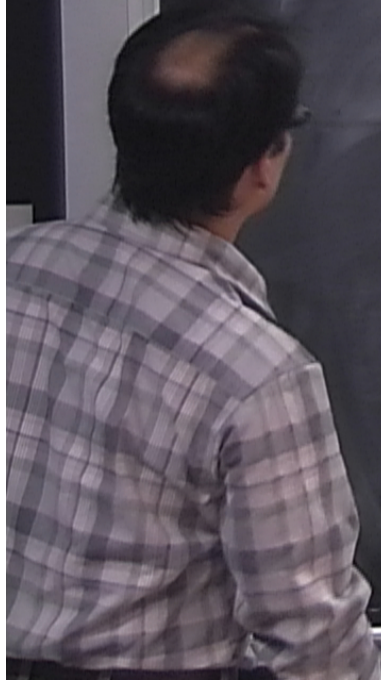
... elements

$$T(z) \Phi_n(\omega_n, \bar{\omega}_n) = \sum_{m < 0} (z - \omega_n)^{-m-2} L_{-m} \Phi_n(\omega_n, \bar{\omega}_n)$$

↙ Primary

(2)

Combine (1) in the limit when $z \rightarrow \omega_n$ & (2)



CAUTION
Do not touch the board when it is hot.
It is dangerous to touch the board when it is hot.
Please do not touch the board when it is hot.

Residues

$$T(z) \Phi_n(\omega_n, \bar{\omega}_n) = \sum_{m < 0} (z - \omega_n)^{-m-2} L_{-m} \Phi_n(\omega_n, \bar{\omega}_n)$$

\downarrow Primary

Combine (1) in the limit when $z \rightarrow \omega_n$ & (2)

LHS $\langle \Phi_1(\omega_1, \bar{\omega}_1) \dots \Phi_n(\omega_n, \bar{\omega}_n) \rangle$



CAUTION
 ALL GASES ARE UNDER HIGH PRESSURE AND SHOULD BE HANDLED WITH CARE.
 ALL OPERATIONS SHOULD BE CONDUCTED IN ACCORDANCE WITH THE SAFETY MANUAL.
 REPORT ANY ACCIDENTS IMMEDIATELY.

$T(z) \Phi_n(\omega_n, \bar{\omega}_n) = \sum_{m=0}^{\infty} (z - \omega_n)^{m-2} \langle \Phi_{n-1} | \hat{\Phi}_n(\omega_n, \bar{\omega}_n) \rangle$

Primary

Combine (1) in the limit when $z \rightarrow \omega_n$ & (2)

LHS $\langle \Phi_1(\omega_1, \bar{\omega}_1) | \dots | \Phi_{n-1}(\omega_{n-1}, \bar{\omega}_{n-1}) | T(z) \Phi_n(\omega_n, \bar{\omega}_n) \rangle = \sum_{m=0}^{\infty} (z - \omega_n)^{m-2} \langle \Phi_1 | \dots | \Phi_{n-1} | \hat{\Phi}_n \rangle$

CAUTION
 ALL GASES ARE UNDER HIGH PRESSURE AND EXTREMELY TOXIC.
 NEVER SMELL OR TASTE THE GASES OR THE LIQUIDS.
 AT ALL TIMES WEAR YOUR SAFETY GOGGLES.
 NEVER WORK ALONE.

$T(z) \Phi_n(\omega_n, \bar{\omega}_n) = \sum_{m=0}^{\infty} (z - \omega_n)^{m-2} \hat{L}_{-m} \Phi_n(\omega_n, \bar{\omega}_n)$

Primary

(2)

Combine (1) in the limit when $z \rightarrow \omega_n$ & (2)

LHS $\langle \Phi_1(\omega_1, \bar{\omega}_1) \dots \Phi_{n-1}(\omega_{n-1}, \bar{\omega}_{n-1}) T(z) \Phi_n(\omega_n, \bar{\omega}_n) \rangle = \sum_{m=0}^{\infty} (z - \omega_n)^{m-2} \langle \Phi_1 \dots \Phi_{n-1} \hat{L}_{-m} \Phi_n \rangle$

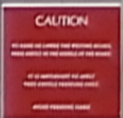
CAUTION
 ALL GAZES ARE LAMPS AND SHOULD BE USED WITH CARE
 ALL IN APPOINTMENT TO AVOID
 FROM BEING HARMED BY THE LIGHT

$\left. \begin{matrix} n < 0 \\ \text{Primary} \end{matrix} \right\} m < 0$
(2)

Combine (1) in the limit when $z \rightarrow \omega_n$ & (2)

$$\langle \Phi_1(\omega_1, \bar{\omega}_1) \dots \Phi_{n-1}(\omega_{n-1}, \bar{\omega}_{n-1}) \underbrace{T(z) \Phi_n(\omega_n, \bar{\omega}_n)}_{\text{Primary}} \rangle = \sum_{m=0}^{\infty} (z - \omega_n)^{m-2} \langle \Phi_1 \dots \Phi_{n-1} \hat{\mathcal{L}}_{-m} \Phi_n \rangle$$

Expand



$\left. \begin{matrix} n < 0 \\ m < 0 \end{matrix} \right\} \text{Primary}$

Combine in the limit when $z \rightarrow \omega_n$ & ②

LHS $\langle \Phi_{n+1}(\bar{\omega}_n, \bar{\omega}_n) T(z) \Phi_n(\bar{\omega}_n, \bar{\omega}_n) \rangle = \sum_{m=0}^{\infty} (z - \omega_n)^{m-2} \langle \Phi_{n-1} \dots \Phi_n \hat{\int}_{-m} \Phi_n \rangle$
 RHS $E \dots z \rightarrow \omega_n \dots m \geq 2$

CAUTION

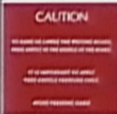
$\left. \begin{matrix} n < 0 \\ m < 0 \end{matrix} \right\} \rightarrow \text{Primary}$

Combine (1) in the limit when $z \rightarrow \omega_n$ & (2)

LHS $\langle \Phi_{n+1}(\bar{\omega}_n, \bar{\omega}_m) T(z) \Phi_n(\bar{\omega}_n, \bar{\omega}_n) \rangle = \sum_{m=0}^{\infty} (z - \omega_n)^{m-2} \langle \Phi_1 \dots \Phi_{n-1} \hat{\Phi}_n \Phi_{n+1} \rangle$

RHS Exp. and $z \rightarrow \omega_n$ $m \geq 2$

$j \neq n$



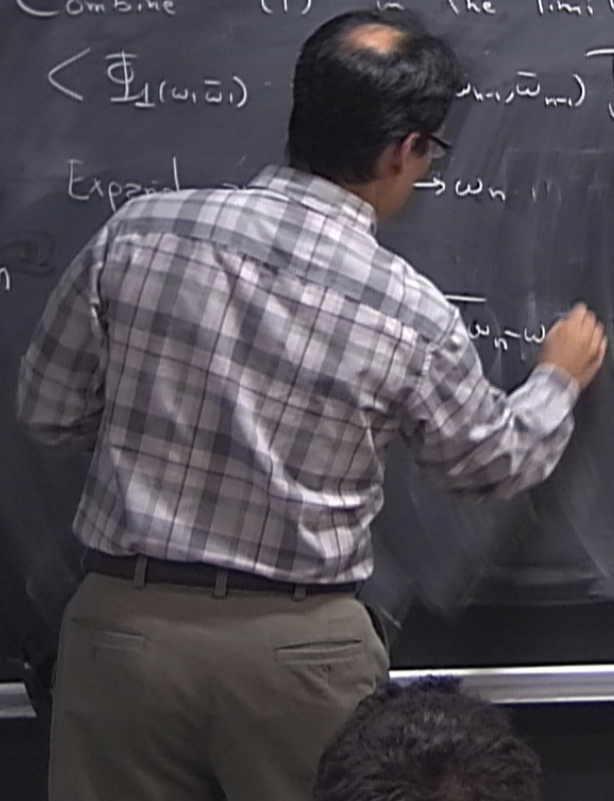
$\left. \begin{matrix} n < 0 \\ m < 0 \end{matrix} \right\} \rightarrow \text{Primary}$

Combine (1) in the limit when $z \rightarrow \omega_n$ & (2)

LHS $\langle \Phi_1(\omega, \bar{\omega}) \dots \Phi_{n-1}(\bar{\omega}_n, \omega_n) T(z) \Phi_n(\bar{\omega}_n, \omega_n) \dots \rangle = \sum_{m=0}^{\infty} (z - \omega_n)^{m-2} \langle \Phi_1 \dots \Phi_{n-1} \hat{\mathcal{L}}_{-m} \Phi_n \rangle$

RHS Expand $\rightarrow \omega_n$ $m \geq 2$

$j \neq n$



CAUTION

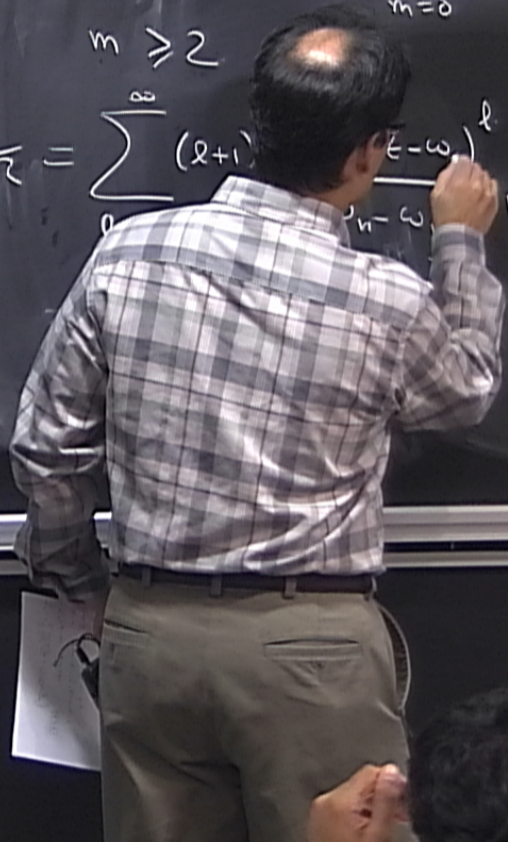
$\underbrace{\dots}_{m < 0}$
 \rightarrow Primary (z)

LHS $\langle \Phi_1(\omega_1, \bar{\omega}_1) \dots \Phi_{n+1}(\omega_{n+1}, \bar{\omega}_{n+1}) T(z) \Phi_n(\bar{\omega}_n, \omega_n) \rangle = \sum_{m=0}^{\infty} (z - \omega_n)^{m-2} \langle \Phi_1 \dots \Phi_{n-1} \int_{-\infty}^{\infty} \Phi_n \rangle$

RHS Expand around $z \rightarrow \omega_n$ $m \geq 2$

$j \neq n$ $\frac{1}{(z - \omega_j)^2} = \frac{1}{((z - \omega_n) + (\omega_n - \omega_j))^2} = \sum_{l=0}^{\infty} (l+1) \frac{(z - \omega_n)^l}{(\omega_n - \omega_j)^{2+l}}$

$\frac{1}{(z - \omega_j)} = \sum$



CAUTION
 ALL POWER AND ENERGY MUST BE OFF AT THE POINT OF THE BOARD
 IF IT IS NECESSARY TO USE THE BOARD
 PLEASE CONTACT THE STAFF

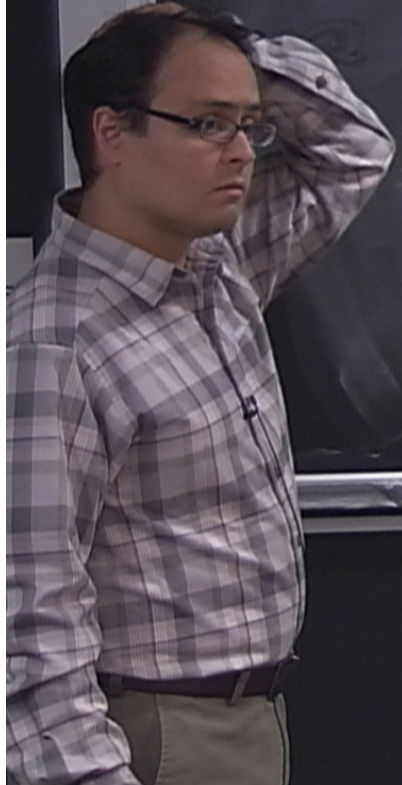
$\underbrace{\quad}_{m < 0}$
 Primary (z)

LHS $\langle \Phi_1(\omega_1, \bar{\omega}_1) \Phi_{n+1}(\omega_{n+1}, \bar{\omega}_{n+1}) T(z) \Phi_n(\bar{\omega}_n, \omega_n) \rangle = \sum_{m=0}^{\infty} (z - \omega_n)^{m-2} \langle \Phi_1 \dots \Phi_{n-1} \int_{-m}^1 \Phi_n \rangle$

RHS Expand around $z \rightarrow \omega_n$ $m \geq 2$

$$\frac{1}{(z - \omega_j)^2} = \frac{1}{\left(\frac{(z - \omega_n) + (\omega_n - \omega_j)}{p} \right)^2} = \sum_{l=0}^{\infty} (l+1) \frac{(-1)^l (z - \omega_n)^l}{(\omega_n - \omega_j)^{l+1}}$$

$$\frac{1}{(z - \omega_j)} = \sum_{l=0}^{\infty} (-1)^l \frac{(z - \omega_n)^l}{(\omega_n - \omega_j)^{l+1}}$$



CAUTION
 ALL POWER CABLES MUST BE PROPERLY SECURED
 BEFORE WORK ON THE FRONT OF THE BOARD.
 IT IS DANGEROUS TO TOUCH
 POWER CABLES WHILE THE BOARD IS ON.
 ALWAYS WEAR YOUR SEATBELT.

$$\langle T(z) \Phi_1(z_1, \bar{z}_1) \dots \Phi_n(z_n, \bar{z}_n) \rangle = \sum_{j=1}^n \left(\frac{h_j}{(z-z_j)^2} + \frac{1}{(z-z_j)} \frac{z}{z_j} \right) \langle \Phi_1(z_1, \bar{z}_1) \dots \Phi_n(z_n, \bar{z}_n) \rangle$$

Descendants :

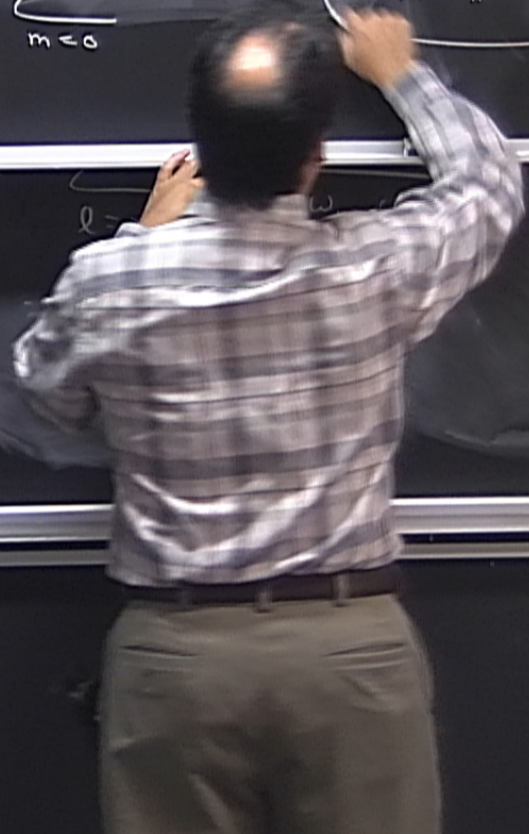
Def. of

$$T(z) \Phi_n(z_n, \bar{z}_n) = \sum_{m=0}^{\infty} (z-z_n)^{m-2} \hat{L}_{-m} \Phi_n(z_n, \bar{z}_n)$$

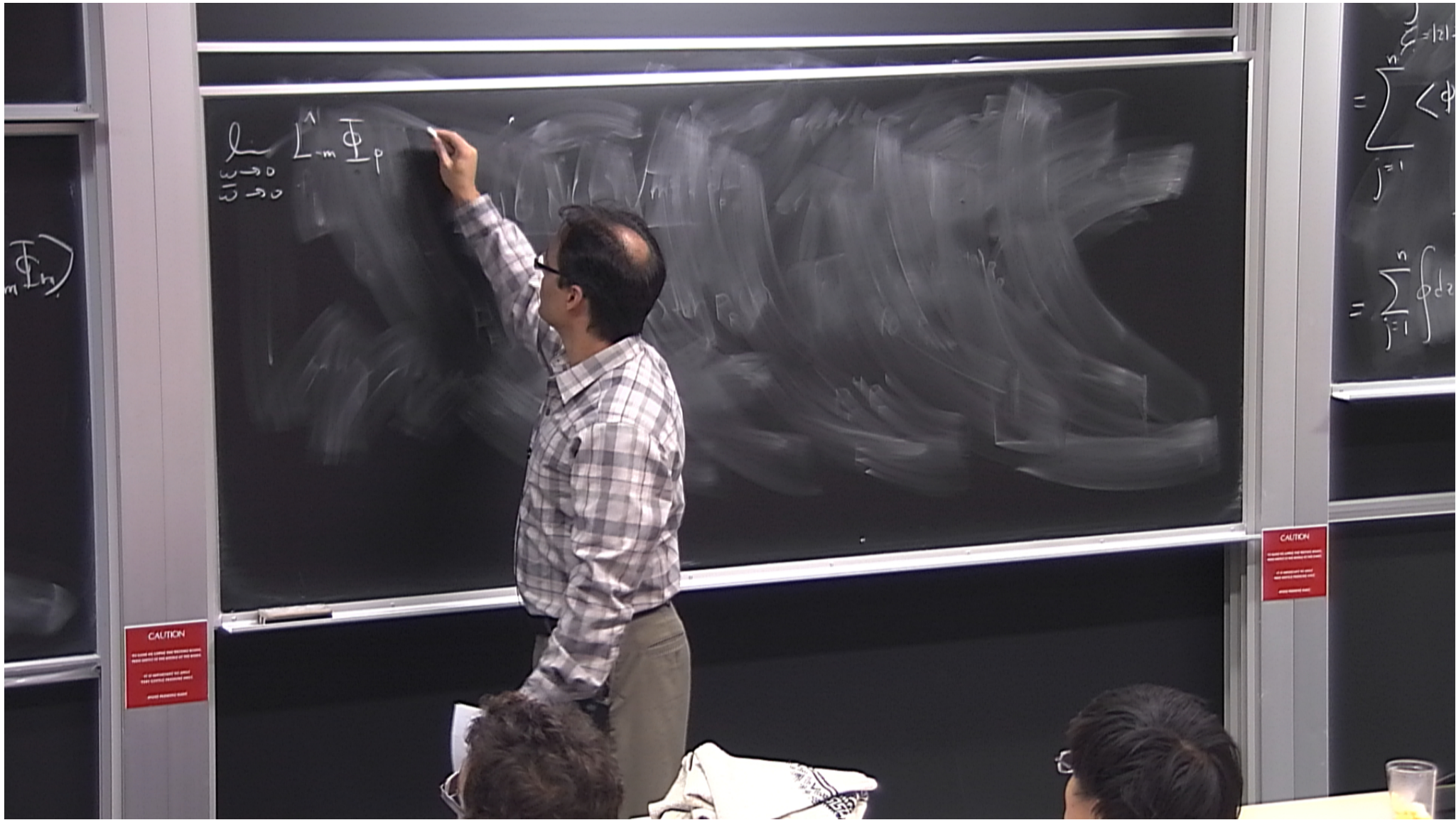
Primary

$$\hat{L}_0 \Phi = \Phi \quad (2)$$

$$\frac{1}{(z-z_j)} = \sum_{l=0}^{\infty} (-1)^l \frac{(z-z_n)^l}{(z_n-z_j)}$$



CAUTION
ALL WORKING SURFACES MUST BE KEPT CLEAR OF ALL OBSTACLES
IF IN CONTACT WITH ANY OF THESE SURFACES, THE USER MUST BE IMMEDIATELY NOTIFIED
PLEASE REPORT ANY DAMAGE TO THE MANUFACTURER



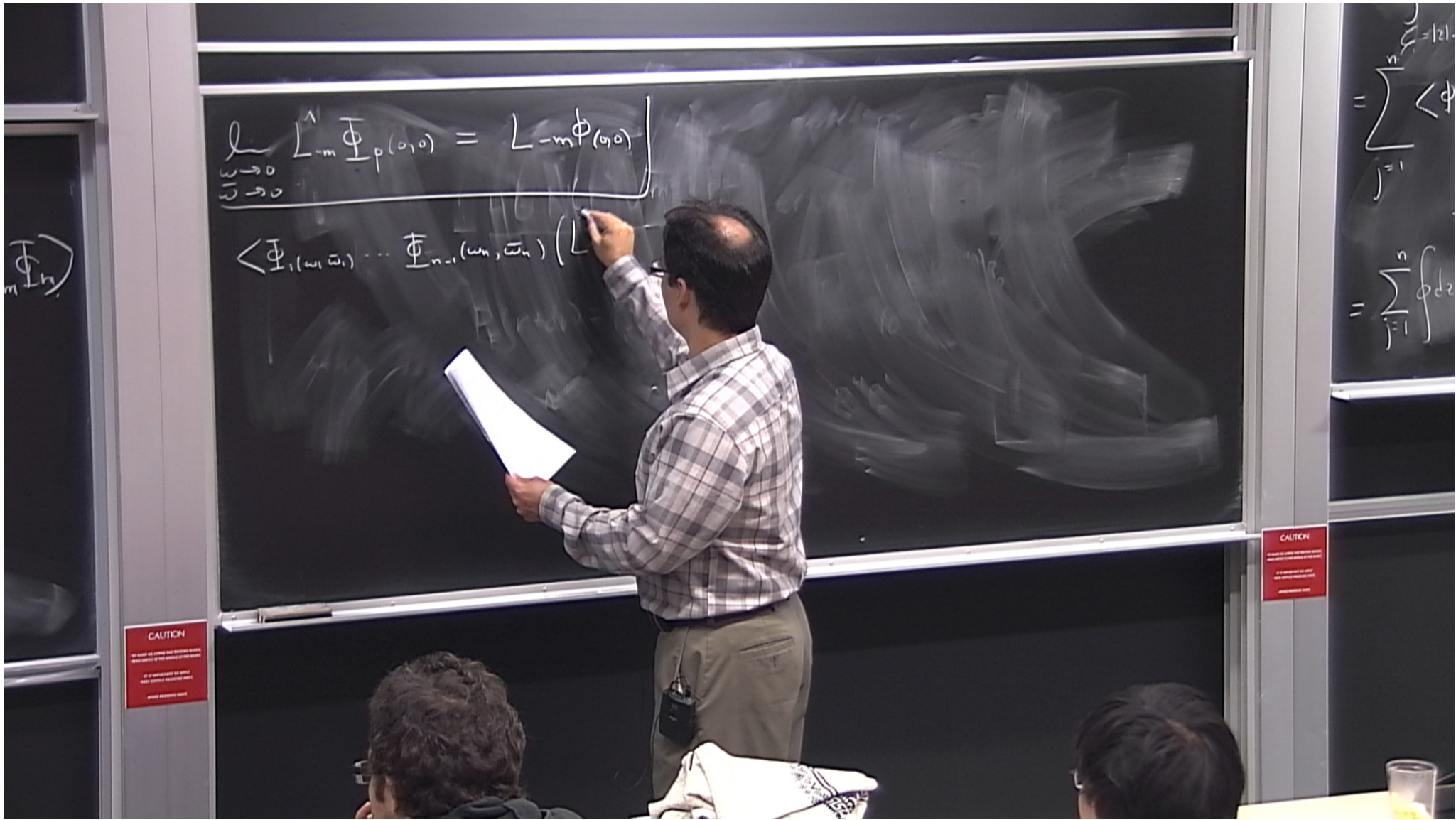
$$\begin{aligned} \ell & \rightarrow 0 \\ \vec{v} & \rightarrow 0 \\ \vec{v} & \rightarrow 0 \end{aligned} \quad L^{-1} \Phi_p$$

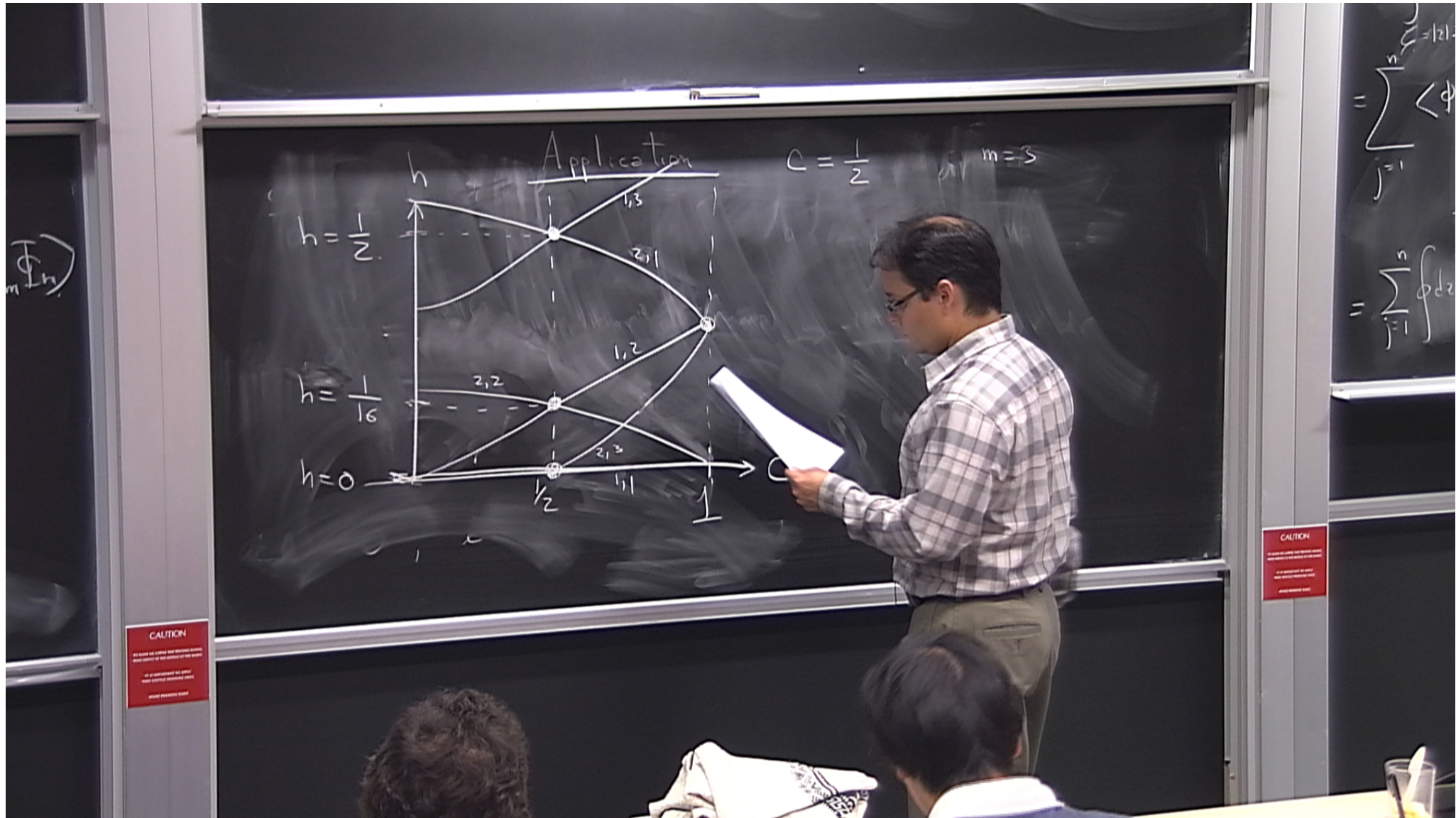
$$= \sum_{j=1}^n \langle \phi_j | \psi \rangle \phi_j$$

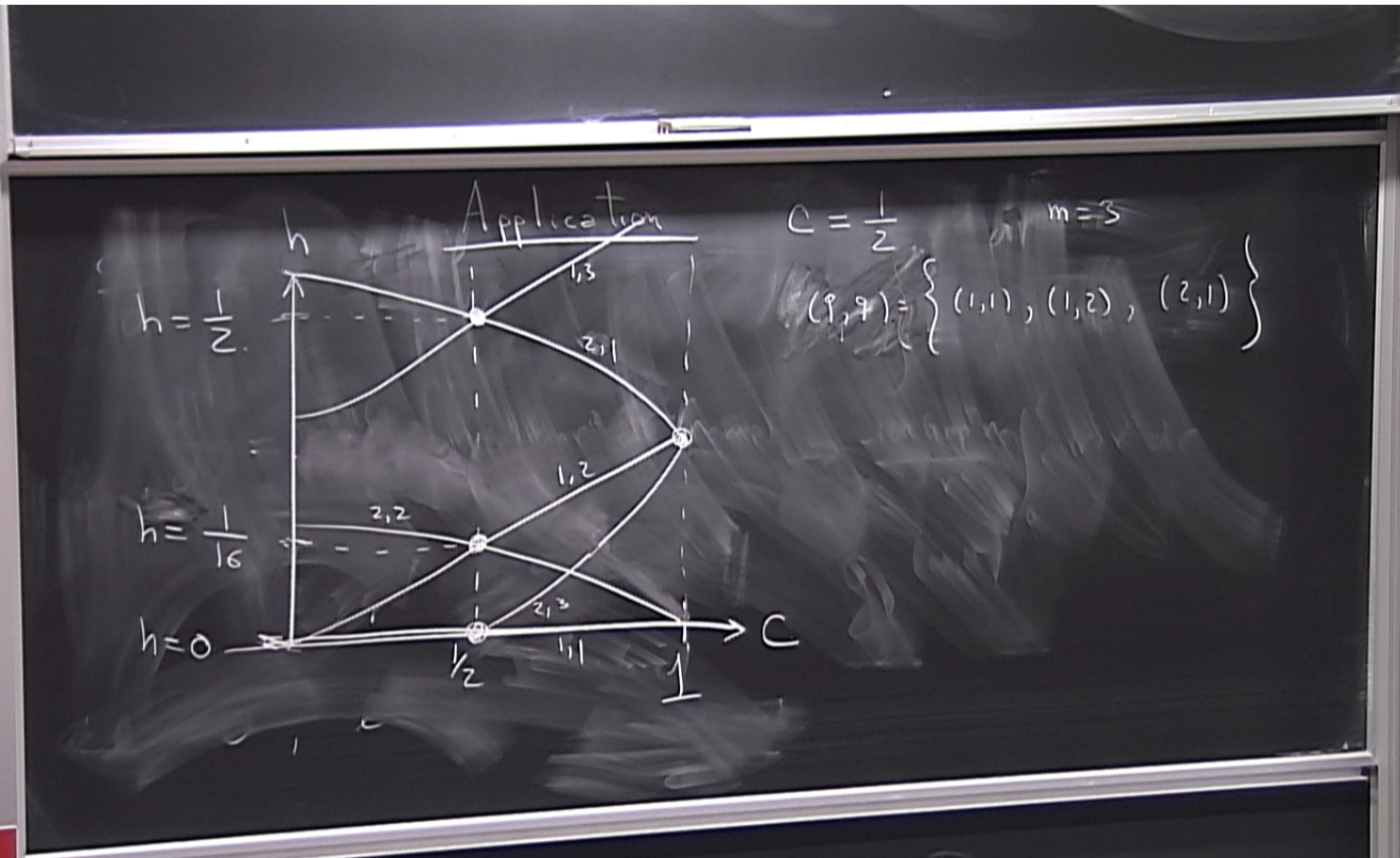
$$= \sum_{j=1}^n \int \phi_j^* \psi dx$$

CAUTION
DO NOT USE CHALK AND ERASERS IN THIS AREA
IF YOU DO, YOU WILL BE FINED
PLEASE REPORT ANY DAMAGE TO THE STAFF

CAUTION
DO NOT USE CHALK AND ERASERS IN THIS AREA
IF YOU DO, YOU WILL BE FINED
PLEASE REPORT ANY DAMAGE TO THE STAFF







$$= \sum_{j=1}^n \langle \phi_j | \phi_j \rangle$$

$$= \sum_{j=1}^n \int \phi_j^2 dx$$

CAUTION
DO NOT USE LIFES AND RESCUE EQUIPMENT
WHILE WORKING IN THE LABORATORY

CAUTION
DO NOT USE LIFES AND RESCUE EQUIPMENT
WHILE WORKING IN THE LABORATORY

Compute a 4-point correlation function

$$\langle \Phi_1(\omega_1, \bar{\omega}_1) \dots \Phi_4(\omega_4, \bar{\omega}_4) \rangle = \left(\frac{\omega_1 \omega_2}{\omega_3 \omega_4} \right)^{1/8} \left(\frac{\omega_1 \omega_2 \omega_3 \omega_4}{\omega_1 \omega_2 \omega_3 \omega_4} \right)^{1/8} \mathbb{F}(x, \bar{x})$$

$$x = \frac{\omega_2 \omega_3}{\omega_1 \omega_4}$$

$$\left. \begin{aligned} & (2,1) \\ & = \frac{1}{16} \\ & \Delta = h - \bar{h} = 0 \\ & \Delta = \frac{1}{8} \end{aligned} \right\}$$

CAUTION
DO NOT TOUCH THE BOARD WHEN IT IS BEING USED BY THE INSTRUCTOR

Compute a 4-point correlation function

$$\langle \Phi_1(\omega_1, \bar{\omega}_1) \dots \Phi_4(\omega_4, \bar{\omega}_4) \rangle = \left(\frac{\omega_1 \omega_2}{\omega_1 \omega_2 \omega_3 \omega_4} \right)^{1/8} \left(\frac{\bar{\omega}_1 \bar{\omega}_2}{\bar{\omega}_1 \bar{\omega}_2 \bar{\omega}_3 \bar{\omega}_4} \right)^{1/8} F(x, \bar{x})$$

$$x = \frac{\omega_2 \omega_3}{\omega_1 \omega_4}$$

Step 1: Null state related to $\Phi_{(2,1)}|0\rangle \equiv |\chi\rangle = (L_{-2} + \alpha L_{-1}^2)|h\rangle = (L_{-2} - \frac{4}{3}L_{-1}^2)|h\rangle$

$$c = \frac{1}{2} \quad \alpha = -\frac{4}{3} \quad \chi$$

$$\left. \begin{aligned} (2,1) \\ \vdots \\ \vdots \end{aligned} \right\}$$

$$= \frac{1}{16}$$

$$\Delta = h - \bar{h} = 0$$

$$\Delta = \frac{1}{8}$$

CAUTION

$$\langle \Phi_1(\omega_1, \bar{\omega}_1) \Phi_2(\omega_2, \bar{\omega}_2) \Phi_3(\omega_3, \bar{\omega}_3) \chi_{(\omega_4, \bar{\omega}_4)} \rangle = 0$$

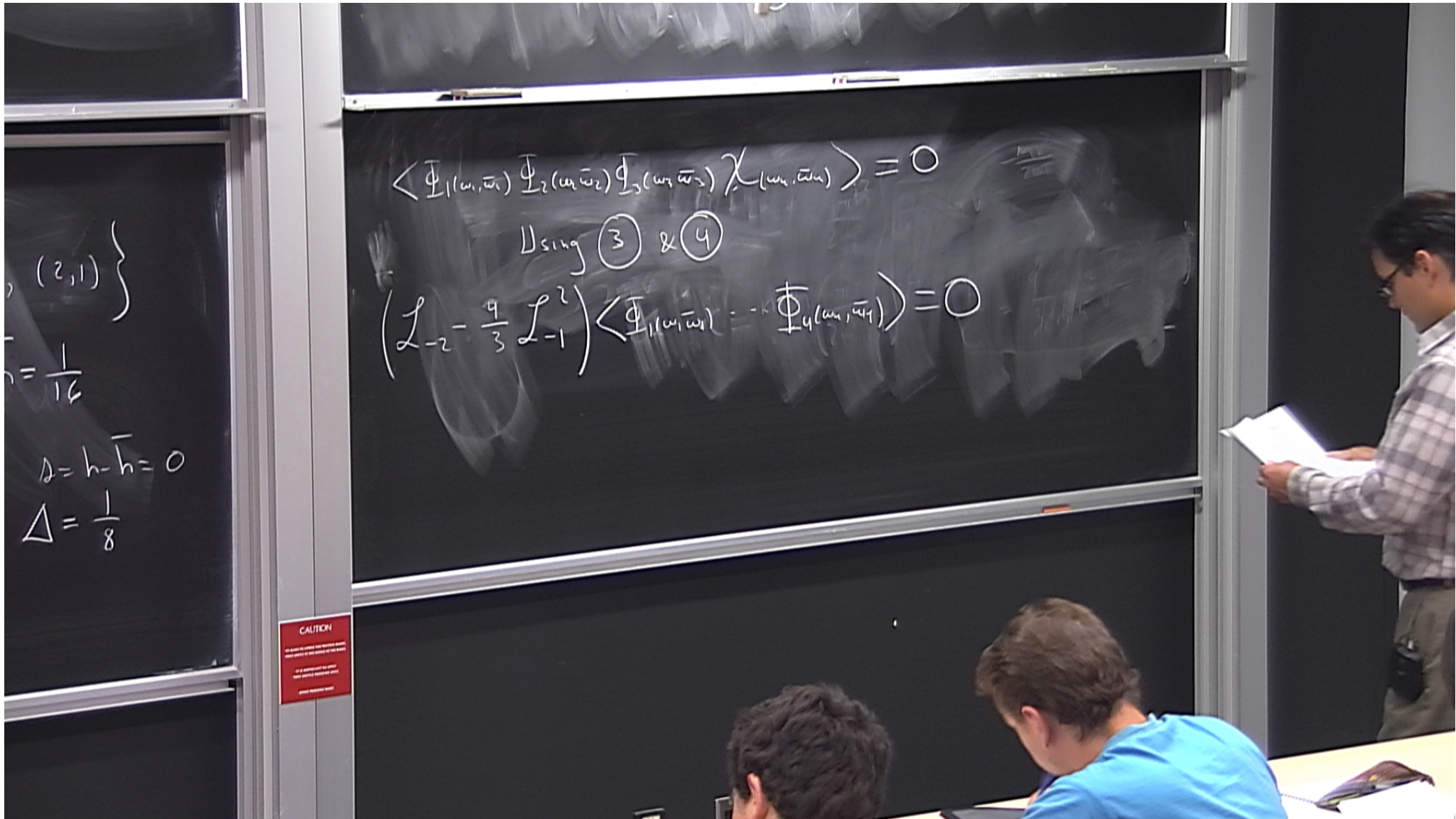
$$(2, 1) \}$$

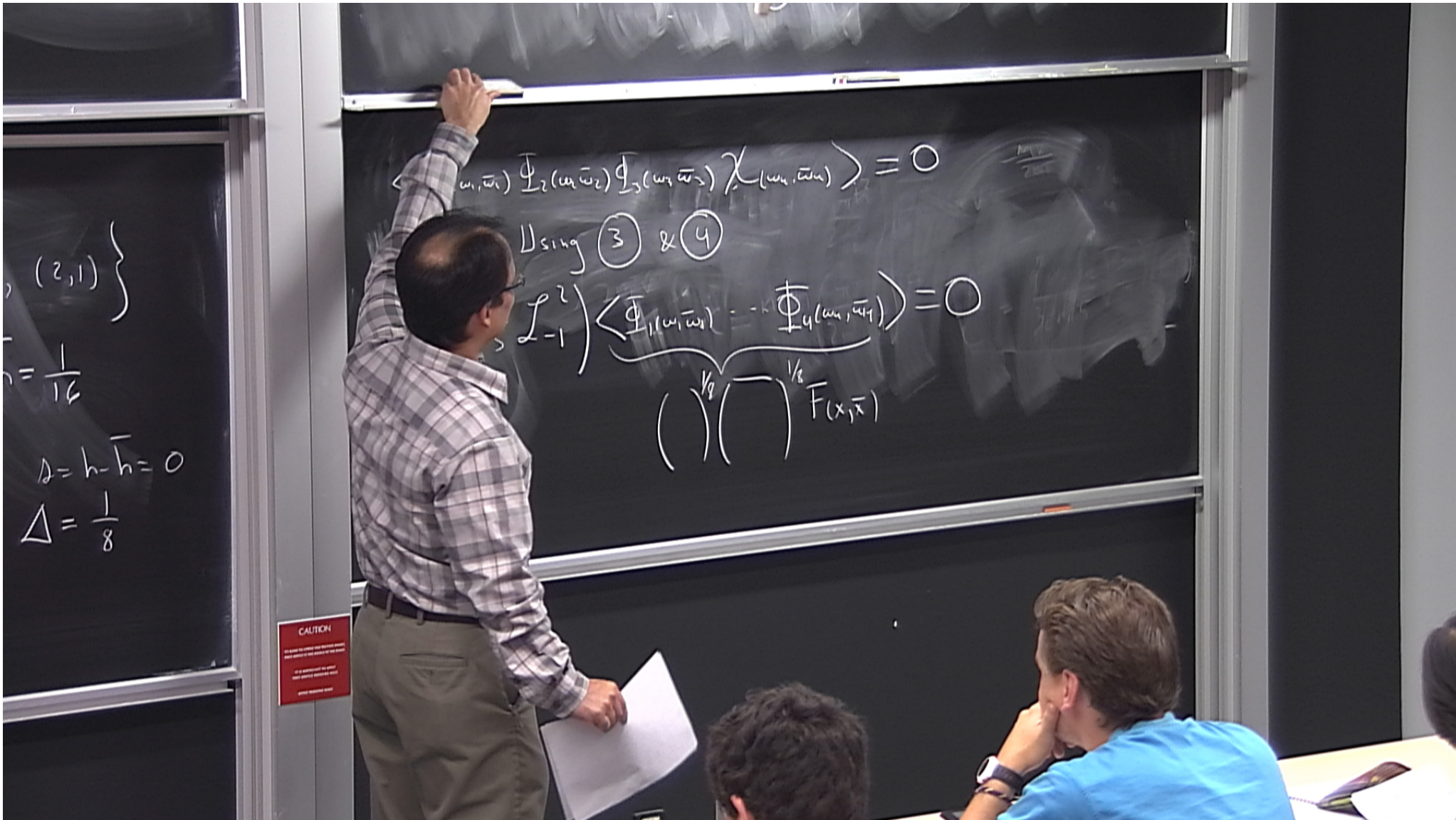
$$= \frac{1}{16}$$

$$\Delta = h - \bar{h} = 0$$

$$\Delta = \frac{1}{8}$$

CAUTION
DO NOT TOUCH THE BOARD WHEN
IT IS BEING USED BY THE LECTURER
OR OTHER STUDENTS

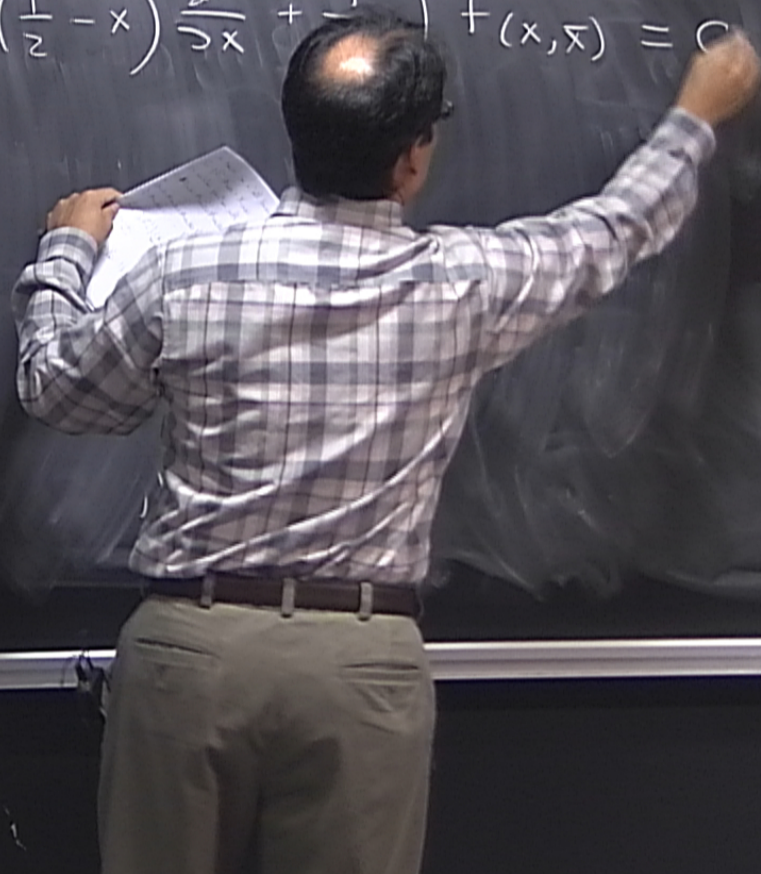




Def. of

Primary $m=0$

$$\Rightarrow \left(x(1-x) \frac{\partial^2}{\partial x^2} + \left(\frac{1}{2} - x \right) \frac{\partial}{\partial x} + \frac{1}{2} \right) F(x, \bar{x}) = 0$$



CAUTION
Do not touch the board when it is hot.
Do not touch the board when it is hot.
Do not touch the board when it is hot.

Def. of

Primary $m=0$

$$\Rightarrow \left(x(1-x) \frac{\partial^2}{\partial x^2} + \left(\frac{1}{2} - x \right) \frac{\partial}{\partial x} + \frac{1}{16} \right) F(x, \bar{x}) = 0$$

Def. of

Primary $m=0$

$$\Rightarrow \left(x(1-x) \frac{d^2}{dx^2} + \left(\frac{1}{2} - x \right) \frac{d}{dx} + \frac{1}{16} \right) F(x, x) = 0$$

2 sols

$$y^{(\pm)}(x) = \sqrt{1 \pm \sqrt{1-x}}$$

Def. of

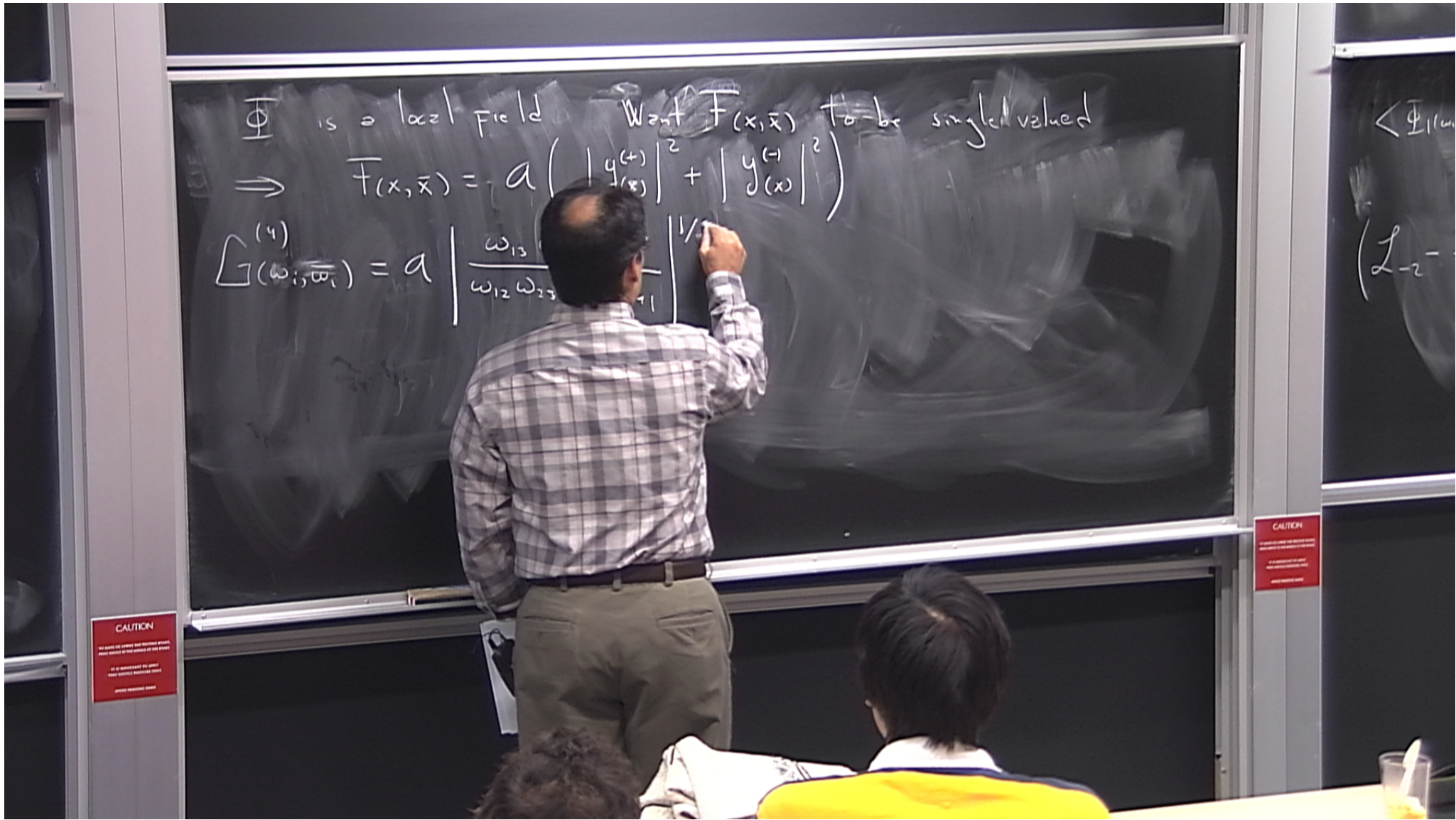
Primary $m=0$

$$\Rightarrow \left(x(1-x) \frac{\partial^2}{\partial x^2} + \left(\frac{1}{2} - x \right) \frac{\partial}{\partial x} + \frac{1}{16} \right) F(x, \bar{x}) = 0$$

2 sols

$$y^{(\pm)}(x) = \sqrt{1 \pm \sqrt{1-x}}$$

$$F(x, \bar{x}) = \sum_{\substack{\epsilon = \pm \\ \bar{\epsilon} = \pm}} a_{\epsilon, \bar{\epsilon}} y^{(\epsilon)}(x) y^{(\bar{\epsilon})}(\bar{x})$$



OPE

Recall : $\Phi_p^{\{k, \bar{k}\}} \equiv \hat{L}_{-k_1} \hat{L}_{-k_2} \dots \hat{L}_{-k_n} \hat{L}_{-\bar{k}_1} \dots \hat{L}_{-\bar{k}_m} \Phi_p$

$$\underbrace{\Phi_n(z, \bar{z})}_{\text{Primaries}} \underbrace{\Phi_m(0, 0)}_{\text{Primary}} = \sum_p \sum_{\{k, \bar{k}\}} C_{nm}^{\{p, \{k, \bar{k}\}, \bar{E}\}} z^{h_p - h_n - h_m + \sum k_i} \bar{z}^{(\dots)} \Phi_p^{\{k, \bar{k}\}}(0, 0)$$

3-pt function Φ

CAUTION

OPE

Recall : $\Phi_p^{\{k, \bar{k}\}} \equiv \hat{L}_{-k_1} \hat{L}_{-k_2} \dots \hat{L}_{-k_n} \hat{L}_{-\bar{k}_1} \dots \hat{L}_{-\bar{k}_m} \Phi_p$ Primary

$$\underbrace{\Phi_n(z, \bar{z})}_{\text{Primaries}} \underbrace{\Phi_m(0, 0)}_{\text{Primary}} = \sum_p \sum_{\{k, \bar{k}\}} C_{nm}^{\{k, \bar{k}\}} z^{h_p - h_n - h_m + \sum k_i} \bar{z}^{(\dots)} \Phi_p^{\{k, \bar{k}\}}(0, 0)$$

3-pt Function $\left(\Phi_p^{\{k, \bar{k}\}}, \Phi_n(z, \bar{z}), \Phi_m(0, 0) \right)$

3-pt Functions $\langle \Phi_p \Phi_n(z, \bar{z}) \Phi_m(0,0) \rangle = \mathcal{O}(\langle \Phi_p \Phi_n \Phi_m \rangle)$

Primaries $\{r, \bar{r}\}$ L_1

$\langle \Phi_p \Phi_m \Phi_n \rangle \sim \frac{C_{pmm}}{mm}$



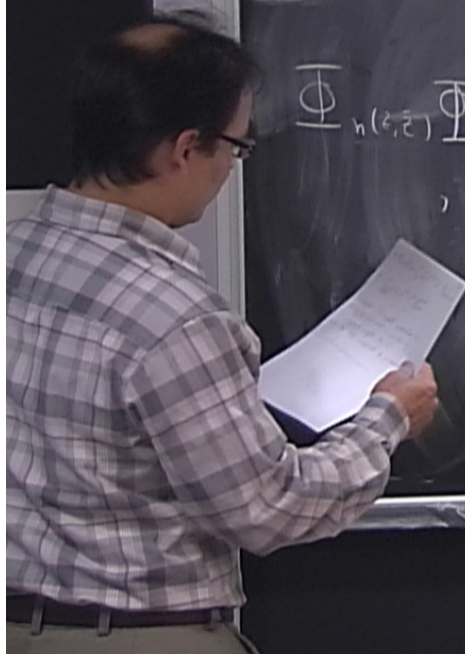
CAUTION
 Do not touch the chalkboard
 when it is hot or when it is
 being cleaned.

Primaries L_1
 3-pt Functions $\langle \Phi_p^{(s, \bar{s})} \Phi_n(z, \bar{z}) \Phi_m(0, 0) \rangle = \mathcal{O} \langle \Phi_p \Phi_n \Phi_m \rangle$

$$\langle \Phi_p \Phi_m \Phi_n \rangle \sim \frac{C_{nm}^p}{C_{nm}^p} \quad C_{nm}^{(s, \bar{s})} = C_{nm}^p \beta_{nm}^{(s, \bar{s})} \beta_{nm}^{(s, \bar{s})}$$

$$\Phi_n(z, \bar{z}) \Phi_m(0, 0) = \sum_p C_{nm}^p z^{h_p - h_n - h_m} \bar{z}^{\dots} \Psi_p(z, \bar{z} | 0, 0)$$

Function of $C, \bar{C}, h's, \bar{h}'s$



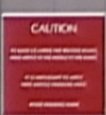
Primaries

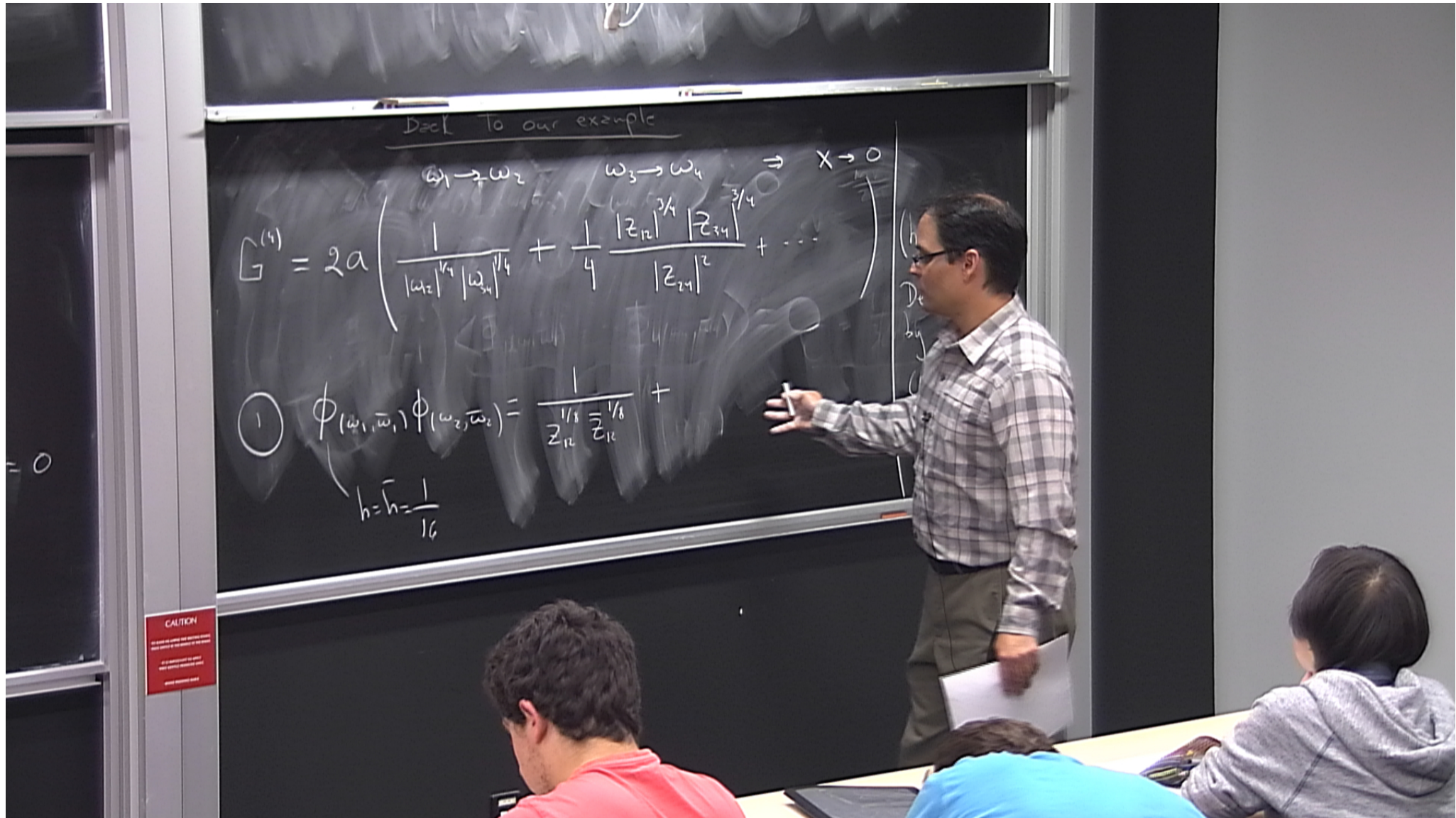
$$\langle \Phi_p(z, \bar{z}) \Phi_n(z, \bar{z}) \Phi_m(0, 0) \rangle = \mathcal{Q} \langle \bar{\Phi}_p \bar{\Phi}_n \bar{\Phi}_m \rangle$$

$$\bar{\Phi}_n(z, \bar{z}) \bar{\Phi}_m(0, 0) = \sum_p C_{nm}^p z^{h_p - h_n - h_m} \bar{z}^{h_p - h_n - h_m} \Psi$$

Function of $C, \bar{C}, h's, \bar{h}'s$

Conclusion: In a 2d CFT all w are the conformal weights of primaries in order to define it.





Back to our example

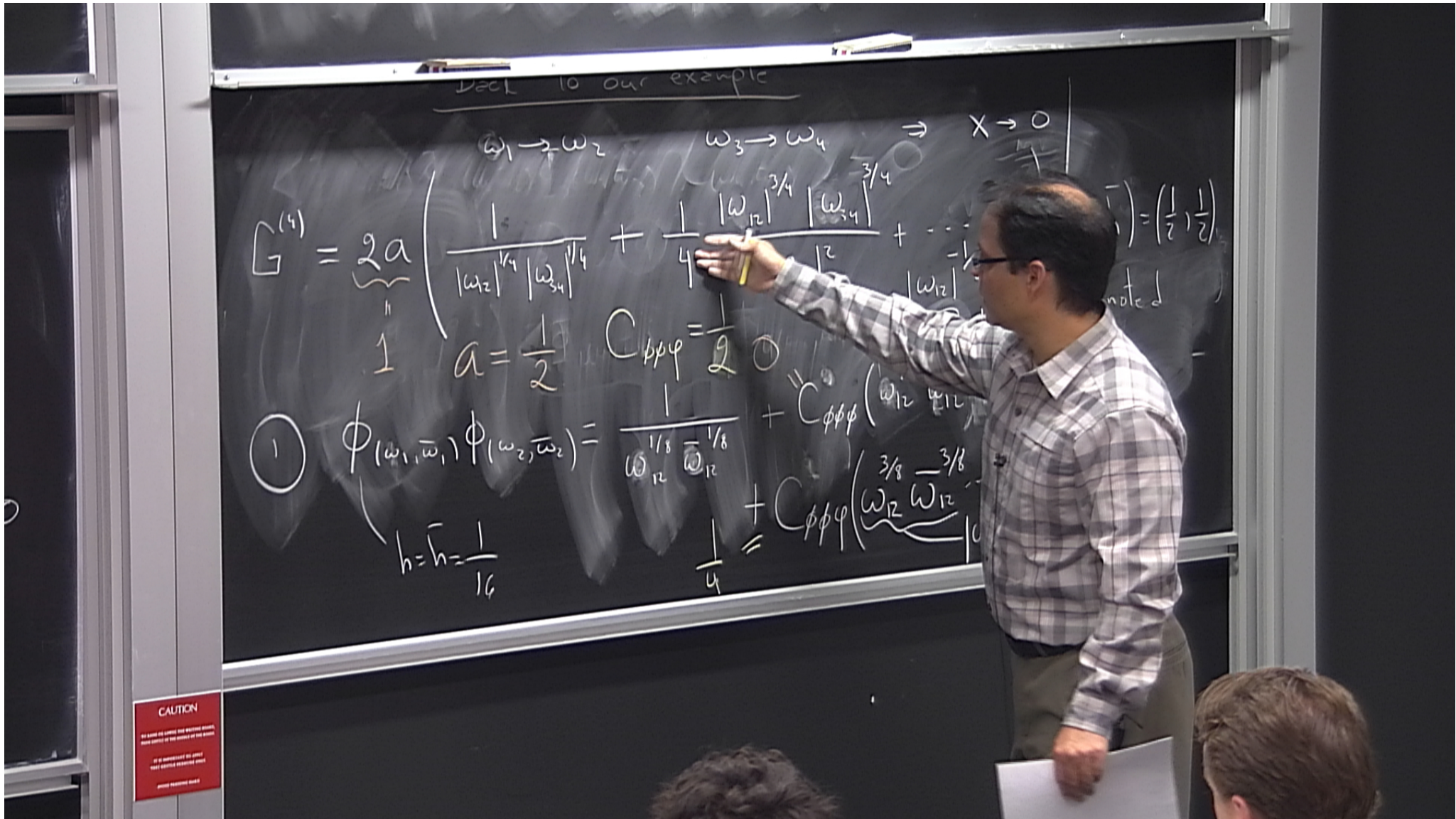
$$\omega_1 \rightarrow \omega_2 \quad \omega_3 \rightarrow \omega_4 \quad \Rightarrow \quad x \rightarrow 0$$

$$G^{(4)} = 2a \left(\frac{1}{|\omega_2|^{1/4} |\omega_{34}|^{1/4}} + \frac{1}{4} \frac{|z_{12}|^{3/4} |z_{34}|^{3/4}}{|z_{24}|^2} + \dots \right)$$

$$\textcircled{1} \quad \phi(\omega_1, \bar{\omega}_1) \phi(\omega_2, \bar{\omega}_2) = \frac{1}{z_{12}^{1/8} \bar{z}_{12}^{1/8}} +$$

$$h = \bar{h} = \frac{1}{16}$$

CAUTION
DO NOT REARMS THE BOARD OR BOARD
AND BOARD OF THE BOARD OF THE BOARD



back to our example

$\omega_1 \rightarrow \omega_2$ $\omega_3 \rightarrow \omega_4$ $\Rightarrow X \rightarrow 0$

$$G^{(4)} = 2a \left(\frac{1}{|\omega_{12}|^{1/4} |\omega_{34}|^{1/4}} + \frac{1}{4} \frac{|\omega_{12}|^{3/4} |\omega_{34}|^{3/4}}{|\omega_{12}|^2} + \dots \right) = \left(\frac{1}{2}, \frac{1}{2} \right)$$

$a = \frac{1}{2}$ $C_{\phi\phi\phi} = \frac{1}{2}$

① $\phi(\omega_1, \bar{\omega}_1) \phi(\omega_2, \bar{\omega}_2) = \frac{1}{\omega_{12}^{1/8} \bar{\omega}_{12}^{1/8}} + C_{\phi\phi\phi}(\omega_{12}, \bar{\omega}_{12}) + C_{\phi\phi\phi}(\omega_{12}, \bar{\omega}_{12})$

$h = \bar{h} = \frac{1}{16}$ $\frac{1}{4} =$

CAUTION
 DO NOT LEAN ON THE BOARD.
 IT IS IMPROPER TO LEAN
 AGAINST THE BOARD.

What is this theory?
2d Ising Model @ criticality
 $T = T_c$

What is this theory?

Ising Model @ criticality

$$T = T_c$$

$$Z = \sum_{\tau} e^{-\beta E(\tau)}$$

$$E(\tau) = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j - h \sum_j \sigma_j$$

What is this theory?

Ising Model @ criticality

$$T = T_c$$

$$Z = \sum_{\tau} e^{-\beta E(\tau)}$$

$$E(\tau) = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j = -h \sum_j \sigma_j$$

Critical exponents; * near T_c

$$t = \frac{(T - T_c)}{T_c}$$

* at T_c

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Critical exponents; * near T_c

$$t = \frac{(T - T_c)}{T_c}$$

* at T_c

$$\langle \sigma_i \sigma_j \rangle_c \sim e^{-|i-j|/\xi(T)}$$

$$\text{at } T = T_c \quad \langle \sigma_i \sigma_{i+n} \rangle = \frac{1}{|n|^\eta}$$

$$G = 2$$

$$\phi$$

CAUTION
 THE BOARD IS LOOSELY AND REVERSIBLY MOUNTED.
 PLEASE BE CAREFUL OF THE BOARD'S POSITION.
 IT IS RESPONSIBLE FOR ANY
 DAMAGE TO THE BOARD'S POSITION.
 PLEASE BE CAREFUL.

CAUTION
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$$\langle \sigma_i \sigma_j \rangle_c \sim e^{-|i-j|/\xi(T)}$$

at $T = T_c$ $\langle \sigma_i \sigma_{i+n} \rangle \sim \frac{1}{|n|}$

In the continuum limit,

$$\epsilon_i \sim \sigma_i \sigma_{i+1}$$

$$G = 2$$

$$\phi$$

CAUTION
 DO NOT TOUCH THE BOARD OR THE BOARDER.
 IT IS PROHIBITED TO TOUCH THE BOARD OR THE BOARDER.
 ALL RIGHTS RESERVED.

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$$\langle \sigma_i \sigma_j \rangle_c \sim e^{-|i-j|/\xi(T)}$$

at $T = T_c$

$$\langle \sigma_i \sigma_{i+n} \rangle = \frac{1}{\ln n}$$

$$\langle \epsilon_i \epsilon_{i+n} \rangle = \frac{1}{\ln |4 - 2/\nu|}$$

In the continuum limit:

$$\epsilon_i \sim \sigma_i \sigma_{i+1}$$

CAUTION

DO NOT LEAN ON THE BOARD OR WRITE ON IT.
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CAUTION

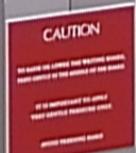
DO NOT LEAN ON THE BOARD OR WRITE ON IT.
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In the limit

$$\mathbb{V}_i \rightarrow \Phi(z, \bar{z})$$

$$\mathcal{E}_i \rightarrow \mathcal{P}(z, \bar{z})$$

$$\langle \Phi(z_1, \bar{z}_1) \Phi(z_2, \bar{z}_2) \rangle \sim \frac{1}{|z_{12}|^2}$$



In the limit

$$\Psi_i \rightarrow \Phi(z, \bar{z})$$

$$\epsilon_i \rightarrow \varphi(z, \bar{z})$$

$$\langle \Phi(z_1, \bar{z}_1) \Phi(z_2, \bar{z}_2) \rangle \sim \frac{1}{|z_{12}|^2}$$

$$\eta = \frac{1}{4} = 4h_{\Phi} \quad h_{\Phi} = \frac{1}{16}$$

CAUTION
DO NOT LEAN ON THE BOARD
OR ON THE WALLS OF THE ROOM
OR ON THE DESKS
OR ON THE CHAIRS
OR ON THE TABLES
OR ON THE FLOORS

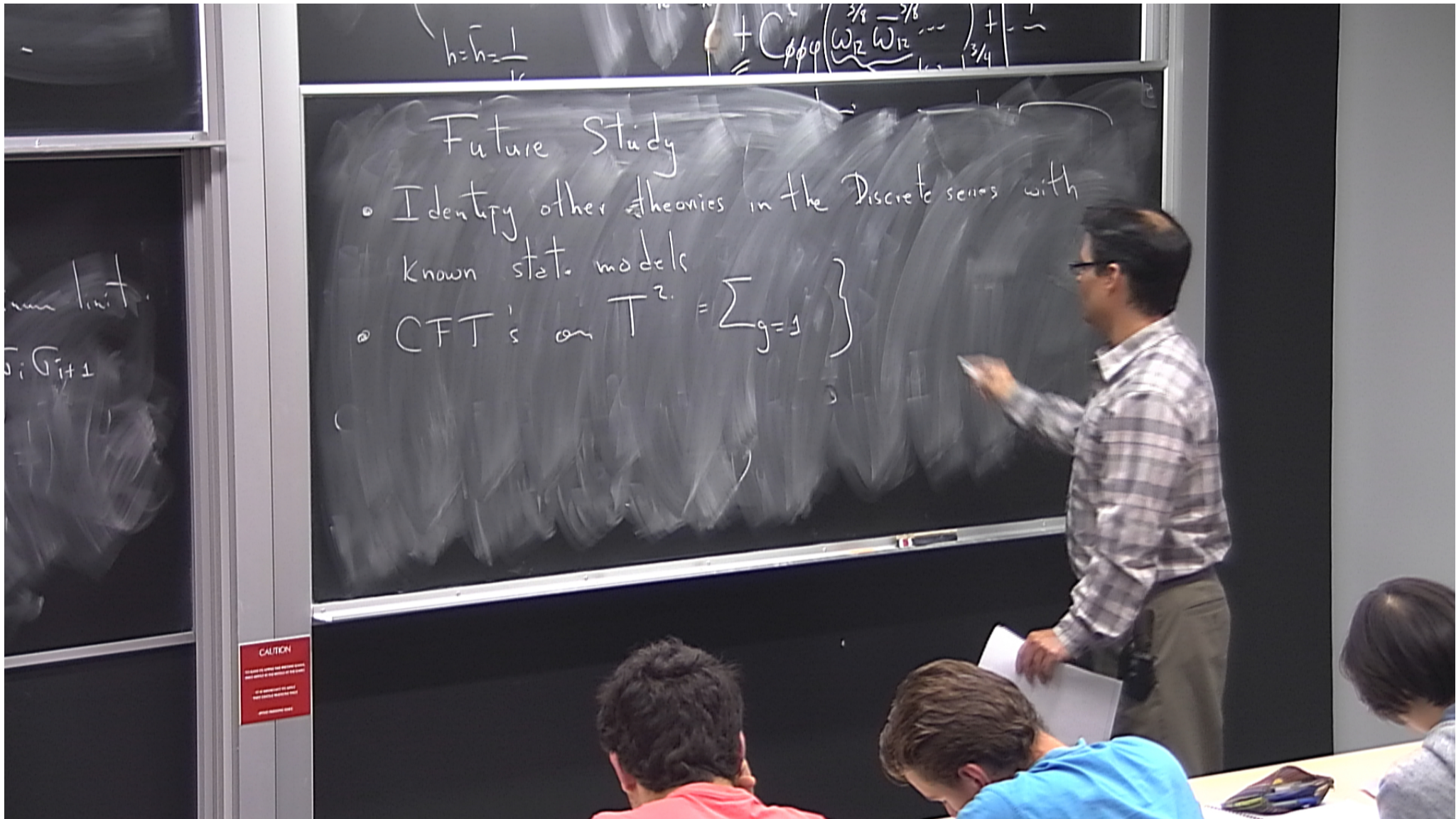
Fusion Rules OPE

$$[\Phi][\Phi] = \mathbb{1} + [\varphi]$$

$$[\Phi][\varphi] = [\Phi]$$

$$[\varphi][\varphi] = \mathbb{1}$$





Future Study

- Identify other theories in the Discrete series with known stat. models
- CFT's on $T^2 = \sum_{g=1}^5$