Title: Conformal field theories at non-zero temperature: operator product expansions, Monte Carlo, and holography

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Abstract: <span>We discuss properties of 2-point functions in CFTs in 2+1D at finite temperature. For concreteness, we focus on those involving conserved flavour currents, in particular on the associated conductivity. At frequencies much greater than the temperature, Ï‰ >> T, the Ï‰ dependence of the conductivity can be computed from the operator product expansion (OPE) between the currents and operators which acquire a non-zero expectation value at T > 0. Such results are found to be in excellent agreement with quantum Monte Carlo studies of the O(2) Wilson-Fisher CFT. Results for the conductivity and other observables are also obtained in vector 1/N expansions. We match these large Ï‰ results to the corresponding correlators of holographic representations of the CFT: the holographic approach then allows us to extrapolate to small Ï‰/T. Other holographic studies implicitly only used the OPE between the currents and the energy-momentum tensor, and this yields the correct leading large Ï‰ behavior for a large class of CFTs. However, for the Wilson-Fisher CFT a relevant â€œthermalâ€• operator must also be considered, and then consistency with the Monte Carlo results is obtained without a previously needed ad hoc rescaling of the T value [1]. We also use the OPE to prove sum rules obeyed by the conductivity. **In collaboration with A. Katz, S. Sachdev and E. SÃ¸rensen** [1] WWK, E. SÃ¸rensen, S. Sachdev, Nat. Phys. 10, 361 (2014)</span>

# CFTs at finite $T$: OPEs, Monte Carlo & holography

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★ Real time response of strongly correlated quantum fluids

- If have *quasiparticles*, get dynamics from Boltzmann...
- Systems w/out qp-s (quantum critical):
  - Metal + gapless boson
  - CFTs

# Plan

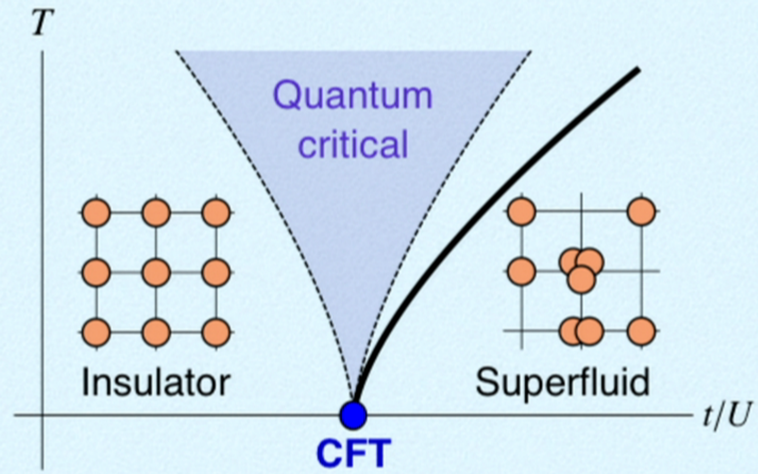
- CFTs, and their relevance
- **Short** times: OPE
- **Long** times: holography
- Proving sum rules on the CFT side
- Summary & Outlook

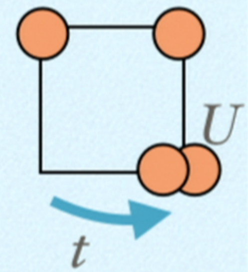
# Conformal Field Theory

- Scale + Lorentz invariant QFT
- Best characterized quantum fluid w/out qp-s
  - Still many open questions: dynamics, etc
- Why care?
  - Experiments!
  - Realistic models: numerics
  - Gravity = CFT? AdS/CFT [Maldacena, etc]

# Bosons in 2+1D

$$H = -t \sum_{\langle i,j \rangle} b_i^\dagger b_j + U \sum_i n_i(n_i - 1) - \mu \sum_i n_i$$



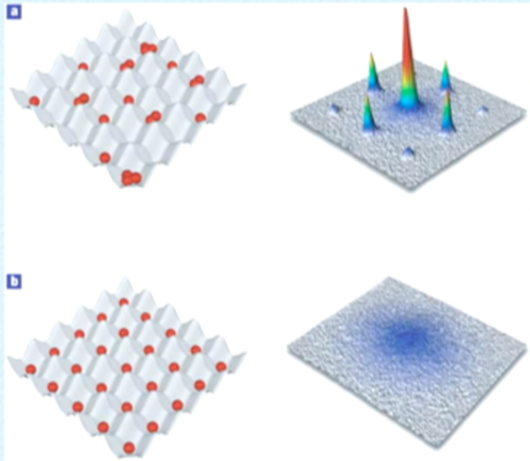

❖ 2+1D O(2) universality: quantum rotors, XY spins, etc

- SSB of O(2) order parameter

O($N$) NLSM:

$$\mathcal{L} = \frac{1}{g}(\partial_\mu \vec{\phi})^2 + i\lambda(\vec{\phi}^2 - 1)$$

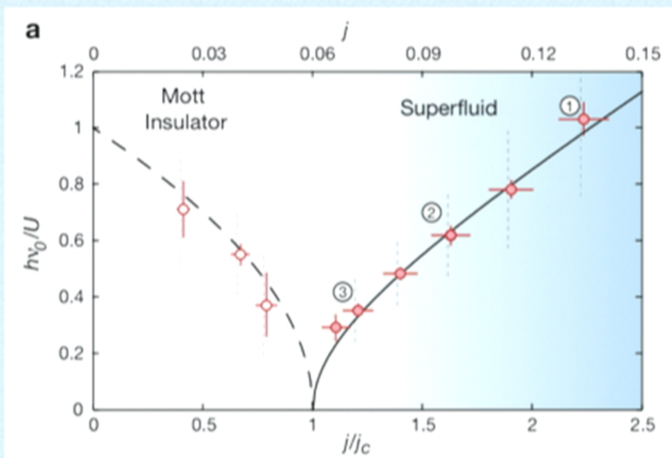
Strongly coupled IR f.p.
in 2+1D (Wilson-Fisher)



Superfluid

Insulator

[Bloch]




[Endres *et al*]

# *Break* conformal symmetry

- Characteristic time scale: 1 / $T$
- *Short* times: probing near vacuum
- *Long* times: excitations interact strongly with background

# Operator Product Expansion

- Consider a scalar primary op $\varphi(x)$, w/ scaling dim $\Delta$:

$$\langle \varphi(x)\, \varphi(0) \rangle = 1 \,/\, x^{2\Delta}$$

- Short time-distance expansion, OPE:

$$\lim_{x \to 0} \varphi(x)\varphi(0) = \sum_O C_O(x)\, x_{\mu_1} \cdots x_{\mu_l} O_{\mu_1 \ldots \mu_l}(0)$$

$$C_O(x) = \frac{\#}{x^{2\Delta + l - \Delta_O}}$$

# Current OPE

• $T = 0$:

$$\lim_{x\to 0} J_\mu(x) J_\nu(0) = \frac{I_{\mu\nu}\mathbb{1}}{x^{2\cdot 2}} + \mathcal{C}_{JJ\mathcal{O}} \frac{x_\mu x_\nu\, \mathcal{O}(0)}{x^{6-\Delta}} + \mathcal{C}_{JJT} \frac{T_{\mu\nu}(0)}{|x|} + \cdots$$

Scalar
dim = $\Delta$

Stress tensor
dim = 3
***Generic**

❖ Get OPE coefficients from $\langle JJ\mathcal{O}\rangle$

# Conductivity via OPE

$$\lim_{\omega_n \gg p} J_x(\boldsymbol{\omega}) J_x(-\boldsymbol{\omega} + \boldsymbol{p}) = -\omega_n \sigma_\infty \delta(\boldsymbol{p}) - \mathcal{C}_{JJ\mathcal{O}} \frac{\mathcal{O}(\boldsymbol{p})}{\omega_n^{\Delta-1}} + \mathcal{C}_{JJT} \left[ T_{xx} - T_{yy} - 12\gamma(T_{xx} + T_{yy}) \right] \Big|_{\boldsymbol{p}} + \cdots$$

Thermal average

$$\langle \mathcal{O} \rangle_T = B T^{\Delta}$$

$$\sigma(i\omega_n) \overset{\omega_n \gg T}{=} \sigma_\infty + b_1 \left( \frac{T}{\omega_n} \right)^{\Delta} + b_2 \left( \frac{T}{\omega_n} \right)^{3} + \cdots$$

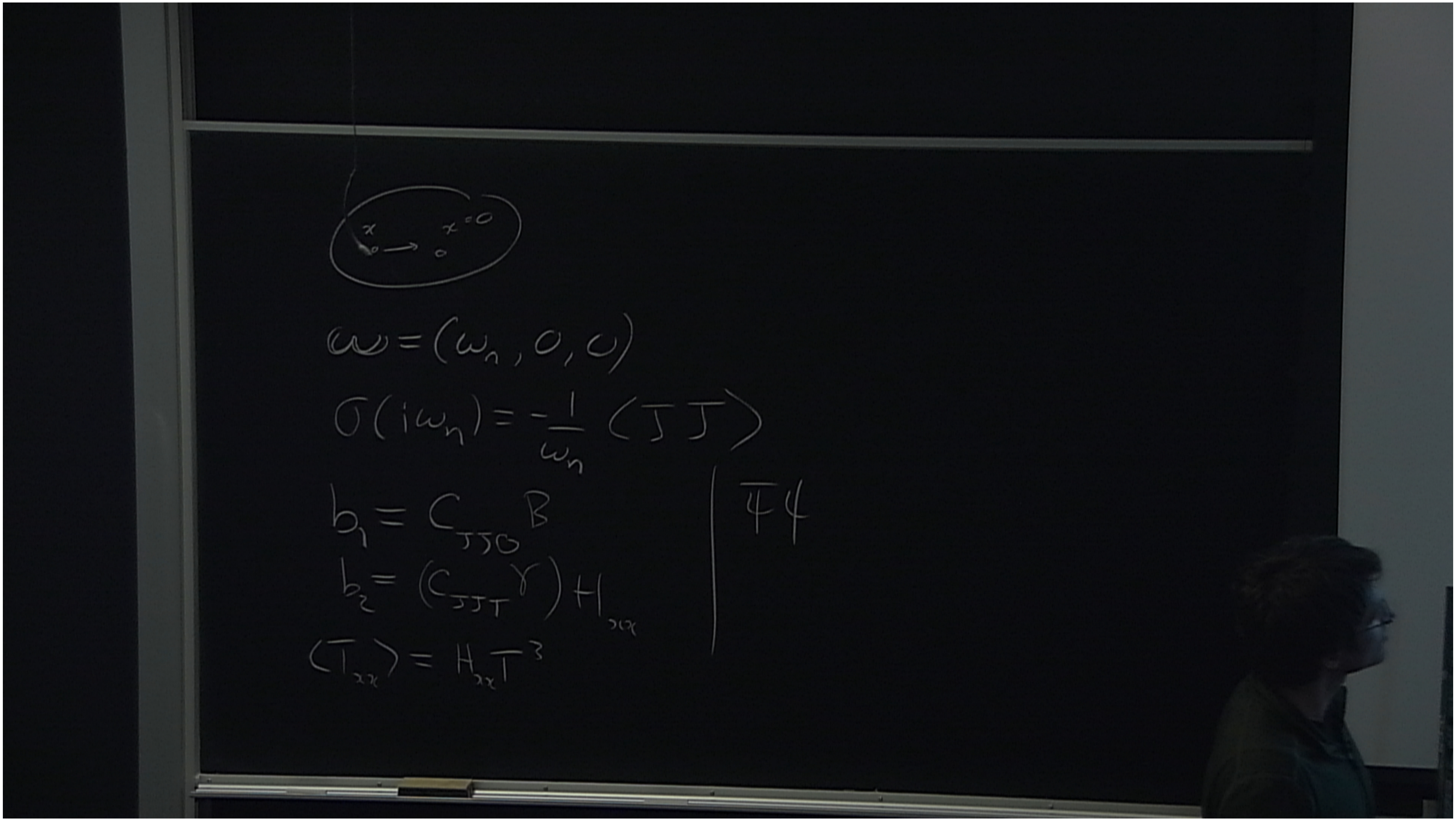
# Dominant op in OPE?

- For O($N$) CFT, it's **SCALAR**:

$$O = \lambda \quad \text{since} \quad \Delta = 3 - 1/\nu$$

- $N = \infty$: $\Delta = 2$
  $N = 2$: $\Delta = 1.5106$

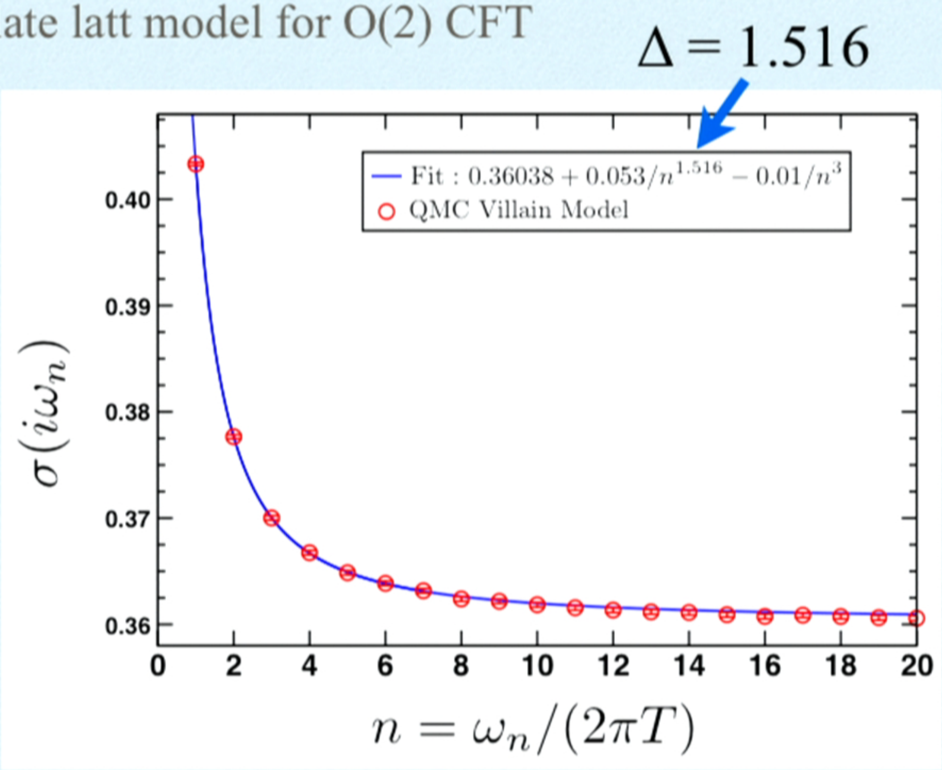
- **All** CFTs have $1/\omega^3$ term because there's always stress tensor
  - Sometimes dominates (Dirac CFT, QED3, etc)

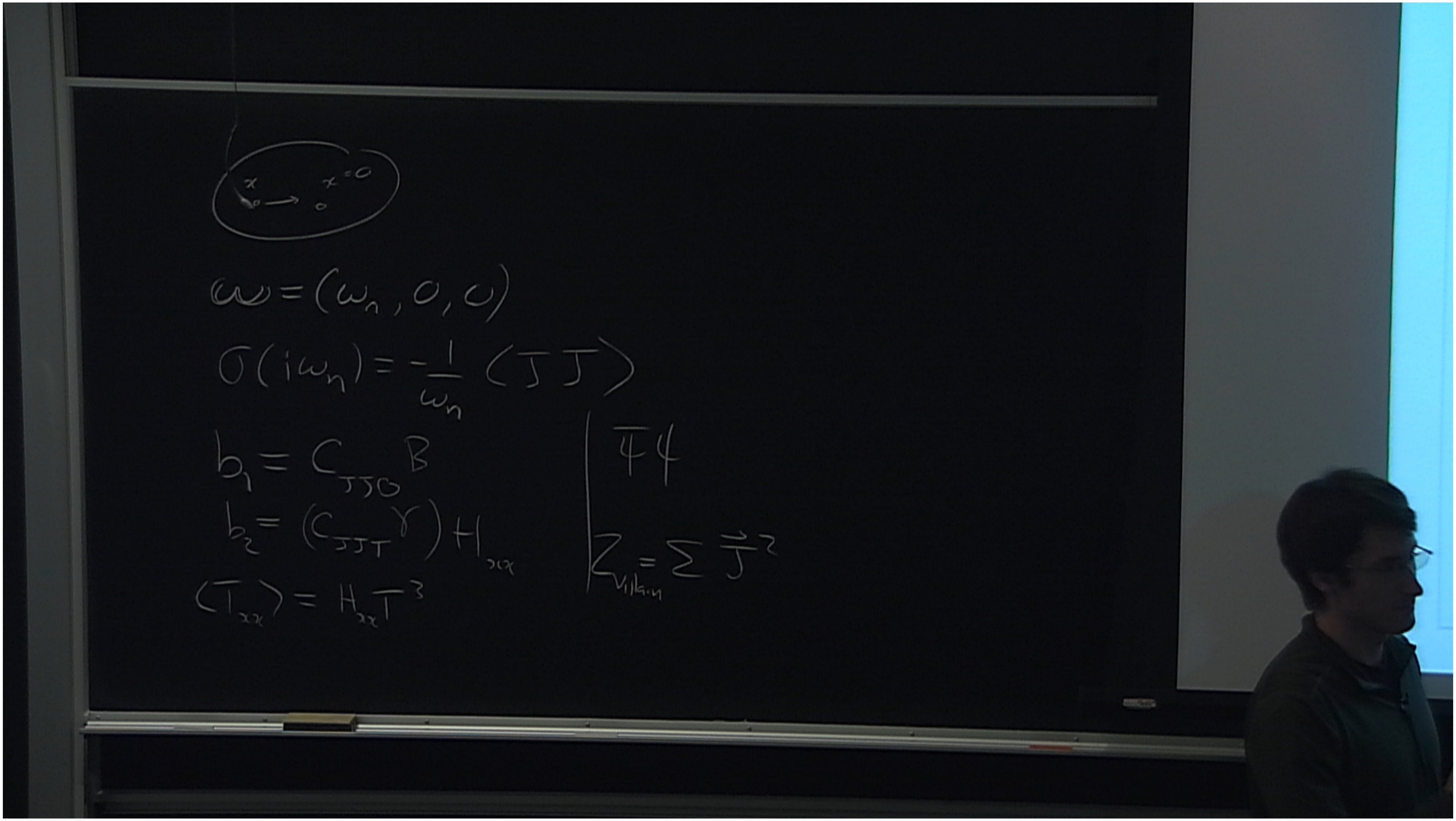

x
x = 0
ω = (ω_n, 0, 0)
σ(iω_n) = − 1/ω_n ⟨J J⟩
b_1 = C_JJO B
b_2 = (C_JJT γ) H_xx
⟨T_xx⟩ = H_xx T^3

# Monte Carlo check

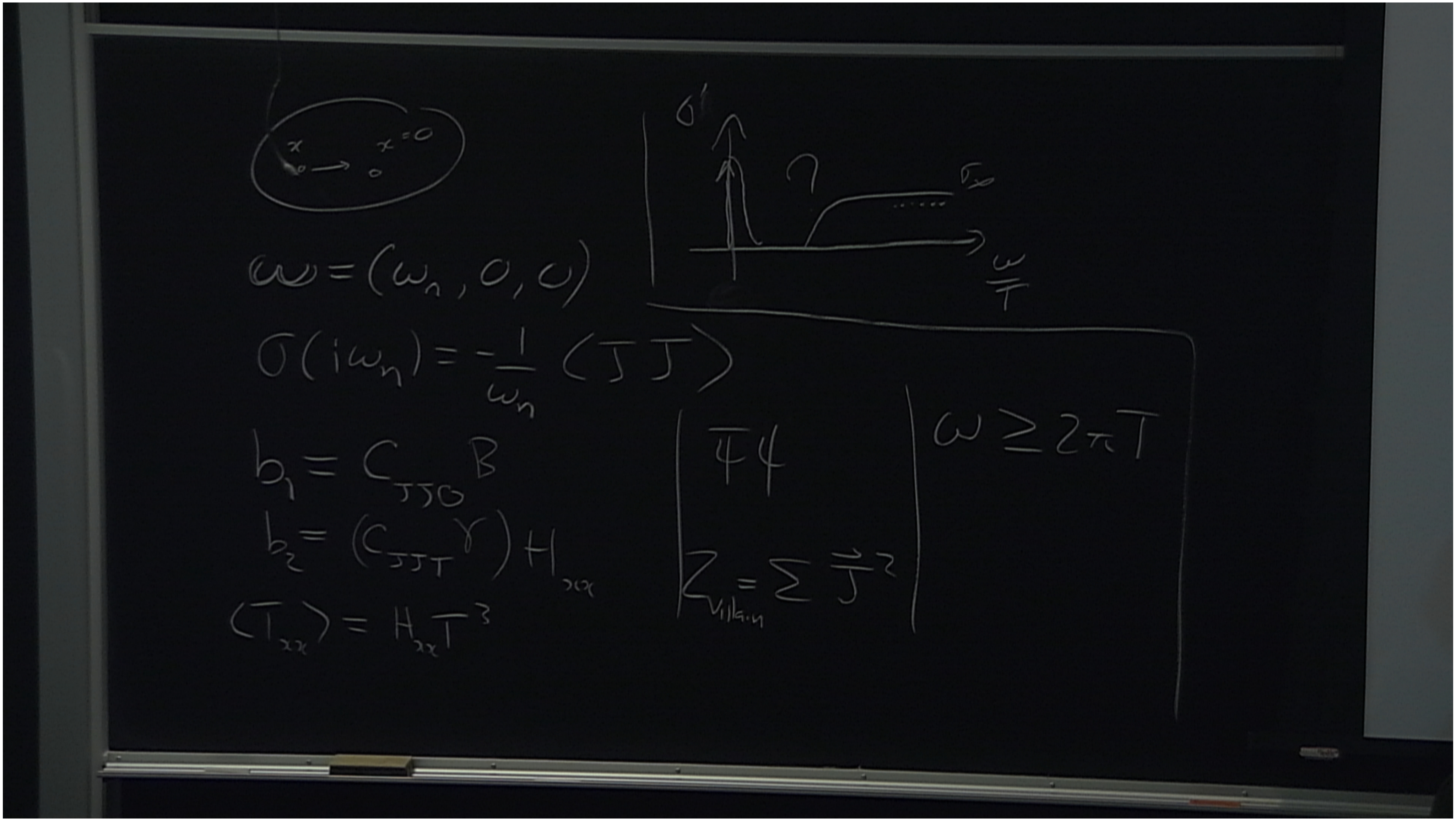
[Katz, Sachdev, **Sorensen**, WWK]

- Simulate latt model for O(2) CFT




ω = (ωn, 0, 0)
σ(iωn) = −1/ωn ⟨J J⟩
b1 = CJJO B
b2 = (CJJT γ) Hxx
⟨Txx⟩ = Hxx T³
ψ̄ ψ
ZVillain = Σ J²

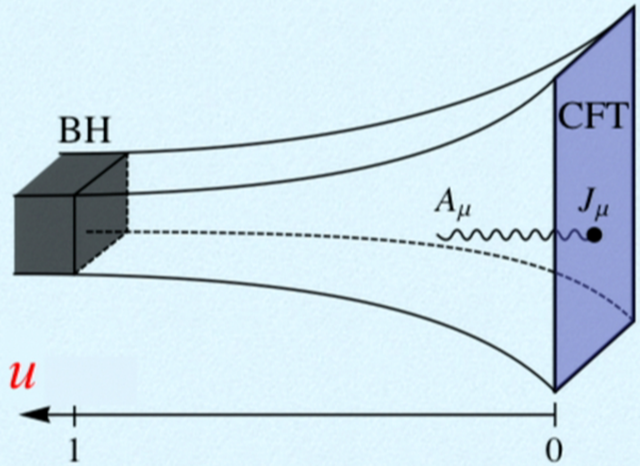
# *Long* times & analytic cont.


ω = (ωn, 0, 0)
σ(iωn) = − 1/ωn ⟨J J⟩
b1 = CJJO B
b2 = (CJJT γ) Hxx
⟨Txx⟩ = Hxx T3
ω/T
ω ≥ 2πT
Z Villain = Σ J2

# $\sigma$ via AdS/CFT

[Maldacena *etc*]

spacetime:
**B-Hole in AdS4**




$$A_\mu(t,x,y;u) \leftrightarrow J_\mu^{\mathrm{CFT}}(t,x,y)$$

- Solve **classical** EoM for $A$ $\rightarrow$ get $J$-correlator in **CFT**

$$\sigma(\omega/T) = \frac{T}{\omega} \left. \frac{\partial_u A_y(\omega,\vec{0};u)}{A_y(\omega,\vec{0};u)} \right|_{u=0}$$

# 1st try

$$S_{\rm bulk}[A_\mu] = \int d^4x\sqrt{-g}\,\frac{1}{g_4^2}\left[F^2 + \gamma C^{abcd}F_{ab}F_{cd}\right]$$

$C$ = traceless part of Riemann

- Derivative expansion
- Asymptotics [WWK 2014]: $\sigma \sim (T/\omega_n)^3$

[Herzog, Kovtun, Sachdev, Son; Ritz, Ward; Myers, Sachdev, Singh]

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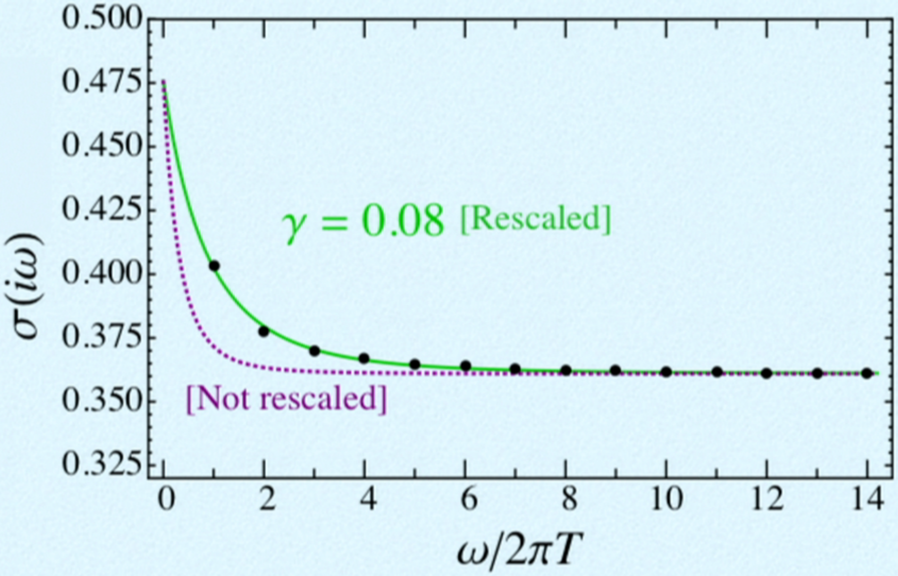
| CFT | AdS |
| --- | --- |
| $T_{\mu\nu}$ | $g_{\mu\nu}$ |

[Herzog, Kovtun, Sachdev, Son; Ritz, Ward; Myers, Sachdev, Singh]

# Holographic fit: take 1




$$\sigma\left(\kappa\,\frac{\omega}{T};\gamma\right)$$

# 2nd try

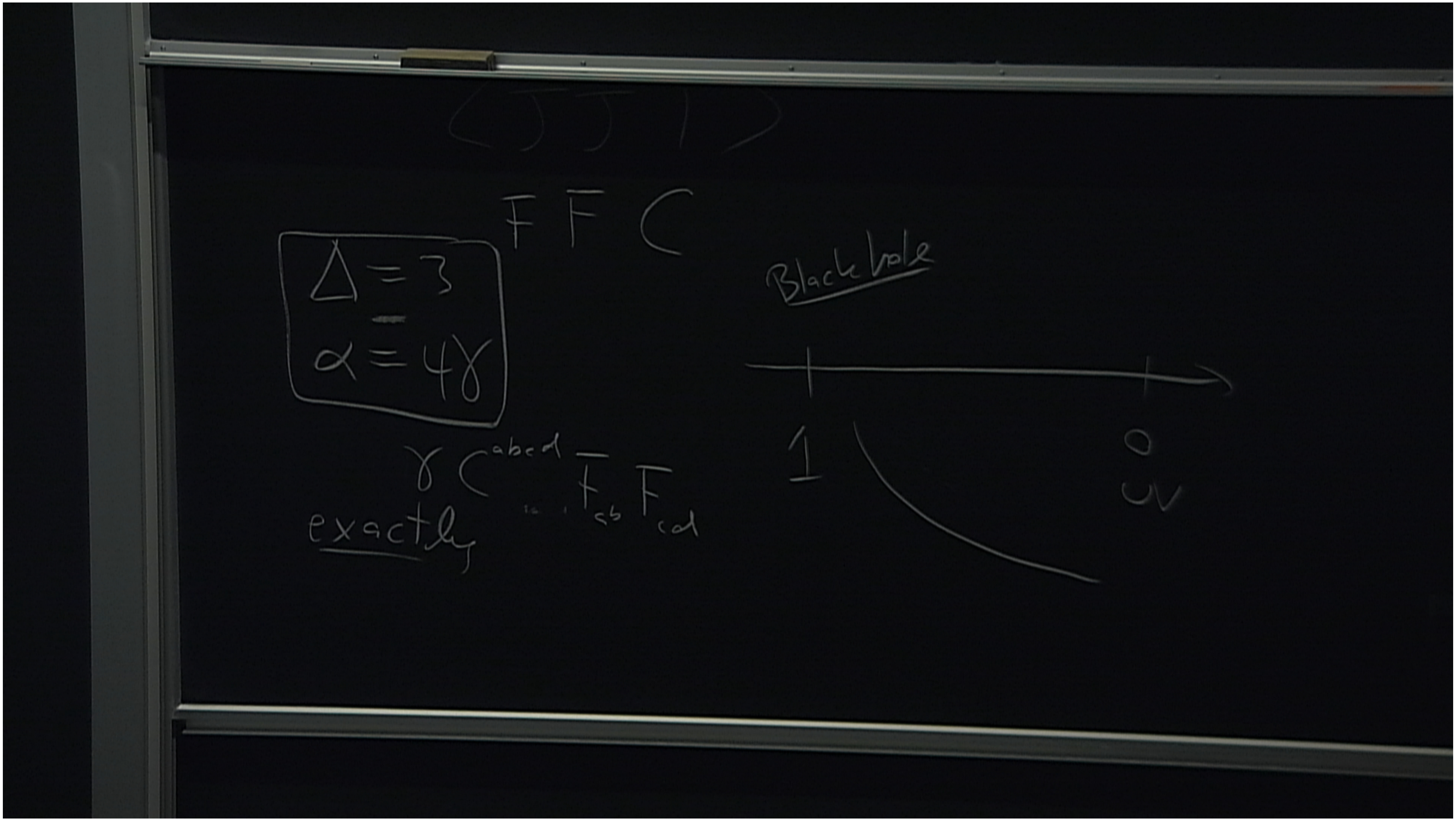
$$S_{\text{bulk}}[A_\mu] = \int d^4x \frac{1}{g_4^2} \left[1 + \alpha\, \varphi(x)\right] F_{ab}F^{ab}$$

- Add scalar field (dilaton)

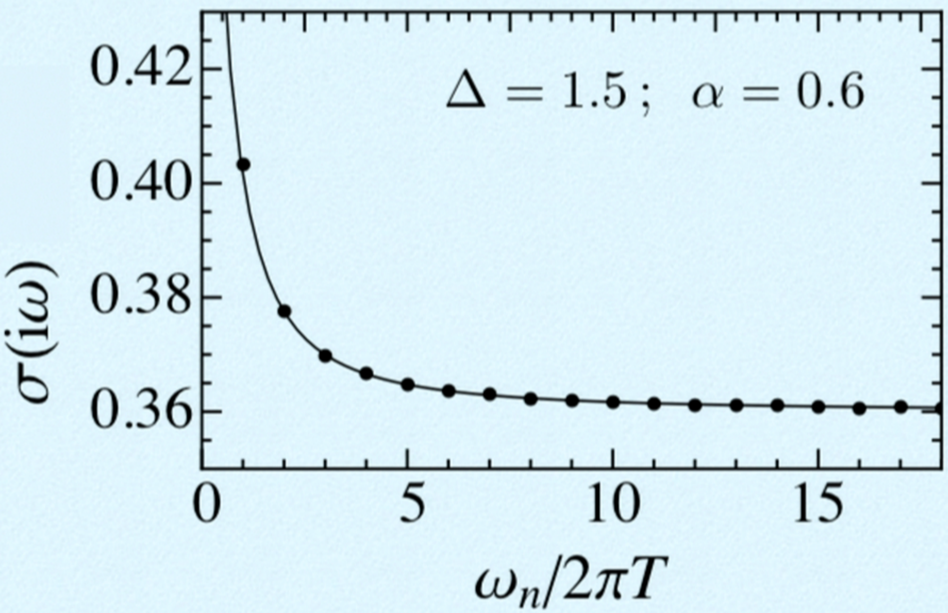
| CFT | AdS |
|---|---|
| $O$ | $\varphi$ |

- Simplest **ansatz**:
  fix profile using OPE of O(2) CFT

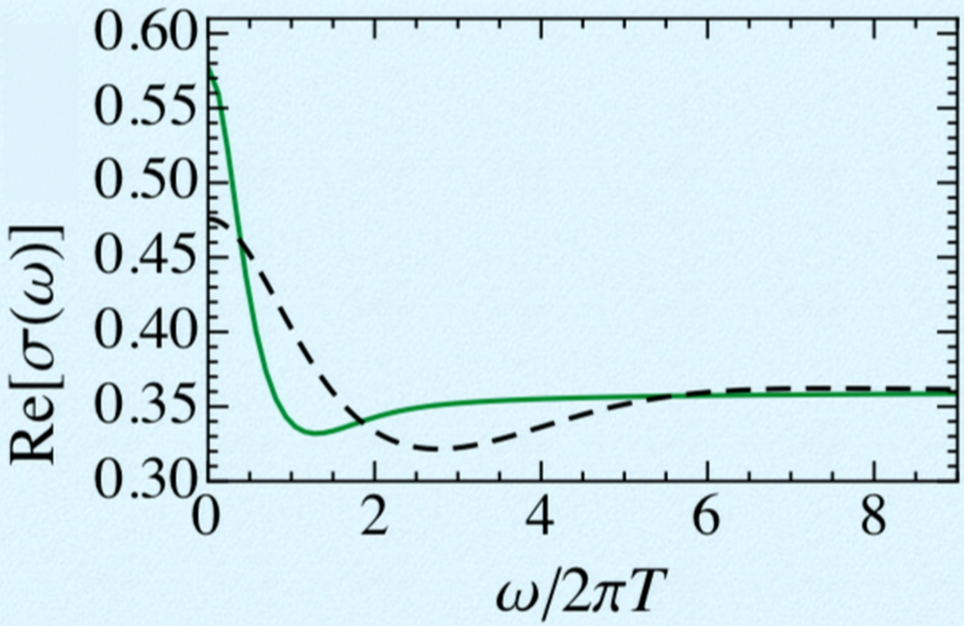
$$\varphi(u) = u^{\Delta}$$


FFC
Δ = 3
α = 4γ
γ C^abcd F_ab F_cd
exactly
Black hole
1
0
UV

# Holographic fit: take 2



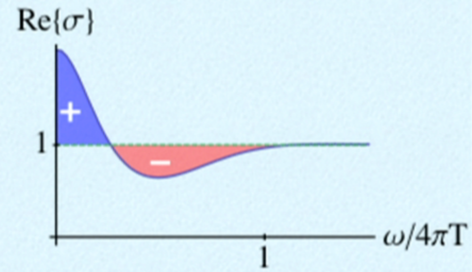
# **Real** frequencies



# Sum rules

[WWK, Sachdev; Gulotta, Herzog, Kaminski]

$$\int_0^\infty d\omega \left[\mathrm{Re}\,\sigma(\omega/T) - \sigma(\infty)\right] = 0$$




❖ S-dual version:

$$\int_0^\infty d\omega \left[\mathrm{Re}\left\{\frac{1}{\sigma(\omega/T)}\right\} - \frac{1}{\sigma(\infty)}\right] = 0$$

- ✔ $\mathcal{N}$=8 super Yang-Mills
- ✔ O($N$) CFT @ $N$=∞
- ✔ Dirac CFT

# Sum rule proof via OPE

$$\sigma(i\omega_n) \overset{\omega_n \gg T}{=} \sigma_\infty + b_1 \left(\frac{T}{\omega_n}\right)^\Delta + b_2 \left(\frac{T}{\omega_n}\right)^3 + \cdots$$

- σ analytic in UHP
- $\Delta > 1 \Rightarrow$ Integrable $\Rightarrow$ Close contour in UHP ✔

# Conclusions

- Dynamics of CFTs in 2+1D
- OPE to constrain short time physics
  - Large ω conductivity $\sigma(i\omega_n) \overset{\omega_n \gg T}{=} \sigma_\infty + b_1 \left(\frac{T}{\omega_n}\right)^\Delta + b_2 \left(\frac{T}{\omega_n}\right)^3 + \cdots$
- Input large-ω data of CFT into simple holographic ansatz
- Can match Monte Carlo data of O(2) CFT w/out unphysical tweaks

# Outlook

- Same analysis (OPE, sum rules, holography) for other correlators & CFTs
- Go beyond simplest holographic ansatz
- Unitarity in proof of sum rules?

# Thanks!

- E.Katz, S.Sachdev, E.Sorensen, **W.Witczak-Krempa**, [1409.3841]

- **W.Witczak-Krempa**, E.Sorensen, S.Sachdev, Nat. Phys. 2014 [1309.2941]

# Long-time tails

- At $T > 0$, correlators of conserved currents decay slowly at long times:

$$\int d^d\boldsymbol{x}\, \langle J^i(\tau, \boldsymbol{x}) J^j(0,0)\rangle \sim \frac{\delta^{ij}}{\tau^{d/2}}$$

- Beyond linear hydro: *interactions* between thermal fluctuations

- 2+1D: $\sigma(\omega/T) \sim \ln(T/|\omega|)\,, \quad |\omega| \ll T$

- In AdS/CFT, $1/N_c$ effect [Caron-Huot & Saremi]