Title: Evolution of quantum field, particle content, and classicality in the three stage universe

Date: Sep 18, 2014 11:00 AM

URL: http://pirsa.org/14090082

Abstract: I will discuss the evolution of a quantum scalar field in a toy universe which has three stages of evolution, viz., (i) an early (inflationary) de Sitter phase (ii) radiation-dominated phase and (iii) late-time (cosmological constant dominated) de Sitter phase. Using the Schr\"odinger picture, the scalar field equations are solved separately for the three stages and matched at the transition points. The boundary conditions are chosen so that field modes in the early de Sitter phase evolve from the Bunch-Davies vacuum state. I shall look the (time-dependent) particle content of this quantum state for the entire evolution of the universe and describe the various features both numerically and analytically. I shall also describe the quantum to classical transition in terms of a classicality parameter which tracks the particle creation and its effect on phase space correlation of the quantum field.

Zip - Zap - Zoom!

Suprit Singh

(Inter-University Centre for Astronomy and Astrophysics, Pune India)

September 18, 2014

Perimeter Institute for Theoretical Physics, Waterloo

Based on:

- *Evolution of quantum field, particle content, and classicality in the three stage universe*
  Suprit Singh, S. K. Modak and T. Padmanabhan
  [arXiv:1308.4976] Phys. Rev. **D 88**, 125020 (2013).

Outline

- Motivational climb – What's the point in doing all this?
- Arena: The three stage Universe with Inflation – Radiation – Λ
- An introduction to the quantum fields – *the players!*
- In the show – Particle creation and handle on quantum-to-classical transition
- Summary and future work.

Outline

- Motivational climb – What's the point in doing all this?
- Arena: The three stage Universe with Inflation – Radiation – Λ
- An introduction to the quantum fields – *the players!*
- In the show – Particle creation and handle on quantum-to-classical transition
- Summary and future work.

Outline

- Motivational climb – What's the point in doing all this?
- Arena: The three stage Universe with Inflation – Radiation – Λ
- An introduction to the quantum fields – *the players!*
- In the show – Particle creation and handle on quantum-to-classical transition
- Summary and future work.

The big picture

- The large scale structures, CMB anisotropies, Cosmic Magnetic Fields ... all have primordial origins
- are believed to be seeded during Inflation through *quantum* fluctuations.
- These fluctuations have substantial amplitudes only on scales close to the Planckian length, but during the inflationary stage they are stretched to galactic scales with nearly unchanged amplitudes.
- Surely, this calls for a quantum-to-classical transition. Mechanisms like decoherence etc are invoked but the feature is not well understood till date.

Important to understand, because the structures exist!

plus there are some more features!

The big picture

- The large scale structures, CMB anisotropies, Cosmic Magnetic Fields ... all have primordial origins
- are believed to be seeded during Inflation through *quantum* fluctuations.
- These fluctuations have substantial amplitudes only on scales close to the Planckian length, but during the inflationary stage they are stretched to galactic scales with nearly unchanged amplitudes.
- Surely, this calls for a quantum-to-classical transition. Mechanisms like decoherence etc are invoked but the feature is not well understood till date.

Important to understand, because the structures exist!

plus there are some more features!

The three stage universe

[Suprit Singh, Modak, Padmanabhan PRD 2013]

- We've got (i) Inflationary de Sitter (ii) Radiation phase and (iii) Late-time accelerated de Sitter phase.

$$a(t) = \begin{cases} e^{H_{\rm inf} t} & t \leqslant t_r \\ (2H_{\rm inf}\, e)^{1/2}\, t^{1/2} & t_r \leqslant t \leqslant t_\Lambda \\ (H_{\rm inf}/H_\Lambda)^{1/2}\, e^{H_\Lambda t} & t \geqslant t_\Lambda \end{cases}$$

  $t_r = (2H_{\rm inf})^{-1}$ and $t_\Lambda = (2H_\Lambda)^{-1}$ transition times fixed by matching.

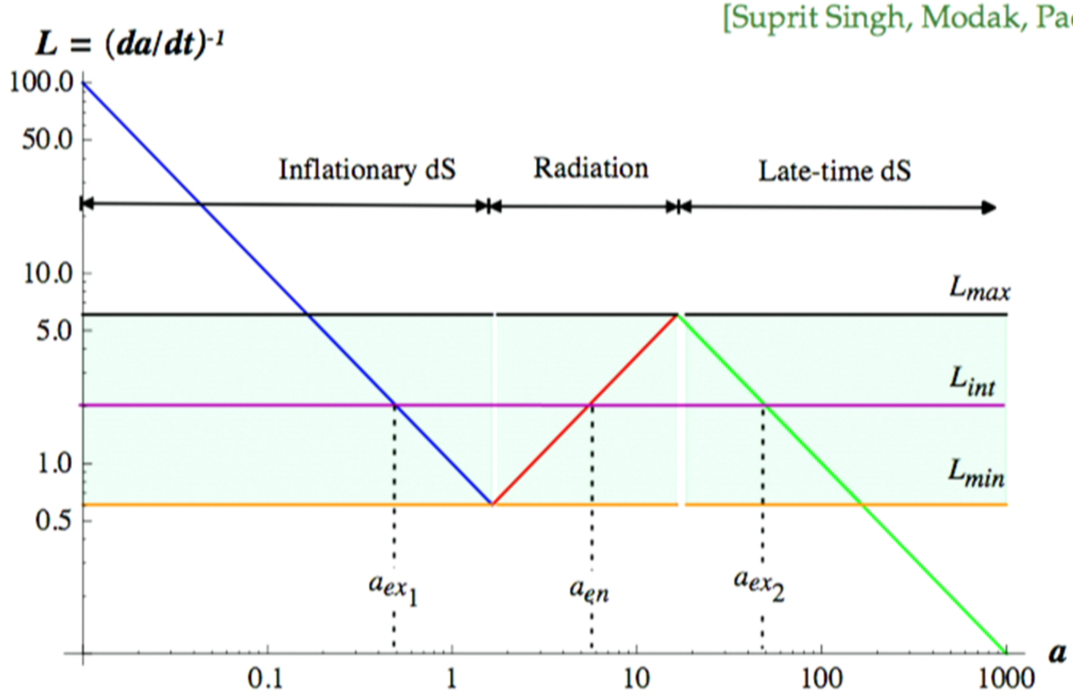
- $H_\Lambda = \epsilon H_{\rm inf} = \epsilon H$ with $\epsilon \ll 1$. For our real universe, $\epsilon \sim 10^{-54}$.

- Better to use scale factor as the time parameter, which gives

$$L(a) = (\dot{a})^{-1} = \begin{cases} (Ha)^{-1} & a \leqslant e^{1/2} \\ (a/He) & e^{1/2} \leqslant a \leqslant (e/\epsilon)^{1/2} \\ (a\epsilon H)^{-1} & a \geqslant (e/\epsilon)^{1/2} \end{cases}$$

The three stage universe – the tracks!

[Suprit Singh, Modak, Padmanabhan PRD 2013]



- Two length scales, $L_{max} = 1/(H\epsilon^{1/2}e^{1/2})$ and $L_{min} = 1/(He^{1/2})$ define a band.
- Provide a rich terrain for non-trivial effects on quantum fields.

Quantum fields, or essentially,

- their vacuum states are unstable under strong external fields and just like any medium show:
  - Dispersive effects or Vacuum polarization
  - Absorptive effects or Particle creation
- Flat spacetime examples: the Schwinger effect and the Casimir effect.
- Put these players on the curved spacetime. Gravity plays the role of an external field.
- With some differences here and there.
- In stationary situations (timelike killing vectors exist), we can define vacuum states uniquely or so called “in” and “out” states. Then, the horizon structures leads to non-trivial effects.

Quantum fields, or essentially,

- their vacuum states are unstable under strong external fields and just like any medium show:
  - Dispersive effects or Vacuum polarization
  - Absorptive effects or Particle creation
- Flat spacetime examples: the Schwinger effect and the Casimir effect.
- Put these players on the curved spacetime. Gravity plays the role of an external field.
- With some differences here and there.
- In stationary situations (timelike killing vectors exist), we can define vacuum states uniquely or so called "in" and "out" states. Then, the horizon structures leads to non-trivial effects.

In the cosmological playground,

- the differences show up. Its expanding! No timelike Killing vectors. So no unique vacuum.
- You can't switch off the background – so no free fields at any time except in the past.
- At best, we can introduce different constructs which probe different aspects of physics in the expanding background and develop an intuitive feel for the various phenomena.
- We have namely two pictures - the Heisenberg picture and the Schrödinger picture.
- It turns out, in what we are interested, the time-dependent particle content or quantifying the quantum-to-classical transition, the Schrödinger picture is better suited.

In the cosmological playground,

- the differences show up. Its expanding! No timelike Killing vectors. So no unique vacuum.
- You can't switch off the background – so no free fields at any time except in the past.
- At best, we can introduce different constructs which probe different aspects of physics in the expanding background and develop an intuitive feel for the various phenomena.
- We have namely two pictures - the Heisenberg picture and the Schrödinger picture.
- It turns out, in what we are interested, the time-dependent particle content or quantifying the quantum-to-classical transition, the Schrödinger picture is better suited.

In the cosmological playground,

- the differences show up. Its expanding! No timelike Killing vectors. So no unique vacuum.
- You can't switch off the background – so no free fields at any time except in the past.
- At best, we can introduce different constructs which probe different aspects of physics in the expanding background and develop an intuitive feel for the various phenomena.
- We have namely two pictures - the Heisenberg picture and the Schrödinger picture.
- It turns out, in what we are interested, the time-dependent particle content or quantifying the quantum-to-classical transition, the Schrödinger picture is better suited.

The Schrödinger framework

[Mahajan and Padmanabhan 2008]

Essence: Fields are essentially harmonic oscillators and vacuum state is gaussian wave functional.

- In an expanding background,

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$$

the frequency and mass become time-dependent.

- Massless, minimally coupled scalar field

$$S[\Phi(\eta, \mathbf{x})] = \frac{1}{2}\int d^3\mathbf{x} \int d\eta\, a^2(\eta)\left(\partial_\eta^2 \Phi - \partial_\mathbf{x}^2 \Phi\right)$$

- Fourier decomposition

$$\Phi(\eta, \mathbf{x}) = \int \frac{d^3 k}{(2\pi)^3}\, \phi_\mathbf{k}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}$$

- So that in terms of $\phi_\mathbf{k}$

$$S[\phi_\mathbf{k}(\eta)] = \frac{1}{2}\int d^3\mathbf{k} \int d\eta\, a^2(\eta)\left(\phi_\mathbf{k}'^2 - k^2\phi_\mathbf{k}^2\right)$$

The Schrödinger framework

[Mahajan and Padmanabhan 2008]

Essence: Fields are essentially harmonic oscillators and vacuum state is gaussian wave functional.

- In an expanding background,

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$$

the frequency and mass become time-dependent.

- Massless, minimally coupled scalar field

$$S[\Phi(\eta, \mathbf{x})] = \frac{1}{2}\int \mathrm{d}^3\mathbf{x} \int \mathrm{d}\eta\, a^2(\eta)\left(\partial_\eta^2\Phi - \partial_\mathbf{x}^2\Phi\right)$$

- Fourier decomposition

$$\Phi(\eta, \mathbf{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\, \phi_\mathbf{k}(\eta)e^{i\mathbf{k}\cdot\mathbf{x}}$$

- So that in terms of $\phi_\mathbf{k}$

$$S[\phi_\mathbf{k}(\eta)] = \frac{1}{2}\int \mathrm{d}^3\mathbf{k} \int \mathrm{d}\eta\, a^2(\eta)\left(\dot\phi_\mathbf{k}^2 - k^2\phi_\mathbf{k}^2\right)$$

- The Schrödinger equation of the wave functional

$$i\frac{\partial\psi(\phi_k,\eta)}{\partial\eta} = -\frac{1}{2a^2(\eta)}\frac{\partial^2\psi(\phi_k,\eta)}{\partial\phi_k^2} + \frac{1}{2}a^2(\eta)k^2\,\phi_k^2\,\psi(\phi_k,\eta)$$

- admits time-dependent, form-invariant, gaussian states with vanishing mean (*squeezed* states)

$$\psi(\phi_{\mathbf{k}},\eta) = N\exp\left[-\alpha_k(\eta)\phi_{\mathbf{k}}^2\right] = N\exp\left[-\frac{a^2(\eta)k}{2}\left(\frac{1-z_k}{1+z_k}\right)\phi_{\mathbf{k}}^2\right]$$

- The dynamics just reduces to the functions, $\alpha_k(\eta)$ or $z_k(\eta)$ (Riccati type equations)

$$i\dot{\alpha}_k = \frac{2\alpha_k^2}{a^2} - \frac{1}{2}a^2k \quad \text{or} \quad \dot{z}_k + 2\,i\,k\,z_k + \left(\frac{\dot{a}}{a}\right)(z_k^2-1) = 0$$

The *excitation parameter* z_k measures the deviation of α_k from the adiabatic value.

- The Schrödinger equation of the wave functional

$$i\frac{\partial\psi(\phi_k,\eta)}{\partial\eta} = -\frac{1}{2a^2(\eta)}\frac{\partial^2\psi(\phi_k,\eta)}{\partial\phi_k^2} + \frac{1}{2}a^2(\eta)k^2\,\phi_k^2\,\psi(\phi_k,\eta)$$

- admits time-dependent, form-invariant, gaussian states with vanishing mean (*squeezed* states)

$$\psi(\phi_{\mathbf{k}},\eta) = N\exp\left[-\alpha_k(\eta)\phi_{\mathbf{k}}^2\right] = N\exp\left[-\frac{a^2(\eta)k}{2}\left(\frac{1-z_k}{1+z_k}\right)\phi_{\mathbf{k}}^2\right]$$

- The dynamics just reduces to the functions, $\alpha_k(\eta)$ or $z_k(\eta)$ (Riccati type equations)

$$i\dot{\alpha}_k = \frac{2\alpha_k^2}{a^2} - \frac{1}{2}a^2k \quad \text{or} \quad \dot{z}_k + 2\,i\,k\,z_k + \left(\frac{\dot{a}}{a}\right)(z_k^2-1) = 0$$

The *excitation parameter* z_k measures the deviation of α_k from the adiabatic value.

- With $\alpha_k = -(ia^2/2)(\dot{\mu}_k/\mu_k)$, non-linear first order equations $\rightarrow$ second-order *linear* differential equation

$$\ddot{\mu}_k + 2\left(\frac{\dot{a}}{a}\right)\dot{\mu}_k + k^2\mu_k = 0$$

Same as the field equation for ϕ_k!

- And z_k is related to μ_k by:

$$z_k = \left(\frac{k\,\mu_k + i\,\dot{\mu}_k}{k\,\mu_k - i\,\dot{\mu}_k}\right)$$

- In general, $\mu_k(\eta) = \mathcal{A}_k s_k(\eta) + \mathcal{B}_k s_k^*(\eta)$.
- But z_k and α_k depend only on the ratio $\dot{\mu}_k/\mu_k$ and so we just need $\mathcal{R}_k = \mathcal{B}_k/\mathcal{A}_k$.
- Solving for μ_k given the boundary conditions completely determines evolution of the system.

Instantaneous particle content

- For $a > a_i$ the state of the field different from the ground state.
- Reasonable to compare it with *instantaneous* eigenstates defined at every instant obtained by adiabatically evolving the eigenstates at some initial epoch.

$$\psi_n(\phi_k, \eta) = F(\phi_k) \exp\left[-i \int_{\eta_i}^{\eta} d\eta \left(n + \frac{1}{2}\right) \omega(\eta)\right]$$

to get $C_n = \langle \psi_n | \psi \rangle$.

- Probability $P_{2n} = |C_{2n}|^2 = P_0 (2n)! \, |z|^{2n}/(n!)^2 2^{2n}$; $P_0 = \sqrt{1 - |z|^2}$.
- This gives the instantaneous particle content to be:

$$\langle n_k \rangle = \sum_{n=0}^{\infty} 2n P_{2n} = \frac{|z_k|^2}{1 - |z_k|^2}$$

- $\langle n_k \rangle$ is not monotonic in general. The mean occupation number can go up and down. It can be accompanied by fluctuating *quantum* noise when the system is far away from classicality.

Let's roll out!

- Solve for the field in the three stages

$$\mu_k^{(1)}(a) = \left(\frac{1}{a} - \frac{iH}{k}\right) \exp\left[\frac{ik}{H} - \frac{ik}{aH}\right]$$

$$\mu_k^{(2)}(a) = \frac{1}{a}\left(C_k e^{-ika/eH} + D_k e^{ika/eH}\right)$$

$$\mu_k^{(3)}(a) = E_k \bar{s}_k(a) + F_k \bar{s}_k^*(a)$$

where

$$\bar{s}_k(a) = \exp\left[\frac{ik}{\epsilon H}\left(1 - \frac{1}{a}\right)\right]\left(\frac{1}{a} - \frac{i\epsilon H}{k}\right)$$

- BC is chosen such that $\mu_k^{(1)}$ evolves from Bunch-Davies vacuum state defined in the asymptotic past.
- C_k, D_k, E_k, F_k are fixed by matching the modes and the first derivatives at the transition points.

Let's roll out!

- Solve for the field in the three stages

$$\begin{aligned} \mu_k^{(1)}(a) &= \left(\frac{1}{a} - \frac{iH}{k}\right) \exp\left[\frac{ik}{H} - \frac{ik}{aH}\right] \\ \mu_k^{(2)}(a) &= \frac{1}{a}\left(C_k e^{-ika/eH} + D_k e^{ika/eH}\right) \\ \mu_k^{(3)}(a) &= E_k \bar{s}_k(a) + F_k \bar{s}_k^*(a) \end{aligned}$$

where

$$\bar{s}_k(a) = \exp\left[\frac{ik}{\epsilon H}\left(1 - \frac{1}{a}\right)\right]\left(\frac{1}{a} - \frac{i\epsilon H}{k}\right)$$

- BC is chosen such that $\mu_k^{(1)}$ evolves from Bunch-Davies vacuum state defined in the asymptotic past.
- C_k, D_k, E_k, F_k are fixed by matching the modes and the first derivatives at the transition points.

Let's roll out!

- Solve for the field in the three stages

$$\mu_k^{(1)}(a) = \left(\frac{1}{a} - \frac{iH}{k}\right) \exp\left[\frac{ik}{H} - \frac{ik}{aH}\right]$$
$$\mu_k^{(2)}(a) = \frac{1}{a}\left(C_k e^{-ika/eH} + D_k e^{ika/eH}\right)$$
$$\mu_k^{(3)}(a) = E_k \bar{s}_k(a) + F_k \bar{s}_k^*(a)$$

where

$$\bar{s}_k(a) = \exp\left[\frac{ik}{\epsilon H}\left(1 - \frac{1}{a}\right)\right]\left(\frac{1}{a} - \frac{i\epsilon H}{k}\right)$$

- BC is chosen such that $\mu_k^{(1)}$ evolves from Bunch-Davies vacuum state defined in the asymptotic past.
- C_k, D_k, E_k, F_k are fixed by matching the modes and the first derivatives at the transition points.

Quantifying the degree of classicality

- Classicality is usually quantified by the use of Wigner distributions and is inferred from the peaking of the distribution on the corresponding classical trajectory.

- Proposal: The classicality parameter,

$$\mathcal{C} \equiv \frac{\langle pq \rangle}{\sqrt{\langle p^2 \rangle \langle q^2 \rangle}}$$

- For our field system with gaussian states,

$$\mathcal{W}(\phi_{\mathbf{k}}, \pi_{\mathbf{k}}, \eta) = \frac{1}{\pi} \exp\left[-\frac{\phi_{\mathbf{k}}^2}{\sigma_k^2} - \sigma_k^2 (\pi_{\mathbf{k}} - \mathcal{J}_k \phi_{\mathbf{k}})^2\right]$$

$$\mathcal{C}_k = \frac{\mathcal{J}_k \sigma_k^2}{\sqrt{1 + (\mathcal{J}_k \sigma_k^2)^2}} = \frac{2\,\mathrm{Im}(z_k)}{1 - |z_k|^2}$$

- For a pure quantum state, $\mathcal{C}_k = 0$ with $\mathcal{J}_k = 0$. Otherwise $\mathcal{C}_k \in [-1, 1]$ when the Wigner function becomes correlated.

Quantifying the degree of classicality

- Classicality is usually quantified by the use of Wigner distributions and is inferred from the peaking of the distribution on the corresponding classical trajectory.

- Proposal: The classicality parameter,

$$\mathcal{C} \equiv \frac{\langle pq \rangle}{\sqrt{\langle p^2 \rangle \langle q^2 \rangle}}$$

- For our field system with gaussian states,

$$\mathcal{W}(\phi_{\mathbf{k}}, \pi_{\mathbf{k}}, \eta) = \frac{1}{\pi} \exp\left[-\frac{\phi_{\mathbf{k}}^2}{\sigma_k^2} - \sigma_k^2(\pi_{\mathbf{k}} - \mathcal{J}_k \phi_{\mathbf{k}})^2\right]$$

$$\mathcal{C}_k = \frac{\mathcal{J}_k \sigma_k^2}{\sqrt{1 + (\mathcal{J}_k \sigma_k^2)^2}} = \frac{2\,\mathrm{Im}(z_k)}{1 - |z_k|^2}$$

- For a pure quantum state, $\mathcal{C}_k = 0$ with $\mathcal{J}_k = 0$. Otherwise $\mathcal{C}_k \in [-1, 1]$ when the Wigner function becomes correlated.

Let's roll out!

- Solve for the field in the three stages

$$\mu_k^{(1)}(a) = \left(\frac{1}{a} - \frac{iH}{k}\right) \exp\left[\frac{ik}{H} - \frac{ik}{aH}\right]$$
$$\mu_k^{(2)}(a) = \frac{1}{a}\left(C_k e^{-ika/eH} + D_k e^{ika/eH}\right)$$
$$\mu_k^{(3)}(a) = E_k \bar{s}_k(a) + F_k \bar{s}_k^*(a)$$

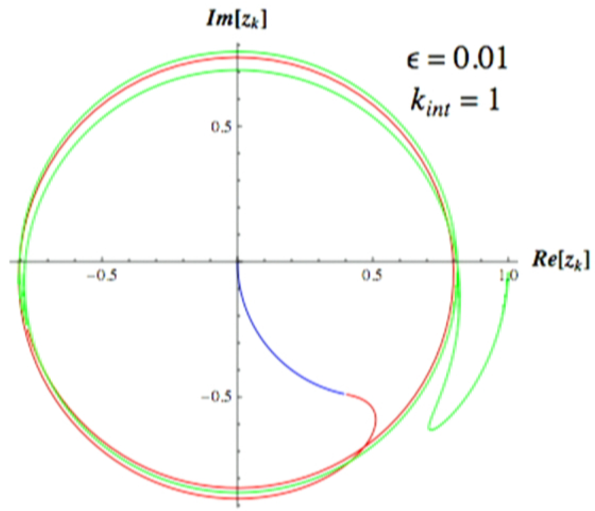
where

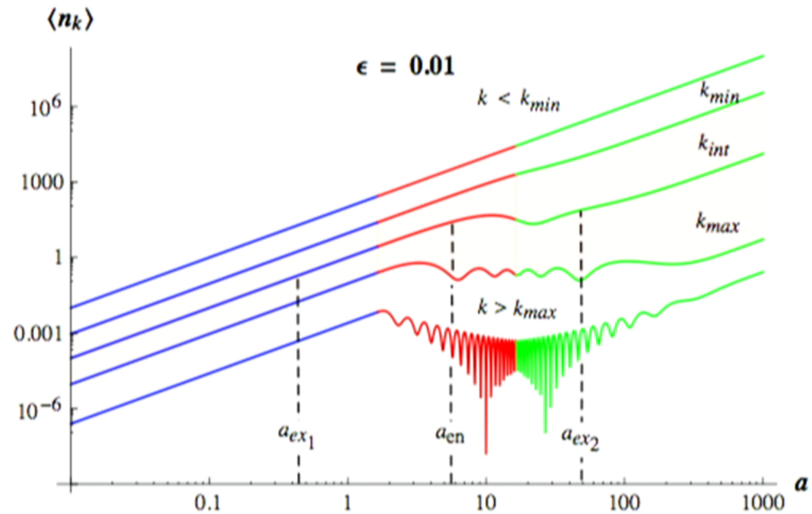
$$\bar{s}_k(a) = \exp\left[\frac{ik}{\epsilon H}\left(1 - \frac{1}{a}\right)\right]\left(\frac{1}{a} - \frac{i\epsilon H}{k}\right)$$

- BC is chosen such that $\mu_k^{(1)}$ evolves from Bunch-Davies vacuum state defined in the asymptotic past.
- C_k, D_k, E_k, F_k are fixed by matching the modes and the first derivatives at the transition points.

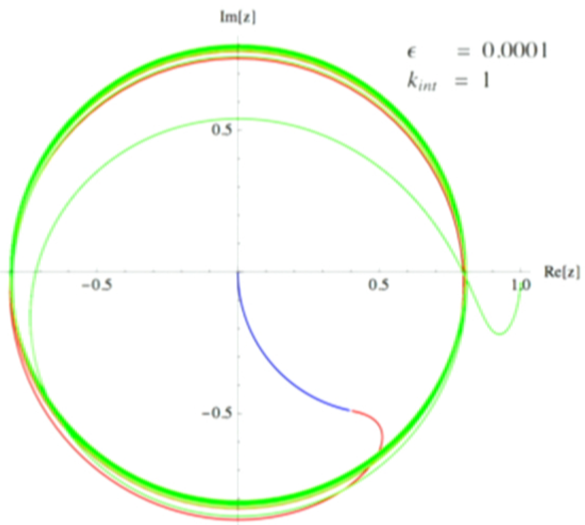
Numerics Gallery

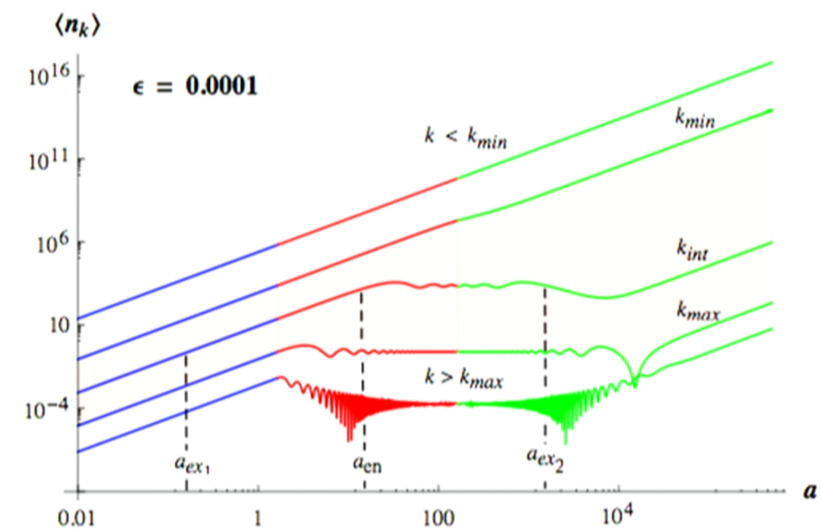
- We choose $H = 1$ and $\epsilon = 0.01, 0.0001$ for the numerics.
- Some very interesting features emerge.



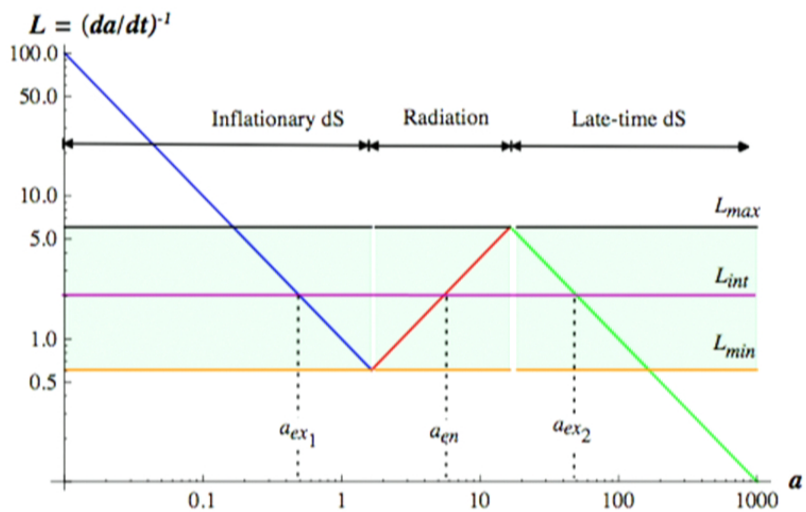


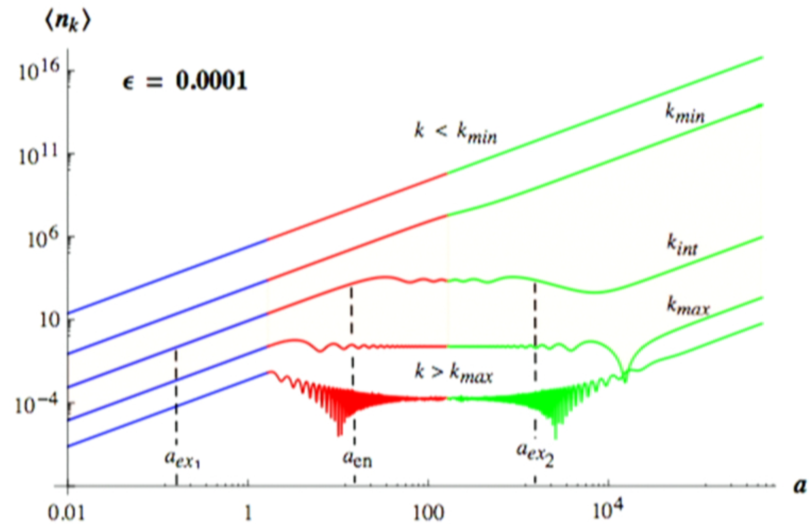
Numerics Gallery

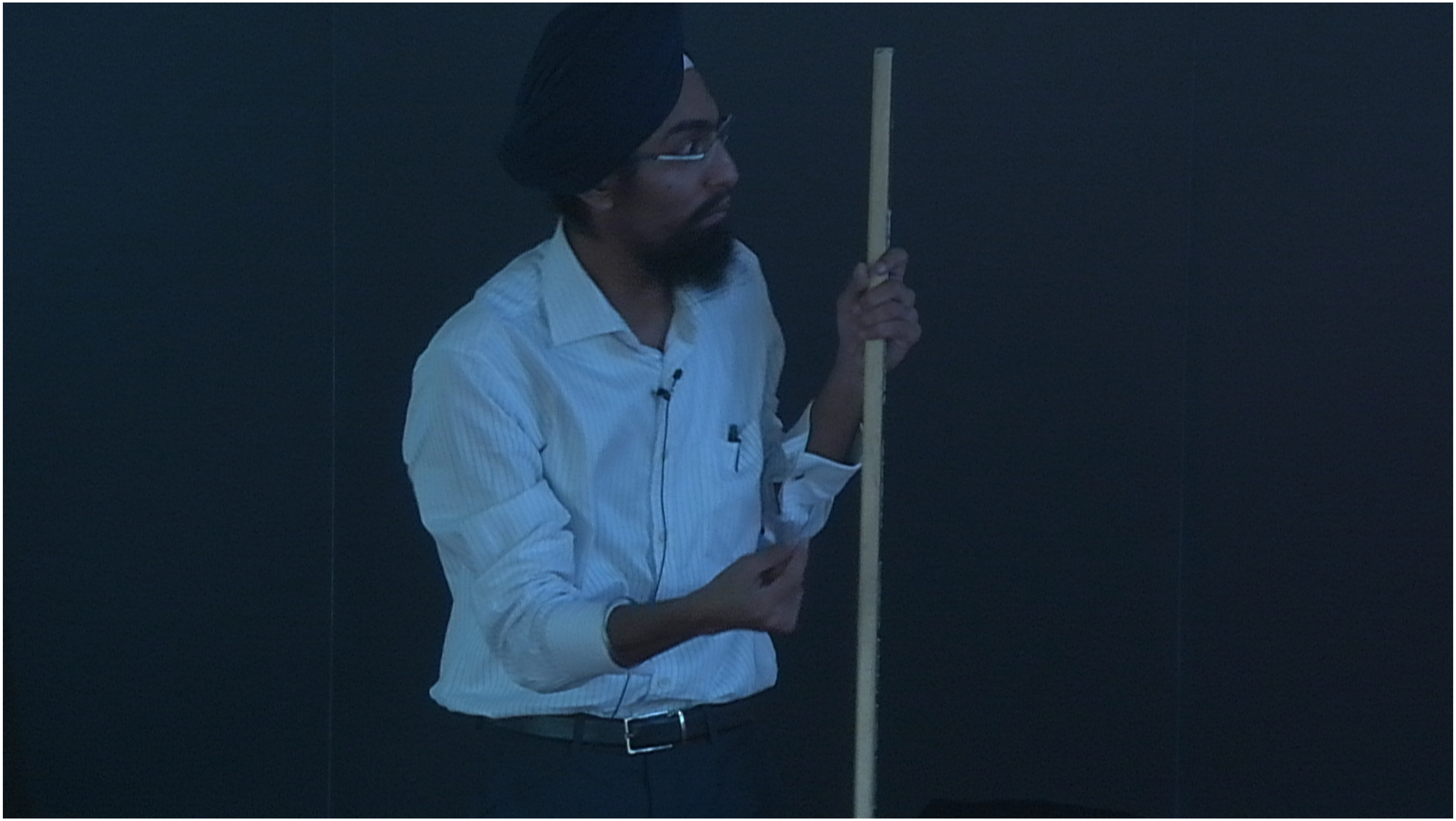




Numerics Gallery







The Schrödinger framework

[Mahajan and Padmanabhan 2008]

Essence: Fields are essentially harmonic oscillators and vacuum state is gaussian wave functional.

- In an expanding background,

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$$

the frequency and mass become time-dependent.

- Massless, minimally coupled scalar field

$$S[\Phi(\eta,\mathbf{x})] = \frac{1}{2}\int d^3x \int d\eta\, a^2(\eta)\left(\partial_\eta^2\Phi - \partial_x^2\Phi\right)$$

- Fourier decomposition

$$\Phi(\eta,\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3}\,\phi_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}}$$

- So that in terms of ϕ_k

$$S[\phi_k(\eta)] = \frac{1}{2}\int d^3k \int d\eta\, a^2(\eta)\left(\phi_k'^2 - k^2\phi_k^2\right)$$

Setting the initial conditions

- The initial state is a ground state with zero particle content at some time say $\eta = \eta_i$ when $a(\eta_i) = a_i$.

- Then, the initial condition of the wave function at $a = a_i$ being the ground state wave function of an harmonic oscillator demands,

$$\alpha_k(a_i) = \frac{a_i^2 k}{2}$$

or equivalently $z_k(a_i) = 0$, implying

$$\left(\frac{\dot{a}}{\mu_k}\frac{\mathrm{d}\mu_k}{\mathrm{d}a}\right)\bigg|_{a_i} = ik$$

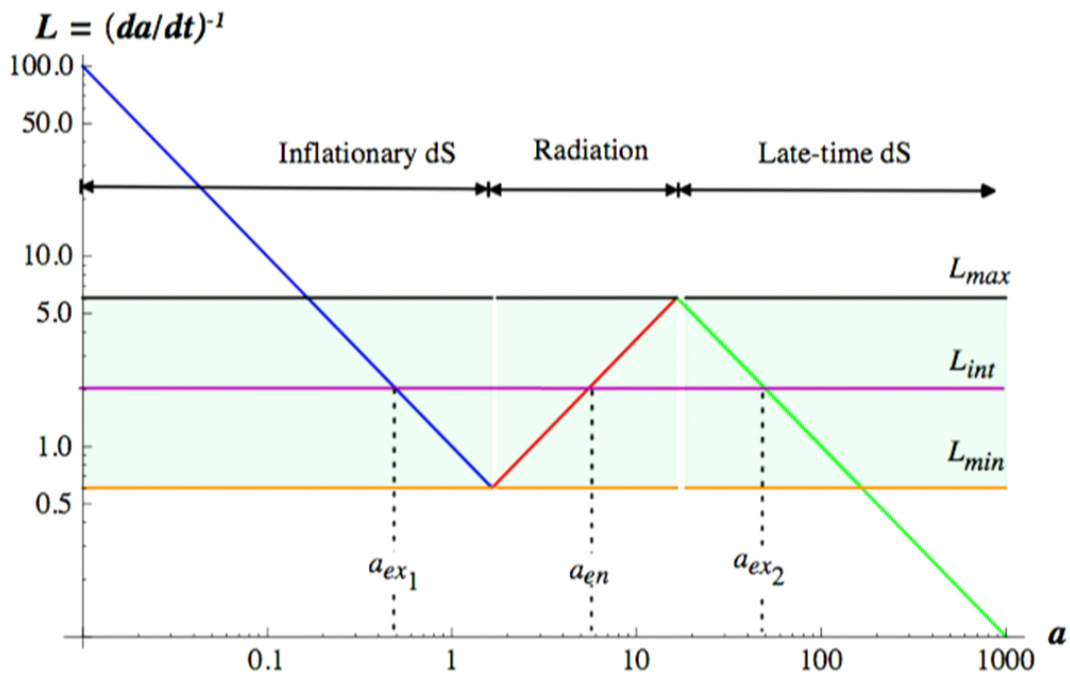
which in turn determines $\mathcal{R}_k(a_i)$ thereby fixing the state.

All set?

$$\ddot{\mu}_k + 2\left(\frac{\dot{a}}{a}\right)\dot{\mu}_k + k^2\mu_k = 0$$

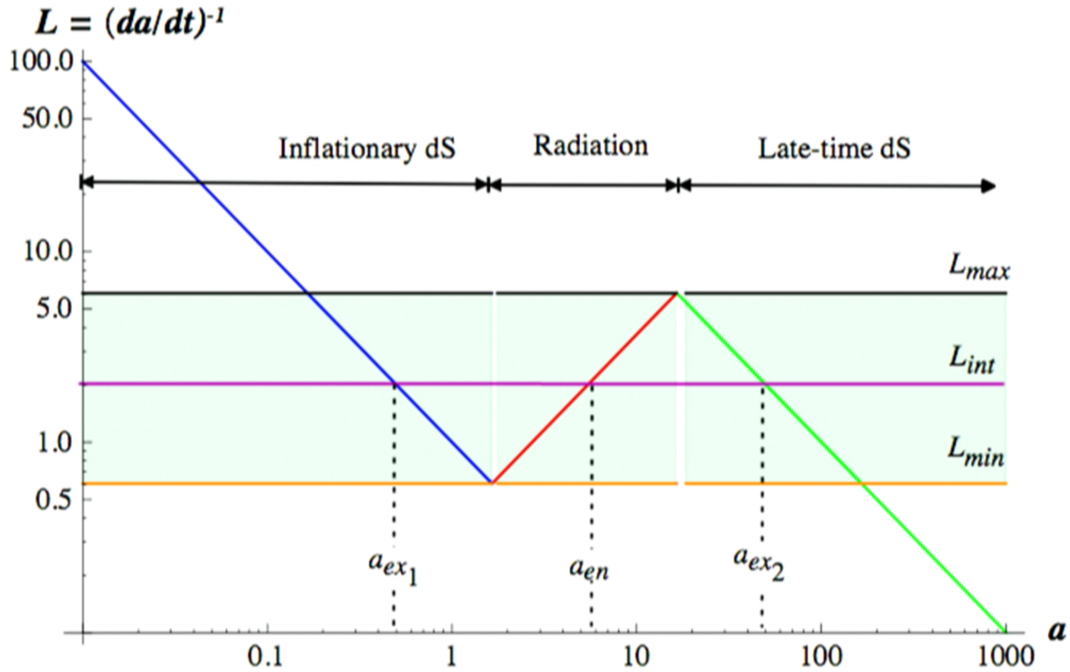
$$z_k = \left(\frac{k\,\mu_k + i\,\dot{\mu}_k}{k\,\mu_k - i\,\dot{\mu}_k}\right)$$

All set?



$L = (da/dt)^{-1}$
100.0
50.0
10.0
5.0
1.0
0.5
Inflationary dS
Radiation
Late-time dS
L_{max}
L_{int}
L_{min}
a_{ex_1}
a_{en}
a_{ex_2}
0.1
1
10
100
1000
a

All set?

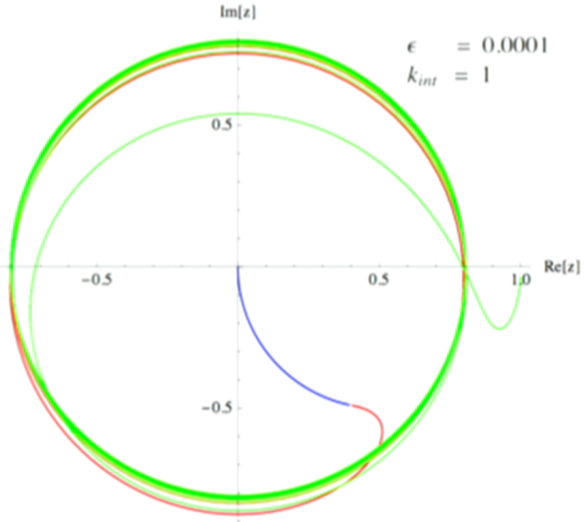


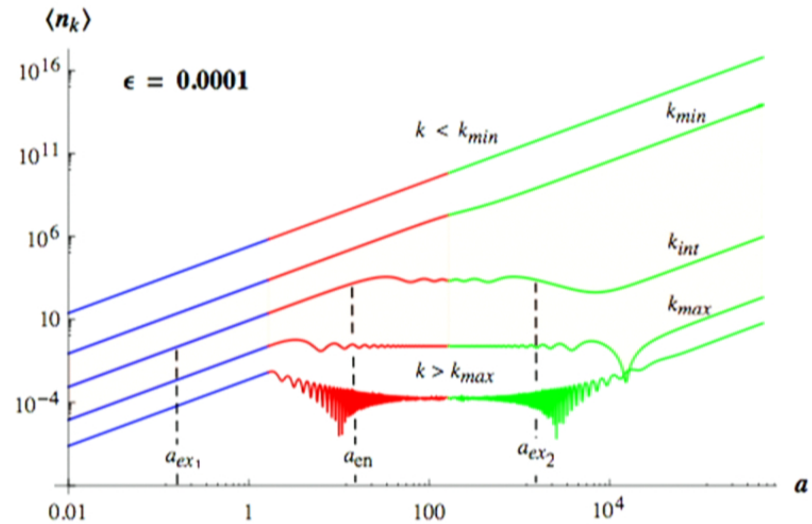
L = (da/dt)-1
100.0
50.0
10.0
5.0
1.0
0.5
Inflationary dS
Radiation
Late-time dS
L_{max}
L_{int}
L_{min}
a_{ex_1}
a_{en}
a_{ex_2}
0.1
1
10
100
1000
a

Numerics Gallery

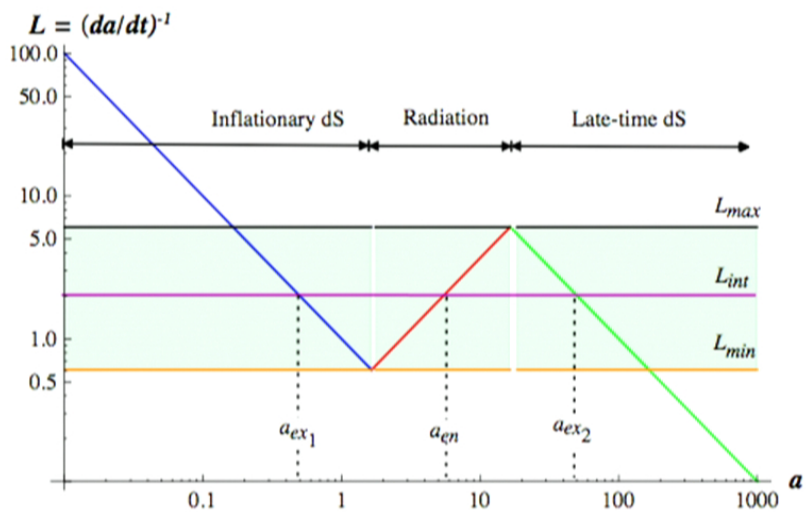
- We choose $H = 1$ and $\epsilon = 0.01, 0.0001$ for the numerics.
- Some very interesting features emerge.

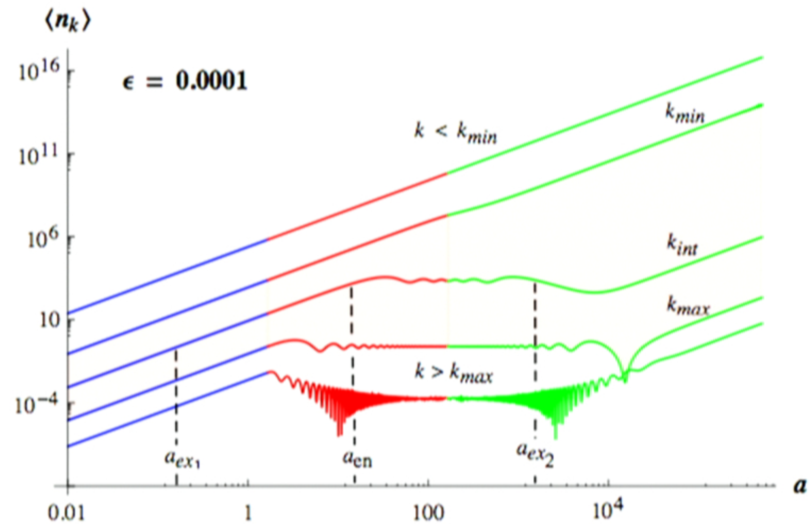
Numerics Gallery



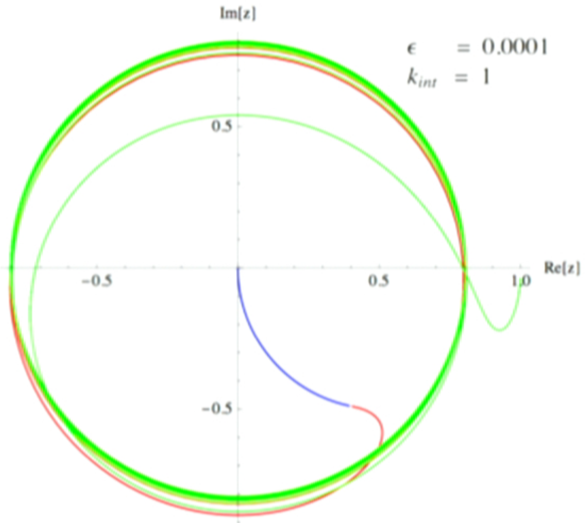


Numerics Gallery





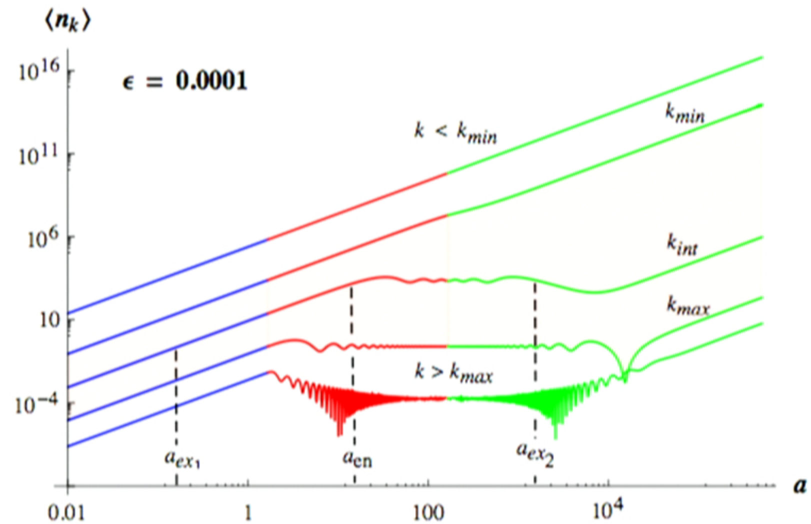
Numerics Gallery



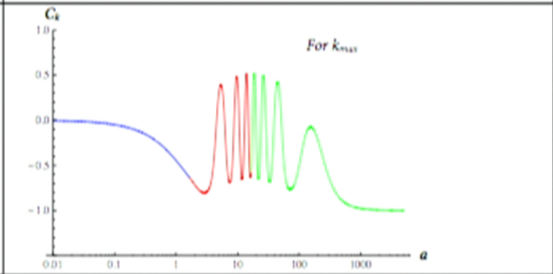
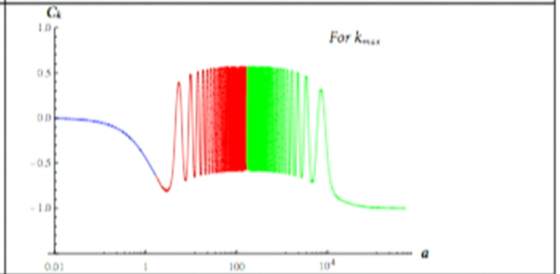
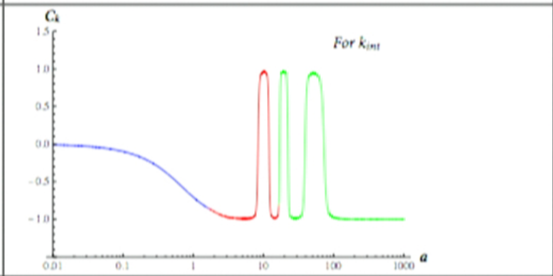
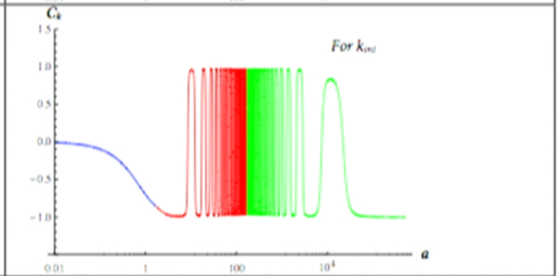
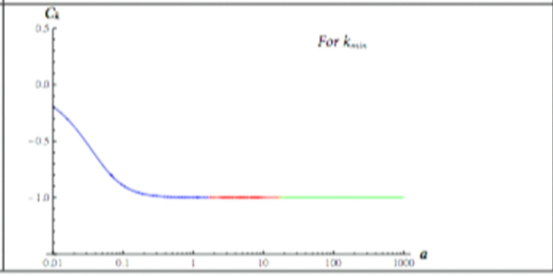
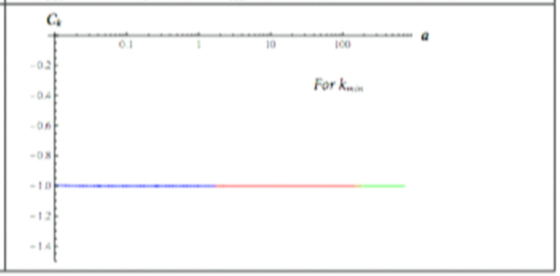
- Non-zero values of z_k imply departure from the adiabatic regime.
- and a *surprising memory effect!* z_k in stage 3 (green) continues to circle a bit! The evolution is non-adiabatic.

Numerics Gallery

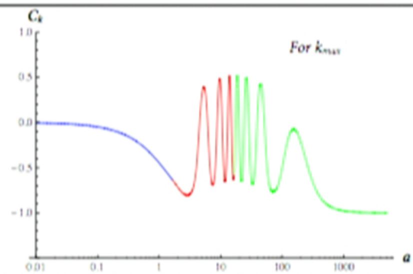
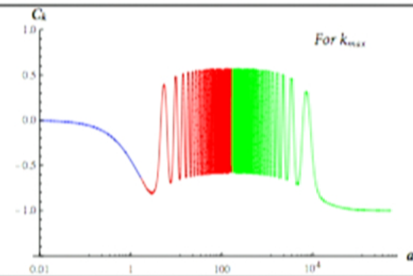
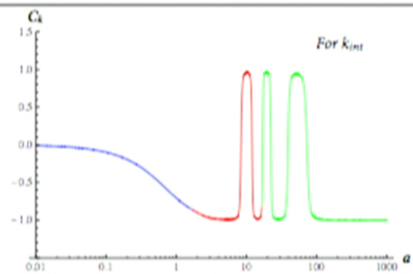
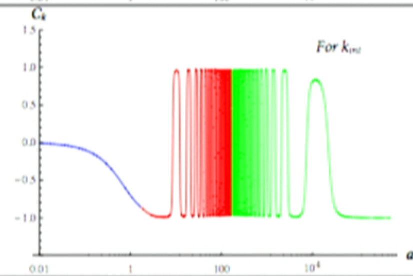
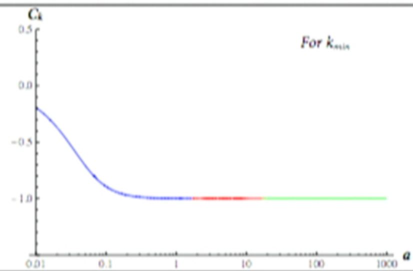
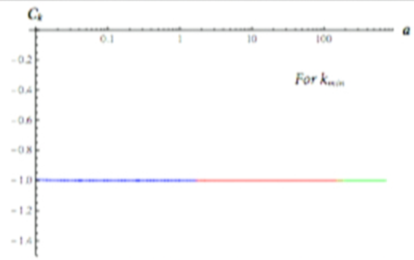
- $\langle n_k \rangle$ follows power law in inflationary dS.
- subsequent behaviour in the radiation phase depends on the scale of mode.
- There is a residual effect for modes that are within the Hubble radius at the end of the first stage.
- $\langle n_k \rangle$ saturates in the radiation phase irrespective of the value of ϵ.



Swings (!) from *Quantum* to *Classical* ...

k	$\epsilon = 0.01$	$\epsilon = 0.0001$
k_{sub}		
k_{int}		
k_{sup}		

Swings (!) from *Quantum* to *Classical* . . .

k	$\epsilon = 0.01$	$\epsilon = 0.0001$
k_{sub}		
k_{int}		
k_{sup}		

For $\epsilon \sim 10^{-54}$, equations are the only pictures!

- The winding number is unmanageably high so we can look at the analytical results.
- For the inflationary dS, we have

$$z_k^{(1)} = \frac{aH}{aH + 2ik}; \qquad \langle n_k^{(1)} \rangle = \frac{a^2H^2}{4k^2}; \qquad \mathcal{C}_k^{(1)} = -\frac{aH}{k\sqrt{\frac{a^2H^2}{k^2} + 1}}$$

- For the radiation phase

$$z_k^{(2)} = \frac{eH\left(e^{\frac{2ik}{\sqrt{e}H}} H(eH + 2iak) - e^{\frac{2iak}{eH}}\left(eH^2 + 2i\sqrt{e}Hk - 2k^2\right)\right)}{-e^{2+\frac{2ik}{\sqrt{e}H}} H^3 + e^{\frac{2iak}{eH}}(eH - 2iak)\left(eH^2 + 2i\sqrt{e}Hk - 2k^2\right)}$$

$$\langle n_k^{(2)} \rangle = \frac{e^2H^2}{8a^2k^6}\Bigg(e^2H^4 + 2\left(a^2H^2k^2 + k^4\right) - \left(e^2H^4 + 4a\sqrt{e}H^2k^2 - 2eH^2k^2\right) \mathrm{Cos}\left[\frac{2\left(-a+\sqrt{e}\right)k}{eH}\right]$$
$$-2kH\left(\left(-a+\sqrt{e}\right)eH^2 + 2ak^2\right)\mathrm{Sin}\left[\frac{2\left(-a+\sqrt{e}\right)k}{eH}\right]\Bigg).$$

- Asymptotic $\left(a \to \sqrt{e/\epsilon}\right)$ saturation: $\langle n_k^{(2)} \rangle_{sat} = e^2H^4/4k^4$

The third stage!

- We can approximate under suitable assumptions, viz., $\epsilon \ll 1$.

$$\langle n_k^{(3)} \rangle \approx \frac{e^2 H^4}{4k^4} - \frac{2H^4 e^{3/2} a \epsilon}{4k^4} + \frac{H^2 \left(e^2 H^4 - e H^2 k^2 + k^4\right) a^2 \epsilon^2}{4k^6}$$

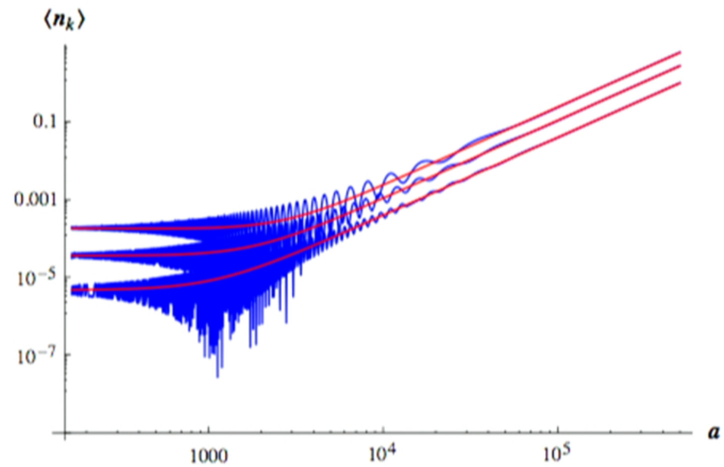
- How good is it?

The third stage!

- We can approximate under suitable assumptions, viz., $\epsilon \ll 1$.

$$\langle n_k^{(3)} \rangle \approx \frac{e^2 H^4}{4k^4} - \frac{2H^4 e^{3/2} a \epsilon}{4k^4} + \frac{H^2 \left(e^2 H^4 - e H^2 k^2 + k^4\right) a^2 \epsilon^2}{4k^6}$$

- How good is it?



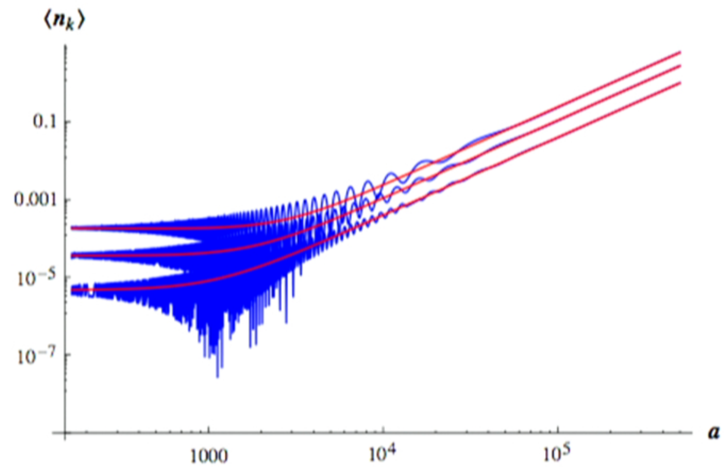
Quite good!

The third stage!

- We can approximate under suitable assumptions, viz., $\epsilon \ll 1$.

$$\langle n_k^{(3)} \rangle \approx \frac{e^2 H^4}{4k^4} - \frac{2H^4 e^{3/2} a \epsilon}{4k^4} + \frac{H^2 \left(e^2 H^4 - eH^2 k^2 + k^4\right) a^2 \epsilon^2}{4k^6}$$

- How good is it?



Quite good!

Implications for cosmology

- Assumptions made in inflationary paradigm:

  (a) quantum fluctuations behave classically on Hubble exit.

  (b) once classical, the fluctuations always stay classical in the subsequent stages.

- What is the notion of this classicality?
- We use the "classicality parameter" to define this. Then (a) is found true but (b) is not.
- Sub-Hubble modes have fluctuations on them.
- Implication: We should have a theory which can admit both the properties of the fields.

Implications for cosmology

- Assumptions made in inflationary paradigm:

  (a) quantum fluctuations behave classically on Hubble exit.

  (b) once classical, the fluctuations always stay classical in the subsequent stages.

- What is the notion of this classicality?
- We use the "classicality parameter" to define this. Then (a) is found true but (b) is not.
- Sub-Hubble modes have fluctuations on them.
- Implication: We should have a theory which can admit both the properties of the fields.