Title: Infinite Chiral Symmetry in Four Dimensions

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Abstract: We describe a new correspondence between four-dimensional conformal field theories with extended supersymmetry and two-dimensional chiral algebras. We explore the resulting chiral algebras in the context of theories of class S. The class S duality web implies nontrivial associativity properties for the corresponding chiral algebras, the structure of which can be summarized by a generalized topological quantum field theory.

Infinite Chiral Symmetry in Four Dimensions

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Based on

arXiv:1312.5344	with C. Beem, M. Lemos, P. Liendo, L. Rastelli and B.C. van Rees
arXiv:1408.6522	with C. Beem, L. Rastelli and B.C. van Rees
(to appear)	with M. Lemos

Introduction

Recently, many new, extended superconformal field theories have been uncovered.

Superconformal theories combine two powerful symmetries.

Mainly supersymmetry is leveraged to study these theories. For example,

- supersymmetric localization techniques for Lagrangian theories (rare)
- out on Coulomb branch: effective low energy effective action

Conformal symmetry's most prominent tools are left unused: convergent OPEs and the conformal bootstrap equations.

The conformal bootstrap approach seems to fall short to produce exact results beyond certain classes of 2d CFTs. Does supersymmetry improve our ability to solve the conformal bootstrap equations?

A new 2d/4d correspondence

More precisely, I want to address the question

Do the conformal bootstrap equations in four dimensions admit a solvable truncation in the case of superconformal field theories?

The answer is *yes* and for four-dimensional SCFTs takes the form of a map

$$\chi : \{\text{4d } \mathcal{N} = 2 \text{ SCFTs/marginal deformations}\} \rightarrow \{\text{2d chiral algebras}\}$$

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Chiral algebras

- contain and organize infinite amount of protected 4d conformal data
- describe the contribution of short multiplets in the double OPE expansion of 4d correlators
- lead to powerful new unitarity bounds

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4d SCFTs

- give rise to interesting new chiral algebras
- predict a generalized TQFT that takes values in chiral algebras

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A new 2d/4d correspondence: symmetries

Consider a four-dimensional $\mathcal{N} = 2$ superconformal field theory.

- its symmetry algebra is

$$\mathfrak{su}(2,2) \oplus \mathfrak{su}(2)_{\mathcal{R}} \oplus \mathfrak{u}(1)_r \subset \mathfrak{su}(2,2|2)$$

Consider a plane $\mathbb{R}^2 \subset \mathbb{R}^4$ ("the chiral algebra plane") parametrized by the complex coordinates $z, \bar{z}$.

- The subalgebra of the conformal algebra keeping this plane fixed is

$$\mathfrak{sl}(2) \oplus \overline{\mathfrak{sl}(2)} \oplus \mathfrak{u}(1)_{\perp} \subset \mathfrak{su}(2,2) ,$$

where $\mathfrak{sl}(2)$ $(\overline{\mathfrak{sl}(2)})$ acts by Möbius transformations on z $(\bar{z})$

There exists nilpotent supercharges $\mathbb{Q}_1, \mathbb{Q}_2$ in $\mathfrak{su}(2,2|2)$ such that

$$[\mathbb{Q}_i, \mathfrak{sl}(2)] = 0 , \qquad \{\mathbb{Q}_i, \ldots\} = \text{diag}[\overline{\mathfrak{sl}(2)} \oplus \mathfrak{sl}(2)_{\mathcal{R}}] \equiv \widehat{\mathfrak{sl}(2)} .$$

A new 2d/4d correspondence: cohomology

We wish to restrict attention to operators $\mathcal{O}$ in the cohomology of $\mathbb{Q}_i$.

Such operators necessarily lie in the chiral algebra plane. At the *origin*

- they are annihilated by $\mathcal{Q}^1_-$ and $\tilde{\mathcal{Q}}_{\dot{-}2}$ and their conjugates
- they must satisfy the chirality condition

$$\frac{E-(j_1+j_2)}{2}-\mathcal{R}=0 \qquad \Longrightarrow \qquad r+(j_1-j_2)=0\,,$$

  where E is the conformal dimension, (j_1, j_2) are the Lorentz spins, and $(\mathcal{R}, r)$ are the quantum numbers under $\mathfrak{su}(2)_\mathcal{R} \oplus \mathfrak{u}(1)_r$
- they necessarily are $\mathfrak{su}(2)_\mathcal{R}$ highest weight states

They are precisely counted by the Schur limit of the superconformal index, hence we call them "Schur operators".

A new 2d/4d correspondence: free hypermultiplets

The scalars in the free hypermultiplet (HM) are $Q, \tilde{Q}$ and their conjugates. They can be organized in $\mathfrak{su}(2)_\mathcal{R}$ and $\mathfrak{su}(2)_F$ doublets as

$$Q^{\mathcal{I}}_{\hat{\mathcal{I}}} = \begin{pmatrix} Q & \tilde{Q} \\ \tilde{Q}^* & -Q^* \end{pmatrix}, \qquad Q^{\mathcal{I}}_{\hat{\mathcal{I}}}(x) Q^{\mathcal{J}}_{\hat{\mathcal{J}}}(0) \sim -\frac{\epsilon^{\mathcal{I}\mathcal{J}}\epsilon_{\hat{\mathcal{I}}\hat{\mathcal{J}}}}{|x|^2}$$

The Schur operators of the free HM are the $\mathfrak{su}(2)_\mathcal{R}$ highest weight states

We then define

$$q_{\hat{\mathcal{I}}}(z) = [u_{\mathcal{I}}(\bar{z}) Q^{\mathcal{I}}_{\hat{\mathcal{I}}}(z,\bar{z})]_{\mathbb{Q}_i} = [Q^1_{\hat{\mathcal{I}}}(z,\bar{z}) + \bar{z} Q^2_{\hat{\mathcal{I}}}(z,\bar{z})]_{\mathbb{Q}_i}$$

and compute the OPEs

$$q_{\hat{\mathcal{I}}}(z) q_{\hat{\mathcal{J}}}(0) \sim -\frac{\bar{z}\ \epsilon^{21}\ \epsilon_{\hat{\mathcal{I}}\hat{\mathcal{J}}}}{z\bar{z}} = \frac{\epsilon_{\hat{\mathcal{I}}\hat{\mathcal{J}}}}{z}$$

We thus find a (non-unitary) pair of symplectic bosons.

- Its 2d stress tensor $T = \frac{1}{2}\epsilon^{\hat{\mathcal{I}}\hat{\mathcal{J}}} q_{\hat{\mathcal{I}}} \partial q_{\hat{\mathcal{J}}}$ has central charge $c_{2d} = -1$.
- Its 2d flavor current $J_{\hat{\mathcal{I}}\hat{\mathcal{J}}} = -\frac{1}{4} q_{(\hat{\mathcal{I}}} q_{\hat{\mathcal{J}})}$ has level $k_{2d} = -\frac{1}{2}$.

A new 2d/4d correspondence: dictionary

Free hypermultiplets $\rightarrow$ Symplectic bosons $q_{\hat{\mathcal{I}}}$

Free vector multiplet $\rightarrow$ small (b, c) ghost system of type $(1, 0)$

Flavor symmetry G $\rightarrow$ affine Kac-Moody symmetry G

- Schur operator: $\mu_A^{(11)}$ (moment map operator)
  - $j_A(z)\, j_B(0) \sim \frac{k_{2d}\, \delta_{AB}}{z^2} + \frac{f_{AB}{}^C\, j_C(0)}{z}$
  - $k_{2d} = -\frac{k_{4d}}{2}$

Stress tensor multiplet $\rightarrow$ Virasoro stress tensor T

- Schur operator: $J^{11}_{+\dot{+}}$ ($\mathfrak{su}(2)_{\mathcal{R}}$ current)
  - $T(z)T(0) \sim \frac{c_{2d}/2}{z^4} + \frac{2T}{z^2} + \frac{\partial T}{z}$
  - $c_{2d} = -12c_{4d}$

Higgs branch chiral ring $\rightarrow$ Virasoro primary generators

- chiral ring relations $\rightarrow$ null relations

A new 2d/4d correspondence: exactly marginal gauging

Consider a 4d $\mathcal{N} = 2$ SCFT $\mathcal{T}$ with flavor symmetry G of level $k_{4d} = 4h^\vee$.
Exactly marginal gauging proceeds in two steps:

- at zero gauge coupling: introduce an $\mathcal{N} = 2$ vector multiplet and restrict to gauge invariant states
- at finite coupling: short multiplets can recombine into long multiplets and acquire an anomalous dimension

Corresponding procedure in chiral algebra?

- at zero gauge coupling: introduce a small adjoint (b, c) ghost system and restrict to gauge invariant states
- at finite coupling: a BRST procedure elegantly captures the one-loop corrections to the supercharge

A new 2d/4d correspondence: exactly marginal gauging

Some comments:

- OPE coefficients are subject to non-renormalization theorem [Baggio, de Boer, Papadodimas]
- chiral algebra is independent of exactly marginal couplings
- the same gauging procedure appears in gauged WZW models via path integral manipulations $\Longrightarrow$ "4d gauging = 2d gauging"
- explanation via localization argument?

A new 2d/4d correspondence: superconformal index

The superconformal index (SCI) is the analogue of the Witten index for the radially quantized theory.

Of particular interest to us, is the *Schur limit* of the SCI:

$$\mathcal{I}^{(Schur)}(q;\mathbf{x}) = \mathrm{Tr}_{\mathcal{H}[S^3]}(-)^F q^{E-R}\prod_i x_i^{f_i} \,,$$

which counts states annihilated by $\mathcal{Q}^1_-$ and $\tilde{\mathcal{Q}}_{\dot{-}2}$ and their conjugates.

Observing that using the chirality condition

$$E - R = \frac{E + j_1 + j_2}{2} = \text{the eigenvalue of } L_0 \,,$$

we find that

$$\mathcal{I}_\chi(q;\mathbf{x}) = \mathrm{Tr}_{\mathcal{H}_\chi}(-)^F q^{L_0}\prod_i x_i^{f_i} = \mathcal{I}^{(Schur)}(q;\mathbf{x}) \,.$$

The chiral algebra provides a 'categorification' of the Schur index.

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New unitarity bounds

The four-point function of twisted moment map operators

$$J_A(z) = \left[u_{\mathcal{I}_1}(\bar{z}) u_{\mathcal{I}_2}(\bar{z}) \, \mu_A^{(\mathcal{I}_1 \mathcal{I}_2)}(z, \bar{z}) \right]_{\mathbb{Q}_i}$$

is completely fixed in terms of the affine level k_{2d}:

$$z_{12}^2 z_{34}^2 \langle J_A(z_1) J_B(z_2) J_C(z_3) J_D(z_4) \rangle = f_{ABCD}(z) = \sum_{R \in \otimes^2 \mathbf{adj}} P^R_{ABCD} f_R(z) \,,$$

where $z = \frac{z_{12} z_{34}}{z_{13} z_{24}}$ is the 2d conformal cross ratio.

For each R we can decompose in $\mathfrak{sl}(2)$ conformal blocks

$$f_R(z) = \sum_{\ell=0}^{\infty} (-)^{\ell} \, a_\ell \, g_\ell(z) \,, \qquad \text{with} \qquad g_\ell(z) = \left(-\frac{1}{2} z \right)^{\ell - 1} z \, {}_2F_1(\ell, \ell; 2\ell; z)$$

New unitarity bounds

The four-point function of four-dimensional moment map operators

$$\mu_A(x;y) \equiv u_{\mathcal{I}_1}(y)u_{\mathcal{I}_2}(y)\ \mu_A^{(\mathcal{I}_1\mathcal{I}_2)}(x)$$

can be decomposed similarly

$$x_{12}^4 x_{34}^4 \langle \mu_A(x_1;y_1)\mu_B(x_2;y_2)\mu_C(x_3;y_3)\mu_D(x_4;y_4)\rangle = \sum_{R\in\otimes^2\mathbf{adj}} P^R_{ABCD}\mathcal{F}_R(z,\bar{z};y_i)\ ,$$

where the 4d conformal cross ratios are written as

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}\ , \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})\ .$$

For each R we can decompose in superconformal blocks

$$\mathcal{F}_R(z,\bar{z};y_i) = \sum_{\text{short}} p_{\Delta,\ell}\ \mathcal{G}^{\text{short}}_{\Delta,\ell}(z,\bar{z};y_i) + \sum_{\text{long}} q_{\Delta,\ell}\ \mathcal{G}^{\text{long}}_{\Delta,\ell}(z,\bar{z};y_i)$$

New unitarity bounds

The p-coefficients are completely determined by the a_ℓ, if we assume no higher spin currents are present.

Four-dimensional unitarity demands that $p \geq 0$, which leads to constraints on the central charges.

For example, unitarity in the singlet channel demands

$$\frac{\dim G_F}{c_{4d}} \geq \frac{24 h^\vee}{k_{4d}} - 12$$

Saturation of this bound implies Higgs branch chiral ring relation:

$$(\mu \otimes \mu)|_{\mathbf{1}} = 0 \ .$$

In the chiral algebra, the stress tensor T equals the Sugawara stress tensor.

More bounds can be obtained in different channels and saturation leads to chiral ring relations $(\mu \otimes \mu)|_R = 0$, and null relations in the chiral algebra.

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New unitarity bounds

G_F		Bound	Representation
$SU(N)$	$N \geqslant 3$	$k_{4d} \geqslant N$	$\mathbf{N^2-1}_{\rm symm}$
$SO(N)$	$N = 4, \ldots, 8$	$k_{4d} \geqslant 4$	$\frac{1}{24}\mathbf{N(N-1)(N-2)(N-3)}$
$SO(N)$	$N \geqslant 8$	$k_{4d} \geqslant N-4$	$\frac{1}{2}\mathbf{(N+2)(N-1)}$
$USp(2N)$	$N \geqslant 3$	$k_{4d} \geqslant N+2$	$\frac{1}{2}\mathbf{(2N+1)(2N-2)}$
G_2		$k_{4d} \geqslant \frac{10}{3}$	$\mathbf{27}$
F_4		$k_{4d} \geqslant 5$	$\mathbf{324}$
E_6		$k_{4d} \geqslant 6$	$\mathbf{650}$
E_7		$k_{4d} \geqslant 8$	$\mathbf{1539}$
E_8		$k_{4d} \geqslant 12$	$\mathbf{3875}$

New unitarity bounds: application

Precisely such Higgs branch relations must be realized in theories whose Higgs branch $\cong G_F$ one-instanton moduli space.

$$(\mu \otimes \mu)|_{\mathcal{I}_2} = 0\ , \qquad \mathrm{Sym}^2(\mathbf{adj}) = (2\mathbf{adj}) \oplus \mathcal{I}_2\ .$$

Known possibilities: SCFTs arising from single $D3$-brane probing F-theory singularity: $G_F = A_1, A_2, D_4, E_6, E_7, E_8$. We can confirm their central charges k_{4d}, c_{4d}.

New possibilities: $G_F = F_4$ with $k_{4d} = 5$ and $c_{4d} = \frac{5}{3}$
$G_F = G_2$ with $k_{4d} = \frac{10}{3}$ and $c_{4d} = \frac{5}{6}$

These are not known to exist.

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$SU(2)$ superconformal QCD

$SU(2)$ superconformal QCD has enhanced $\mathfrak{so}(8)$ flavor symmetry.

In fact, it is precisely the D_4 theory of the previous slide.

Therefore, its chiral algebra must contain an $\widehat{\mathfrak{so}(8)}_{-2}$ current algebra and its stress tensor equals the Sugawara stress tensor.

Could this be the complete description?

- the theory can be realized by gluing two theories of eight free half hypermultiplets. Explicit cohomology computations reveal that no new generators appear up to dimension 5.
- explicit null relation computations verify the Joseph relations
- the Schur index was checked against the affine vacuum character up to dimension 5. (Note that the computation of affine characters at negative level is hard and involves Kazhdan Lusztig technology)
- S-duality is implemented automatically by $\mathfrak{so}(8)$ triality

More (conjectural) results

- The chiral algebra for $\mathcal{N} = 4$ SYM theory with gauge group G is isomorphic to an $\mathcal{N} = 4$ super $\mathcal{W}$-algebra with rank$G$ generators given by chiral primaries of dimensions $\frac{d_i}{2}$, where d_i are the degrees of the Casimir invariants of G.
- The chiral algebras associated to SCFTs engineered in F-theory, are conjectured to be the affine current algebra at the appropriate level. In particular: $\chi[T_3] = \widehat{(\mathfrak{e}_6)}_{-3}$.

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Chiral algebras of class $\mathcal{S}$

The chiral algebra is independent of marginal couplings.

Up to marginal couplings, four-dimensional SCFTs of class $\mathcal{S}$ are fully specified by

- a decorated Riemann surface $\mathcal{C}_{g,s}$
- a choice of simply laced Lie algebra $\mathfrak{g}$

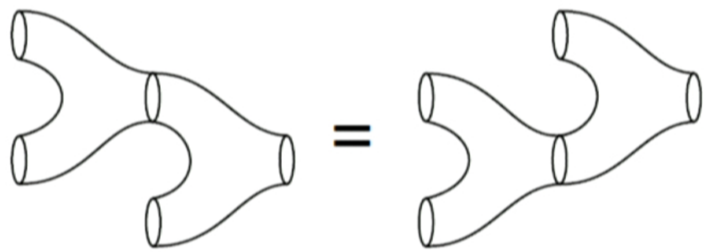
Chiral algebras of class $\mathcal{S}$ are thus described by a generalized TQFT that takes values in 2d chiral algebras.

The gluing prescription was already outlined earlier, thus it suffices to specify the chiral algebras associated to

and their generalizations with submaximal punctures.

Chiral algebras of class $\mathcal{S}$: associativity

Generalized S-duality translates into associativity of the TQFT:



Generalized S-duality completely determines the SCI [Gaiotto, Rastelli, Razamat].

Is a similar theory space bootstrap possible for its categorification?

Chiral algebras of class $\mathcal{S}$: trinions

The chiral algebra associated to

- T_2 is the theory of trifundamental symplectic bosons:

$$q_{abc}(z)q_{a'b'c'}(0) \sim \frac{\epsilon_{aa'}\epsilon_{bb'}\epsilon_{cc'}}{z}$$

- T_3 is conjectured to be the $\widehat{(\mathfrak{e}_6)}_{-3}$ affine current algebra

$$J_A(z)J_B(0) \sim \frac{-3\delta_{AB}}{z^2} + \frac{f_{AB}{}^C J_C(0)}{z}$$

- $T_{n\geq 4}$ is conjectured to be a $\mathcal{W}$-algebra generated by
  - a stress tensor T with Virasoro central charge $c_{2d} = -12c_{4d}$
  - three commuting $\widehat{\mathfrak{su}(n)}_{-n}$ critical current algebras
  - one current of dimension $\frac{1}{2}\ell(n-\ell)$ transforming in the $(\wedge^\ell, \wedge^\ell, \wedge^\ell)$ of $\mathfrak{su}(n)^3$ for $\ell = 1, \ldots, n-1$

Chiral algebras of class $\mathcal{S}$: trinions

The existence of such a $\mathcal{W}$-algebra is nontrivial.

For a random choice of generators one does not expect to be able to solve the associated Jacobi identities.

If one succeeds, one gains information about an infinite amount of protected CFT data (spectral data and OPE coefficients), and moreover a tool to predict Higgs branch relations.

For T_4 we showed that there exists a unique choice of singular OPEs such that the algebra is associative. [Lemos, WP (to appear)]

Chiral algebras of class $\mathcal{S}$: T_4 chiral algebra

generator	dimension	$\mathfrak{su}(4)_1$	$\mathfrak{su}(4)_2$	$\mathfrak{su}(4)_3$
$(J^1)_{a_1}^{\ b_1}$	1	**15**	**1**	**1**
$(J^2)_{a_2}^{\ b_2}$	1	**1**	**15**	**1**
$(J^3)_{a_3}^{\ b_3}$	1	**1**	**1**	**15**
T	2	**1**	**1**	**1**
$W_{a_1 a_2 a_3}$	$\frac{3}{2}$	**4**	**4**	**4**
$\tilde{W}^{b_1 b_2 b_3}$	$\frac{3}{2}$	$\bar{\mathbf{4}}$	$\bar{\mathbf{4}}$	$\bar{\mathbf{4}}$
$V_{[a_1 c_1][a_2 c_2][a_3 c_3]}$	2	**6**	**6**	**6**

Example OPE:

$$W_{a_1 a_2 a_3}(z) W_{c_1 c_2 c_3}(0) \sim \frac{\omega V_{[a_1 c_1][a_2 c_2][a_3 c_3]}(0)}{z}$$

Example null relation:

$$(J^1)_{(a_1}^{\ \ b} V_{[c_1)b][a_2 c_2][a_3 c_3]} = \frac{2}{\omega} W_{(a_1[a_2[a_3} W_{c_1)c_2]c_3]}$$

Chiral algebras of class $\mathcal{S}$: closing punctures

A partially closed puncture

- is determined by an embedding $\Lambda : \mathfrak{su}(2) \hookrightarrow \mathfrak{su}(n)$
- carries flavor symmetry $\mathfrak{h}_\Lambda$, the centralizer of $\Lambda(\mathfrak{su}(2)) \subset \mathfrak{su}(n)$
- can be obtained by giving a Higgs branch vev to the moment map μ_A^{11}

$$\langle \mu \rangle = v\, \Lambda(t_-)$$

and removing the resulting free Goldstone modes in the IR.

Chiral algebra analogue:

- impose quantum constraints

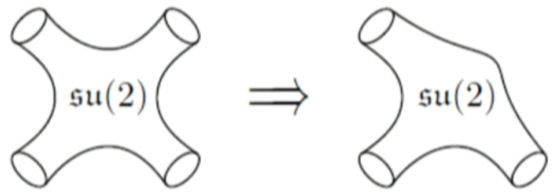
$$J_- = 1$$

(and some additional constraints)

- such constraints are precisely implemented by quantum Drinfeld Sokolov reduction of the affine current algebra *with modules*

Chiral algebras of class $\mathcal{S}$: closing punctures example

Let us start from the chiral algebra associated to the A_1 four-punctured sphere, *i.e.* the $\widehat{\mathfrak{so}(8)}_{-2}$ current algebra, and fully close one puncture.



The $\mathfrak{so}(8)$ current algebra can be reduced as

$$J_A(z) \;\to\; \{J^{(1)}_{(a_1 b_1)}(z)\,,\; J^{(2)}_{(a_2 b_2)}(z)\,,\; J^{(3)}_{(a_3 b_3)}(z)\,,\; J^{(4)}_{(a_4 b_4)}(z)\,,\; J_{a_1 a_2 a_3 a_4}(z)\}\,,$$

In an $\widehat{\mathfrak{su}(2)}_{-2}$ subalgebra, we impose the constraint that $J^{(1)}_{-} = 1$ via a BRST procedure with current

$$d(z) = \left(c^{-}\left[J^{(1)}_{-} - 1)\right]\right)(z)\,,$$

and ghosts (c^{-}, b_{-}) with the standard OPE $c^{-}(z)b_{-}(0) \sim \frac{1}{z}$

Chiral algebras of class $\mathcal{S}$: closing punctures example

This BRST problem can be solved via a (terminating) spectral sequence for the double complex (extension of [de Boer, Tjin])

$$d_0(z) = -c^-(z)\,, \qquad d_1(z) = \left(c^- J_-^{(1)}\right)(z)\,,$$

with $d = d_0 + d_1$ and $d_0^2 = d_1^2 = \{d_0, d_1\} = 0$.

As a result we find the (tic-tac-toed) generators of dimensions

$$[\hat{\mathcal{J}}_+^{(1)}] = 2\,, \qquad [\mathcal{J}_{a_I b_I}^{(I)}] = 1\,,$$
$$[(\mathcal{J}_1)_{a_2 a_3 a_4}] = 3/2\,, \quad [(\mathcal{J}_2)_{a_2 a_3 a_4}] = 1/2\,.$$

Finally, at the specific level $k_{2d} = -2$ one can show that

- all generators can be written in terms of $(\mathcal{J}_2)_{a_2 a_3 a_4}$ via null relations
- $(\mathcal{J}_2)_{a_2 a_3 a_4}$ satisfies the correct symplectic boson OPE

$$(\mathcal{J}_2)_{a_2 a_3 a_4}(z)(\mathcal{J}_2)_{b_2 b_3 b_4}(0) \sim \frac{\epsilon_{a_2 b_2}\epsilon_{a_3 b_3}\epsilon_{a_4 b_4}}{z} + d(\ldots)$$

Chiral algebras of class $\mathcal{S}$: cylinder and cap chiral algebra

The cylinder and cap chiral algebras do not descend from a physical 4d SCFT. They are formally required to complete the TQFT structure.

The cylinder chiral algebra is conjectured to be generated by

- an $\widehat{\mathfrak{su}(n)}_{-n}$ affine current algebra
- bifundamental dimension zero currents $\{g_{ab}(z),\ a, b = 1, \ldots, n\}$
  - that are acted upon on the left by the affine currents
  - that obey a determinant condition $\det g = 1$

The cap chiral algebra is somewhat exotic, since it features negative dimension fields.

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Outlook

Many interesting future directions:

- obtain more unitarity bounds by studying other correlators
- characterize the chiral algebras that can descend from 4d SCFTs
- include surface operators $\rightarrow$ vertex operator?
- can we implement a theory space bootstrap?
- for Lagrangian theories: can we relate to localization, holography?
- 1d/3d correspondence