Title: Entanglement entropy of Wilson loops: Holography and matrix models

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Abstract: A half-BPS circular Wilson loop in maximally supersymmetric SU(N) Yang-Mills theory in an arbitrary representation is described by a Gaussian matrix model with a particular insertion. The additional entanglement entropy of a spherical region in the presence of such a loop was recently computed by Lewkowycz and Maldacena using exact matrix model results. In this talk I will utilise the supergravity solutions that are dual to such Wilson loops in large representations to calculate this entropy holographically. Employing the results of Gomis, Matsuura, Okuda and Trancanelli to express this holographic entanglement entropy in a matrix model language, I will demonstrate complete agreement with the formula derived by Lewkowycz and Maldacena.

Entanglement entropy of Wilson loops: Holography and matrix models

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28 October 2014

Based on [1407.5629] with Michael Gutperle

Goal

- Consider the entanglement entropy in the presence of a half-BPS Wilson loop in a large representation using holography
- Demonstrate agreement with a general formula derived by [Lewkowycz & Maldacena]
- Utilise the bubbling geometries of [D'Hoker, Estes & Gutperle] and the matrix model results of [Gomis, Matsuura, Okuda & Trancanelli]

Outline

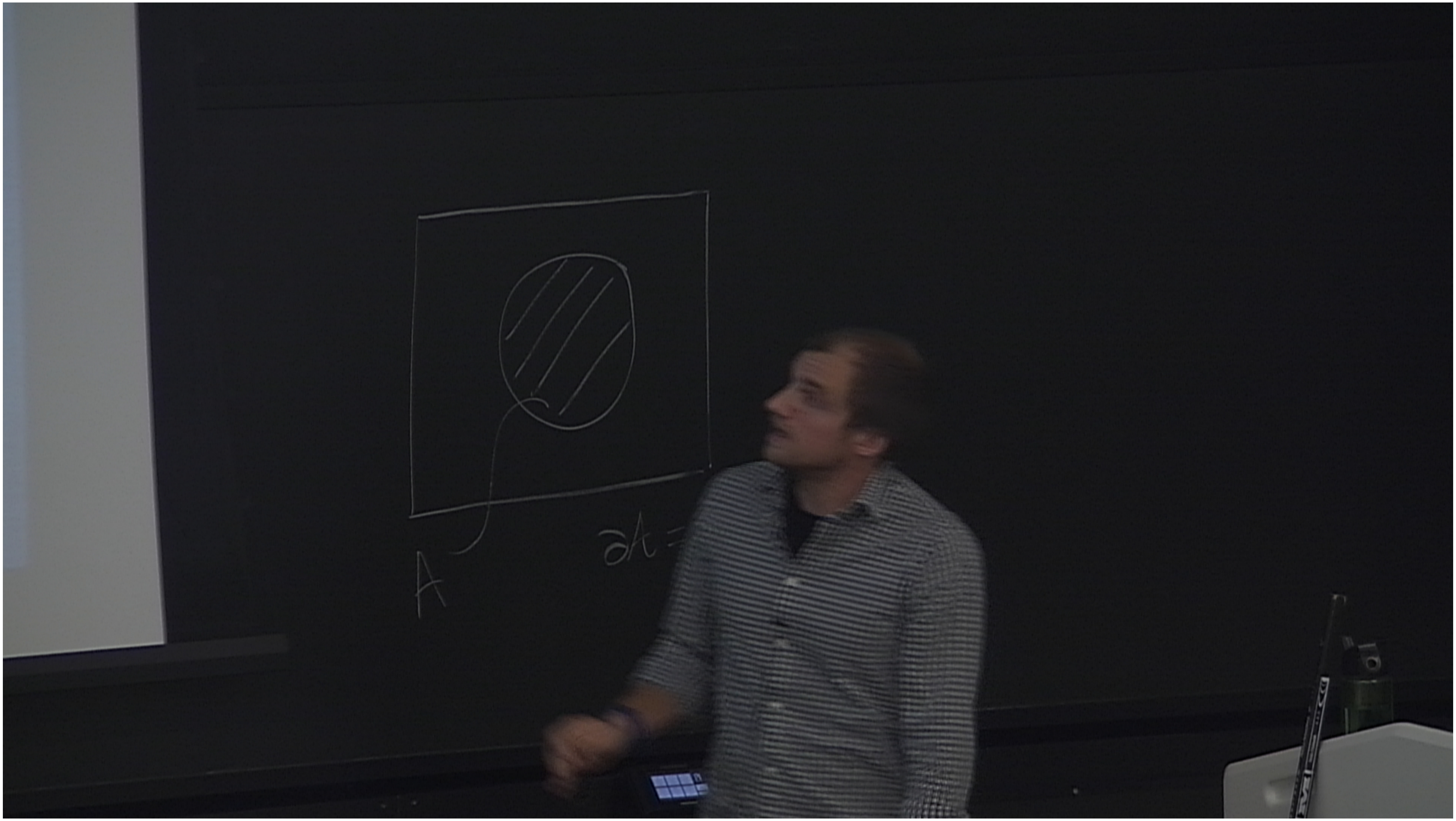
1. LM formula
2. Matrix model description of half-BPS Wilson loops
3. Bubbling geometries
4. Holographic computation
5. Summary

Entanglement entropy of ball-shaped regions

- Entanglement entropy:

$$S_{\mathcal{A}} = -\operatorname{tr} \rho_{\mathcal{A}} \log \rho_{\mathcal{A}}$$

  von Neumann entropy of the reduced density matrix $\rho_{\mathcal{A}}$ associated with a spatial region $\mathcal{A}$
- Consider a region with spherical boundary $\partial\mathcal{A}$ with radius R in a CFT_d
- $S_{\mathcal{A}}$ is given by the thermodynamic entropy of the same theory at inverse temperature $\beta = 2\pi R$ defined on $S^1_\beta \times H^{d-1}$ [Casini, Huerta & Myers]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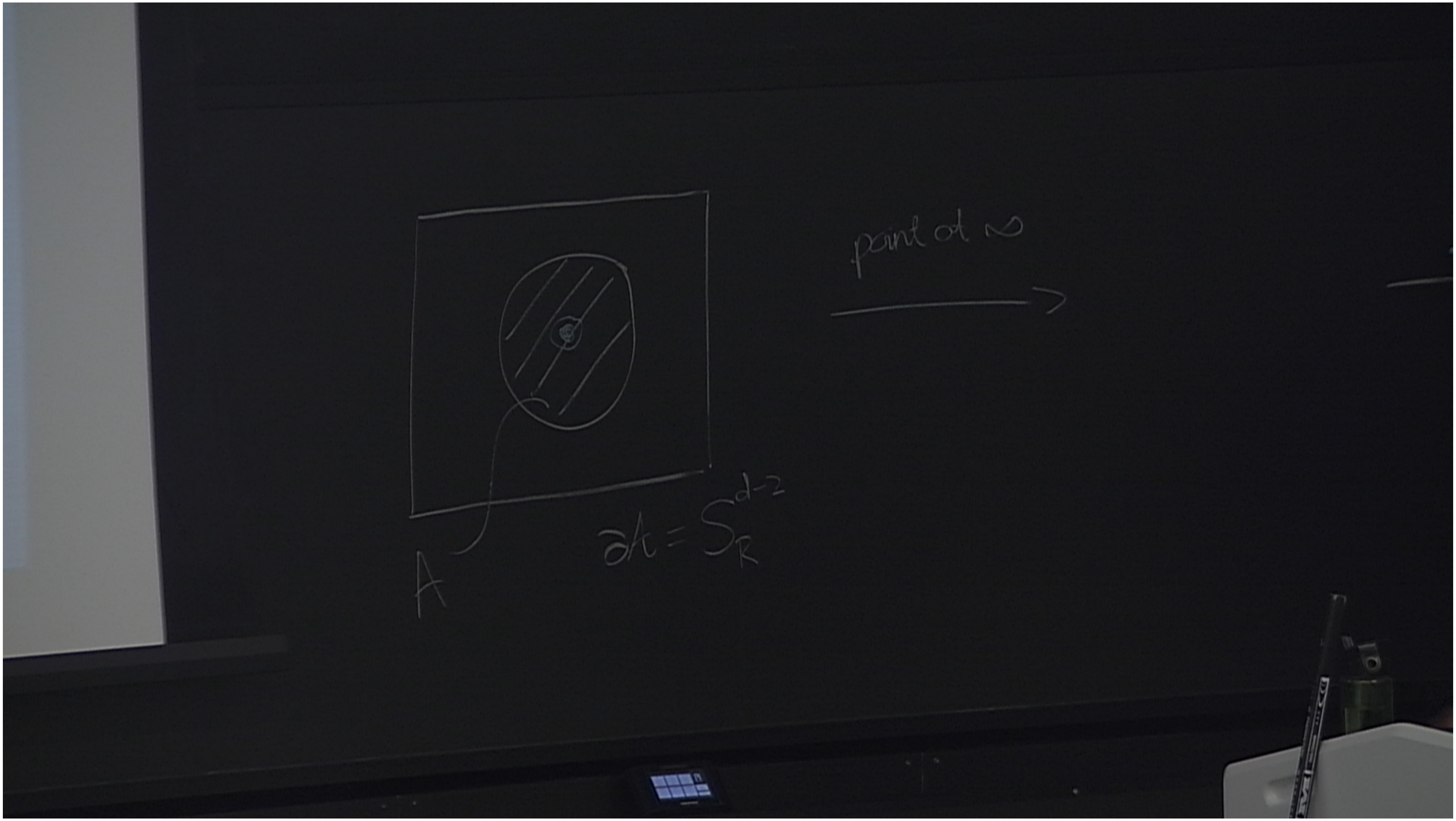
Entanglement entropy in the presence of a Wilson loop

- Consider $\Delta S_{\mathcal{A}}$: the additional entanglement entropy in the presence of a circular Wilson loop relative to the vacuum
- Need the free energy and its first derivative with respect to temperature in the presence of the loop
- Field theory result [Lewkowycz & Maldacena]:

$$\Delta S_{\mathcal{A}} = \log \langle W_{\mathcal{R}} \rangle + 2\pi \, \mathrm{Vol}(S^{d-2}) \, h_W$$

where h_W is the scaling weight of the loop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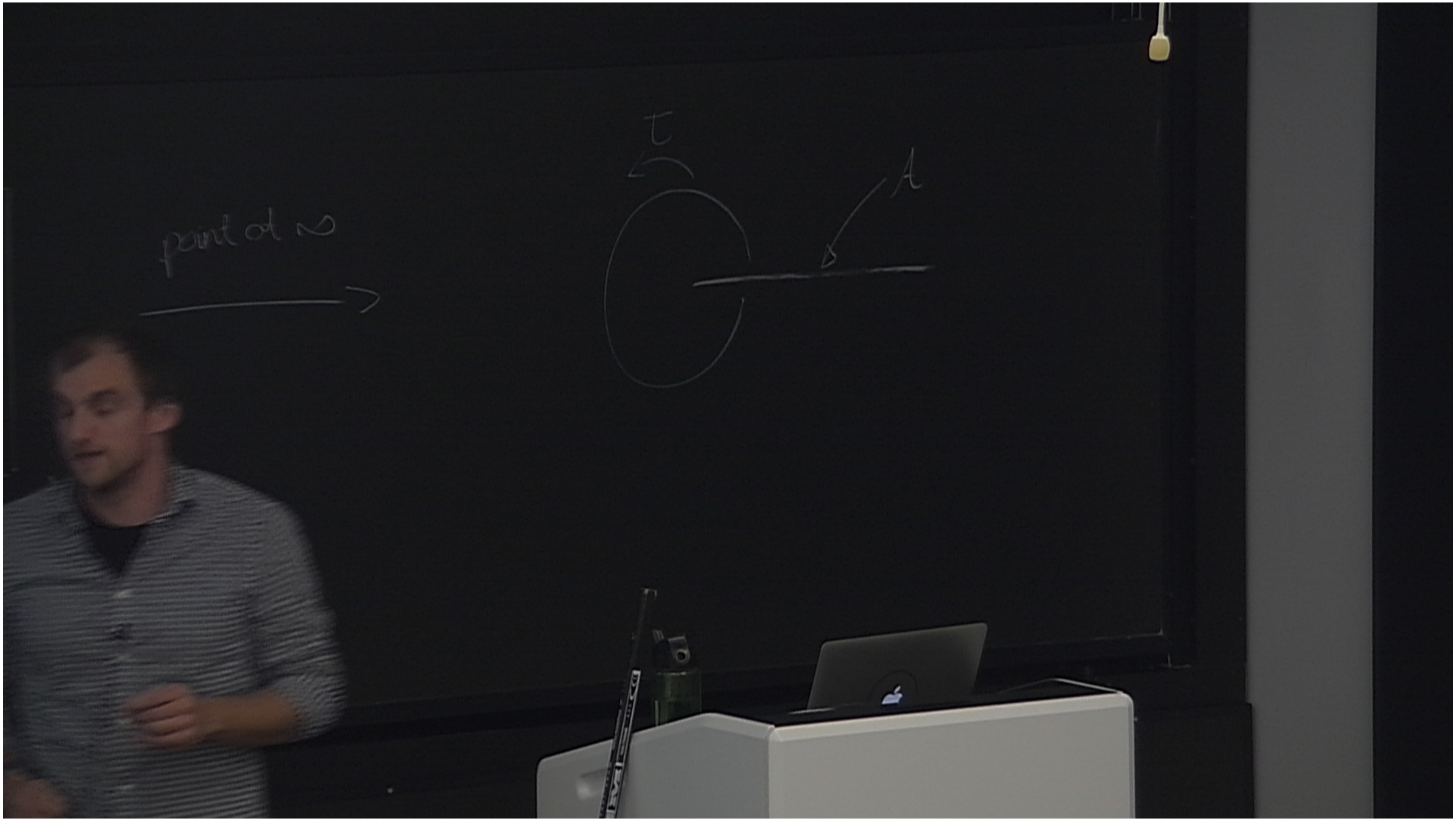
∂A = S_R^{d-2}



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A sketch of the derivation

- Thermodynamic entropy and some definitions:

$$S = (1 - \beta\partial_\beta)\log Z$$

$$Z_W = \int \mathcal{D}\phi\, W\, e^{-S} \quad \text{and} \quad \langle X\rangle_W = \frac{\langle XW\rangle}{\langle W\rangle}$$

- Key step:

$$\beta\partial_\beta \log Z_W = -\beta \int d^dx \left\langle \frac{\partial\sqrt{g}\mathcal{L}}{\partial g^{\mu\nu}}\, \partial_\beta g^{\mu\nu} \right\rangle_W = -\int d^dx \sqrt{g}\, \langle T_{\tau\tau}\rangle_W$$

- Conformal invariance, tracelessness and conservation fix $\langle T_{\mu\nu}\rangle_W$ up to a constant h_W and the result follows from $S_W - S$

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Our Wilson loop as a matrix model

- We consider the half-BPS circular Wilson loop in $d = 4$, $\mathcal{N} = 4$ $SU(N)$ supersymmetric Yang-Mills theory in representation $\mathcal{R}$

$$W_{\mathcal{R}} = \mathrm{tr}_{\mathcal{R}} \, \mathrm{P} \exp i \oint ds \left(A_\mu \dot{x}^\mu + i \Phi_I n^I |\dot{x}| \right)$$

- The expectation value of this loop localises to the expectation value of a specific operator in the Gaussian matrix model [Pestun]:

$$\langle W_{\mathcal{R}} \rangle = \frac{1}{\mathcal{Z}} \int [dM] \, \mathrm{tr}_{\mathcal{R}} \, e^{M'} \exp\left(-\frac{2N}{\lambda} \, \mathrm{tr} \, M^2 \right)$$

  where $M' \equiv M - \frac{1}{N} (\mathrm{tr} \, M) \mathbb{1}_{N \times N}$.
- Our focus: large representations with $|\mathcal{R}| \sim O(N^2)$

Matrix models

- Quantum gauge theory in zero dimensions of an $N \times N$ Hermitian matrix M
- Work in eigenvalue formulation: $M \to \operatorname{diag}(m_1, m_2, \ldots, m_N)$
- Saddle-point approximation: fix coupling λ and take $N \to \infty$
- Assume distribution of eigenvalues is continuous with compact support on an interval $\mathcal{C}$:

$$\frac{1}{N}\sum_{i=1}^{N} f(m_i) \to \int_{\mathcal{C}} dx\, \rho(x) f(x)$$

Partition function

$$
\begin{aligned}
\mathcal{Z} &\propto \int [dM] \exp\left(-\frac{2N}{\lambda}\,\mathrm{tr}\,M^2\right) \\
&= \int \prod_{i=1}^{N} dm_i\, \Delta(m)^2 \exp\left(-\frac{2N}{\lambda}\sum_{j=1}^{N} m_j^2\right), \quad \Delta(m)^2 \equiv \prod_{k<l} |m_k - m_l|^2 \\
&\equiv \int \prod_{i=1}^{N} dm_i\, \exp\left(-\mathcal{S}_0\right)
\end{aligned}
$$

$$
\begin{aligned}
-\mathcal{S}_0 &= -\frac{2N}{\lambda}\sum_{i=1}^{N} m_i^2 + 2\sum_{i<j} \log|m_i - m_j| \\
&\to -\frac{2N^2}{\lambda}\int_{\mathcal{C}} dx\, \rho(x)\, x^2 + N^2 \int_{\mathcal{C}\times\mathcal{C}} dx\, dy\, \rho(x)\, \rho(y) \log|x-y|
\end{aligned}
$$

Matrix model resolvent

- Solve saddle-point equation by introducing an auxiliary function: the resolvent

$$\omega(z) = \lambda \int_{\mathcal{C}} dx \, \frac{\rho(x)}{z - x}$$

- Analytic on the whole complex z plane except a discontinuity across the interval $\mathcal{C}$
- Express via contour deformations as

$$\rho(x) = \frac{i}{2\pi\lambda} \left(\omega_+(x) - \omega_-(x) \right) \quad \text{where} \quad \omega_\pm(x) \equiv \omega(x \pm i\epsilon)$$

- In the case of $\mathcal{S}_0$ the eigenvalues follow the Wigner semicircle law:

$$\rho_{(0)}(x) = \frac{2}{\pi\lambda} \sqrt{\lambda - x^2} \quad \text{for} \quad \mathcal{C} = \left[-\sqrt{\lambda}, \sqrt{\lambda} \right]$$

Our Wilson loop as a matrix model (II)

- Compute our Wilson loop using saddle-point methods as before, but now with the insertion of $\mathrm{tr}_{\mathcal{R}}\, e^{M'}$:

$$\log \langle W_{\mathcal{R}} \rangle = -\left(\mathcal{S}_{\text{Wilson}} - \mathcal{S}_0\right)$$

- Divide Young tableau $\mathcal{R}$ into g blocks, where the I^{th} block has n_I rows of length K_I and we define $\hat{K}_I \equiv K_I - |\mathcal{R}|/N$
- The effective action includes a linear shift and multiple intervals and the interactions simplify at large λ [Okuda & Trancanelli]:

$$-\mathcal{S}_{\text{Wilson}} = N \sum_{I=1}^{g+1} \int_{\mathcal{C}_I} dx\, \rho(x) \left(-\frac{2N}{\lambda}\, x^2 + \hat{K}_I\, x\right) + N^2 \int_{\mathcal{C}\times\mathcal{C}} dx\, dy\, \rho(x)\, \rho(y) \log|x-y|$$

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Return to LM formula

- Recall for $d = 4$:

$$\Delta S_{\mathcal{A}} = \log \langle W_{\mathcal{R}} \rangle + 8\pi^2 h_W$$

- The coefficient h_W can be related to the second moment of the eigenvalue distribution [Gomis, Matsuura, Okuda & Trancanelli]:

$$h_W = -\frac{N^2}{3\pi^2 \lambda} \Delta\rho_2 \quad \text{where} \quad \Delta\rho_2 \equiv \rho_2 - \rho_2^{(0)}$$

$$\rho_2 \equiv \int_{\mathcal{C}} dx\, \rho(x)\, x^2 \quad \text{and} \quad \rho_2^{(0)} = \frac{\lambda}{4}$$

- Our computation: holographic entanglement entropy agrees precisely with the LM result in this form

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Holographic description of Wilson loops

Depends on the size of the representation $\mathcal{R}$:

- fundamental: probe string [Rey & Yee; Maldacena]
- k^{th} (anti-) symmetric: probe (D5-) D3-brane with k units of worldvolume electric flux [Gomis & Passerini]
- $|\mathcal{R}| \sim O(N^2)$: probe description breaks down and is replaced by a fully backreacted bubbling geometry [D'Hoker, Estes & Gutperle]

Symmetry ansatz in IIB supergravity

- Bosonic symmetry group:

$$SO(2,1) \times SO(3) \times SO(5)$$

- Fibration of $AdS_2 \times S^2 \times S^4$ over a Riemann surface Σ with boundary $\partial\Sigma$:

$$ds^2 = f_1^2\, ds^2_{AdS_2} + f_2^2\, ds^2_{S^2} + f_4^2\, ds^2_{S^4} + 4\sigma^2 d\Sigma^2$$

$$ds^2_{AdS_2} = \frac{dv^2 + d\tau^2}{v^2} \quad \text{and} \quad d\Sigma^2 = |dw|^2$$

- All functions depend on $w, \bar{w}$
- Example: $AdS_5 \times S^5$ with $w = x + i\theta$

$$f_1 = L\cosh x, \quad f_2 = L\sinh x, \quad f_4 = L\sin\theta, \quad \sigma = L/2$$

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Harmonic functions and boundary conditions

- Solution characterised by two real harmonic functions h_1, h_2
- Take $\Sigma =$ LHP and $\partial\Sigma = \mathbb{R}$
- Regularity imposes boundary conditions on h_1, h_2 at $\partial\Sigma$:

$$h_2 \sim i(w - \bar{w}) \Rightarrow h_2|_{\partial\Sigma} = 0$$

$$h_1|_{\partial\Sigma} \; : \; \text{alternating Neumann and Dirichlet BCs}$$

- Genus g solution: $2g + 2$ real numbers (branch points)
- Higher genus solutions admit more nontrivial 3- and 5-cycles (bubbles), which support non-zero 3- and 5-form flux

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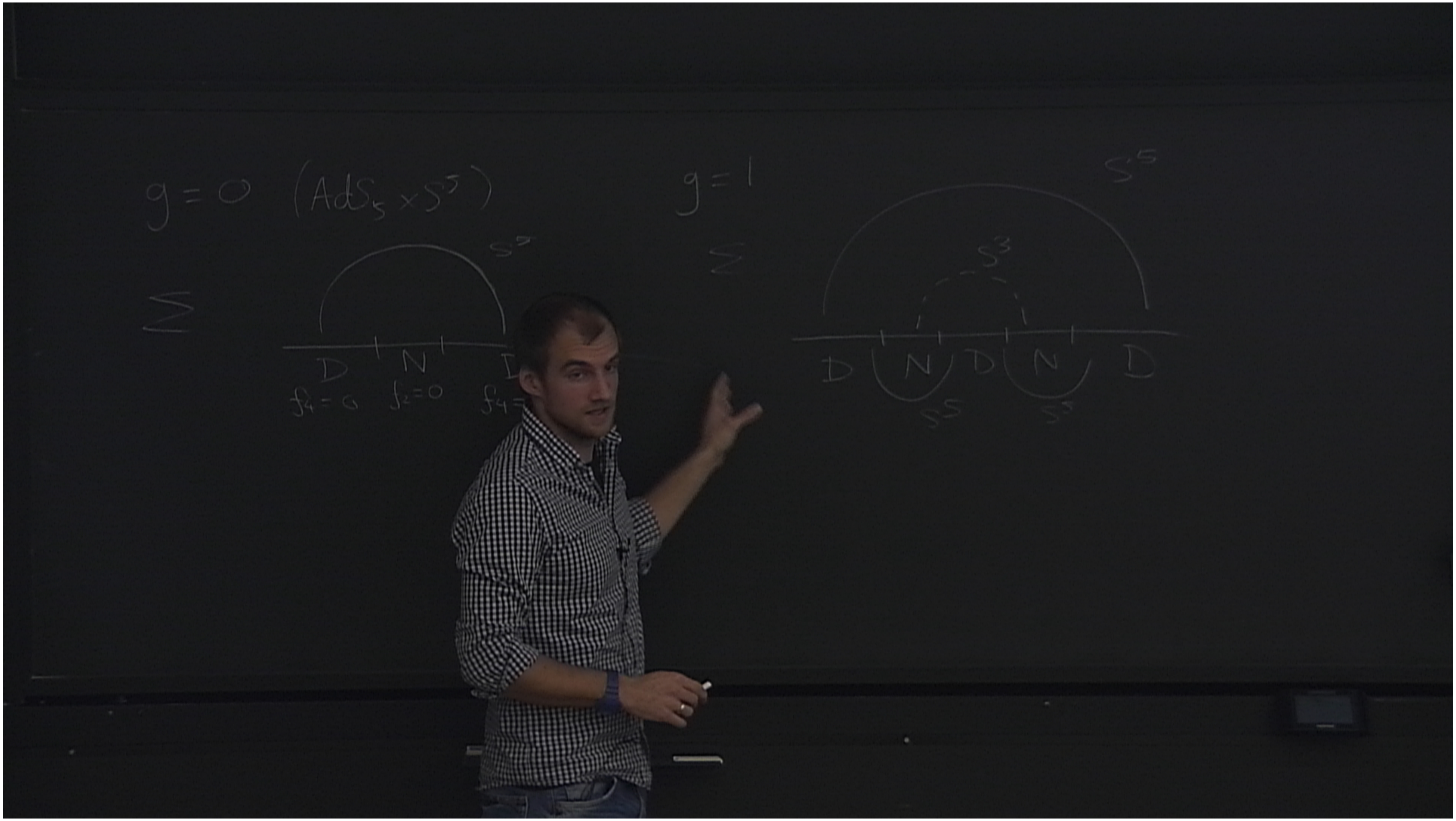
Some examples

- $g = 0$: $AdS_5 \times S^5$
- $g = 1$: expressed in terms of elliptic integrals
- Explicit formulae not known for $g \geq 2$


g = 0 (AdS5 × S5)
g = 1
S5
S3
D
N
D
D
N
D
N
D
S5
S5

Map to field theory and matrix model

- Integrals of fluxes through nontrivial cycles for genus g are given by the n_I, K_I of a Young tableau [Okuda & Trancanelli]
- The resolvent $\omega(z)$ of the matrix model can be related to the branch points that characterise the bubbling solution [Okuda & Trancanelli]
- Ansatz and constraints for $\omega(z)$ match regularity constraints and flux integrals for the geometry if we identify

$$h_1 = \frac{i\alpha'}{8g_s}\left[2(z-\bar{z}) - (\omega - \bar{\omega})\right] \quad \text{and} \quad h_2 = \frac{i\alpha'}{4}(z-\bar{z})$$

- We also identify z with the coordinate on Σ: $z \equiv w$
- Use this to phrase the holographic entanglement entropy in matrix model language

Holographic entanglement entropy

- Minimal surface prescription [Ryu & Takayanagi]:

$$S_{\mathcal{A}} = \frac{A_{\text{min}}}{4G_N}$$

  where A_{min} is the area of a co-dimension two minimal surface anchored at the AdS boundary on $\partial\mathcal{A}$

- Recall: fibration of $AdS_2 \times S^2 \times S^4$ over Σ
- Choose $v = v(z, \bar{z})$ at constant τ and find the area functional

$$A = 2\,\mathrm{Vol}(S^2)\,\mathrm{Vol}(S^4) \int d^2z\, f_2^2\, f_4^4\, \sigma^2 \sqrt{1 + \frac{f_1^2}{v^2\sigma^2}\frac{\partial v}{\partial z}\frac{\partial v}{\partial \bar{z}}}$$

- Minimised by $v =$ constant

Holographic entanglement entropy (II)

- Consider $AdS_5 \times S^5$ with metric

$$ds^2 = L^2 \left[\frac{1}{u^2} \left(du^2 + d\tau^2 + dr^2 + r^2 ds^2_{S^2} \right) + ds^2_{S^5} \right]$$

- The minimal surface at constant τ anchored at $u = 0$ on a sphere of radius $r = R$ is

$$u(r)^2 + r^2 = R^2$$

- In AdS_2 slicing, which we can write as

$$ds^2 = L^2 \left[dx^2 + \cosh^2 x \, ds^2_{AdS_2} + \sinh^2 x \, ds^2_{S^2} + ds^2_{S^5} \right]$$

this surface is given by a constant AdS_2 radial coordinate:

$$v = R$$

- For a general bubbling geometry (asymptotic to $AdS_5 \times S^5$), the surface $v = R$ in AdS_2 slicing ends on a sphere of radius R at the boundary in Poincaré slicing.

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Express minimal area in matrix model language

- Rewrite minimal area in terms of h_1, h_2
- Could just evaluate (numerically), but only for $g = 1$
- Instead, write in terms of $\omega(z)$ using OT map:

$$A_{\text{min}} = -\frac{\pi^3 \alpha'^4}{6 g_s^2} \int d^2 z \, \{ 2(z-\bar{z})^2 (\partial_z \omega + \partial_{\bar{z}} \bar{\omega}) - 4(z-\bar{z})(\omega - \bar{\omega}) \\ -2(z-\bar{z})^2 \partial_z \omega \, \partial_{\bar{z}} \bar{\omega} + (z-\bar{z})(\omega-\bar{\omega})(\partial_z \omega + \partial_{\bar{z}} \bar{\omega}) \}$$

- Strategy: exchange order of integrals using

$$\omega(z) = \lambda \int_{\mathcal{C}} dx \, \frac{\rho(x)}{z - x}$$

- Need to regulate at large $|z|$ (boundary of $AdS_5 \times S^5$)

Result

- Result for a general number of intervals, describing a Wilson loop in a general large representation $\mathcal{R}$:

$$S_{\mathcal{A}} = N^2 \left[\frac{R^2}{\varepsilon^2} - \log \frac{R}{\varepsilon} - \log \sqrt{\lambda} + \frac{3}{4} - \frac{2\rho_2}{3\lambda} + \int_{\mathcal{C}\times\mathcal{C}} dx\, dy\, \rho(x)\, \rho(y) \log|x-y| \right]$$

- The coefficient of the logarithmic divergence is independent of the UV cut-off ε and equals N^2
- Subtract off the vacuum result ($g = 0$) to find

$$\Delta S_{\mathcal{A}} = N^2 \left[\int_{\mathcal{C}\times\mathcal{C}} dx\, dy\, \rho(x)\, \rho(y) \log|x-y| - \frac{2\Delta\rho_2}{3\lambda} - \left(\log \sqrt{\lambda} - \log 2 - \frac{1}{4} \right) \right]$$

Comparison

- RHS of LM formula:

$$
\begin{aligned}
\log \langle W_{\mathcal{R}} \rangle + 8\pi^2 h_W &= N \sum_{I=1}^{g+1} \int_{\mathcal{C}_I} dx\, \rho(x) \left(-\frac{2N}{\lambda} x^2 + \hat{K}_I\, x \right) \\
&\quad + N^2 \int_{\mathcal{C}\times\mathcal{C}} dx\, dy\, \rho(x)\, \rho(y) \log |x-y| \\
&\quad + N^2 \left(-\log\sqrt{\lambda} + \log 2 + \frac{3}{4} \right) - \frac{8N^2}{3\lambda}\, \Delta\rho_2 \\
&= \Delta S_{\mathcal{A}} + N \sum_I \int_{C_I} dx\, \rho(x)\, \hat{K}_I\, x - \frac{4N^2}{\lambda}\, \Delta\rho_2
\end{aligned}
$$

✓ The final two terms sum to zero once we impose the saddle-point equation

Summary

- ✓ Agreement with LM formula for large representations
- ✓ Confirmation of OT map and consistency of Ryu-Takanyanagi in a more general situation
- ► Next step: explore other bubbling geometries
- ► Example: solutions of eleven-dimensional supergavity found by [D'Hoker, Estes, Gutperle & Krym] that describe Wilson surfaces in the six-dimensional $(2, 0)$ superconformal theory

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