Title: Lifshitz Field Theories, Anomalies and Hydrodynamics

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Abstract: I will review various aspects of field theories that posses a Lifshitz scaling symmetry. I will detail our study of the cohomological structure of anisotropic Weyl anomalies (the equivalent of trace anomalies in relativistic scale invariant field theories). I will also analyze the hydrodynamics of Lifshitz field theories and in particular of Lifshitz superfluids which may give insights into the physics of high temperature superconductors.

Lifshitz Field Theories, Anomalies and Hydrodynamics

Perimeter Institute - Theory Group Seminar

Shira Chapman - Tel Aviv University

With Igal Arav, Yaron Oz – to appear.
With Carlos Hoyos, Yaron Oz – 1402.2981.

Outline

- Lifshitz Scaling Symmetry
  – Strange Metals
  – Field Theory Realizations
- Scaling Anomalies
  – Conformal Weyl Anomalies - Review
  – Anisotropic Weyl anomalies
- Lifshitz Hydrodynamics
  – Normal Fluids
  – Superfluids

Lifshitz Scaling Symmetry

Lifshitz Scaling

$$t \to \lambda^z t, \qquad x^i \to \lambda x^i, \qquad i = 1, \ldots, d$$

z - dynamical critical exponent, measures anisotropy.

- Broken boost invariance.
- For $z = 1$ symmetry can be enhanced to full conformal group.
- For $z = 2$ symmetry can be enhanced to full non-relativistic conformal invariance (Schrodinger conformal group).
- Lifshitz scaling shows up in condensed matter systems (quantum critical points).

Strange Metals

- High T_c superconductors, heavy fermion compounds and other materials have a **strange metallic phase** (non LFL).
- Universal behavior, e.g linear temperature dependence of resistivity $\rho \sim T$.
- Believed to be a consequence of **Quantum Criticality**.

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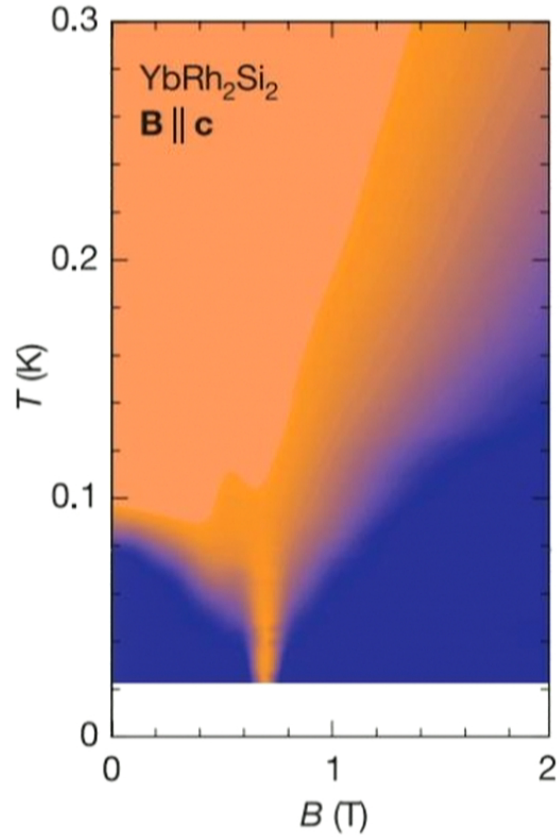
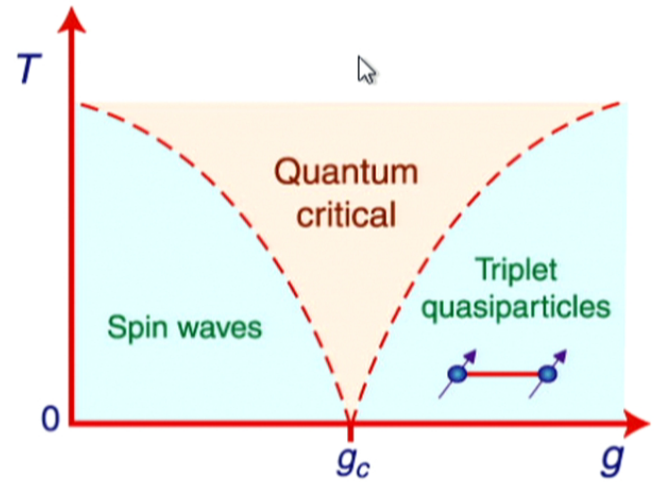


Quantum Critical Points - QCP

- **Zero temperature second order phase transitions**, induced by tuning parameter (magnetic field, doping).
- Driven by quantum fluctuations.
- Infinite correlation length $\rightarrow$ scale invariance
- Influence of QCP felt way above $T = 0$.
- At the quantum QCP there is a **Lifshitz scaling symmetry**.



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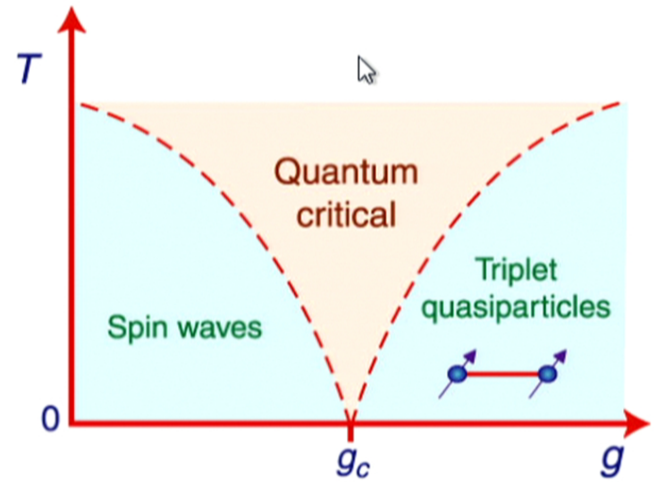


Lifshitz Algebra

- Generators:
  - Time, space translations: $P_0 = \partial_t$, $P_i = \partial_i$
  - Scaling $D = zt\partial_t + x^i\partial_i$
  - Spatial rotations $M_{ij} = x_i\partial_j - x_j\partial_i$
- Commutation relations:
  - $[D, P_i] = -P_i \qquad [D, P_0] = -zP_0$
  - $[D, M_{ij}] = 0$
- There seem to be no Casimir operators.
- Questions: Representations? Spin-statistics theorem?

Field Theory Realizations

- Free scalar (z even):

$$S = \int dtd^dx \left[\frac{1}{2}(\partial_t\phi)^2 - \frac{\kappa}{2z}((\nabla^2)^{\frac{z}{2}}\phi)^2\right]$$

Scaling dimensions: $[\phi] = \frac{z-d}{2}$, $[k] = 0$.

- Example: $2+1$ dimensions, $z = 2$:

$$\mathcal{L} = \frac{1}{2}(\partial_t\phi)^2 - \frac{\kappa}{4}(\nabla^2\phi)^2$$

– Dimensionless scalar, Dispersion relation $\omega^2 \sim p^4$.

- Free fermions in $2+1$ dimensions:

$$\mathcal{L} = \bar{\psi}\gamma^0\partial_t\psi + \bar{\psi}\nabla^2\psi$$

– ψ two component real fermion, $\gamma^0 = i\sigma_2$.

- Can combine the scalar and fermion to a WZ-type supersymmetric model.

Conformal Weyl Anomalies - Review

- Relativistic (z=1) Ward identity:

$$T^{\mu}_{\mu} = 0$$

- Quantum anomalies modify Ward identity on curved spacetime of even dimension D:

$$\langle T^{\mu}_{\mu} \rangle_g = -(-)^{\frac{D}{2}} a E_D + \Sigma_i c_i I_i$$

where E_D is the Euler density (**A-type Anomaly**) and I_i are strictly Weyl invariant densities constructed from the Weyl tensor and its covariant derivatives (**B-type anomalies**).

- The problem of finding all the allowed contributions to the trace anomaly is cohomological.
- In $(2+4k)$ dimensions can also have contribution from the Weyl "partner" of gravitational anomaly.

Weyl Cohomology

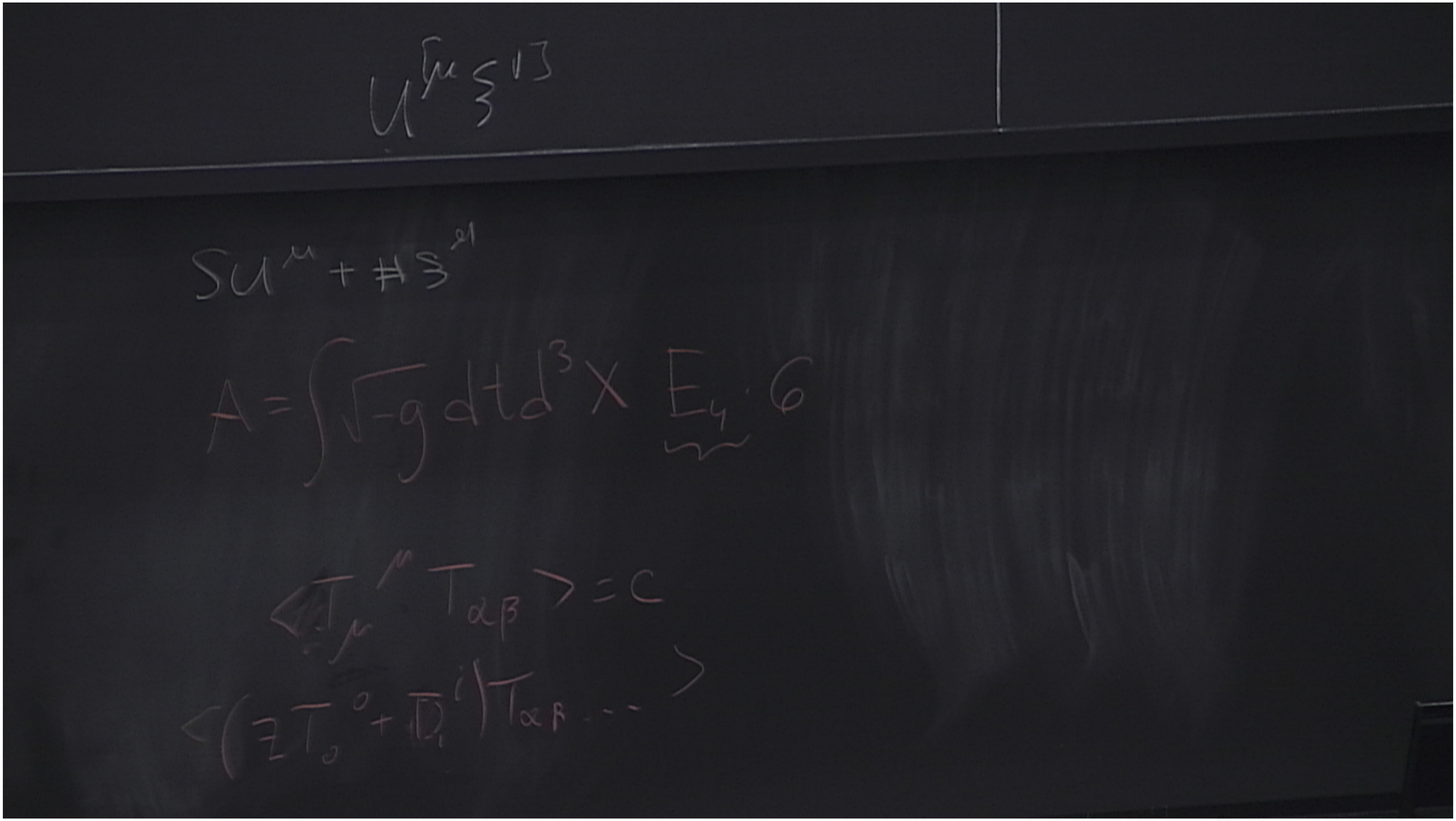
- To derive the most general allowed trace anomalies, one requires the Wess-Zumino consistency conditions:

$$\delta^W_{\sigma 1} A_{\sigma 2} - \delta^W_{\sigma 2} A_{\sigma 1} = 0, \qquad A_\sigma = \delta^W_\sigma W(g_{\mu\nu})$$

- Trivial terms $A_\chi = \delta^W_\chi G(g_{\mu\nu})$ where G is a local functional of the background fields can be cancelled by appropriate counter terms.
- Alternative description in terms of a BRST-like ghost. Can replace transformation parameter by a Grassmannian ghost. The **Wess-Zumino consistency condition** then becomes:

$$\boxed{\delta^W_\chi A_\chi = 0, \qquad A_\chi \neq \delta^W_\chi G(g_{\mu\nu})}$$

- We are looking for the cocycles of the Weyl operator which are not coboundaries (terms which are closed but not exact).
- Relative Cohomology w.r.t diffeomorphisms.


u^[μ ξ^ν]
A = ∫√-g dt d³x E₄ · 6
<T_μ^μ T_αβ> = c

Weyl Cohomology - Examples

- In 2 dimensions only one term in the cohomology:

$$\int (\sqrt{-g}dtdx)\ \sigma R$$

  and no coboundary terms.

- In 4 dimensions - Possible scalar expressions of dimension 4:

$$R^2, \quad R_{\mu\nu}R^{\mu\nu}, \quad R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}, \quad \epsilon^{\alpha\beta\gamma\delta}R_{\mu\nu\alpha\beta}R^{\mu\nu}{}_{\gamma\delta}, \quad \Box R$$

- Combinations that vanish under the Weyl operator (cocycles) are the **integrated** Weyl tensor squared, Euler density, Pontryagin term:

$$W^2 = (R_{\mu\nu\rho\sigma})^2 - 2(R_{\mu\nu})^2 + \frac{1}{3}R^2 \quad E_4 = (R_{\mu\nu\rho\sigma})^2 - 4(R_{\mu\nu})^2 + R^2$$

$$P_1 = \epsilon^{\alpha\beta\gamma\delta}R_{\mu\nu\alpha\beta}R^{\mu\nu\gamma\delta}$$

  and $\Box R$ which turns out to be trivial (coboundary).

Scaling Anomalies in Lifshitz Field Theories

The Lifshitz Ward identity

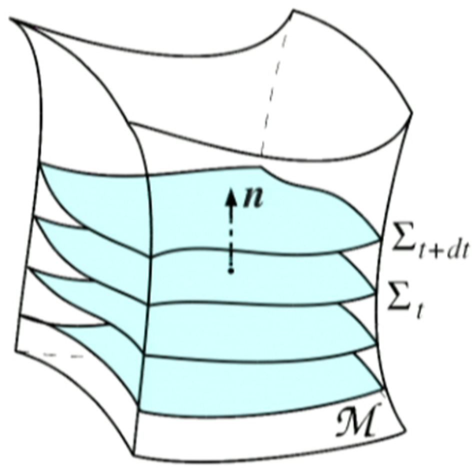
$$zT^0_0 + T^i_i = 0$$

Can be modified by quantum anomalies.

- Question - what are the allowed quantum corrections to the Lifshitz ward identity consistent with the Wess-Zumino consistency conditions?

The Geometric Structure

- Since time scales differently than space, one has to consider the time direction separately, by foliating spacetime into equal-time slices.
- When considering a theory defined over a general curved manifold, this structure is generalized to a codimension-one foliation defined over the manifold.
- The foliation structure over a manifold can be locally encoded by a 1-form t_α.
- Vector V^α tangent to foliation if $t_\alpha V^\alpha = 0$.
- By the Frobenius theorem, such a 1-form (locally) defines a codimension-1 foliation if it satisfies the condition: $t_{[\alpha}\partial_\beta t_{\gamma]} = 0$



More Geometric Structure

Background Fields of Lifshitz Field Theory

$g_{\mu\nu}$ - Metric, t_α - Foliation 1-form.

- On a curved manifold the lack of boost invariance translates into foliation preserving diffeomorphisms $\mathcal{L}_\xi\, t_\alpha \sim t_\alpha$ or in foliation adapted coordinates:

$$t \to f(t), \qquad x \to g(x,t)$$

- Equivalently can extend to any ξ by having the foliation one form transforming appropriately:

$$\delta^D_\xi g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu, \quad \delta^D_\xi t_\alpha = \mathcal{L}_\xi\, t_\alpha,$$

- All FPD invariants can be built by contracting the background fields and their covariant derivatives.

Anisotropic Weyl Invariance

- The second relevant symmetry is anisotropic Weyl invariance:

$$\begin{aligned}
&\delta^W_\sigma t_\alpha = 0 \ , \\
&\delta^W_\sigma (g^{\alpha\beta} t_\alpha t_\beta) = -2\sigma z (g^{\alpha\beta} t_\alpha t_\beta) \ , \\
&\delta^W_\sigma P_{\alpha\beta} = 2\sigma P_{\alpha\beta} \ , \\
&\delta^W_\sigma n_\alpha = z\sigma n_\alpha \qquad \delta^W_\sigma n^\alpha = -z\sigma n^\alpha \ ,
\end{aligned}$$

where n_α is the normalized foliation one form:

$$\boxed{n_\alpha = t_\alpha / \sqrt{|g^{\beta\gamma} t_\beta t_\gamma|}}$$

and $P_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$ is the projector transverse to the foliation.

- We look for the relative cohomology of the anisotropic Weyl operator with respect to foliation preserving diffeomorphisms.

Ward identities

- Using the stress energy tensor: $T^{\mu\nu}_{(g)} \equiv \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}}$, $T_{(e)}{}^{\mu\nu} \equiv \frac{1}{e} e^{a\nu} \frac{\delta S}{\delta e^a{}_\mu}$ and the current associated to the foliation 1-form $J^\alpha \equiv \frac{1}{\sqrt{-g}}\frac{\delta S}{\delta t_\alpha}$ we can derive the following identities:
- Foliation preserving diffeomorphisms:

$$\nabla_\mu T^\mu_{(e)\nu} = J^\mu \nabla_\nu t_\mu; \qquad \nabla_\mu T^\mu_{(g)\nu} = J^\mu \nabla_\nu t_\mu - \nabla_\mu (J^\mu t_\nu).$$

- Local Lorentz transformations:

$$T_{(e)[\mu\nu]} = J_{[\mu} t_{\nu]}.$$

- Anisotropic Weyl scaling:

$$T^{\mu\nu}_{(e)} P_{\mu\nu} - z T^{\mu\nu}_{(e)} n_\mu n_\nu = T^{\mu\nu}_{(g)} P_{\mu\nu} - z T^{\mu\nu}_{(g)} n_\mu n_\nu = 0.$$

- Where the two stress tensors are related as follows:

$$T^{\mu\nu}_{(e)} = T^{\mu\nu}_{(g)} + J^\mu t^\nu.$$

Anisotropic Weyl Anomalies

- All the Lifshitz scale anomalies can be built using the following:

Basic Tangent Tensors

- $P_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$ – Projector,
- $a_\mu \equiv \mathcal{L}_n n_\mu = n^\nu \nabla_\nu n_\mu$ – Acceleration,
- $K_{\mu\nu} \equiv \frac{1}{2}\mathcal{L}_n P_{\mu\nu}$ – Extrinsic curvature ,
- $\tilde{R}_{\mu\nu\rho\sigma}$ – Intrinsic Riemann curvature ,
- $\mathcal{L}_n$ – Lie derivatives in the direction of n^α (temporal derivatives),
- $\widetilde{\nabla}_\mu$ – Spatial (foliation projected) covariant derivatives,
- $\tilde{\epsilon}^{\mu\nu\rho\dots} = n_\alpha \epsilon^{\alpha\mu\nu\rho\dots}$ – Intrinsic Levi-Civita tensor .

- Naive scaling dimensions:

$$d_\sigma[P_{\mu\nu}] = 2 \quad d_\sigma[P^{\mu\nu}] = -2 \quad d_\sigma[n_\alpha] = z \quad d_\sigma[\tilde{R}_{\mu\nu\rho\sigma}] = 2$$

$$d_\sigma[a_\alpha] = 0 \quad d_\sigma[K_{\mu\nu}] = 2 - z \quad d_\sigma[\mathcal{L}_n] = -z \quad d_\sigma[\widetilde{\nabla}_\alpha] = 0$$

Anisotropic Weyl Anomalies

- Total scaling dimension should satisfy:

$$\boxed{\begin{array}{c} zn_T + n_S = d + z \ , \\ n_S + dn_\epsilon \quad \text{is even} \ . \end{array}}$$

where $n_T \equiv n_\mathcal{L} + n_K$ is the total number of time derivatives, $n_S \equiv n_\nabla + n_a + 2n_R$ is the total number of spatial derivatives.

Type A and Type B anomalies

Classification of Anomalies

Type B - Strictly Weyl invariant scalar densities.
Type A - All the rest (Euler density for conformal anomalies).

- For conformal (parity even) anomalies (Deser-Schwimmer):
  Type A $\leftrightarrow$ total derivative (topological invariant).
- In Lifshitz theories we use the first classification.

Results in 1+1 Dimensions

- for $z = 1$:
- Possible Terms:

$$K^2, \quad \mathcal{L}_n K \quad , a^2, \quad \widetilde{\nabla}_\mu a^\mu$$

$$K \tilde{\epsilon}^\mu a_\mu, \quad \tilde{\epsilon}^\mu \widetilde{\nabla}_\mu K, \quad \tilde{\epsilon}^\mu \mathcal{L}_n a_\mu$$

- Cocycles:

$$K^2 + \mathcal{L}_n K, \qquad a^2 + \widetilde{\nabla}_\mu a^\mu,$$

$$K \tilde{\epsilon} a_\mu + \tilde{\epsilon}^\mu \widetilde{\nabla}_\mu K, \quad \tilde{\epsilon}^\mu \mathcal{L}_n a_\mu$$

- Anomaly:

$$\mathcal{A}_1 = \tilde{\epsilon}^\mu \mathcal{L}_n a_\mu$$

- We find the same single anomaly for integer $z = 1, \ldots, 12$.

Anomalies in Two and Three Dimensions

dim	z	n_{an}	Anomalies
1+1	1-12	1	$\tilde{\epsilon}^{\mu}\mathcal{L}_n\, a_{\mu}$
2+1	1	1	$\tilde{\epsilon}^{\alpha\beta}K_{\alpha}^{\gamma}\mathcal{L}_n\, K_{\gamma\beta}$
	2	2	$\mathrm{Tr}(K^2) - \frac{1}{2}K^2,\ \left(\tilde{R} + \widetilde{\nabla}_{\alpha}a^{\alpha}\right)^2$
	3	0	–
	4	2	$\left(2\tilde{R} + \widetilde{\nabla}_{\alpha}a^{\alpha}\right)^3, \ldots$
	2/3	2	$\mathrm{Tr}\, K^4 + \ldots,\ \ (\mathcal{L}_n\, K)^2 + \ldots$
	3/2	0	–
3+1	1	12	$\left(\nabla_{\alpha}a^{\alpha} - \frac{1}{2}a^2 + \frac{1}{4}\tilde{R}\right)^2, \ \ \ldots$
	2	3	$\tilde{\epsilon}_{\beta\gamma\delta}K^{\alpha\beta}\widetilde{\nabla}^{\delta}K_{\alpha}{}^{\gamma}, \ \ \ldots$
	3	7	$K^2 - 3\,\mathrm{Tr}(K^2),\ \ \left(\widetilde{\nabla}_{\alpha}a^{\alpha} - \frac{1}{6}a^2 + \frac{3}{4}\tilde{R}\right)^3$
	3/2	2	$K^3 - \frac{9}{2}K\,\mathrm{Tr}(K^2) + \frac{9}{2}\,\mathrm{Tr}(K^3), \ \ \ldots$

- General structure: $(\tilde{\nabla} a - \frac{d-2}{2z}a^2 + \frac{z}{2(d-1)}\tilde{R})^{(d+z)/2}$

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Comparison to the Conformal Case

- The conformal theory can be regarded as Lifshitz theory with $z = 1$.
- Any diffeomorphism invariant term that may appear in the conformal cohomology may also appear in the Lifshitz one.
- The conformal cocycles and coboundaries should be contained in the Lifshitz cocycles and couboundaries respectively.
- For example:

$$\int \sqrt{-g}\sigma R = \int \sqrt{-g}\sigma \left[2(K^2 + \mathcal{L}_n K) - 2(a^2 + \widetilde{\nabla}_\mu a^\mu)\right],$$

- Can be removed by appropriate counter term that is not diffeomorphism invariant:

$$W_{c.t.} = \int \sqrt{-g}(K^2 - a^2).$$

- Both in 1+1 and 3+1 dimensions, the Euler density becomes a coboundary term. **All anomalies are B-type**.

Relation to 1+1 dimensional Gravitational Anomalies

- In 1+1 dimensional conformal theories can always "shift" the gravitational anomaly to a pure Lorentz anomaly.
- $W_{\text{conf}}[e^a{}_\mu] = W_{\text{conf}}[e^0{}_\mu, e^1{}_\mu]$, where W_{conf} is invariant under spacetime coordinates transformations but not under local Lorentz tranformations.
- The anomalous Ward identities that correspond to the Lorentz anomaly in conformal theories:

$$T_{(e)[\mu\nu]} = aR\epsilon_{\mu\nu}, \quad \nabla_\mu T^{\mu\nu}_{(e)} = aR\epsilon^{ab}\omega^\nu{}_{ab},$$
$$T^\mu_{(e)\mu} = -2a\epsilon^{ab}\nabla_\mu\omega^\mu{}_{ab}.$$

where $\omega^\mu{}_{ab} = e_{a\nu}\nabla^\mu e_b{}^\nu$ is the spin connection.

- Weyl partner of gravitational anomaly.

Relation to 1+1 dimensional Gravitational Anomalies

- Can redefine the conformal theory as a Lifshitz theory by adding foliation dependence:

$$W_{\text{Lif}}[e^a{}_\mu, t^a] \equiv W_{\text{conf}}[e^a{}_\mu n_a, e^a{}_\mu \epsilon_{ab} n^b],$$

- This action is FPD invariant. The Lorentz anomaly has been "shifted" into the foliation dependence.
- Coincides with the conformal action $W_{\text{conf}}[e^a{}_\mu] = W_{\text{conf}}[e^0{}_\mu, e^1{}_\mu]$, for flat foliation.
- By the above rotation:

$$\begin{aligned} T^\mu_{(e)\mu} = & -2a\epsilon^{ab}\nabla_\mu \omega^\mu{}_{ab} = -4a\nabla_\mu \left[e_{0\nu}\nabla^\mu e_1{}^\nu\right] \to \\ & -4a\nabla_\mu \left[n_\nu \nabla^\mu \tilde{n}^\nu\right] = 4a\tilde{\epsilon}^\rho(\mathcal{L}_n a_\rho - (a_\rho K + \widetilde{\nabla}_\rho K)). \end{aligned}$$

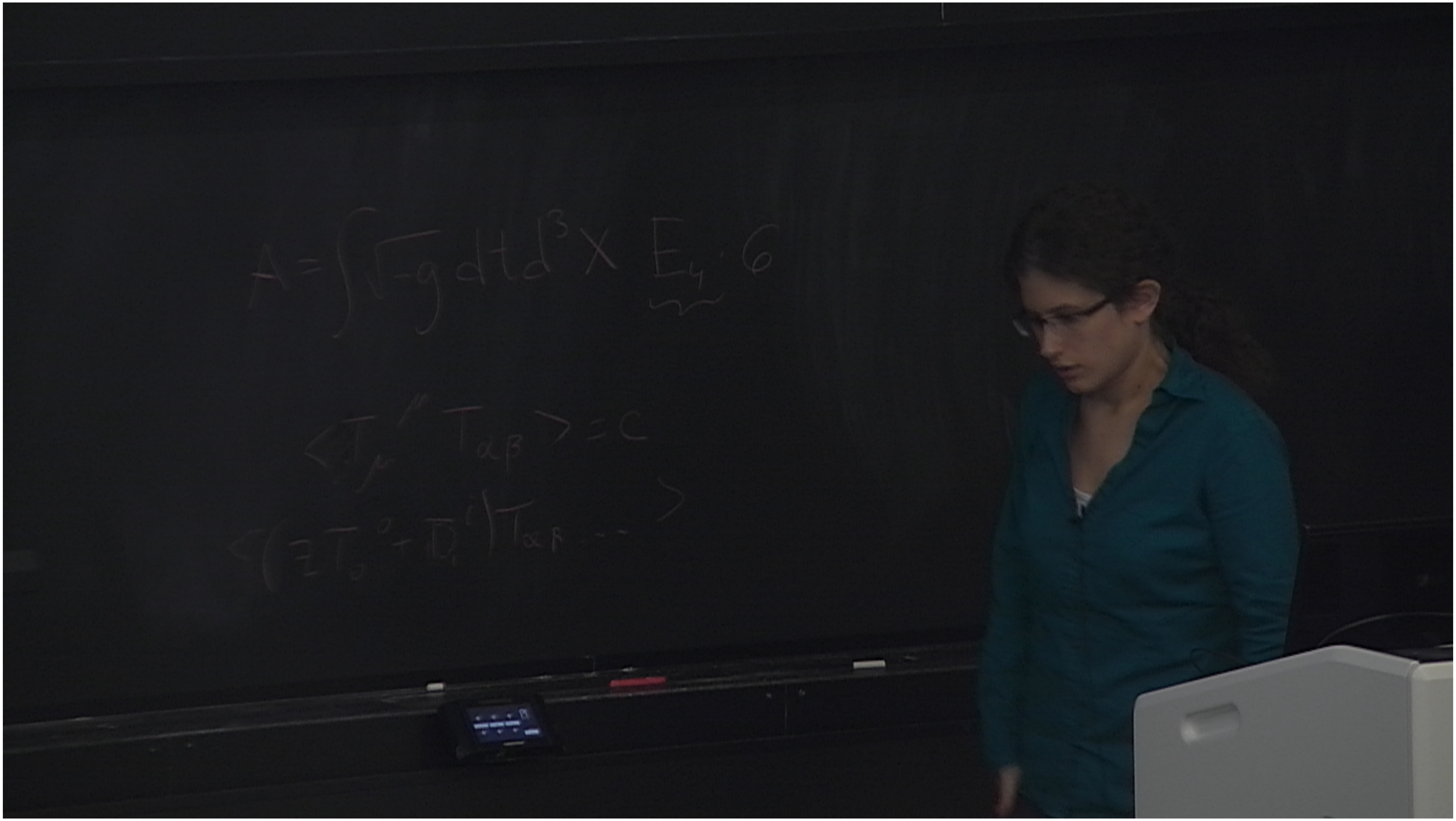
- Our parity odd Weyl anomaly.

General Conclusions and Open Questions

- All anomalies turn out to be B-type up to the addition of coboundary terms. Is this always the case?
- Can we prove that in $1+1$ dimensions for any z the form of the anomaly is $\tilde{\epsilon}^{\mu}\mathcal{L}_n a_{\mu}$?
- Can we find a general structure for anomalies in other dimensions?
- a/f/c-theorems.
- Computing the anomaly coefficients in field theoretical and holographic models.
- Entanglement entropy

Hydrodynamic Regime of Lifshitz Field theories

- Systems with ordinary critical points have a hydrodynamic description with transport coefficients whose temperature dependence is determined by the scaling at the critical point.
- Quantum critical systems can also have a hydrodynamic description.
- At quantum critical regime the hydrodynamic description will be appropriate if the characteristic length of thermal fluctuations $\ell_T \sim \frac{1}{T^{1/z}}$ is much smaller than the size of the system $L \gg \ell_T$ and both are smaller than the correlation length of quantum fluctuations $\xi \gg L \gg \ell_T$.



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Hydrodynamics

- The hydrodynamic description should take into account the effects due to lack of boost invariance.
- Since boost invariance is explicitly broken in Lifshitz field theories, the conserved stress-energy tensor is not necessarily symmetric. The Lorentz current is $J^{\mu\alpha\beta} = x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha}$, and

$$\partial_\mu J^{\mu\alpha\beta} = T^{\alpha\beta} - T^{\beta\alpha},$$

- In Lorentz invariant theories the energy-momentum tensor is symmetric. If boost or rotational symmetries are broken this condition can be relaxed.
- Allow for an antisymmetric contribution:

$$T^{[\mu\nu]} = u^{[\mu} V_A^{\nu]},$$

  Assuming boost invariance is broken in the rest frame of the normal fluid.
- In order to see the asymmetric part, should construct the stress tensor as a response of the action to a change in the vielbein.

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- In order to see the asymmetric part, should construct the stress tensor as a response of the action to a change in the vielbein.

- The conservation of the energy-momentum tensor determines the hydrodynamic equations $\partial_\mu T^{\mu\nu}$.
- Hydrodynamic variables: 4-velocity u^μ, Temperature T, for charged fluid chemical potential μ. For a superfluid there is also a superfluid velocity.
- The new terms from $\pi_A^{[\mu\nu]}$ should be compatible with the laws of thermodynamics, in particular with the local second law $\partial_\mu j_s^\mu \geq 0$ (positive divergence of the entropy current).
- We present the correction to the stress tensor up to first order in the derivative expansion. The results are universal up to the value of the coefficients in the hydrodynamic expansion, which depend on the details of the critical point.

Contribution to the Normal Fluid

- In the neutral fluid a single new transport coefficient is allowed by the absence of boost invariance. The effect of the new coefficient is a production of dissipation when the fluid accelerates.

$$\pi_A^{[\mu\nu]} = -\alpha u^{[\mu} a^{\nu]}$$

Superfluids

- Two fluid flows.
- Spontaneous symmetry breaking
- S-wave superfluid - condensate - complex scalar operator
- Phase of scalar ϕ - Goldstone mode - participates in the hydrodynamics.
- In the relativistic system:
  u^μ - normal fluid 4-velocity.
  $\xi^\mu = -\partial_\mu \phi$ - Goldstone phase gradient - superfluid velocity.
- In non-relativistic systems $\vec{v}_n$, $\vec{v}_s$.
  Counterflow $\vec{w} = \vec{v}_n - \vec{v}_s$.
- Superconductors - broken gauge symmetry - $\xi^\mu \equiv -\partial_\mu \phi + \mathcal{A}_\mu$.

Constitutive relations

- Stress-tensor:

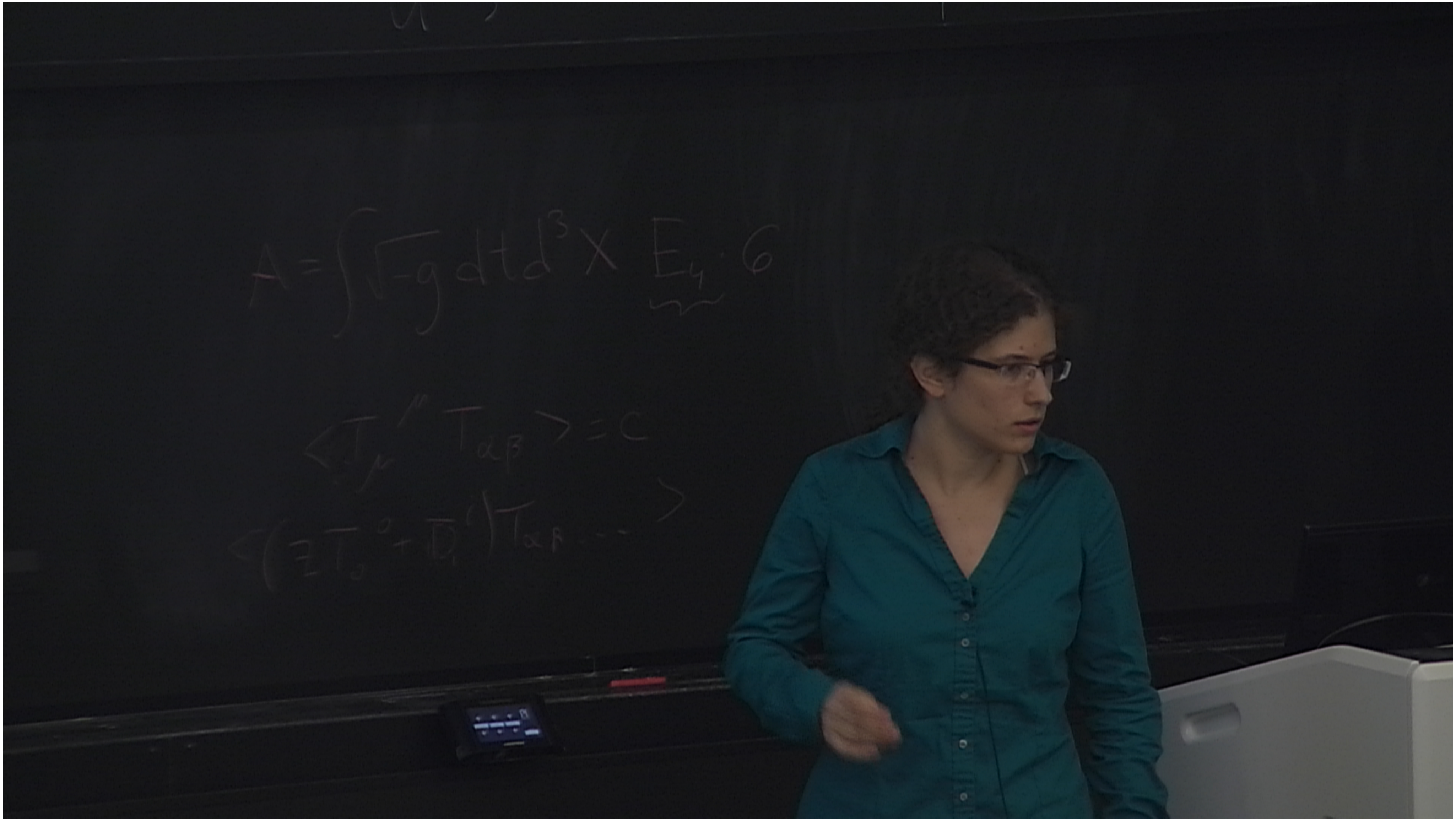
$$T^{\mu\nu} = (\varepsilon_n + p)u^\mu u^\nu + p\eta^{\mu\nu} + f\xi^\mu\xi^\nu + \pi^{(\mu\nu)} + \pi_A^{[\mu\nu]} \ .$$

- Decompose:

$$\pi^{(\mu\nu)} = (Q^\mu u^\nu + Q^\nu u^\mu) + \Pi P^{\mu\nu} + \Pi_t^{\mu\nu} \ ,$$

Q^μ represent the heat flow.

- Can determine dissipative corrections $\Pi_t^{\mu\nu}, \Pi, Q^\mu, V_{A\mu}$ from the requirement of locally positive entropy production rate.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⟨T_μ^μ T_αβ⟩ = c

- equations of motion in the non-relativistic limit:
  - mass conservation:

$$\partial_t(\rho_n + \rho_s) + \partial_i(\rho_n v_n^i + \rho_s v_s^i) = 0$$

  - Navier-Stokes:

$$\partial_t(\rho_n v_n^i + \rho_s v_s^i) + \partial_k(\rho_n v_n^i v_n^k + \rho_s v_s^i v_s^k) + \partial^i p + \nu^i = 0$$

  - Energy conservation:

$$\partial_t E + \partial_i[Q^i + Q'^i] + \nu_e = 0$$

- Dissipative corrections:

$$Q'^i \sim \pi^{i0}_{(1)} =$$
$$= \omega^i (Q_1 \omega^j \partial_j T + Q_2 \omega^j D_t v_{nj}) + Q_3 P^{ij}_\omega \partial_j T + Q_4 P^{ij}_\omega D_t v_{nj} + \dots$$

- scaling of transport coefficients

$$\sim T^{\frac{\Delta}{z}} F\left(\frac{\mu}{T^{\frac{2(z-1)}{z}}}, \frac{w^2}{T^{\frac{2(z-1)}{z}}} \right) .$$

$$[Q_1] = d - z; \quad [Q_2] = d - 2(z-1); \quad [Q_3] = z + d - 2; \quad [Q_4] = d$$

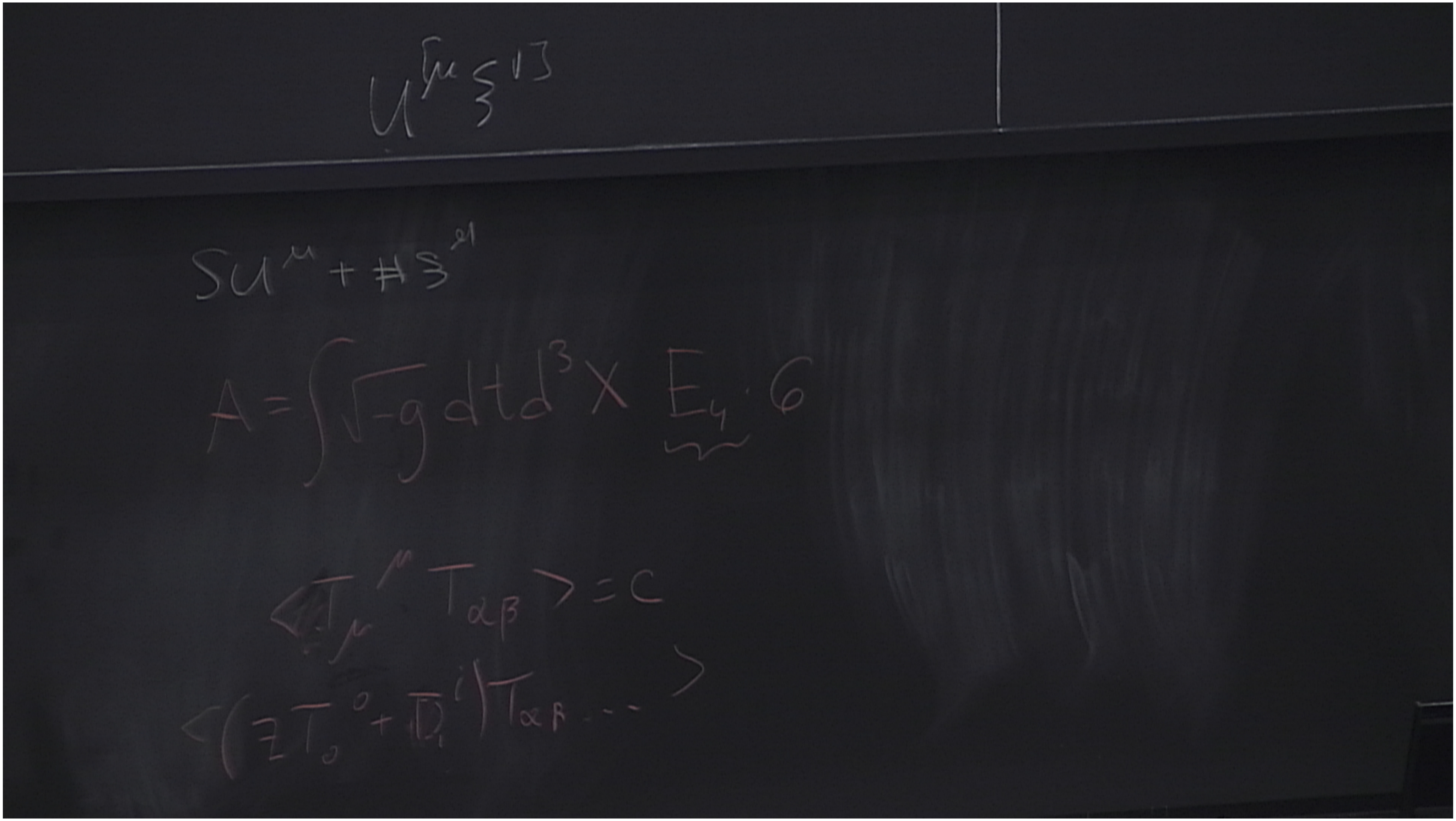
Experimental Implications

- Part of heat flow proportional to acceleration
- Anisotropy between the direction of the counterflow and the transverse direction.
- Hard to disentangle from effect of shear viscosity
- In superconductors alternating current create phase between normal and super components
- Perhaps some unusual frequency dependence would reveal the effect in superconductors

Outlook

- Suggest a measurement in superconductors.
- Include parity violating effects.
- Construct holographic model


Thank
You


A = ∫√-g dtd³x E4 · 6