Title: 14/15 PSI - Relativity- Lecture 13

Date: Sep 24, 2014 09:00 AM

URL: http://pirsa.org/14090062

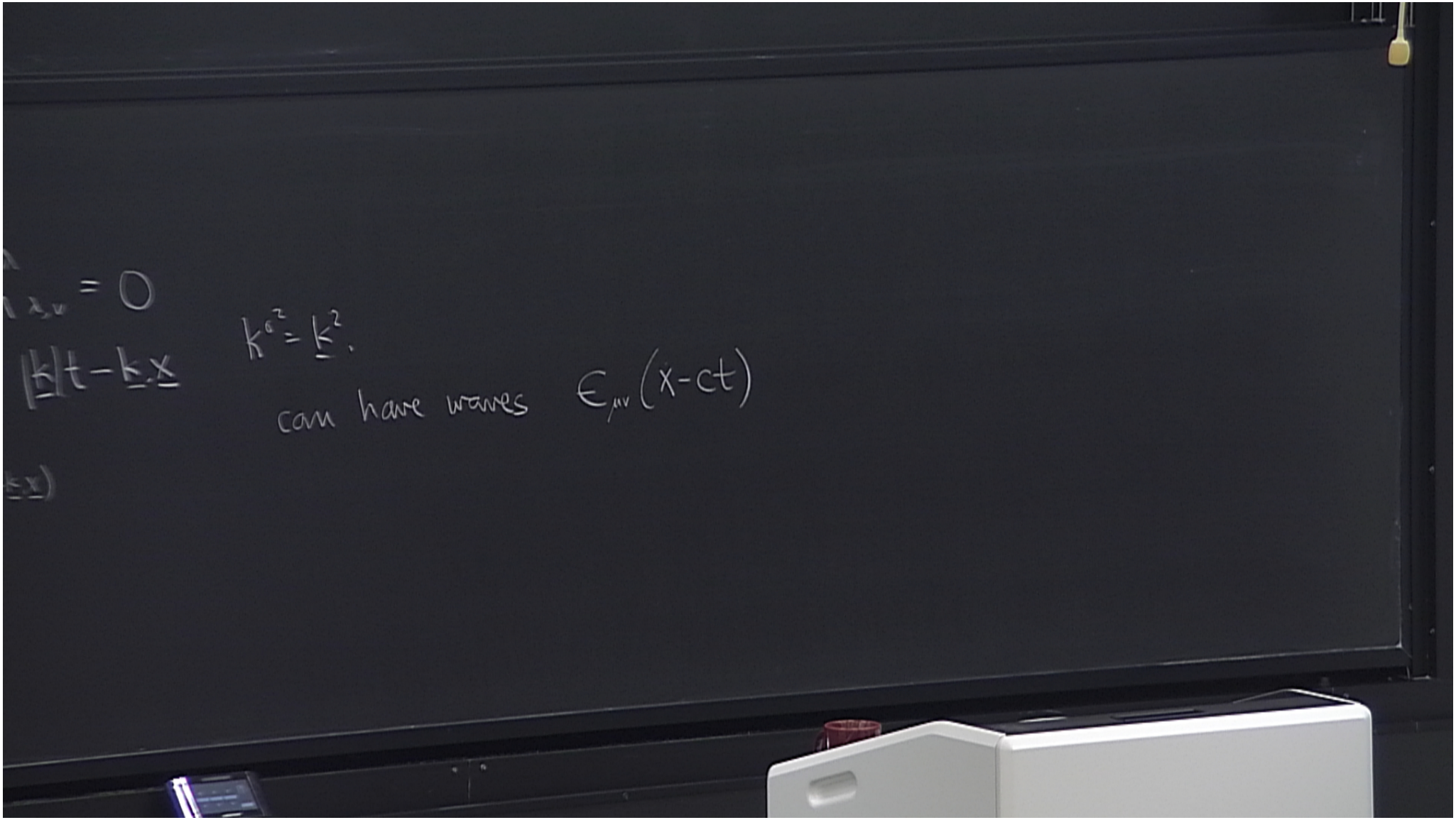
Abstract:

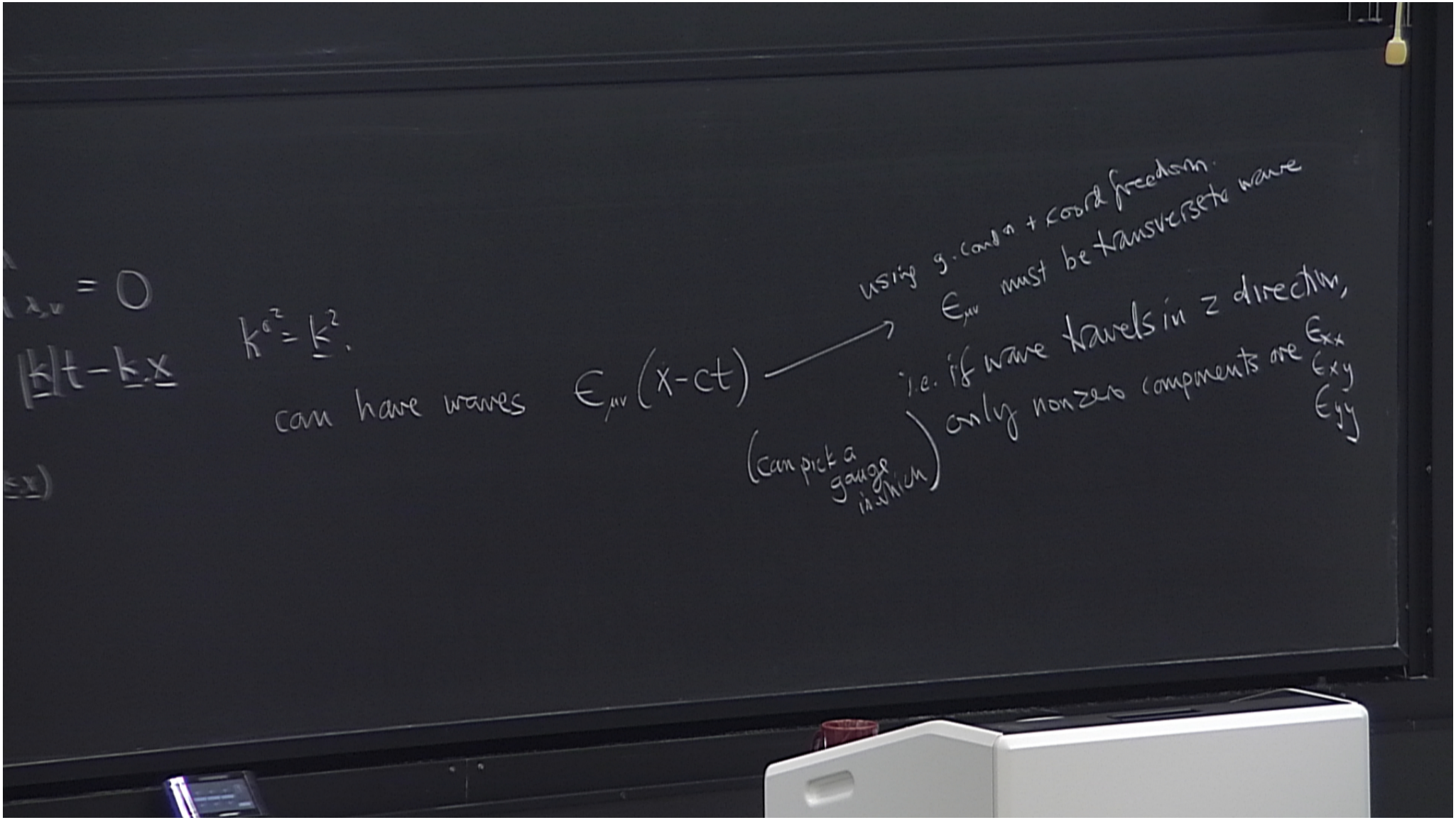
last time, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

pick gauge / coordinates $g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} = 0 \Rightarrow h^{\lambda}_{\nu,\lambda} - \frac{1}{2} h^{\lambda}_{\lambda,\nu} = 0$

$i(k\cdot x) \longrightarrow |\underline{k}|t - \underline{k}\cdot\underline{x}$ $\quad k^0 = \underline{k}$.

$R_{\mu\nu} = 0 \Rightarrow \underbrace{\Box h_{\mu\nu}}_{k^0 = \underline{k}} = 0$, $h_{\mu\nu} = \sum_{\underline{k}} \epsilon_{\mu\nu}(\underline{k}) \underbrace{e}_{e^{i(|\underline{k}|t - \underline{k}\cdot\underline{x})}}$


$= 0$
$k^{0^2} = \underline{k}^2$.
$|\underline{k}|t - \underline{k}.\underline{x}$
can have waves $\epsilon_{\mu\nu}(x-ct)$


= 0
$k^{0^2} = \underline{k}^2$.
$\underline{k}t - \underline{k}\cdot\underline{x}$
can have waves $\epsilon_{\mu\nu}(x-ct)$
(can pick a gauge in which)
using g. cond$^n$ + coord freedom.
$\epsilon_{\mu\nu}$ must be transverse & traceless wave
i.e. if wave travels in z direction,
only nonzero compments are $\epsilon_{xx}$ $\epsilon_{xy}$ $\epsilon_{yy}$

$$h_{\mu\nu} = \sum \epsilon_{\mu\nu}(k)\, e^{ik\cdot x} + \text{cc.}$$

d$^{n}$: $\quad k_\mu \epsilon^{\mu}_{\ \nu} - \frac{1}{2} k_\nu \epsilon^{\mu}_{\ \mu} = 0$

e to change coords as long as we maintain gauge cond$^{n}$

$$h_{\mu\nu} \to h'_{\mu\nu} = h_{\mu\nu} - \epsilon_{\mu\nu} - \epsilon_{\nu\mu} \qquad \epsilon_{\mu\nu} \to \epsilon'_{\mu\nu} = \epsilon_{\mu\nu} + k_\mu C_\nu + k_\nu C_\mu$$

$$\text{if } \epsilon_\mu(x) = \sum_k \left( i C_\mu^{(k)} e^{ikx} + \text{c.c.} \right) \qquad k^0 = |\vec{k}|$$

+ cc.

as we maintain gauge cond$^n$

$\epsilon_{\mu\nu} - \epsilon_{\nu\mu}$ } $\epsilon_{\mu\nu} \to \epsilon'_{\mu\nu} = \epsilon_{\mu\nu} + k_\mu C_\nu + k_\nu C_\mu$

cc )

$k_\mu \epsilon'^{\mu}_{\ \nu} - \frac{1}{2} k_\nu \epsilon'^{\mu}_{\ \mu} = (k.C) k_\nu - \frac{1}{2} k_\nu (2k.C) = 0.$

e maintain gauge cond$^n$:

$$\epsilon_{\mu\nu} \to \epsilon'_{\mu\nu} = \epsilon_{\mu\nu} + k_\mu C_\nu + k_\nu C_\mu$$

gc: $k_\mu \epsilon'^{\mu}_{\ \nu} - \frac{1}{2} k_\nu \epsilon'^{\mu}_{\ \mu} = (k\cdot C) k_\nu - \frac{1}{2} k_\nu (2 k\cdot C) = 0.$

reduced to 6 by gauge cond$^n$, and down to 2 by 4 remaining coord transfm$^n$s which preserve the gauge condition

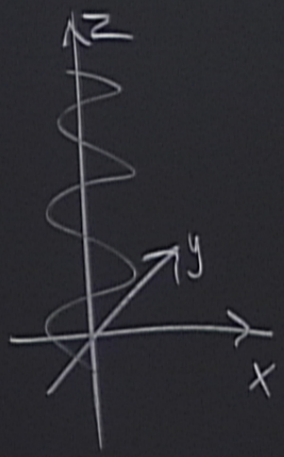
i.e. 10 components of $h_{\mu\nu}$ get reduced to 6 b

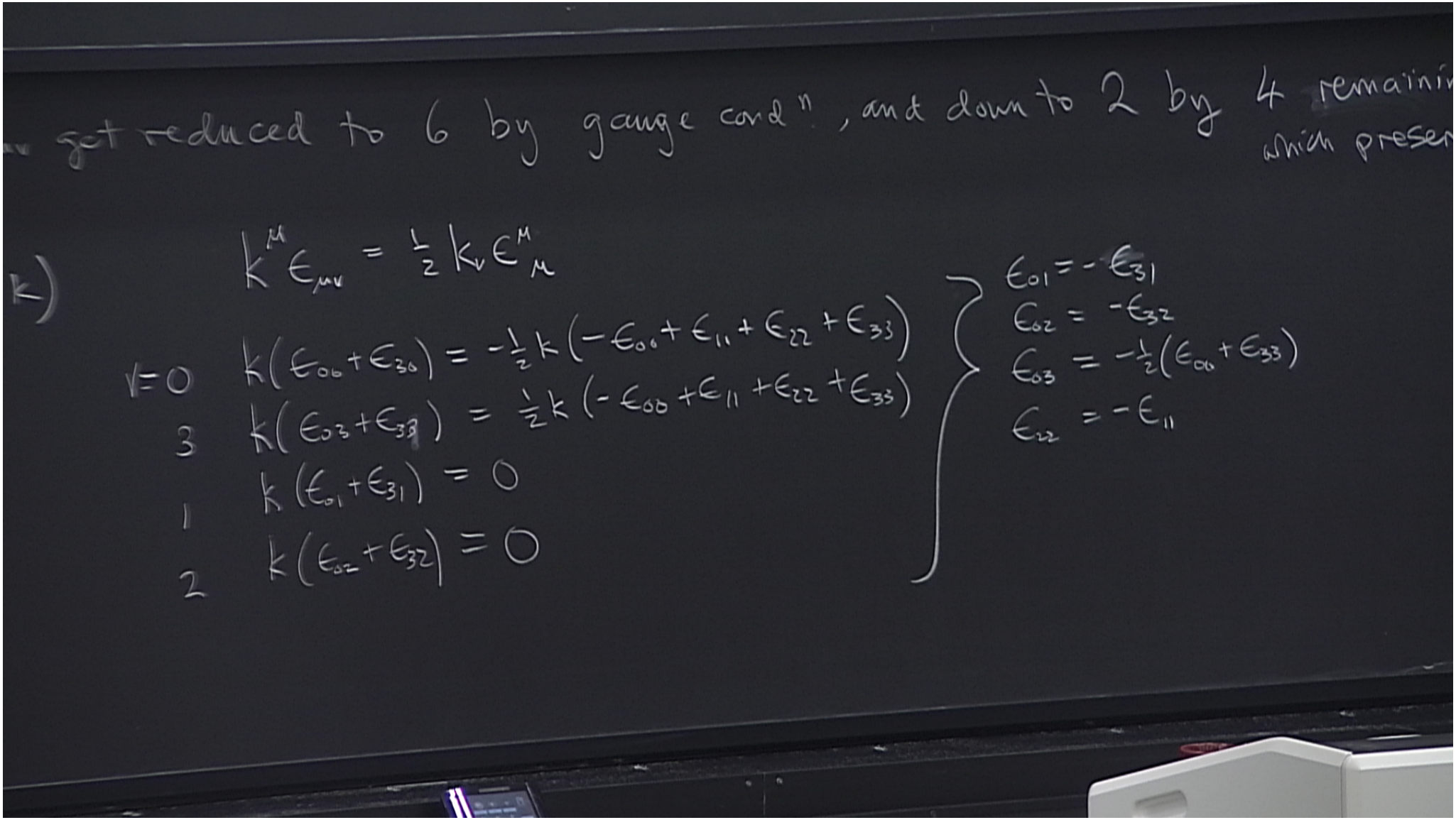
e.g. $k^\mu = (k, 0, 0, k)$

$$e^{ik\cdot x} = e^{-ikt+ikz}$$

$$= e^{ik(z-t)}$$

wave propagating in z-dir!




get reduced to 6 by gauge cond", and down to 2 by 4 remainin
which preser
$k^\mu \epsilon_{\mu\nu} = \frac{1}{2} k_\nu \epsilon^\mu{}_\mu$
$\nu=0$ $k(\epsilon_{00}+\epsilon_{30}) = -\frac{1}{2}k(-\epsilon_{00}+\epsilon_{11}+\epsilon_{22}+\epsilon_{33})$
3 $k(\epsilon_{03}+\epsilon_{33}) = \frac{1}{2}k(-\epsilon_{00}+\epsilon_{11}+\epsilon_{22}+\epsilon_{33})$
1 $k(\epsilon_{01}+\epsilon_{31}) = 0$
2 $k(\epsilon_{02}+\epsilon_{32}) = 0$
$\epsilon_{01} = -\epsilon_{31}$
$\epsilon_{02} = -\epsilon_{32}$
$\epsilon_{03} = -\frac{1}{2}(\epsilon_{00}+\epsilon_{33})$
$\epsilon_{22} = -\epsilon_{11}$

## residual coordinate freedom ($C^\mu$)

$$\epsilon'_{\mu\nu} = \epsilon_{\mu\nu} + k_\mu C_\nu + k_\nu C_\mu$$

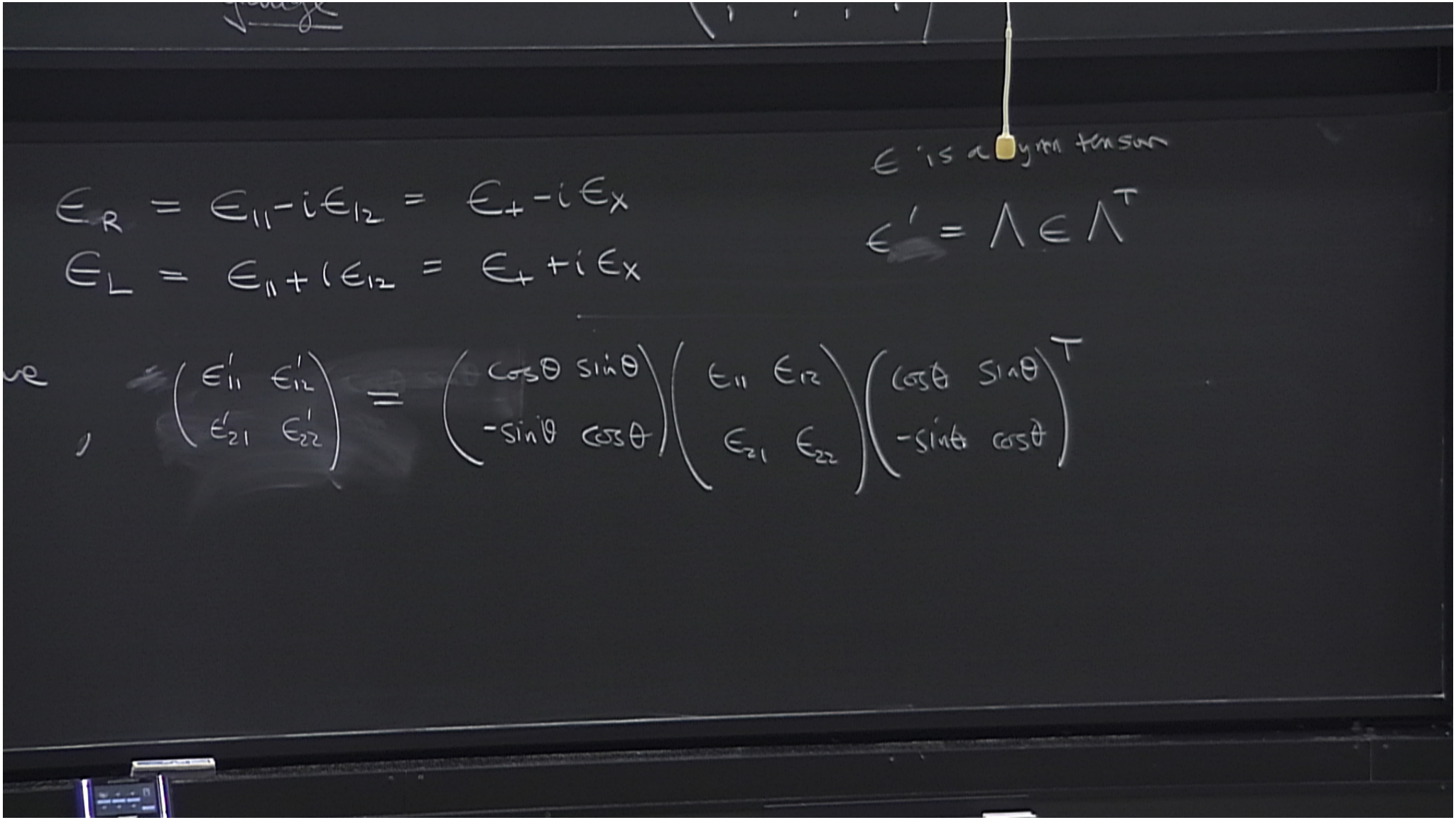
$k_\mu = (-k, 0, 0, k)$

$$\left.\begin{aligned} \epsilon'_{00} &= \epsilon_{00} - 2kC_0 \\ \epsilon'_{13} &= \epsilon_{13} + kC_1 \\ \epsilon'_{23} &= \epsilon_{23} + kC_2 \\ \epsilon'_{33} &= \epsilon_{33} + 2kC_3 \end{aligned}\right\} \Rightarrow \text{choose } C^\mu \text{ so that}$$

$$\epsilon'_{00} = \epsilon'_{13} = \epsilon'_{23} = \epsilon'_{33} = 0. \Rightarrow \epsilon_{\mu\nu} =$$

"Transverse, traceless" or TT gauge

$$\equiv \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \epsilon_+ & \epsilon_\times & \cdot \\ \cdot & \epsilon_\times & -\epsilon_+ & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$


$\epsilon_R = \epsilon_{11} - i\epsilon_{12} = \epsilon_+ - i\epsilon_\times$
$\epsilon_L = \epsilon_{11} + i\epsilon_{12} = \epsilon_+ + i\epsilon_\times$
$\epsilon$ is a symm tensor
$\epsilon' = \Lambda \epsilon \Lambda^T$
$\begin{pmatrix} \epsilon'_{11} & \epsilon'_{12} \\ \epsilon'_{21} & \epsilon'_{22} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^T$

also define $\epsilon_R = \epsilon_{11} - i\epsilon_{12} = \epsilon_+ - i\epsilon_\times$

$\epsilon_L = \epsilon_{11} + i\epsilon_{12} = \epsilon_+ + i\epsilon_\times$

$\epsilon$ is a sym tensor

$\epsilon' = \Lambda \epsilon \Lambda^T$

...ons which leave fixed

$$\begin{pmatrix} \epsilon'_{11} & \epsilon'_{12} \\ \epsilon'_{21} & \epsilon'_{22} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^T$$

$\Rightarrow \epsilon'_R = e^{2i\theta} \epsilon_R$

$\epsilon'_L = e^{-2i\theta} \epsilon_L$

(means that the graviton has spin 2)

circularity

left and right polarised gravitational waves.



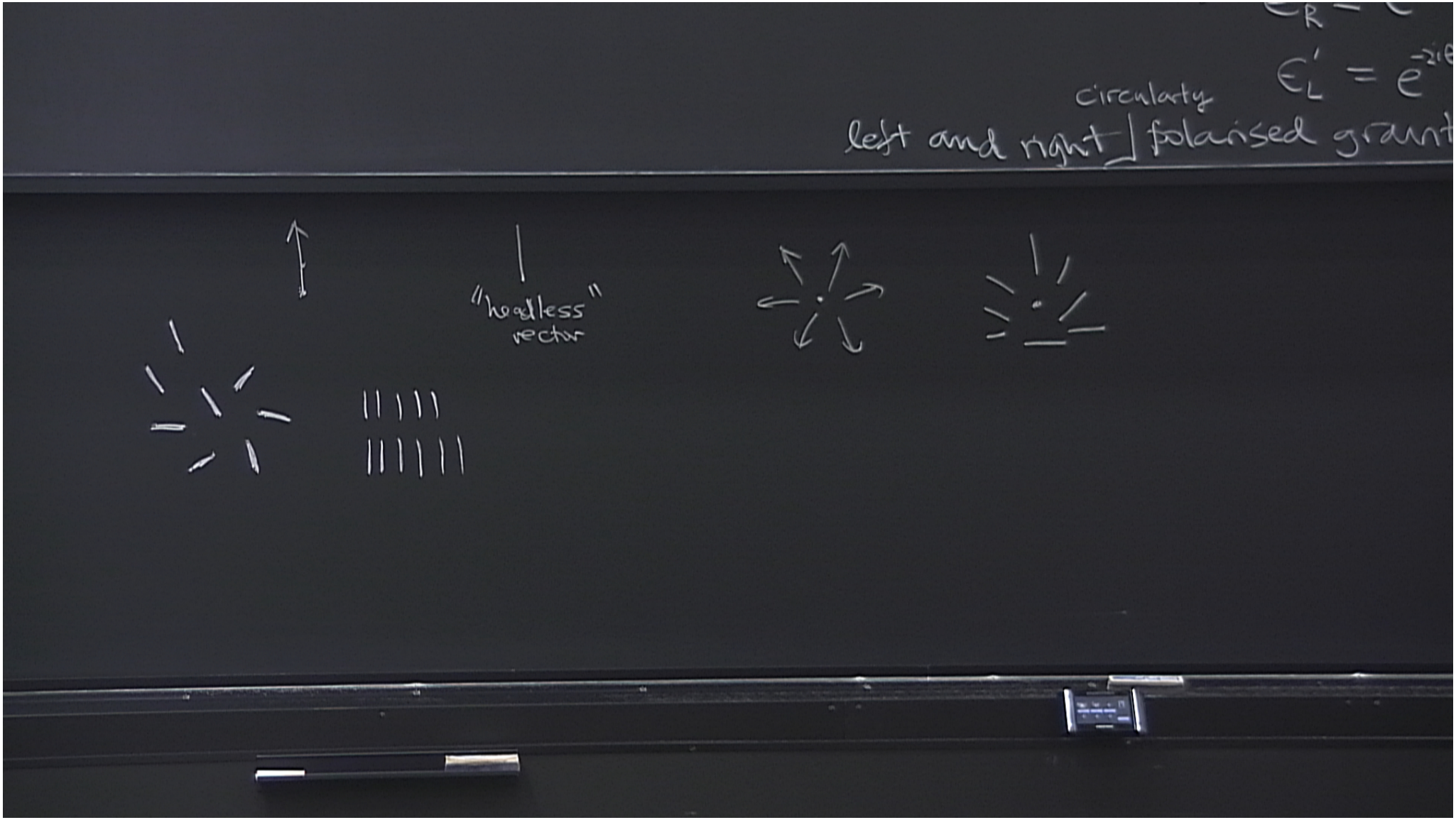

Can also define $\epsilon_R = \epsilon_{11}$

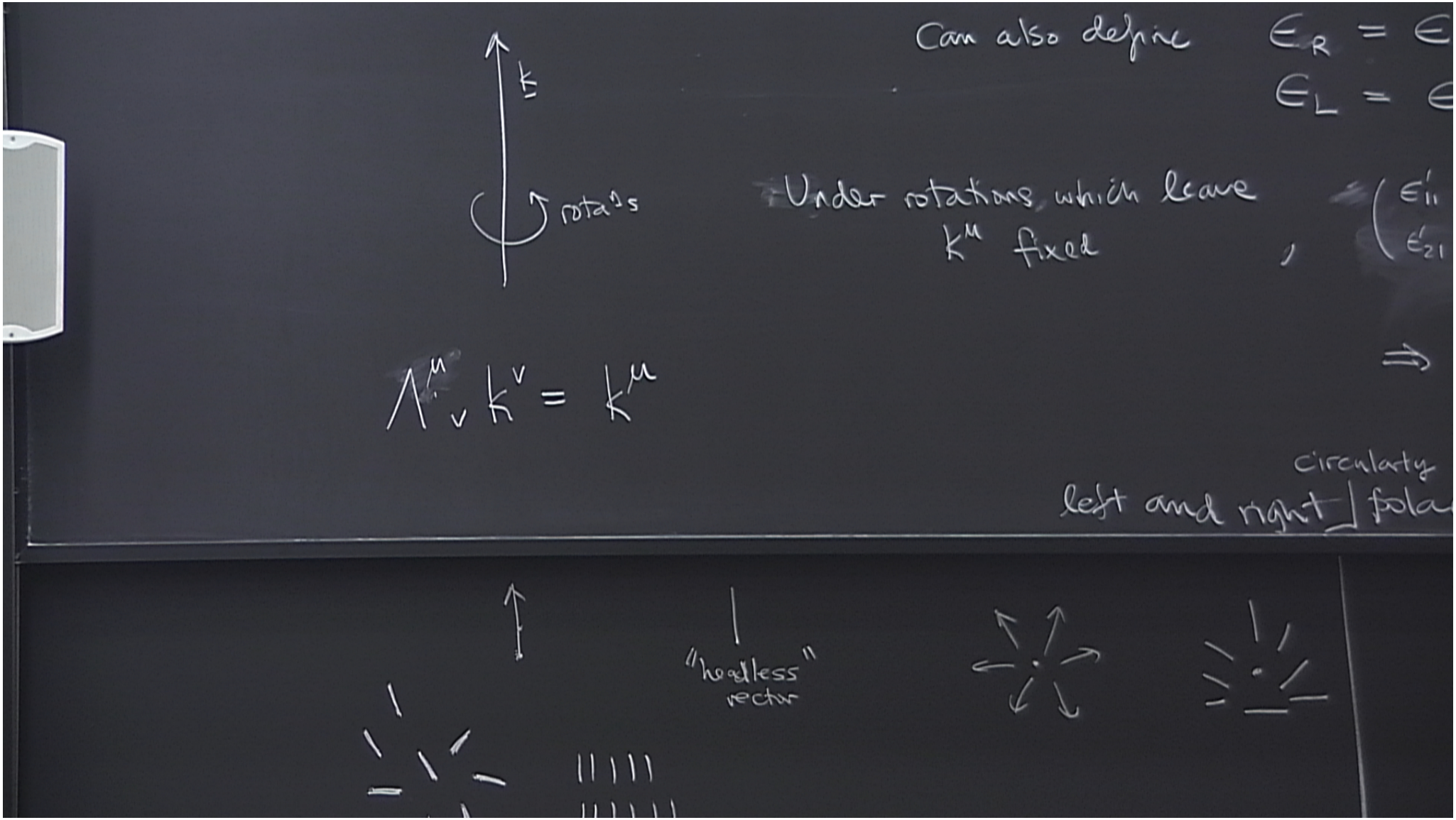
$\epsilon_L = \epsilon_{11}$

Under rotations which leave $k^\mu$ fixed , $\begin{pmatrix} \epsilon'_{11} & \\ \epsilon'_{21} & \end{pmatrix}$

$\Rightarrow$

circularly left and right polari


circularity
left and right ] polarised
"headless" vector


k
rotations
Can also define
Under rotations which leave
$k^\mu$ fixed
$\Lambda^\mu{}_\nu k^\nu = k^\mu$
circularity
left and right
"headless"

$\epsilon_R' = e^{-2i\theta}\epsilon_R$

$\epsilon_L' = e^{-2i\theta}\epsilon_L$

circularity

spin 2

right ] polarised gravitational waves.

$\underline{J} \rightarrow J = 0, \frac{1}{2}, 1 \ldots$

## Higher dimensions

in $d$ spacetime dimensions $h_{\mu\nu} = \frac{d(d+1)}{2}$ components

$\epsilon_R' = e^{2i\theta}\epsilon_R$ (means that the graviton has spin 2)

circularity $\epsilon_L' = e^{-2i\theta}\epsilon_L$

right / polarised gravitational waves.



$\underline{J} \rightarrow J = 0, \frac{1}{2}, 1 \ldots$

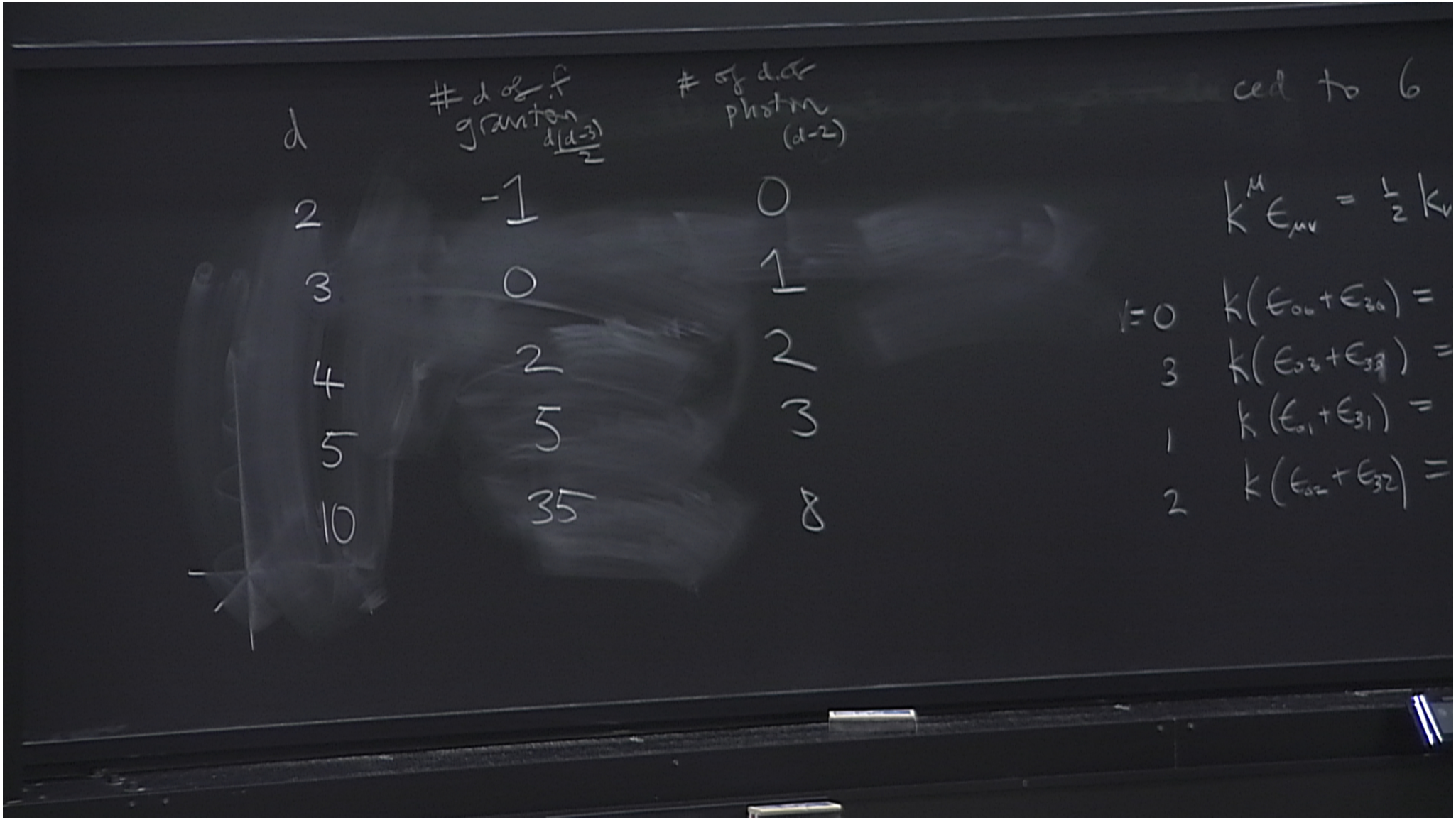
Higher dimensions

in $d$ spacetime dimensions $h_{\mu\nu} = \frac{d(d+1)}{2}$ components (symmetric)

$d$ gauge cond$^{n}$s $\frac{d(d+1)}{2} - d$

$d$ residual coord transf$^{n}$s $\frac{d(d-1)}{2} - 2d = \frac{d(d-3)}{2} = \frac{(d-2)(d-3)}{2} + d - 3$

# of indept components of symm, traceless $(d-2)\times(d-2)$ matrix


d
# d of f graviton d(d-3)/2
# of d.of photon (d-2)
2 -1 0
3 0 1
4 2 2
5 5 3
10 35 8
ced to 6
$k^\mu \epsilon_{\mu\nu} = \frac{1}{2} k_\nu$
$\nu = 0$ $k(\epsilon_{00} + \epsilon_{30}) =$
3 $k(\epsilon_{03} + \epsilon_{33}) =$
1 $k(\epsilon_{01} + \epsilon_{31}) =$
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$\epsilon_R = \epsilon_R$

$\epsilon_L' = e^{-2i\theta}\epsilon_L$ circularity

spin 2

right ] polarised gravitational waves.

$\underline{J} \rightarrow J = 0, \frac{1}{2}, 1 \ldots$

## Higher dimensions

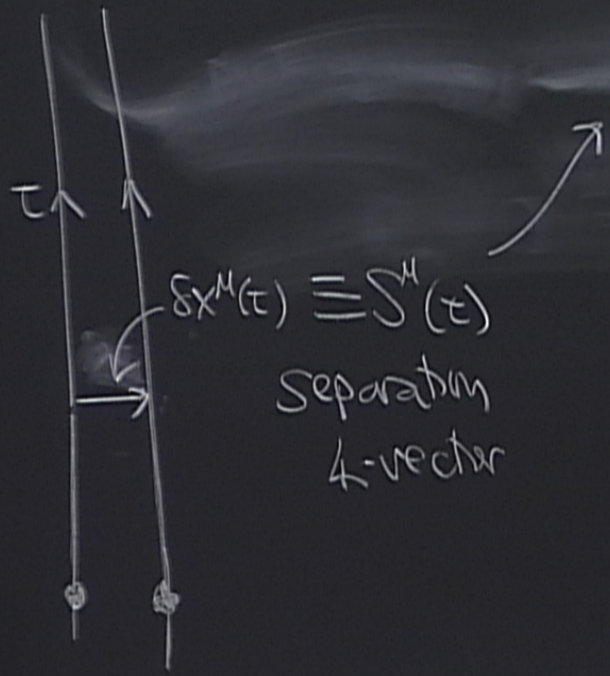
in $d$ spacetime dimensions $h_{\mu\nu}$ = $\frac{d(d+1)}{2}$ components (symmetric)

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# of indept components of Symm, traceless $(d-2)\times(d-2)$ Matrix

# Detecting gravitational waves




$$\frac{D^2 S^\mu}{D\tau^2} = R^\mu{}_{\nu\rho\sigma} U^\nu U^\rho S^\sigma$$

calculate to leading order in $h_{\mu\nu}$, assuming $U^\nu = (1,$

## ng gravitational waves

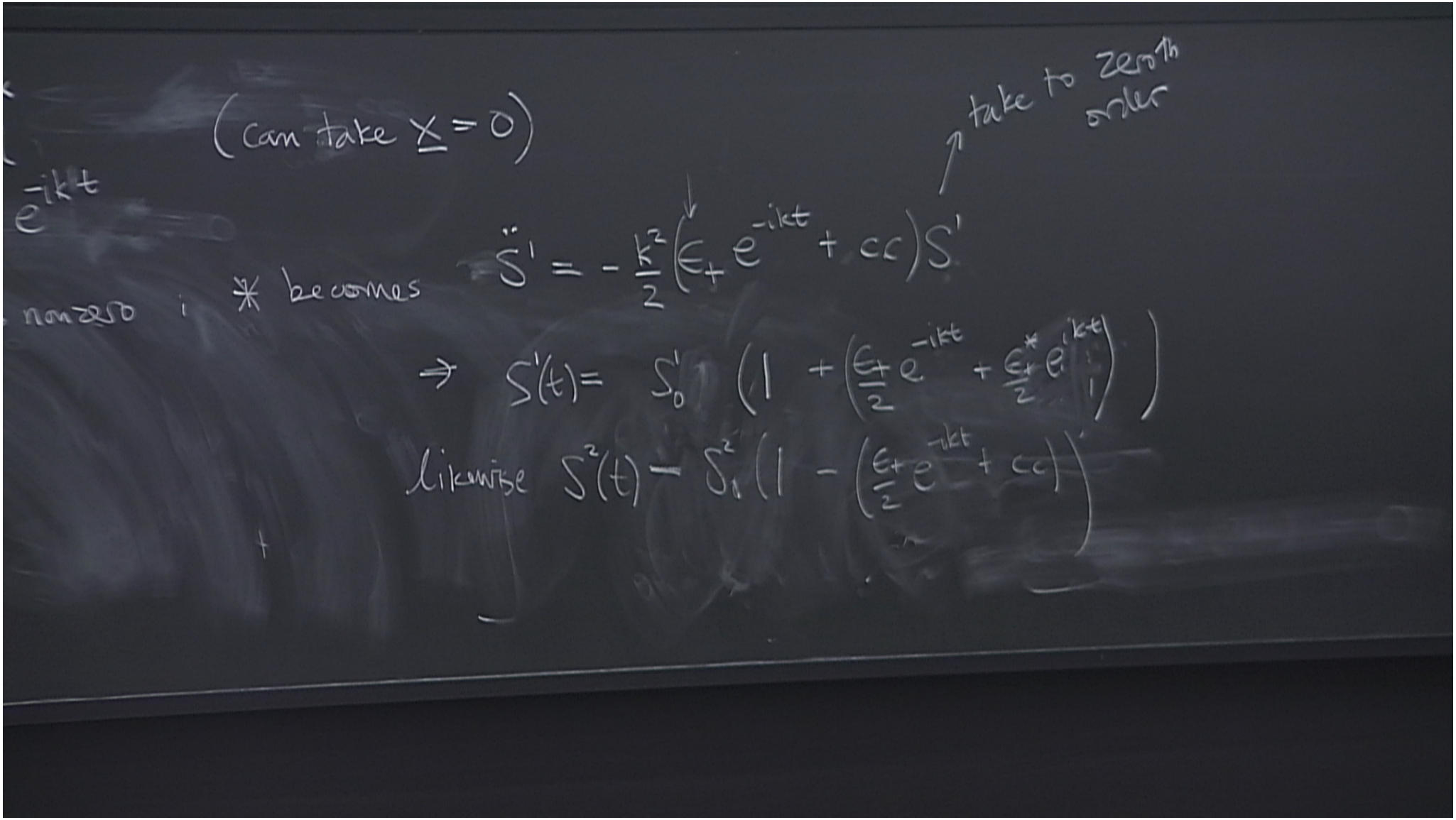
$$\frac{D^2 S^\mu}{D\tau^2} = \underbrace{R^\mu{}_{\nu\rho\sigma}}_{\text{first order in } h} \underbrace{U^\nu U^\rho S^\sigma}_{\text{zeroth order in } h}$$

calculate to leading order in $h_{\mu\nu}$, assuming $U^\nu = (1, \underline{0})$ to zeroth order,

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}\left(h_{\mu\sigma,\nu\rho} - h_{\nu\sigma,\mu\rho} - h_{\rho\nu,\mu\sigma} + h_{\rho\mu,\nu\sigma}\right)$$

TT gauge $h_{0\mu} = 0$

$$R_{\mu 00\sigma} = \frac{1}{2} h_{\mu\sigma,00} \Rightarrow \boxed{\frac{d^2 S^i}{dt^2} = \frac{1}{2} h_{ij,00} S^j}$$

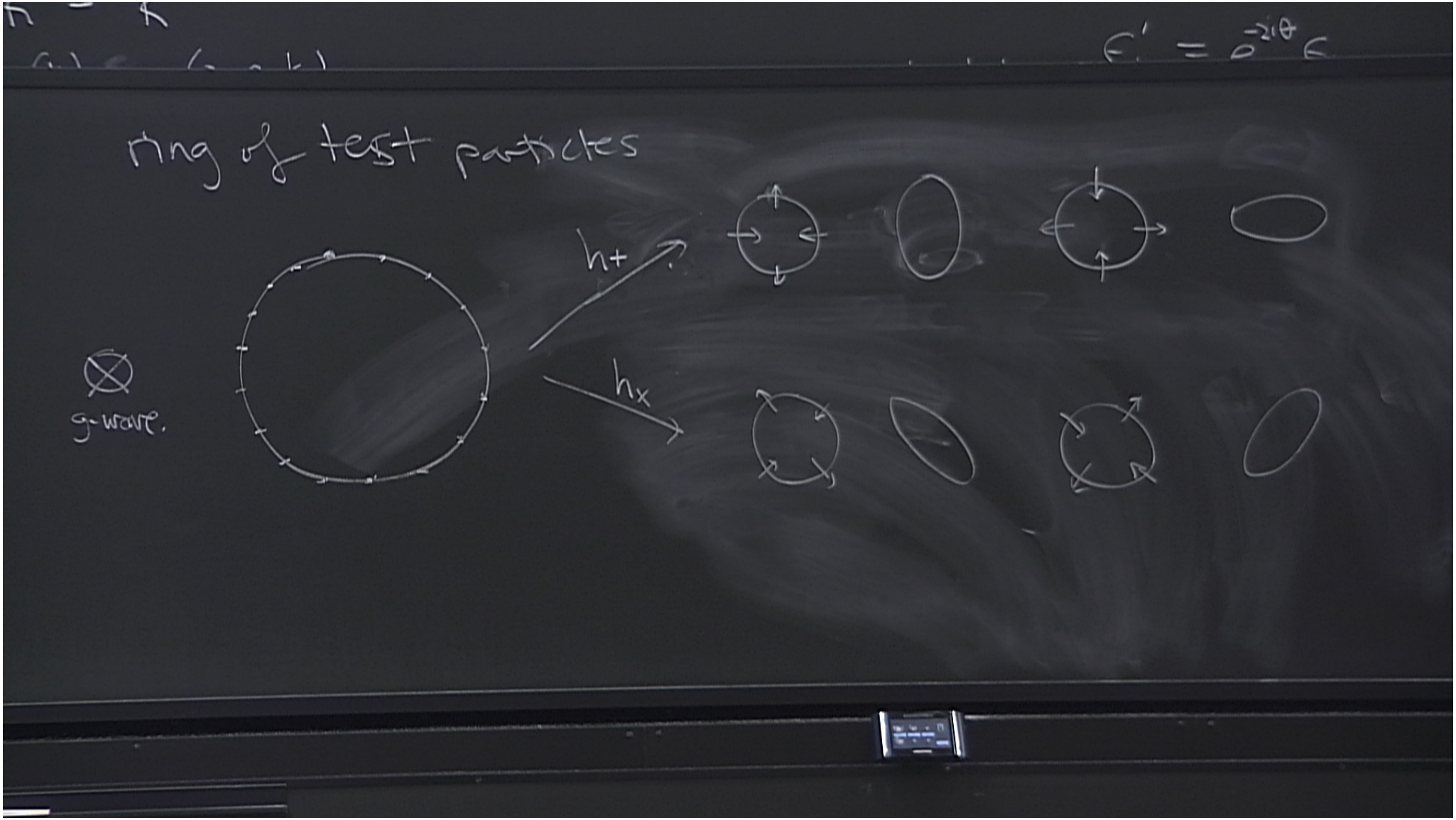

(can take $\underline{x}=0$)
take to zeroth order
$e^{-ikt}$
$\ddot{S}^1 = -\frac{k^2}{2}(\epsilon_+ e^{-ikt} + cc)S^1$
* becomes
nonzero
$\Rightarrow S^1(t) = S_0^1\left(1 + \left(\frac{\epsilon_+}{2}e^{-ikt} + \frac{\epsilon_+^*}{2}e^{ikt}\right)\right)$
likewise $S^2(t) = S_1^2\left(1 - \left(\frac{\epsilon_+}{2}e^{-ikt} + cc\right)\right)$

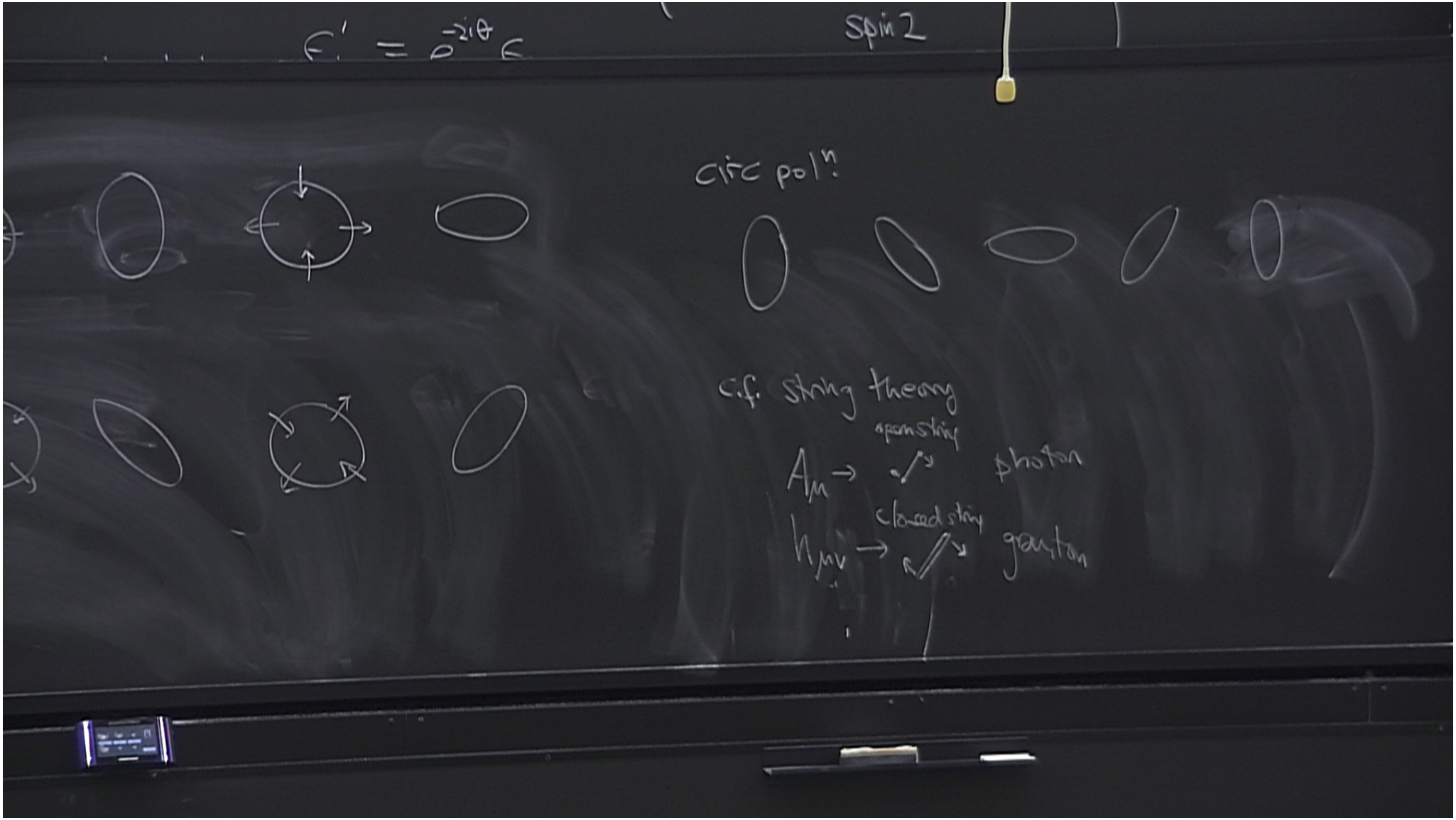
Next, assume $\epsilon_x$ is nonzero:

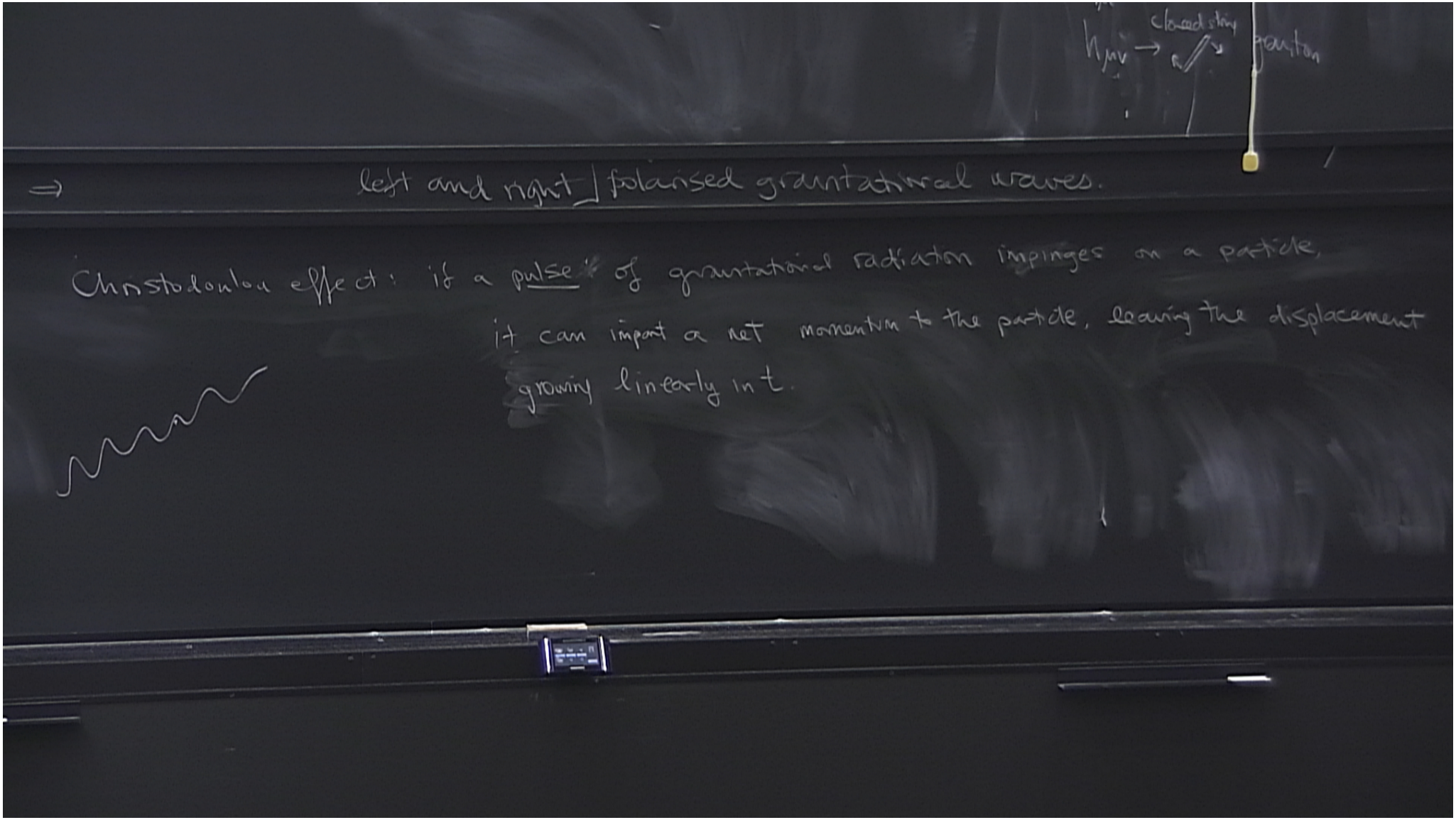
$$\ddot{S}^1 = -\frac{k^2}{2}\left(\epsilon_+ e^{-ikt} + c.c.\right) S^2$$

$$\Rightarrow \quad S^1(t) = S_0^2\left(1 + \frac{\epsilon_x}{2} e^{-ikt} + c.c.\right)$$

$$S^2(t) = S_0^1\left(1 + \frac{\epsilon_x}{2} e^{-ikt} + c.c.\right)$$


ring of test particles
h+
hx
g-wave.


$\epsilon' = e^{-2i\theta}\epsilon$
spin 2
circ pol$^n$
c.f. string theory
open string
$A_\mu \rightarrow$
photon
closed string
$h_{\mu\nu} \rightarrow$
graviton




$\Rightarrow$ left and right | polarised gravitational waves.

Christodoulou effect: if a pulse of gravitational radiation impinges on a particle, it can impart a net momentum to the particle, leaving the displacement growing linearly in $t$.

# gravitational waves

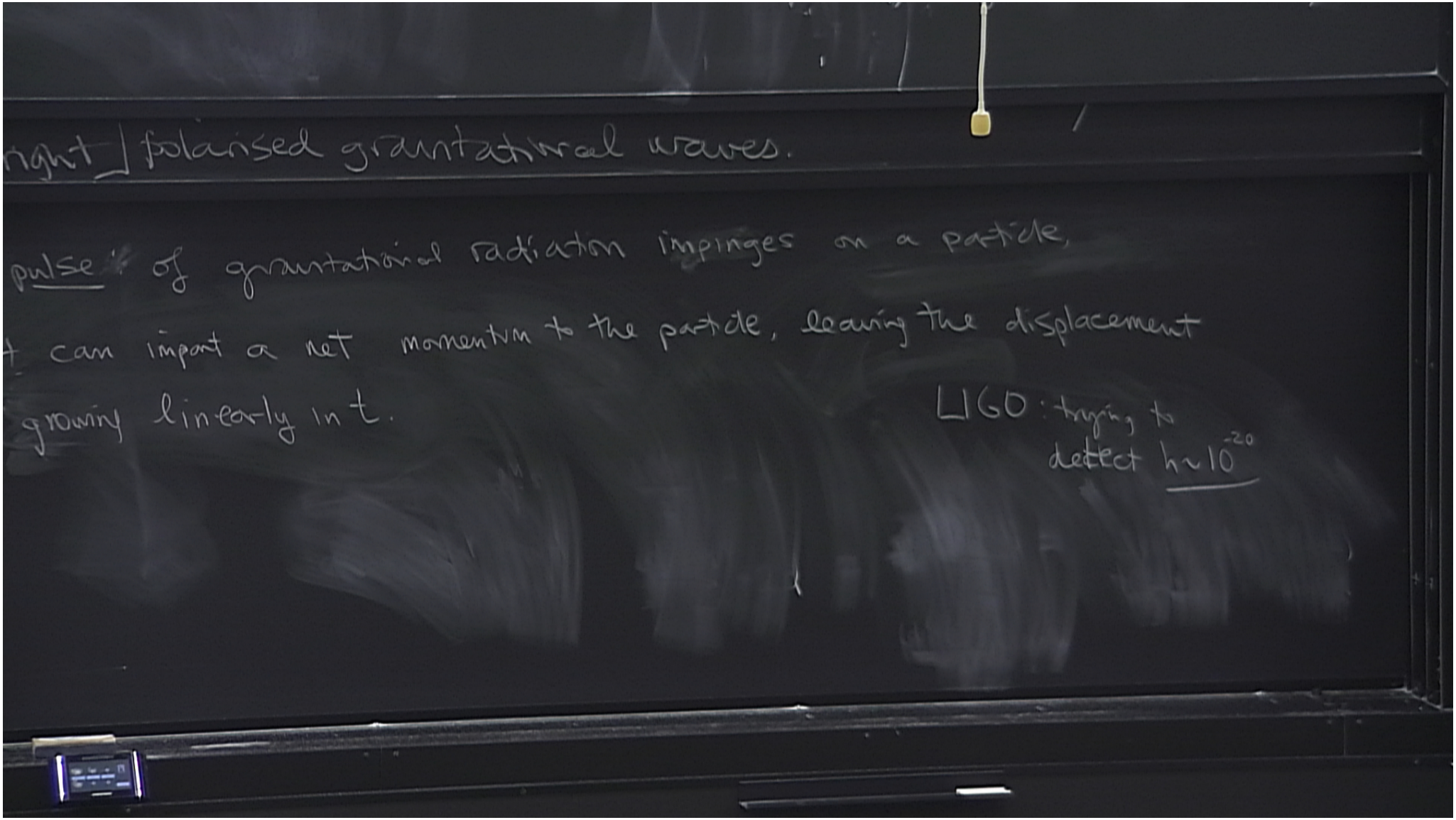
$$\frac{D^2 S^\mu}{D\tau^2} = \underbrace{R^\mu{}_{\nu\rho\sigma}}_{\text{first order in } h} \underbrace{U^\nu U^\rho S^\sigma}_{\text{zeroth order in } h}$$

calculate to leading order in $h_{\mu\nu}$, assuming $U^\nu = (1, \underline{0})$ to zeroth order,

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}\left(h_{\mu\sigma,\nu\rho} - h_{\nu\sigma,\mu\rho} - h_{\rho\nu,\mu\sigma} + h_{\rho\mu,\nu\sigma}\right)$$

TT gauge $h_{0\mu} = 0$

$$R_{\mu 00\sigma} = \frac{1}{2} h_{\mu\sigma,00} \Rightarrow \boxed{\frac{d^2 S^i}{dt^2} = \frac{1}{2} h^i_{j,00} S^j} \quad *$$


right] polarised gravitational waves.
pulse of gravitational radiation impinges on a particle,
it can impart a net momentum to the particle, leaving the displacement
growing linearly in t.
LIGO: trying to
detect $h \sim 10^{-20}$