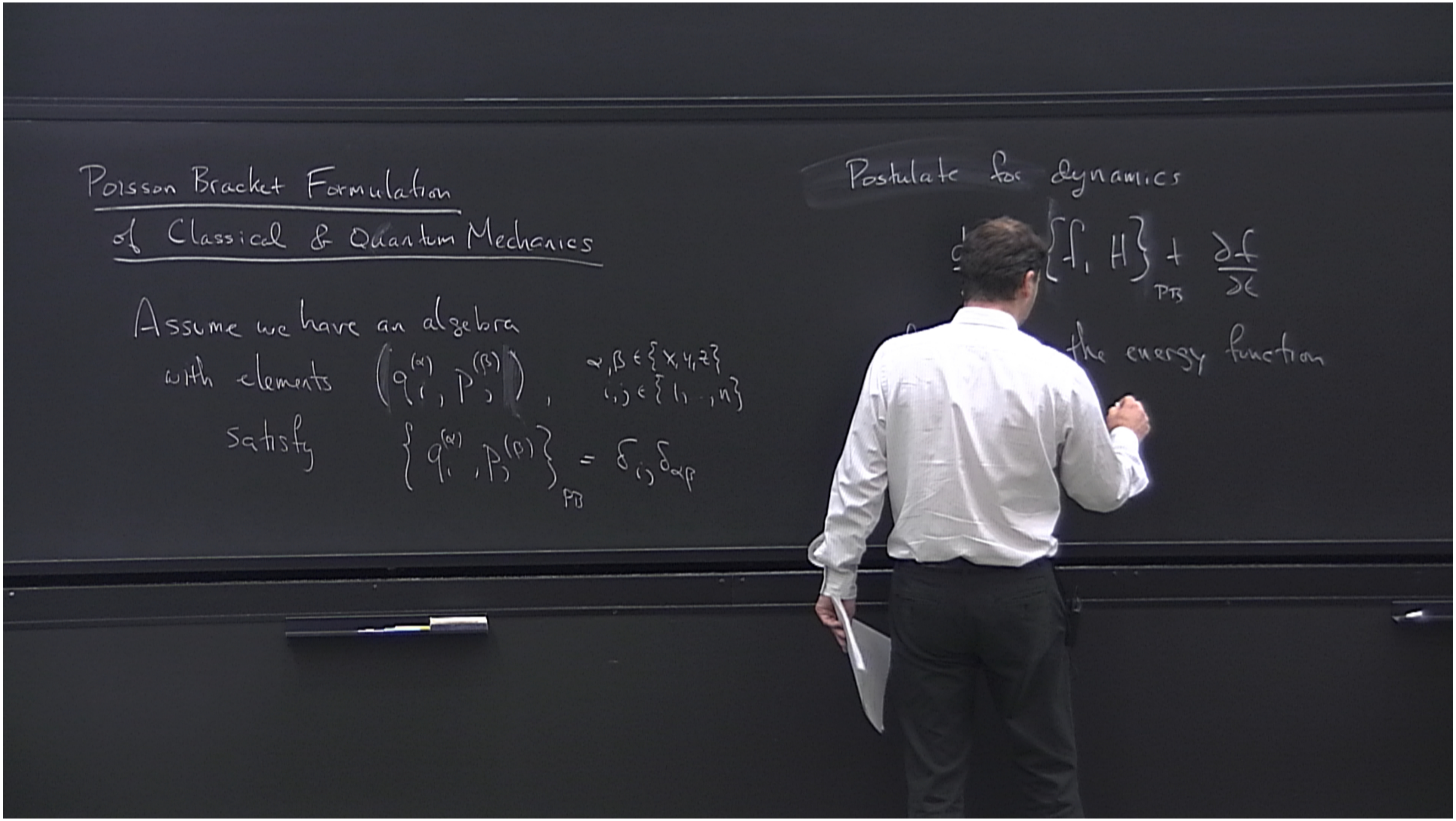
Title: 14/15 PSI Quantum Theory-15

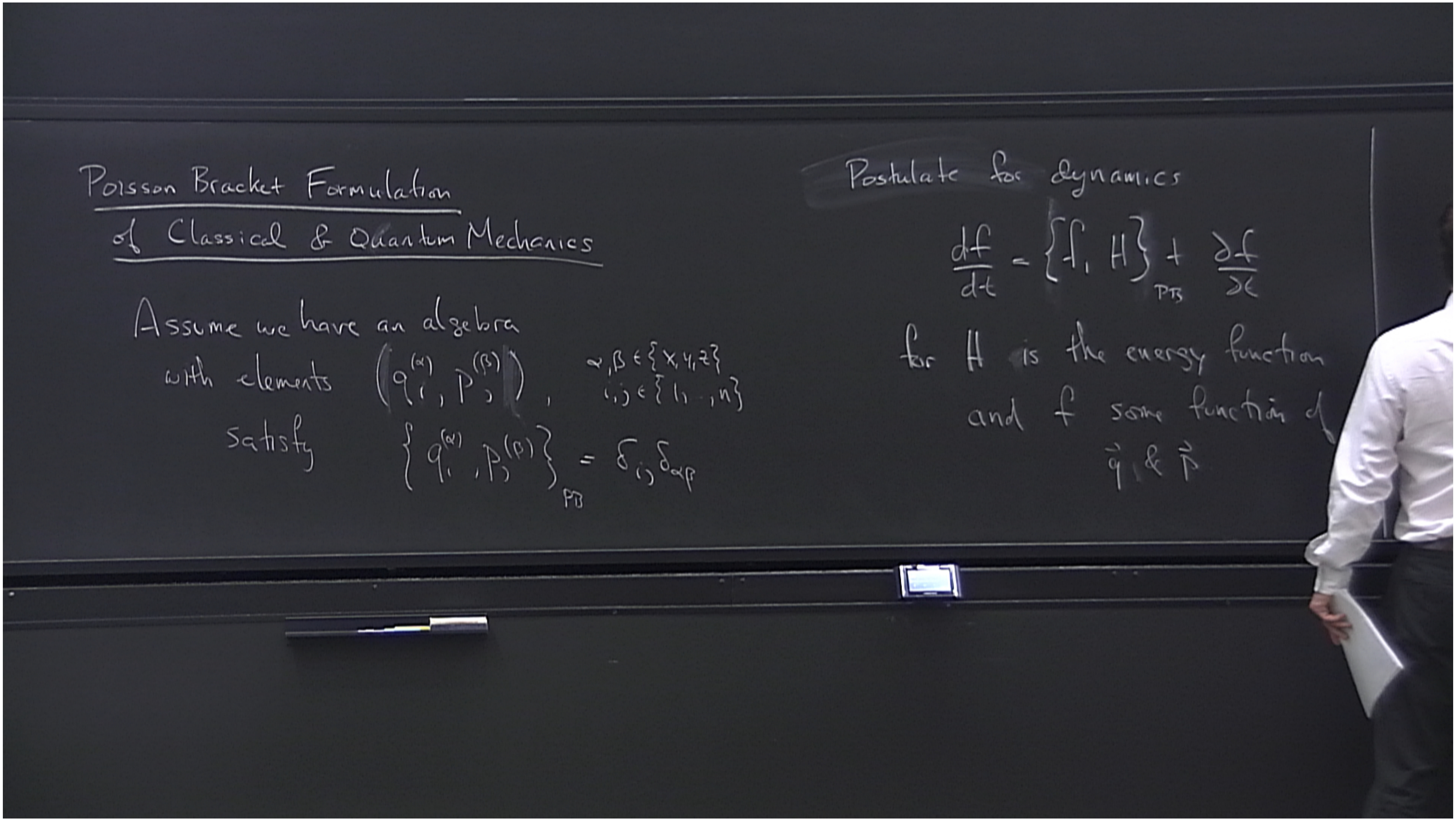
Date: Sep 26, 2014 10:45 AM

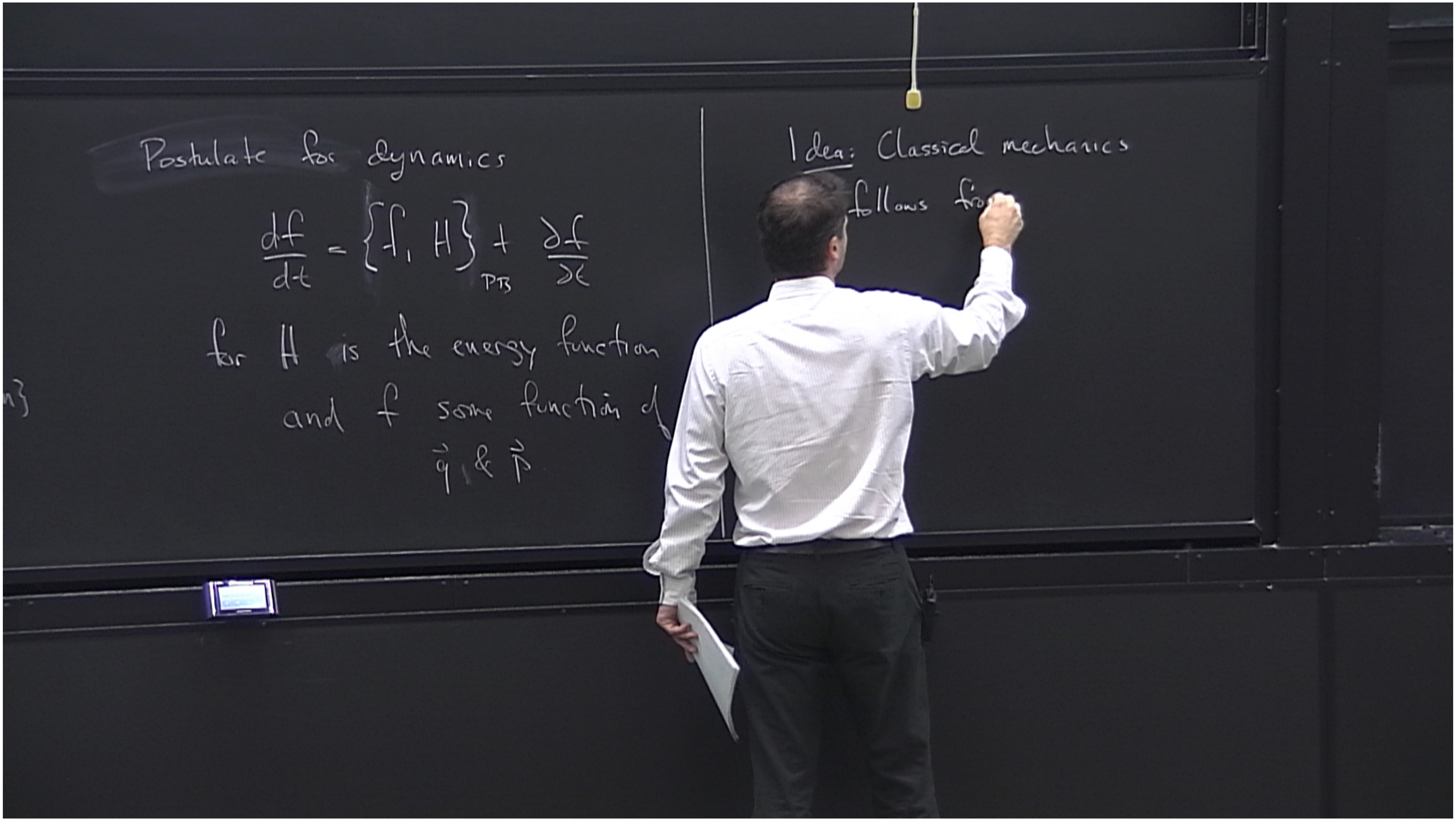
URL: http://pirsa.org/14090060

Abstract:



Poisson Bracket Formulation
of Classical & Quantum Mechanics
Assume we have an algebra
with elements $\left(q_i^{(\alpha)}, p_j^{(\beta)}\right)$, $\alpha,\beta \in \{x,y,z\}$ $i,j \in \{1,\ldots,n\}$
satisfy $\{q_i^{(\alpha)}, p_j^{(\beta)}\}_{PB} = \delta_{ij}\delta_{\alpha\beta}$
Postulate for dynamics
$\{f, H\}_{PB} + \frac{\partial f}{\partial t}$
the energy function


Poisson Bracket Formulation of Classical & Quantum Mechanics
Assume we have an algebra with elements $(q_i^{(\alpha)}, p_j^{(\beta)})$, $\alpha,\beta \in \{x,y,z\}$, $i,j \in \{1,\dots,n\}$
satisfy $\{q_i^{(\alpha)}, p_j^{(\beta)}\}_{PB} = \delta_{ij}\delta_{\alpha\beta}$
Postulate for dynamics
$\frac{df}{dt} = \{f, H\}_{PB} + \frac{\partial f}{\partial t}$
for H is the energy function and f some function of $\vec{q}$ & $\vec{p}$

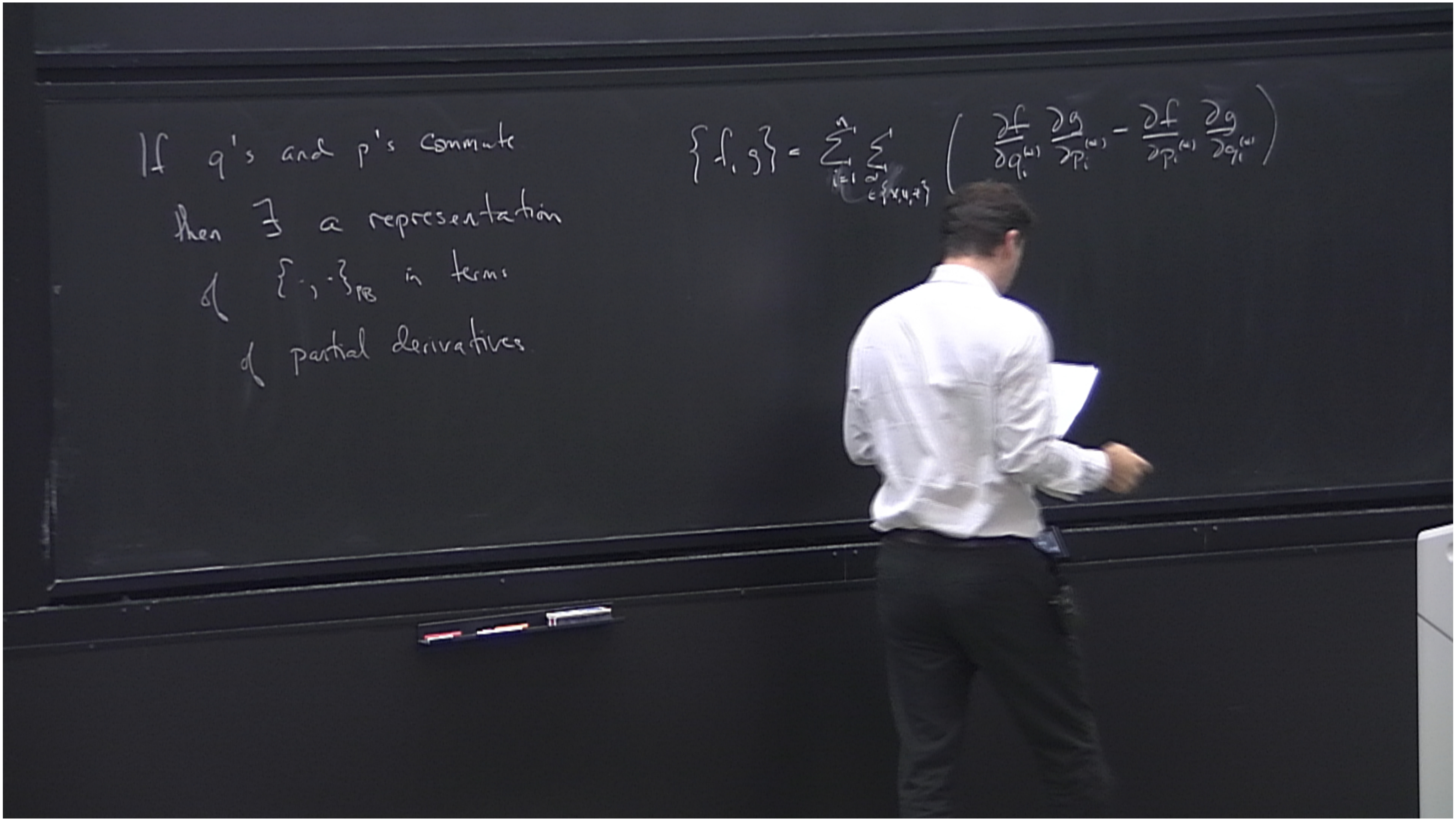

Postulate for dynamics
$\frac{df}{dt} = \{f, H\}_{PB} + \frac{\partial f}{\partial t}$
for H is the energy function
and f some function of
$\vec{q}$ & $\vec{p}$
Idea: Classical mechanics
follows from

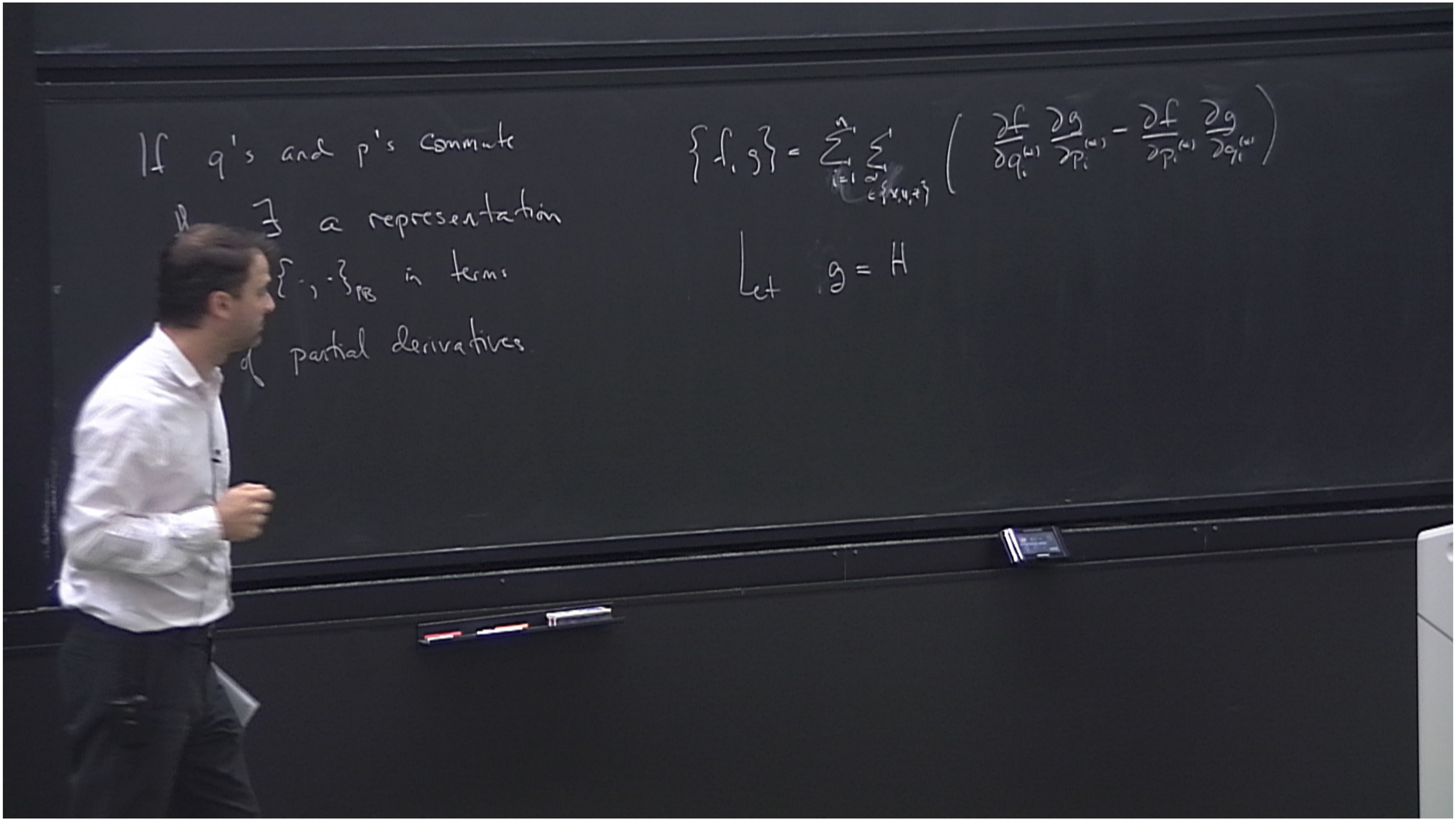
Postulate for dynamics

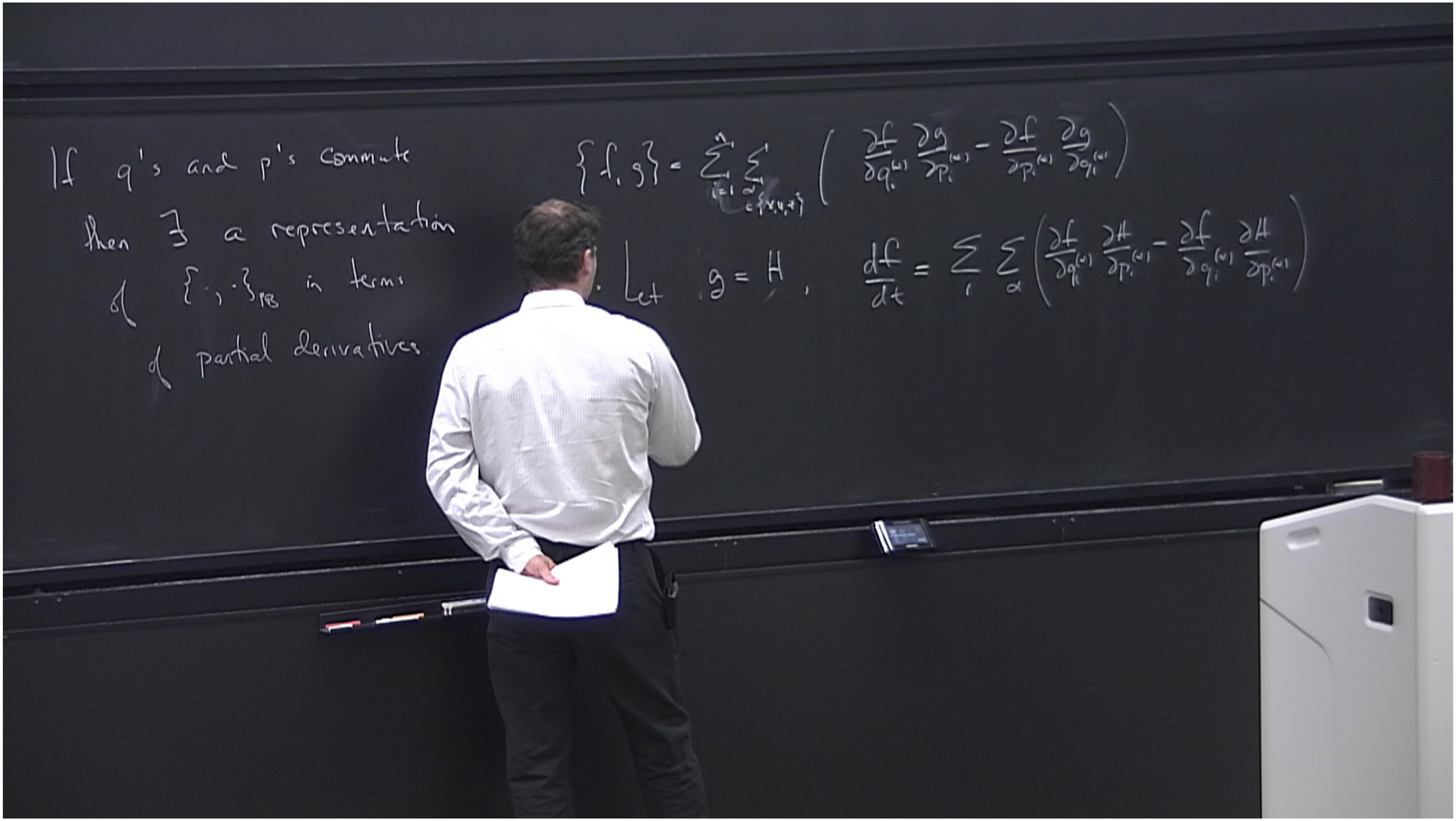
$$\frac{df}{dt} = \{f, H\}_{PB} + \frac{\partial f}{\partial t}$$

for H is the energy function and f some function of $\vec{q}$ & $\vec{p}$.

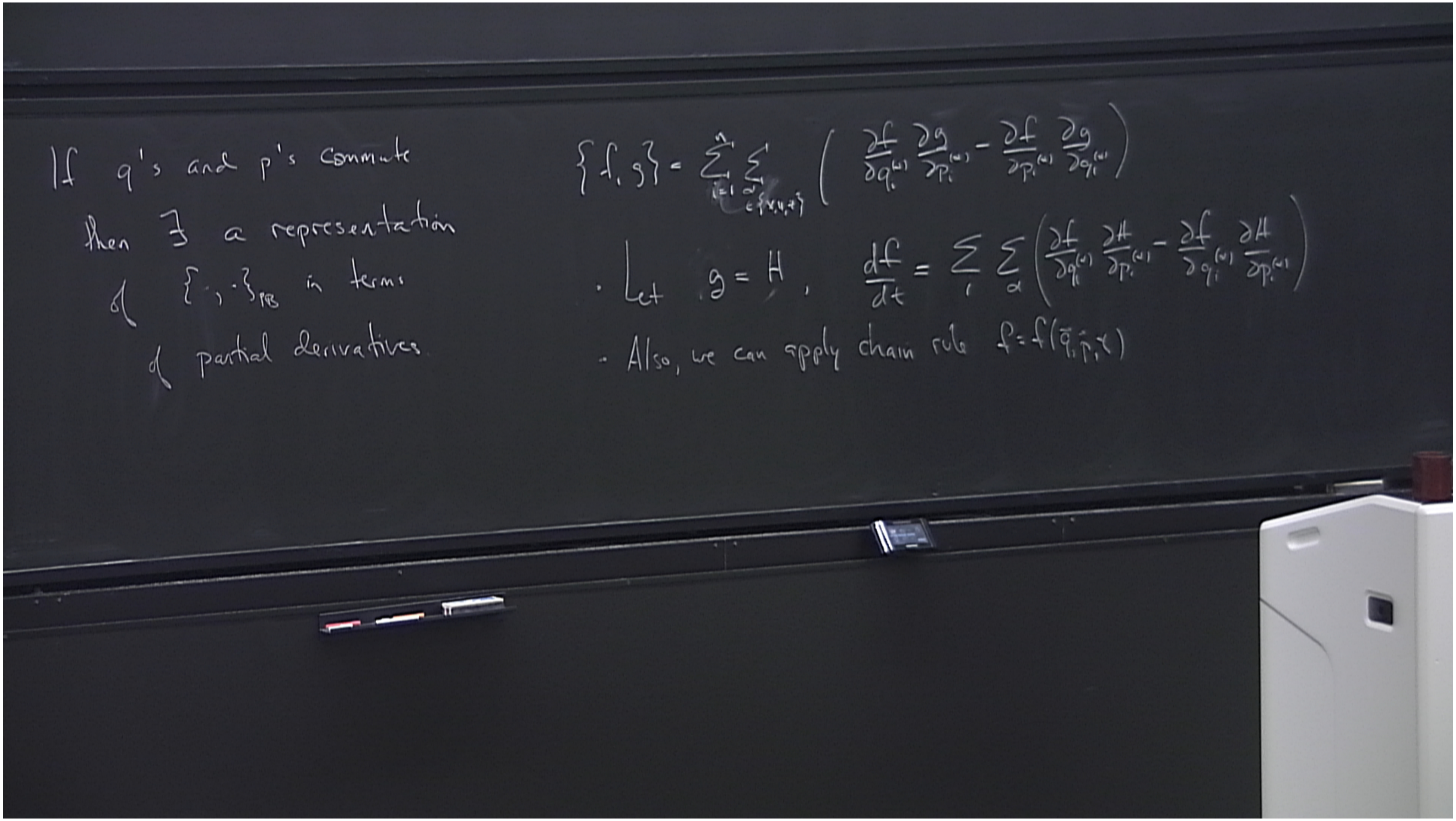
Idea: Classical mechanics follows from assuming a commutative algebra & Q. Mehanics follows from assuming a non-commutative algebra.

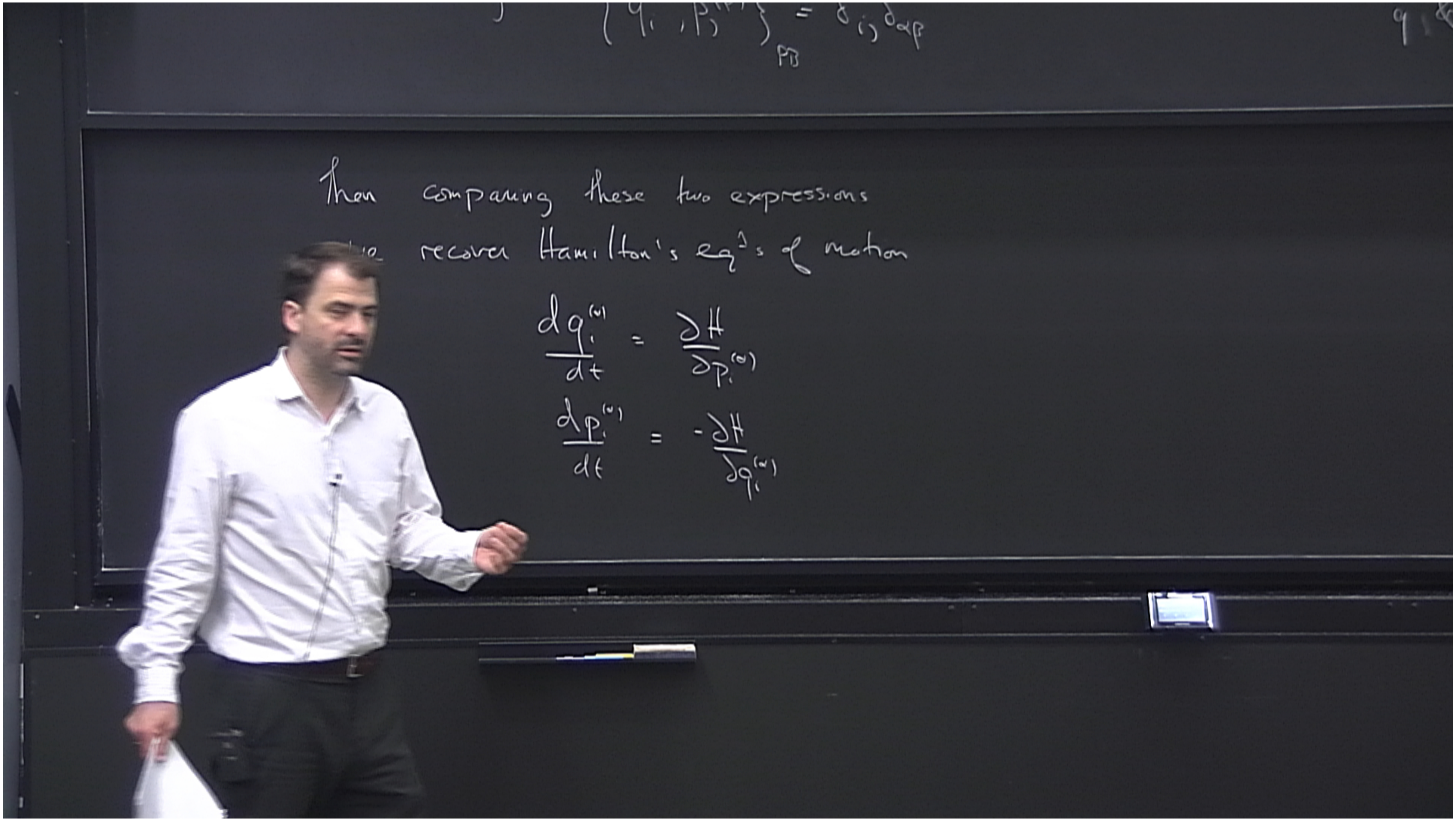

If q's and p's commute
then $\exists$ a representation
of $\{\cdot,\cdot\}_{PB}$ in terms
of partial derivatives
$\{f,g\} = \sum_{i=1}^{n} \sum_{\alpha \in \{x,y,z\}} \left(\frac{\partial f}{\partial q_i^{(\alpha)}} \frac{\partial g}{\partial p_i^{(\alpha)}} - \frac{\partial f}{\partial p_i^{(\alpha)}} \frac{\partial g}{\partial q_i^{(\alpha)}} \right)$


If q's and p's commute
∃ a representation
$\{\cdot,\cdot\}_{PB}$ in terms
partial derivatives
$\{f,g\} = \sum_{i=1}^{n} \sum_{\alpha \in \{x,y,z\}} \left(\frac{\partial f}{\partial q_i^{(\alpha)}} \frac{\partial g}{\partial p_i^{(\alpha)}} - \frac{\partial f}{\partial p_i^{(\alpha)}} \frac{\partial g}{\partial q_i^{(\alpha)}} \right)$
Let $g = H$



If q's and p's commute
then ∃ a representation
of $\{\cdot,\cdot\}_{PB}$ in terms
of partial derivatives
$\{f,g\} = \sum_{i=1}^{n} \sum_{\alpha \in \{x,y,z\}} \left(\frac{\partial f}{\partial q_i^{(\alpha)}} \frac{\partial g}{\partial p_i^{(\alpha)}} - \frac{\partial f}{\partial p_i^{(\alpha)}} \frac{\partial g}{\partial q_i^{(\alpha)}} \right)$
Let $g = H$, $\frac{df}{dt} = \sum_i \sum_\alpha \left(\frac{\partial f}{\partial q_i^{(\alpha)}} \frac{\partial H}{\partial p_i^{(\alpha)}} - \frac{\partial f}{\partial q_i^{(\alpha)}} \frac{\partial H}{\partial p_i^{(\alpha)}} \right)$


If q's and p's commute
then ∃ a representation
of $\{\cdot,\cdot\}_{PB}$ in terms
of partial derivatives
$\{f,g\} = \sum_{i=1}^{n} \sum_{\alpha \in \{x,y,z\}} \left(\frac{\partial f}{\partial q_i^{(\alpha)}} \frac{\partial g}{\partial p_i^{(\alpha)}} - \frac{\partial f}{\partial p_i^{(\alpha)}} \frac{\partial g}{\partial q_i^{(\alpha)}} \right)$
· Let $g = H$, $\frac{df}{dt} = \sum_i \sum_\alpha \left(\frac{\partial f}{\partial q_i^{(\alpha)}} \frac{\partial H}{\partial p_i^{(\alpha)}} - \frac{\partial f}{\partial q_i^{(\alpha)}} \frac{\partial H}{\partial p_i^{(\alpha)}} \right)$
· Also, we can apply chain rule $f = f(\vec{q}, \vec{p}, t)$

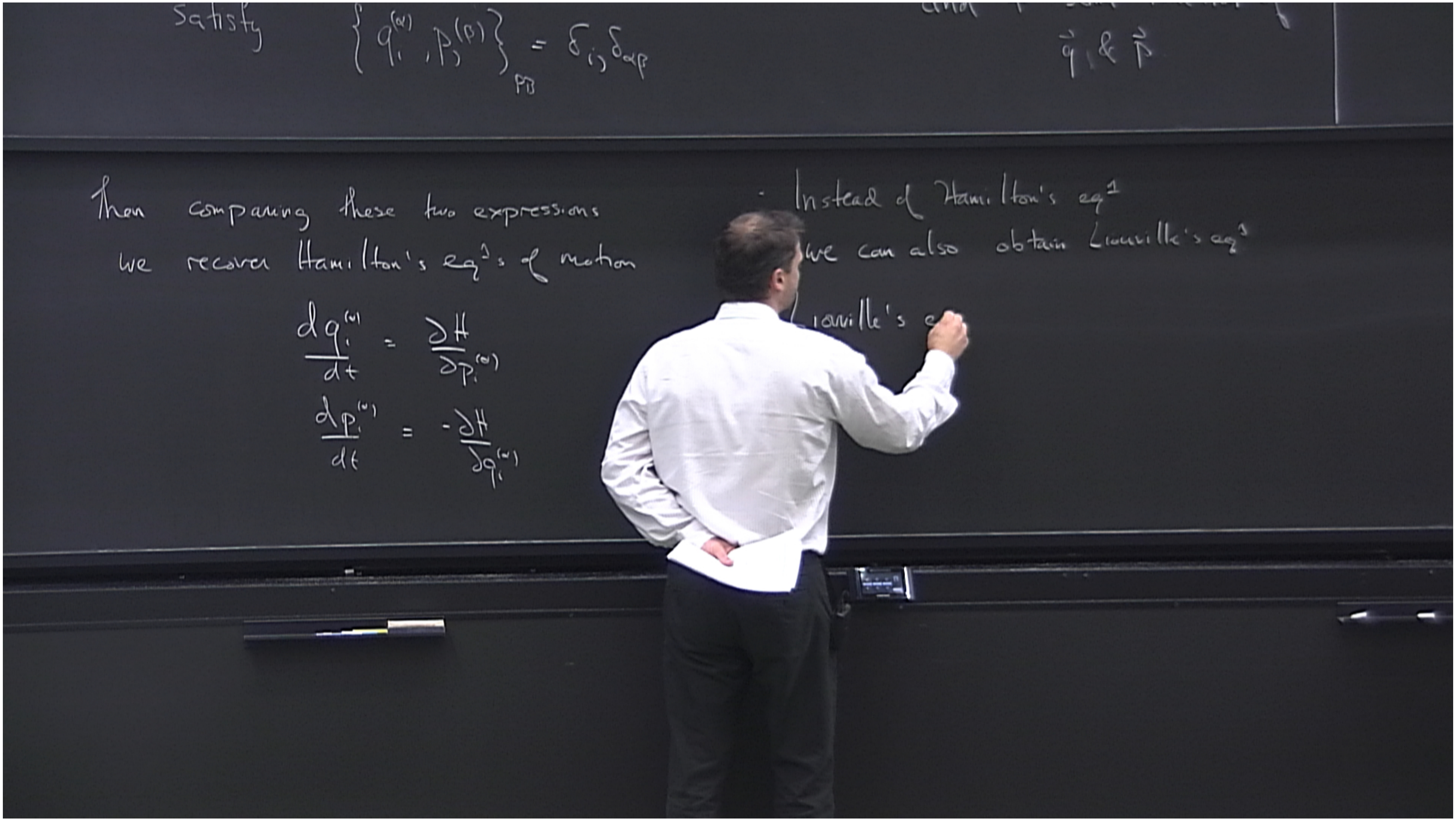

$\{q_i, p_j\}_{PB} = \delta_{ij}\delta_{\alpha\beta}$
Then comparing these two expressions
we recover Hamilton's eq^{n}s of motion
$\frac{dq_i^{(\alpha)}}{dt} = \frac{\partial H}{\partial p_i^{(\alpha)}}$
$\frac{dp_i^{(\alpha)}}{dt} = -\frac{\partial H}{\partial q_i^{(\alpha)}}$

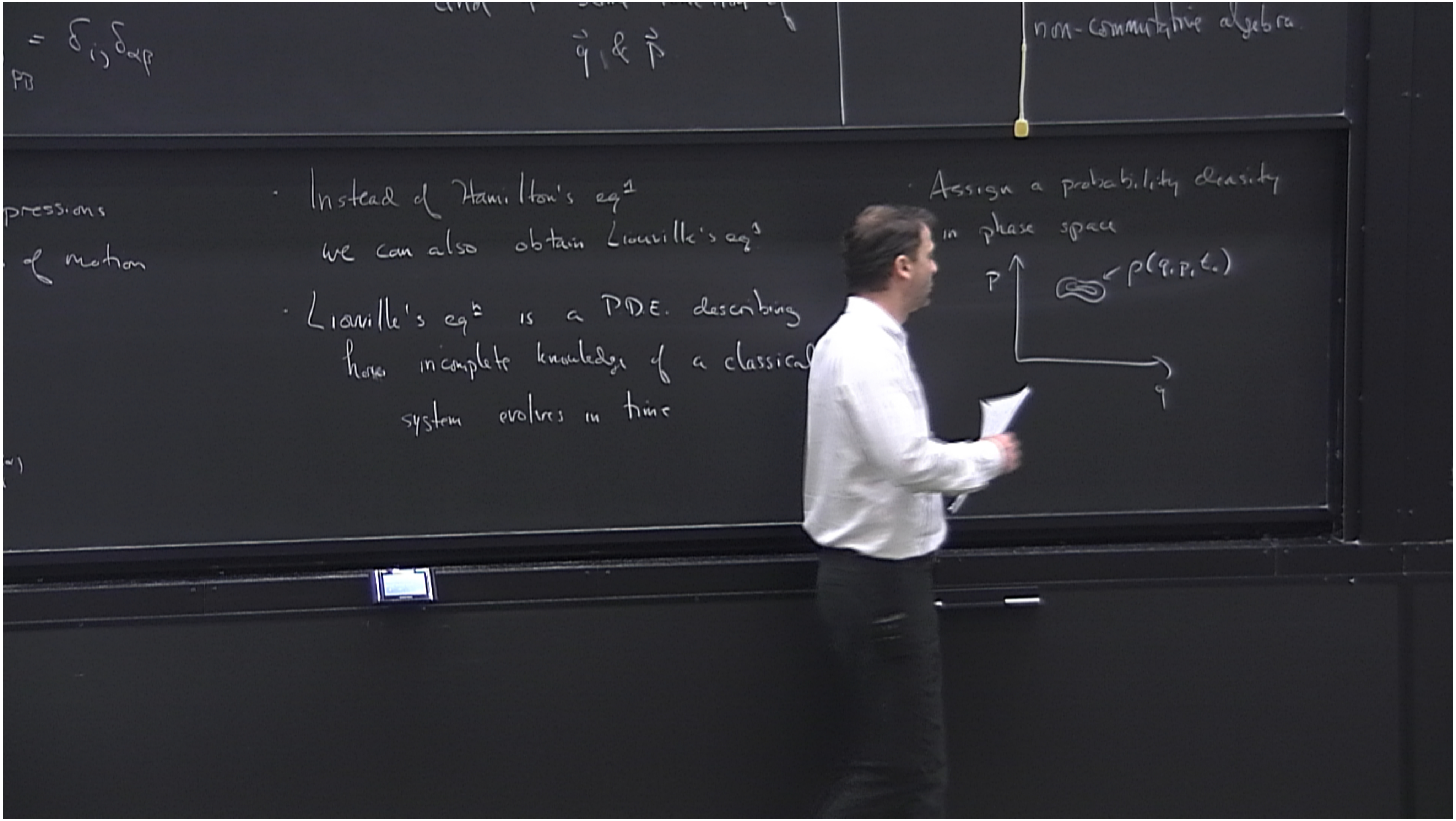
$$\{q_i^{(\alpha)}, p_j^{(\beta)}\}_{PB} = \delta_{ij}\delta_{\alpha\beta}$$

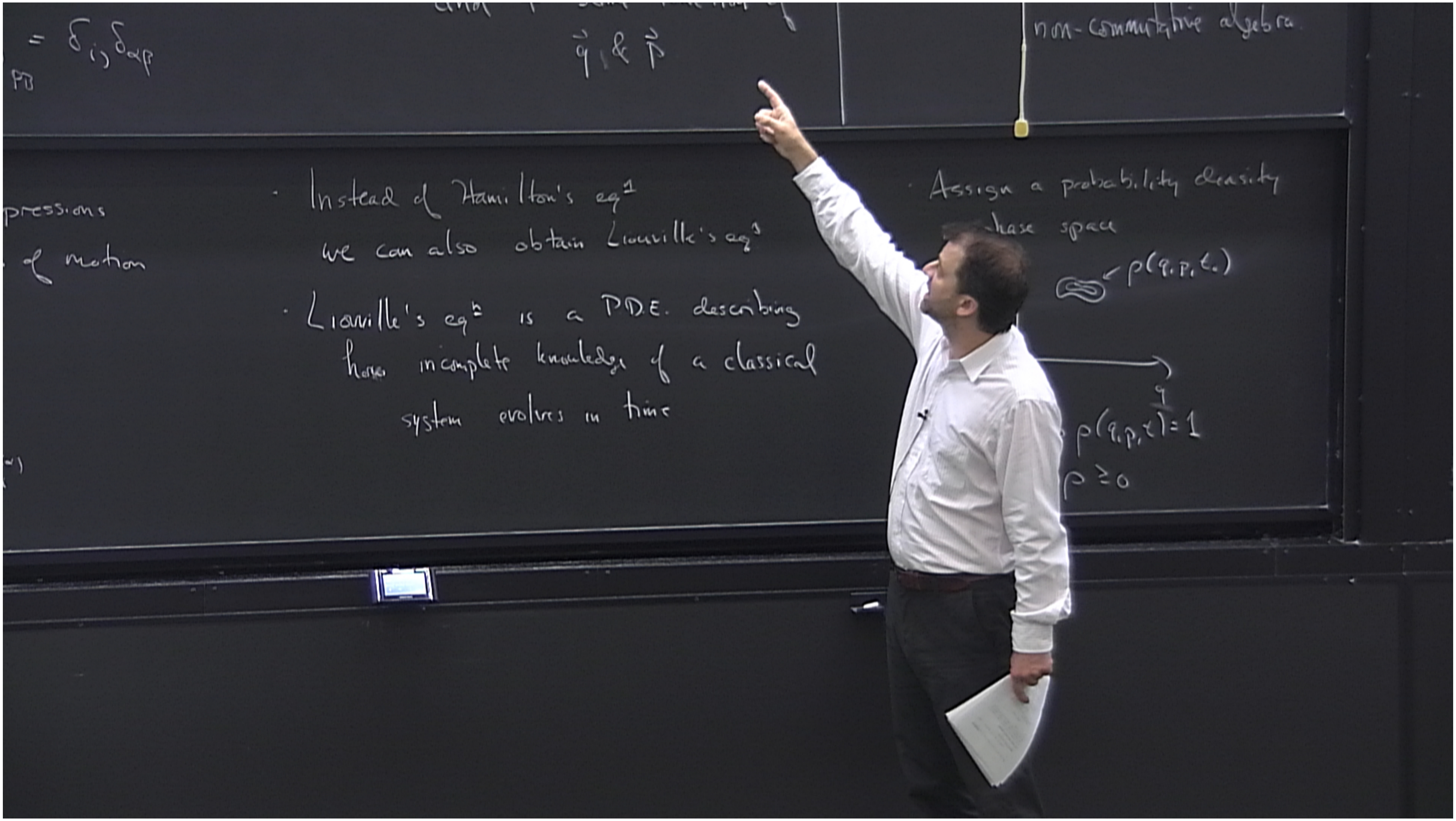
Then comparing these two expressions we recover Hamilton's eq^{n}s of motion

$$\frac{dq_i^{(\alpha)}}{dt} = \frac{\partial H}{\partial p_i^{(\alpha)}}$$

$$\frac{dp_i^{(\alpha)}}{dt} = -\frac{\partial H}{\partial q_i^{(\alpha)}}$$

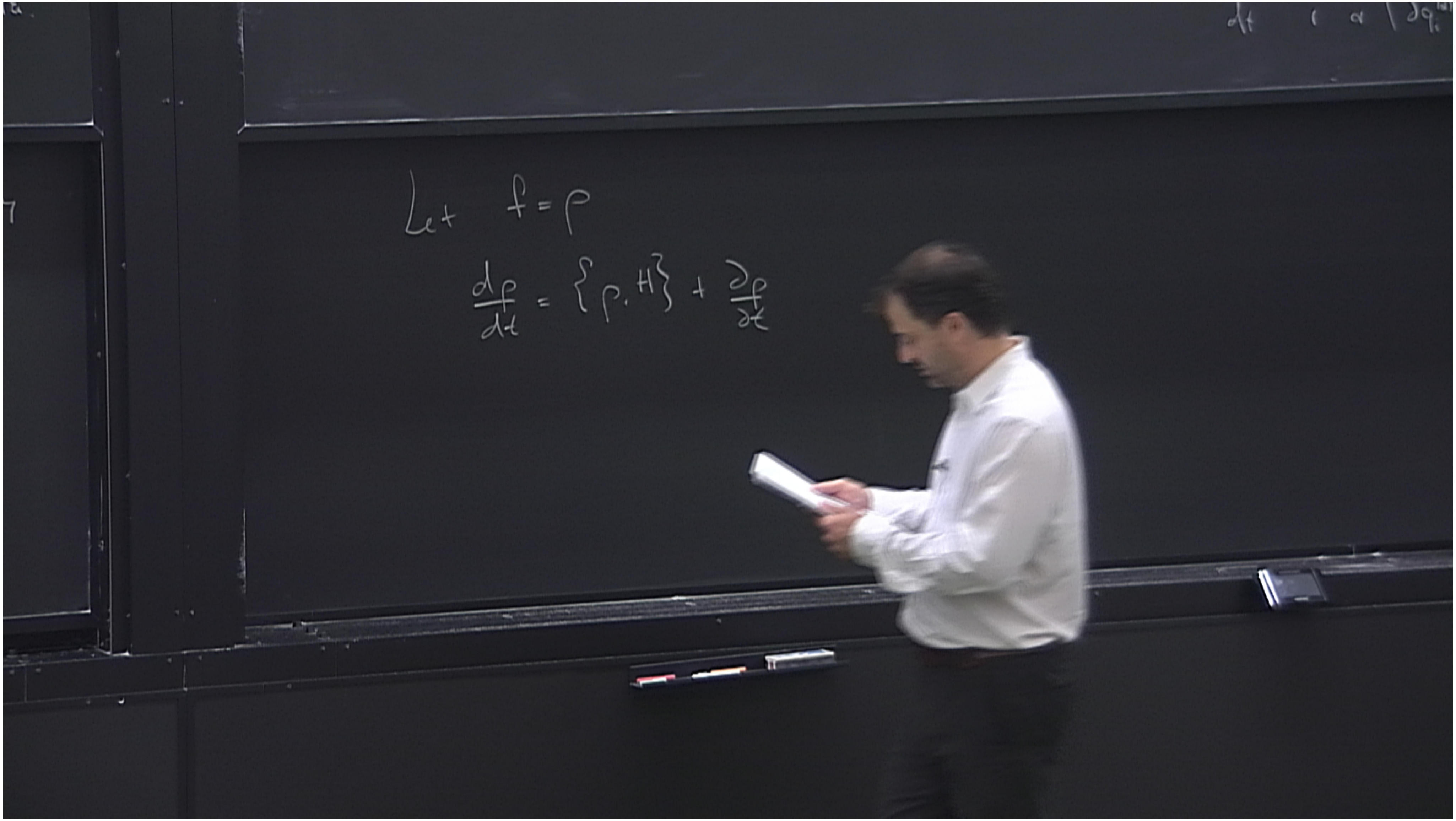

satisfy $\{q_i^{(\alpha)}, p_j^{(\beta)}\}_{PB} = \delta_{ij}\delta_{\alpha\beta}$
$\vec{q}$ & $\vec{p}$
Then comparing these two expressions
we recover Hamilton's eq^{n}s of motion
$\frac{dq_i^{(\alpha)}}{dt} = \frac{\partial H}{\partial p_i^{(\alpha)}}$
$\frac{dp_i^{(\alpha)}}{dt} = -\frac{\partial H}{\partial q_i^{(\alpha)}}$
Instead of Hamilton's eq^n
we can also obtain Liouville's eq^n
Liouville's e


non-commutative algebra
· Instead of Hamilton's eq^n
we can also obtain Liouville's eq^n
· Liouville's eq^n is a P.D.E. describing
how incomplete knowledge of a classical
system evolves in time
· Assign a probability density
in phase space
ρ(q, p, t₀)
p
q



non-commutative algebra
q & p
Instead of Hamilton's eq^n
we can also obtain Liouville's eq^n
Liouville's eq^n is a P.D.E. describing
how incomplete knowledge of a classical
system evolves in time
Assign a probability density
phase space
ρ(q,p,t₀)
ρ(q,p,t) = 1
ρ ≥ 0
pressions
of motion

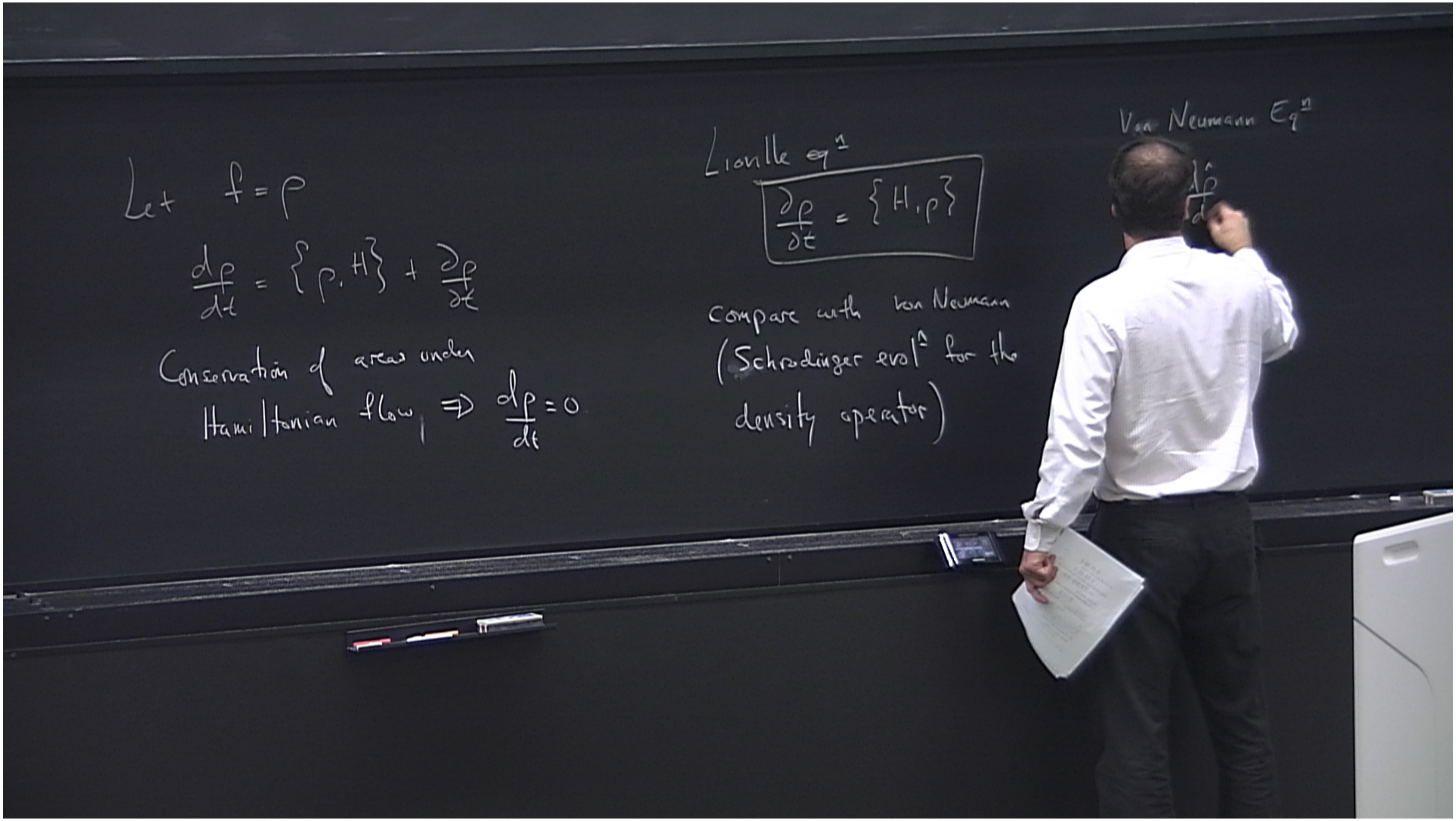

non-commutative algebra
Instead of Hamilton's eqn
we can also obtain Liouville's eqn
Liouville's eqn is a P.D.E. describing
how incomplete knowledge of a classical
system evolves in time
Assign a probability density
space
$\rho(q,p,t_0)$

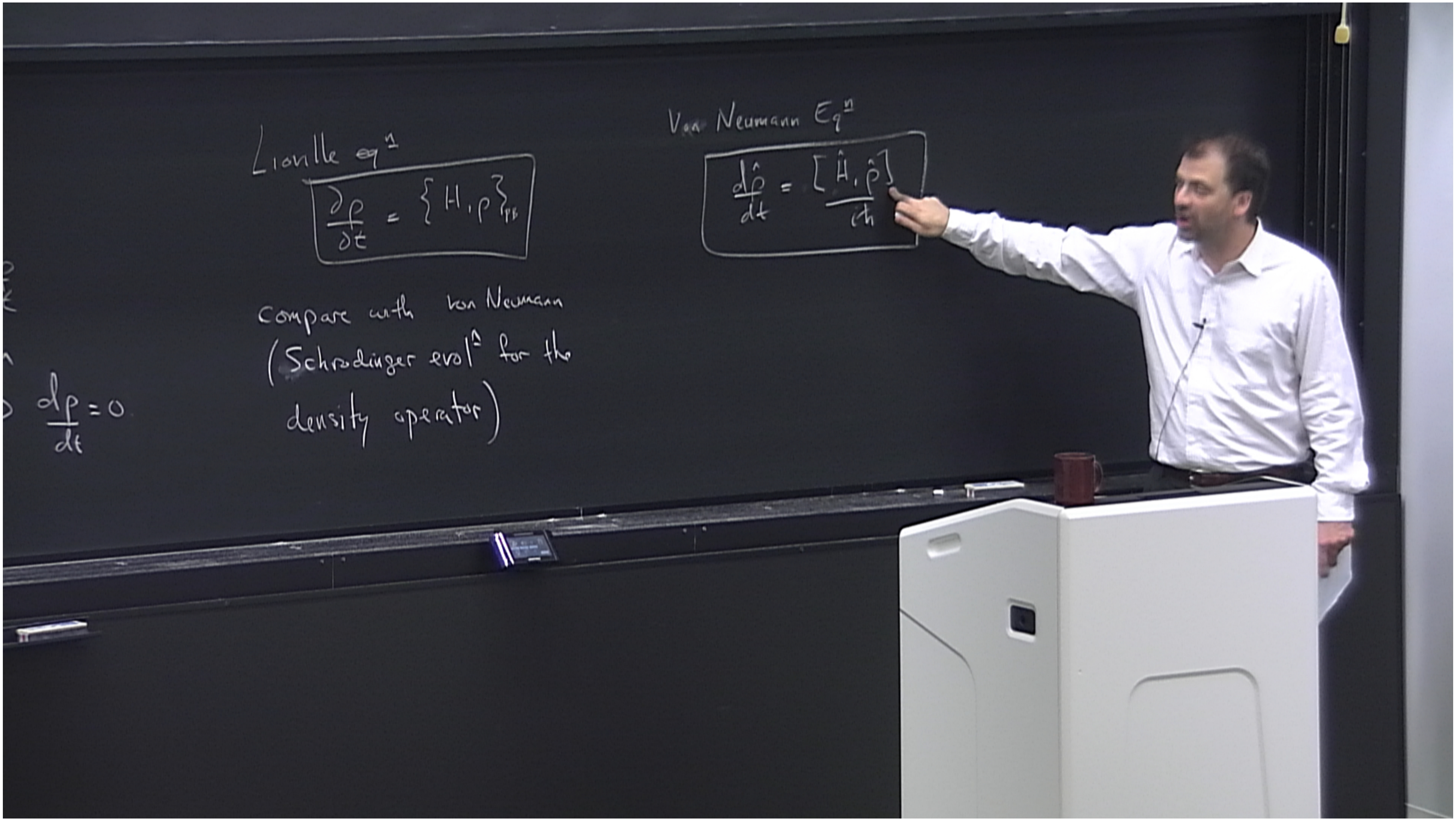

Let f = ρ
dρ/dt = {ρ, H} + ∂ρ/∂t

Let $f = \rho$

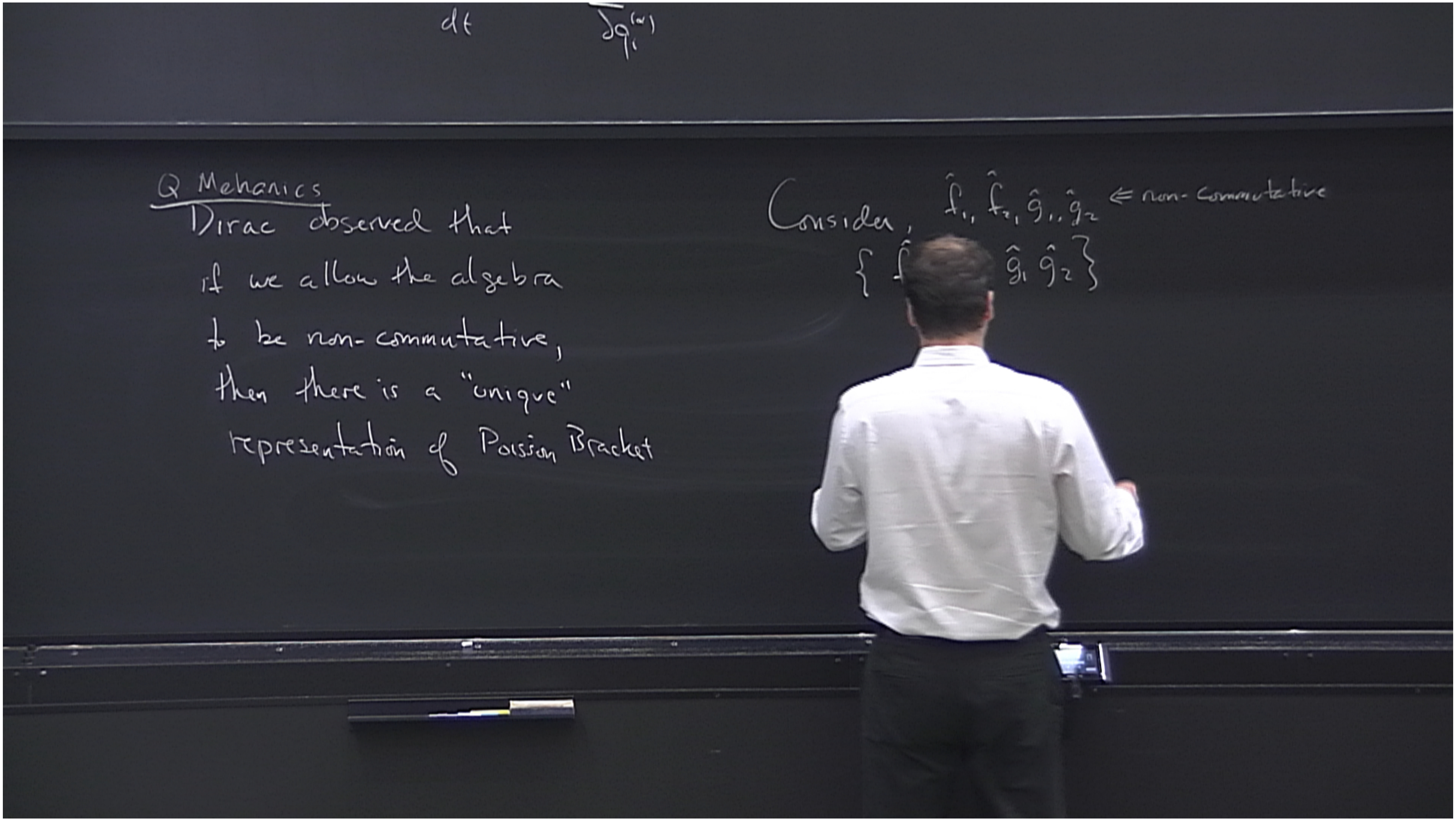
$$\frac{d\rho}{dt} = \{\rho, H\} + \frac{\partial \rho}{\partial t}$$

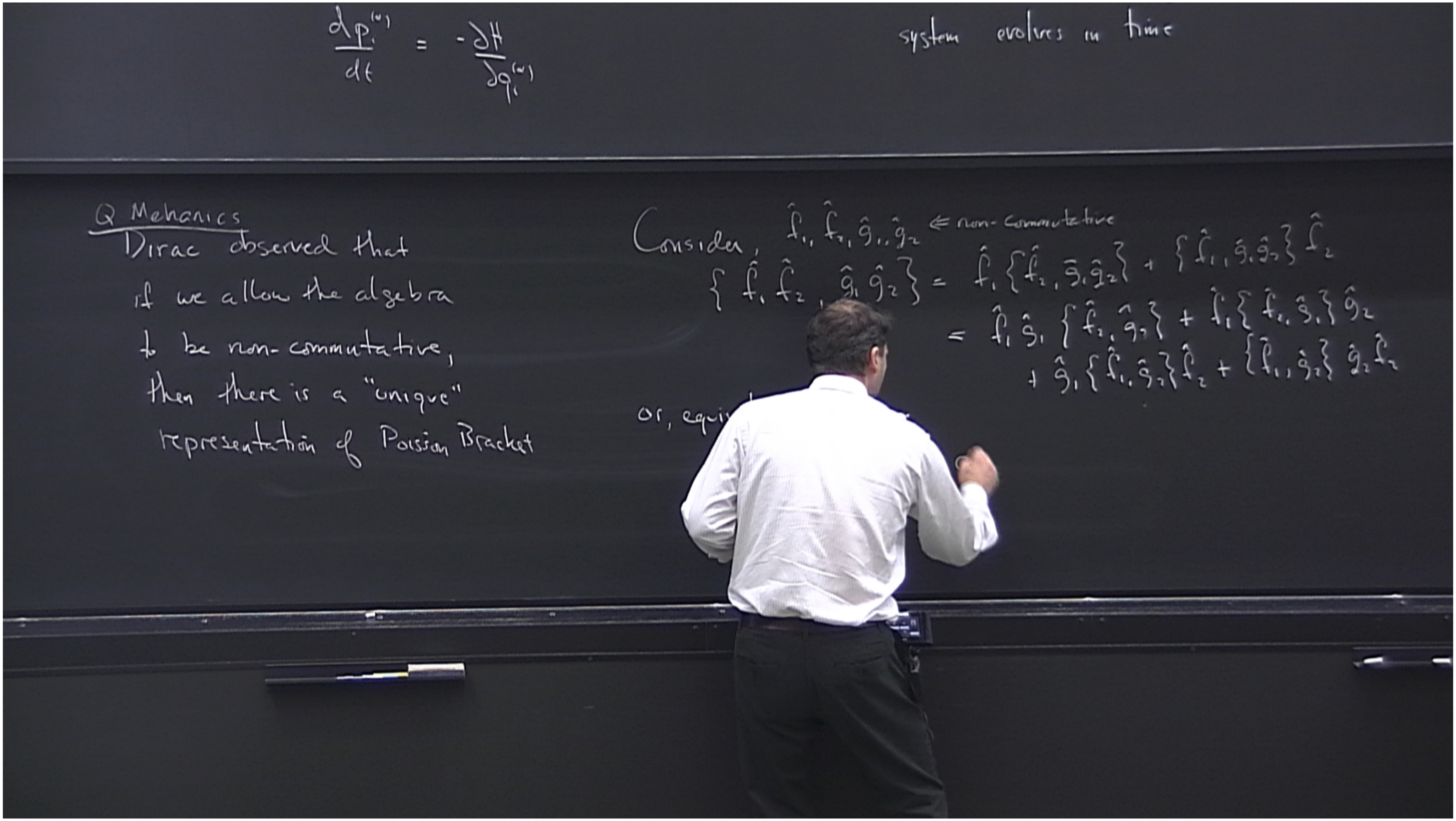
Conservation of areas under Hamiltonian flow, $\Rightarrow \frac{d\rho}{dt} = 0$


Let f = ρ
dρ/dt = {ρ, H} + ∂ρ/∂t
Conservation of areas under Hamiltonian flow ⇒ dρ/dt = 0
Liouville eqⁿ
∂ρ/∂t = {H, ρ}
Compare with von Neumann
(Schrodinger evolⁿ for the density operator)
Von Neumann Eqⁿ

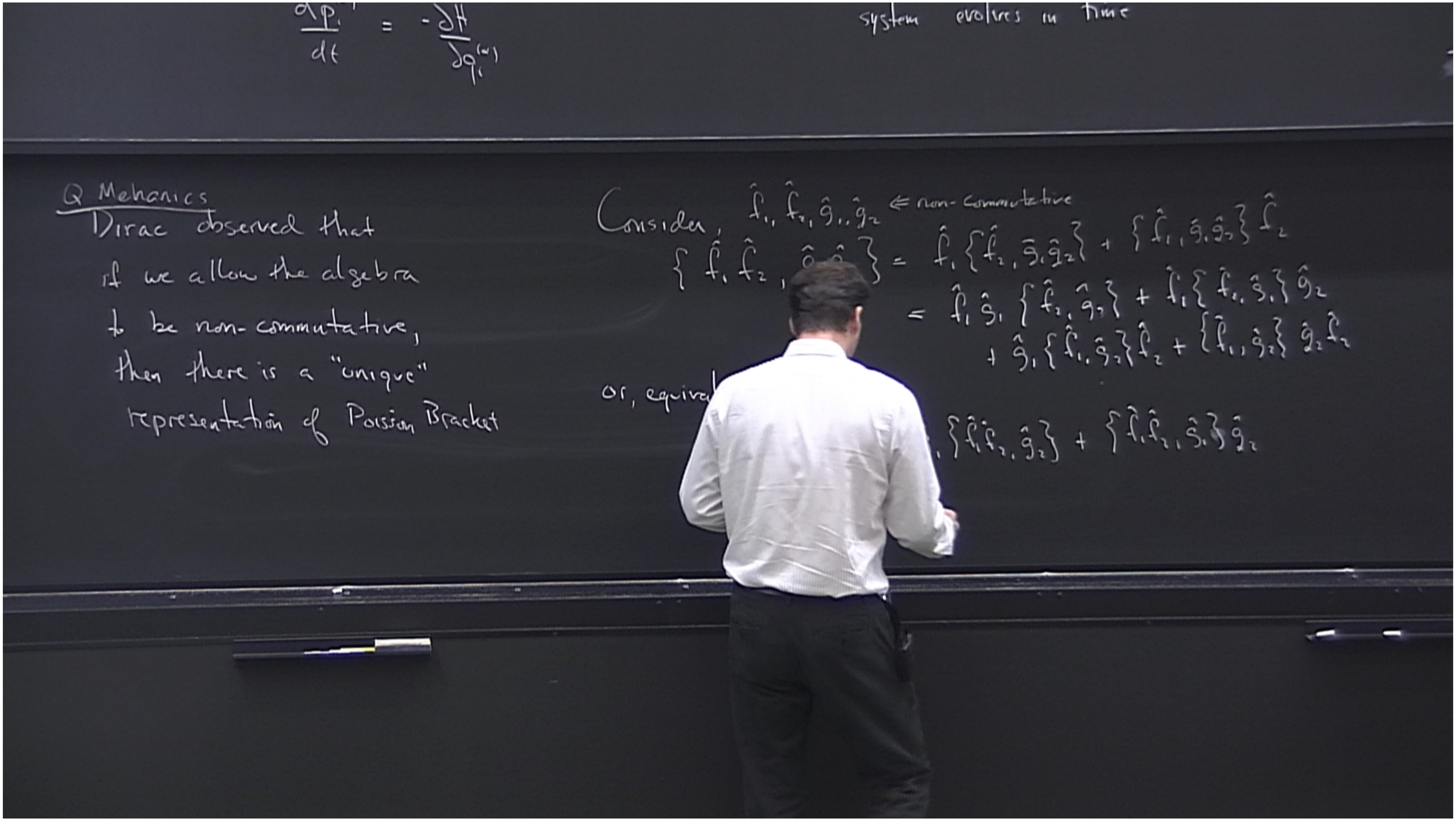


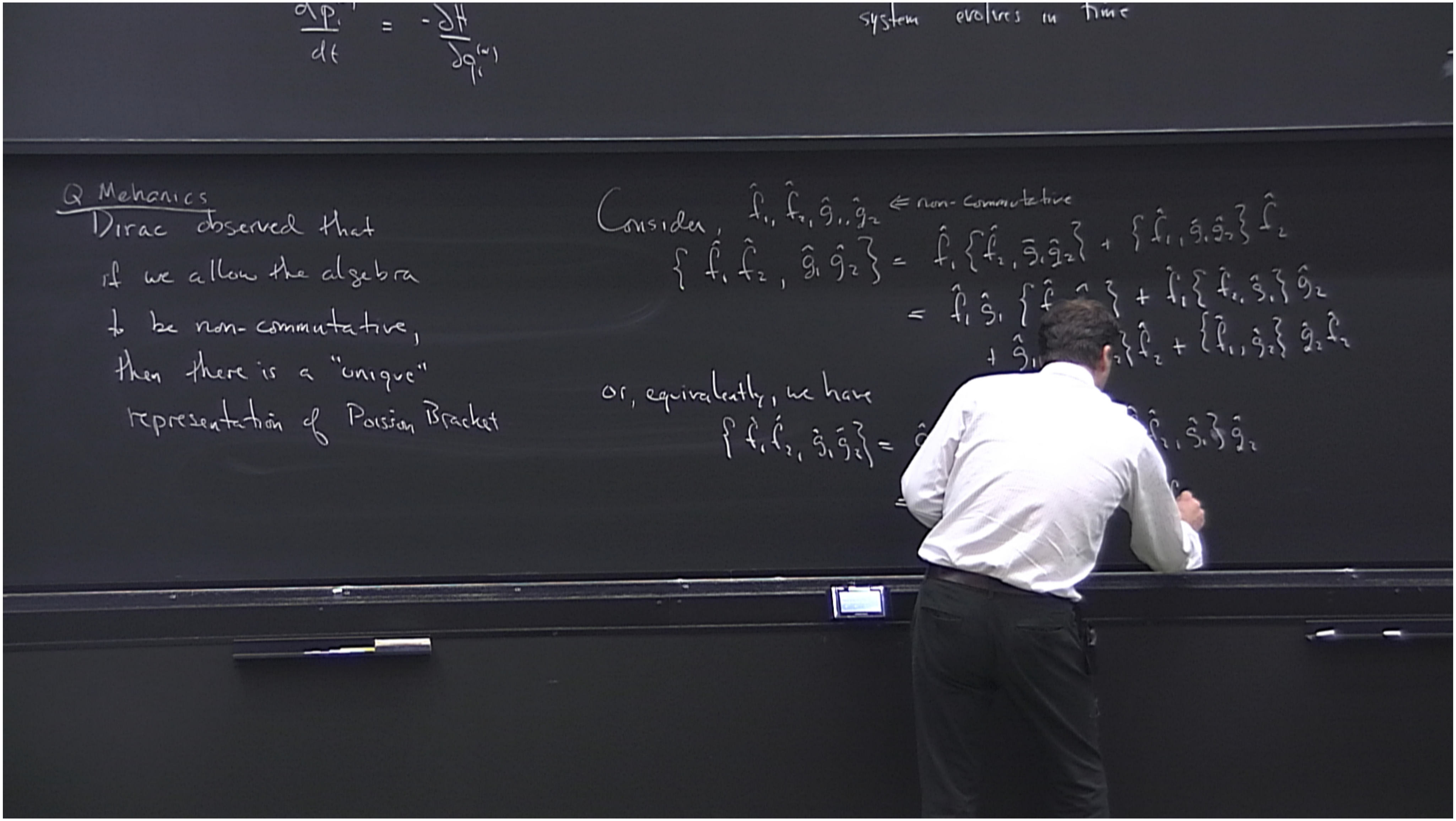
Von Neumann Eq^n
$\frac{\partial \rho}{\partial t} = \{H, \rho\}_{PB}$
$\frac{d\hat{\rho}}{dt} = \frac{[\hat{H}, \hat{\rho}]}{i\hbar}$
compare with von Neumann
(Schrodinger evol^n for the
density operator)
$\frac{d\rho}{dt} = 0$


Q Mehanics
Dirac observed that
if we allow the algebra
to be non-commutative,
then there is a "unique"
representation of Poission Bracket
Consider, $\hat{f}_1, \hat{f}_2, \hat{g}_1, \hat{g}_2 \Leftarrow$ non-commutative



$\frac{dp_i^{(\alpha)}}{dt} = -\frac{\partial H}{\partial q_i^{(\alpha)}}$
system evolves in time
Q Mehanics
Dirac observed that
if we allow the algebra
to be non-commutative,
then there is a "unique"
representation of Poisson Bracket
Consider, $\hat{f}_1, \hat{f}_2, \hat{g}_1, \hat{g}_2$ ⇐ non-Commutative
$\{\hat{f}_1\hat{f}_2, \hat{g}_1\hat{g}_2\} = \hat{f}_1\{\hat{f}_2, \hat{g}_1\hat{g}_2\} + \{\hat{f}_1, \hat{g}_1\hat{g}_2\}\hat{f}_2$
$= \hat{f}_1\hat{g}_1\{\hat{f}_2, \hat{g}_2\} + \hat{f}_1\{\hat{f}_2, \hat{g}_1\}\hat{g}_2$
$+ \hat{g}_1\{\hat{f}_1, \hat{g}_2\}\hat{f}_2 + \{\hat{f}_1, \hat{g}_1\}\hat{g}_2\hat{f}_2$
or, equi


system evolves in time
Q Mehanics
Dirac observed that
if we allow the algebra
to be non-commutative,
then there is a "unique"
representation of Poisson Bracket
Consider, $\hat{f}_1, \hat{f}_2, \hat{g}_1, \hat{g}_2$ ⇐ non-commutative
or, equiva


system evolves in time
Q Mechanics
Dirac observed that
if we allow the algebra
to be non-commutative,
then there is a "unique"
representation of Poisson Bracket
Consider, $\hat{f}_1, \hat{f}_2, \hat{g}_1, \hat{g}_2$ ⇐ non-commutative
$\{\hat{f}_1\hat{f}_2, \hat{g}_1\hat{g}_2\} = \hat{f}_1\{\hat{f}_2, \hat{g}_1\hat{g}_2\} + \{\hat{f}_1, \hat{g}_1\hat{g}_2\}\hat{f}_2$
or, equivalently, we have

$$\frac{dp_i^{(\alpha)}}{dt} = -\frac{\partial H}{\partial q_i^{(\alpha)}}$$

system evolves in time

$$\int dq\, dp\, \rho(q,p,t) = 1$$

$\&\ \rho \geq 0$

...bserved that
...llow the algebra
...on-commutative,
...ere is a "unique"
...ntation of Poisson Bracket

Consider, $\hat{f}_1, \hat{f}_2, \hat{g}_1, \hat{g}_2 \Leftarrow$ non-commutative

$$\{\hat{f}_1\hat{f}_2, \hat{g}_1\hat{g}_2\} = \hat{f}_1\{\hat{f}_2, \hat{g}_1\hat{g}_2\} + \{\hat{f}_1, \hat{g}_1\hat{g}_2\}\hat{f}_2$$

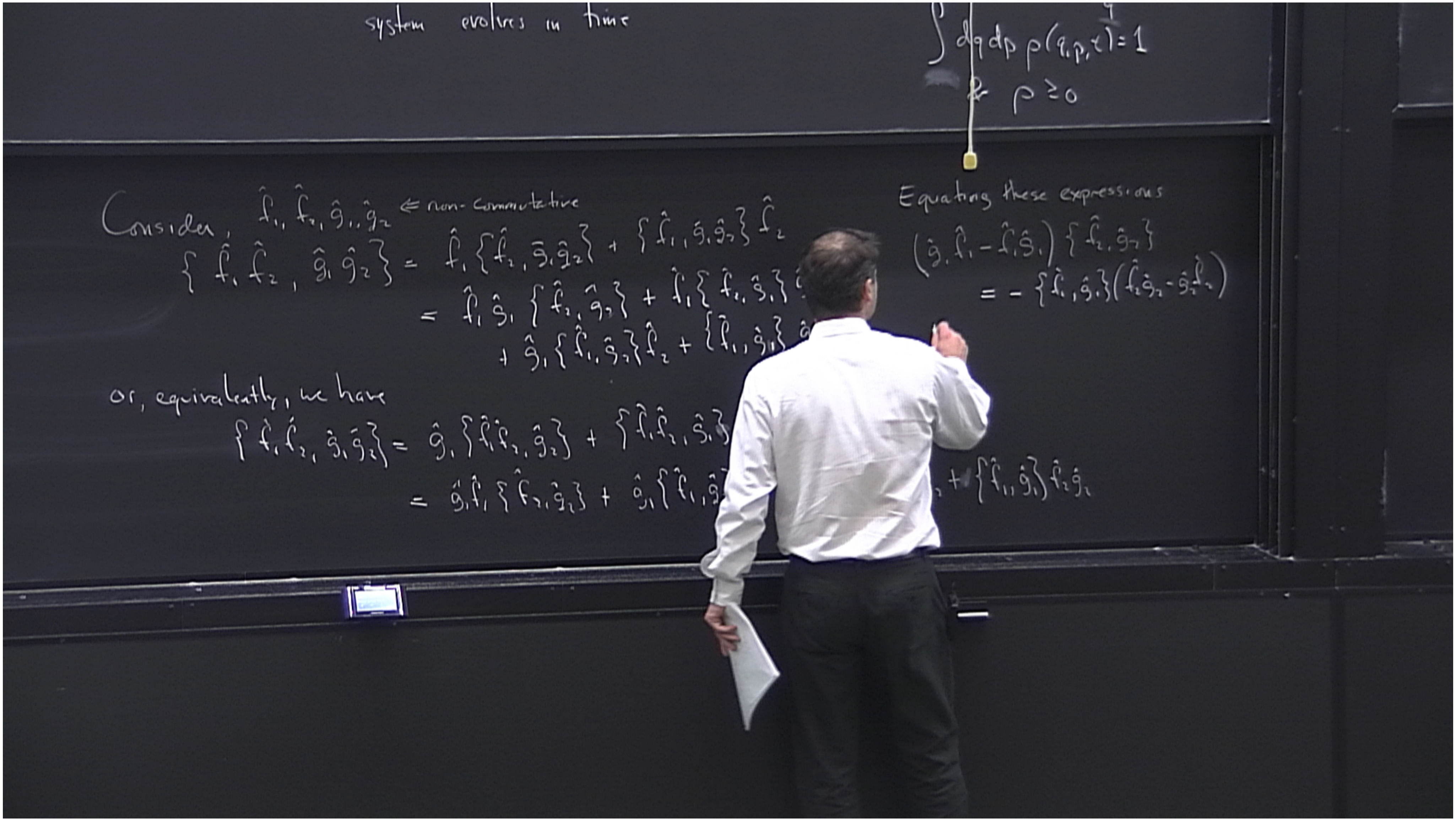
$$= \hat{f}_1\hat{g}_1\{\hat{f}_2, \hat{g}_2\} + \hat{f}_1\{\hat{f}_2, \hat{g}_1\}\hat{g}_2 + \hat{g}_1\{\hat{f}_1, \hat{g}_2\}\hat{f}_2 + \{\hat{f}_1, \hat{g}_1\}\hat{g}_2\hat{f}_2$$

or, equivalently, we have

$$\{\hat{f}_1\hat{f}_2, \hat{g}_1\hat{g}_2\} = \hat{g}_1\{\hat{f}_1\hat{f}_2, \hat{g}_2\} + \{\hat{f}_1\hat{f}_2, \hat{g}_1\}\hat{g}_2$$

$$= \hat{g}_1\hat{f}_1\{\hat{f}_2, \hat{g}_2\} + \hat{g}_1\{\hat{f}_1, \hat{g}_2\}\hat{f}_2 + \ldots \{\hat{f}_1, \hat{g}_1\}\hat{f}_2\hat{g}_2$$

Equating ...


system evolves in time
$\int dq\, dp\, \rho(q,p,t) = 1$
& $\rho \geq 0$
Consider, $\hat{f}_1, \hat{f}_2, \hat{g}_1, \hat{g}_2$ ⇐ non-commutative
Equating these expressions
or, equivalently, we have

system evolves in time

$$\int dq\,dp\,\rho(q,p,t) = 1$$

$\&\ \rho \geq 0$

Consider, $\hat{f}_1, \hat{f}_2, \hat{g}_1, \hat{g}_2 \Leftarrow$ non-commutative

$$\{\hat{f}_1\hat{f}_2, \hat{g}_1\hat{g}_2\} = \hat{f}_1\{\hat{f}_2, \hat{g}_1\hat{g}_2\} + \{\hat{f}_1, \hat{g}_1\hat{g}_2\}\hat{f}_2$$
$$= \hat{f}_1\hat{g}_1\{\hat{f}_2, \hat{g}_2\} + \hat{f}_1\{\hat{f}_2, \hat{g}_1\}\hat{g}_2 + \hat{g}_1\{\hat{f}_1, \hat{g}_2\}\hat{f}_2 + \{\hat{f}_1, \hat{g}_1\}\hat{g}_2\hat{f}_2$$

or, equivalently, we have

$$\{\hat{f}_1\hat{f}_2, \hat{g}_1\hat{g}_2\} = \hat{g}_1\{\hat{f}_1\hat{f}_2, \hat{g}_2\} + \{\hat{f}_1\hat{f}_2, \hat{g}_1\}\hat{g}_2$$
$$= \hat{g}_1\hat{f}_1\{\hat{f}_2, \hat{g}_2\} + \hat{g}_1\{\hat{f}_1, \hat{g}_2\}\hat{f}_2 + \hat{f}_1\{\hat{f}_2, \hat{g}_1\}\hat{g}_2 + \{\hat{f}_1, \hat{g}_1\}\hat{f}_2\hat{g}_2$$

Equating these expressions

$$(\hat{g}_1\hat{f}_1 - \hat{f}_1\hat{g}_1)\{\hat{f}_2, \hat{g}_2\} = -\{\hat{f}_1, \hat{g}_1\}(\hat{f}_2\hat{g}_2 - \hat{g}_2\hat{f}_2)$$

Must hold $\forall\ f_1, g_1, f_2\ \&\ g_2$,

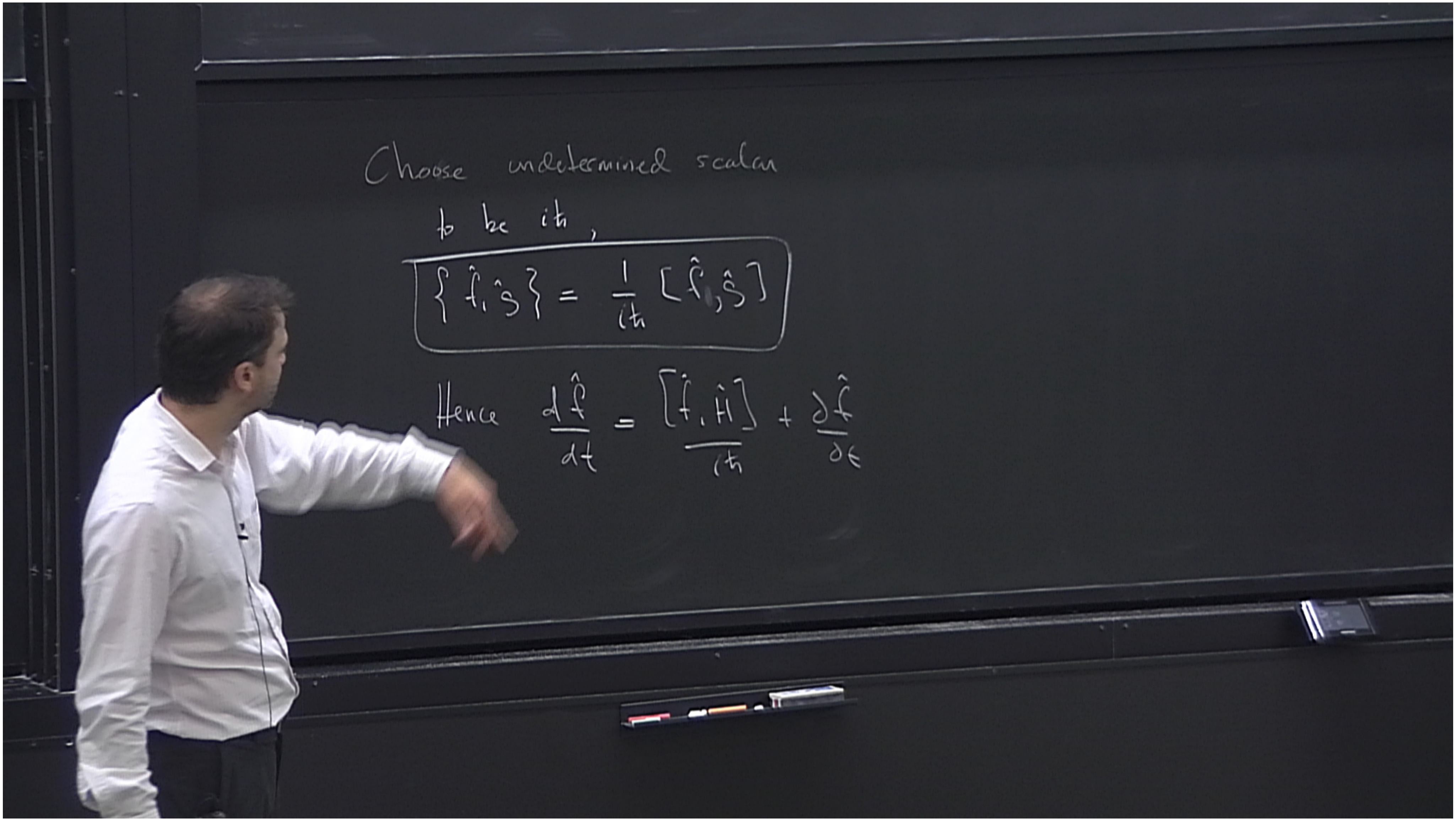
$$\Rightarrow \boxed{\{\hat{f}_1, \hat{g}_1\} \propto \hat{f}_1\hat{g}_1 - \hat{g}_1\hat{f}_1}$$

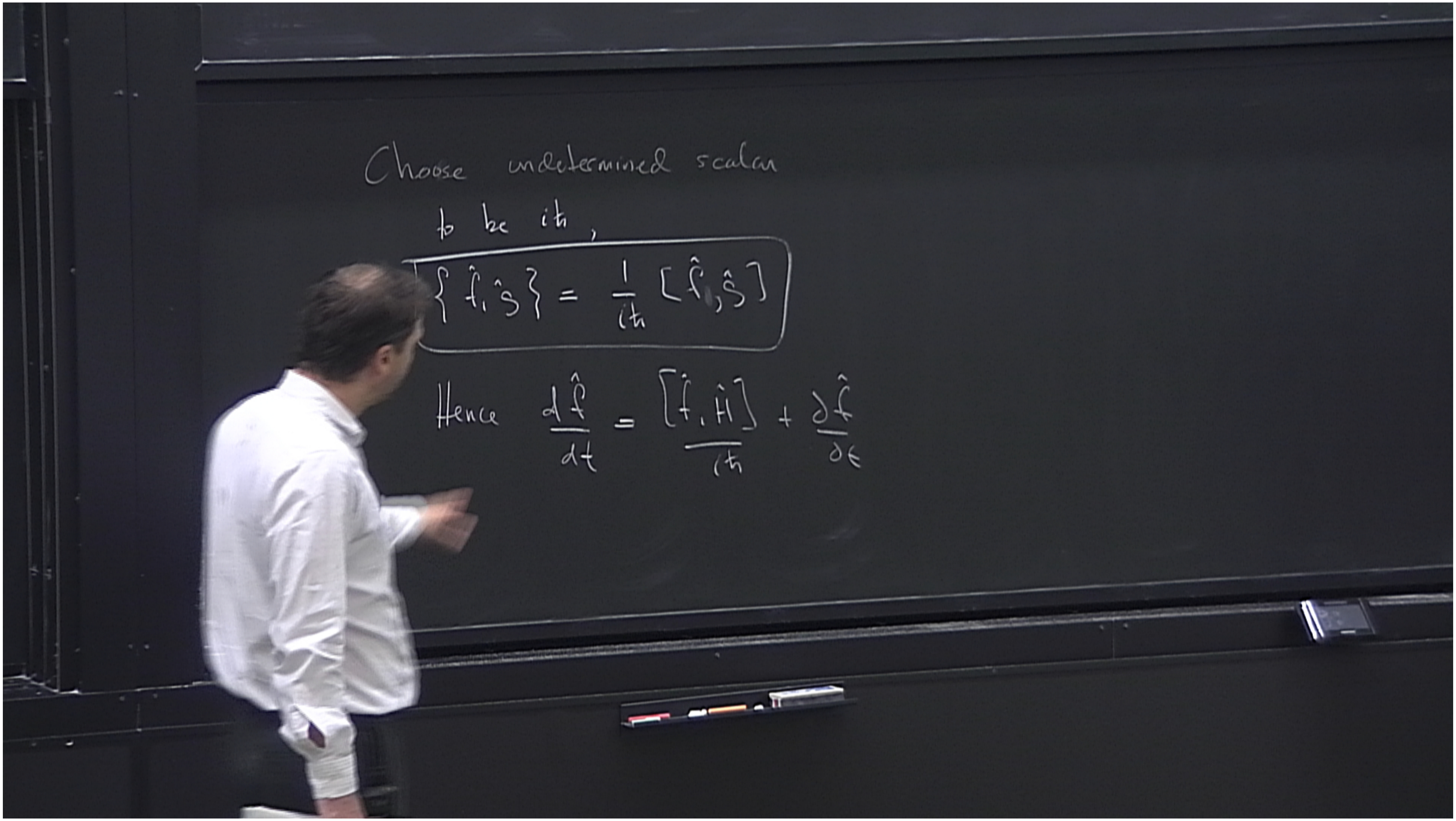
Choose undetermined scalar

to be $i\hbar$,

$$\boxed{\{\hat{f}, \hat{g}\} = \frac{1}{i\hbar}[\hat{f}, \hat{g}]}$$

Hence $$\frac{d\hat{f}}{dt} = \frac{[\hat{f}, \hat{H}]}{i\hbar} + \frac{\partial \hat{f}}{\partial t}$$


Choose undetermined scalar
to be $i\hbar$,
$\{\hat{f}, \hat{s}\} = \frac{1}{i\hbar}[\hat{f}, \hat{s}]$
Hence $\frac{d\hat{f}}{dt} = \frac{[\hat{f}, \hat{H}]}{i\hbar} + \frac{\partial \hat{f}}{\partial t}$

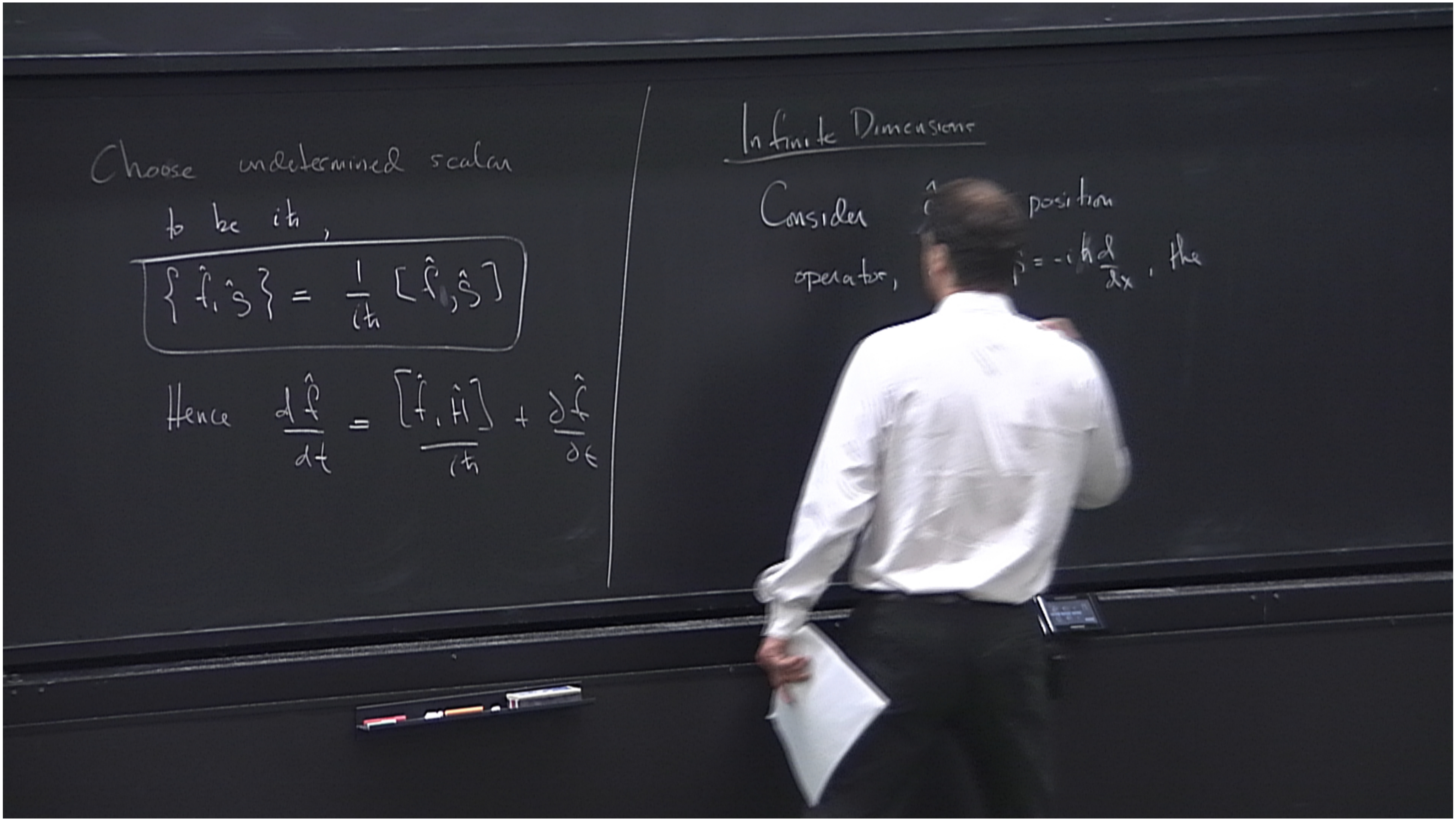

Choose undetermined scalar
to be $i\hbar$,
$\boxed{\{\hat{f},\hat{g}\} = \frac{1}{i\hbar}[\hat{f},\hat{g}]}$
Hence $\frac{d\hat{f}}{dt} = \frac{[\hat{f},\hat{H}]}{i\hbar} + \frac{\partial\hat{f}}{\partial t}$

Choose undetermined scalar

to be $i\hbar$,

$$\boxed{\{\hat{f},\hat{g}\} = \frac{1}{i\hbar}[\hat{f},\hat{g}]}$$

Hence $$\frac{d\hat{f}}{dt} = \frac{[\hat{f},\hat{H}]}{i\hbar} + \frac{\partial \hat{f}}{\partial t}$$


Choose undetermined scalar
to be $i\hbar$,
$\boxed{\{\hat{f},\hat{g}\} = \frac{1}{i\hbar}[\hat{f},\hat{g}]}$
Hence $\frac{d\hat{f}}{dt} = \frac{[\hat{f},\hat{H}]}{i\hbar} + \frac{\partial\hat{f}}{\partial t}$
Infinite Dimensions
Consider
position
operator,
$\hat{p} = -i\hbar\frac{d}{dx}$, the

Choose undetermined scalar to be $i\hbar$,

$$\boxed{\{\hat{f},\hat{g}\} = \frac{1}{i\hbar}[\hat{f},\hat{g}]}$$

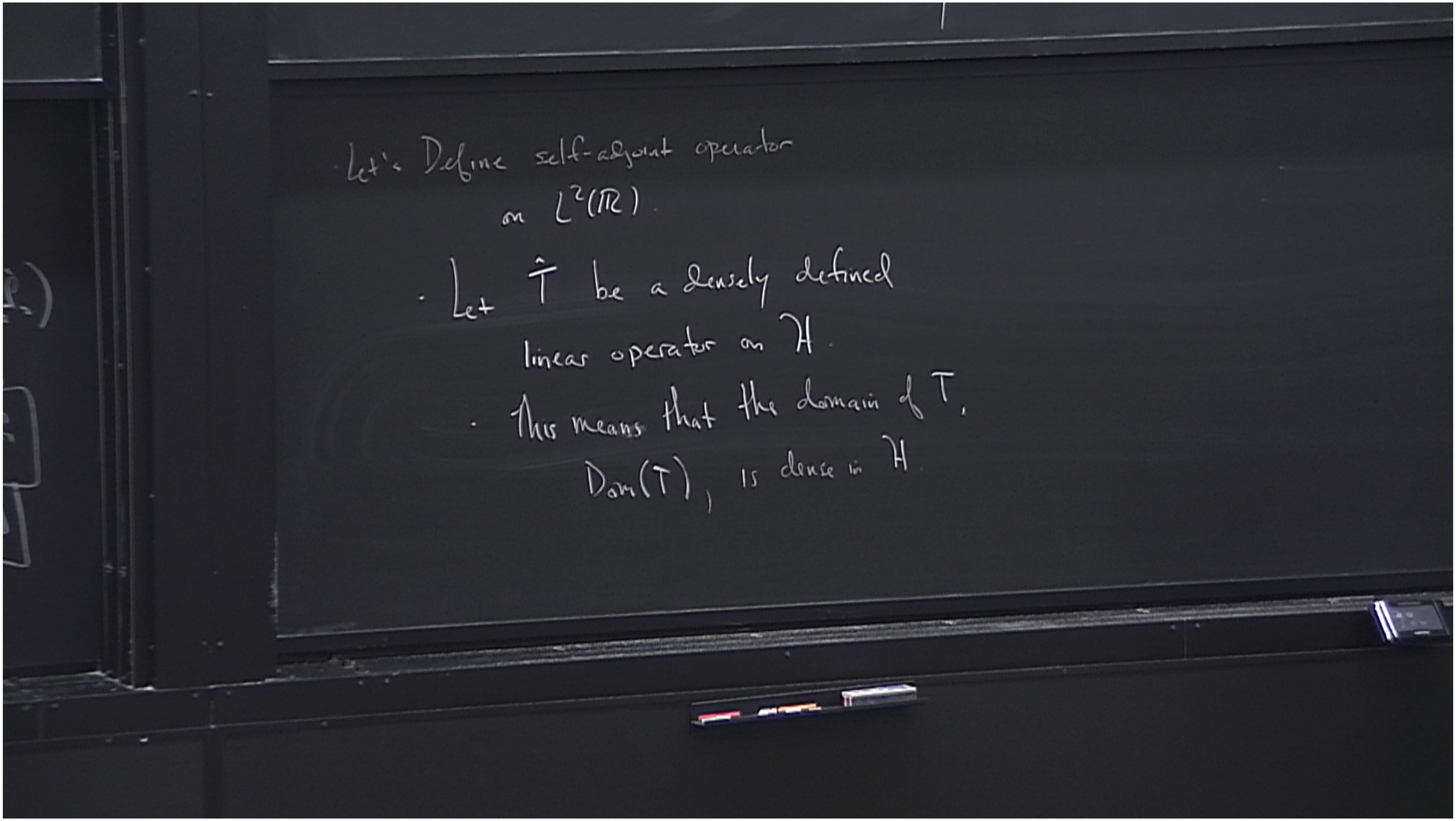
Hence $\dfrac{d\hat{f}}{dt} = \dfrac{[\hat{f},\hat{H}]}{i\hbar} + \dfrac{\partial \hat{f}}{\partial t}$

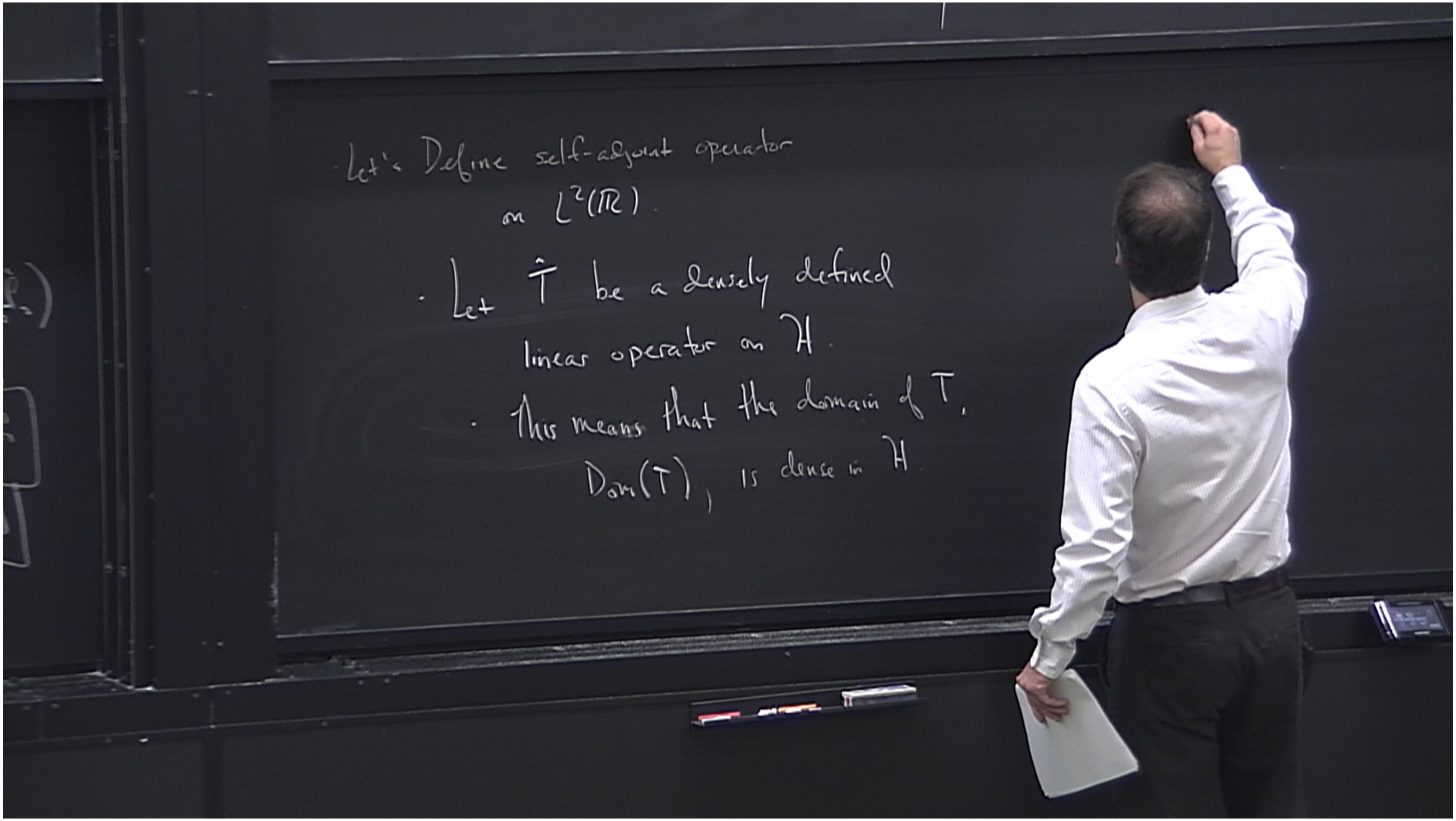
Infinite Dimensions

Consider $\hat{q}$, the position operator, and $\hat{p} = -i\hbar\dfrac{d}{dx}$, the momentum operator

Let's Define self-adjoint operator

on $L^2(\mathbb{R})$.


Let's Define self-adjoint operator
on $L^2(\mathbb{R})$.
· Let $\hat{T}$ be a densely defined
linear operator on $\mathcal{H}$.
· This means that the domain of T,
$\mathrm{Dom}(T)$, is dense in $\mathcal{H}$.


Let's Define self-adjoint operator
on $L^2(\mathbb{R})$.
· Let $\hat{T}$ be a densely defined
linear operator on $\mathcal{H}$.
· This means that the domain of T,
Dom(T), is dense in $\mathcal{H}$.

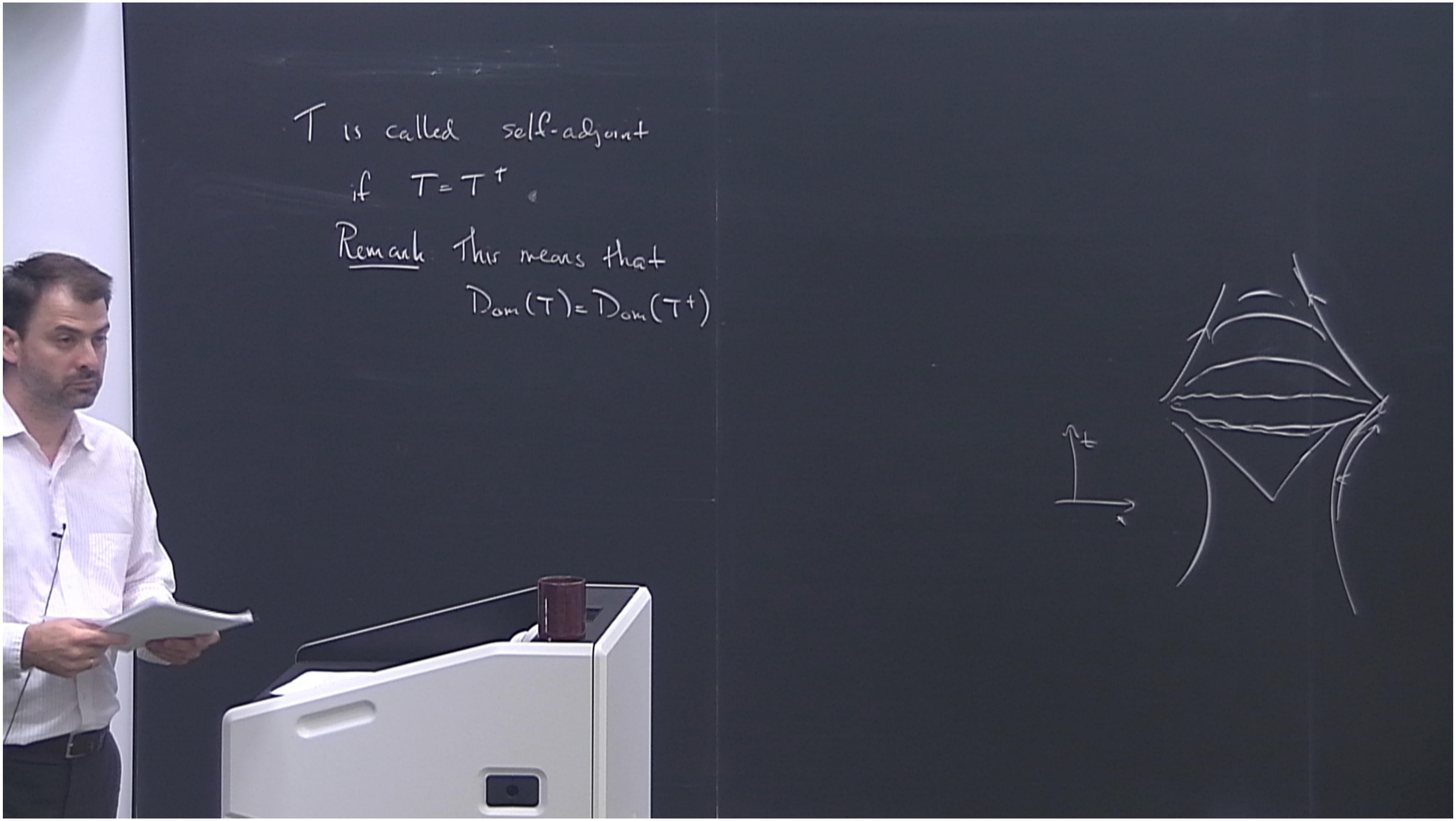
Let's Define self-adjoint operator on $L^2(\mathbb{R})$

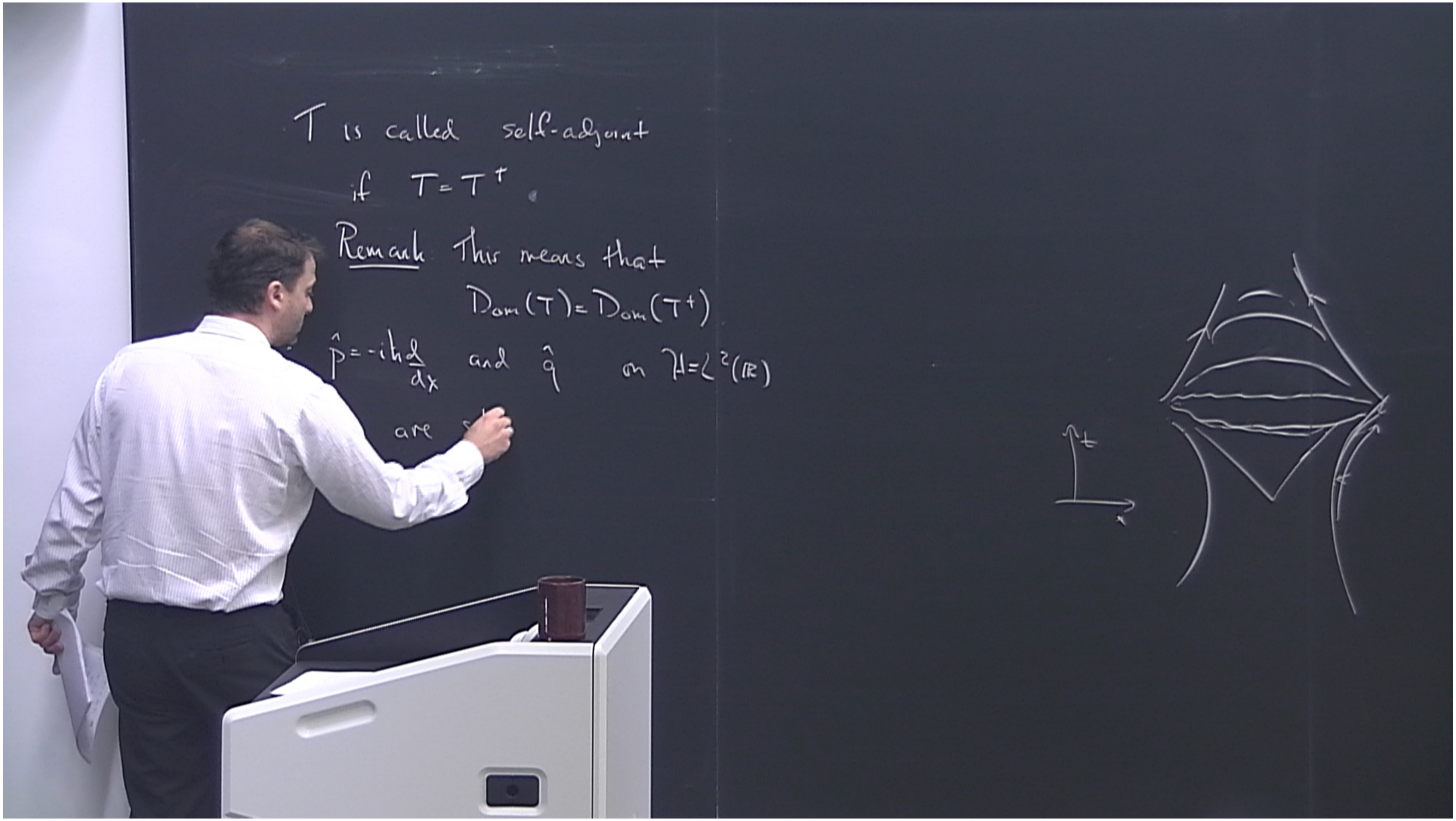
- Let $\hat{T}$ be a densely defined linear operator on $\mathcal{H}$.
- This means that the domain of T, $\mathrm{Dom}(T)$, is dense in $\mathcal{H}$

The adjoint of T, denote $T^\dagger$, is defined by 2 conditions

(i) $\mathrm{Dom}(T^\dagger)$ is all $\varphi \in \mathcal{H}$ s.t. $\exists\ \eta \in \mathcal{H}$, satisfying

$$\langle T\psi | \varphi\rangle = \langle \psi | \eta\rangle \quad \forall\ |\psi\rangle \in \mathrm{Dom}(T)$$


T is called self-adjoint
if T = T†.
Remark: This means that
Dom(T) = Dom(T†)
t


T is called self-adjoint
if $T = T^\dagger$.
Remark This means that
$Dom(T) = Dom(T^\dagger)$
$\hat{p} = -i\hbar \frac{d}{dx}$ and $\hat{q}$ on $\mathcal{H} = L^2(\mathbb{R})$
are
t
x

There are two rigorous approaches to calculate using $\hat{p}$ & $\hat{q}$ even though their eigenstates are not in $L^2(\mathbb{R})$

(1) Dirac (justifies standard textbook approach)

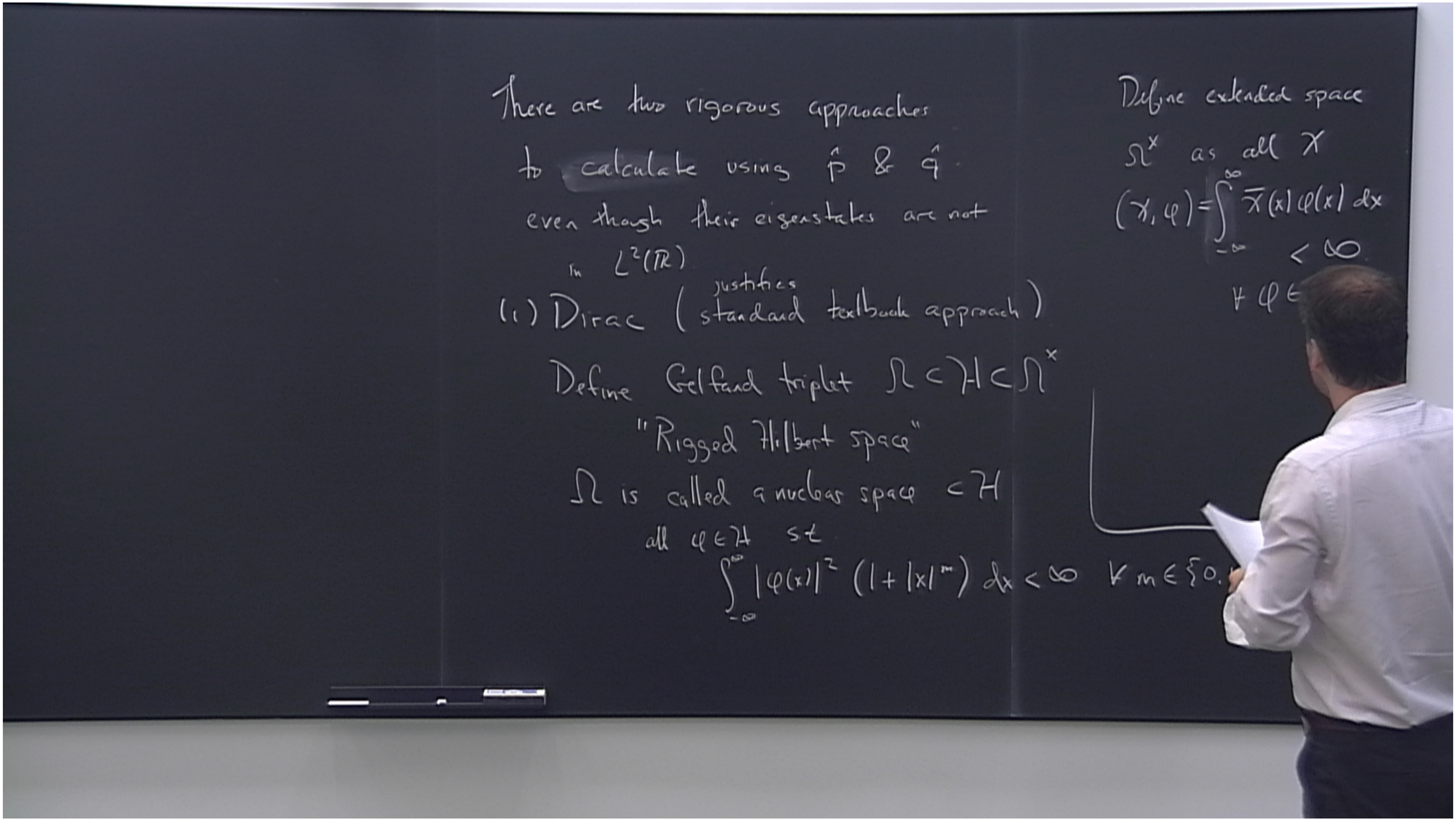
Define Gelfand triplet $\Omega \subset \mathcal{H} \subset \Omega^*$

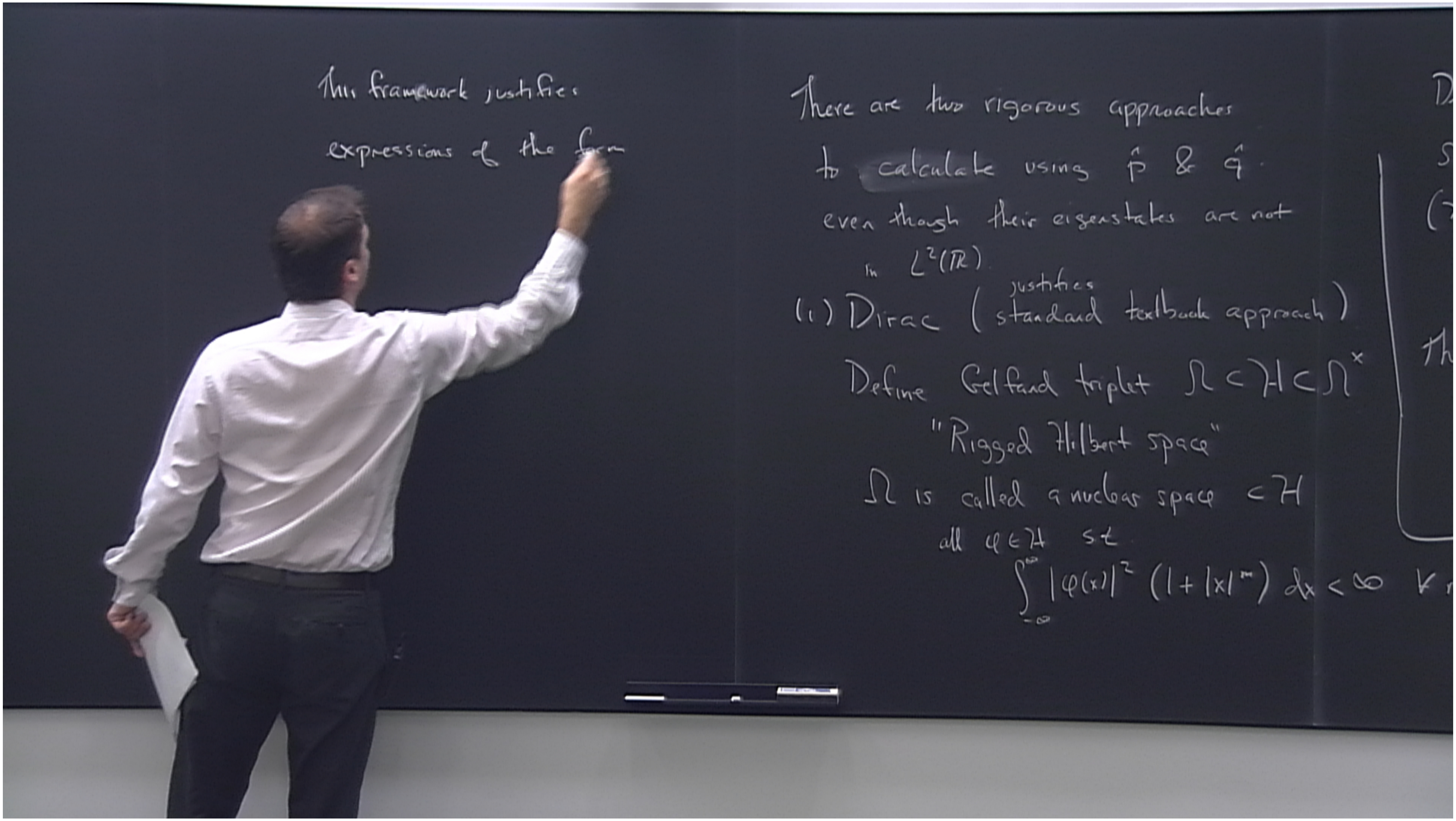
"Rigged Hilbert space"

Ω is called a nuclear space $\subset \mathcal{H}$

all $\varphi \in \mathcal{H}$ s.t.

$$\int_{-\infty}^{\infty} |\varphi(x)|^2 (1 + |x|^m)\, dx < \infty \quad \forall m \in \{0, 1, 2, \ldots\}$$


There are two rigorous approaches
to calculate using $\hat{p}$ & $\hat{q}$
even though their eigenstates are not in $L^2(\mathbb{R})$
(1) Dirac (justifies standard textbook approach)
Define Gelfand triplet $\Omega \subset \mathcal{H} \subset \Omega^\times$
"Rigged Hilbert space"
Ω is called a nuclear space $\subset \mathcal{H}$
all $\varphi \in \mathcal{H}$ s.t.
$\int_{-\infty}^{\infty} |\varphi(x)|^2 (1+|x|^m)\, dx < \infty \quad \forall m \in \{0,$
Define extended space
$\Omega^\times$ as all χ
$(\chi, \varphi) = \int_{-\infty}^{\infty} \bar{\chi}(x) \varphi(x)\, dx < \infty$
$\forall \varphi \in$



This framework justifies
expressions of the form
There are two rigorous approaches
to calculate using $\hat{p}$ & $\hat{q}$.
even though their eigenstates are not
in $L^2(\mathbb{R})$
justifies
(i) Dirac (standard textbook approach)
Define Gelfand triplet $\Omega \subset \mathcal{H} \subset \Omega^*$
"Rigged Hilbert space"
Ω is called a nuclear space $\subset \mathcal{H}$
all $\varphi \in \mathcal{H}$ s.t.
$\int_{-\infty}^{\infty} |\varphi(x)|^2 (1+|x|^m)\, dx < \infty$

This framework justifies expressions of the form

$$\langle q|p\rangle = \frac{e^{ikx}}{(2\pi\hbar)^{1/2}}$$

$$\hat{\mathbb{1}} = \int_{-\infty}^{\infty} dx\, |x\rangle\langle x| = \int_{-\infty}^{\infty} dp\, |p\rangle\langle p|$$

There are two rigorous approaches to calculate using $\hat{p}$ & $\hat{q}$ even though their eigenstates are not in $L^2(\mathbb{R})$

(1) Dirac (justifies standard textbook approach)

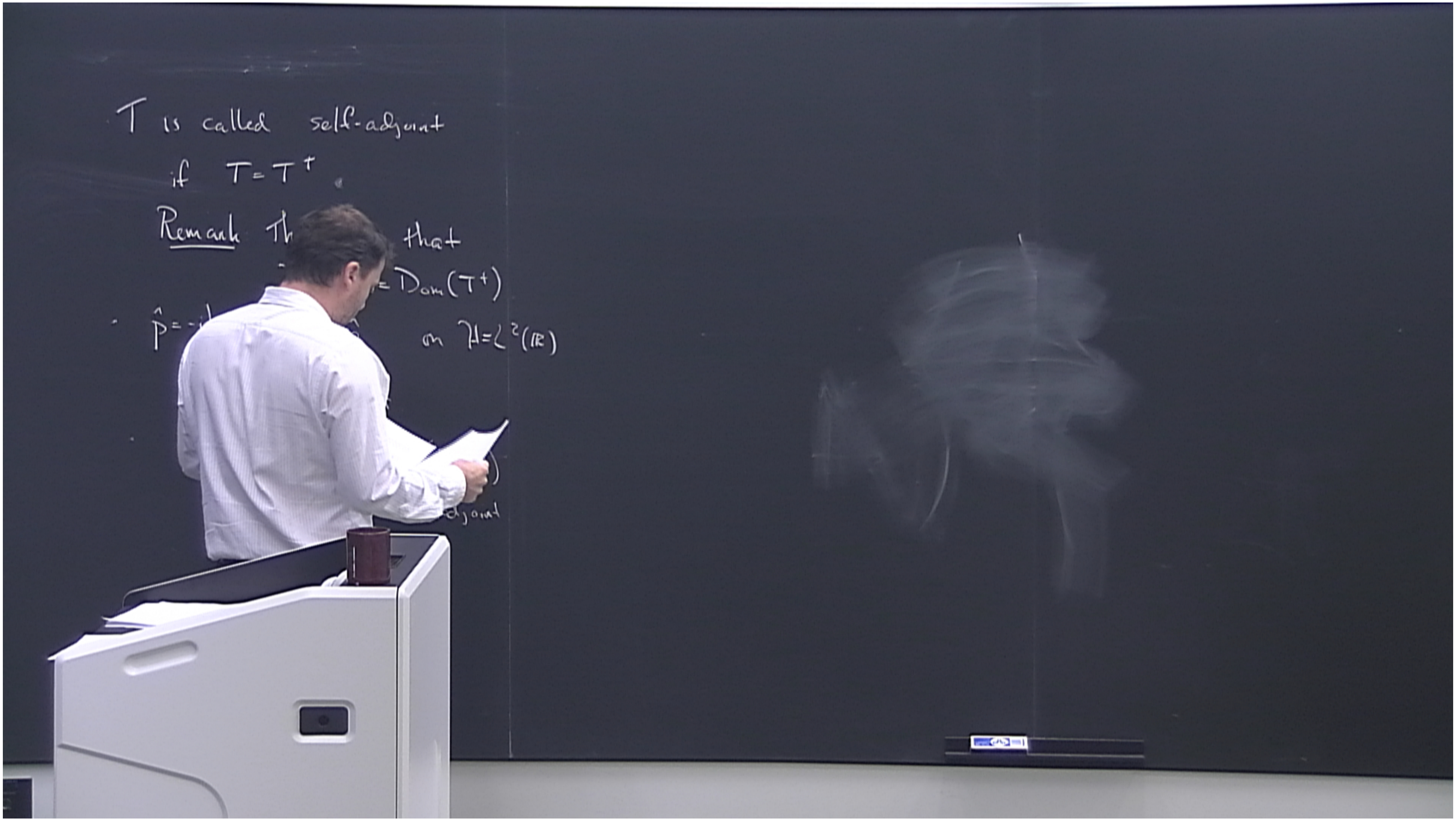
Define Gelfand triplet $\Omega \subset \mathcal{H} \subset \Omega^{\times}$

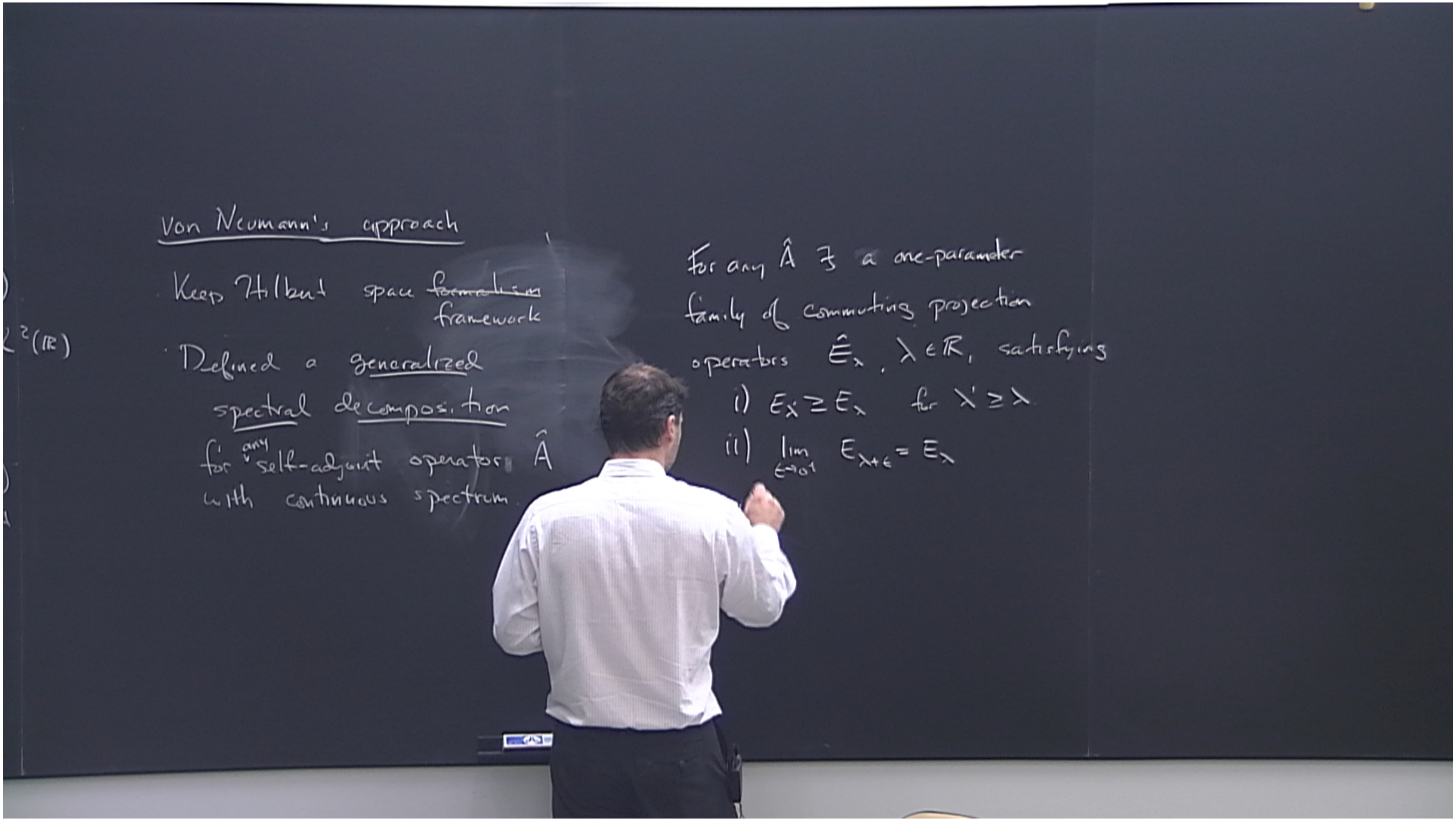
"Rigged Hilbert space"

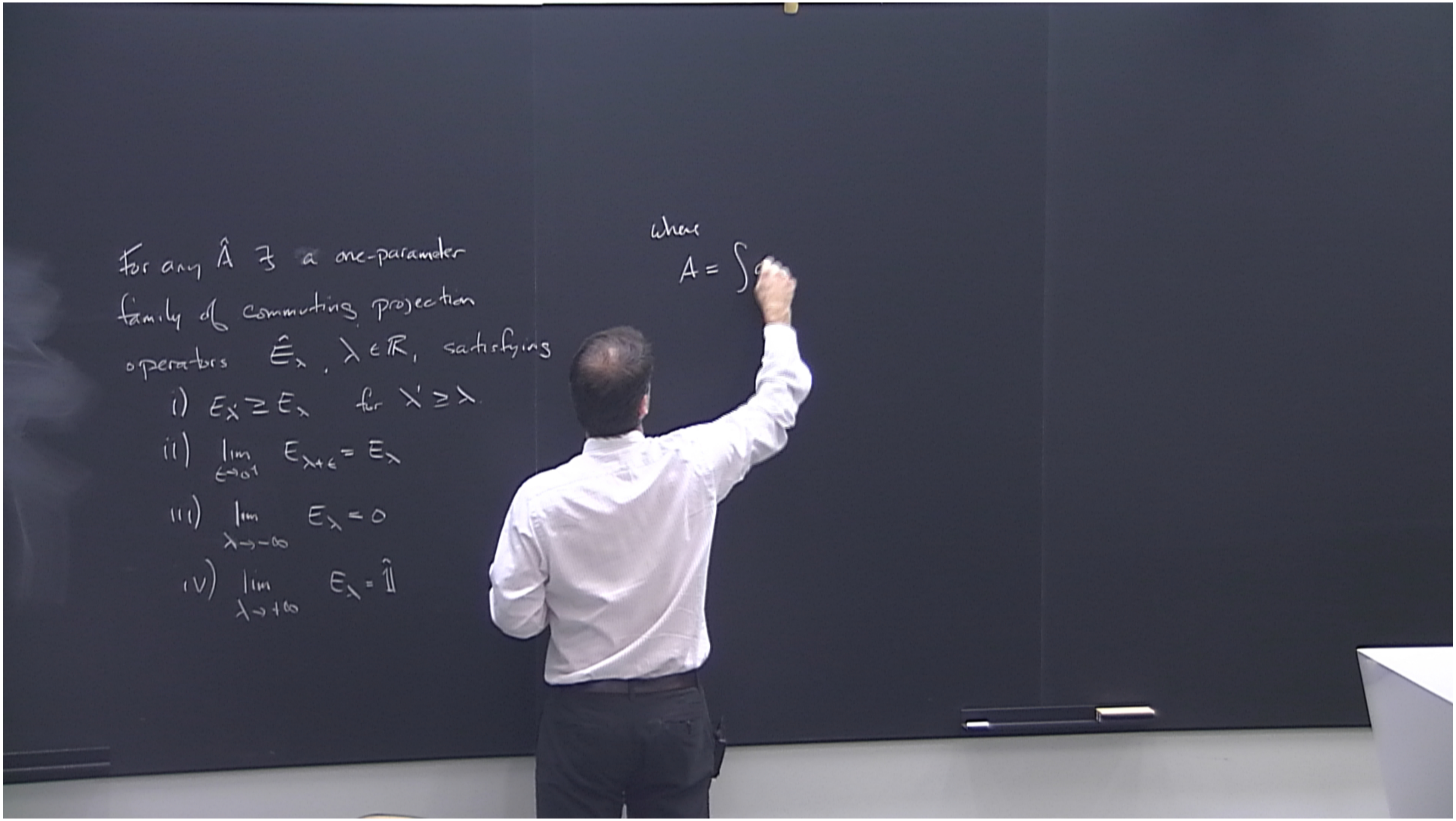
Ω is called a nuclear space $\subset \mathcal{H}$

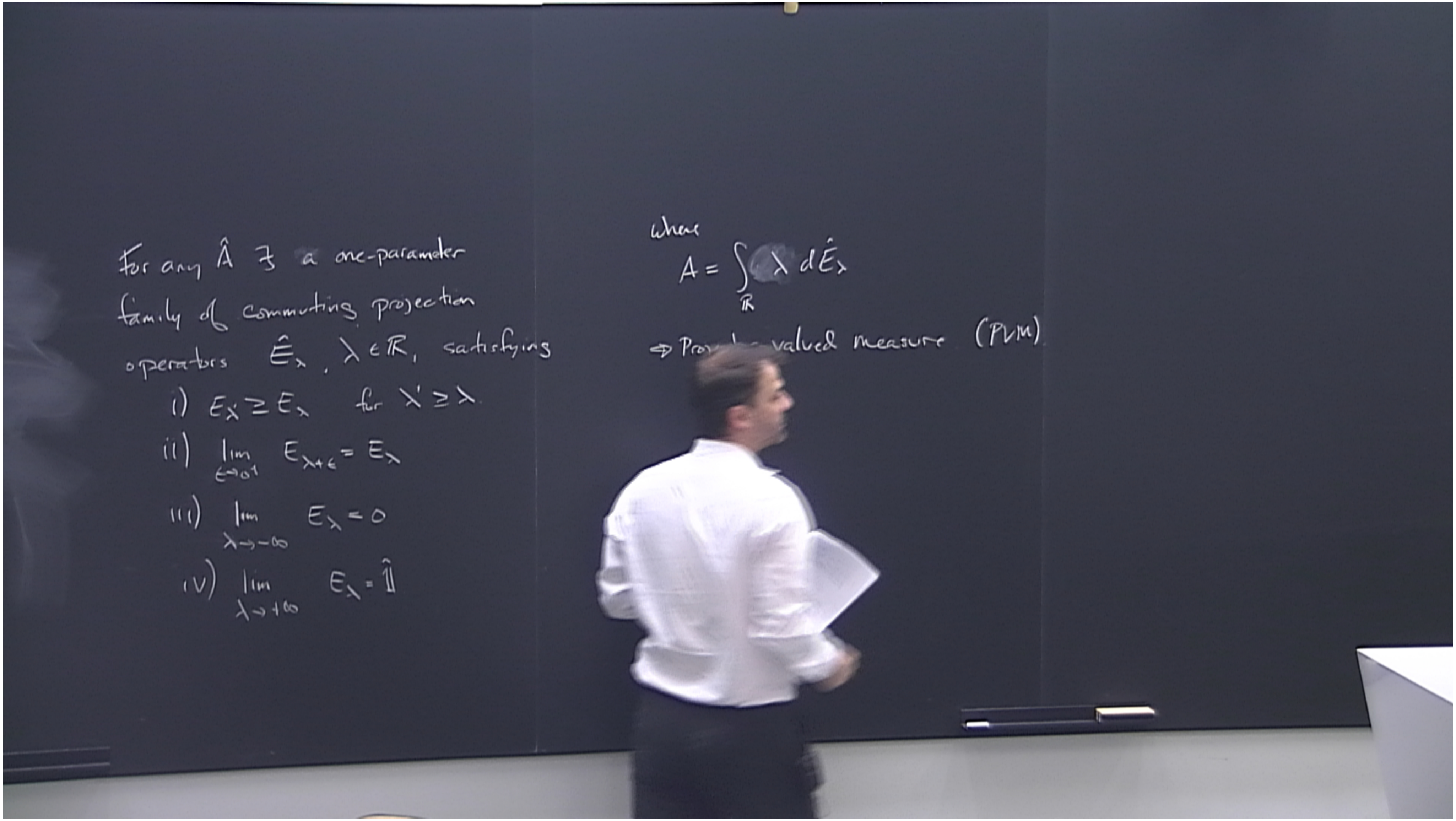
all $\varphi \in \mathcal{H}$ s.t.

$$\int_{-\infty}^{\infty} |\varphi(x)|^2 \left(1 + |x|^m\right) dx < \infty$$


T is called self-adjoint
if T = T†
Remark
that
= Dom(T†)
on H = L²(R)
adjoint


$^2(\mathbb{R})$
Von Neumann's approach
· Keep Hilbert space ~~formalism~~ framework
· Defined a generalized spectral decomposition
for any self-adjoint operator $\hat{A}$
with continuous spectrum.
For any $\hat{A}$ $\exists$ a one-parameter
family of commuting projection
operators $\hat{E}_\lambda$, $\lambda \in \mathbb{R}$, satisfying
i) $E_{\lambda'} \geq E_\lambda$ for $\lambda' \geq \lambda$
ii) $\lim_{\epsilon \to 0^+} E_{\lambda+\epsilon} = E_\lambda$


For any $\hat{A}$ ∃ a one-parameter
family of commuting projection
operators $\hat{E}_\lambda$, $\lambda \in \mathbb{R}$, satisfying
i) $E_{\lambda'} \geq E_\lambda$ for $\lambda' \geq \lambda$
ii) $\lim_{\epsilon \to 0^+} E_{\lambda+\epsilon} = E_\lambda$
iii) $\lim_{\lambda \to -\infty} E_\lambda = 0$
iv) $\lim_{\lambda \to +\infty} E_\lambda = \hat{1}$
where
$A = \int$


For any $\hat{A}$ $\exists$ a one-parameter
family of commuting projection
operators $\hat{E}_\lambda$, $\lambda \in \mathbb{R}$, satisfying
i) $E_{\lambda'} \geq E_\lambda$ for $\lambda' \geq \lambda$
ii) $\lim_{\epsilon \to 0^+} E_{\lambda+\epsilon} = E_\lambda$
iii) $\lim_{\lambda \to -\infty} E_\lambda = 0$
iv) $\lim_{\lambda \to +\infty} E_\lambda = \hat{1}$
where
$A = \int_{\mathbb{R}} \lambda \, d\hat{E}_\lambda$
valued measure (PVM)

For any $\hat{A}$ $\exists$ a one-parameter family of commuting projection operators $\hat{E}_\lambda$, $\lambda \in \mathbb{R}$, satisfying

i) $E_{\lambda'} \geq E_\lambda$ for $\lambda' \geq \lambda$

ii) $\lim\limits_{\epsilon \to 0^+} E_{\lambda+\epsilon} = E_\lambda$

iii) $\lim\limits_{\lambda \to -\infty} E_\lambda = 0$

iv) $\lim\limits_{\lambda \to +\infty} E_\lambda = \hat{\mathbb{1}}$

where

$$A = \int_{\mathbb{R}} \lambda \, d\hat{E}_\lambda$$

$\Rightarrow$ Projector-valued measure (PVM)

For any $\hat{A}$ $\exists$ a one-parameter family of commuting projection operators $\hat{E}_\lambda$, $\lambda \in \mathbb{R}$, satisfying

i) $E_{\lambda'} \geq E_\lambda$ for $\lambda' \geq \lambda$

ii) $\lim_{\epsilon \to 0^+} E_{\lambda+\epsilon} = E_\lambda$

iii) $\lim_{\lambda \to -\infty} E_\lambda = 0$

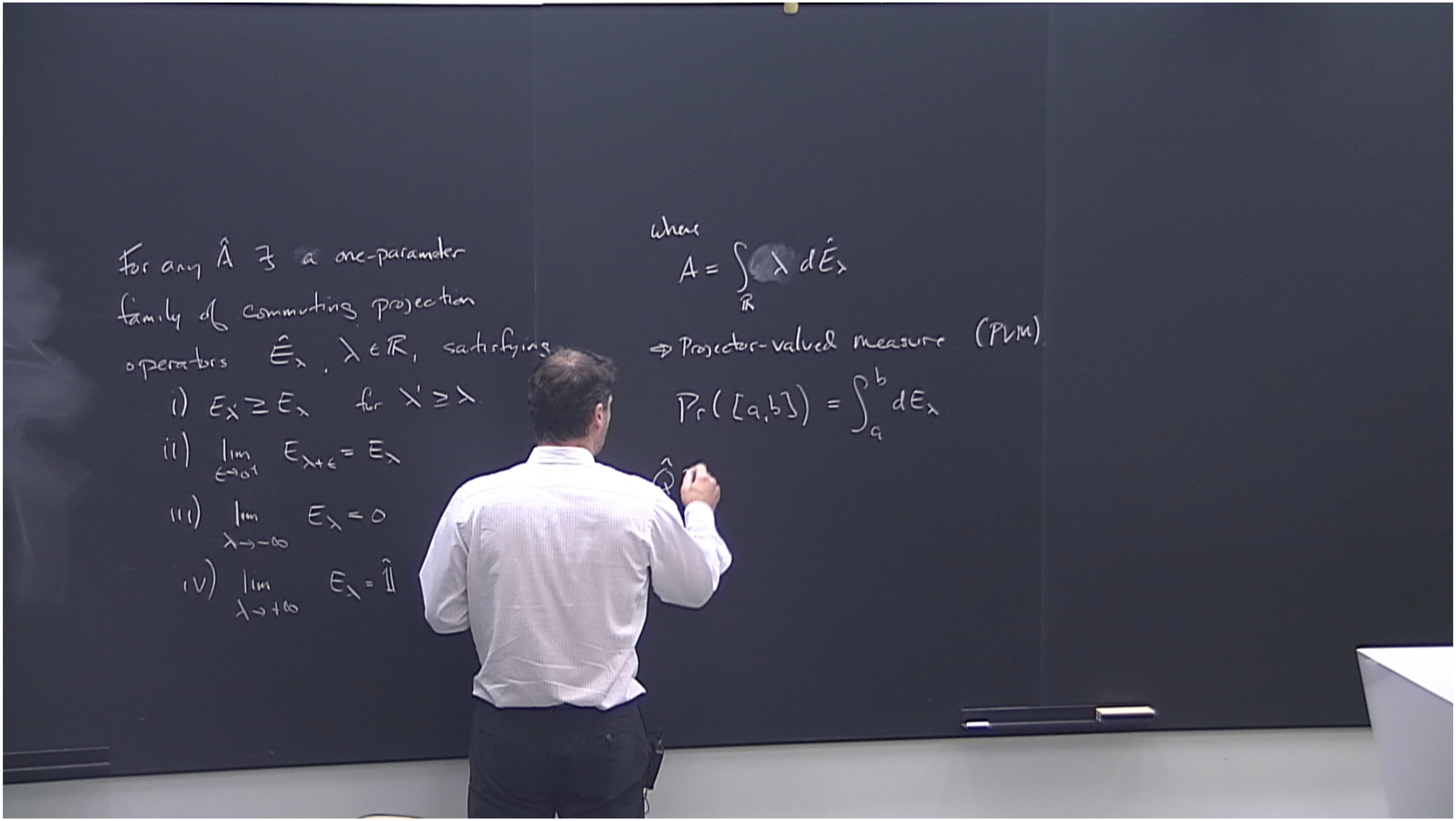
iv) $\lim_{\lambda \to +\infty} E_\lambda = \hat{\mathbb{1}}$

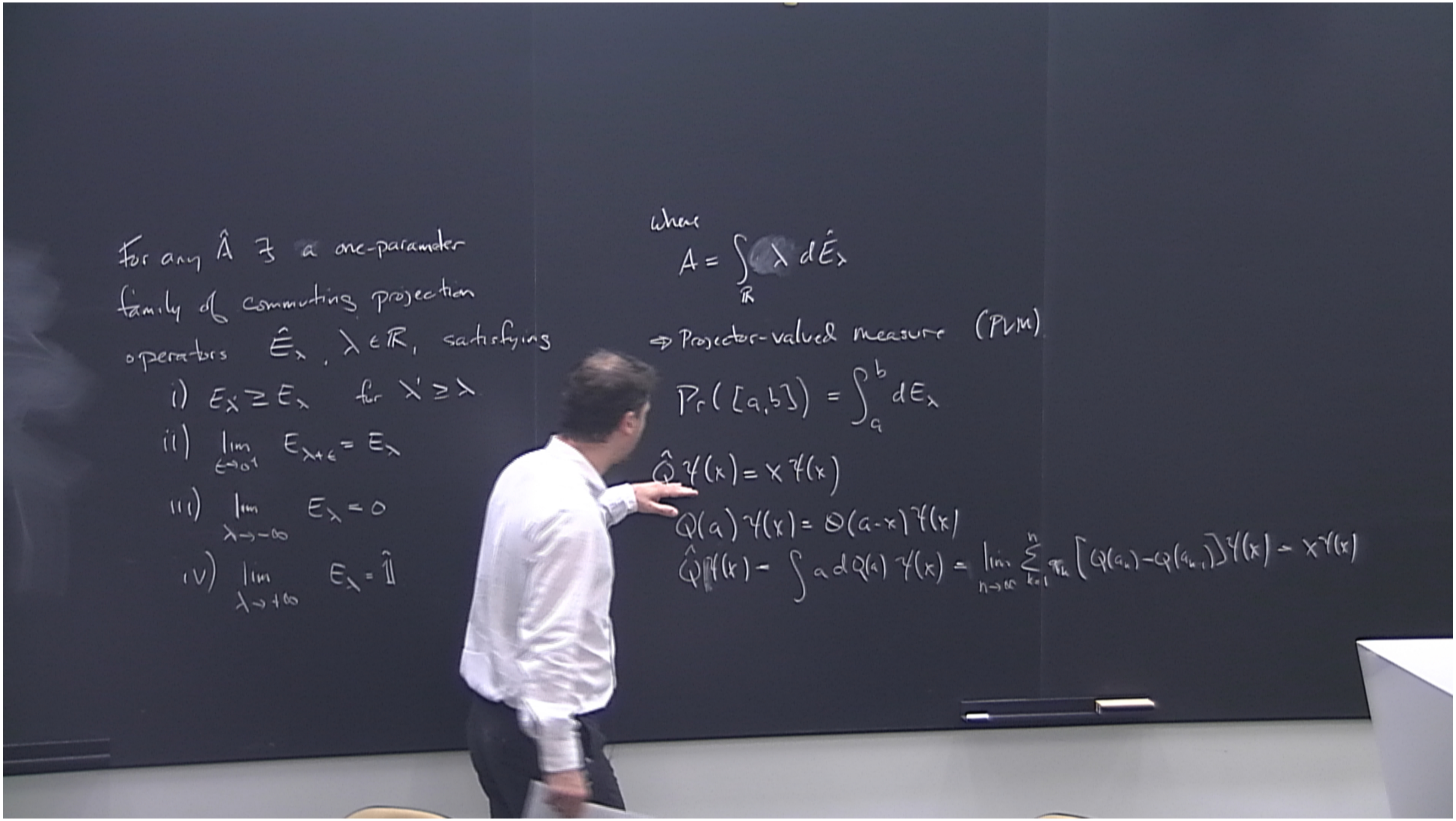
where

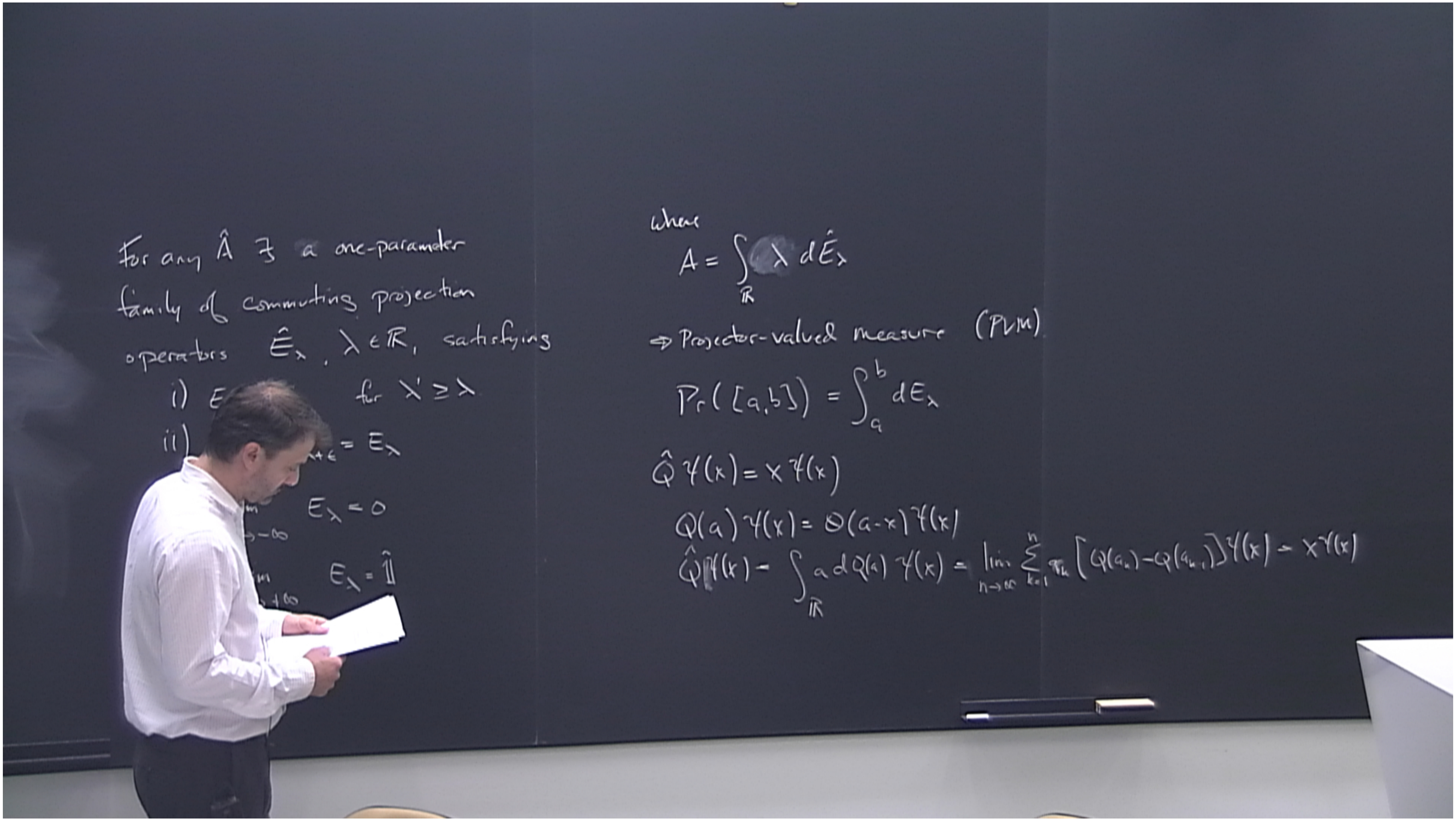
$$A = \int_{\mathbb{R}} \lambda \, d\hat{E}_\lambda$$

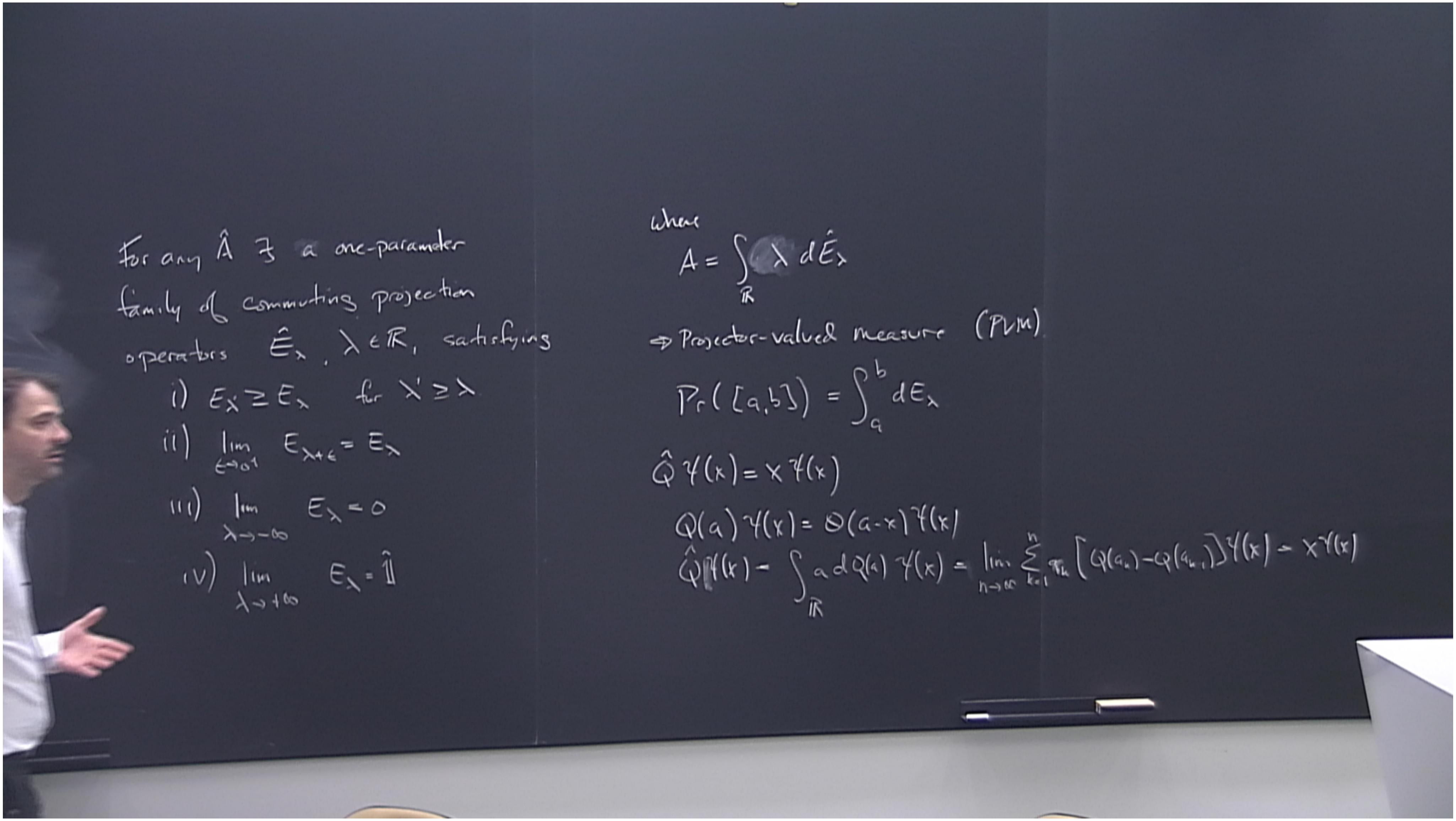
$\Rightarrow$ Projector-valued measure (PVM)

$$\Pr([a,b]) = \int_a^b dE_\lambda$$


For any $\hat{A}$ $\exists$ a one-parameter
family of commuting projection
operators $\hat{E}_\lambda$, $\lambda \in \mathbb{R}$, satisfying
i) $E_{\lambda'} \geq E_\lambda$ for $\lambda' \geq \lambda$
ii) $\lim_{\epsilon \to 0^+} E_{\lambda+\epsilon} = E_\lambda$
iii) $\lim_{\lambda \to -\infty} E_\lambda = 0$
iv) $\lim_{\lambda \to +\infty} E_\lambda = \hat{1}$
where
$A = \int_{\mathbb{R}} \lambda \, d\hat{E}_\lambda$
$\Rightarrow$ Projector-valued measure (PVM)
$Pr([a,b]) = \int_a^b dE_\lambda$
$\hat{Q}$


For any $\hat{A}$ $\exists$ a one-parameter family of commuting projection operators $\hat{E}_\lambda$, $\lambda \in \mathbb{R}$, satisfying
i) $E_{\lambda'} \geq E_\lambda$ for $\lambda' \geq \lambda$
ii) $\lim_{\epsilon \to 0^+} E_{\lambda+\epsilon} = E_\lambda$
iii) $\lim_{\lambda \to -\infty} E_\lambda = 0$
iv) $\lim_{\lambda \to +\infty} E_\lambda = \hat{\mathbb{1}}$
where
$A = \int_{\mathbb{R}} \lambda \, d\hat{E}_\lambda$
⇒ Projector-valued measure (PVM)
$P_r([a,b]) = \int_a^b dE_\lambda$
$\hat{Q}\psi(x) = x\psi(x)$
$Q(a)\psi(x) = \Theta(a-x)\psi(x)$
$\hat{Q}\psi(x) = \int a \, dQ(a) \, \psi(x) = \lim_{n \to \infty} \sum_{k=1}^{n} a_k [Q(a_k) - Q(a_{k-1})]\psi(x) = x\psi(x)$


For any Â ∃ a one-parameter
family of commuting projection
operators Ê_λ , λ ∈ ℝ, satisfying
i) E
for λ' ≥ λ
ii)
= E_λ
E_λ = 0
E_λ = 𝟙
where
A = ∫_ℝ λ dÊ_λ
⇒ Projector-valued measure (PVM)
Pr([a,b]) = ∫_a^b dE_λ
Q̂ ψ(x) = x ψ(x)
Q(a) ψ(x) = Θ(a-x) ψ(x)
Q̂ ψ(x) = ∫_ℝ a dQ(a) ψ(x) = lim_{n→∞} Σ_{k=1}^n a_k [Q(a_k) - Q(a_{k-1})] ψ(x) = x ψ(x)


For any $\hat{A}$ $\exists$ a one-paramater family of commuting projection operators $\hat{E}_\lambda$, $\lambda \in \mathbb{R}$, satisfying
i) $E_{\lambda'} \geq E_\lambda$ for $\lambda' \geq \lambda$
ii) $\lim_{\epsilon \to 0^+} E_{\lambda+\epsilon} = E_\lambda$
iii) $\lim_{\lambda \to -\infty} E_\lambda = 0$
iv) $\lim_{\lambda \to +\infty} E_\lambda = \hat{1}$
where
$A = \int_{\mathbb{R}} \lambda \, d\hat{E}_\lambda$
$\Rightarrow$ Projector-valued measure (PVM)
$Pr([a,b]) = \int_a^b dE_\lambda$
$\hat{Q}\psi(x) = x\psi(x)$
$Q(a)\psi(x) = \Theta(a-x)\psi(x)$
$\hat{Q}\psi(x) = \int_{\mathbb{R}} a \, dQ(a)\, \psi(x) = \lim_{n\to\infty} \sum_{k=1}^{n} a_k [Q(a_k) - Q(a_{k-1})]\psi(x) = x\psi(x)$