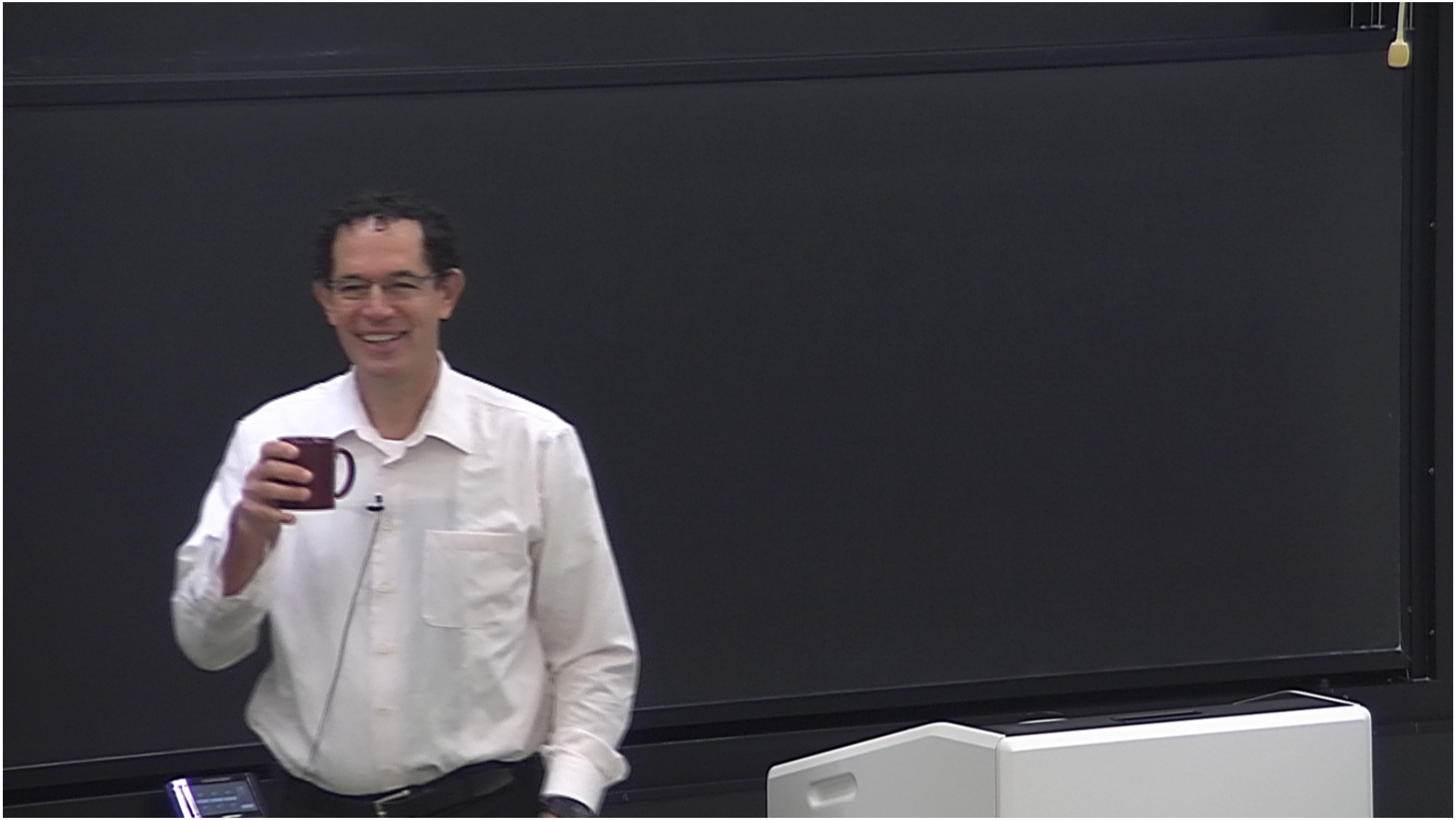
Title: 14/15 PSI - Relativity- Lecture 12

Date: Sep 23, 2014 09:00 AM

URL: http://pirsa.org/14090061

Abstract:

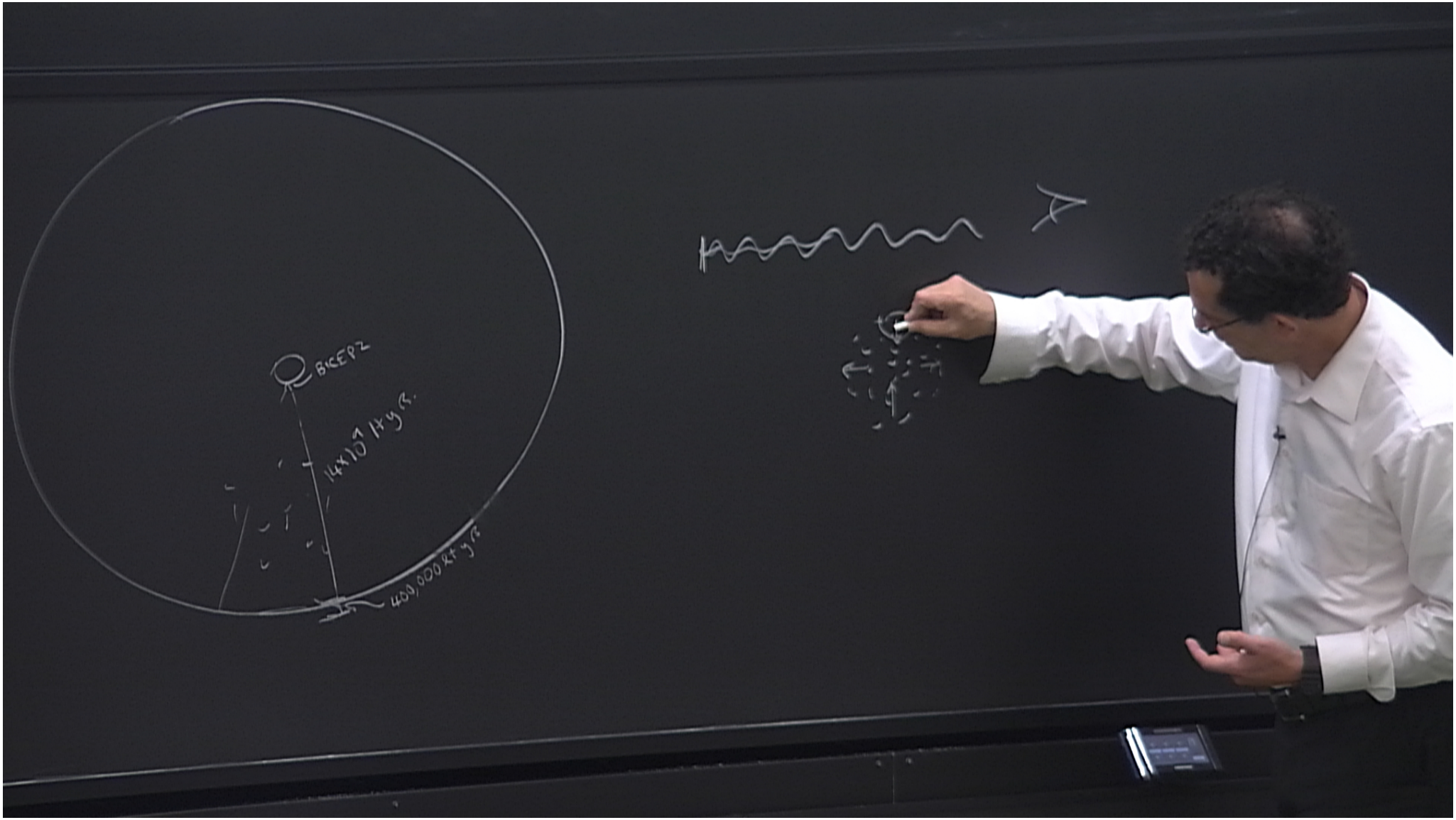


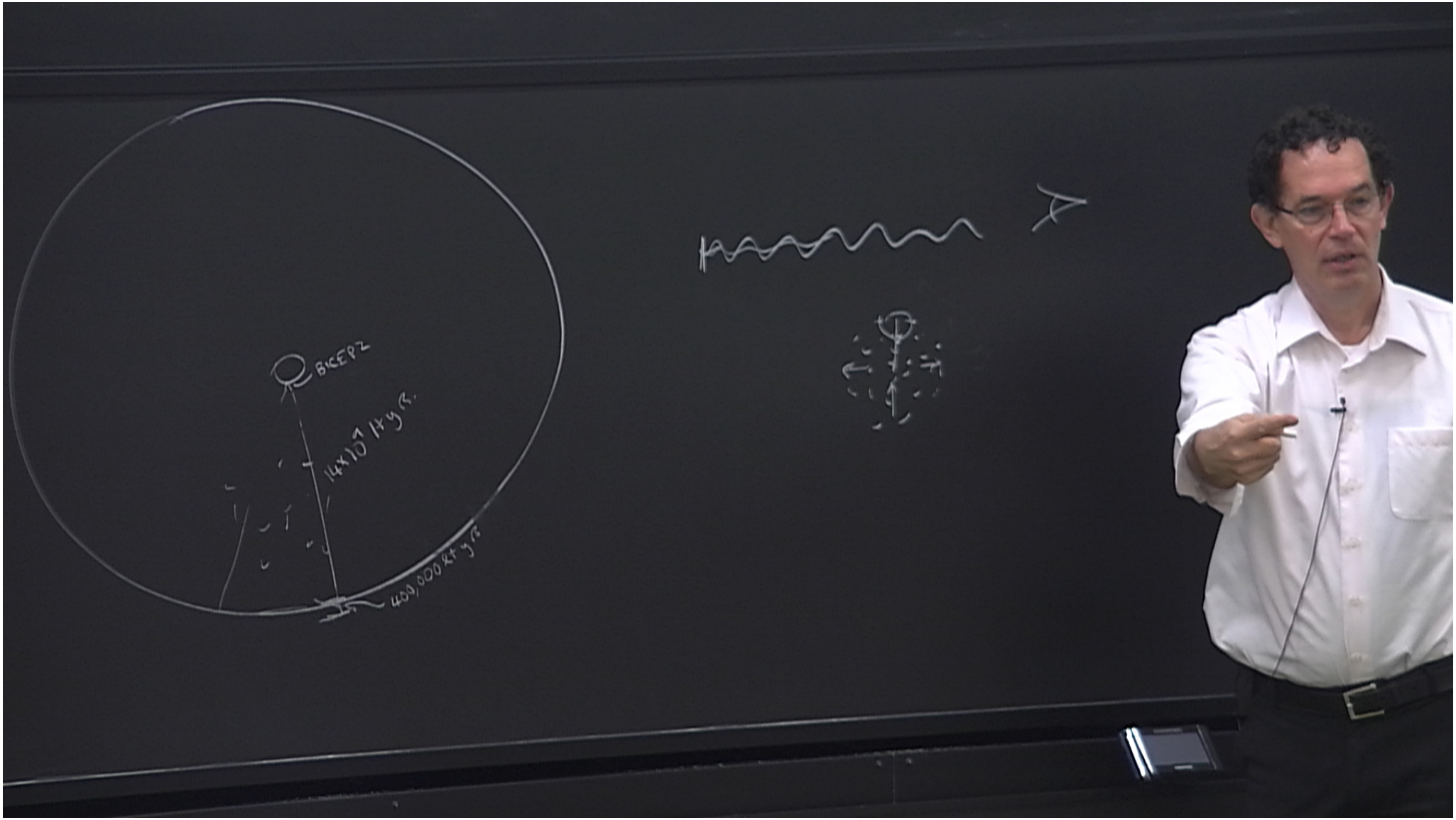

BICEPS

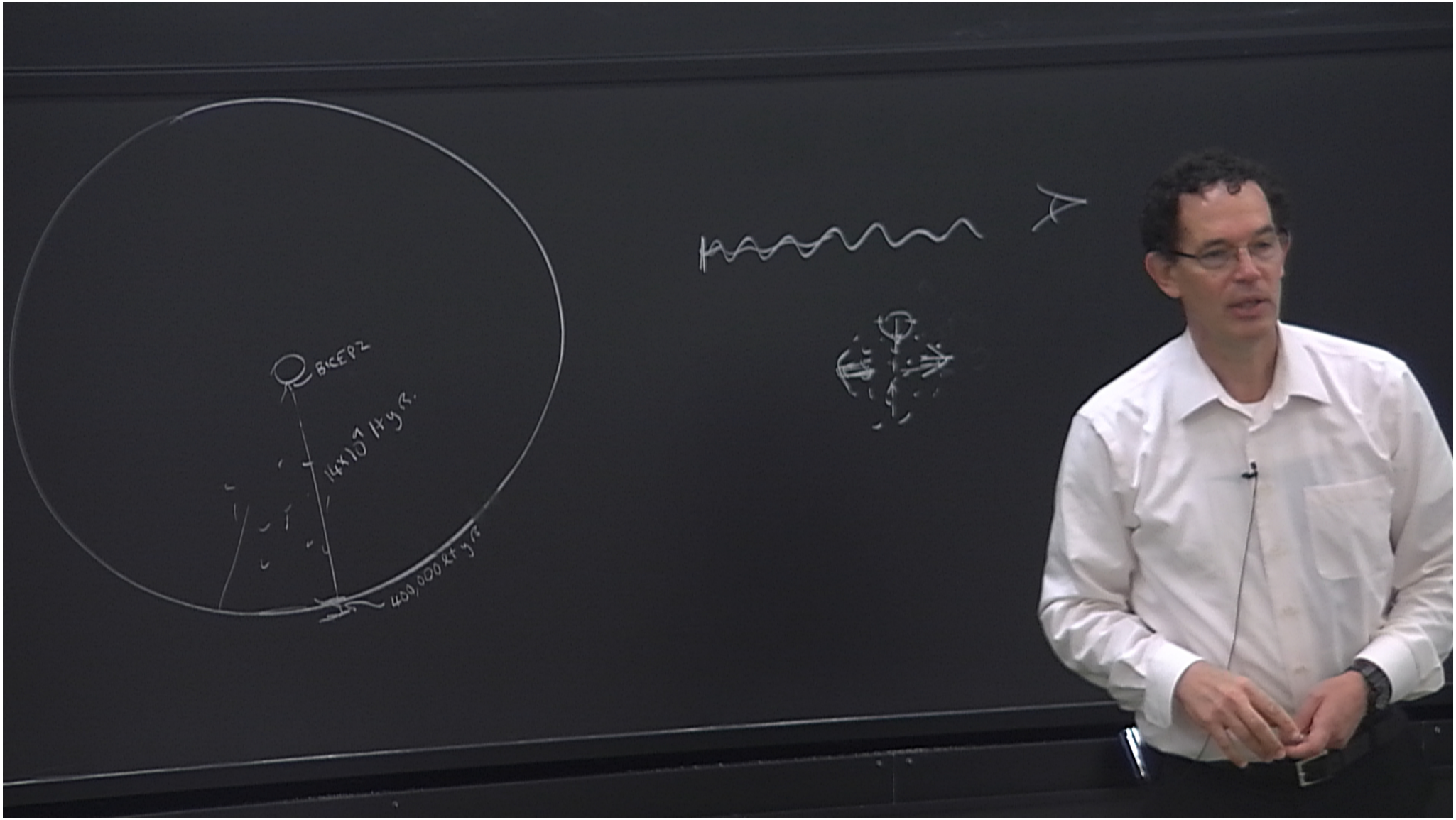

BICEPS
14x10 ft yr.
450,000 ft yr

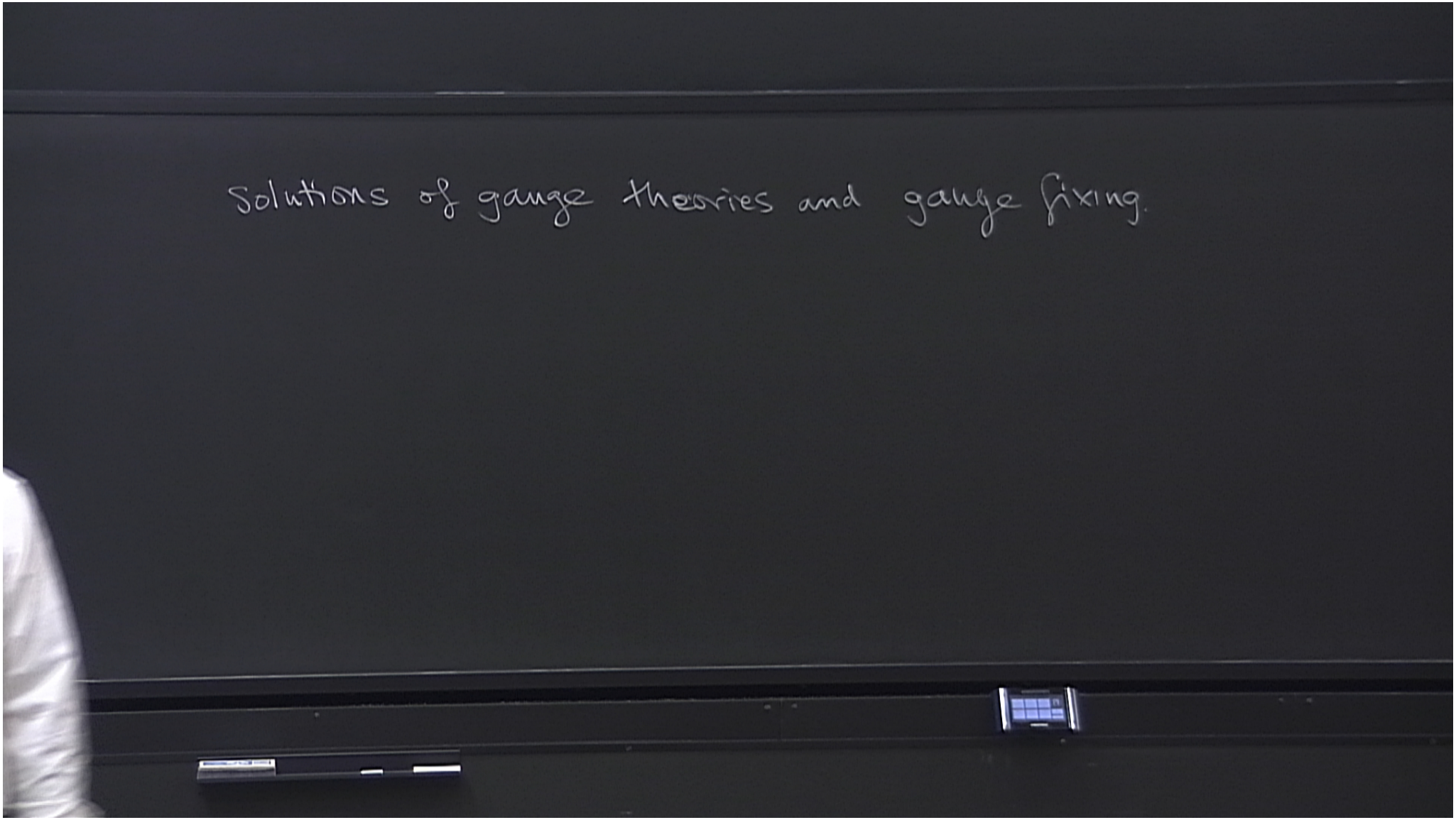

BICEPS
14x10^7 Hyg.
450,000 Hyg


BICEPS
14x10 ft yr.
400,000 ft yr


BICEPS
14x10^7 light yrs.
400,000 light yrs

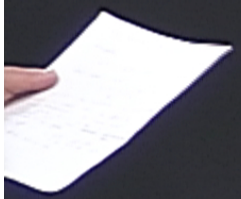

BICEPS
14x10 ly yr.
450,000 ly yr

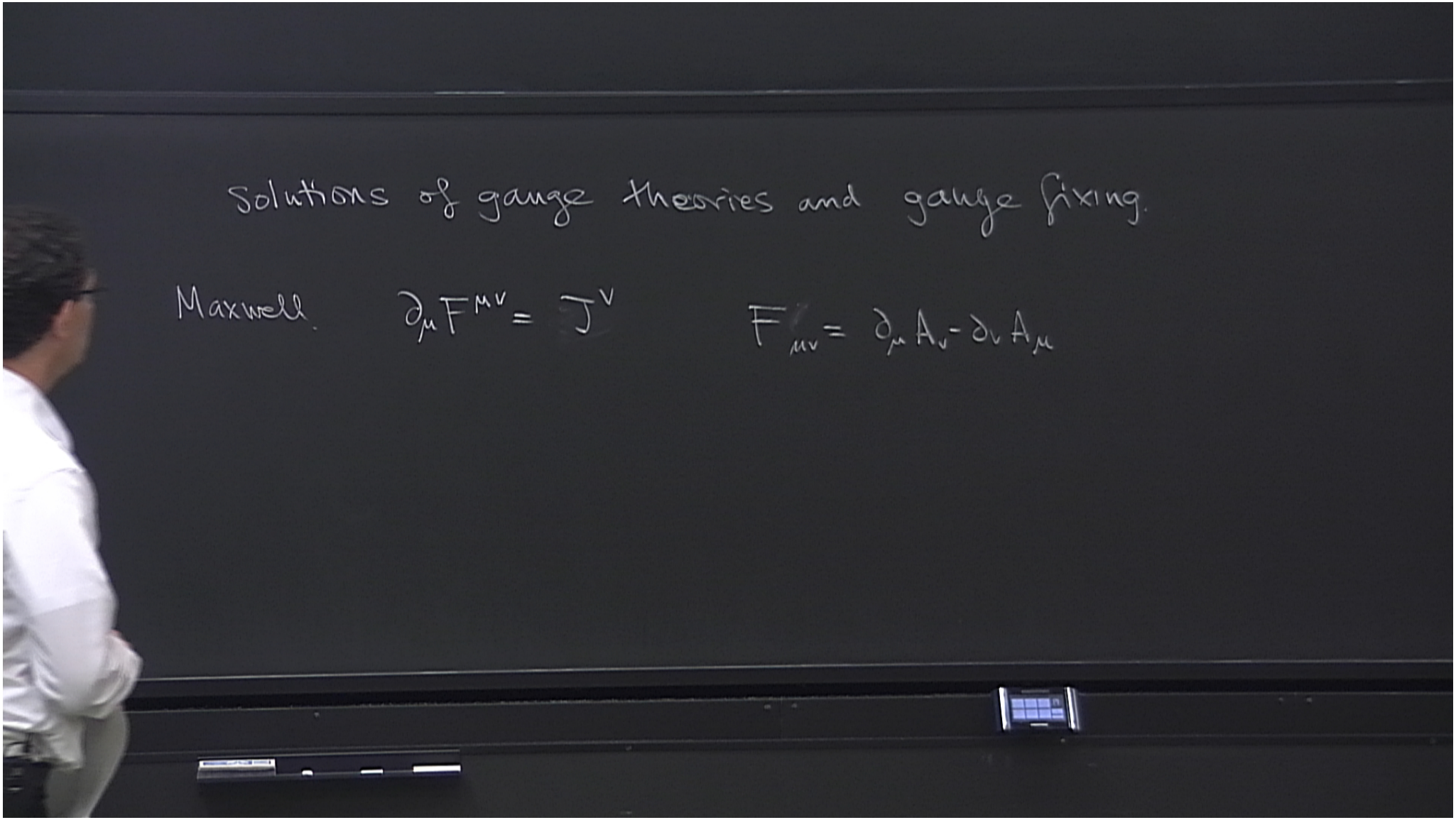

BICEPS
14x10 ly yr.
459,000 ly yr

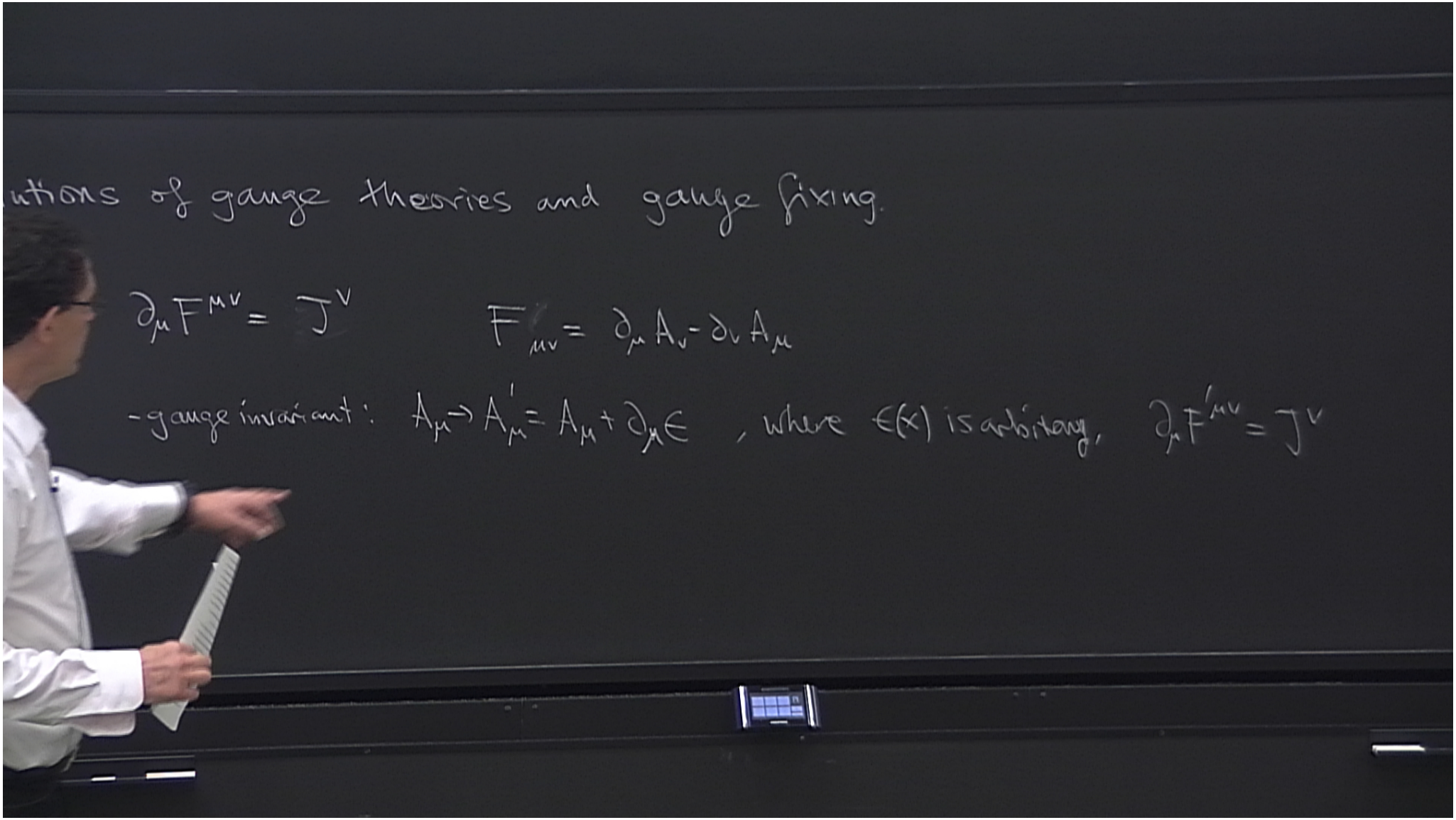

Solutions of gauge theories and gauge fixing.

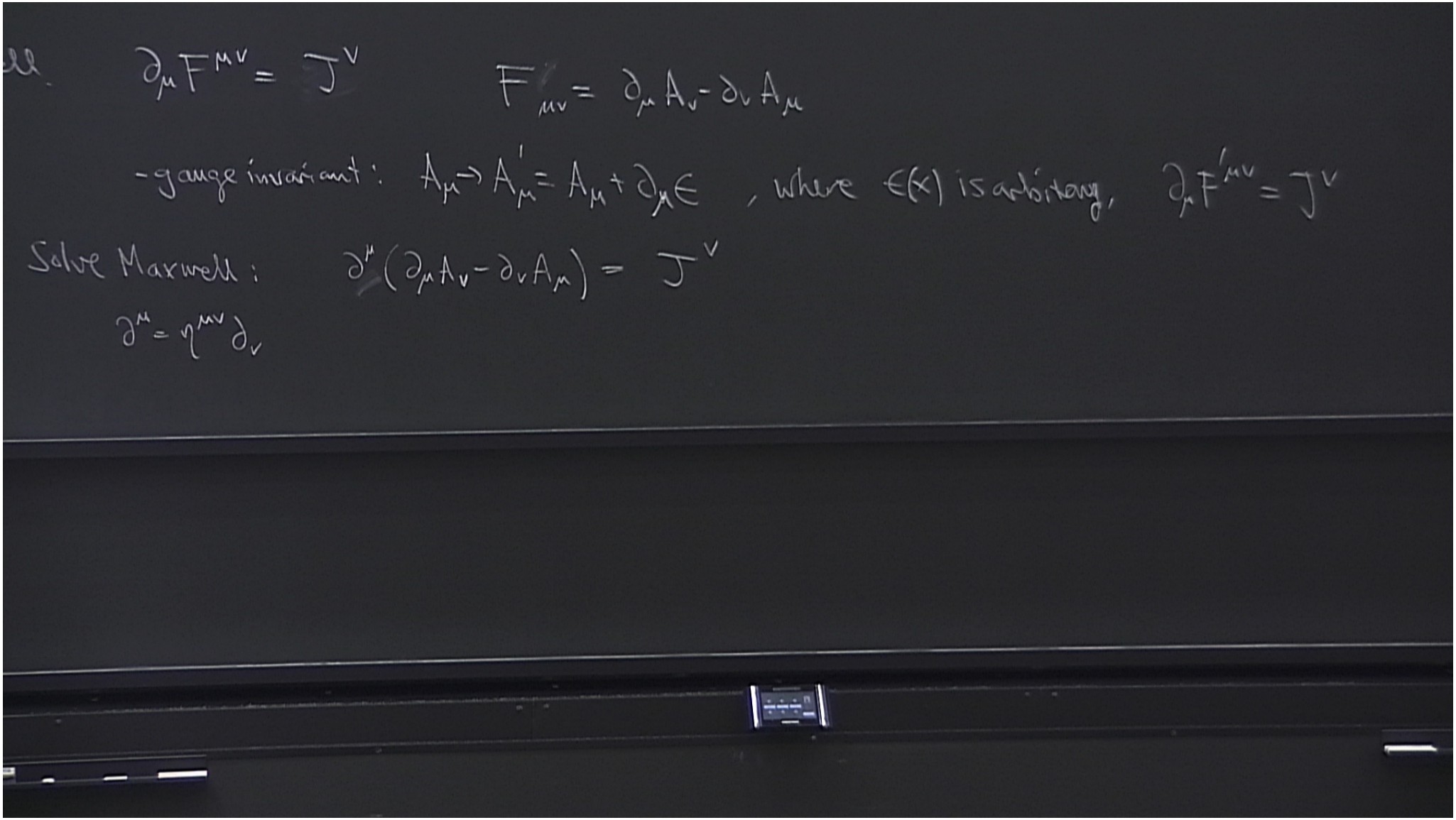
Solutions of gauge theories and gauge fixing.

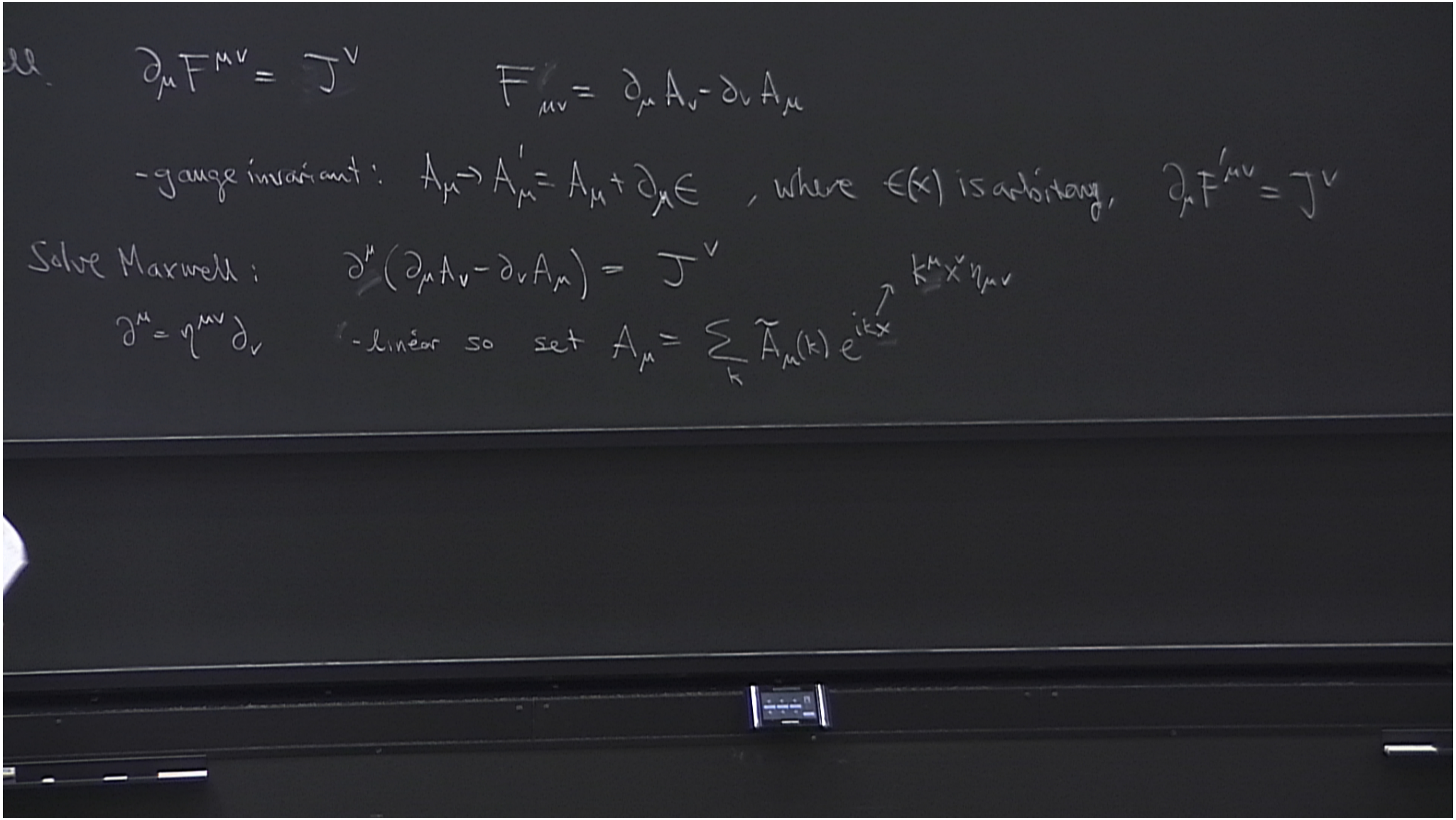
Maxwell $\partial_\mu F^{\mu\nu} = J^\nu$

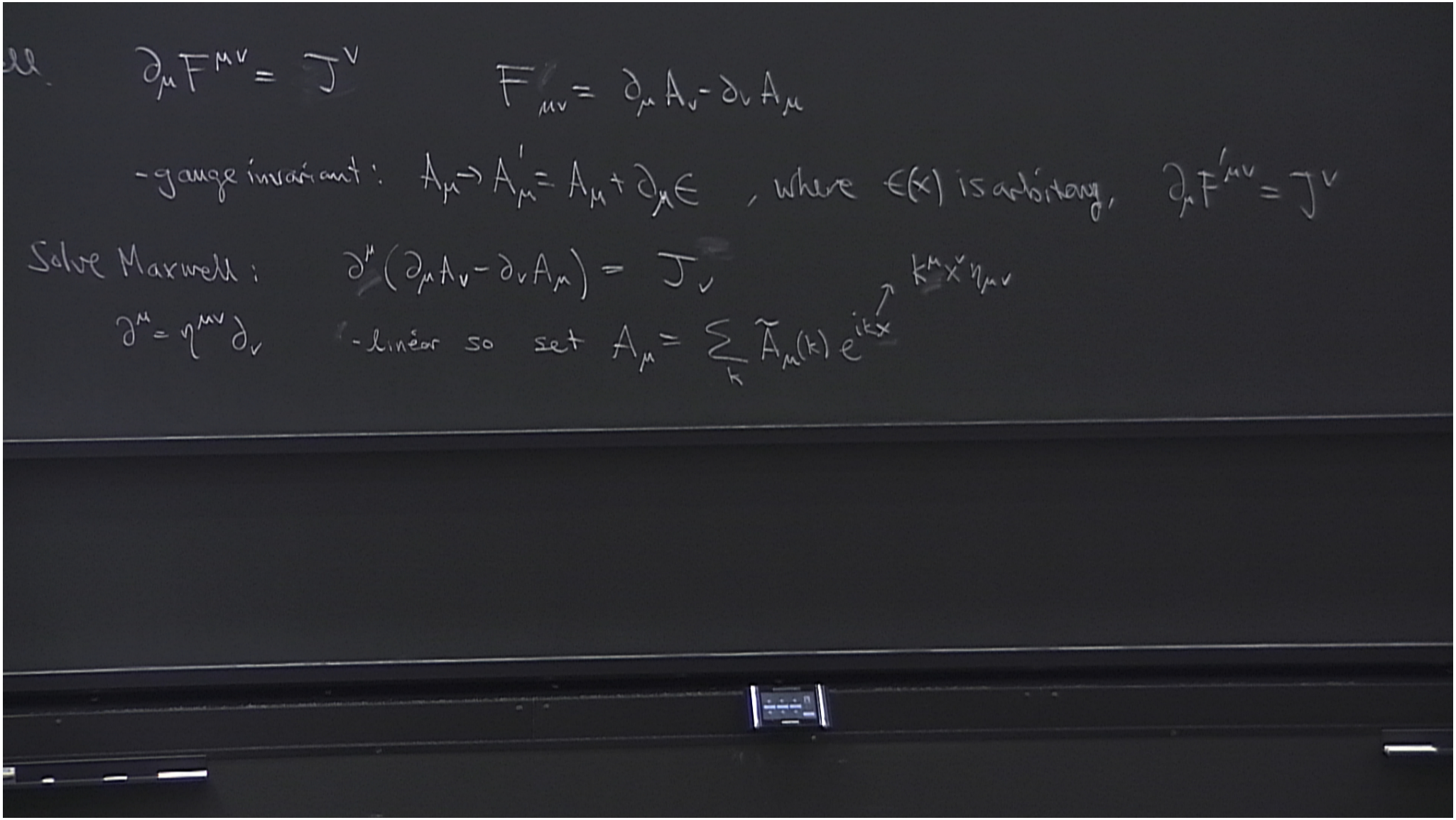



Solutions of gauge theories and gauge fixing.
Maxwell
$\partial_\mu F^{\mu\nu} = J^\nu$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$


utions of gauge theories and gauge fixing.
$\partial_\mu F^{\mu\nu} = J^\nu$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
- gauge invariant: $A_\mu \to A'_\mu = A_\mu + \partial_\mu \epsilon$ , where $\epsilon(x)$ is arbitrary, $\partial_\mu F'^{\mu\nu} = J^\nu$


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- gauge invariant: $A_\mu \to A'_\mu = A_\mu + \partial_\mu \epsilon$, where $\epsilon(x)$ is arbitary, $\partial_\mu F'^{\mu\nu} = J^\nu$
Solve Maxwell: $\partial^\mu(\partial_\mu A_\nu - \partial_\nu A_\mu) = J^\nu$
$\partial^\mu = \eta^{\mu\nu}\partial_\nu$


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$k^\mu x^\nu \eta_{\mu\nu}$


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$\partial^\mu = \eta^{\mu\nu}\partial_\nu$ - linear so set $A_\mu = \sum_k \tilde{A}_\mu(k) e^{ikx}$, $k^\mu x^\nu \eta_{\mu\nu}$, sim' for $J_\nu$

$(-k$



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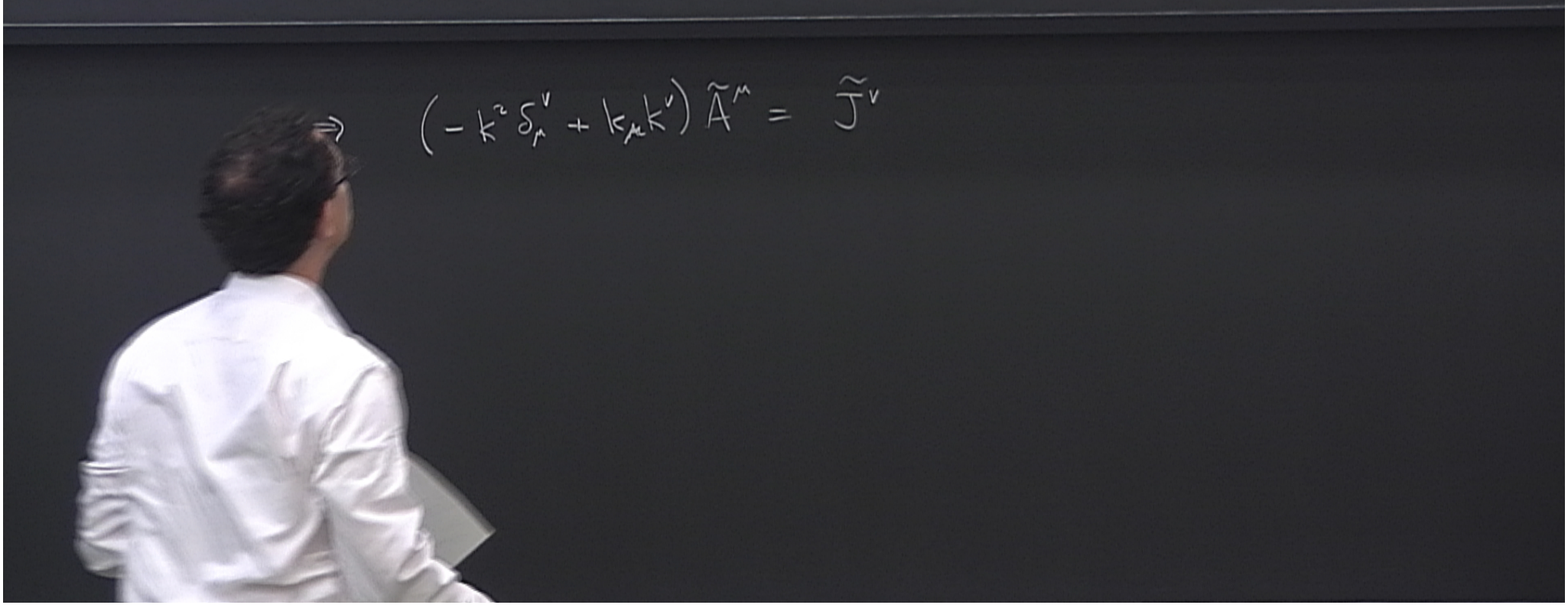
$$\Rightarrow \quad \left(-k^2 \delta^\nu_\mu + k_\mu k^\nu\right) \tilde{A}^\mu = \tilde{J}^\nu$$

- gauge invariant: $A_\mu \to A_\mu = A_\mu + \partial_\mu \epsilon$, where $\epsilon(x)$ is arbitrary. $\partial_\mu F^{\mu\nu} = J^\nu$

Solve Maxwell: $\partial^\mu (\partial_\mu A^\nu - \partial^\nu A_\mu) = J^\nu$

$\partial^\mu = \eta^{\mu\nu}\partial_\nu$ - linear so set $A_\mu = \sum_k \tilde{A}_\mu(k) e^{ikx}$, $k^\mu x^\nu \eta_{\mu\nu}$, sim' for $J_\nu$

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$$\Rightarrow \underbrace{\left(-k^2 \delta^\nu_\mu + k_\mu k^\nu\right)}_{M^\nu_\mu} \tilde{A}^\mu = \tilde{J}^\nu$$

$$\Rightarrow \tilde{A}^\mu = \left(M^{-1}\right)^\mu_\nu \tilde{J}^\nu$$

- gauge invariant: $A_\mu \to A_\mu = A_\mu + \partial_\mu \epsilon$, where $\epsilon(x)$ is arbitrary. $\partial_\mu F^{\mu\nu} = J^\nu$

Solve Maxwell: $\partial^\mu (\partial_\mu A^\nu - \partial^\nu A_\mu) = J^\nu$

$\partial^\mu = \eta^{\mu\nu}\partial_\nu$ - linear so set $A_\mu = \sum_k \tilde{A}_\mu(k) e^{ikx}$, $k^\mu x^\nu \eta_{\mu\nu}$, sim' for $J_\nu$

$$\Rightarrow \underbrace{\left(-k^2 \delta^\nu_\mu + k_\mu k^\nu\right)}_{M^\nu_\mu} \tilde{A}^\mu = \tilde{J}^\nu \Rightarrow \tilde{A}^\mu = (\Lambda^{-1})^\mu_\nu \tilde{J}^\nu$$

$$M^\nu_\mu k^\mu = 0$$

- gauge invariant: $A_\mu \to A_\mu = A_\mu + \partial_\mu \epsilon$, where $\epsilon(x)$ is arbitrary. $\partial_\mu F^{\mu\nu} = J^\nu$

Solve Maxwell: $\partial^\mu (\partial_\mu A^\nu - \partial^\nu A_\mu) = J^\nu$

$\partial^\mu = \eta^{\mu\nu}\partial_\nu$ - linear so set $A_\mu = \sum_k \tilde{A}_\mu(k) e^{ikx}$, $k^\mu x^\nu \eta_{\mu\nu}$, sim$^y$ for $J_\nu$

$$\Rightarrow \underbrace{\left(-k^2 \delta^\nu_\mu + k_\mu k^\nu\right)}_{M^\nu_\mu} \tilde{A}^\mu = \tilde{J}^\nu$$

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$M^\nu_\mu k^\mu = 0$ : $k^\mu$ is an eigenvector with eigenvalue zero.

- gauge invariant: $A_\mu \to A_\mu = A_\mu + \partial_\mu \epsilon$, where $\epsilon(x)$ is arbitrary. $\partial_\mu F^{\mu\nu} = J^\nu$

Solve Maxwell: $\partial^\mu (\partial_\mu A^\nu - \partial^\nu A_\mu) = J^\nu$

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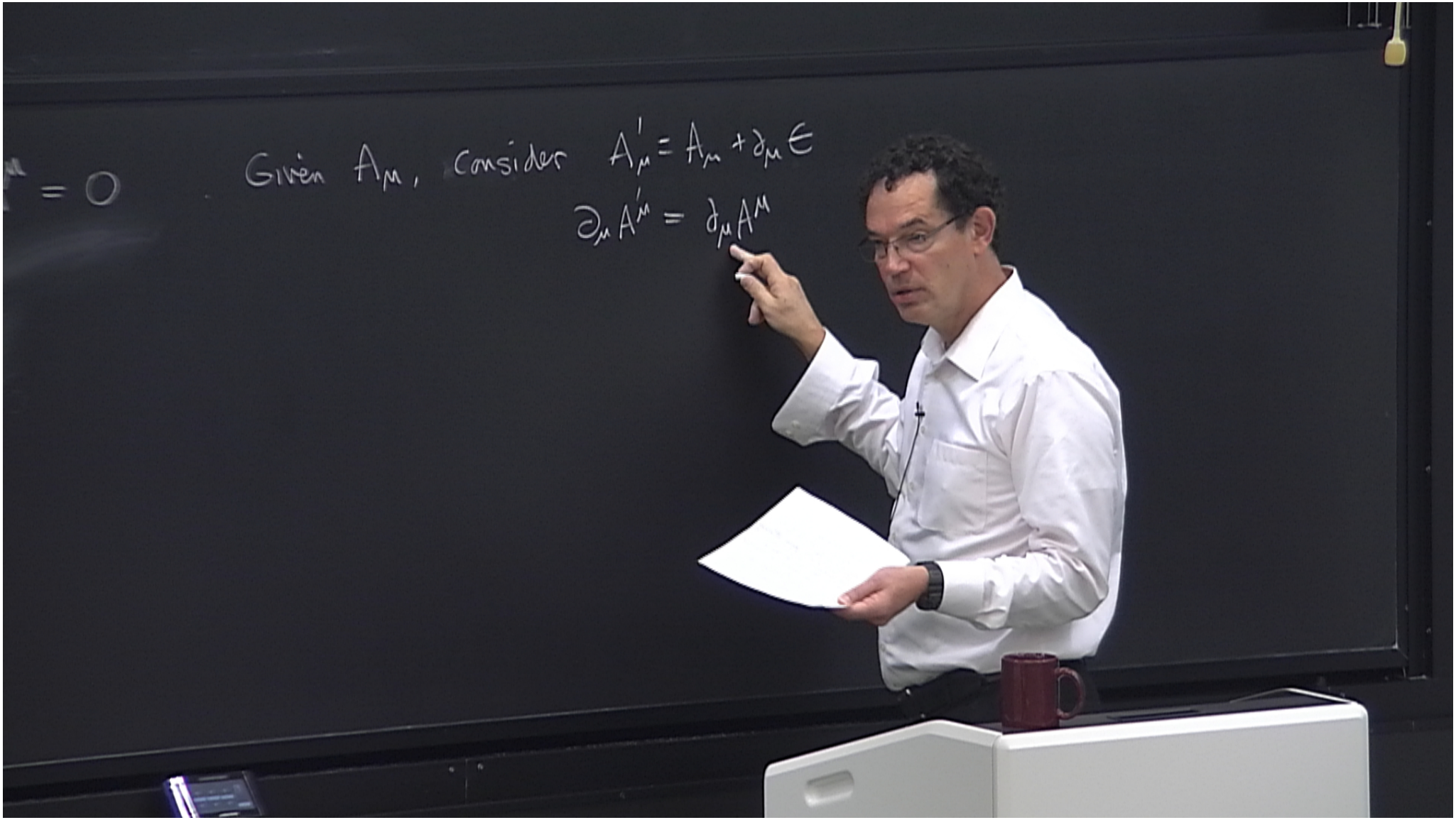
$$\Rightarrow \underbrace{\left(-k^2 \delta^\nu_\mu + k_\mu k^\nu\right)}_{M^\nu_\mu} \tilde{A}^\mu = \tilde{J}^\nu$$

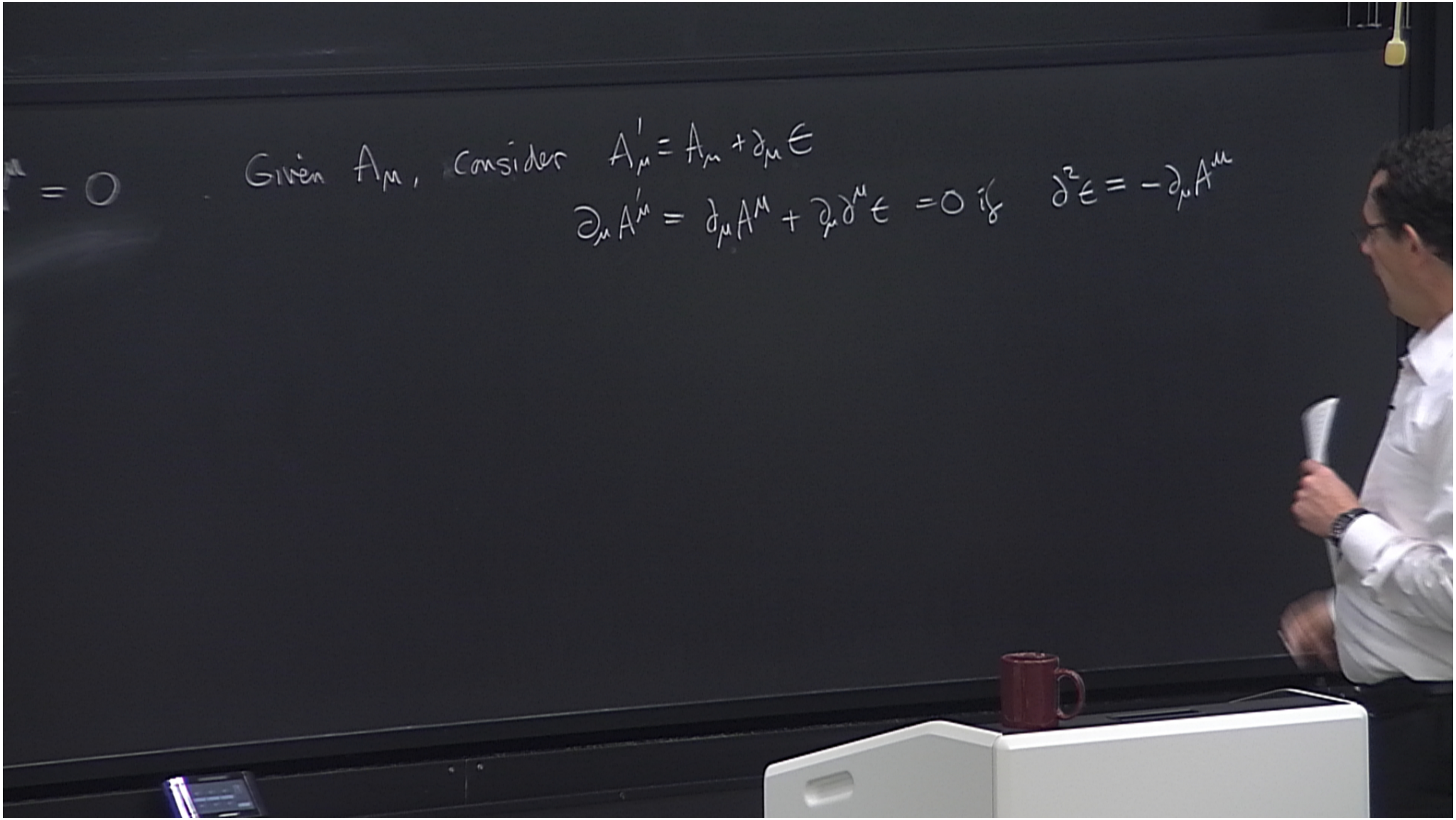
$$\Rightarrow \tilde{A}^\mu = (M^{-1})^\mu_\nu \tilde{J}^\nu$$

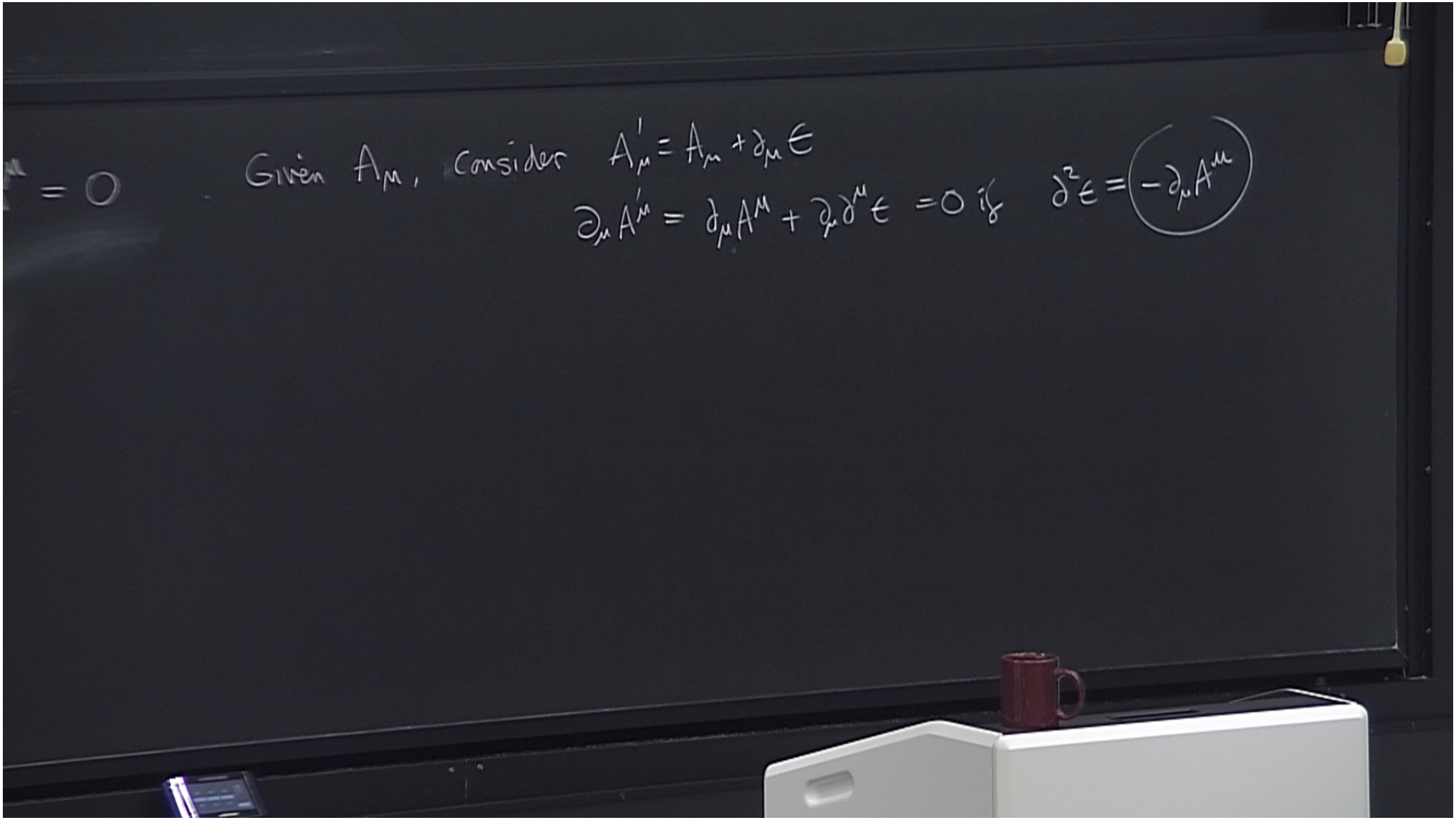
$M^\nu_\mu k^\mu = 0$ : $k^\mu$ is an eigenvector with eigenvalue zero.

$\Rightarrow M^\mu_\nu$ not invertible $\Rightarrow$ no unique sol$^{\text{n}}$ to eq$^{\text{n}}$

to find solutions, need to pick a gauge; e.g. $\partial_\mu A^\mu = 0$


$= 0$
Given $A_\mu$, consider $A'_\mu = A_\mu + \partial_\mu \epsilon$
$\partial_\mu A'^\mu = \partial_\mu A^\mu$


$= 0$
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$\partial_\mu A'^\mu = \partial_\mu A^\mu + \partial_\mu \partial^\mu \epsilon = 0$ if $\partial^2 \epsilon = -\partial_\mu A^\mu$


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$= 0$

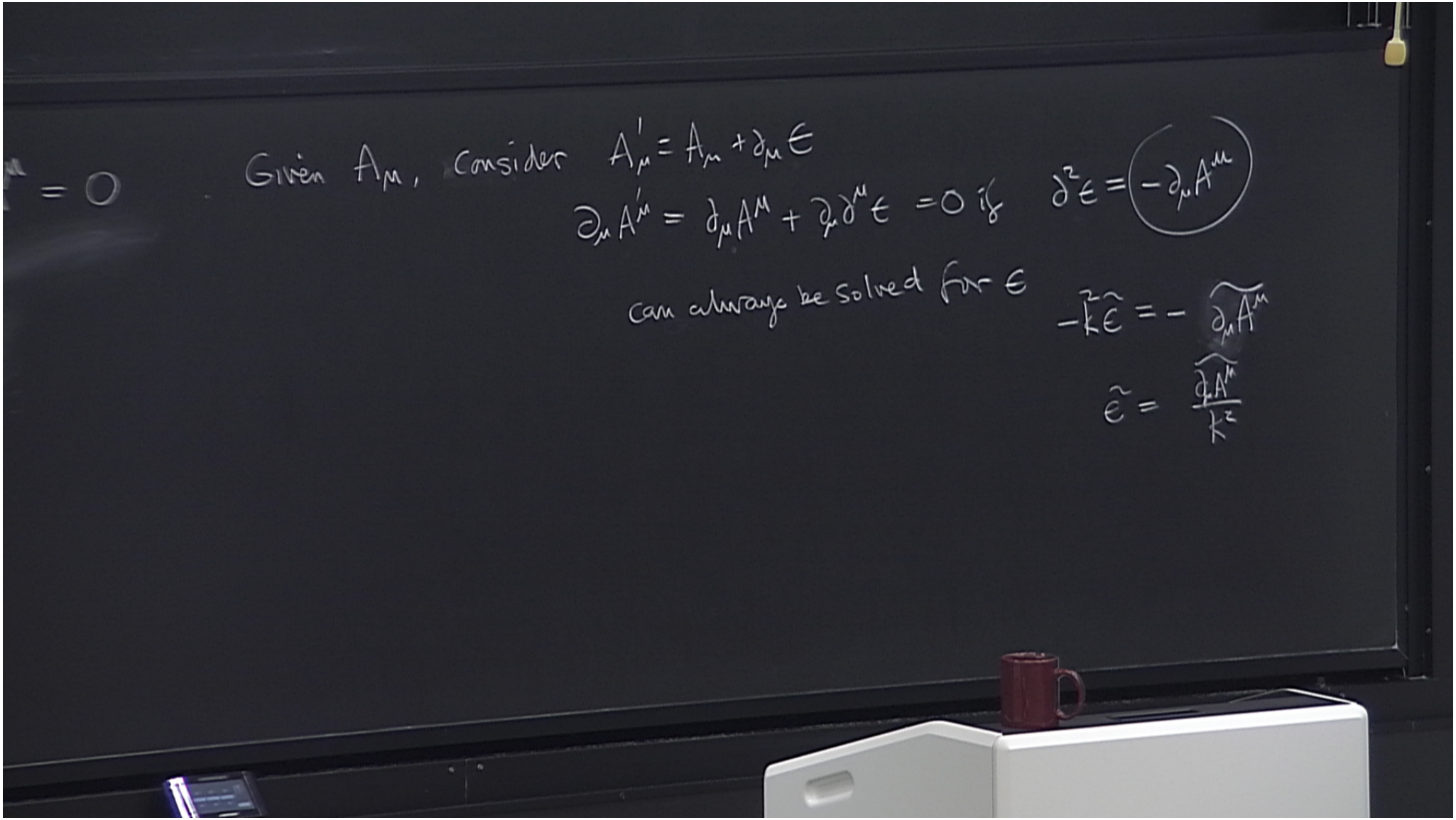
Given $A_\mu$, consider $A'_\mu = A_\mu + \partial_\mu \epsilon$

$\partial_\mu A'^\mu = \partial_\mu A^\mu + \partial_\mu \partial^\mu \epsilon = 0$ if $\partial^2 \epsilon = -\partial_\mu A^\mu$

can always be solved for $\epsilon$

$-\widetilde{k^2 \epsilon} = -\widetilde{\partial_\mu A^\mu}$

$\tilde{\epsilon} = \frac{\widetilde{\partial_\mu A^\mu}}{k^2}$



to find solutions, need to pick a gauge; e.g. $\partial_\mu A^\mu = 0$ Give
Lorentz gauge

temporal gauge

(or $A^0 = 0$ or $\underline{\nabla} \cdot \underline{A} = 0$ , etc.)

Coulomb gauge

to find solutions, need to pick a gauge; e.g. $\partial_\mu A^\mu = 0$
Lorentz gauge

temporal gauge

(or $A^0 = 0$ or $\underline{\nabla} \cdot \underline{A} = 0$ , etc.)
Coulomb gauge

$\sum_k A_\mu(k) e^{ikx}$, sim$^{ly}$ for $J_\nu$

$= \tilde{J}^\nu$

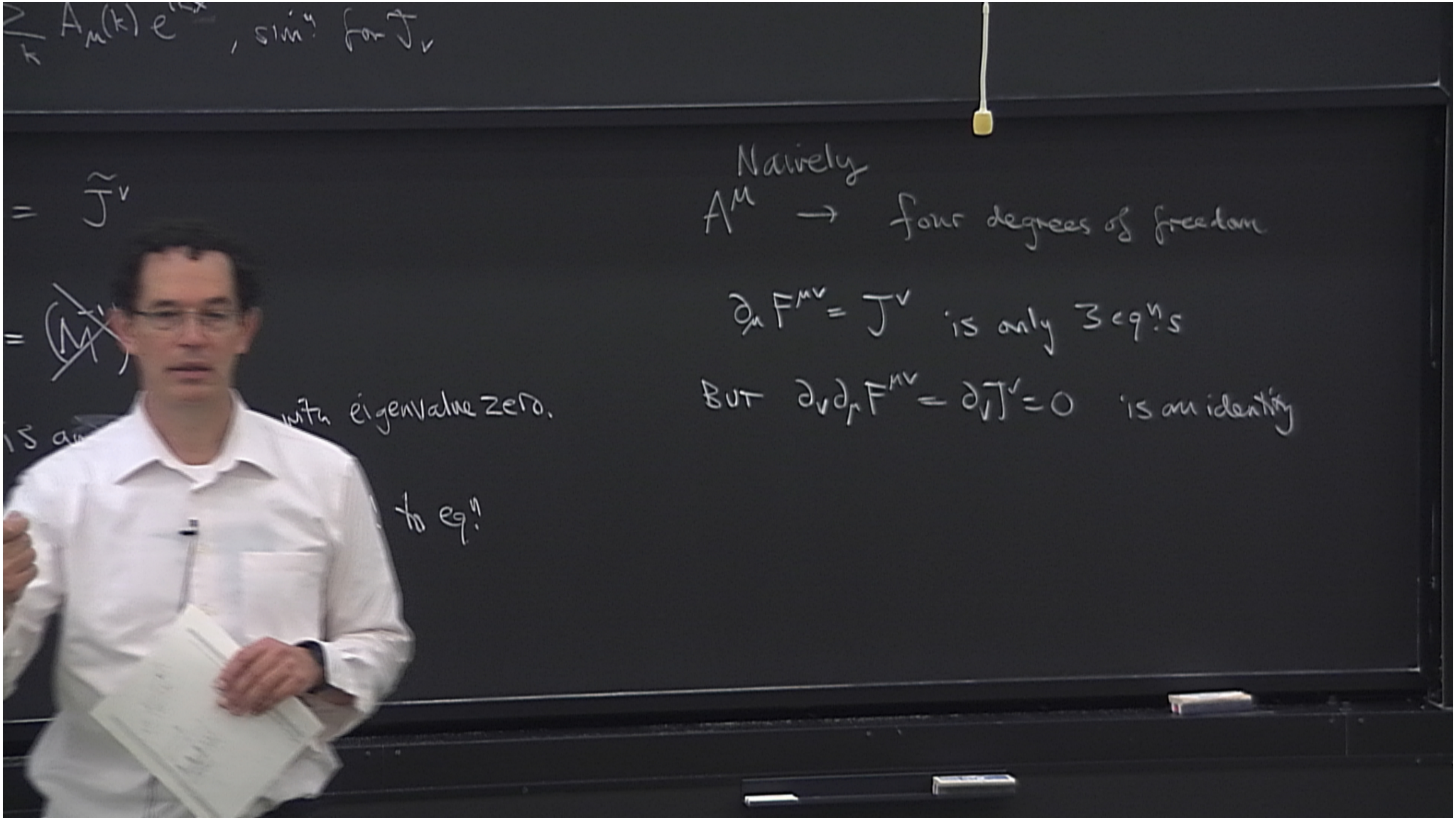
$= (\Lambda^{-1})^\mu{}_\nu \tilde{J}^\nu$

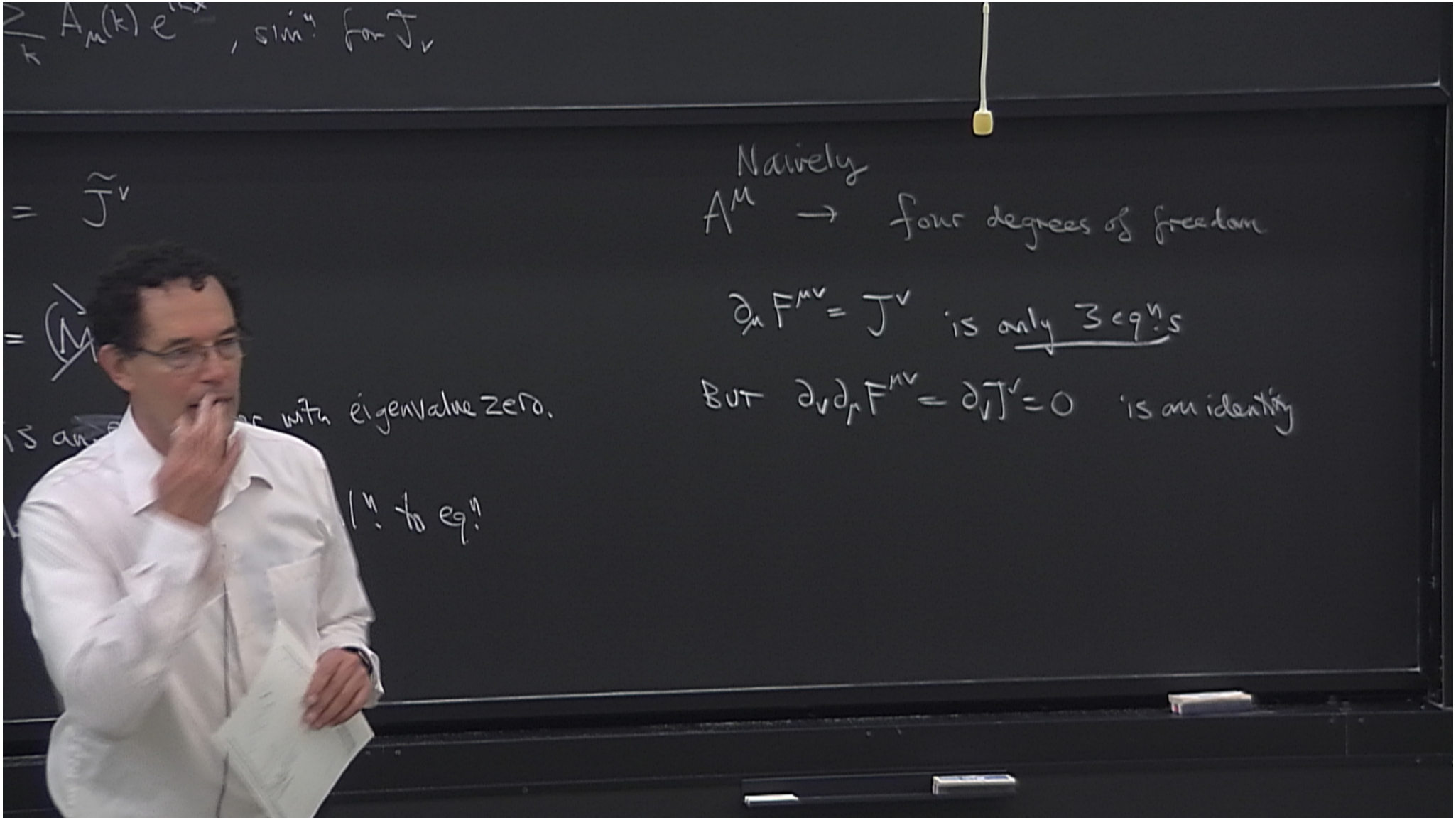
is an eigenvector with eigenvalue zero.

ble $\Rightarrow$ no unique sol$^n$ to eq$^n$

Naively

$A^\mu \rightarrow$ four degrees of freedom


$\sum_k A_\mu(k) e^{ikx}$, sum for $J_\nu$
$= \tilde{J}^\nu$
with eigenvalue zero.
to eq!
Naively
$A^\mu \rightarrow$ four degrees of freedom
$\partial_\mu F^{\mu\nu} = J^\nu$ is only 3 eq$^n$s
BUT $\partial_\nu \partial_\mu F^{\mu\nu} = \partial_\nu J^\nu = 0$ is an identity


$A_\mu(k)\, e^{ikx}$, sum for $J_\nu$
$= \tilde{J}^\nu$
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$\sum_k A_\mu(k) e^{ikx}$, sim$^y$ for $J_\nu$

$= \tilde{J}^\nu$

$= (\Lambda^{-1})^\mu{}_\nu \tilde{J}^\nu$

is an eigenvector with eigenvalue zero.

ble $\Rightarrow$ no unique sol$^n$ to eq$^n$

Naively

$A^\mu \rightarrow$ four degrees of freedom

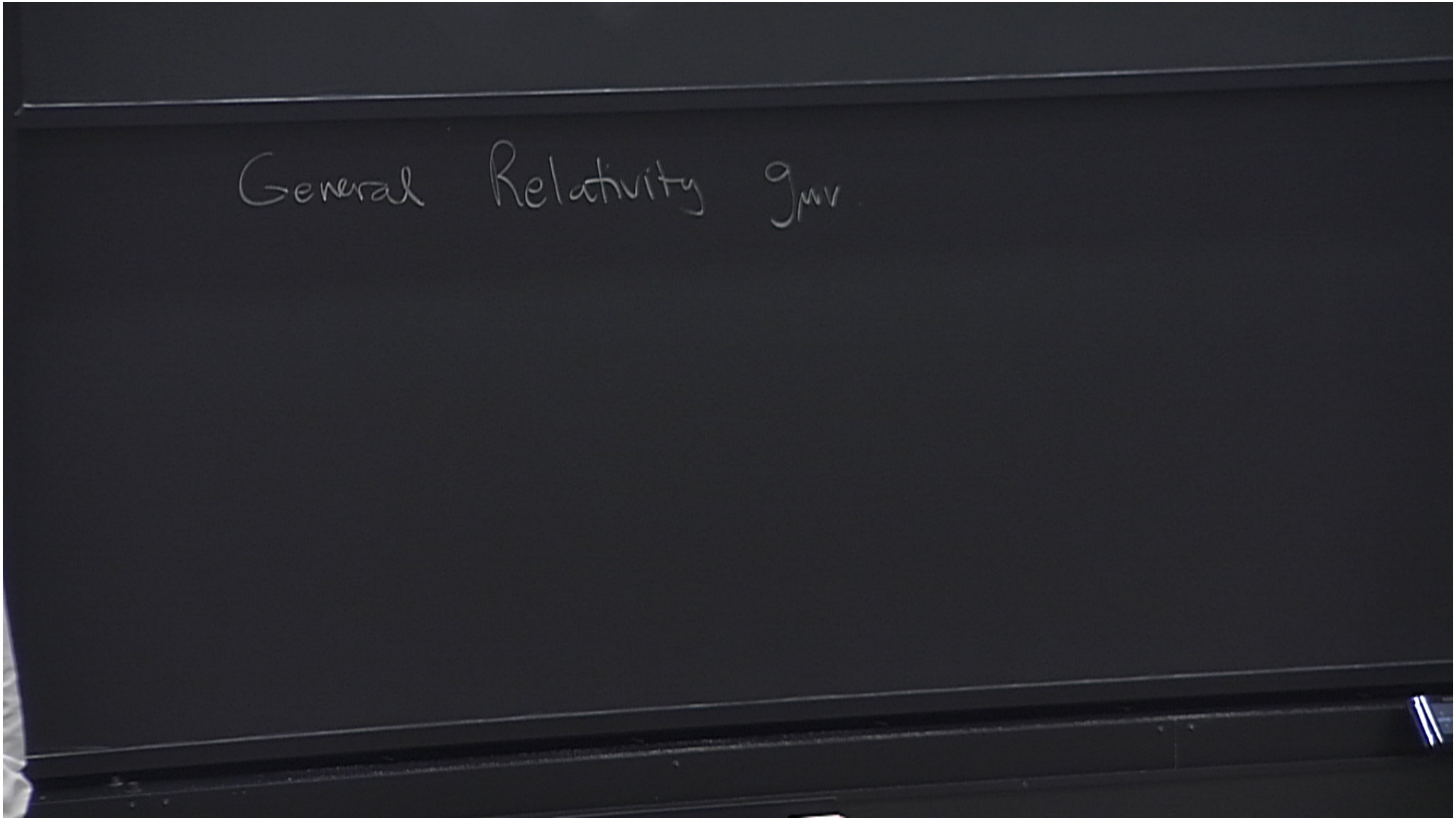
$\partial_\mu F^{\mu\nu} = J^\nu$ is only 3 eq$^n$s

BUT $\partial_\nu \partial_\mu F^{\mu\nu} = \partial_\nu J^\nu = 0$ is an identity

$\Rightarrow$ dof $= 3$

BUT $A'_\mu = A_\mu + \partial_\mu \epsilon$ is also a sol$^n$

$\Rightarrow$ dof $= 2$.


General Relativity $g_{\mu\nu}$

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$G_{\mu\nu} = 8\pi G T_{\mu\nu}$ looks like ten equations

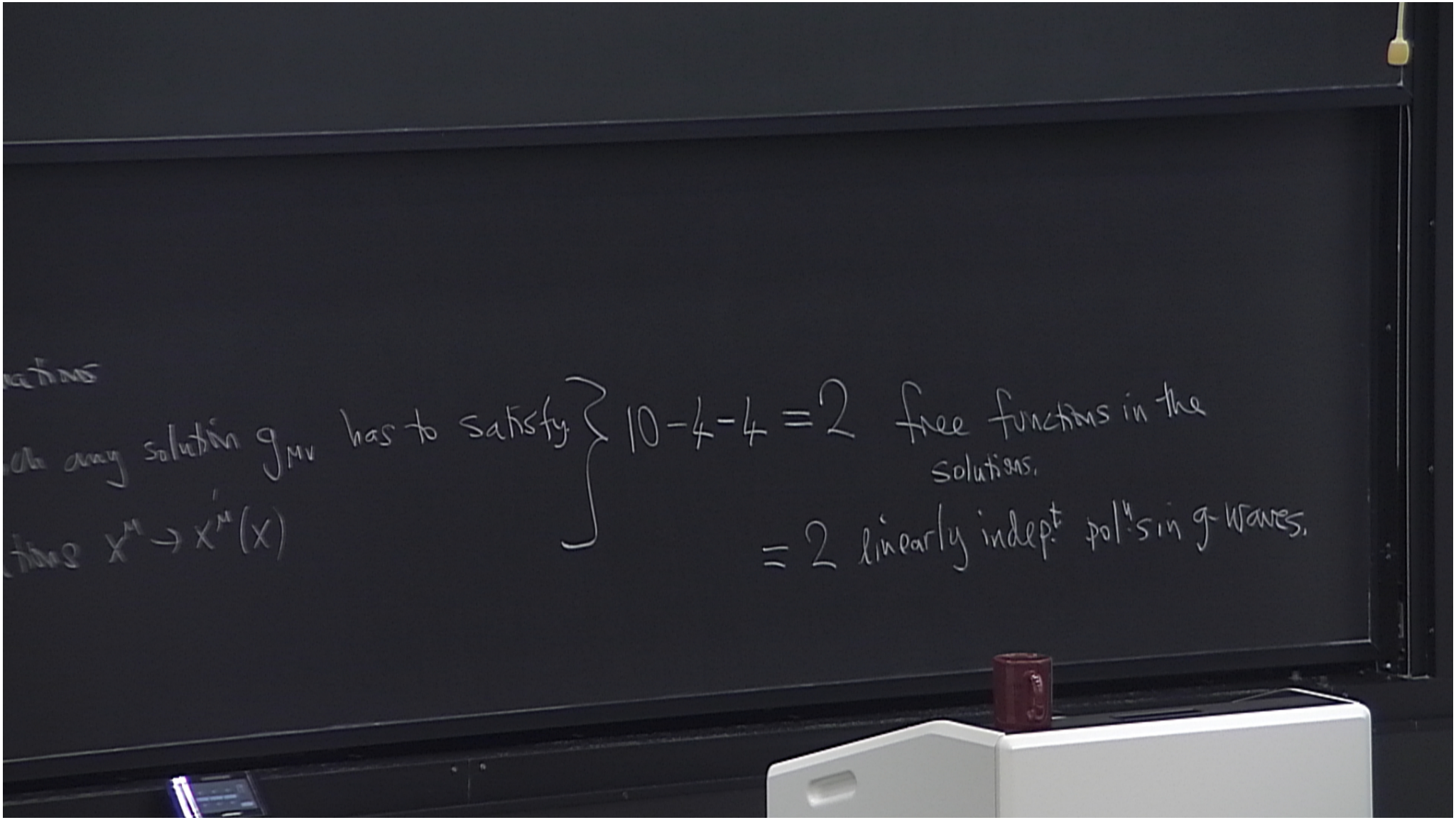
but we have Bianchi Identity $\nabla_\mu G^{\mu\nu} = 0$, which any solution $g_{\mu\nu}$ has to satisfy

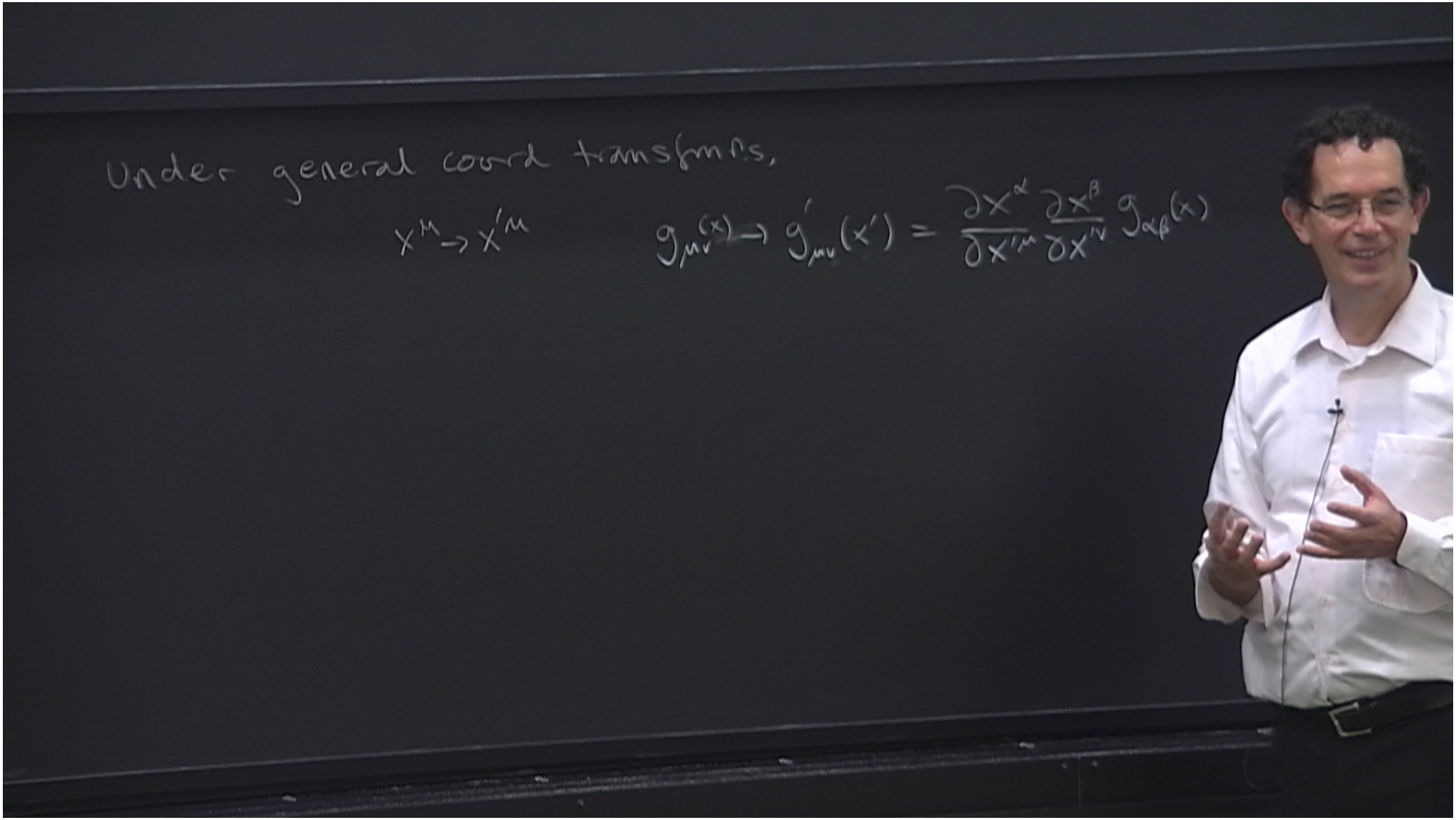
General Relativity $g_{\mu\nu}$

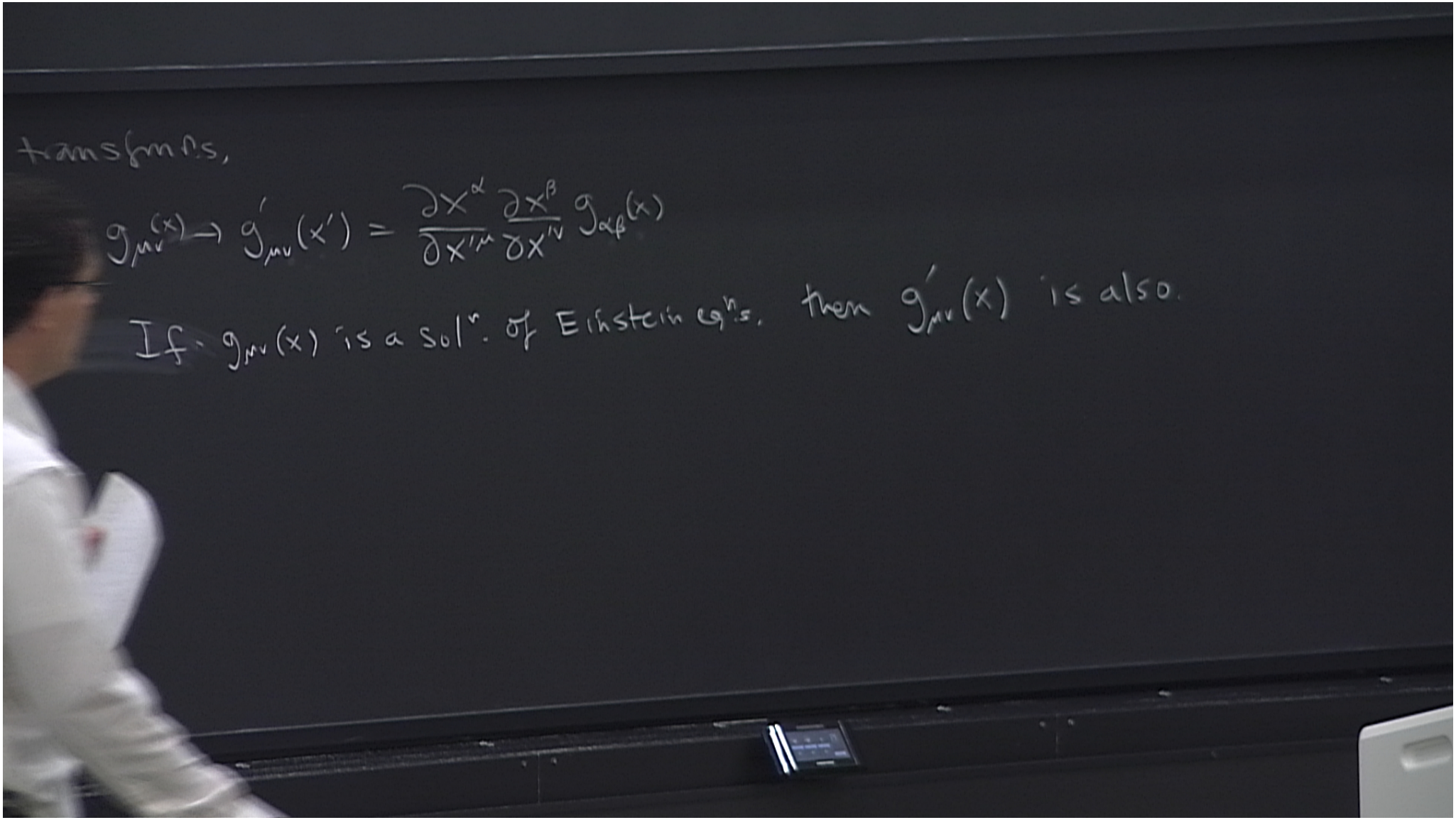
$G_{\mu\nu} = 8\pi G T_{\mu\nu}$ looks like ten equations

but we have Bianchi Identity $\nabla_\mu G^{\mu\nu} = 0$, which any solution $g_{\mu\nu}$ has to satisfy

and we have 4 general coordinate transformations $x^\mu \to x'^\mu(x)$


ations
ch any solution $g_{\mu\nu}$ has to satisfy
time $x^\mu \to x'^\mu(x)$
$10-4-4=2$ free functions in the solutions.
$=2$ linearly indep.t pol.n's in g. waves.


Under general coord transfmns,
$x^\mu \to x'^\mu$
$g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$


transfmns.
$g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$
If $g_{\mu\nu}(x)$ is a sol$^n$. of Einstein eq$^n$s. then $g'_{\mu\nu}(x)$ is also.

transfmns.

$$g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$$

If $g_{\mu\nu}(x)$ is a sol$^n$. of Einstein eq$^n$s. then $g'_{\mu\nu}(x')$ (relabel $x' \to x$) is also.

general coord transfmns.

$x^\mu \to x'^\mu$ $\qquad g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$

If $g_{\mu\nu}(x)$ is a sol$^n$. of Einstein eq$^{ns}$. then $g'_{\mu\nu}(x')$ is

relabel $x' \to x$

Infinitesimal coordinate transfmn: $|\epsilon^\mu| \ll 1$

$x'^\mu = x^\mu + \epsilon^\mu(x)$

- general coord transfmns,

$x^\mu \to x'^\mu$ $\qquad g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$

If $g_{\mu\nu}(x)$ is a sol$^n$. of Einstein eq$^{n}$s. then $g'_{\mu\nu}(x')$ (relabel $x' \to x$) is

Infinitesimal coordinate transfmn: $|\epsilon^\mu| \ll 1$

$x'^\mu = x^\mu + \epsilon^\mu(x) \Rightarrow x^\mu = x'^\mu - \epsilon^\mu(x')$

- general coord transfmns,

$x^\mu \to x'^\mu$ $\qquad g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$ (*)

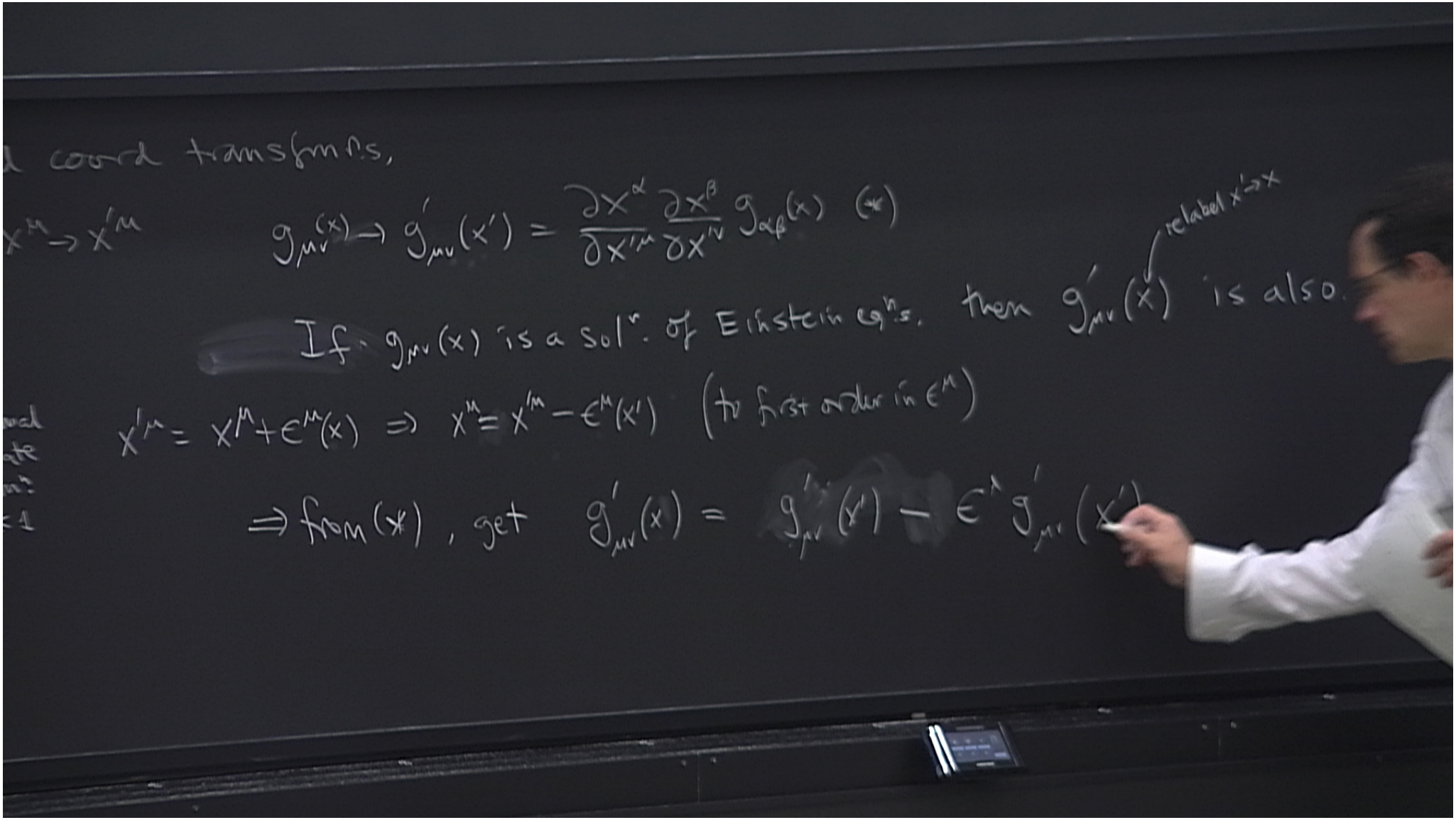
If $g_{\mu\nu}(x)$ is a sol$^n$. of Einstein eq$^{n}$s. then $g'_{\mu\nu}(x')$ is

relabel $x' \to x$

Infinitesimal coordinate transfmn: $|\epsilon^\mu| \ll 1$

$x'^\mu = x^\mu + \epsilon^\mu(x) \Rightarrow x^\mu = x'^\mu - \epsilon^\mu(x')$ (to first order in $\epsilon^\mu$)

$\Rightarrow$ from (*), get $g'_{\mu\nu}(x)$


coord transfmns.
$x^\mu \to x'^\mu$
$g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$ (*)
If $g_{\mu\nu}(x)$ is a sol$^n$. of Einstein eq$^n$s. then $g'_{\mu\nu}(x')$ is also.
relabel $x' \to x$
$x'^\mu = x^\mu + \epsilon^\mu(x) \Rightarrow x^\mu = x'^\mu - \epsilon^\mu(x')$ (to first order in $\epsilon^\mu$)
$\Rightarrow$ from (*), get $g'_{\mu\nu}(x) = g'_{\mu\nu}(x') - \epsilon^\lambda g'_{\mu\nu}(x')$

d coord transfmn s,

$x^\mu \to x'^\mu$

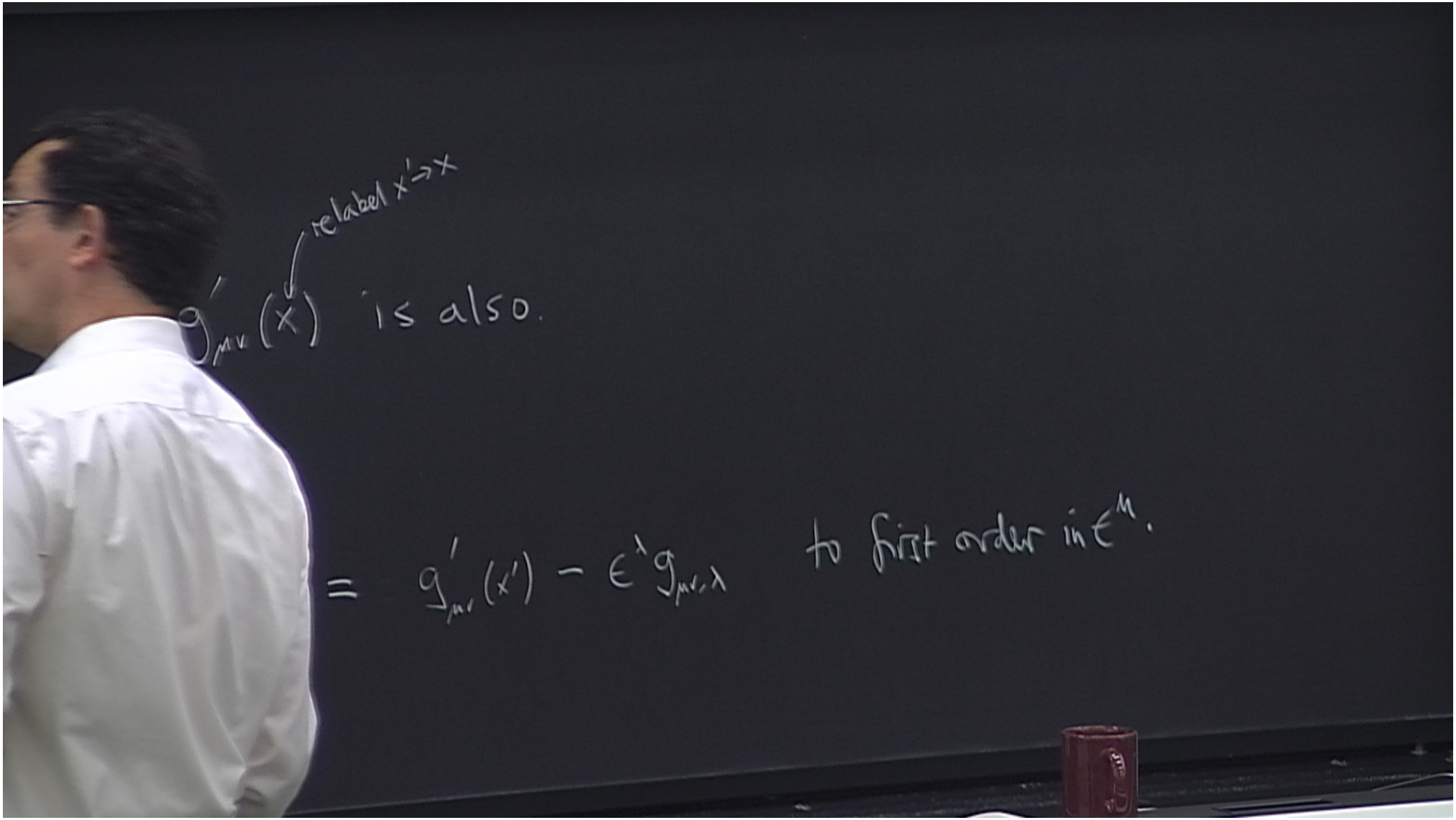
$$g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x) \quad (*)$$

If $g_{\mu\nu}(x)$ is a sol$^n$. of Einstein eq$^n$s. then $g'_{\mu\nu}(x')$ is also.

(relabel $x' \to x$)

$x'^\mu = x^\mu + \epsilon^\mu(x) \Rightarrow x^\mu = x'^\mu - \epsilon^\mu(x')$ (to first order in $\epsilon^\mu$)

$\Rightarrow$ from $(*)$, get $g'_{\mu\nu}(x) = g'_{\mu\nu}(x') - \epsilon^\lambda g'_{\mu\nu,\lambda}(x')$


relabel x'->x
$g'_{\mu\nu}(x)$ is also.
$= g'_{\mu\nu}(x') - \epsilon^\lambda g_{\mu\nu,\lambda}$ to first order in $\epsilon^\mu$.

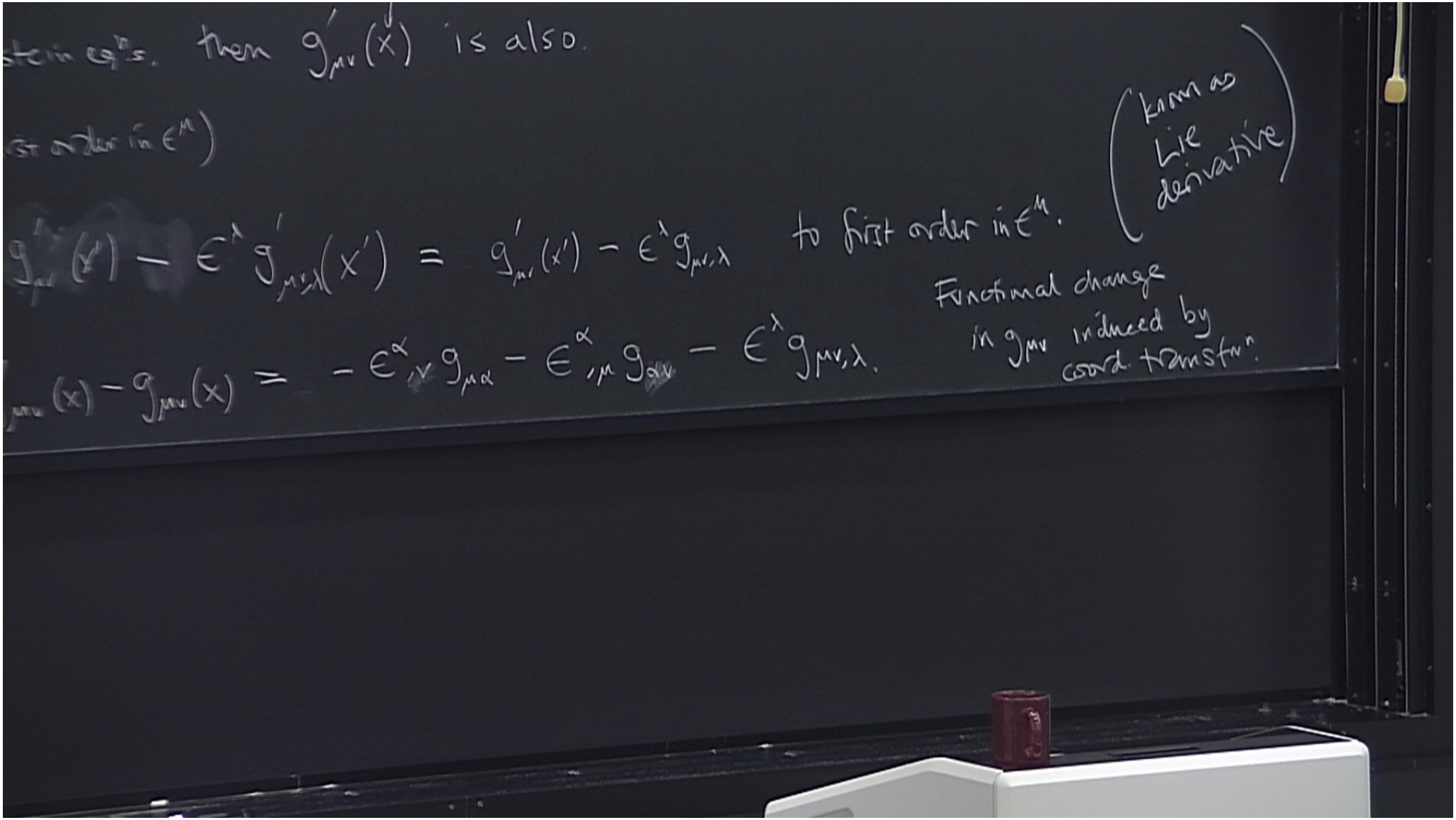
$\frac{\partial x'^\alpha}{\partial x^\mu}\frac{\partial x'^\beta}{\partial x^\nu}$

$g_{\mu\nu}(x)$ is a sol$^n$. of Einstein eq$^n$s. then $g'_{\mu\nu}(x')$ is also.

$x'^\mu = x^\mu - \epsilon^\mu(x')$ (to first order in $\epsilon^\mu$)

, get $g'_{\mu\nu}(x) = g'_{\mu\nu}(x') - \epsilon^\lambda g'_{\mu\nu,\lambda}(x') = g'_{\mu\nu}(x') - \epsilon^\lambda g_{\mu\nu,\lambda}$ to first order in $\epsilon^\mu$.

$\Rightarrow \delta g_{\mu\nu}(x) = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = -\epsilon^\alpha{}_{,\nu}\, g_{\mu\alpha} - \epsilon^\alpha{}_{,\mu}\, g_{\alpha\nu} - \epsilon^\lambda g_{\mu\nu,\lambda}$.

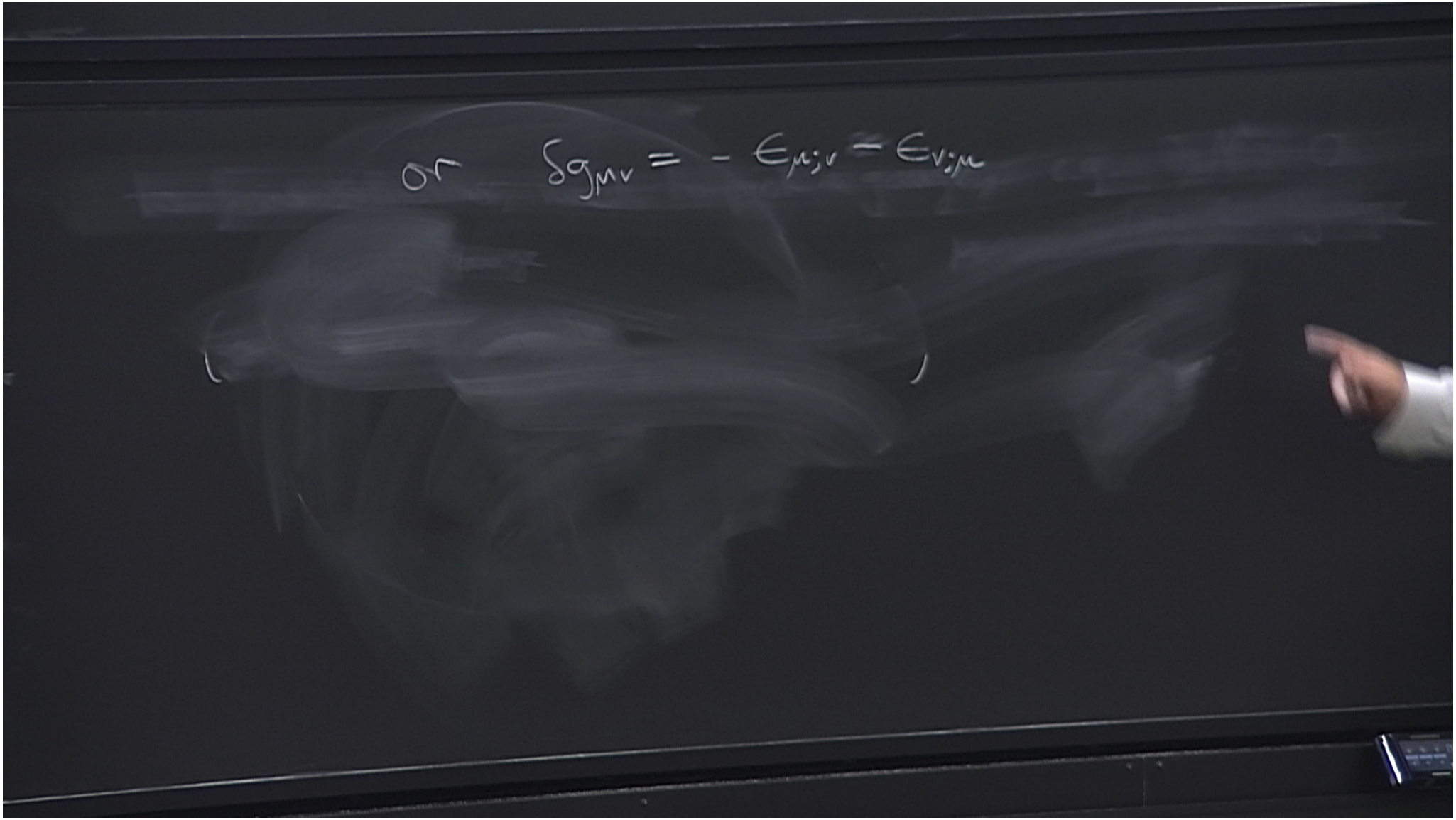

stein eq's. then $g'_{\mu\nu}(x')$ is also.
(1st order in $\epsilon^\mu$)
$g'_{\mu\nu}(x') - \epsilon^\lambda g'_{\mu\nu,\lambda}(x') = g'_{\mu\nu}(x') - \epsilon^\lambda g_{\mu\nu,\lambda}$ to first order in $\epsilon^\mu$.
(known as Lie derivative)
$g_{\mu\nu}(x) - g_{\mu\nu}(x) = -\epsilon^\alpha{}_{,\nu} g_{\mu\alpha} - \epsilon^\alpha{}_{,\mu} g_{\alpha\nu} - \epsilon^\lambda g_{\mu\nu,\lambda}$.
Functional change in $g_{\mu\nu}$ induced by coord. transfn.

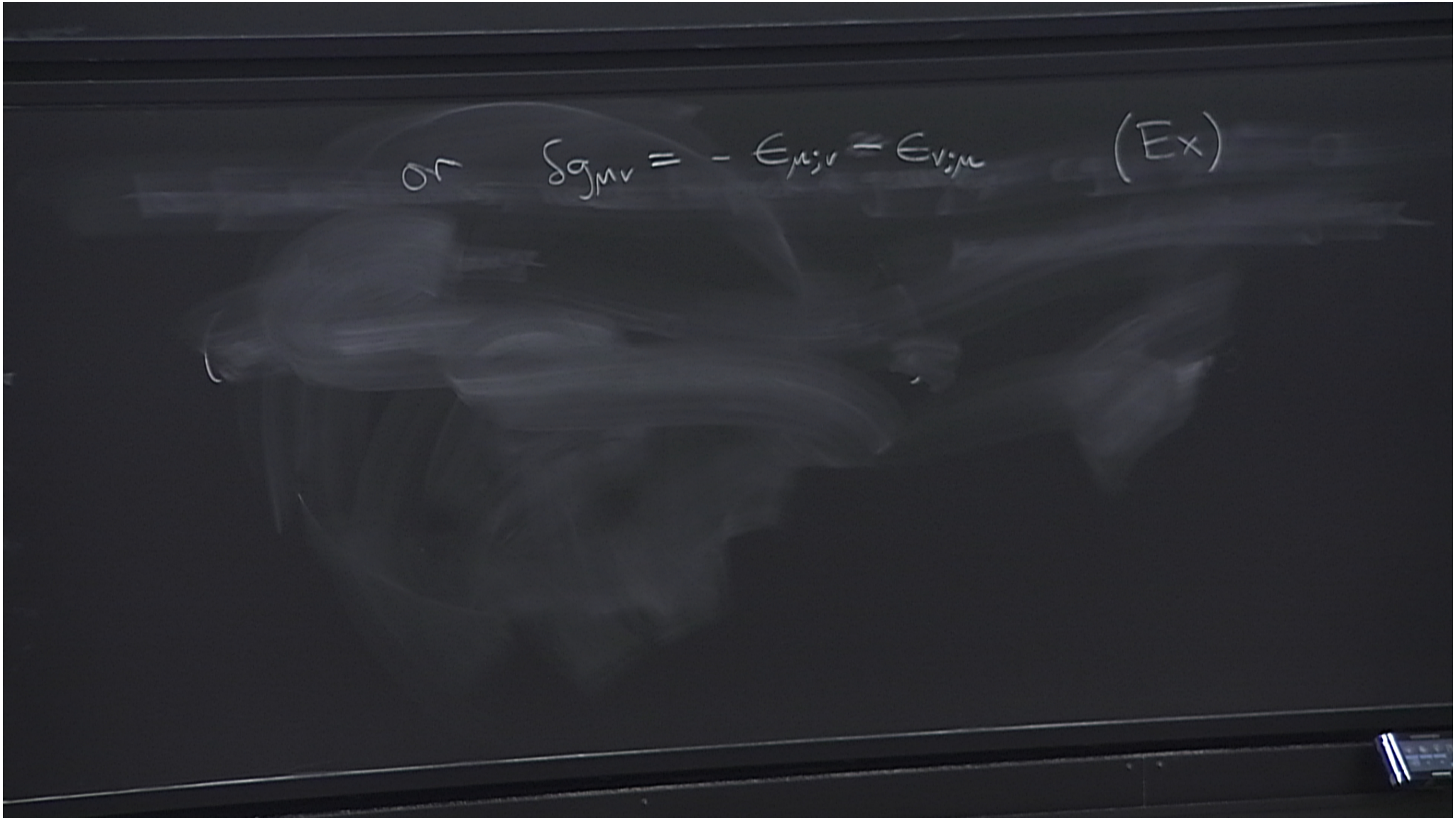
Infinitesimal coordinate transfm's $|\epsilon^\mu| \ll 1$

$x'^\mu = x^\mu + \epsilon^\mu(x) \Rightarrow x^\mu = x'^\mu - \epsilon^\mu(x')$ (to first order in $\epsilon^\mu$)

$\Rightarrow$ from (*), get $g'_{\mu\nu}(x) = g'_{\mu\nu}(x') - \epsilon$

$\frac{\partial x^\mu}{\partial x'^\nu} = \delta^\mu_\nu - \epsilon^\mu_{,\nu}$ $\Rightarrow \delta g_{\mu\nu}(x) = g'_{\mu\nu}(x) - g_{\mu\nu}(x)$


or $\delta g_{\mu\nu} = -\epsilon_{\mu;\nu} - \epsilon_{\nu;\mu}$


or $\delta g_{\mu\nu} = -\epsilon_{\mu;\nu} - \epsilon_{\nu;\mu}$ (Ex)

$$\frac{\partial x'^{\mu}}{\partial x^{\nu}} = \delta^{\mu}_{\nu} - \epsilon^{\mu}_{,\nu} \qquad \Rightarrow \quad \delta g_{\mu\nu}(x) = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = -\epsilon^{\alpha}_{,\nu} g_{\mu\alpha} - \epsilon^{\alpha}_{,\mu} g_{\alpha\nu} - \epsilon^{\lambda} g_{\mu\nu,\lambda}.$$

or $\delta g_{\mu\nu} = -\epsilon_{\mu;\nu} - \epsilon_{\nu;\mu}$ (Ex)

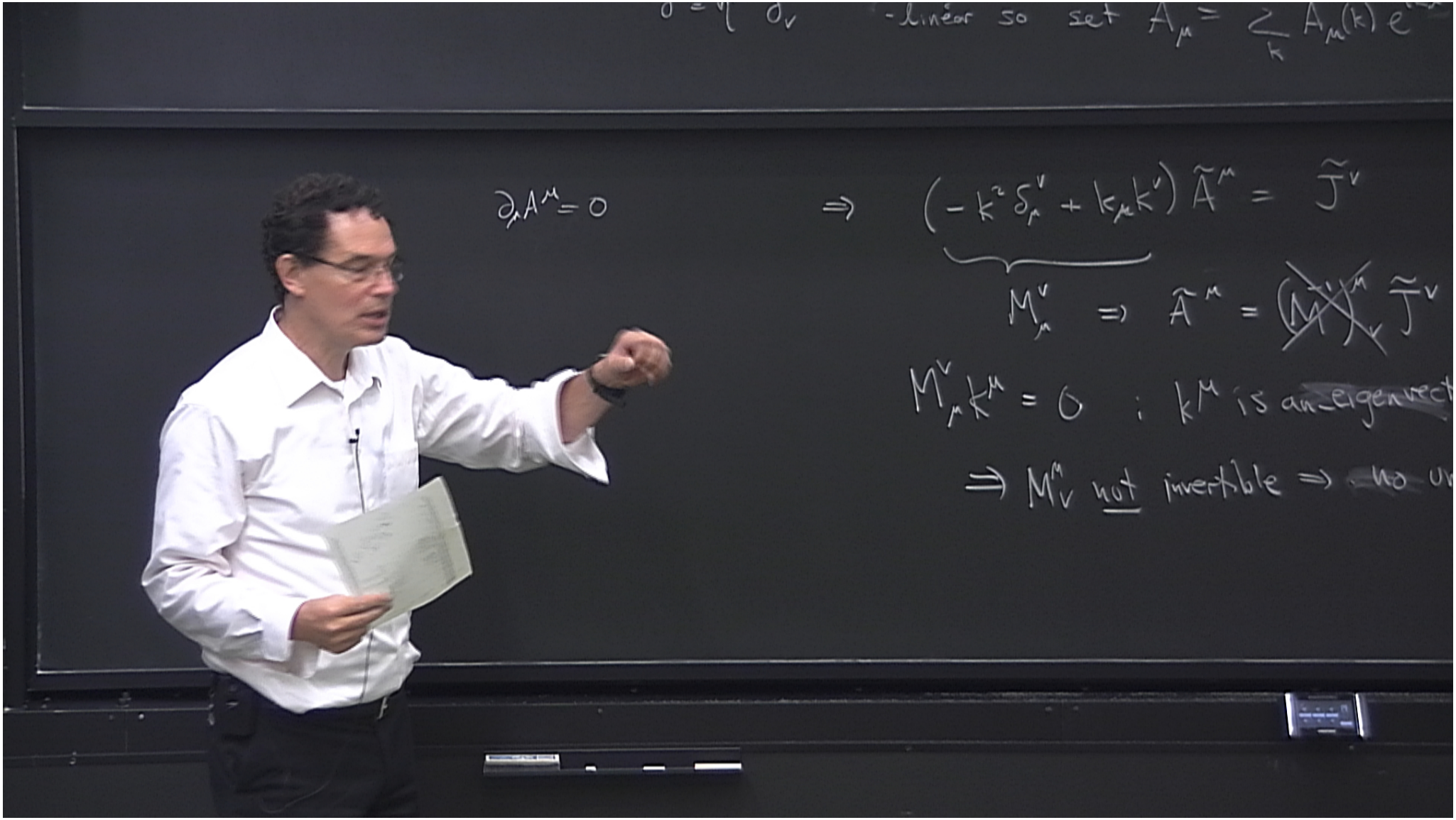
When we solve the Einstein equations, we need to pick a gauge $\Leftrightarrow$ pick coordinates

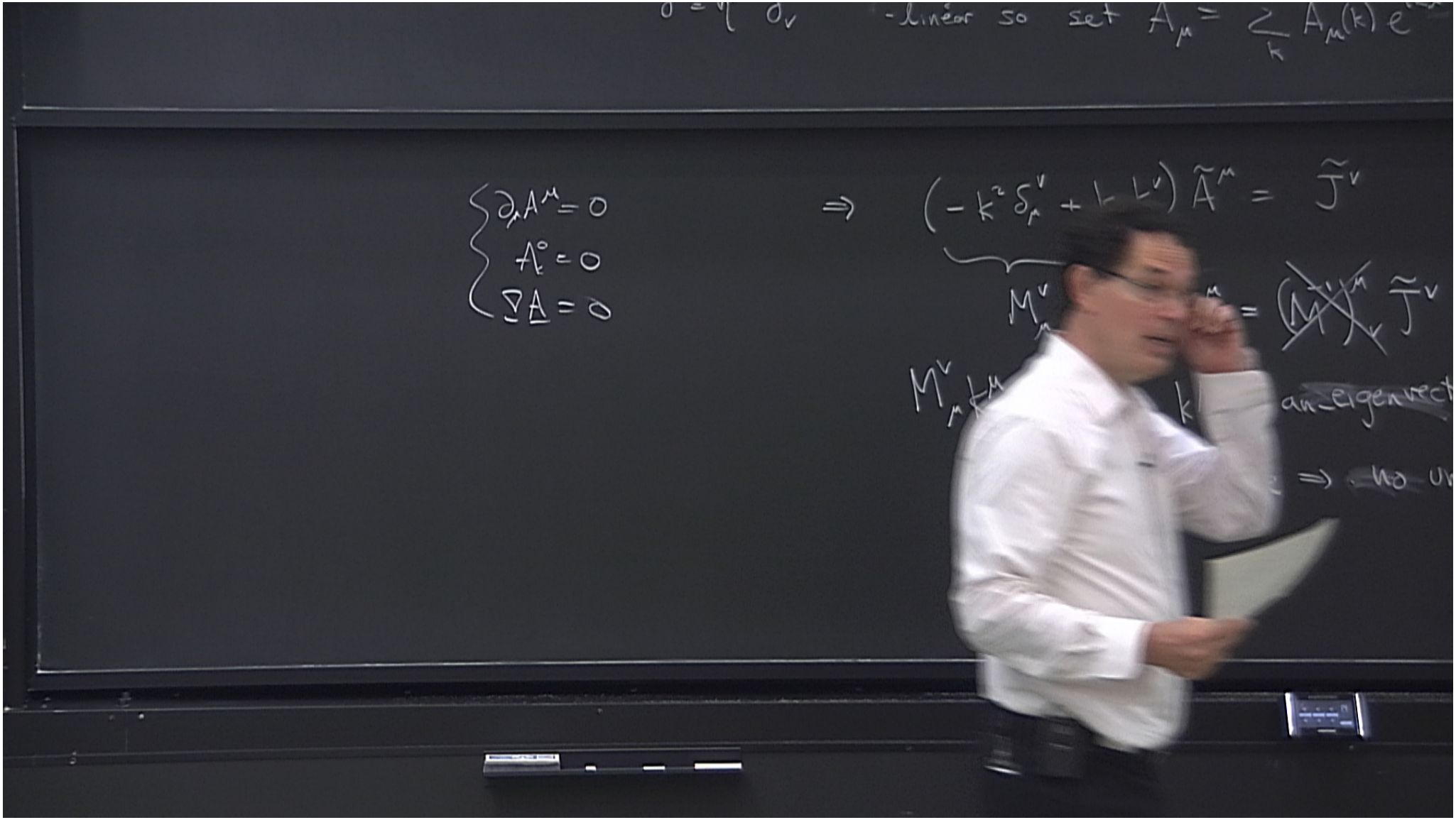
$$\frac{\partial x'^{\mu}}{\partial x^{\nu}} = \delta^{\mu}_{\nu} - \epsilon^{\mu}_{,\nu} \qquad \Rightarrow \quad \delta g_{\mu\nu}(x) = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = -\epsilon^{\alpha}_{,\nu} g_{\mu\alpha} - \epsilon^{\alpha}_{,\mu} g_{\alpha\nu} - \epsilon^{\lambda} g_{\mu\nu,\lambda}.$$

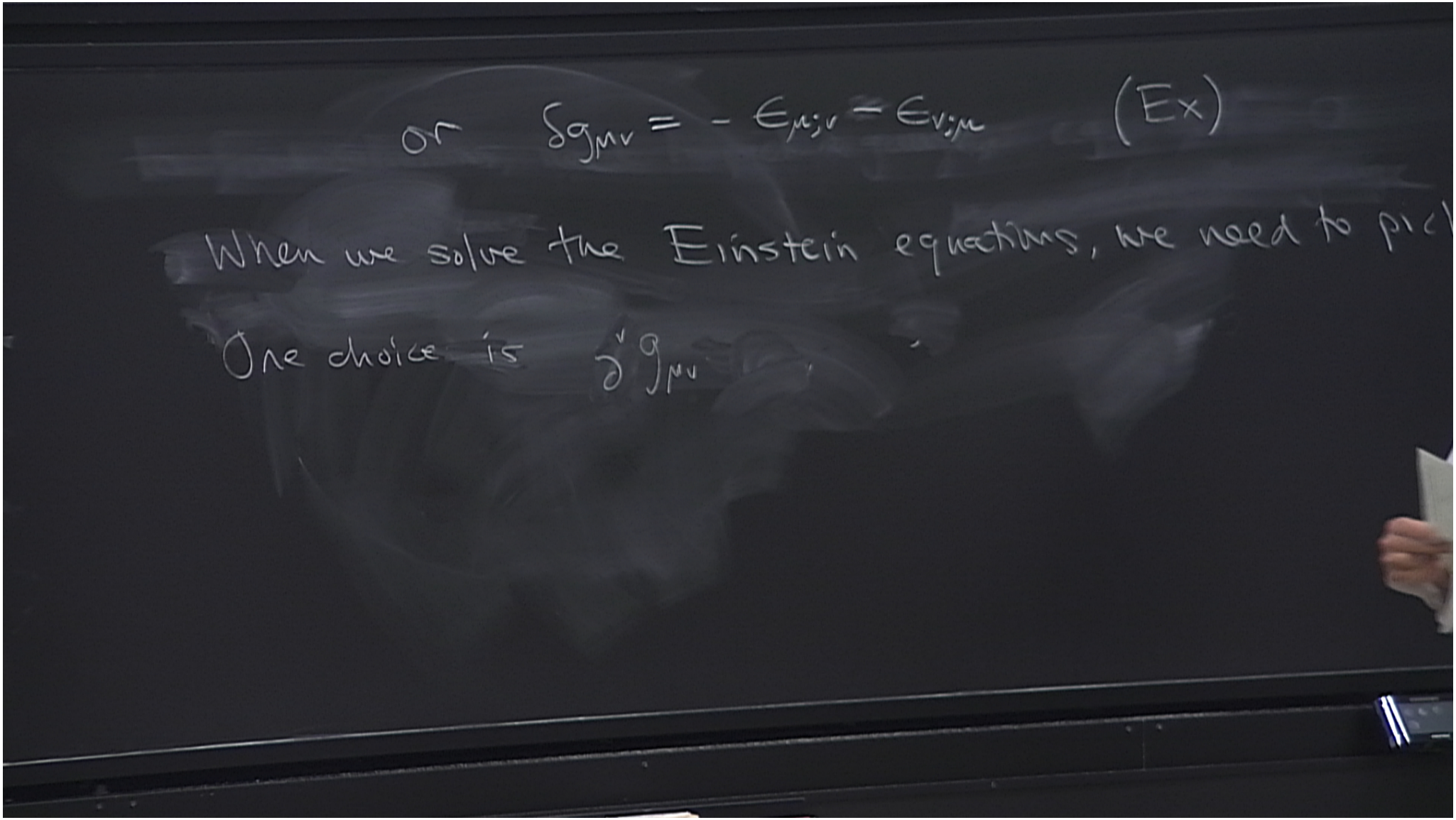
or $\delta g_{\mu\nu} = -\epsilon_{\mu;\nu} - \epsilon_{\nu;\mu}$ (Ex)

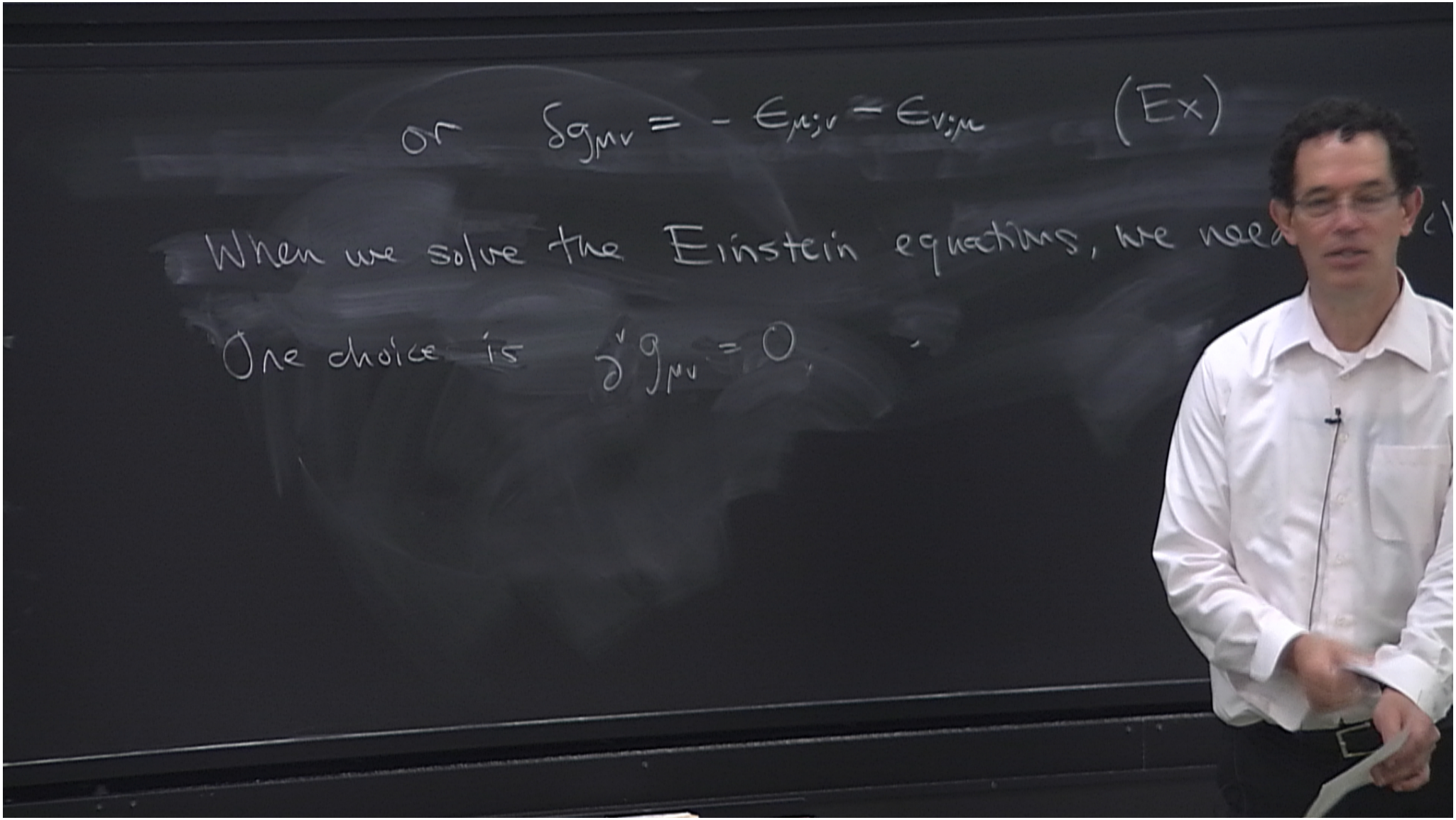
When we solve the Einstein equations, we need to pick a gauge $\Leftrightarrow$ pick coordinates

One choice is


$\partial_\mu A^\mu = 0$
$\Rightarrow$
$(-k^2 \delta^\nu_\mu + k_\mu k^\nu) \tilde{A}^\mu = \tilde{J}^\nu$
$M^\nu_\mu \Rightarrow \tilde{A}^\mu = (\Lambda^{-1})^\mu_\nu \tilde{J}^\nu$
$M^\nu_\mu k^\mu = 0$ ; $k^\mu$ is an eigenvect
$\Rightarrow M^\mu_\nu$ not invertible $\Rightarrow$ no u


$\partial_\mu A^\mu = 0$
$A^0 = 0$
$\nabla \underline{A} = 0$
$\Rightarrow$
$(-k^2 \delta^\nu_\mu + k_\mu k^\nu)\tilde{A}^\mu = \tilde{J}^\nu$
$M^\nu_\mu$
$M^\nu_\mu k^\mu$
an_eigenvect
$\Rightarrow$ no

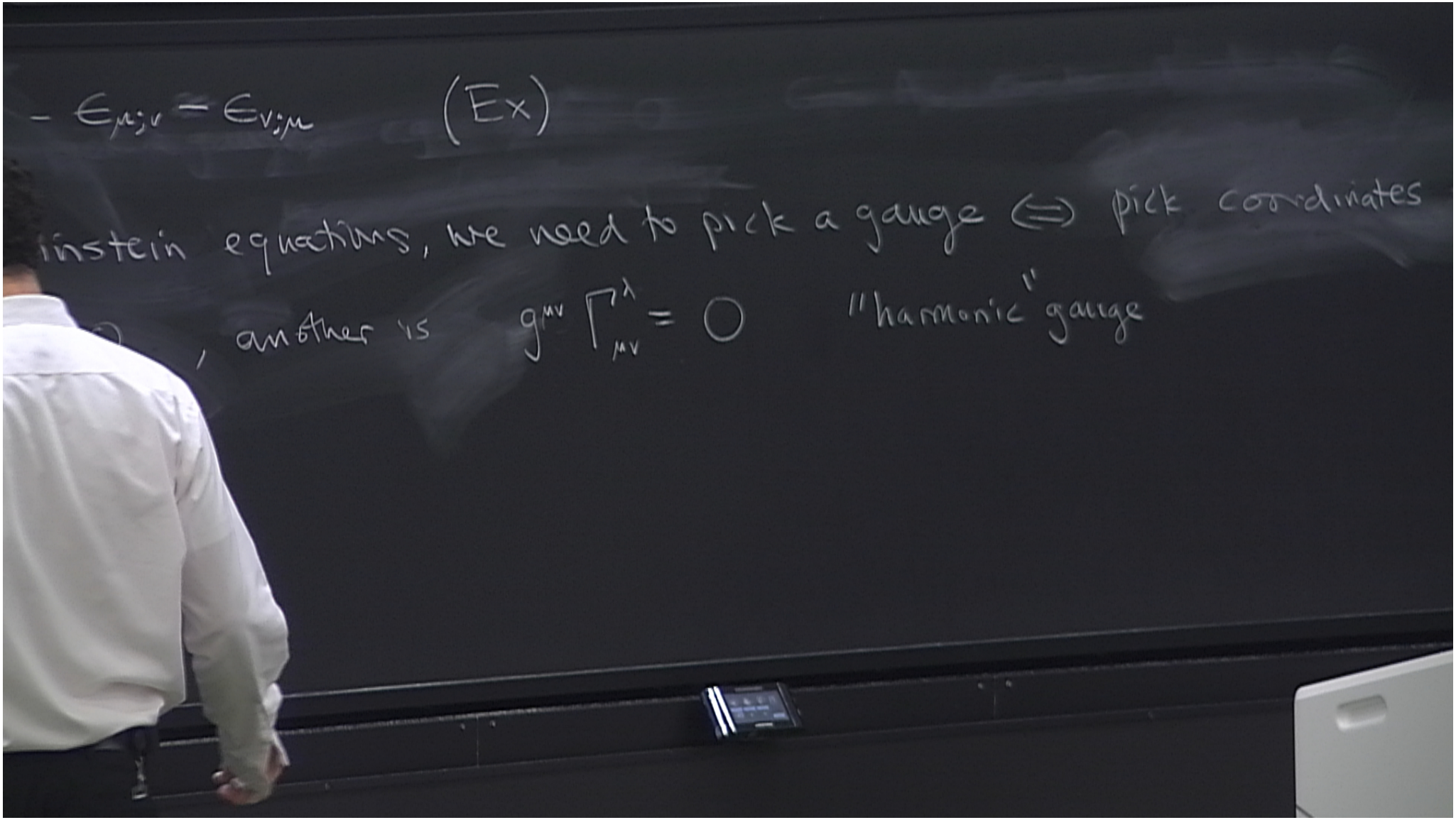

or $\delta g_{\mu\nu} = -\epsilon_{\mu;\nu} - \epsilon_{\nu;\mu}$ (Ex)
When we solve the Einstein equations, we need to pi
One choice is $\delta^* g_{\mu\nu}$

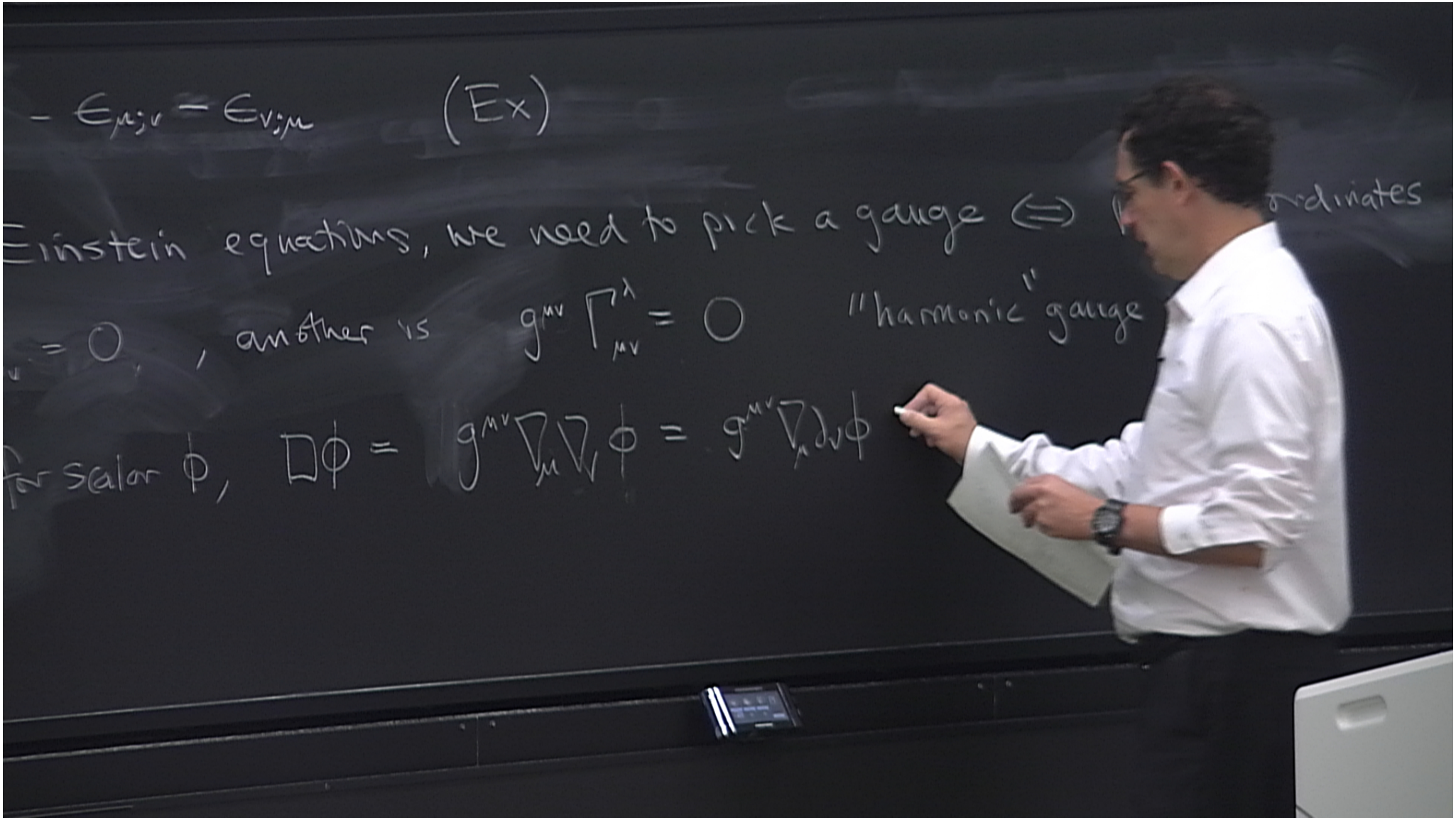

or $\delta g_{\mu\nu} = -\epsilon_{\mu;\nu} - \epsilon_{\nu;\mu}$ (Ex)
When we solve the Einstein equations, we need
One choice is $\partial^\nu g_{\mu\nu} = 0$

or $\delta g_{\mu\nu} = -\epsilon_{\mu;\nu} - \epsilon_{\nu;\mu}$ (Ex)

When we solve the Einstein equations, we need to pick a g

One choice is $\partial^\nu g_{\mu\nu} = 0$, another is $g^{\mu\nu}\Gamma^\lambda_{\mu\nu} = 0$


$-\epsilon_{\mu\nu\rho} = \epsilon_{\nu\rho\mu}$ (Ex)
instein equations, we need to pick a gauge $\Leftrightarrow$ pick coordinates
, another is $g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} = 0$ "harmonic" gauge

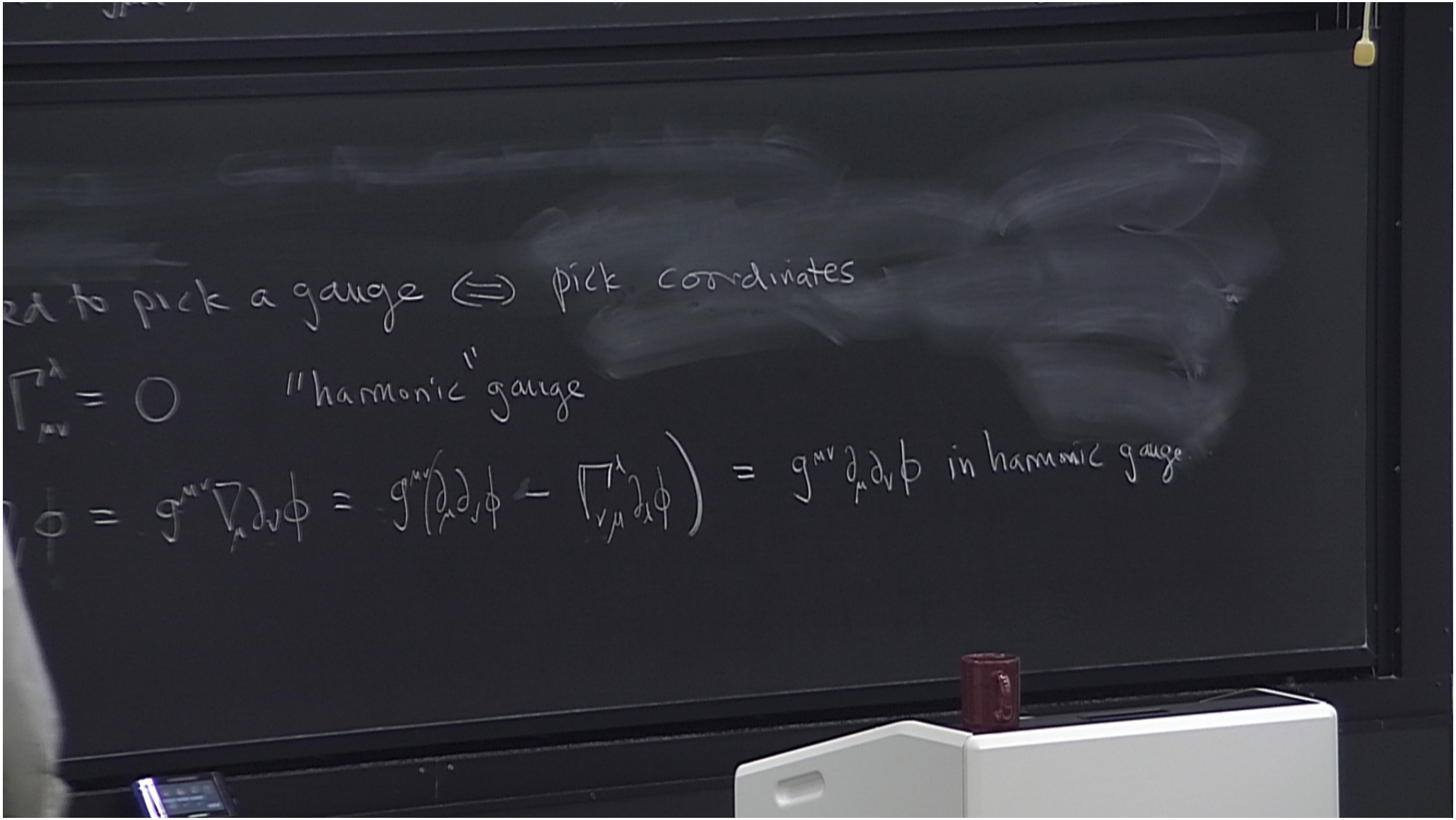

$-\epsilon_{\mu\nu} = \epsilon_{\nu\mu}$ (Ex)
Einstein equations, we need to pick a gauge ($\equiv$) coordinates
$= 0$, another is $g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} = 0$ "harmonic" gauge
for scalar $\phi$, $\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = g^{\mu\nu}\partial_\mu\partial_\nu\phi$

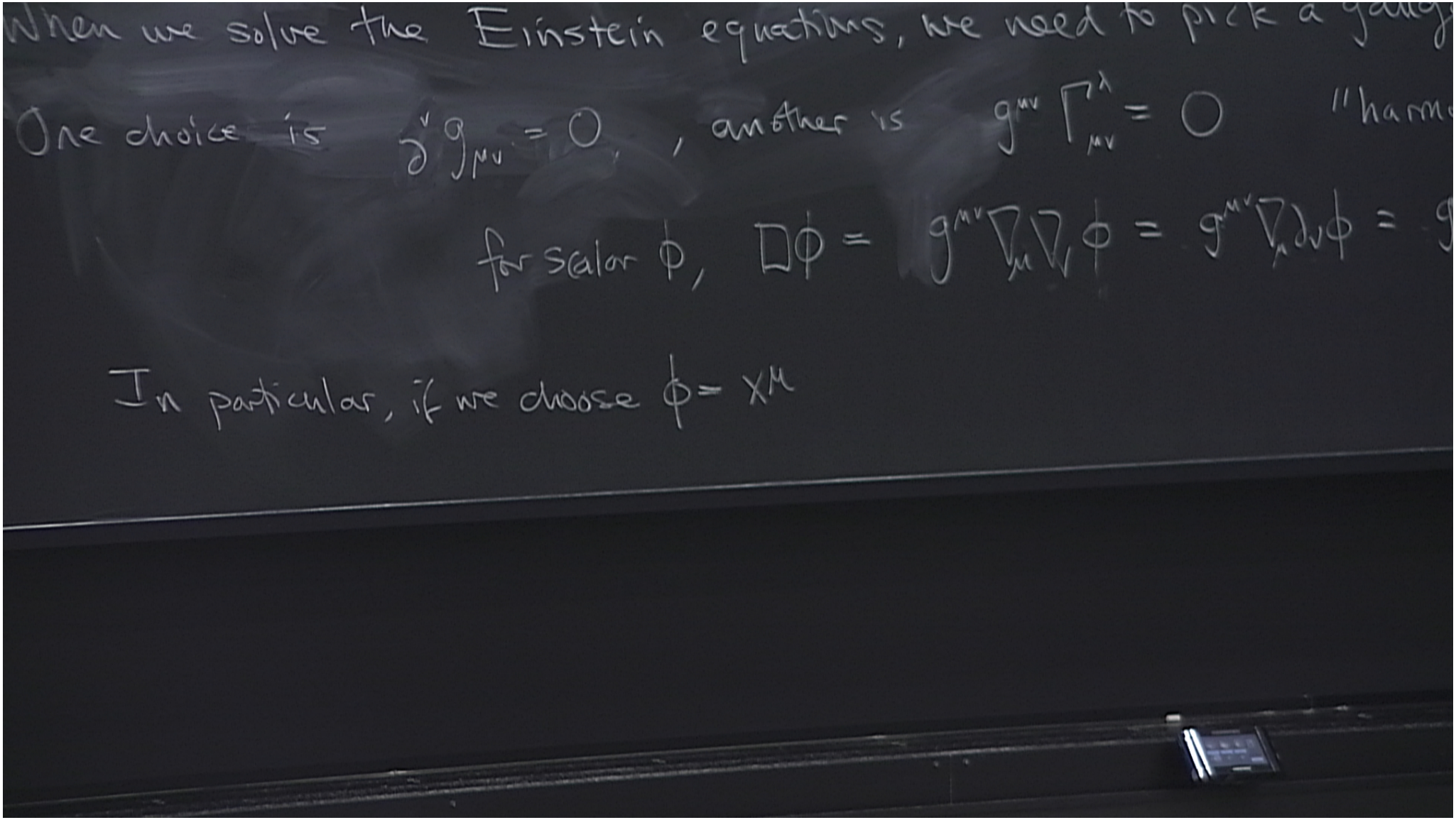
$= -\epsilon_{\mu;\nu} = \epsilon_{\nu;\mu}$ (Ex)

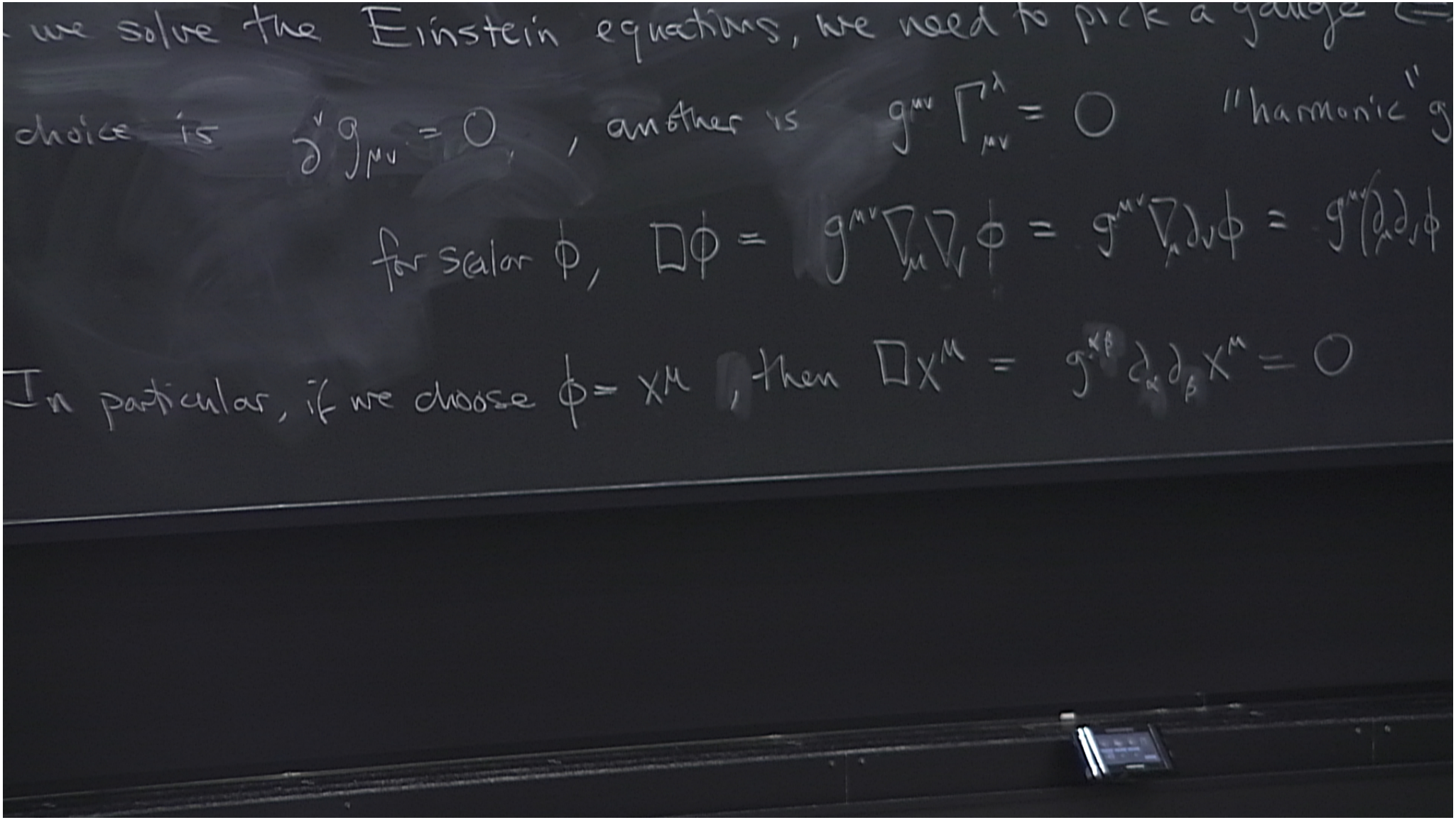
Einstein equations, we need to pick a gauge $\Leftrightarrow$ pick coordinat

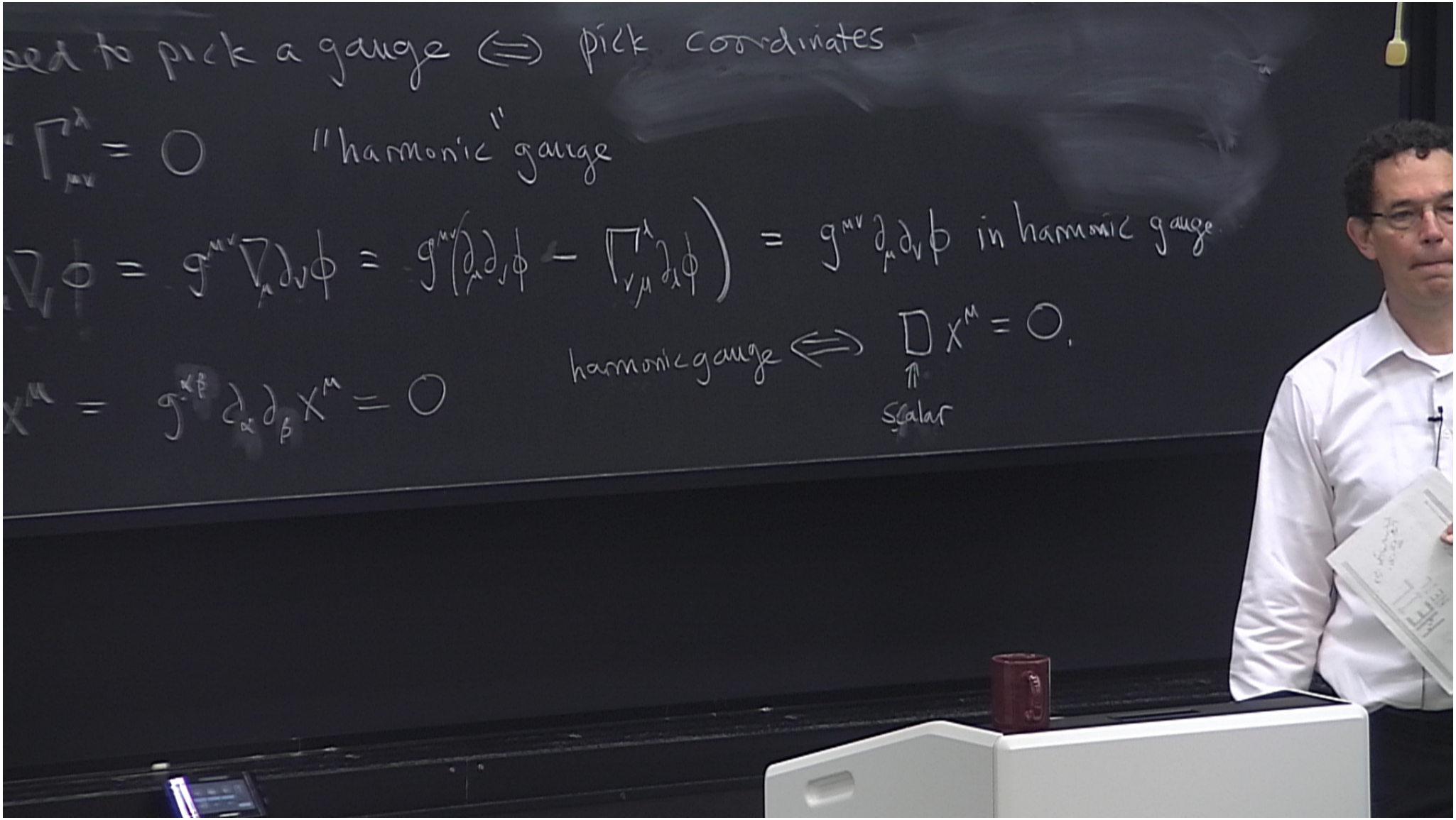
$g_{\mu\nu} = 0$, another is $g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} = 0$ "harmonic" gauge

for scalar $\phi$, $\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = g^{\mu\nu}\nabla_\mu\partial_\nu\phi = g^{\mu\nu}\left(\partial_\mu\partial_\nu\phi - \Gamma^{\lambda}_{\nu\mu}\partial_\lambda\phi\right) =$


d to pick a gauge ⟺ pick coordinates
$\Gamma^\lambda_{\mu\nu} = 0$ "harmonic" gauge
$\phi = g^{\mu\nu}\nabla_\mu\partial_\nu\phi = g^{\mu\nu}\left(\partial_\mu\partial_\nu\phi - \Gamma^\lambda_{\nu\mu}\partial_\lambda\phi\right) = g^{\mu\nu}\partial_\mu\partial_\nu\phi$ in harmonic gauge


When we solve the Einstein equations, we need to pick a gauge
One choice is $\partial^\nu g_{\mu\nu} = 0$, another is $g^{\mu\nu}\Gamma^\lambda_{\mu\nu} = 0$ "harmo
for scalar $\phi$, $\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = g^{\mu\nu}\partial_\mu\partial_\nu\phi$ =
In particular, if we choose $\phi = x^\mu$


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In particular, if we choose $\phi = x^\mu$, then $\Box x^\mu = g^{\alpha\beta}\partial_\alpha\partial_\beta x^\mu = 0$


need to pick a gauge ⟺ pick coordinates
$\Gamma^\lambda_{\mu\nu} = 0$ "harmonic" gauge
$\nabla_\mu \partial_\nu \phi = g^{\alpha\beta} \nabla_\alpha \partial_\beta \phi = g^{\alpha\beta}\left(\partial_\alpha \partial_\beta \phi - \Gamma^\lambda_{\alpha\beta} \partial_\lambda \phi\right) = g^{\alpha\beta} \partial_\alpha \partial_\beta \phi$ in harmonic gauge
$X^\mu = g^{\alpha\beta} \partial_\alpha \partial_\beta X^\mu = 0$
harmonic gauge ⟺ $\Box X^\mu = 0$.
scalar

One choice is $\partial^\nu g_{\mu\nu} = 0$, another is $g^{\mu\nu}\Gamma^\lambda_{\mu\nu} = 0$

for scalar $\phi$, $\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi =$

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[Another common choice of coords is $g_{\mu\nu} = \delta_{\mu\nu}$

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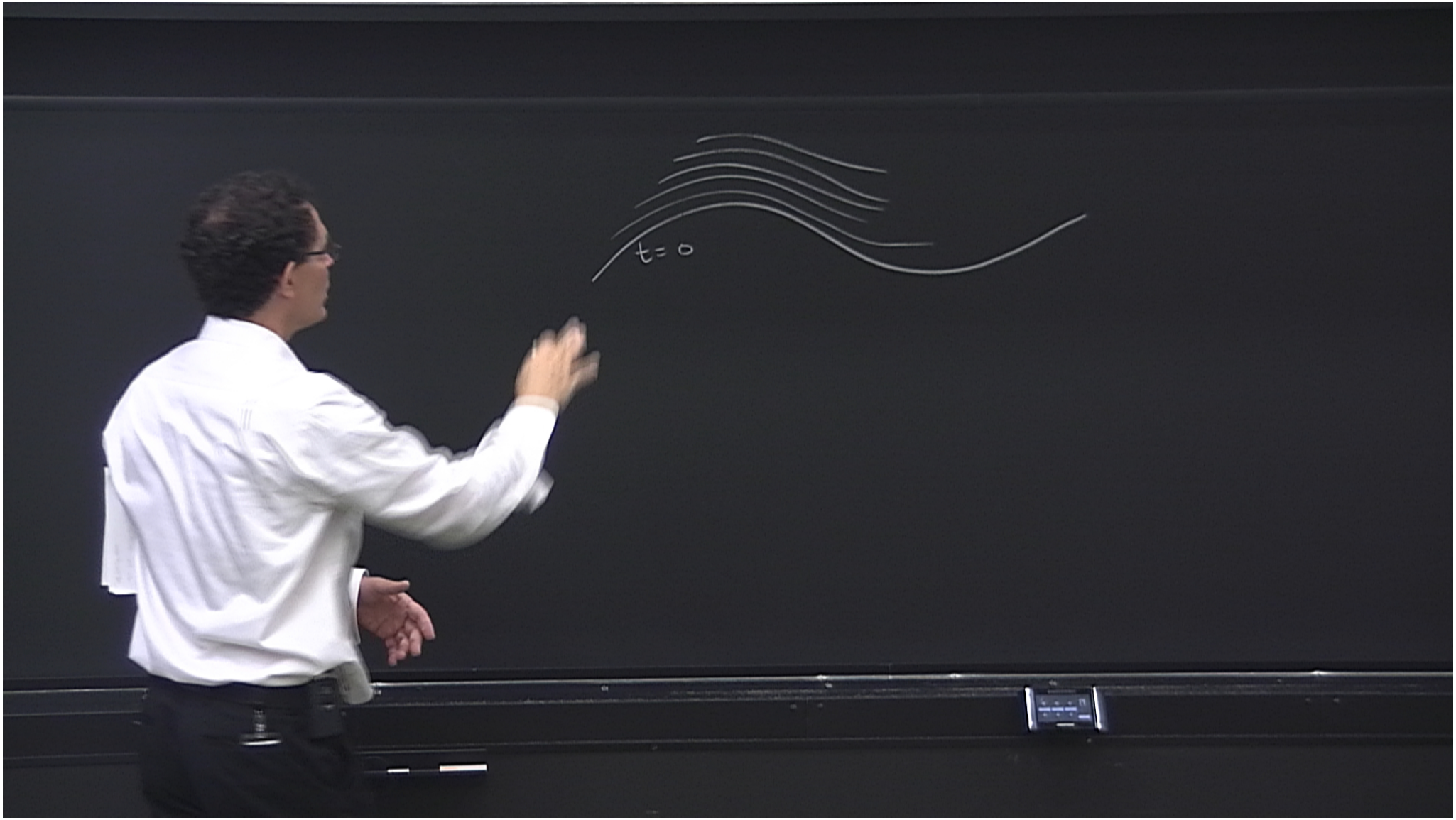
One choice is $\partial^\nu g_{\mu\nu} = 0$, another is $g^{\mu\nu}\Gamma^\lambda_{\mu\nu} = 0$

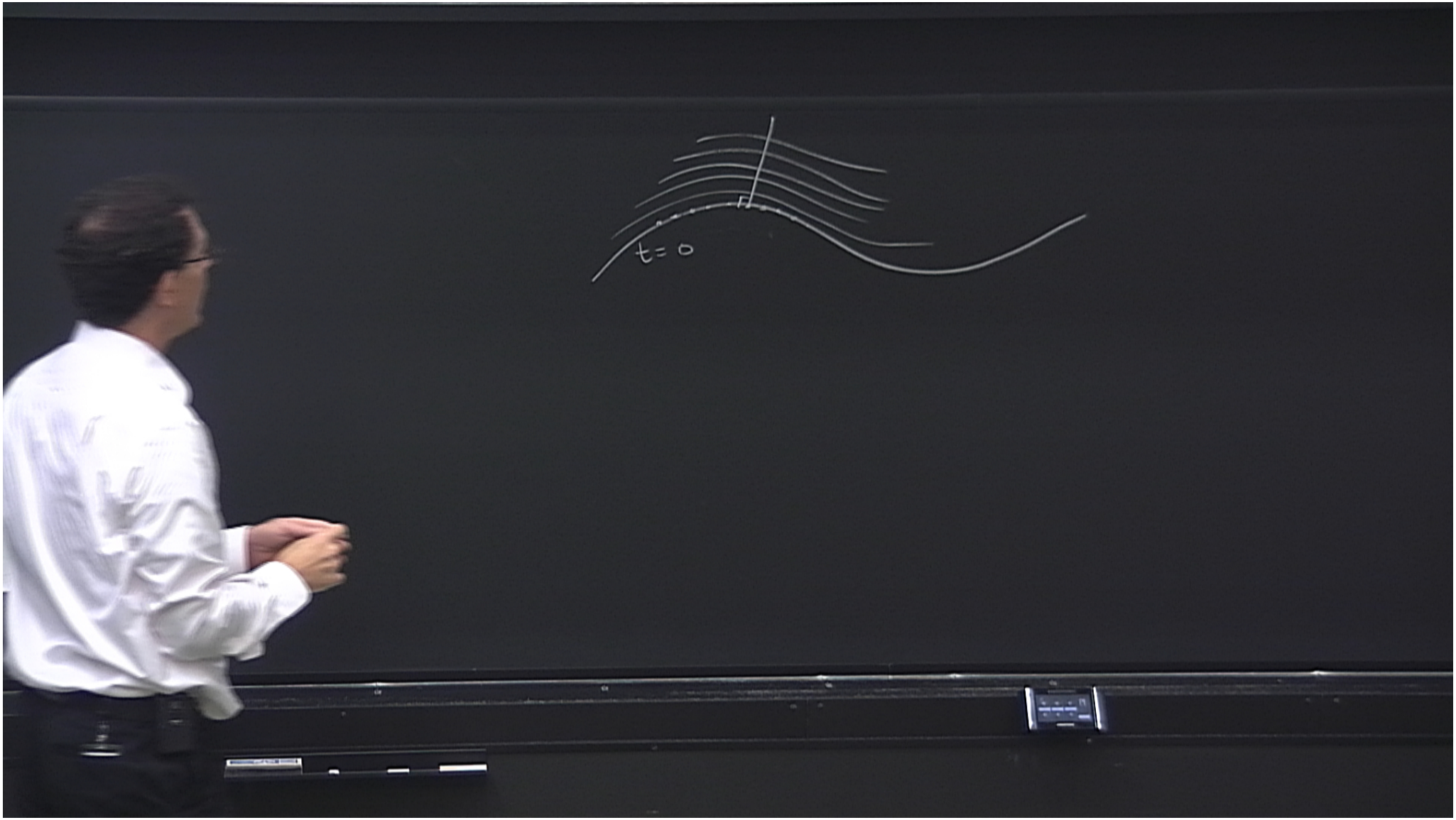
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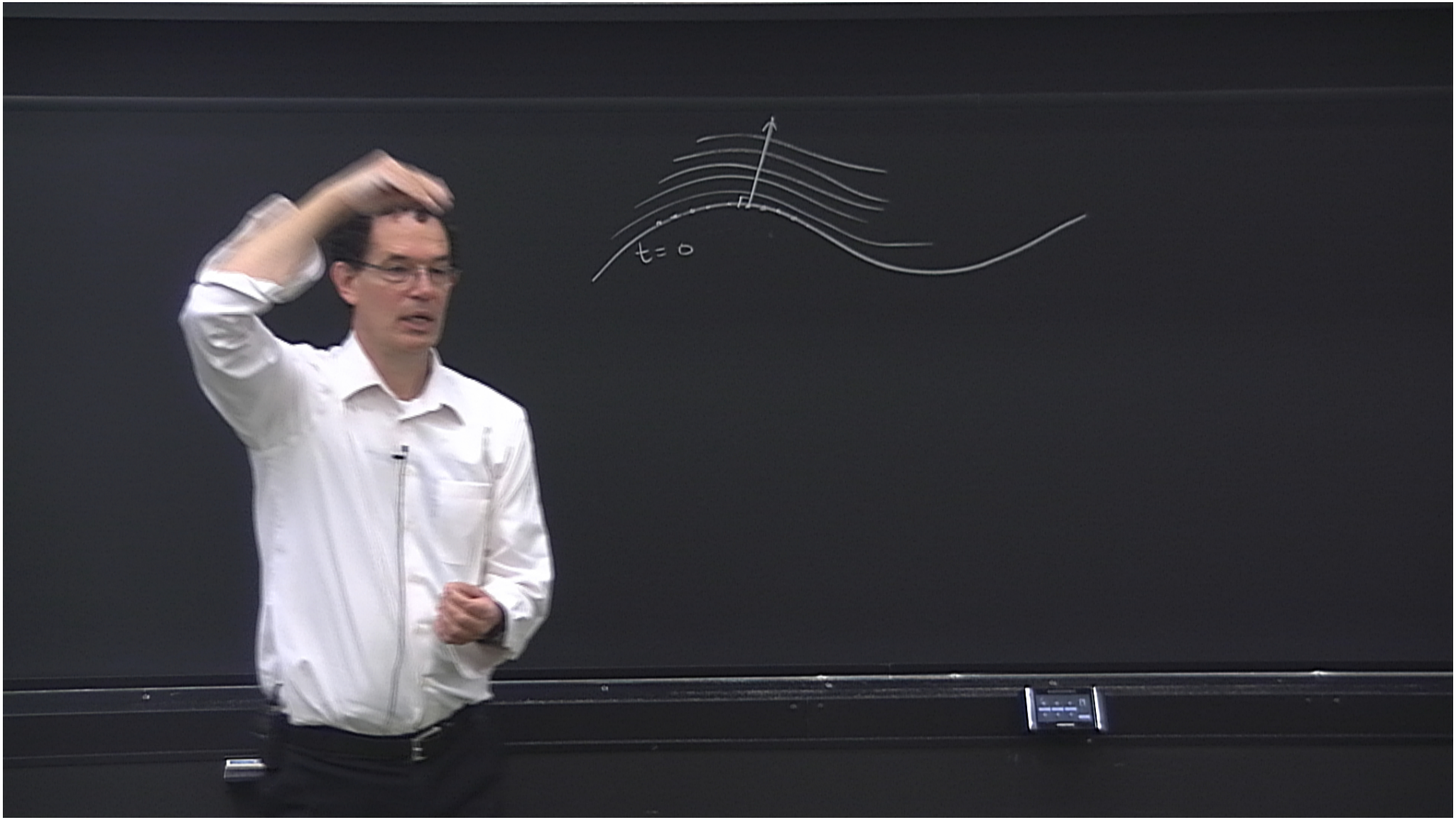
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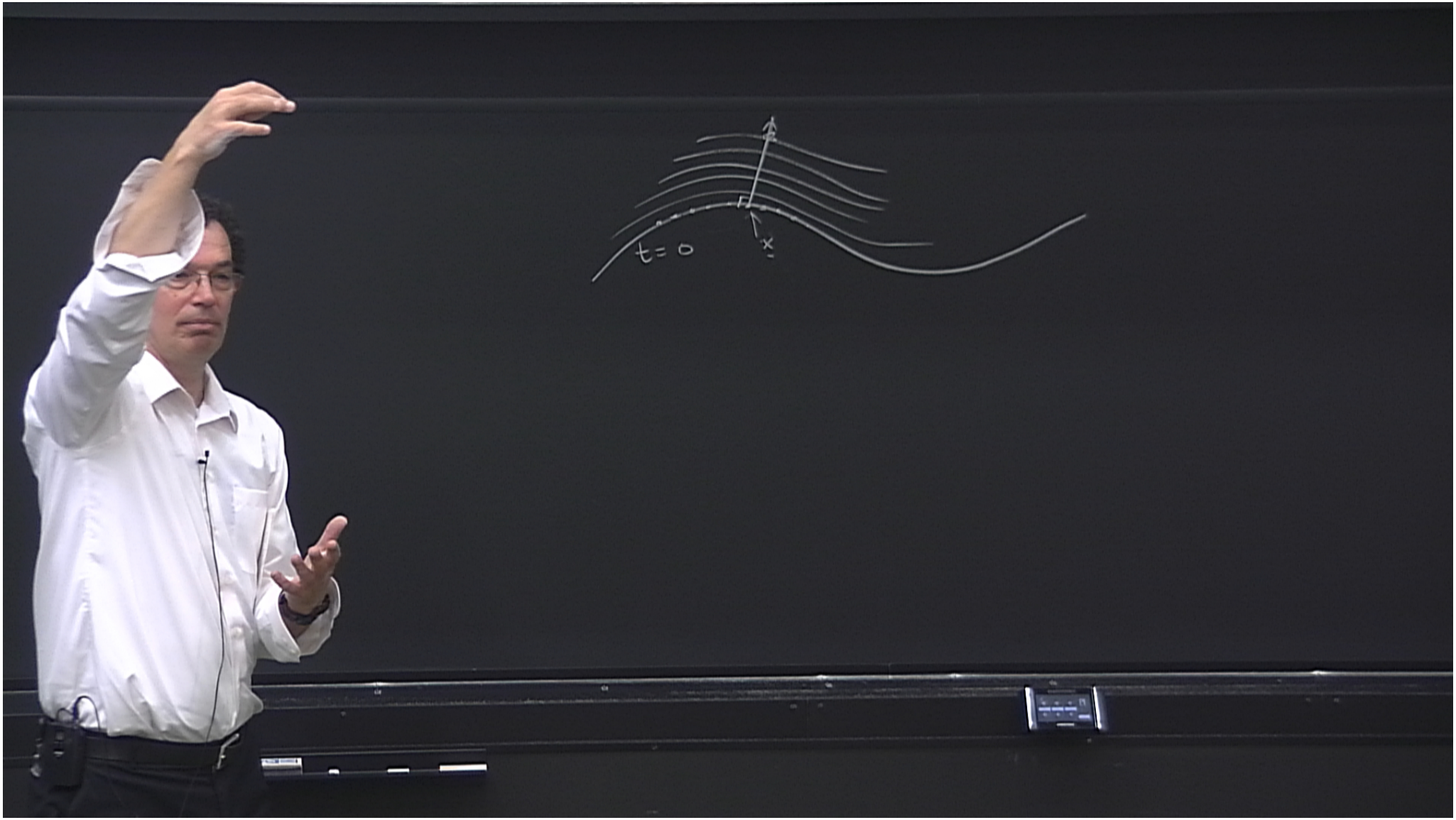
[Another common choice of coords is $g_{\mu\nu} = -\delta_{\mu\nu}$ "synchronous" gauge]


t = 0

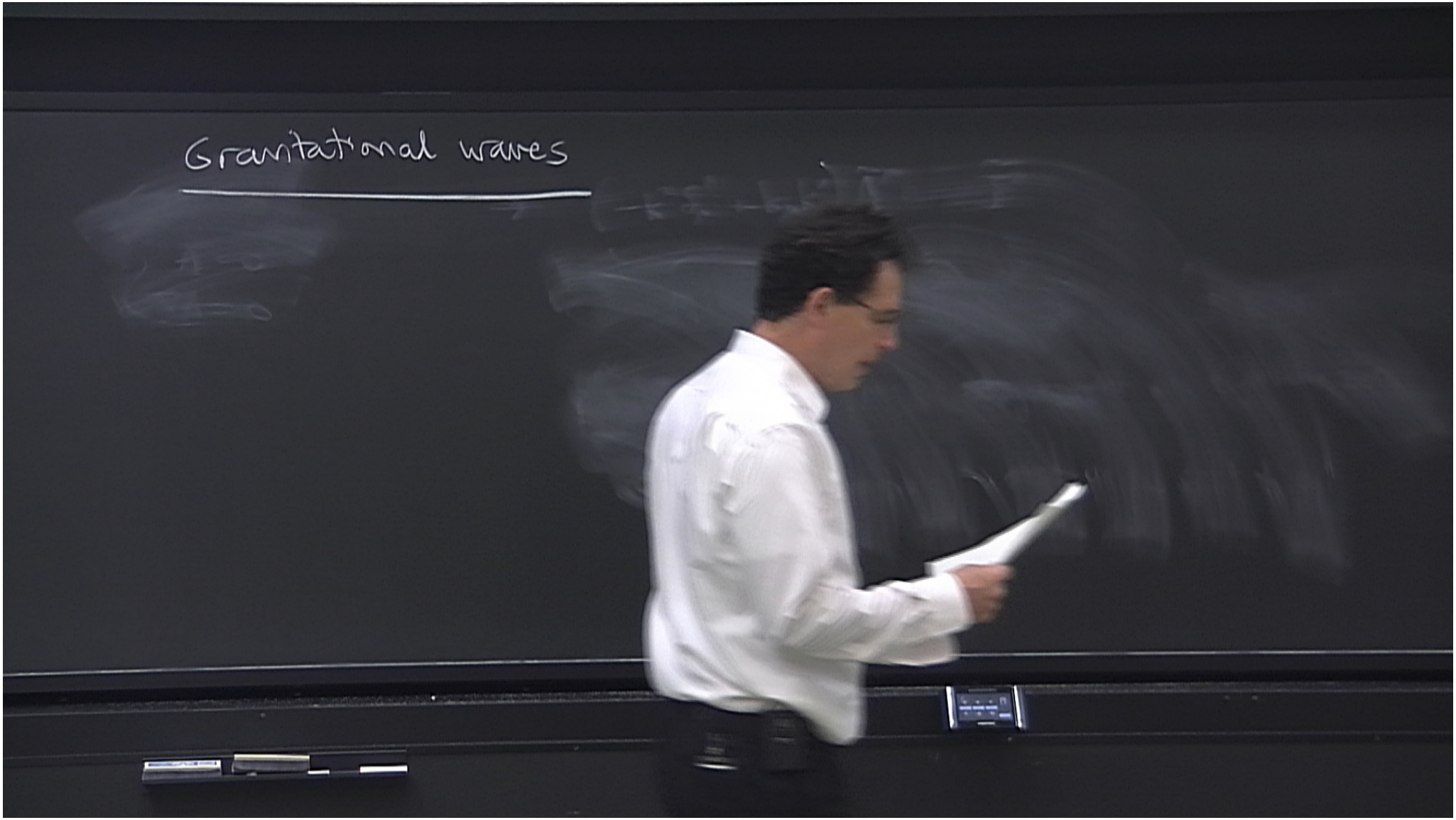

t = 0


t=0


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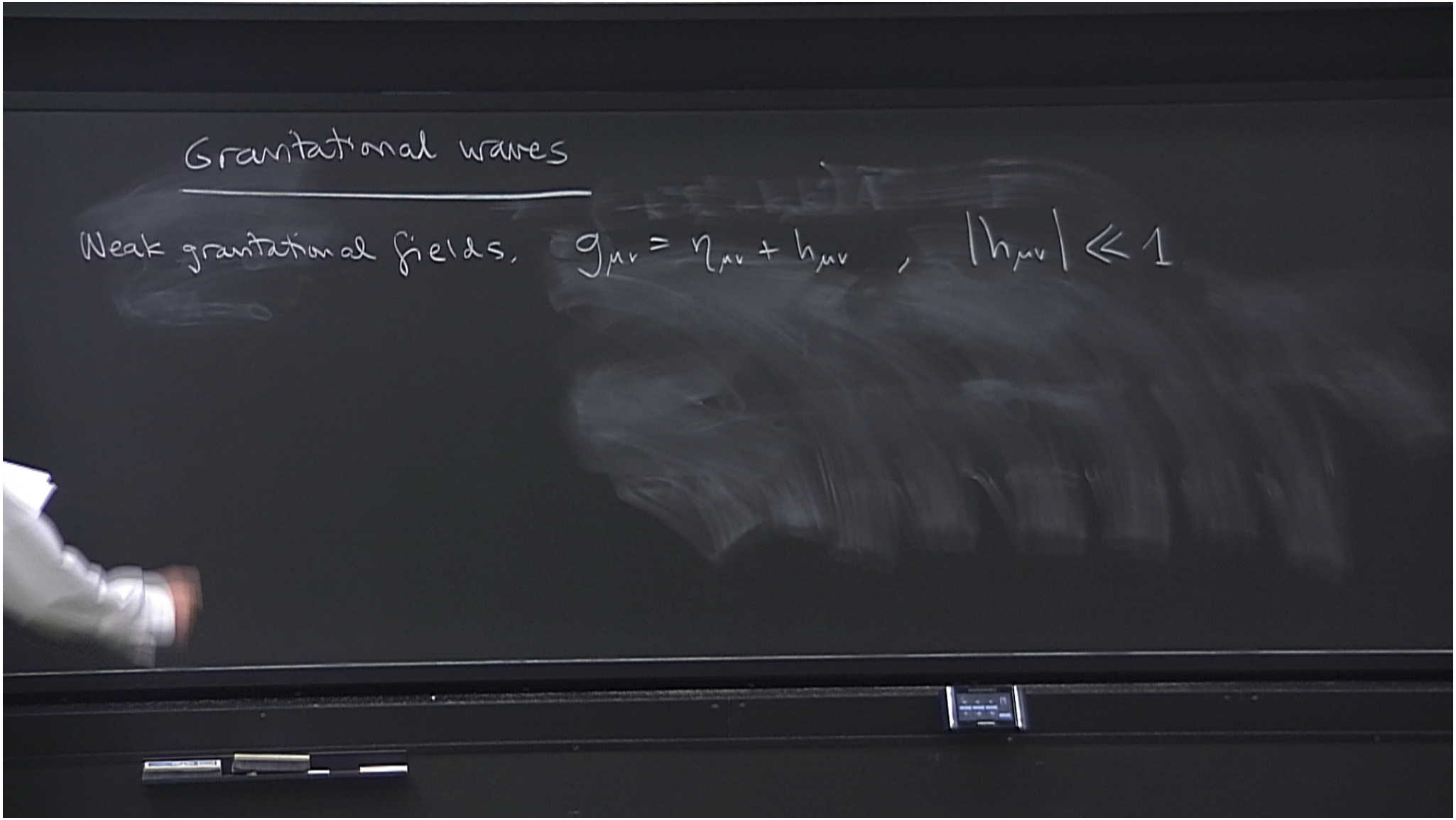

t = 0
x

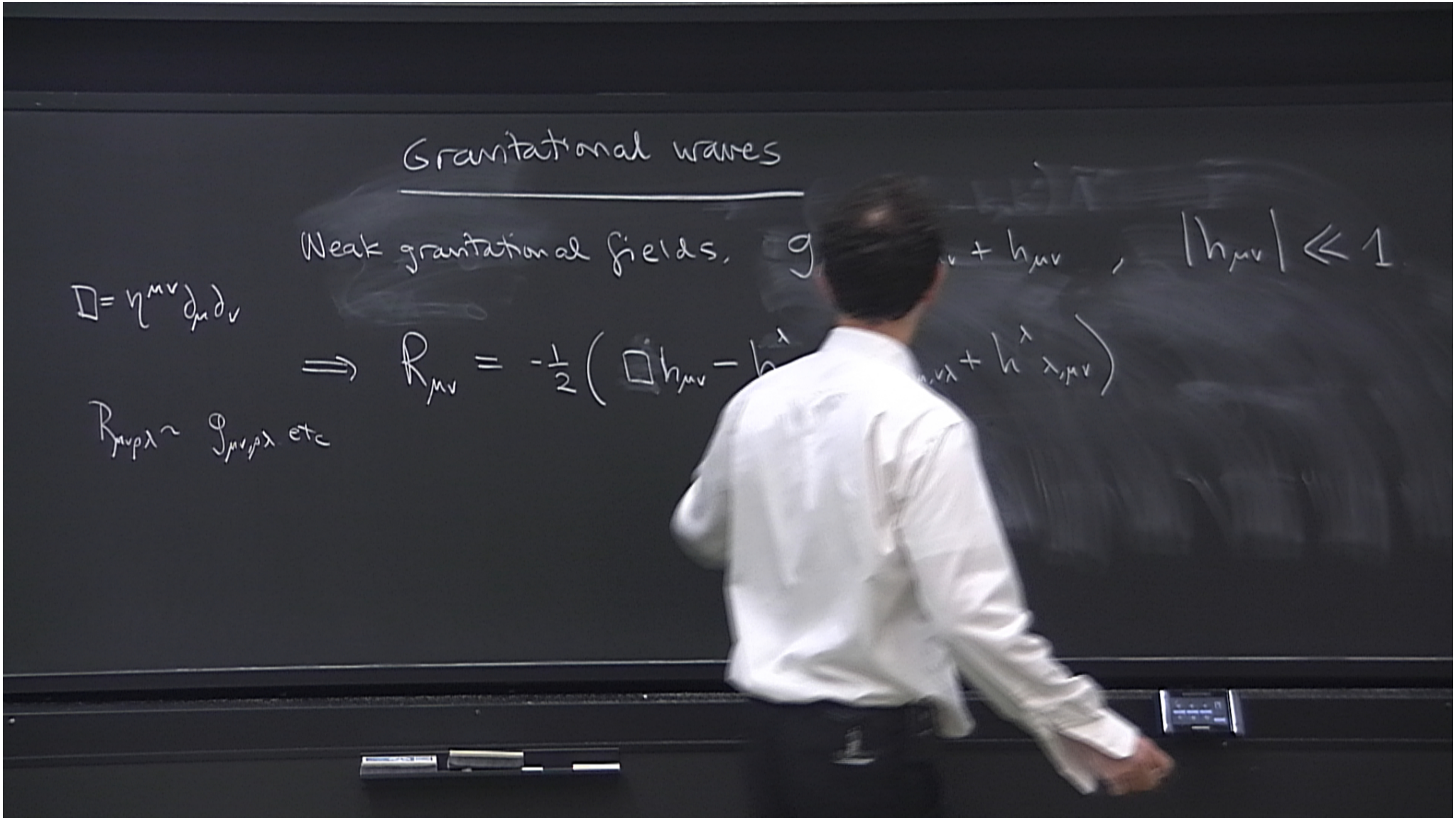

t = 0
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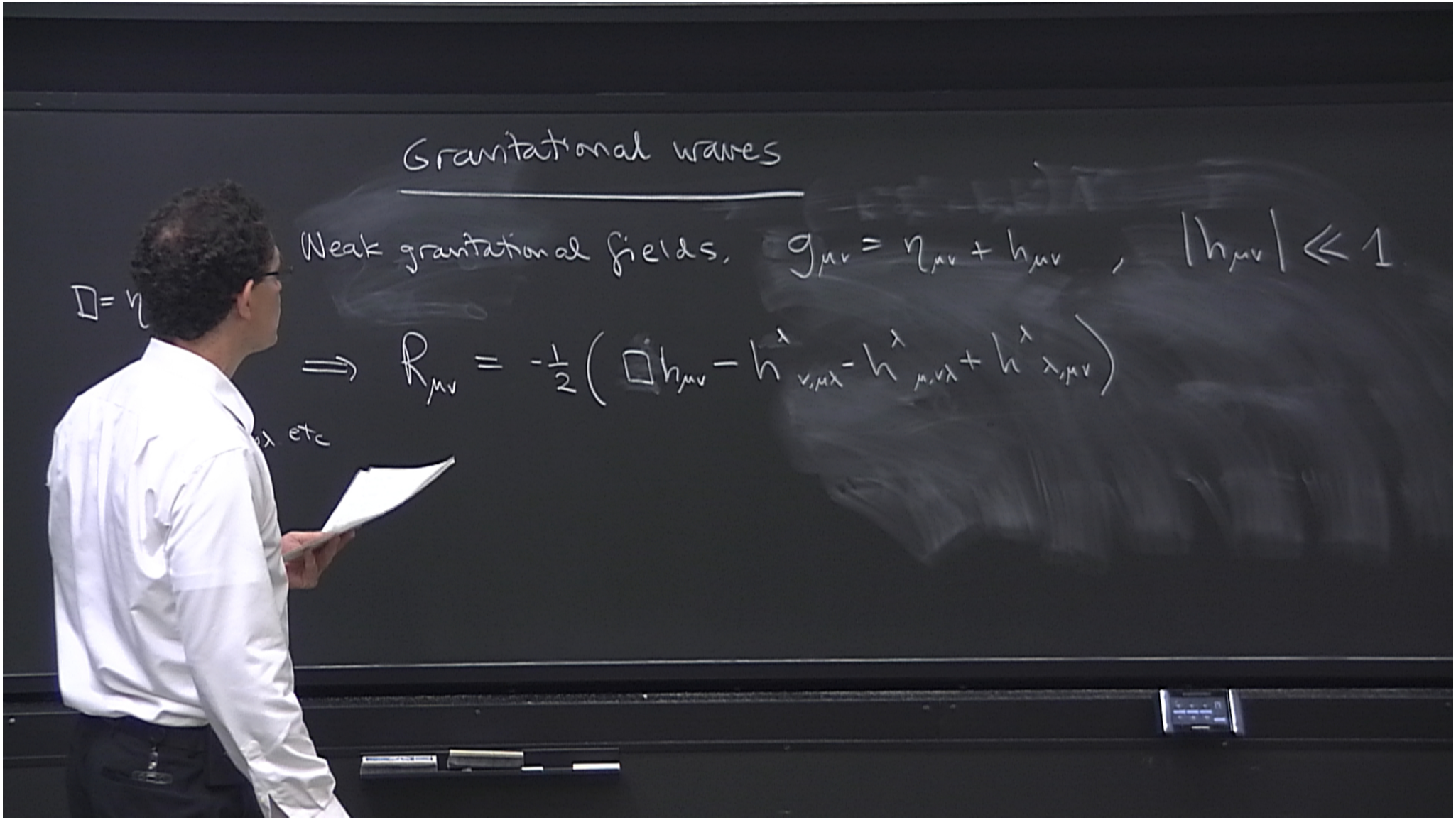

Gravitational waves

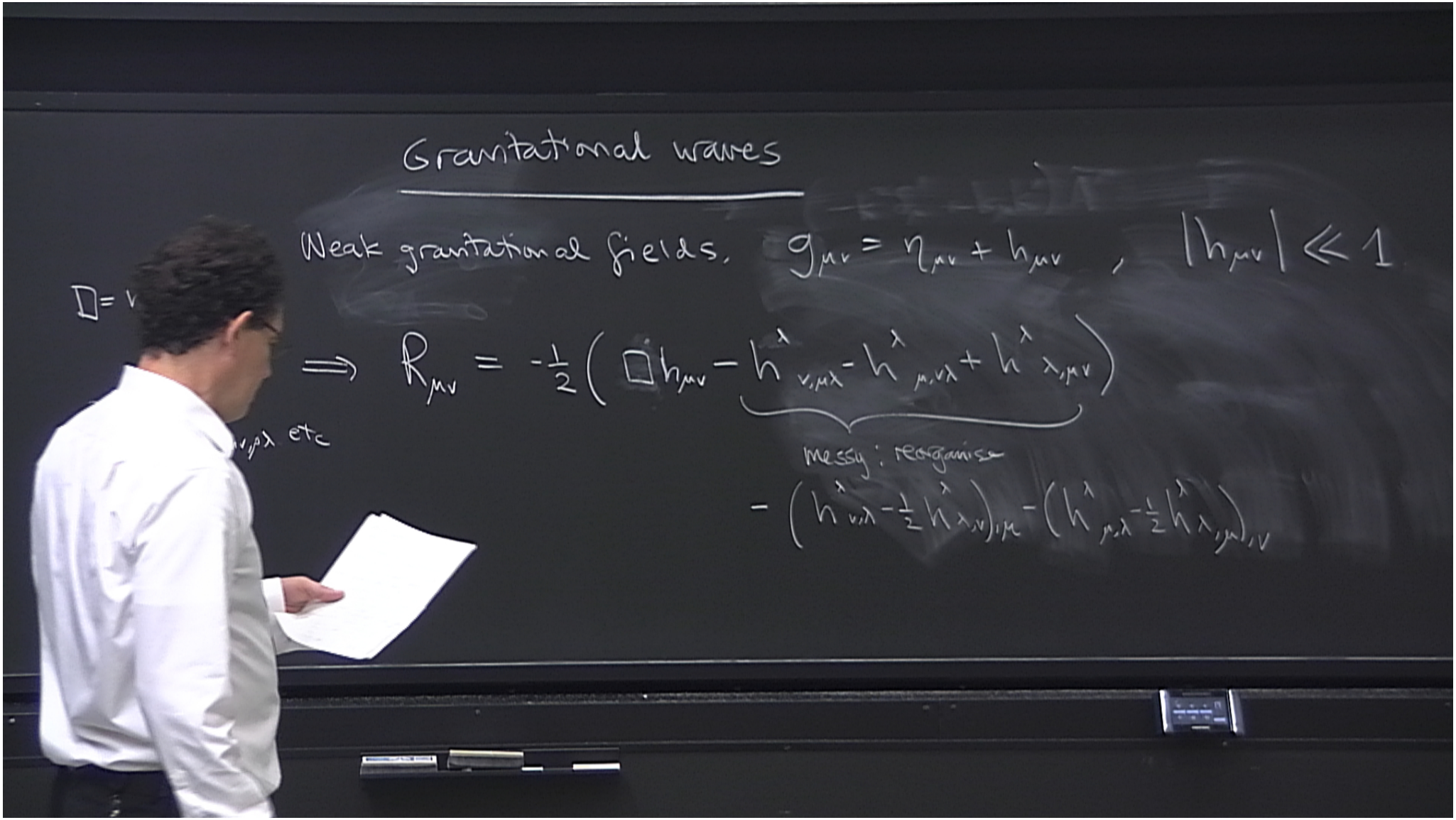
# Gravitational waves

Weak gravitational fields. $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$


Gravitational waves
Weak gravitational fields, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ , $|h_{\mu\nu}| \ll 1$


Gravitational waves
Weak gravitational fields.
$|h_{\mu\nu}| \ll 1$
$\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$
$\Rightarrow R_{\mu\nu} = -\frac{1}{2}(\Box h_{\mu\nu}$
$R_{\mu\nu\rho\lambda} \sim g_{\mu\nu,\rho\lambda}$ etc


Gravitational waves
Weak gravitational fields. $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ , $|h_{\mu\nu}| \ll 1$
$\Box = \eta$
$\Longrightarrow R_{\mu\nu} = -\frac{1}{2}\left( \Box h_{\mu\nu} - h^{\lambda}{}_{\nu,\mu\lambda} - h^{\lambda}{}_{\mu,\nu\lambda} + h^{\lambda}{}_{\lambda,\mu\nu} \right)$
etc


Gravitational waves
Weak gravitational fields. $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ , $|h_{\mu\nu}| \ll 1$
$\Box = \nu$
$\Rightarrow R_{\mu\nu} = -\frac{1}{2}\left( \Box h_{\mu\nu} - h^{\lambda}{}_{\nu,\mu\lambda} - h^{\lambda}{}_{\mu,\nu\lambda} + h^{\lambda}{}_{\lambda,\mu\nu} \right)$
messy: reorganise
$- \left( h^{\lambda}{}_{\nu,\lambda} - \frac{1}{2} h^{\lambda}{}_{\lambda,\nu} \right)_{,\mu} - \left( h^{\lambda}{}_{\mu,\lambda} - \frac{1}{2} h^{\lambda}{}_{\lambda,\mu} \right)_{,\nu}$

# Gravitational waves

Weak gravitational fields. $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ , $|h_{\mu\nu}| <$

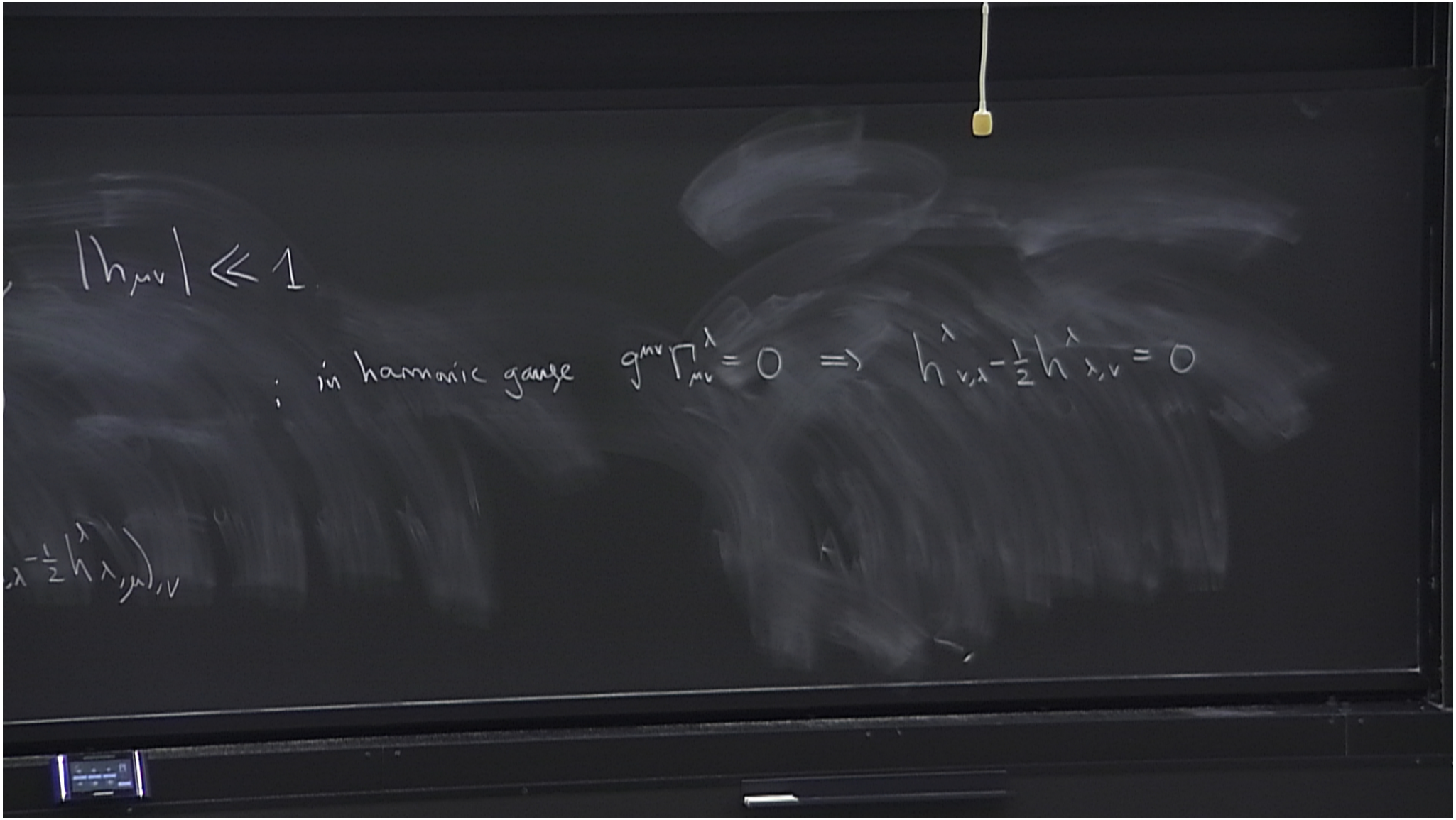
$\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$

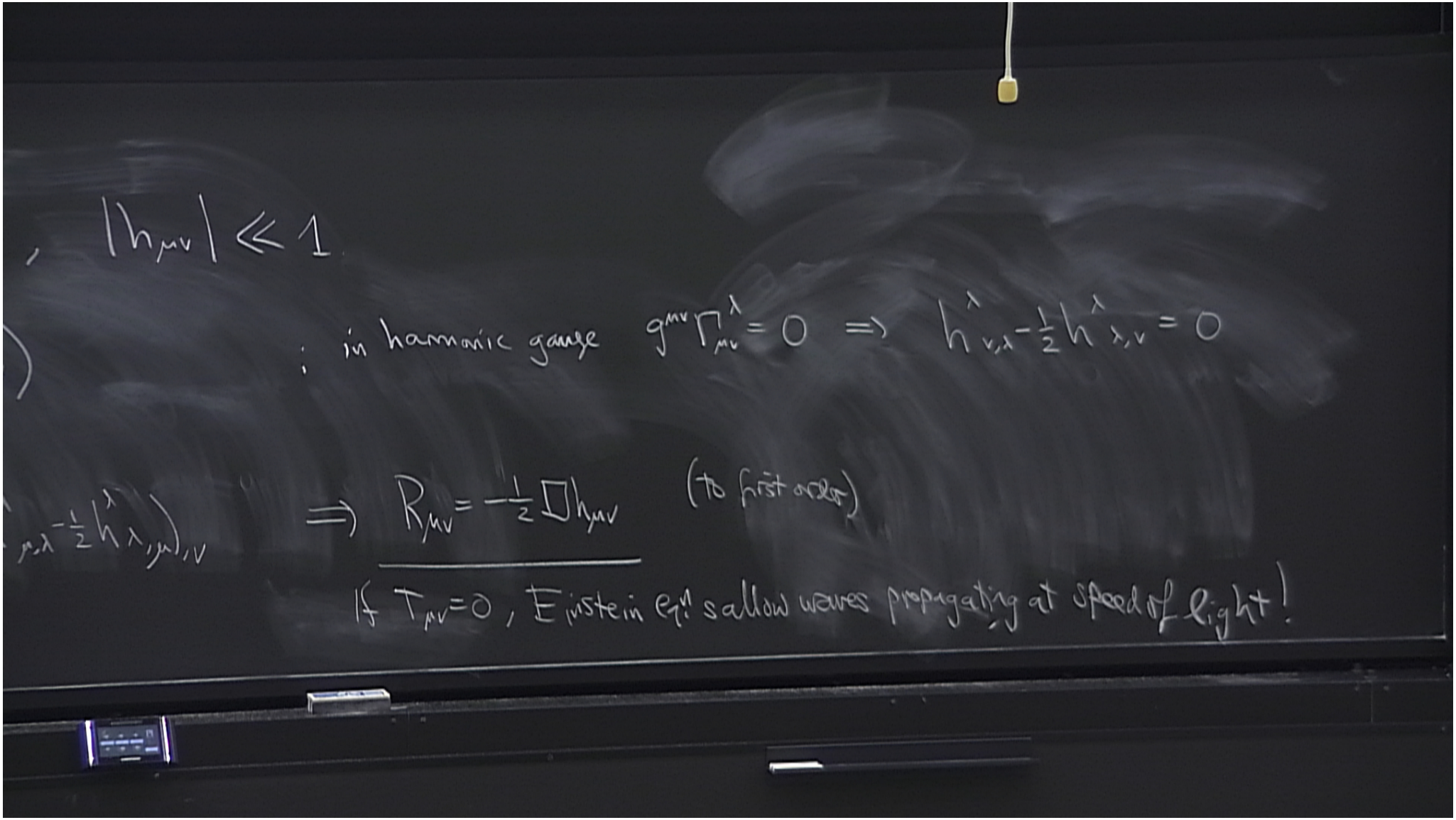
$$\Longrightarrow R_{\mu\nu} = -\frac{1}{2}\left(\Box h_{\mu\nu} - \underbrace{h^{\lambda}_{\nu,\mu\lambda} - h^{\lambda}_{\mu,\nu\lambda} + h^{\lambda}_{\lambda,\mu\nu}}\right)$$

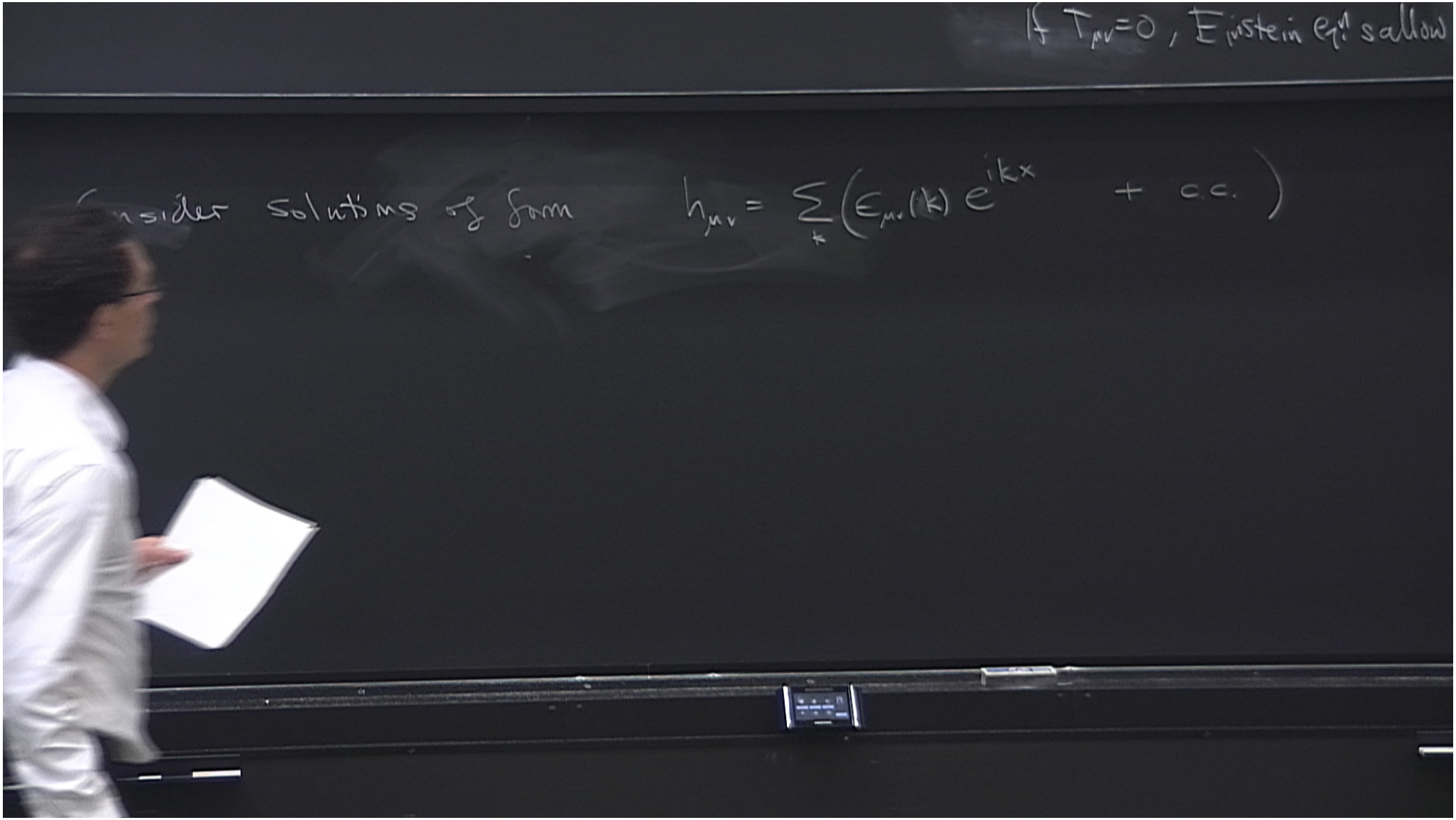
messy: reorganise

$$- \left(h^{\lambda}_{\nu,\lambda} - \frac{1}{2}h^{\lambda}_{\lambda,\nu}\right)_{,\mu} - \left(h^{\lambda}_{\mu,\lambda} - \frac{1}{2}h^{\lambda}_{\lambda,\mu}\right)_{,\nu}$$

$R_{\mu\nu\rho\lambda} \sim g_{\mu\nu\rho\lambda}$ etc

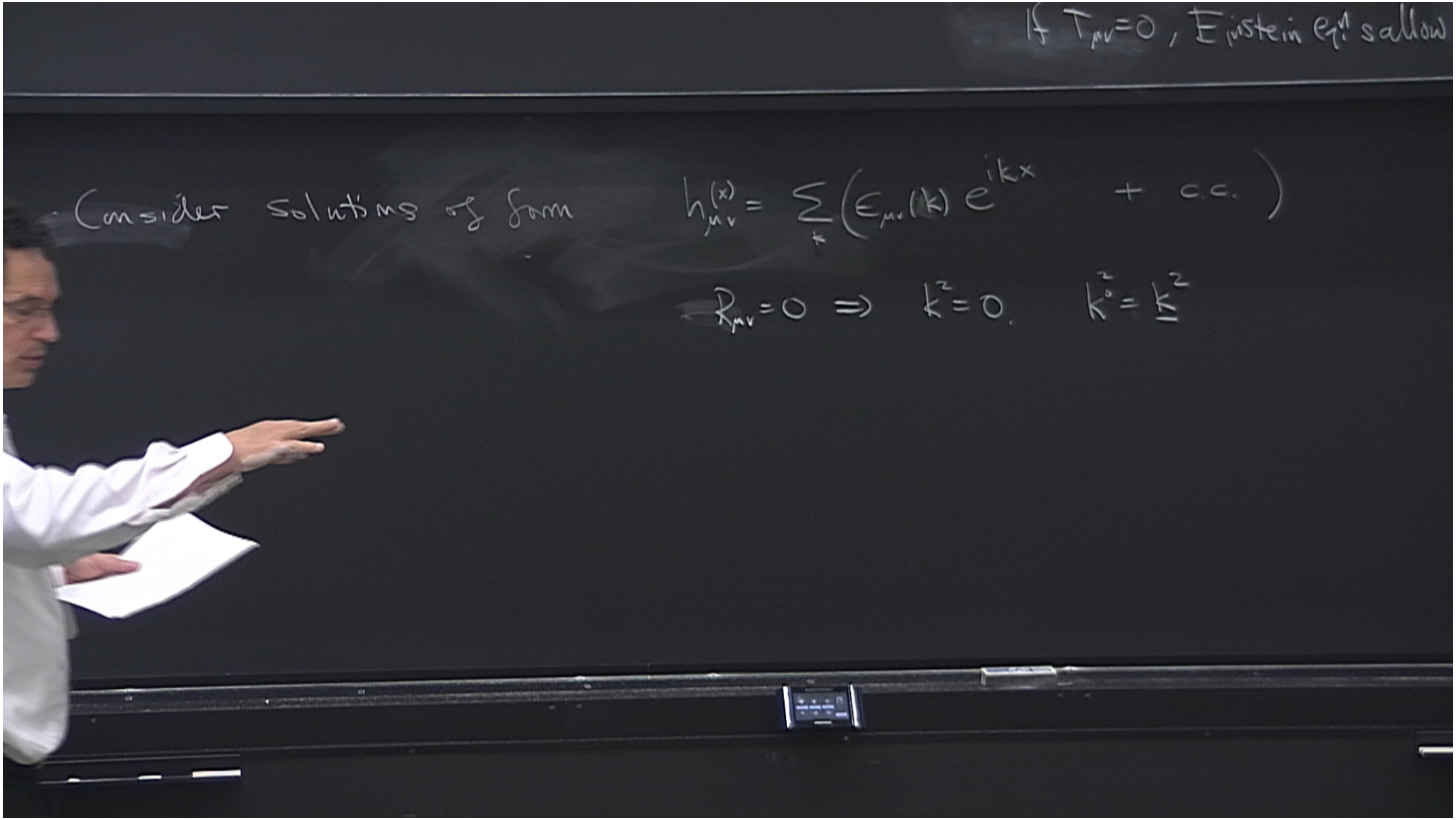

$|h_{\mu\nu}| \ll 1$
: in harmonic gauge $g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} = 0 \Rightarrow h^{\lambda}_{\nu,\lambda} - \frac{1}{2} h^{\lambda}_{\lambda,\nu} = 0$


$|h_{\mu\nu}| \ll 1$
; in harmonic gauge $g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} = 0 \Rightarrow h^{\lambda}_{\nu,\lambda} - \frac{1}{2}h^{\lambda}_{\lambda,\nu} = 0$
$\Rightarrow R_{\mu\nu} = -\frac{1}{2}\Box h_{\mu\nu}$ (to first order)
If $T_{\mu\nu}=0$, Einstein eqn's allow waves propagating at speed of light!


If $T_{\mu\nu}=0$, Einstein eq$^{n}$ s allow
nsider solutions of form $h_{\mu\nu} = \sum_{k} \left( \epsilon_{\mu\nu}(k)\, e^{ikx} + \text{c.c.} \right)$

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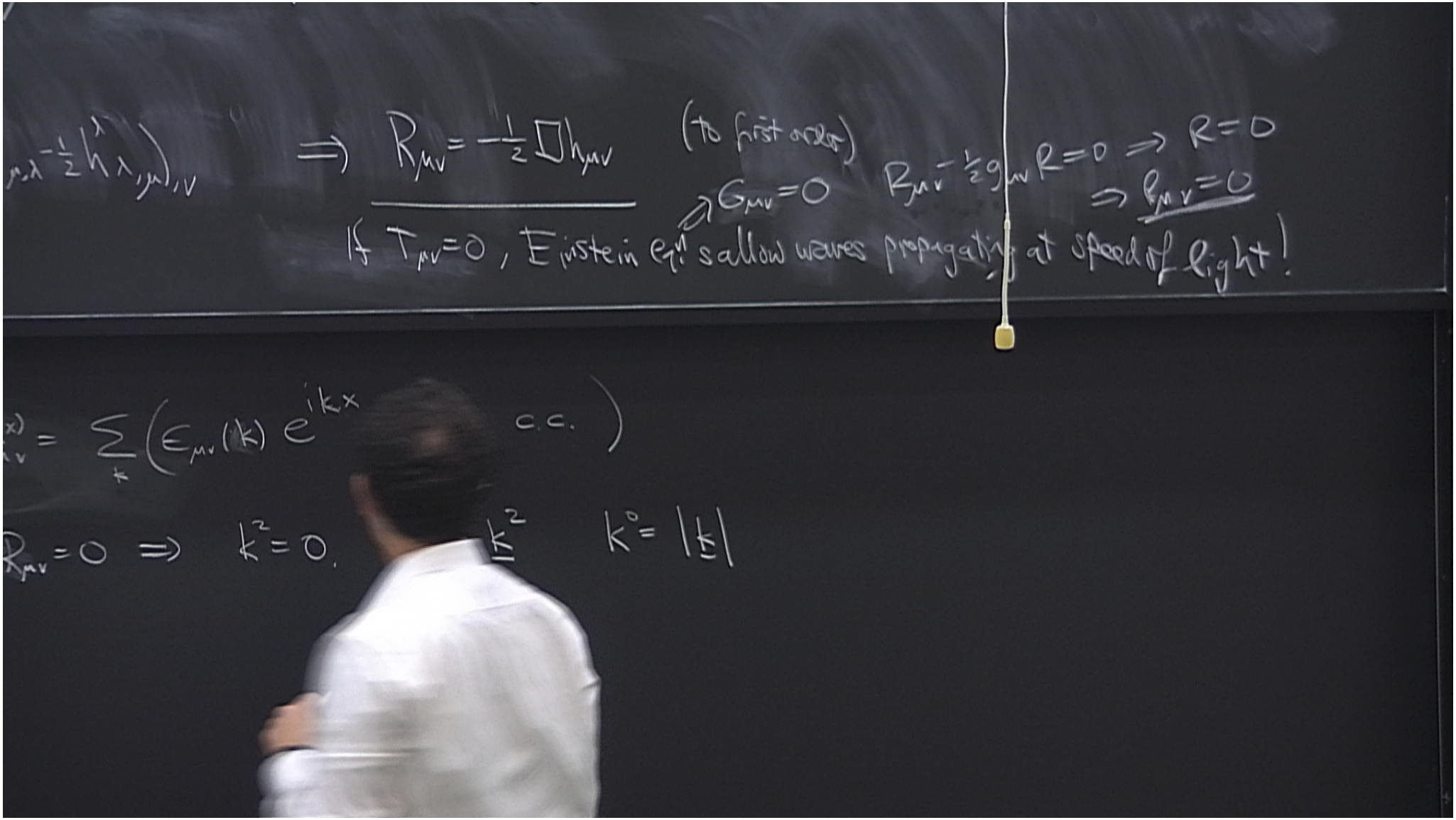
Consider solutions of form $h^{(x)}_{\mu\nu} = \sum_{k} \left( \epsilon_{\mu\nu}(k)\, e^{ikx} + \text{c.c.} \right)$

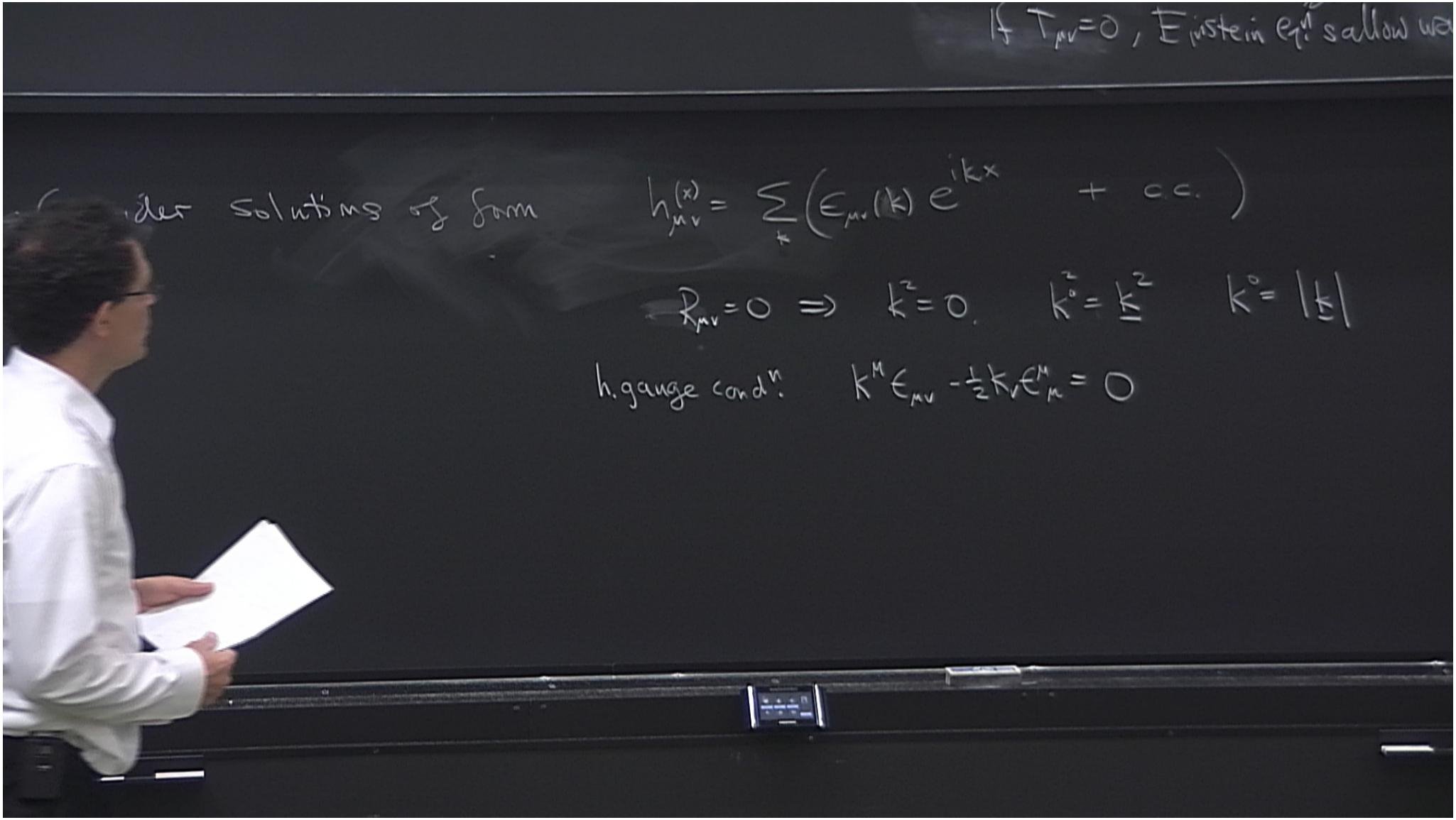

If $T_{\mu\nu}=0$, Einstein eq$^{n}$ s allow
Consider solutions of form $h^{(x)}_{\mu\nu} = \sum_{k} \left( \epsilon_{\mu\nu}(k)\, e^{ikx} + \text{c.c.} \right)$
$R_{\mu\nu}=0 \Rightarrow k^2=0$. $k^0 = |\vec{k}|$

If $T_{\mu\nu}=0$, Einstein eq$^{n}$s allow

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$\Rightarrow R_{\mu\nu} = -\frac{1}{2}\Box h_{\mu\nu}$ (to first order)
$\Rightarrow G_{\mu\nu} = 0$
$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \Rightarrow R = 0$
$\Rightarrow R_{\mu\nu} = 0$
If $T_{\mu\nu} = 0$, Einstein eqn's allow waves propagating at speed of light!
$= \sum_k \left( \epsilon_{\mu\nu}(k)\, e^{ik\cdot x} \quad \text{c.c.} \right)$
$R_{\mu\nu} = 0 \Rightarrow k^2 = 0.$
$\underline{k}^2$
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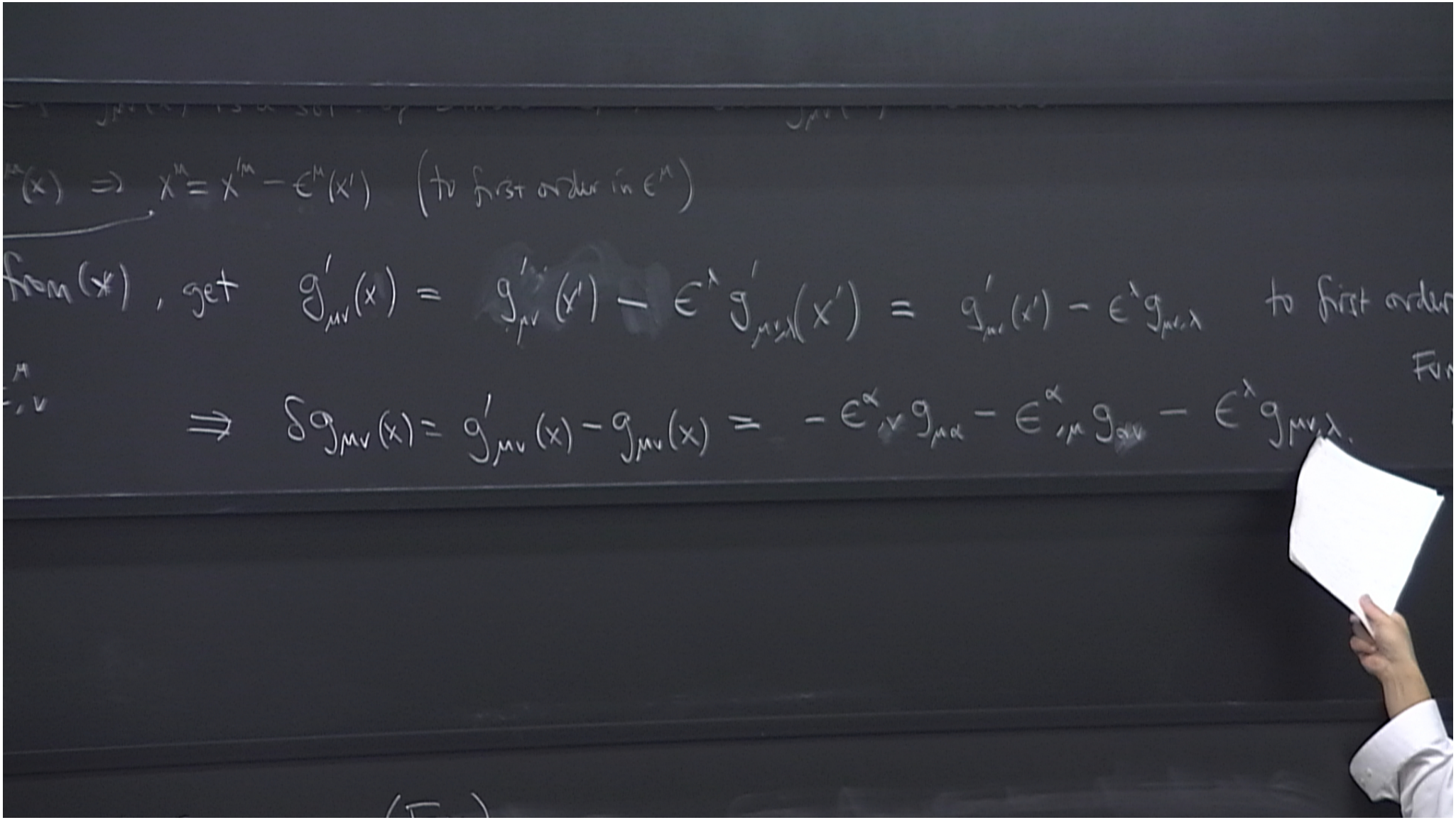
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h. gauge cond$^{n}$: $k^\mu \epsilon_{\mu\nu} - \frac{1}{2} k_\nu \epsilon^\mu{}_\mu = 0$

Still have gauge freedom: $h'_{\mu\nu} = h_{\mu\nu} - \epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}$


$x^\mu(x) \Rightarrow x^\mu = x'^\mu - \epsilon^\mu(x')$ (to first order in $\epsilon^\mu$)
from (*), get $g'_{\mu\nu}(x) = g'_{\mu\nu}(x') - \epsilon^\lambda g'_{\mu\nu,\lambda}(x') = g'_{\mu\nu}(x') - \epsilon^\lambda g_{\mu\nu,\lambda}$ to first order
$\Rightarrow \delta g_{\mu\nu}(x) = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = -\epsilon^\alpha{}_{,\nu} g_{\mu\alpha} - \epsilon^\alpha{}_{,\mu} g_{\alpha\nu} - \epsilon^\lambda g_{\mu\nu,\lambda}$.

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(treat $\epsilon$ as of order $h$) $\quad - \epsilon^{\lambda}{}_{,\nu\lambda}$

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$- \epsilon^{\lambda}{}_{\nu,\lambda} - \epsilon$

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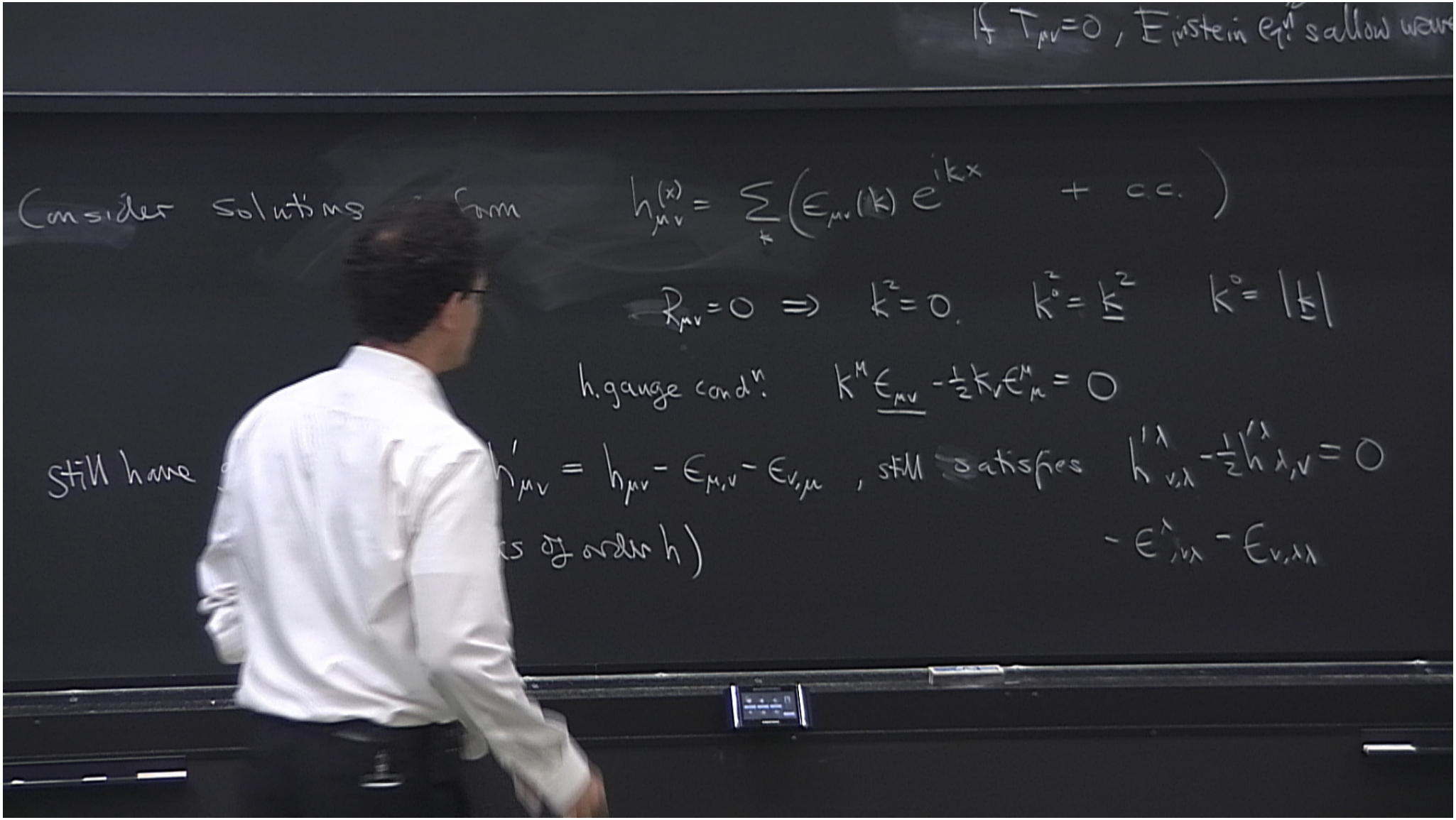
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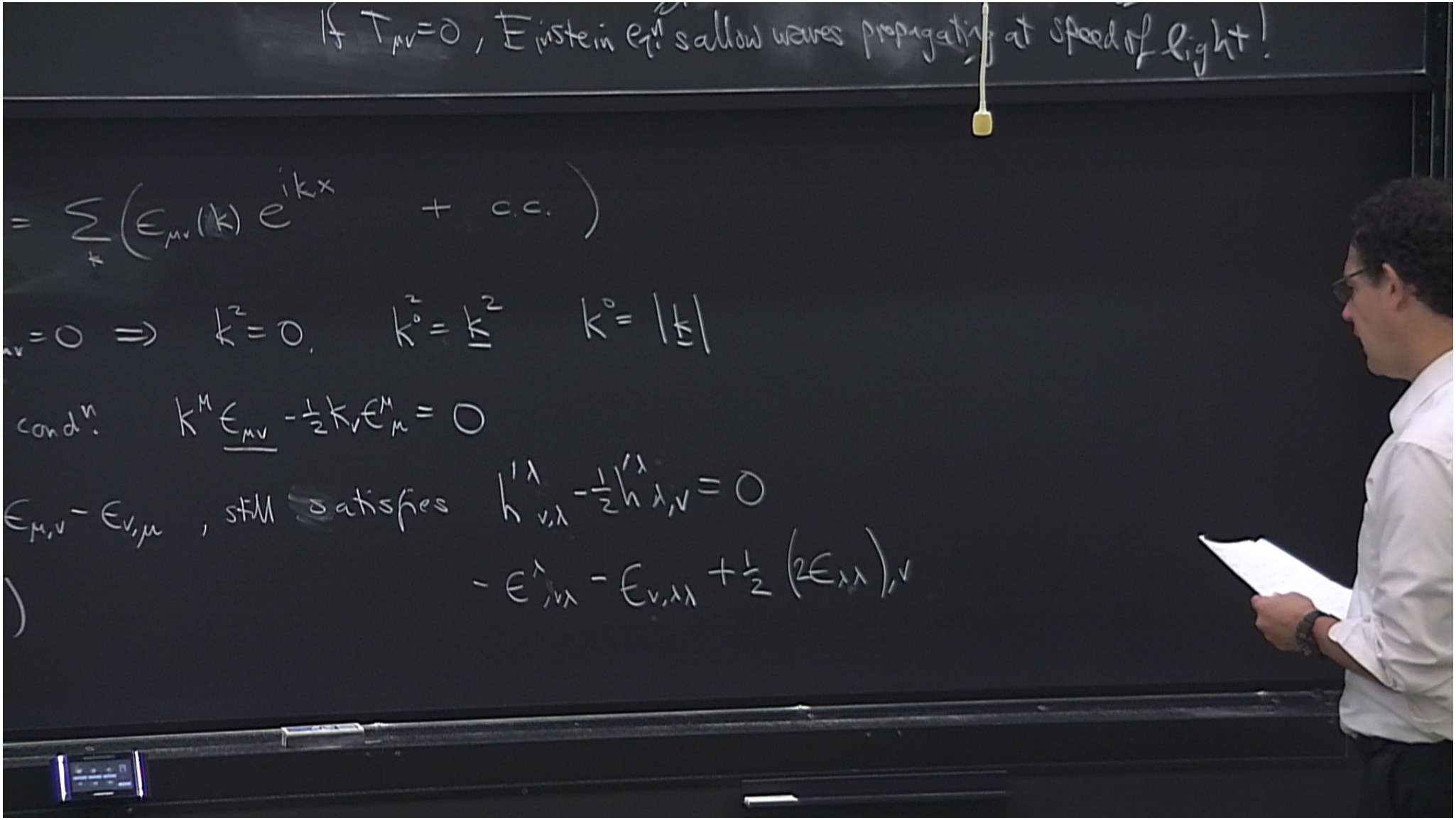
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If $T_{\mu\nu}=0$, Einstein eq$^{n}$ allow wave
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s of order h)


If $T_{\mu\nu}=0$, Einstein eq$^{n}$: sallow waves propagating at speed of light!
$= \sum_{k} \left( \epsilon_{\mu\nu}(k)\, e^{ikx} + \text{c.c.} \right)$
$\ldots = 0 \Rightarrow k^2 = 0, \quad k_0^2 = \underline{k}^2 \quad k^0 = |\underline{k}|$
cond$^{n}$: $k^{\mu}\epsilon_{\mu\nu} - \frac{1}{2}k_{\nu}\epsilon^{\mu}{}_{\mu} = 0$
$\epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}$, still satisfies $h'^{\lambda}_{\nu,\lambda} - \frac{1}{2}h'^{\lambda}_{\lambda,\nu} = 0$
$- \epsilon^{\lambda}_{\nu,\lambda} - \epsilon_{\nu,\lambda\lambda} + \frac{1}{2}(2\epsilon_{\lambda\lambda})_{,\nu}$

If $T_{\mu\nu}=0$, Einstein eq$^{n}$: allow waves propagating at speed of light!

$$= \sum_{k} \left( \epsilon_{\mu\nu}(k)\, e^{ikx} + \text{c.c.} \right)$$

$\ldots = 0 \Rightarrow k^2 = 0, \quad k_0^2 = \underline{k}^2 \quad k^0 = |\underline{k}|$

cond$^{n}$: $k^\mu \epsilon_{\mu\nu} - \frac{1}{2} k_\nu \epsilon^\mu{}_\mu = 0$

$\epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}$, still satisfies $h'^{\lambda}_{\nu,\lambda} - \frac{1}{2} h'^{\lambda}_{\lambda,\nu} = 0$

$$-\epsilon'_{,\nu\lambda} - \epsilon_{\nu,\lambda\lambda} + \tfrac{1}{2}(2\epsilon_{\lambda\lambda})_{,\nu} = -\Box \epsilon_\nu$$

If $T_{\mu\nu}=0$, Einstein eq$^{n}$: allow waves propagating at speed

of form $h^{(x)}_{\mu\nu} = \sum_{k} \left( \epsilon_{\mu\nu}(k)\, e^{ikx} + \text{c.c.} \right)$

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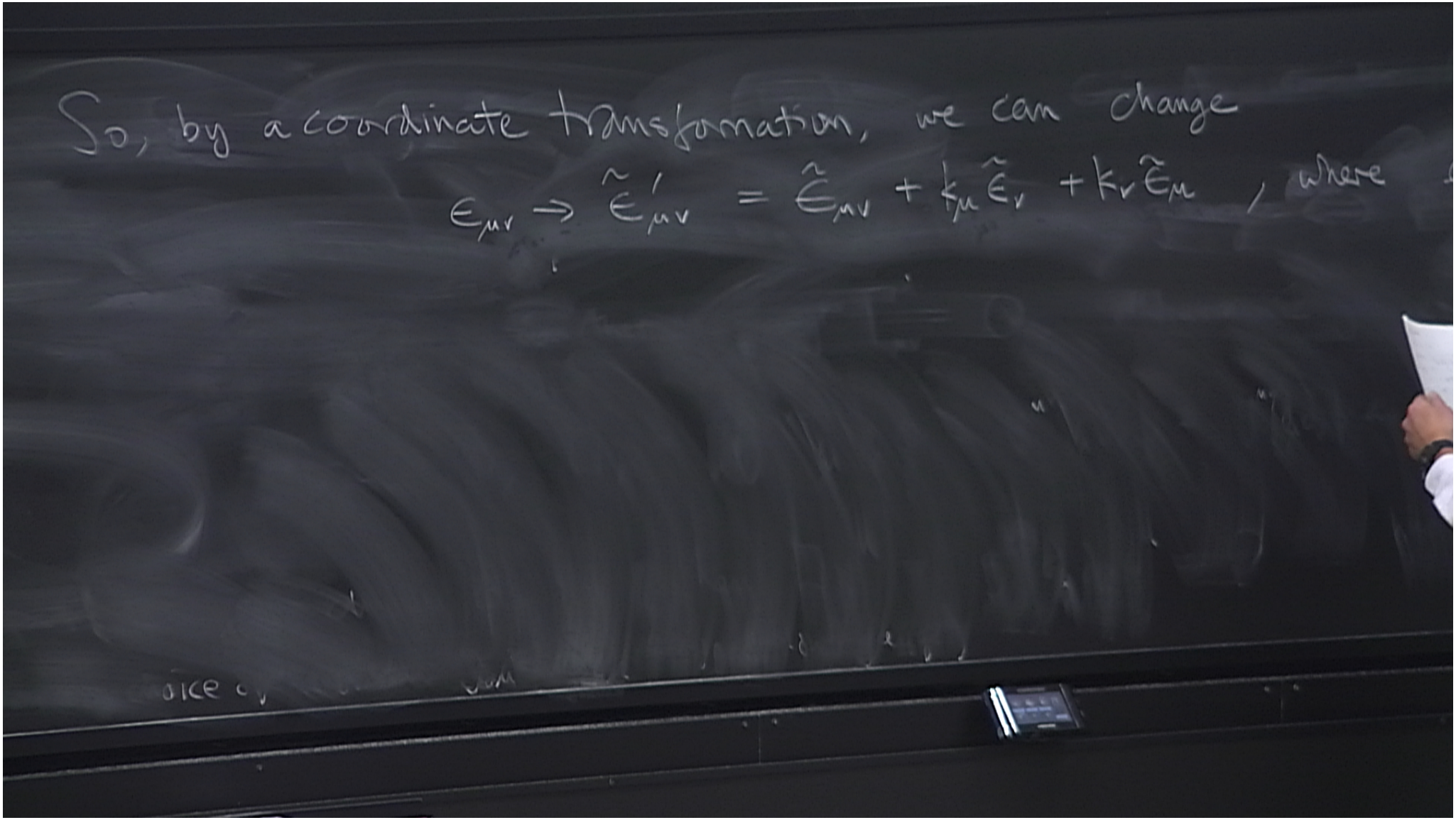
h. gauge cond$^{n}$: $k^\mu \underline{\epsilon_{\mu\nu}} - \frac{1}{2} k_\nu \epsilon^\mu{}_\mu = 0$

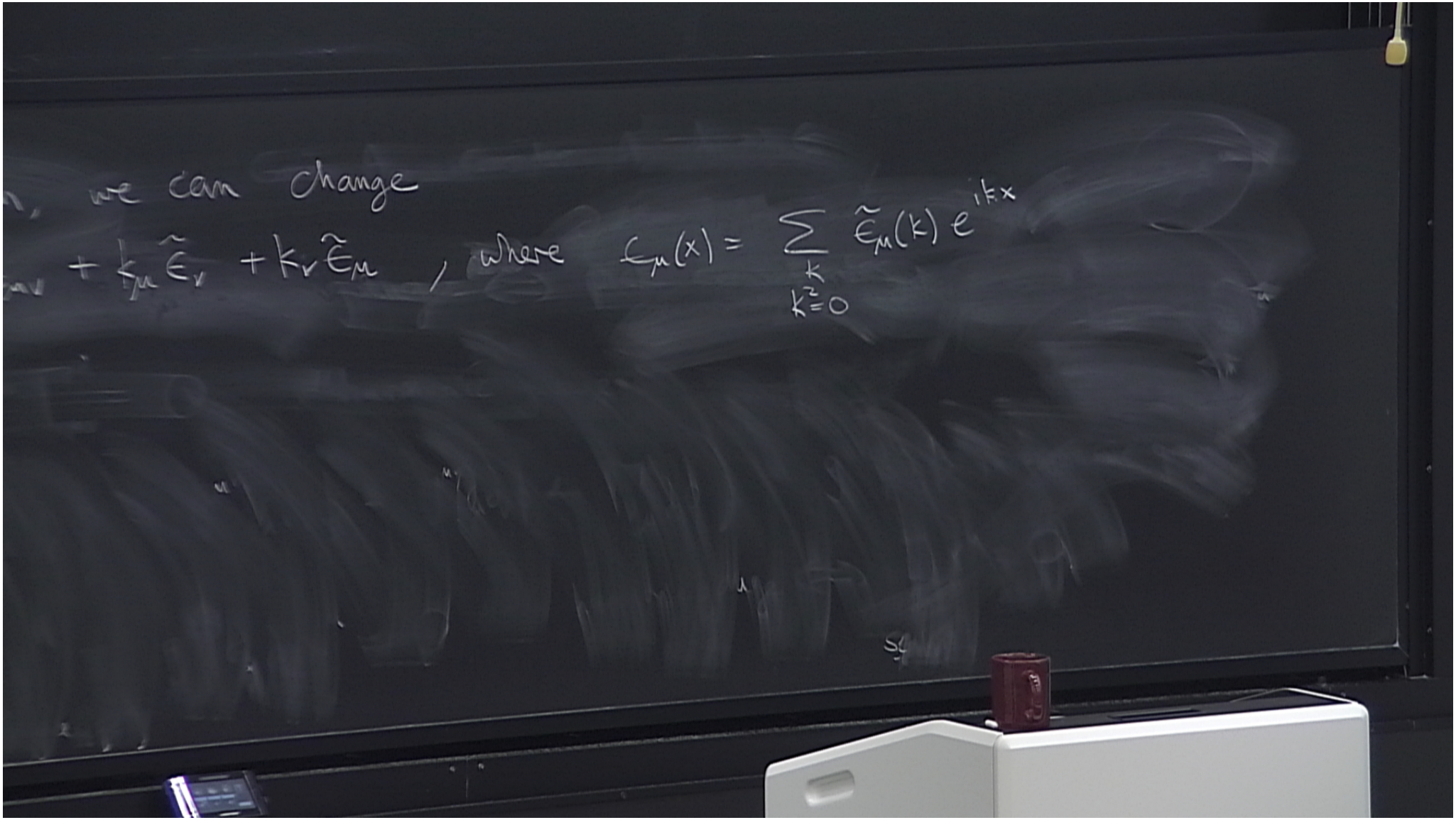
$h'_{\mu\nu} = h_{\mu\nu} - \epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}$, still satisfies $h'^{\lambda}_{\nu,\lambda} - \frac{1}{2} h'^{\lambda}_{\lambda,\nu} = 0$

(at $\epsilon$ as of order $h$)

$- \epsilon^{\lambda}_{\nu,\lambda\lambda} - \epsilon_{\nu,\lambda\lambda} + \frac{1}{2} (2\epsilon_{\lambda\lambda})_{,\nu} = - \Box \epsilon_\nu$

if $\underline{\Box \epsilon_\nu = 0}$


So, by a coordinate transformation, we can change
$\epsilon_{\mu\nu} \Rightarrow \tilde{\epsilon}'_{\mu\nu} = \tilde{\epsilon}_{\mu\nu} + k_\mu \tilde{\epsilon}_\nu + k_\nu \tilde{\epsilon}_\mu$ , where


we can change
$+k_\mu \tilde{\epsilon}_\nu + k_\nu \tilde{\epsilon}_\mu$ , where $\epsilon_\mu(x) = \sum_{\substack{k \\ k^2=0}} \tilde{\epsilon}_\mu(k)\, e^{ik\cdot x}$

If $T_{\mu\nu}=0$, Einstein eq.ⁿ: sallow waves propagating at speed of

form $h^{(x)}_{\mu\nu} = \sum_k \left( \epsilon_{\mu\nu}(k)\, e^{ikx} + \text{c.c.} \right)$ (polarisation tensor)

$R_{\mu\nu}=0 \Rightarrow k^2=0 \quad k^0{}^2 = \underline{k}^2 \quad k^0 = |\underline{k}|$

h. gauge condⁿ: $k^\mu \epsilon_{\mu\nu} - \frac{1}{2} k_\nu \epsilon^\mu{}_\mu = 0$

$h'_{\mu\nu} = h_{\mu\nu} - \epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}$, still satisfies $h'^{\lambda}_{\nu,\lambda} - \frac{1}{2} h'^{\lambda}_{\lambda,\nu} = 0$

$\epsilon$ as of order $h$)

$-\epsilon^{\lambda}_{\nu,\lambda} - \epsilon_{\nu,\lambda\lambda} + \frac{1}{2}(2\epsilon_{\lambda\lambda})_{,\nu} = -\Box \epsilon_\nu$

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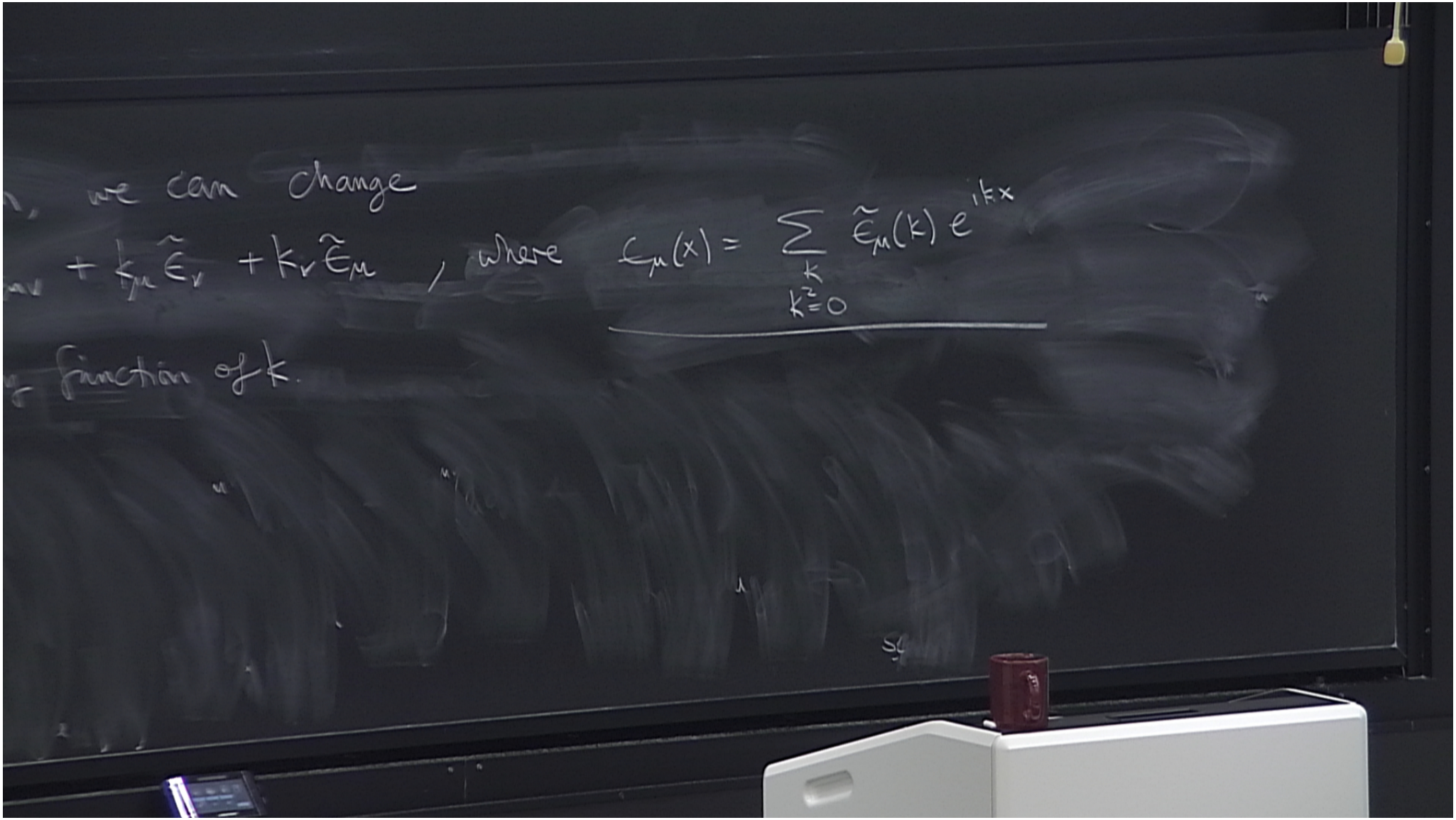
$-\epsilon^{\lambda}_{\nu,\lambda} - \epsilon_{\nu,\lambda\lambda} + \tfrac{1}{2}(2\epsilon_{\lambda\lambda})_{,\nu} = -\Box \epsilon_\nu$

if $\Box \epsilon_\nu = 0$

a coordinate transformation, we can change

$$\epsilon_{\mu\nu} \Rightarrow \tilde{\epsilon}'_{\mu\nu} = \tilde{\epsilon}_{\mu\nu} + k_\mu \tilde{\epsilon}_\nu + k_\nu \tilde{\epsilon}_\mu \,, \quad \text{where} \quad \zeta_\mu(x) = \sum_{k^2=0}$$

with $\tilde{\epsilon}_\mu$ being an arbitrary function of $k$.


we can change
$+ k_\mu \tilde{\epsilon}_\nu + k_\nu \tilde{\epsilon}_\mu$ , where $\epsilon_\mu(x) = \sum_{\substack{k \\ k^2=0}} \tilde{\epsilon}_\mu(k)\, e^{ikx}$
function of $k$.

binary pulsar PSR 1913+16
Hulse+Taylor

So, by a coordinate transformation, we can

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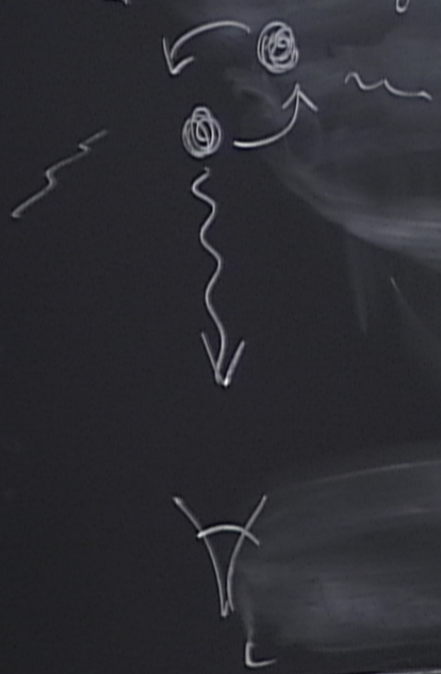


So, by a coordinate transformation, we can

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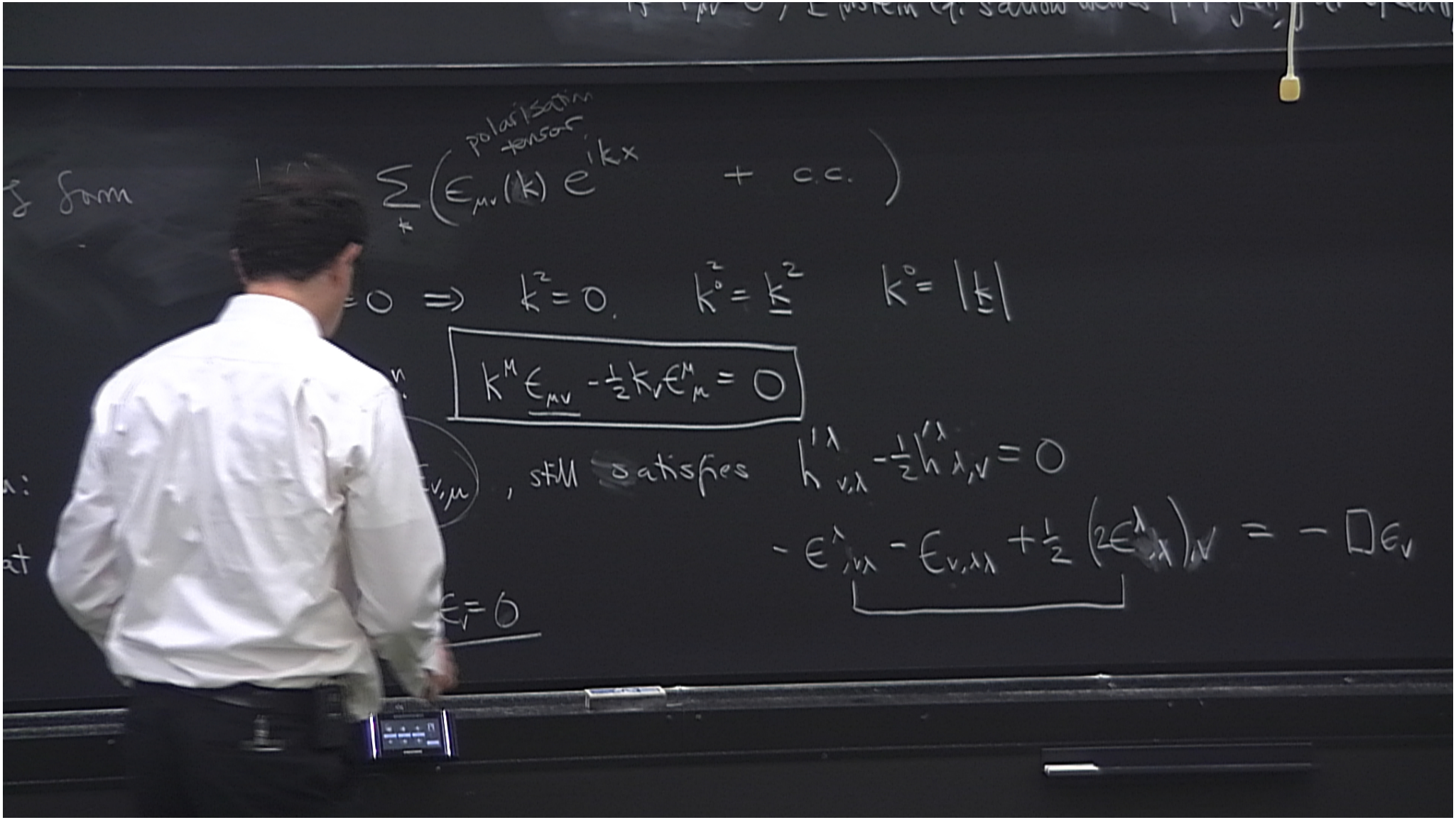
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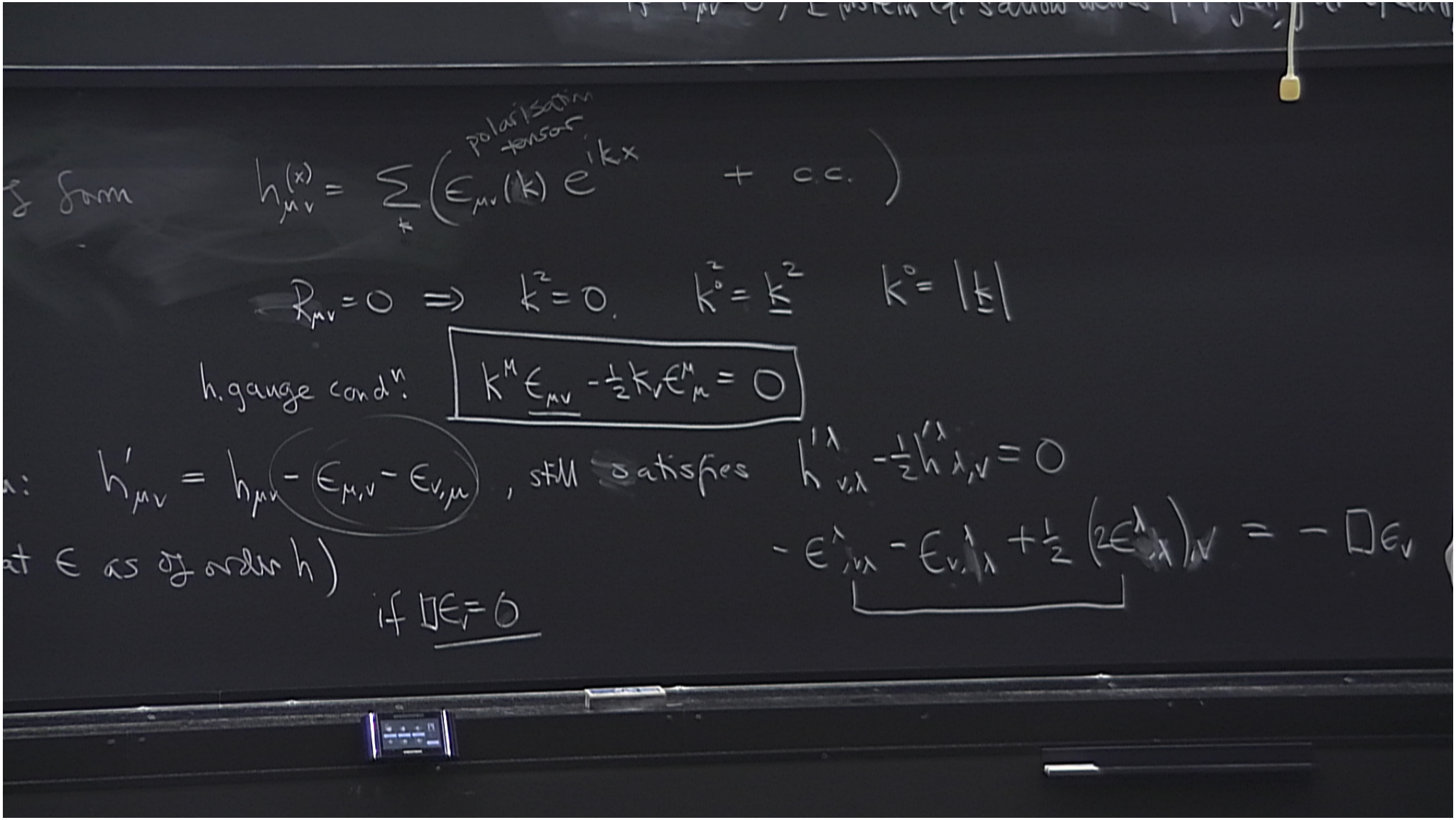


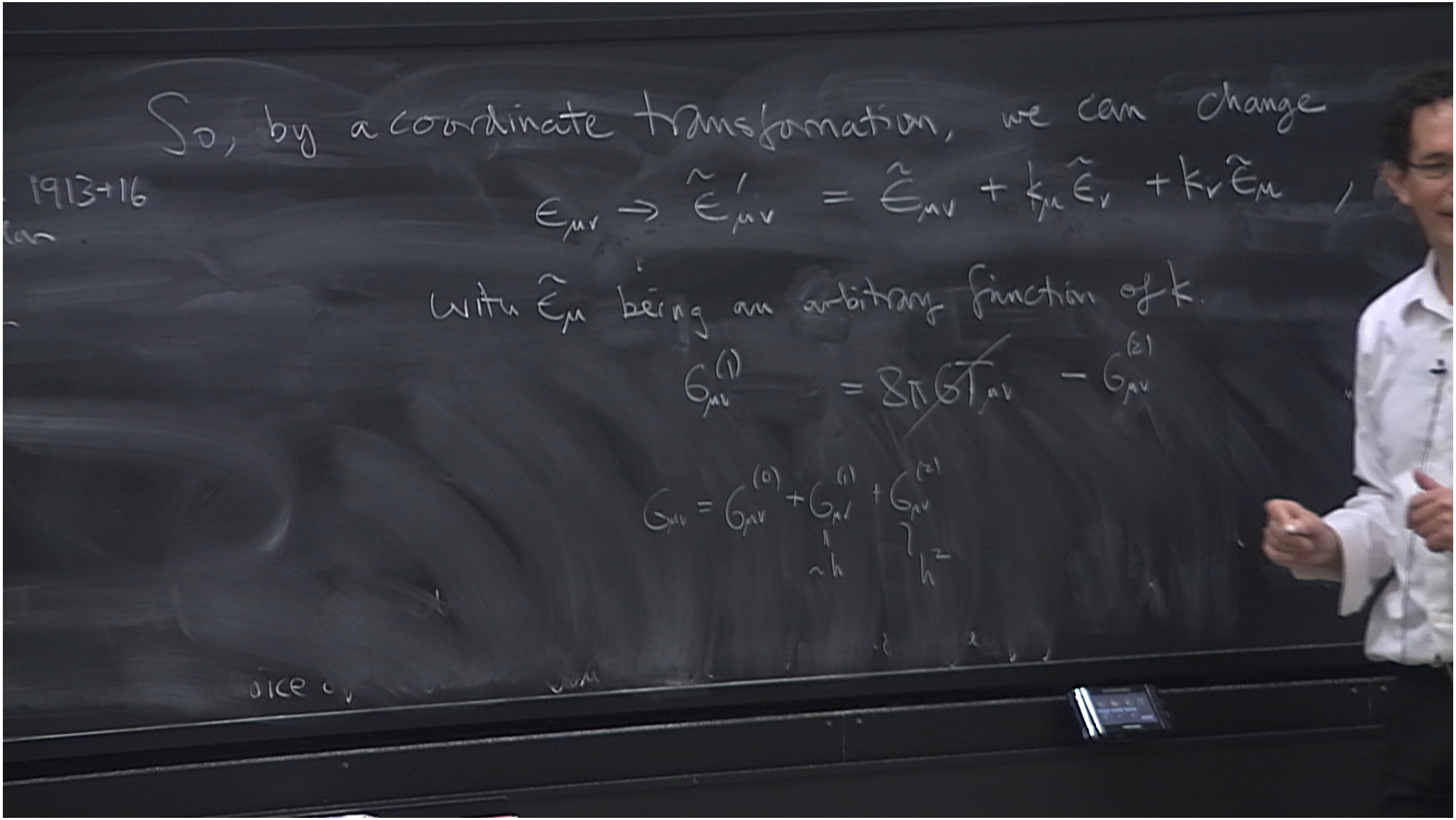
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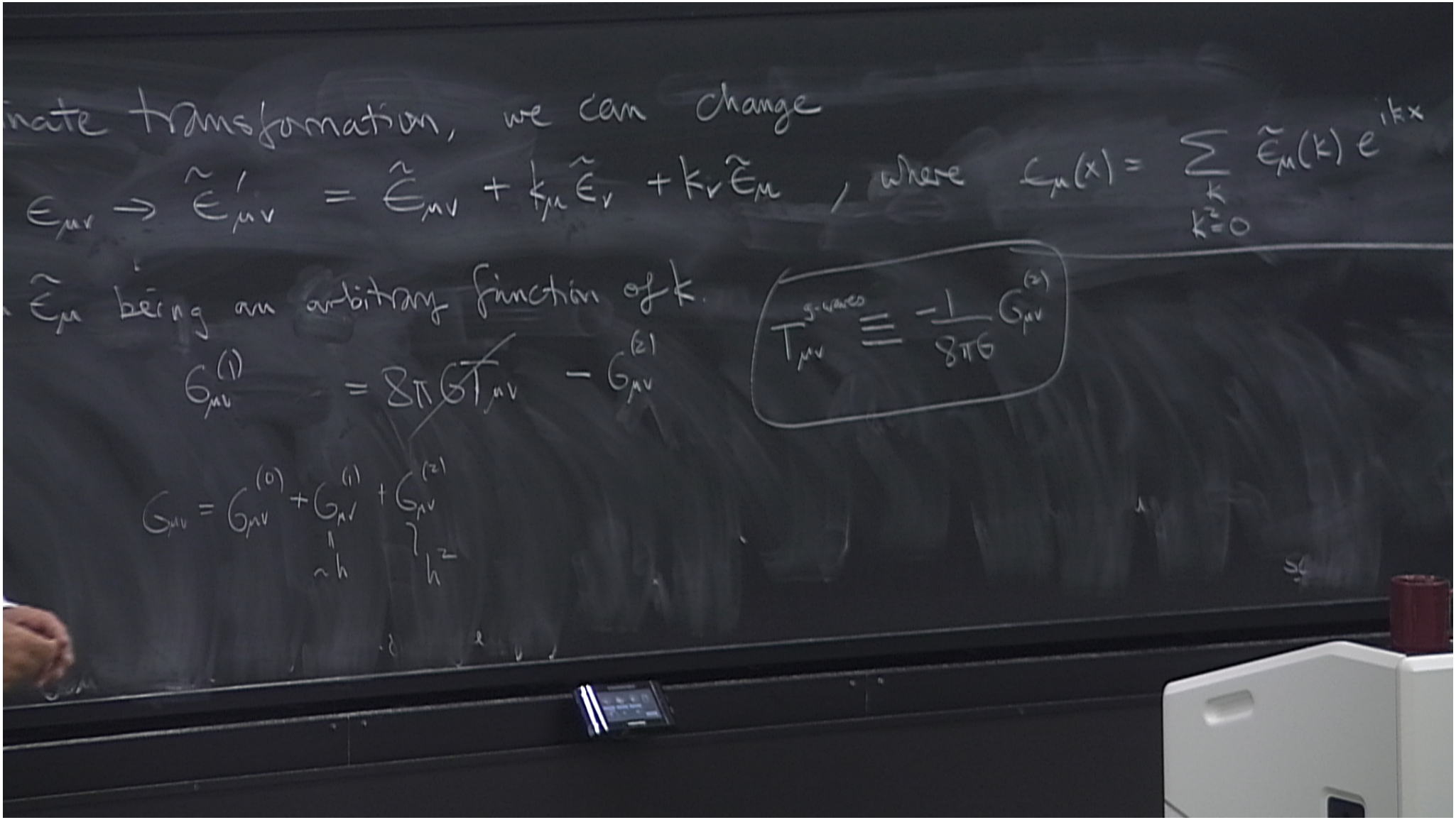
$$\epsilon_{\mu\nu} \Rightarrow \tilde{\epsilon}'_{\mu\nu} = \tilde{\epsilon}_{\mu\nu} + k_\mu \tilde{\epsilon}_\nu$$

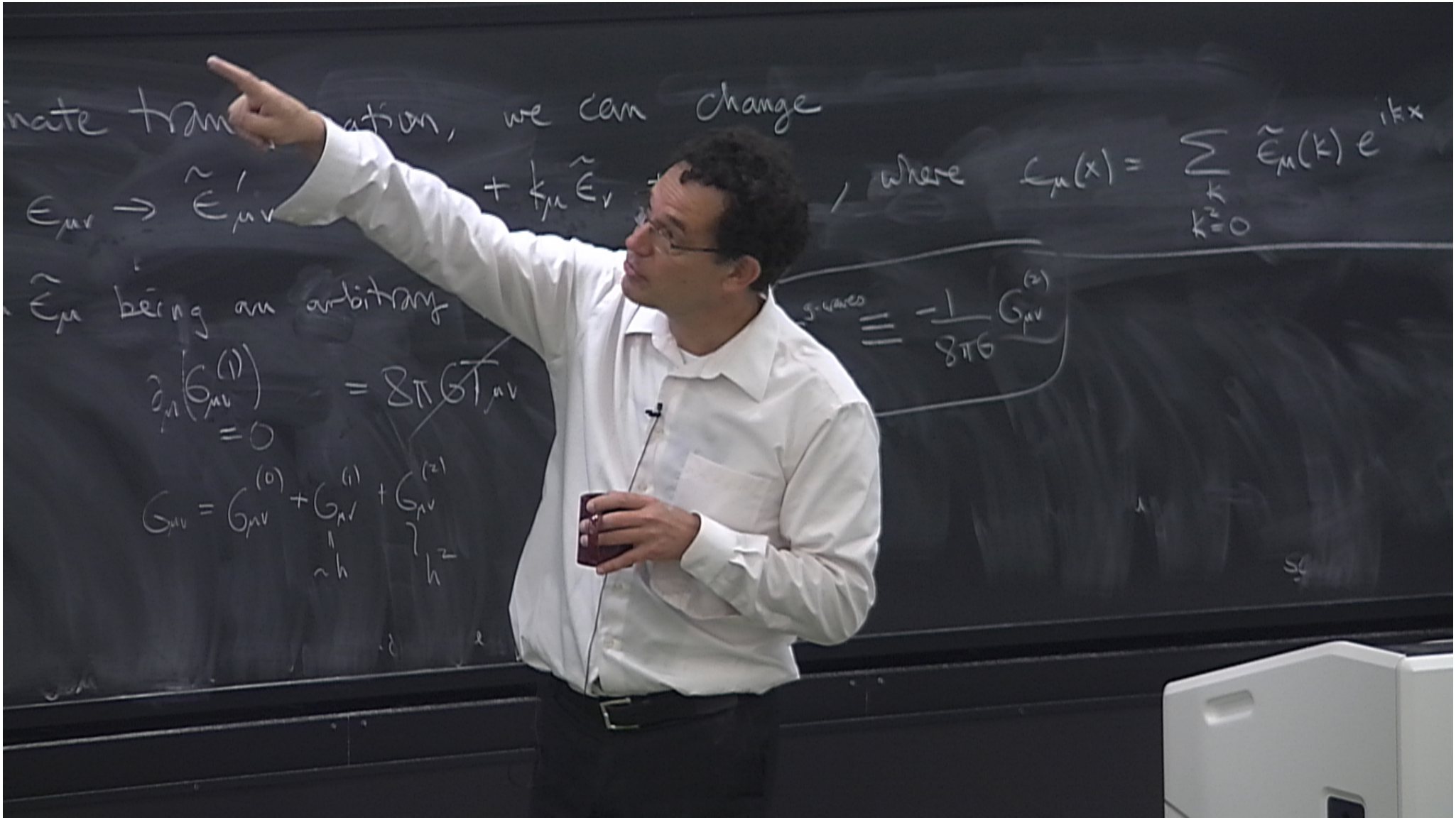
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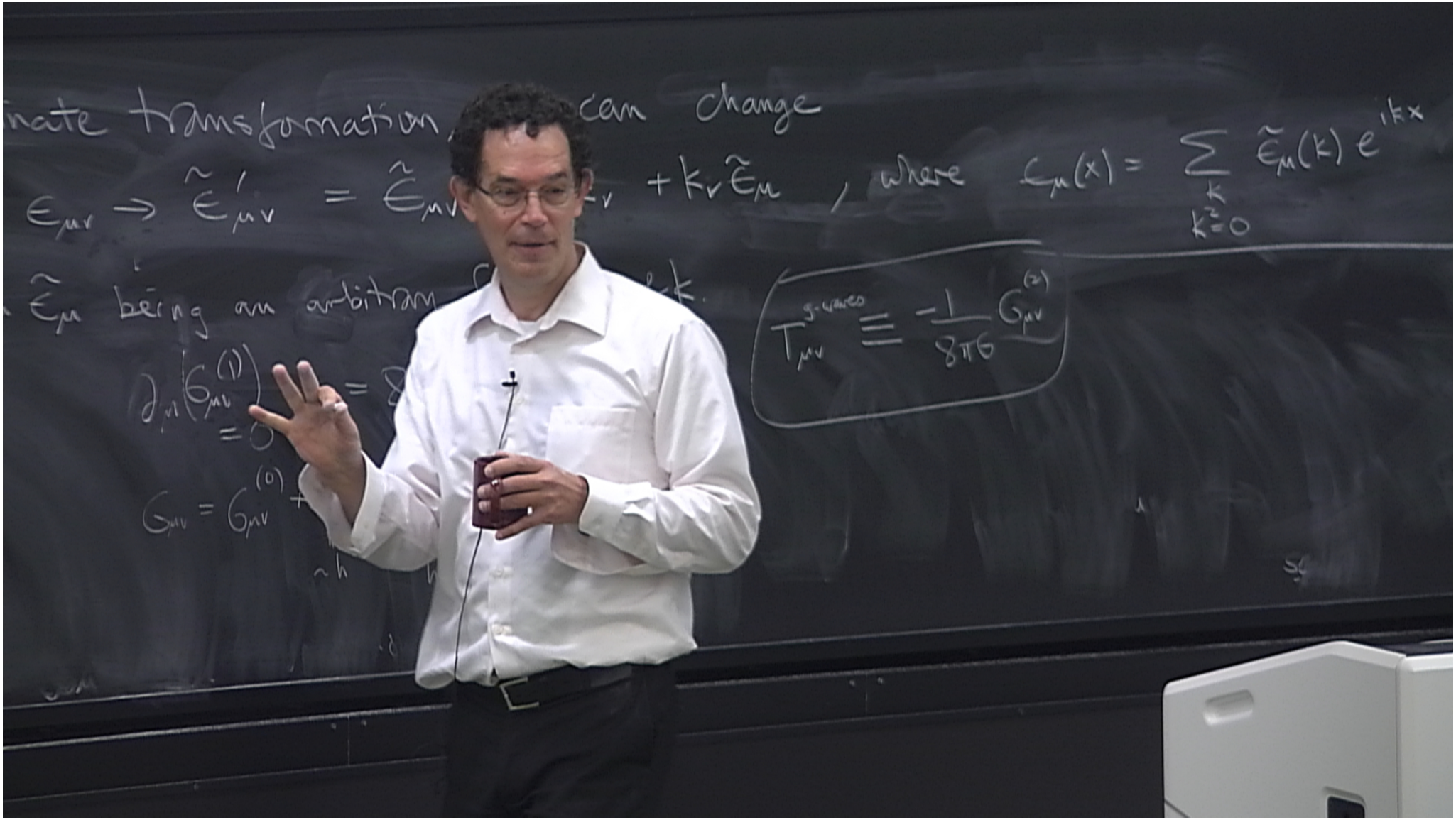

polarisation tensor
$\sum_k \left( \epsilon_{\mu\nu}(k)\, e^{ikx} + \text{c.c.} \right)$
$k^2 = 0 \Rightarrow k^2 = 0 \quad k^0 = |\underline{k}| \quad k^0 = |\underline{k}|$
$k^\mu \epsilon_{\mu\nu} - \frac{1}{2} k_\nu \epsilon^\mu{}_\mu = 0$
, still satisfies
$h'^\lambda_{\nu,\lambda} - \frac{1}{2} h'^\lambda_{,\lambda\nu} = 0$
$-\epsilon^\lambda_{\nu,\lambda\lambda} - \epsilon_{\nu,\lambda\lambda} + \frac{1}{2}\left(2\epsilon^\lambda_{\nu,\lambda}\right)_{,\nu} = -\Box \epsilon_\nu$

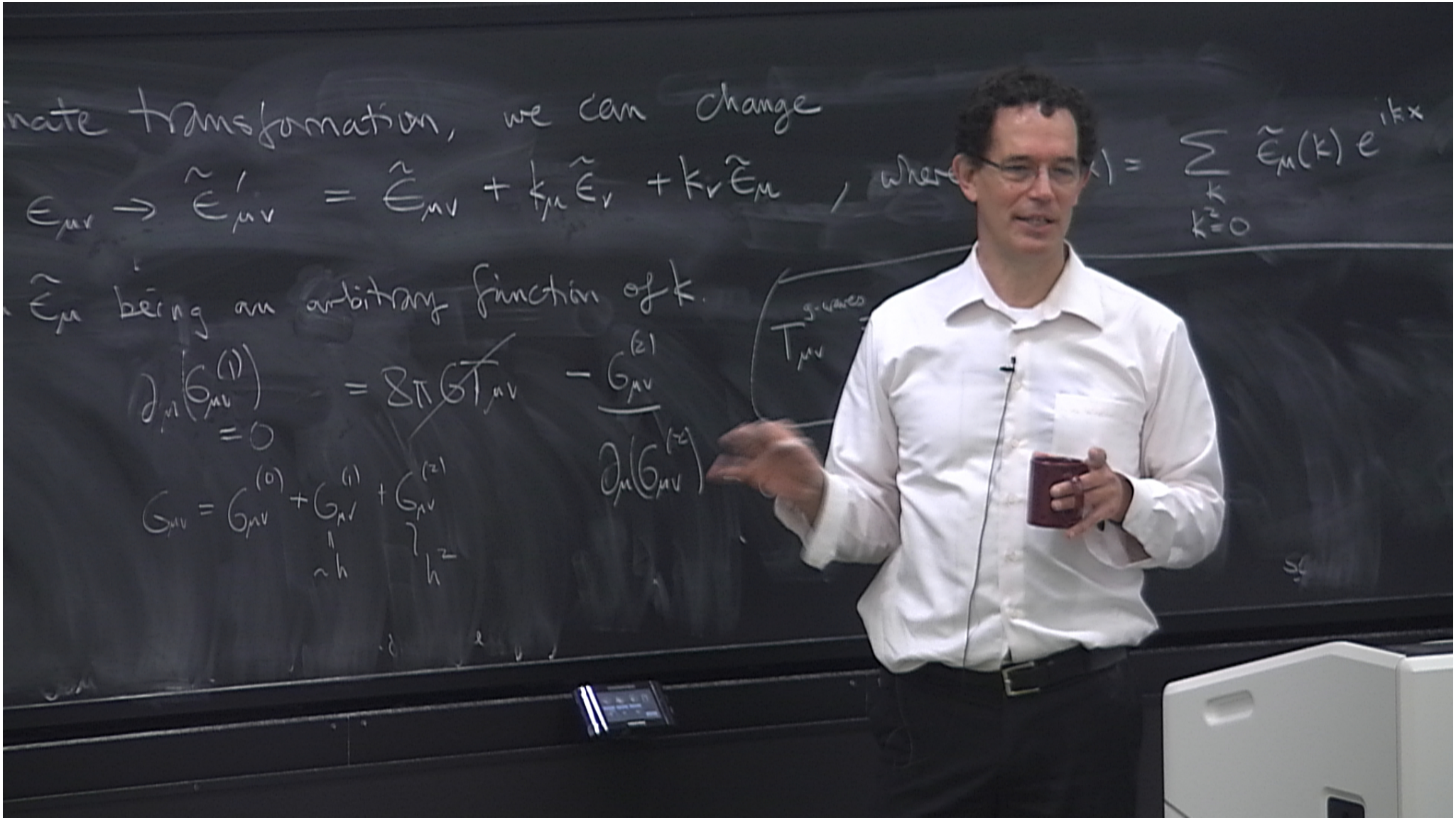

polarisation tensor
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$-\epsilon^\lambda{}_{,\nu\lambda} - \epsilon_{\nu,\lambda}{}^{,\lambda} + \frac{1}{2}(2\epsilon^\lambda{}_{,\lambda})_{,\nu} = -\Box \epsilon_\nu$
if $\Box \epsilon_\nu = 0$

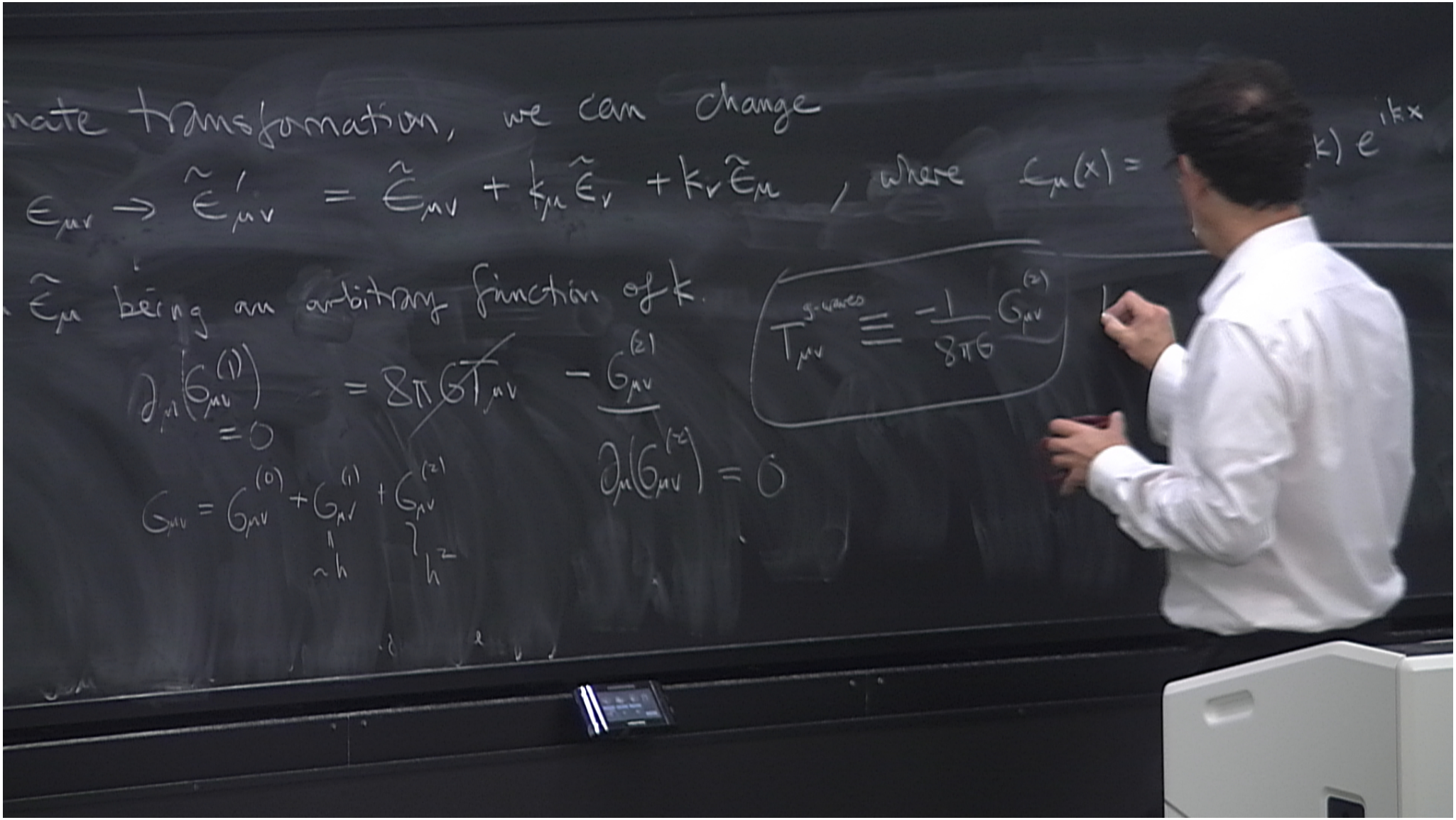

So, by a coordinate transformation, we can change
1913+16
$\epsilon_{\mu\nu} \Rightarrow \tilde{\epsilon}'_{\mu\nu} = \tilde{\epsilon}_{\mu\nu} + k_\mu \tilde{\epsilon}_\nu + k_\nu \tilde{\epsilon}_\mu$ ,
with $\tilde{\epsilon}_\mu$ being an arbitrary function of k.
$G^{(1)}_{\mu\nu} = 8\pi G T_{\mu\nu} - G^{(2)}_{\mu\nu}$
$G_{\mu\nu} = G^{(0)}_{\mu\nu} + G^{(1)}_{\mu\nu} + G^{(2)}_{\mu\nu}$
$\sim h$
$h^2$

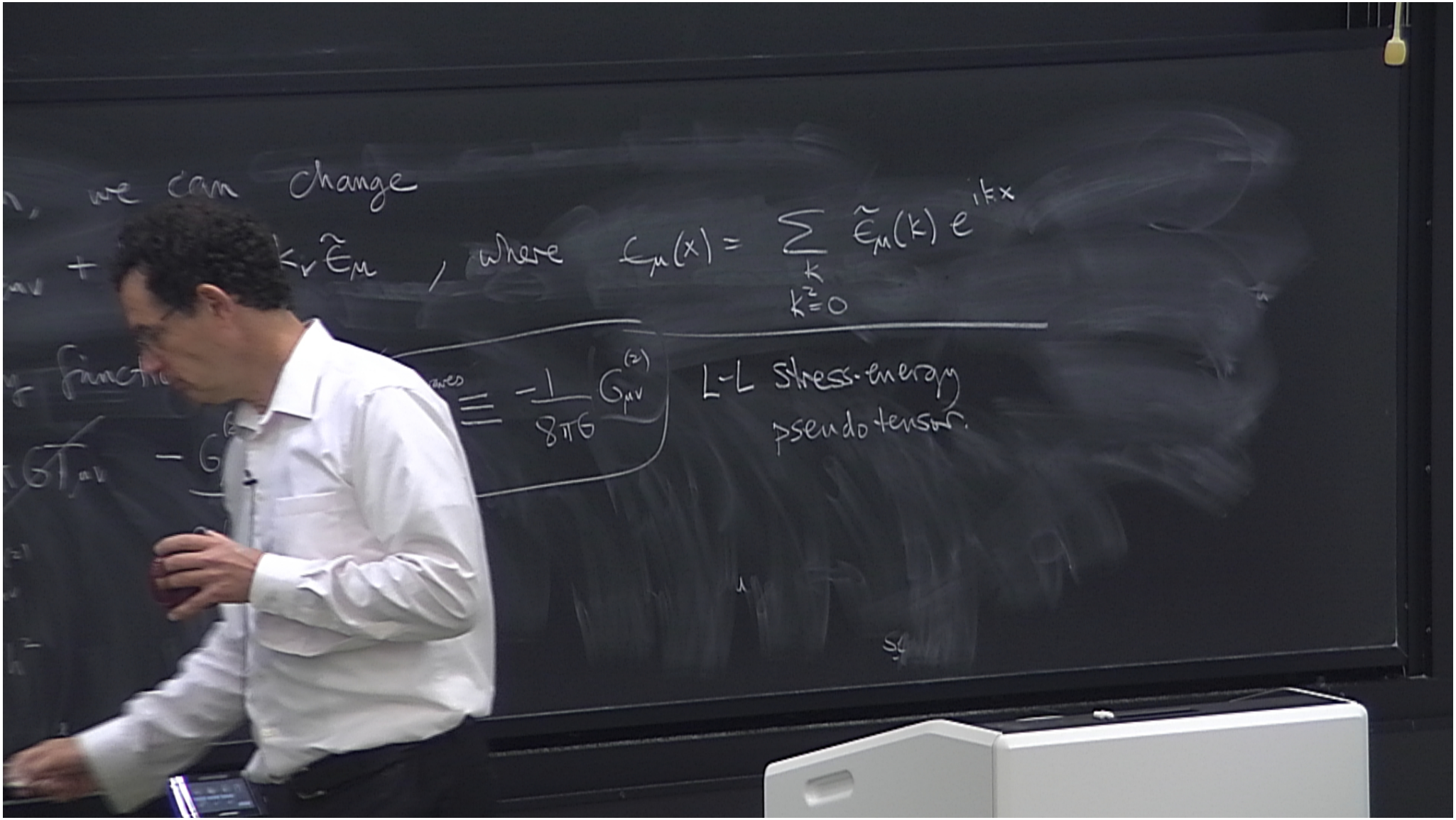

nate transformation, we can change
$\epsilon_{\mu\nu} \to \tilde{\epsilon}'_{\mu\nu} = \tilde{\epsilon}_{\mu\nu} + k_\mu \tilde{\epsilon}_\nu + k_\nu \tilde{\epsilon}_\mu$ , where $\epsilon_\mu(x) = \sum_{k,\ k^2=0} \tilde{\epsilon}_\mu(k)\, e^{ikx}$
$\tilde{\epsilon}_\mu$ being an arbitrary function of $k$.
$G^{(1)}_{\mu\nu} = 8\pi G T_{\mu\nu} - G^{(2)}_{\mu\nu}$
$T^{g\text{-waves}}_{\mu\nu} \equiv -\frac{1}{8\pi G} G^{(2)}_{\mu\nu}$
$G_{\mu\nu} = G^{(0)}_{\mu\nu} + G^{(1)}_{\mu\nu} + G^{(2)}_{\mu\nu}$
$\sim h$
$h^2$


nate tran ation, we can change
$\epsilon_{\mu\nu} \Rightarrow \tilde{\epsilon}'_{\mu\nu}$
$+ k_\mu \tilde{\epsilon}_\nu$
where $\epsilon_\mu(x) = \sum_{k^2=0} \tilde{\epsilon}_\mu(k) e^{ikx}$
$\tilde{\epsilon}_\mu$ being an arbitrary
$\partial_\mu (G^{(1)}_{\mu\nu}) = 8\pi G T_{\mu\nu}$
$= 0$
g-waves $\equiv -\frac{1}{8\pi G} G^{(2)}_{\mu\nu}$
$G_{\mu\nu} = G^{(0)}_{\mu\nu} + G^{(1)}_{\mu\nu} + G^{(2)}_{\mu\nu}$
$\sim h$
$h^2$


$\epsilon_{\mu\nu} \to \tilde{\epsilon}'_{\mu\nu} = \tilde{\epsilon}_{\mu\nu} + k_\nu \tilde{\epsilon}_\mu$ , where $\epsilon_\mu(x) = \sum_k \tilde{\epsilon}_\mu(k) e^{ikx}$
$k^2 = 0$
$T^{\text{g-waves}}_{\mu\nu} \equiv \frac{-1}{8\pi G} G^{(2)}_{\mu\nu}$


nate transformation, we can change
$\epsilon_{\mu\nu} \to \tilde{\epsilon}'_{\mu\nu} = \tilde{\epsilon}_{\mu\nu} + k_\mu \tilde{\epsilon}_\nu + k_\nu \tilde{\epsilon}_\mu$ , where
$\sum_{k^2=0} \tilde{\epsilon}_\mu(k) e^{ikx}$
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$\partial_\mu(G^{(1)}_{\mu\nu}) = 8\pi G T_{\mu\nu} - G^{(2)}_{\mu\nu}$
$= 0$
$\partial_\mu(G^{(2)}_{\mu\nu})$
$G_{\mu\nu} = G^{(0)}_{\mu\nu} + G^{(1)}_{\mu\nu} + G^{(2)}_{\mu\nu}$
$\sim h$
$h^2$


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$\partial_\mu (G^{(1)}_{\mu\nu}) = 8\pi G T_{\mu\nu} - G^{(1)}_{\mu\nu}$
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$T^{\text{g-waves}}_{\mu\nu} \equiv \frac{-1}{8\pi G} G^{(2)}_{\mu\nu}$
$G_{\mu\nu} = G^{(0)}_{\mu\nu} + G^{(1)}_{\mu\nu} + G^{(2)}_{\mu\nu}$
$\sim h$ $h^2$
$\partial_\mu (G^{(2)}_{\mu\nu}) = 0$


we can change
where
L-L stress-energy
pseudo-tensor.