Title: 14/15 PSI - Conformal Field Theory- Lecture 1

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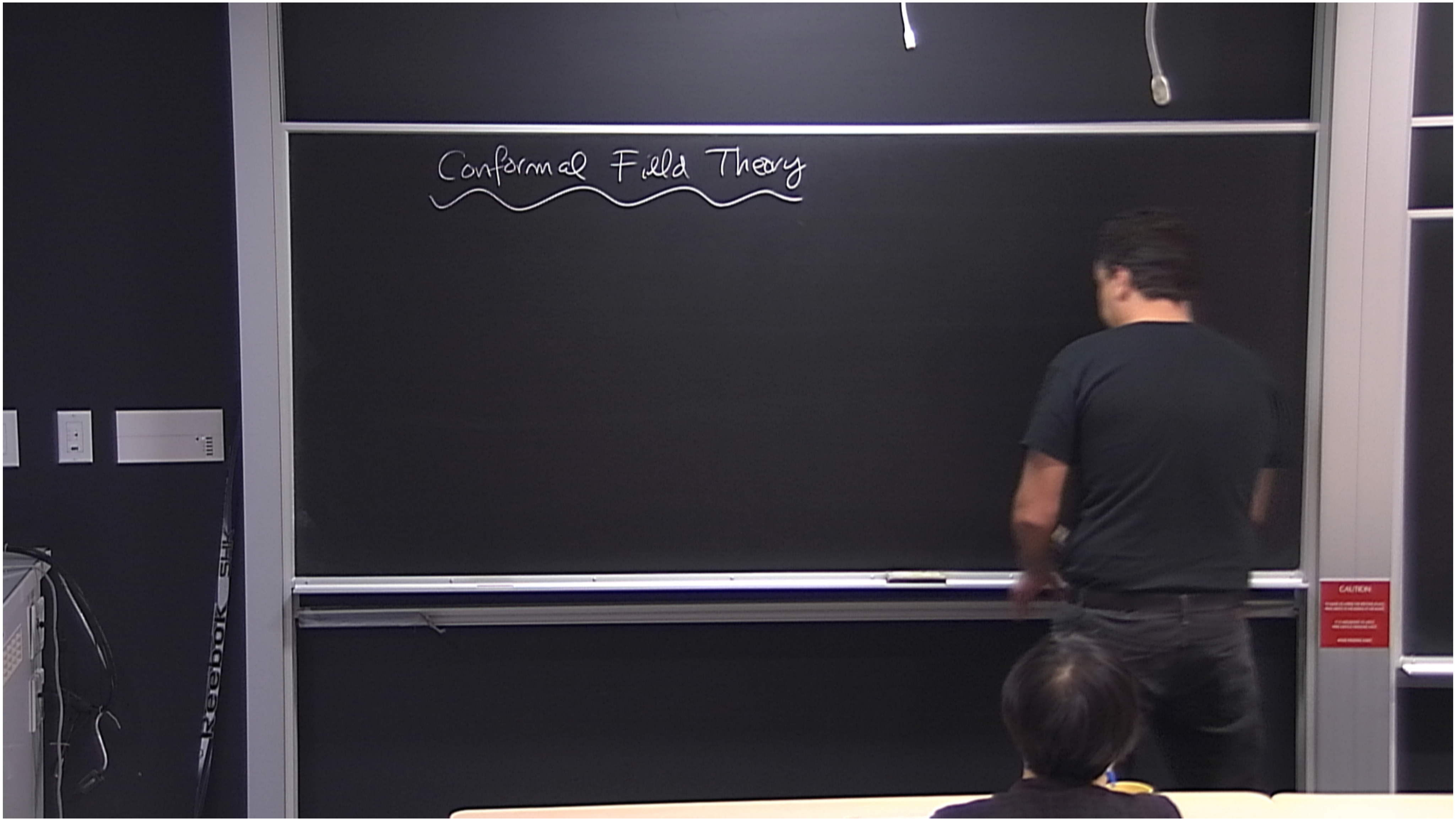
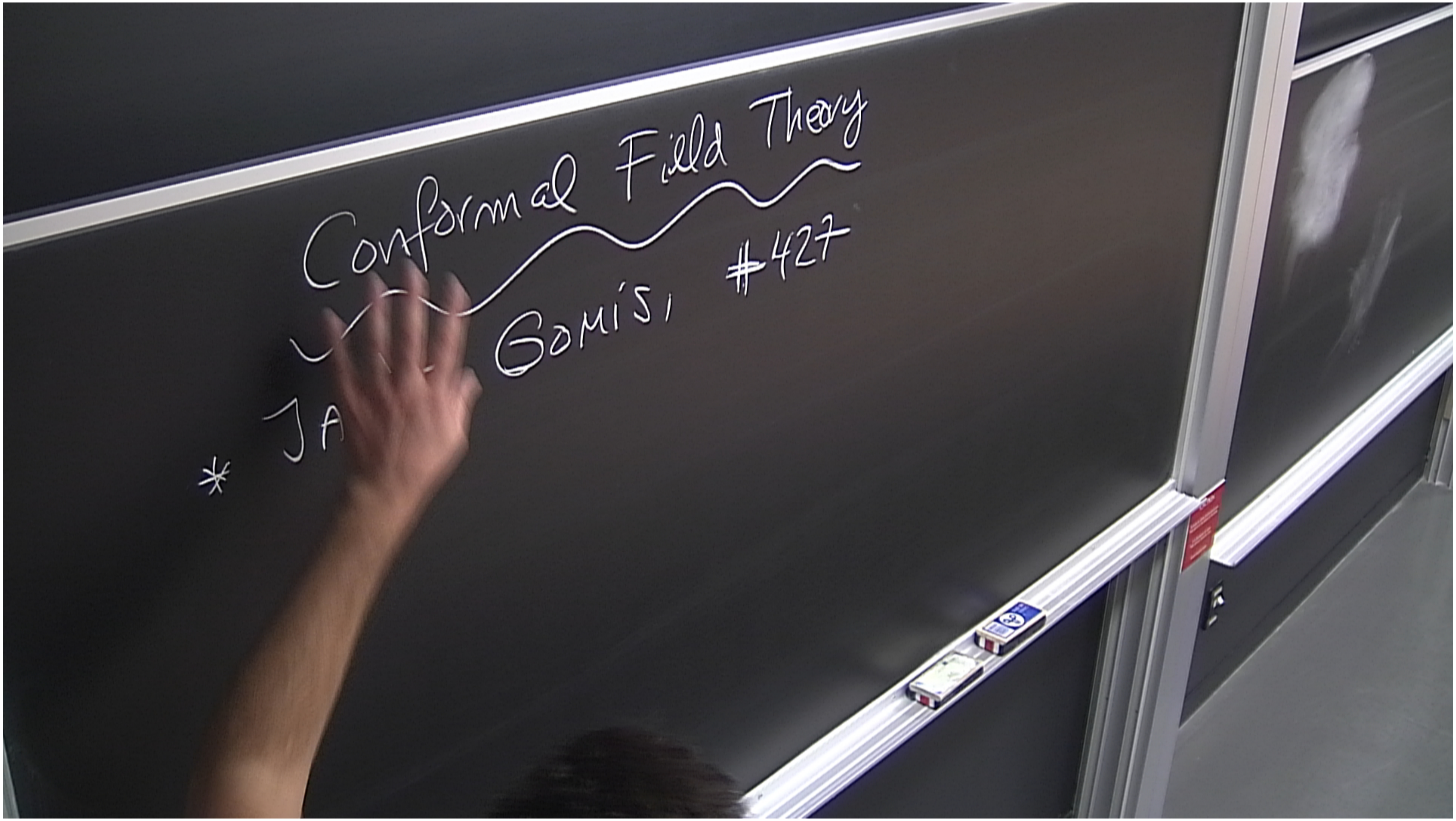
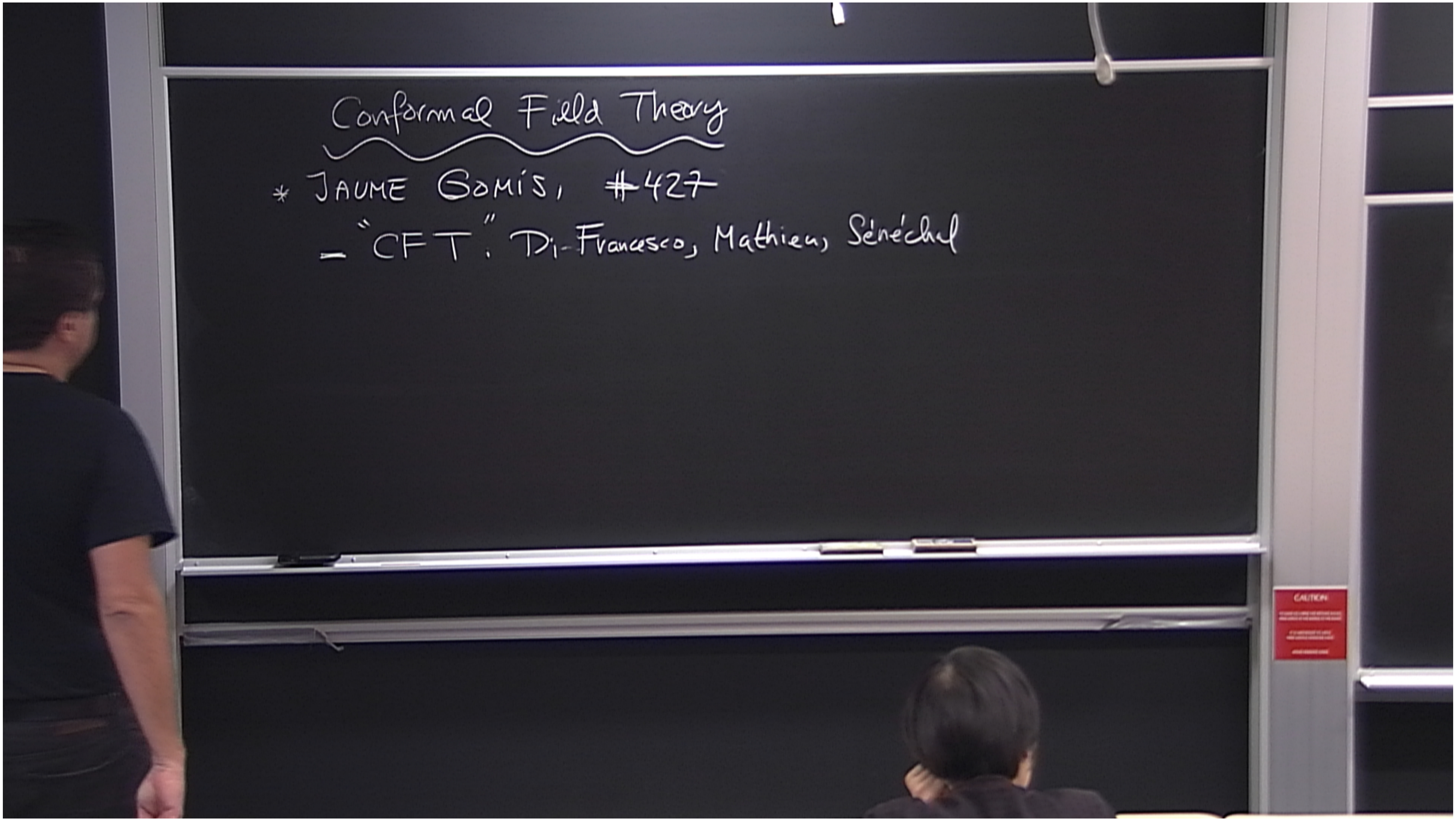
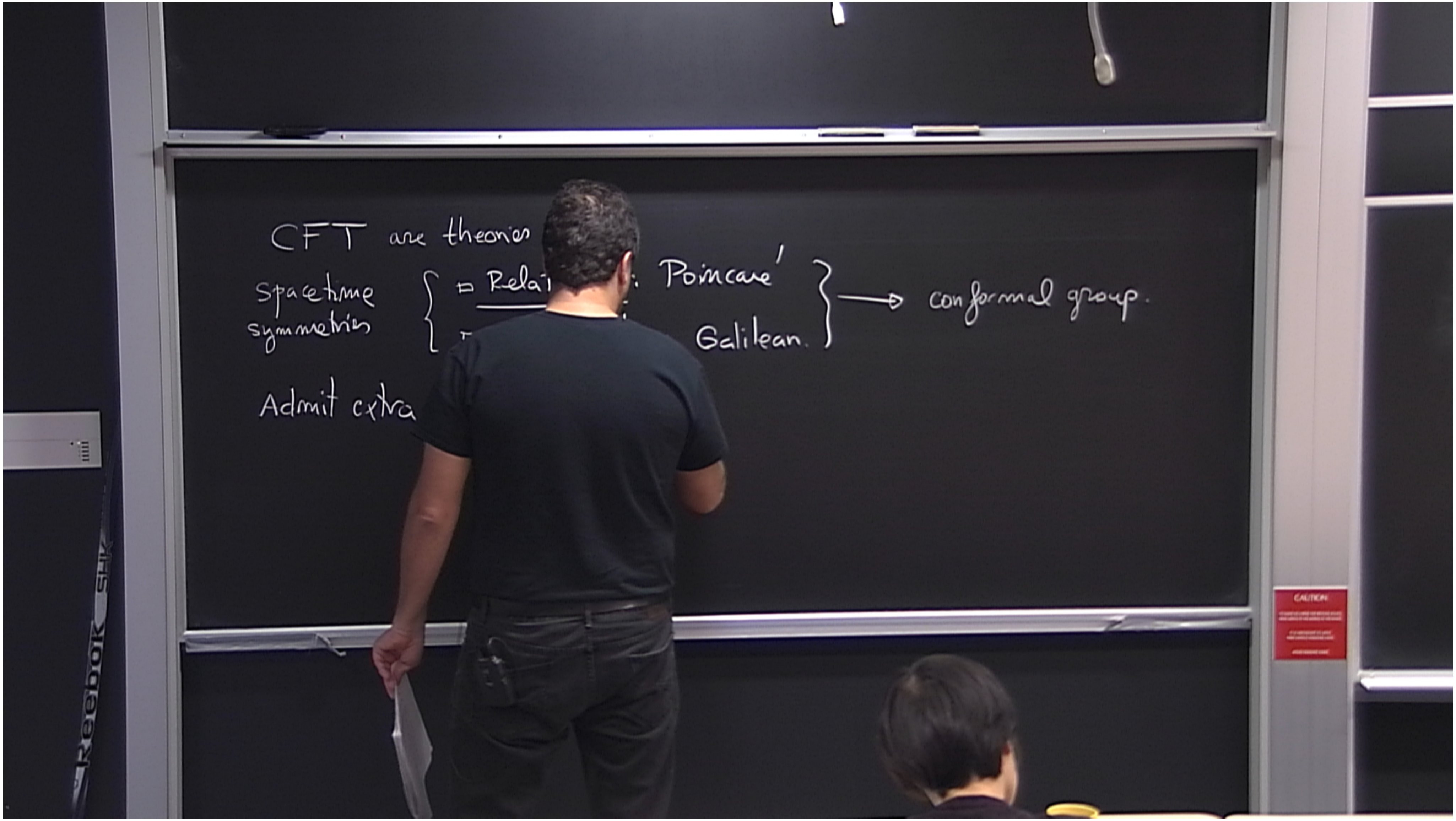
URL: http://pirsa.org/14100008

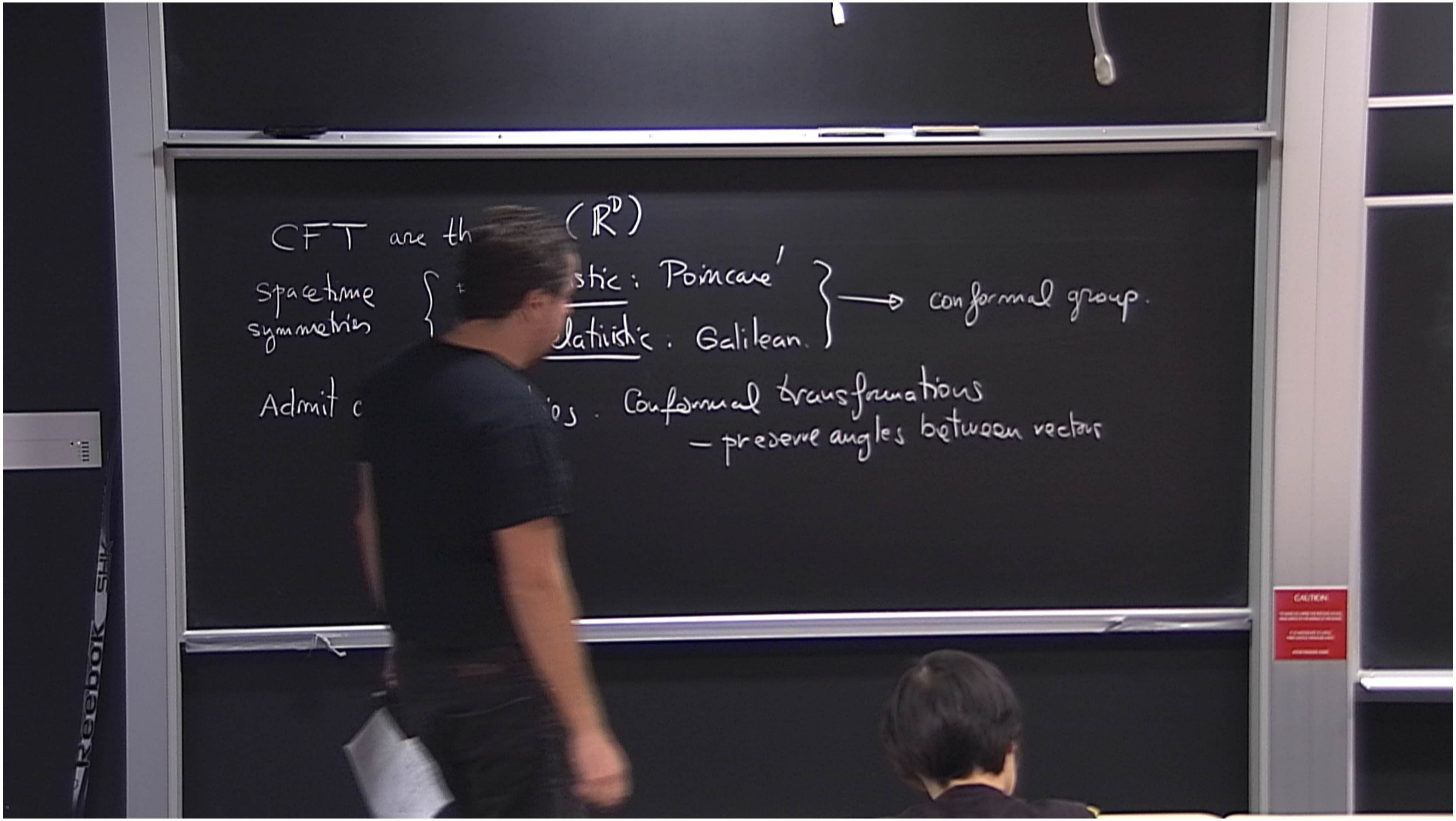
Abstract:

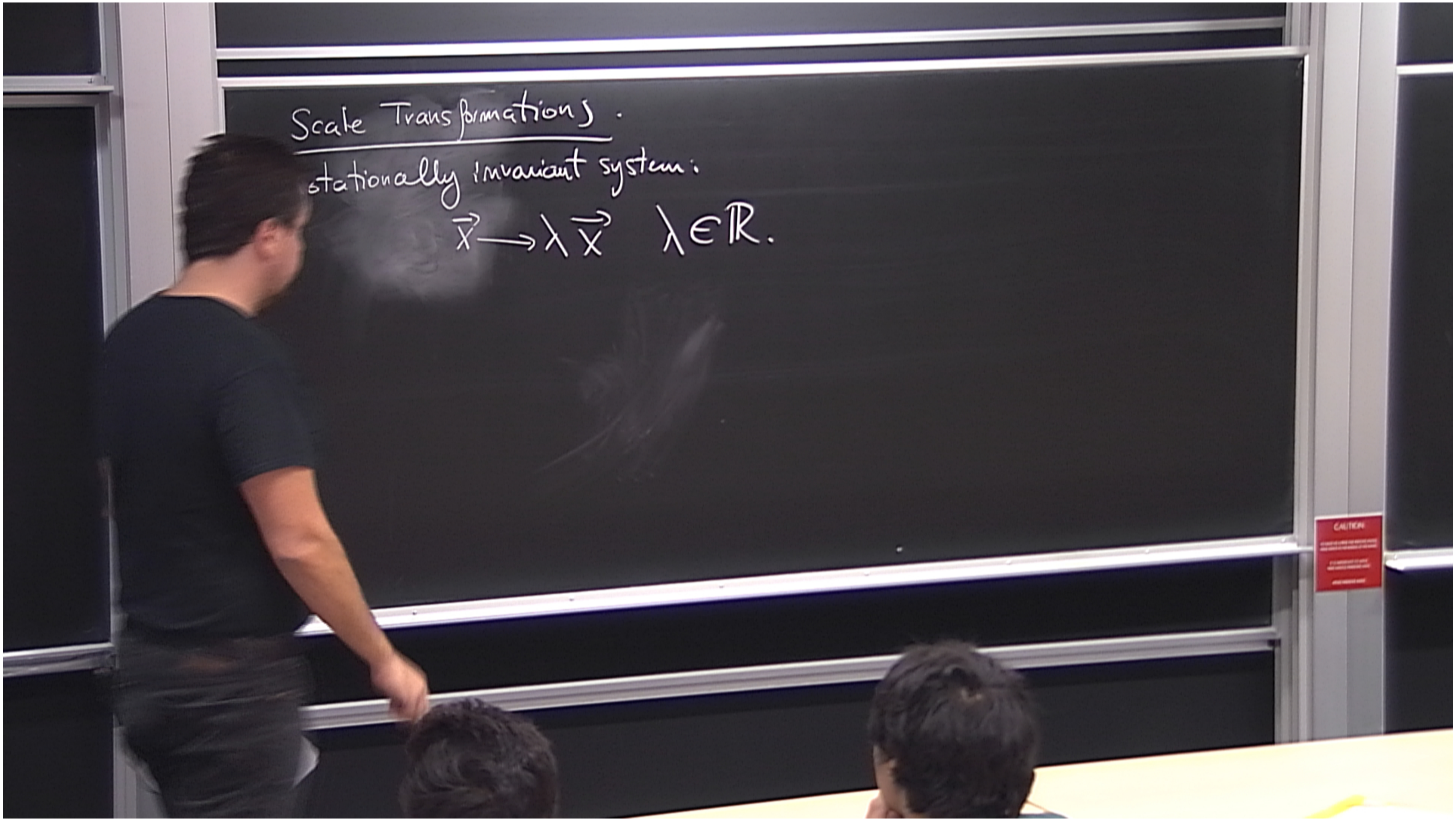

Conformal Field Theory
Reebok
CAUTION

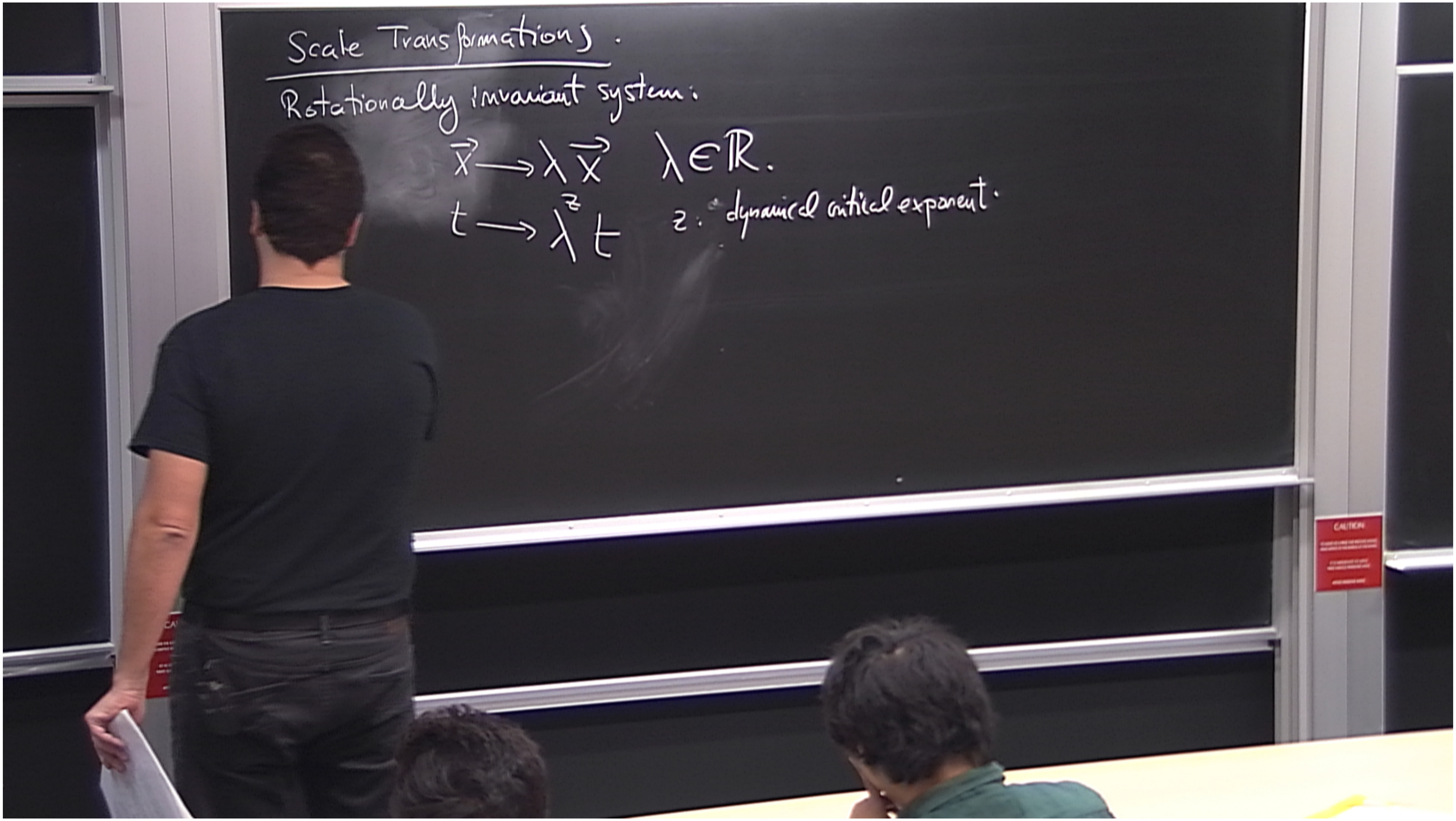

Conformal Field Theory
Gomis, #427
* JA

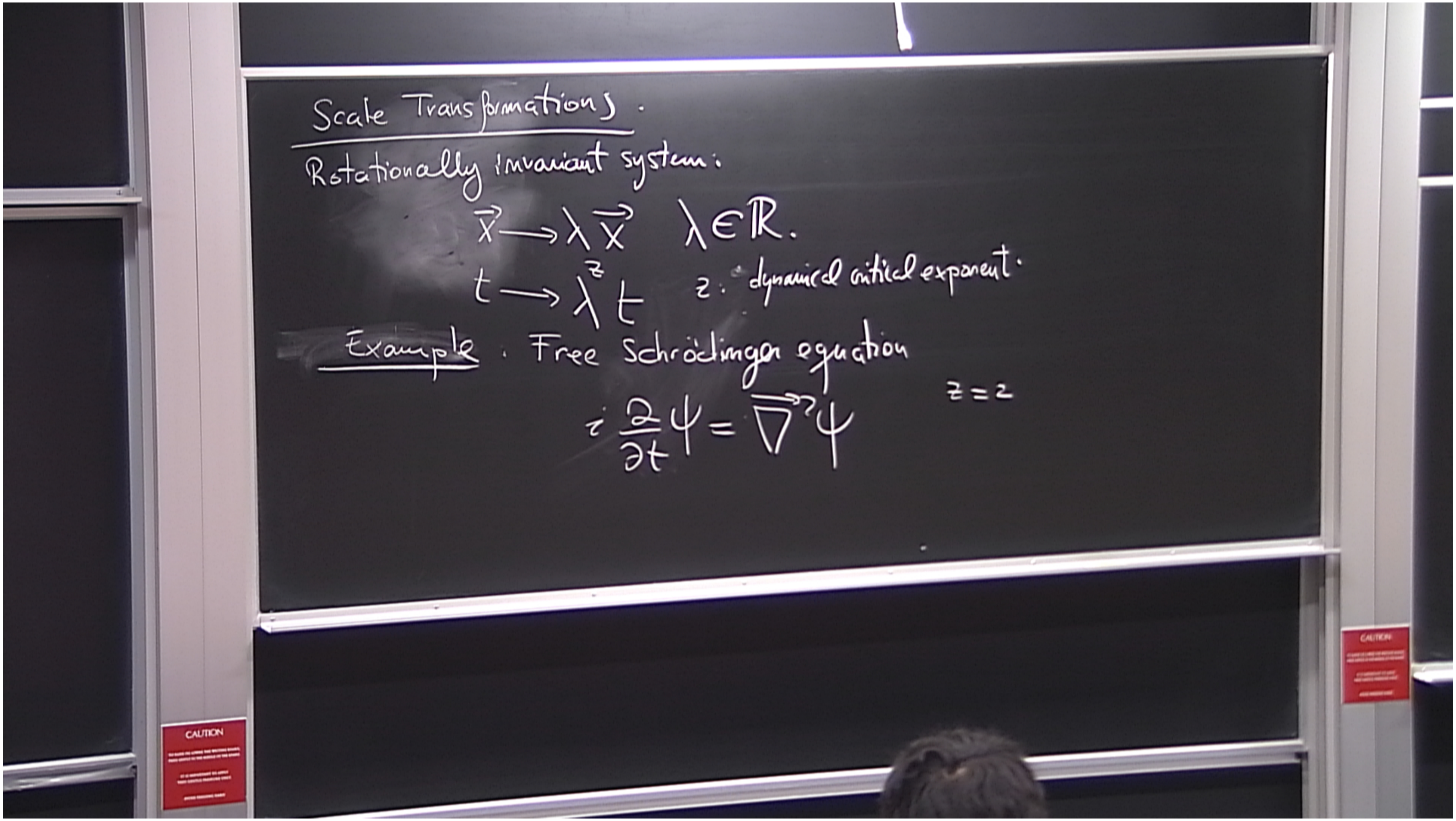

Conformal Field Theory
* JAUME GOMIS, #427
– "CFT": Di-Francesco, Mathieu, Sénéchal

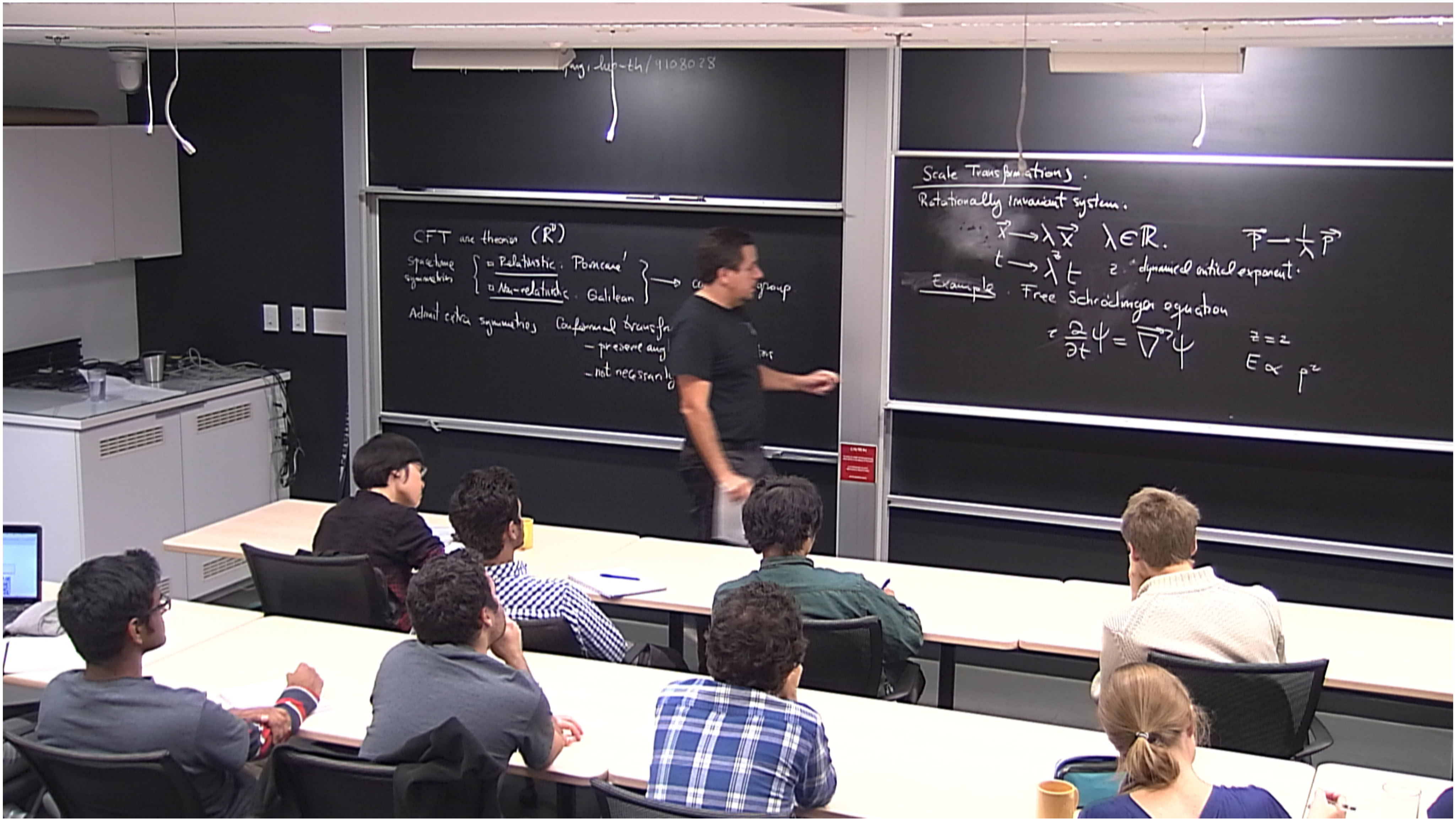

CFT are theories
spacetime symmetries
Poincaré
Galilean.
conformal group.
Admit extra

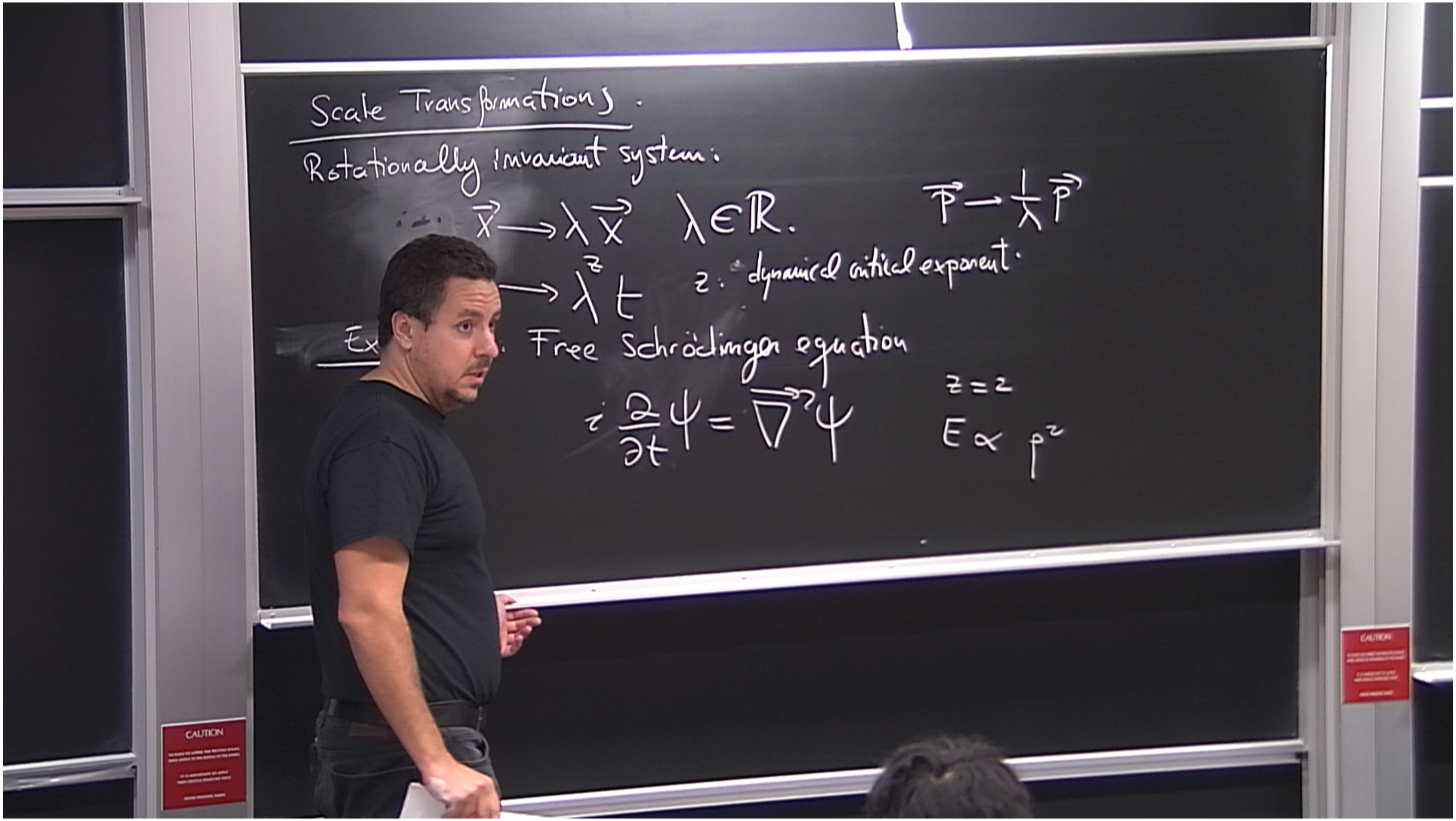

CFT are th
(ℝ^D)
spacetime symmetries
stic : Poincaré
lativistic : Galilean
→ conformal group.
Admit c
es . Conformal transformations
– preserve angles between vectors
CAUTION
Reebok

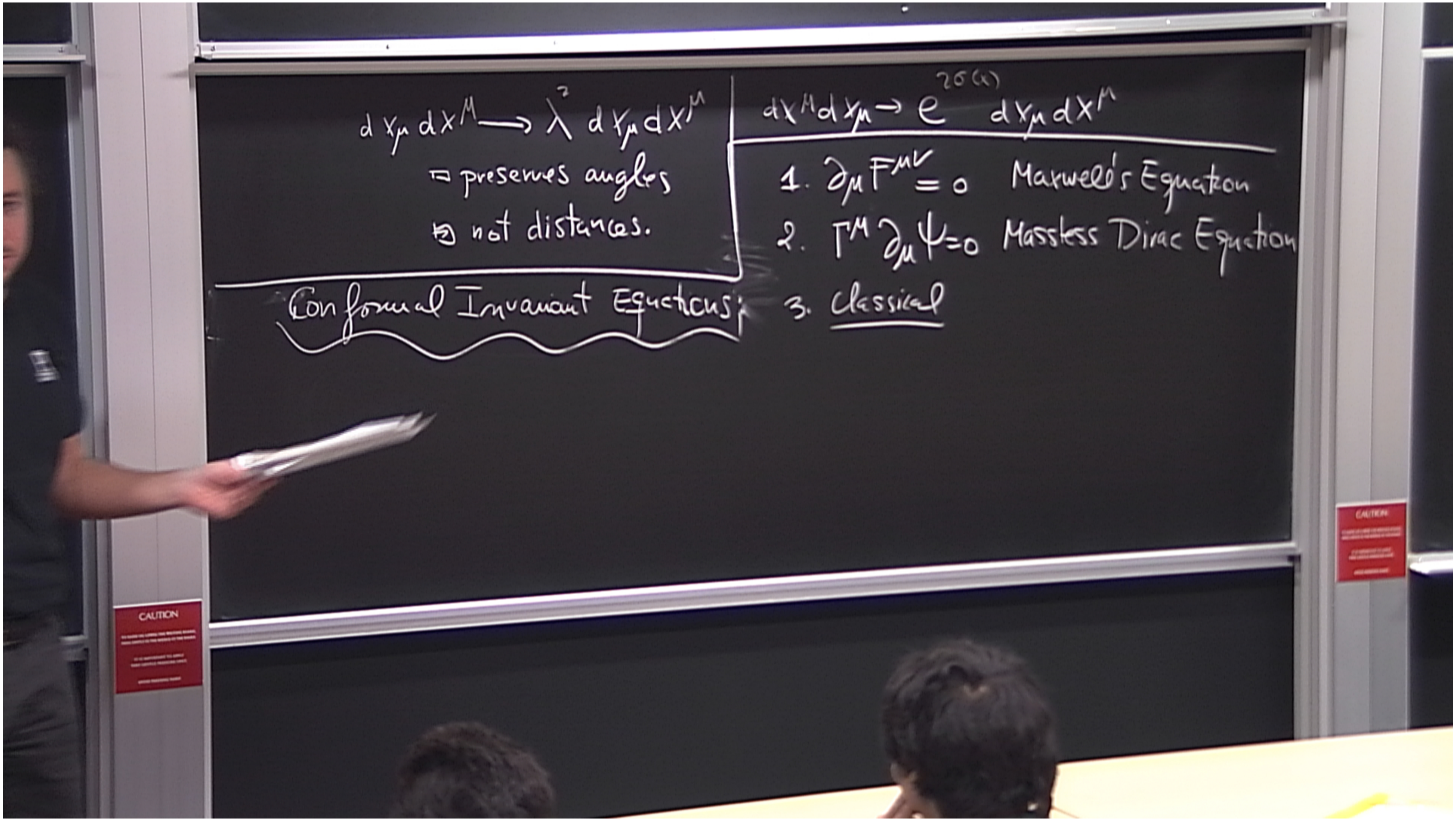

Scale Transformations.
stationally invariant system:
$\vec{x} \to \lambda \vec{x}$ $\lambda \in \mathbb{R}$.

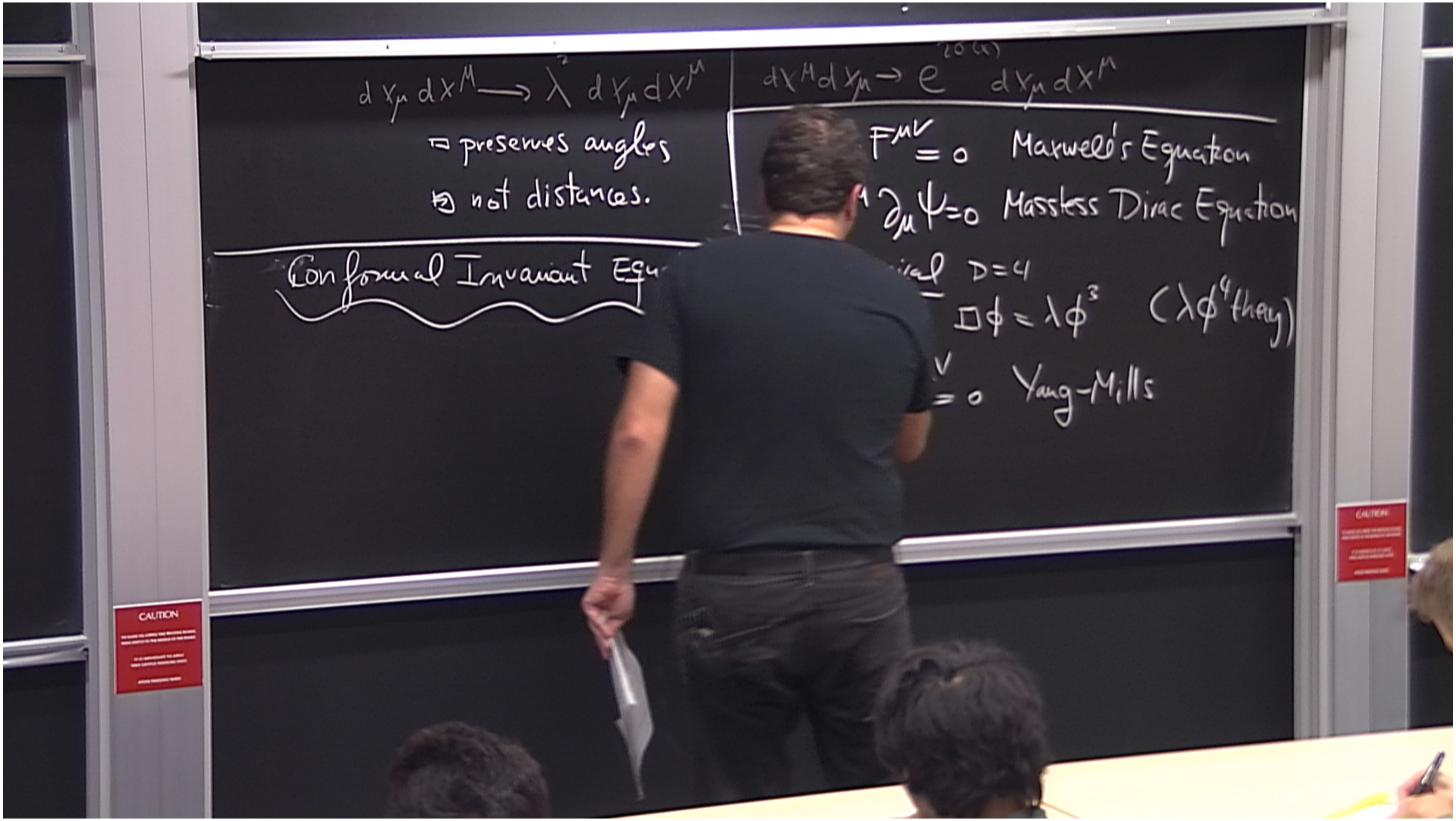

Scale Transformations.
Rotationally invariant system:
$\vec{x} \longrightarrow \lambda \vec{x}$ $\lambda \in \mathbb{R}$.
$t \longrightarrow \lambda^{z} t$ z: dynamical critical exponent.
CAUTION


Scale Transformations.
Rotationally invariant system:
$\vec{x} \longrightarrow \lambda \vec{x}$ $\lambda \in \mathbb{R}$.
$t \longrightarrow \lambda^{z} t$ z: dynamical critical exponent.
Example: Free Schrödinger equation
$i \frac{\partial}{\partial t}\psi = \vec{\nabla}^2 \psi$
$z = 2$

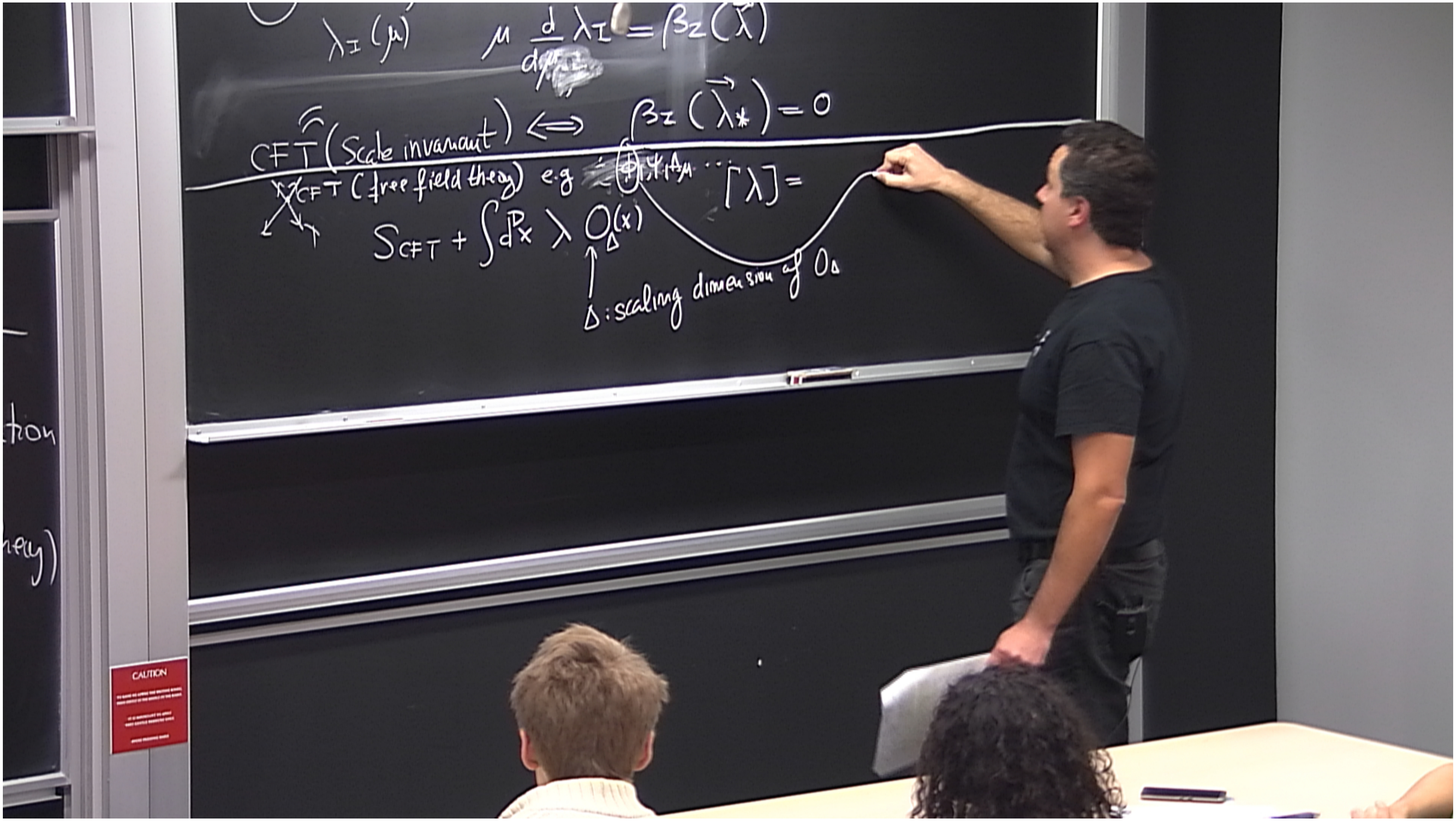

CFT are theories
Relativistic. Poincaré
Non-relativistic. Galilean
group
Admit extra symmetries. Conformal transf
- preserve ang
- not necessarily
Scale Transformations.
Rotationally invariant system.
λ ∈ R.
z : dynamical critical exponent.
Example. Free Schrödinger equation
z = 2


Scale Transformations.
Rotationally invariant system:
x → λx   λ ∈ ℝ.
p → 1/λ p
→ λ^z t   z: dynamical critical exponent.
Free Schrödinger equation
i ∂/∂t ψ = ∇²ψ
z = 2
E ∝ p²

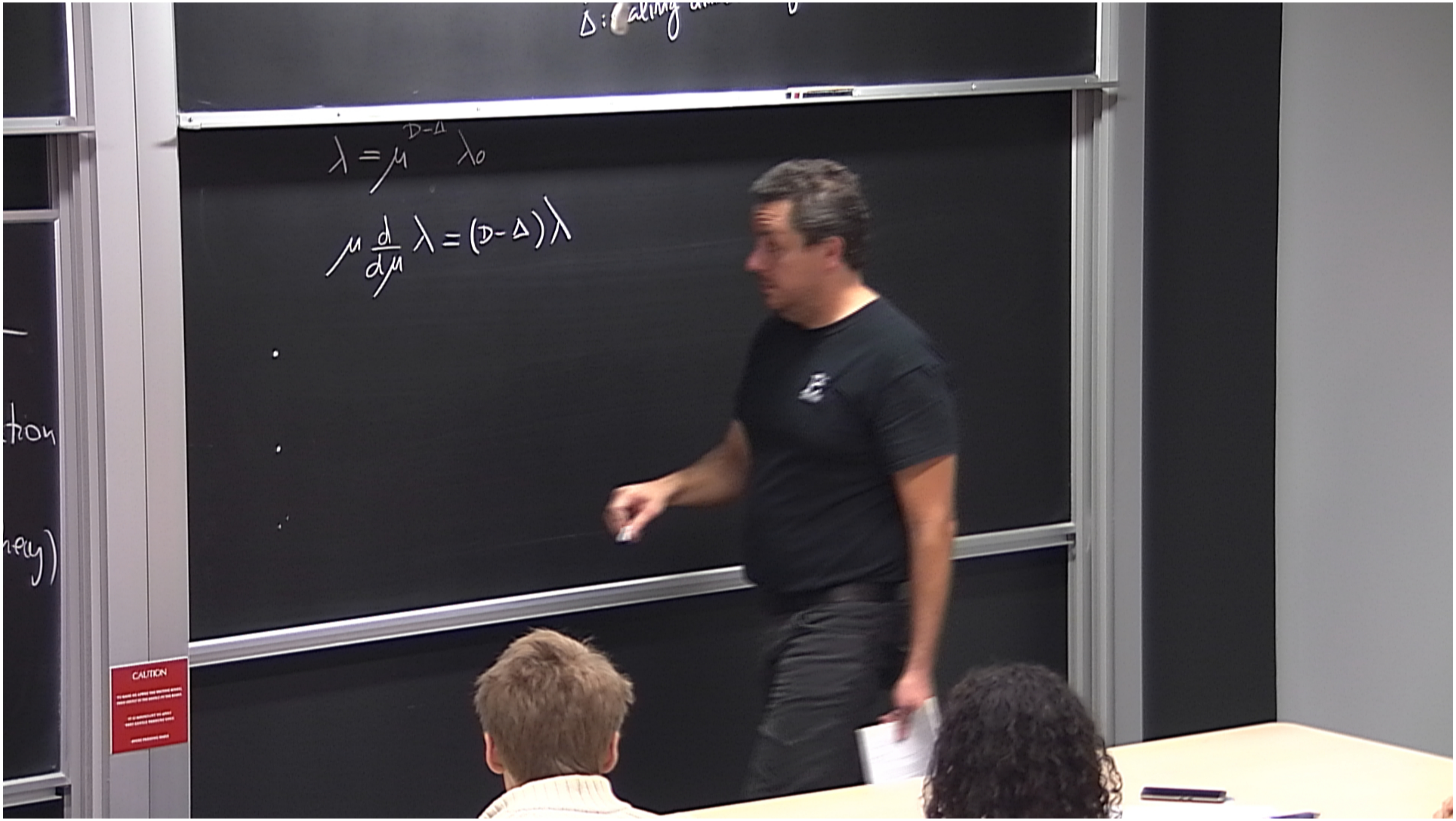

$dx_\mu dx^\mu \longrightarrow \lambda^2 dx_\mu dx^\mu$
⇒ preserves angles
⇒ not distances.
$dx^\mu dx_\mu \rightarrow e^{2\sigma(x)} dx_\mu dx^\mu$
Conformal Invariant Equations:
1. $\partial_\mu F^{\mu\nu} = 0$ Maxwell's Equation
2. $\Gamma^\mu \partial_\mu \psi = 0$ Massless Dirac Equation
3. Classical

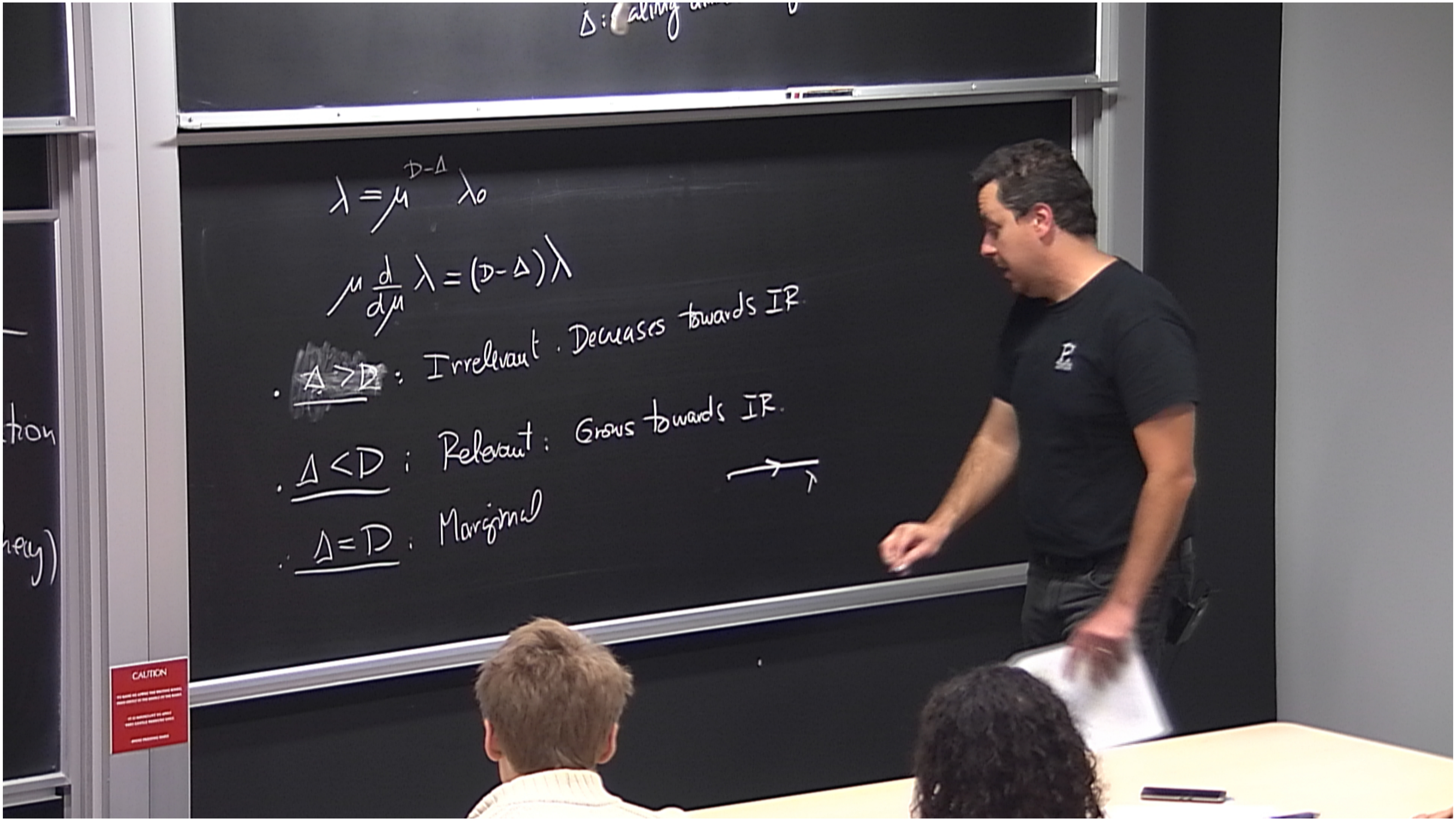


$dx_\mu dx^\mu \to \lambda^2 dx_\mu dx^\mu$
⇒ preserves angles
⇒ not distances.
Conformal Invariant Equ
$dx^\mu dx_\mu \to e^{2\sigma(x)} dx_\mu dx^\mu$
$F^{\mu\nu} = 0$ Maxwell's Equation
$\partial_\mu \psi = 0$ Massless Dirac Equation
D=4
$\Box\phi = \lambda\phi^3$ ($\lambda\phi^4$ theory)
$= 0$ Yang-Mills
CAUTION

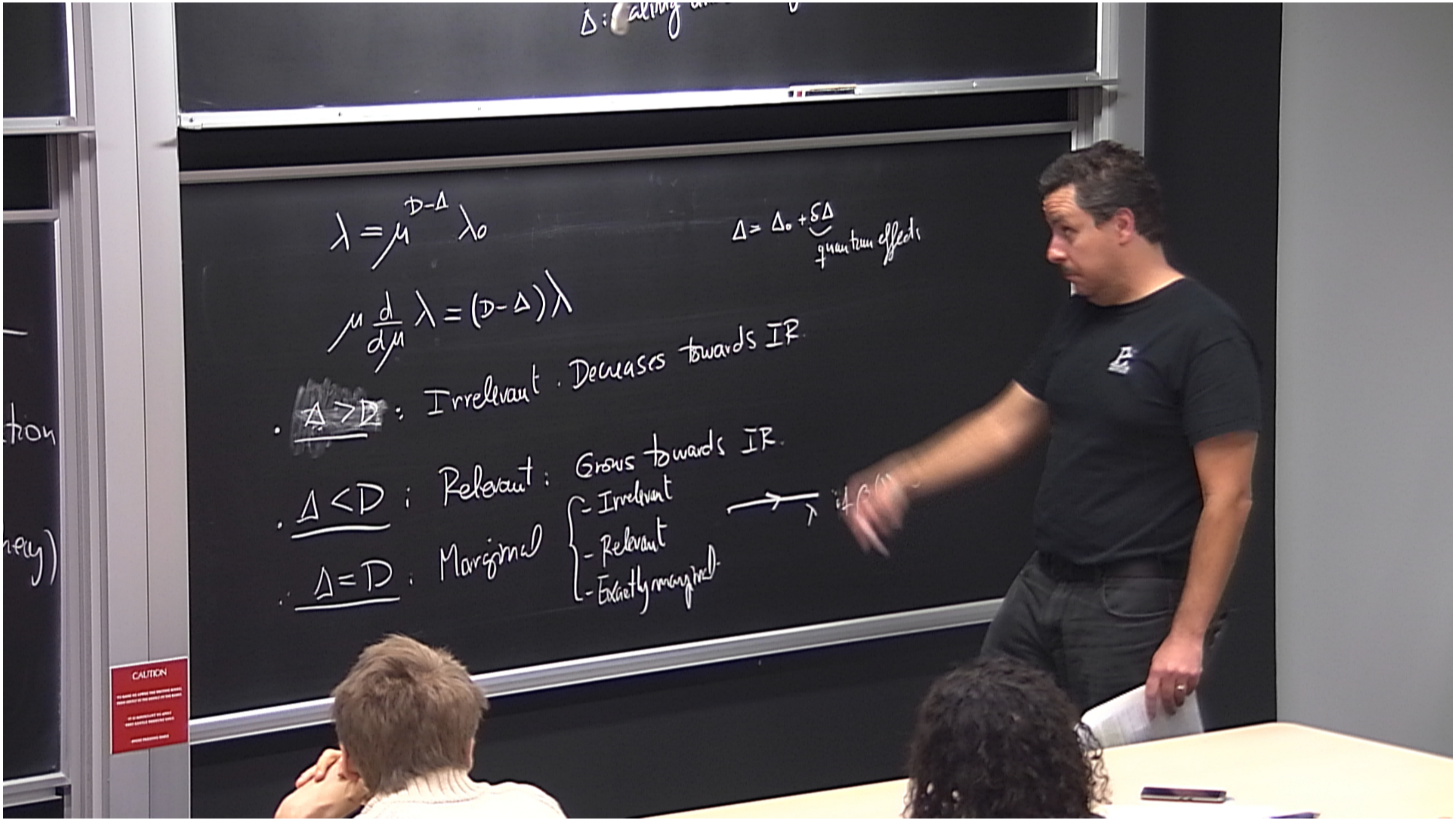


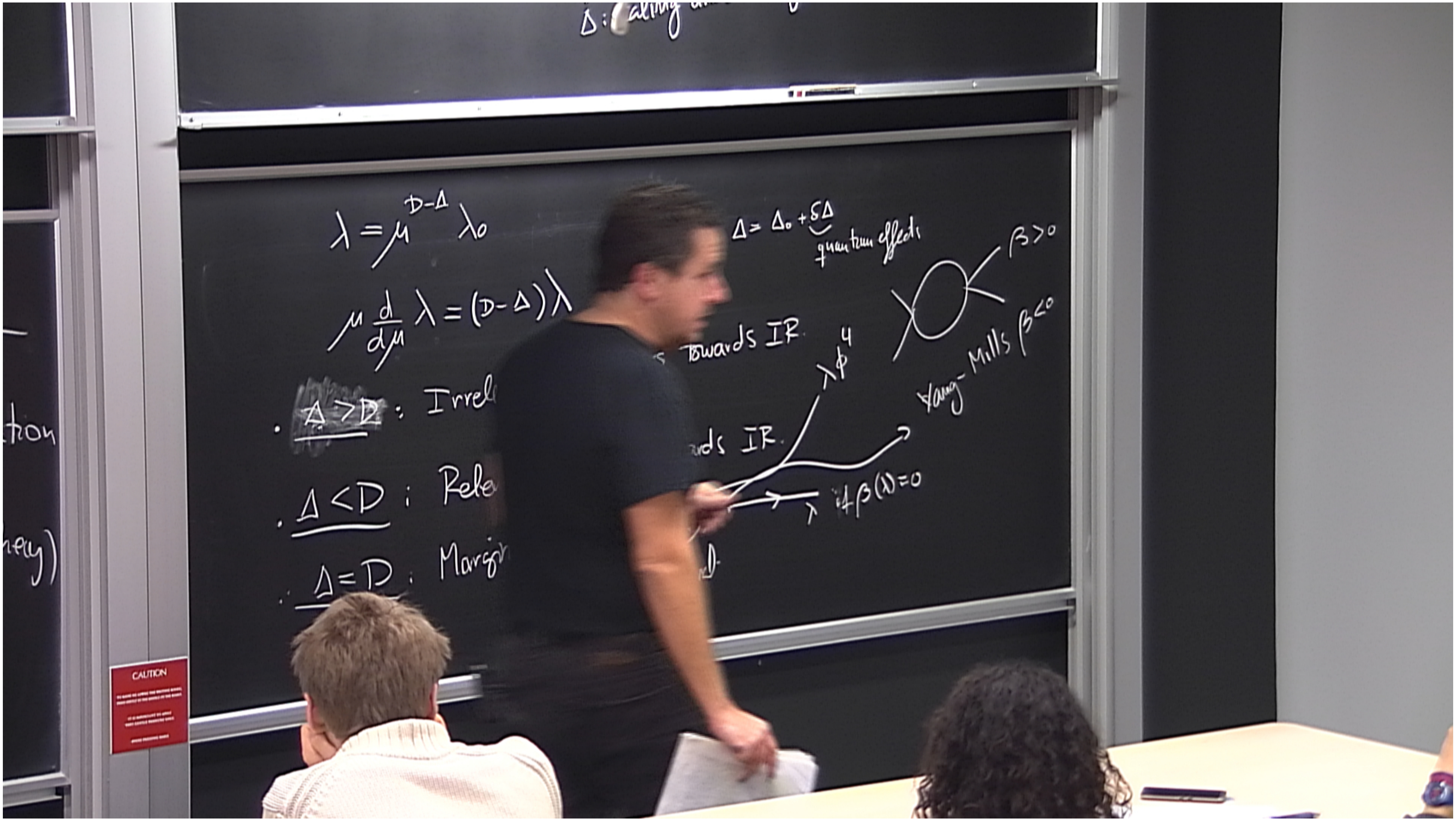
CFT (Scale invariant) ⟺ β_I(λ*) = 0
CFT (free field theory) e.g. φ, ψ, A_μ ...
S_CFT + ∫d^D x λ O_Δ(x)
[λ] =
Δ: scaling dimension of O_Δ
CAUTION

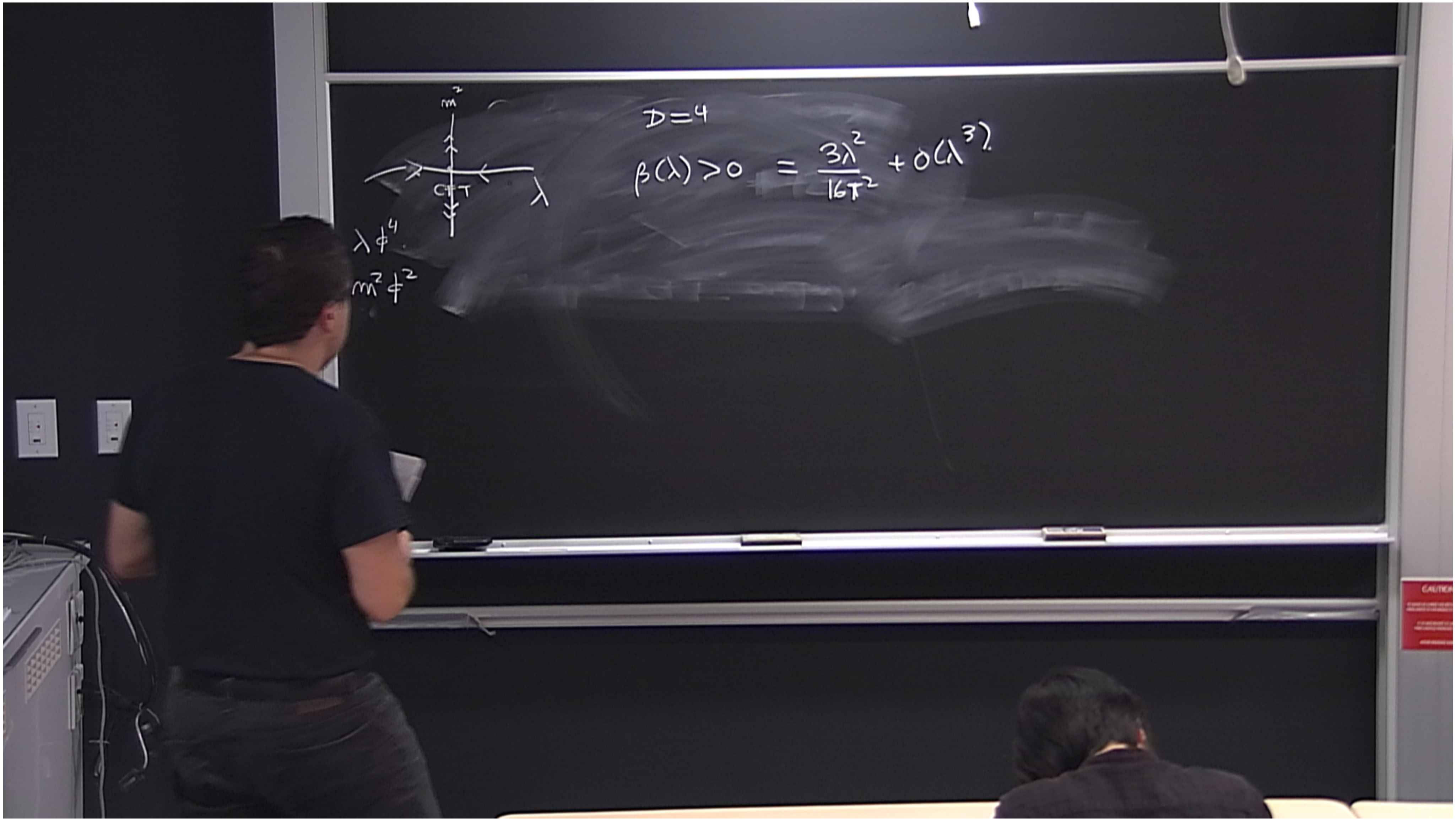

$\lambda = \mu^{D-\Delta} \lambda_0$
$\mu \frac{d}{d\mu} \lambda = (D-\Delta)\lambda$
CAUTION


λ = μ^(D-Δ) λ0
μ d/dμ λ = (D-Δ) λ
Δ > D : Irrelevant . Decreases towards IR.
Δ < D : Relevant : Grows towards IR.
Δ = D : Marginal

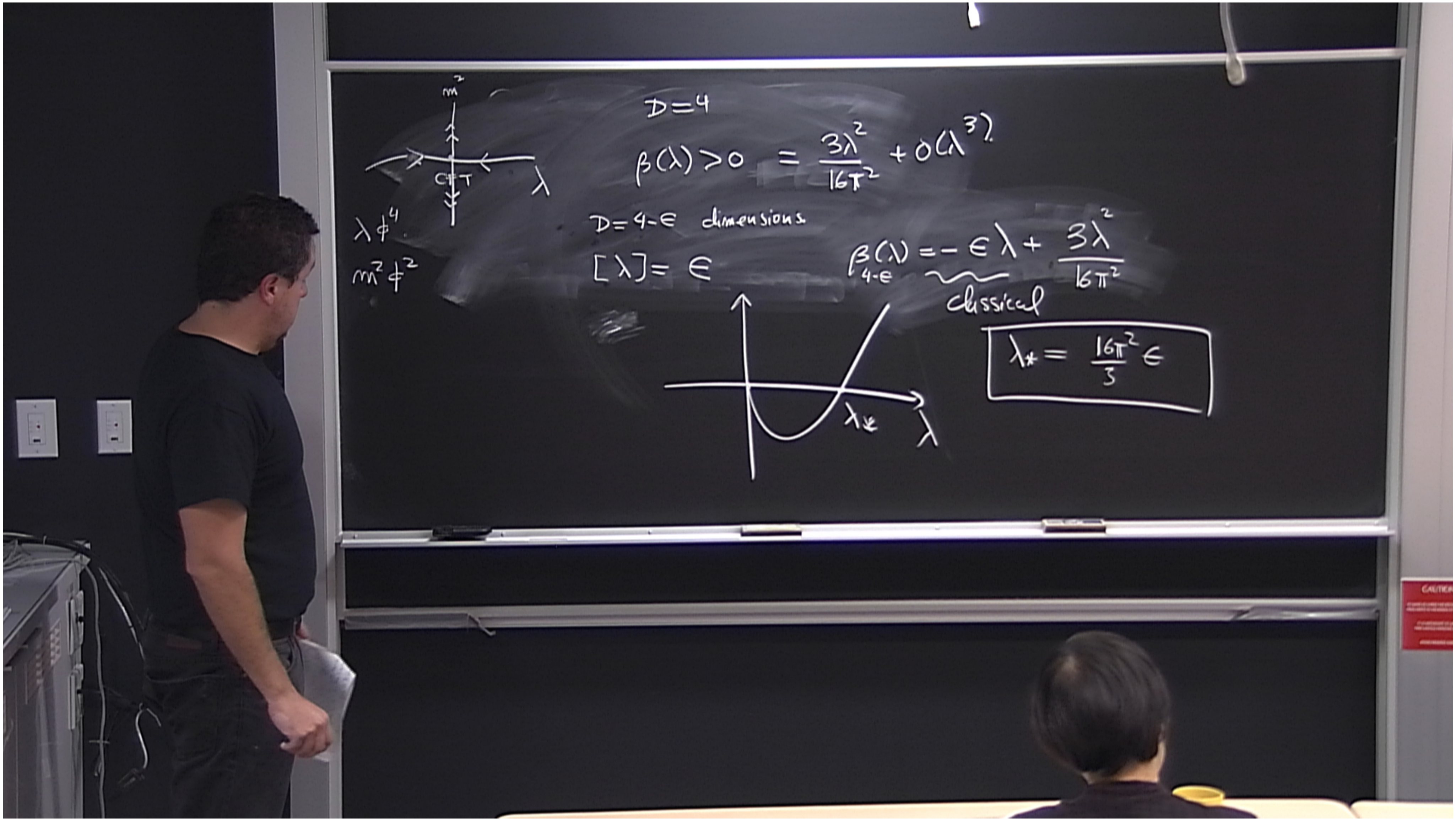
λ = μ^(D-Δ) λ0
Δ
μ d/dμ λ = (D-Δ)λ
Δ>D : Irrelevant . Decreases towards
Δ<D : Relevant : Grows towards IR.
Δ=D : Marginal
CAUTION


$\lambda = \mu^{D-\Delta} \lambda_0$
$\Delta = \Delta_0 + \delta\Delta$
quantum effects
$\mu \frac{d}{d\mu} \lambda = (D-\Delta)\lambda$
• $\Delta > D$: Irrelevant . Decreases towards IR.
• $\Delta < D$: Relevant : Grows towards IR.
• $\Delta = D$: Marginal
- Irrelevant
- Relevant
- Exactly marginal
CAUTION

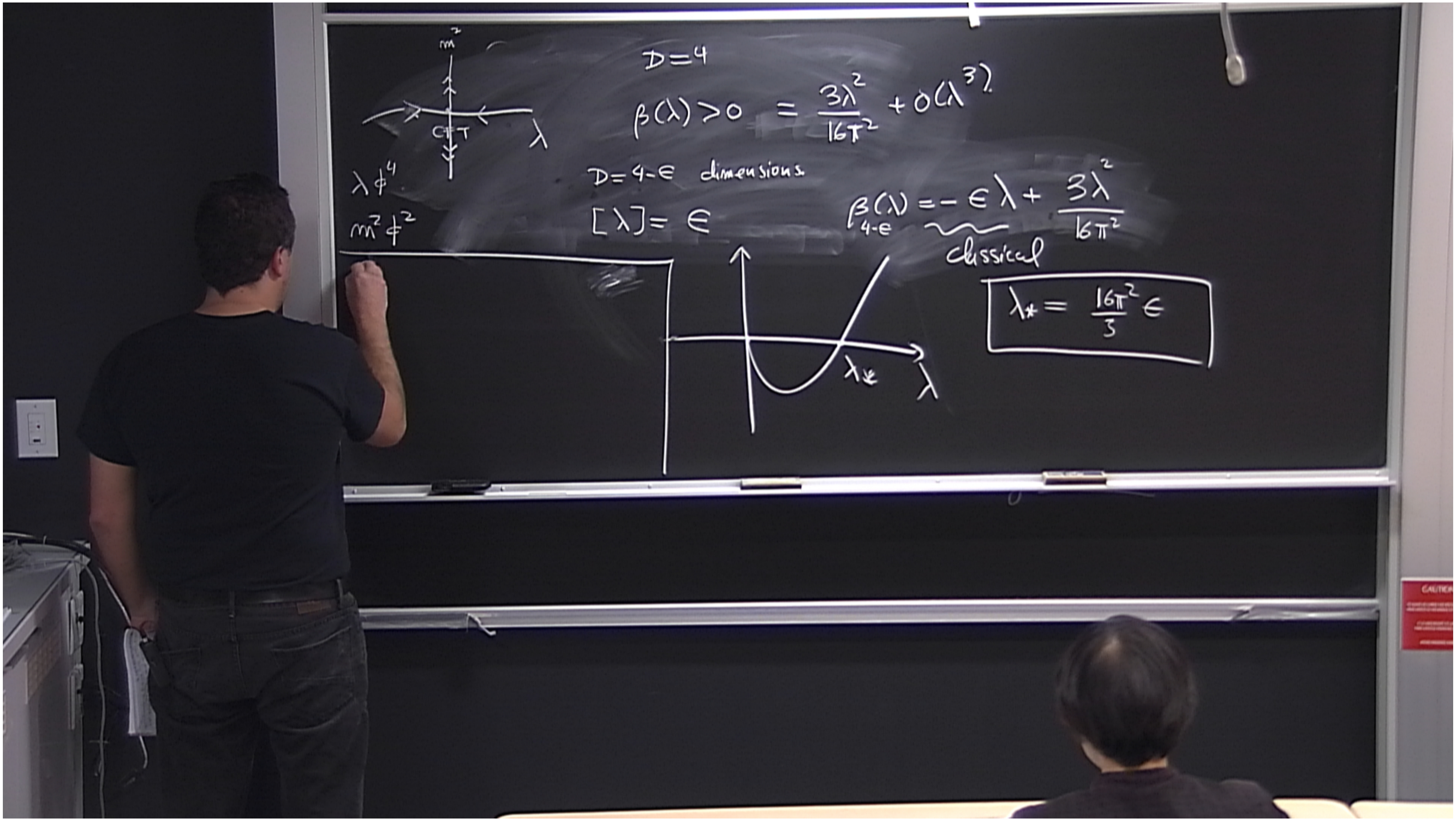

λ = μ^{D-Δ} λ₀
μ d/dμ λ = (D-Δ)λ
Δ>D : Irrel
Δ<D : Rele
Δ=D : Margin
Δ = Δ₀ + δΔ
quantum effects
β>0
Yang-Mills β<0
towards IR
λφ⁴
if β(λ)=0

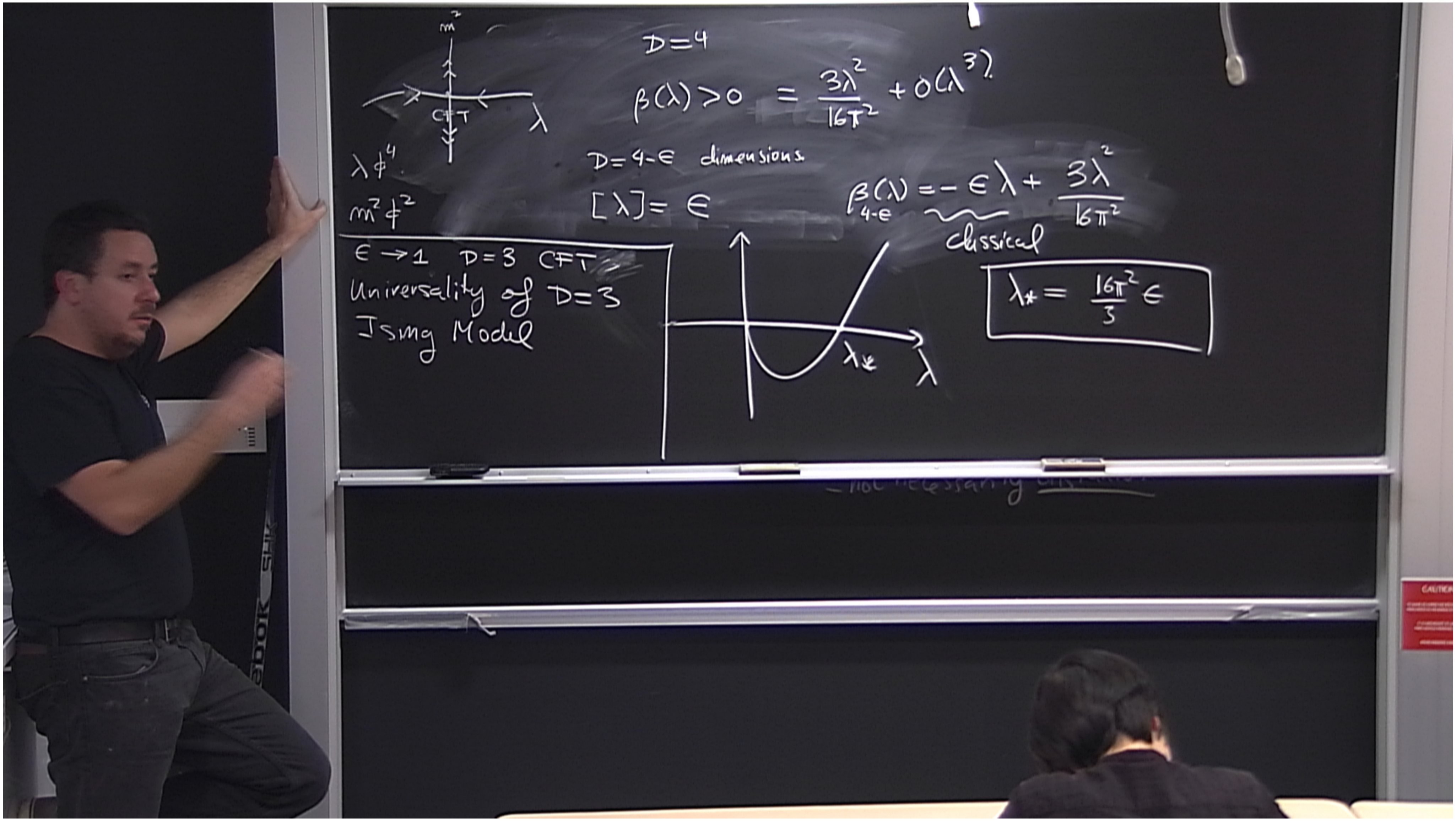


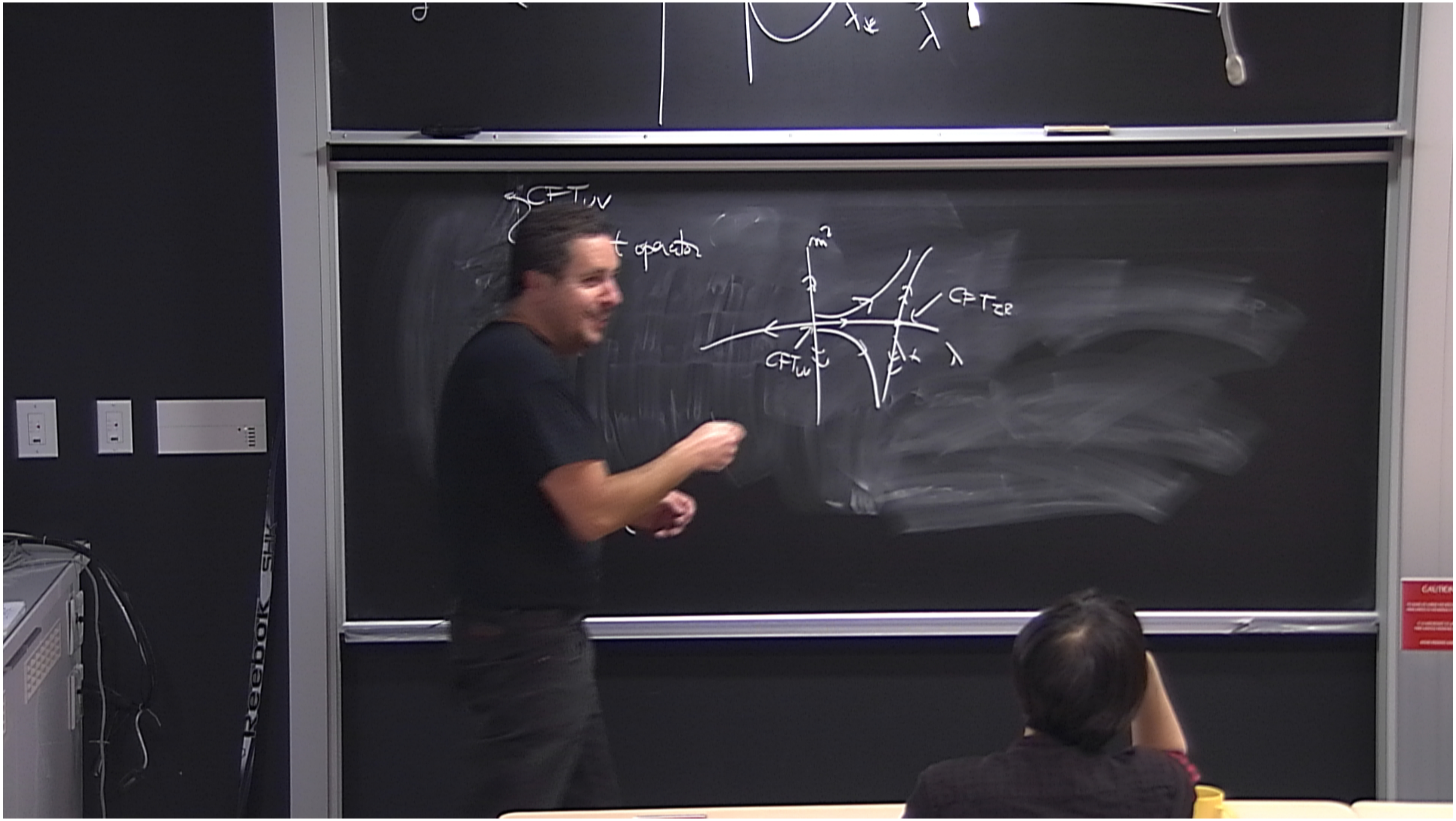
m^2
CFT
λ
$\lambda\phi^4$
$m^2\phi^2$
$D=4$
$\beta(\lambda) > 0 = \frac{3\lambda^2}{16\pi^2} + O(\lambda^3)$

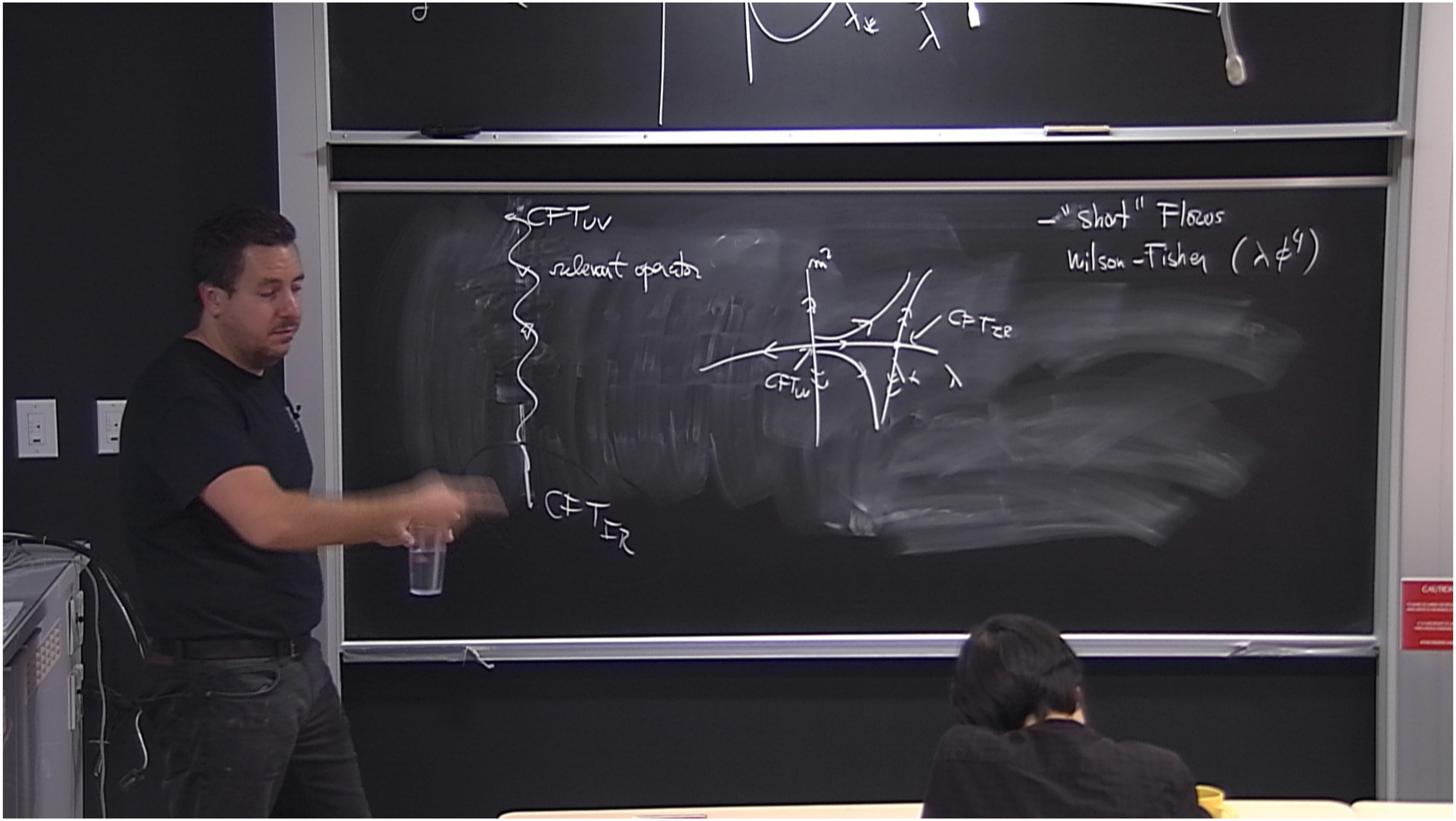


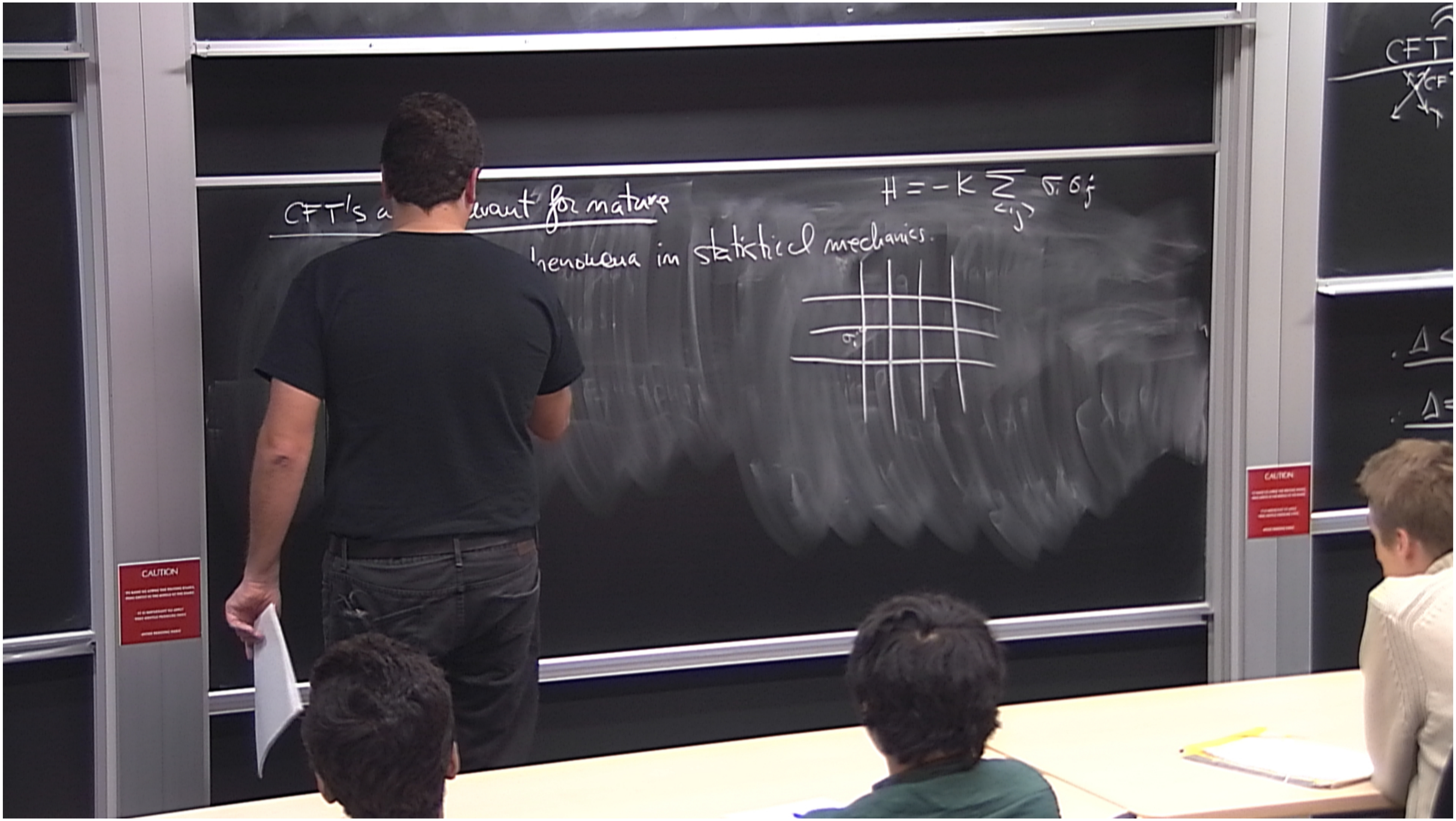
m^2
CFT
λ
$\lambda\phi^4$
$m^2\phi^2$
D=4
$\beta(\lambda) > 0 = \frac{3\lambda^2}{16\pi^2} + O(\lambda^3)$
D=4-ε dimensions
$[\lambda] = \epsilon$
$\beta_{4-\epsilon}(\lambda) = -\epsilon\lambda + \frac{3\lambda^2}{16\pi^2}$
classical
$\lambda_* = \frac{16\pi^2}{3}\epsilon$
λ_*
λ

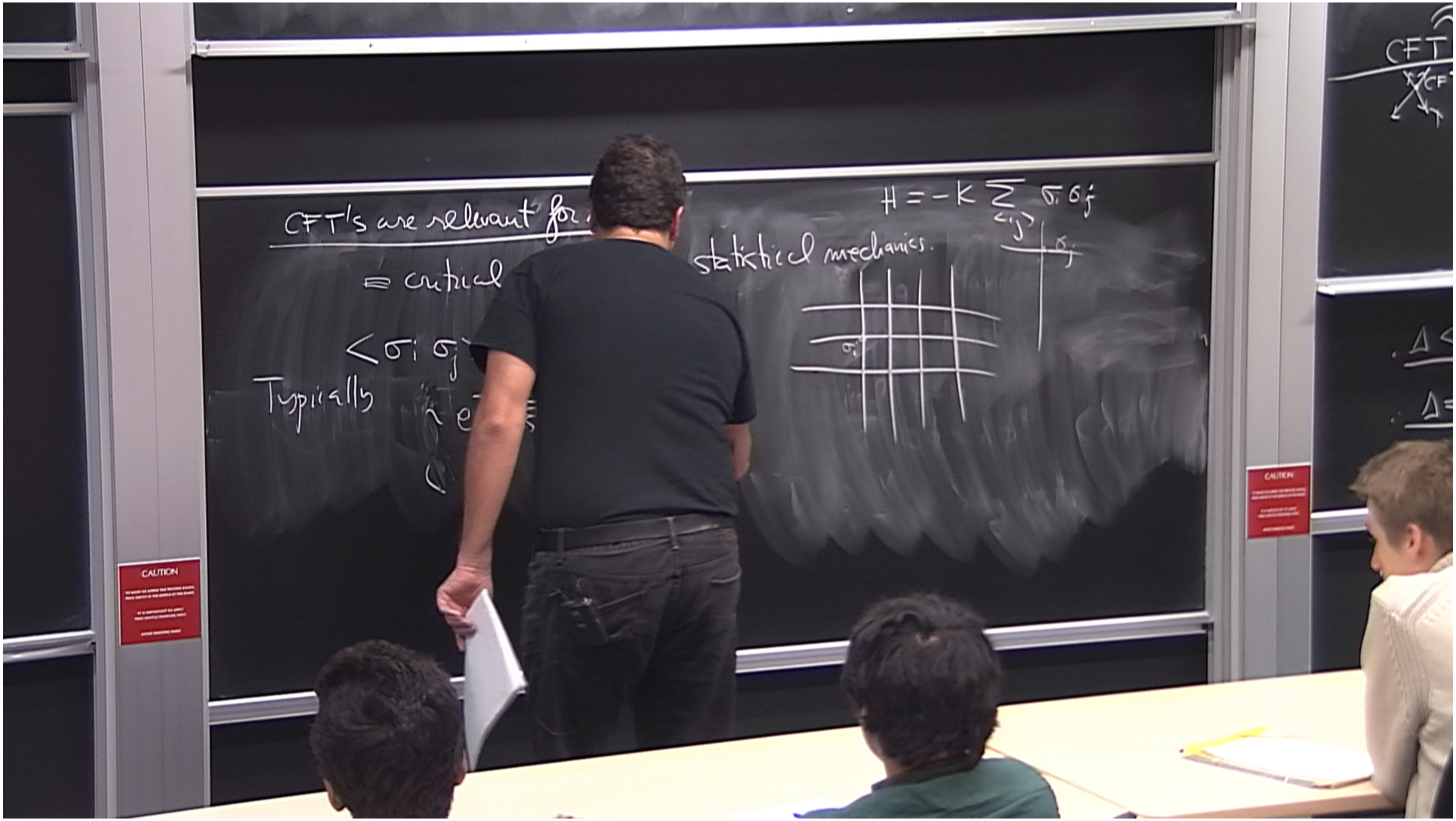

D=4
β(λ) > 0 = 3λ²/16π² + O(λ³)
D = 4-ε dimensions
[λ] = ε
β_{4-ε}(λ) = -ελ + 3λ²/16π²
classical
λ* = 16π²/3 ε
λφ⁴
m²φ²
CFT
λ
λ*


D=4
β(λ)>0 = 3λ²/16π² + O(λ³)
CFT
λ
λφ⁴
m²φ²
D=4-ε dimensions
[λ]= ε
β(λ) = -ελ + 3λ²/16π²
4-ε
classical
λ* = 16π²/3 ε
ε→1 D=3 CFT
Universality of D=3
Ising Model
λ*

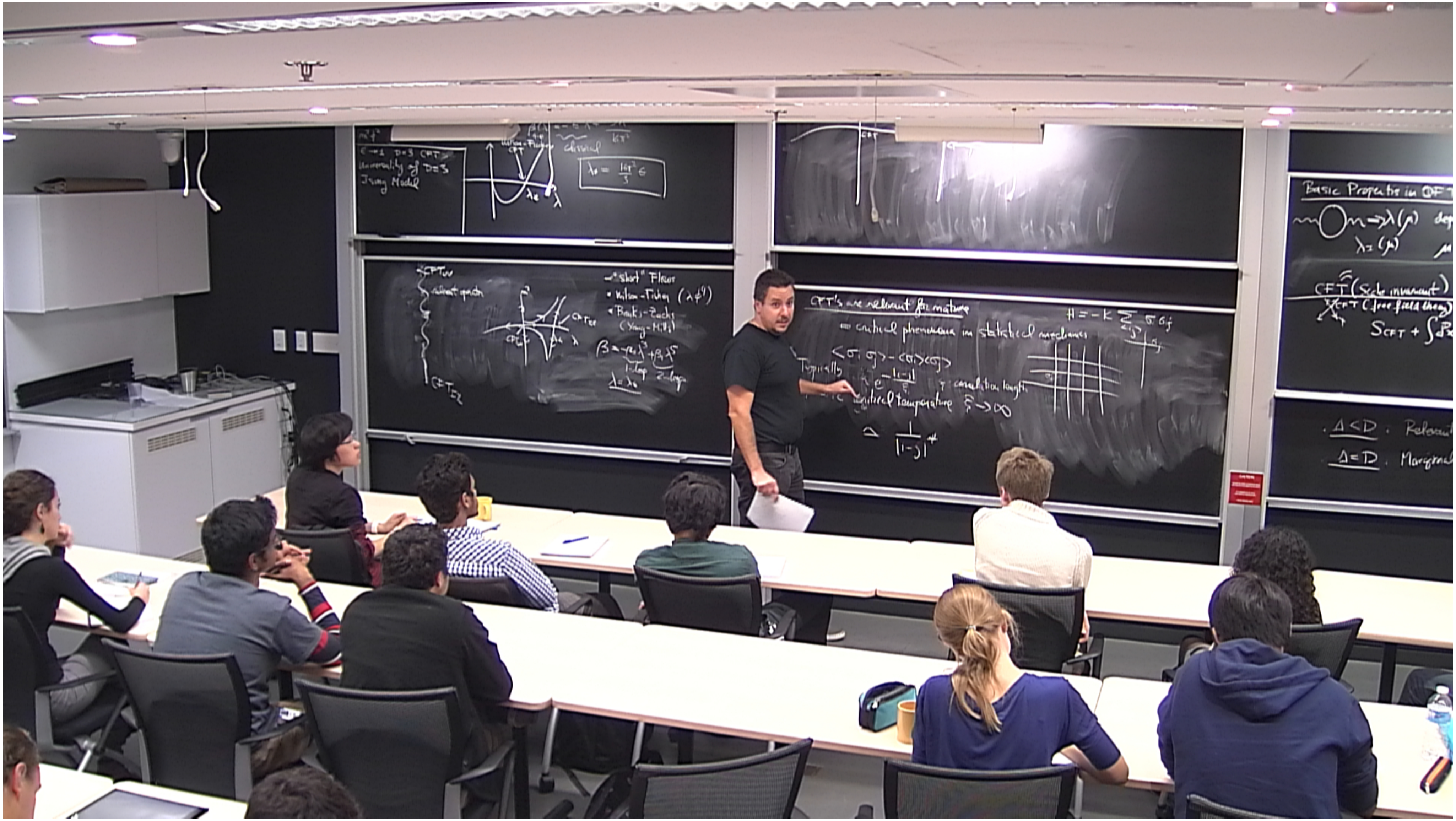

CFT_UV
operator
m²
CFT_IR
CFT_UV
λ
λ*
Reebok

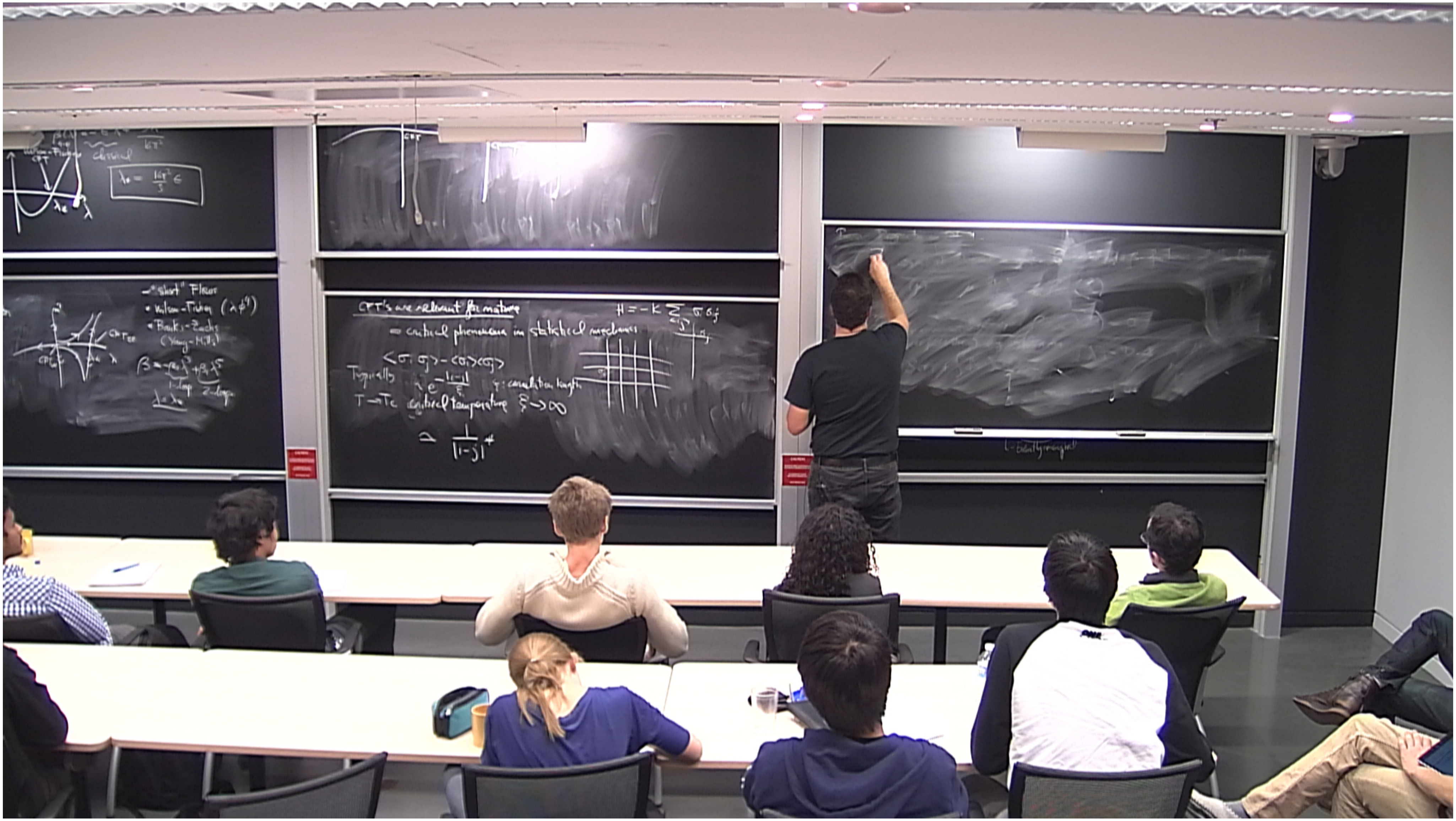

CFT_UV
relevant operator
CFT_IR
m²
CFT_IR
CFT_UV
λ
– "short" Flows
Wilson-Fisher (λφ⁴)

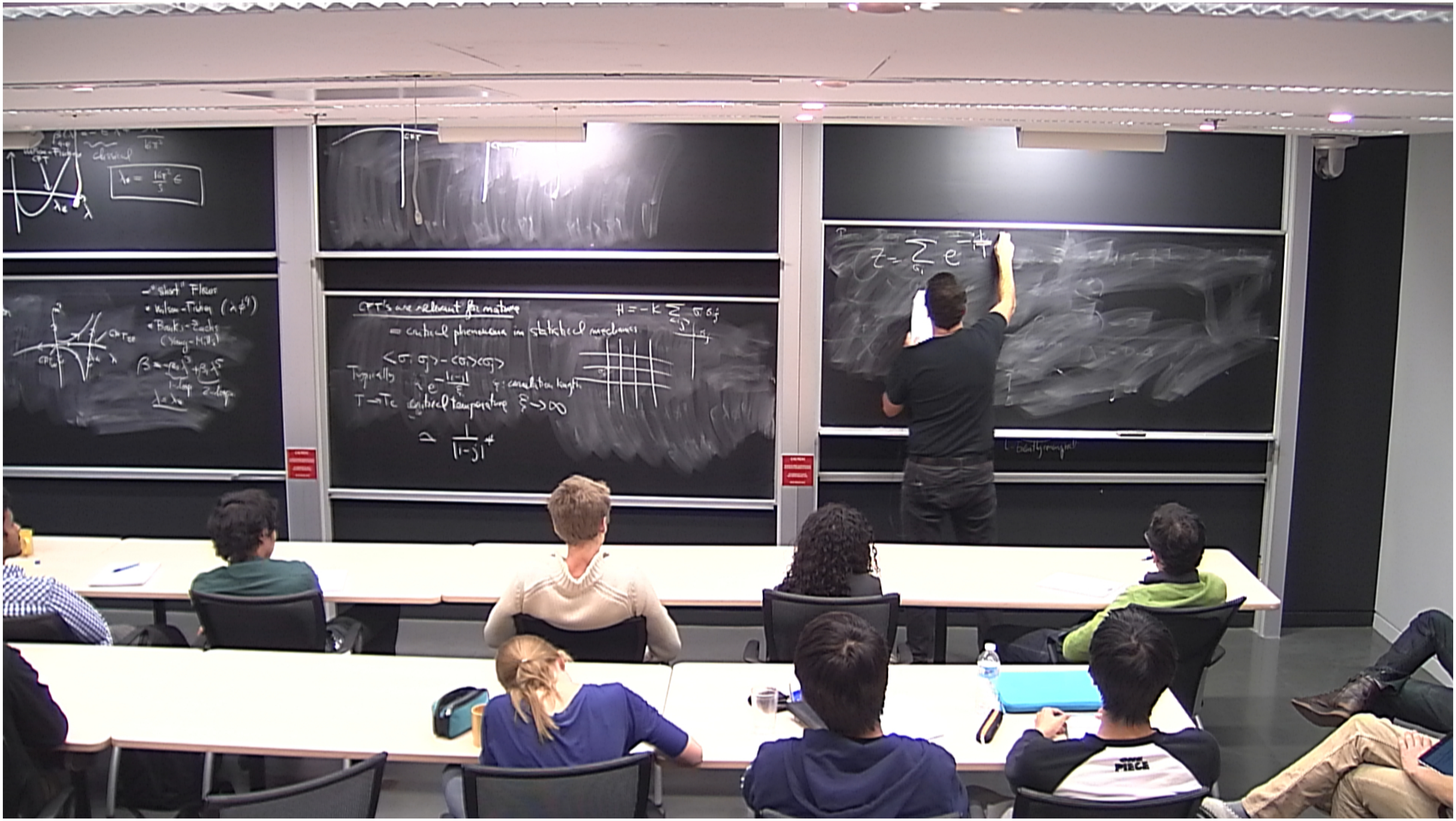

CFT's a   evant for nature
henomena in statistical mechanics.
$H = -K \sum_{<ij>} \sigma_i \sigma_j$
CFT



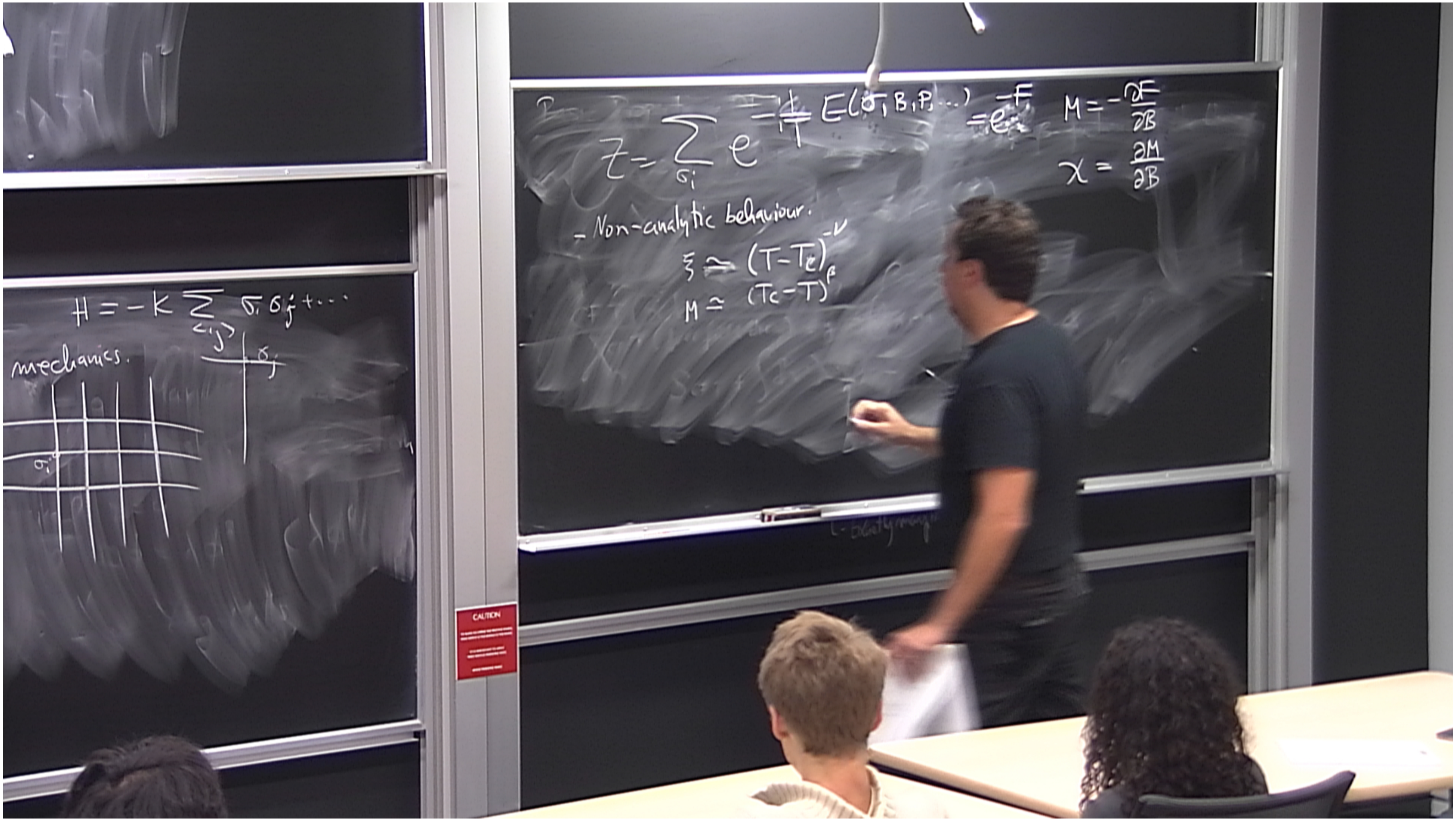
CFT's are relevant for
statistical mechanics.
critical
Typically
H = -K Σ σi σj
<i,j>
σj
σi
CFT

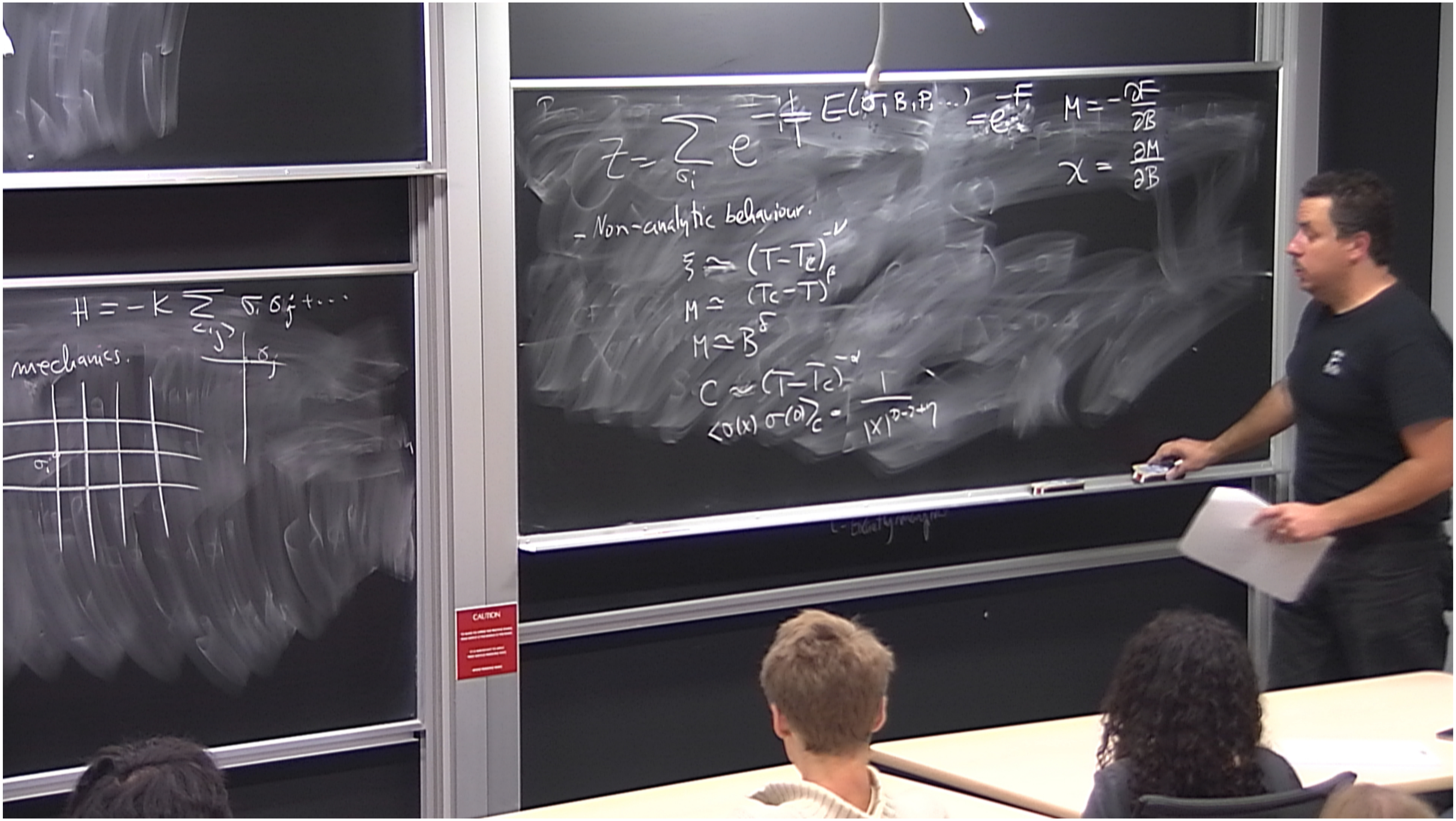

CFT's are relevant for nature
critical phenomena in statistical mechanics
Typically
critical temperature


CFT's are relevant for matter
= critical phenomena in statistical mechanics
ξ: correlation length
T→Tc critical temperature ξ→∞
"Short" Flows
Wilson-Fisher
Banks-Zaks

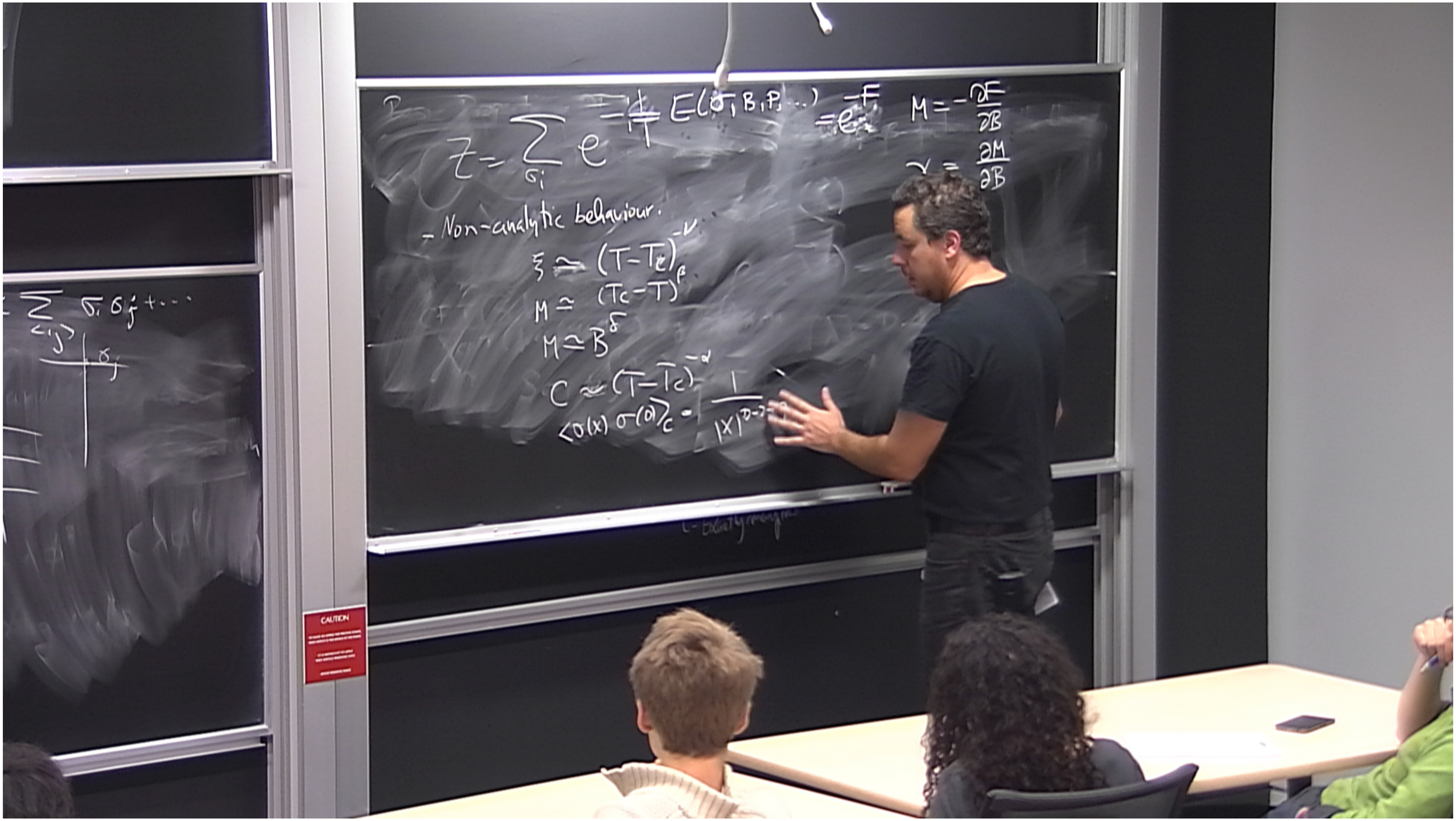

CFT's are relevant for nature
= critical phenomena in statistical mechanics
Typically
critical temperature
"Short" Flows
Wilson-Fisher


H = -K Σ σi σj
mechanics.
Z = Σ e

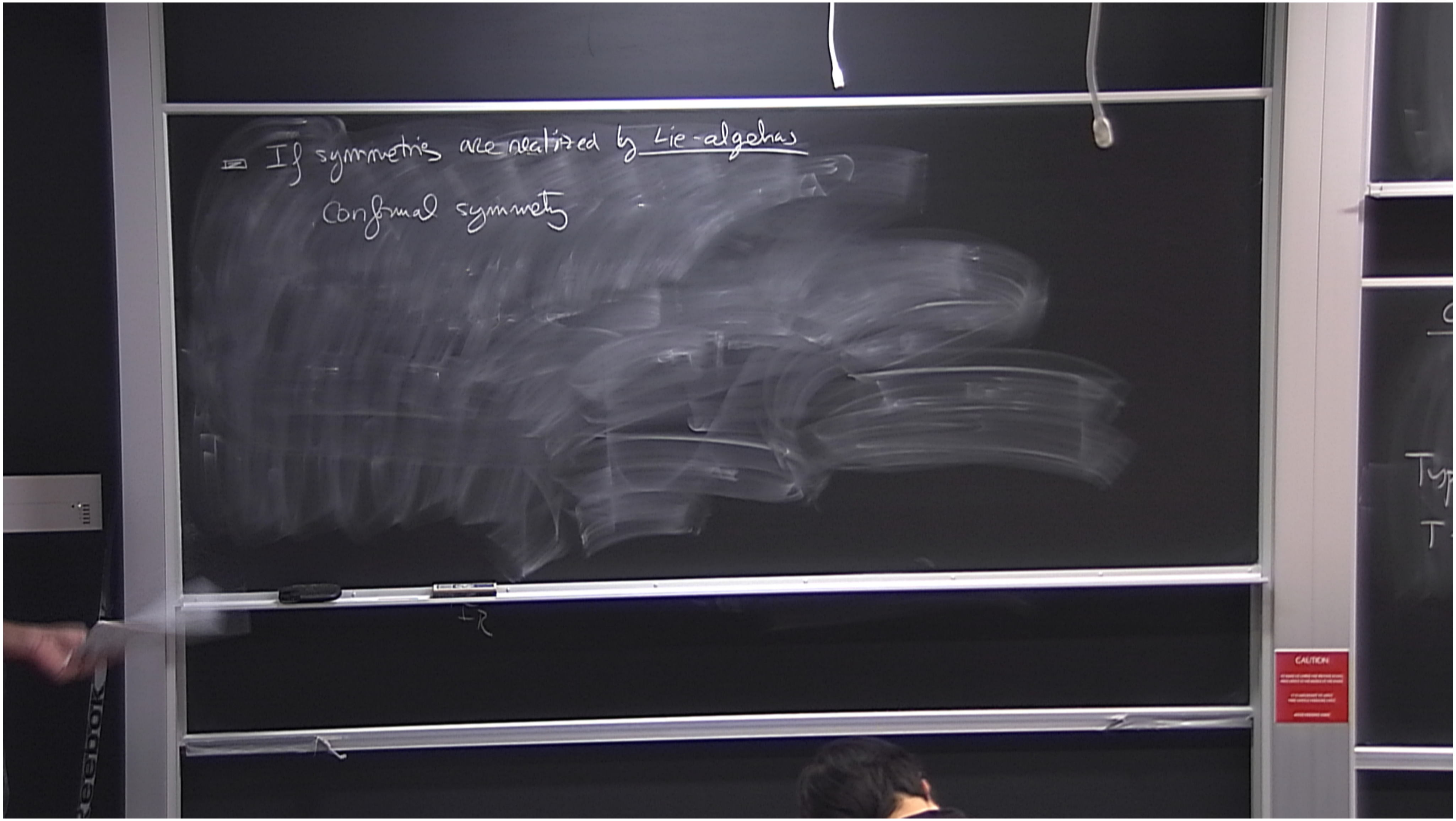

H = -K Σ σ_i σ_j + ...
mechanics.
Z = Σ e^{-1/T E(σ_i, B, P, ...)} = e^{-F}
M = -∂F/∂B
χ = ∂M/∂B
- Non-analytic behaviour.
ξ ≃ (T - T_c)^{-ν}
M ≃ (T_c - T)^β

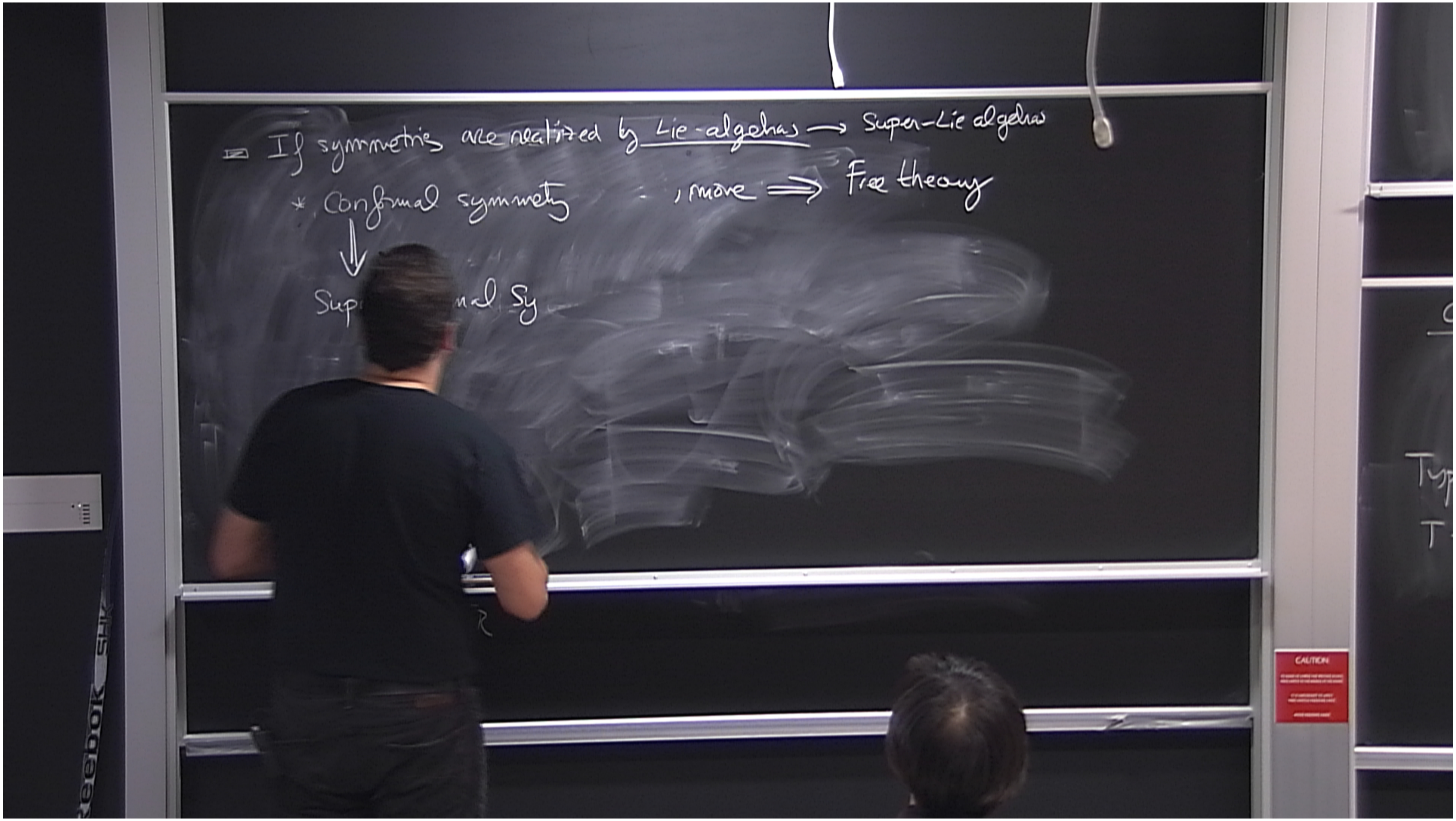


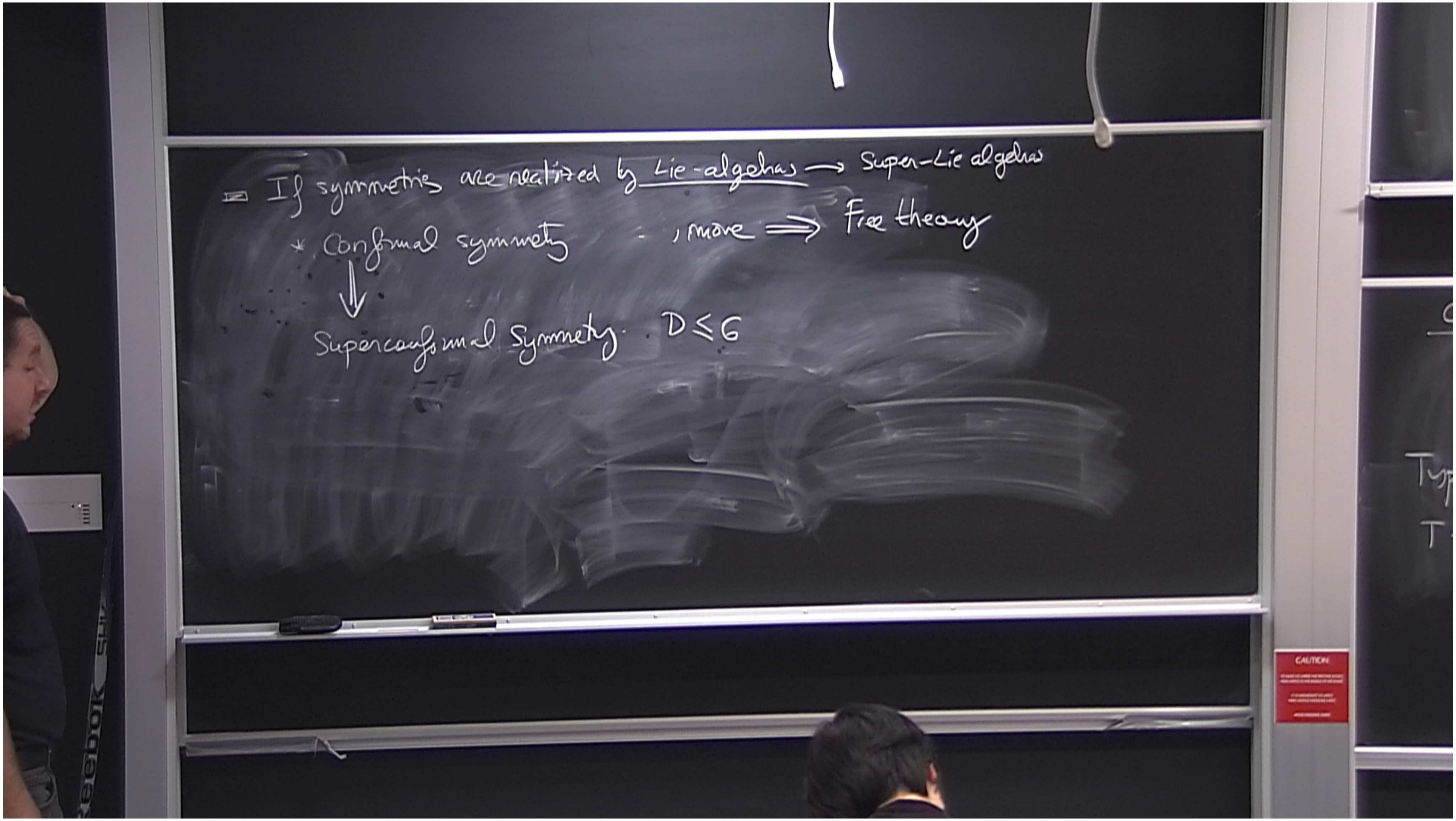
H = -K Σ σ_i σ_j + ...
mechanics.
Z = Σ e^{-1/T E(σ, B, P, ...)} = e^{-F}
M = -∂F/∂B
χ = ∂M/∂B
- Non-analytic behaviour.
ξ ≃ (T-Tc)^{-ν}
M ≃ (Tc-T)^β
M ≃ B^δ
C ≃ (T-Tc)^{-α}
<σ(x) σ(0)>_c = 1/|x|^{D-2+η}
CAUTION

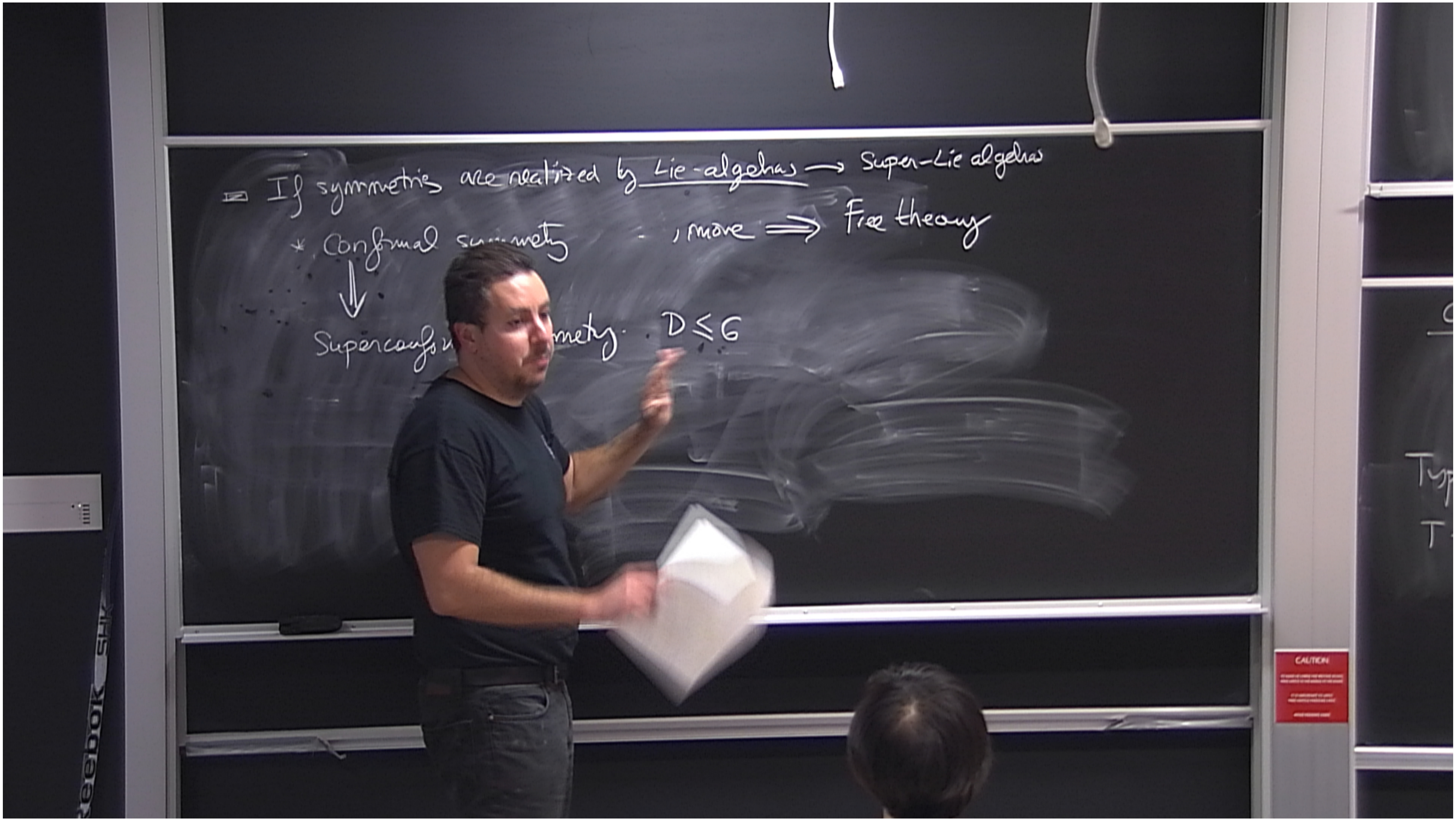

Z = Σ e
M = - ∂F/∂B
∂M/∂B
- Non-analytic behaviour.
ξ ≃ (T - Tc)^-ν
M ≃ (Tc - T)^β
M ≃ B^δ
C ≃ (T - Tc)^-α
CAUTION

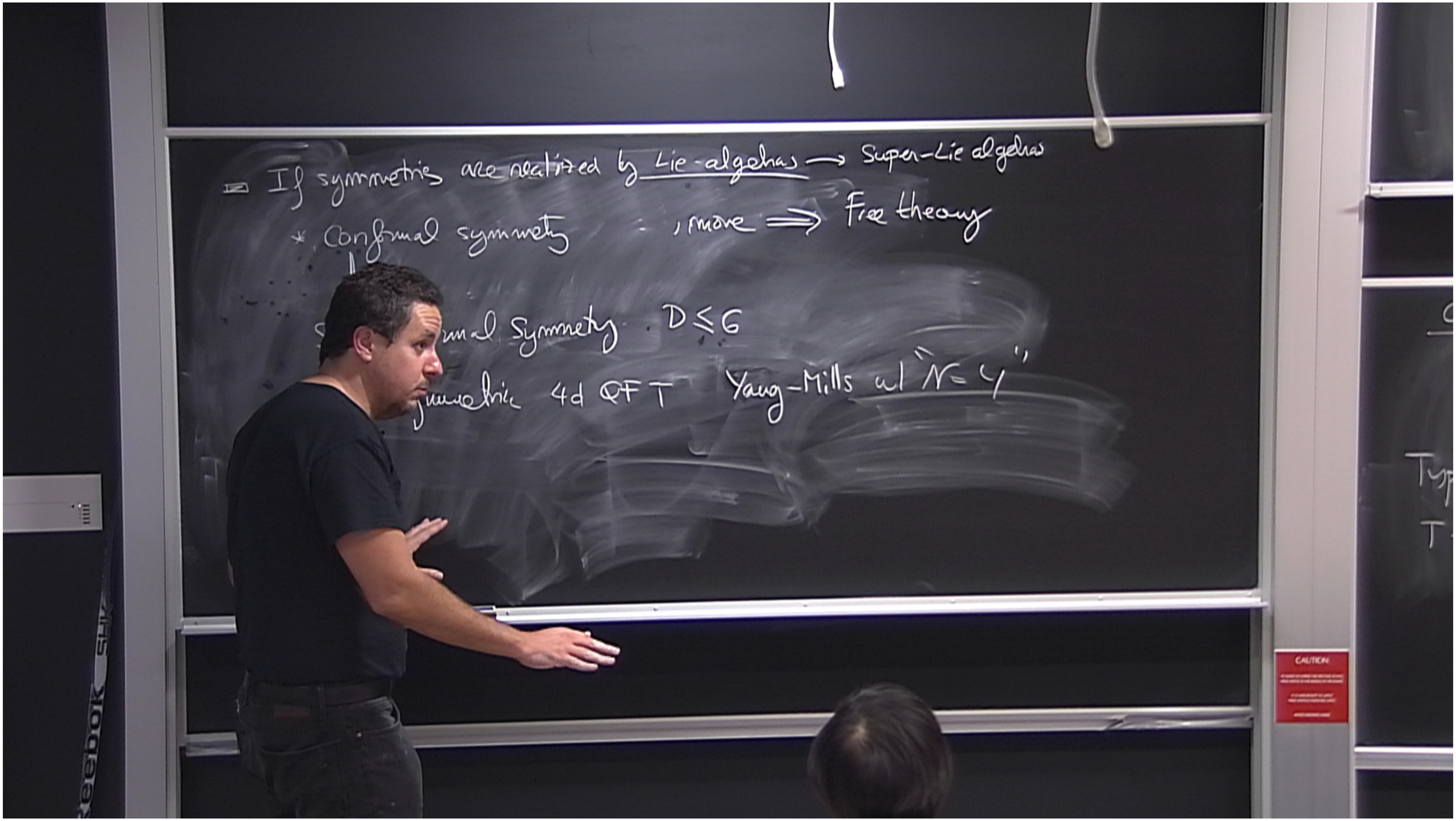

Quantum phase Transitions at T=0
H(λ)
λ→λ*
E1
CAUTION

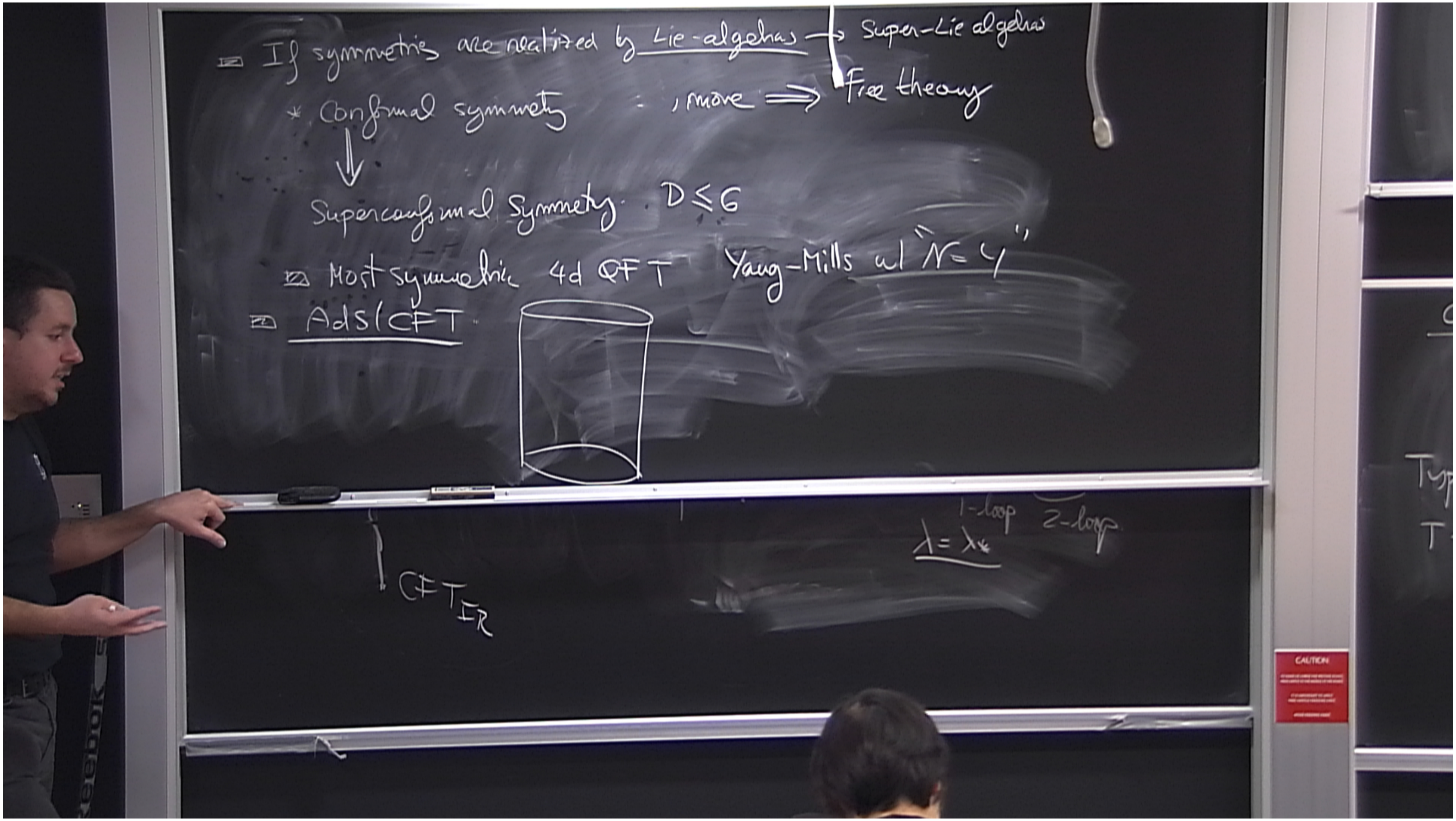

If symmetries are realized by Lie-algebras
Conformal symmetry

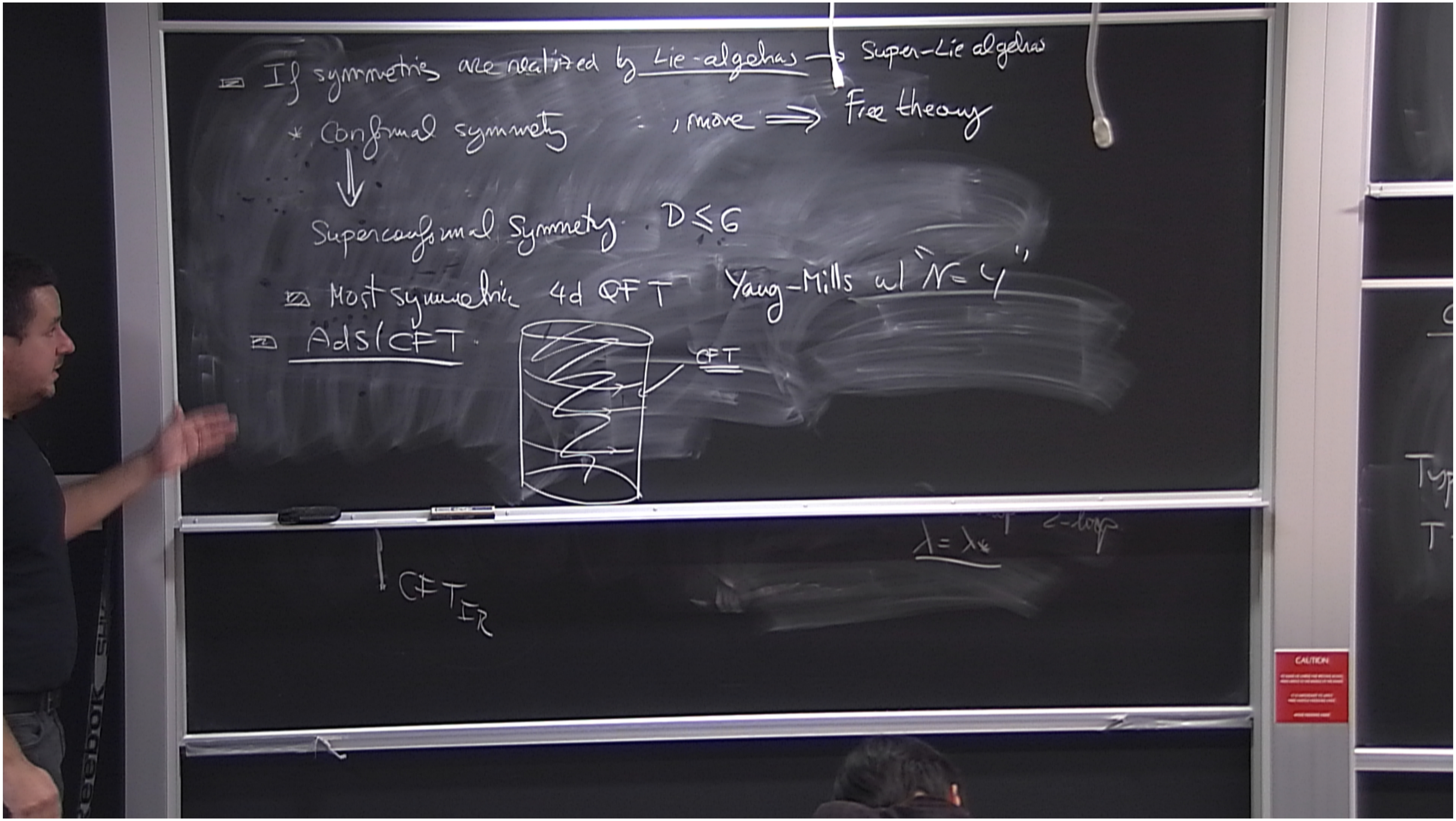

If symmetries are realized by Lie-algebras → Super-Lie algebras
* Conformal symmetry , more ⟹ Free theory
CAUTION

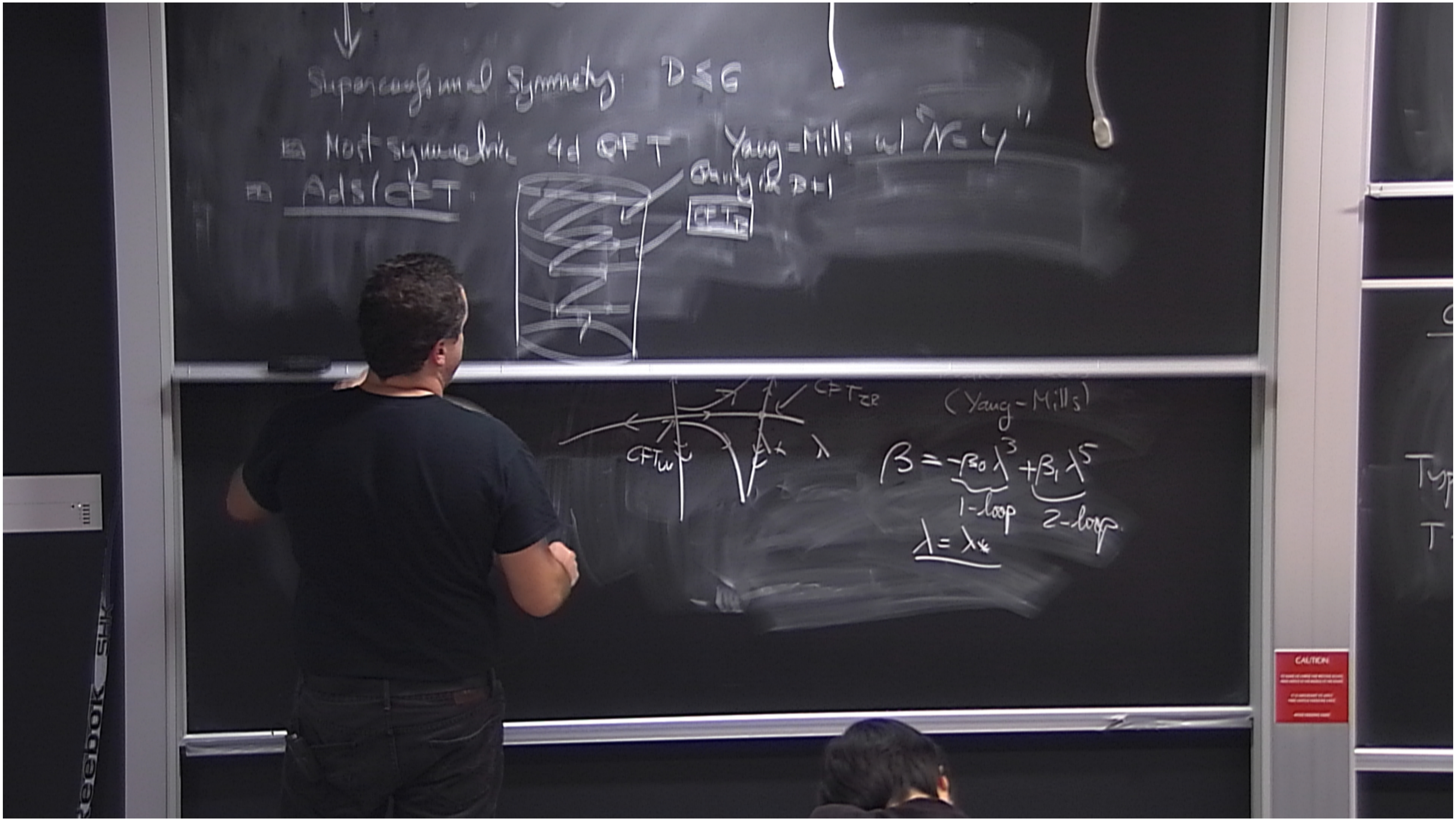

If symmetries are realized by Lie-algebras → Super-Lie algebras
* Conformal symmetry , more ⟹ Free theory
Superconformal Symmetry . D ≤ 6
CAUTION

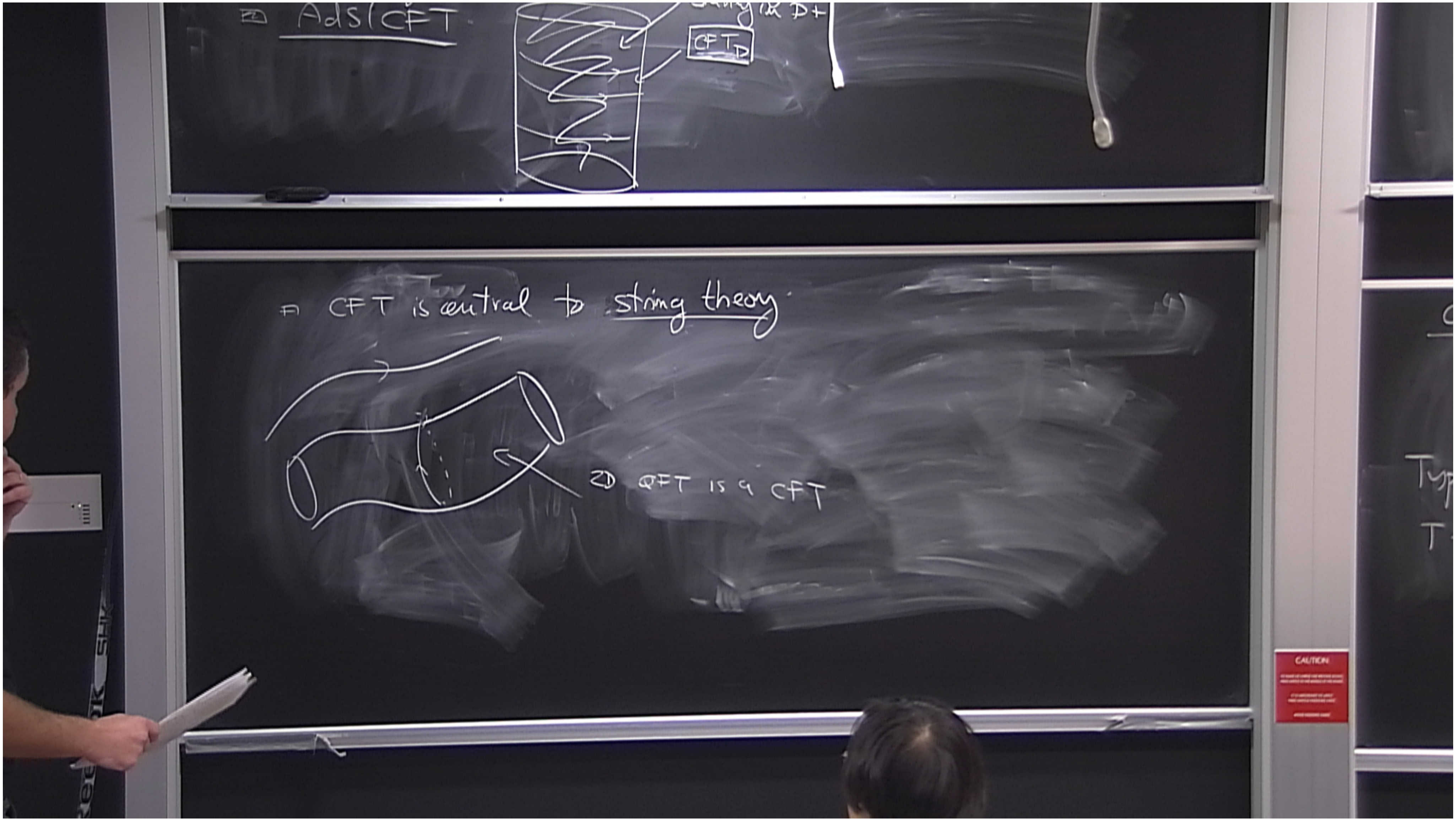

If symmetries are realized by Lie-algebras → Super-Lie algebras
* Conformal symmetry , more ⇒ Free theory
D ≤ 6
CAUTION

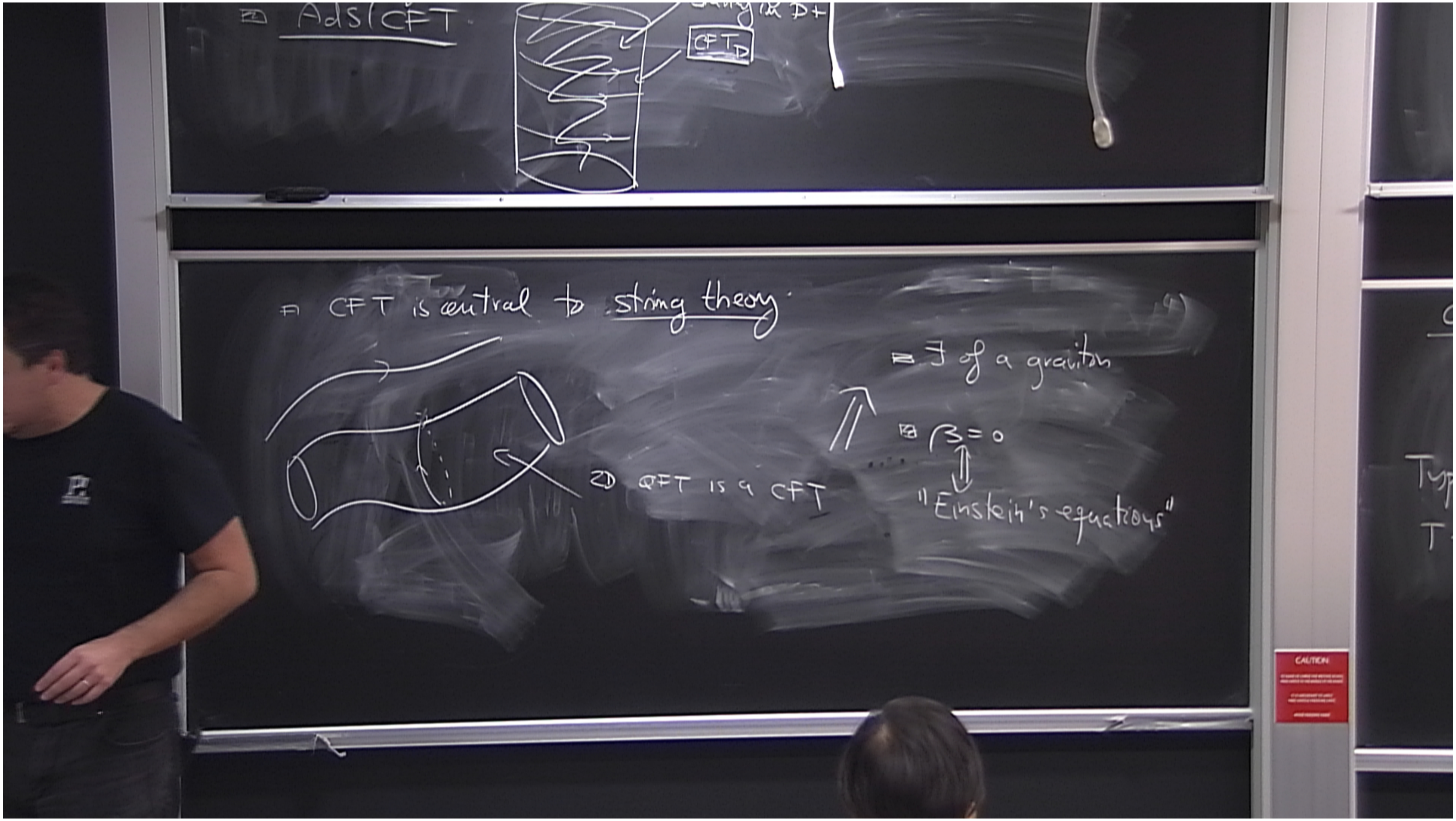

If symmetries are realized by Lie-algebras → Super-Lie algebras
* Conformal symmetry , more ⟹ Free theory
Symmetry D ≤ 6
4d QFT Yang-Mills w/ "N=4"


If symmetries are realized by Lie-algebras → Super-Lie algebras
* Conformal symmetry , more ⟹ Free theory
Superconformal Symmetry D ≤ 6
Most symmetric 4d QFT Yang-Mills w/ "N=4"
AdS/CFT
1-loop 2-loop
λ = λ*
CFT IR


If symmetries are realized by Lie-algebras → Super-Lie algebras
* Conformal symmetry
, more ⟹ Free theory
Superconformal Symmetry D ≤ 6
Most symmetric 4d QFT
Yang-Mills w/ "N=4"
AdS/CFT
CFT
CFT IR
λ = λ*
2-loop


Superconformal Symmetry: D ≤ 6
Most symmetric 4d QFT
Yang-Mills w/ "N=4"
AdS/CFT
Gravity in D+1
CFT_D
CFT_IR
(Yang-Mills)
CFT_UV
λ
β = -β_0 λ^3 + β_1 λ^5
1-loop
2-loop
λ = λ*
CAUTION


AdS/CFT
CFT
CFT is central to string theory
2D QFT is a CFT


AdS/CFT
CFT_D
CFT is central to string theory
≅ ∃ of a graviton
β = 0
"Einstein's equations"
2D QFT is a CFT