

Title: PHYS 781 - Lecture 2

Date: Sep 25, 2014 02:00 PM

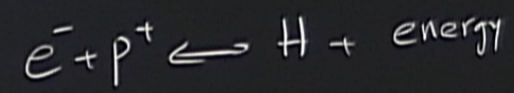
URL: <http://pirsa.org/14090086>

Abstract:



$$\mu_e + \mu_p = \mu_H = E_0$$

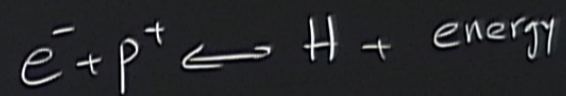
13.6 eV



$$\frac{n_e n_p}{n_H} = \left(\frac{m_e k_B T}{2\pi \hbar^2} \right)^{3/2} e^{-E_0/k_B T}$$

$$\mu_e + \mu_p = \mu_H = E_0$$

13.6 eV



$$\frac{n_e n_p}{n_H} = \left(\frac{m_e k_B T}{2\pi \hbar^2} \right)^{3/2} e^{-E_0/kT}$$

$$X = \frac{n_e}{n_H} = \frac{n_p}{n_H}$$

$$X_e^2 \propto \frac{m_e^{3/2}}{n_H} T^{3/2} e^{-E_0/kT}$$

$$\sim \left(\frac{m_e}{T} \right)^{3/2} \frac{T^3}{n_H} e^{-E/kT}$$

$$n_\gamma = \frac{1}{e^{E/kT} - 1}$$

Spontaneous em. induced em.

$$j_\gamma \propto (1 + n_\gamma) A$$

$$\alpha_\gamma I_\gamma \propto n_\gamma A$$

$$\frac{n_2}{n_1} = e^{-\frac{E_2 - E_1}{kT}}$$

$$n_\gamma = \frac{1}{e^{\frac{E_\gamma}{kT}} - 1}$$

Spontaneous em. $\nearrow$ induced em. $\nwarrow$

$$j_\nu \propto (1 + n_\gamma) A$$

$$\alpha_\nu I_\nu \propto n_\gamma A$$

$$\frac{dI_\nu}{d\ell} = j_\nu - \underbrace{\alpha I_\nu}_{n\sigma_{1\rightarrow 2}}$$

Oscillator strength

$$A = 4\pi^2 \left(\frac{q^2}{mc} \right) \left(\frac{2\nu^2}{c^2} \right) f_{12}$$

$$\left. \frac{dP}{dt} \right|_{\text{em/abs}} = B I_\nu$$

X-section

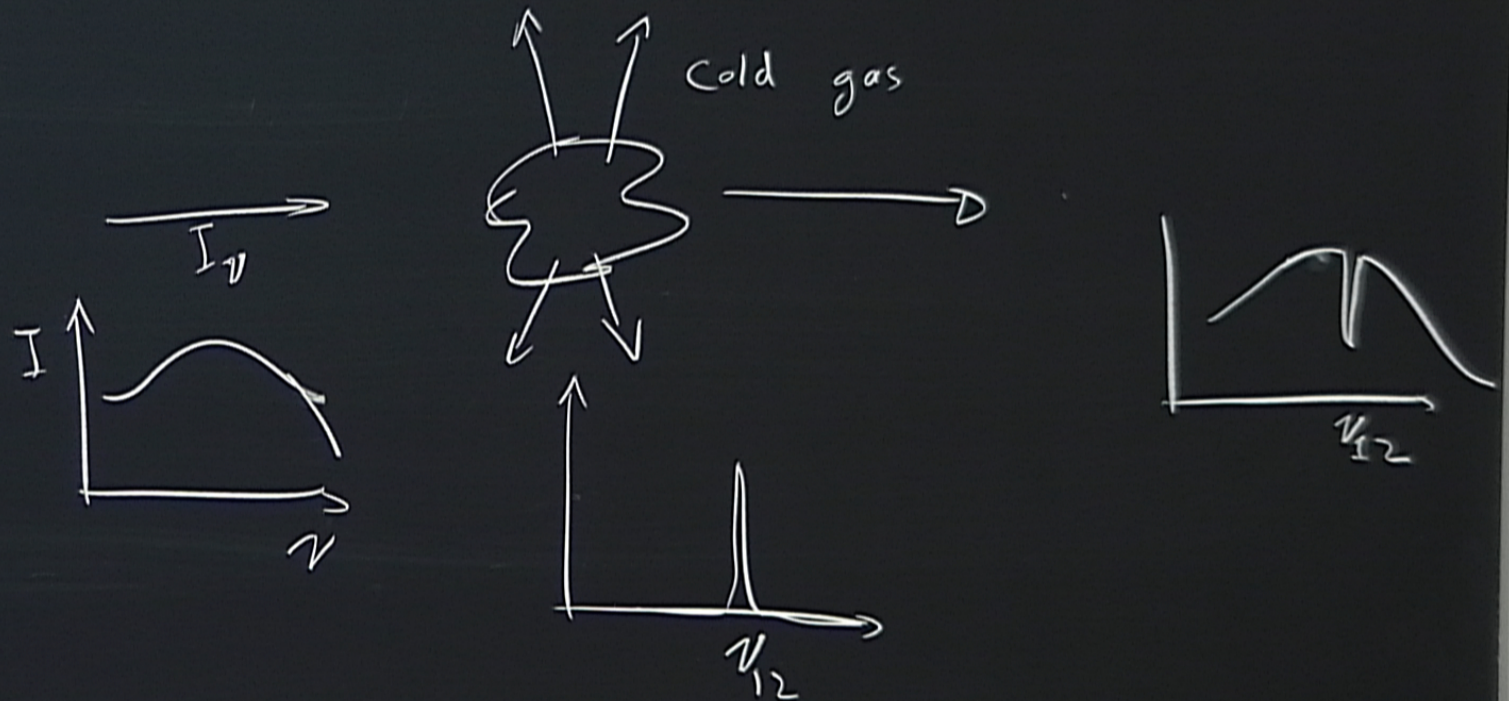
$$\sigma_{1\rightarrow 2} = \frac{\pi q^2}{mc} f_{12} \delta(\nu - \nu_{12})$$

$$\frac{B}{A} = \frac{2h\nu^3}{c^2}$$

$$\begin{aligned}
 n_\gamma &= \frac{1}{e^{E_\gamma/kT} - 1} \\
 j_\nu &\propto \overset{\text{Spontaneous em.}}{(1+n_\gamma)} \overset{\text{induced em.}}{A} \\
 \alpha_\nu I_\nu &\propto n_\gamma A \\
 A &= 4\pi^2 \left(\frac{q^2}{mc}\right) \left(\frac{2\nu^2}{c^2}\right) \rho_{12} \\
 \text{X-section } \sigma_{1 \rightarrow 2} &= \frac{\pi q^2}{mc} \rho_{12} \delta(\nu - \nu_{12}) \\
 \frac{dI_\nu}{d\ell} &= j_\nu - \underbrace{n\sigma_{1 \rightarrow 2}}_{\text{Oscillator strength}} I_\nu \\
 \left. \frac{d\hat{p}}{dt} \right|_{\text{em/abs}} &= \hbar I_\nu \\
 \frac{B}{A} &= \frac{2h\nu^3}{c^2}
 \end{aligned}$$

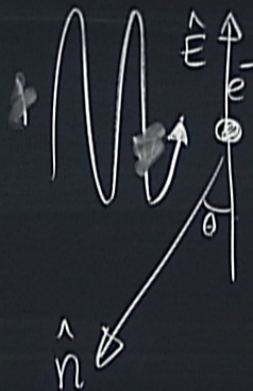
$\frac{n_2}{n_1} = e^{-\frac{E_2 - E_1}{kT}}$

$x = \frac{1}{\nu}$



Agenda:

Thomson scattering
H. Oscillator
Compton
Radiation drag



$$\vec{E} = \hat{E} e^{-i\omega t}$$

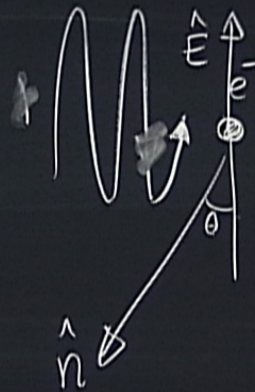
$$m_e \ddot{\vec{x}} = q \vec{E}$$

$$\frac{d\mathcal{E}}{dt} = \frac{2}{3} \frac{q^2 a^2}{c^3}$$

$$\frac{d\mathcal{E}}{dt d\Omega} = \frac{q^2}{4\pi c^3} \frac{|\vec{a} \times \hat{n}|^2}{a^2 (\sin\theta)^2}$$

Agenda:

Thomson scattering
H. Oscillator
Compton
Radiation drag



$$\vec{E} = \hat{E} e^{-i\omega t}$$

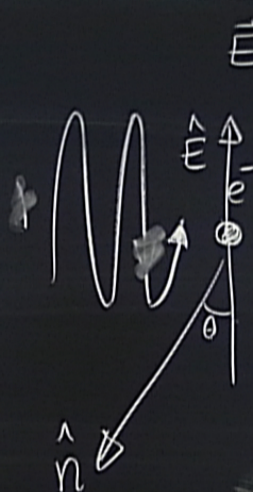
$$m_e \ddot{\vec{x}} = q\vec{E} \rightarrow a = \frac{q\hat{E}}{m_e}$$

$$\frac{d\mathcal{E}}{dt} = \frac{2}{3} \frac{q^2 a^2}{c^3}$$

$$\left| \frac{d\mathcal{E}}{dt d\Omega} \right| = \frac{q^2}{4\pi c^3} \frac{|\vec{a} \times \hat{n}|^2}{a^2 (\sin\theta)^2}$$

Agenda:

Thomson scattering
H. Oscillator
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Radiation



$$\vec{E} = \hat{E} e^{-i\omega t}$$

$$m_e \ddot{\vec{x}} = q \vec{E} \rightarrow a = \frac{q \hat{E}}{m_e}$$

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$$\left| \frac{d\mathcal{E}}{dt d\Omega} \right| = \frac{q^2}{4\pi c^3} \frac{|\vec{a} \times \hat{n}|^2}{a^2 (\sin\theta)^2}$$

$$= \frac{d\sigma}{dt d\Omega} \frac{c}{4\pi} \vec{E} \times \vec{B}$$

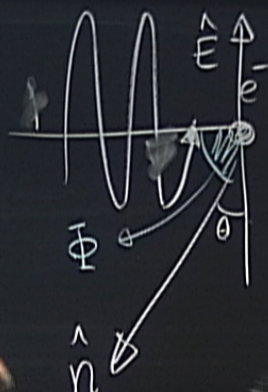
Agenda:

Thomson scattering

H. Oscillator

Compton

Radiation drag



$$\vec{E} = \hat{E} e^{-i\omega t}$$

$$m_e \ddot{\vec{x}} = q \vec{E} \rightarrow a = \frac{q \hat{E}}{m_e}$$

$$\frac{d\mathcal{E}}{dt} = \frac{2}{3} \frac{q^2 a^2}{c^3}$$

$$\left| \frac{d\mathcal{E}}{dt d\Omega} \right| = \frac{q^2}{4\pi c^3} \underbrace{|\vec{a} \times \hat{n}|^2}_{a^2 (\sin\theta)^2}$$

$$= \frac{d\sigma}{d\Omega} \frac{c}{4\pi} |\hat{E}|^2 \frac{1}{2}$$

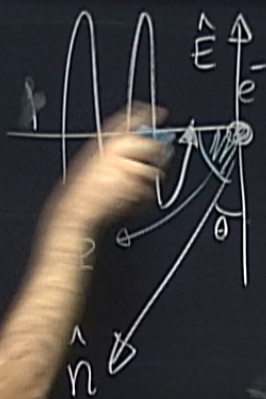
$$\sigma_T = \frac{8\pi r_e^2}{3} = 6.7 \times 10^{-25} \text{ cm}^2$$

$$\frac{d\sigma_T}{d\Omega} = \frac{r_e^2}{2} (1 + \cos^2\theta)$$

Agenda:

Thomson scattering
H. Oscillator
C

drag



$$\vec{E} = \hat{E} e^{-i\omega t}$$

$$m_e \ddot{\vec{x}} = q \vec{E} \rightarrow a = \frac{q \hat{E}}{m_e}$$

$$\frac{d\mathcal{E}}{dt} = \frac{2}{3} \frac{q^2 a^2}{c^3}$$

$$\left| \frac{d\mathcal{E}}{dt d\Omega} = \frac{q^2}{4\pi c^3} \frac{|\vec{a} \times \hat{n}|^2}{a^2 (\sin\theta)^2} \right|$$

$$= \frac{d\sigma}{d\Omega} \frac{c}{4\pi} |\hat{E}|^2 \frac{1}{2}$$

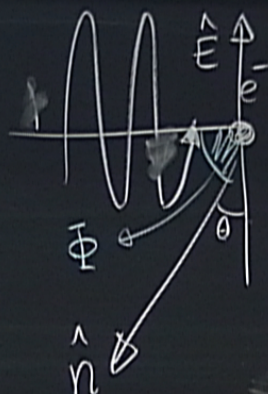
$$\sigma_T = \frac{8\pi r_e^2}{3} = 6.7 \times 10^{-25} \text{ cm}^2$$

$$\frac{d\sigma_T}{d\Omega} = \frac{r_e^2}{2} (1 + \cos^2\theta)$$

$$\frac{r^2}{mc^2}$$

Agenda:

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H. Oscillator
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$$\vec{E} = \hat{E} e^{-i\omega t}$$

$$m_e \ddot{\vec{x}} = q \vec{E} \rightarrow a = \frac{q \hat{E}}{m_e}$$

$$\frac{d\mathcal{E}}{dt} = \frac{2}{3} \frac{q^2 a^2}{c^3}$$

$$\left| \frac{d\mathcal{E}}{dt d\Omega} \right| = \frac{q^2}{4\pi c^3} \underbrace{|\vec{a} \times \hat{n}|^2}_{a^2 (\sin\theta)^2}$$

$$r_e \equiv \frac{q^2}{mc^2}$$

$$= \frac{d\sigma}{d\Omega} \frac{c}{4\pi} |\hat{E}|^2 \frac{1}{2}$$

$$\sigma_T = \frac{8\pi r_e^2}{3} = 6.7 \times 10^{-25} \text{ cm}^2$$

$$\frac{d\sigma_T}{d\Omega} = \frac{r_e^2}{2} (1 + \cos^2\theta)$$

No Freq. dep.

$$\ddot{\vec{x}} = q\vec{E} \rightarrow a = \frac{q\hat{E}}{m_e}$$

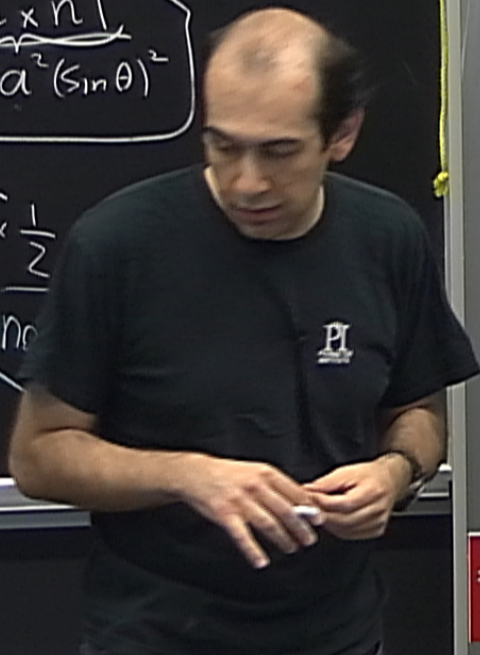
$$= \frac{2}{3} \frac{q^2 a^2}{c^3}$$

$$= \frac{q^2}{4\pi c^3} \frac{|\vec{a} \times \hat{n}|^2}{a^2 (\sin\theta)^2}$$

$$\frac{d\Omega}{4\pi} \frac{c}{4\pi} |\hat{E}|^2 \times \frac{1}{2}$$

$$7 \times 10^{-25} \text{ cm}^2$$

$$(\Phi)$$

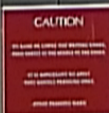


H.O.

$$m_e \ddot{\vec{x}} = q\vec{E} - m_e \omega_0^2 \vec{x}$$

$$m_e (-\omega^2 + \omega_0^2) \hat{x} = q\hat{E}$$

$$\hat{a} = -\omega^2 \hat{x} = \frac{-\omega^2 q \hat{E}}{m_e (\omega_0^2 - \omega^2)}$$



H.O.

$$m_e \ddot{\vec{x}} = q \vec{E} - m_e \omega_0^2 \vec{x} - m_e \gamma \dot{\vec{x}} \quad \rightarrow \text{damping coeff}$$

$$m_e (-\omega^2 + \omega_0^2) \hat{x} = q \hat{E}$$

$$\hat{a} = -\omega^2 \hat{x} = \frac{-\omega^2 q \hat{E}}{m_e [(\omega_0^2 - \omega^2) + i \gamma \omega]}$$

Lorentzian profile

$$\sigma_T = \frac{8 \pi r_0^2}{3}$$

$$\frac{\omega^4}{(\omega^2 - \omega_0^2)^2 + \gamma^2 \omega^2}$$

$$\gamma = \frac{2}{3} \frac{\omega_0^2 r_0}{c}$$

$$\omega \approx \omega_0 \rightarrow$$

$$\frac{\omega_0^4}{(\omega - \omega_0)^2 + \gamma^2 \omega_0^2} = \frac{\omega_0^2}{4(\omega - \omega_0)^2 + \gamma^2}$$

H.O.

$$m_e \ddot{\vec{x}} = q \vec{E} - m_e \omega_0^2 \vec{x} - m_e \gamma \dot{\vec{x}} \quad \rightarrow \text{damping coeff.}$$

$$m_e (-\omega^2 + \omega_0^2) \hat{x} = q \hat{E}$$

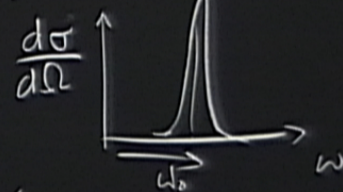
$$\hat{a} = -\omega^2 \hat{x} = \frac{-\omega^2 q \hat{E}}{m_e [(\omega_0^2 - \omega^2) + i \gamma \omega]}$$

$$\sigma_T = \frac{8 \pi r_0^2}{3} \left(\frac{\omega^4}{(\omega^2 - \omega_0^2)^2 + \gamma^2 \omega^2} \right)$$

$$\gamma = \frac{2}{3} \frac{\omega_0^2 r_0}{c}$$

$$\frac{\gamma}{\omega_0} \sim \omega_0 r_0$$

Lorentzian profile $\sim \gamma$



$$\begin{aligned} & \frac{\omega_0^4}{(\omega - \omega_0)^2 + \gamma^2} \\ & \sim \frac{\omega_0^2}{4(\omega - \omega_0)^2 + \gamma^2} \end{aligned}$$

H.O.

$$m_e \ddot{\vec{x}} = q \vec{E} - m_e \omega_0^2 \vec{x} - m_e \gamma \dot{\vec{x}} \quad \rightarrow \text{damping coeff.}$$

$$m_e (-\omega^2 + \omega_0^2) \hat{x} = q \hat{E}$$

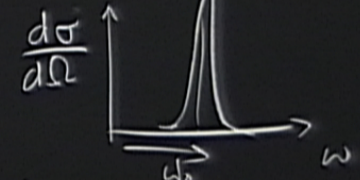
$$\hat{a} = -\omega^2 \hat{x} = \frac{-\omega^2 q \hat{E}}{m_e [(\omega_0^2 - \omega^2) + i \gamma \omega]}$$

$$\sigma_T = \frac{8 \pi r_0^2}{3} \left[\frac{\omega^4}{(\omega^2 - \omega_0^2)^2 + \gamma^2 \omega^2} \right]$$

$$\gamma = \frac{2}{3} \frac{\omega_0^2 r_0}{c}$$

$$\frac{\gamma}{\omega_0} \sim \frac{\omega_0 r_0}{c}$$

Lorentzian profile $\sim \gamma$



$$\begin{aligned} & \frac{\omega_0^4}{(\omega - \omega_0)^2 + \gamma^2 \omega^2} \\ & \sim \frac{\omega_0^2}{4(\omega - \omega_0)^2 + \gamma^2} \end{aligned}$$

dep.

CAUTION
BE CAREFUL AND AVOID THE EXCESSIVE HEAT
WHICH MAY BE CAUSED BY THE LASER OR THE SPARK
IT IS PROHIBITED TO SMOK
WHILE WORKING WITH LASER
WHILE WORKING WITH LASER

$$\int \sigma_{H0} d\omega \sim r_0^2 \times \omega \int \frac{1}{4(\omega - \omega_0)^2 + \gamma^2}$$

H.O.

$$m_e \ddot{\vec{x}} = q \vec{E} - m_e \omega_0^2 \vec{x} - m_e \gamma \dot{\vec{x}} \quad \rightarrow \text{damping coeff.}$$

$$m_e (-\omega^2 + \omega_0^2) \hat{x} = q \hat{E}$$

$$\hat{a} = -\omega^2 \hat{x} = \frac{-\omega^2 q \hat{E}}{m_e [(\omega_0^2 - \omega^2) + i \gamma \omega]}$$

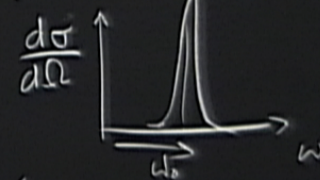
$$\sigma_{H0} = \frac{8 \pi r_0^2}{3}$$

$$\frac{\omega^4}{(\omega^2 - \omega_0^2)^2 + \gamma^2 \omega^2}$$

$$\gamma = \frac{2}{3} \frac{\omega_0^2 r_0}{c}$$

$$\frac{\gamma}{\omega_0} \sim \frac{\omega_0 r_0}{c}$$

Lorentzian profile



$$\frac{\omega_0^4}{(\omega - \omega_0)^2 + \gamma^2} \sim \frac{\omega_0^4}{4(\omega - \omega_0)^2 + \gamma^2}$$

$$= \frac{\omega_0^4}{4(\omega - \omega_0)^2 + \gamma^2}$$

$$d_{12} \propto (1+n_{\gamma}) A$$

$$\alpha_{\nu} I_{\nu} \propto n_{\gamma} A$$

$$\frac{dI_{\nu}}{d\ell} = d_{12} - \alpha_{\nu} I_{\nu}$$

oscillator strength

single atom

$$\left. \frac{dP}{dt} \right|_{\text{em/abs}} = B I_{\nu}$$

$$A = 4\pi^2 \left(\frac{q^2}{mc} \right) \left(\frac{2\nu^2}{c^2} \right) f_{12}$$

x-section

$$\sigma_{1 \rightarrow 2} = \frac{\pi q^2}{mc} f_{12} \delta(\nu - \nu_{12})$$

$$f_{12} \lesssim 1$$

$$\frac{B}{A} = \frac{2h\nu^3}{c^2}$$

$f_{12} \sim 1$ for classical H.O.

$$\int \sigma_{1 \rightarrow 2} d\nu \sim \frac{\alpha f_{1 \rightarrow 2}}{m_e}$$

$$\int \sigma_{HO} d\omega \sim r_0^2 \times \omega_0^2 \int \frac{d\omega}{4(\omega - \omega_0)^2 + \gamma^2} \sim \frac{r_0^2 \omega_0^2}{\gamma} \sim \frac{r_0^2 \omega_0^2}{\omega_0^2 r_0} \sim r_0 \sim \frac{q^2}{m_e}$$

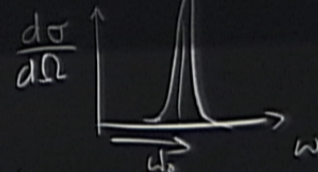
$$\sigma_{HO} = \frac{8\pi r_0^2}{3}$$

$$\frac{\omega^4}{(\omega^2 - \omega_0^2)^2 + \gamma^2 \omega_0^2}$$

$$\gamma = \frac{2}{3} \frac{\omega_0^2 r_0}{c}$$

$$\frac{\gamma}{\omega_0} \sim \frac{\omega_0 r_0}{c}$$

Lorentzian profile



$$\frac{\omega_0^4}{(\omega - \omega_0)^2 + \gamma^2 \omega_0^2} \approx \frac{\omega_0^4}{4\gamma^2 \omega_0^2} = \frac{\omega_0^2}{4(\omega - \omega_0)^2 + \gamma^2}$$

$$\int \sigma_{12} d\omega \sim \frac{m_e}{r_0^2 \times \omega^2} \int \frac{d\omega}{4(\omega - \omega_0)^2 + \gamma^2} \sim \frac{r_0^2 \omega_0^2}{\gamma} \sim \frac{r_0^2 \omega_0}{\omega_0^2 r_0} \sim r_0 \sim \frac{1}{m_e}$$

H.O.

$$m_e \ddot{\vec{x}} = q \vec{E} - m_e \omega_0^2 \vec{x} - m_e \gamma \dot{\vec{x}} \quad \rightarrow \text{damping coeff.}$$

$$m_e (-\omega^2 + \omega_0^2) \hat{x} = q \hat{E}$$

$$\hat{a} = -\omega^2 \hat{x} = \frac{-\omega^2 q \hat{E}}{m_e [(\omega_0^2 - \omega^2) + i \gamma \omega]}$$

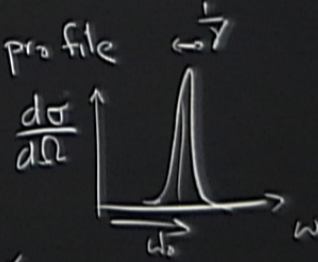
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$$\frac{\gamma}{\omega_0} \sim \frac{\omega_0 r_0}{c}$$

Lorentzian profile

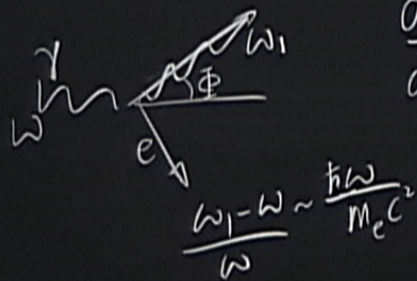


$$\frac{\omega_0^4}{(\omega - \omega_0)^2 + \gamma^2 \omega_0^2} \approx \frac{\omega_0^2}{4(\omega - \omega_0)^2 + \gamma^2}$$

$$\hbar\omega \lesssim \frac{m_e c^2}{0.5 \text{ MeV}}$$

you can use classical E&M

Compton scattering Klein-Nishina

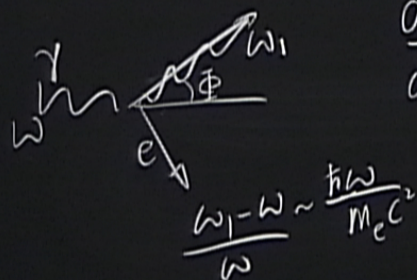


$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{unpolarized}} = \frac{1}{2} r_0^2 \left(\frac{\omega_1}{\omega} \right)^2 \left(\frac{\omega}{\omega_1} + \frac{\omega_1}{\omega} - \sin^2 \Phi \right)$$

$$\hbar\omega \lesssim \frac{m_e c^2}{0.5 \text{ MeV}}$$

you can use classical E&M

Compton scattering Klein-Nishina

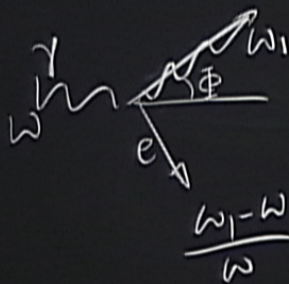


$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{unpolarized}} = \frac{1}{2} r_0^2 \left(\frac{\omega_1}{\omega} \right)^2 \left(\frac{\omega}{\omega_1} + \frac{\omega_1}{\omega} - \sin^2 \Phi \right)$$

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you can use classical E&M

Compton scattering Klein-Nishina



$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{unpolarized}} = \frac{1}{2} r_0^2 \left(\frac{\omega_1}{\omega} \right)^2 \left(\frac{\omega}{\omega_1} + \frac{\omega_1}{\omega} - \sin^2 \Phi \right)$$

Radiation Drag:

$$\vec{F} = -\frac{4}{3} \sigma_T U_{\text{rad}} \gamma^2 \left(\frac{\vec{v}}{c} \right)$$

Fluctuation-Dissip. thm

$$\frac{dE}{dt} = -\frac{4}{3} \sigma_T U_{\text{rad}} \gamma^2 \left(\frac{v}{c} \right)^2 c = \vec{F} \cdot \vec{v}$$

$$T_e = T_\gamma$$

$$f(\vec{p}) \propto e^{-\frac{p^2}{2m_e k T_e}}$$