Title: 14/15 PSI - Relativity- Lecture 14

Date: Sep 25, 2014 09:00 AM

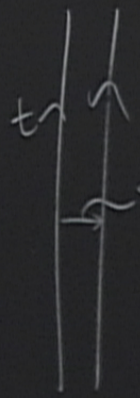
URL: http://pirsa.org/14090063

Abstract:

Correction:

Christodoulou memory effect

(See article by K. Thorne)



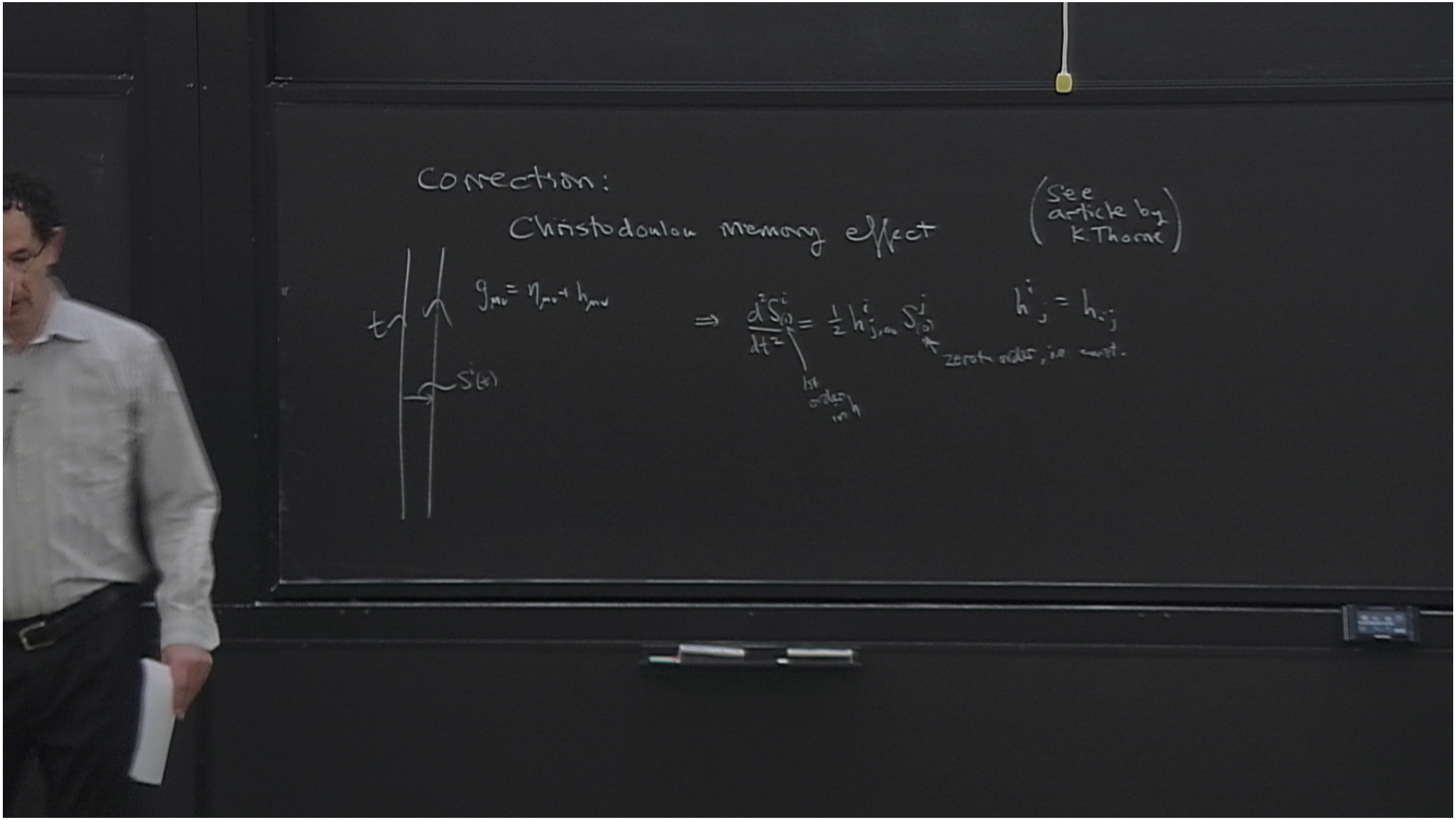

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

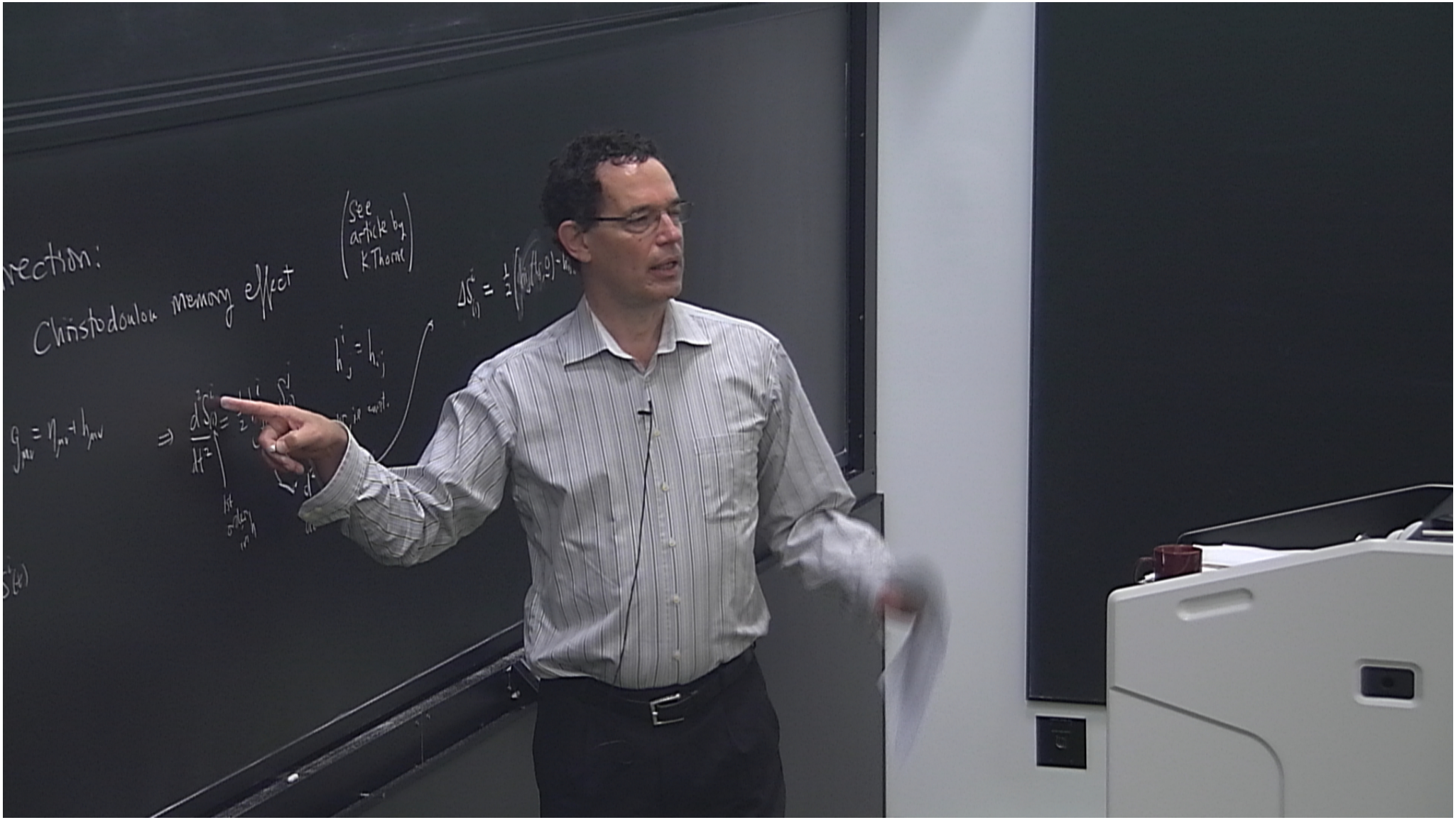
$\Rightarrow \frac{d^2 S^i_{(1)}}{dt^2} = \frac{1}{2} h^i_{j,00} S^j_{(0)}$

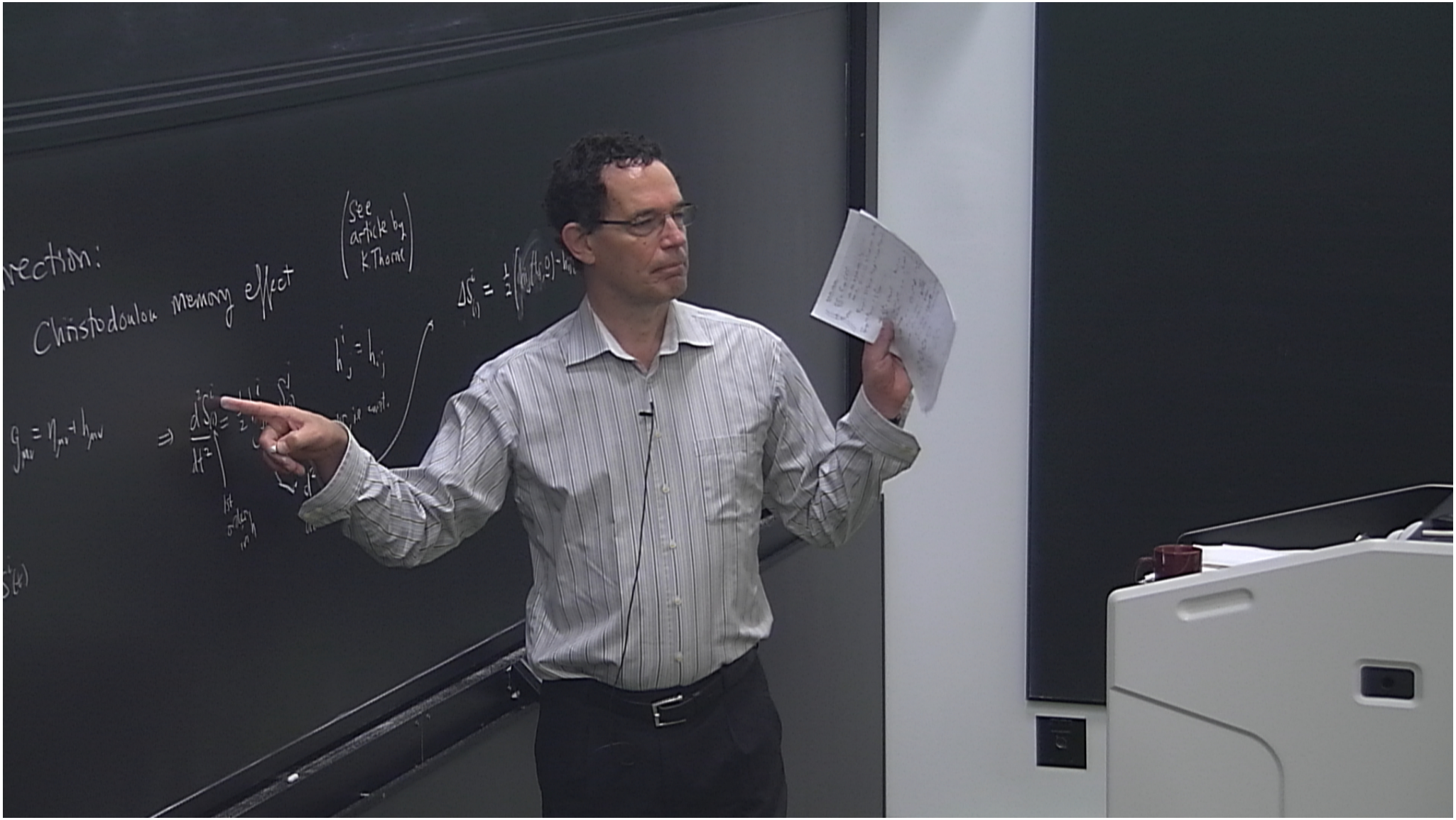
1st order in $h$

$S^j_{(0)}$ zeroth order, i.e. const.

$h^i_j = h_{ij}$

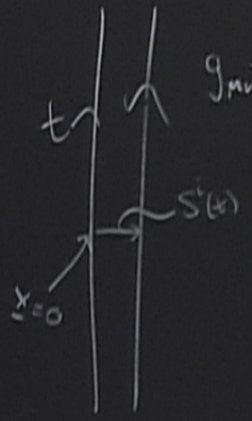

Correction:
Christodoulou memory effect
(See article by K. Thorne)
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
$S^i(t)$
$\Rightarrow \frac{d^2 S^i_{(1)}}{dt^2} = \frac{1}{2} h^i_{j,00} S^j_{(0)}$
1st order in $h$
zeroth order, i.e. const.
$h^i_{\ j} = h_{\cdot j}$


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Christodoulou memory effect
(See article by K Thorne)


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Christodoulou memory effect (See article by K. Thorne)




$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$\Rightarrow \frac{d^2 S^i_{(1)}}{dt^2} = \frac{1}{2} h^i_{j,\mu\nu} S^j_{(0)}$

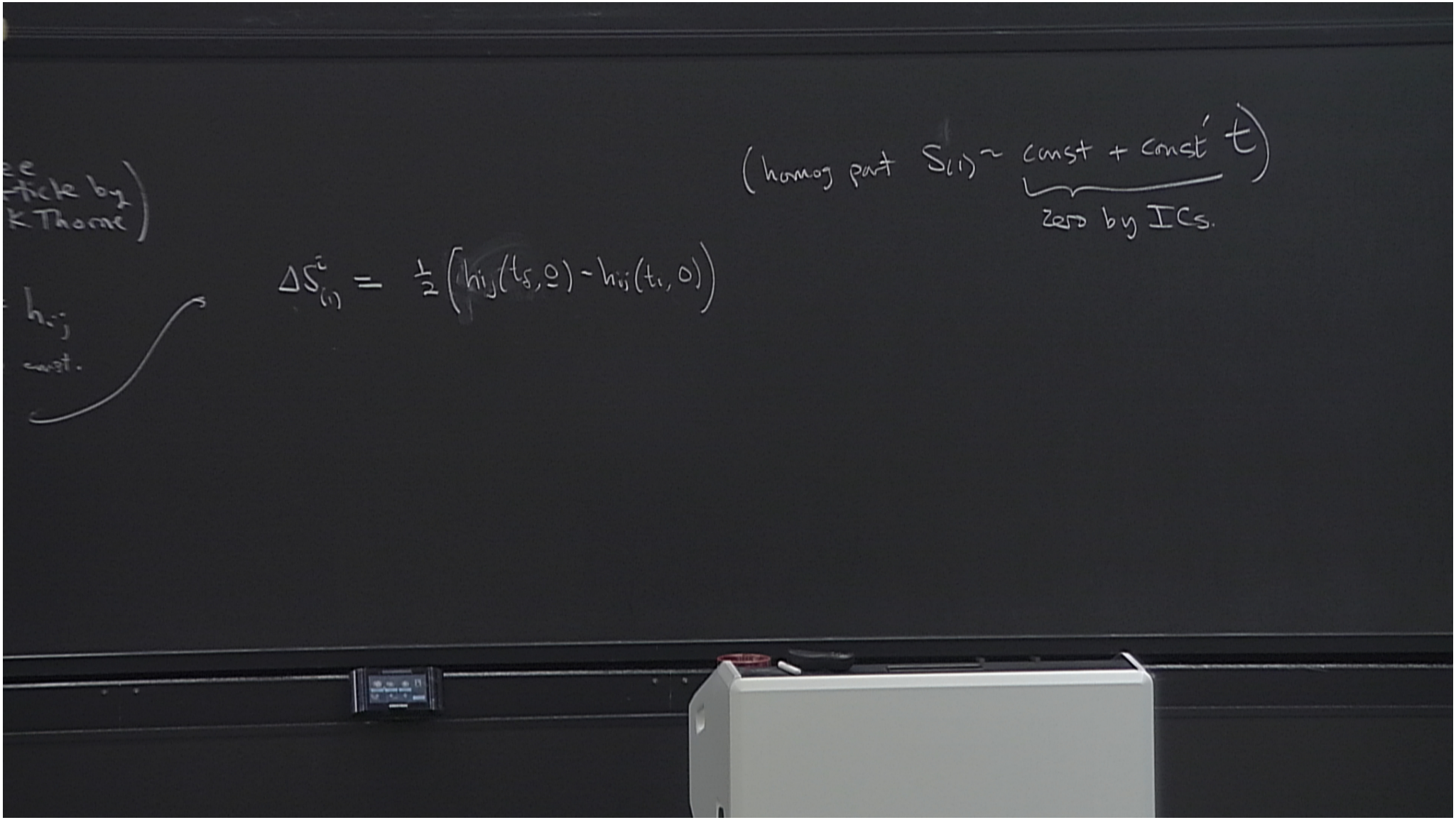
$h^i_{\ j} = h_{\cdot\cdot j}$

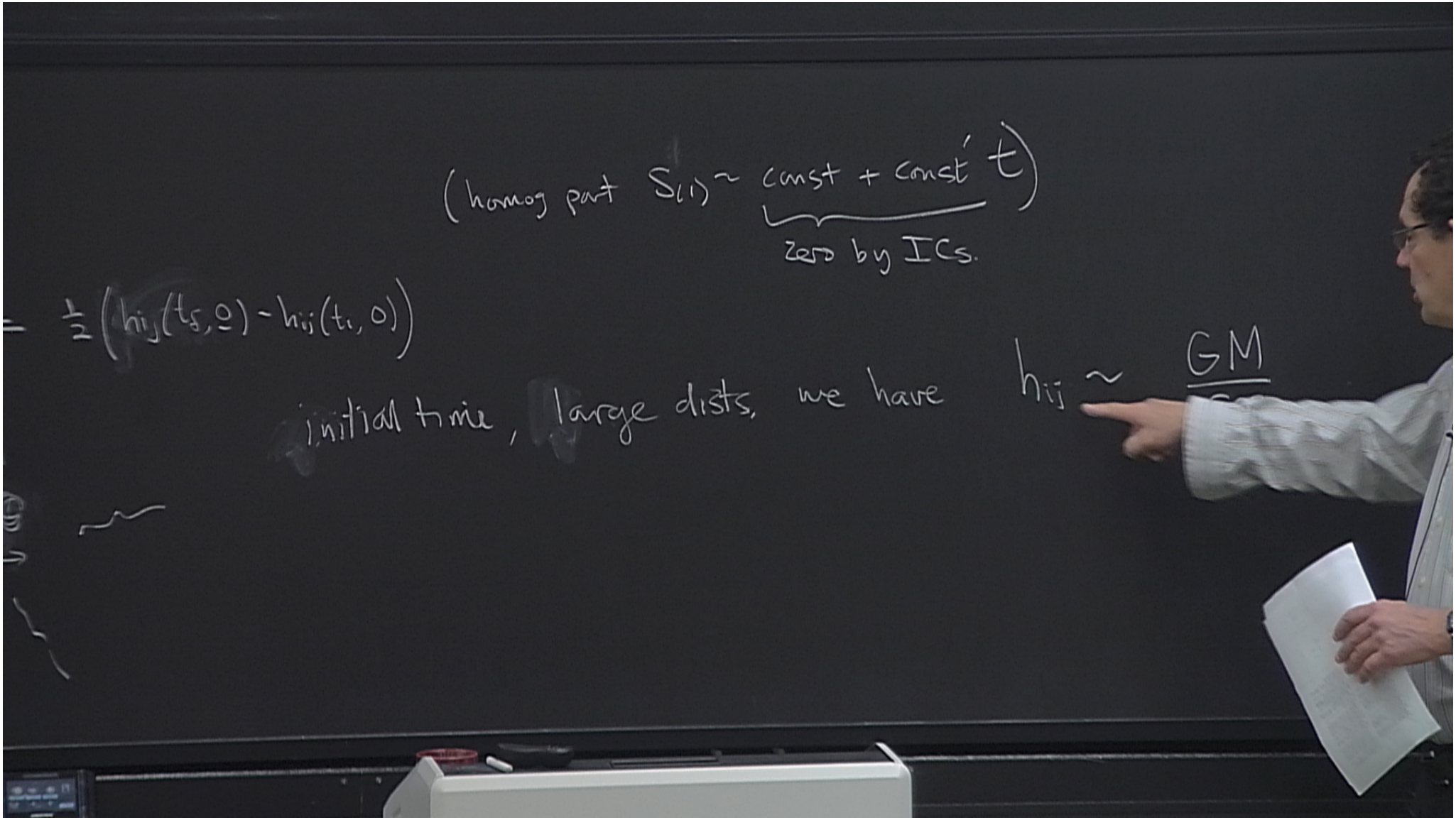
$S^j_{(0)}$ zeroth order, i.e. const.

1st order in $h$

$\frac{d^2}{dt^2} h_{ij}(t, \underline{0})$

$\Delta S^i_{(1)} = \frac{1}{2}\left(h_{ij}(t_f, \underline{0}) - h_{ij}(t_i, 0)\right)$


(homog part $S_{(1)} \sim$ const + const' $t$)
zero by ICs.
$\Delta S^2_{(1)} = \frac{1}{2}\left(h_{ij}(t_f, 0) - h_{ij}(t_i, 0)\right)$
$h_{ij}$
const.


(homog part $S_{(1)} \sim$ const + const' $t$)
zero by ICs.
$= \frac{1}{2}\left(h_{ij}(t_f, \underline{0}) - h_{ij}(t_i, 0)\right)$
initial time, large dists, we have $h_{ij} \sim \frac{GM}{}$

(See article by K. Thorne)

$= h_{ij}$

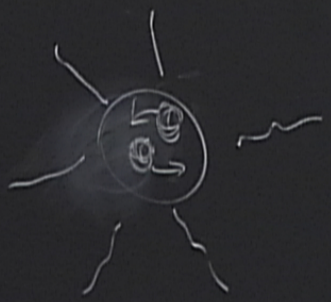
, i.e. const.

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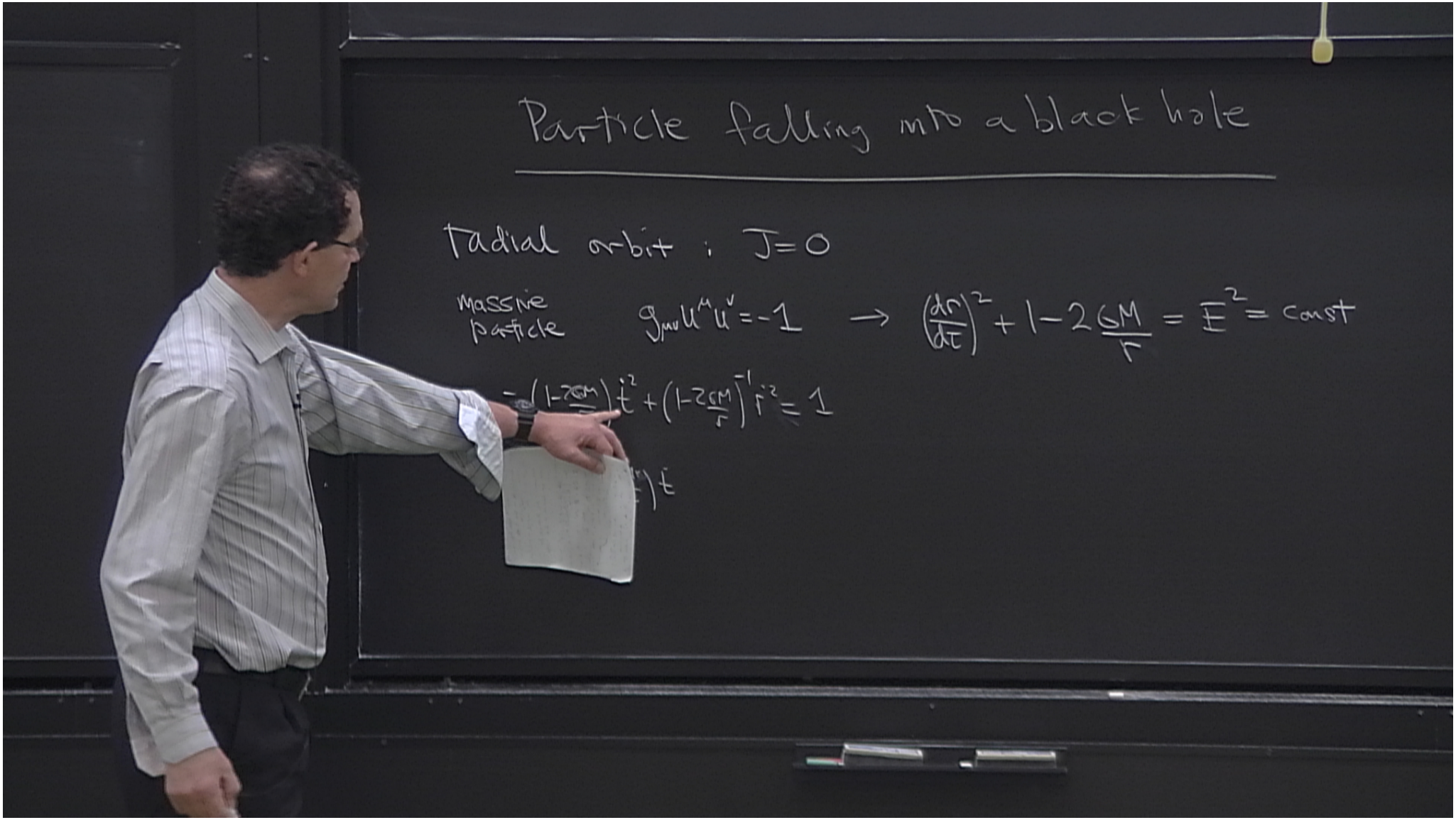
$$\Delta S^i_{(1)} = \frac{1}{2}\left(h_{ij}(t_f, 0) - h_{ij}(t_i, 0)\right)$$




initial time, $t_i$, large dists, we have $h_{ij} \sim \frac{GM(t_i)}{rc^2}$

total "mass" related to total energy

later time, $t_f$, " " $h_{ij} \sim \frac{GM(t_f)}{rc^2}$


Particle falling into a black hole
radial orbit : J=0
massive particle
$g_{\mu\nu}u^\mu u^\nu = -1 \rightarrow \left(\frac{dr}{d\tau}\right)^2 + 1 - \frac{2GM}{r} = E^2 = \text{const}$
$-\left(1-\frac{2GM}{r}\right)\dot{t}^2 + \left(1-\frac{2GM}{r}\right)^{-1}\dot{r}^2 = 1$

# Particle falling into a black hole

Radial orbit : $J=0$

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$$-\left(1-\frac{2GM}{r}\right)\dot{t}^2 + \left(1-\frac{2GM}{r}\right)^{-1}\dot{r}^2 = 1$$

$$E = \left(1-\frac{2GM}{r}\right)\dot{t}$$

$$\Rightarrow \Delta S^i \sim \frac{G(M(t_i) - M(t_f))}{r\ c^2}$$

# Particle falling into a black hole

Radial orbit : $J = 0$

massive particle $\quad g_{\mu\nu} u^\mu u^\nu = -1 \rightarrow \left(\frac{dr}{d\tau}\right)^2 + 1 - 2\frac{GM}{r} = E^2 = \text{const}$

$$-\left(1-\frac{2GM}{r}\right)\dot{t}^2 + \left(1-\frac{2GM}{r}\right)^{-1}\dot{r}^2 = 1$$

$$E = \left(1-\frac{2GM}{r}\right)\dot{t}$$

parametric solution :

$$r = \frac{R}{2}\left(1+\cos\theta\right)$$

$$\tau = \left(\frac{R^3}{8GM}\right)^{1/2}\left(\theta + \sin\theta\right)$$

, where $R$ is the initial radius

$$\Rightarrow \Delta S' \sim \frac{G(M(t_i) - M(t_f))}{r\, c^2}$$

# Particle falling into a black hole

Radial orbit : $J = 0$

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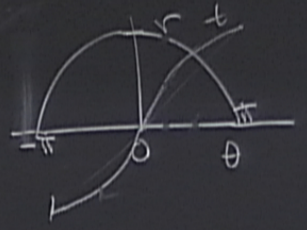
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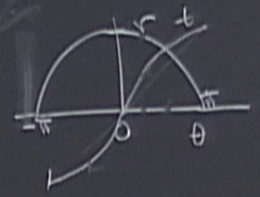
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particle starts at radius $r=R$ at $\theta=0$

hits $r=0$ at $\theta=\pi$, $\tau = \frac{\pi}{2}\left(\frac{R^3}{2GM}\right)^{1/2}$ ; if $R=2GM$, then $\boxed{\tau = \pi GM}$ $\left(\tau \approx 10^{-5}\left(\frac{M}{M_\odot}\right)\text{s}\right)$



$-\left(1-\frac{2GM}{r}\right)\dot{t}^2+\left(1-\frac{2GM}{r}\right)^{-1}\dot{r}^2=1$

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## Geometry of Schwarzschild metric

$ds^2=-\alpha dt^2+\alpha^{-1}dr^2+r^2\underbrace{d\Omega^2}_{\text{metric on } S^2}$ , $\alpha\equiv 1-\frac{r_s}{r}$ , $r_s=\frac{2GM}{c^2}$

gular at 2 places: $r=r_s$ and $r=0$

it is a solution of $R_{\mu\nu}=0$ $\forall$ $r\neq 0$ or $r_s$

nt ... at the solution for $r>r_s$ to solution for $r<r_s$ ; note that for $r<r_s$, $r$ is "timelike"

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## Geometry of Schwarzschild metric

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Singular at 2 places: $r=r_s$ and $r=0$

but it is a solution of $R_{\mu\nu}=0$ $\forall$ $r\neq 0$ or $r_s$

we want to connect the solution for $r>r_s$ to solution for $r<r_s$ ; note that for $r<r_s$, $r$ is "timelike"

$E = \left(1-\frac{2GM}{r}\right)\dot{t}$

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## Geometry of Schwarzchild metric

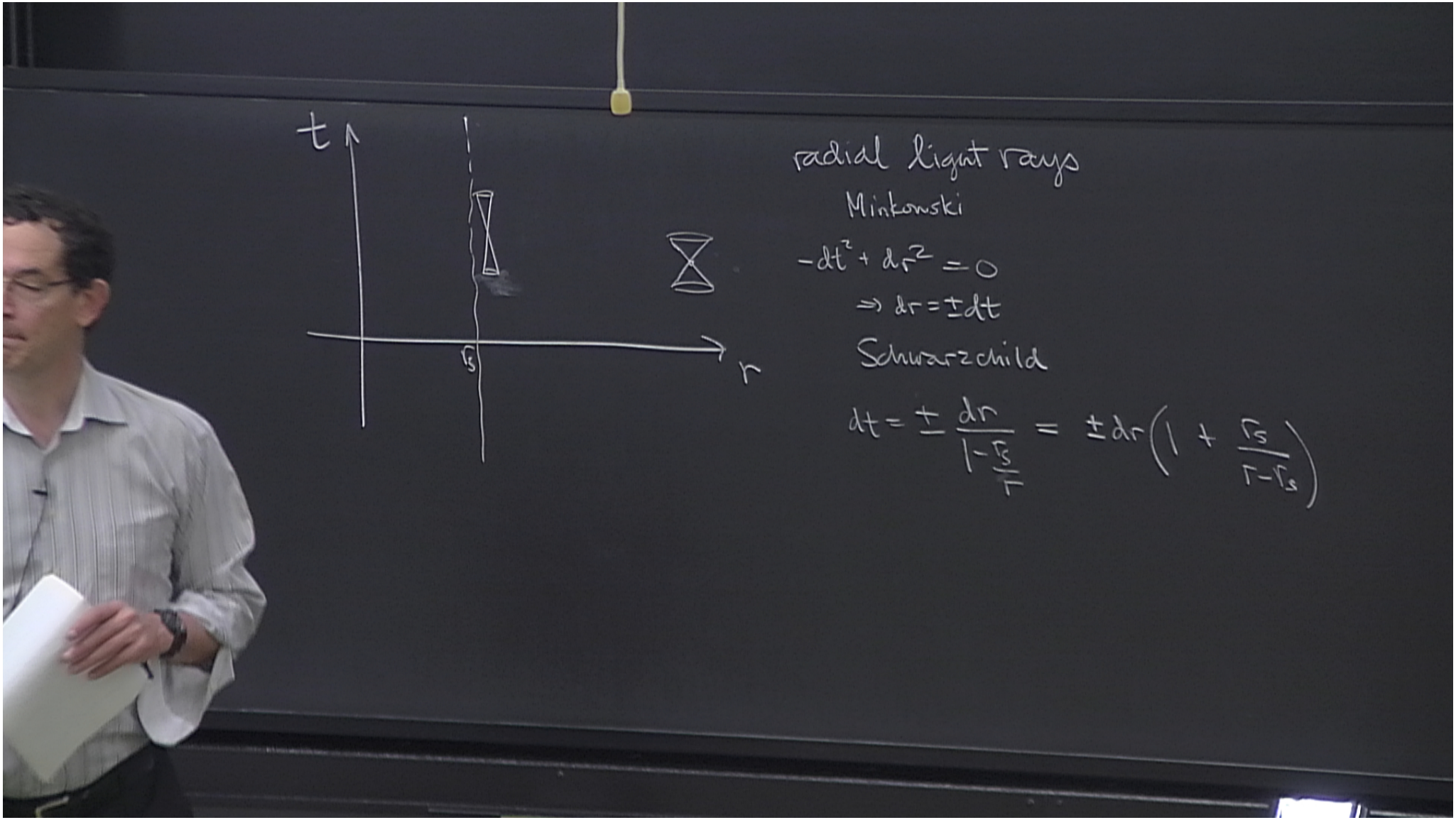
$$ds^2 = -\alpha dt^2 + \alpha^{-1}dr^2 + r^2 d\Omega^2 \quad , \quad \alpha \equiv 1 - \frac{r_s}{r} \quad , \quad r_s = \frac{2GM}{c^2}$$

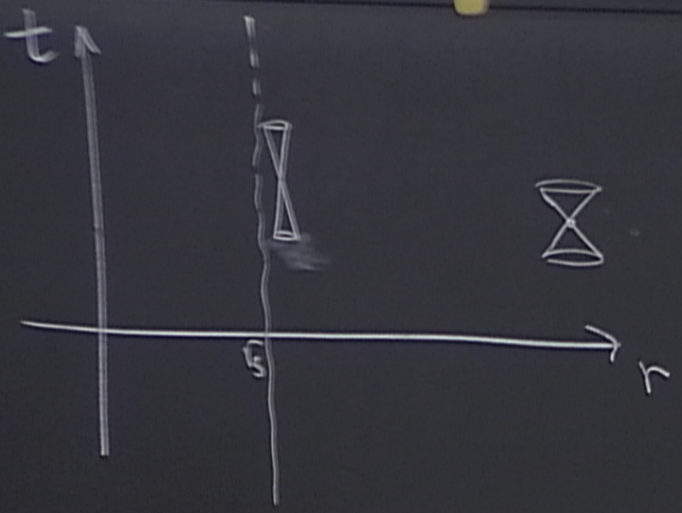
($d\Omega^2$: metric on $S^2$)

Singular at 2 places: $r = r_s$ and $r = 0$

but it is a solution of $R_{\mu\nu} = 0$ $\forall$ $r \neq 0$ or $r_s$

we want to connect the solution for $r > r_s$ to solution for $r < r_s$ ; note that for $r < r_s$, $r$ is "timelike" $t$ is "spacelike".


t
r
$r_s$
radial light rays
Minkowski
$-dt^2 + dr^2 = 0$
$\Rightarrow dr = \pm dt$
Schwarzchild
$dt = \pm \frac{dr}{1 - \frac{r_s}{r}} = \pm dr \left(1 + \frac{r_s}{r - r_s}\right)$




radial light rays

Minkowski

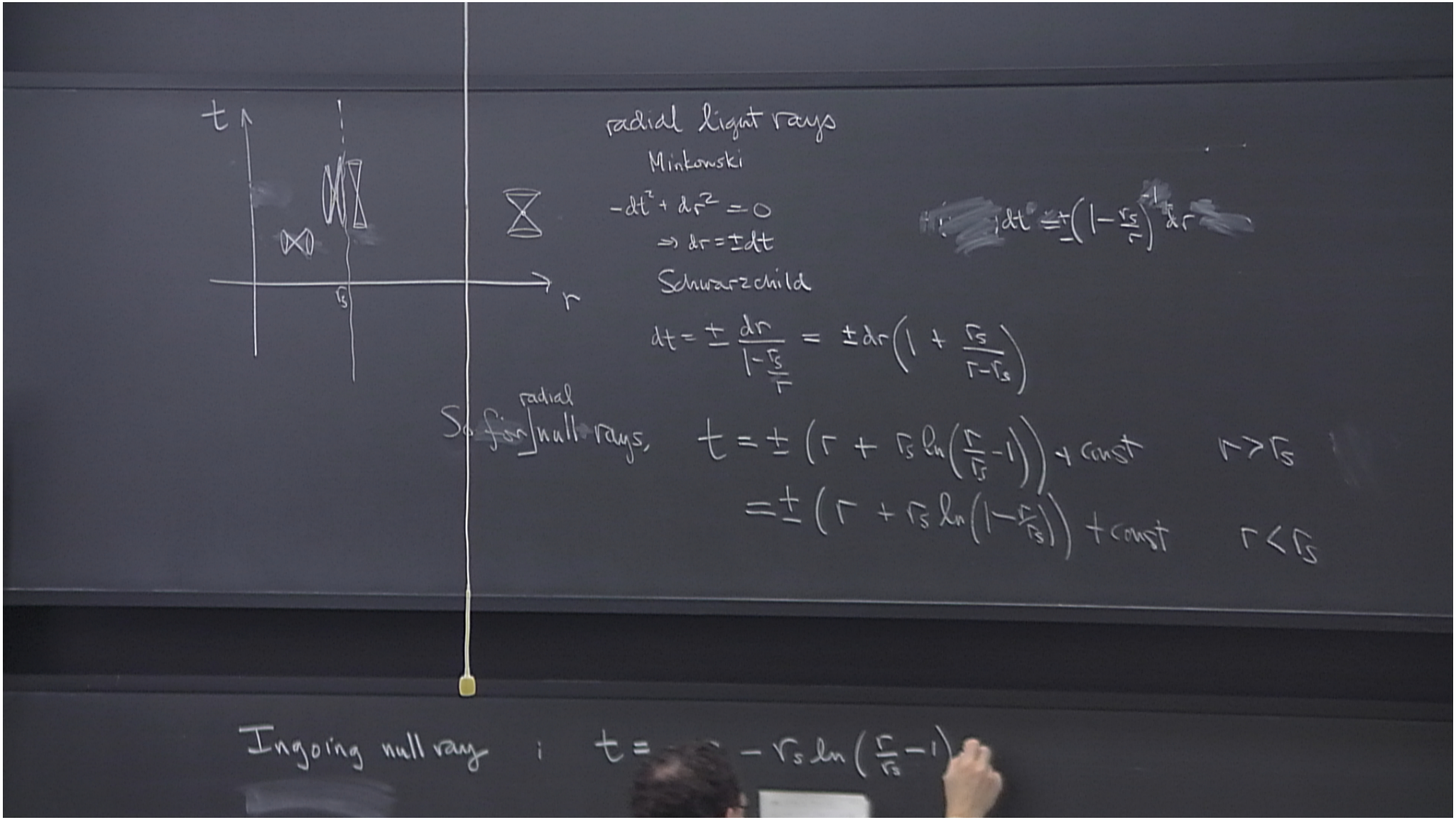
$-dt^2 + dr^2 = 0$

$\Rightarrow dr = \pm dt$

$dt = \pm \left(1 - \frac{r_s}{r}\right)^{-1} dr$

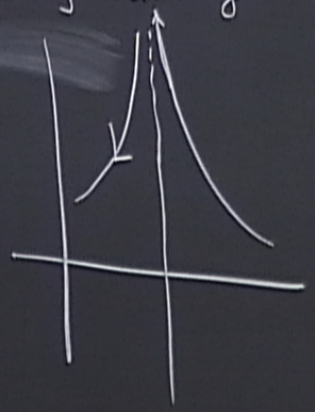
Schwarzschild

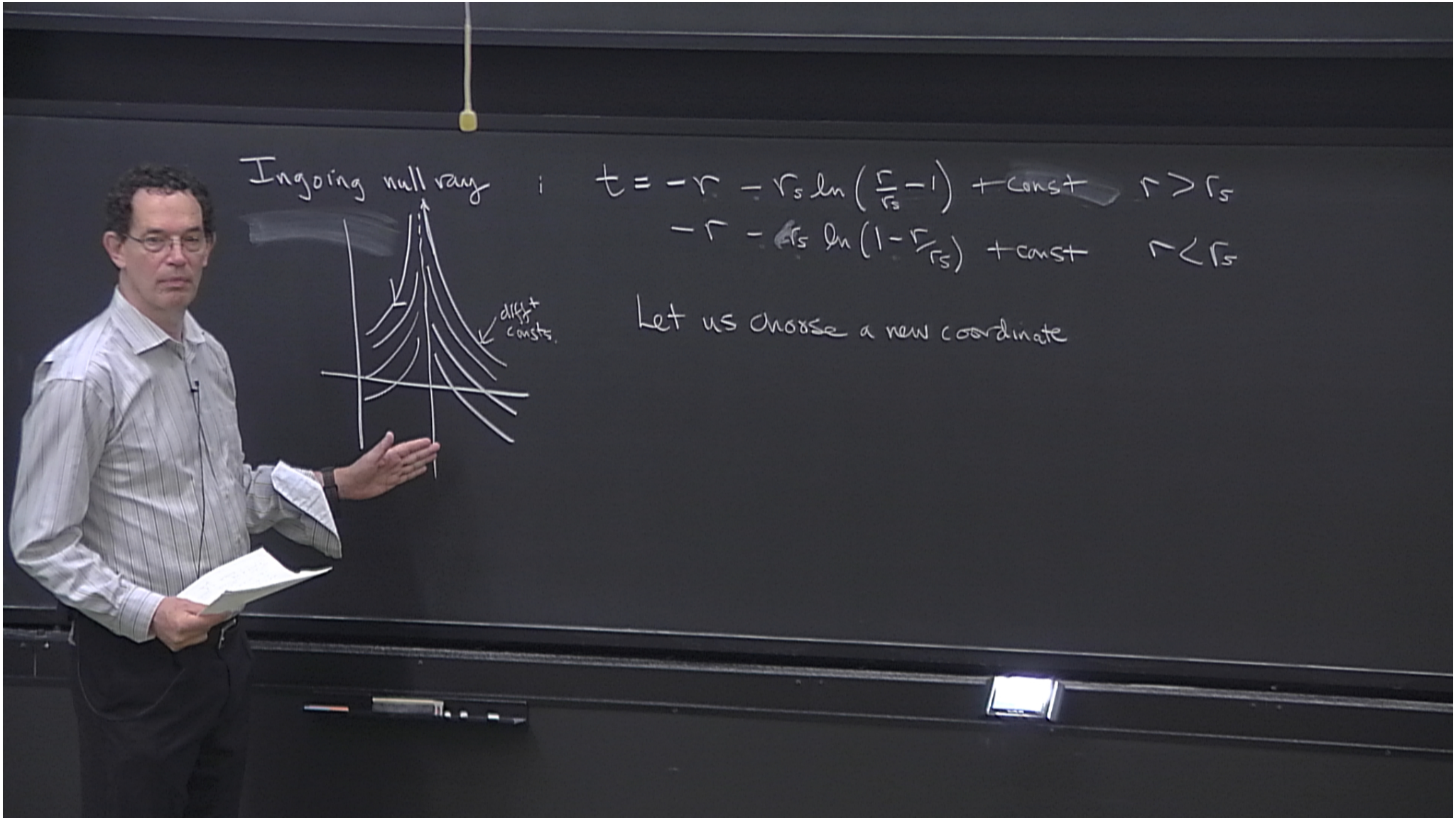
$$dt = \pm \frac{dr}{1 - \frac{r_s}{r}} = \pm dr \left(1 + \frac{r_s}{r - r_s}\right)$$


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r
$r_s$
radial light rays
Minkowski
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Schwarzschild
$dt = \pm \frac{dr}{1-\frac{r_s}{r}} = \pm dr\left(1 + \frac{r_s}{r-r_s}\right)$
So for radial null rays, $t = \pm\left(r + r_s \ln\left(\frac{r}{r_s} - 1\right)\right) + \text{const}$ $r > r_s$
$= \pm\left(r + r_s \ln\left(1 - \frac{r}{r_s}\right)\right) + \text{const}$ $r < r_s$
Ingoing null ray : $t =$ $- r_s \ln\left(\frac{r}{r_s} - 1\right)$

Ingoing null ray : $t = -r - r_s \ln\left(\frac{r}{r_s} - 1\right) + \text{const}$ $r > r_s$

$-r - r_s \ln\left(1 - \frac{r}{r_s}\right) + \text{const}$ $r < r_s$

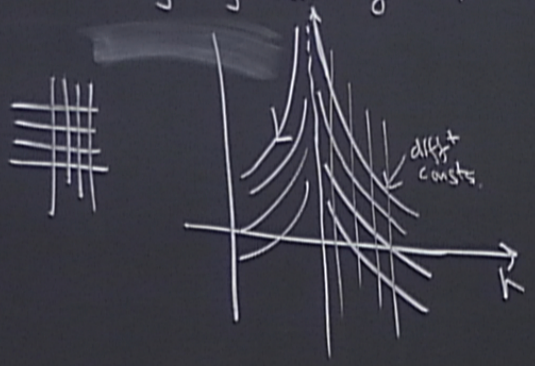



Ingoing null ray ; $t = -r - r_s \ln\left(\frac{r}{r_s} - 1\right) + \text{const}$ $r > r_s$
$-r - r_s \ln\left(1 - \frac{r}{r_s}\right) + \text{const}$ $r < r_s$
diff't consts.
Let us choose a new coordinate

$+\text{const} \qquad r < r_s$

Ingoing null ray : $t = -r - r_s \ln\left(\frac{r}{r_s} - 1\right) + \text{const} \qquad r > r_s$

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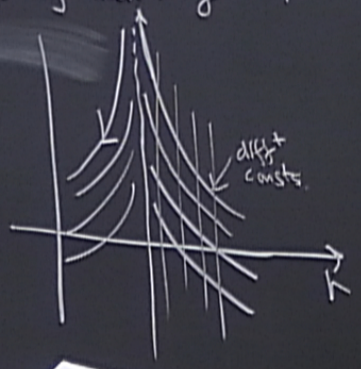

Let us choose a new coordinate

$v = t + r + r_s \ln\left(\frac{r}{r_s} - 1\right)$ , which is constant on ingoing null rays.

$+ \text{const} \qquad r < r_s$

Ingoing null ray : $t = -r - r_s \ln\left(\frac{r}{r_s} - 1\right) + \text{const} \qquad r > r_s$

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Let us choose a new coordinate

$$v = t + r + r_s \ln\left|\frac{r}{r_s} - 1\right|$$

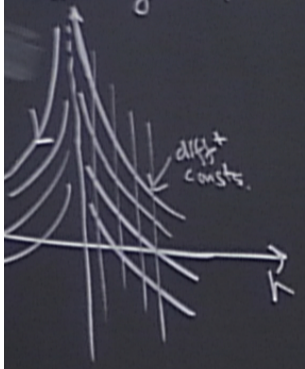
, which is constant on ingoing null rays.

$dv = dt + dr$

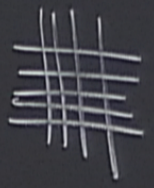
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null ray : $t = -r - r_s \ln\left(\frac{r}{r_s} - 1\right) + \text{const} \qquad r > r_s$

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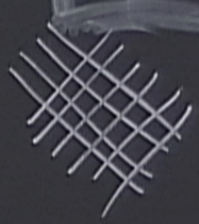

$-dt^2 + dr^2$



$u = t - r$

$v = t + r$

$-du\,dv$



Let us choose a new coordinate

$$v = t + r + r_s \ln\left|\frac{r}{r_s} - 1\right|$$

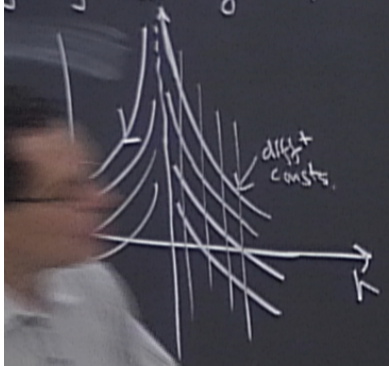
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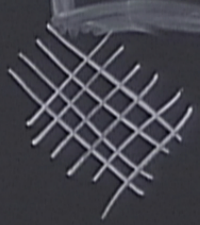
$-r - r_s \ln\left(1 - \frac{r}{r_s}\right) + \text{const} \qquad r < r_s$




$-dt^2 + dr^2$



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$-du\,dv$



Let us choose a new coordinate

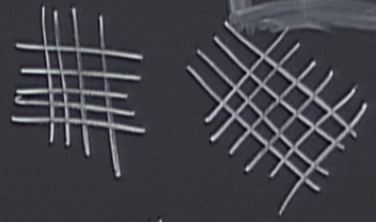
$$v = t + r + r_s \ln\left|\frac{r}{r_s} - 1\right|$$ , which is constant on ingoing null rays.

$r \neq r_s$ : $dv = dt + \frac{dr}{1 - \frac{r_s}{r}} \Rightarrow ds^2 = -\alpha dv^2 + 2\,dv\,dr + r^2 d\Omega^2$

$t = -r - r_s \ln\left(\frac{r}{r_s} - 1\right) + \text{const} \quad r > r_s$

$\quad -r - r_s \ln\left(1 - \frac{r}{r_s}\right) + \text{const} \quad r < r_s$

$-dt^2 + dr^2$ $\quad u = t - r$, $v = t + r$ $\quad -du\,dv$



Let us choose a new coordinate

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$r \neq r_s$: $\quad dv = dt + \frac{dr}{1 - r_s/r}$ $\quad \Rightarrow \quad ds^2 = -\alpha\,dv^2 + 2\,dv\,dr + r^2 d\Omega^2$

$-\alpha\left(dt + \frac{dr}{\alpha}\right)^2 + 2\left(dt + \frac{dr}{\alpha}\right)dr$

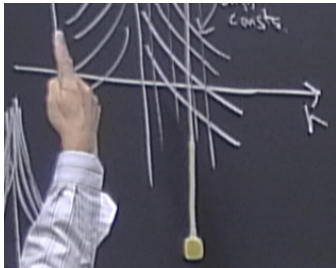
completely nonsingular at $r = r_s$ !

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$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Eddington - Finkelstein coordinates.




Let us choose a new coordinate

$$v = t + r + r_s \ln\left|\frac{r}{r_s} - 1\right|$$ , which is constant on ingoing null rays.

$r \neq r_s$ : $dv = dt + \frac{dr}{1 - r_s/r}$ $\Rightarrow$ $ds^2 = \frac{-\alpha dv^2 + 2dvdr + r^2 d\Omega^2}{-\alpha\left(dt + \frac{dr}{\alpha}\right)^2 + 2\left(dt + \frac{dr}{\alpha}\right)dr}$ completely nonsingular at $r = r_s$ !

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Eddington-Finkelstein coordinates.

In these coords, ingoing light rays satisfy $v = \text{const}$.

Note: choice of 'ingoing coordinate' $v$ determines the direction of 'time' $r$ for $r < r_s$ ; $r$ must decrease.

$-\alpha\left(dt+\frac{dr}{\alpha\beta}\right)^2+2\left(dt+\frac{dr}{\alpha}\right)dr$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

# Eddington-Finkelstein coordinates.

In these coords, ingoing light rays satisfy $v = \text{const}$.
(as opposed to outgoing)

Note: choice of 'ingoing' coordinate $v$ determines the direction of 'time' $r$ for $r < r_s$ ;
(this is the physical case for gravitational collapse)

$E = \left(1 - \frac{2GM}{r}\right)\dot{t}$

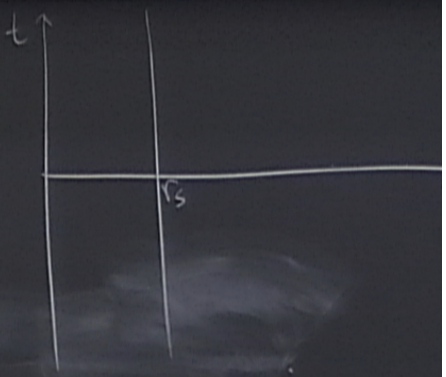
$\tau = \left(\frac{R^3}{8GM}\right)^{1/2}(\theta + \sin\theta)$ , where $R$ is the initial radius

particle starts at radius $r = R$ at $\theta = 0$

hits $r = 0$ at $\theta = \pi$, $\tau = \frac{\pi}{2}\left(\frac{R^3}{2GM}\right)^{1/2}$ ; if $R = 2GM$ then $\boxed{\tau = \pi GM}$ $(\tau \approx 10^{-}$

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$$u = t - r - r_s \ln\left|\frac{r}{r_s} - 1\right| = \text{const on outgoing light rays.}$$



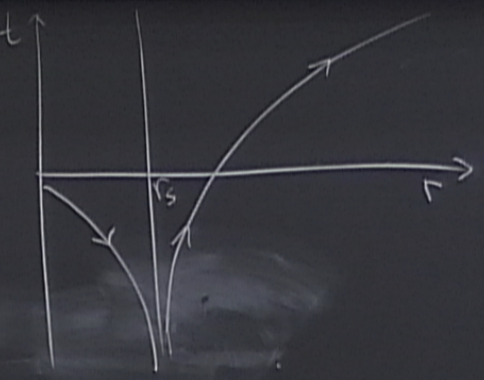
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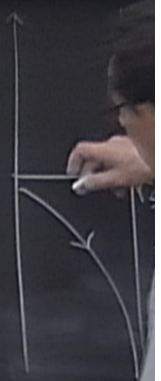
Consider instead outgoing light rays.

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$dt = du + dr/\alpha$

$\Rightarrow ds^2 = -\alpha du^2 - 2du dr + r^2 d\Omega^2$, also fine at $r = r_s$.

BUT: direction of 'time', $r$ is the opposite!



# Particle falling into a black hole

Radial orbit : $J=0$

massive particle $g_{\mu\nu}u^\mu u^\nu = -1 \rightarrow \left(\frac{dr}{d\tau}\right)^2 + 1 - \frac{2GM}{r} = E^2 = \text{const}$

$-\left(1-\frac{2GM}{r}\right)\dot{t}^2 + \left(1-\frac{2GM}{r}\right)^{-1}\dot{r}^2 = 1$

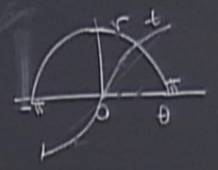
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resolution : there are two versions of the region $r<r_s$ ; one is the black hole, and the other is a white hole.

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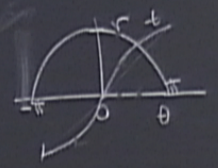
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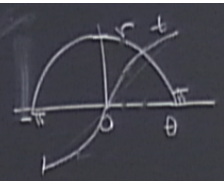
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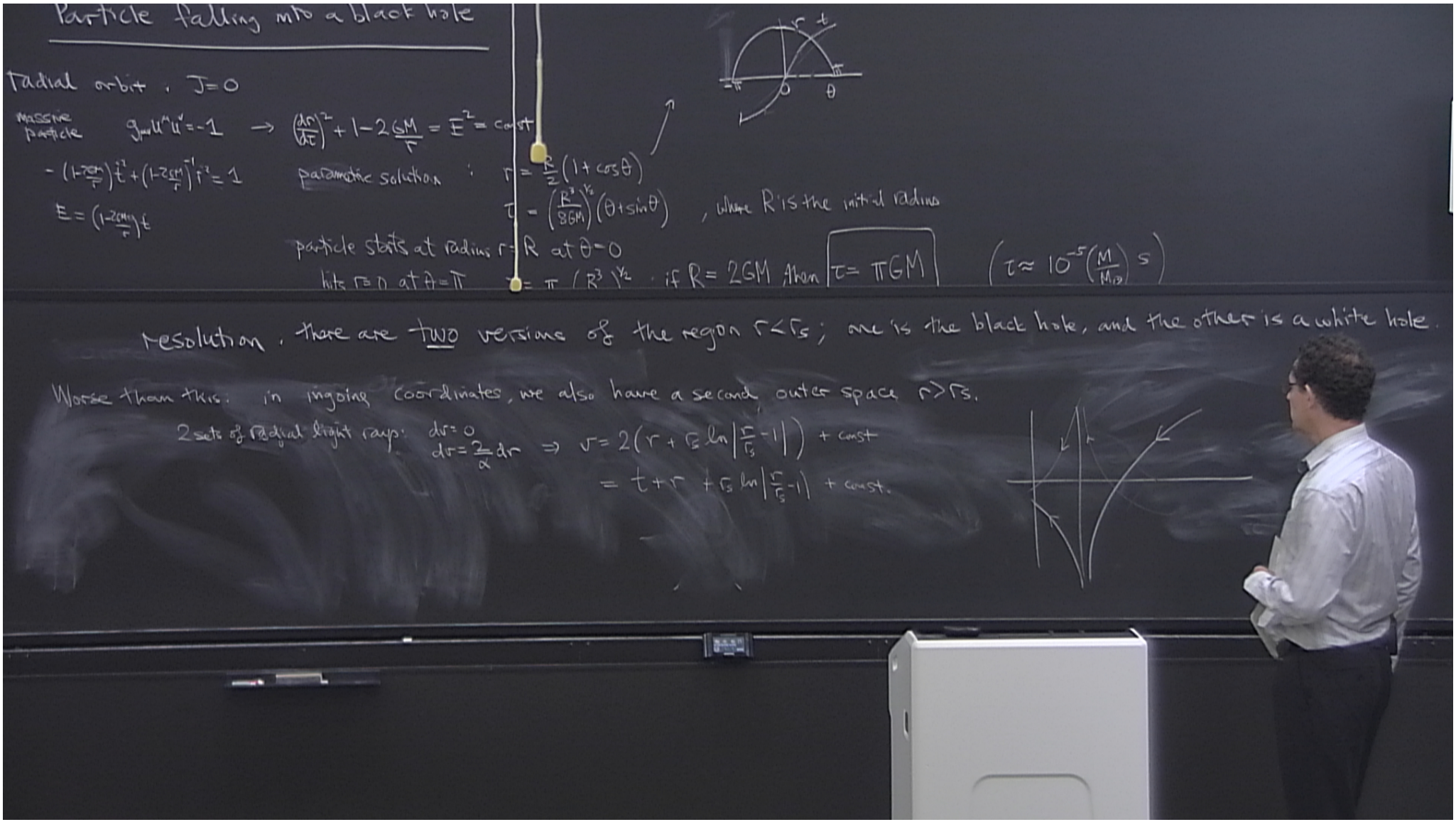

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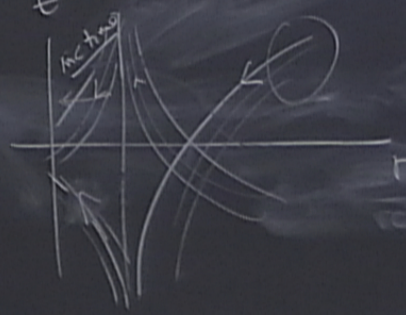

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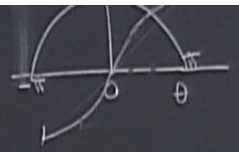
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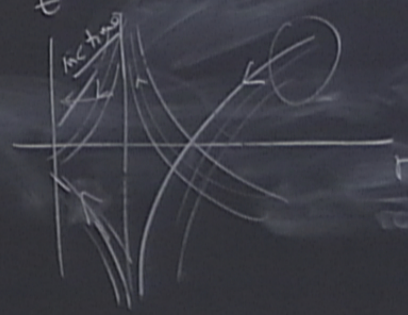

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$\Rightarrow$ a whole new universe !